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ERROR STUDIES FOR GROUND TRACKING OF SYNCHRONOUS SATELLITES

J. L. COOLEY

JANUARY 1972

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GODDARD SPACE FLIGHT CENTER
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ABSTRACT

The results of various sets of tracking error analysis studies of the ability of ground stations to determine the position and velocity of synchronous satellites are summarized. The effects of varying 1) the ground station configuration from 1 to 6 tracking stations in differing locations, 2) the ground station measurement type — such as S-Band, C-Band, VHF, and lasers —, and 3) the uncertainties in ground station location are investigated. The linear error analysis computer program used includes the effects of ground tracking station location uncertainties, measurement noise and biases, and station timing bias.

The results of the study show that two ground trackers are needed if at least 2000 meters (1σ) position accuracy is desired, with a favorable two-station solution giving less than 500 meters (1σ) position accuracy. Under favorable circumstances, a multi-station laser solution gives a synchronous satellite position accuracy of less than 100 meters (1σ). The various cases illustrate features of synchronous satellite tracking from ground stations, such as the importance of measurement type, the importance of tracking geometry, the importance of ground station location uncertainty, and the importance of determining various tracking vector components.

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ERROR STUDIES FOR GROUND TRACKING OF SYNCHRONOUS SATELLITES

INTRODUCTION

This report summarizes the results of various sets of error analysis studies of the ability of ground stations to determine the position and velocity of synchronous satellites, and draws some general conclusions therefrom. Some of these studies, made originally for inclusion in a study report on the feasibility of using synchronous satellites to track other orbiting spacecraft (see Reference 1), have been expanded considerably as described below. Other studies were made for specific synchronous missions, and still others explored the possible benefits of tracking synchronous satellites by precise laser measurements.

A linear error analysis computer program based on the minimum variance statistical filter (Kalman-Schmidt filter) was used to transform the expected measurement and location uncertainties into spacecraft orbital uncertainties (see Reference 2). Error analysis is a pre-flight tool to simulate the real-time or post-flight orbit determination process. The optimal error analysis version simulates post-flight processing of the tracking data; it assumes that data can be optimally weighted and edited over the data span. The nonoptimal (or neglect) error analysis version simulates operational orbit determination where data is processed with less editing and less optimal weighting due to time constraints. In each version it is assumed that the biases in the error model (station location uncertainty, measurement bias, and station time bias) are not to be solved for in the orbit determination process, and the biases themselves are given conservative values. No gravitational uncertainties are modeled.

TRACKING FROM GROUND STATIONS

Ground tracking of synchronous satellites presents a nearly static tracking situation. That is, the tracking link between a ground station and a synchronous satellite is nearly constant due to the small relative motion of the satellite with respect to the tracking station. The station-satellite range rate is nearly zero, and the range and angular measurements are nearly constant. The station-satellite geometry is nearly static.

The first set of runs to be presented compare 1 and 2 station solutions for S-Band tracking. The advantages of having a second ground station, giving a second geometric tracking view of the synchronous satellite, will be investigated.

Tracking schemes and station location uncertainty will also be varied. The next set of runs compare 2 and 3 station combinations for range and range rate tracking of an ATS spacecraft. These cases show the effect of ground station latitude and longitude separation. The effect of time bias is also investigated. Then VHF tracking of the SIRIO and SMS satellite orbits is presented. Again, 2 and 3 station combinations are considered. Solutions are obtained for range measurements only, range rate measurements only, and range and range rate in order to compare the effect of the measurement types. Maximum ionospheric effects on the VHF measurements are also modeled. Finally laser tracking, with precise range measurements, for 2, 3, 4, 5, and 6 station combinations are given for various ground station location undertainties.

S-Band Tracking

The original group of studies (see Reference 1) assumed simultaneous tracking of a synchronous satellite by two ground stations, the satellite sub-longitude points and the ground stations tracking each being as follows:

16° West	Rosman and Madrid
88° West	Mojave and Rosman
180	Toowomba and Mojave
112° East	Toowomba and Carnarvon

Station location uncertainties (conservative) were assumed as follows (in meters):

	Latitude	Longitude	Altitude
Rosman	35	35	35
Madrid	39	31	37
Mojave	34	37	35
Toowomba	63	60	61
Carnarvon	60	64	60

Measurements were made each minute for 24 hours using S-Band tracking. Uncertainties assumed for the S-Band system were as follows (Reference 5): range noise — 10 meters; range bias — 20 meters; range rate noise — 0.1 cm/sec; range rate bias — 1 cm/sec (increased to account for various modelling errors not specifically included, such as those related to gravitational harmonics); angular noise — 0.8 milliradians; and angular bias — 1.6 milliradians. A minimum elevation angle of 5° was assumed.

Additional studies were made assuming a station switching plan, where the stations alternated in tracking, one at a time, for 3 hour periods (see Reference 3). This mode of tracking has the advantage of freeing a station periodically for other tracking while still retaining some of the benefits of two-station geometry

Position and velocity uncertainties (1σ) for the synchronous satellites at selected intervals are presented in Table 1. Position uncertainties vary considerably, from 340 to 1200 meters after 24 hours of tracking, depending on the combination of tracking stations. The greater the longitude and latitude separation, the better the results. The best combination with either mode of tracking is Toowomba (Australia) and Mojave (California). With simultaneous tracking, two combinations (Rosman-Madrid and Toowomba-Mojave) have reduced the position uncertainty (1σ) to less than 500 meters by 6 hours; in contrast, the poorest combination of Toowomba and Carnarvon (also in Australia) after 24 hours of tracking still gives a position uncertainty of 1200 meters. With station switching it takes more than 6 hours, or one tracking period from each station, to overcome the a-priori uncertainty; even after 9 hours the position uncertainty is greater than 2000 meters for all but the Toowomba-Mojave combination. The uncertainties are not reduced nearly as quickly for the switching mode as for the simultaneous mode, but probably more frequent station switching would reduce the undertainties sooner. The velocity uncertainties behave in similar fashion.

A further refinement was then made, assuming that one way to reduce the position uncertainty for the synchronous satellite was to improve the ground station location uncertainties. Reference 4 states that, by using tracking data from the GEOS satellites, these uncertainties have been reduced to approximately 10 meters. Accordingly, studies were made assuming a-priori 10-meter uncertainty in each location component.

Table 2 shows the results when the reduced station location uncertainties (of 10 meters in each component) were used. With the reduced station location uncertainties, the position uncertainty for the synchronous satellite ranges from 160 to 440 meters after 24 hours — a significant decrease over the results in Table 1. It is noticed that 24 hours of tracking in the switching mode are required to achieve the accuracies obtained by 12 hours of simultaneous tracking. With the best combination of simultaneous trackers (Toowomba-Mojave) it is possible to reduce the position uncertainty to less than 500 meters in 3 hours. Thus, ground station location uncertainty is a very important error source for synchronous tracking, and decreasing the station location uncertainty makes a significant difference in the synchronous satellite position uncertainty.

Some studies were also made with one station tracking for the entire period. Results, when the standard station location uncertainties are used, as shown in

Table 1

1σ Uncertainties in Synchronous Satellite Position and Velocity, S-Band Tracking, 2 Stations, Standard Location Uncertainties

ASSUMPTIONS

Tracking Uncertainties (1σ)

	Noise	Bias
Range	10 meters	20 meters
Range rate	0.1 cm/sec	1 cm/sec
Angles	0.8 mrad	1.6 mrad
Sampling rate	1/minute	

Satellite A Priori (1σ)

Position	17.3 km
Velocity	547 m/s

Station Location Uncertainties (1σ) – meters

	Lat.	Long.	Alt.
Rosman	35	35	35
Madrid	39	31	37
Mojave	34	37	35
Toowomba	63	60	61
Carnarvon	60	64	60

Satellite Location	Ground Trackers	Tracking Time (hrs)	Simultaneous		Switching	
			Position (m)	Velocity (m/s)	Position (m)	Velocity (m/s)
16° West	Rosman and Madrid	3	1100	0.06	68,000	4.96
		6	490	0.03	10,040	0.31
		9	450	0.03	2420	0.07
		12	410	0.03	640	0.04
		24	360	0.03	450	0.03
88° West	Mojave and Rosman	3	1500	0.18	60,000	4.35
		6	850	0.06	14,700	0.60
		9	780	0.06	4810	0.14
		12	750	0.06	1330	0.07
		24	730	0.05	800	0.06
180°	Toowomba and Mojave	3	580	0.04	54,200	3.90
		6	400	0.03	2080	0.16
		9	370	0.03	530	0.05
		12	350	0.03	380	0.03
		24	340	0.03	340	0.02
112° East	Toowomba and Carnarvon	3	2600	0.28	55,700	4.02
		6	2720	0.12	19,280	0.56
		9	1890	0.12	6480	0.15
		12	1300	0.12	1780	0.10
		24	1200	0.12	900	0.07

Table 2

1σ Uncertainties in Synchronous Satellite Position and Velocity, S-Band Tracking, 2 Stations, Improved Location Uncertainties

ASSUMPTIONS

Tracking Uncertainties (1σ)	Noise	Bias
Range	10 meters	20 meters
Range rate	0.1 cm/sec	1 cm/sec
Angles	0.8 mrad	1.6 mrad
Sampling rate	1/minute	

Station Location Uncertainties (1σ)

All stations — 10 meters each component

Satellite A Priori (1σ)	
Position	17.3 km
Velocity	547 m/s

Satellite Location	Ground Trackers	Tracking Time (hrs)	Simultaneous Position (m)	Simultaneous Velocity (m/s)	Switching Position (m)	Switching Velocity (m/s)
16° West	Rosman and Madrid	3	1010	0.06	68,000	4.95
		6	310	0.02	10,000	0.31
		9	240	0.02	2400	0.07
		12	220	0.02	520	0.02
		24	190	0.02	250	0.02
88° West	Mojave and Rosman	3	1360	0.17	59,900	4.35
		6	550	0.04	14,800	0.60
		9	440	0.03	4730	0.13
		12	410	0.03	1140	0.04
		24	400	0.03	440	0.03
180°	Toowomba and Mojave	3	450	0.03	54,150	3.89
		6	170	0.01	2080	0.16
		9	160	0.01	400	0.04
		12	160	0.01	180	0.01
		24	160	0.01	160	0.01
112° East	Toowomba and Carnarvon	3	1900	0.18	55,700	4.02
		6	630	0.04	19,290	0.55
		9	520	0.03	6220	0.11
		12	430	0.03	1210	0.04
		24	370	0.03	420	0.03

Table 3, are poor — never as good as 10 kilometers (1σ) and even after 24 hours of tracking varying from 11.1 to 16.8 kilometers as compared with 340 to 1200 meters for the two-station solutions. Even using the reduced station location undertainties brings little improvement — after 24 hours, using the better (from Table 3) station in each case, the position uncertainty still ranges from 11.1 to 12.3 kilometers. The static tracking geometry then limits the one station solutions. Thus it is evident that two trackers are needed if at least 2 kilometers (1σ) position accuracy is desired.

Comparison with Real Data

These theoretical results were compared with ATS-3 orbit determination experience, which confirms that two ground stations are adequate for tracking with a position accuracy of 2 kilometers or better and that one-station tracking gives large errors in position (Reference 6). Using range and range rate data from Mojave and Rosman over various 24- and 48-hour arcs in a 2-1/2 day interval and comparing the orbit determination overlap differences, the following position uncertainties were obtained: 0.35 to 1.9 with tracking data from both stations, 2.2 to 18.6 kilometers with Mohave data only, and 2.8 to 29.6 kilometers with Rosman data only. These figures represent the maximum and minimum for the many arcs considered. Thus both the theoretical cases and the real data case show the great differences between the 1-station and 2-station solutions for the position and velocity of synchronous satellites.

Range and Range Rate Tracking

For the ATS F&G project, error analysis studies were made to determine the best 3-station combination for tracking an ATS spacecraft (References 7 and 8). For these cases, two stations were fixed, the problem being to determine which of several possible 3rd stations would give the best orbital solution. The 2-station cases reported here are for 1) Gander, Newfoundland, and Ahmedabad, India; 2) Gander and Winkfield, England; and 3) Winkfield and India. The 3-station cases are 1) Gander, India, and Tananarive; 2) Gander, India, and Madrid; and 3) Winkfield, Gander, and India. Other 2- and 3-station solutions are discussed in References 7 and 8.

For all these studies, the ATS was assumed to be located at synchronous height over the equator at 15° E longitude. Tracking uncertainties (1σ) were assumed as follows: range noise — 10 meters; range bias — 20 meters; range rate noise — 3 cm/sec; and range rate bias — 1 cm/sec (this figure reflects bias due to spin and effects of modelling uncertainties). Measurements were simulated at one per second for 2-1/2 minutes every hour for 24 hours. Ground station location uncertainties (in meters) were assumed as follows:

Table 3

1σ Uncertainties in Synchronous Satellite Position and Velocity, S-Band Tracking, 1 Station, Standard Location Uncertainties

ASSUMPTIONS

Tracking Uncertainties (1σ)

	Noise	Bias
Range	10 meters	20 meters
Range rate	0.1 cm/sec	1 cm/sec
Angles	0.8 mrad	1.6 mrad
Sampling rate	1/minute	

Satellite A Priori (1σ)

Position	17.3 km
Velocity	547 m/s

Station Location Uncertainties (1σ) — meters

	Lat.	Long.	Alt.
Rosman	35	35	35
Madrid	39	31	37
Mojave	34	37	35
Toowomba	63	60	61
Carnarvon	60	64	60

Satellite Location	Tracking Time (hrs)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)
16° West		Rosman Tracking		Madrid Tracking	
	3	68.0	4.96	61.0	4.45
	12	17.6	3.62	25.0	4.09
	24	13.0	3.25	11.7	3.87
88° West		Rosman Tracking		Mojave Tracking	
	3	60.9	4.42	59.9	4.35
	12	24.0	4.09	27.5	4.17
	24	12.2	3.92	16.8	3.45
180°		Toowomba Tracking		Mojave Tracking	
	3	54.2	3.90	66.6	4.83
	12	21.5	3.68	15.4	3.73
	24	15.1	3.19	12.3	3.31
112° East		Toowomba Tracking		Carnarvon Tracking	
	3	55.7	4.02	59.8	4.32
	12	14.3	3.44	19.8	3.95
	24	11.1	3.10	12.6	3.84

	Latitude	Longitude	Altitude
Gander	36	36	36
Ahmedabad, India	40	40	40
Madrid	39	31	37
Tananarive	30	30	30
Winkfield	35	35	35

As shown in Table 4, the 2-station solutions using 1) Gander and India and 2) Winkfield and India are both good, with position uncertainties (1σ) after 24 hours of tracking of 292 meters and 372 meters, respectively. The effects of good latitude and longitude separation are evident when compared with the position uncertainty of 860 meters from Gander and Winkfield tracking.

Table 5 presents the results when a 3rd station is added to each of the above combinations — in all 3 cases the position uncertainty after 24 hours is less than 300 meters. The Gander-India-Tananarive combination is the best, however, because of greater latitude separation. The position uncertainty is improved faster initially — to 395 meters by 3 hours and 251 meters by 12 hours.

For all the cases, after 24 hours, by far the largest part of the position uncertainty is along the longitudinal component.

The Winkfield-India tracking combination was used also for several cases to determine the effect of station time bias. When the inclination of the synchronous orbit was small (0.13 degrees), even a large time bias of 1 second had little effect (Reference 9). However, when the orbit was inclined more (4.95 degrees), time bias in the magnitude of 100 milliseconds begins to have some effect, as follows:

1σ Uncertainties in Position (meters)

Tracking Time (hrs)	No Time Bias	With Time Bias of		
		10 msec	100 msec	1 sec
4	928	928	928	939
8	402	402	403	495
16	402	402	403	461
24	372	372	378	533

Table 4

1σ Uncertainties in ATS Position and Velocity, 2 Stations, Range and Range Rate Tracking

ASSUMPTIONS

Tracking Uncertainties (1σ)	Noise	Bias
Range	10 meters	20 meters
Range rate	3 cm/sec	1 cm/sec
Sampling rate:	1/sec for 2-1/2 min every hour	

Station Location Uncertainties (1σ)	
Gander	53 meters
Ahmedabad, India	69 meters
Madrid	62 meters
Winkfield	51 meters

A Priori (1σ)	
Position	20 km
Velocity	20 m/s

ATS Location: 15° E

Tracking Time (hrs)	Gander and India		Gander and Winkfield		Winkfield and India	
	Position (meters)	Velocity (m/s)	Position (meters)	Velocity (m/s)	Position (meters)	Velocity (m/s)
1	10,450	1.68	18,570	3.04	10,250	1.13
2	4,590	0.56	17,610	0.92	5,550	0.49
3	1,410	0.17	3,535	0.17	1,960	0.19
6	350	0.03	1,170	0.07	450	0.04
12	305	0.02	965	0.07	392	0.03
18	292	0.02	965	0.07	374	0.03
24	292	0.02	860	0.06	372	0.03
After 24 hours, by directional components*						
N	19	0.02	49	0.06	14	0.03
V	275	0.001	830	0.003	347	0.001
W	96	0.007	218	0.02	131	0.009

*N = $\vec{V} \times (\vec{R} \times \vec{V})$ (in-plane $\perp$ to $\vec{V}$)

V = $\vec{V}$ (in-plane along velocity vector)

W = $\vec{R} \times \vec{V}$ (out-of-plane)

Table 5

1σ Uncertainties in ATS Position and Velocity, 3 Stations, Range and Range Rate Tracking

ASSUMPTIONS

Tracking Uncertainties (1σ)	Noise	Bias
Range	10 meters	20 meters
Range rate	3 cm/sec	1 cm/sec
Sampling rate:	1/sec for 2-1/2 min every hour	

A Priori (1σ)	
Position	20 km
Velocity	20 m/s

Station Location Uncertainties (1σ)	
Gander	53 meters
Ahmedabad, India	69 meters
Tananarive	52 meters
Madrid	62 meters
Winkfield	51 meters

ATS Location: 15° E

Tracking Time (hrs)	Gander, India, Tananarive		Gander, India, Madrid		Gander, Winkfield, India	
	Position (meters)	Velocity (m/s)	Position (meters)	Velocity (m/s)	Position (meters)	Velocity (m/s)
1	651	0.11	7280	1.11	2330	0.42
2	491	0.03	3600	0.45	1030	0.12
3	395	0.03	1050	0.13	585	0.06
6	266	0.02	325	0.03	305	0.02
12	251	0.02	295	0.02	286	0.02
18	244	0.02	280	0.02	280	0.02
24	244	0.02	280	0.02	280	0.02
After 24 hours, by directional components*						
N	8	0.02	26	0.02	25	0.02
V	228	0.001	269	0.001	272	0.001
W	24	0.004	73	0.004	61	0.004

*$N = \vec{V} \times (\vec{R} \times \vec{V})$

$V = \vec{V}$

$W = \vec{R} \times \vec{V}$

VHF Tracking

Studies were also made for specific missions using VHF tracking. These studies covered transfer orbits (described in the Appendix) in addition to the synchronous orbits.

For a set of studies for the Sirio project (Reference 10), the synchronous satellite was located over the equator at 73° west longitude. It was tracked for 2-1/2 minutes each two hours by Santiago, Rosman, and Mojave in rotation, with a sampling rate of one measurement per second. The following system uncertainties were assumed: range noise — 15 meters, range bias — 30 meters, range rate noise — 10 cm/sec, and range rate bias — 20 cm/sec. Three tracking modes were considered: 1) range rate measurements only, 2) range measurements plus range rate measurements with perfect ionosphere modelling assumed, and 3) range and range rate measurements including an additional range bias of 300 meters to allow for maximum uncertainties due to propagation of the VHF signal through the ionosphere. This added bias assumes corrections for only about 50% of the ionospheric effect (Reference 11).

Conservative station location uncertainties (1σ) assumed were Santiago — 71 meters, Rosman — 60 meters, and Mojave — 61 meters. A-priori uncertainties in position and velocity of the synchronous orbit were obtained from the error analysis studies for the transfer orbit (Appendix).

The nonoptimal (neglect) version of the error propagation program was used. The statistical filter used in this version assumes that real-time orbit determination is being modelled — that the biases in the error model (ground station location uncertainty and measurement bias) are not to be solved for, that measurement data are weighted in the process on the basis of noise variance only, and that no data editing is possible during the short data spans.

Table 6 presents the spacecraft position and velocity uncertainties (1σ) at four-hour intervals and also the breakdown of these uncertainties after 24 hours. As expected, range rate measurements alone are not sufficient; with range and range rate measurements, the position uncertainty after 24 hours is 6-8 km. In terms of components, practically all of the position uncertainty is in the longitudinal component (perpendicular to the line of sight) due to the factors of nearly stationary geometry and large station location and measurement biases.

For studies made for the SMS project (Reference 12), the synchronous satellite was assumed to be located over 95° West longitude. Tracking was simulated from Rosman and Santiago (and in one case, from Fairbanks) at the rate of one measurement per second for five minutes during each hour.

Table 6

1σ Uncertainties in Synchronous Satellite Position and Velocity, VHF Tracking for Sirio

ASSUMPTIONS

Tracking Uncertainties (1σ)	Noise	Bias	Station Location Uncertainties (1σ)	
Range	15 meters	30 meters	Santiago	71 meters
Range rate	10 cm/sec	20 cm/sec	Rosman	60 meters
Sampling rate:	1/sec for 2-1/2 min each 2 hours, rotating stations		Mojave	61 meters
$\epsilon \geq 5°$				

Tracking Time (hrs)	Range Rate Only		Range and Range Rate: No Ionospheric Range Bias		Range and Range Rate: Ionospheric Range Bias of 300 Meters	
	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)
0	9.3	1.01	1.6	0.22	5.4	0.27
4	14.1	0.92	6.2	0.37	5.4	0.48
8	27.4	2.85	12.6	0.79	17.9	1.22
12	113.3	8.39	9.9	0.87	14.1	1.20
16	91.8	6.99	8.3	0.80	10.4	0.96
20	112.1	8.82	6.6	0.55	8.1	0.67
24	73.0	5.40	6.3	0.52	7.5	0.60
After 24 hours by directional components						
N (Line of sight)	14.6	5.38	0.1	0.51	0.4	0.59
V (Longitudinal)	71.5	0.15	6.3	0.06	7.4	0.08
W (Out of plane)	2.1	0.51	0.6	0.07	0.6	0.08

Conservative station location uncertainties were assumed as follows: Rosman — 60 meters, Santiago — 71 meters, and Fairbanks — 60 meters. Minimum elevation angles were 10° for Rosman and Santiago and 5° for Fairbanks.

Various combinations were studied: 2 stations, range and range rate, range only, and range rate only; and 3 stations, range and range rate. Measurement uncertainties assumed were: range noise — 15 meters; range bias — 150 meters for Rosman and Santiago and 300 meters for Fairbanks; range rate noise — 5 cm/sec; and range rate bias — 1 cm/sec. The basic range bias of 30 meters was increased considerably to allow for maximum uncertainties due to the propagation of the VHF signal through the ionosphere assuming corrections for only about 50% of the total effect. The amount of the additional range bias is dependent on the elevation angle and is therefore greater for Fairbanks, whose elevation angle is approximately 6°.5 as compared with 47° for Rosman and 43° for Santiago.

Both optimal and nonoptimal (neglect) versions of the error propagation program were used, and additional studies were made using the optimal version and reducing the station location uncertainties to 10 meters in each component.

Results from the nonoptimal version cases (real-time processing) are given in Table 7 at 4-hour intervals and, after 24 hours of tracking, broken down by components, assuming that the standard station location errors are used. Similarly, Table 8 shows results with the reduced station location uncertainties. After 24 hours, the position uncertainty (1σ) is 5.7 kilometers when two stations (with standard location errors) track simultaneously using range and range rate measurements, or 2.7 kilometers when the three stations track. And again the range solution is far better than the range rate solution — practically all of the position uncertainty after 24 hours is along the longitudinal component due to geometry and biases. Reducing the station location errors has little effect in this case, due to the large measurement biases assumed.

When post-flight processing is modeled, uncertainties can be improved somewhat, as shown in Table 9. After 24 hours, the position uncertainty is reduced from 2.7 to 2.4 kilometers for three stations tracking with range and range rate measurements.

<u>Tracking by Lasers</u>

Studies were made to simulate the tracking of a synchronous satellite (such as ATS-F) at 66° West by lasers at various locations (Reference 13). A laser tracking system provides precise range measurements as well as angular measurements; measurement uncertainties assumed were — range noise of

Table 7

1σ Uncertainties in Synchronous Satellite Position and Velocity, VHF Tracking for SMS, Neglect Filter, Standard Location Uncertainties

ASSUMPTIONS

Tracking Uncertainties (1σ)

	Noise	Bias
Range	15 meters	150 meters — Rosman and Santiago* 300 meters — Fairbanks*
Range rate	5 cm/sec	1 cm/sec
Sampling rate:	1/sec for 5 min each hour (simultaneous)	

Satellite A Priori (1σ)

Position	20 km
Velocity	20 m/s

Station Location Uncertainties (1σ)

Rosman	50 meters
Santiago	71 meters
Fairbanks	60 meters

Tracking Time (hrs)	2 Stations R&$\dot{R}$		2 Stations R		2 Stations $\dot{R}$		3 Stations R&$\dot{R}$	
	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)
4	7.1	0.52	7.1	0.52	14.6	1.01	4.7	0.34
8	6.9	0.50	6.9	0.50	12.4	0.90	4.3	0.34
12	6.6	0.48	6.6	0.48	14.8	1.07	3.6	0.28
16	6.4	0.46	6.4	0.46	32.3	2.35	3.0	0.22
20	6.1	0.44	6.1	0.44	55.6	4.05	2.8	0.20
24	5.7	0.42	5.8	0.42	68.1	4.95	2.7	0.19
After 24 hours, by directional components†								
N (Line of sight)	0.3	0.42	0.3	0.42	1.5	4.95	0.2	0.19
V (Longi-tudinal)	5.7	0.01	5.7	0.01	68.1	0.05	2.7	0.01
W (Out of plane)	0.1	0.01	0.1	0.01	0.1	0.01	0.1	0.01

*Maximum bias included for ionospheric effects (daytime tracking).

†$N = \vec{V} \times (\vec{R} \times \vec{V})$ $\quad V = \vec{V}$ $\quad W = \vec{R} \times \vec{V}$

Table 8

1σ Uncertainties in Synchronous Satellite Position and Velocity, VHF Tracking for SMS, Neglect Filter, Improved Location Uncertainties

ASSUMPTIONS

Tracking Uncertainties (1σ)

	Noise	Bias
Range	15 meters	150 meters — Rosman and Santiago* 300 meters — Fairbanks*
Range rate	5 cm/sec	1 cm/sec
Sampling rate:	1/sec for 5 min each hour (simultaneous)	

Satellite A Priori (1σ)

Position	20 km
Velocity	20 m/s

Station Location Uncertainties (1σ)

All stations — 10 meters in each component

Tracking Time (hrs)	2 Stations R&$\dot{R}$		2 Stations R		2 Stations $\dot{R}$		3 Stations R&$\dot{R}$	
	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)
4	6.9	0.50	6.9	0.50	14.6	1.01	4.7	0.34
8	6.7	0.49	6.7	0.49	12.4	0.90	4.2	0.33
12	6.5	0.47	6.5	0.47	14.8	1.07	3.5	0.28
16	6.2	0.45	6.2	0.45	32.3	2.35	3.0	0.22
20	6.0	0.43	6.0	0.43	55.6	4.05	2.7	0.20
24	5.6	0.40	5.6	0.40	68.1	4.95	2.7	0.19
After 24 hours, by directional components†								
N (line of sight)	0.3	0.40	0.3	0.40	1.5	4.95	0.1	0.19
V (Longitudinal)	5.6	0.01	5.6	0.01	68.1	0.05	2.6	0.01
W (Out of plane)	0.1	0.01	0.1	0.01	0.1	0.01	0.1	0.01

*Maximum bias included for ionospheric effects (daytime tracking).

†$N = \vec{V} \times (\vec{R} \times \vec{V}) \qquad V = \vec{V} \qquad W = \vec{R} \times \vec{V}$

Table 9

1σ Uncertainties in Synchronous Satellite Position and Velocity, VHF Tracking for SMS, Optimal Filter

ASSUMPTIONS

Tracking Uncertainties (1σ)

	Noise	Bias
Range	15 meters	150 meters — Rosman and Santiago* 300 meters — Fairbanks*
Range rate	5 cm/sec	1 cm/sec
Sampling rate:	1/sec for 5 min each hour (simultaneous)	

Satellite A Priori (1σ)

Position	20 km
Velocity	20 m/s

Station Location Uncertainties (1σ)

Rosman	50 meters
Santiago	71 meters
Fairbanks	60 meters

Tracking Time (hrs)	2 Stations R&$\dot{R}$		2 Stations R		2 Stations $\dot{R}$		3 Stations R&$\dot{R}$	
	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)
Standard Location Errors								
4	8.8	0.64	8.6	0.63	19.2	1.27	3.5	0.26
8	6.1	0.45	6.5	0.47	18.0	1.27	2.8	0.20
12	5.5	0.40	5.6	0.41	18.4	1.32	2.7	0.19
16	5.1	0.37	5.1	0.37	20.2	1.46	2.6	0.19
20	4.7	0.34	4.7	0.34	22.0	1.59	2.5	0.18
24	4.4	0.32	4.4	0.32	23.8	1.72	2.4	0.17
After 24 hours, by directional components†								
N (Line of sight)	0.2	0.32	0.2	0.32	1.9	1.72	0.1	0.17
V (Longitudinal)	4.3	0.01	4.3	0.01	23.7	0.07	2.4	0.01
W (Out of plane)	0.3	0.02	0.4	0.01	0.4	0.04	0.2	0.02

*Maximum bias included for ionospheric effects (daytime tracking).

† $N = \vec{V} \times (\vec{R} \times \vec{V})$ $\qquad V = \vec{V}$ $\qquad W = \vec{R} \times \vec{V}$

1.2 meters, range bias of 0.15 meters, and angular noise and bias of 0.5 milliradians (Reference 14). A sampling rate of one per minute was taken with a minimum elevation angle of 5°. The following stations and conservative location uncertainties (in meters) were used:

		Location Uncertainties (1σ)		
Station No.	Name	Latitude	Longitude	Altitude
1	Rosman	35	35	35
2	Mojave	34	37	35
3	Santiago	41	41	41
4	Ascension	43	103	105
5	Quito	38	38	38
6	Canary	77	138	127
7	Madrid	39	31	37

These large location uncertainties allow comparison to be made with the S-band results and show the full impact of station location uncertainty.

Several two-station cases were run, using combinations of the first three stations. Position uncertainties (1σ) at various intervals for two combinations using laser ranging measurements follow (in meters):

Time (hours)	Rosman-Mojave	Mojave-Santiago
3	1530	690
6	1140	650
12	910	620

After 12 hours of tracking from Rosman and Mojave, the position uncertainty is approximately 910 meters, which is nearly the same as with S-band tracking. However a better combination would be tracking from Mojave and Santiago (which has latitude as well as longitude separation), with a resulting position uncertainty of 620 meters.

The 3-station combination of Rosman, Mojave, and Santiago was used for several studies to determine what angle measurements contribute. Position uncertainties (1σ), in meters, at various intervals are as follows:

Tracking Time (hours)	Range and Angles with Angle Uncertainties of		Range Only — No Angle Measurements
	0.5 mrad	0.1 mrad	
3	670	660	690
6	610	580	620
9	610	580	610
12	600	580	600

As expected, the effect of angle measurements is relatively slight. Thus the following cases will utilize laser ranging measurements only.

Two other situations were explored, 1) adding lasers at other locations to vary the number of trackers from 3 to 6 in order to analyze the effect of geometry and 2) using the standard station location uncertainties, reducing the station location uncertainties to 10 meters in each component, and using no station location uncertainty in order to analyze the effect of station location uncertainty in this case. Results are combined in Table 10.

When the standard station location uncertainties are used, four stations are more than adequate to reduce the satellite position uncertainty to less than 500 meters after 12 hours of tracking. In fact, with the 3-station combination of Rosman, Mojave, and Santiago the uncertainty is only 600 meters; this is easily improved by only small reductions in the station location uncertainties. The second 3-station case is poorer because of the large location uncertainties for Ascension, but better with reduced location uncertainties.

By adding additional lasers, the uncertainty can be reduced to the 200-meter level; and for improved station location uncertainties to the 100-meter level and less. The last column in Table 10, with laser range measurements only and perfect station locations, re-emphasizes the importance of the station location uncertainties. With the precise laser range measurements, then, the uncertainties in the synchronous satellite position will almost directly depend on the station location uncertainties; hence any improvement in reducing the ground station location uncertainty will help to improve laser tracking of synchronous satellites.

Table 10

1σ Uncertainties in Synchronous Satellite Position (meters)
After 12 Hours of Tracking by Lasers

ASSUMPTIONS

Tracking Uncertainties (1σ)

	Noise	Bias
Range	1.2 meters	0.15 meters
Angles	0.5 mrad	0.5 mrad
Sampling rate:	1/minute	

Satellite A Priori (1σ)

Position	20 km
Velocity	20 m/s

Standard Station Location Uncertainties (1σ) – meters

	Lat.	Long.	Alt.
1. Rosman	35	35	35
2. Mojave	34	37	35
3. Santiago	41	41	41
4. Ascension	43	103	105
5. Quito	38	38	38
6. Canary	77	138	127
7. Madrid	39	31	37

Number of Trackers	Station Numbers	Station Location Uncertainties Used		
		Standard	10 m/10 m/10 m	None
3	1, 2, 3	600	159	2.9
	1, 3, 4	658	86	1.4
4	1, 2, 3, 4	423	62	1.1
5	1, 2, 3, 4, 5	424	61	1.1
	1, 2, 3, 4, 6	376	56	0.9
	1, 2, 3, 4, 7	239	58	0.9
6	1, 2, 3, 4, 5, 6	383	56	0.9
	1, 2, 3, 4, 5, 7	227	58	0.9

Table 11 shows directly the synchronous satellite position uncertainty (1σ) resulting from several station location uncertainties. Most cases give less than 100 meters (1σ) uncertainty. The improvement for 4 stations over 3 and the improvement with reduced station location uncertainty is clearly shown. Table 12 gives the same result for slightly different laser noise and bias values. No change is evident for the station error in each component of 10 meters, and only slight changes due to the reduced noise but increased bias. Thus the laser system itself is precise enough to be outweighed by the station location uncertainties until those uncertainties become extremely small. The important error parameter for laser tracking of synchronous satellites is station location uncertainty.

CONCLUSIONS

Tracking a synchronous satellite from a single ground station leaves large errors in the determination of synchronous satellite position (2 to 30 kilometers). Two (or more) trackers are needed if at least 2 kilometers (1σ) position accuracy is desired. A favorable two-station solution gives a 500 meter (1σ) or less position accuracy. With more than 2 stations tracking, the uncertainty is reduced more quickly; and under favorable circumstances (laser tracking with small range bias from stations with less than 20-meter total position uncertainty), a synchronous satellite position accuracy of less than 100 meters (1σ) may be achieved.

The importance of measurement type is shown. Angular measurements are only of slight benefit at synchronous height, while range rate measurements are not effective due to the extremely small station-satellite range rate. Thus the most important measurement is range. Secondly, the importance of tracking geometry is shown. The greater the longitude and latitude separation between the ground tracking stations, the better the results. Additional ground stations add a different look at the synchronous satellite due to a different line of sight. In general, the orbit determination will be more effective according to how well the synchronous satellite is covered by having line-of-sight tracking from each direction. Third, the importance of ground station location uncertainty is shown. As an example, Toowomba and Mojave determine a synchronous satellite position to 130 meters when they have 20-meter location uncertainty, but this more than doubles to 320 meters with 60-100 meter ground location uncertainty. Fourth, the importance of considering tracking components is shown. The synchronous orbit is well determined along a station-satellite line of sight, being approximately as good as the range bias and station location uncertainty in that direction. However, by far the largest part of the position uncertainty is along the longitudinal component, in the orbit plane but perpendicular to the

Table 11

Laser Tracking Position Errors (meters)
Versus Station Location Errors

Tracking Stations Used \ Station Error In Each Component (m)	10	3	0
Rosman, N.C. Mojave, Cal. Santiago, Chile	159	45	2.9
Same and Ascension	61	19	1.1

Laser Errors: Noise 1.2 m
Bias 0.15 m

Sampling Rate: 1/min

Synchronous Satellite: 66° W longitude

Tracking Time: 12 hours

Table 12

Laser Tracking Position Errors (meters)
Versus Station Location Errors, Modified Laser Errors

Tracking Stations Used \ Station Error In Each Component (m)	10	3	0
Rosman, N.C. Mojave, Cal. Santiago, Chile	159	42	5.0
Same and Ascension	61	18	1.9

Laser Errors: Noise 0.3 m
Bias 0.3 m

Sampling Rate: 1/min

Synchronous Satellite: 66° W longitude

Tracking Time: 12 hours

Earth-satellite line of sight. Any station location uncertainty has a direct effect on producing synchronous satellite location uncertainty in the longitudinal direction.

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APPENDIX

TRACKING SYNCHRONOUS TRANSFER ORBITS

Sirio

Tracking was simulated from injection into the transfer orbit (for orbital parameters, see Reference 10) to second apogee, at which time the synchronous orbit insertion maneuver is assumed to occur. The spacecraft was tracked for 2-1/2 minutes during each 15-minute period, using Tananarive and Carnarvon alternately for the first ten hours and Santiago and Rosman alternately thereafter to 2nd apogee. A sampling rate of one measurement per second was used, with the following tracking uncertainties (1σ):

Range noise	15 meters
Range bias	30 meters
Range rate noise*	10 cm/sec
Range rate bias†	20 cm/sec

Conservative station location uncertainties (1σ) were assumed as:

Tananarive	50 meters
Carnarvon	106 meters
Santiago	71 meters
Rosman	60 meters

Types of measurements used were: 1) range rate only, 2) range and range rate with perfect ionosphere modelling assumed, and 3) range and range rate with an added range bias of 300 meters to allow for maximum uncertainties due to propagation of the VHF signal through the ionosphere. This latter figure assumes ionospheric predictions correct for about 50% of the ionospheric effect. The statistical filter used assumes that real-time orbit determination is modeled.

At second apogee, 1σ uncertainties in position and velocity are:

*Propagation path uncertainties treated separately.
†Includes effects of model uncertainties.

	Range and Range Rate				Range Rate Only	
	No Ionospheric Range Bias		Ionospheric Range Bias of 300 Meters			
	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)
Total	1.6	0.22	5.4	0.27	9.3	1.01
By directional component*						
N (Line of sight)	0.1	0.17	0.4	0.22	2.9	0.31
V (Longitudinal)	1.2	0.04	4.2	0.06	3.0	0.15
W (Out of plane)	1.1	0.13	3.4	0.14	8.4	0.95

*$N = \vec{V} \times (\vec{R} \times \vec{V})$ $\quad W = \vec{R} \times \vec{V}$

$V = \vec{V}$

Most of the position uncertainty at second apogee is in the longitudinal and out-of-plane directions and is due to the range rate bias except in the case where the 300-meter range bias is included.

SMS

Tracking was simulated from injection into the transfer orbit (for orbital parameters, see Reference 12) to first apogee, at which time it is assumed the synchronous orbit maneuver will be made. Range and range rate measurements are made each second for 2-1/2 minutes in each 15-minute interval, with Tananarive and Carnarvon alternating each 15 minutes. Tracking uncertainties and station location uncertainties were the same as used for Sirio.

Two cases were studied: 1) assuming an additional range bias for maximum ionospheric effects of 300 meters for each station, and 2) assuming a reduced range bias for Carnarvon (making the total 150 meters), because the refraction corrections are less for higher elevation angles. Resulting position and velocity uncertainties (1σ) at 1st apogee are presented below.

	300-Meter Added Range Bias Both Stations (Total 330 meters)		Reduced Range Bias for Carnarvon (Total 150 meters)	
	Position (km)	Velocity (m/s)	Position (km)	Velocity (m/s)
1 σ Uncertainties at 1st Apogee (Total)	5.0	0.24	3.7	0.20
By directional component				
N (Line of sight)	0.6	0.14	0.4	0.08
V (Longitudinal)	3.6	0.05	1.8	0.03
W (Out of plane)	3.4	0.19	3.2	0.18