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LAUNCH WINDOW ANALYSIS IN A NEW PERSPECTIVE WITH EXAMPLES OF DEPARTURES FROM EARTH TO MARS

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16. Abstract

Earth-departure windows are investigated for two round trip stopover missions to Mars. These are the 1981 inbound Venus swingby mission and the 1986 direct minimum-energy mission. The secular effects of planetary oblateness are used to predict the motion of the parking orbit. A procedure is developed for matching the motion of the parking orbit and the escape asymptote. Earth-departure velocity penalties, caused by orbital plane misalinement, are reduced by synchronizing the motion of the parking orbit and the escape trajectory.

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SUMMARY

A technique relying upon the effects of gravitational harmonics for parking-orbit alinement is analyzed. This technique is applied to the problem of injection from an assembly parking orbit around the Earth to an escape trajectory toward Mars. The initial orientation of the assembly orbit is chosen so that resulting nodal and apsidal perturbations will maintain continually a nearly ideal alinement with the required departure asymptote during succeeding days of the departure opportunity. Thus, the motion of the parking orbit is synchronized with the motion of the escape asymptote. Injection velocity penalties caused by misalinement are greatly reduced or eliminated.

Parking orbits that track the escape asymptote have moderate-to-high apoapsis altitudes, typically in a range from 2000 to 8000 nautical miles. This method may be useful for planning round trip interplanetary missions. In relation to the high apoapsis altitudes, the method could be used for defining parking orbits at a target planet by considering the departure window for the return trip to Earth.

INTRODUCTION

Conditions for an ideal minimum ΔV impulse to transfer from a circular parking orbit to a hyperbolic escape orbit occur when the orbits are coplanar and the impulse is applied at periapsis of the escape hyperbola. An additional condition for an elliptical orbit is that the periapsis position vectors of the parking orbit and the escape hyperbola must be colinear. For preliminary analysis of an interplanetary mission, it is often economical and judicious to assume implicitly that these ideal alinement conditions will exist at the time of injection and then to calculate the ideal or characteristic velocity of an escape trajectory.

Unfortunately, these conditions seldom exist when dynamic effects are simulated in an analysis of the injection window. First, the oblateness of the planet perturbs the ascending node and the argument of periapsis of the orbit. The secular effects of these perturbations are that the ascending node and argument of periapsis change linearly with time. Second, the hyperbolic excess velocity vector is also changing with time. These motions are independent and invariably result in misalinement of the orbital plane and the departure trajectory. The escape maneuver no longer can be applied at periapsis, and, further, a large plane change may be necessary. Substantial, and sometimes severe, velocity penalties may be required to accomplish the required turning.

Many authors have studied this problem (refs. 1 and 2). All of these authors have approached the problem by choosing the inclination, eccentricity, and periapsis radius of the parking orbit, and then waiting for the ascending node and argument of periapsis to change into a configuration such that the velocity impulse required to inject into the escape orbit is a minimum. This procedure results in long time periods during a departure opportunity when injection is impossible because of spacecraft fuel limitations.

Efficient propulsive techniques have been devised to accommodate this problem and to buy more time for scheduling the injection maneuver. Even though these techniques can reduce the velocity requirements dramatically, there still may be relatively long periods during a departure opportunity when injection is impossible because of a large plane-change velocity penalty.

A technique that predicts the parking-orbit inclination and eccentricity required to keep the motion of the parking orbit and the escape orbit in near synchronization is presented in this paper. The initial orientation of the assembly orbit is chosen so that the resulting nodal and apsidal perturbations will shift the original orbit so that it continually maintains a nearly ideal alinement with the required departure asymptote during succeeding days of the departure opportunity. Plane-change velocity penalties caused by orbit-plane misalinement are reduced greatly or are eliminated.

Using assembly orbits that track the escape trajectory is an interesting mechanism for buying more time for scheduling the injection maneuver. Ideally, this approach could also afford the most flexible schedule for launch and orbital assembly of the spacecraft and the propulsion systems. Unexpected difficulties can delay the launch of a vehicle or the assembly of several vehicles in Earth orbit. In addition to built-in holds in the assembly schedule, delays can be accommodated by letting the injection date slip to a later time within the injection window. Slipping the injection date no longer has a consequence because there is an opportunity to initiate the escape maneuver during every revolution of the parking orbit. Therefore, in the ideal situation, this technique offers flexibility as a principal advantage.

Booster performance and the geographic location of the launch site largely dictate the characteristics of the assembly orbit; the general problem is to maximize the payload that can be delivered to the assembly orbit. Ordinarily, the spacecraft and propulsion systems would be launched directly into an assembly orbit, which not only maximizes the payload but also affords a timely schedule for launching several vehicles. These considerations lead to the principal (but unproven) disadvantage of the technique presented in this report. That is, the characteristics of assembly orbits that track the escape asymptote may or may not be compatible with the characteristics dictated by booster performance. This problem cannot be answered until the characteristics of orbits that are dictated by the two methods are discovered. The ideal situation would be that the characteristics of the assembly orbit dictated by the two methods would be the same or at least compatible. This report investigates the following two questions and presents a partial resolution of the problem.

1. What are the orbital elements of the parking orbits that track the escape asymptote and permit a departure at any time during the launch opportunity?

2. How long does the motion of the orbit remain synchronous with the motion of the departure asymptote?

The authors thank Ellis W. Henry and Gregory A. Zambo, NASA Manned Spacecraft Center, Houston, Texas, for contributing their work on the multiple-impulse heliocentric transfer trajectories.

SYMBOLS

ALFA	right ascension of excess hyperbolic velocity vector, deg
APO ALT	apoapsis altitude, n. mi.
a	semimajor axis
C	constant defined by equation (22)
CIR DV	circular-to-elliptical transfer ΔV, fps
c	auxiliary constant for the differential $\dot{\omega}_\infty$
DELTA	declination of excess hyperbolic velocity vector, deg
ECC	eccentricity (FORTRAN symbol in tables III, IV, and V)
EMOS	Earth mean orbital speed
e	eccentricity
f	auxiliary variable for the differential $\dot{\Omega}_\infty$
g	auxiliary variable for the differential $\dot{\omega}_\infty$
h	auxiliary variable in equation (16)
INCL	inclination, deg (FORTRAN symbol in tables III, IV, and V)
i	inclination, deg
i_L	critical inclination equal to 63.43 deg

J_2	oblateness coefficient
k	indexing parameter ($k = \pm 1$)
NODE	right ascension of ascending node, deg
n	mean motion
OMEGA	argument of periapsis, deg
PC ANG	plane-change angle, deg
PERIOD	orbital period, hr
R_B	equatorial planet radius
r_π	radius of periapsis
SMA	semimajor axis, ft
SUM DV	sum of CIR DV and TMI DV
TEI DV	injection ΔV at the time of ideal alinement
TJD	elliptical orbit departure Julian date, days
TMI DV	trans-Mars injection ΔV, fps
TRUAN	true anomaly of departure maneuver
t	time
V_∞	excess hyperbolic velocity
VMAG	excess hyperbolic velocity magnitude, fps
V PERI	velocity at periapsis, fps
Z	variable defined by equation (20)
α, β, ϕ	variables introduced for solution of cubic equation (A2)
α_∞	right ascension of the V_∞ vector
$\alpha_0, \alpha_1, \alpha_2$	coefficients of ξ in cubic equation (A2)
ΔV	impulsive-velocity increment

δ	declination of the V_∞ vector
η	true anomaly of hyperbolic escape asymptote
μ	gravitational parameter
ξ	trigonometric variable equal to $\tan^2 i$
σ	auxiliary variable used in defining Ω_∞
Ω_∞	ascending node as computed from escape asymptote
$\dot{\Omega}_s$	secular rate of ascending node computed from planetary oblateness
$\dot{\Omega}_\infty$	secular rate of ascending node computed from motion of escape asymptote
ω_∞	argument of periapsis as computed from escape asymptote
$\dot{\omega}_s$	secular rate of argument of periapsis computed from planetary oblateness
$\dot{\omega}_\infty$	secular rate of argument of periapsis as computed from motion of escape asymptote

ANALYSIS

The method that will be presented is similar to the one used for alining a parking orbit about an oblate planet with arrival and departure hyperbolic excess velocity vectors as presented in reference 3. In that reference, alining an orbit at an oblate planet requires that the total change in the ascending node and argument of periapsis of the parking orbit be equal to the corresponding quantities of the escape orbit. The method used here requires that the ascending node rate and argument of periapsis rate of the parking orbit match instantaneously with the corresponding rates of the escape orbit.

Derivation of Synchronous i and e

The requirement that the two orbits be synchronized at any time t may be expressed mathematically as

$$\dot{\Omega}_s(t) = \dot{\Omega}_\infty(t) \tag{1}$$

$$\dot{\omega}_s(t) = \dot{\omega}_\infty(t) \tag{2}$$

It will be assumed in this derivation that the radius of periapsis r_π is specified and that the hyperbolic excess-velocity vector $(V_\infty, \alpha_\infty, \delta_\infty)$ and its rate $(\dot{V}_\infty, \dot{\alpha}_\infty, \dot{\delta}_\infty)$ as functions of time are known.

From reference 4, $\dot{\Omega}_s$ and $\dot{\omega}_s$ are found from

$$\dot{\Omega}_s = \frac{-3nJ_2R_B{}^2}{2a^2\left(1-e^2\right)^2} (\cos i) \tag{3}$$

$$\dot{\omega}_s = \frac{-3nJ_2R_B{}^2}{2a^2\left(1-e^2\right)^2} \left(\frac{5}{2} \sin^2 i - 2\right) \tag{4}$$

It must be noted that these are secular perturbations (as implied by the subscript s). Because i, a, and e are not secularly perturbed, $\dot{\Omega}_s$ and $\dot{\omega}_s$ are constants, when i, a, and e are known.

From reference 5, the ascending node and argument of periapsis of the hyperbola are given at any time by

$$\Omega_\infty = \alpha_\infty + \pi + \sigma \tag{5}$$

or

$$\Omega_\infty = \alpha_\infty - \sigma \tag{6}$$

where

$$\sigma = \sin^{-1}\left(\frac{\tan \delta_\infty}{\tan i}\right) \tag{7}$$

As shown in reference 5, equation (5) is used for a maximum periapsis declination and equation (6) for a minimum periapsis declination.

The argument of periapsis of the hyperbola is given by

$$\omega_\infty = -\eta + \tan^{-1}\left[\frac{\tan \delta_\infty}{\sin i \cos\left(\Omega_\infty - \alpha_\infty\right)}\right] \tag{8}$$

where

$$\eta = \cos^{-1}\left[-\left(1 + \frac{V_\infty^{2} r_\pi}{\mu}\right)^{-1}\right] \tag{9}$$

The time derivatives of Ω_∞ and ω_∞ are found by differentiating equations (5) and (6) and equation (8). These results are

$$\dot{\Omega}_\infty = \dot{\alpha}_\infty + f\dot{\delta}_\infty \tag{10}$$

$$\dot{\omega}_\infty = c\dot{V}_\infty + g\dot{\delta}_\infty \tag{11}$$

where

$$f = \frac{k}{\cos^2 \delta_\infty \cos \sigma \tan i} \tag{12}$$

$$g = \frac{-k}{\sin i \cos \sigma} \tag{13}$$

$$c = \frac{2 \cos^2 \eta}{\sin \eta} \frac{V_\infty r_\pi}{\mu} \tag{14}$$

and $k = \pm 1$, depending upon whether maximum or minimum periapsis declination is desired.

From equations (1) and (2)

$$\frac{\dot{\Omega}_\infty}{\dot{\omega}_\infty} = \frac{\dot{\Omega}_s}{\dot{\omega}_s} \tag{15}$$

By using equations (3), (4), (10), and (11) in equation (15)

$$\frac{\dot{\alpha}_\infty + \dot{f}\delta_\infty}{c\dot{V}_\infty + g\dot{\delta}_\infty} = h = \frac{\cos i}{\frac{5}{2}\sin^2 i - 2} \tag{16}$$

Note that equation (16) is independent of the eccentricity e. Thus, it may be solved by iteration for the inclination i, because V_∞ and $\dot{V}_\infty$ are known. An interesting particular solution of equation (16), when $\dot{V}_\infty = 0$, is given in the appendix. This solution is analytic and is useful in providing initial guesses for the numerical solution of equation (16).

Three restrictions on the inclination should be stated. The first restriction is that

$$\pi - |\delta| > i > |\delta| \tag{17}$$

which is seen by inspection of equation (7). The second restriction is that if

$$\dot{\Omega}_\infty < 0$$

then $$i < \frac{\pi}{2}$$

and if $$\dot{\Omega}_\infty > 0$$

then $$i > \frac{\pi}{2} \tag{18}$$

which is seen by inspection of equation (3). The third restriction is that if

$$\dot{\omega}_\infty < 0$$

then $$i_L < i < \pi - i_L$$

and if $\dot{\omega}_\infty > 0$

then
$$0 < i < i_L \tag{19}$$

where $i_L = \sin^{-1}\left(\sqrt{\frac{4}{5}}\right) \approx 63.43°$, which is seen by inspection of equation (4).

Violation of the restriction of equation (17) will cause the iterative solution of equation (16) for i to fail immediately. The restrictions of equations (18) and (19) must be tested before the determination of the eccentricity is begun.

When the inclination has been found, the eccentricity may be found from equations (1) and (3).

$$\frac{\dot{\Omega}_S}{\cos i} = \frac{\dot{\Omega}_\infty}{\cos i} = -\frac{3nJ_2R_B^2}{2a^2\left(1 - e^2\right)^2} = Z \tag{20}$$

Because $a = r_\pi/(1 - e)$ and $n = \frac{\mu}{a^3}$, equation (20) becomes

$$\frac{Z}{C} = \frac{(1 - e)^{3/2}}{(1 + e)^2} \tag{21}$$

where

$$C = -\frac{3}{2}J_2R_B^2\sqrt{\frac{\mu}{r_\pi^7}} \tag{22}$$

Equation (21) may now be solved by iteration for the eccentricity e and, in fact, is identical to the iteration performed in reference 3. Note from equation (21) that the quantity Z/C must always be positive for a solution to exist. The requirement that Z/C be positive is simply a restatement of the restriction given by equation (18).

The computation of the inclination and eccentricity can now be summarized.

1. Given V_∞, $\dot{V}_\infty$ at time t, r_π, and Z, solve equations (16) for the inclination.

2. Compute $\dot{\Omega}_\infty$ and $\dot{\omega}_\infty$ from equations (10) and (11); then test this inclination to see that it meets the restrictions of equations (18) and (19).

3. Now solve equation (21) for the eccentricity.

PROCEDURE

The techniques discussed in this report have been programed in FORTRAN V for the UNIVAC 1108 digital computer. Two approaches were investigated. In the first approach, instantaneous choices of the orbital elements were calculated as functions of time. By making the proper choices of inclination and eccentricity (ideally, updating these elements continuously by small midcourse maneuvers), the orbit would be forced to maintain ideal alinement. Although the individual midcourse maneuvers are small (from 80 to 100 fps to as large as 500 fps every 5 days), their sum rapidly accumulates; consequently, this approach was abandoned.

In the second approach, the parking orbit is not forced to remain ideally alined. Instead, the orbit is required to be ideally alined at some time during the injection window and to be approximately alined during the remainder of the injection window. For this approach, there are no guidance and targeting corrections; it is assumed that all the accumulated errors are removed at the time of the injection maneuver (or maneuvers as in the case of a multiple-impulse departure sequence). Thus, the orbit merely drifts and, in a general sense, follows the motion of the escape asymptote. The orbit is no longer ideally alined. The injection maneuver may again include an off-periapsis thrust as well as a plane change. The following are important differences.

1. The orbit can be alined during any portion of the injection window.

2. The plane-change misalinement is greatly reduced or eliminated during the remainder of the window.

The second approach has been used to determine the effective Earth-orbital departure windows for two types of Mars round trip stopover missions: the 1981 Mars stopover with inbound Venus swingby and the 1986 direct Mars stopover with near-Hohmann transfers. The heliocentric profiles for these missions are shown in figures 1 and 2.

The Mars mission flight plans were calculated by using a patched-conic trajectory model. Successive conic trajectories were pieced together to form a reference mission or flight plan; a linear weighted least-squares iteration technique was used to produce the launch dates, flight times, and impulsive characteristic velocity requirements for flight plans that exhibit a minimum in the sum of the velocity impulses. Patched-conic solutions were calculated at 5-day steps through 100-day Earth departure periods in 1981 for the swingby mission and in 1986 for the direct mission. The hyperbolic excess velocity vectors at Earth departure were then used to compute the characteristics of

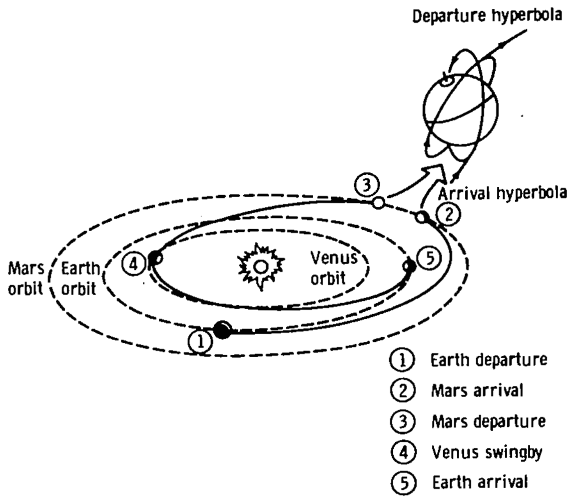


Figure 1. - Illustration of the 1981 Mars stopover/Venus swingby mission.

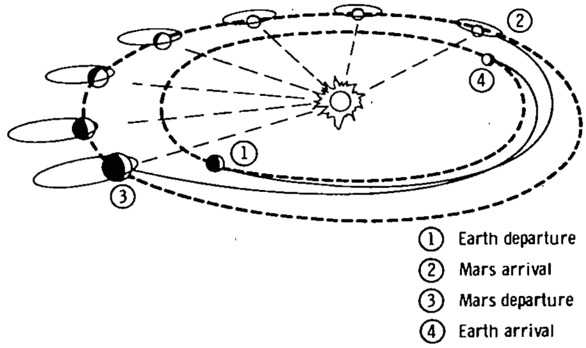


Figure 2. - Illustration of the conjunction class mission and the parking orbit about Mars.

the parking orbits at Earth and the injection-velocity requirements for the Earth escape maneuver.

The Earth escape maneuver is modeled by a single, minimum ΔV, impulsive thrust that includes a plane change as well as an off-periapsis thrust component, as shown in figure 3. The general formulation for the injection ΔV is given in reference 5. The ΔV is minimized by a one-dimensional search on the parking orbit true anomaly.

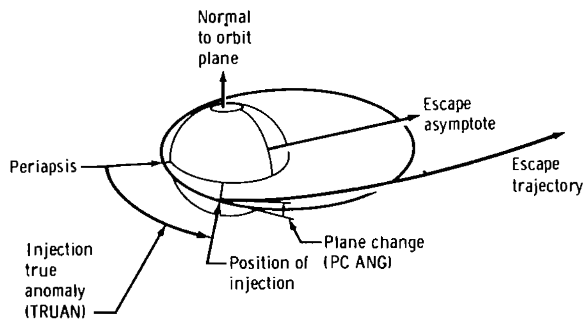


Figure 3. - Geometry of single-impulse injection maneuver.

RESULTS

The motion of the escape asymptote during the course of a launch opportunity dramatically affects the solutions that can be obtained for the tracking orbit. There is no question that solutions can be found. Whether these solutions are useful or desirable is the question that must be investigated.

A desirable solution may have restrictions on the range of acceptable orbital inclinations and apoapsis altitudes. Because these parameters are the dependent variables of a deterministic system of equations, there is no discrete control over them. Two methods to control the inclination and eccentricity of the tracking orbit can be investigated.

1. Trajectory shaping can be used to control the orientation and motion of the escape asymptote. This approach is demonstrated by the use of two- and three-impulse

2. Given the time history of the escape asymptote during the course of a launch opportunity, solutions can be obtained at discrete times during the launch opportunity. This approach amounts to shifting the epoch of the solution (e.g., the time of ideal alinement) and provides a mechanism for "fine tuning" the orbital elements.

Both approaches have been applied in an analysis of the 1981 Venus swingby and the 1986 direct minimum-energy Mars stopover missions. The results for the two-impulse trajectories are discussed in the following paragraphs, and the results for the three-impulse trajectories are discussed in a later section.

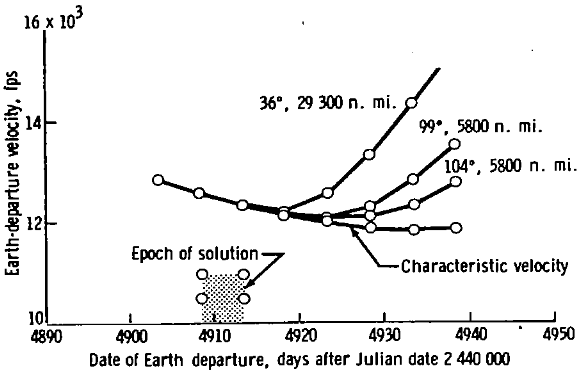
The analysis for the 1981 Venus swingby mission is included as an example to show the effect of shifting the time of ideal alinement (e.g., the epoch of the solution). The results presented for this mission are typical. The orbits display high apoapsis altitudes and both posigrade and retrograde inclinations. The solutions for the orbit inclinations and apoapsis altitudes are found to be slightly different when the solutions are evaluated at different epochs. Typically, three solutions are found at any epoch. The reasons for this are discussed in the appendix. The orbits track for approximately 50 percent of the total available window.

The 1981 Mars Stopover With Venus Swingby

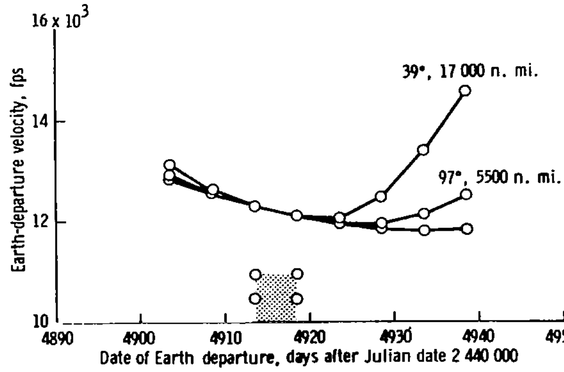
The 1981 Mars stopover with a Venus swingby mission consists of a direct interplanetary transfer from Earth to Mars, a short orbital stopover at Mars, and a return transfer from Mars to Earth that is also a free flyby of Venus. The direct transfer from Earth to Mars can be either a type I (heliocentric transfer angle $<180^\circ$) or a type II (heliocentric transfer angle $>180^\circ$) trajectory. Both transfer trajectories have the same arrival dates at Mars. The Earth-departure dates for these trajectories are separated by 40 to 60 days. The result is that there are two separate Earth-departure opportunities, one in November of 1981 and the other in January 1982. A summary of the characteristics of these missions is presented in tables I and II. The stay time at Mars for all swingby missions analyzed in this report is 60 days.

The Earth-departure velocity contour for this mission displays a characteristic double minimum, and there is a rise in the departure velocity at the transition from the type I to the type II solutions. The double minimum in the Earth-departure velocity contour prevents the use of the total available launch window. Tracking orbits can follow the motion of the escape asymptote only if the motion is continuous. Thus, the type I and type II departure windows must be considered separately. (Using broken-plane heliocentric trajectories with an intermediate impulse applied between Earth and Mars can eliminate the discontinuity. This will be discussed later.)

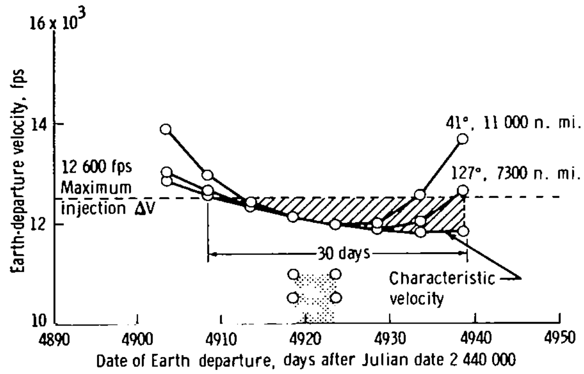
The Earth-departure velocity requirements for the type II window are shown in figures 4(a), 4(b), and 4(c). The effect of shifting the epoch of the solution is found by comparing these figures. The time of ideal alinement is indicated in the figure where the curves converge and is marked by asterisks and a crosshatched region at the bottom of the grid. Three solutions are plotted in figure 4(a). These solutions have inclinations and apoapsis altitudes of 36° and 29 300 nautical miles, 99° and 5800 nautical miles, and 104° and 5800 nautical miles.



(a) Alinement date of October 31, 1981 (Julian date 2 444 908.65).



(b) Alinement date of November 5, 1981 (Julian date 2 444 913.65).



(c) Alinement date of November 10, 1981 (Julian date 2 444 918.65).

Figure 4. - Earth-departure velocity requirements for the 1981 Venus swingby with Earth-to-Mars transfer angles greater than 180°.

The fourth and bottom curve displays the ideal or characteristic velocity for departure from a 262-nautical-mile altitude circular-parking orbit. This curve is simply the scalar difference between the periapsis speed of the escape hyperbola and the circular speed of the 262-nautical-mile altitude parking orbit, and it represents the absolute minimum injection ΔV. The bottom curve thus provides a standard of comparison to measure the performance of different solutions. The bottom curve, used as a basis for comparison, is indicative that the 104° orbit, which appears to maintain alinement best, tracks almost ideally the first 15 days before gradually drifting out of alinement. Likewise, the 36° inclined orbit appears to track the first 15 days but rapidly diverges during the remainder of the launch window.

If the time of alinement is shifted by 10 days, there are two solutions as shown in figure 4(b); one with an inclination of 97° and apoapsis altitude of 5500 nautical miles, and the other with an inclination of 39° and 17 000-nautical-mile altitude.

The apoapsis altitude of 17 000 nautical miles for the 39° solution is considerably lower than the corresponding solution of the 29 000-nautical-mile altitude and 36° inclination, as shown in figure 4(a).

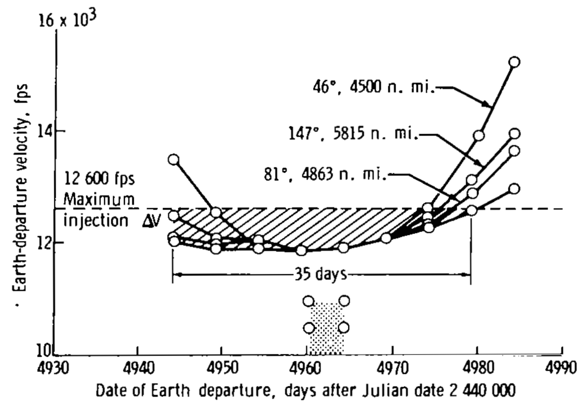
If the time of alinement is again shifted by 10 days, two more solutions, which are shown in figure 4(c), are found. The solutions are at 41° inclination with an 11 000-nautical-mile apoapsis altitude and at 127° inclination with a 7300-nautical-mile altitude. When the epoch is shifted forward through the launch period, the apoapsis altitude decreases, and different solutions are found at each new epoch. The curves of figure 4 are typical of solutions that can be found. If an artificial limit is drawn to represent a maximum permissible injection ΔV of 12 600 fps (represented by the broken line in fig. 4(c)), it can be seen that there are 30 days during the type II window in which the ideal injection ΔV is less than 12 600 fps. The solution with the 41° inclination provides 22 days during which the injection ΔV is less than 12 600 fps. This solution tracks approximately 67 percent of the available launch window.

The time histories of the solutions in figure 4 are presented in tables III(a), III(b), and III(c). The plane change and off-periapsis thrust requirements, the orbital elements, and other parameters that describe the solution are presented. These parameters are included for future reference and possible comparison with other methods.

In table III, the time of ideal alinement is indicated when the off-periapsis thrust angle (TRUAN) and the plane-change angle (PC ANG) are simultaneously zero. Two values of ΔV (TMI DV and SUM DV) are shown in table III. The TMI DV is the vector ΔV required for a single-impulse injection maneuver to escape from the elliptical tracking orbit into a trajectory toward Mars. The SUM DV considers the additional ΔV that is required to transfer from the initial 262-nautical-mile altitude circular parking-orbit to the elliptical orbit. The circular-to-elliptical orbit transfer ΔV is printed under the heading CIR DV.

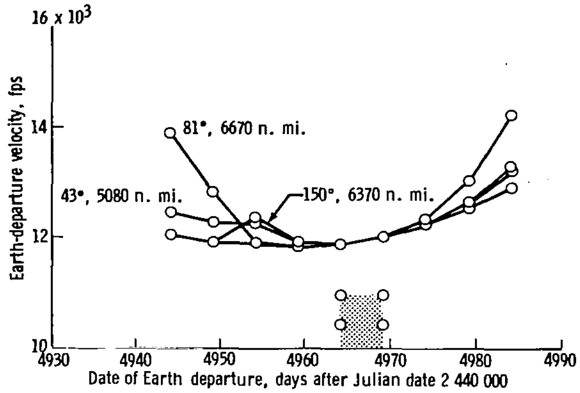
The time history of the motion of the escape asymptote is also shown in table III. The excess hyperbolic velocity vector is printed in terms of speed in feet per second (VMAG), and right ascension (ALFA) and declination (DELTA) in degrees. The declination is measured with respect to the equatorial plane of the Earth. The right ascension is measured from an inertial X-axis through Aries; this is also the system in which orbital elements are measured.

The Earth-departure velocity requirements for the type I launch window are shown in figures 5(a), 5(b), and 5(c). The time histories of the solutions are presented in tables IV(a), IV(b), and IV(c). Three solutions are shown in figure 5(a); one is posigrade with a 46° inclination and 4500-nautical-mile apoapsis altitude, one is nearly polar with an 81° inclination and 4863-nautical-mile altitude, and one is retrograde with a 147° inclination and 5815-nautical-mile altitude. An examination of figures 5(b) and 5(c) indicates the effect of shifting the epoch forward through this launch window. The inclinations remain nearly the same, and the altitudes of the solutions increase.

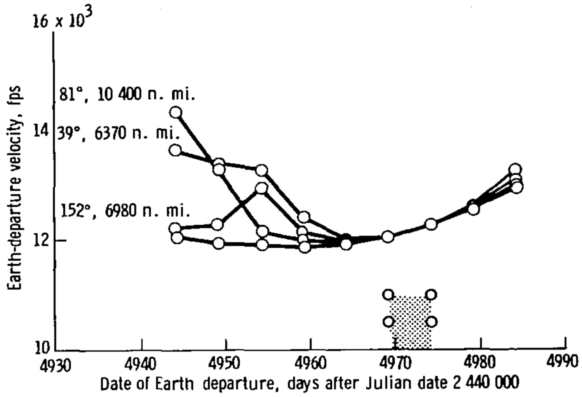


(a) Alinement date of December 20, 1981 (Julian date 2 444 959.31).

Figure 5. - Earth-departure velocity requirements for the 1981 Venus swingby with Earth-to-Mars transfer angles less than 180°.



(b) Alinement date of December 25, 1981 (Julian date 2 444 964. 31).



(c) Alinement date of December 30, 1981 (Julian date 2 444 969. 31).

Figure 5. - Concluded.

Also in figure 5(a), an artificial limit can be drawn to represent a maximum permissible injection ΔV of 12 600 fps (represented by the broken line in fig. 5(a)). There are 35 days in the type I window during which the ideal injection ΔV is less than 12 600 fps. The solution with the 46° inclination provides 30 days during which the velocity requirements are less than 12 600 fps. The 46° inclined orbit permits departure any time (i. e., at each periapsis passage) during 86 percent of the available launch window.

The injection maneuver is a sequence of two maneuvers, the CIR DV and the TEI DV. Often these are of nearly equal magnitude. There are several methods of handling the implied logistical problems of launch, Earth-orbit assembly, and scheduling of these two maneuvers. For example, if the altitude of the solution is low enough, the spacecraft could be launched directly into a tracking orbit; if the altitude is high, the spacecraft could be launched into an intermediate phasing orbit and the CIR DV could be scheduled after the launch and assembly and before the opening of the injection window.

The CIR DV is introduced to form a relative basis for comparison of different solutions and it may or may not be scheduled discretely in an operational sequence.

The 1986 Direct Minimum-Energy Mars Stopover

The direct minimum-energy Mars stopover mission consists of a near-Hohmann transfer from Earth to Mars, a stopover at Mars of 300 to 500 days, and a near-Hohmann return transfer from Mars to Earth. The total trip times for these missions are approximately 750 to 1000 days.

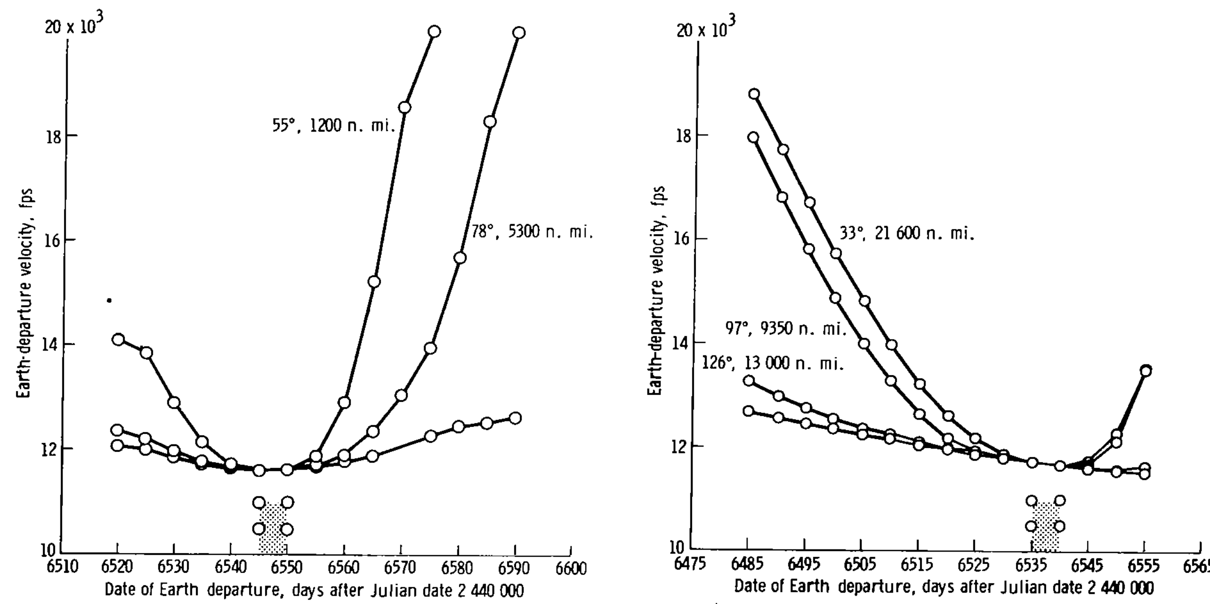
The Earth-departure opportunity in 1986 allows 110 days during which the ideal injection ΔV (for departure from a 262-nautical-mile altitude circular orbit) is less than 12 600 fps. Both type I and type II transfers simultaneously take on minimum injection ΔV's during this departure period, and trajectories with 180° Earth-Mars transfer angles do not present a problem as in the 1981 Venus swingby mission.

The 1986 direct mission opportunity contains unique problems. The type I transfers (transfer angles less than 180°) have rather high departure asymptote declinations. The asymptote declination is greater than 40° for more than half the departure opportunity and is as high as 62° in the early part of the opportunity. It is not possible to use an assembly-parking orbit with an inclination less than these declinations without planning for significant plane-change velocity penalties. A parking orbit that will track these asymptotes without a plane-change requirement must have a high orbital inclination.

The use of type II trajectories eliminates the problem of high asymptote declinations. In fact, for most of the type II window, the asymptote declination is within 5° of the Earth's equatorial plane. However, the type II trajectories introduce a new problem. The motion of the escape asymptote is such that only parking orbits with high retrograde inclinations will track them for very long.

The solutions for the type I window are shown in figure 6(a) and table V(a). Two solutions are shown in figure 6(a), one at 78° inclination with a 5300-nautical-mile apoapsis altitude and the other at 55° inclination with a 1200-nautical-mile apoapsis altitude. The 78° inclined orbit provides 50 days during which the velocity requirement is less than 12 600 fps. The 55° inclined orbit provides almost 30 days during which the departure velocity is less than 12 600 fps. As can be seen in figure 6(a), the orbits track moderately well for one-third to one-half the total available injection window. These two solutions are shown in more detail in table V(a).

The solutions for the type II window are shown in figure 6(b) and table V(b). Three solutions are shown in figure 6(b), and they all display very high apoapsis



(a) Type I heliocentric transfers. (b) Type II heliocentric transfers.

Figure 6. - Earth-departure velocity requirements for the two-impulse direct mission in 1986.

altitudes: 33° and 21 600 nautical miles, 97° and 9350 nautical miles, and 126° and 13 000 nautical miles. The 33° inclined solution allows a 30-day departure period during which the departure velocity is less than 12 600 fps. The 97° inclined solutions yield 40 days in which the departure velocity is less than 12 600 fps. These two solutions track for about 30 to 40 percent of the total available injection window. The third solution at 126° provides nearly 60 days during which the departure velocity is less than 12 600 fps, or about 75 percent of the total available injection window. These solutions are shown in more detail in table V(b).

Multiple-Impulse Heliocentric Transfer Trajectories

The Mars orbit, which is slightly elliptical and inclined to the Earth's orbital plane, precludes the possibility of a true Hohmann trajectory. More important, the injection geometry and ΔV requirements change during the course of the departure opportunity. There is flexibility for controlling these requirements by use of multiple-impulse heliocentric trajectories, particularly three-impulse trajectories in the transition from type I to type II solutions.

For single-impulse ballistic trajectories, the heliocentric paths are represented by conic arcs that are solutions of Lambert's problem. The conic arc will have the Sun at its focus and will lie in a plane containing the Sun and the position vectors of the Earth at the time of departure, and the target-planet Mars at the time of arrival. When the difference in heliocentric longitude between the Earth at departure and the target planet at arrival approaches 180°, the plane containing the heliocentric transfer becomes highly inclined to the ecliptic and, in the limit, becomes polar. Although the inertial velocity has not increased for these limiting solutions, the relative Earth departure ΔV increases tremendously (on the order of one Earth mean orbital speed which is equal to approximately 100 000 fps). The effect of the Earth's orbital speed around the Sun is lost and nearly all the inertial velocity must be obtained through the spacecraft propulsion system.

High-Earth-departure ΔV's can be avoided by permitting an intermediate impulse on the Earth-to-Mars trajectory. With the intermediate impulse, the heliocentric trajectory is broken into two legs. The legs of the new trajectory intersect and have a common position and time in space. In the general problem, four independent variables are introduced: three coordinates to describe the position of the impulse and one to describe the time of the impulse. The problem ceases to be deterministic, and many solutions can be found.

Three kinds of three-impulse trajectories were investigated, one for the 1981 Venus swingby and two for the 1986 direct mission. A minimum-sum ΔV three-impulse trajectory was used for the Venus swingby with no control over the geometry of either leg. The method used is presented in reference 6.

A minimum-sum ΔV three-impulse trajectory was also used for the 1986 direct mission. For these trajectories, the geometry of Earth-departure and the Mars-arrival trajectories was controlled. In one kind, the Earth-departure trajectory is constrained to lie in the ecliptic plane; in the other, the Mars-arrival trajectory is constrained to lie in the Mars-orbital plane.

The 1981 Venus Swingby With Three-Impulse Outbound Trajectory

The 1981 Venus swingby mission is somewhat improved using three-impulse trajectories. The best orbit, shown in figure 7 and table VI, tracks for nearly 45 days with the ΔV for Earth departure less than 12 600 fps or for more than half the total available injection window of 70 days. This orbit has an inclination of 75° and an apoapsis altitude of 1900 nautical miles. Other solutions found for this mission are summarized in table VII.

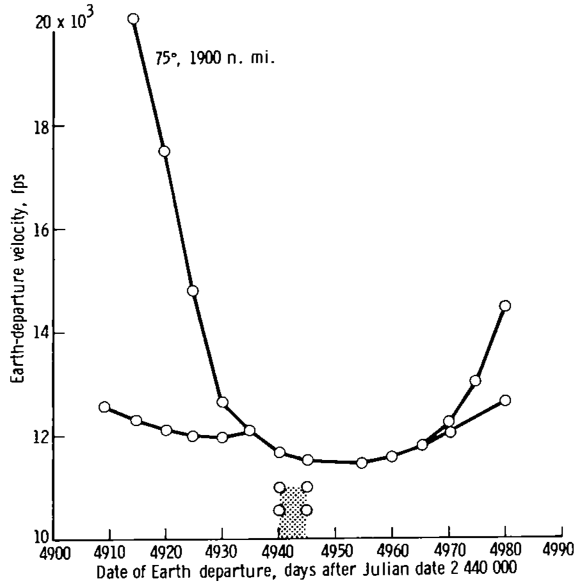
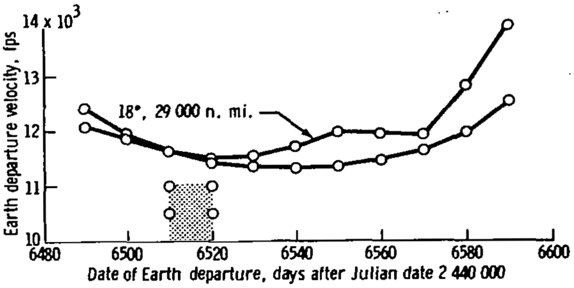


Figure 7.- Earth-departure velocity requirements for the 1981 Venus swingby with minimum $\Sigma\Delta V$ three-impulse trans-Mars trajectories.

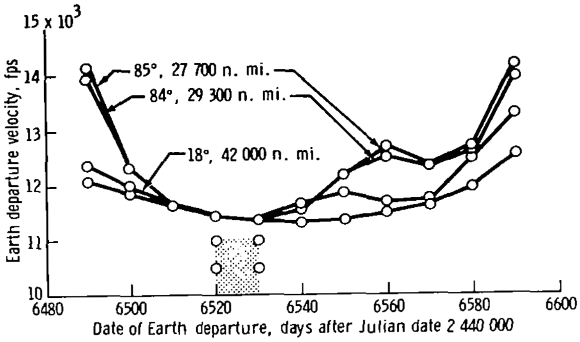
One may note that there still is a discontinuity in the velocity requirements for Earth departure and that the orbit does not track the full injection window in figure 7 partly because of the discontinuity. It is not understood why this discontinuity exists, because the Hohmann spike has been eliminated. The clue to the problem is that, in the mission analysis program, the Mars orbit stay time was forced to be 60 days and the Venus flyby was forced to be a free flyby with a positive flyby altitude. The result is that the return leg from Mars to Earth is nearly always the same, and changing the outbound flight time from Earth to Mars accounts for the variability in the Earth departure date. (This is a result and not a restriction in the model because the flight times were independent variables chosen to minimize the sum of all the velocity impulses.) A similar result can be seen in the two-impulse trajectories shown in tables I and II. These missions are minimum sum ΔV mission plans with fixed Mars stay times. Note that the Venus swingby date is always the same.

The 1986 Direct Mission With Three-Impulse Trajectories

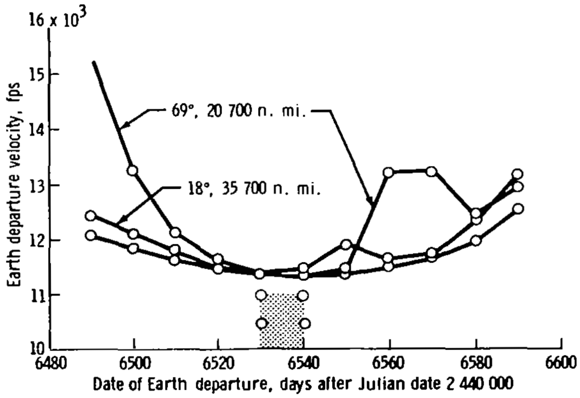
The results for the 1986 direct mission using three-impulse trajectories are shown in figures 8 and 9. Constraining the Earth-departure asymptote to the ecliptic plane for the intermediate impulse trajectory in 1986 has the effect of arresting the motion of the escape asymptote. The end effect is that very slow regression rates are needed to track the slow-moving asymptote. The result is that extremely high apoapsis altitudes are found, as is shown in figure 8. One orbit at 18° inclination tracks extremely well, tracking nearly 90 days with the departure velocity below 12 600 fps. The high apoapsis altitudes are similar to those in the type II trajectories shown in figure 6(b).



(a) Date of alinement at 2 446 510.

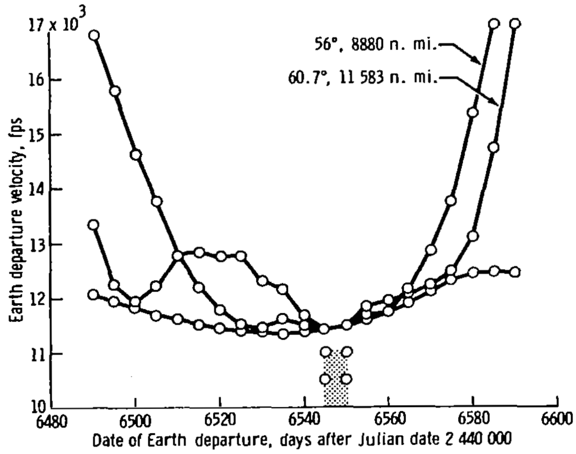


(b) Date of alinement at 2 446 520.

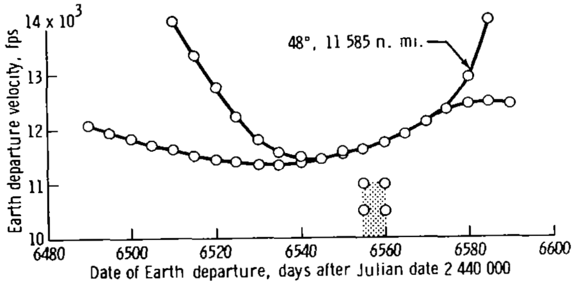


(c) Date of alinement at 2 446 530.

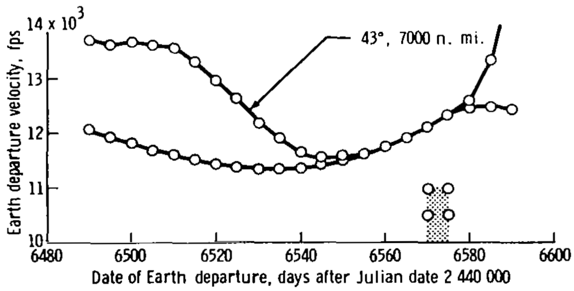
Figure 8. - Earth-departure velocity requirements for the 1986 direct Mars mission using three-impulse trans-Mars trajectories with the Earth-departure asymptotes in the ecliptic plane.



(a) Date of alinement at 2 446 545.

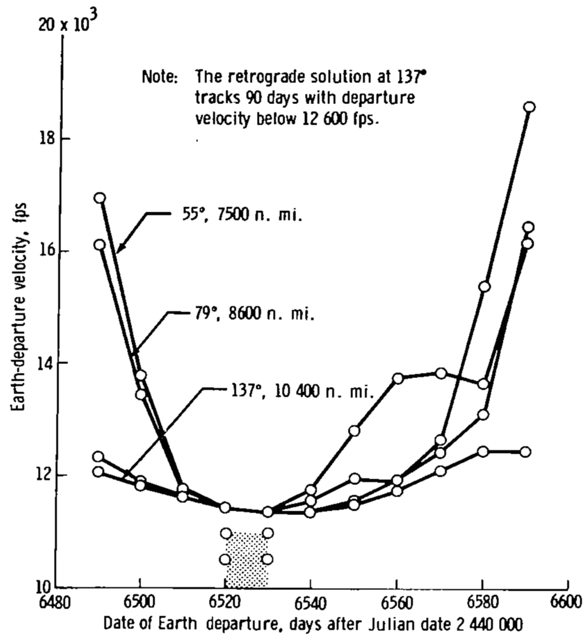


(b) Date of alinement at 2 446 555.



(c) Date of alinement at 2 446 570.

Figure 9. - Earth-departure velocity requirements for the 1986 direct Mars mission using three-impulse trans-Mars trajectories with the Mars-arrival asymptotes in Mars-orbital plane.



(d) Date of alinement at 2 446 520.

Figure 9. - Concluded.

The results of constraining the Mars-arrival asymptote in the Mars-orbit plane are shown in figure 9. These results are essentially the same as those found in the type I launch window shown in figure 6(a). There is a somewhat wider variation in the choice of orbital inclinations than there is for the type I window. The range of apoapsis altitudes is slightly higher than the type I window. A retrograde orbit, as shown in figure 9(d), of 137° and 10 400-nautical-mile altitude tracks nearly 90 days with the departure velocity less than 12 600 fps.

The three-impulse trajectories give generally the same result as the two-impulse trajectories. The orbits appear to track for longer periods of time when the three-impulse trajectories are used.

CONCLUDING REMARKS

A preliminary injection window analysis was performed for two round trip Mars stopover missions: the 1981 Mars stopover with inbound Venus swingby and the 1986 direct minimum-energy Mars stopover.

The results showed that by properly choosing the orbital elements of the Earth launch and assembly parking orbit, orbits can be found to follow the motion of the required trans-Mars departure asymptote.

For the direct mission using near-Hohmann transfers, the Earth-departure opportunity in 1986 provides nearly 100 days during which the ideal scalar or characteristic velocity requirements are less than 12 600 fps for the trans-Mars injection maneuver. When parking orbits were alined properly for this mission, they were found to track from 50 days to nearly the full 100 days with little or no velocity penalties caused by misalinement of the parking orbit and the departure asymptote. The vector-velocity requirements can be maintained within one-half of 1 percent of the ideal characteristic velocity for periods of as much as 90 days within the departure opportunity. Further, solutions are found with orbital inclinations of approximately 30°, 40°, 50°, and 70°. These solutions have high apoapsis altitudes, often from 5000 to 20 000 nautical miles.

For the Mars stopover with the inbound Venus swingby, the Earth-departure opportunity in 1981 provides 70 days during which the ideal scalar ΔV requirements are less than 12 600 fps for the trans-Mars injection.

Two solutions for a moderately high elliptical parking orbit maintained alinement for periods as long as 40 days during this injection opportunity. The solutions have inclinations of 53° and 74° with apoapsis altitudes near 2400 nautical miles. In relation to the high apoapsis altitudes, the technique may be useful for defining parking orbits at a target planet by considering the departure window for the return trip to Earth.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, September 27, 1971
981-30-00-00-72

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6. Willis, Edward A., Jr: Optimization of Double-Conic Interplanetary Trajectories. NASA TN D-3184, 1966.

TABLE I. - LAUNCH DATES AND FLIGHT TIMES FOR MINIMUM-SUM ΔV MARS STOPOVER/VENUS SWINGBY MISSIONS

LAUNCHED IN 1981 and 1982

Calendar date of launch, month/day/yr	Julian date of launch (add 2 440 000)	Outbound flight time, days	Mars arrival date (add 2 440 000)	Orbital stay time, days	Intermediate flight time, days	Venus flyby[a] Julian date (add 2 440 000)	Return flight time, days	Total trip time, days	Outbound trajectory type
11/19/81	4928	297	5225	20	144	5389	157	618	II
1/10/82	4980	233	5213	20	154	5387	155	562	I
11/12/81	4920	297	5217	30	142	5389	157	626	II
1/8/82	4978	228	5209	30	149	5388	156	563	I
11/5/81	4914	294	5208	40	141	5389	157	632	II
1/12/82	4981	224	5205	40	143	5388	156	563	I
11/10/81	4919	278	5197	60	133	5390	152	623	II
12/30/81	4969	224	5193	60	137	5390	160	581	I
11/9/81	4918	267	5185	80	126	5391	172	645	II
12/23/81	4961	219	5180	80	131	5391	165	595	I

[a]The Venus flyby altitudes vary between 2500 and 2600 n. mi. above the planet's surface.

TABLE II. - VELOCITY REQUIREMENTS FOR MINIMUM-SUM ΔV MARS STOPOVER/VENUS SWINGBY MISSIONS

LAUNCHED IN 1981 and 1982

Calendar date of launch, month/day/yr	Julian date of launch (add 2 440 000)	Orbital stay time, days	ΔV of Earth departure,[a] fps	ΔV of Mars arrival,[b] fps	ΔV of Mars departure,[b] fps	ΔV of Earth entry,[c] fps	ΔV total,[d] fps	ΔV spacecraft,[e] fps	Outbound trajectory type
11/19/81	4928	20	11 900	4100	11 300	40 000	27 300	15 400	II
1/10/82	4980	20	12 800	5500	11 400	40 200	29 700	16 900	I
11/12/81	4920	30	12 000	4200	11 400	40 000	27 600	15 600	II
1/11/82	4978	30	12 600	5600	11 300	40 000	29 500	16 900	I
11/5/81	4914	40	12 300	4500	11 400	40 000	28 200	15 900	II
1/12/82	4981	40	12 400	5800	11 400	40 400	29 600	17 200	I
11/10/81	4919	60	12 200	5000	11 800	40 300	29 000	16 800	II
12/30/81	4969	60	12 000	6390	11 560	39 800	29 950	17 950	I
11/9/81	4918	80	12 300	5800	12 600	39 800	30 700	18 400	II
12/23/81	4961	80	11 800	7555	12 000	39 700	31 200	19 555	I

[a]Parking orbit = 262-n. mi. altitude circular.
[b]Parking orbit = 200 by 10 000 n. mi.
[c]400 000-ft entry altitude.
[d]Sum of columns 4, 5, and 6.
[e]Sum of columns 5 and 6.

TABLE III. - THE 1981 VENUS SWINGBY WITH EARTH-TO-MARS TRANSFER ANGLES GREATER THAN 180°

(a) Date of ideal alinement is October 31, 1981 (Julian date 2 444 908.65)

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 12700.00 RA= -175.86 DEL= 11.31
OUTGOING V-INFINITY= 11978.00 RA= -174.22 DEL= 13.02

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.110739+09	.796661	36.5630	168.5000	236.4696	17.1429	4057.	6511.	33513.	29301.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444903.65	169.38	235.35	1.50	4371.18	12881.98	13487.00	-177.76	10.13	.95
2444908.65	168.50	236.57	.00	4057.22	12568.02	12700.00	-175.86	11.31	.00
2444913.65	167.62	237.79	.00	3819.35	12330.15	11978.00	-174.22	13.02	.01
2444918.65	166.74	239.01	4.00	3701.18	12211.98	11341.00	-172.85	15.39	1.22
2444923.65	165.86	240.22	17.50	4023.22	12534.01	10816.00	-171.75	18.55	3.07
2444928.65	164.98	241.44	36.50	4802.68	13313.47	10442.00	-170.89	22.61	5.29
2444933.65	164.10	242.64	57.50	5806.43	14317.22	10276.00	-170.16	27.68	8.33
2444938.65	163.23	243.84	79.00	6878.55	15389.34	10405.00	-169.35	33.80	12.91

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
2	1.0	.393875+08	.428304	104.6520	167.1366	229.0459	3.6364	7690.	4878.	29881.	5815.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444903.65	185.07	231.82	356.00	7967.59	12846.03	13487.00	-177.76	10.13	.24
2444908.65	187.14	229.05	.00	7689.58	12568.02	12700.00	-175.86	11.31	.00
2444913.65	189.20	226.27	5.50	7458.07	12336.51	11978.00	-174.22	13.02	.07
2444918.65	191.27	223.49	12.50	7285.81	12164.24	11341.00	-172.85	15.39	.02
2444923.65	193.33	220.72	22.00	7196.72	12075.16	10816.00	-171.75	18.55	.06
2444928.65	195.40	217.94	34.00	7227.04	12105.48	10442.00	-170.89	22.61	.04
2444933.65	197.46	215.16	48.00	7429.54	12307.98	10276.00	-170.16	27.68	.27
2444938.65	199.53	212.39	64.50	7882.81	12761.25	10405.00	-169.35	33.80	1.06

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
3	3.0	.393875+08	.428304	99.4770	2.2273	75.4880	3.6364	7690.	4878.	29881.	5815.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444903.65	.88	29.42	-.50	7969.47	12847.90	13487.00	-177.76	10.13	.54
2444908.65	2.23	25.89	.00	7689.58	12568.02	12700.00	-175.86	11.31	.00
2444913.65	3.57	22.36	.00	7451.71	12330.15	11978.00	-174.22	13.02	.00
2444918.65	4.92	18.83	-.50	7263.36	12141.80	11341.00	-172.85	15.39	.68
2444923.65	6.26	15.30	-1.00	7204.46	12082.90	10816.00	-171.75	18.55	2.10
2444928.65	7.60	11.77	.50	7411.57	12290.01	10442.00	-170.89	22.61	3.95
2444933.65	8.95	8.24	2.50	7925.22	12803.66	10276.00	-170.16	27.68	5.82
2444938.65	10.29	4.72	3.00	8619.90	13498.34	10405.00	-169.35	33.80	7.51

TABLE III. - THE 1981 VENUS SWINGBY WITH EARTH-TO-MARS TRANSFER ANGLES GREATER THAN 180° - Continued

(b) Date of ideal alinement is November 5, 1981 (Julian date 2 444 913.65)

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 11976.00 RA= -174.22 DEL= 13.02
OUTGOING V-INFINITY= 11341.00 RA= -172.85 DEL= 15.39

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.732947+08	.692779	39.9540	169.7529	236.1198	9.2308	4803.	7520.	32530.	16976.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444903.65	173.26	231.68	7.50	5623.48	13150.99	13487.00	-177.76	10.13	2.89
2444908.65	171.51	233.90	2.00	5089.83	12617.34	12700.00	-175.66	11.31	1.15
2444913.65	169.75	236.12	.00	4602.64	12330.14	11976.00	-174.22	13.02	.00
2444918.65	168.00	238.34	.00	4603.22	12130.73	11341.00	-172.85	15.39	.04
2444923.65	166.24	240.56	4.50	4546.24	12073.75	10816.00	-171.75	18.55	1.51
2444928.65	164.49	242.78	17.50	4977.90	12505.40	10442.00	-170.89	22.61	3.71
2444933.65	162.73	245.00	35.00	5892.89	13420.40	10276.00	-170.16	27.68	6.28
2444938.65	160.98	247.21	54.00	7052.02	14579.53	10405.00	-169.35	33.80	9.53

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
2	3.0	.386114+08	.416812	97.0280	4.1461	22.4537	3.5299	7572.	4758.	29761.	5559.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444903.65	2.05	30.38	-1.50	8144.65	12902.63	13487.00	-177.76	10.13	1.67
2444908.65	3.10	26.41	-.50	7818.09	12576.07	12700.00	-175.66	11.31	.60
2444913.65	4.15	22.46	.00	7572.16	12330.14	11976.00	-174.22	13.02	.00
2444918.65	5.19	18.50	.00	7372.71	12130.69	11341.00	-172.85	15.39	.02
2444923.65	6.24	14.54	-1.00	7225.33	11983.32	10816.00	-171.75	18.55	.63
2444928.65	7.28	10.59	-1.50	7202.79	11960.78	10442.00	-170.89	22.61	1.94
2444933.65	8.33	6.64	-1.50	7395.42	12153.40	10276.00	-170.16	27.68	3.56
2444938.65	9.38	2.68	-1.00	7773.60	12531.59	10405.00	-169.35	33.80	4.91

TABLE III. - THE 1981 VENUS SWINGBY WITH EARTH-TO-MARS TRANSFER ANGLES GREATER THAN 180° - Concluded

(c) Date of ideal alinement is November 10, 1981 (Julian date 2 444 918.65)

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 11341.00 RA= -172.85 DEL= 15.39
OUTGOING V-INFINITY= 10816.00 RA= -171.75 DEL= 18.55

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.549981+08	.590574	41.3200	168.8974	237.6743	6.0000	5600.	6530.	31533.	10953.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444903.65	177.89	226.78	17.00	7356.53	13886.71	13487.00	-177.76	10.13	5.88
2444908.65	174.89	230.41	8.50	6429.99	12960.17	12700.00	-175.86	11.31	3.53
2444913.65	171.89	234.04	2.50	5862.04	12392.22	11978.00	-174.22	13.02	1.39
2444918.65	168.90	237.67	.00	5600.49	12130.67	11341.00	-172.85	15.39	.00
2444923.65	165.90	241.31	.00	5443.85	11974.03	10816.00	-171.75	18.55	.10
2444928.65	162.90	244.94	5.00	5466.31	11996.49	10442.00	-170.89	22.61	1.92
2444933.65	159.91	248.57	17.50	6043.82	12574.00	10276.00	-170.16	27.68	4.53
2444938.65	156.91	252.20	33.50	7141.88	13672.06	10405.00	-169.35	33.80	7.50

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
2	1.0	.439470+08	.487617	127.7580	199.4636	213.5869	4.2857	6638.	5493.	30495.	7316.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444903.65	187.73	225.21	13.00	7540.44	13033.00	13487.00	-177.76	10.13	2.61
2444908.65	191.64	228.00	7.00	7149.58	12642.14	12700.00	-175.86	11.31	1.63
2444913.65	195.55	230.79	2.50	6850.71	12343.27	11978.00	-174.22	13.02	.69
2444918.65	199.46	233.59	.00	6638.11	12130.67	11341.00	-172.85	15.39	.00
2444923.65	203.37	236.38	.00	6481.31	11973.88	10816.00	-171.75	18.55	.08
2444928.65	207.28	239.17	3.00	6392.61	11885.17	10442.00	-170.89	22.61	.81
2444933.65	211.19	241.97	11.00	6544.77	12037.33	10276.00	-170.16	27.68	2.71
2444938.65	215.10	244.76	25.00	7140.32	12652.89	10405.00	-169.35	33.80	5.20

TABLE IV. - THE 1981 VENUS SWINGBY WITH EARTH-TO-MARS TRANSFER ANGLE LESS THAN 180°

(a) Date of ideal alinement is December 20, 1981 (Julian date 2 444 959.31)

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 10440.00 RA= 151.70 DEL= -18.56
OUTGOING V-INFINITY= 10629.00 RA= 149.38 DEL= -13.77

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.352364+08	.360954	46.6030	170.2105	185.6412	3.0769	7700.	4165.	29168.	4448.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	192.12	163.96	19.50	7978.10	12143.53	11099.00	163.95	-27.05	.60
2444949.31	184.81	171.19	10.50	7815.85	11981.27	10671.00	159.39	-25.35	1.17
2444954.31	177.51	178.41	1.50	7810.17	11975.59	10614.00	153.95	-24.15	1.68
2444959.31	170.21	185.64	.00	7700.05	11865.48	10440.00	151.70	-18.56	.00
2444964.31	162.91	192.87	.00	7754.62	11920.05	10629.00	149.38	-13.77	.17
2444969.31	155.61	200.10	2.00	7926.27	12091.70	11090.00	147.57	-9.48	1.31
2444974.31	148.31	207.32	7.50	8456.56	12621.98	11789.00	146.32	-5.90	4.02
2444979.31	141.00	214.55	15.00	9509.02	13674.44	12681.00	145.76	-2.92	7.28
2444984.31	133.70	221.78	23.50	11030.52	15195.94	13739.00	145.80	-.55	10.82

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
2	1.0	.393875+08	.428304	147.7540	119.5438	174.9974	3.6364	6967.	4878.	29881.	5815.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	98.83	143.44	9.00	7608.03	12486.47	11099.00	163.95	-27.05	4.11
2444949.31	105.73	153.96	4.00	7222.16	12100.60	10671.00	159.39	-25.35	2.58
2444954.31	112.64	164.48	4.00	7186.22	12064.66	10614.00	153.95	-24.15	2.44
2444959.31	119.54	175.00	.00	6987.04	11865.48	10440.00	151.70	-18.56	.00
2444964.31	126.45	185.52	.00	7041.12	11919.56	10629.00	149.38	-13.77	.06
2444969.31	133.35	196.03	1.00	7201.85	12080.28	11090.00	147.57	-9.48	1.03
2444974.31	140.26	206.55	3.00	7589.99	12468.43	11789.00	146.32	-5.90	2.89
2444979.31	147.16	217.07	5.50	8217.38	13095.81	12681.00	145.76	-2.92	4.72
2444984.31	154.07	227.59	6.50	9055.85	13934.28	13739.00	145.80	-.55	6.60

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
3	3.0	.364950+08	.382993	81.1800	328.7136	50.4116	3.2432	7465.	4401.	29403.	4863.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	333.20	63.34	10.00	9089.03	13489.68	11099.00	163.95	-27.05	7.65
2444949.31	331.71	59.03	5.50	8124.92	12525.56	10671.00	159.39	-25.35	4.96
2444954.31	330.21	54.72	3.50	7520.32	11920.97	10614.00	153.95	-24.15	.40
2444959.31	328.71	50.41	.00	7464.83	11865.48	10440.00	151.70	-18.56	.00
2444964.31	327.22	46.10	.00	7518.83	11919.47	10629.00	149.38	-13.77	.03
2444969.31	325.72	41.80	2.50	7663.80	12064.45	11090.00	147.57	-9.48	.63
2444974.31	324.22	37.49	7.50	7951.60	12352.24	11789.00	146.32	-5.90	1.85
2444979.31	322.73	33.18	16.00	8462.37	12863.02	12681.00	145.76	-2.92	3.55
2444984.31	321.23	28.87	25.50	9222.69	13623.34	13739.00	145.80	-.55	5.54

TABLE IV. - THE 1981 VENUS SWINGBY WITH EARTH-TO-MARS TRANSFER ANGLE LESS THAN 180° - Continued

(b) Date of ideal alinement is December 25, 1981 (Julian date 2 444 964.31)

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 10629.00 RA= 149.38 DEL= -13.77
OUTGOING V-INFINITY= 11090.00 RA= 147.57 DEL= -9.48

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.371677+08	.394161	43.1500	164.5348	191.7525	3.3333	7400.	4519.	29522.	5084.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	191.82	160.69	30.00	7963.81	12482.93	11099.00	163.95	-27.05	3.59
2444949.31	185.00	168.46	19.50	7789.76	12308.90	10671.00	159.39	-25.35	3.78
2444954.31	178.17	176.22	8.50	7764.58	12283.71	10614.00	153.95	-24.15	3.91
2444959.31	171.35	183.99	2.50	7397.31	11916.43	10440.00	151.70	-18.56	1.48
2444964.31	164.53	191.75	.00	7400.32	11919.44	10629.00	149.38	-13.77	.00
2444969.31	157.71	199.52	.00	7536.13	12055.26	11090.00	147.57	-9.48	.14
2444974.31	150.89	207.28	2.00	7838.42	12357.54	11789.00	146.32	-5.90	1.99
2444979.31	144.07	215.05	7.50	8548.95	13068.07	12681.00	145.76	-2.92	4.78
2444984.31	137.25	222.81	15.00	9746.80	14265.93	13739.00	145.80	-.55	7.90

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
2	1.0	.410639+08	.451643	150.1860	124.0602	183.5163	3.8710	6798.	5122.	30124.	6367.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	98.29	142.47	5.00	6959.44	12081.02	11099.00	163.95	-27.05	.87
2444949.31	104.73	152.73	2.00	6817.73	11939.30	10671.00	159.39	-25.35	.52
2444954.31	111.18	162.99	8.50	7277.27	12398.85	10614.00	153.95	-24.15	4.23
2444959.31	117.62	173.26	1.00	6786.06	11907.64	10440.00	151.70	-18.56	1.28
2444964.31	124.06	183.52	.00	6797.86	11919.44	10629.00	149.38	-13.77	.00
2444969.31	130.50	193.78	.00	6933.52	12055.09	11090.00	147.57	-9.48	.10
2444974.31	136.94	204.04	.50	7165.87	12287.45	11789.00	146.32	-5.90	.85
2444979.31	143.39	214.30	.50	7553.54	12675.12	12681.00	145.76	-2.92	2.18
2444984.31	149.83	224.56	.50	8133.31	13254.89	13739.00	145.80	-.55	3.71

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
3	3.0	.419714+08	.463500	81.0750	327.1743	46.0619	4.0000	6675.	5244.	30247.	6665.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	331.56	58.50	24.00	8673.50	13917.86	11099.00	163.95	-27.05	8.41
2444949.31	330.46	55.39	17.00	7606.72	12851.07	10671.00	159.39	-25.35	5.75
2444954.31	329.37	52.28	9.50	6704.92	11949.27	10614.00	153.95	-24.15	.81
2444959.31	328.27	49.17	3.00	6635.02	11879.38	10440.00	151.70	-18.56	.69
2444964.31	327.17	46.06	.00	6675.09	11919.44	10629.00	149.38	-13.77	.00
2444969.31	326.08	42.95	.00	6810.44	12054.80	11090.00	147.57	-9.48	.02
2444974.31	324.98	39.84	3.00	7038.27	12282.63	11789.00	146.32	-5.90	.67
2444979.31	323.88	36.73	9.00	7432.52	12676.87	12681.00	145.76	-2.92	2.07
2444984.31	322.79	33.62	17.50	8056.78	13301.14	13739.00	145.80	-.55	3.77

TABLE IV. - THE 1981 VENUS SWINGBY WITH EARTH-TO-MARS TRANSFER ANGLE LESS THAN 180° - Concluded

(c) Date of ideal alinement is December 30, 1981 (Julian date 2 444 969.31)

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 11090.00 RA= 147.57 DEL= -9.48
OUTGOING V-INFINITY= 11769.00 RA= 146.32 DEL= -5.90

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.410639+08	.451643	39.0260	159.4629	198.1531	3.8710	6933.	5122.	30124.	6367.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	188.30	160.70	45.50	8510.27	13631.85	11099.00	163.95	-27.05	7.97
2444949.31	182.54	168.19	34.00	8274.41	13395.99	10671.00	159.39	-25.35	7.58
2444954.31	176.77	175.68	21.50	8166.32	13287.90	10614.00	153.95	-24.15	7.15
2444959.31	171.00	183.17	11.00	7277.86	12399.44	10440.00	151.70	-18.56	4.40
2444964.31	165.23	190.66	2.50	6883.48	12005.05	10629.00	149.38	-13.77	1.83
2444969.31	159.46	198.15	.00	6933.20	12054.78	11090.00	147.57	-9.48	.00
2444974.31	153.69	205.64	.00	7148.43	12270.01	11789.00	146.32	-5.90	.06
2444979.31	147.93	213.13	2.00	7489.59	12611.17	12681.00	145.76	-2.92	1.45
2444984.31	142.16	220.62	7.00	8154.57	13276.15	13739.00	145.80	-.55	3.83

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
2	1.0	.429308+08	.475489	152.1850	129.1117	192.6452	4.1379	6687.	5368.	30371.	6981.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	99.38	143.71	7.00	6858.60	12226.60	11099.00	163.95	-27.05	2.49
2444949.31	105.33	153.50	6.50	6902.51	12270.51	10671.00	159.39	-25.35	3.47
2444954.31	111.27	163.29	13.50	7573.52	12941.51	10614.00	153.95	-24.15	5.98
2444959.31	117.22	173.07	4.00	6761.19	12129.18	10440.00	151.70	-18.56	3.06
2444964.31	123.17	182.86	.50	6583.40	11951.40	10629.00	149.38	-13.77	1.09
2444969.31	129.11	192.65	.00	6686.78	12054.78	11090.00	147.57	-9.48	.00
2444974.31	135.06	202.43	.00	6901.96	12269.96	11789.00	146.32	-5.90	.03
2444979.31	141.00	212.22	-1.00	7202.89	12570.88	12681.00	145.76	-2.92	.61
2444984.31	146.95	222.01	-2.00	7627.06	12995.06	13739.00	145.80	-.55	1.61

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
3	3.0	.532380+08	.577037	80.9190	326.0401	42.9257	5.7143	5659.	6396.	31398.	10374.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444944.31	329.40	52.25	46.00	7930.66	14326.37	11099.00	163.95	-27.05	9.18
2444949.31	328.73	50.39	36.00	6865.09	13260.81	10671.00	159.39	-25.35	6.48
2444954.31	328.06	48.52	24.00	5746.10	12151.81	10614.00	153.95	-24.15	2.16
2444959.31	327.39	46.66	12.50	5596.25	11991.97	10440.00	151.70	-18.56	1.79
2444964.31	326.71	44.79	4.00	5542.44	11938.16	10629.00	149.38	-13.77	.71
2444969.31	326.04	42.93	.00	5659.07	12054.78	11090.00	147.57	-9.48	.00
2444974.31	325.37	41.06	.00	5874.23	12269.94	11789.00	146.32	-5.90	.01
2444979.31	324.69	39.19	3.50	6192.94	12588.66	12681.00	145.76	-2.92	.92
2444984.31	324.02	37.33	10.00	6699.06	13094.78	13739.00	145.80	-.55	2.30

TABLE V. - SOLUTIONS EVALUATED FOR THE TWO-IMPULSE DIRECT MARS STOPOVER IN 1986

(a) Type I heliocentric transfers

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 9475.00 RA= -63.58 DEL= -52.78

OUTGOING V-INFINITY= 9507.00 RA= -67.28 DEL= -49.47

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.253269+08	.110917	55.6160	.6903	134.2475	1.8750	10254.	1350.	26353.	1187.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2446520.00	74.60	95.34	28.50	12763.46	14113.61	11225.00	-42.35	-61.69	11.70
2446525.00	59.82	103.12	26.00	12495.14	13845.29	10950.00	-43.31	-62.92	11.04
2446530.00	45.03	110.90	17.00	11557.90	12908.06	10336.00	-48.69	-60.92	8.34
2446535.00	30.25	118.68	9.00	10613.18	12163.33	9887.00	-54.05	-58.58	5.41
2446540.00	15.47	126.47	2.50	10367.56	11717.71	9600.00	-59.10	-55.65	2.36
2446545.00	.69	134.25	.00	10253.77	11603.92	9475.00	-63.58	-52.78	.00
2446550.00	345.91	142.03	.00	10262.07	11612.22	9507.00	-67.28	-49.47	.02
2446555.00	331.13	149.81	3.00	10521.92	11872.07	9691.00	-70.12	-46.46	3.78
2446560.00	316.35	157.59	11.50	11560.85	12911.00	10017.00	-72.11	-42.73	8.63
2446565.00	301.57	165.38	22.50	13988.04	15238.19	10475.00	-73.30	-39.57	15.00
2446570.00	286.78	173.16	33.50	17214.48	18564.64	11050.00	-73.79	-36.68	22.35
2446575.00	272.00	180.94	44.00	21226.33	22576.49	11733.00	-73.67	-34.08	30.87
2446580.00	257.22	188.72	57.00	26157.23	27507.38	12259.00	-70.93	-31.12	42.18
2446585.00	242.44	196.50	77.50	31338.97	32689.12	12552.00	-65.85	-27.08	56.50
2446590.00	227.66	204.28	110.50	34950.36	36300.52	12862.00	-60.88	-22.31	70.29

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
2	3.0	.378724+08	.405431	77.9330	100.0745	83.5415	3.4286	6966.	4638.	29641.	5316.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2446520.00	109.42	101.31	.00	7749.94	12388.17	11225.00	-42.35	-61.69	3.46
2446525.00	107.55	97.52	6.50	7576.13	12214.35	10950.00	-43.31	-62.92	2.89
2446530.00	105.68	94.02	4.50	7328.03	11966.25	10336.00	-48.69	-60.92	2.31
2446535.00	103.81	90.53	2.50	7128.33	11766.56	9887.00	-54.05	-58.58	1.49
2446540.00	101.94	87.04	.50	7007.06	11645.29	9600.00	-59.10	-55.65	.61
2446545.00	100.07	83.54	.00	6965.69	11603.92	9475.00	-63.58	-52.78	.00
2446550.00	98.20	80.05	.00	6974.07	11612.30	9507.00	-67.28	-49.47	.05
2446555.00	96.34	76.55	2.00	7034.35	11672.58	9691.00	-70.12	-46.46	.70
2446560.00	94.47	73.06	6.00	7267.73	11905.96	10017.00	-72.11	-42.73	2.51
2446565.00	92.60	69.56	12.50	7731.04	12369.27	10475.00	-73.30	-39.57	4.45
2446570.00	90.73	66.07	21.00	8434.21	13072.44	11050.00	-73.79	-36.68	6.52
2446575.00	88.86	62.57	30.50	9335.98	13974.21	11733.00	-73.67	-34.08	8.73
2446580.00	86.99	59.08	43.00	11050.01	15688.23	12259.00	-70.93	-31.12	12.99
2446585.00	85.12	55.59	57.00	13639.78	18278.01	12552.00	-65.85	-27.08	19.58
2446590.00	83.25	52.09	70.00	16304.65	20942.87	12862.00	-60.88	-22.31	26.80

TABLE V. - SOLUTIONS EVALUATED FOR THE TWO-IMPULSE DIRECT MARS STOPOVER IN 1986 - Concluded

(b) Type II heliocentric transfers

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 9940.94 RA= -42.62 DEL= 1.31
OUTGOING V-INFINITY= 9698.51 RA= -41.76 DEL= -.20

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.501052+08	.550592	97.7830	317.5579	211.6136	5.2174	5596.	6131.	31134.	9343.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2446485.36	311.07	233.38	11.00	11830.85	17962.19	13011.58	-61.27	1.63	13.65
2446490.36	311.72	231.19	10.50	10695.91	16827.25	12670.61	-58.85	2.50	11.91
2446495.36	312.37	229.00	9.50	9650.63	15781.97	12337.08	-56.53	3.22	10.29
2446500.36	313.03	226.80	8.50	9703.59	14834.93	12010.06	-54.32	3.78	8.71
2446505.36	313.68	224.61	7.00	7852.13	13983.47	11687.43	-52.15	4.02	7.14
2446510.36	314.34	222.42	4.50	7114.97	13246.31	11370.38	-50.09	4.05	5.67
2446515.36	314.99	220.23	2.00	6505.68	12637.02	11059.04	-48.12	3.82	4.14
2446520.36	315.64	218.03	-1.00	6047.42	12178.76	10754.71	-46.25	3.29	2.58
2446525.00	316.25	216.00	357.00	5785.19	11916.53	10480.66	-44.65	2.55	1.07
2446530.00	316.90	213.81	-1.50	5671.54	11802.88	10202.54	-43.56	2.17	.34
2446535.00	317.56	211.61	.00	5595.94	11727.28	9940.94	-42.62	1.31	.00
2446540.00	318.21	209.42	.00	5531.06	11662.41	9698.51	-41.76	-.20	.00
2446545.00	318.67	207.23	-2.50	5476.54	11607.88	9479.56	-40.87	-2.70	.23
2446550.00	319.52	205.04	351.50	5446.45	11577.79	9295.74	-39.71	-6.86	.42
2446555.00	320.17	202.84	338.00	5513.18	11644.52	9203.27	-37.65	-14.27	.66

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
2	3.0	.873041+08	.742078	33.8620	139.3356	27.9319	12.0000	3729.	7998.	33000.	21587.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2446485.36	153.05	7.73	59.00	10802.20	18799.99	13011.58	-61.27	1.63	17.01
2446490.36	151.67	9.76	53.00	9731.58	17729.38	12670.61	-58.85	2.50	14.27
2446495.36	150.28	11.80	46.50	8700.16	16697.96	12337.08	-56.53	3.22	11.88
2446500.36	148.90	13.83	40.00	7713.35	15711.15	12010.06	-54.32	3.78	9.78
2446505.36	147.52	15.87	33.50	6798.35	14796.15	11687.43	-52.15	4.02	7.97
2446510.36	146.14	17.90	27.50	5956.69	13954.49	11370.38	-50.09	4.05	6.32
2446515.36	144.76	19.94	21.00	5216.22	13214.02	11059.04	-48.12	3.82	4.87
2446520.36	143.38	21.97	15.50	4610.12	12607.91	10754.71	-46.25	3.29	3.54
2446525.00	142.10	23.86	11.00	4193.17	12190.96	10480.66	-44.65	2.55	2.43
2446530.00	140.72	25.90	3.00	3856.56	11854.36	10202.54	-43.56	2.17	1.05
2446535.00	139.34	27.93	.00	3729.48	11727.27	9940.94	-42.62	1.31	.00
2446540.00	137.95	29.97	.00	3664.62	11662.42	9698.51	-41.76	-.20	.02
2446545.00	136.57	32.00	6.50	3721.70	11719.49	9479.56	-40.87	-2.70	1.44
2446550.00	135.19	34.04	25.50	4299.79	12297.58	9295.74	-39.71	-6.86	3.66
2446555.00	133.81	36.07	56.00	5528.54	13526.33	9203.27	-37.65	-14.27	7.34

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
3	3.0	.612917+08	.632615	126.0250	136.4235	29.6652	7.0588	4743.	6944.	31947.	13025.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2446485.36	117.63	17.01	51.50	6313.44	13257.63	13011.58	-61.27	1.63	.07
2446490.36	119.52	18.18	44.00	6033.77	12977.96	12670.61	-58.85	2.50	.17
2446495.36	121.41	19.36	37.00	5792.61	12736.80	12337.08	-56.53	3.22	.26
2446500.36	123.31	20.53	30.50	5587.04	12531.23	12010.06	-54.32	3.78	.37
2446505.36	125.20	21.70	24.00	5410.69	12354.68	11687.43	-52.15	4.02	.30
2446510.36	127.09	22.88	18.00	5260.16	12204.35	11370.39	-50.09	4.05	.15
2446515.36	128.99	24.05	13.00	5133.88	12078.07	11059.04	-48.12	3.82	.14
2446520.36	130.88	25.23	9.00	5039.00	11983.19	10754.71	-46.25	3.29	.62
2446525.00	132.64	26.32	6.50	4988.15	11932.34	10480.66	-44.65	2.55	1.16
2446530.00	134.53	27.49	2.00	4864.52	11808.71	10202.54	-43.56	2.17	.50
2446535.00	136.42	28.67	.00	4783.08	11727.27	9940.94	-42.62	1.31	.00
2446540.00	138.32	29.84	.00	4718.63	11662.82	9698.51	-41.76	-.20	.10
2446545.00	140.21	31.01	4.50	4731.33	11675.52	9479.56	-40.87	-2.70	1.32
2446550.00	142.10	32.19	19.00	5184.01	12128.20	9295.74	-39.71	-6.86	3.66
2446555.00	144.00	33.36	45.50	6573.32	13517.51	9203.27	-37.65	-14.27	7.67

TABLE VI. - THE 1981 VENUS SWINGBY WITH 60-DAY MARS STOPOVER, THREE-IMPULSE TRANS-MARS TRAJECTORY WITH MINIMUM-SUM ΔV $(\Sigma\Delta V)$

(Date of ideal alinement is December 1, 1981 (Julian date 2 444 940.00)

ORBITAL STAYTIME = 5.00 PERIAPSIS ALTITUDE = 262.00

INCOMING V-INFINITY= 9678.20 RA= -175.46 DEL= 25.78
OUTGOING V-INFINITY= 9064.60 RA= 177.97 DEL= 20.68

N	M	SMA	ECC	INCL	NODE	OMEGA	PERIOD	TEI DV	CIR DV	V PERI	APO ALT
1	1.0	.276849+08	.186643	75.6450	177.4213	236.3475	2.1929	9424.	2234.	27236.	1963.

TJD	NODE	OMEGA	TRUAN	TMI DV	SUM DV	VMAG	ALFA	DELTA	PC ANG
2444910.00	207.20	277.98	325.50	21120.55	23354.07	12550.00	-175.59	11.42	28.68
2444915.00	202.30	271.01	331.50	18190.83	20424.36	11886.00	-174.04	13.24	23.04
2444920.00	197.33	264.05	337.00	15319.01	17552.53	11325.00	-172.63	15.94	17.66
2444925.00	192.35	257.10	342.50	12612.92	14846.44	10917.00	-171.26	19.87	12.26
2444930.00	187.37	250.14	349.50	10417.40	12650.92	10766.00	-169.56	25.51	6.14
2444935.00	182.40	243.20	7.50	9867.48	12101.01	11118.00	-166.78	33.66	1.38
2444940.00	177.42	236.25	.00	9423.51	11657.04	9678.20	-175.48	25.78	.00
2444945.00	172.44	229.30	.00	9266.34	11499.87	9064.60	177.97	20.68	.03
2444955.00	162.49	215.39	-1.50	9233.08	11466.61	8824.00	163.65	6.78	1.24
2444960.00	157.52	208.44	.00	9336.77	11570.30	9276.40	157.03	.07	1.03
2444965.00	152.54	201.49	5.00	9533.26	11766.79	10071.00	151.24	-5.12	.03
2444970.00	147.56	194.54	15.50	9938.86	12172.38	11050.00	147.10	-7.95	2.48
2444975.00	142.59	187.59	31.50	10752.51	12986.03	12015.00	145.04	-7.51	5.07
2444980.00	137.61	180.64	53.50	12120.30	14353.83	12832.00	145.51	-2.96	10.19

TABLE VII. - SUMMARY OF EARTH DEPARTURE PARKING ORBITS FOUND FOR THE THREE-IMPULSE VENUS SWINGBY MISSION IN 1981

Epoch, month/day/yr	Inclination, deg	Apoapsis altitude, n. mi.	Tracking time below 12 600 fps, days
11/01/81	36.81 99.02	23 676 6 084	15 25
11/06/81	38.67 96.84	13 857 5 316	20 25
11/11/81	39.21 130.00	8 050 5 815	15 20
11/16/81	39.36 132.61	4 651 3 718	20 15
11/21/81	40.51 132.71 96.10	2 186 1 752 2 811	15 15 25
11/26/81	77.87 57.01 141.20	779 856 3 100	30 10 10
12/06/81	75.79 53.17	1 752 1 752	35 25
12/16/81	74.46 50.25 159.10	2 424 2 074 3 889	40 30 30
12/21/81	71.00 49.10 161.50	3 552 2 811 4 250	45 30 25

APPENDIX

SOLUTION FOR THE INCLINATION AT THE MINIMUM-ENERGY POINT ($\dot{V}_\infty = 0$) DURING THE LAUNCH OPPORTUNITY

Normally, during a launch opportunity, the magnitude of the hyperbolic excess velocity goes through a minimum. The solution for the inclination at this time is analytic and provides a useful insight into the number of solutions available. It also provides convenient initial guesses for the inclination when the complete solution is required at some other time during the launch opportunity.

With the condition $\dot{V}_\infty = 0$, equation (16) becomes

$$\frac{\dot{\alpha}_\infty + f\dot{\delta}_\infty}{g\dot{\delta}_\infty} = h \tag{A1}$$

This equation reduces to

$$\xi^3 + \alpha_2\xi^2 + \alpha_1\xi + \alpha_0 = 0 \tag{A2}$$

This is a cubic equation in ξ where

$$\xi = \tan^2 i \tag{A3}$$

$$\alpha_2 = -\left[8 + \tan^2\delta_\infty + \frac{1}{x^2}\left(2 + \sec^2\delta_\infty\right)^2\right]$$

$$\alpha_1 = 16 + 8\tan^2\delta_\infty + \frac{4}{x^2}\left(2 + \sec^2\delta_\infty\right)\left(2\sec^2\delta - 1\right)$$

$$\alpha_0 = -\left[16\tan^2\delta_\infty + \frac{4}{x^2}\left(1 - 2\sec^2\delta_\infty\right)^2\right]$$

$$x = \dot{\alpha}_\infty/\dot{\delta}_\infty \tag{A4}$$

By inspection of these coefficients, α_2 and α_0 are always negative and α_1 is always positive. This indicates that there are three real positive roots for ξ. The three solutions for ξ may be written (consult any mathematical handbook)

$$\xi_1 = 2\sqrt{\frac{-\alpha}{3}} \cos \frac{\phi}{3}$$

$$\xi_2 = 2\sqrt{\frac{-\alpha}{3}} \cos \left(\frac{\phi}{3} + 120^\circ\right)$$

$$\xi_3 = 2\sqrt{\frac{-\alpha}{3}} \cos \left(\frac{\phi}{3} + 240^\circ\right) \tag{A5}$$

where

$$\phi = \cos^{-1}\left[\frac{-\beta}{2}\left(-\frac{\alpha^3}{27}\right)^{-1/2}\right]$$

$$\alpha = \frac{1}{3}\left(3\alpha_1 - \alpha_2^{\,2}\right)$$

$$\beta = \frac{1}{27}\left(2\alpha_2^{\,3} - 9\alpha_1\alpha_2 + 27\alpha_o\right) \tag{A6}$$

Each part of equation (A5) actually represents two solutions for the tangent of the inclination because

$$\tan i_j = \pm\sqrt{\xi_j^{\,2}}$$

For the posigrade orbital inclination, the solution is

$$i_{j+} = \tan^{-1}\left(\sqrt{\xi_j^{\,2}}\right)$$

For the retrograde orbital inclination, the solution is

$$i_{j-} = \pi - \tan^{-1}\left(\sqrt{\xi_j^{\,2}}\right) \tag{A7}$$

For each solution for i_j which is posigrade, there is a solution which is retrograde. However, from the restriction given by equation (18), only one is valid; that is, if $\dot{\Omega}_\infty$ is positive, i can only be retrograde. Thus, there are at most three values of the inclination that will satisfy equation (A2).