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WIND TUNNEL SIMULATION OF STORE JETTISON WITH THE AID OF MAGNETIC ARTIFICIAL GRAVITY

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16. Abstract

A method employed in the simulation of jettison of stores from aircraft involves small scale wind-tunnel drop tests from a model of the parent aircraft. Proper scaling of such experiments generally dictates that the gravitational acceleration should ideally be a test variable. A method of introducing a controllable artificial component of gravity by magnetic means has been proposed by Covert. The use of a "magnetic artificial gravity" facility based upon this idea, in conjunction with small scale wind-tunnel drop tests, would thereby improve the accuracy of simulation. The work reported here includes a review of the scaling laws as they apply to the design of such a facility. The design constraints involved in the integration of such a facility with a wind tunnel are defined. A detailed performance analysis procedure applicable to such a facility is developed. A practical magnet configuration is defined which is capable of controlling the strength and orientation of the magnetic artificial gravity field in the vertical plane, thereby allowing simulation of store jettison from a diving or climbing aircraft. The factors involved in the choice between continuous or intermittent operation of the facility, and the use of normal or superconducting magnets, are defined.

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LIST OF SYMBOLS

A	Cross-sectional area of coil winding (Eq. 11.10-13)
a,b,c	Principal axes of magnetization of a body
$\vec{B}$	Magnetic field strength
B	Coil winding buildup (Fig. 7)
$D_{a,b,c}$	Demagnetizing factors of a body (Eqs. 4.7-9)
E	Electrical energy (Eq. 12.8)
$\vec{F}_{mag}$	Magnetic force (Eq. 4.5)
F_p	Coil winding packing factor (Eq. 11.35)
g	Gravitational acceleration
g_M	Acceleration due to combined gravitational and magnetic forces acting on store model (Eq. 4.1)
g_{mag}	Acceleration of store model due to magnetic force (Eqs. 4.2,3)
I	Electrical current
J	Electrical current density (amps/unit area)
$K_{x,y,z}$	Radii of gyration of store about x,y,z axes (Eq. 3.2)
k_t	Conversion factor in magnetic force and moment relations (Eq. 4.19, 23)
$k_{xx/zz}$	Coefficient of inductive coupling between axial gradient and vertical gradient coil systems (Eqs. 12.16,17)
L	Characteristic length of store (Eq. 3.2)
L	Self inductance of coil or coils (Eqs. 11.26-29)
$\bar{\ell}$	Length of coil mean turn (Eqs. 11.7-9)
ℓ	Mean length of one side of a square coil Eq. 11.49)
M	Mach number (Eq. 3.1)

LIST OF SYMBOLS

(Continued)

$\vec{M}$	Magnetic moment of a magnetized body (Eq. 4.5)
$M_{xx/zz}$	Mutual inductance between axial gradient and vertical gradient coil system (Eq. 11.30)
$\vec{m}$	Magnetization (magnetic moment/unit volume) (Eq. 4.6)
M_{sat}	Saturation magnetization of ferromagnetic material (Eq. 4.15)
m	Coil winding mass (Eq. 11.74)
N	Number of turns of conductor in a coil system
n	Number of turns of conductor on a single coil
P	Electrical power (Eq. 12.1)
Q	Magnetic performance parameter (Eqs. 11.18-21)
q	Dynamic pressure (Eq. 3.3)
R	Electrical resistance (Eqs. 11.22-25)
R	Radius of iron sphere in store model (Eq. 1e.1)
R_e	Reynolds number (Eqs. 3.5,6)
R_o	One half of clear inside dimension of coil assembly (Fig. 7)
r	Coil corner radii (Fig. 7)
r	Radial measure in spherical coordinates (Eqs. 13.1,2)
S	Coil resistance parameters (Eqs. 11.22-25)
s	Outside length of a square coil (Eq. 11.41)
s	Laplace transform variable (Eqs. 12.14-25)
T	Absolute temperature
T	Self-inductance parameters (Eqs. 11.26-29)
$\vec{T}_{mag}$	Magnetic torque on a ferromagnetic body (Eq. 4.13)
V_{mag}	Volume of magnetic material (Eq. 4.6)
V	Volume of coil windings (Eq. 11.72)

LIST OF SYMBOLS (Continued)

V	Coil or coil system terminal voltage (Eqs. 11.26-29)
$W_{xx/zz}$	Mutual inductance parameter-axial and vertical gradient coil system (Eq. 11.30)
w	Weight
x,y,z	Principal axes of store
X,Y,Z	Tunnel-fixed coordinates (Fig. 1b)
X',Y',Z'	Earth-fixed coordinates (Fig. 1a)
$\alpha(\ )$	Coil winding buildup ratios (Eqs. 11.1,2)
$\beta(\ )$	Coil winding buildup ratios (Eqs. 11.3-6)
γ	Geometric parameter used in formula for self-inductance of a square coil (Eq. 11.41)
δ	Ratio of mean axial spacing of square coils to near length of one side (Eq. 11.49)
η	$\sqrt{1+\alpha^2}$ (Eq. 11.49)
Θ	Dive angle of parent aircraft (Eqs. 4.2,3)
Θ	Elevation component in spherical coordinates (Eq. 13.1)
μ_0	Permeability of free space
ρ_s	Mass density of store (Eqs. 3.3,6,7)
ρ	Electrical resistivity (Eq. 11.35)
τ	Coil or coil system time constant (L/R) (Eqs. 12.14-17)
ϕ	Angle parameters related to coil system geometry (Fig. 7)
ϕ	Azimuthal component in spherical coordinates (Eq. 13.3)
χ	Magnetic susceptibility (Eqs. 4.7-9)
χ	$\sqrt{2-\alpha^2}$ (Eq. 11.49)
ψ	Direction of magnetic force on a ferromagnetic sphere due to adjacent spheres, relative to direction of applied saturating field (Fig. 13, Eq. 13.13)
$\vec{\nabla}$	Vector gradient operator (Eq. 4.5)

LIST OF SYMBOLS

(Concluded)

SUBSCRIPTS

a,b,c	Principal magnetic axes of ferromagnetic body
mag	Magnetic
M	Conditions for model
P	Conditions for prototype
S	Conditions for store
x,y,z	Measured in the x,y,z direction
x	Quantities related to axial ambient field coil system (e.g. I_x)
z	Quantities related to vertical ambient field coil system
xx	Quantities related to axial gradient field coil system
zz	Quantities related to vertical gradient field coil system

WIND TUNNEL SIMULATION OF STORE JETTISON WITH THE AID OF MAGNETIC ARTIFICIAL GRAVITY

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1.0 INTRODUCTION

An important component of the problem of simulation of store jettison by means of small scale drop tests in a wind tunnel arises from the appearance of gravity in the scaling relationships. Generally, for accurate reproduction of the full scale trajectory, the ratio of gravity force to aerodynamic force must be the same for the model as for the full scale store (Reference 1). When other necessary conditions for simulation are imposed, it is generally found that a greater than normal gravity is required for small scale models.

A basic method of providing the required increment of body force corresponding to the "gravity" needed for such tests has been proposed by Covert (Reference 2). This method involves the use of magnet coils surrounding the wind tunnel test section which interact with ferromagnetic material imbedded in the model of the jettisonable store. In this manner, a magnetic "artificial gravity" field is provided which is approximately uniform throughout the test section. It is feasible to extend this method to provide control of the angulation of the resultant "gravity" field, thereby allowing simulation of diving or climbing attitudes.

Since a "magnetic artificial gravity" facility appears to offer a solution to the difficulties encountered in conventional store jettison test methods, a study was undertaken for the design of such a facility.

2.0 SUMMARY OF PRESENT STUDY

Included in this study were the following items:

1. Review of the scaling laws applicable to the wind tunnel simulation of store jettison, in terms of a controllable artificial gravity field.
2. Definition of the design constraints involved in the integration of the facility with a wind tunnel.
3. Development of a detailed performance analysis procedure. The performance analysis is applicable to air-core magnet systems and provides a detailed distribution of the strength and uniformity of the artificial gravity field.
4. Establishment of a practical magnet configuration and analysis of the magnetic performance of the configuration.
5. Extension of the performance analysis procedure to include the effects of residual nonuniformities in the artificial gravity field on typical store trajectories. This therefore provides an evaluation of the facility in terms of the end use.
6. Exploration of the relative merits of iron-core and air-core magnet configurations.
7. Determination of factors involved in the choice of the mode of operation of the facility. The alternatives considered are:
   - (A) Continuous operation of a normal (non-superconducting) coil system,
   - (B) intermittent operation of a normal coil system, and
   - (C) continuous operation of a superconducting coil system.

   The following are some of the factors involved:
   - (a) Since the store-dropping procedure is intermittent, intermittent operation of the magnets may be feasible.

(b) Intermittent operation of the magnets reduces the average power required to operate "normal" magnets. This is beneficial for two reasons:

   i) Reduces the coil cooling system requirements, and

   ii) reduces the cost of electrical power.

(c) In contrast to intermittent operation, it has been determined that continuous operation of a normal system would typically require approximately 10^2 to 10^3 megawatts for a wind tunnel facility in the 3' to 4' size range. This is typically at least one order of magnitude higher than the power required to run the wind tunnel itself.

(d) Intermittant operation requires relatively sophisticated intermittent power supply systems, which incorporate means of storing and controlling the release of large quantities of electrical energy.

(e) Continuous operation of the magnet system is feasible with the use of superconducting coils. In this case, power costs are relatively small and the power supplies need be of only modest capacity.

(f) Use of superconducting coils involves the use of more elaborate and expensive materials and construction techniques. The engineering of such a system is more complicated, because additional factors are involved:

   i) Thermal design - The coils are immersed in a triple-walled container of complex shape designed to contain liquid helium, liquid nitrogen and thermal insulation.

   ii) Structural design - The magnet system must support the self-induced magnetic stresses and gravity forces, in a manner which is compatible with the thermal design.

iii) Material selection - The superconducting material that is selected must be capable of stable and reliable superconducting operation at the maximum design magnetic field levels.

(g) A superconducting facility will require a supply of liquid helium and liquid nitrogen. This will either be provided in batches and the "boiloff" discarded, or by a closed cycle using a refrigeration system to conserve the helium and nitrogen.

8. It has been determined that under certain conditions, it is feasible to simulate multiple simultaneous store launches in a facility of this kind. The main limitations stem from errors introduced by the mutual interaction of the stores. Since these magnetic interaction forces vary inversely with the fourth power of the separation between the centers of gravity, it may be assumed that negligible perturbations to the trajectories are incurred if the separation is large enough. (Typically on the order of two store diameters.)

Under the current work, general specifications for magnetic artificial gravity facilities are being considered for both intermittent and continuous operation. It is necessary to accumulate additional technical information, particularly in the area of current design practice involving large multi-component superconducting magnet systems, before it is possible to make the selection between the intermittent operation (normal conductor) case and the continuous operation (superconductor) case. In view of the present uncertainty as to the mode of operation, it appears premature at this time to attempt to prepare detailed cost and time estimates for the design of a facility for a medium-sized wind tunnel.

Since the emphasis of the present work has been on the development of the basic design of the magnetic artificial gravity facility, the detailed study of additional equipment requirements has been deferred to a future time. Included in this category are such items as cameras, timing units, stroboscopic flash units, etc., necessary to perform store jettison tests in a transonic/supersonic wind tunnel. It is considered that specification of such items at this point is premature; however, it is expected that conventional wind tunnel store jettison test techniques using such items may be employed with the artificial gravity facility, and no important restrictions on their use will be incurred because of any particular characteristics of the facility itself.

3.0 SUMMARY OF SCALING LAWS FOR STORE JETTISON WITH ARTIFICIAL GRAVITY

The following are relationships among the test conditions occurring in the simulation of store jettison and similar problems in the wind tunnel with the "free drop" method (References 1 - 4). The value of gravity, "g_M", under the test conditions is considered to be a variable to allow for a magnetic component of body force.

1. Model and prototype are geometrically similar.
2. Mach number is the same for model and prototype.

i.e., $M_M = M_p$ (3.1)

3. Mass distribution is the same for model and prototype.

i.e., $\left(\frac{K_x}{L}\right)_M = \left(\frac{K_x}{L}\right)_p$; $\left(\frac{K_y}{L}\right)_M = \left(\frac{K_y}{L}\right)_p$; $\left(\frac{K_z}{L}\right)_M = \left(\frac{K_z}{L}\right)_p$ (3.2)

4. Ratio of aerodynamic acceleration to "gravitational" acceleration is the same for model and prototype, at geometrically similar points in the trajectory.

i.e., $\frac{g_M}{g} = \frac{(\rho_s)_p}{(\rho_s)_M} \frac{(L)_p}{(L)_M} \frac{(q)_M}{(q)_p}$ (3.3)

5. Induced angles of attack are the same for model and prototype.

i.e., $\frac{g_M}{g} = \frac{L_p}{L_M} \frac{T_M}{T_p}$ (3.4)

(Assuming 2 and 4 hold)

6. Reynolds Number ratio (for air) (Reference 5):

$$\frac{Re_M}{Re_p} = \frac{L_M}{L_p} \frac{(\rho_s)_M}{(\rho_s)_p} \left[\frac{T_p}{T_M}\right]^{1.26} \tag{3.5}$$

or, with conditions 3.1 - 3.5 satisfied,

$$\frac{Re_M}{Re_p} = \frac{L_M}{L_p} \frac{(\rho_s)_M}{(\rho_s)_p} \left[\frac{T_p}{T_M}\right]^{0.26} \tag{3.6}$$

7. Store density ratio (from 3.4, 3.5)

$$\frac{(\rho_s)_M}{(\rho_s)_p} = \frac{T_p}{T_M} \frac{q_M}{q_p} \tag{3.7}$$

<u>Limits on Reynolds Number Scaling</u>

Equation 3.6 illustrates a fundamental limitation to Reynolds Number scaling due to the practical limits available for the store density ratio $(\rho_s)_M/(\rho_s)_p$, and the temperature ratio T_p/T_M. Since the Reynolds Number ratio is only weakly dependent on T_p/T_M, the strongest compensation for small scale factor is provided by the store density ratio. However, it is not always feasible to increase the density of the model store to such an extent as to fully compensate for the scale factor (L_M/L_p), and produce full scale Reynolds Number. In general therefore, the Reynolds Number ratio will be related most strongly to the scale factor.

4.0 MAGNETIC FORCES AND ARTIFICIAL GRAVITY

The following is a summary of the relationships governing the scaled gravity obtained by magnetic forces acting on a store model containing ferromagnetic material.

The total body force acting on the store model is the sum of the gravity and magnetic forces. In terms of the scaled gravity $\vec{g}_M$, this is:

$$\vec{g}_M = \vec{g} + \vec{g}_{mag} \tag{4.1}$$

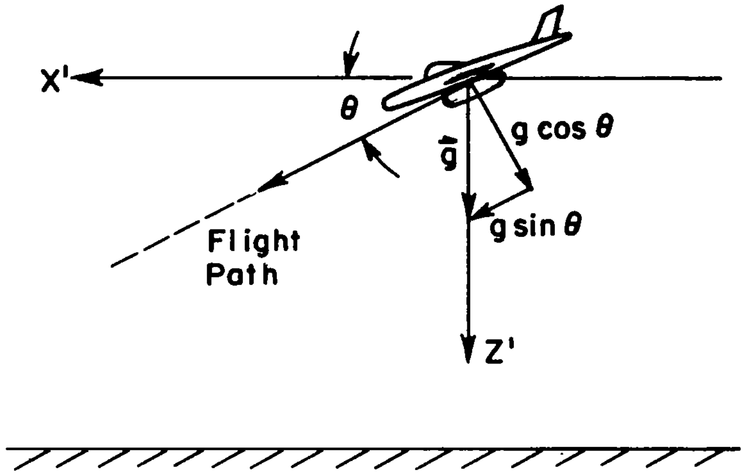


Figure 1a. Prototype Parent Aircraft and Store in Earth-Fixed Reference Frame.

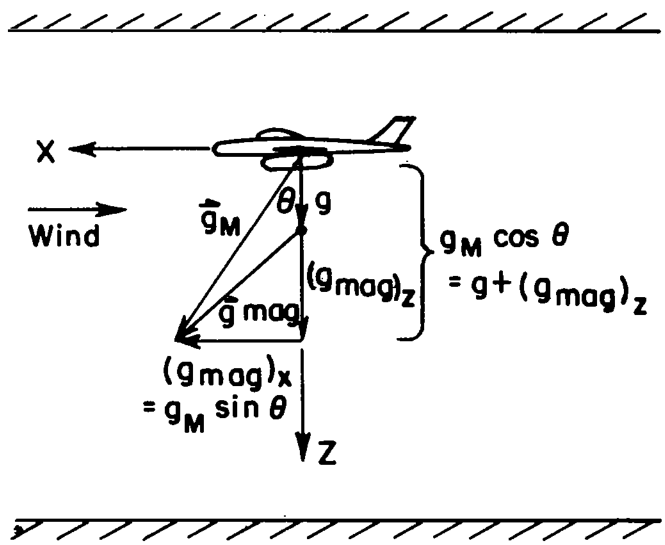


Figure 1b. Model Parent Aircraft and Store in Wind-Tunnel Reference Frame.

In terms of the dive angle, θ, relative to the horizontal, the magnetic gravity components are: (See Figs. 1a, b)

$$(g_{mag})_z = g_M \cos \theta - g \tag{4.2}$$

$$(g_{mag})_x = g_M \sin \theta \tag{4.3}$$

The "magnetic gravity" component, $\vec{g}_{mag}$, is given by

$$\vec{g}_{mag} = \frac{\vec{F}_{mag}}{(w_s)_M} g \tag{4.4}$$

The magnetic force component $\vec{F}_{mag}$ is:

$$F_{mag} = (\vec{M} \cdot \vec{\nabla})\vec{B} \tag{4.5}$$

where $\vec{M}$ is the total magnetic moment of the magnetized material imbedded in the store model and $\vec{\nabla}\vec{B}$ is the magnetic field gradient tensor.

<u>Magnetization of Model Core</u> (Reference 6)

The magnetic moment $\vec{M}$ is given by

$$\vec{M} = \int^{V_{mag}} \vec{m}\, dv \simeq \bar{\vec{m}}\, V_{mag} \tag{4.6}$$

The average magnetization $\bar{\vec{m}}$, for "soft" magnetic materials such as iron, can be related to the applied magnetic field $\vec{B}$ as follows:

$$\bar{m}_a = \frac{M_a}{V_{mag}} \simeq \left(\frac{\chi}{1 + \chi D_a}\right) B_a \tag{4.7}$$

$$\bar{m}_b = \frac{M_b}{V_{mag}} \simeq \left(\frac{\chi}{1 + \chi D_b}\right) B_b \tag{4.8}$$

$$\bar{m}_c = \frac{M_c}{V_{mag}} \simeq \left(\frac{\chi}{1 + \chi D_c}\right) B_c \tag{4.9}$$

Where χ is the magnetic susceptibility of the material, the factors D_a, D_b, D_c are the demagnetizing factors associated with the three principal magnetic axes a, b, and c of the ferromagnetic body, and depend upon the external shape of the body, and B_a, B_b, B_c are the a, b, c components of $\vec{B}$.

The demagnetizing factors are related as follows:

$$D_a + D_b + D_c = 1 \tag{4.10}$$

If the material is magnetically saturated, the relationship between the applied field and the resultant magnetization is more complicated and the components are no longer uncoupled. For the special case of equal demagnetizing factors, however, the magnetization components reduce to:

$$m_a = \frac{B_a}{|B|} \cdot m_{sat}; \; m_b = \frac{B_b}{|B|} \cdot m_{sat}; \; m_c = \frac{B_c}{|B|} m_{sat} \tag{4.11}$$

where

$$|B| = \sqrt{B_a^{\,2} + B_b^{\,2} + B_c^{\,2}} \tag{4.12}$$

The average magnetization $\vec{m}$, (and the magnetic moment $\vec{M}$) are thus parallel to the applied field $\vec{B}$.

Torque-Free Condition

In the particular case of interest, it is a requirement that no extraneous torques be introduced by the magnetic field. The magnetic torque T_{mag} is given by:

$$T_{mag} = \vec{M} \times \vec{B} \tag{4.13}$$

Thus for $\vec{T}_{mag}$ to be zero, it is necessary that $\vec{M}$ be parallel to $\vec{B}$. For unsaturated or saturated material, this condition is satisfied if the three demagnetizing factors, D_a, D_b, and D_c are equal, and the material possesses low rotational hysteresis (Reference 7).

i.e., $T_{mag} = 0$ if $D_a = D_b = D_c = 1/3$

This condition is satisfied by a sphere, or other shapes such as a cube or a short cylinder. For this case ($D_a = D_b = D_c$), the average magnetization component in the tunnel-fixed frame $\bar{m}_x$, $\bar{m}_y$, $\bar{m}_z$ are related directly to the field components B_x, B_y, and B_z, without the necessity of a transformation involving the attidude of the iron core relative to the tunnel.

i.e., for ($D_a = D_b = D_c = 1/3$)

i) Unsaturated core, $3|B| < M_{sat}$

$$\bar{m}_x = 3B_x;\ \bar{m}_y = 3B_y;\ \bar{m}_z = 3B_z \qquad (4.14a,b,c)$$

ii) Saturated core, $3|B| \geq M_{sat}$

$$\bar{m}_x = \frac{B_x}{|B|} \cdot m_{sat};\ \bar{m}_y = \frac{B_y}{|B|} \cdot m_{sat};\ \bar{m}_z = \frac{B_z}{|B|} \cdot m_{sat} \qquad (4.15a,b,c)$$

<u>Magnetic Force Components</u>

The magnetic force components in the rectangular coordinates (x,y,z), for equal demagnetizing factors ($D_a = D_b = D_c$), are as follows:*

$$\frac{F_x}{V_{mag}} = K[\frac{B_x}{|B|} B_{xx} = \frac{B_y}{|B|} B_{yx} = \frac{B_z}{|B|} B \qquad (4.16)$$

$$\frac{F_y}{V_{mag}} = K[\frac{B_x}{|B|} B_{xy} + \frac{B_y}{|B|} B_{yy} + \frac{B_z}{|B|} B_{zy}] \qquad (4.17)$$

$$\frac{F_z}{V_{mag}} = K[\frac{B_x}{|B|} B_{zz} = \frac{B_y}{|B|} B_{yz} + \frac{B_z}{|B|} B_{zz}] \qquad (4.18)$$

The coefficient K has the following values depending upon the level of magnetization: (from Eqs. 4.14, 15)

i) Unsaturated ($|\vec{B}| < \frac{m_{sat}}{3}$)

$$K = k_t\ 3|\vec{B}| \qquad (4.19)$$

---

*The following notation is used to represent the gradient components:

$$B_{xx} = \frac{\partial B_x}{\partial x},\ B_{xy} = \frac{\partial B_x}{\partial y}, \text{ etc.}$$

ii) Saturated $\left(|B| > \frac{m_{sat}}{3}\right)$

$$K = k_t \, \overline{m}_{sat} \tag{4.20}$$

Magnetic Field Gradient Interrelations

The gradient components of the steady field B in free space (or air) are related through Maxwell's Equations as follows:

$$B_{xx} + B_{yy} + B_{zz} = 0 \tag{4.21}$$

and

$$B_{xy} = B_{yx}; \; B_{xz} = B_{zx}; \; B_{yz} = B_{zy} \tag{4.22}$$

Force Units

In order to use common units of measurement, a conversion factor "k_t" is required in the force equation.

i.e., $$\frac{F_{mag}}{V_{mag}} = k_t \, M \cdot \nabla B \tag{4.23}$$

for F = pounds

M = kilogauss

B = kilogauss

∇B = kilogauss/in.

V_{mag} = $(in)^3$

k_t = 1.14 $(in\text{-}lb)(in)^{-3}(Kgauss)^{-2}$

Example

(a) Consider an iron sphere of diameter d_{mag} = 1" having a saturation magnetization m_{sat} = 21 kilogauss (typical for iron), immersed in a magnetic field B_z = 10 kilogauss, with a gradient B_{zz} = 0.1 kilogauss/in. The density ρ_{mag} of the sphere is 0.280 lb/in^3. Calculate the total magnetic force, and the force per unit weight, on the sphere.

Since $3|B| > M_{sat}$, the sphere is saturated, and

$$F_z = V_{mag}\, k_t\, M_{sat}\, B_{zz}$$
$$= (\pi/6)(1)^3(1.4)(21)(0.1)$$
$$= 1.25 \text{ lb}$$

Total magnetic force = 1.25 lb.

Weight of sphere, $w_{mag} = \rho_{mag} V_{mag}$

$$= (0.280)((\pi/6)1^3)$$

i.e., $w_{mag} = 0.148$ lb.

Magnetic force/unit weight $= F_z/w_{mag} = \frac{1.25g}{0.148} = 8.45g$

Thus, the total force (magnetic plus gravity) acting on the iron sphere is 9.45 times the weight of the sphere, and with no additional mass, the sphere would be accelerated in the z-direction (downwards) with 9.45 g's.

For a saturated iron sphere, the acceleration due to magnetic force is

$$|g_{mag}| \simeq 84\left(\frac{w_{mag}}{w_s}\right) \nabla B \; g \qquad (4.24)$$

where $\frac{w_{mag}}{w_s}$ = ratio of magnetic mass to total mass of the store model.

or

$$\nabla B = 0.019\left(\frac{g_m}{g} - 1\right)\left(\frac{w_s}{w_{mag}}\right) \qquad (4.25)$$

4.1 FORCE FIELD UNIFORMITY

It is not possible to solve the equations relating the forces and steady magnetic fields to find a magnetic field configuration which produces a uniform force field over an extended three dimensional region of space and also satisfies the equations relating the field gradients to one another.

It is possible, however, to produce a magnetic force which is uniform along a line. Two cases are discussed below:

Uniform Force on an Unsaturated Iron Sphere

Consider the F_z components along the z-axis, and assume that at x = 0, y = 0, the gradient components B_{xy}, B_{xz}, B_{yz} are zero. From Eqs. (4.18,19) (unsaturated case)

assume $$\frac{F_z}{V_{mag}} = k_t 3\ B_z\ B_{zz} = \text{const.} \qquad (4.26)$$

i.e. $$B_z = k_1\ z^{1/2} \qquad (4.27)$$

It is feasible to produce a magnetic field distribution approximately according to Eq. (4.27) over a limited distance, by means of an axisymmetric coil arrangement.

This field distribution is shown in Figure 2. The maximum field strength is governed by the saturation of the sphere and the usable range is limited by the practical problem of producing the increasingly large gradient in the negative z-direction.

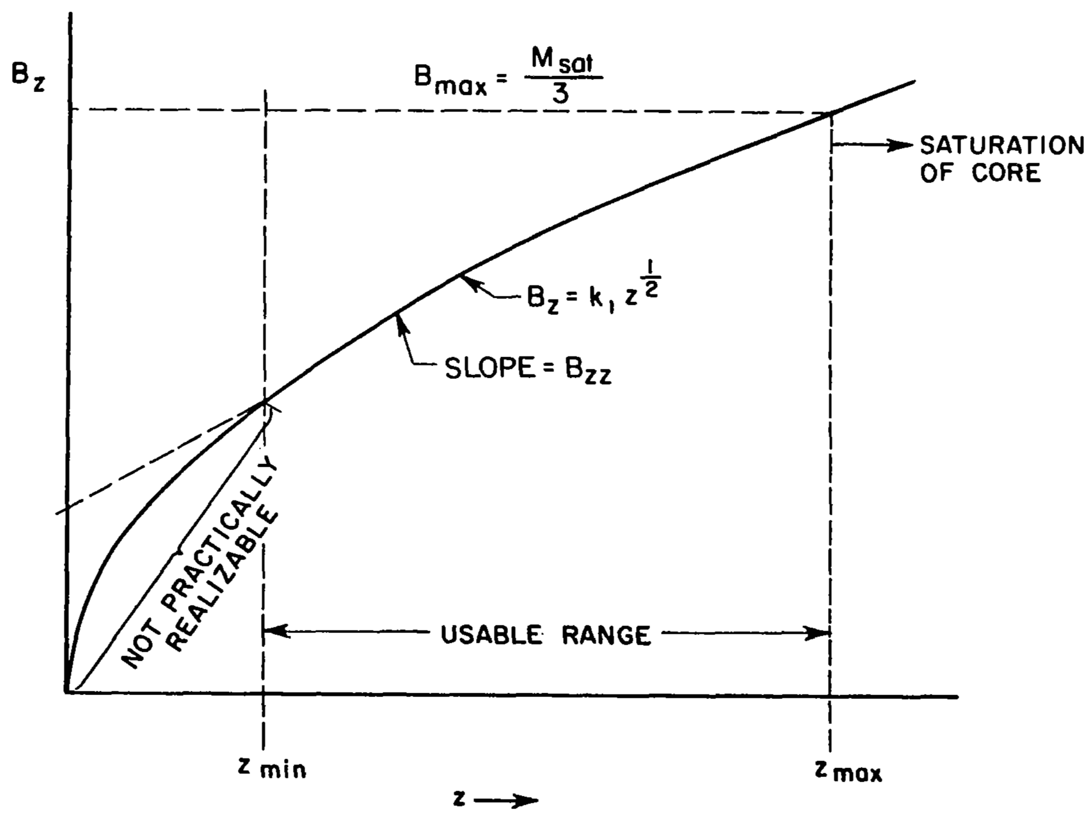


Figure 2. Distribution of Vertical Field Strength for Uniform Vertical Force Along the Vertical Axis, for an Unsaturated Ferromagnetic Sphere of High Permeability.

If it is assumed that the maximum practical gradient is twice that corresponding to the peak (saturation limited) field, B_z(max), then it can be shown that the usable range is given by

$$z_{max} - z_{min} = \frac{1}{8} \frac{m_{sat}}{B_{zz}(z_{max})}$$

<u>Example</u>

The usable range of unsaturated operation for a typical situation is calculated here. If m_{sat} = 21 kilogauss, gravity scale factor g_M/g = 20, length scale factor L_M/L_P = 1/20, mass ratio (w_s/w_{mag}) = 2, prototype store length L_p = 100", then from Eq. (4.25)

$$B_{zz} = (0.0119)(20-1)(2)$$

$$= 0.452 \text{ kilogauss/in}$$

$$\therefore z_{max} - z_{min.} = (1/8)\frac{(21)}{(0.452)}$$

$$= 5.8 \text{ inches}$$

but, model length L_m = 5.0 inches.
Therefore, in this example, the maximum useful length of uniform-force region is only slightly greater than the length of the store model. Note that if all factors remain the same other than the gravity and length scale factors, the ratio of the useful range to store length is approximately constant.

Due to the severe limitations on the useful range of uniform force inherent in this method (unsaturated core), this approach was not pursued further. The following approach, which is based upon a saturated core, was found to be more suitable.

<u>Uniform Force on a Saturated Iron Sphere</u>

Consider the F_z component at locations along the z-axis and assume that at x = 0, y = 0 the gradient components B_{xy}, B_{xz}, B_{yz} are zero, and B_y, B_z are also zero.

From Eqs. (4.18,20)(saturated case)

$$\frac{F_z}{V_{mag}} = k_t\, m_{sat}\, B_{zz} \tag{4.28}$$

$$\therefore B_z = a_0 + a_1 z \tag{4.29}$$

$$\text{and } B_z > \frac{m_{sat}}{3}$$

Thus, the vertical magnetic field strength varies linearly with vertical distance, and in the region of uniform force must be greater than that required to saturate the sphere. This field distribution is shown in Figure 3.

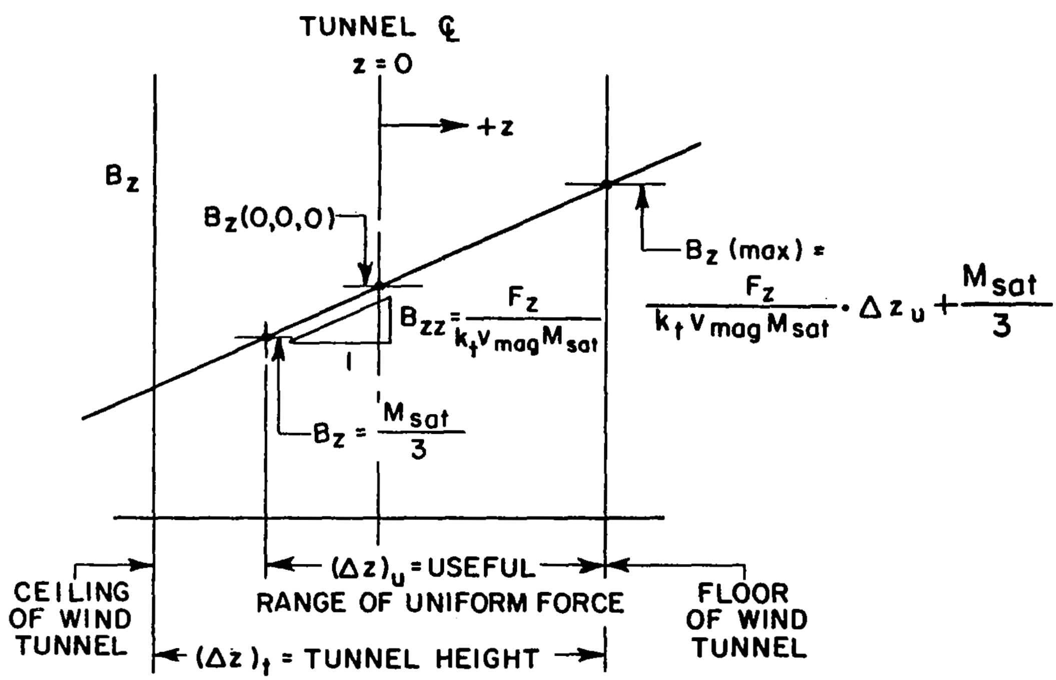


Figure 3. Distribution of Vertical Field Strength for Uniform Vertical Force Along the Vertical Axis, for a Saturated Ferromagnetic Sphere.

Example

The field requirements for saturated operation in a typical situation are calculated here.

$$m_{sat} = 21 \text{ kilogauss}; \quad \frac{L_M}{L_P} = 1/20$$

$$\frac{g_M}{g} = 20; \quad \left(\frac{w_s}{w_{mag}}\right) = 2$$

Tunnel test section height $(\Delta z)_t = 48"$

Useful range of z, $(\Delta z)_u = .75\ (\Delta z)_t = 36"$

From Eq. (4.25)

$B_{zz} = 0.452$ kilogauss/in.

$B_z = 7.0$ kilogauss @ $z = -12"$

$B_z = 7.0 + (36)(0.452) = 23.3$ kilogauss @ $z = +24"$
(floor of tunnel)

5.0 IRON-CORE MAGNET SYSTEMS

In the course of the preliminary layout and analysis of possible magnet configurations, a decision was made to exclude from further consideration systems employing iron or other ferromagnetic material in the magnetic circuit. This decision was based upon the following factors:

a) "Material effects," namely variations in magnetic properties of the ferromagnetic material, would make prediction and control of the magnetic force field extremely difficult, since the material would be partly or wholly saturated.
b) Since the required magnetic fields are well above the saturation level of the best magnetic materials, and the effective air gaps would be large, the use of iron offers only marginal reduction in magnet power (or amp-turn) requirements.
c) Geometrical considerations for this particular application

(for example, requirements of model visibility and space for the wind tunnel test section) prevent the iron from being used to advantage.

The performance of iron-core magnet systems with large air gaps is difficult to analyze with accuracy; it is usually necessary to build preliminary small-scale models and measure in detail the magnetic field configurations for a range of magnet currents.

Since it is possible, on the other hand, to analyze air-core magnet systems in a straightforward manner by using methods of linear superposition to obtain very accurate estimates of magnetic performance of arbitrary coil configurations, there appeared to be no advantage in building small scale working models of coil systems in the preliminary design evaluation phase. In fact, the process of design optimization may be performed quite readily for air-core systems using purely analytical methods.

For the reasons outlined above, no small scale working coil system models were constructed.

6.0 SINGLE-AXIS, CONSTANT GRADIENT AIR-CORE COIL SYSTEM

The field configuration shown in Figure 3 can be produced approximately by superposition of the field contributions from four coaxial coils. (See Fig. 4.) The basic approach is as follows:

(i) Two identical circular coils, coaxial with the z-axis, are arranged symmetrically above and below the x-y plane and spaced a distance $2z_*$ apart. Each coil has N_{z_o} turns, and both coils are in series electrically and connected such that the current I_z passes through the coils in the same sense so as to produce a net vertical field $B_z(0,0,0)$ at the center of symmetry. If $2z_* = R_z$, such an arrangement is known as a "Helmholtz pair," and by virtue of the particular choice of spacing, will produce a uniform B_z field over a large volume of space surrounding the center of symmetry.

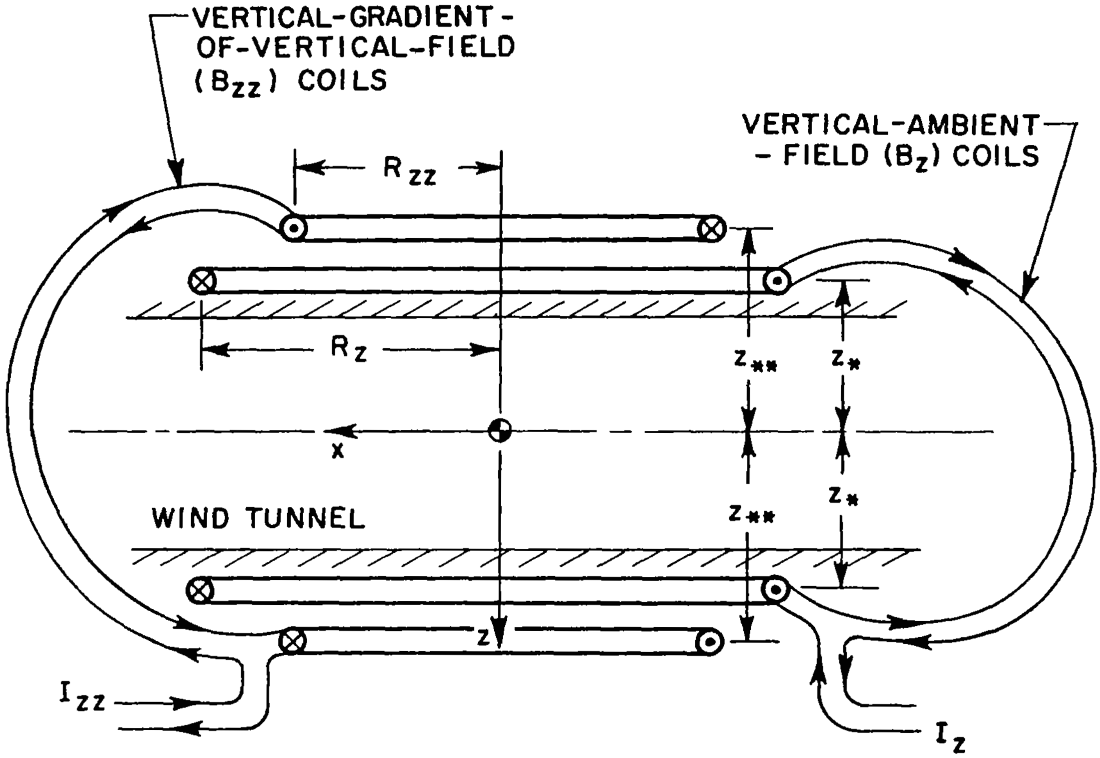


Figure 4. Arrangement of Circular Coils to Provide a Vertical Gradient of the Vertical Field and an Ambient Vertical Field Along the Vertical Axis.

(ii) Added to the Helmholtz pair is a pair of "gradient coils," of radius R_{zz}, coaxial with the z-axis, and arranged symmetrically above and below the x-y plane. These coils are spaced a distance $2z_{**}$ apart. Each coil has N_{zz_o} turns and both coils are in series electrically, and are connected such that the upper coil produces a negative B_z, and the lower coil produces a positive B_z. The net effect of these two coils is a magnetic field which is zero at the center of symmetry, and which has a gradient B_{zz} along the z-axis. If $z_{**} = \frac{\sqrt{3}}{2}R_{zz}$, this gradient is constant over an appreciable distance.

Thus, the Helmholtz coils produce a uniform ambient vertical field, and the gradient coils produce a uniform gradient of the vertical field. This arrangement is analyzed quantitatively below.

Analysis

1. Circular Coils

The vertical field B_z on the z-axis due to the coil arrangement shown in Figure 4 is:

$$B_z(0,0,z) = \frac{\mu_o}{2}\left\{\frac{N_{z_o} I_z}{R_z}\left[\left(1+\left(\frac{z_* - z}{R_z}\right)^2\right)^{-3/2} + \left(1+\left(\frac{z_*+z}{R_z}\right)^2\right)^{-3/2}\right]\right.$$

$$\left. + \frac{N_{zz_o} I_z}{R_{zz}}\left[\left(1+\left(\frac{z_{**} - z}{R_{zz}}\right)^2\right)^{-3/2} - \left(1+\left(\frac{z_{**}+z}{R_{zz}}\right)^2\right)^{-3/2}\right]\right\} \tag{6.1}$$

$$B_{zz}(0,0,z) = \frac{3}{2}\mu_o\frac{N_{z_o} I_z}{R_z}\left[\left(1+\left(\frac{z_*-z}{R_z}\right)^2\right)^{-5/2}\left(\frac{z_*-z}{R_z}\right) - \left(1+\left(\frac{z_*+z}{R_z}\right)^2\right)^{-5/2}\left(\frac{z_*+z}{R_z}\right)\right]$$

$$+\frac{3}{2}\mu_o\frac{N_{zz_o} I_z}{R_{zz}}\left[\left(1+\left(\frac{z_{**}-z}{R_{zz}}\right)^2\right)^{-5/2}\left(\frac{z_{**}-z}{R_{zz}}\right) + \left(1+\left(\frac{z_{**}+z}{R_{zz}}\right)^2\right)^{-5/2}\left(\frac{z_{**}+z}{R_{zz}}\right)\right] \tag{6.2}$$

where μ_o is the permeability of free space.

For the Helmholtz pair:

$$\text{if} \quad \frac{\partial^2 B_z}{\partial z^2} = 0 \text{ @ } z = 0; \; z_* = Rz \tag{6.3}$$

For the gradient coils,

$$\text{if} \quad \frac{\partial^3 B_z}{\partial z^3} = 0 \text{ @ } z = 0; \; z_{**} = \frac{\sqrt{3}}{2} R_z \tag{6.4}$$

2. Square Coils

If the circular coils are replaced by square coils of dimension $2R_z$ and $2R_{zz}$ on a side, the field equations are:

$$B_z = \frac{\sqrt{2}}{\pi}\mu_o \sum_{i,j=1}^{4} \left(\frac{N_o I}{R}\right)_i [(1+\tfrac{1}{2}u_j^2)^{-1/2}(1+u_j^2)^{-1}] \qquad (6.5)$$

$$(i=z,z,zz,zz)$$

$$\text{let } B(u_j) = [(1+\tfrac{1}{2}u_j^2)^{-1/2}(1+u_j^2)^{-1}] \qquad (6.6)$$

$$u_1 = \frac{z_* - z}{R_z};\ u_2 = \frac{z_* - z}{R_z};\ u_3 = \frac{z_{**} - z}{R_{zz}};\ u_4 = \frac{z_{**} + z}{R_{zz}} \qquad (6.7)$$

$$\frac{\partial B_z}{\partial z} = \sum_{i,j=1}^{4} \frac{\sqrt{2}}{\pi}\mu_o \left(\frac{N_o I}{R}\right)_i \frac{\partial(Bu_j)}{\partial z} \qquad (6.8)$$

$$\frac{\partial Bu_j}{\partial z} = -[2(1+\tfrac{1}{2}u_j^2)^{-1/2}(1+u_j^2)^{-2}+\tfrac{1}{2}(1+\tfrac{1}{2}u_j^{\,2})^{-3/2}(1+u_j^2)^{-1}]u_j\frac{\partial u_j}{\partial z} \qquad (6.9)$$

$$\text{For } \frac{\partial^2 B_z}{\partial z^2} = 0 \text{ @ } x,y,z = 0;\ \frac{z_x}{R_z} = 0.55 \qquad (6.10)$$

$$\text{and } \left(\frac{B_z}{N_z I_z/R_z}\right) = \frac{\mu_o}{\pi} \qquad (6.11)$$

where $N_z I_z$ = total ampturns in z-çoils

$$\text{and for } \frac{\partial^3 B_z}{\partial z^3} = 0 \text{ @ } x,y,z = 0;\ \frac{z_{**}}{R_{xx}} = 0.94 \qquad (6.12)$$

$$\text{and } \left(\frac{B_{zz}R_{zz}}{N_{zz}I_{zz}/R_{zz}}\right) = 0.81\frac{\mu_o}{\pi} \qquad (6.13)$$

where $N_{zz}I_{zz}$ = total ampturns in zz-coils.

Example:

As an example of the magnitudes involved, consider the following case, which is an extension of the example on page 17.

Square coils: $Z_{*} = 30''$ $R_z = 54.6''$

$Z_{**} = 42''$ $R_{zz} = 44.7''$

$B_z = 13.42$ kilogauss $(@x_3y, z=0)$

$B_{zz}(0,0,0) = 0.452$ k.gauss/in.

From 6.11.

$$B_z(0,0,0) = \frac{\mu_o}{\Pi} N_z I_z / R_z$$

$$N_z I_z = \frac{(12.42)\ (54.6)\ (\Pi)}{(39.4\text{in/m})(4\Pi\text{x}10^{-6})}$$

$$= 4.31\text{x}10^6 \text{ (total ampturns in Helmholtz coils.)}$$

And from Equation 6.13,

$$B_{zz}(0,0,0) = 0.81\frac{\mu_o}{\Pi}\ \frac{N_{zz}I_{zz}}{R_{zz}}$$

$$\text{i.e. } N_{zz}I_{zz} = \frac{B_{zz}(0,0,0)R_{zz}^{\ 2}}{0.81\mu_o/\Pi}$$

$$= \frac{(0.452)\ (54.6)^2}{(0.81\text{x}4\Pi\text{x}10^{-6}/\Pi)(39.4)}$$

$$= 10.5\text{x}10^6 \text{ (total ampturns in gradient coils.)}$$

7.0 COMBINED VERTICAL AND HORIZONTAL FORCES

In order to simulate store jettison from a diving climbing aircraft, it is necessary to provide a magnetic force component along the tunnel axis in addition to the vertical component, as defined by Equations 24, 26. The axial force component F_x can be provided by a set of four coils coaxial with the x-axis and spaced symmetrically about the center of symmetry of the z-coils. Thus, at the center of symmetry, for a saturated iron sphere,

$$\frac{F_x}{V_{mag}} = k_t m_{sat} \frac{B_x}{|B|} B_{xx} \tag{7.1}$$

$$\frac{F_z}{V_{mag}} = k_t m_{sat} \frac{B_z}{|B|} B_{zz} \tag{7.2}$$

but, rom 4.21 $B_{xx}(0,0,0) = K_{xx}I_{xx} - \frac{1}{2} K_{zz}I_{zz}$ (7.3)

$$B_{zz}(0,0,0) = K_{zz}I_{zz} - \frac{1}{2} K_{xx}I_{xx} \tag{7.4}$$

and $B_x(0,0,0) = K_xI_x$ (7.5)

$$B_z(0,0,0) = K_zI_z \qquad (K_x, K_z, K_{xx}, K_{zz} = \text{const}) \tag{7.6}$$

$$\frac{F_x(0,0,0)}{V_{mag}} = k_t m_{sat} \frac{K_xI_x}{((K_xI_x)^2 + (K_zI_z)^2)^{1/2}} (K_{xx}I_{xx} - \frac{1}{2} K_{zz}I_{zz}) \tag{7.7}$$

$$\frac{F_z(0,0,0)}{V_{mag}} = k_t m_{sat} \frac{K_zI_z}{((K_xI_x)^2 + (K_zI_z)^2)^{1/2}} (K_{zz}I_{zz} - \frac{1}{2} K_{xx}I_{xx}) \tag{7.8}$$

From Equations (5.6, 5.7), it is seen that the maximum combined magnetic force, F_{mag}, is obtained if the direction of the

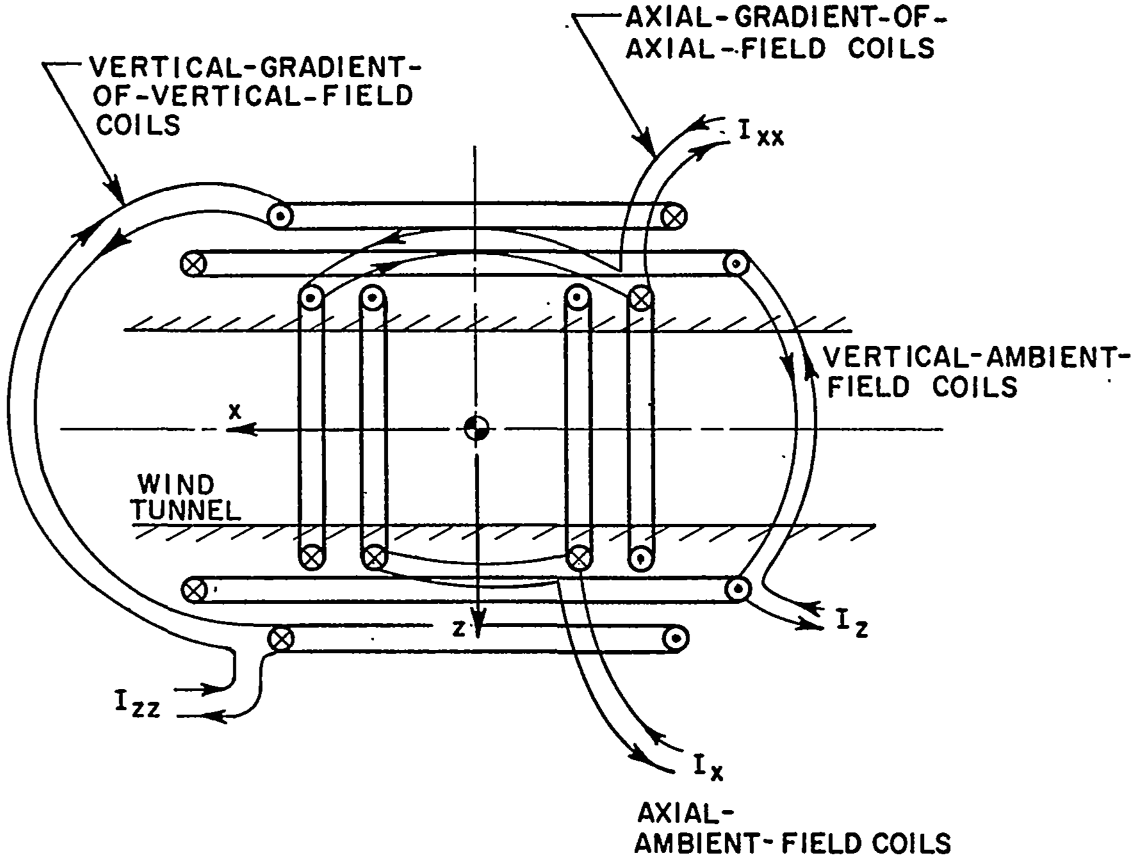


Figure 5. Arrangement of Coils to Provide Combined Axial and Vertical Forces.

x- currents is opposite in sign to the direction of the z-currents.

i.e.,

$$\frac{I_x}{|I_x|} = - \frac{I_z}{|I_z|} \quad ; \quad \frac{I_{xx}}{|I_{xx}|} = - \frac{I_{zz}}{|I_{zz}|} \qquad (7.9,10)$$

This situation is shown schematically in Figure 5, in which the x-currents have been chosen to be negative (i.e., the x-field and x-field gradient coil in the positive x half-space produce negative B_x along the x-axis).

In addition to enhancing the strength of the resultant force, it can be shown that the gradient component B_{yy} is reduced, as are the corresponding y-directed forces at locations outside the y=0 plane.

8.0 FORCE FIELD NONUNIFORMITIES

Consider the force field due to a uniform vertical gradient of the vertical field superimposed on a uniform ambient vertical field, acting on a saturated iron sphere.

Assume that B_{xy}, B_{xz}, B_{yz} are negligible.

Note that:

$$B_{xx} = - \frac{1}{2} B_{zz} = \text{const} \; ; \quad B_x = - \frac{1}{2} B_{zz} \cdot x \qquad (8.1,2)$$

$$B_{yy} = - \frac{1}{2} B_{zz} = \text{const} \; ; \quad B_y = - \frac{1}{2} B_{zz} \cdot y \qquad (8.3,4)$$

$$B_z = B_{zo} + B_{zz} \cdot z. \qquad (8.5)$$

Then

$$\frac{F_x}{V_{mag}} = k_t m_{sat} \frac{B_x}{|B|} \cdot B_{xx} \qquad (7.1)$$

or

$$\frac{F_x}{V_{mag}} = (k_t m_{sat} B_{zz}) \frac{B_{zz} \cdot x}{4B_{zo}} \left[\left(1 + \frac{B_{zz} \cdot z}{B_{zo}}\right)^2 + \frac{B_{zz}(x^2+y^2)^{1/2}}{2B_{zo}} \right]^{-1/2} \tag{8.6}$$

$$\frac{F_y}{V_{mag}} = (k_t m_{sat} B_{zz}) \frac{B_{zz} \cdot y}{4B_{zo}} \left[\left(1 + \frac{B_{zz} \cdot z}{B_{zo}}\right)^2 + \frac{B_{zz}(x^2+y^2)^{1/2}}{2B_{zo}} \right]^{-1/2} \tag{8.7}$$

$$\frac{F_z}{V_{mag}} = (k_t m_{sat} B_{zz}) \left[1 + \left(\frac{\frac{B_{zz}(x^2+y^2)^{1/2}}{B_{zo}}}{1 + \frac{B_{zz} \cdot z}{B_{zo}}}\right)^2\right] \tag{8.8}$$

The z-force is uniform along the z-axis. The x and y force components are approximately proportional to the product of the z-force and the x or y displacements. These forces thus appear to be repulsive, away from the z-axis, and increasing in strength with distance from the z-axis. Also, for a given displacement from the z-axis, the x and y forces vary approximately inversely with z displacement, and likewise, they vary approximately with the ambient field level, B_{z_o}. Thus, specifications of the desired degree of uniformity of the force field will be directly reflected in the required level of the ambient field.

TABLE - Example of Off-Axis Forces, for B_{z_o} = 12.42 kilogauss B_{zz} = 0.452 kilogauss/in. Normalized with Respect to the Force at the Center of Symmetry, F_{z_o}

x	y	z	F_x/F_{z_o} (%)	F_y/F_{z_o} (%)	$(1-F_z/F_{z_o})$ (%)
0	0	0	0%	0%	0%
12"	0	0	+10.65%	0%	+2.4%
12"	12"	-12"	+18.5%	+18.5%	+12.2%
12"	12"	0	+10.17%	+10.17%	+4.7%
12"	12"	+12"	+7.4%	+7.4%	+2.3%
12"	12"	+24"	+5.8%	+5.8%	+1.4%

9.0 STABILITY OF SATURATION MAGNETIZATION

The magnetic force on a saturated body is proportional to the saturation magnetization of the body. Thus, it is of interest to examine the accuracy with which this quantity can be estimated, and also the variation of this quantity under the conditions experienced in the wind tunnel environment.

The most important effect on m_{sat} is due to changes in temperature. If $m_{sat}(T_{ref})$ is the saturation magnetization measured at a reference temperature T_{ref}, the saturation magnetization $m_{sat}(T)$ for iron at another temperature T is related to the Curie temperature T_c approximately as follows: (Reference 7)

$$\frac{m_{sat}(T)}{m_{sat}(T_{ref})} = \frac{1 - k\left(\frac{T}{T_c}\right)^{3/2}}{1 - k\left(\frac{T_{ref}}{T_c}\right)^{3/2}} \qquad (9.1)$$

where T, T_{ref}, T_c = absolute temperature
k = 0.11 (empirical, for iron)
T_c = 1870°R for iron

The temperature coefficient of magnetization can be written

$$\frac{\partial(m_{sat})}{\partial T} = -m_{sat}(T_{ref})\,\frac{1}{T_c}\,\frac{3}{2}\left[\frac{k\left(\frac{T}{T_c}\right)^{1/2}}{1 - k\left(\frac{T_{ref}}{T_c}\right)^{3/2}}\right] \qquad (9.2)$$

e.g. $\frac{\partial m_{sat}}{m_{sat}} / T_{\partial} = 48\text{ppm}/°\text{F}$ @ $T = T_{ref} = 530°R\,(70°F.)$ for iron.

10.0 FIELD AND FORCE ANALYSIS OF ARBITRARY AIR-CORE COIL CONFIGURATIONS

In the cases described above, each magnet coil has been represented by a concentrated current element. This leads to great simplification in the initial design and allows promising configurations to be readily conceived and evaluated approximately. However, in order to evaluate the force field due to a magnet system composed of coils having large winding buildup, over a large region of space, greater detail is required in the analysis. In this case, it is most convenient to use a digital computer, since the number of computation steps in the evaluation procedure becomes very large.

In the analysis that has been developed for this application, the magnet coils have been approximated by a series of straight-line current elements. This choice was made since it appeared at the outset that the most probable configuration, to be compatible with a typical wind tunnel test section, would involve square or rectangular coils.

The total magnetic field and field gradient components at a point in space due to an array of current carrying elements surrounded by a medium of uniform magnetic susceptibility can be evaluated by linear superposition of the effects of each individual current element.

The magnetic force field on a ferromagnetic sphere is calculated from the total field and field gradient components using Equations (4.16-18).

The relations used in the computation of the magnetic field, field gradient components, and the magnetic forces, are outlined in Appendix A.

A computer program has been developed to provide a tabulation of the magnetic field components, component derivatives, and magnetic force components for a set of field positions. This program is called "TABLE" and is described in detail in Appendix B.

A computer program has been developed as an extension of TABLE which provides a qualitative graphical display of the distribution of the magnetic force field. This program is called "PLOT" and is described in detail in Appendix C.

11.0 PRACTICAL COIL CONFIGURATIONS

A family of practical coil configurations has been developed from the basic arrangement of Figure 5. An example showing the basic elements and the proportions that are expected to be typical of a magnetic artificial gravity facility is outlined in Figure 6.

In general, the wind tunnel test section structure will most probably be of closed jet design. In the case of a porous-wall transonic test section, designed to operate over a range of Mach Number, the clearance between the inner and outer walls must be larger than required for purely structural reasons, since plenum volume must be provided, and provision must be made for controlled mass removal from the plenum in order to control the test section Mach Number.

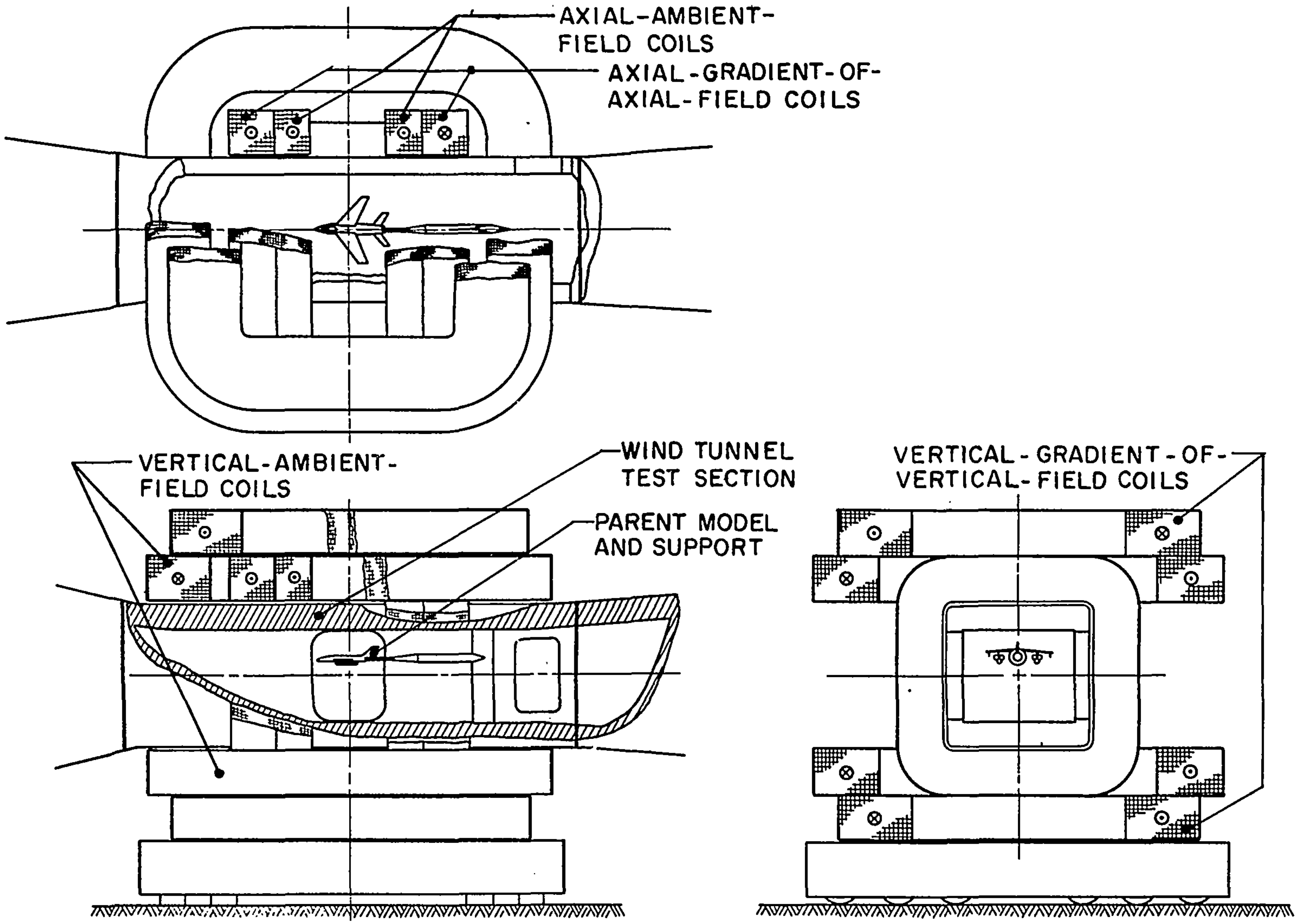


Figure 6. Practical Arrangement for a Working Two-Component Magnetic Artificial Gravity Facility.

The acquisition of store jettison data will most probably be by photographic means. This requires at least one large viewing window in the test section wall.

Access to the test section can be provided a short distance from the center of the test section.

The parent model support structure is supported itself by the test section, and means must be provided for remote control of the release of the store models, as in conventional facilities.

Operations may call for sharing of the tunnel circuit with other types of facilities, for example conventional force and moment balance. This may dictate that the magnetic system be removable without major disassembly. This would in turn require that the test section be removable along with the magnet system, as indicated in Figure 6.

11.1 ANALYSIS AND OPTIMIZATION OF COIL SYSTEMS

In order to analyze the coil geometry shown in Figure 6 in a systematic way, the configuration is defined in terms of parameters and constraints. Such parameters and constraints can be categorized as follows:

(a) Geometric parameters

These apply to a particular configuration, and are dimensionless "shape factors" or length ratios sufficient to define the shape of the configuration.

(b) Geometric constraints

These apply to a particular configuration, and define limits to the geometric parameters imposed by mechanical interference.

(c) Performance parameters

These apply to a particular configuration, and define the relevant performance characteristics of the configuration in terms of the geometric parameters, material parameters, and a characteristic linear dimension of the configuration.

(d) Material parameters

These are related to the material and operating temperature chosen for the coil conductor, and may be contained in the performance parameters.

(3) Cost-related parameters

Approximate cost factors can be estimated from the geometric, performance, and material parameters, for some parts of the system.

11.2 GEOMETRIC PARAMETERS AND CONSTRAINTS

The coil geometry outlined in Figure 6 is shown in greater detail in Figure 7 in which all relevant dimensions are identified by symbols. The geometric parameters relating these dimensions are defined below.

Geometric Parameters: (Square coils)

(a) Angles to winding centroids (in vertical plane through x-axis)

$$\phi_x, \phi_{xx}, \phi_z, \phi_{zz}$$

(b) Buildup factors $\alpha_1 = \frac{B_1}{R_o}$; $\alpha_2 = \frac{B_2}{R_o}$ (11.1.2)

$$\beta_x = \frac{w_x}{B_1} \; ; \; \beta_{xx} = \frac{w_{xx}}{B_1} \qquad (11.3,4)$$

$$\beta_x = \frac{w_z}{B_1} \; ; \; \beta_{zz} = \frac{w_{zz}}{B_2} \qquad (11.5,6)$$

(c) Internal radius ratios

$$\left(\frac{r_{x_1}}{R_o}\right) \; ; \; \left(\frac{r_{z_1}}{R_o}\right) \; ; \; \left(\frac{r_{zz_1}}{R_o}\right)$$

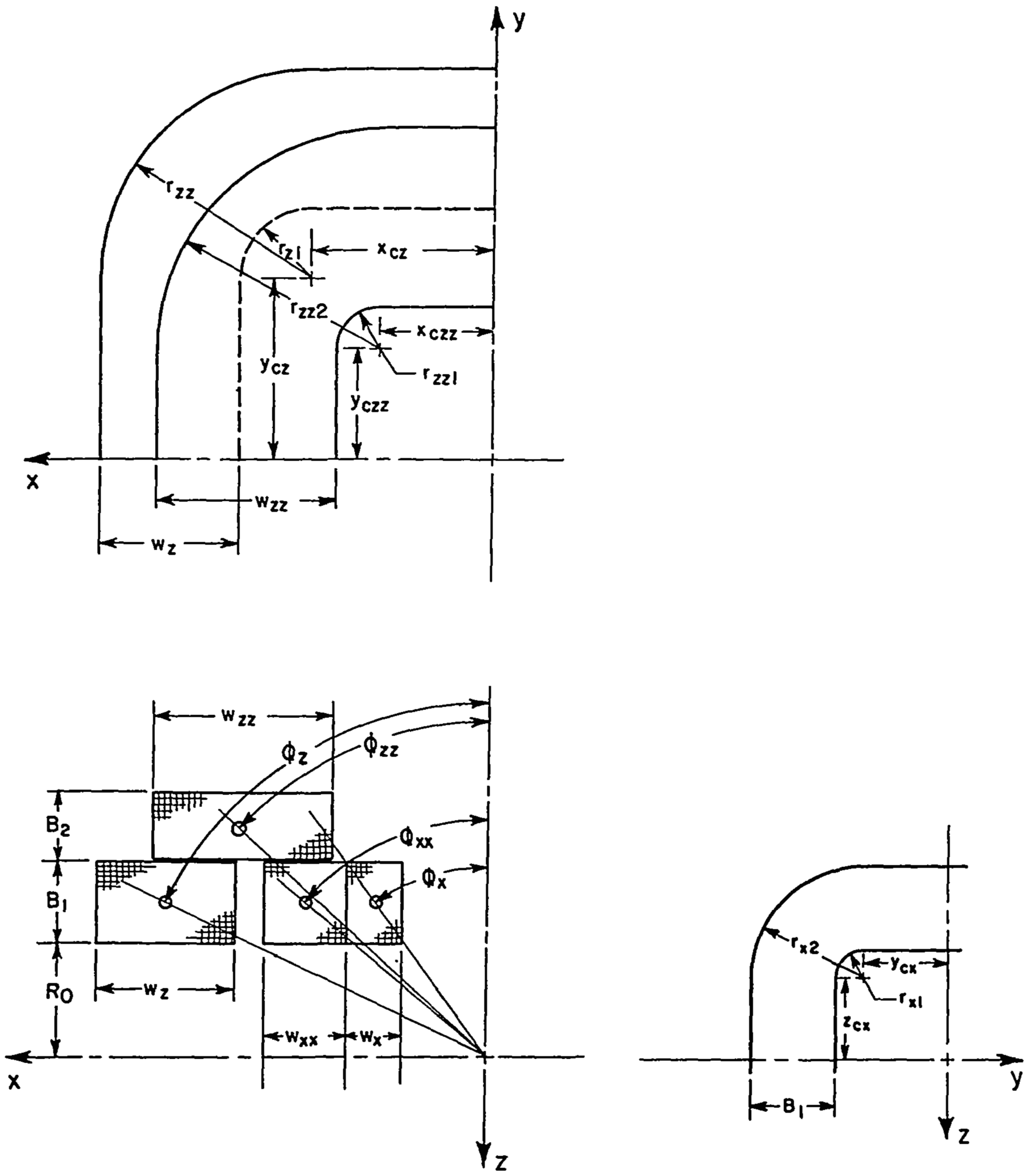


Figure 7. Generalized Dimensions of Practical Air Core Coil Configuration.

(d) Mean-turn length parameters

Define $\bar{\ell}_{jo}$ as the length of the mean turn (located at the winding centroid) of a single j-coil.

Then,

$$\frac{\bar{\ell}_{x_o}}{R_o} = \frac{\bar{\ell}_{xx_o}}{R_o} = 8\left[1+ \frac{\Pi}{8}\alpha_1 - \left(1 - \frac{\Pi}{4}\right)\left(\frac{r_{x_1}}{R_o}\right)\right] \qquad (11.7)$$

$$\frac{\bar{\ell}_{z_o}}{R_o} = 8\left[\left(1+\frac{\alpha_1}{2}\right)\tan\phi_z - \left(1- \frac{\Pi}{4}\right)\left(\frac{r_{z_1}}{R_o} + \frac{\alpha_1\beta_z}{2}\right)\right] \qquad (11.8)$$

$$\frac{\bar{\ell}_{zz_o}}{R_o} = 8\left[\left(1+\alpha_1+\frac{\alpha_2}{2}\right)\tan\phi_{zz} - \left(1- \frac{\Pi}{4}\right)\left(\frac{r_{zz_1}}{R_o} + \frac{\alpha_2\beta_{zz}}{2}\right)\right] \qquad (11.9)$$

(e) Winding area parameters

Define A_{jo} as the area of a single j-coil.

Then

$$\frac{A_{x_o}}{R_o^{\,2}} = \alpha_1^{\,2}\beta_x \qquad (11.10)$$

$$\frac{A_{xx_o}}{R_o^{\,2}} = \alpha_1^{\,2}\beta_{xx} \qquad (11.11)$$

$$\frac{A_{z_o}}{R_o^{\,2}} = \alpha_1^{\,2}\beta_z \qquad (11.12)$$

$$\frac{A_{zz_o}}{R_o^{\,2}} = \alpha_2^{\,2}\beta_{zz} \qquad (11.13)$$

Geometric Constraints (Square Coils)

(a) $$\frac{\alpha_1}{2(1+\frac{\alpha_1}{2})}(\beta_x + \beta_{xx}) \leq \tan\phi_{xx} - \tan\phi_x$$

(Interference of x-coil and xx coil) (11.13a)

(b) $$\frac{\alpha_1(\beta_z+\beta_{zz}) + \sqrt{2}\ (\frac{r_{z_1}}{R_O})}{2(1+\frac{\alpha_1}{2})} < \tan\phi_z - \tan\phi_{zz}$$

(Interference of z-coil and xx-coil) (11.13b)

11.3 EFFECTS OF NON-ZERO WINDING CROSS-SECTIONAL AREA

The magnetic field distribution from a system of coils of non-zero cross sectional area is different from that due to a similar system of concentrated current elements located at the winding cross section centroids of the former system. The analysis of the fields due to a system of coils of non-zero cross-sectional area may be carried out by dividing each coil cross-section into zones and representing the current flowing through each zone by a concentrated current element located at the centroid of the zone. With the coil geometry specified in terms of the parameters shown in Figure 7, it is most convenient to determine the end-points of the current elements using a computer program, since the number of elements can become quite large. A program to accomplish this is outlined in Appendix D. In this program, the rounded coil corners are approximated in the field computations by forty-five degree bevels.

11.4 OPTIMIZATION OF ϕ-PARAMETERS FOR UNIFORM FORCE FIELD

A force-field-uniformity maximization procedure based upon the analysis outlined above proceeds as follows: First, a set of buildup factors (α's and β's) are selected compatible with a set of initial values of the ϕ's chosen to provide approximately uniform fields and gradients (i.e., $\phi_x = 29°$, $\phi_{xx} = 43°$, $\phi_z = 61°$, $\phi_{zz} = 47°$) based upon single-current-element square loops. The field property B_x is computed at the center of symmetry of the coil system, and at two points on the x-axis ($+\Delta x$) and ($-\Delta x$) from the center of symmetry, due to currents flowing only in the x-coils. The angle, ϕ_x, is adjusted as follows:

$$\phi_x(\alpha_1,\beta_x)_{\text{optimum}} \rightarrow B_x(+\Delta x,0,0,I_x)+B_x(-\Delta x,0,0,I_x)=2B_x(0,0,0,I_x) \tag{11.14}$$

Similarly, the other angles ϕ_{xx}, ϕ_z, ϕ_{zz} are optimized as follows:

$$\phi_{xx}(\alpha_1,\beta_{xx})_{\text{opt}} \rightarrow B_{xx}(+\Delta x,0,0,I_{xx})+B_{xx}(-\Delta x,0,0,I_{xx})=2B_{xx}(0,0,0,I_{xx}) \tag{11.15}$$

$$\phi_z(\alpha_1,\beta_z)_{\text{opt}} \rightarrow B_z(0,0,+\Delta z,I_z)+B_z(0,0,-\Delta z,I_z)=2B_z(0,0,0,I_z) \tag{11.16}$$

$$\phi_{zz}(\alpha_2,\beta_{zz})_{\text{opt}} \rightarrow B_{zz}(0,0,+\Delta z,I_{zz})+B_{zz}(0,0,-\Delta z,I_{zz})=2B_{zz}(0,0,0,I_{zz}) \tag{11.17}$$

So far, this adjustment procedure has been performed by trial and error (not directly by the computer program); however, it is evidently a simple matter to implement this step automatically by iteration. The variations in the values of the optimum ϕ's are typically small, of the order of two or three degrees, to achieve the maximum force field uniformity.

11.5 PERFORMANCE PARAMETERS OF MAGNET SYSTEM

The performance of the magnet system can be defined in terms of parameters relating the magnetic and electrical properties of the system. Of interest are the following items:

(a) Magnetic performance parameters (Q_j)

These parameters relate the magnetic field properties at the center of symmetry to the ampere-turns in each coil subsystem and are computed from the geometric parameters, as defined below:

define:

$$Q_x = \left[\frac{B_x(0,0,0)}{(N_x I_x / R_o)}\right] \quad (11.18) \qquad Q_{xx} = \left[\frac{B_{xx}(0,0,0) R_o}{(N_{xx} I_{xx} / R_o)}\right] \quad (11.19)$$

$$Q_z = \left[\frac{B_z(0,0,0)}{N_z I_z / R_o}\right] \quad (11.20) \qquad Q_{zz} = \left[\frac{B_{zz}(0,0,0) R_o}{(N_{zz} I_{zz} / R_o)}\right] \quad (11.21)$$

(b) Electrical performance parameters.

These parameters relate the electrical properties of the coil system to the geometric parameters, and the material parameters. Included in this category are the electrical resistance and self-inductance of each coil subsystem, and the mutual inductance of coil subsystems which are magnetically coupled.

(i) Resistance parameters. (S_j)

$$S_x = \frac{R_o R_x}{N_x^2} \quad (11.22) \qquad S_{xx} = \frac{R_o R_{xx}}{N_{xx}^2} \quad (11.23)$$

$$S_z = \frac{R_o R_z}{N_z^2} \quad (11.24) \qquad S_{zz} = \frac{R_o R_{zz}}{N_{zz}^2} \quad (11.25)$$

where R_x, R_{xx}' etc. are the resistances of the x, xx, etc. coil subsystem respectively, as described below.

(ii) Self-inductance parameters. (T_{xi})

$$T_x = \frac{L_x}{R_o N_x^2} \quad (11,26) \qquad T_{xx} = \frac{L_{xx}}{R_o N_{xx}^2} \quad (11.27)$$

$$T_z = \frac{L_z}{R_o N_z^2} \quad (11,28) \qquad T_{zz} = \frac{L_{zz}}{R_o N_{zz}^2} \quad (11.29)$$

where L_x, L_{xx}, etc. are the self-inductances of the x, xx, etc. coil subsystems respectively, as described below.

(iii) Mutual-inductance parameter $(W_{xx/zz})$

$$W_{xx/zz} = \frac{M_{xx/zz}}{R_o N_{xx} N_{zz}} \quad (11.30)$$

where $M_{xx/zz}$ is the mutual inductance between the xx coil subsystem and the zz coil subsystem, as described below. Due to the summetry of this particular coil configuration, the xx and zz subsystems are the only pair with non-zero net magnetic coupling.

Magnetic Performance Parameters

(i) Approximate Values

These may be estimated from the mean-turn geometry, with acceptable accuracy for the purposes of preliminary analysis, from Equations 6.11,13, as follows:

$$Q_x \simeq \frac{B_x(0,0,0)}{N_x I_x / R_o} = \frac{\mu_o}{\Pi} \left[\frac{1}{1+\frac{\alpha_1}{2}}\right] \quad (11.31)$$

$$Q_{xx} \simeq \frac{B_{xx}(0,0,0) R_o}{N_{xx} I_{xx} / R_o} = \frac{0.81\mu_o}{\Pi} \left[\frac{1}{1+\frac{\alpha_1}{2}}\right] \quad (11.32)$$

$$Q_z \simeq \frac{B_z(0,0,0)}{N_z I_z/R_o} = \frac{0.55\mu_o}{\Pi}\left[\frac{1}{1+\frac{\alpha_1}{2}}\right] \qquad (11.33)$$

$$Q_{zz} \simeq \frac{B_{zz}(0,0,0)R_o}{N_{zz} I_{zz}/R_o} = \frac{0.81\mu_o}{\Pi}\left[\frac{1}{1+\alpha_1+\frac{\alpha_2}{2}}\right] \qquad (11.34)$$

(ii) Accurate Values

These may be calculated using the field analysis computer program described in detail in Appendix B, in conjunction with the current element location program detailed in Appendix D.

Resistance Parameters

The resistance parameter S_{jo} of a single coil "j_o" of constant current density, of the configuration defined in Figure 7, can be estimated as follows:

$$S_{jo} = \frac{R_{jo} R_o}{n_{jo}^2} = \frac{\rho_j}{(\overline{F}_p)_{jo}} \left(\frac{R_o^2}{\overline{A}_{jo}}\right) \left(\frac{\overline{\ell}_{jo}}{R_o}\right) \qquad (11.35)$$

where R_{jo} = resistance of jo coil

$\overline{\rho}_j$ = average resistivity of j-coil conductor material

$\overline{\ell}_{jo}$ = length of mean turn of single coil (or controid filament)

$\overline{A}_{jo}$ = total cross sectional area of single j-coil

$(\overline{F}_p)_{jo}$ = average packing area factor of conductor (ratio of conductor cross section to total winding cross section)

n_{jo} = number of turns in single j coil

The resistivity ρ_j depends upon the conductor material and the operating temperature, the packing factor F_p depends upon the construction used, and the number of turns depends upon the desired impedance level. The remaining factors are seen to be the reciprocal of the winding area parameter, and the mean length parameter. (See Eqs. 11.7-13).

For the particular configuration, $N_j = 2n_{jo}$, and the total resistance parameters are:

$$S_j = 2S_{jo}$$

$$S_j = \frac{R_o R_j}{N_j^{\,2}} = \frac{1}{2}\,\frac{R_o R_j}{n_{jo}^{\,2}} \qquad (11.36)$$

i.e.

$$S_x = \frac{R_o R_x}{N_x^{\,2}} = \frac{1}{2}\,\frac{\rho_x}{(F_p)_x}\left(\frac{\bar{\ell}_{xo}}{R_o}\right)\left(\frac{R_o^{\,2}}{A_{xo}}\right) \qquad (11.37)$$

$$S_{xx} = \frac{R_o R_{xx}}{N_{xx}^{\,2}} = \frac{1}{2}\,\frac{\rho_{xx}}{(F_p)_{xx}}\left(\frac{\bar{\ell}_{xxo}}{R_o}\right)\left(\frac{R_o^{\,2}}{A_{xxo}}\right) \qquad)11.38)$$

$$S_z = \frac{R_o R_z}{N_z^{\,2}} = \frac{1}{2}\,\frac{\rho_z}{(F_p)_{zz}}\left(\frac{\bar{\ell}_z}{R_o}\right)\left(\frac{R_o^{\,2}}{A_z}\right) \qquad (11.39)$$

$$S_{zz} = \frac{R_o R_{zz}}{N_{zz}^{\,2}} = \frac{1}{2}\,\frac{\rho_{zz}}{(F_p)_{zz}}\left(\frac{\bar{\ell}_{zz}}{R_o}\right)\left(\frac{R_o^{\,2}}{A_{zz}}\right) \qquad (11.40)$$

Self-Inductance Parameters

The self-inductance of a symmetric pair of coils comprising one of the coil subsystems is found from the self-inductance of an isolated coil and the mutual inductance between the two coils.

(i) Self-inductance of individual coils. (Reference 8)

The self-inductance L_{jo} of an isolated square coil is given by the formula:

$$\frac{L_{jo}}{R_o n_j^{\,2}} = (1.475\times10^{-2})\left(\frac{s_j}{R_o}\right)\left[1 + \frac{3.023}{\gamma_j} + 7.30\ln\gamma_j\right] \quad (11.41)$$

(microhenries/inch turn^2)

where: s_j = outside length of one side of j-coil (inches)

$$\gamma_j = \frac{s_j}{t_j+V_j}$$

t_j = radial thickness of j-coil

V_j = axial thickness of j-coil

n_j = number of turns in j-coil

For the particular coil configuration:

$$\frac{s_x}{R_o} = \frac{s_{xx}}{R_o} = 2(1+\alpha_1) \quad (11.42)$$

$$\frac{s_z}{R} = 2\left[(1+\frac{\alpha_1}{2})\tan\phi_z + \frac{\alpha_1\beta_z}{2}\right] \quad (11.43)$$

$$\frac{s_z}{R_o} = 2\left[(1+\alpha_1+\frac{\alpha_2}{2})\tan\phi_{zz} + \frac{\alpha_2\beta_{zz}}{2}\right] \quad (11.44)$$

$$\gamma_x = \left(\frac{s_x}{R_o}\right)\left(\frac{1}{\alpha_1(1+\beta_x)}\right) \quad (11.45)$$

$$\gamma_{xx} = \left(\frac{s_{xx}}{R_o}\right)\left(\frac{1}{\alpha_1(1+\beta_{xx})}\right) \quad (11.46)$$

$$\gamma_z = \left(\frac{s_z}{R_o}\right)\left(\frac{1}{\alpha_1(1+\beta_z)}\right) \quad (11.47)$$

$$\gamma_{zz} = \left(\frac{s_{zz}}{R_o}\right)\left(\frac{1}{\alpha_2(1+\beta_{zz})}\right) \quad (11.48)$$

(ii) Mutual inductance between two identical parallel, square, coaxial coils. (Reference 8)

The mutual inductance $M_{j/j}$ between the two identical j-coils is given by the formula:

$$\frac{M_{j/j}}{R_o n_j} = (10.77\times10^{-2})\left(\frac{\ell_j}{R_o}\right)\left[\ell n\frac{\eta_j(1+\eta_j)}{f_j(1+\chi_j)} + 0.1886(\delta_j+\chi_j-\eta_j)\right] \tag{11.49}$$

(microhenries/inch turn^2)

where: ℓ_j = mean length of one side of j-coil

n_j = number of turns in j-coil

δ_j = ratio of mean axial spacing of coils to mean length "ℓ_j"

$\eta_j = \sqrt{1+\delta_j}$

$\chi_j = \sqrt{2+\delta_j}$

For the particular coil configuration:

$$\frac{\ell_x}{R_o} = \frac{\ell_{xx}}{R_o} = 2\left(1+\frac{\alpha_1}{2}\right) \tag{11.50}$$

$$\frac{\ell_z}{R_o} = 2\left(1+\frac{\alpha_1}{2}\right)\tan\phi_z \tag{11.51}$$

$$\frac{\ell_{zz}}{R_o} = 2\left(1+\alpha_1+\frac{\alpha_2}{2}\right)\tan\phi_{zz} \tag{11.52}$$

$\delta_x = \tan\phi_x$; $\delta_{xx} = \tan\phi_{xx}$

$\delta_z = \cot\phi_z$; $\delta_{zz} = \cot\phi_{zz}$

(iii) Total self-inductance parameters

The total self-inductance L_j of each coil pair is given by:

$$L_j = 2[L_{jo} \pm M_{j/j}] \tag{11.53}$$

for the particular coil configuration, $n_j = \frac{N_j}{2}$

$$T_x = \frac{L_x}{R_o N_x^2} = \frac{1}{2}[(\frac{L_{xo}}{R_o(N_x/2)^2}) + (\frac{M_{x/x}}{R_o(N_x/2)^2})] \tag{11.54}$$

$$T_{xx} = \frac{L_{xx}}{R_o N_{xx}^2} = \frac{1}{2}[(\frac{L_{xxo}}{R_o(N_{xx}/2)^2}) - (\frac{M_{xx/xx}}{R_o(N_{xx}/2)^2})] \tag{11.55}$$

$$T_z = \frac{L_z}{R_o N_z^2} = \frac{1}{2}[(\frac{L_{zo}}{R_o(N_z/2)^2}) + (\frac{M_{z/z}}{R_o(N_z/2)^2})] \tag{11.56}$$

$$T_{zz} = \frac{L_{zz}}{R_o N_{zz}^2} = \frac{1}{2}[(\frac{L_{zzo}}{R_o(N_{zz}/2)^2}) - (\frac{M_{zz/zz}}{R_o(N_{zz}/2)^2})] \tag{11.57}$$

Mutual Inductance Parameter (Gradient Coils) (Ref. 8)

The axial gradient coil pair is magnetically coupled with the vertical gradient coil pair due to the way in which the terminals of these coils are necessarily connected. As a result, changes in current through one pair of coils will result in net induced voltages in the other pair. This effect must be considered since it influences the specifications of the power supplies required to regulate the currents in the two systems.

The particular gradient coil configuration can be approximated by the arrangement shown in Figure 8 below.

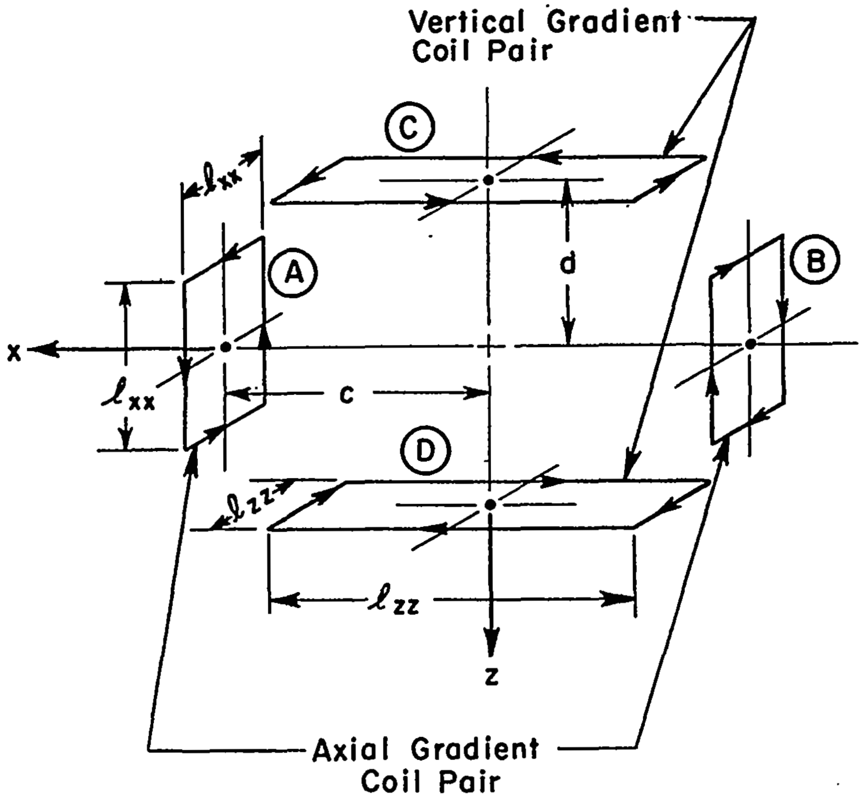


Figure 8. Approximation of the Axial Gradient and the Vertical Gradient Coil Pairs for Estimation of the Mutual Inductance Between Each Pair of Coils.

From the symmetry of the figure, further simplification of the analysis can be made by observing that the mutual inductance, M_{AC}, between coils A and C is equal to the mutual inductances M_{AD}, M_{BC}, M_{BD}. Therefore, it is necessary only to find an expression for M_{AC}, as shown in Figure 9.

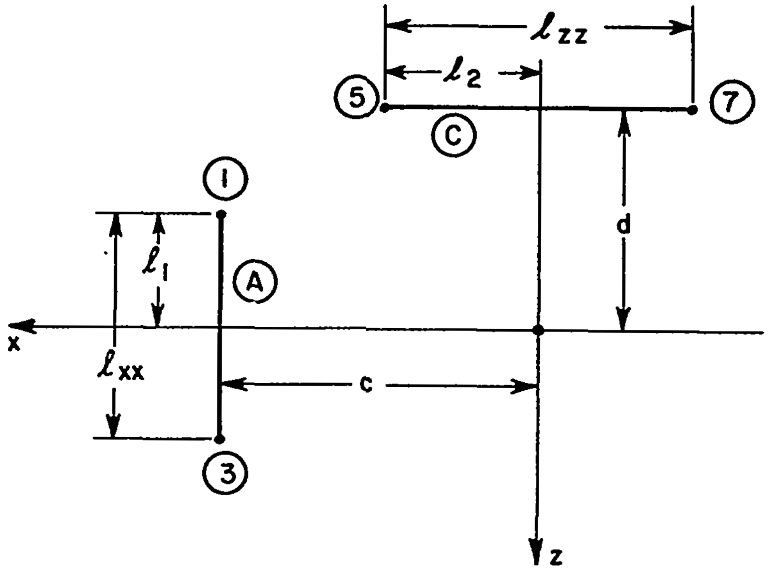


Figure 9. Coil Pair Geometry for Calculation of Mutual Inductance.

$$M_{AC} = M_{15} + M_{37} - M_{17} - M_{35} \tag{11.58}$$

$$M=0.00508 n_1 n_2 \ell_1 [2\ell n A+(1+\frac{\ell_2}{1})\ell n B+C-E] \tag{11.59}$$
(Ref. 8)

where:

$$A = (\frac{\ell_1}{D})[(1+\frac{\ell_2}{\ell_1})+\sqrt{(1+\frac{\ell_2}{\ell_1})^2+(\frac{D}{\ell_1})^2}] \tag{11.60}$$

$$B = \frac{(1+\frac{\ell_2}{\ell_1})+\sqrt{(1+\frac{\ell_2}{\ell_1})^2+(\frac{D}{\ell_1})^2}}{-(1-\frac{\ell_2}{\ell_1})+\sqrt{(1-\frac{\ell_2}{\ell_1})^2+(\frac{D}{\ell_1})^2}} = \frac{(1+\frac{\ell_2}{\ell_1})+E}{-(1-\frac{\ell_2}{\ell_1})+C} \tag{11.61}$$

$$C = \sqrt{\left(1-\frac{\ell_2}{\ell_1}\right)^2 + \left(\frac{D}{\ell_1}\right)^2} \quad ; \qquad E = \sqrt{\left(1+\frac{\ell_2}{\ell_1}\right)^2 + \left(\frac{D}{\ell_1}\right)^2} \qquad (11.62)$$

$$M = \sum_{j=1}^{4} a_j M_j \left(\ell_1, \frac{\ell_1}{D_j}, \frac{\ell_2}{\ell_1}\right) \qquad (11.63)$$

where:

$a_1 = +1$, $\quad D_1 = \sqrt{(d-\ell_1)^2 + (c-\ell_2)^2}$

$a_2 = -1$, $\quad D_2 = \sqrt{(d-\ell_1)^2 + (c+\ell_2)^2}$

$a_3 = -1$, $\quad D_3 = \sqrt{(d+\ell_1)^2 + (c-\ell_2)^2}$

$a_4 = +1$, $\quad D_4 = \sqrt{(d+\ell_1)^2 + (c+\ell_2)^2}$

$$\ell_1 = R_o\left(1+\frac{\alpha_1}{2}\right)$$

$$\ell_2 = R_o\left(1+\alpha_1+\frac{\alpha_2}{2}\right)\tan\phi_{zz}$$

$$c = R_o\left(1+\frac{\alpha_1}{2}\right)\tan\phi_{xx}$$

$$d = R_o\left(1+\alpha_1+\frac{\alpha_2}{2}\right)$$

$$W_{xx/zz} = \left(\frac{M_{xx/zz}}{R_o N_{xx} N_{zz}}\right) = 0.00508\left(1+\frac{\alpha_1}{2}\right)\sum_{j=1}^{4}\left(2\ln A_j + \left(1+\frac{\ell_2}{\ell_1}\right)\ln B_j + \rho_1 - E_j\right) \qquad (11.64)$$

(microhenries/inch turn^2)

Current Density

The current density is of interest, particularly with superconductors.

(i) The overall current density J_{jo} in each winding core is related to the performance parameters as follows:

$$\left(\frac{N_j I_j}{R_o}\right) = J_{jo}\left(2R_o\left(\frac{A_{jo}}{R_o^2}\right)\right) \qquad (11.65)$$

$$= \frac{B_j}{Q_j} \qquad (11.66)$$

i.e. $$J_{jo} = \frac{B_j}{R_o}\left[\frac{1}{2Q_j\left(\frac{A_{jo}}{R_o^2}\right)}\right] \qquad (11.67)$$

where $B_j = B_x$; $R_o B_{xx}$; B_z; $R_o B_{zz}$

(ii) The current density J_c in the conductor itself is related to the overall current density J_{jo} by the packing factor $(F_p)_j$:

$$J_c = \frac{J_{jo}}{(F_p)_j} \qquad (11.68)$$

Peak Magnetic Field Strength Inside Coil Conductor (Ref.9)

In the case of superconducting coil material, the current density is constrained by the properties of the particular material and the local magnetic field strength. The properties of typical superconducting materials are shown in Figure 10 below.

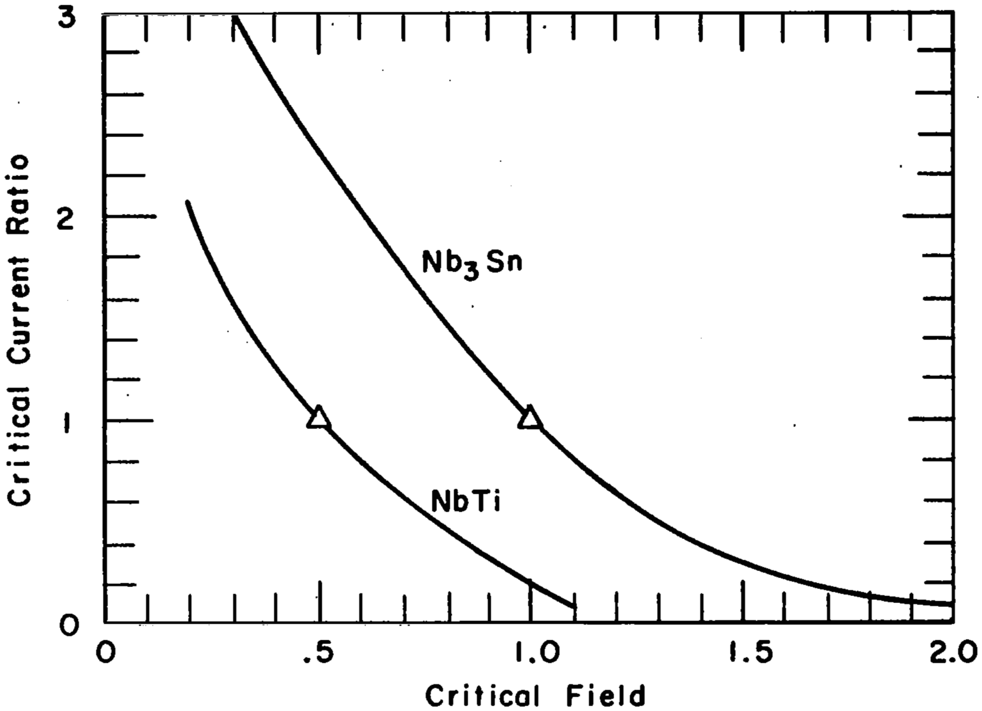


Figure 10. Critical Properties of Typical Superconducting Coil Material.

For discussion purposes, the properties of a typical superconducting material can be approximated by the relation:

$$J<J_{crit} = J_{ref} + \frac{dJ_{crit}}{dB_{crit}} (B_{ref}-B) \qquad (11.69)$$

If superconducting material is used, it is necessary to find the maximum value of B within the coil winding $(B_{max})_{cond.}$ and the corresponding current density $J_{(B=B_{max})_{cond.}}$, in order to assume that the critical current density is not exceeded. The constraint can be stated alternately as follows:

$$\frac{J_{(B=B_{max})\,cond}}{J_{crit}} = \frac{J_{(B=B_{max})\,cond}}{J_{ref}+\frac{dJ_{crit}}{dB_{crit}}(B_{max})_{cond}} < 1 \quad (11.70)$$

For the coil geometry under consideration, it has been found that the location of the maximum field point is in the positive-z, negative-x quadrant of the system, in the y = 0 plane. This condition is shown in Figure 11.

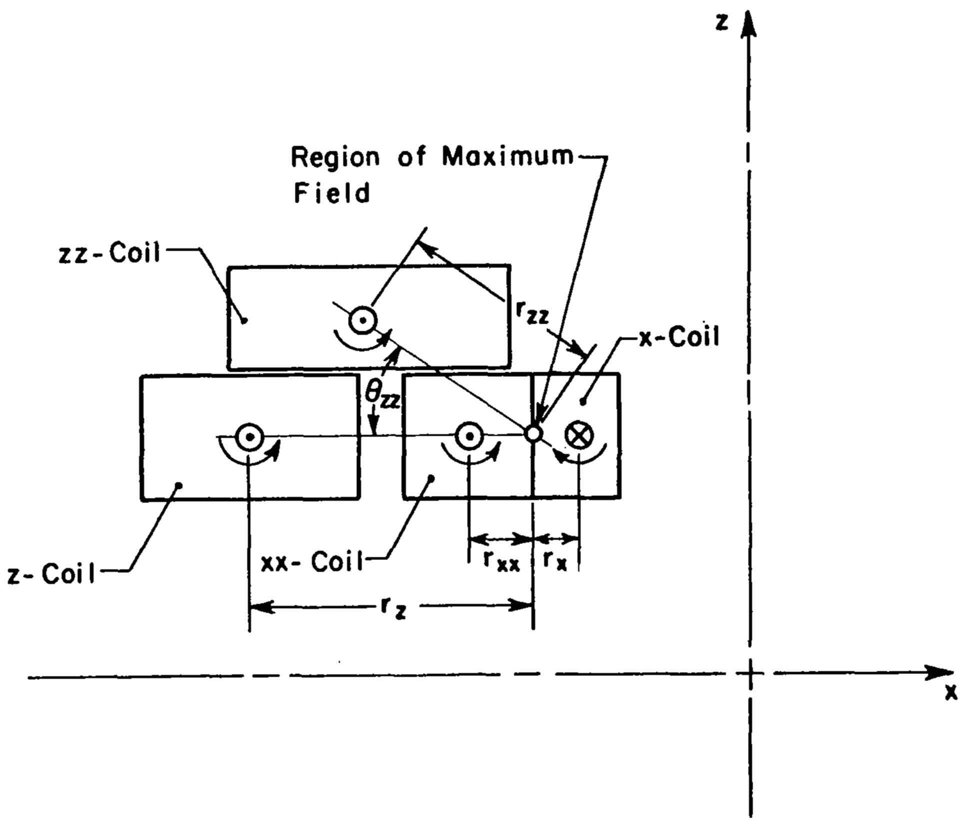


Figure 11. Region of Maximum Field Within Coil Conductor.

This field strength can be estimated with satisfactory accuracy (5%) by assuming that the windings are equivalent to infinite filaments located at the winding centroids, and carrying the corresponding currents. The effect of other quadrants is ignored.

The net magnetic field $(B_{max})_{cond.}$ at the point of interest is:

$$(B_{max})_{cond} = \frac{\mu_o\sqrt{a^2+b^2+2ab\cos\Theta_{zz}}}{2\Pi} \tag{11.71}$$

where

$$a = \left(\frac{|J_x|\alpha_1{}^2\beta_x R_o{}^2}{r_x} + \frac{|J_{xx}|\alpha_1{}^2\beta_{xx} R_o{}^2}{r_{xx}} + \frac{|J_z|\alpha_1{}^2\beta_z R_o{}^2}{r_z}\right)$$

$$b = \left(\frac{|J_{zz}|\alpha_2{}^2\beta_{zz} R_o{}^2}{r_{zz}}\right)$$

$$r_x = R_o\frac{\alpha_1\beta_x}{2} \quad ; \quad r_{xx} = R_o\frac{\alpha_1\beta_{xx}}{2}$$

$$r_z = R_o(1+\frac{\alpha_1}{2})(\tan\phi_z-\tan\phi_{xx})+\frac{\alpha_1\beta_{xx}}{2}$$

$$r_{zz} = R_o\sqrt{\left((1+\alpha_1+\frac{\alpha_2}{2})\tan\phi_{zz}-(1-\frac{\alpha_1}{2})\tan\phi_z+\frac{\alpha_1\beta_{xx}}{2}\right)^2+\left(\frac{\alpha_1}{2}+\frac{\alpha_2}{2}\right)^2}$$

11.6 COST-RELATED PARAMETERS

Several parameters which may be expected to be closely related to the cost of the facility can be evaluated from the geometric, material, and performance parameters of the coil system. Included among these are the following:

(a) Winding volume parameters (V_j/R_o^3)

The total volume V_w of coil windings is

$$\frac{(V_w)_{total}}{R_o^3} = \sum_{j=1}^{4} \frac{V_j}{R_o^3} \tag{11.72}$$

$$\frac{V_{w\ total}}{R_o^3} = \sum_{j=1}^{4} 2\left(\frac{A_{jo}}{R_o^2}\right)\left(\frac{\bar{\ell}_{jo}}{R_o}\right) \tag{11.73}$$

$(j=x, xx, z, zz)$

(b) Winding mass (weight) parameters (m_j/R_o^3)

The mass of conductor (exclusive of structural systems) included in each coil pair is:

$$\frac{m_j}{R_o^3} = \left(\frac{m}{V}\right)_j (F_p)_j \left(\frac{V_j}{R_o^3}\right) \tag{11.74}$$

where $\left(\frac{m}{V}\right)_j$ = density of conductor in j-coils.

$(F_p)_j$ = winding packing factor - j-coils.

(c) Current-length product

The cost of superconducting material is often quoted in terms of (price)/(unit current x unit length) (eg., dollars/kiloamp. foot). Thus, it is desirable to derive a parameter in terms of this quantity.

The total length $\ell_{j_{tot}}$ of conductor in the j-coil pair is

$$\ell_{j_{tot}} = R_o N_j \left(\frac{\bar{\ell}_j}{R_o}\right) \tag{11.75}$$

The current-length product $\ell_{j_{tot}} I_j$, for the j-coils can be related to the performance parameter Q_j as follows:

$$\Sigma \ell_j I_j = B_j R_o^2 \left(\frac{\bar{\ell}_{jo}}{R_o}\right)\left(\frac{1}{Q_j}\right) \qquad (11.76)$$

where $B_j = B_x,\ R_o B_{xx},\ B_z,\ R_o B_{zz}.$

12.0 GENERAL POWER SUPPLY REQUIREMENTS

Electrical power supplies are required to provide controlled currents through each of the four coil pairs.

The fields (B_{xo}, B_{zo}) may be controlled independently of the gradients (B_{xxo}, B_{zzo}) by use of four separate power supplies, one for each of B_x, B_{xx}, B_z, and B_{zz}. Considerations that are involved are as follows:

(a) Control of force amplitude while maintaining saturation.

The operation of the system requires that the spherical iron core of the store model be saturated at all points within the useful volume of the tunnel test section. This in turn places a lower limit on the net magnetic field strength at any point. With the combined axial and vertical ambient field strength held at a level adequate to saturate the sphere, the magnetic force may then be varied by adjustment of the axial and vertical field gradient coil currents.

(b) Minimization of force field nonuniformities.

It has been demonstrated that the nonuniformities in the force field due to the gradient fields can be reduced by increases in the uniform ambient field level (Sect. 8). Thus, there is an advantage in operating the ambient field at the maximum level, independent of the force required. Alternately stated, the maximum installed power available for the ambient field (B_x, B_z) coil should be used at all times (in a ratio consistent with the required force field inclination, of course).

The alternative would be to have only two independent power supplies, with the B_{xx} coils connected in series with the B_x coils to one power supply, and the B_{zz} and B_z connected in series to the other power supply. Adjustment in the force level can be accomplished to some degree by adjustment of the currents, variation of the size of the spherical core relative to the store model, or choice of magnetic material having different saturation magnetization level.

12.1 POWER REQUIREMENTS FOR STEADY OPERATION

The d.c. power required for steady operation of the system can be found from the performance parameters as follows :

$$\text{Total d.c. power} \quad P_{(d.c.)_{total}} = \Sigma P_j(dc) \qquad (12.1)$$

$$P_{j(d.c.)} = I_j^{2} R_j$$

or

$$P_{x(d.c.)} = R_o B_{xo}^{2} \left(\frac{S_x}{Q_x^{2}}\right) \quad (12.3) \qquad ; \qquad P_{xx(d.c.)} = R_o (B_{xxo} R_o)^{2} \left(\frac{S_{xx}}{Q_{xx}^{2}}\right) \quad (12.4)$$

$$P_{z(d.c.)} = R_o B_{zo}^{2} \left(\frac{S_z}{Q_z^{2}}\right) \quad (12.5) \qquad ; \qquad P_{zz(d.c.)} = R_o (B_{zzo} R_o)^{2} \left(\frac{S_{zz}}{Q_{zz}^{2}}\right) \quad (12.6)$$

or $$P_{(dc)\,(total)} = R_o\left\{B_{xo}^2\left[\left(\frac{S_x}{Q_x^2}\right) + \left(\frac{B_{xxo}R_o}{B_{xo}}\right)^2\left(\frac{S_{xx}}{Q_{xx}^2}\right)\right] + B_{zo}^2\left[\left(\frac{S_z}{Q_z^2}\right) + \left(\frac{B_{zzo}R_o}{B_{zo}}\right)^2\left(\frac{S_{zz}}{Q_{zz}^2}\right)\right]\right\} \qquad (12.7)$$

Discussion

Several assumptions may be justified concerning the probable values of the parameters in the expression 12.7 above.

(i) $(B_{x_o})_{max} = \text{const.}$

$(B_{z_o})_{max} = \text{const.}$

(depend upon saturation magnetization of spherical core in store model)

(ii) $\dfrac{R_o(B_{xxo})_{max}}{(B_{xo})_{max}} = \text{const.}$

$\dfrac{R_o(B_{zzo})_{max}}{(B_{zo})} = \text{const.}$

(gradients (and forces) inversely proportional to scale factor, due to aerodynamic scaling requirements)

Note that the other factors $\left(\frac{S_j}{Q_j^2}\right)$ are related to shape, material, and construction method.

On the basis of the assumptions and observations above, note that the maximum d.c. power required for the coil system scales as the first power of the linear dimension.

12.2 ENERGY REQUIREMENTS FOR STARTUP OR INTERMITTENT OPERATION

The magnet system not only dissipates energy due to ohmic losses, but also stores energy by way of the magnetic fields which are produced. Thus, the electrical energy required to produce change in the field level will be a function of the time taken to effect the desired change, in addition to the difference in stored energy between the two levels.

$$\text{i.e. } \Delta E_j = \underbrace{\int_{t_1}^{t_2} I_j^{\,2} R_j \,dt}_{\text{(dissipation)}} + \underbrace{\tfrac{1}{2} L_j (I_2^{\,2} - I_1^{\,2})}_{\text{(storage)}} \qquad (12.8)$$

In particular,

$$\Delta E_x = R_o \frac{1}{Q_x^{\,2}} \int_{t_1}^{t_2} S_x\, B_{xo}^{\,2} dt + \frac{1}{2} R_o^{\,3} \left(\frac{T_x}{Q_x^{\,2}}\right) \left(B_{xo}^{\,2}(t_2) - B_{xo}^{\,2}(t_1)\right) \qquad (12.9)$$

$$\Delta E_{xx} = R_o \frac{1}{Q_{xx}^{\,2}} \int_{t_1}^{t_2} S_{xx} (R_o B_{xxo})^2 dt + \frac{1}{2} R_o^{\,3} \left(\frac{T_x}{Q_{xx}^{\,2}}\right) \left((R_o B_{xxo})_2^2 - (R_o B_{xxo})_1^2\right) \qquad (12.10)$$

$$\Delta E_z = R_o \frac{1}{Q_z^{\,2}} \int_{t_1}^{t_2} S_z (B_{zo})^2 dt + \frac{1}{2} R_o^{\,3} \left(\frac{T_z}{Q_z^{2}}\right) \left((B_{zo})_2^2 - (B_{zo})_1^2\right) \qquad (12.11)$$

$$\Delta E_{zz} = R_o \frac{1}{Q_{zz}^{\,2}} \int_{t_1}^{t_2} S_{zz} (R_o B_{zz})^2 dt + \frac{1}{2} R_o^{\,3} \left(\frac{T_{zz}}{Q_{zz}^{\,2}}\right) \left((R_o B_{zzo})_2^2 - (R_o B_{zzo})_1^2\right) \qquad (12.12)$$

Note that the resistance parameters (S_j) are included under the integral sign. Recall that S_j is proportional to the resistivity ρ_j of the conductor material which in general may be affected by changes in temperature of the conductor, due in turn to the dissipation itself. The exception to this of course is the case of superconducting material, in which the dissipative term may be negligible.

12.3 RESPONSE OF MAGNET SYSTEM TO POWER INPUT VARIATIONS

If it is assumed that the resistance parameters are constant, the four coil systems respond to voltage inputs as follows below:

(a) Four independent power supplies V_x, V_{xx}, V_z, V_{zz}

(i) Non-superconducting coils

$$I_x(s) = \left(\frac{1}{R_x}\right)\left(\frac{1}{1 + \tau_x s}\right) V_x(s) \qquad (12.13)$$

$$I_z(s) = \left(\frac{1}{R_z}\right)\left(\frac{1}{(1 + \tau_z s)}\right) V_z(s) \qquad (12.14)$$

$$I_{xx}(s) = \left(\frac{1}{R_{xx}}\right)\left(\frac{1+\tau_{zz}s}{D}\right)V_{xx}(s) + \left(\frac{1}{R_{xx}R_{zz}}\right)^{\frac{1}{2}}\left(\frac{-k_{xx/zz}\sqrt{\tau_{xx}\tau_{zz}}}{D}\right)V_{zz}(s) \qquad (12.15)$$

$$I_{zz}(s) = \left(\frac{1}{R_{xx}R_{zz}}\right)^{\frac{1}{2}}\left(\frac{-k_{xx/zz}\sqrt{\tau_{xx}\tau_{zz}}}{D}\right)V_{xx}(s) + \left(\frac{1}{R_{zz}}\right)\left(\frac{1+\tau_{xx}s}{D}\right)V_{zz}(s) \qquad (12.16)$$

where:

$$D = 1 + (\tau_{xx}+\tau_{zz})s + (1-k^2_{xx/zz})\tau_{xx}\tau_{zz}s^2$$

$$\tau_j = \left(\frac{L_j}{R_j}\right) = R_o^2 \frac{T_j}{S_j}$$

$$k_{xx/zz} = \frac{M_{xx/zz}}{\sqrt{L_{xx}L_{zz}}} = \frac{W_{xx/zz}}{\sqrt{T_{xx}T_{zz}}}$$

s = Laplace transform operator.

(ii) Superconducting coils ($R_j = 0$)

$$I_x(s) = \left[\frac{1}{L_x s}\right] V_x(s) \qquad (12.17)$$

$$I_z(s) = \left[\frac{1}{L_z s}\right] V_z(s) \qquad (12.18)$$

$$I_{xx}(s) = \left[\frac{1}{(1-k^2_{xx/zz})L_{xx}s}\right] V_{xx}(s) + \left[\frac{-k_{xx/zz}}{(1-k^2_{xx/zz})\sqrt{L_{xx}L_{zz}}}\right] V_{zz}(s)$$

(12.19)

$$I_{zz}(s) = \left[\frac{-k_{xx/zz}}{(1-k^2_{xx/zz})\sqrt{L_{xx}L_{zz}}}\right] V_{xx}(s) + \left[\frac{1}{(1-k^2_{xx/zz})L_{zz}s}\right] V_{zz}(s)$$

(12.20)

(b) Two independent power supplies $(V_x + V_{xx})$, $(V_z + V_{zz})$ with x and xx coils in series $(I_x = I_{xx})$ and z and zz coils in series $(I_z = I_{zz})$.

(i) Non-superconducting coils.

$$I_x = I_{xx}(s) = \left[\frac{1}{R_x+R_{xx}}\right]\left[\frac{1+\tau_{z*}s}{D_*}\right](V_x+V_{xx})(s)$$
$$+\left(\frac{1}{\sqrt{(R_x+R_{xx})(R_z+R_{zz})}}\right)\left[\frac{-k_{xx/zz}}{D_*}\sqrt{\frac{\tau_{xx}\tau_{zz}}{\left(1+\frac{L_x}{L_{xx}}\right)\left(1+\frac{L_z}{L_{zz}}\right)}}\,s\right](V_z+V_{zz})(s)$$

(12.21)

$$I_z = I_{zz}(s) = \left[\frac{1}{R_z+R_{zz}}\right]\left[\frac{1+\tau_{x*}s}{D_*}\right](V_z+V_{zz})(s)$$

$$+\frac{1}{\sqrt{(R_x+R_{xx})(R_z+R_{zz})}}\left[\frac{-k_{xx/zz}}{D_*}\sqrt{\frac{\tau_{xx}\tau_{zz}}{\left(1+\frac{L_x}{L_{xx}}\right)\left(1+\frac{L_z}{L_{zz}}\right)}}\,s\right](V_x+V_{xx})(s)$$

(12.22)

where $\tau_{x*} = \dfrac{L_x+L_{xx}}{R_x+R_{xx}}$; $\tau_{z*} = \dfrac{L_z+L_{zz}}{R_z+R_{zz}}$

$$D_* = 1+(\tau_{x*}+\tau_{z*})s+\frac{(1-k^2_{xx/zz})}{\left(1+\frac{L_x}{L_{xx}}\right)\left(1+\frac{L_z}{L_{zz}}\right)}\tau_{x*}\tau_{z*}s^2$$

(ii) Superconducting coils ($R_j = 0$)

$$I_x(s) = I_{xx}(s) = \left[\frac{1}{(L_x+L_{xx})\left(1-\dfrac{k^2_{xx/zz}}{(1+\frac{L_x}{L_{xx}})(1+\frac{L_z}{L_{zz}})}\right)s}\right](V_x+V_{xx})(s)$$

$$+\left[\frac{-k_{xx/zz}}{\sqrt{L_{xx}L_{zz}}\left((1+\frac{L_x}{L_{xx}})(1+\frac{L_z}{L_{zz}})-k^2_{xx/zz}\right)s}\right](V_z+V_{zz})(s) \qquad (12.23)$$

$$I_z(s) = I_{zz}(s) = \left[\frac{1}{(L_z+L_{zz})\left(1-\dfrac{k^2_{xx/zz}}{(1+\frac{L_x}{L_{xx}})(1+\frac{L_z}{L_{zz}})}\right)s}\right](V_z+V_{zz})(s)$$

$$+\left[\frac{-k_{xx/zz}}{\sqrt{L_{xx}L_{zz}}\left((1+\frac{L_x}{L_{xx}})(1+\frac{L_z}{L_{zz}})-k^2_{xx/zz}\right)s}\right](V_x+V_{xx})(s) \qquad (12.24)$$

13.0 MULTIPLE SIMULTANEOUS STORE JETTISON TESTS

When two or more stores are jettisoned simultaneously in the magnetic artificial gravity facility, perturbations to the trajectories will be experienced due to mutual magnetic attraction or repulsion between the store models. Consequently, it is of interest to estimate the magnitude of these interaction forces and their variation with the spacing and relative orientation of the store models in the ambient magnetic fields.

Analysis

a) "Two-body problem."

The problem can be illustrated by analyzing the forces between two spherical iron bodies immersed in a saturating magnetic field. The situation is shown in Figure 12.

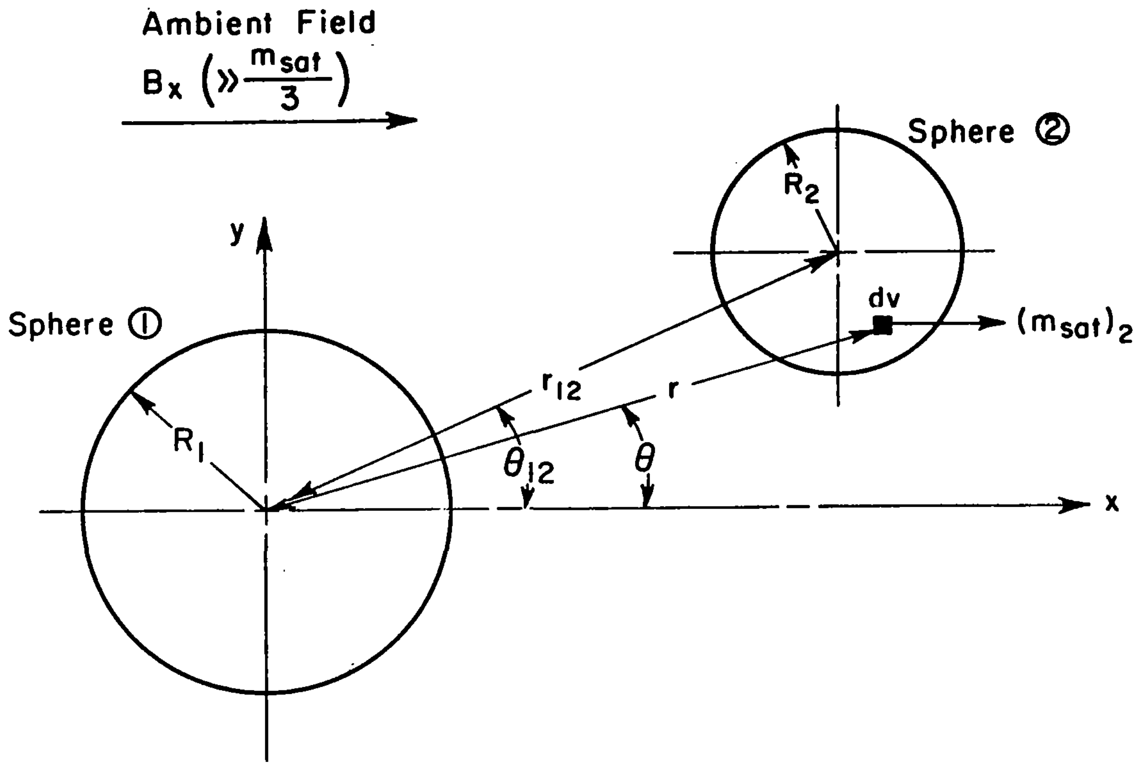


Figure 12. Two Iron Spheres Immersed in a Saturating Ambient Magnetic Field.

Sphere 1 is magnetically saturated, and the magnetization is assumed to be parallel to the ambient field B_x. (i.e. The perturbation to the magnetization of sphere 1, due to sphere 2, is neglected.)

The external field due to sphere 1, in spherical coordinates, is

$$B_r)_{(1)} = 2\left[\frac{(m_{sat})_{(1)} R_1^3}{3}\right] \frac{\cos\theta}{r^3} \tag{13.1}$$

$$B_\theta)_{(1)} = \left[\frac{m_{sat(1)} R_1^3}{3}\right] \frac{\sin\theta}{r^3} \tag{13.2}$$

$$B_{\phi}\}_{(1)} = 0 \tag{13.3}$$

$$\frac{dF}{dV}\}_{(2)} = k_t\, \vec{m}_{(2)} dV_{(2)} \cdot (\vec{\nabla}\vec{B})_{1)} \tag{13.4}$$

$$\frac{dF_r}{dV}\Big)_{(2)} = \frac{-k_t (m_{sat}\}_{(1)} (m_{sat}\}_{(2)} R_1^3}{r^4}[2-3\sin^2\theta] \tag{13.5}$$

$$\frac{dF_\theta}{dV}\}_{(2)} = \frac{-k_t (m_{sat}\}_{(1)} (m_{sat}\}_{(2)} R_1^3}{r^4}\ [\sin 2\theta(3-5\sin^2\theta)] \tag{13.6}$$

$$\frac{dF_\phi}{dV}\}_{(2)} = 0 \tag{13.7}$$

or, for $z = 0$,

$$\frac{dF_x}{dV} = \frac{-k_t (m_{sat}\}_{(1)} (m_{sat}\}_{(2)} R_1^3}{r^4}[\cos\theta(2-5\sin^2\theta)] \tag{13.8}$$

$$\frac{dF_y}{dV} = \frac{-k_t (m_{sat}\}_{(1)} (m_{sat}\}_{(2)} R_1^3}{r^4}[\sin\theta\ \ (4-5\sin^2\theta)] \tag{13.9}$$

The magnitude of the total force $\left|\frac{dF}{dV}\right|$ on an element is:

$$\left|\frac{dF}{dV}\right| = \sqrt{\left(\frac{dF_x}{dV}\right)^2 \left(\frac{dF_y}{dV}\right)^2} + (\text{or } = \sqrt{\left(\frac{dF_r}{dV}\right)^2 + \left(\frac{dF_\theta}{dV}\right)^2}\ ) \tag{13.10}$$

i.e.

$$\left|\frac{dF}{dV}\right| = \frac{k_t (m_{sat}\}_{(1)} (m_{sat}\}_{(2)} R_1^3}{r^4}[\cos\theta\sqrt{(2-5\sin^2\theta)^2 + (4-5\sin^2\theta)^2}] \tag{13.11}$$

Define

$$K_{|dF|}(\theta) = [\cos\theta\sqrt{(2-5\sin^2\theta)^2 + (4-5\sin^2\theta)}\ ] \qquad (13.12)$$

The direction $\psi_{|dF|}$ of the total elemental force is

$$\psi_{dF} = \tan^{-1}[\tan\theta\ [\frac{4-5\sin^2\theta}{2-5\sin^2\theta}]] - \Pi \qquad (13.13)$$

The angle-dependent terms $K_{|dF|}(\theta)$ and $\psi_{|dF|}$ are plotted in Figures 13 and 14.

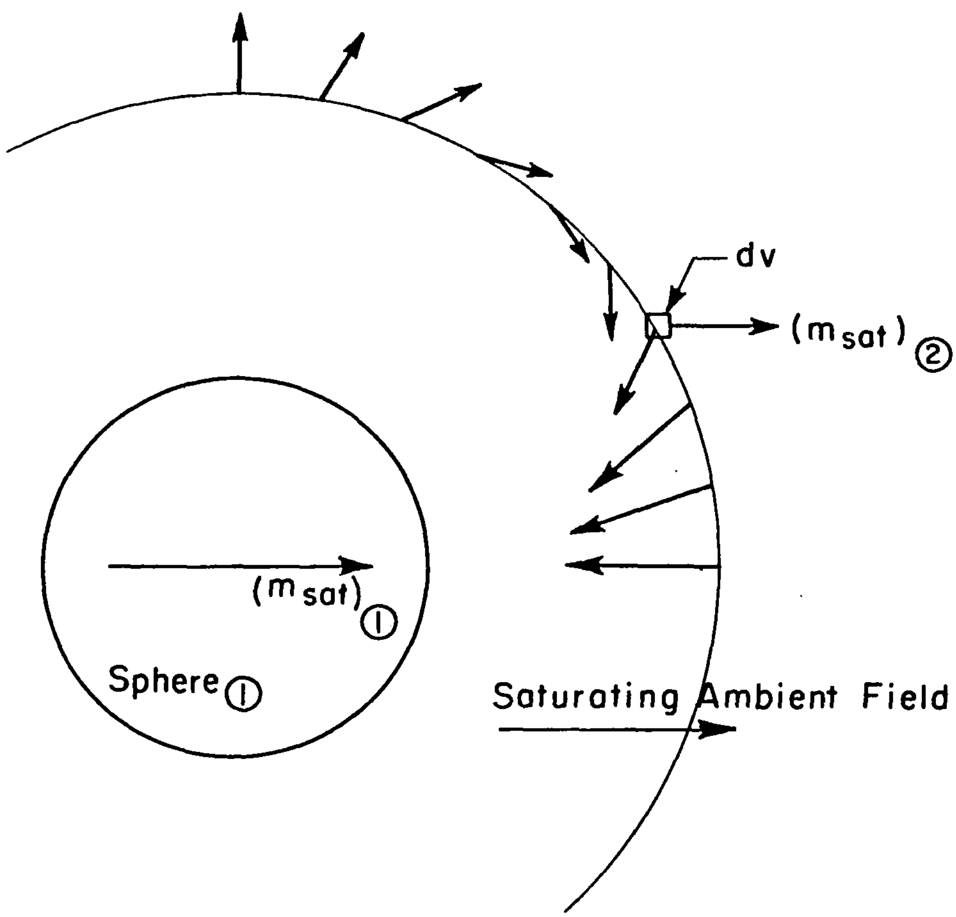


Figure 13. Variation of Strength and Direction of Magnetic Force on a Volume Element dV Due to a Saturated Sphere and a Saturating Ambient Field, at a Given Radius.

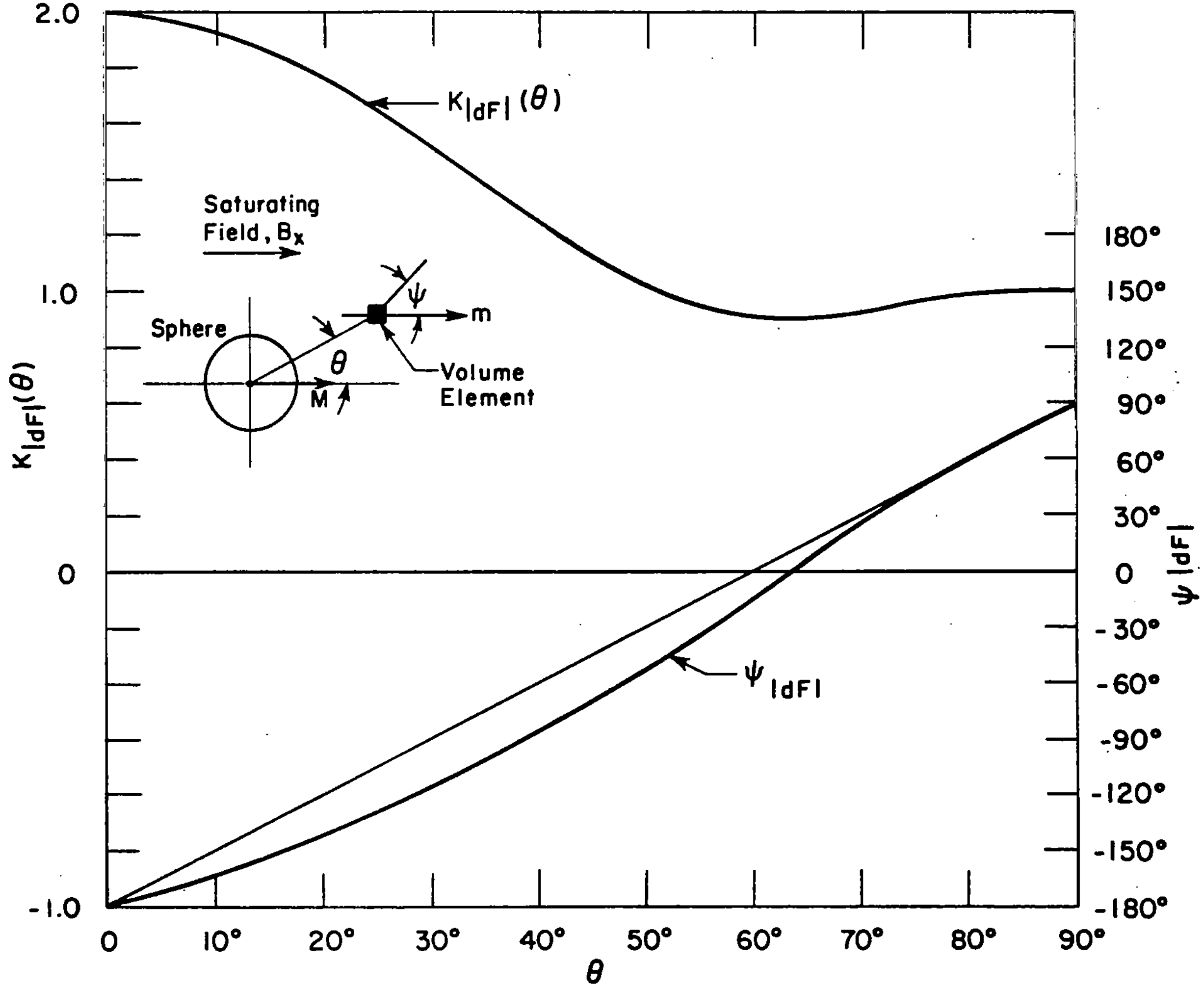


Figure 14. Angle Dependent Factor $K_{|dF|}(\theta)$ and Direction $\psi_{|dF|}$ of Total Magnetic Force on a Magnetized Volume Element, Due to a Sphere Magnetized Parallel to the Element.

The force components on a volume element (dV_2) thus vary inversely with the fourth power of the distance from the center of sphere 1, are functions of the angle to the volume element from the center of sphere 1, relative to the ambient field, and are proportional to the product of the saturation magnetization of sphere 1 and that of the volume element.

The total forces F_r, F_θ exerted on sphere 2 due to the gradients of the field from sphere 1 are found by integrating the elemental forces over the volume of sphere 2.

i.e.,

$$F_{r(2)} = \int^{V(2)} \left(\frac{dF_r}{dV}\right)_{(2)} dV \tag{13.14}$$

$$F_{\theta(2)} = \int^{V(2)} \left(\frac{dF_\theta}{dV}\right)_{(2)} dV \tag{13.15}$$

$$F_{\phi(2)} = 0 \tag{13.16}$$

The integrations indicated in 13.14,15, for the general case, are not evaluated here in their exact form. Instead, a conservative approximate form is presented which illustrates the situation in a relatively simple way. The following assumptions are made:

(i) The direction of the total force corresponds to that of the elemental force calculated for $\theta = \theta_{12}$.

(ii) The magnitude of the total force is estimated by integration of a volume force which varies with $(1/s^4)$ over the volume of sphere 2, where s is a distance in a rectangular coordinate system.

The results are as follows:

$$|F|_{(2)} \simeq \frac{k_t (m_{sat})(1)(m_{sat})(2) R_1^3 \left(\frac{4}{3}\Pi R_2^3\right)}{r_{12}^4} \left[\frac{1}{\left(1-\left(\frac{R_2}{r_{12}}\right)^2\right)^2}\right] (K_{dF}(\theta_{12})) \tag{13.17}$$

and, angle of $F_{12} = \psi_{12}$

$$\psi_{12} \simeq \tan^{-1}\left[\tan\theta_{12}\left[\frac{4-5\sin^2\theta_{12}}{2-5\sin^2\theta_{12}}\right]\right]-\Pi \tag{13.18}$$

The total force is thus identical to the elemental force at the center of sphere 2 multiplied by the volume of sphere 2, and weighted by a function of (R_2/r_{12}) which approaches unity as (R_2/r_{12}) approaches zero, as is expected.

If it is assumed that the two spheres are identical, of radius R, and saturation magnetization m_{sat}, and their centers separated by a distance r,
i.e.,

$$R_1 = R_2 = R$$

$$m_{sat_1} = m_{sat_2} = m_{sat}$$

$$r_{12} = r$$

then:

$$|F| \simeq \frac{k_t}{R}\left(\frac{4}{3}\Pi R^3\right)(m_{sat})^2 \left[\left(\frac{R}{r}\right)^4 \left(\frac{1}{1-\left(\frac{R}{r}\right)^2}\right)^2\right] (K_{dF}(\theta)) \tag{13.19}$$

Observe that $\left(\frac{R}{r}\right)_{max} = 0.5$ (contact)

Values of the weighting function of $\left(\frac{R}{r}\right)$ in Eq. 13.19 are tabulated below.

$(\frac{r}{R})$	$[(\frac{R}{r})^4 \frac{1}{(1-(\frac{R}{r})^2)^2}]$	$(\frac{r}{R})$	$[(\frac{R}{r})^4 \frac{1}{(1-(\frac{R}{r})^2)^2}]$
2	1.11×10^{-1}	7	4.33×10^{-4}
3	1.57×10^{-2}	8	2.52×10^{-4}
4	4.45×10^{-3}	9	1.58×10^{-4}
5	8.03×10^{-4}	10	1.01×10^{-4}

The acceleration $(a_s)_M$ of the store model due to the perturbing force estimated above is as follows:

$$(a_s)_M = \frac{|F|}{(w_s)_M} g \tag{13.20}$$

$$= (\frac{w_{mag}}{w_s}) \frac{F}{w_{mag}} g \tag{13.21}$$

but $$w_{mag} = \frac{4}{3}\Pi R^3 \rho_{mag} \tag{13.22}$$

$$\therefore (a_s)_M = (\frac{w_{mag}}{w_s}) \frac{k_t m_{sat}^2}{R\rho_{mag}} [f(\frac{R}{r})][K_{dF}(\theta)] g \tag{13.23}$$

<u>Example</u>:

Let $(\frac{w_{mag}}{w_s}) = 0.5$

$m_{sat} = 21$ kilogauss

$\rho_{mag} = 0.283$ lb/in^3

and $k_t = 1.14$ (in.lb)(in)$^{-3}$ (K.gauss)$^{-2}$

$$(a_s)_m = \frac{(0.5)(1.14)(21)^2}{R(0.283)} \left[f\left(\frac{R}{r}\right)\right]\left[K_{dF}(\theta)\right] \; g$$

$$= \frac{(8.9 \times 10^2)}{R} \left[f\left(\frac{R}{r}\right)\right]\left[K_{dF}(\theta)\right] \; g$$

Let $R = 0.5" \; ; \quad \left(\frac{r}{R}\right) = 6 \; ; \; \theta = 0 \; ,$

then $$(a_s)_m = \frac{(8.9 \times 10^2)}{(0.5)} \left[8.03 \times 10^{-4}\right]\left[2.0\right) \; g$$

$$= 2.9 \; g$$

i.e., For two store models, each containing 1" diameter iron spheres, with a separation 3" between centers of gravity, the mutual perturbing acceleration is of the order of 1.5 to 3 g's, depending upon the orientation of the line between the centers relative to the applied field.

For greater separation, the perturbation is less. For example, by increasing the separation from (r/R) = 6 to (r/R) = 8, the perturbing acceleration diminishes from 2.9 g's to 0.88 g.

For this example, the probable gravity scale factor would be of the order of 20. Thus, the perturbation acceleration is a significant fraction of the "normal" acceleration at short separation distances, but diminishes rapidly with separation.

Note the effect of scale: the ratio of perturbation acceleration to "normal" acceleration is independent of scale to a first approximation, due to the reduced normal acceleration required with increasing model size.

(b) Multi-body problem.

The total force on a single sphere is the vector sum of the forces produced by all surrounding spheres as calculated for the two-body problem. That is, the forces add linearly.

APPENDIX A

MAGNETIC FIELDS DUE TO AN ARRAY OF STRAIGHT LINE CURRENT ELEMENTS AND CORRESPONDING FORCES ON A FERROMAGNETIC SPHERE

The following is an outline of the relations required to compute the magnetic field strength and gradient components and the corresponding forces on a ferromagnetic sphere, produced by an array of straight-line current elements, and by extension, by an assembly of square or rectangular magnet coils. These relations apply for regions of constant permeability, and therefore do not allow for ferromagnetic material in the vicinity, as would be the case if iron cores were to be used in association with the coil assembly.

The relationships are based upon the principle of linear superposition. That is, the field strength at a particular point in space is found by adding the effects of individual current elements.

Field and Gradient Components from a Single Current Element

The field strength at a point in space due to a straight-line current element is found by application of the Biot-Savart Law, viz.

$$\vec{B} = \frac{\mu_o I}{4\Pi} \oint \frac{d\vec{\ell} \times \vec{r}}{r^3} \qquad \text{(A-1)}$$

In terms of rectangular coordinates and the quantities illustrated in Figure A-1, the field components are as follows:

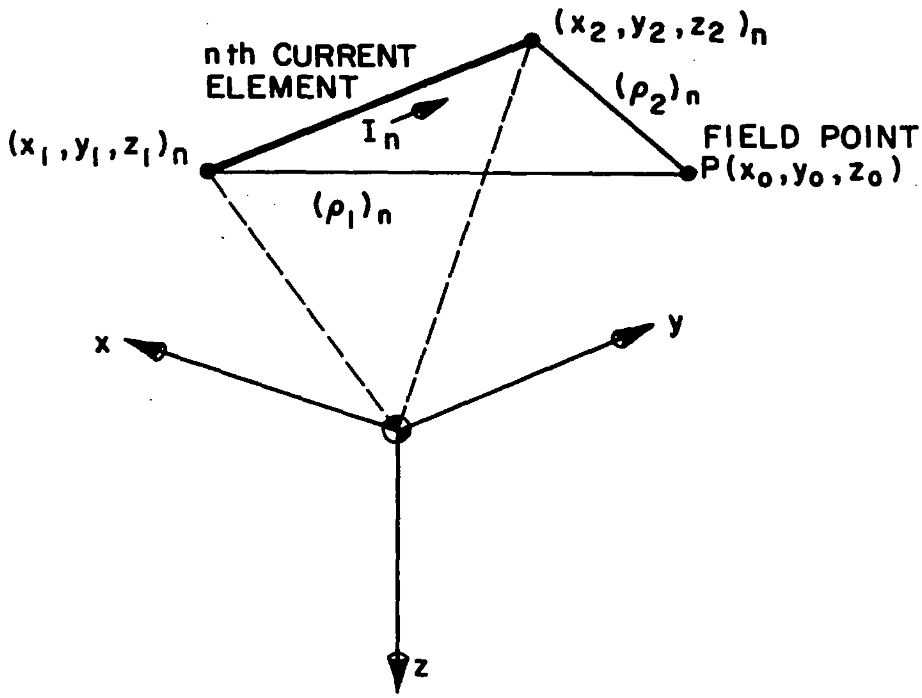


Figure A-1. Definition of Current Element and Field Point Positions.

$$B_{x_n} = \frac{\mu_o I_n}{4\Pi} G_n U_n \tag{A-2}$$

$$B_{Y_n} = \frac{\mu_o I_n}{4\Pi} G_n V_n \tag{A-3}$$

$$B_{z_n} = \frac{\mu_o I_n}{4\Pi} G_n W_n \tag{A-4}$$

where:

$$U_n = [(y_1-y_0)(z_2-z_0) - (y_2-y_0)(z_1-z_0)] \tag{A-5}$$

$$V_n = [(z_1-z_0)(x_2-x_0) - (z_2-z_0)(x_1-x_0)] \tag{A-6}$$

$$W_n = [(x_1-x_0)(y_2-y_0) - (x_2-x_0)(y_1-y_0)] \tag{A-7}$$

let $$H_n = 1 + \frac{\vec{\rho}_1 \cdot \vec{\rho}_2}{\rho_1\rho_2} \qquad \text{(A-8)}$$

if $H_n \geq 10^{-2}$, let $$G_n = \left[\frac{(\rho_1 + \rho_2)}{\rho_1\rho_2}\left(\frac{1}{\rho_1\rho_2 \quad \vec{\rho}_1\cdot\vec{\rho}_2}\right)\right] \qquad \text{(A-9)}$$

if $H_n < 10^{-2}$, let $$G_n = \left[\frac{(\rho_1+\rho_2)}{\rho_1\rho_2}\,\frac{(\rho_1\rho_2 - \vec{\rho}_1\cdot\vec{\rho}_2)}{\vec{\rho}_1 \times \vec{\rho}_2}\right] \qquad \text{(A-10)}$$

$$\rho_1 = [(x_1-x_0)^2 + (y_1-y_0)^2 + (z_1-z_0)^2]^{1/2} \qquad \text{(A-11)}$$

$$\rho_2 = [(x_2-x_0)^2 + (y_2-y_0)^2 + (z_2-z_0)^2]^{1/2} \qquad \text{(A-12)}$$

$$\vec{\rho}_1\cdot\vec{\rho}_2 = [(x_1-x_0)(x_2-x_0)+(y_1-y_0)(y_2-y_0)+(z_1-z_0)(z_2-z_0)] \qquad \text{(A-13)}$$

$$\rho_1 \times \rho_2 = U + V + W \qquad \text{(A-14)}$$

by letting $Y_1 = U$, $Y_2 = V$, $Y_3 = W$,. (A-15)

Equations (A-2,4,6) can be reduced by index notation to the form

$$(B_{xi})_n = \frac{\mu_o I_n}{4\Pi} G_n Y_i \qquad i = 1,2,3 \qquad \text{(A-16)}$$

The gradient components are thus,

$$\left(\frac{\partial B_{xi}}{\partial_{xj}}\right)_n = \frac{\mu_o I_n}{4\Pi}\left[G_n \frac{\partial Y_i}{\partial_{xj}} + Y_i \frac{\partial G_n}{\partial_{xj}}\right] \qquad j = 1,2,3 \qquad \text{(A-17)}$$

for $H \geq 10^{-2}$, $$\frac{\alpha G_n}{\alpha_{xj}} = \frac{\alpha}{\alpha_{xj}} \left\{ \frac{(\rho_1+\rho_2)}{\rho_1\rho_2} \frac{1}{(\rho_1\rho_2 \; \vec{\rho}_1 \cdot \vec{\rho}_2)} \right\} \qquad \text{(A-18)}$$

let
$$\rho_1 + \rho_2 = P$$
$$\rho_1\rho_2 = Q$$
$$\rho_1 \cdot \rho_2 = R$$
$$[\vec{\rho}_1 x \vec{\rho}_2] = S$$

expanding (A-18),

$$\frac{\partial G}{\partial_{xj}} = \left[\frac{1}{Q(Q+R)}\right] \frac{\partial P}{\partial_{xj}} + \left[\frac{-P(2Q+R)}{Q^2(Q+R)^2}\right] \frac{\partial Q}{\partial_{xj}} + \left[\frac{P}{Q(Q+R)^2}\right] \frac{\partial R}{\partial_{xj}} \qquad \text{(A-19)}$$

for $H<10^{-2}$, $$\frac{\partial G}{\partial_{xj}} = \frac{\partial}{\partial_{xj}} \left[\frac{(\rho_1+\rho_2)(\rho_1\rho_2-\rho_1 \cdot \rho_2)}{\rho_1\rho_2 \quad (\vec{\rho}_1 \times \vec{\rho}_2)^2}\right] \qquad \text{(A-20)}$$

Expanding (A-20)

$$\frac{\partial G}{\partial_{xj}} = \left[\frac{Q - R}{QS^2}\right] \frac{\partial P}{\partial_{xj}} + \left[\frac{P(Q-S(Q-R))}{Q^2S^3}\right] \frac{\partial Q}{\partial_{xj}}$$

$$+ \left[\frac{P}{QS^3}\right] \frac{\partial R}{\partial_{xj}} + \left[\frac{-2P(Q-R)}{QS^3}\right] \frac{\partial S}{\partial_{xj}} \qquad \text{(A-21)}$$

The total field properties at a point (x_o, y_o, z_o) due to N current elements are thus given by the following:

$$(B_{xi}) = \frac{\mu o}{4\Pi} \sum_{n=1}^{N} (IGY_i)_n \qquad i=1,2,3 \qquad \text{(A-22)}$$

$$\left(\frac{\partial B_{xi}}{\partial_{xj}}\right) = \frac{\mu_o}{4\Pi} \sum_{n=1}^{N} I_n \left(G\frac{\partial Y_i}{\partial_{xj}} + Y_1 \frac{\partial G}{\partial_{xj}}\right)_n \qquad i=1,2,3 \qquad \text{(A-23)}$$

<u>Magnetic Forces</u>

The magnetic force components can be written in index notation as follows:

$$\frac{F_{xj}}{V_{mag}} = k_t\ K \sum_{i=1}^{3} \left[\frac{B_{xi}}{|B|} \frac{\partial B_{xi}}{\partial_{xj}}\right] \qquad j=1,2,3 \qquad \text{(A-24)}$$

where $K = 3|B|$, if $3|B| < m_{sat}$

or $K = m_{sat}$, if $3|B| > m_{sat}$.

APPENDIX B

COMPUTER PROGRAM FOR CALCULATING AND TABULATING MAGNETIC FIELD AND FIELD GRADIENT COMPONENTS DUE TO AN ARRAY OF STRAIGHT LINE CURRENT ELEMENTS, AND THE CORRESPONDING FORCES ON A FERROMAGNETIC SPHERE

The following is a brief description of a computer program "TABLE" written in Fortran IV language which is used to compute and tabulate the magnetic field properties and the magnetic force components on a ferromagnetic sphere; using the relations outlined in Appendix A.

Tabulating Program ("TABLE")

TABLE was developed to provide a tabulation of the magnetic field components, component derivatives, and magnetic force components for a set of field positions.

The input data for this program includes the end points of the current elements as defined in Figure A-1, the current flowing in each element, and the saturation magnetization of the sphere material. Variable names associated with the input data can be found in the "Input Variable List" of the Program Listing on page 72.

Sample input and output data are shown on pages 81 and 82.

```
C                         ARTIFICIAL GRAVITY - TABLE
C
C A CODE TO CALCULATE THE MAGNETIC FORCES ON A BODY IN A FIELD PRODUCED
C BY COILS CONSISTING OF STRAIGHT LINE CURRENT ELEMENTS. .  THE FORCES
C AND FIELD CHARACTERISTICS ARE TABULATED AT CORRESPONDING POINTS IN
C THREE DIMENSIONAL SPACE.
C CURRENT ELEMENTS ARE COUNTED COUNTER-CLOCKWISE ABOUT THE CORRESPONDING
C COORDINATE DIRECTIONS.  ALL CURRENTS ARE POSITIVE COUNTER-CLOCKWISE.
C
C
C                    INPUT VARIABLE LIST
C
C VARIABLE NAME                          DEFINITION
C       DA            DEMAGNETIZING CONSTANT FOR THE MODEL.
C       AMS           SATURATION MAGNETIZATION FOR THE MODEL.
C       XKT           MAGNETIC FORCE CONSTANT.
C       XMU           MAGNETIC PERMEABILITY OF FREE SPACE.
C       INM           THE NUMBER OF DATA SETS.
C       INPOPT        INPUT OPTION.
C       CONFG         DESCRIPTION OF ARTIFICIAL GRAVITY CONFIGURATION.
C       IM            NUMBER OF INCREMENTS IN THE X-DIRECTION.
C       JM            NUMBER OF INCREMENTS IN THE Y-DIRECTION.
C       KM            NUMBER OF INCREMENTS IN THE Z-DIRECTION.
C       LM            TOTAL NUMBER OF CURRENT ELEMENTS
C       DX            'DELTA' X.
C       DY            'DELTA' Y.
C       DZ            'DELTA' Z.
C       X(1)          X COORDINATE OF STARTING POINT FOR INCREMENTING
C       Y(1)          Y COORDINATE OF STARTING POINT FOR INCREMENTING.
C       Z(1)          Z COORDINATE OF STARTING POINT FOR INCREMENTING.
C       X1,Y1,Z1      COORDINATES OF THE END POINTS OF THE STRAIGHT LINE
C       X2,Y2,Z2      CURRENT ELEMENTS MAKING UP THE COILS.
C       CUR           MAGNITUDE OF THE CURRENT IN AMPERES,+ FROM 1 TO 2.
C       CURT          CURRENT FLOWING IN A LOOP OF FOUR CURRENT ELEMENTS.
C       CURX1         THE TOTAL CURRENT IN THE +X FIELD COIL.
C       CURX2         THE TOTAL CURRENT IN THE +X GRADIENT COIL.
```

```
C       CURX3          THE TOTAL CURRENT IN THE -X FIELD COIL.
C       CURX4          THE TOTAL CURRENT IN THE -X GRADIENT COIL.
C       CURZ1          THE TOTAL CURRENT IN THE +Z FIELD COIL.
C       CURZ2          THE TOTAL CURRENT IN THE +Z GRADIENT COIL.
C       CURZ3          THE TOTAL CURRENT IN THE -Z FIELD COIL.
C       CURZ4          THE TOTAL CURRENT IN THE -Z GRADIENT COIL.
C
C
      DIMENSION X(100),Y(100),Z(100),S(3),T(3),DP(3),DS(3),DD(3),DC(3),
     1DG(3),X1(500),Y1(500),Z1(500),X2(500),Y2(500),Z2(500),CUR(500),CON
     2FG(18),CURT(500)
  100 FORMAT(4I4,6F8.4)
  110 FORMAT(6F8.4,F10.0)
  111 FORMAT(3F6.1,9F8.1,3F8.3)
  112 FORMAT(F5.3,F9.2,2F12.11,2I5)
  113 FORMAT(3X,1HX,5X,1HY,4X,1HZ,6X,2HBX,6X,2HBY,6X,2HBZ,4X,6HDBX/DX,2X
     1,6HDBX/DY,2X,6HDBX/DZ,2X,6HDBY/DY,2X,6HDBY/DZ,2X,6HDBZ/DZ,4X,2HFX,
     26X,2HFY,6X,2HFZ)
  114 FORMAT(//)
  115 FORMAT(///)
  116 FORMAT(6X,6HINCHES,16X,5HGAUSS,29X,9HGAUSS/IN.,27X,10HLBF/CU.IN.)
  118 FORMAT(1H1)
  127 FORMAT(24X,10HINPUT DATA)
  128 FORMAT(2X,6HX1(IN),2X,6HY1(IN),2X,6HZ1(IN),2X,6HX2(IN),2X,6HY2(IN)
     1,2X,6HZ2(IN),1X,10HCURRENT(A))
  129 FORMAT(1X,6F8.4,F10.0)
  150 FORMAT(4F10.0)
  151 FORMAT(3X,32HTOTAL CURRENT IN +X FIELD COIL= ,F10.0,6H AMPS.,3X,35
     1HTOTAL CURRENT IN +X GRADIENT COIL= ,F10.0,6H AMPS.)
  152 FORMAT(3X,32HTOTAL CURRENT IN -X FIELD COIL= ,F10.0,6H AMPS.,3X,35
     1HTOTAL CURRENT IN -X GRADIENT COIL= ,F10.0,6H AMPS.)
  153 FORMAT(3X,32HTOTAL CURRENT IN +Z FIELD COIL= ,F10.0,6H AMPS.,3X,35
     1HTOTAL CURRENT IN +Z GRADIENT COIL= ,F10.0,6H AMPS.)
  154 FORMAT(3X,32HTOTAL CURRENT IN -Z FIELD COIL= ,F10.0,6H AMPS.,3X,35
     1HTOTAL CURRENT IN -Z GRADIENT COIL= ,F10.0,6H AMPS.)
  155 FORMAT(18A4)
```

```
 159  FORMAT(24X,18A4)
 165  FORMAT(F10.0)
 166  FORMAT(6F8.4)
C INPUT THE MAGNETIZATION CONSTANT FOR THE GEOMETRY OF THE BODY, THE
C MAGNITUDE OF THE SATURATION MAGNETIZATION FOR THE MATERIAL, THE
C MAGNETIC FORCE CONSTANT,THE PERMEABILITY OF THE MEDIUM, THE NUMBER
C DATA SETS, AND THE INPUT OPTION.
C INPOPT=1 CORRESPONDS TO INPUTING THE CURRENT IN EACH ELEMENT.  INPOPT
C =2 CORRESPONDS TO INPUTING THE CURRENT  IN EACH LOOP OF FOUR ELEMENTS.
      READ(5,112) CA,AMS,XKT,XMU,INM,INPOPT
      XMP=XMU/(4.*3.1416)
      IF(INPOPT.EQ.1) GO TO 171
C INPOPT=2
C INPUT THE DESCRIPTION OF THE CONFIGURATION (LE. 72 CHARACTERS).
      READ(5,155) (CONFG(IC),IC=1,18)
C INPUT THE MAXIMUM NUMBER OF X INCREMENTS, THE MAXIMUM NUMBER OF Y
C INCREMENTS, THE MAXIMUM NUMBER OF Z INCREMENTS, THE NUMBER OF CURRENT
C ELEMENTS, DELTA X, Y, AND Z, AND THE CORNER POINT OF THE PLOT (MAXIMUM
C VALUE OF X,MINIMUM VALUE OF Z, AND Y POSITION).
C NOTE THAT DX IS NEGATIVE AND DZ IS POSITIVE.
      READ(5,100) IM,JM,KM,LM,DX,DY,DZ,X(1),Y(1),Z(1)
C INPUT THE END POINTS OF THE CURRENT ELEMENTS.
      READ(5,166) (X1(NI),Y1(NI),Z1(NI),X2(NI),Y2(NI),Z2(NI),NI=1,LM)
      NTM=LM/4
C INPOPT=1
 171  DO 200 INP=1,INM
      IF(INPOPT.EQ.2) GO TO 174
C INPUT THE DESCRIPTION OF THE CONFIGURATION (LE. 72 CHARACTERS).
      READ(5,155) (CONFG(IC),IC=1,18)
C INPUT THE MAXIMUM NUMBER OF X INCREMENTS, THE MAXIMUM NUMBER OF Y
C INCREMENTS, THE MAXIMUM NUMBER OF Z INCREMENTS, THE NUMBER OF CURRENT
C ELEMENTS, DELTA X, Y, AND Z, AND THE CORNER POINT OF THE PLOT (MAXIMUM
C VALUE OF X,MINIMUM VALUE OF Z, AND Y POSITION).
C NOTE THAT DX IS NEGATIVE AND DZ IS POSITIVE.
      READ(5,100) IM,JM,KM,LM,DX,DY,DZ,X(1),Y(1),Z(1)
C INPUT THE END POINTS OF THE CURRENT ELEMENTS AND THE CURRENT FLOWING
```

```
C IN EACH ELEMENT.
      READ(5,110) (X1(N),Y1(N),Z1(N),X2(N),Y2(N),Z2(N), CUR(N),N=1,LM)
      GO TO 173
C INPOPT=2
C INPUT THE CURRENT FLOWING IN EACH LOOP OF FOUR CURRENT ELEMENTS.
 174  READ(5,165) (CURT(NT),NT=1,NTM)
      KL=-3
      KL1=0
C ASSIGN CURRENT MAGNITUDES TO EACH CURRENT ELEMENT.
      DO 170 JT=1,NTM
      KL=KL+4
      KL1=KL1+4
      DO 170 NT1=KL,KL1
      CUR(NT1)=CURT(JT)
 170  CONTINUE
C INPUT THE TOTAL CURRENT IN BOTH GRADIENT AND FIELD COILS(TOTAL
C AMPERE TURNS).
 173  READ(5,150) CURX1,CURX2,CURX3,CURX4
      READ(5,150) CURZ1,CURZ2,CURZ3,CURZ4
      WRITE(6,118)
C OUTPUT THE CHARACTERISTICS OF THE CURRENT ELEMENTS IN THIS CALCULATION
      WRITE(6,159) (CONFG(IOC),IOC=1,18)
      WRITE(6,115)
      WRITE(6,127)
      WRITE(6,128)
      WRITE(6,129) (X1(N1),Y1(N1),Z1(N1),X2(N1),Y2(N1),Z2(N1),CUR(N1),
     1N1=1,LM)
      WRITE(6,118)
      WRITE(6,159) (CONFG(IOC),IOC=1,18)
      WRITE(6,115)
      WRITE(6,151) CURX1,CURX2
      WRITE(6,152) CURX3,CURX4
      WRITE(6,153) CURZ1,CURZ2
      WRITE(6,154) CURZ3,CURZ4
      WRITE(6,114)
      WRITE(6,113)
```

```
      WRITE(6,116)
      IP=0
C CALCULATE AND STORE THE SPACIAL COORDINATES FOR THIS CALCULATION.
      ITLM=IM-1
      JTLM=JM-1
      KTLM=KM-1
      DO 2002 ITL=1,ITLM
 2002 X(ITL+1)=X(ITL)+DX
      DO 2000 JTL=1,JTLM
 2000 Y(JTL+1)=Y(JTL)+DY
      DO 2001 KTL=1,KTLM
 2001 Z(KTL+1)=Z(KTL)+DZ
C BEGIN THE MAGNETIC FORCE CALCULATION.
      DO 200 J=1,JM
      DO 300 I=1,IM
      IF(I-1) 25,25,26
 26   WRITE(6,114)
 25   DO 300 K=1,KM
C INITUALIZE THE VALUES TO BE SUMMED.
      BX=0.0
      BY=0.
      BZ=0.0
      BXX=0.
      BXY=0.
      BXZ=0.
      BYX=0.
      BYY=0.
      BYZ=0.
      BZX=0.
      BZY=0.
      BZZ=0.
      DO 210 L=1,LM
C CALCULATE A,B,C,D,E,AND F
      A=(X1(L)-X(I))/39.37
      B=(X2(L)-X(I))/39.37
      C=(Y1(L)-Y(J))/39.37
```

```
      D=(Y2(L)-Y(J))/39.37
      E=(Z1(L)-Z(K))/39.37
      F=(Z2(L)-Z(K))/39.37
C SUBSCRIPT A,C,E,B;D,F FCR LATER USE
      S(1)=A
      S(2)=C
      S(3)=E
      T(1)=B
      T(2)=D
      T(3)=F
C CALCUALTE U, V, AND W.
      U=C*F-D*E
      V=E*B-F*A
      W=A*D-B*C
C CALCULATE RHO1 AND RHO2.
      R1=(A*A+C*C+E*E)**0.5
      R2=(B*B+D*D+F*F)**0.5
C CALCULATE THE SUM, PRODUCT, DOT PRODUCT, AND CROSS PRODUCT OF RHO1
C AND RHO2.
      RS=R1+R2
      RM=R1*R2
      RDR=A*B+C*D+E*F
      RXR=U+V+W
C CALCULATE THE DERIVITIVES OF THE SUM, ETC. OF RHO1 AND RHO2.
      DO 220 M=1,3
      DP(M)=-(S(M)*R2/R1+T(M)*R1/R2)
      DS(M)=-(S(M)/R1+T(M)/R2)
      DC(M)=-(S(M)+T(M))
 220  CONTINUE
      CC(1)=F-E+C-D
      CC(2)=E-F+B-A
      DC(3)=D-C+A-B
C CALCULATE AND TEST H TO DETERMINE EQUATION FOR G TO BE USED.
      H=(RM+RDR)/RM
      IF(H-0.01) 2,1,1
 1    G=RS/(RM*(RM+RDR))
```

```
C CALCULATE G AND ITS DERIVITIVES IN THE X,Y,Z DIRECTIONS.
      DC 23C M1=1,3
      DGA=RM*(RM+RDR)*DS(M1)
      DGB=RS*(RM*(DP(M1)+DD(M1))+DP(M1)*(RM+RDR))
      DG(M1)=(DGA-DGB)/(RM*(RM+RDR))**2
 230  CCNTINUE
      GO TC 3
 2    G=((RS)*(RM-RDR))/(RM*RXR*RXR)
      DO 240 M2=1,3
      DGA=(RS*(DP(M2)-DD(M2))+DS(M2)*(RM-RDR))*RM*RXR**2
      DGB=RS*(RM-RDR)*(RM*2.*RXR*DC(M2)+DP(M2)*RXR**2)
      DG(M2)=(DGA-DGB)/(RM*RXR**2)**2
 240  CCNTINUE
C CALCUALTE THE FIELD CCNTRIBUTICNS OF EACH CURRENT ELEMENT.
 3    DGX=DG(1)
      DGY=DG(2)
      DGZ=DG(3)
      CURP=XMP*CUR(L)*10000./39.37
      CURM=XMP*CUR(L)*G*10000.
      BX1=CURM*U
      BY1=CURM*V
      BZ1=CURM*W
C CALCULATE THE GRADIENT CCNTRIBUTICNS OF EACH CURRENT ELEMENT.
      BXX1=CURP*U*DGX
      BXY1=CURP*(G*(E-F)+U*DGY)
      BXZ1=CURP*(G*(D-C)+U*DGZ)
      BYY1=CURP*V*DGY
      BYZ1=CURP*(G*(A-B)+V*DGZ)
      BZZ1=CURP*DGZ*W
C SUM THE INDIVUAL CONTRIBUTIONS TO THE FIELD AND GRADIENT TC GET THE
C TOTAL FIELD AND GRADIENTS.
      BX=BX+BX1
      BY=BY+BY1
      BZ=BZ+BZ1
      BXX=BXX+BXX1
      BXY=BXY+BXY1
```

```
      BXZ=BXZ+BXZ1
      BYY=BYY+BYY1
      BYZ=BYZ+BYZ1
      BZZ=BZZ+BZZ1
  210 CONTINUE
C CALCULATE AND TEST THE MAGNETIZATION OF THE BODY FOR SATURATION.
      XDK=XKT/DA
      RB=(BX**2+BY**2+BZ**2)**0.5
      AM=(1/DA)*RB
      IF(AM-AMS) 10,10,11
C CALCULATE THE FORCES PRODUCED ON THE BODY.
  10  FX=XDK*(BX*BXX+BY*BXY+BZ*BXZ)
      FZ=XDK*(BX*BXZ+BY*BYZ+BZ*BZZ)
      FY=XDK*(BX*BXY+BY*BYY+BZ*BYZ)
      GO TO 12
C CALCULATE THE COMPONENTS OF THE MAGNETIZATION AT SATURATION.
  11  BMY=(BY/RB)*AMS
      BMX=(BX/RB)*AMS
      BMZ=(BZ/RB)*AMS
C CALCULATE THE FORCES PRODUCED ON THE BODY.
      FX=XKT*(BMX*BXX+BMY*BXY+BMZ*BXZ)
      FZ=XKT*(BMX*BXZ+BMY*BYZ+BMZ*BZZ)
      FY=XKT*(BMX*BXY+BMY*BYY+BMZ*BYZ)
  12  CONTINUE
  20  IP=IP+1
C IF AT THE BOTTOM OF THE PAGE, SKIP TO NEXT PAGE AND WRITE
C THE HEADING FOR THE NUMERICAL TABULATION.
      IF(IP-40) 50,51,51
  51  WRITE(6,118)
      WRITE(6,159) (CONFG(IOC),IOC=1,18)
      WRITE(6,115)
      WRITE(6,151) CURX1,CURX2
      WRITE(6,152) CURX3,CURX4
      WRITE(6,153) CURZ1,CURZ2
      WRITE(6,154) CURZ3,CURZ4
      WRITE(6,114)
```

```
      WRITE(6,113)
      WRITE(6,116)
      IP=0
 50   CONTINUE
C OUTPUT THE TABLE OF NUMERICAL VALUES OF FIELD CHARACTERISTICS AND
C FORCE MAGNITUDES.
 300  WRITE(6,111) X(I),Y(J),Z(K),BX,BY,BZ,BXX,BXY,BXZ,BYY,BYZ,BZZ,FX
     1,FY,FZ
 200  CONTINUE
      CALL EXIT
      END
```

INPUT DATA

X1(IN)	Y1(IN)	Z1(IN)	X2(IN)	Y2(IN)	Z2(IN)	CURRENT(A)
48.0000	48.0000	48.0000	48.0000	-48.0000	48.0000	5000000.
48.0000	-48.0000	48.0000	48.0000	-48.0000	-48.0000	5000000.
48.0000	-48.0000	-48.0000	48.0000	48.0000	-48.0000	5000000.
48.0000	48.0000	-48.0000	48.0000	48.0000	48.0000	5000000.
35.2000	64.0000	64.0000	35.2000	-64.0000	64.0000	6690000.
35.2000	-64.0000	64.0000	35.2000	-64.0000	-64.0000	6690000.
35.2000	-64.0000	-64.0000	35.2000	64.0000	-64.0000	6690000.
35.2000	64.0000	-64.0000	35.2000	64.0000	64.0000	6690000.
-35.2000	64.0000	64.0000	-35.2000	-64.0000	64.0000	6690000.
-35.2000	-64.0000	64.0000	-35.2000	-64.0000	-64.0000	6690000.
-35.2000	-64.0000	-64.0000	-35.2000	64.0000	-64.0000	6690000.
-35.2000	64.0000	-64.0000	-35.2000	64.0000	64.0000	6690000.
-48.0000	48.0000	48.0000	-48.0000	-48.0000	48.0000	-5000000.
-48.0000	-48.0000	48.0000	-48.0000	-48.0000	-48.0000	-5000000.
-48.0000	-48.0000	-48.0000	-48.0000	48.0000	-48.0000	-5000000.
-48.0000	48.0000	-48.0000	-48.0000	48.0000	48.0000	-5000000.
80.0000	80.0000	80.0000	-80.0000	80.0000	80.0000	-4165000.
-80.0000	80.0000	80.0000	-80.0000	-80.0000	80.0000	-4165000.
-80.0000	-80.0000	80.0000	80.0000	-80.0000	80.0000	-4165000.
80.0000	-80.0000	80.0000	80.0000	80.0000	80.0000	-4165000.
87.3000	87.3000	48.0000	-87.3000	87.3000	48.0000	-4550000.
-87.3000	87.3000	48.0000	-87.3000	-87.3000	48.0000	-4550000.
-87.3000	-87.3000	48.0000	87.3000	-87.3000	48.0000	-4550000.
87.3000	-87.3000	48.0000	87.3000	87.3000	48.0000	-4550000.
87.3000	87.3000	-48.0000	-87.3000	87.3000	-48.0000	-4550000.
-87.3000	87.3000	-48.0000	-87.3000	-87.3000	-48.0000	-4550000.
-87.3000	-87.3000	-48.0000	87.3000	-87.3000	-48.0000	-4550000.
87.3000	-87.3000	-48.0000	87.3000	87.3000	-48.0000	-4550000.
80.0000	80.0000	-80.0000	-80.0000	80.0000	-80.0000	4165000.
-80.0000	80.0000	-80.0000	-80.0000	-80.0000	-80.0000	4165000.
-80.0000	-80.0000	-80.0000	80.0000	-80.0000	-80.0000	4165000.
80.0000	-80.0000	-80.0000	80.0000	80.0000	-80.0000	4165000.

TABLE Sample Input - Current Element End Points and Currents.

TOTAL CURRENT IN +X FIELD COIL= 6690000. AMPS.
TOTAL CURRENT IN -X FIELD COIL= 6690000. AMPS.
TOTAL CURRENT IN +Z FIELD COIL= -4550000. AMPS.
TOTAL CURRENT IN -Z FIELD COIL= -4550000. AMPS.

TOTAL CURRENT IN +X GRADIENT COIL= 5000000. AMPS.
TOTAL CURRENT IN -X GRADIENT COIL= -5000000. AMPS.
TOTAL CURRENT IN +Z GRADIENT COIL= -4165000. AMPS.
TOTAL CURRENT IN -Z GRADIENT COIL= 4165000. AMPS.

X	Y	Z	BX	BY	BZ	DBX/DX	DBX/DY	DBX/DZ	DBY/DY	DBY/DZ	DBZ/DZ	FX	FY	FZ
INCHES			GAUSS			GAUSS/IN.						LBF/CU.IN.		
30.0	0.0	-40.0	56894.7	0.0	8269.9	1292.1	0.0	43.9	-18.9	0.0	-1273.2	21.973	0.0	-2.389
30.0	0.0	-38.0	56894.0	0.0	5765.6	1251.4	0.0	-45.1	-25.8	0.0	-1225.6	21.212	0.0	-2.980
30.0	0.0	-36.0	56718.2	0.0	3386.3	1182.5	0.0	-128.6	-32.5	0.0	-1150.0	20.055	0.0	-3.367
30.0	0.0	-34.0	56390.9	0.0	1176.8	1096.6	0.0	-195.4	-39.1	0.0	-1057.5	18.678	0.0	-3.719
30.0	0.0	-32.0	55949.9	0.0	-839.7	1003.9	0.0	-242.1	-45.3	0.0	-958.6	17.227	0.0	-3.894
30.0	0.0	-30.0	55435.2	0.0	-2658.6	912.3	0.0	-269.7	-51.2	0.0	-861.1	15.803	0.0	-3.901
30.0	0.0	-28.0	54881.9	0.0	-4288.2	826.5	0.0	-281.2	-56.7	0.0	-769.8	14.466	0.0	-3.769
30.0	0.0	-26.0	54318.5	0.0	-5744.0	749.4	0.0	-280.3	-61.8	0.0	-687.5	13.247	0.0	-3.530
30.0	0.0	-24.0	53766.8	0.0	-7045.0	681.7	0.0	-270.2	-66.5	0.0	-615.2	12.159	0.0	-3.214
30.0	0.0	-22.0	53242.1	0.0	-8211.5	623.7	0.0	-253.6	-70.8	0.0	-552.9	11.201	0.0	-2.845
30.0	0.0	-20.0	52755.2	0.0	-9262.8	574.6	0.0	-232.8	-74.6	0.0	-499.9	10.366	0.0	-2.442
30.0	0.0	-18.0	52312.8	0.0	-10217.0	533.7	0.0	-209.2	-78.0	0.0	-455.7	9.643	0.0	-2.018
30.0	0.0	-16.0	51919.2	0.0	-11090.7	500.2	0.0	-184.1	-81.0	0.0	-419.2	9.022	0.0	-1.582
30.0	0.0	-14.0	51576.7	0.0	-11898.3	473.1	0.0	-158.3	-83.5	0.0	-389.5	8.491	0.0	-1.140
30.0	0.0	-12.0	51286.2	0.0	-12652.8	451.6	0.0	-132.3	-85.7	0.0	-365.9	8.040	0.0	-0.697
30.0	0.0	-10.0	51047.5	0.0	-13365.7	435.2	0.0	-106.4	-87.4	0.0	-347.8	7.659	0.0	-0.254
30.0	0.0	-8.0	50860.3	0.0	-14047.0	423.1	0.0	-80.9	-88.8	0.0	-334.3	7.342	0.0	0.188
30.0	0.0	-6.0	50723.5	0.0	-14705.8	415.0	0.0	-55.9	-89.8	0.0	-325.2	7.082	0.0	0.630
30.0	0.0	-4.0	50636.2	0.0	-15350.3	410.4	0.0	-31.5	-90.4	0.0	-319.9	6.872	0.0	1.071
30.0	0.0	-2.0	50597.1	0.0	-15988.0	409.0	0.0	-7.7	-90.8	0.0	-318.3	6.709	0.0	1.515
30.0	0.0	0.0	50605.1	0.0	-16625.6	410.8	0.0	15.6	-90.9	0.0	-320.0	6.591	0.0	1.962
30.0	0.0	2.0	50659.2	0.0	-17269.9	415.6	0.0	38.4	-90.6	0.0	-324.9	6.514	0.0	2.414
30.0	0.0	4.0	50758.4	0.0	-17927.4	423.3	0.0	60.7	-90.2	0.0	-333.1	6.479	0.0	2.876
30.0	0.0	6.0	50901.8	0.0	-18604.5	434.0	0.0	82.6	-89.5	0.0	-344.5	6.486	0.0	3.349
30.0	0.0	8.0	51088.4	0.0	-19307.8	448.0	0.0	104.0	-88.6	0.0	-359.4	6.537	0.0	3.836
30.0	0.0	10.0	51317.4	0.0	-20044.4	465.4	0.0	125.0	-87.5	0.0	-377.9	6.636	0.0	4.341
30.0	0.0	12.0	51587.8	0.0	-20821.9	486.7	0.0	145.4	-86.3	0.0	-400.4	6.787	0.0	4.868
30.0	0.0	14.0	51898.4	0.0	-21648.9	512.2	0.0	165.1	-84.9	0.0	-427.3	6.997	0.0	5.418
30.0	0.0	16.0	52247.4	0.0	-22534.6	542.6	0.0	183.7	-83.3	0.0	-459.3	7.275	0.0	5.995
30.0	0.0	18.0	52632.4	0.0	-23489.8	578.5	0.0	200.9	-81.6	0.0	-496.9	7.634	0.0	6.601
30.0	0.0	20.0	53049.7	0.0	-24526.5	620.8	0.0	215.9	-79.8	0.0	-540.9	8.085	0.0	7.233
30.0	0.0	22.0	53493.9	0.0	-25658.3	670.1	0.0	227.7	-77.9	0.0	-592.1	8.647	0.0	7.889
30.0	0.0	24.0	53957.3	0.0	-26900.2	727.2	0.0	234.7	-76.0	0.0	-651.2	9.337	0.0	8.561
30.0	0.0	26.0	54428.4	0.0	-28268.6	792.5	0.0	235.1	-73.9	0.0	-719.6	10.174	0.0	9.231
30.0	0.0	28.0	54891.4	0.0	-29780.3	866.1	0.0	226.1	-71.8	0.0	-794.3	11.174	0.0	9.877
30.0	0.0	30.0	55324.7	0.0	-31450.9	946.9	0.0	204.8	-69.6	0.0	-877.3	12.345	0.0	10.458
30.0	0.0	32.0	55700.0	0.0	-33292.4	1032.2	0.0	167.6	-67.4	0.0	-964.8	13.690	0.0	10.925
30.0	0.0	34.0	55992.8	0.0	-35309.8	1117.2	0.0	111.8	-65.2	0.0	-1052.0	15.138	0.0	11.214
30.0	0.0	36.0	56134.2	0.0	-37495.1	1194.2	0.0	36.5	-63.0	0.0	-1131.1	16.634	0.0	11.262

TABLE Sample Output.

APPENDIX C

COMPUTER PROGRAM FOR PLOTTING THE MAGNITUDE AND DIRECTION OF THE MAGNETIC FORCE ON A FERROMAGNETIC SPHERE DUE TO AN ARRAY OF STRAIGHT LINE CURRENT ELEMENTS

The following is a brief description of a computer program "PLOT" written in Fortran IV language which is used to plot the magnitude and direction of magnetic force on a ferromagnetic sphere using the relations outlined in Appendix A.

Plotting Program ("PLOT")

PLOT was developed to provide a qualitative graphical display of the distribution of the magnetic force field on a ferromagnetic sphere due to an array of straight-line current elements.

Each display is in two parts, consisting of:

a) A plot of the magnitude of the magnetic force, and

b) A plot of the angle of the magnetic forces for an array of field points in a y = const. plane.

The magnetic forces to be plotted are calculated as described in Appendix B, for TABLE. A symbol representing the force range in which the magnetic force lies is then matched to the force. This symbol is placed in the J^{th} and K^{th} spacial position (corresponding to Jx increments and Kz increments from a starting point) of the force display, and the entire display array is then printed. In this way, the magnetic force at discrete points in the xz plane is represented by a symbol in the output field. Detailed examples of the use of this technique can be found in Ref. 10.

Since the range of force magnitude is generally large, and a multitude of symbols would be required to represent a typical range of constant force increments, a logarithmic increment has been used instead. Thus, each force range represents a constant percentage of the value of the upper (or lower) limit of the range.

By this method, the force magnitude is first reduced to an exponent of a base number, i.e.,

$$F = A^n \tag{C-1}$$

$$n = \frac{\log F}{\log A} \tag{C-2}$$

Then, the exponent is reduced to a positive integer

$$n \rightarrow N \qquad N \text{ is an integer greater than zero}$$

Now, the symbol representing F will be that symbol in the array having subscript N, i.e.,

$$F(K,J) = \text{FSymbol } (N) \tag{C-3}$$

where FSymbol(N) is the N^{th} symbol in the array representing force magnitudes.

Reducing n to a positive integer can be accomplished in several ways. In PLOT, n was reduced by adding 1 and truncating in the case of positive exponents or adding 1 plus the magnitude of the largest negative exponent to be considered in the case of negative exponents.

$$\text{for } n > 0, \qquad N = \text{truncated } (n+1) \tag{C-4}$$

$$\text{for } n < 0, \qquad N = \text{truncated } (n + 1 + |a|) \tag{C-5}$$

a = magnitude of largest negative exponent
(i.e., $F = 0$ when $n < -a$)

Thus, the force represented by FSymbol(N) has a magnitude between $A^{n+\varepsilon}$ and $A^{N+1-\varepsilon}$, where ε is small. (See the sample output for PLOT, page 102 for an example.)

The array representing angle magnitudes is similarly constructed. In this case, a constant angle increment is used so that the symbol subscript is calculated by dividing the angle by the increment, adding one, and truncating, i.e.,

$$\theta = \tan^{-1} \frac{F_x}{F_z} \qquad \text{(C-6)}$$

$$M = \text{truncated } \left(\frac{\theta}{\Delta\theta} + 1\right) \qquad \text{(C-7)}$$

$$\theta(K,J) = \text{Symbol } (M) \qquad \text{(C-8)}$$

Again, the θ array is printed so that a symbol is associated with the angle at each point in the xz plane.

```
C                          ARTIFICAL GRAVITY-PLOT
C
C
C A CODE TO CALCULATE THE FORCE ON A MAGNETIC BODY IN A FIELD PRODUCED
C BY COILS CONSISTING OF STRAIGHT LINE CURRENT ELEMENTS.  THE MAGNETIC
C FORCE AND ANGLE ARE FIELD PLOTTED IN THE XZ PLANE.
C CURRENT ELEMENTS ARE COUNTED COUNTER-CLOCKWISE ABOUT THE CORRESPONDING
C COORDINATE DIRECTIONS.  ALL CURRENTS ARE POSITIVE COUNTER-CLOCKWISE.
C
C
C                    INPUT VARIABLE LIST
C
C VARIABLE NAME                            DEFINITION
C
C       FMT1            FORMAT STATEMENTS FOR INPUT AND OUTPUT OPERATIONS
C       FMT2            REQUIRING VARIABLE FORMAT.
C       FMT3             ''      ''     ''     ''     ''     ''     ''
C       FMT4             ''      ''     ''     ''     ''     ''     ''
C       ANLG            THE BASE FOR LOGARITHMIC FORCE PLOTS.
C       ANMX            ABSOLUTE VALUE OF THE LARGEST NEGATIVE POWER
C                       OF ANLG TO BE USED IN FORCE PLOTS, PLUS 1.
C       LBNMX           THE NUMBER OF PLOTTING SYMBOLS REPRESENTING FORCES
C                       OF MAGNITUDE GREATER THAN 1.
C       SYMB1           PLOTTING SYMBOLS FOR FORCE PLOTS.
C       SYMB2            ""     ""     ""     ""     ""     ""
C       SYMB3            ""     ""     ""     ""     ""     ""
C       SYMB4            ""     ""     ""     ""     ""     ""
C       DCT,ASTER        ""     ""     ""     ""     ""     ""
C       DTHETA          ANGLE INCREMENT FOR ANGLE PLOT.
C       NANG            NUMBER OF SYMBOLS FOR ANGLE PLOT.
C       ASYMB1          PLOTTING SYMBOLS FOR ANGLE PLOTS.
C       ASYMB2           ""     ""     ""     ""     ""     ""
C       DA              DEMAGNETIZING CONSTANT FOR THE MODEL.
C       AMS             SATURATION MAGNETIZATION FOR THE MODEL.
C       XKT             MAGNETIC FORCE CONSTANT.
C       XMU             MAGNETIC PERMEABILITY OF FREE SPACE.
```

```
C       INM            NUMBER CF CATA SETS FOR THIS RUN.
C       INPOPT         INPUT OPTICN.
C       CONFG          A DESCRIPTION OF THE MAGNET CONFIGURATION.
C       IM             NUMBER OF INCREMENTS IN THE X-DIRECTION.
C       JM             NUMBER OF INCREMENTS IN THE Y-DIRECTION.
C       KM             NUMBER OF INCREMENTS IN THE Z-DIRECTION.
C       LM             TOTAL NUMBER OF CURRENT ELEMENTS
C       DX             'DELTA' X.
C       DY             'DELTA' Y.
C       DZ             'DELTA' Z.
C       X(1)           X COORDINATE OF STARTING POINT FOR INCREMENTING
C       Y(1)           Y COORDINATE OF STARTING POINT FOR INCREMENTING.
C       Z(1)           Z COORDINATE OF STARTING POINT FOR INCREMENTING.
C       X1,Y1,Z1       COORDINATES OF THE END POINTS OF THE STRAIGHT LINE
C       X2,Y2,Z2       CURRENT ELEMENTS MAKING UP THE COILS.
C       CUR            MAGNITUDE OF THE CURRENT IN AMPERES,+ FROM 1 TO 2.
C       CURT           CURRENT FLOWING IN A LOOP OF FOUR CURRENT ELEMENTS.
C       CURX1          THE TOTAL CURRENT IN THE +X FIELD COIL.
C       CURX2          THE TOTAL CURRENT IN THE +X GRADIENT COIL.
C       CURX3          THE TOTAL CURRENT IN THE -X FIELD COIL.
C       CURX4          THE TOTAL CURRENT IN THE -X GRADIENT COIL.
C       CURZ1          THE TOTAL CURRENT IN THE +Z FIELD COIL.
C       CURZ2          THE TOTAL CURRENT IN THE +Z GRADIENT COIL.
C       CURZ3          THE TOTAL CURRENT IN THE -Z FIELD COIL.
C       CURZ4          THE TOTAL CURRENT IN THE -Z GRADIENT COIL.
C
      DIMENSION CONFG(18),ANGL(200,200),FORCE(100,100),ASYMB1(50),ASYMB2
     1(50),ANG1(50),ANG2(50),X(100),Y(100),Z(100),S(3),T(3),DP(3),DS(3),
     2DC(3),DG(3),X1(500),Y1(500),Z1(500),X2(500),Y2(500),Z2(500),CUR(50
     30),CURT(400),SYMB1(20),SYMB2(80),SYMB3(80),SYMB4(80),FMIN1(12),FMA
     4X1(12),FMIN2(80),FMAX2(80),FORCY(100,100),DD(3),FMT1(3),FMT2(2),FM
     5T3(3),FMT4(2)
  100 FORMAT(4I4,6F8.4)
  110 FORMAT(6F8.4,F10.0)
  112 FORMAT(F5.3,F9.2,2F12.11,2I5)
  114 FORMAT(//)
```

```
115  FORMAT(///)
116  FORMAT(6X,6HINCHES,16X,5HGAUSS,29X,9HGAUSS/IN.,27X,10HLBF/CU.IN.)
118  FORMAT(1H1)
119  FORMAT(3A4,2A3,3A4,2A3)
127  FORMAT(24X,10HINPUT DATA)
128  FORMAT(2X,6HX1(IN),2X,6HY1(IN),2X,6HZ1(IN),2X,6HX2(IN),2X,6HY2(IN)
    1,2X,6HZ2(IN),1X,10HCURRENT(A))
129  FORMAT(1X,6F8.4,F10.0)
131  FORMAT(2F10.0)
132  FORMAT(3X,6HSYMBOL,5X,24HFORCE RANGE (LBF/CU.IN.))
133  FORMAT(5X,1A2,7X,F9.3,2X,2HTO,1X,F9.3)
135   FORMAT(3X,29HPLOT OF FX IN X-Z PLANE AT Y=,F6.1,1X,3HIN.)
136  FORMAT(3X,27HHORIZONTAL COORDINATE IS X ,F5.1,1X,3HIN.,4H TO ,F5.1
    1,1X,3HIN.)
137  FORMAT(3X,25HVERTICAL COORDINATE IS Z ,F5.1,1X,3HIN.,4H TO ,F5.1,1
    1X,3HIN.)
138  FORMAT(3X,29HPLOT OF FY IN X-Z PLANE AT Y=,F6.1,1X,3HIN.)
142  FORMAT(1X,120H. . . . . . . . . . . . . . . . . . . . . . . . .
    1. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .
    2. )
143  FORMAT(2F8.4,I5)
144  FORMAT(2A2)
145  FORMAT(3X,25H* DENOTES SATURATION LINE)
150  FORMAT(4F10.0)
151  FORMAT(3X,32HTOTAL CURRENT IN +X FIELD COIL= ,F10.0,6H AMPS.,3X,35
    1HTOTAL CURRENT IN +X GRADIENT COIL= ,F10.0,6H AMPS.)
152  FORMAT(3X,32HTOTAL CURRENT IN -X FIELD COIL= ,F10.0,6H AMPS.,3X,35
    1HTOTAL CURRENT IN -X GRADIENT COIL= ,F10.0,6H AMPS.)
153  FORMAT(3X,32HTOTAL CURRENT IN +Z FIELD COIL= ,F10.0,6H AMPS.,3X,35
    1HTOTAL CURRENT IN +Z GRADIENT COIL= ,F10.0,6H AMPS.)
154  FORMAT(3X,32HTOTAL CURRENT IN -Z FIELD COIL= ,F10.0,6H AMPS.,3X,35
    1HTOTAL CURRENT IN -Z GRADIENT COIL= ,F10.0,6H AMPS.)
155  FORMAT(18A4)
159  FORMAT(24X,18A4)
160  FORMAT(F6.3,I5)
162  FORMAT(3X,6HSYMBOL,9X,18HANGLE RANGE (DEG.))
```

```
 163  FORMAT(3X,47HPLOT OF TOTAL MAGNETIC FORCE IN X-Z PLANE AT Y=,F6.1,
     11X,3HIN.)
 164  FORMAT(3X,47HPLOT OF MAGNETIC FORCE ANGLE IN X-Z PLANE AT Y=,F6.1,
     11X,3HIN.)
 165  FORMAT(F10.0)
 166  FORMAT(6F8.4)
C INPUT THE FORMAT CODES FOR VARIABLE FORMATTED STATEMENTS.
C FMT1 SPECIFIES THE SIZE OF THE FORCE FIELD PLOT.  FOR EXAMPLE,
C FMT1=(1H .,31A2) CORRESPONDS  TO A PLOTTING REGION X=-30. TO X=+30.
C WITH AN INCREMENT (I.E. DX) OF 2. INCHES.
C FMT2 IS THE INPUT CODE FOR PLOTTING SYMBOLS REPRESENTING FORCES LESS
C THAN 1. AND FMT3 IS FOR SYMBOLS REPRESENTING  FORCES GREATER THAN 1.
C EXAMPLES ARE FMT2=(12A2) AND FMT3=(36A2/22A2) FOR 12 AND 58 SYMBOLS
C RESPECTIVIELY.  FMT4 IS FOR ANGLE SYMBOLS, FMT4=(NANGMA2).
      READ(5,119) FMT1,FMT2,FMT3,FMT4
C INPUT THE BASE FOR FORCE PLOTTING AND ABSOLUTE VALUE OF THE LARGEST
C NEGATIVE POWER OF THE BASE, PLUS 1, AND THE NUMBER OF SYMBOLS FOR
C GREATER THAN ONE.
      READ(5,143) ANLG,ANMX,LBNMX
      LANMX=ANMX-1.
C INPUT LETTER VALUES FOR THE FORCE AND ANGLE MAGNITUDES FOR PLOTTING.
C SYMB1 AND SYMB2 ARE SYMBOLS FOR POSITIVE FORCE MAGNITUDES LESS THAN
C AND  GREATER THAN 1 RESPECTIVELY.
      READ(5,FMT2) (SYMB1(N1),N1=1,LANMX)
      READ(5,FMT3) (SYMB2(N2),N2=1,LBNMX)
C SYMB3 AND SYMB4 ARE SYMBOLS FOR NEGATIVE FORCE MAGNITUDES LESS THAN
C AND GREATER THAN ONE RESPECTIVELY.
      READ(5,FMT2) (SYMB3(N3),N3=1,LANMX)
      READ(5,FMT3) (SYMB4(N4),N4=1,LBNMX)
      READ(5,144) DOT,ASTER
C INPUT THE MAGNITUDE OF THE ANGLE INCREMENT AND THE NUMBER OF ANGLE
C INCREMENTS.
      READ(5,160) DTHETA,NANGM
C ASYMB1 AND ASYMB2 ARE SYMBOLS FOR POSITIVE AND NEGATIVE ANGLE
C MAGNITUDES RESPECTIVELY.
      READ(5,FMT4) (ASYMB1(NTH1),NTH1=1,NANGM)
```

```
      READ(5,FMT4) (ASYMB2(NTH2),NTH2=1,NANGM)
C INPUT THE MAGNETIZATION CONSTANT FOR THE GEOMETRY OF THE BODY, THE
C MAGNITUDE OF THE SATURATION MAGNETIZATION FOR THE MATERIAL, THE
C MAGNITIC FORCE CONSTANT, THE PERMEABILITY OF FREE SPACE, THE NUMBER OF
C DATA SETS, AND THE INPUT OPTION.
C INPOPT=1 CORRESPONDS TO INPUTING THE CURRENT IN EACH ELEMENT.  INPOPT
C =2 CORRESPONDS TO INPUTING THE CURRENT  IN EACH LOOP OF FOUR ELEMENTS.
      READ(5,112) DA,AMS,XKT,XMU,INM,INPOPT
      XMP=XMU/(4.*3.1416)
C CALCULATE THE LOGARITHM OF THE BASE FOR FORCE PLOTS.
      XLG=ALOG(ANLG)
      AM1=1.05*AMS
C CALCULATE THE MAGNITUDES OF FORCE RANGES FOR FORCE PLOTS.
      DO 400 JO=1,LANMX
      J1=JO-(LANMX+1)
      J2=J1+1
      FMIN1(JO)=ANLG**J1
      FMAX1(JO)=ANLG**J2
  400 CONTINUE
      DO 900 KO=1,LBNMX
      K1=KO-1
      K2=KO
      FMIN2(KO)=ANLG**K1
      FMAX2(KO)=ANLG**K2
  900 CONTINUE
      FMIN1(1)=0.
      FMIN2(1)=1.0
C CALCULATE THE ANGLE RANGES FOR ANGLE PLOTING.
      ANG1(1)=0.
      ANG2(1)=DTHETA
      NANGN=NANGM-1
      DO 504 NANG=1,NANGN
      ANG1(NANG+1)=ANG1(NANG)+DTHETA
      ANG2(NANG+1)=ANG2(NANG)+DTHETA
  504 CONTINUE
      IF(INPOPT.EQ.1) GO TO 171
```

```
C INPOPT=2
C INPUT THE DESCRIPTION OF THE CONFIGURATION (LE. 72 CHARACTERS).
      READ(5,155) (CONFG(IC),IC=1,18)
C INPUT THE MAXIMUM NUMBER OF X INCREMENTS, THE MAXIMUM NUMBER OF Y
C INCREMENTS, THE MAXIMUM NUMBER OF Z INCREMENTS, THE NUMBER OF CURRENT
C ELEMENTS, DELTA X, Y, AND Z, AND THE CORNER POINT OF THE PLOT (MAXIMUM
C VALUE OF X,MINIMUM VALUE OF Z, AND Y POSITION).
C NOTE THAT DX IS NEGATIVE AND DZ IS POSITIVE.
      READ(5,100) IM,JM,KM,LM,DX,DY,DZ,X(1),Y(1),Z(1)
C INPUT THE END POINTS OF THE CURRENT ELEMENTS.
      READ(5,166) (X1(NI),Y1(NI),Z1(NI),X2(NI),Y2(NI),Z2(NI),NI=1,LM)
      NTM=LM/4
C INPOPT=1
 171  DO 200 INP=1,INM
      IF(INPOPT.EQ.2) GO TO 174
C INPUT THE DESCRIPTION OF THE CONFIGURATION (LE. 72 CHARACTERS).
      READ(5,155) (CONFG(IC),IC=1,18)
C INPUT THE MAXIMUM NUMBER OF X INCREMENTS, THE MAXIMUM NUMBER OF Y
C INCREMENTS, THE MAXIMUM NUMBER OF Z INCREMENTS, THE NUMBER OF CURRENT
C ELEMENTS, DELTA X, Y, AND Z, AND THE CORNER POINT OF THE PLOT (MAXIMUM
C VALUE OF X,MINIMUM VALUE OF Z, AND Y POSITION).
C NOTE THAT DX IS NEGATIVE AND DZ IS POSITIVE.
      READ(5,100) IM,JM,KM,LM,DX,DY,DZ,X(1),Y(1),Z(1)
C INPUT THE END POINTS OF THE CURRENT ELEMENTS AND THE CURRENT FLOWING
C IN EACH ELEMENT.
      READ(5,110) (X1(N),Y1(N),Z1(N),X2(N),Y2(N),Z2(N), CUR(N),N=1,LM)
      GO TO 173
C INPOPT=2
C INPUT THE CURRENT FLOWING IN EACH LOOP OF FOUR CURRENT ELEMENTS.
 174  READ(5,165) (CURT(NT),NT=1,NTM)
      KL=-3
      KL1=0
C ASSIGN CURRENT MAGNITUDES TO EACH CURRENT ELEMENT.
      DO 170 JT=1,NTM
      KL=KL+4
      KL1=KL1+4
```

```
      DO 170 NT1=KL,KL1
      CUR(NT1)=CURT(JT)
  170 CONTINUE
C INPUT THE TOTAL CURRENT IN BOTH GRADIENT AND FIELD COILS(TOTAL
C AMPERE TURNS).
  173 READ(5,150) CURX1,CURX2,CURX3,CURX4
      READ(5,150) CURZ1,CURZ2,CURZ3,CURZ4
      WRITE(6,118)
C OUTPUT THE CHARACTERISTICS OF THE CURRENT ELEMENTS IN THIS CALCULATION
      WRITE(6,159) (CONFG(IOC),IOC=1,18)
      WRITE(6,115)
      WRITE(6,127)
      WRITE(6,128)
      WRITE(6,129) (X1(N1),Y1(N1),Z1(N1),X2(N1),Y2(N1),Z2(N1),CUR(N1),
     1N1=1,LM)
C OUTPUT THE SYMBOL AND CORRESPONDING FORCE RANGE FOR PLOTTING.
      WRITE(6,118)
      WRITE(6,132)
      WRITE(6,133) (SYMB1(IO),FMIN1(IO),FMAX1(IO),IO=1,LANMX)
      WRITE(6,133) (SYMB2(IN),FMIN2(IN),FMAX2(IN),IN=1,LBNMX)
C OUTPUT THE SYMBOL AND CORRESPONDING ANGLE RANGE FOR ANGLE PLOTTING.
      WRITE(6,115)
      WRITE(6,162)
      WRITE(6,133) (ASYMB1(NANGO),ANG1(NANGO),ANG2(NANGO),NANGO=1,NANGM)
C CALCULATE AND STORE THE SPACIAL COORDINATES FOR THIS CALCULATION.
      ITLM=IM-1
      JTLM=JM-1
      KTLM=KM-1
      DO 2002 ITL=1,ITLM
 2002 X(ITL+1)=X(ITL)+DX
      DO 2000 JTL=1,JTLM
 2000 Y(JTL+1)=Y(JTL)+DY
      DO 2001 KTL=1,KTLM
 2001 Z(KTL+1)=Z(KTL)+DZ
C BEGIN THE MAGNETIC FORCE CALCULATION.
      DO 200 J=1,JM
```

```
      DO 300 I=1,IM
      DO 300 K=1,KM
C CLACULATE THE MAGNETIC FORCES DUE TO EACH CURRENT ELEMENT.
C INITUALIZE THE VALUES TO BE SUMMED.
      BX=0.0
      BY=0.
      BZ=0.0
      BXX=0.
      BXY=0.
      BXZ=0.
      BYX=0.
      BYY=0.
      BYZ=0.
      BZX=0.
      BZY=0.
      BZZ=0.
      DO 210 L=1,LM
C CALCULATE A,B,C,D,E,AND F
      A=(X1(L)-X(I))/39.37
      B=(X2(L)-X(I))/39.37
      C=(Y1(L)-Y(J))/39.37
      D=(Y2(L)-Y(J))/39.37
      E=(Z1(L)-Z(K))/39.37
      F=(Z2(L)-Z(K))/39.37
C SUBSCRIPT A,C,E,B,D,F FOR LATER USE
      S(1)=A
      S(2)=C
      S(3)=E
      T(1)=B
      T(2)=D
      T(3)=F
C CALCUALTE U, V, AND W.
      U=C*F-D*E
      V=E*B-F*A
      W=A*D-B*C
C CALCULATE RHO1 AND RHO2.
```

```
      R1=(A*A+C*C+E*E)**0.5
      R2=(B*B+D*D+F*F)**0.5
C CALCULATE THE SUM, PRODUCT, DOT PRODUCT, AND CROSS PRODUCT OF RHO1
C AND RHO2.
      RS=R1+R2
      RM=R1*R2
      RDR=A*B+C*D+E*F
      RXR=U+V+W
C CALCULATE THE DERIVITIVES OF THE SUM, ETC. OF RHO1 AND RHO2.
      DO 220 M=1,3
      DP(M)=-(S(M)*R2/R1+T(M)*R1/R2)
      DS(M)=-(S(M)/R1+T(M)/R2)
      DD(M)=-(S(M)+T(M))
  220 CONTINUE
      DC(1)=F-E+C-D
      DC(2)=E-F+B-A
      DC(3)=D-C+A-B
C CALCULATE AND TEST H TO DETERMINE EQUATION FOR G TO BE USED.
      H=(RM+RDR)/RM
      IF(H-0.01) 2,1,1
 1    G=RS/(RM*(RM+RDR))
C CALCULATE G AND ITS DERIVITIVES IN THE X,Y,Z DIRECTIONS.
      DO 230 M1=1,3
      DGA=RM*(RM+RDR)*DS(M1)
      DGB=RS*(RM*(DP(M1)+DD(M1))+DP(M1)*(RM+RDR))
      DG(M1)=(DGA-DGB)/(RM*(RM+RDR))**2
  230 CONTINUE
      GO TO 3
 2    G=((RS)*(RM-RDR))/(RM*RXR*RXR)
      DO 240 M2=1,3
      DGA=(RS*(DP(M2)-DD(M2))+DS(M2)*(RM-RDR))*RM*RXR**2
      DGB=RS*(RM-RDR)*(RM*2.*RXR*DC(M2)+DP(M2)*RXR**2)
      DG(M2)=(DGA-DGB)/(RM*RXR**2)**2
  240 CONTINUE
C CALCUALTE THE FIELD CONTRIBUTIONS OF EACH CURRENT ELEMENT.
 3    DGX=DG(1)
```

```
      DGY=DG(2)
      DGZ=DG(3)
      CURP=XMP*CUR(L)*10000./39.37
      CURM=XMP*CUR(L)*G*10000.
      BX1=CURM*U
      BY1=CURM*V
      BZ1=CURM*W
C CALCULATE THE GRADIENT CONTRIBUTIONS OF EACH CURRENT ELEMENT.
      BXX1=CURP*U*DGX
      BXY1=CURP*(G*(E-F)+U*DGY)
      BXZ1=CURP*(G*(D-C)+U*DGZ)
      BYY1=CURP*V*DGY
      BYZ1=CURP*(G*(A-B)+V*DGZ)
      BZZ1=CURP*DGZ*W
C SUM THE INDIVUAL CONTRIBUTIONS TO THE FIELD AND GRADIENT TO GET THE
C TOTAL FIELD AND GRADIENTS.
      BX=BX+BX1
      BY=BY+BY1
      BZ=BZ+BZ1
      BXX=BXX+BXX1
      BXY=BXY+BXY1
      BXZ=BXZ+BXZ1
      BYY=BYY+BYY1
      BYZ=BYZ+BYZ1
      BZZ=BZZ+BZZ1
  210 CONTINUE
C CALCULATE AND TEST THE MAGNETIZATION OF THE BODY FOR SATURATION.
      XDK=XKT/CA
      RB=(BX**2+BY**2+BZ**2)**0.5
      AM=(1/CA)*RB
      IF(AM-AMS) 10,10,11
C CALCULATE THE FORCES PRODUCED ON THE BODY.
 10   FX=XDK*(BX*BXX+BY*BXY+BZ*BXZ)
      FZ=XDK*(BX*BXZ+BY*BYZ+BZ*BZZ)
      FY=XDK*(BX*BXY+BY*BYY+BZ*BYZ)
      GO TO 12
```

```
C CALCULATE THE COMPONENTS OF THE MAGNETIZATION AT SATURATION.
 11    BMY=(BY/RB)*AMS
       BMX=(BX/RB)*AMS
       BMZ=(BZ/RB)*AMS
C CALCULATE THE FORCES PRODUCED ON THE BODY.
       FX=XKT*(BMX*BXX+BMY*BXY+BMZ*BXZ)
       FZ=XKT*(BMX*BXZ+BMY*BYZ+BMZ*BZZ)
       FY=XKT*(BMX*BXY+BMY*BYY+BMZ*BYZ)
C DEFINE THE SATURATION LINE BY ASSIGNING ASTER (*) TO THE FORCES AND
C ANGLES ALONG THE SATURATION LINE.
       IF(AM-AM1) 710,710,12
 710   FORCE(K,I)=ASTER
       FORCY(K,I)=ASTER
       ANGL(K,I)=ASTER
       GO TO 300
 12    CONTINUE
C DEFINE THE X AND Z AXES BY ASSIGNING DOT (.) TO THE FORCES AND ANGLES
C ALONG X=0. AND Z=0.
       IF(Z(K).EQ.0..OR.X(I).EQ.0.) GO TO 14
       GO TO 15
 14    FORCE(K,I)=DOT
       FORCY(K,I)=DOT
       ANGL(K,I)=DOT
       GO TO 300
C CALCULATE THE TOTAL MAGNETIC FORCE AND ANGLE IN THE X-Z PLANE.
 15    FT=(FX*FX+FZ*FZ)**0.5
       ETA=FX/FZ
       THETA=ATAN(ETA)*180./3.1416
C MATCH A SYMBOL WITH THE CORRESPONDING FORCE AND ANGLE MAGNITUDES.
C FORCE MATCHING IS ACCOMPLISHED BY FIRST CALCULATING THE EXPONENT OF
C THE BASE NUMBER, THEN THE EXPONENT IS TRUNCATED TO AN INTEGER
C CORRESPONDING TO A POSITION IN SYMBOL ARRAY.  ANGLE MATCHING IS
C ACCOMPLISHED BY TRUNCATING THE DIVIDEND OF THE ANGLE AND THE ANGLE
C INCREMENT TO DETERMINE THE ARRAY POSITION.
       IF(THETA) 501,502,502
 501   NOA2=1.-(THETA/DTHETA)
```

```
      ANGL(K,I)=ASYMB2(NOA2)
      GO TO 503
502   NOA1=THETA/DTHETA+1.
      ANGL(K,I)=ASYMB1(NOA1)
503   CONTINUE
72    IF(FY) 73,74,74
73    IF(FY+0.001) 75,75,76
75    YN=ALOG(-FY)/XLG
      IF(YN) 77,77,78
77    NUY1=YN+ANMX
      IF(NUY1) 76,76,79
76    NUY1=1
79    FORCY(K,I)=SYMB3(NUY1)
      GO TO 505
78    NUY2=YN+1.
      FORCY(K,I)=SYMB4(NUY2)
      GO TO 505
74    IF(FY-0.001) 80,81,81
81    YP=ALOG(FY)/XLG
      IF(YP) 82,82,83
82    NUY1=YP+ANMX
      IF(NUY1) 80,80,84
80    NUY1=1
84    FORCY(K,I)=SYMB1(NUY1)
      GO TO 505
83    NUY2=YP+1.
      FORCY(K,I)=SYMB2(NUY2)
505   IF(FT) 506,507,507
506   IF(FT+0.001) 508,508,509
508   FN=ALOG(-FT)/XLG
      IF(FN) 510,510,511
510   NUF1=FN+ANMX
      IF(NUF1) 509,509,513
509   NUF1=1
513   FORCE(K,I)=SYMB3(NUF1)
      GO TO 300
```

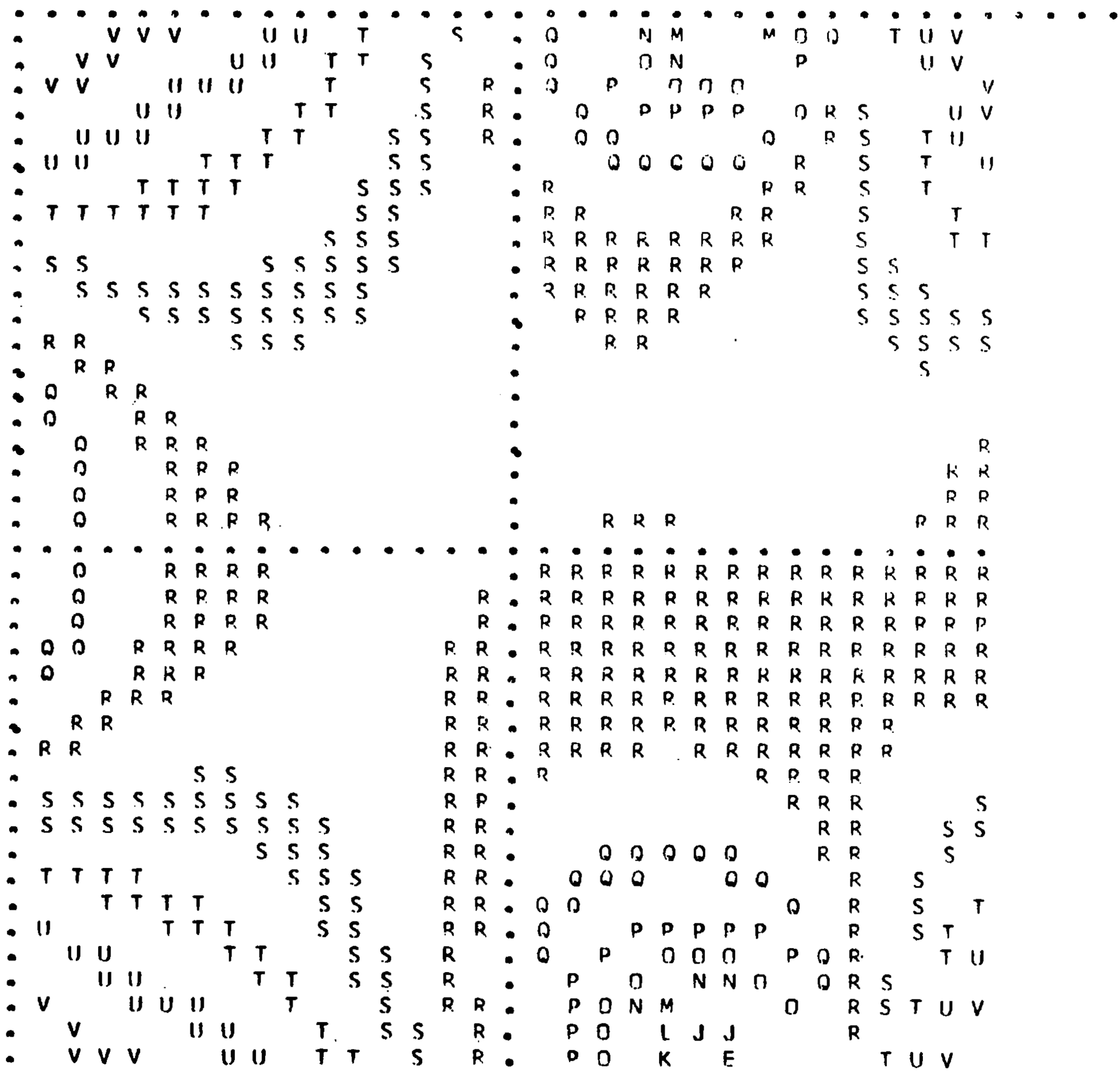
```
 511   NUF2=FN+1.
       FORCE(K,I)=SYMB4(NUF2)
       GO TO 300
 507   IF(FT-0.001) 514,515,515
 515   FP=ALOG(FT)/XLG
       IF(FP) 516,516,517
 516   NUF1=FP+ANMX
       IF(NUF1) 514,514,518
 514   NUF1=1
 518   FORCE(K,I)=SYMB1(NUF1)
       GO TO 300
 517   NUF2=FP+1.
       FORCE(K,I)=SYMB2(NUF2)
 300   CONTINUE
       WRITE(6,118)
C WRITE HEADINGS FOR TOTAL FORCE PLOT.
       WRITE(6,159) (CONFG(IOC),IOC=1,18)
       WRITE(6,115)
       WRITE(6,163) Y(J)
       WRITE(6,136) X(1),X(IM)
       WRITE(6,137) Z(1),Z(KM)
       WRITE(6,151) CURX1,CURX2
       WRITE(6,152) CURX3,CURX4
       WRITE(6,153) CURZ1,CURZ2
       WRITE(6,154) CURZ3,CURZ4
       WRITE(6,114)
       WRITE(6,142)
C PLOT TOTAL FORCE IN X-Z PLANE.
       WRITE(6,FMT1) ((FORCE(KT,IT),IT=1,IM),KT=1,KM)
       WRITE(6,118)
C WRITE HEADINGS FOR ANGLE PLOT.
       WRITE(6,159) (CONFG(IOC),IOC=1,18)
       WRITE(6,115)
       WRITE(6,164) Y(J)
       WRITE(6,136) X(1),X(IM)
       WRITE(6,137) Z(1),Z(KM)
```

```
      WRITE(6,151) CURX1,CURX2
      WRITE(6,152) CURX3,CURX4
      WRITE(6,153) CURZ1,CURZ2
      WRITE(6,154) CURZ3,CURZ4
      WRITE(6,114)
      WRITE(6,142)
C PLOT FORCE ANGLE IN X-Z PLANE.
      WRITE(6,FMT1) ((ANGL(KA,IA),IA=1,IM),KA=1,KM)
      WRITE(6,118)
C WRITE HEACINGS FOR THE FY PLCT.
      WRITE(6,159) (CCNFG(ICC),IOC=1,18)
      WRITE(6,115)
      WRITE(6,138) Y(J)
      WRITE(6,136) X(1),X(IM)
      WRITE(6,137) Z(1),Z(KM)
      WRITE(6,145)
      WRITE(6,151) CURX1,CURX2
      WRITE(6,152) CURX3,CLRX4
      WRITE(6,153) CURZ1,CURZ2
      WRITE(6,154) CURZ3,CURZ4
      WRITE(6,114)
      WRITE(6,142)
C PLOT FY IN THE X-Z PLANE.
      WRITE(6,FMT1) ((FORCY(KY,IY),IY=1,IM),KY=1,KM)
  200 CONTINUE
C END OF PROGRAM.
      CALL EXIT
      END
```

SYMBOL	FORCE RANGE (LBF/CU.IN.)						
	0.0	TO	0.350				
A	0.350	TO	0.386	P	6.116	TO	6.727
	0.386	TO	0.424		6.727	TO	7.400
B	0.424	TO	0.467	Q	7.400	TO	8.140
	0.467	TO	0.513		8.140	TO	8.954
C	0.513	TO	0.564	R	8.954	TO	9.850
	0.564	TO	0.621		9.850	TO	10.834
D	0.621	TO	0.683	S	10.834	TO	11.918
	0.683	TO	0.751		11.918	TO	13.110
E	0.751	TO	0.826	T	13.110	TO	14.421
	0.826	TO	0.909		14.421	TO	15.863
F	0.909	TO	1.000	U	15.863	TO	17.449
	1.000	TO	1.100		17.449	TO	19.194
G	1.100	TO	1.210	V	19.194	TO	21.113
	1.210	TO	1.331		21.113	TO	23.224
H	1.331	TO	1.464	W	23.224	TO	25.547
	1.464	TO	1.611		25.547	TO	28.102
I	1.611	TO	1.772	X	28.102	TO	30.912
	1.772	TO	1.949		30.912	TO	34.003
J	1.949	TO	2.144	Y	34.003	TO	37.403
	2.144	TO	2.358		37.403	TO	41.143
K	2.358	TO	2.594	Z	41.143	TO	45.258
	2.594	TO	2.853		45.258	TO	49.783
L	2.853	TO	3.138	1	49.783	TO	54.762
	3.138	TO	3.452		54.762	TO	60.238
M	3.452	TO	3.797	2	60.238	TO	66.261
	3.797	TO	4.177		66.261	TO	72.888
N	4.177	TO	4.595	3	72.888	TO	80.176
	4.595	TO	5.054		80.176	TO	88.194
O	5.054	TO	5.560	4	88.194	TO	97.013
	5.560	TO	6.116		97.013	TO	106.714

PLOT Symbols - Force Magnitude Ranges.

PLOT Sample Output, Force Angle.



PLOT Sample Output, Force Magnitude.

APPENDIX D

COMPUTATION OF CURRENT ELEMENT END POINTS

```
C
C
C          THIS SUBPROGRAM WILL DETERMINE THE COORDINATES OF THE
C         END POINTS OF AN ARRAY OF LINE ELEMENTS DESIGNED SO THAT
C          THE FIELDS PRODUCED BY THIS ARRAY OF ELEMENTS MODELS
C        THE FIELDS OF AN ACTUAL ELECTROMAGNETIC COIL CONFIGURATION
C
C
C     INPUT PARAMETER DEFINITION  (SEE FIGURES D-1,2,3 )
C
C          FROM          FROM          *       FROM          FROM
C         PROGRAM        FIGURE        *      PROGRAM        FIGURE
C                                      *
C        WIDTX(1)  =  WX/B1            *    WIDTX(2)  =  WXX/B1
C                  =  BETAX            *              =  BETAXX
C        WIDTZ(1)  =  WZ/B1            *    WIDTZ(2)  =  WZZ/B2
C
C        ALPHAZ(1)=  PHIZ              *    ALPHAZ(2)=  PHIZZ
C        BUILD(1)  =  B1/R0            *    BUILD(2)  =  B2/R0
C                  =  ALPHA1           *              =  ALPHA2
C        RCURX(1)  =  RX1/R0           *    RCURX(2)  =  RXX1/R0
C        RCURZ(1)  =  RZ1/R0           *    RCURZ(2)  =  RZZ1/R0
C
C        ALPHAC    =  ALPHAC
C        R0        =  THE INNER RADIUS FOR THE ARTIFICIAL GRAVITY
C                               CONFIGURATION AND THE SCALE FACTOR
C        AMPDX(1)  =  THE CURRENT DENSITY FOR THE AXIAL FIELD COILS
C                               (AMP/SQUARE INCH)
C        AMPDX(2)  =  THE CURRENT DENSITY FOR THE AXIAL GRADIENT
C                               COILS (AMP/SQUARE INCH)
C        AMPDZ(1)  =  THE CURRENT DENSITY FOR THE VERTICAL FIELD
C                               COILS (AMP/SQUARE INCH)
C        AMPDZ(2)  =  THE CURRENT DENSITY FOR THE VERTICAL GRADIENT
C                               COILS (AMP/SQUARE INCH)
C
C        SUBDIVISION INTO MULTIPLE LOOPS
C
C          FROM          FORM          *       FROM          FROM
C         PROGRAM        FIGURE        *      PROGRAM        FIGURE
C                                      *
C        NXX(1)    =  NLX(X)           *    NXX(2)    =  NLXX(X)
C        NXZ(1)    =  NLX(Z)           *    NXZ(2)    =  NLXX(Z)
C        NZX(1)    =  NLZ(X)           *    NZX(2)    =  NLZZ(X)
C        NZZ(1)    =  NLZ(Z)           *    NZZ(2)    =  NLZZ(Z)
C
C
C     OUTPUT PARAMETER DEFINITION
C
C        P(I,J,K,L,M,N) = THE COORDINATES OF THE END POINTS OF
C                               THE LINE ELEMENTS
C        CUR(I,J,K,L)   = THE CURRENT FLOWING IN THE PARTICULAR
C                               ELEMENT
C
C      WHERE THE DUMMY VARIABLES MEAN,
C
C        I - DESIGNATES THE VERTICAL OR AXIAL FIELD COILS
C        J - DESIGNATES THE COIL NUMBER
C        K - DESIGNATES THE LOOP NUMBER
C        L - DESIGNATES THE ELEMENT NUMBER
```

```
C               M - DESIGNATES THE STARTING OR END POINT
C               N - DESIGNATES THE COORDINATE DIRECTION
C
C
      DIMENSION WIDTX(2),WIDTZ(2),ALPHAX(2),BUILD(2),RADIUS(2),ALPHAZ(2)
     1     ,AMPS(2),BETAX(2),BETAZ(2),RCURX(2),RCURZ(2),CONF(20),
     2     P(2,4,32,8,2,3),CUR(2,4,32,8)
 2001 FORMAT ( 9F8.1/10F8.1/20A4/8I1/F10.1)
C
C               FUNCTION DEFINITIONS
C
      TANG(A)=SIN(A*0.017453)/COS(A*0.017453)
      XRRC(K,L)=ABS(FLOAT(((K+L-(K/5)*4)/2)-2)
      YRRC(K,L)=ABS(XRRC(K,L)-1.0)
      YELM(K,L)=FLOAT(1-((K+L-9*((K+L)/9))/5)**2)
      XELM(K,L)=YELM(K+2,L)
      DCR(N,K)=FLOAT(N+1-2*K)/FLOAT(2*N)
      INDEX(K)=1+IABS((K/2)-1)
      IREP(J,K)=J-K*((J-1)/K)
      JREP(J,K)=1+((J-1)/K)
C
C               INPUT PARAMETERS ALREADY DEFINED AND A CONFIGURATION
C                                DESCRIPTION
C
      READ (5,2001)  (WIDTX(IN),ALPHAX(IN),AMPDX(IN),RCURX(IN),WIDTZ(IN),
     1     ALPHAZ(IN),AMPDZ(IN),RCURZ(IN),BUILD(IN),IN=1,2),ALPHAC,
     1     (CONF(IN),IN=1,20),(NXX(IN),NXZ(IN),NZX(IN),NZZ(IN),IN=1,2),R0
C
C               CONVERSION OF INPUT PARAMETERS TO QUANTIES USED BY THE
C                                           PROGRAM
C
      DO 1000 I=1,2
      BUILD(I)=BUILD(I)*R0
      RCURX(I)=RCURX(I)*R0
      RCURZ(I)=RCURZ(I)*R0
      WIDTX(I)=WIDTX(I)*BUILD(1)
      WIDTZ(I)=WIDTZ(I)*BUILD(I)
      BETAX(I)=ALPHAC
      BETAZ(I)=ALPHAC
 1000 CONTINUE
      RADIUS(1)=R0+(BUILD(1)/2.0)
      RADIUS(2)=R0+BUILD(1)+(BUILD(2)/2.0)
C
C
C          CALCULATION OF THE COORDINATES
C
C               AXIAL FIELD CALCULATIONS
C
      DO 1001 J=1,4
      NLOOP=NXX(INDEX(J))*NXZ(INDEX(J))
      DO 1001 K=1,NLOOP
      DO 1001 L=1,9
      DO 1001 M=1,2
C
      P(1,J,K,L,M,1)=   RADIUS(1)*TANG(ALPHAX(INDEX(J)))+WIDTX(INDEX(J))
     1      *DCR(NXX(INDEX(J)),IREP(NLOOP,NXX(INDEX(J))))
C
      P(1,J,K,L,M,2)=((RADIUS(1)+BUILD(1)*DCR(NXZ(INDEX(J)),JREP(NLOOP,
     1     NXX(INDEX(J)))))*(1.0+YRRC(L,M)*(TANG(BETAX(INDEX(J)))-1.0))-
     2     RCURX(INDEX(J))*YRRC(L,M)*(TANG(BETAX(INDEX(J)))-1.0))*
```

```
      3     YELM(L,M)
C
      P(1,J,K,L,M,3)=((RADIUS(1)+BUILD(1))*DCR(NXZ(INDEX(J)),JREP(NLOOP,
     1     NXX(INDEX(J))))))*(1.0+XRRC(L,M)*(TANG(BETAX(INDEX(J)))-1.0))-
     2     RCURX(INDEX(J))*XRRC(L,M)*(TANG(BETAX(INDEX(J)))-1.0))*
     3     XELM(L,M)
C
      IF (J.NE.4) CUR(1,J,K,L)=AMPDX(INDEX(J))*WIDTX(INDEX(J))*BUILD(1)/
     1     FLOAT(NLOOP)
      IF (J.EQ.4) CUR(1,J,K,L)=(-AMPDX(INDEX(J))*WIDTX(INDEX(J))*
     1     BUILD(1)/FLOAT(NLOOP))
 1001 CONTINUE
C
C
C               VERTICAL FIELD COILS
C
      DO 1002 J=1,4
      NLOOP=NZX(INDEX(J))*NZZ(INDEX(J))
      DO 1002 K=1,NLOOP
      DO 1002 L=1,8
      DO 1002 M=1,2
C
      P(2,J,K,L,M,1)=((RADIUS(INDEX(J))*TANG(ALPHAZ(INDEX(J)))+WIDTZ(
     1     INDEX(J))*DCR(NZX(INDEX(J)),IREP(NLOOP,NZX(INDEX(J))))))*(1.0+
     2     XRRC(L,M)*(TANG(BETAZ(INDEX(J)))-1.0))-RCURZ(INDEX(J))*
     3     XRRC(L,M)*(TANG(BETAZ(INDEX(J)))-1.0))*XELM(L,M)
C
      P(2,J,K,L,M,2)=((RADIUS(INDEX(J))*TANG(ALPHAZ(INDEX(J)))+WIDTZ(
     1     INDEX(J))*DCR(NZX(INDEX(J)),IREP(NLOOP,NZX(INDEX(J))))))*(1.0+
     2     YRRC(L,M)*(TANG(BETAZ(INDEX(J)))-1.0))-RCURZ(INDEX(J))*
     3     YRRC(L,M)*(TANG(BETAZ(INDEX(J)))-1.0))*YELM(L,M)
C
      P(2,J,K,L,M,3)=  RADIUS(INDEX(J))+BUILD(INDEX(J))*
     1     DCR(NZZ(INDEX(J)),JREP(NLOOP,NZX(INDEX(J))))
C
      IF (J.NE.4) CUR(2,J,K,L)=AMPDZ(INDEX(J))*WIDTZ(INDEX(J))*
     1     BUILD(INDEX(J))/FLOAT(NLOOP)
      IF (J.EQ.4) CUR(2,J,K,L)=(-AMPDZ(INDEX(J))*WIDTZ(INDEX(J))*
     1     BUILD(INDEX(J))/FLOAT(NLOOP))
 1002 CONTINUE
C
C               THIS IS THE END OF THE SUBPROGRAM
C
```

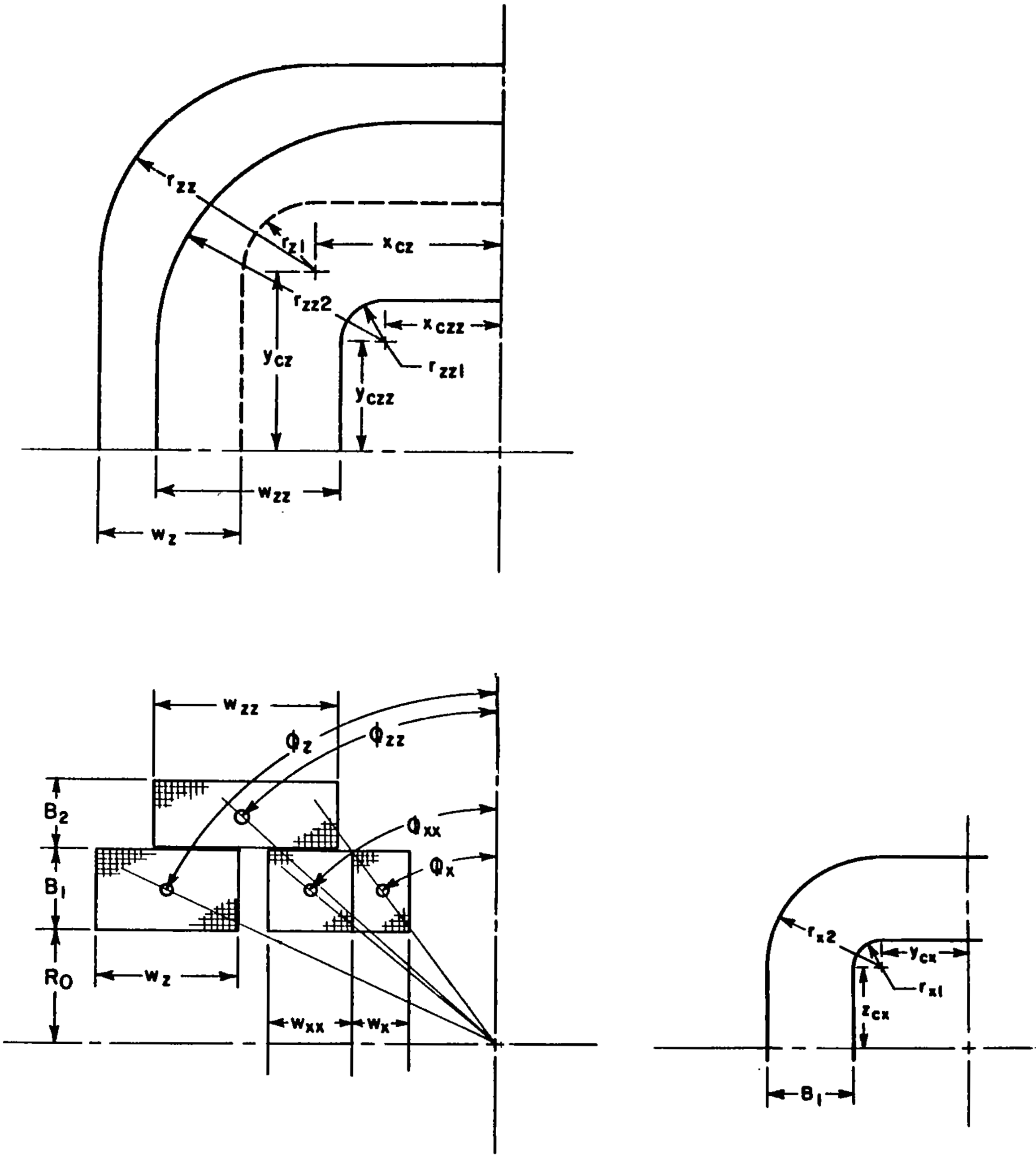


Figure D-1. Generalized Dimensions of Practical Air Core Coil Configuration.

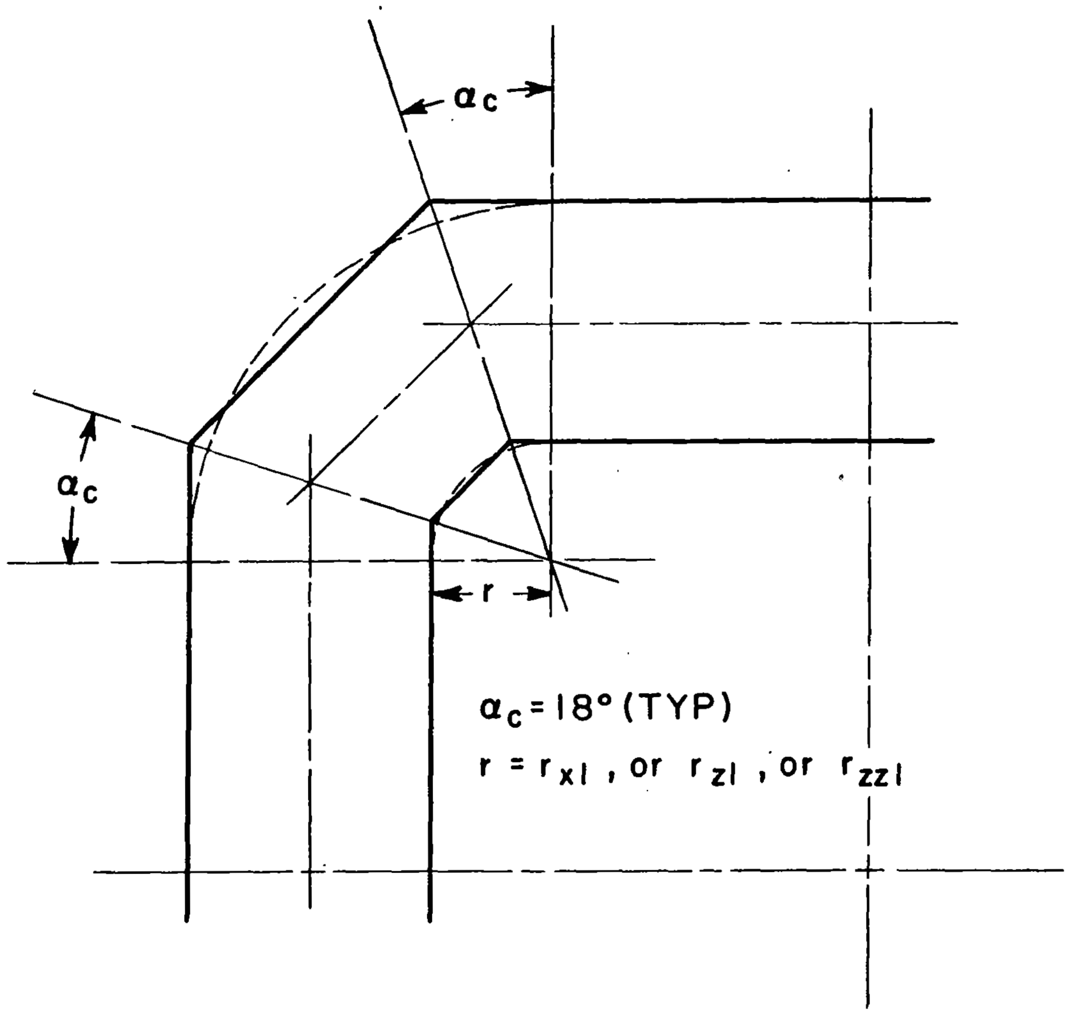


Figure D-2. Straight-Line Approximation to Rounded Corner.

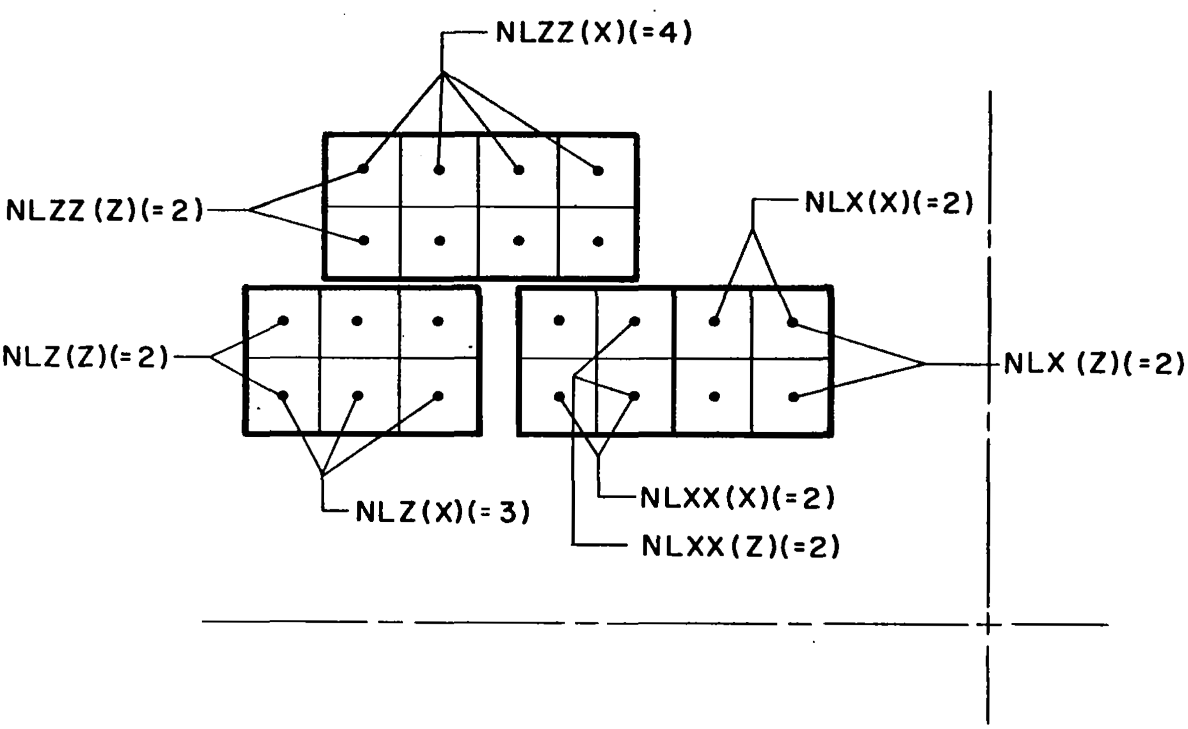


Figure D-3. Illustration of Nomenclature Used in Subdivision of Windings into Multiple Loops.

APPENDIX E

COMPUTER SIMULATION OF STORE DROP IN A MAGNETIC ARTIFICIAL GRAVITY FACILITY

A computer code has been developed for use in the evaluation of coil configurations considered in the artificial gravity program. This code, designated STORE, provides a measure of the correlation of non-uniformities in the artificial gravity field with trajectory errors by calculating ideal or constant gravity trajectories and comparing with the trajectories calculated for a store released in the artificial gravity field. All aerodynamic forces and moments are considered. The code itself consists of a main controlling program (MAIN), and the following four subroutines: INP1 (for input), ARGRAV (calculates the artificial gravity components), TRAJ (provides the trajectory coordinates), and OUTPUT (controls the output). A general flow chart depicting the interrelation of these four routines and MAIN appears in Figure E-1. Input for STORE includes characteristics of the artificial gravity coil system as well as dynamic and aerodynamic characteristics of the store model used in the evaluation. A complete listing of input variables can be found in the "Input Variable List" of the input subroutine (INP1) listed on page 125. A sample output sheet is on page 141.

LIST OF SYMBOLS

D	Store diameter
I_x, I_y, I_z	Moments of inertia about principle axes of store
L	Store length
m	Mass of store
q	Dynamic pressure
r	Relative distance between store and aircraft
S	Reference area of store
t	Time
Δt	Time increment
α	Angle of attack
β	Angle of side slip
$()_x, ()_y, ()_z$	Coordinate components
$()_S$	Referred to store coordinate system
$()_E$	Referred to earth coordinate system

Theory

The store trajectories are calculated by determining the linear and angular accelerations through application of Newton's Second Law for a rotating frame (the store coordinate system of Figure E-3), then integrating twice by Simpson's Rule. From Ref. 11, the vector equations of motion are,

$$\vec{a}_s = \frac{\vec{C}_f q_s}{m} - \vec{\omega}_s \times \vec{V}_s + [C]^T \vec{A}_g \tag{E-1}$$

$$\dot{\vec{\omega}} = [1/I]\{\vec{C}_m q_s - \vec{K}\} \tag{E-2}$$

where a_s and $\dot{\vec{\omega}}_s$ are the linear and angular accelerations in a frame fixed to the principle axes of the store and $[C]^T$ is the transpose of the rotation matrix defined below. (See Appendices A, B, C).

In this convenient vector form, the aerodynamic force coefficients are:

$$\vec{C}_F = \begin{Bmatrix} C_x \\ C_y \\ C_z \end{Bmatrix} \tag{E-3}$$

The aerodynamic moment coefficients, as defined in Fig. E-2, are:

$$\vec{C}_m = \begin{Bmatrix} C_p D \\ C_q L \\ C_r L \end{Bmatrix} \tag{E-4}$$

The moments of inertia about the principle axes are:

$$[1/I] = \begin{bmatrix} 1/I_x & 0 & 0 \\ 0 & 1/I_y & 0 \\ 0 & 0 & 1/I_z \end{bmatrix} \tag{E-5}$$

The vector $\vec{K}$ arising from the rotating frame is:

$$\vec{K} = \left\{ \begin{matrix} \omega_y \omega_z (I_z - I_y) \\ \omega_x \omega_z (I_x - I_z) \\ \omega_x \omega_y (I_y - I_x) \end{matrix} \right\} \qquad \text{(E-6)}$$

The aerodynamic force coefficients are calculated by first calculating lift, drag, and side force coefficients by the conventional definitions (i.e., perpendicular and parallel to the wind vector), then the angles of attack and side slip are used to determine the resulting forces in the store coordinate system (see Fig. E-2). Thus,

$$C_x = (C_L \sin\alpha - C_D \cos\alpha)\cos\beta + C_s \sin\beta \qquad \text{(E-7)}$$

$$C_y = (C_D \cos\alpha \sin\beta + C_s \cos\beta) \qquad \text{(E-8)}$$

$$C_z = -(C_D \sin\alpha + C_L \cos\alpha) \qquad \text{(E-9)}$$

where,

$$C_L = C_{Lo} + C_{L_\alpha}\alpha \qquad \text{(E-10)}$$

$$C_D = C_{Do} + C_{D_{\alpha^2}}\alpha^2 + C_{D_{\beta^2}}\beta^2 \qquad \text{(E-11)}$$

$$C_s = C_{so} + C_{s_\beta}\beta \qquad \text{(E-12)}$$

The moment coefficients are calculated from the static and dynamic derivatives as,

$$C_p = 0 \qquad \text{(E-13)}$$

$$C_q = C_{qo} + C_{q_\alpha}\alpha + C_{q_{\dot\alpha}}\dot\alpha + C_{q_{\omega_Y}}\omega_Y \qquad \text{(E-14)}$$

$$C_r = C_{r_o} + C_{r_\beta}\beta + C_{r_{\dot\beta}}\dot\beta + C_{r_{\omega_z}}\omega_z \qquad \text{(E-15)}$$

At this point, the static and dynamic derivatives for moment coefficients and the derivatives for force coefficients ($C_{L\alpha}$, $C_{S\beta}$, etc.) form part of the input for the calculation and must be either estimated or determined from experimental data. The 'subzero' factors (C_{L_o}, C_{q_o}, C_{r_o}, C_{s_o}) are generally a result of aerodynamic interference from the releasing body (i.e., aircraft model) since the stores are otherwise symmetric. For simplicity, these are approximated by a power series in $1/r$, i.e.,

$$C_{\xi_o} = a_\xi + \frac{b_\xi}{r} + \frac{c_\xi}{r^2} + \frac{d_\xi}{r^3} \qquad \xi = L,S,q,r \qquad \text{(E-16)}$$

where, again, a, b, c, and d can be determined from a 'curve fit' of experimental data, or estimated. The series can be extended to higher order terms with minor modifications to the input and trajectory subroutines. It is noted that the goal of the calculation is a reasonable evaluation of the coil system so that a set of coefficients which produces a representative trajectory is sufficient.

The first integration of the dynamical equations is performed in the store coordinate system. Using Simpson's Rule based on half the time increment, the store velocity is

$$\vec V_{s_{n+1}} = \vec V_{s_n} + \frac{\Delta_t}{6}\left(a_{s_n} + 4a_{s_{n+\frac{1}{2}}} + a_{s_{n+1}}\right) \qquad \text{(E-17)}$$

where $a_{s_{n+\frac{1}{2}}} = a_s(t_n + \Delta t/2)$. In order to determine both the acceleration and velocity (for the next integration) at the half interval point, $t_n + \Delta t/2$, the velocity is approximated as a quadratic in time between t_n and t_{n+1},

$$\vec{V}_{s_n} = c_1 t_n^2 + c_2 t_n + c_3 \tag{E-18}$$

$$\vec{a}_{s_n} = \frac{d\vec{V}_{s_n}}{dt} = 2c_1 t_n + c_2 \tag{E-19}$$

where the coefficients are

$$c_1 = (\vec{a}_{s_{n+1}} - \vec{a}_{s_n})/2\Delta t \tag{E-20}$$

$$c_2 = \vec{a}_{s_n} - 2c_1 t_n \tag{E-21}$$

$$c_3 = \vec{V}_{s_n} - (c_1 t_n^2 + c_2 t_n) \tag{E-22}$$

So that

$$\vec{a}_{s_{n+\frac{1}{2}}} = 2c_1 \ (t_n + \Delta t/2) + c_2 \tag{E-23}$$

$$\vec{V}_{s_{n+\frac{1}{2}}} = c_1 \ (t_n + \Delta t/2)^2 + c_2 \ (t_n + \Delta t/2) + c_3 \tag{E-24}$$

The angular acceleration is integrated in the same manner. Next, the linear and angular velocities are transferred to the non-rotating earth axis (see Figure E-3) by application of the proper rotation matrices, i.e.,

$$\vec{V}_{E_{n+1}} = [C]_n \ \vec{V}_{s_{n+1}} \tag{E-25}$$

$$\dot{\vec{\theta}}_{n+1} = [D]_n \ \vec{\omega}_{s_{n+1}} \tag{E-26}$$

where $[C]_n$ and $[D]_n$ are as follows:

$$[C]_n = \begin{bmatrix} \cos\theta_n\cos\psi_n & \sin\phi_n\sin\theta_n\cos\psi_n - \cos\phi_n\sin\psi_n & \cos\phi_n\sin\theta_n\cos\psi_n \\ \cos\theta_n\sin\psi_n & \sin\phi_n\sin\theta_n\sin\psi_n + \cos\phi_n\cos\psi_n & \cos\phi_n\sin\theta_n\sin\psi_n \\ -\sin\theta_n & \sin\phi_n\cos\theta_n & \cos\phi_n\cos\theta_n \end{bmatrix} \tag{E-27}$$

$$[D]_n = \begin{bmatrix} 1 & \sin\phi_n\tan\theta_n & \cos\phi_n\tan\theta_n \\ 0 & \cos\phi_n & -\sin\phi_n \\ 0 & \sin\phi_n\sec\theta_n & \cos\phi_n\sec\theta_n \end{bmatrix} \tag{E-28}$$

and ϕ, θ, and ψ are the Euler angles defined in Figure E-3.

Now, the linear velocities and Euler rates are integrated, again by Simpson's Rule, to determine the next coordinates of the store c.g. and the Euler angles locating the store principal axes, i.e,

$$\vec{X}_{E_{n+1}} = \vec{X}_{E_n} + \frac{\Delta t}{6}\left(\vec{V}_{E_n} + 4\vec{V}_{E_{n+\frac{1}{2}}} + \vec{V}_{E_{n+1}}\right) \tag{E-29}$$

$$\theta_{n+1} = \begin{Bmatrix} \phi \\ \theta \\ \psi \end{Bmatrix} = \vec{\theta}_n + \frac{\Delta t}{6}\left(\dot{\theta}_n + 4\dot{\theta}_{n+\frac{1}{2}} + \dot{\theta}_{n+1}\right) \tag{E-30}$$

where

$$\vec{V}_{E_{n+\frac{1}{2}}} = [C]_n \vec{V}_s (t_n+\Delta t/2) \tag{E-31}$$

$$\dot{\vec{\theta}}_{n+\frac{1}{2}} = [C]_n \vec{\omega}_s (t_n+\Delta t/2) \tag{E-32}$$

The computer code developed for the trajectory calculation has been tested for the cases of a non-rotating sphere with constant acceleration and for a purely rotating sphere. Under these conditions, the dynamical equations reduce to

$$\vec{X}_{E_n} = \frac{1}{2}\,\vec{a}t_n^2; \quad \vec{a} = \text{const} \tag{E-33}$$

and

$$\vec{\theta}_{n+1}=\theta_n+(\dot{\theta}_n\Delta t+\ddot{\vec{\theta}}_n\frac{(\Delta t)^2}{2}) \tag{E-34}$$

respectively, where

$$\dot{\theta}_n = [D]\vec{b}t_n, \quad \ddot{\theta}_n = [D]\vec{b} \qquad \vec{b} = \text{const.} \tag{E-35}$$

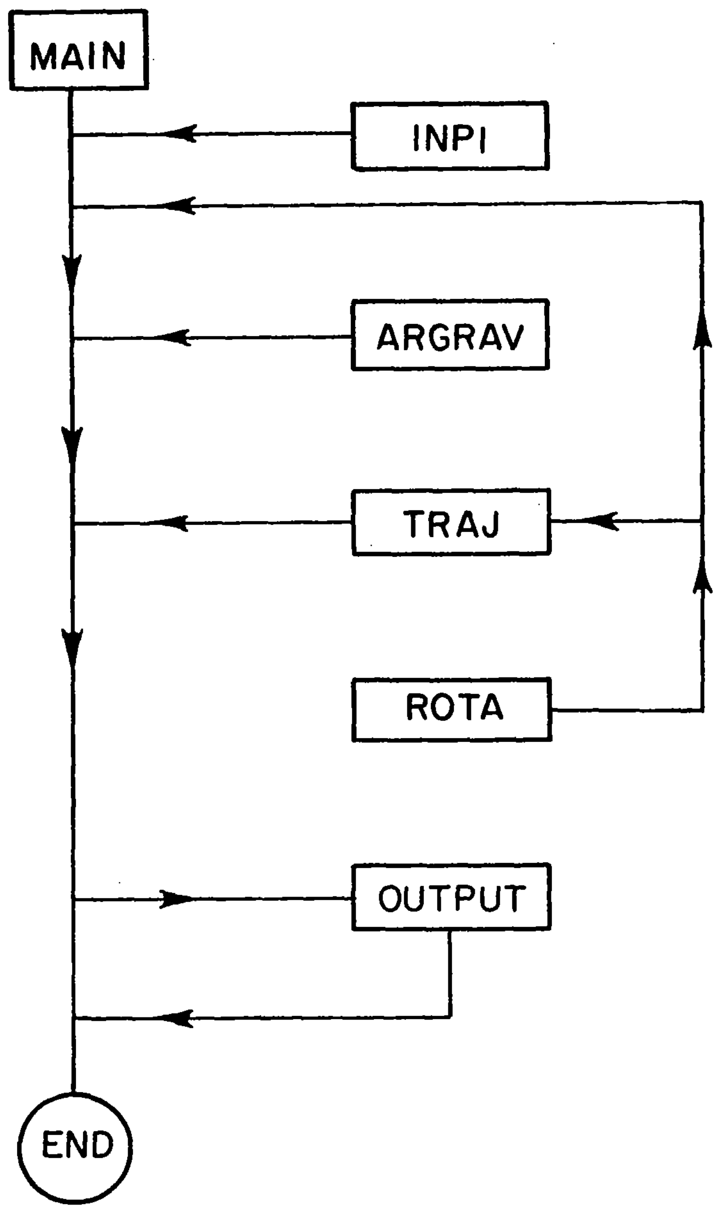


Figure E-1. General Flow Chart for Store.

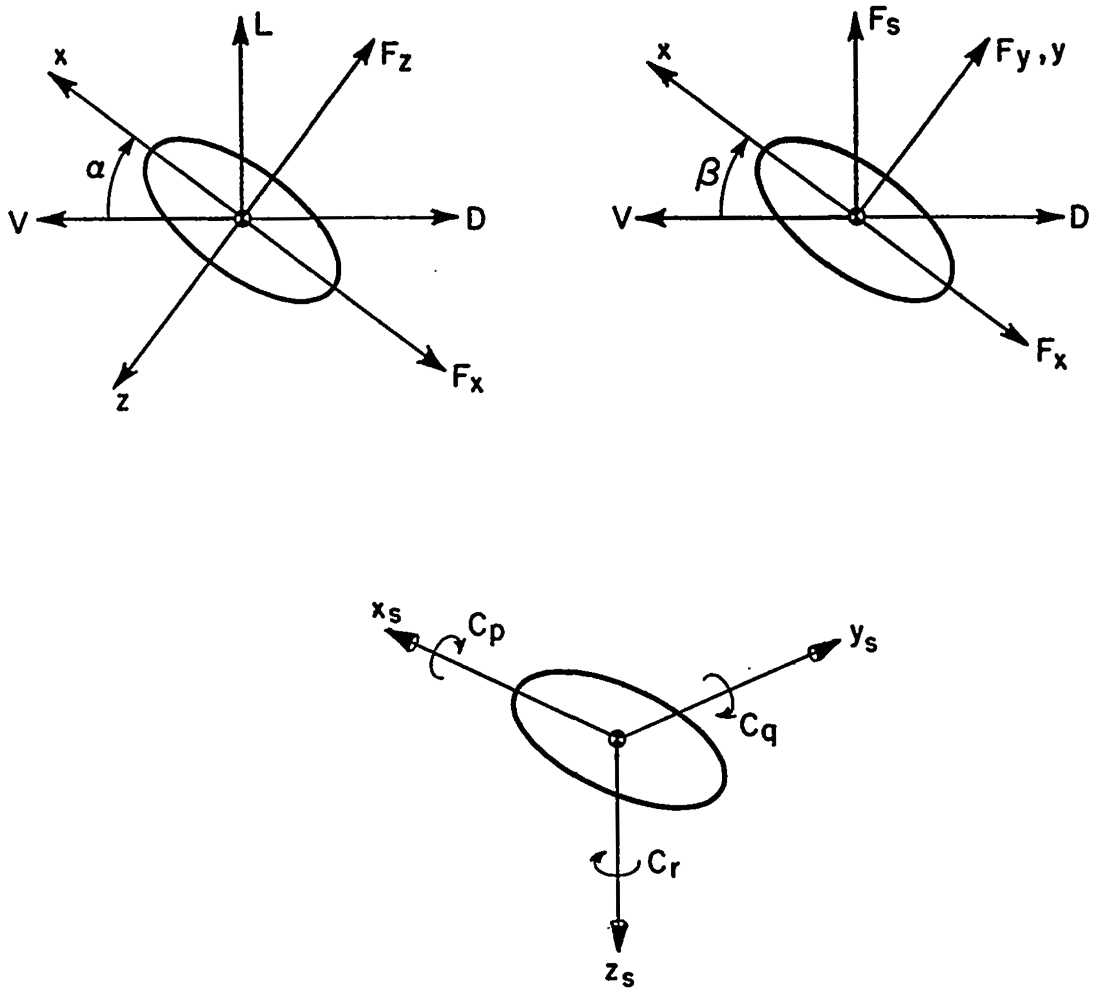


Figure E-2. Aerodynamic Coefficients in Store Axis.

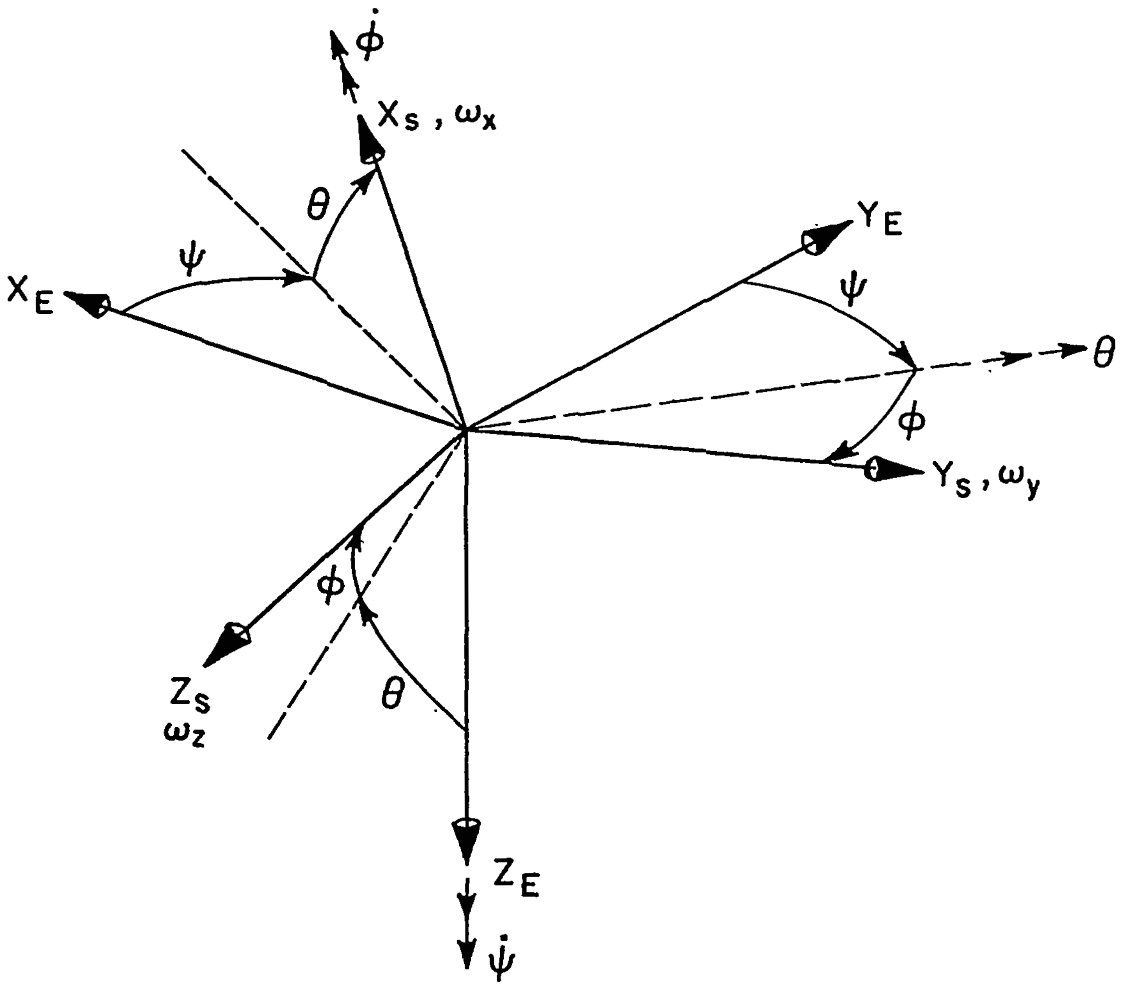


Figure E-3. Coordinate Systems and Euler Angles.

```
C                          ARTIFICIAL GRAVITY-STORE
C
C
C A CODE TO CALCULATE THE FORCES ON A MAGNETIC BODY IN A FIELD PRODUCED
C BY COILS CONSISTING OF STRAIGHT LINE CURRENT ELEMENTS.
C
C THE MAGNETIC ACCELERATIONS ARE USED TO PREDICT STORE TRAJECTORIES AND
C TRAJECTORY ERRORS RESULTING FROM THE NON-UNIFORMITY OF THE ARTIFICIAL
C GRAVITY FIELD.
C
      DIMENSION XTR(1000,3),XR(1000,3),AX(1000),AY(1000),AZ(1000),X(1000
     1),ERRX(1000,3),CONFG(18),X1(500),Y1(500),Z1(500),X2(500),Y2(500),Z
     22(500),CUR(500),CONFS(18),GES1(1000),GES2(1000),ZETA2(1000),XRN(10
     300,3),XTRN(1000,3),ALPHA(1000),ALPHAR(1000),ALPHAI(1000),ERRAL(100
     40),ZETA1(1000),XE(3),THETA(3),XAC(3),VAC(3),XM(2,3),CF(2,3),AG(2,3
     5),C(2,3,3),WXV(2,3),TIME(1000),CS(3),SN(3),GS(2,3),ACS(1000,2,3),C
     61(2,3),C2(2,3),VSH(2,3),ACSH(2,3),VS(1000,2,3),VE(1000,2,3),VEH(2,
     73),BETA(1000),XREL(1000,3),XNE(1000,3),CFO(4),AF(4,4),C3(2,3),XE1(
     83),THETA1(3),XAC1(3)
C
C INPUT THE CHARACTERISTICS OF THE STORE AND ARTIFICIAL GRAVITY
C CONFIGURATIONS FOR THIS CALCULATION.
      CALL       INP1(DA,AMS,XKT,XMU,RHOI,X1,Y1,Z1,X2,Y2,Z2,CUR,LM,CONFG
     1,CURX1,CURX2,CURX3,CURX4,CURZ1,CURZ2,CURZ3,CURZ4,CONFS,XE1,THETA1,
     2XAC1,VAC,XM,XNS,DELT,RHOZE,BD,BL,XMIN,XMAX,ZMIN,ZMAX,VS,VE,AF,CELA
     3,CESB,CDO,CDA2,CMA,CMAD,CMTD,CNB,CNBD,CNPSID,BSTL,ASTL,BETSTL,ALPS
     4TL,G)
C
C CALCULATE CONSTANT VARIABLES.
      CIN=12.*BL
      SR=.3927*RHOZE*BD**2
      DELT6=DELT/6.
      TVEL=VAC(1)
C
C DEFINE STARTING CONDITIONS.
      TIME(1)=0.
```

```
      ITR=1
      ITRA=0
      ITRB=0
      VMAG=(VS(1,1,1)*VS(1,1,1)+VS(1,1,2)*VS(1,1,2)+VS(1,1,3)*VS(1,1,3))
     1**.5
      IF(VMAG.EQ.0.)  GO TO 961
      ALPHA(1)=ARSIN(VS(1,1,3)/VMAG)
      BETA(1)=ARSIN(VS(1,1,2)/VMAG)
      GO TO 962
  961 ALPHA(1)=0.
      BETA(1)=0.
  962 ALPHAI(1)=ALPHA(1)*57.3
      ALPHAR(1)=ALPHAI(1)
C CALCULATE THE INITIAL ROTATION MATRIX.
      CALL ROTA(THETA1,C)
      DO 941 NI=1,3
      AG(2,NI)=0.
      XTR(1,NI)=-12.*(XAC1(NI)-XE1(NI))
      XTRN(1,NI)=XTR(1,NI)+12.*C(1,NI,1)*XNS
      XR(1,NI)=XTR(1,NI)
      XRN(1,NI)=XTRN(1,NI)
      THETA(NI)=THETA1(NI)
      XAC(NI)=XAC1(NI)
  941 XE(NI)=XE1(NI)
C
C CALCULATE THE MAGNETIC FORCE AT THE STORE RELEASE POINT.
      CALL ARGRAV(DA,AMS,XKT,XMU,XTR,X1,Y1,Z1,X2,Y2,Z2,CUR,LM,RHOI,AX,AY
     1,AZ,ITR)
      GES1(1)=((AX(1)**2+(AZ(1)+G)**2)**.5)/G
      ZETA1(1)=ATAN(AX(1)/(AZ(1)+G))*57.3
      AG(1,1)=AX(ITR)
      AG(1,2)=AY(ITR)
      AG(1,3)=AZ(ITR)+G
C
C CALCULATE THE IDEAL OR CONSTANT GRAVITY TRAJECTORY.
  921 CALL       TRAJ(XTR,XTRN,XE,THETA,XAC,VAC,XM,XNS,DELT,SR,DELT6,ALP
```

```
     1HA,BETA,C,AG,VS,VE,TIME,AF,CELA,CESB,CDO,CDA2,CMA,CMAD,CMTD,CNB,CN
     2BD,CNPSID,ITR,ITRA,ITRB,BETSTL,ALPSTL,BSTL,ASTL,BL,VMAG)
C
      ALPHAI(ITR)=ALPHA(ITR)*57.3
      TIME(ITR)=TIME(ITR-1)+DELT
C
C CALCULATE THE LOCAL G ANGLE AND MAGNITUDE.
      GES1(ITR)=GES1(1)
      ZETA1(ITR)=ZETA1(1)
C
C IS THE STORE OUTSIDE OF THE LIMITS OF THE REGION OF INTEREST.
      IF(XTR(ITR,1).GE.XMAX.OR.XTR(ITR,1).LE.XMIN.OR.XTR(ITR,3).GE.ZMAX.
     1OR.XTR(ITR,3).LE.ZMIN)  GO TO 940
      GO TO 921
C
 940  IMAX=ITR
C
C CALCULATE THE ACTUAL TRAJECTORY.
C DEFINE STARTING CONDITIONS.
      DO 950 NM=1,3
      THETA(NM)=THETA1(NM)
      XAC(NM)=XAC1(NM)
 950  XE(NM)=XE1(NM)
      ITRA=0
      ITRB=0
      ITR=1
      CALL ROTA(THETA,C)
C
 926  CALL ARGRAV(DA,AMS,XKT,XMU,XR,X1,Y1,Z1,X2,Y2,Z2,CUR,LM,RHOI,AX,AY,
     1AZ,ITR)
      AG(1,1)=AX(ITR)
      AG(1,2)=AY(ITR)
      AG(1,3)=AZ(ITR)+G
C
C CALCULATE THE LOCAL G ANGLE AND MAGNITUDE.
      GES2(ITR)=((AX(ITR)**2+(AZ(ITR)+G)**2)**.5)/G
```

```
      ZETA2(ITR)=ATAN(AX(ITR)/(AZ(ITR)+G))*57.3
C
      CALL         TRAJ(XR,XRN,XE,THETA,XAC,VAC,XM,XNS,DELT,SR,DELT6,ALP
     1HA,BETA,C,AG,VS,VE,TIME,AF,CELA,CESB,CDO,CDA2,CMA,CMAD,CMTD,CNB,CN
     2BD,CNPSID,ITR,ITRA,ITRB,BETSTL,ALPSTL,BSTL,ASTL,BL,VMAG)
C
      ALPHAR(ITR)=ALPHA(ITR)*57.3
      IF(ITR.EQ.IMAX) GO TO 924
      GO TO 926
C
  924 CALL ARGRAV(DA,AMS,XKT,XMU,XR,X1,Y1,Z1,X2,Y2,Z2,CUR,LM,RHOI,AX,AY,
     1AZ,IMAX)
C
      GES2(IMAX)=((AX(IMAX)**2+(AZ(IMAX)+G)**2)**.5)/G
      ZETA2(IMAX)=ATAN(AX(IMAX)/(AZ(IMAX)+G))*57.3
C
C CALCULATE THE TRAJECTORY ERRORS BASED ON IDEAL CONDITIONS.
      DO 925 JL=1,IMAX
      DO 943 IL=1,3
  943 ERRX(JL,IL)=((XTR(JL,IL)-XR(JL,IL))/CIN)*100.
      IF(ALPHAI(JL).EQ.0.) GO TO 936
      ERRAL(JL)=((ALPHAI(JL)-ALPHAR(JL))/ALPHAI(JL))*100.
      GO TO 925
  936 IF(ALPHAR(JL).EQ.ALPHAI(JL)) GO TO 937
      ERRAL(JL)=((ALPHAR(JL)-ALPHAI(JL))/ALPHAR(JL))*100.
      GO TO 925
  937 ERRAL(JL)=0.
  925 CONTINUE
C
C OUTPUT THE TRAJECTORIES AND TRAJECTORY ERRORS.
      CALL        OUPUT(CONFG,X1,Y1,Z1,X2,Y2,Z2,CUR,LM,CONFS,CURX1,CURX2,
     1CURX3,CURX4,CURZ1,CURZ2,CURZ3,CURZ4,TIME,XTR,XR,ERRX,IMAX,GES1,GES
     22,ZETA1,ZETA2,TVEL,ALPHAR,XRN,ALPHAI,XTRN,ERRAL)
      CALL EXIT
      END
```

```
      SUBROUTINE INP1(DA,AMS,XKT,XMU,RHOI,X1,Y1,Z1,X2,Y2,Z2,CUR,LM,CONFG
     1,CURX1,CURX2,CURX3,CURX4,CURZ1,CURZ2,CURZ3,CURZ4,CONFS,XE1,THETA1,
     2XAC1,VAC,XM,XNS,DELT,RHOZE,BD,BL,XMIN,XMAX,ZMIN,ZMAX,VS,VE,AF,CELA
     3,CESB,CDO,CDA2,CMA,CMAD,CMTD,CNB,CNBD,CNPSID,BSTL,ASTL,BETSTL,ALPS
     4TL,G)
C
C                             INPUT SUBROUTINE
C
C                      INPUT VARIABLE LIST
C
C VARIABLE NAME                            DEFINITION
C
C      CONFS            DESCRIPTION OF STORE CONFIGURATION.
C      XE1              INITIAL STORE POSITION IN THE EARTH SYSTEM (X,Y,Z).
C      THETA1           INITIAL EULER ANGLES (PHI,THETA,PSI).
C      XAC1             INITIAL POSITION OF AIRCRAFT (X,Y,Z).
C      VAC              VELOCITY COMPONENTS OF TUNNEL WIND (VX,VY,VZ).
C      XM               STORE MASS AND MOMENTS OF INERTIA.  XM(1,1)=XM(1,2)=
C                       XM(1,3)=MASS, XM(2,1)=IX, XM(2,2)=IY, AND XM(2,3)=IZ
C      AF               CONSTANTS FOR DETERMINIG AERODYNAMIC INTERFERENCE
C                       FIELD.  AF(1,1),AF(1,2),AF(1,3) AND AF(1,4) ARE FOR
C                       SIDE FORCE, AF(2,1),AF(2,2),AF(2,3), AND AF(2,4) ARE
C                       FOR NORMAL FORCE, AF(3,I) FOR PITCHING MOMENT, AND
C                       AF(4,I) FOR YAWING MOMENT.
C      CELA             LIFT CURVE CLOPE (DCL/DALPHA)
C      CESB             SIDE FORCE DIRIVITIVE (DCS/DBETA).
C      CDO              BASE DRAG.
C      CDA2             DCD/DALPHA**2
C      CMA              DCM/DALPHA (PITCHING MOMENT)
C      CMAD             DCM/DALPHADOT  ''  ''
C      CMTD             DCM/DWYDOT  ''  ''
C      CNB              DCN/DBETA  (YAWING MOMENT)
C      CNBD             DCN/DBETADOT  ''  ''
C
      DIMENSION CONFG(18),X1(500),Y1(500),Z1(500),X2(500),Y2(500),Z2(500
     1),CUR(500),CONFS(18),XE1(3),THETA1(3),XAC1(3),VAC(3),XM(2,3),VS(10
```

```
     200,2,3),VE(1000,2,3),AF(4,4),CURT(100)
C
C      CNPSID          DCN/DWZDOT  ''  ''
C      XNS             DISTANCE FROM STORE C.G. TO NOSE.
C      DELT            TIME INCREMENT FOR TRAJECTORY CALCULATIONS.
C      RHOZE           DENSITY OF TUNNEL ATMOSPHERE.
C      BD              STORE DIAMETER.
C      BL              STORE LENGTH.
C      XMIN,XMAX       X LIMITS OF REGION OF INTEREST.
C      ZMIN,ZMAX       Z LIMITS OF REGION OF INTEREST.
C      BSTL            CONSTANT FOR SIDE FORCE COEFFICIENT CALCULATION PAST
C                      STALL.
C      ASTL            CONSTANT FOR LIFT COEFFICIENT CALCULATION PAST STALL
C      BETSTL          STALLING ANGLE OF SIDE SLIP.
C      ALPSTL          STALLING ANGLE OF ATTACK.
C      G               ACCELERATION DUE TO GRAVITY.
C      VS              INITIAL VELOCITY COMPONENTS AND ROTATION RATES OF
C                      STORE RELATIVE TO STORE C.S.  VS(1,I) CORRESPONDS
C                      TO VELOCITIES AND VS(2,I) TO ROTATION RATES.
C      VE              CORRESPONDING VELOCITIES AND ROTATION RATES IN
C                      EARTH AXIS.
C
 164  FORMAT(18A4)
C
C      DA              DEMAGNETIZING CONSTANT FOR THE MODEL.
C      AMS             SATURATION MAGNETIZATION FOR THE MODEL.
C      XKT             MAGNETIC FORCE CONSTANT.
C      XMU             MAGNETIC PERMEABILITY OF FREE SPACE.
C      RHOI            DENSITY OF MAGNETIC MATERIAL OF SPHERE.
C      LM              TOTAL NUMBER OF CURRENT ELEMENTS
C      INPOPT          INPUT OPTION.
C      CONFG           A DESCRIPTION OF THE MAGNET CONFIGURATION.
C      X1,Y1,Z1        COORDINATES OF THE END POINTS OF THE STRAIGHT LINE
C      X2,Y2,Z2        CURRENT ELEMENTS MAKING UP THE COILS.
C      CURT            CURRENT FLOWING IN A LOOP OF FOUR CURRENT ELEMENTS.
C      CUR             MAGNITUDE OF THE CURRENT IN AMPERES,+ FROM 1 TO 2.
```

```
C       CURX1        THE TOTAL CURRENT IN THE +X FIELD COIL.
C       CURX2        THE TOTAL CURRENT IN THE +X GRADIENT COIL.
C       CURX3        THE TOTAL CURRENT IN THE -X FIELD COIL.
C       CURX4        THE TOTAL CURRENT IN THE -X GRADIENT COIL.
C       CURZ1        THE TOTAL CURRENT IN THE +Z FIELD COIL.
C       CURZ2        THE TOTAL CURRENT IN THE +Z GRADIENT COIL.
C       CURZ3        THE TOTAL CURRENT IN THE -Z FIELD COIL.
C       CURZ4        THE TOTAL CURRENT IN THE -Z GRADIENT COIL.
C
C NOTE THAT DOUBLE SUBSCRIPTED VARIABLES SUCH AS XM(I,J) ARE READ IN THE
C ORDER XM(1,1),XM(2,1),XM(3,1),...,XM(1,2),XM(2,2),..., ETC. UNLESS
C SPECIFIED AS IN STATEMENT READ(5,162).
C
      READ(5,164) CONFS
      READ(5,160) XE1,THETA1,XAC1,VAC,XM,AF
 160  FORMAT(3F14.8/3F14.8/3F14.8/3F14.8/6F12.6/8F9.5/8F9.5)
      READ(5,161) CELA,CESB,CDO,CDA2,CMA
      READ(5,161) CMAD,CMTD,CNB,CNBD,CNPSID
      READ(5,161) XNS,DELT,RHOZE,BD,BL
      READ(5,161) XMIN,XMAX,ZMIN,ZMAX,BSTL
 161  FORMAT(5F14.8)
      READ(5,163) ASTL,BETSTL,ALPSTL,G
 163  FORMAT(4F14.8)
      READ(5,162) ((VS(1,IN,IM),IM=1,3),IN=1,2)
      READ(5,162) ((VE(1,JN,KM),KM=1,3),JN=1,2)
 162  FORMAT(6F12.6)
C
      READ(5,112) DA,AMS,XKT,XMU,RHOI,LM,INPOPT
 112  FORMAT(F5.3,F9.2,2F12.11,F6.5,2I5)
      READ(5,164) CONFG
C INPOPT=1 CORRESPONDS TO INPUTING THE CURRENT IN EACH ELEMENT.  INPOPT
C =2 CORRESPONDS TO INPUTING THE CURRENT  IN EACH LOOP OF FOUR ELEMENTS.
      IF(INPOPT.EQ.1) GO TO 171
      READ(5,166) (X1(NI),Y1(NI),Z1(NI),X2(NI),Y2(NI),Z2(NI),NI=1,LM)
 166  FORMAT(6F8.4)
      NTM=LM/4
```

```
      READ(5,165) (CURT(NT),NT=1,NTM)
  165 FORMAT(F10.0)
C ASSIGN CURRENT MAGNITUDES TO EACH CURRENT ELEMENT.
      KL=-3
      KL1=0
      DO 170 JT=1,NTM
      KL=KL+4
      KL1=KL1+4
      DO 170 NT1=KL,KL1
  170 CUR(NT1)=CURT(JT)
      GO TO 173
  171 READ(5,910) (X1(N),Y1(N),Z1(N),X2(N),Y2(N),Z2(N),CUR(N),N=1,LM)
  910 FORMAT(6F8.4,F10.0)
  173 READ(5,950) CURX1,CURX2,CURX3,CURX4
      READ(5,950) CURZ1,CURZ2,CURZ3,CURZ4
  950 FORMAT(4F10.0)
      RETURN
      END
```

```
      SUBROUTINE ARGRAV(DA,AMS,XKT,XMU,X,X1,Y1,Z1,X2,Y2,Z2,CUR,LM,RHOI,A
     1X,AY,AZ,ITR)
C
C                       ARTIFICIAL GRAVITY SUBROUTINE
C
      DIMENSION X(1000,3),SGR(3),TGR(3),DP(3),DS(3),DD(3),DC(3),DG(3),X1
     1(500),Y1(500),Z1(500),X2(500),Y2(500),Z2(500),CUR(500),AX(1000),AY
     2(1000),AZ(1000)
C
      XMP=XMU/(4.*3.1416)
C
C INITUALIZE THE VALUES TO BE SUMMED.
      BX=0.0
      BY=0.
      BZ=0.0
      BXX=0.
      BXY=0.
      BXZ=0.
      BYX=0.
      BYY=0.
      BYZ=0.
      BZX=0.
      BZY=0.
      BZZ=0.
C
      DO 210 L=1,LM
C CALCULATE A,B,C,D,E,AND F
      AGR=(X1(L)-X(ITR,1))/39.37
      BGR=(X2(L)-X(ITR,1))/39.37
      CGR=(Y1(L)-X(ITR,2))/39.37
      DGR=(Y2(L)-X(ITR,2))/39.37
      EGR=(Z1(L)-X(ITR,3))/39.37
      FGR=(Z2(L)-X(ITR,3))/39.37
C SUBSCRIPT A,C,E,B,D,F FOR LATER USE
      SGR(1)=AGR
      SGR(2)=CGR
```

```
      SGR(3)=EGR
      TGR(1)=BGR
      TGR(2)=DGR
      TGR(3)=FGR
C CALCUALTE U, V, AND W.
      UGR=CGR*FGR-DGR*EGR
      VGR=EGR*BGR-FGR*AGR
      WGR=AGR*DGR-BGR*CGR
C CALCULATE RHO1 AND RHO2.
      RG1=(AGR*AGR+CGR*CGR+EGR*EGR)**.5
      RG2=(BGR*BGR+DGR*DGR+FGR*FGR)**.5
C CALCULATE THE SUM, PRODUCT, DOT PRODUCT, AND CROSS PRODUCT OF RHO1
C AND RHO2.
      RS=RG1+RG2
      RM=RG1*RG2
      RDR=AGR*BGR+CGR*DGR+EGR*FGR
      RXR=UGR+VGR+WGR
C CALCULATE THE DERIVITIVES OF THE SUM, ETC. OF RHO1 AND RHO2.
      DO 220 M=1,3
      DP(M)=-(SGR(M)*RG2/RG1+TGR(M)*RG1/RG2)
      DS(M)=-(SGR(M)/RG1+TGR(M)/RG2)
      DD(M)=-(SGR(M)+TGR(M))
  220 CONTINUE
      DC(1)=FGR-EGR+CGR-DGR
      DC(2)=EGR-FGR+BGR-AGR
      DC(3)=DGR-CGR+AGR-BGR
C CALCULATE AND TEST H TO DETERMINE EQUATION FOR G TO BE USED.
      H=(RM+RDR)/RM
      IF(H-0.01) 2,1,1
C CALCULATE G AND ITS DERIVITIVES IN THE X,Y,Z DIRECTIONS.
 1    GGR=RS/(RM*(RM+RDR))
      DO 230 M1=1,3
      DGA=RM*(RM+RDR)*DS(M1)
      DGB=RS*(RM*(DP(M1)+DD(M1))+DP(M1)*(RM+RDR))
      DG(M1)=(DGA-DGB)/(RM*(RM+RDR))**2
  230 CONTINUE
```

```
      GO TO 3
 2    GGR=((RS)*(RM-RDR))/(RM*RXR*RXR)
      DO 240 M2=1,3
      DGA=(RS*(DP(M2)-DD(M2))+DS(M2)*(RM-RDR))*RM*RXR**2
      DGB=RS*(RM-RDR)*(RM*2.*RXR*DC(M2)+DP(M2)*RXR**2)
      DG(M2)=(DGA-DGB)/(RM*RXR**2)**2
 240  CONTINUE
C CALCUALTE THE FIELD CONTRIBUTIONS OF EACH CURRENT ELEMENT.
 3    DGX=DG(1)
      DGY=DG(2)
      DGZ=DG(3)
      CURP=XMP*CUR(L)*10000./39.37
      CURM=XMP*CUR(L)*GGR*10000.
      BX1=CURM*UGR
      BZ1=CURM*WGR
      BY1=CURM*VGR
C CALCULATE THE GRADIENT CONTRIBUTIONS OF EACH CURRENT ELEMENT.
      BXX1=CURP*UGR*DGX
      BXY1=CURP*(GGR*(EGR-FGR)+UGR*DGY)
      BXZ1=CURP*(GGR*(DGR-CGR)+UGR*DGZ)
      BYY1=CURP*VGR*DGY
      BYZ1=CURP*(GGR*(AGR-BGR)+VGR*DGZ)
      BZZ1=CURP*DGZ*WGR
C SUM THE INDIVUAL CONTRIBUTIONS TO THE FIELD AND GRADIENT TO GET THE
C TOTAL FIELD AND GRADIENTS.
      BX=BX+BX1
      BY=BY+BY1
      BZ=BZ+BZ1
      BXX=BXX+BXX1
      BXY=BXY+BXY1
      BXZ=BXZ+BXZ1
      BYY=BYY+BYY1
      BYZ=BYZ+BYZ1
      BZZ=BZZ+BZZ1
 210  CONTINUE
C CALCULATE AND TEST THE MAGNETIZATION OF THE BODY FOR SATURATION.
```

```
      RB=(BX**2+BY**2+BZ**2)**0.5
      XDK=XKT/DA
      AM=(1/DA)*RB
      IF(AM-AMS) 10,10,11
C CALCULATE THE FORCES PRODUCED ON THE BODY.
 10   FX=XDK*(BX*BXX+BY*BXY+BZ*BXZ)
      FY=XDK*(BX*BXY+BY*BYY+BZ*BYZ)
      FZ=XDK*(BX*BXZ+BY*BYZ+BZ*BZZ)
      GO TO 12
C CALCULATE THE COMPONENTS OF THE MAGNETIZATION AT SATURATION.
 11   BMY=(BY/RB)*AMS
      BMX=(BX/RB)*AMS
      BMZ=(BZ/RB)*AMS
C CALCULATE THE FORCES PRODUCED ON THE BODY.
      FX=XKT*(BMX*BXX+BMY*BXY+BMZ*BXZ)
      FY=XKT*(BMX*BXY+BMY*BYY+BMZ*BYZ)
      FZ=XKT*(BMX*BXZ+BMY*BYZ+BMZ*BZZ)
 12   CONTINUE
C
      AX(ITR)=FX/RHOI
      AY(ITR)=FY/RHOI
      AZ(ITR)=FZ/RHOI
C
      RETURN
      END
```

```
      SUBROUTINE TRAJ(XREL,XNE,XE,THETA,XAC,VAC,XM,XNS,DELT,SR,DELT6,ALP
     1HA,BETA,C,AG,VS,VE,TIME,AF,CELA,CESB,CDO,CDA2,CMA,CMAD,CMTD,CNB,CN
     2BD,CNPSID,ITR,ITRA,ITRB,BETSTL,ALPSTL,BSTL,ASTL,BL,VMAG)
C
C                          TRAJECTORY SUBROUTINE
C
      DIMENSION XE(3),THETA(3),XAC(3),VAC(3),XM(2,3),CF(2,3),AG(2,3),C(2
     1,3,3),CS(3),SN(3),WXV(2,3),TIME(1000),GS(2,3),ACS(1000,2,3),C1(2,3
     2),C2(2,3),C3(2,3),VSH(2,3),ACSH(2,3),VS(1000,2,3),VE(1000,2,3),VEH
     3(2,3),ALPHA(1000),BETA(1000),XREL(1000,3),XNE(1000,3),CFO(4),AF(4,
     44)
C
C CALCULATE THE DYNAMIC PRESSURE AND THE RATES OF ANGLE OF ATTACK AND
C SIDE SLIP.
      QS=SR*VMAG**2
      IF(ITR.EQ.1) GO TO 210
      ALPHAD=(ALPHA(ITR)-ALPHA(ITR-1))/DELT
      BETAD=(BETA(ITR)-BETA(ITR-1))/DELT
      GO TO 211
  210 ALPHAD=0.
      BETAD=0.
C
C CALCULATE THE INTERFERENCE FORCE AND MOMENT COEFFICIENTS.
  211 RV=(XREL(ITR,1)*XREL(ITR,1)+XREL(ITR,2)*XREL(ITR,2)+XREL(ITR,3)*XR
     1EL(ITR,3))**0.5
      IF(RV.EQ.0.)  KRM=1
      IF(RV.NE.0.) KRM=4
      DO 300 JR=1,4
      CFO(JR)=0.
      DO 300 KR=1,KRM
      IF(KR.EQ.1)  RK=1.
      IF(KR.NE.1) RK=RV**(KR-1)
  300 CFO(JR)=CFO(JR)+AF(JR,KR)/RK
C
C CALCULATE THE LIFT, DRAG, AND SIDE FORDE COEFFICIENTS.
      CDE=CDO*(1.+CDA2*(BETA(ITR)*BETA(ITR)+ALPHA(ITR)*ALPHA(ITR)))
```

```
      CES=CFO(1)+BETA(ITR)*CESB
      CEL=CFO(2)+CELA*ALPHA(ITR)
C
C TEST FOR STALL AND CALCULATE THE LIFT AND SIDE FORCE AT STALL.
      IF(BETA(ITR).GE.BETSTL)  GO TO 200
      GO TO 201
 200  ITRB=ITRB+1
      IF(ITRB.EQ.1) CESSTL=CES
      CES=CESSTL-BSTL*(BETA(ITR)-BETSTL)**2
 201  IF(ALPHA(ITR).GE.ALPSTL)  GO TO 202
      GO TO 203
 202  ITRA=ITRA+1
      IF(ITRA.EQ.1) CELSTL=CEL
      CEL=CELSTL-ASTL*(ALPHA(ITR)-ALPSTL)**2
C
C CALCULATE THE FORCE COEFFICIENTS RELATIVE TO THE STORE C.S.
 203  CF(1,1)=(CEL*SIN(ALPHA(ITR))-CDE*COS(ALPHA(ITR)))*COS(BETA(ITR))+C
     1ES*SIN(BETA(ITR))
      CF(1,2)=CDE*COS(ALPHA(ITR))*SIN(BETA(ITR))+CES*COS(BETA(ITR))
      CF(1,3)=-(CDE*SIN(ALPHA(ITR))+CEL*COS(ALPHA(ITR)))
C
C CALCULATE THE MOMENT COEFFICIENTS.
      CF(2,1)=0.
      CF(2,3)=(CFO(4)+CNB*BETA(ITR)+CNBD*BETAD+CNPSID*VS(ITR,2,3))*BL
      CF(2,2)=(CFO(3)+CMA*ALPHA(ITR)+CMAD*ALPHAD+CMTD*VS(ITR,2,2))*BL
C
C
C CALCULATE THE ACCELERATIONS DUE TO THE ROTATING STORE C.S.
      WXV(1,1)=VS(ITR,2,2)*VS(ITR,1,3)-VS(ITR,2,3)*VS(ITR,1,2)
      WXV(1,2)=VS(ITR,2,3)*VS(ITR,1,1)-VS(ITR,2,1)*VS(ITR,1,3)
      WXV(1,3)=VS(ITR,2,1)*VS(ITR,1,2)-VS(ITR,2,2)*VS(ITR,1,1)
C
      WXV(2,1)=VS(ITR,2,2)*VS(ITR,2,3)*(XM(2,3)-XM(2,2))/XM(2,1)
      WXV(2,2)=VS(ITR,2,1)*VS(ITR,2,3)*(XM(2,1)-XM(2,3))/XM(2,2)
      WXV(2,3)=VS(ITR,2,2)*VS(ITR,2,1)*(XM(2,2)-XM(2,1))/XM(2,3)
C
```

```
      TIMH=TIME(ITR)+DELT/2.
C
      DO 120 NF=1,2
      DO 140 MF=1,3
      GS(NF,MF)=0.
      DO 130 LF=1,3
C TRANSFER THE MAGNETIC AND GRAVITY FORCES TO THE STORE C.S.
 130  GS(NF,MF)=GS(NF,MF)+C(NF,LF,MF)*AG(NF,LF)
C CALCULATE THE ACCELERATION OF THE STORE IN THE STORE C.S.
      ACS(ITR+1,NF,MF)=CF(NF,MF)*QS/XM(NF,MF)-WXV(NF,MF)+GS(NF,MF)
      IF(ITR.EQ.1) ACS(1,NF,MF)=ACS(2,NF,MF)
C CALCULATE THE CONSTANTS FOR THE POWER SERIES EXPANSION IN TIME FOR
C VELOCITY (I.E. V=C1*T**2+C2*T+C3 ).
      C1(NF,MF)=(ACS(ITR+1,NF,MF)-ACS(ITR,NF,MF))/(DELT*2.)
      C2(NF,MF)=ACS(ITR,NF,MF)-2.*C1(NF,MF)*TIME(ITR)
      C3(NF,MF)=VS(ITR,NF,MF)-(C1(NF,MF)*TIME(ITR)**2+C2(NF,MF)*TIME(ITR
     1))
C CALCULATE THE ACCELERATION AND VELOCITY AT THE CENTER OF THE INTERVAL
      VSH(NF,MF)=C1(NF,MF)*TIMH**2+C2(NF,MF)*TIMH+C3(NF,MF)
      ACSH(NF,MF)=2.*C1(NF,MF)*TIMH+C2(NF,MF)
C USE SIMPSON'S RULE TO CALCULATE THE STORE VELOCITY AND ROTATION RATES.
 140  VS(ITR+1,NF,MF)=VS(ITR,NF,MF)+(ACS(ITR,NF,MF)+4.*ACSH(NF,MF)+ACS(I
     1TR+1,NF,MF))*DELT6
      DO 120 NE=1,3
      VE(ITR+1,NF,NE)=0.
      VEH(NF,NE)=0.
      DO 150 ME=1,3
C TRANSFER VELOCITIES AND ROTATION RATES TO THE EARTH COORDINATE SYSTEM
      VE(ITR+1,NF,NE)=VE(ITR+1,NF,NE)+C(NF,NE,ME)*VS(ITR+1,NF,ME)
 150  VEH(NF,NE)=VEH(NF,NE)+C(NF,NE,ME)*VSH(NF,ME)
C USE SIMPSON'S RULE TO CALCULATE STORE POSITION (SPACIAL COORDINATES
C AND EULER ANGLES).
      IF(NF.EQ.1) XE(NE)=XE(NE)+(VE(ITR,1,NE)+4.*VEH(1,NE)+VE(ITR+1,1,NE
     1))*DELT6
      IF(NF.EQ.2) THETA(NE)=THETA(NE)+(VE(ITR,2,NE)+4.*VEH(2,NE)+VE(ITR+
     11,2,NE))*DELT6
```

```
 120  CONTINUE
C CALCULATE THE ROTATION MATRIX FOR AXIS ROTATION.
      CALL ROTA(THETA,C)
      DO 180 LE=1,3
C CALCULATE THE RELATIVE POSITION OF STORE C.G. AND NOSE IN THE EARTH
C COORDINATE SYSTEM.
      XAC(LE)=XAC(LE)+VAC(LE)*DELT
      XREL(ITR+1,LE)=-12.*(XAC(LE)-XE(LE))
 180  XNE(ITR+1,LE)=XREL(ITR+1,LE)+12.*C(1,LE,1)*XNS
C
      ITR=ITR+1
C
C CALCULATE THE MAGNITUDE OF THE STORE VELOCITY, STORE ANGLE OF ATTACK,
C AND ANGLE OF SIDE SLIP.
      VMAG=(VS(ITR,1,1)**2+VS(ITR,1,2)**2+VS(ITR,1,3)**2)**.5
      IF(VMAG.EQ.0.) GO TO 220
      BETA(ITR)=ARSIN(VS(ITR,1,2)/VMAG)
      ALPHA(ITR)=ARSIN(VS(ITR,1,3)/VMAG)
      GO TO 221
 220  BETA(ITR)=0.
      ALPHA(ITR)=0.
 221  RETURN
      END
```

```
      SUBROUTINE ROTA(THETA,C)
C
C                          ROTATION MATRIX SUBROUTINE
C
      DIMENSION C(2,3,3),THETA(3),CS(3),SN(3)
C
C CALCULATE TRIG. FUNCTIONS OF THE EULER ANGLES.
      DO 100 KT=1,3
      SN(KT)=SIN(THETA(KT))
 100  CS(KT)=COS(THETA(KT))
C
C CALCULATE THE MATRIX FOR LINEAR VELOCITY TRANSFER.
      C(1,1,1)=CS(2)*CS(3)
      C(1,1,2)=SN(1)*SN(2)*CS(3)-CS(1)*SN(3)
      C(1,1,3)=CS(1)*SN(2)*CS(3)+SN(1)*SN(3)
C
      C(1,2,1)=CS(2)*SN(3)
      C(1,2,2)=SN(1)*SN(2)*SN(3)+CS(1)*CS(3)
      C(1,2,3)=CS(1)*SN(2)*SN(3)-SN(1)*CS(3)
C
      C(1,3,1)=-SN(2)
      C(1,3,2)=SN(1)*CS(2)
      C(1,3,3)=CS(1)*CS(2)
C
C CALCULATE THE MATRIX FOR ANGULAR VELOCITY TRANSFER.
      C(2,1,1)=1.
      C(2,1,2)=SN(1)*(SN(2)/CS(2))
      C(2,1,3)=CS(1)*(SN(2)/CS(2))
C
      C(2,2,1)=0.
      C(2,2,2)=CS(1)
      C(2,2,3)=-SN(1)
C
      C(2,3,1)=0.
      C(2,3,2)=SN(1)/CS(2)
      C(2,3,3)=CS(1)/CS(2)
```

```
C
      RETURN
      END
```

```
      SUBROUTINE OUPUT(CONFG,X1,Y1,Z1,X2,Y2,Z2,CUR,LM,CONFS,CURX1,CURX2,
     1CURX3,CURX4,CURZ1,CURZ2,CURZ3,CURZ4,TIME,XTR,XR,ERRX,IMAX,GES1,GES
     22,ZETA1,ZETA2,TVEL,ALPHAR,XRN,ALPHAI,XTRN,ERRAL)
C
C                              OUTPUT SUBROUTINE
C
      DIMENSION TIME(1000),XTR(1000,3),XR(1000,3),ERRX(1000,3),GES1(1000
     1),GES2(1000),ZETA1(1000),ZETA2(1000),ALPHAI(1000),XTRN(1000,3),ALP
     2HAR(1000),XRN(1000,3),ERRAL(1000),X1(500),Y1(500),Z1(500),X2(500),
     3Y2(500),Z2(500),CUR(500),CONFG(18),CONFS(18)
C
 814  FORMAT(//)
C
 815  FORMAT(///)
 818  FORMAT(1H1)
 827  FORMAT(24X,10HINPUT DATA)
 828  FORMAT(2X,6HX1(IN),2X,6HY1(IN),2X,6HZ1(IN),2X,6HX2(IN),2X,6HY2(IN)
     1,2X,6HZ2(IN),1X,10HCURRENT(A))
 829  FORMAT(1X,6F8.4,F10.0)
 851  FORMAT(3X,32HTOTAL CURRENT IN +X FIELD COIL= ,F10.0,6H AMPS.,3X,35
     1HTOTAL CURRENT IN +X GRADIENT COIL= ,F10.0,6H AMPS.)
 852  FORMAT(3X,32HTOTAL CURRENT IN -X FIELD COIL= ,F10.0,6H AMPS.,3X,35
     1HTOTAL CURRENT IN -X GRADIENT COIL= ,F10.0,6H AMPS.)
 853  FORMAT(3X,32HTOTAL CURRENT IN +Z FIELD COIL= ,F10.0,6H AMPS.,3X,35
     1HTOTAL CURRENT IN +Z GRADIENT COIL= ,F10.0,6H AMPS.)
 854  FORMAT(3X,32HTOTAL CURRENT IN -Z FIELD COIL= ,F10.0,6H AMPS.,3X,35
     1HTOTAL CURRENT IN -Z GRADIENT COIL= ,F10.0,6H AMPS.)
 859  FORMAT(24X,18A4)
 951  FORMAT(17X,91HCONSTANT GRAVITY AND ACTUAL TRAJECTORIES FOR WIND TU
     1NNEL STORE DROP WITH ARTIFICIAL GRAVITY)
 952  FORMAT(17X,34HARTIFICIAL GRAVITY CONFIGURATION: ,18A4)
 953  FORMAT(17X,20HSTORE CONFIGURATION:,18A4)
 960  FORMAT(4X,F6.3,10F8.4,3F10.4)
 970  FORMAT(4X,7HTIME(S),2X,5HX(IN),3X,5HY(IN),3X,5HZ(IN),1X,7HG'S(XZ),
     11X,7HG ANGLE,3X,5HX(IN),3X,5HY(IN),3X,5HZ(IN),1X,7HG' S(XZ),1X,7HG
     2ANGLE,3X,7H%ERRORX,3X,7H%ERRORY,3X,7H%ERRORZ)
```

```
 971  FORMAT(20X,21HCONSTANT ACCELERATION,27X,6HACTUAL,23X,18H% TRAJECTO
     1RY ERROR)
 972  FORMAT(10F12.4)
 973  FORMAT(1X,7HTIME(S),7X,13HANG OF ATTACK,3X,5HX(IN),7X,5HY(IN),7X,5
     1HZ(IN),4X,13HANG OF ATTACK,2X,5HX(IN),7X,5HY(IN),7X,5HZ(IN),12H%ER
     2ROR ALPHA)
 974  FORMAT(12X,40HCONSTANT ACCELERATION NOSE/TAIL POSITION,24X,25HACTU
     1AL NOSE/TAIL POSITION)
 980  FORMAT(3X,22HTUNNEL WIND VELOCITY= ,F9.4,4H FPS)
 981  FORMAT(17X,38H% ERROR IS NORMALIZED TO STORE LENGTH.)
C
C OUTPUT THE CHARACTERISTICS OF THE ARTIFICIAL GRAVITY CONFIGURATION.
      WRITE(6,859) (CONFG(IOC),IOC=1,18)
      WRITE(6,815)
      WRITE(6,814)
      WRITE(6,827)
      WRITE(6,828)
      WRITE(6,829) (X1(N1),Y1(N1),Z1(N1),X2(N1),Y2(N1),Z2(N1),CUR(N1),
     1N1=1,LM)
C
C OUTPUT THE ACTUAL AND IDEAL TRAJECTORIES AND TRAJECTORY ERRORS.
      WRITE(6,818)
      WRITE(6,951)
      WRITE(6,815)
      WRITE(6,952) (CONFG(JO),JO=1,18)
      WRITE(6,953) (CONFS(KO),KO=1,18)
      WRITE(6,814)
      WRITE(6,851) CURX1,CURX2
      WRITE(6,852) CURX3,CURX4
      WRITE(6,853) CURZ1,CURZ2
      WRITE(6,854) CURZ3,CURZ4
      WRITE(6,980) TVEL
      WRITE(6,981)
      WRITE(6,815)
      WRITE(6,971)
      WRITE(6,979)
```

```
      WRITE(6,960) (TIME(IO),XTR(IO,1),XTR(IO,2),XTR(IO,3),GES1(IO),ZETA
     11(IO),XR(IO,1),XR(IO,2),XR(IO,3),GES2(IO),ZETA2(IO),ERRX(IO,1),ERR
     2X(IO,2),ERRX(IO,3),IO=1,IMAX)
      WRITE(6,818)
      WRITE(6,974)
      WRITE(6,973)
      WRITE(6,972) (TIME(IN),ALPHAI(IN),XTRN(IN,1),XTRN(IN,2),XTRN(IN,3)
     1,ALPHAR(IN),XRN(IN,1),XRN(IN,2),XRN(IN,3),ERRAL(IN),IN=1,IMAX)
      RETURN
      END
```

CONSTANT GRAVITY AND ACTUAL TRAJECTORIES FOR WIND TUNNEL STORE DROP WITH ARTIFICIAL GRAVITY

ARTIFICIAL GRAVITY CONFIGURATION: EIGHT ORTHOGONAL HELMHOLTZ COILS, SUPERCOND. (ORTH8-5.4)
STORE CONFIGURATION: TEST THREE OF DEBUG--MODEL OF 15 FT. STORE

TOTAL CURRENT IN +X FIELD COIL= 0. AMPS. TOTAL CURRENT IN +X GRADIENT COIL= 0. AMPS.
TOTAL CURRENT IN -X FIELD COIL= 0. AMPS. TOTAL CURRENT IN -X GRADIENT COIL= 0. AMPS.
TOTAL CURRENT IN +Z FIELD COIL= 4550000. AMPS. TOTAL CURRENT IN +Z GRADIENT COIL= 4165000. AMPS.
TOTAL CURRENT IN -Z FIELD COIL= 4550000. AMPS. TOTAL CURRENT IN -Z GRADIENT COIL= -4165000. AMPS.
TUNNEL WIND VELOCITY= 500.0000 FPS
% ERROR IS NORMALIZED TO STORE LENGTH.

	CONSTANT ACCELERATION					ACTUAL					% TRAJECTORY ERROR		
TIME(S)	X(IN)	Y(IN)	Z(IN)	G'S(XZ)	G ANGLE	X(IN)	Y(IN)	Z(IN)	G'S(XZ)	G ANGLE	%ERRORX	%ERRORY	%ERRORZ
0.0	0.0	0.0	1.5000	10.4805	0.0	0.0	0.0	1.5000	10.4805	0.0	0.0	0.0	0.0
0.001	-0.0002	0.0000	1.5020	10.4805	0.0	-0.0002	0.0000	1.5020	10.4805	-0.0000	0.0	0.0000	0.0
0.002	-0.0007	0.0000	1.5081	10.4805	0.0	-0.0007	0.0000	1.5081	10.4805	-0.0001	-0.0000	0.0000	0.0
0.003	-0.0015	0.0001	1.5181	10.4805	0.0	-0.0015	0.0001	1.5181	10.4806	-0.0002	0.0	0.0000	-0.0000
0.004	-0.0026	0.0003	1.5322	10.4805	0.0	-0.0026	0.0002	1.5322	10.4807	-0.0003	0.0	0.0000	-0.0000
0.005	-0.0041	0.0004	1.5503	10.4805	0.0	-0.0041	0.0004	1.5503	10.4808	-0.0005	0.0	0.0000	-0.0000
0.006	-0.0058	0.0006	1.5724	10.4805	0.0	-0.0058	0.0006	1.5724	10.4808	-0.0006	0.0	0.0000	-0.0000
0.007	-0.0080	0.0009	1.5985	10.4805	0.0	-0.0080	0.0009	1.5985	10.4809	-0.0009	0.0	0.0000	-0.0000
0.008	-0.0104	0.0011	1.6286	10.4805	0.0	-0.0104	0.0011	1.6286	10.4811	-0.0011	0.0	0.0000	-0.0000
0.009	-0.0132	0.0015	1.6627	10.4805	0.0	-0.0132	0.0015	1.6627	10.4613	-0.0014	0.0	0.0000	-0.0000
0.010	-0.0163	0.0016	1.7008	10.4805	0.0	-0.0163	0.0018	1.7008	10.4814	-0.0018	0.0	0.0000	-0.0000
0.011	-0.0197	0.0022	1.7430	10.4805	0.0	-0.0197	0.0022	1.7430	10.4816	-0.0021	0.0	0.0000	-0.0000
0.012	-0.0234	0.0026	1.7891	10.4805	0.0	-0.0234	0.0025	1.7891	10.4818	-0.0025	0.0	0.0000	-0.0000
0.013	-0.0275	0.0031	1.8392	10.4805	0.0	-0.0275	0.0031	1.8392	10.4820	-0.0030	0.0	0.0000	-0.0001
0.014	-0.0319	0.0036	1.8933	10.4805	0.0	-0.0319	0.0036	1.8933	10.4823	-0.0035	0.0	0.0000	-0.0001
0.015	-0.0366	0.0041	1.9514	10.4805	0.0	-0.0366	0.0041	1.9514	10.4825	-0.0040	0.0	0.0000	-0.0001
0.016	-0.0417	0.0047	2.0135	10.4805	0.0	-0.0417	0.0047	2.0135	10.4828	-0.0045	0.0	0.0000	-0.0001
0.017	-0.0470	0.0052	2.0796	10.4805	0.0	-0.0470	0.0052	2.0796	10.4831	-0.0051	0.0	0.0000	-0.0001
0.018	-0.0527	0.0058	2.1497	10.4805	0.0	-0.0527	0.0058	2.1497	10.4835	-0.0057	0.0	0.0000	-0.0002
0.019	-0.0588	0.0065	2.2237	10.4805	0.0	-0.0588	0.0065	2.2238	10.4838	-0.0064	0.0	0.0000	-0.0002
0.020	-0.0651	0.0071	2.3018	10.4805	0.0	-0.0651	0.0071	2.3018	10.4841	-0.0070	0.0	0.0000	-0.0003
0.021	-0.0718	0.0078	2.3838	10.4805	0.0	-0.0718	0.0078	2.3838	10.4845	-0.0078	0.0	0.0000	-0.0003
0.022	-0.0786	0.0084	2.4698	10.4805	0.0	-0.0788	0.0084	2.4698	10.4849	-0.0085	0.0	0.0000	-0.0004
0.023	-0.0861	0.0091	2.5597	10.4805	0.0	-0.0861	0.0091	2.5598	10.4853	-0.0093	0.0	0.0000	-0.0004
0.024	-0.0937	0.0098	2.6537	10.4805	0.0	-0.0937	0.0098	2.6538	10.4857	-0.0101	0.0	0.0000	-0.0005
0.025	-0.1017	0.0105	2.7516	10.4805	0.0	-0.1017	0.0105	2.7517	10.4862	-0.0110	0.0001	0.0000	-0.0006
0.026	-0.1100	0.0113	2.8535	10.4805	0.0	-0.1100	0.0113	2.8536	10.4866	-0.0119	0.0001	0.0000	-0.0007
0.027	-0.1186	0.0120	2.9593	10.4805	0.0	-0.1186	0.0120	2.9595	10.4870	-0.0126	0.0001	0.0000	-0.0008
0.028	-0.1275	0.0127	3.0691	10.4805	0.0	-0.1276	0.0127	3.0693	10.4875	-0.0138	0.0001	0.0000	-0.0009
0.029	-0.1368	0.0135	3.1829	10.4805	0.0	-0.1368	0.0134	3.1831	10.4880	-0.0148	0.0001	0.0000	-0.0011
0.030	-0.1464	0.0142	3.3006	10.4805	0.0	-0.1464	0.0142	3.3009	10.4885	-0.0159	0.0001	0.0000	-0.0012
0.031	-0.1563	0.0149	3.4223	10.4805	0.0	-0.1563	0.0149	3.4226	10.4890	-0.0169	0.0002	0.0000	-0.0014
0.032	-0.1666	0.0156	3.5480	10.4805	0.0	-0.1666	0.0156	3.5482	10.4894	-0.0181	0.0002	0.0000	-0.0016
0.033	-0.1772	0.0164	3.6775	10.4805	0.0	-0.1772	0.0164	3.6779	10.4900	-0.0192	0.0	0.0000	-0.0018
0.034	-0.1880	0.0171	3.8111	10.4805	0.0	-0.1880	0.0171	3.8114	10.4905	-0.0204	0.0	0.0000	-0.0020
0.035	-0.1992	0.0178	3.9486	10.4805	0.0	-0.1992	0.0179	3.9490	10.4910	-0.0216	0.0	-0.0000	-0.0022
0.036	-0.2106	0.0184	4.0900	10.4805	0.0	-0.2106	0.0184	4.0904	10.4915	-0.0229	0.0010	-0.0000	-0.0025
0.037	-0.2223	0.0191	4.2353	10.4805	0.0	-0.2225	0.0191	4.2358	10.4921	-0.0243	0.0010	-0.0000	-0.0028
0.038	-0.2344	0.0198	4.3846	10.4805	0.0	-0.2346	0.0198	4.3852	10.4926	-0.0256	0.0010	-0.0000	-0.0031
0.039	-0.2468	0.0204	4.5378	10.4805	0.0	-0.2470	0.0204	4.5385	10.4931	-0.0270	0.0010	-0.0000	-0.0034

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