

CASE FILE COPY

N 72 - 18895

ENVELOPES IN ECLIPSING BINARY STARS

Su-Shu Huang

Department of Astronomy, Northwestern University

NC 7-14-007-041

Abstract

The present paper consists of three parts: (1) the relevance of envelopes to the study of the light curves of eclipsing binaries; (2) the disk envelope, ^{and} (3) the spherical envelope. They are separately considered in three sections, each of which is further divided into sub sections. In the fourth section a brief concluding remark is presented.

1. RELEVANCE OF THE ENVELOPE TO THE LIGHT CURVE OF THE ECLIPSING BINARY

(1.1) Two Types of Eclipsing Binaries

It is ironic that eclipsing stars which provide the astronomer with some invaluable stellar data that cannot be obtained in other ways do not represent always the prototype of ordinary stars. Because of the selection effect, the two components are usually very close. As a result they interact with each other not only dynamically through the mutual gravitational attraction, but also physically in terms of mass exchange. The mass exchange in turn greatly complicates the analysis of light curves because the simple model of two spherical stars eclipsing each other no longer represents a good approximation of the problem.

It follows that we must divide the eclipsing stars into two classes: Those whose light curves can be understood, after proper

rectification, by two spherical stars eclipsing each other and those which cannot be so easily understood. The first group of stars has provided the astronomer with the basic data as regards the stellar masses and radii. Thus the study of such objects becomes one of the foundation blocks of stellar astronomy. Indeed it is this group of binaries whose light curves have been extensively analysed since the time of Henry Norris Russell who may rightfully be regarded as one of the founders of modern astrophysics. Finding myself inadequate I have no intention of reviewing such a great wealth of data accumulated in the past for this group of binaries, especially here in the University of Pennsylvania, one of the centers for such activities, and before many experts in the audience. The other class of eclipsing stars defies such a clear-cut interpretation and has provided the astronomer a challenge as to his imagination. While the light curves of binaries belonging to this peculiar group have been observationally studied, their physical interpretation often proves to be elusive. In this present paper we will discuss some attempts to understand the nature of this second group of eclipsing stars. However, I have no illusion that those mysterious systems which have baffled so many outstanding astronomers can be understood overnight. Indeed it would be presumptuous for anyone, most of all myself, to do so. What we propose to do is only to point out the importance of certain facts which may help our search for the proper interpretation for each peculiar system.

(1.2) Consequence of Mass Exchange in Binary Stars

The basic cause of the peculiar behavior of stars in close binary systems is the mass exchange between the two components or the loss of mass from either or both components. For the single star the path of evolution is well known. The three stages of ~~their~~^{its} evolution, namely the pre-main-sequence contraction, the stationary state of the main sequence and the rapid evolution of post-main-sequence, are clearly defined and each has its own natural time scale. This is not so for stars in close binaries because of the mass exchange between the two components or the forced mass loss resulting from the presence of the companion. When the mass of a star changes, its physical characteristics as well as its evolutionary path change accordingly. Consequently the evolutionary path of stars in close binaries will be quite different from that ~~for~~^{of} single stars. A main sequence star or a post-main-sequence star that acquires a great deal of mass from its companion has no parallel in single stars. Indeed the evolutionary track of the star on the H-R diagram depends not only on how much mass is acquired but also how fast it is acquired. Also mass loss or mass gain makes it impossible to define the natural time scale of evolution ~~in~~^{of stars} close binaries. While all these problems belong to the field of stellar interior and evolution and fortunately do not concern us because our interest is confined only to the outermost layer of the star that conditions the light curve, it is nevertheless important to remember that we must be cautious in applying the evolutionary track and time scale derived by calculation for single stars without either mass loss or mass gain to components

of close binary systems where evidence is overwhelmingly strong in the variation of the stellar mass (e.g., Koch 1970). I must stress this point because we do often find in literature that the evolution of close binary stars has been discussed in the light of evolution computed for single stars. Thus the structure of close binary stars must be computed as close binary stars per se, such as Lucy (1971) has done in his common convective envelope model of W UMa systems presented earlier in this colloquium and their evolutionary course must be considered together with the change of the orbit of binaries in consequence of mass exchange or loss (Huang 1956, 1963b, Kruszewski 1966), such as in studies made by Paczyński, Kippenham, Plavec, and others (for reference see Paczyński 1971).

Actually we are convinced that the study of stellar evolution by theoretical calculations for the constant stellar mass has already seen its peak. The future of this study now lies in the calculations for stars of variable mass of which the component stars in close binaries provide important cases.

Most peculiarities found in some close binary systems from direct observation arise from the effects of the mass loss and/or the mass exchange on the outer layers of the star. In the first place there exists gaseous streams in the system as a result of this exchange or loss. These streams can be observed spectroscopically but perhaps do not greatly distort the light curve because they are optically thick only in lines ^{but} ~~and~~ optically thin in the continuum. An example is \bar{U} Cephei (e.g. Struve 1949).
However depending upon the mode

of mass ejection the matter may also form an envelope of which the continuous opacity is appreciable. It is this envelope, we believe, that is the culprit of the abnormal behavior of the light curve, whether in or out of eclipses.

(1.3) Idealization of Envelopes

What differentiates an envelope from the star is the lack of a sharp edge. Look at the sun and see how sharp its edge is. But its corona which may be regarded as a very tenuous envelope is amorphous without a permanent shape. It is changing and shifting around continuously. This very simple example gives us a good idea of how difficult, if not impossible, it is to deal with the envelope in any precise way. What we can do under such a circumstance is to idealize, rather drastically, the envelope before we make a quantitative study. With this idealization in mind we divide the envelope into three classes according to their shape. Furthermore we may specify its variability by assigning it several time scales, such as the time scale of fluctuation, the time scale of existence, etc, for the sake of interpreting time variations of the observed results.

Consider, in general, a gaseous medium permeating in a binary system. Because of the gravitational attraction of two stars as well as their orbital motion, the distribution of matter in the medium will be seen as composed of three parts: that inside each of two lobes of the innermost contact surface and that outside this surface. We define them respectively as the envelope around each of the components and the envelope around the entire system.

In dealing with the envelopes around the individual stars, we have found that their effect on the light curve depends greatly upon their shape. In reality there are infinite varieties of shapes that are possible for the envelope, just as for the solar corona mentioned a moment ago. However in order to make a mathematical analysis, we must introduce some simplifying assumptions. For this purpose it may be useful to take a look at the shape of the galaxies which can be either spiral, elliptical or irregular. Perhaps we may similarly classify envelopes, not simply around binary stars but around stars in general, along this same line. Accordingly we divide the stellar envelope, as regards its shape, into three types: the lenticular, the spheroidal, and the irregular. Such a division for the sake of interpreting observed data of a close binary is important only for the envelope around the individual component or components in the system. For the envelope around the entire system, perhaps it does not make any great difference to the light curve whether it is disk-like or spheroidal because we only observed the system close to its orbital plane. Figure 1 summarizes the general scheme for the interpretation of light curves of eclipsing binaries.

(1.4) Opacity in the Envelope

The nature of opacity in the stellar envelope depends directly upon the temperature and the chemical compositions in the medium. Consequently it is sensitive to the luminosities of both component stars and the distances from them. In the neighborhood of a hot and luminous star, electron scattering is obviously the most important source of opacity. For example Huang (see Struve 1957) suggested that electron scattering played a role in broadening the

spectral line in β Lyrae before it was taken as the source of opacity in the disk envelope for explaining the light curve (Huang 1963a, also Woolf 1965) of this system. This suggestion was confirmed by Appenzeller (1965) who has found that the change of polarization with phase in this system seems to be consistent with the disk model. Since electron scattering is associated with a high^{ly} ionized medium, we would expect to observe the line emission, as in the H II region in interstellar space.

As we shall discuss later, Hall (1971a) has found it necessary to have a disk around the secondary of BM Orionas in order to understand both its photometric and spectroscopic data. Since the primary component of this system is a B star, the opacity in the disk envelope is likely to be due to electron scattering also. Consequently we would expect a similar behavior in the polarization ⁱⁿ with this binary as ⁱⁿ with β Lyrae.

If the temperature is low, solid and dust particles could become the source of opacity. Since the cross-section due to solid particles whose linear dimensions are much greater than wavelengths is purely geometrical, it is wavelengthⁱⁿ independent just as electron scattering. The characteristic feature of solid particles is of course the emission of radiation in the infrared region. When the envelope composed of these particles are eclipsed by the companion star, an infrared eclipse will result, thereby yielding a critical test ^{for} ~~to~~ observation.

In the intermediate range of temperaturesⁱⁿ, the opacity will be atomic and molecular just like in the atmospheres of stars from

type A to type M Their characteristic feature is wave length^h dependant. Also they produce line and band absorptions. Figure 2 summarizes the sources of opacity in the envelope and their observable characteristics.

While we have discussed the different sources of opacity separately, it should be noted that two or more sources may be operative in the same medium. This is especially true if we consider two different regions of wave lengths. For example, the far ultra-violet observation in the range between 3300 - 3380A of β Lyrae performed in the orbiting astronomical observatory indicates the appreciable variations in the light curve (Kondo, McCluskey and Hauck 1971). It seems that in the far ultraviolet region, electron scattering may not play an important role in producing opacity. While the exact reason for the behavior remains to be investigated, it clearly indicates that the eclipse is caused by an envelope because it is impossible to have stellar eclipse varying so greatly with wavelength^h.

(1.5) Photometric Characteristics of Envelopes

A critical characteristic of the binary whose component star is losing mass to or gaining mass from its companion is the variation in period, although in many cases the variation may have been too small to be detected observationally. The variation may be either systematic or erratic depending upon the mode of mass exchange and loss. In general the systematic variation is likely related to mass change arising from evolution in the interior

structure of the component star and should have a long time scale. Such is perhaps the case of γ Lyrae (Huang 1963). But the erratic variations are likely caused by the sporadic ejections and accumulations of mass by component stars or by gaseous flow in the system.

As an approximation we may assume that the envelope around the entire binary system produces the variation in the maximum light, while the envelope around individual stars distort the light curves only during eclipses. The slow drift of the maximum in the light curve of β Lyrae shown clearly in the report on photometric results from the 1959 international campaign on Beta Lyrae by Larsson-Leander (1969) seems to indicate the gradual changes in the optical thickness of the envelope around the entire system.

On the other hand the cyclic variations of light curves during eclipses is mainly due to the change in the envelope around the eclipsing star. Either the shape of the envelope or the density distribution inside it may be unsteady thereby making the eclipse never repeat itself. Again this can be most clearly illustrated by Figure 8 in Larsson-Leander's paper. Another example of the same nature is RT Lacertae whose light curve was known to be variable (Koch, Sobieski and Wood 1963). ^{By a} comparison of data obtained in 1968 and 1969 with those obtained in 1965, Hall and Milone (1971) found that primary eclipse had become deeper by 0.2 mag, that the maxima had become very nearly equal, and that an asymmetry at secondary minimum apparent in 1965 had vanished.

The extreme case of the non-repeatability of light curve during eclipse has been found in C V Serpentis -- a Wolf-Rayet

binary. It was ^{discovered} ~~found~~ to be an eclipsing binary by Gaposchkin (1949) with a light curve showing a primary eclipse of about 0.15 mag and a secondary of about 0.10 mag. Later Hjellming and Hiltner (1963) found photoelectrically a primary eclipse of about 0.55 mag but no data were obtained ~~at~~ the expected time of secondary eclipse. But recently observations by Stepien (1970) and Kuhl and Schweizer (1970) yielded no evidence for either a secondary or a primary eclipse ^{at all}. Perhaps this is the most drastic case that shows the unpredictability of light curves of eclipsing binaries with envelopes.

Another of the most intriguing phenomena concerning the light curves of some binaries is the unequal durations of primary and secondary eclipse of the same system. Such a result is untenable if eclipses are caused by a star moving in a circular orbit. But it can be easily understood if an envelope exists around ^e of the components in the binary system. We can see this point by a simple example in which pure absorption plays the dominant role in the envelope. Since the envelope absorbs radiation, eclipse by the envelope shows up clearly in the light curve.

But the envelope produces no observational effect when it is eclipsed by the companion star because it is dark in optical radiation. ^t It follows that one eclipse lasts longer than the other one. Of course observed in infrared radiation the two eclipses will last an equal time.

The situation will be different if the scattering process dominates the envelope. The envelope attenuates the radiation coming from the companion star behind it in the same way as

pure absorption. However, the scattering process also make the envelope luminous like the faint flow of the fog around a street lamp. Intuitively it seems that the faint glow of the envelope may escape detection in the light curve, even ~~the~~ the obscuration effect is quite appreciable. If so, primary and secondary eclipses will have ^u unequal durations. While our intuition has yet to be proven by actual calculations now in progress, the observed results of V444 Cygni (Huang 1970) and RT Lacertae (Hall and Milone 1971) appear to indicate that this explanation is in the right direction. *Figure 3 outlines the photometric characteristics of eclipsing binaries.*

2. THE DISK STRUCTURE

(2.1) Basic Physics

If the disk should be introduced solely for the interpretation of the peculiar behavior of the light curves of some eclipsing binaries, it would be simply an ad hoc hypothesis without much physical significance. This is especially true as the interpretation of observed data is not unique once we depart from the spherical model of stars. For this reason we must first examine the problem of disk formation in general before we can consider seriously the disk model for interpreting the photometric and spectroscopic peculiarities of some binaries.

It may first be pointed out that a gaseous or dust medium with a net angular momentum around a star has its final stable state in the form of a rotating ring or disk. The reason for this is very simple. Collisions among the gaseous particles in the medium dissipate energy that belongs to the random motion while the ^{kinetic} energy ^{that} belongs to the circular motion which is controlled by the law of conservation of angular momentum cannot

be dissipated. This is why the frequency of appearance of the disk structure is second only to that of the spherical structure for large aggregates of matter in the astronomical world.

(2.2) Observational Evidence

Let us now take a look at the celestial objects that possess the disk structure. The most impressive and obvious case of such a disk structure in the universe is provided by the galaxies. Our own milky system ~~was~~ ^{as well} ~~well~~ as other spiral galaxies all possess a flat disk where luminous stars shine. Indeed the collapse of the galaxy into a disk is a clear-cut example of the behavior of a bulk of particles with a net angular momentum in a central gravitational field.

Coming back to our own solar system we have rings around the planet Saturn. They are thin, flat and concentric in the plane of the planet's equator. Observations show that they are formed by a swarm of individual particles each exercising independent circular motion around the planet. Indeed it is the only possible way to have stable rings, because | neither a solid nor a liquid ring could continue permanently to revolve around a planet.

Next we have the general structure of our planetary system whose main objects, the planets and asteroids, are approximately confined to a plane and are revolving around the sun all in the same sense. Therefore we may regard the system basically a disk. This naturally leads to the suggestion that they were formed out of a rotating disk ^{of} ~~of~~ dust and gaseous particles, which has frequently been termed the solar nebula. Hence it is very likely

that similar planetary systems will be found around many stars because the formation of rotating disks of dust and gas is so natural (e.g. Huang 1968).

While we have not unambiguously proved by direct observations the existence of the disk structure that preceded the formation of any planetary system, the appearance of a rotating ring in Be stars has been unequivocally accepted. It was Struve (1931) who proposed that the emission lines in Be stars were produced by a gaseous ring revolving around the central star. The two components of the emission line simply represent the approaching and receding portions of the circular motion as seen by the observer.

Then we have Joy's (1942, 1947) discovery of gaseous emission rings around the ⁱprimary component of the eclipsing binary, RW Tauri. The observational evidence for the existence of such a ring is definitive because the ring is eclipsed by the secondary component. Since Joy's discovery gaseous emission rings of this kind have been found in more than twenty binary stars (e.g., Sahade 1960). The observation of gaseous rings around a component of an eclipsing system led us to investigate the idea (Huang 1963^{a, b} 1965) that the peculiar^u behavior, both photometrically and spectroscopically, of some binaries may be understood by assuming an opaque or semi-transparent disk structure around one or both of the individual components, because the difference between an emission ring and a disk that is opaque or semi-transparent in the continuum lies only in the density of the medium -- a difference in degrees rather than in fundamentals. *Indeed*
~~Consequently~~ ^{one} it should expect that a structure may be found that bridges up the emission ring and the opaque disk. Hall's (1971b)

paper on RS Cephei presented earlier in this colloquium seems to show that this system indeed represents an intermediate case between the emission ring without showing obscuration and the opaque disk without showing emission lines, for it shows emission lines as well as indicates the effect of obscuration. Presumably the emission lines are produced near the outer edge of the disk which is opaque enough in the inner region for effecting the light curve.

(2.3) Formation of Rings or Disk in Binary Systems

There are several ways that rotating rings and disk may be formed in binary systems. We shall discuss them separately:

(i) Rotational instability -- Just as the emission rings are formed around Be stars as a result of rotational breakup at the equator, similar rings can be formed around the components of binaries if the stars are rotating rapidly. However, most emission rings found in binaries do not appear to be formed in this way, because the component stars in the center of rings are not rotating at such a rapid rate as to become rotationally unstable. However, there may be cases in which rotation plays a role in producing a disk or ring.

(ii) Angular momentum transport through magnetic braking -- It has been suggested that T Tauri or T Tauri-like stars eject mass through magnetic activities. In the process the stellar angular momentum is being transferred to the surrounding medium which collapses into a rotating disk when enough angular momentum has been acquired (See references in Huang 1968). Ejection of mass ~~formed~~^{found} in both Be stars and T Tauri-like stars is believed to be related to their gravitational contraction, with Be stars in the post-main-sequence stage (Crampin and Hoyle 1960) and T Tauri-like stars in the pre-main-sequence stage (Herbig 1957). Hall (1971a) has suggested that the

secondary component of BM Orionis is a ^c ~~face~~ of disk formation in the pre-main sequence stage. His argument is very convincing as the trapezium to which this system belongs is extremely young. Thus, while the primary B star may have already reached the main sequence, the secondary component could very well be still in the stage of gravitational contraction leading to the main sequence. If so, this will put the secondary of this system to the class of T Tauri-like star. We will come back to this system later.

(111) Cataclysmic ejection -- when a star explodes, it leaves some debris around. If the star originally possesses a large amount of angular momentum, such a debris will easily collapse into a disk. Furthermore, if the explosion is of the supernova nature, this debris will be rich in heavy elements which may condense into solid particles. At the same time the implosion of the central core that accompanies the explosion may lead to the formation of a black hole. This is obviously the line of argument taken by Cameron (1971) and ^{wfuck} Stothers (1971) for the ϵ Aurigae system. ~~This point~~ will be discussed more extensively later.

(1v) Ejection is a result of stellar evolution reaching the innermost [^] contact surface -- Observationally this is perhaps the most important case of formation of envelopes. Whenever an emission ring is observed in a binary, the less massive component always fills the innermost contact surface while the ring always revolved around the more massive component. This fact seems at first puzzling but appears natural after a little reflection on the division of angular

momentum between two components of a binary system. A simple consideration shows that the angular momentum per unit mass of the less massive component is greater than that of the more massive component. Thus when the mass ejected by the less massive component reaches the neighborhood of the more massive component, it finds itself possessing an extra angular momentum. According to what we have mentioned before, a gaseous medium around a central field (in this case, produced by the massive component) with a net angular momentum will simply collapse into a rotating ring or disk. This was the explanation advanced at the Victoria conference on binary stars for the existence of the ring in binary systems (Huang 1957).

This explanation has its serious flaw, as in the process of moving from the neighborhood of the less massive star to the more massive star, the angular momentum of the ejected mass is not conserved. So we really cannot say that when it reaches the neighborhood of the less massive star, it still possesses a greater angular momentum per unit mass than that of the more massive star itself. To resolve this uncertainty by straight calculations according to the three-body problem is difficult. However it becomes less forbidding if one considers the problem statistically. Indeed it can be easily shown on the basis of the restricted three-body problem that the rate of change of angular momentum of the third body depends only upon its coordinates but is independent of its velocity. If we divide the plane of orbit into four quadrants by the line joining the two stars and another bisecting it we find that the rate is positive in two while negative in the rest (Huang 1965). Thus while an individual particle may lose or gain

any component of a binary has its maximum size, which may be estimated by the stability calculation within the framework of the restricted three-body problem (Hill 1886, Darwin 1897, Goudas 1963, Deprit and Price 1965, Heron 1965) At the same time the inner radius of the disk or ring must be larger than the star around which it revolves. Here we see a criterion for the ring formation, namely the space between the stellar surface and corresponding lobe of the innermost contact surface. The greater this space, the larger the a priori chance of forming a ring there (Huang and Struve 1956) Now the size of the lobe surrounding a component increases with the ratio of the mass of this component to that of its companion Hence from this consideration the ring formation is also preferred to appear around the more massive component that is in the main sequence stage while the less massive component is ejecting mass through the innermost contact surface and is therefore likely to be a subgiant or a giant star Indeed most binaries where gaseous rings have been observed fulfill this expectation

(11) Shape of the disk -- If the disk or ring is tenuous and has reached its equilibrium state, it will be geometrically thin, as in the case of Saturn's rings where all particles seem to revolve in one and the same orbital plane. Conversely a geometrically thin disk has to be optically thin too. Perhaps the name "ring" is better suited for such a structure than the name "disk," although we use both in an interchangeable way.

If the medium is not extremely tenuous, the disk is likely to have a thickness, a fact that can be testified by our own galactic plane which has a thickness of a few hundred parsecs at the sun. Such a structure could be better named "disk" than "ring," as in the case of

the galatic ^c_^ disk. Statistically the orbital planes of different particles around the gravitational center will make different small inclinations with the plane of symmetry and the tilt can be found in all possible ways of azimuthal orientation simply as a result of randomness. The net effect of these orbital motions will make the distribution of particles at any given instant look like a lens bound by two concave surfaces. It is for this reason that the solar nebular has often been assumed to be of this shape (e.g. Chandrasekhar 1946).

In order to obtain an appreciable optical thickness for the disk, it requires a large amount of material, which will make the disk geometrically thick. It is especially true, if one remembers that the density in the disk cannot be large because of the component of gravitational attraction along the direction perpendicular to the disk is small. In other words, there is no effective mechanism to compress the medium in the disk and an optically thick ^c_^ disk has to be geometrically thick, too.

(2.5) Calculation of Light Curves Based on the Disk Model

(i) Thin disk -- Let us examine first the calculation of the light curve according to the thin disk (or ring) model. When a component possesses a ring, the entire entity is determined by the following parameters:

r = radius of the star,

r' = radius of the inner boundary of the disk around the star,

r'' = radius of the outer boundary of the disk around the star

In addition we have to specify two parameters in order to define completely the orientation of the disk:

j = inclination of the disk,

Ω = angle between the nodal lines of the binary orbit and the disk,

τ_0 = optical thickness of the disk assumed to be constant over the entire disk so that the intensity of light coming from the eclipsed star is cut off by a factor of $\exp(-\tau_0/\cos j)$ in passing the disk.

Therefore even the simplest case of the disk structure contains 6 parameters, r , r' , r'' , j , Ω and τ_0 instead of only one, namely r for the eclipse by a simple spherical star.

(ii) Thick disk -- The geometric parameters that enter into a thick disk are those defined for the thin disk, namely r , r' , r'' , j , Ω , τ_0 plus the two additional ones:

t' = half thickness at the inner boundary of the disk,

t'' = half thickness at the outer boundary.

Also there are the unknown equations defining the two concave surfaces as well as the inner and outer boundary surfaces. Here we have assumed that the optical thickness in the direction parallel to the symmetric axis of the disk is constant given by τ_0 , just as in the thin disk. Actually it may vary with the distance from this axis. Even with this simplifying assumption, it is still very tedious to calculate the variation of optical depths across the disk when the latter is viewed at an inclination other than $j = 0$. A less formidable case obtains when j is equal or close to $\pi/2$. When projected on the celestial sphere, it will be simply or nearly a rectangle. If the opacity is high in the disk, we may consider it completely dark to make the light curve easily calculable (Hall 1971a, Huang 1965).

The problem in calculating the light curve for the thick disk model

does not lie solely in the geometrical projection. However tedious it is, the geometry can be handled with no fundamental difficulty. The most troublesome point occurs when the opacity in the disk is due to the scattering process, such as in β Lyrae and perhaps $\widehat{B M}$ Orionis too, as we have a very nasty problem of radiative transfer in the disk. It is somehow like the well known reflection effect, but is more complicated. The basic difficulty lies in the fact that the medium has only a cylindrical symmetry while the transfer equation has so far been extensively studied only in plane-parallel and spherical cases. The result of the scattering of light from both stars in a thick disk is to make it luminous. Obviously the brightness varies from point to point, over its four surfaces, and it is not easy to write down the emergent intensity of scattered radiation from them. In any case we expect this scattered radiation produces changes in luminosity of the disk with phase, which distorts the light curve both in and out of eclipse.

However, if the opacity is due largely to large particles, then *because in such cases the light is purely absorbed.* the scattering problem does not arise [^] and is converted into infrared radiation. In such cases, one would expect to observe some infrared excess and a quite pronounced eclipse in infrared radiation when the disk is being eclipsed. When the light curve is not complicated by the scattered light from the disk it may be calculated by considering only the effect of obscuration by the disk. The projected area on the celestial sphere [^] of a thick disk is bounded by two half ellipses and two straight (assumed) lines with a central opening. As a first approximation ^{we} ~~we~~ may assume that light from the companion star is cut down everywhere within the projected area by a constant factor, $\exp(-\tau_0 / \cos j)$, except the central opening which is transparent. Calculations based on this simplified model have been carried out with a view to understanding the light curve of ϵ Aurigae, a system which will be discussed later on.

(2.6) Illustrative Cases with the Disk Structure

So far only a few peculiar binaries have been subject to analysis on the basis of the disk model; they are β Lyrae (Huang 1963), ϵ Aurigae (Huang 1965, Wilson 1971) and B M Orionis (Hall 1971a, also Hall and Garrison 1969). We shall discuss here only two systems -- B M Orionis and ϵ Aurigae. Beta Lyrae is not discussed here because of the difficulty in treating the scattered radiation in the disk mentioned in the previous sub-sections.

(1) B M Orionis -- Perhaps one of the best illustrations of the presence of the disk structure is B M Orionis. The system has a period of about 6.5 days and its eclipse from the first to the last contact lasts about 16 hours and that from second to third contacts lasts about $8\frac{1}{2}$ hours. Thus the ratio of radii had to be 0.35 or less. If primary eclipse were annular, its depth could be at most 18% of maximum light, according to Hall (1971a). But the depth of primary eclipse ~~was~~ ^{is} 45% of maximum light (0.7 mag) and has a flat bottom. Consequently it cannot be annular eclipse even if the secondary is completely dark and the eclipses must be total with the secondary about equal in luminosity as the primary component. But on the spectrogram taken two hours after the onset of totality (Doremus 1970) there is not a trace of the spectrum of the secondary component. Only the primary spectrum could be seen. This compelled Hall (1971a) to suggest a disk model surrounding the secondary component. In order to derive the flat bottom, Hall assumed a thick disk looking edge-on, i.e. nearly rectangular when projected on the celestial sphere, because only in such a configuration can one insure the observed constancy of light during the time of minimum light. The light curve calculated according to this model indicates a good agreement with observation, as we can see in Hall's paper.

(11) ϵ Aurigae -- The ring or disk structure around the secondary of ϵ Aurigae had been mentioned by Kopel (1955) and by Hack (1961) but serious attempts to understand its light curve based on this structure were made by Huang (1965b) and Wilson (1971). Cameron (1971) applied also the concept of a disk to explain the eclipse when he assumed the secondary component as a black hole. The opacity in the envelope was assumed (Schönberg and Jung 1938) to be due to solid particles because the light curve of ϵ Aurigae has long been known to be wave-length independent (e.g. Kuiper, Struve, and Strömgren 1937). More recent observation by Huruata and Kitamura (1958) and by Thiessen (1957) indicated that while the small and irregular light variations amounting to 0.1 - 0.2 mag. in amplitude are wave-length dependent, the light changes due to eclipse are not. Also it has been quoted in Zopal's (1955) paper that no intrinsic polarization has been found. ^{Since there} ~~There~~ are not enough electrons (Kraft 1954) to produce necessary opacity in the envelope, ^{indeed} ~~hence~~ it is likely that the opacity in the envelope of ϵ Aurigae is due to solid particles whose dimensions are large compared with the wavelength in the optical region, ^a ~~Although~~ this is by no means a conclusion, as we will see later on.

It appears that Cameron (1971) introduced the obscuring disk practically perpendicular to the line of sight. The origin of such a disk is difficult to understand. How could we envisage all particles neatly collapsed into a disk with a large inclination to the orbital plane? One may argue that the secondary star might rotate in this sense and the shedded matter, as a result of rotational instability or other causes, might have coalesced into a ring in the equatorial plane. However such an argument has failed to note the difference between a single rotating star and a component in a binary. In the single star there

is a unique symmetric plane. Hence the collapse of matter will lead to a ring in that unique plane as we have mentioned earlier. But it is not so in the case of the component in a binary system. Unless the rotational vector of the component is nearly parallel to the vector of the orbital angular momentum, the equatorial plane is not the unique symmetric plane in the system. The dominant ^{moment} ~~moment~~ symmetric plane is still the orbital plane. Hence it is hard to envisage the collapse in any plane at a large angle with the orbital plane.

Even if the disk or ring were formed with a large inclination to the orbital plane, it is difficult to see that the disk will maintain its shape for a long time. For as has been pointed out by Wilson (1971), any disk with a large inclination to the orbital plane is not stable. The simplest way to see that is a consideration of the precessional motion of the particles' orbits as a result of the tidal perturbation of the stars. Since different orbits precess at different rates, the disk will soon spread into a spherical envelope. It follows that we do not have to consider this model seriously.

That leaves only models by Wilson (1971) and Huang (1965b). Both of them have assumed that the plane of the disk is close to the plane of orbit. *The large difference between these things is the geometric thickness of the disk.*
 A thick, both geometrically and optically, disk was assumed by Huang and a geometrically thin but optically thick disk, viewed at an inclination slightly different from $\pi/2$, by Wilson. As we have mentioned in subsection (2.4), an optically thick disk composed of gaseous and dust particles must be geometrically thick too, Wilson's model is not realistic.

Wilson also presented the light curves based on his ^{thin} ~~three~~ ring model. But when his parameters were fed into our own program for computing light curves described in subsection (2.5), it was found that his calculated results could not be verified. Since Wilson did not present his mathematical analysis in his preprint, which was sent to me at the time we had

just completed our program, it is not feasible to locate the cause of the discrepancy. Tentatively we may assume that there are some printing errors in his preprint, or, though unlikely, either of us might have made some calculational errors.

Wilson objected to the large radius for the primary obtained by Huang. According to Wilson's calculation, Huang's value for the radius of the primary amounts to $M_V = -9 - -9.5$ mag., depending upon the estimated mass of the secondary. Actually we are not overly disturbed by these values. In the first place ~~Stothers~~ (1971) found from the distance (1.34 kpc) of Aur OB1 association (which \in Aurigae seems to belong to) that this star has $M_V = -8.7$ mag. which is not too far from the lower estimate based on Huang's model. Secondly the mass estimate made for the secondary component is tentative at best, and so is the estimate of M_V for the primary. Actually if we examine the most recent light curve given by Gyldenkerke (1970), it is very difficult to make the primary radius much less than 0.65 of the separation between two components that Huang assumed. Smaller radii than this value would put the first (3rd) and the second (4th) contact too close together as compared with the observed data, unless one is willing to accept some very unusual configuration for the disk.

We are more concerned with the discrepancy about the inclination. Both Wilson and Huang assumed it close to $\pi/2$ but astrometrically (Strand 1959) it was found to be 72° . The discrepancy of the nature has happened before. For example the eccentricity of τ^1 Cep was found to be near zero from the light curve but the velocity curve yielded a large value. Struve (1949) suggested that the velocity measurement was distorted by the line formed in the gaseous stream, thereby resolving the conflict. In the present case both determinations may be viewed as tentative at best and further study should be made to iron out this discrepancy.

Next let us consider the fine structure in the minimum light. It is simple to see that the slight maximum inside the minimum in the light curve given by Gyldenkerke^{2v} is caused by the opening in the middle of the disk (Wilson 1971). However there is the problem of the asymmetry of the curve as well as fluctuation and the addition dip near ^{but} not exactly in the center. The asymmetry may be caused by ^{or/and} Ω_K fluctuation by ^{an uneven} the distribution of matter in the disk. However, we must emphasize the fact that the fluctuation in the minimum light does not reflect directly the fluctuation of density in the disk at any given time, because the matter in the disk is revolving with different angular velocities at different radii from the secondary component.

In addition to the ^f fluctuation we may advance two speculative possibilities. The one or two small dips that we may detect from the observed points in the light curve during the broad minimum may represent one or two condensations associated with the disk. If, as has been speculated (Huang 1965b, Kopal 1971), the disk is going to become a planetary system, we may even take these condensations as proto-planets. Such a structure would very much resemble Saturn and its rings and satellites. Indeed, like the satellites in the Saturn system, the condensation or condensations around the secondary can be shown to be close to the outer edge of the disk or even outside the disk. Assuming that the secondary have the same mass as the primary component, we can easily calculated^d that the period of a condensation near the inner edge of the disk would have a period of about 100 days. Thus in the total duration of eclipse that lasts 2 years we would observe several passages of the condensations over the primary's surface, a prediction that contradicts observed data. On the other hand we find no difficulty in locating these condensations near the outer region of the ring. At a distance of 0.17 of the separation that has been assumed to be the outer radius of the disk, the period of revolution of such a condensation around the secondary will be of the order of

1000 days, if we again assume equal masses for two stars. Hence the period is longer than the time of eclipse by the disk. That makes the transit^h by such a condensation over the primary's surface only once in each eclipse. According to this interpretation, the durations of the little eclipse inside major eclipse measures the time that the condensation keeps projected on the stellar surface of the primary component as viewed by the observer. This duration varies according to its size and the position in its orbit around the secondary. From a rough estimate of the depth of the eclipse by the condensation, we find that the size of condensation is of the order 0.04 of the separation between two components. According to this interpretation, the phases of these little eclipses inside the main eclipse change from cycle to cycle. Glydenkerⁿ's data and results obtained in earlier eclipses seem to confirm this point. While that does not necessarily mean the correctness of interpretation, further investigations along this line seem warranted.

An alternative and equally speculative explanation of the fine structure in the minimum light is that the ring is not completely circular but ~~it~~ is elliptical with the secondary at one of its foci. Because the motion of particles are slow near the apastron, the density is higher there than that near the periastron where the solid particles move rapidly. This creates an uneven distribution of matter which in turn produces the asymmetry of the eclipse. From the light curve we must assume that the periastrum of the ring is in front so it is the less dense part of the ring that eclipses the primary first. According to this interpretation the dip near the center represents the glancing eclipse of the primary by the secondary component. It agrees with the prediction that this glancing eclipse ^{does not} occurs ~~not~~ in the middle but is slightly ahead of the mid point of eclipse. However according to Wilson (1971) the secondary thus assumed is too big. He favored the idea that the dip is due to a cloud around the black hole.

If the ^{so} solid particles obscure the radiation from the primary

component by absorption, they must re-emit the absorbed energy in the form of infrared radiation. Mitchell (1964, see also Wildey and Murray 1964) found a large excess of radiation in this system as compared with a normal star of the same spectral and luminosity type. It is this result that Cameron (1971) and Kopal (1971) quoted in their papers. However, Low and Mitchell (1965), with more extensive observations, derived no infrared excess. Thus Low and Mitchell's findings casts a serious blow to all theories that require the presence of a disk or an envelope composed of dust and solid particles. To all of us who have advocated such an envelope it is very disturbing indeed. However, one may argue that infrared radiation may be deficient in the star thereby obliterating the infrared radiation from the envelope. Hence a more critical test will be the detection of eclipses in infrared when the primary component obscures the disk from the observer. For this reason it is important to know what is the time of this infrared eclipse.

Since the eccentricity of the orbit is not zero, the infrared eclipse does not lie in the mid way between two optical (primary) eclipses. The exact time for the infrared eclipse depends upon the eccentricity, e , and the longitude of periastron, ω . Now there are two solutions for the spectroscopic orbit for ξ Aurigae. From these solutions we can derive the times that the next infrared eclipse is expected to occur. The result is given in Table 1.

Whether the secondary is a black hole or not is a problem that should be investigated from the point of view of stellar evolution. A recent study by Demarque and Morris (1971) along this line seems to suggest that there is no compelling evidence for a black hole in the system. They maintain a view, first advanced by Morris (1963) that the system

itself in spherical symmetry, if only statistically. Also, some of the particles of less velocities may be trapped in the potential well of its companion. Hence we may expect a less dense envelope to form around its companions in such binaries.

(11) The component star has an intrinsically extended envelope, or the atmospheres of the component star is so tenuous such that simply as a result of thermal motion, the matter escapes the atmosphere to form an envelope inside the lobe of the innermost contact surface. Perhaps we may include in this category Zeta Aurigae and similar objects with extended atmospheres. Wood's (1971) study presented earlier in this conference seems to apply to this group of stars.

(3.2) Complications and Characteristics

In order to understand the light curve affected by spherical envelopes we must consider two aspects: (i) the attenuation of the *light from the* eclipsed star by the envelope of the eclipsing star, and (ii) the surface brightness distribution of the eclipsed star that possesses an envelope. As in the case of disks, the attenuation can be caused either by scattering (whether coherent or incoherent) or by pure ~~ab~~absorption. In general the two components of an eclipsing binary are close, so the envelope must be too hot to form solid particles. Hence the case of pure absorption is not important in most spherical envelopes.

The attenuation of radiation in the envelope of the eclipsing star is easy to compute. The difficulty lies in the calculation of the brightness distribution of projected disks of both eclipsing and eclipsed stars when they possess envelopes. The envelope of one component star

scatters the light from its own star inside, producing a sharp discontinuity in the brightness distribution of the stellar disk. This problem can be mathematically formulated by applying the equation of radiative transfer in a spherical coordinate system (Huang, unpublished). Thus the limb darkening law of the eclipsed star is no longer given by a simple formula but can only be tabulated numerically.

A far more difficult problem concerns the transfer in the envelope of radiation coming from the companion. The difficulty lies in the fact that the incident radiation has no spherical symmetry with respect to the star with the envelope. No mathematical solution is yet available.

The computation of eclipsing light curves for stars with spherical envelopes, like that for stars with thin disk, is a straight forward but tedious. While the eclipse by a simple star has only three different configurations, namely partial eclipse, total eclipse and annular eclipse, that by a star with spherical envelope create 8 configurations (Huang 1970). Therefore in the computation we must choose the right configuration in each interval between two ^{consecutive} contacts, ^{according to} ~~depending upon~~ the relative values of the radii of the eclipsing star and its envelope and the radius of the eclipsed star. If the eclipsed star also has an envelope, the brightness distribution cannot be given by a simple formula but is presentable only in the form of a table, as has been mentioned before, which may be stored in the computer. That part of scattered radiation which comes originally from the incident radiation on the envelope from the

companion star has to be treated in an approximate way as the usual reflection effect ^{(e.g. Chen and Rhain 1971),} until a satisfactory mathematical treatment of this problem becomes available.

A preliminary computation (Huang 1970) has been performed based on the model of a spherical envelope with a view to understanding V444 Cygni. We have assumed that the scattering coefficient in the envelope is constant everywhere and that the disk of the eclipsed star is uniformly bright. While the result is quite encouraging, it appears that the constancy of the scattering coefficient is not correct. At present we are trying to apply different laws of scattering coefficient to the computation.

(3.3) Illustrative Cases with Spherical Envelope

Binary systems whose light curves may be understood by spherical envelopes may be more numerous than those with the disk structure. The following three systems represent only those which have been studied recently.

(1) V 356 Sagittarii -- It is composed of a A2 II and a B 3 V star and has well defined total and annular eclipse. Popper (1955) found that in order to obtain a satisfactory interpretation of the principal feature of the light curve, it is necessary to assume a distribution of light over the disk of the A star considerably more concentrated toward the center than is given by the usual cosine law of limb darkening. In other words, Popper has assumed a scattering envelope around the A2 II component. Since the A star is a giant, its envelope likely belongs to the second case discussed in sub-section (3.1). We also have no difficulty in assuming that the opacity in the envelope is due to electron scattering because it is in the vicinity of two

early - type stars. Consequently Popper's model is a good example of the general picture of spherical envelopes described in this article.

(11) V 444 Cygni -- The behavior of the light curve was found to be unusual and does not fit the model of two simple stars eclipsing each other. Several investigators have tried to interpret it in terms of spherical envelopes around component stars formed by the ejected matter from the W component (see references in Huang 1970). Again there is no difficulty in assigning electron scattering as the source of opacity in the envelopes.

(111) R T Lacertae -- Recently studied by Hall and Milone (1971) the system is an Algol-type eclipsing binary composed of two sub-giants of spectral types $K 1$ and $G 9$ respectively. From the color study they have concluded that the hot star is obscured during the secondary eclipse. This is obviously a very unusual case and led them to introduce what may be called an opaque envelope around the hot star. This opaque envelope is supposed to be dark but it does not affect the color indices arising from the luminous area which is brighter than the other component. This additional opaque area eclipses the companion and produces a minimum deeper than it otherwise could be. While this model enables them to explain also the unequal durations of primary and secondary eclipses, there remains the unresolved problem of the source of opacity in the envelope. It cannot be due to electron scattering because of the low temperatures of the two stars. Nor can it be due to solid particles in a spherical envelope. On the other hand, solid particles might appear in a disk envelope which dissipates energy more effectively than a spherical envelope. For this reason Hall and Taylor's (1971) earlier suggestion of a disk model deserves further investigation.

4. CONCLUDING REMARKS

Beer (1958) quoted from an unidentified source a passage which says: "By crossing the borderland from definite realm of geometry into the wilderness of astrophysics, we may run the risk of losing our way -- but there is also the happy prospect of converting the wilderness gradually into a paradise." More than a decade after Beer's paper, we now find that astrophysics is not exactly a wilderness, as there remain some rules in playing the game of light curves of binaries, even after the realm of geometry has been crossed. This, perhaps, is the point that the present article tries to emphasize. On the other hand, it is still a long way before our reaching the paradise.

It is my pleasure to acknowledge my sincere thanks to Dr. R. H. Koch who invited me to present this review, thereby enabling me to examine the problem of envelopes in an overall manner as is given here, to Drs. D.S. Hall, E.F. Milone and R.E. Wilson for their kindness in sending me the preprint of their unpublished papers, which, especially ones by Dr. Hall, have benefitted me greatly in the course of my investigations and have been quoted many times here. I would also like to thank Mrs. Vida M. Wach^kerling, who has carried out the computations of light curves based on the disk model at the Vogelback Computing Center of the university. The present study was supported by a grant from the National Aeronautics and Space Administration.

REFERENCES

- Appenzeller, I 1965, Ap. J., 141, 1390.
- Batten, A H. and Plavec, M. 1971, Sky and Telescope, 42, 147, 213.
- Beer, A 1958, Comm. Coll. Intern. d'Astrop. (Etoiles a Raies d'emission),
8, 387.
- Buerger, P. F. and Collins, G. W. II. 1970, Ap. J., 161, 1025.
- Cameron, A. G. W. 1971, Nature, 229, 178.
- Chandrasekhar, S. 1946, Rev. Mod. Phys. 18, 94.
- Chen, K -Y. and Rhein, W. J. 1971, Pub A.S.P., 83, 449.
- Coyne, G. V. 1970, Ap. J., 161, 1011.
- Crampin, J and Hoyle, F. 1960, M.N.R.A.S., 120, 33.
- Darwin, G. H 1897. Acta Mathematica, 21, 99.
- Demarque, P. and Morris, S. C. 1971, Nature, 230, 516.
- Deprit, A and Price, J. F., 1965, A. J., 70, 836.
- Doremus, C. 1970, Pub. A.S.P., 82, 745
- Gaposchkin, S. 1949, Peremennye Zvezdy, 7, 36.
- Goudas, C. L. 1963, Icarus, 2, 1.
- Gyldenkerne, K. Vistas in Astronomy, 12, 199.
- Hack, M. 1961, Mem. Soc. Astr. Italiana, 32, No. 4.
- Hall, D. S. 1971a, presented at the Bamberg Variable Star Colloquium.
----- 1971b, IAU Colloquium No. 16.
- Hall, D. S. and Garrison, L. M. Jr. 1969, Pub. A.S.P., 81, 771.
- Hall, D. S. and Milone, E. F. 1971, preprint "An Interpretation of the
Complicated Eclipsing Binary RT Lacertae."
- Hall, D. S. and Taylor, M. C. 1971, Bull A.A.S., 3, No. 1 pt. 1 p. 12.
- Herbig, G. H. 1957, "Non-Stable Stars", Cambridge University Press p. 3.
- Héron, M. 1965 Ann. d'ap, 28, 992.
- Hill, G. W. 1886, Acta Mathematica, 8, 1.
- Hjellming, R. M. and Hiltner, W. A. 1963, Ap. J., 137, 1080.

- Huang, S -S. 1956, A. J., 61, 49.
- 1957, J.R.A.S. Canada, 51, 91.
- 1963a, Ap. J., 138, 342.
- 1963b, Ap. J., 138, 471.
- 1965a, Ap. J., 141, 201.
- 1965b, Ap. J., 141, 976
- 1967, Ap. J., 148, 793.
- 1968, Vistas in Astronomy, 11, 217.
- 1970, Ap. J., 161, 1033.
- Huang, S.-S. and Struve, O. 1956, A. J., 61, 300.
- Huruhata, M. and Kitamura, M. 1958, Tokyo Obs. Bull No. 102, 1103.
- Joy, A. H., 1942, Pub A.S.P., 54, 35.
- 1947, Pub. A.S.P., 59, 171.
- Koch, R. H., 1970, in Proc. IAU Colloquium No. 6., Mass Loss and Evolution
in Close Binaries, Ed. K. Gyldenkerne and R. M. West, Copenhagen:
Copenhagen University, 65.
- Koch, R. H., Sobieski, S. and Wood, F. B., 1963, Pub. University Pennsylv-
vania, Astro. Series, Vol. 9.
- Kondo, Y, McCluskey, G., and Houck, T. E., 1971, presented at Orbiting
Astronomical Observatory Symposium, Amherst, Mass. Aug. 22-23, 1971.
- Kopal, Z. 1955, Comm. Coll. Intern. d'Ap. Liege (Les particules solides
dans les astres) 6, 241.
- 1971, Astrop. and Space Science 10, 332.
- Kraft, R. P. 1954, Ap. J., 120, 391.
- Kruszewski, A. 1966, Advances Astron. Astrop., 4, 233.
- Kuhi, L. V. and Schweizer, F. 1970, Ap. J., 160, L185.
- Kuiper, G. P. 1941, Ap. J., 93, 133.
- Kuiper, G. P., Struve, O., and Stromgren, B. 1937, Ap. J., 86, 570.

- Larsson-Leander, G. 1969, Ark. f. Astro. 5, 253.
- Low, F J , and Mitchell, R. I. 1965, Ap. J., 141, 327.
- Lucy, L B. 1971, IAU Colloquium No. 16.
- Mitchell, R I. 1964, Ap. J , 140, 1607.
- Morris, S. C. 1962, J.R.A.S. Canada, 56, 210.
 ----- . 1963, Thesis, University of Toronto.
- Paczński, B. 1971 Ann. Rev. Astron. Astrop. 9, 183.
- Popper, D. M. 1955, Ap. J., 121, 56.
- Prendergast, K. H. 1960, Ap. J., 132, 162.
- Sahade, J 1960, in Stellar Atmospheres, ed. J. L. Greenstein, (Chicago: University of Chicago Press), Chap. 12.
- Schönberg, E. and Jung, B. 1938, A. N., 265, 221.
- Stępień, K 1970, Acta Astr., 20, 13.
- Stothers, R. 1971, Nature, 229, 180.
- Strand, K. Aa. 1959, A. J., 64, 346.
- Struve, O. 1931, Ap. J., 73, 94.
 ----- . 1949, M.N.R.A.S., 109, 487.
 ----- . 1957, "Non-Stable Stars" (ed. G H. Herbig) Cambridge: Cambridge U. Press p. 93.
- Thiessen, G. 1957, Zs. f. Ap., 43, 233.
- Wildey, R. L. and Murray, B. C. 1964, Ap. J., 139, 435.
- Wilson, R. E. 1971, preprint, "A model of ϵ Aurigae."
- Wood, D. B. 1971, IAU Colloquium No. 16.
- Woolf, N. J. 1965, Ap. J., 141, 155.

LEGENDS

Figure 1 -- Classification and interpretation of eclipsing binaries.

The eclipsing binaries are divided into two classes according to whether either or both stars are undergoing mass change. The mass ejected by the evolving star may form an envelope which complicates the analysis of the light curve in such a way that the usual model of two spherical stars eclipsing each other becomes invalid. The figure illustrates the general scheme of how the envelopes may play a part in the interpretation of light curves.

Figure 2 -- Sources of opacity in the envelope and their observable indications.

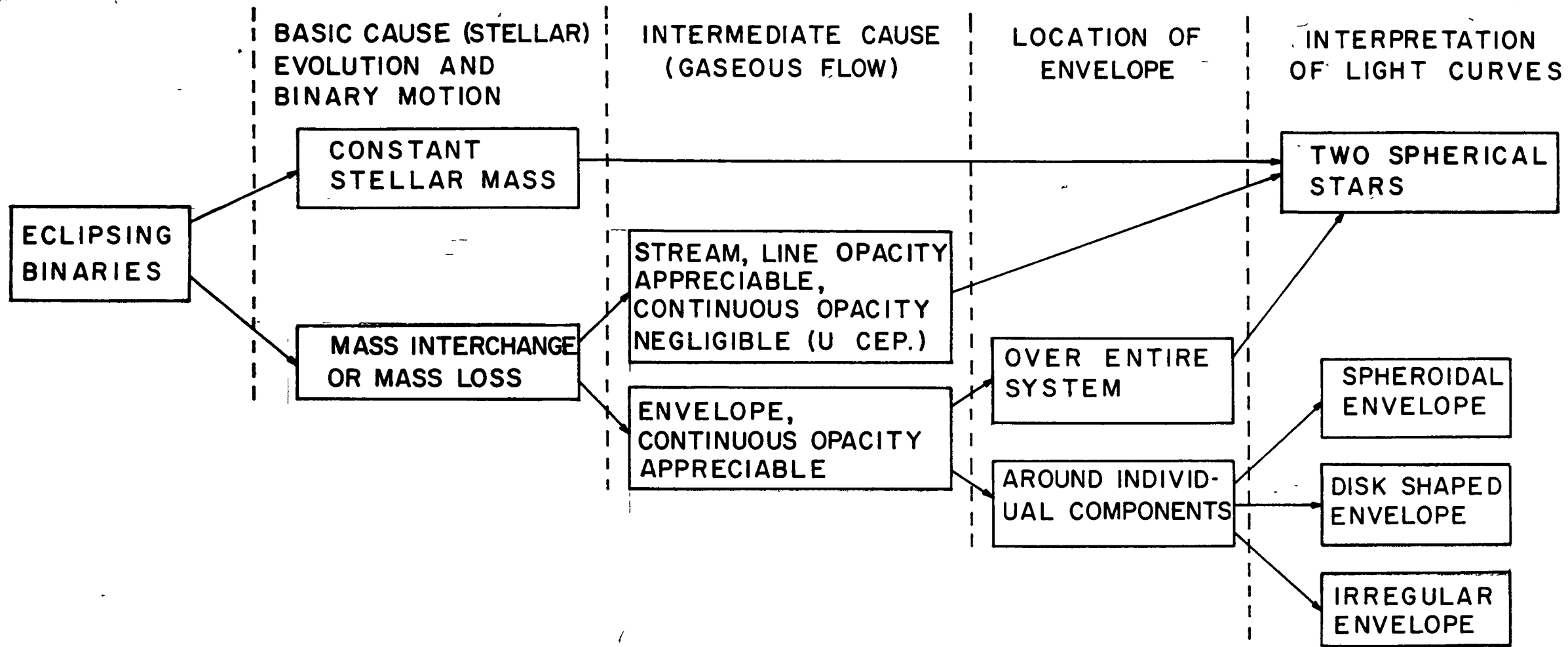
Figure 3 -- Photometric characteristics of the eclipsing binary that possesses one or more envelopes. The characteristics of eclipsing binaries that are detectable by photometric observations are summarized in this diagram. It shows that these characteristics depend greatly upon the location of the envelopes.

TABLE 1
TIMES OF NEXT INFRARED ECLIPSE BASED ON SPECTROSCOPIC DATA

SP. DATA		INVESTIGATORS	TIMES OF INFRARED ECLIPSE
e	ω		
33	350°	Kuiper, etc. (1937)	June 1974 -- June 1976
.172	348°	Morris (1962)	Oct. 1971 -- Oct. 1973

Figure 1

CLASSIFICATION OF ECLIPSING BINARIES AND INTERPRETATION OF THEIR LIGHT CURVES



SOURCES OF OPACITY AND THEIR OBSERVABLE INDICATORS

INCREASING TEMPERATURE

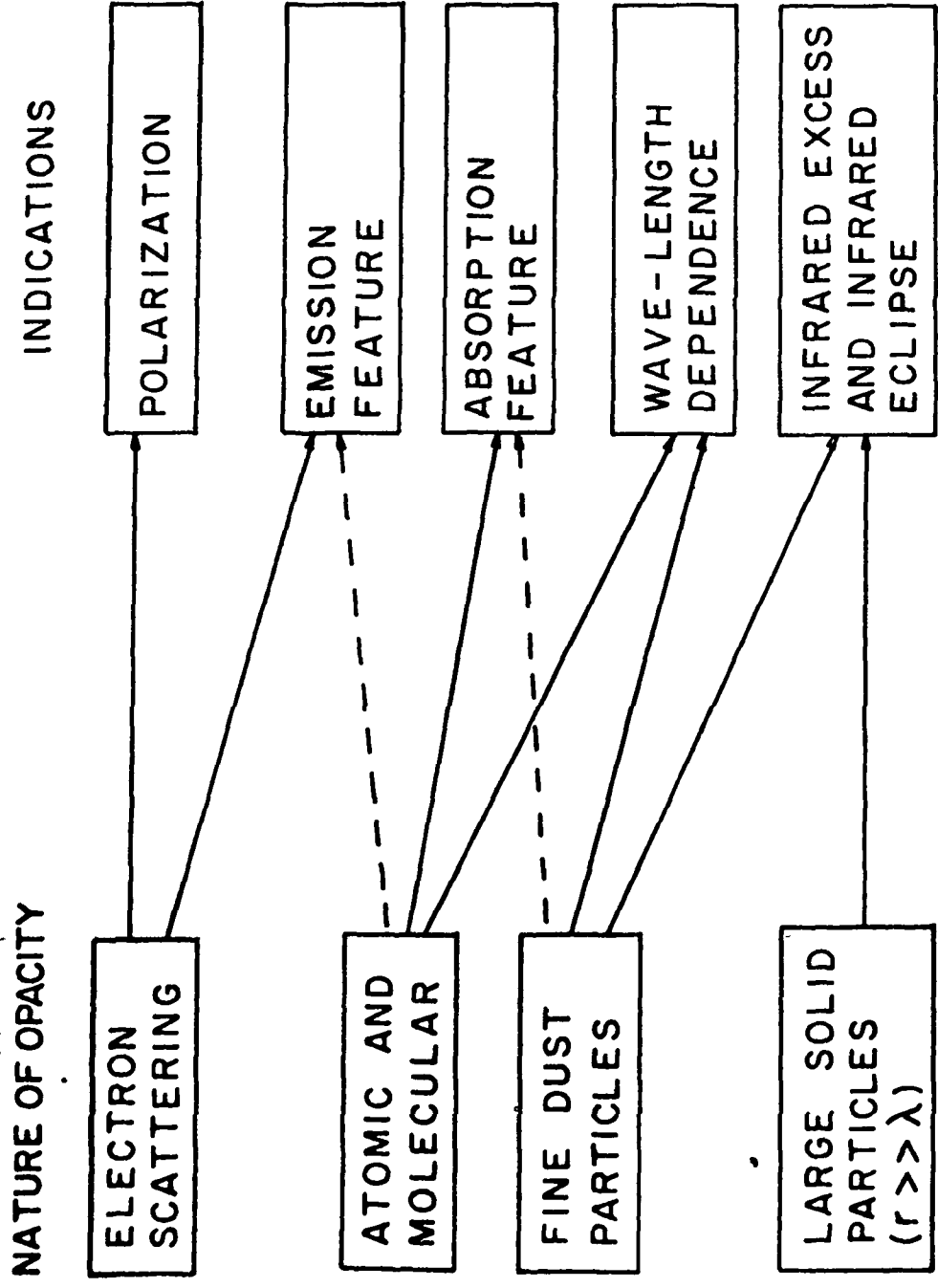


Figure 3

PHOTOMETRIC CHARACTERISTICS OF ENVELOPES

