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Collisionless Solar Wind Protons:
A Comparison of Kinetic and Hydrodynamic Descriptions

by

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ABSTRACT

Kinetic and hydrodynamic descriptions of a collisionless solar wind proton gas are compared. Heat conduction and viscosity are neglected in the hydrodynamic formulation but automatically included in the kinetic formulation. The results of the two models are very nearly the same, indicating that heat conduction and viscosity are not important in the solar wind proton gas beyond about 0.1 AU. It is concluded that the hydrodynamic equations provide a valid description of the collisionless solar wind protons, and hence that future models of the quiet solar wind should be based on a hydrodynamic formulation.

I. Introduction

A hydrodynamic description of the solar wind is generally much more convenient than a kinetic description. When using hydrodynamic equations it is relatively easy to distinguish between various physical effects and to neglect unimportant effects (e.g. viscosity, thermal conduction) in order to simplify the description. Various types of collisional and wave particle interactions can be included in a hydrodynamic treatment by simply adding suitable source and sink terms to the conservation equations. However, when a gas is not collision-dominated, it becomes difficult, if not impossible, to derive realistic transport coefficients, and if viscous or thermal conduction effects are important, the hydrodynamic equations may no longer provide a suitable description of the gas. Solar wind protons are not collision-dominated beyond about 0.1 AU (Hundhausen, 1968), and the question has been raised as to the validity of hydrodynamic equations in this region. It is this question which we shall consider here.

The first theoretical description of the dynamic behaviour of the solar wind plasma was given by Parker (1958, 1963), who used the continuity and momentum equations of hydrodynamics, along with a polytropic temperature law, in developing a one-fluid model of the solar wind. Parker's hydrodynamic description was subsequently extended by several authors to include an energy equation in place of the polytropic temperature law (Chamberlain, 1961, 1965; Noble and Scarf, 1963; Scarf and Noble, 1965; Whang and Chang, 1965; Whang et al., 1966; Parker, 1964, 1965; Meyer and Schmidt, 1966, 1968; Konyukov, 1969). A two-fluid hydrodynamic description was developed by Sturrock and

Hartle (1966; Hartle and Sturrock, 1968) and later extended by Hartle and Barnes (1970) to include the effects of proton heating. A two-fluid model including proton heating, as well as a proton thermal anisotropy, has recently been developed by Leer and Axford (1971), while Whang (1971a) has given an elegant description of the proton thermal anisotropy and the proton heat flux by adding a fourth moment equation to the standard hydrodynamic equations.

In contrast, many authors have taken the view that since collisions are infrequent in the proton gas beyond about 0.1 AU, the solar wind protons should be described by a kinetic equation (Jensen, 1963; Brandt and Casinelli, 1966; Jockers, 1968, 1970; Griffel and Davis, 1969; Hollweg, 1970, 1971; Eviatar and Schulz, 1970; Lemaire and Scherer, 1971; Schulz and Eviatar, 1971; Chen et al., 1972). Griffel and Davis (1969) and Eviatar and Schulz (1970; Schulz and Eviatar, 1971) have attempted to include the effects of collisional and wave-particle interactions, while the other authors have considered the protons to be collisionless. All of these workers have either stated or implied that the observed state of the solar wind cannot be fully understood on the basis of hydrodynamic theory, but only in terms of kinetic theory.

In the present report, we compare equivalent kinetic and hydrodynamic models of a collisionless solar wind proton gas, in an attempt to discover the degree of validity of the hydrodynamic description. Viscosity and thermal conduction are neglected in the hydrodynamic treatment, but a proton thermal anisotropy is included. The differences in the results of the kinetic and hydrodynamic models are found to be so small that we are led to conclude that

the effects of proton thermal conduction and viscosity are relatively unimportant, and hence that the hydrodynamic description of the collisionless proton gas is quite adequate. The same conclusions were reached with regard to the polar wind, following a similar study comparing kinetic and hydrodynamic models (Holzer et al., 1971).

2. Kinetic and Hydrodynamic Descriptions

For a kinetic description, we shall use the model of Hollweg (1970) and its extension to include a non-radial magnetic field (Chen et al., 1972). Our hydrodynamic description will be based on the model developed by Leer and Axford (1971) but will involve assumptions slightly different from those used by Leer and Axford. Since we wish to compare the results of the two descriptions, it is necessary to use exactly the same boundary conditions and to treat the electron gas in the same manner in both cases. Consequently, as we shall be making direct use of the results obtained by Hollweg (1970) and Chen et al. (1972), the hydrodynamic description must be constructed carefully, to ensure that it is entirely consistent with the kinetic description.

Let us begin by outlining the assumptions which are common to both the kinetic and hydrodynamic treatments. The solar wind is assumed to be a steady, radial, spherically symmetric flow of a proton-electron plasma, with the electron component collision-dominated and highly subsonic throughout the region of interest. The electron gas can thus be described by the hydrodynamic radial momentum equation:

$$\frac{d}{dr} (n_e k T_e) = -n_e e E_r \quad (1)$$

where inertial, magnetic, and gravitational terms are neglected. We assume that the electron temperature is given by a polytropic law:

$$T_e = T_{eo} \left(\frac{n_e}{n_{eo}} \right)^{\alpha-1} \quad (2)$$

where α is the polytrope index. Evidently this is not the best description of the electron temperature, but it is the one chosen by Hollweg ($\alpha=1$) and Chen et al. A better description would involve the solution of the electron energy equation with the effects of heat conduction included (Chapman, 1957; Hartle and Sturrock, 1968; Holzer and Axford, 1970; Forslund, 1970; Leer and Axford, 1971).

The protons are taken to be collision-dominated in the region $r \leq r_o$, where $r_o \approx 0.1$ AU, and the boundary conditions at r_o are determined by the results of the two fluid model of Hartle and Sturrock (1968). In $r > r_o$, we assume quasi-neutrality ($n_e \approx n_p \approx n$) and take the interplanetary field to be of the form

$$\underline{B} = B_o \left(\frac{r_o}{r} \right)^2 \left(1, \frac{r\omega}{v}, 0 \right) \quad (3)$$

In Hollweg's model $\omega = 0$, corresponding to a radial magnetic field, while in the model of Chen et al. $\omega = 2.7 \times 10^{-6}$ radians sec^{-1} , corresponding to a spiral field resulting from a 27 day solar equatorial rotation period. (Note that Chen et al. should have used $\omega = 2.9 \times 10^{-6}$ radians sec^{-1} , corresponding to the 25 day equatorial rotation period of the sun, since the motion of the earth about the sun has no effect on the interplanetary magnetic field.) We shall

consider both the cases where $\omega = 0$ and $\omega = 2.7 \times 10^{-6}$ radians sec^{-1} .

All of the above assumptions are common to both the kinetic and hydrodynamic models which we are considering here. The difference between the models appears in the treatment of the proton gas in $r > r_0$. In the kinetic approach (cf. Hollweg, 1970; Chen et al., 1972) one must begin by describing proton trajectories in a magnetic field, with electric and gravitational forces present. Using Liouville's theorem, it is then possible to write the proton distribution function at r in terms of the known distribution function at r_0 . Thus $n(r)$ is determined, and from (1) and (2), $E(r)$ can be calculated, so that a self-consistent description is obtained. Since the distribution function is calculated directly in $r > r_0$, the effects of thermal conduction and viscosity are included automatically in the kinetic description. We shall not reproduce any part of the mathematical description of the kinetic model, but the interested reader is referred to the papers of Hollweg (1970) and Chen et al. (1972). However, we must write down the equations describing the hydrodynamic model, since they are slightly different from those used by Leer and Axford (1971).

The steady hydrodynamic equations describing the proton gas are given by Chew et al. (1956):

$$\nabla \cdot (n \underline{v}) = 0 \quad (4)$$

$$m \underline{v} \cdot (\nabla \underline{v}) + \frac{1}{n} \nabla \cdot \underline{P} + e \left(\underline{E} + \frac{1}{c} \underline{v} \times \underline{B} \right) + m \frac{GM_{\odot}}{r^2} \hat{r} = 0 \quad (5)$$

$$\underline{v} \cdot \nabla \left(\frac{B^2 T^{\parallel}}{n^2} \right) = 0 \quad (6)$$

$$\underline{v} \cdot \nabla \left(\frac{T^{\perp}}{B} \right) = 0 \quad (7)$$

where \underline{v} is the proton flow velocity, m is the proton mass, \underline{P} is the diagonal proton pressure tensor, and T^{\parallel} and T^{\perp} are the proton temperatures parallel and perpendicular to the magnetic field. The equations have been closed by neglecting thermal conduction, and the pressure tensor has been taken to be of the form $\underline{P} = p_{\parallel} \hat{b}\hat{b} + p_{\perp} (\hat{a}\hat{a} + \hat{c}\hat{c})$, where \hat{a} , \hat{b} , and \hat{c} are unit vectors of a local Cartesian coordinate system, with \hat{b} parallel to the magnetic field.

Thus $p_{\parallel} = nk T^{\parallel}$ and $p_{\perp} = nk T^{\perp}$. Since \underline{P} is defined in this way, the effects of viscosity have also been neglected.

For a radial, spherically symmetric expansion, (4) - (7) reduce to

$$\frac{d}{dr} (n v r^2) = 0 \quad (8)$$

$$m v \frac{dv}{dr} + \frac{1}{n} \frac{d}{dr} (nk T_e) + \frac{\hat{r}}{n} \cdot \nabla \underline{P} + m \frac{GM_{\odot}}{r^2} = 0 \quad (9)$$

$$\frac{d}{dr} \left(\frac{T^{\parallel} B^2}{n^2} \right) = 0 \quad (10)$$

$$\frac{d}{dr} \left(\frac{T^{\perp}}{B} \right) = 0 \quad (11)$$

The neglect of magnetic forces in (1), (9) - (11) appears to be reasonable in $r \gtrsim 0.1$ AU on the basis of the work of Whang (1971b) (see also Weber and Davis, 1967, 1970; Weber, 1970). It will be useful to rewrite equation (9), making use of (2), (3), (6), (10), and (11):

$$\begin{aligned}
 \frac{1}{v} \frac{dv}{dr} & \left\{ v^2 - \frac{k}{m} \left[\frac{1}{1 + \Omega^2} \left(3 T^{\parallel} + \Omega^2 T^{\perp} \right) + \alpha T_e \right] \right\} \\
 & = \frac{2k}{rm} \left[\frac{1}{1 + \Omega^2} \left(3 T^{\parallel} + \Omega^2 T^{\perp} \right) + \alpha T_e \right] \\
 & + \frac{k}{rm} \frac{1}{(1 + \Omega^2)^2} \left[\left(2 + \Omega^2 \right) \left(\Omega^2 T^{\perp} - 2 T^{\parallel} \right) \right. \\
 & \quad \left. + \left(2 - \Omega^2 - \Omega^4 \right) \left(T^{\perp} - T^{\parallel} \right) \right] \tag{12}
 \end{aligned}$$

where $\Omega = \frac{r\omega}{v} = \tan \psi$, and ψ is the spiral angle of the magnetic field. Evidently (2), (3), (8), (10) - (12) provide a closed set of hydrodynamic equations which describe the protons in $r > r_0$. These equations can be numerically integrated simultaneously to yield results which can be compared with those of the kinetic model.

3. Results

We can now compare results of the kinetic and hydrodynamic descriptions by examining radial profiles of v , $T_p = \frac{1}{3} (T^{\parallel} + 2 T^{\perp})$, and T^{\parallel}/T^{\perp} , the proton thermal anisotropy. Figures 1 and 2 show the results of Chen et al. (1972), and Figures 3 and 4 show the equivalent results of our hydrodynamic description. The boundary conditions used in obtaining these results are listed in the figure captions.

Flow speed. Comparing Figures 1a and 2 with Figures 3a and 4, we see that the radial profile of v is nearly the same in the kinetic and hydrodynamic models for various electron temperature models. At 1 AU the flow speeds predicted by our hydrodynamic equations are only 5 - 8% smaller than those given by the kinetic model. Evidently, in both descriptions, the flow speed at 1 AU is strongly dependent on the electron temperature model, so one should not compare the results of two descriptions where different electron temperature models are used (viz. the results of Hollweg (1970) and Chen et al. (1972) should not be compared with those of Hartle and Sturrock (1968)). Also, it has been shown by Hartle and Barnes (1970), Barnes et al. (1971), and Leer and Axford (1971) that heating the proton gas in the collision-dominated region leads to larger flow speeds at 1 AU, so the increase in v with increasing $T_p(r_o)$ reported by Chen et al. (1972) is not at all surprising.

Proton temperature. Again comparing Figures 1a and 2 with Figures 3a and 4, we see that the radial profile of T_p is nearly the same in the kinetic and hydrodynamic models for various electron temperature models. At 1 AU the proton temperatures predicted by the hydrodynamic equations are only

2 - 3% larger than those given by the kinetic equations. It is seen in both the kinetic and hydrodynamic treatments that one effect of a spiral magnetic field is a decrease of T_p , primarily through a decrease of T^{\parallel} .

Proton thermal anisotropy. A comparison of Figures 1b and 3b indicates that there is only a very small difference between the anisotropy (T^{\parallel}/T^{\perp}) predicted by the kinetic equations and that predicted by the hydrodynamic equations. However, we note that from the hydrodynamic formulation (cf. equations (10) and (11)) we can immediately write the form of the anisotropy,

$$\frac{T^{\parallel}}{T^{\perp}} = \left(\frac{n}{n_o} \right)^2 \left(\frac{B_o}{B} \right)^3 \quad (13)$$

and hence can gain a feeling for the physical processes important in determining the anisotropy. Since most of the solar wind acceleration takes place in $r < 0.1$ AU, in $r > 0.1$ AU the density varies nearly as r^{-2} , while near the sun $B \sim r^{-2}$ and far from the sun $B \sim r^{-1}$. Evidently at some distance $r = r_{\max}$, the anisotropy has a maximum value given by

$$\left(\frac{T^{\parallel}}{T^{\perp}} \right)_{\max} \approx \frac{2}{3\sqrt{3}} \cot^2 \psi_o \left(1 + \tan^2 \psi_o \right)^{3/2} \quad (14)$$

where $\psi_o = \psi(r_o)$. For $\omega = 2.7 \times 10^{-6}$ radians sec^{-1} , a solar wind speed at 1 AU of $v = 285$ km sec^{-1} leads to $r_{\max} = 1$ AU, while the more appropriate choice of $\omega = 2.9 \times 10^{-6}$ radians sec^{-1} leads to $r_{\max} = 1$ AU if $v = 305$ km sec^{-1} . In the absence of proton collisions in $r > 0.1$ AU, the maximum anisotropy is

about 11, while for a radial field the anisotropy at 1 AU is about 45. Leer and Axford (1971) have shown that the combination of a spiral magnetic field and proton-proton Coulomb collisions is sufficient to reduce the anisotropy to observed values (Hundhausen et al., 1967, 1970; Hundhausen, 1968, 1970; Axford, 1968).

By introducing an artificial, constant electron anisotropy, $T_e^{\parallel}/T_e^{\perp} = 1.2$, in $r > r_0$ the flow velocity is reduced some 3% at the orbit of the earth. This is a somewhat smaller reduction than was obtained by Hollweg (1971).

4. Conclusions

From the comparison of a kinetic model of collisionless solar wind protons with a hydrodynamic model in which heat conduction and viscous effects are neglected, we conclude that heat conduction and viscosity are not important processes in the solar wind proton gas in $0.1 \text{ AU} < r < 1 \text{ AU}$. Consequently, the hydrodynamic equations provide a valid description of the solar wind in $r < 1 \text{ AU}$. The protons are not in reality collisionless, but the hydrodynamic equations can be modified easily to include the effects of collisions (or wave-particle interactions) in the proton gas (e.g. Hartle and Sturrock, 1968; Nishida, 1969; Holzer and Axford, 1970; Leer and Axford, 1971; Toichi, 1971). Since heat conduction and viscosity are not important for the protons, these modified hydrodynamic equations should also be valid.

Hence it is clear that models of the quiet solar wind in $r < 1 \text{ AU}$ should be described by hydrodynamic equations, since a kinetic formulation adds nothing but complexity.

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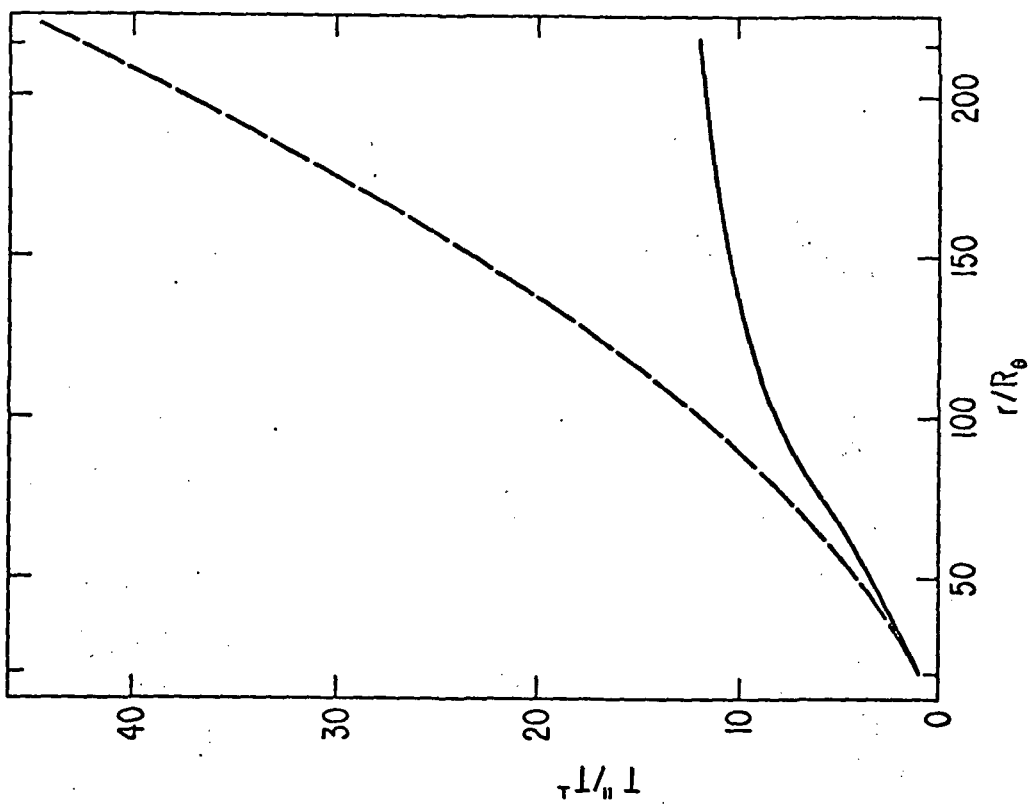
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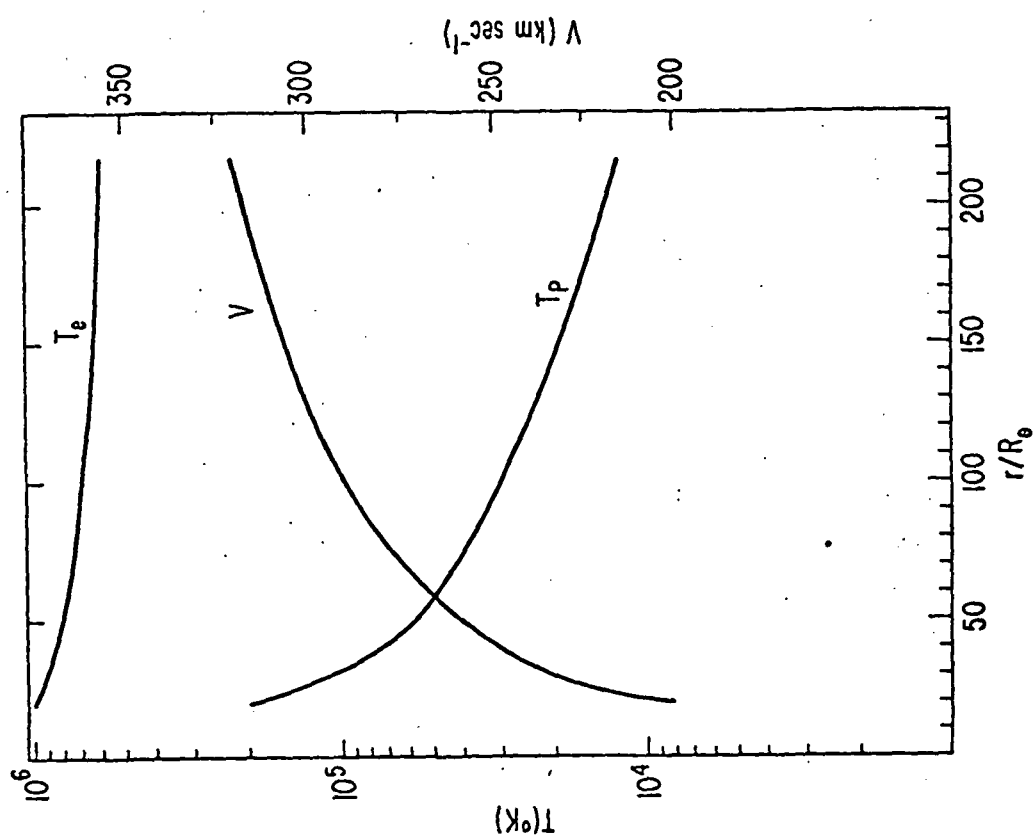
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FIGURE CAPTIONS

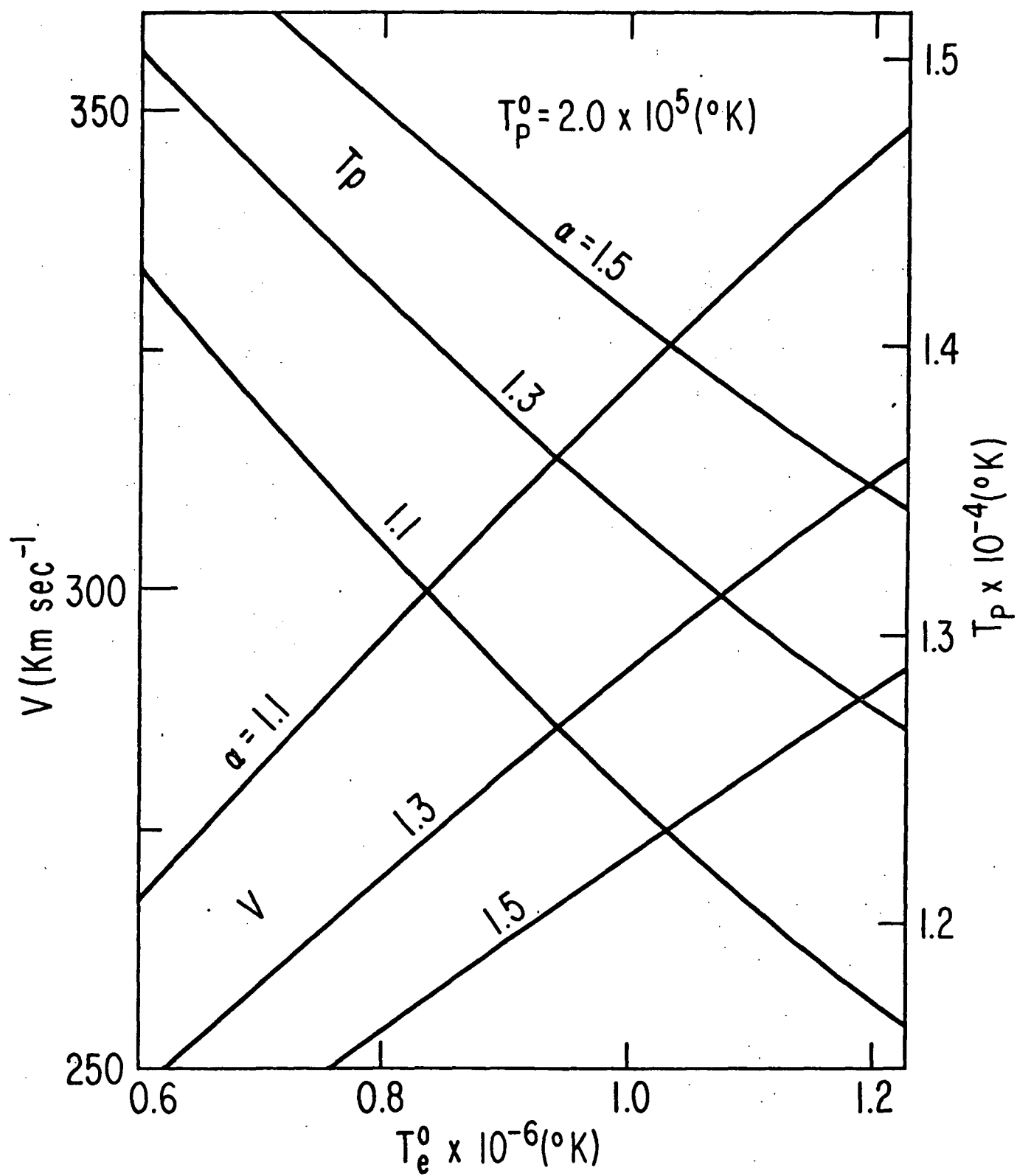
- Figure 1. Kinetic description of the solar wind protons: a.) Electron-temperature, T_e , mean proton-temperature, T_p , and flow velocity, v ; and b.) anisotropy, T^{\parallel}/T^{\perp} , versus heliocentric distance, r , in the ecliptic plane. The polytropic index for the electron gas, α , is 1.1, and the initial conditions are $T_e = 10^6$ °K, $T^{\parallel} = T^{\perp} = T_p = 2 \times 10^5$ °K and $v = 200$ km sec $^{-1}$ at $r = 20R_{\odot}$. The solid curves are for an angular velocity of the sun, $\omega = 2.7 \times 10^{-6}$ sec $^{-1}$. The broken curve in Figure 1b is for $\omega = 0$.
- Figure 2. Kinetic description of solar wind protons: The flow velocity, v , and the mean proton temperature T_p , at 1 AU, as a function of the electron temperature, T_e^0 , at $r = 20R_{\odot}$ for different polytropic indices, α .
- Figure 3. Hydrodynamic description of the solar wind: a.) Electron temperature, T_e , mean proton temperature, T_p , and flow velocity, v ; and b.) anisotropy, T^{\parallel}/T^{\perp} , versus heliocentric distance, r , in the ecliptic plane. The polytropic index for the electron gas, α , is 1.1, and the initial conditions are $T_e = 10^6$ °K, $T^{\parallel} = T^{\perp} = T_p = 2 \times 10^5$ °K and $v = 200$ km sec $^{-1}$ at $r = 20R_{\odot}$. The solid curves are for an angular velocity of the sun $\omega = 2.7 \times 10^{-6}$ sec $^{-1}$. The broken curve in Figure 3b is for $\omega = 0$.
- Figure 4. Hydrodynamic description of the solar wind: The flow velocity, v , and the mean proton temperature, T_p , at 1 AU, as a function of the electron temperature, T_e^0 , at $r = 20R_{\odot}$ for different polytropic indices, α .

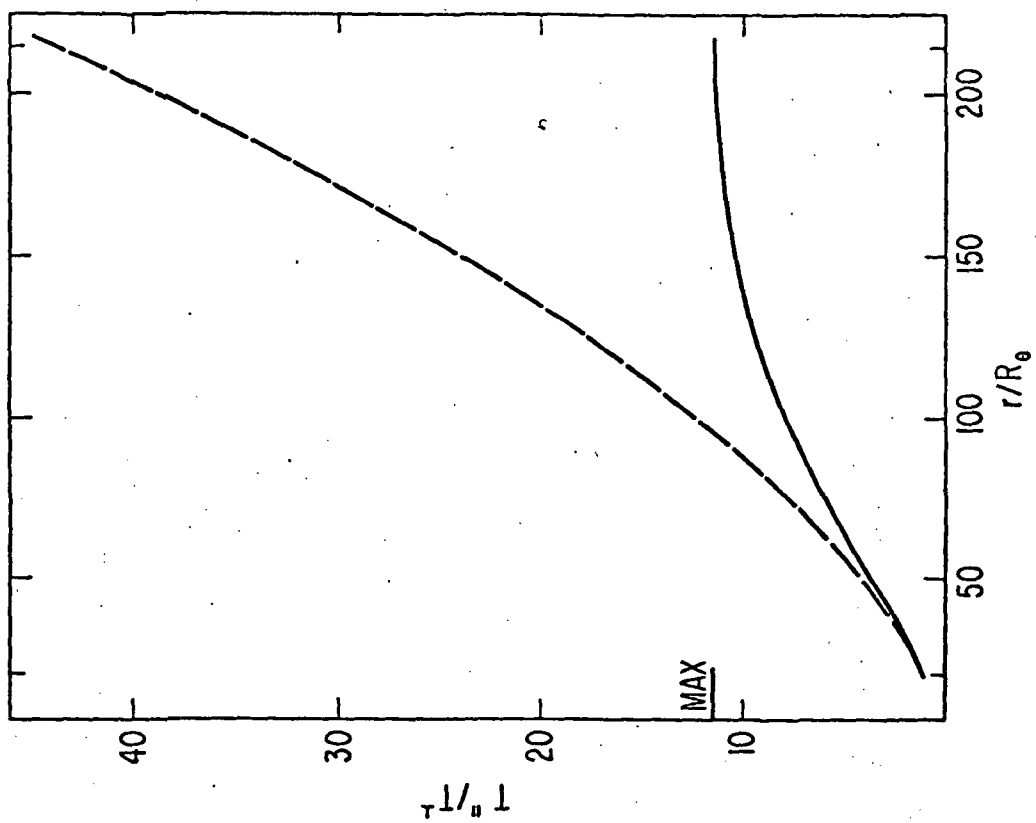


b

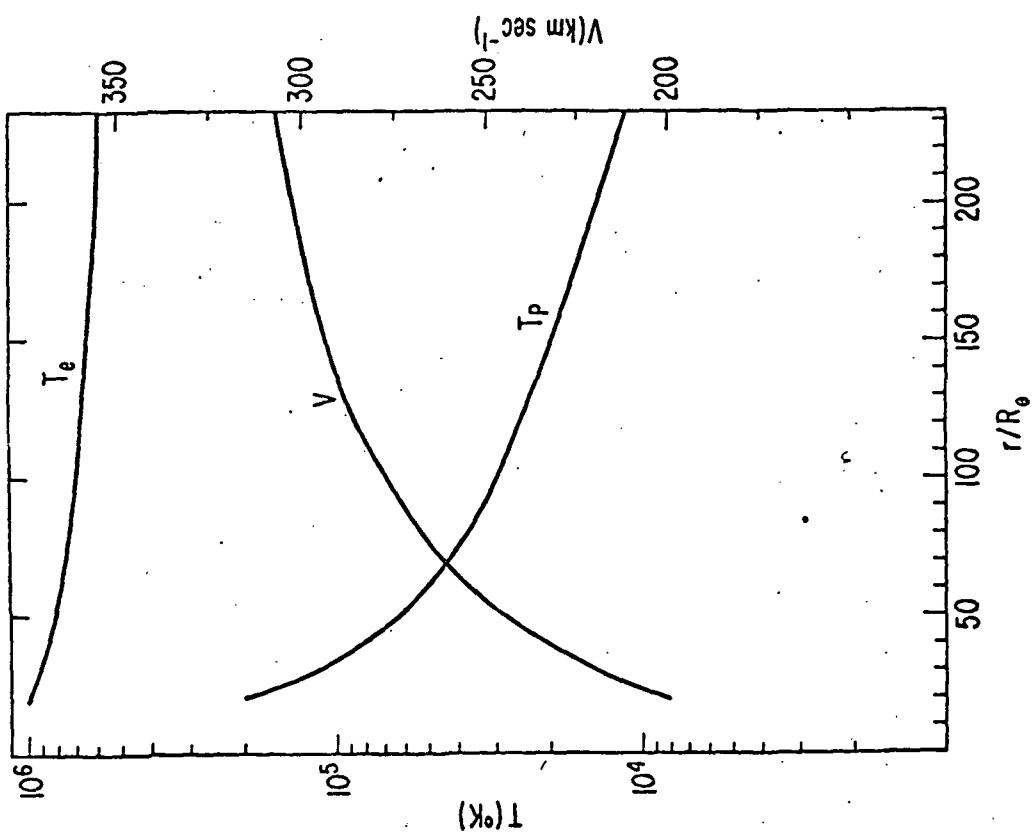


a





b



a

