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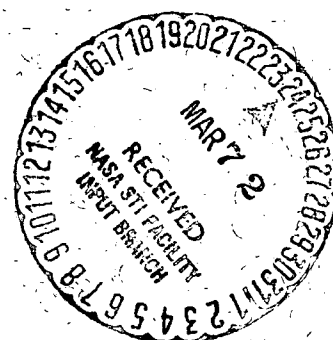
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# THE EFFECT OF MULTIPLE ENCOUNTERS ON SHORT PERIOD COMET ORBITS

BARBARA E. LOWREY

FEBRUARY 1972



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**GODDARD SPACE FLIGHT CENTER**  
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Barbara E. Lowrey

Astrodynamics and Geodynamics Division

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ABSTRACT

The observed orbital elements of short period comets are found to be consistent with the hypothesis of derivation from long period comets as long as two assumptions are made. First, the distribution of short period comets has been randomized by multiple encounters with Jupiter and second, the short period comets have lower velocities of encounter with Jupiter than is generally expected. Some 16% of the observed short period comets have lower encounter velocities than is allowed mathematically using Laplace's method. This may be due to double encounter processes with Jupiter and Saturn, or as a result of prolonged encounters. The distribution of unobservable short period comets can be inferred in part from the observed comets. Many have orbits between Jupiter and Saturn with somewhat higher inclinations than those with perihelions near the earth. Debris from those comets may form the major component of the zodiacal dust. Comets with very low velocity will not normally be observable, Comet Schwassmann-Wachmann 1 being the only known exception. Therefore, it is suggested that data on the distribution of dust in the solar system be

obtained experimentally from a mission to the outer planets.  
These data may allow further conclusions on the origin of  
other material in the solar system, such as meteors or  
meteorites.

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# THE EFFECT OF MULTIPLE ENCOUNTERS ON SHORT PERIOD COMET ORBITS

## INTRODUCTION

Classically, the distribution of cometary orbits in  $1/a$  has been studied in an attempt to determine the origin of comets. (Porter, 1964; Richter, 1963). More recently, investigations by analytic and numerical integration methods have been made in order to match the distributions of cometary orbits in  $1/a$  and other classical orbital elements. Everhart (1969) has computed on a random basis the orbits of short period comets that have resulted from close encounters of random parabolic orbits with Jupiter. He finds that the distribution of orbits does not agree with those of the known short-period orbits, even after allowing for observational selection. He concludes that the capture of near-parabolic comets is apparently not the source of the observable short period comets, at least not without considering multiple encounters.

Instead, the encounter velocity and the angle of approach are computed in this paper. These variables have several advantages. The encounter velocity  $u$  is nearly invariant with regards to the passages through Jupiter's sphere of influence; it preserves a memory of the original energy relative to Jupiter. The angle of approach,  $\theta$ , may be expected to become random after several encounters with Jupiter. Thus, expensive numerical computations may be avoided and the physical processes which have affected comet orbits may be inferred through a direct comparison of these quantities computed for short period comets with the mathematical constraints on  $u$  and  $\theta$ .

The approach utilized in this paper is not presented as a complete substitute for numerical integration, but rather as a complementary analysis. It reduces

the number of variables to be studied by numerical integration by identifying the variables of major importance. It shows directly that a classical quandary concerning the lack of retrograde short period comets is not significant to the origin of short period comets. And it frames in a simple manner a number of problems concerning the origin of comets, meteors, meteorites and the zodiacal light which require further investigation.

## ANALYSIS

The formulas to be used are a combination of formulas derived by Opik (1963, 1965, 1966) and previously by this author (1966). The formulas are obtained by simplifying Laplace's method, where the orbit is assumed to be controlled by Jupiter only while in Jupiter's sphere of influence and by the sun only outside of Jupiter's influence. The further assumption is that the heliocentric orbital elements are specified by three parameters describing the velocity vector relative to Jupiter at the exit from Jupiter's sphere of influence (assumed to be so far from Jupiter that the energy due to Jupiter's gravitational field is negligible). Further, Jupiter is assumed to be moving in a circle in the plane of the ecliptic.

The first variable,  $u$ , is the magnitude of the velocity vector at the entrance to or exit from Jupiter's sphere of influence (or "infinite" distance from Jupiter, but at the heliocentric distance of Jupiter). It is hereafter called the encounter velocity, following Opik's terminology. In this approximation,  $u$  is an invariant in the encounter process. The angle of approach,  $\theta$ , is the angle between Jupiter's velocity vector and the comet's velocity vector before or after passage through Jupiter's sphere of influence. This angle will change during a close

encounter, the maximum change occurring when the comet is closest to Jupiter's surface. The third variable,  $\phi$ , specifies the angular distance of the velocity vector from the ecliptic. This coordinate system is analogous to latitude and longitude, only the "pole" is in the direction of Jupiter's velocity vector. A major reason for this choice of pole is that the heliocentric energy is independent of  $\phi$ ; also, the condition for retrograde orbits is independent of  $\phi$ .

Since  $u$  is an invariant in this approximation, the orbital elements can be calculated as a function of  $\theta$  and  $\phi$  for a given  $u$ . Those orbital elements which have the same value of  $u$  but different values of  $\theta$  and  $\phi$  will be termed "cognate orbits." Thus, following an encounter with Jupiter, the new orbit will be a cognate orbit of the old orbit.

The encounter velocity may be calculated from the heliocentric orbital elements before or after encounter:

$$u^2 = 3 - 2 \sqrt{a(1-e^2)} \cos i - 1/a$$

where  $a$ ,  $e$ ,  $i$  are the semimajor axis, eccentricity and inclination of the heliocentric orbit. Here we shall take  $a$  normalized to Jupiter's orbit,  $a_J=1$ , so that  $u$  is expressed in units of  $u_J$ . From  $u$  and the heliocentric semimajor axis, the angle of approach can be obtained

$$\cos \theta = \frac{1 - u^2 - 1/a}{2u}$$

( $\theta$  is the same as  $\alpha$  in Opik's notation.)

Conversely, from  $u$  and  $\theta$ , the semimajor axis of the heliocentric orbit can be obtained

$$a = \frac{-1}{u^2 + 2u \cos \theta - 1}$$



where  $a$  is again normalized to  $a_J$ . Then the condition for the parabolic orbit ( $a=\infty$ ) occurs when  $u^2 + 2 u \cos \theta - 1 = 0$  or, for a given value of  $u$ ,  $\theta_p = \cos^{-1}((1 - u^2)/2u)$ . In order for any orbits to be hyperbolic,  $u$  must be greater than  $\sqrt{2} - 1. = .414$  and when  $u$  is greater than  $\sqrt{2} + 1. = 2.414$  all orbits will be hyperbolic. For  $.414 < u < 2.414$ , those orbits with  $\theta < \theta_p$  will be hyperbolic. In other words, if the comet leaves Jupiter's sphere of influence in the forward direction of Jupiter's motion, the velocity of the particle relative to Jupiter will add to Jupiter's velocity to yield the most energetic orbits; if the comet leaves Jupiter's sphere of influence so that the comet velocity opposes Jupiter, the velocities will subtract and a small heliocentric orbit will result.

Typical values of the heliocentric semimajor axis are shown in Fig. 1. It will be noted that as the value of  $\theta$  increases from  $\theta_p$ , the value of  $a$  decreases sharply. The smallest possible value of the semimajor axis is 2.6 au, that, 1/2 of Jupiter's semimajor axis, corresponding to an orbit with an aphelion,  $Q=5.2$  au and  $q=0.0$ , and occurring when  $\theta = 180.^\circ$ . The steepness of the change in  $a$  following a small change in  $\theta$  after an encounter when  $\theta$  is near  $\theta_p$  is one reason why  $1/a$  has been used to study the distribution of cometary orbits. The difference  $\Delta\theta = \theta - \theta_p$  will be termed the scattering angle.

The boundary between retrograde and direct orbits is obtained from  $\theta_r = \cos^{-1}(-1./u)$  where retrograde orbits occur if  $\theta > \theta_r$ . For this condition to occur  $u$  must be greater than 1.0; otherwise all orbits will be direct.

From these two formulas, the distribution of direct and retrograde and hyperbolic and elliptic orbits can be mapped as a function of  $u$  and  $\theta$ . The ordinate  $\theta$  is mapped in units of cosine  $\theta$  in order to be proportional to the probability of  $\theta$  occurring if a random distribution of encounter velocities is assumed. (Fig. 2).

The angle  $\phi$ , which measures the out-of-plane component, can be determined from the inclination of the heliocentric orbit

$$\sin \phi = \frac{u \cos \theta + 1}{u \sin \theta} \tan i$$

It is this angle which measures the out-of-plane component. Once  $\phi$  is obtained, then the eccentricity of the heliocentric orbit can be expressed:

$$e^2 = u^4 (1 - z) + 2 u^3 \cos \theta (2 - z) + u^2 (4 \cos \theta - z)$$

where

$$z = \sin^2 \theta \cos^2 \phi$$

It is important to note that the eccentricity is dependent upon  $\phi$ . That is, although the semimajor axis is not affected by the out-of-plane component, the eccentricity is. For example, the line which separates the direct and retrograde orbits in Fig. 2 gives a perihelion = 0.0 when the orbit is in the plane of the ecliptic but not when the orbit has any inclination to the ecliptic (although it still separates direct and retrograde orbits.)

From the eccentricity and the semimajor axis, the values of  $\theta$  and  $\phi$  which yield a specified value of perihelion  $q$  can be found. The dotted lines in Fig. 2a show the values of  $\theta$  and  $\phi$  where  $q = .5, 1., 2., 3.$  au when  $\phi = 0^\circ$  ( $i = 0^\circ$ , also). Figs. 2b and 2c show the lines of constant  $q$  for the cases  $\phi = 45^\circ$  and  $\phi = 85^\circ$ , respectively. The higher values of  $\phi$  produce fewer values of  $q < 2.$  au, proportionately, that is, the minimum perihelions occur more commonly when the comet orbit is close to the ecliptic.

These formulas are sufficiently accurate for high velocity orbits. As  $u$  becomes lower, the accuracy becomes less, and the validity of the assumption

of the invariance of  $u$  will be discussed later in this paper. Still, for low velocity objects, it is useful to compute  $u$  as a first approximation and to demonstrate the statistical distribution in  $u$  and  $\theta$ .

## RESULTS

The values of  $u, \theta, \phi$  were calculated for the short period comet orbits in the lists of Porter (1964). The results are tabulated in Table 1.

The asterisks in the table means "Not Calculated". For  $u$ , this happens when  $u^2 < 0.0$ , implying that the comet cannot approach Jupiter. For one of the four comets, P/Oterma, this was a consequence of the simplifications introduced and not true in fact, because  $u$  is computed at Jupiter's mean radius, 5.2 au, rather than at the edge of Jupiter's sphere of influence. For example, P/Oterma was placed into and removed from the listed orbit by Jupiter. P/Wilson - Harrington and P/Kulin are to be deleted from the next orbital catalogue due to insufficient observations (Marsden, private communication).

Only the 4th comet, P/Encke, is currently free from large perturbations by close approaches to Jupiter, a condition which was apparently a consequence of the secular acceleration by non-gravitational forces; however, it might be of interest to determine by numerical studies the smallest aphelion an encounter with Jupiter could yield.

Some comets have  $u^2 > 0.0$ , but  $\cos \theta < -1$ . This condition occurs when  $2a > 5.2\text{au}$  but  $Q < 5.2\text{ au}$ . However, if the aphelion is not much less than 5.2 au, the comet may still approach close enough to Jupiter to be scattered into a larger orbit (or to have been derived from a larger orbit). Similarly,  $\phi$  is not calculated when the  $\sin \phi$  is greater than +1.

### Distribution in $u$

The frequency distribution in  $u$  for short period comet orbits is shown in Fig. 3. The concentration of orbits with low velocity in  $u$  is apparent, the greatest number occurring for  $u=.5$ . In part, this may be explained by the greater chance of capture as  $u$  decreases.

However, the large amount (16%) of orbits having  $u < .414$ , the minimum encounter velocity allowed by direct capture from a parabolic orbit, is surprising. Havnes (1970) called attention to this problem in a somewhat less precise manner, when he postulated the existence of a "primary field" of comets having orbits between Jupiter and Saturn. That is, the observed orbits which have low values of  $u$  have  $Q \sim 5.2$  and  $q \sim 1$  au; these are cognate to orbits which have  $q \sim 5.2$  and  $Q \sim 10-20$  au.

The large percentage of  $u < .414$  is the cause of many confusions in trying to determine the origin of short period comets. It is not expected from the hypothesis of direct capture by Jupiter and therefore computations assuming direct capture do not produce the observed distribution of orbital elements. At the same time, the present study shows that the frequency distribution is continuous into much higher values of  $u$  and therefore the postulation of two origins of comets to account for short and long period comets seems unnecessary. Instead, the mechanisms for producing values of  $u < .414$  need to be examined more carefully.

One plausible mechanism for producing comets with  $u < .414$  is that they are captured first by Saturn. This would lower the jovicentric relative velocity. In particular, those comets placed by Saturn into an orbit with  $q \sim 5.2$  au would be most susceptible to being "stolen" by an encounter with Jupiter. Comets of

Saturn family captured by Jupiter would look like the traditional Jupiter family-orbits of low inclination and aphelia near Jupiter - due to the small value of  $u$  and to the observational selection effects dependent on the small perihelia. Multiple encounters cause the orbits to change between the observable "Jupiter family" comets and the comets with aphelia between Jupiter and Saturn and perihelia close to Jupiter. Although the latter orbits are not directly observable, transitions between cognate orbits have been computed on commonly observed comet orbits. P/Brooks 2 is one example. When it was discovered in 1889 it had an orbital period of 7 years and an aphelion of 5.4au, near Jupiter. Dubiago showed that in July 1886 it had passed within 2 radii of Jupiter's surface and that prior to this its period was 31 years with an aphelion distance of 14 au. (Richter 1963).

There are other mechanisms for producing the small values of  $u$  besides prior capture by Saturn. Arnold (1964) has discussed the importance of the "Fermi" effect on the evolution of meteor orbits. This is the decrease or increase in  $u$  because the scattering planet is in an eccentric orbit and therefore, if random effects of the long period perturbations on the meteor orbit are assumed, the meteor will encounter the planet with differing relative velocities, producing a net acceleration or deceleration. This mechanism requires substantial time to produce a significant effect on  $u$ ; several orders of magnitude longer than is commonly assumed for comet lifetimes in the solar system. In contrast, Kazimirchak-Polonskaya's studies on comet orbits during a 400 year span show large changes in  $u$  for Comet Shain-Schaldach. During prolonged approaches to Saturn, its encounter velocity with Jupiter increased from  $-.06$  to  $+.2$ .

Further, the accuracy of Laplace's method for low velocity passages near Jupiter is not as good as for higher velocities which have been well studied due to space flight applications. For example, it was found that during the 1961 passage of Comet Kearns-Kwee by Jupiter  $u$  decreased from .61 to .48, while  $\theta$  increased from  $71^\circ$  to  $114^\circ$ , according to the orbital elements computed by Kazimirchak-Polonskaya. This encounter sharply reduced the probability of hyperbolic ejection by Jupiter and nearly reduced  $u$  to below the limit for hyperbolic ejection according to Laplace's method.

The non-gravitational forces have observable effect on the orbit. These are strongly dependent on the heliocentric distance and do not seem to be significant beyond 3 au. (Marsden, 1969). However, to reduce  $u$  substantially by non-gravitational forces, a major portion of the mass of the comet must be removed.

The character of the comet orbits is further brought out when the comets are plotted on a diagram as in Fig. 2. It can be seen from this diagram (Fig. 4) that there are two distinct groups of comets, a group with large values of  $\theta$  and small values of  $u$ , and a group with large values of  $u$  and values of  $\theta$  tending to cluster near  $\theta_p$ . For convenience, those orbits of  $u < 1.0$  will be termed low velocity comets, and those with values of  $u > 1.0$  will be termed high velocity objects. The low velocity comets are those with low inclinations to the ecliptic and aphelions near Jupiter, often called "Jupiter family" comets.

### High Velocity

About 20 of the short period orbits are classed as high velocity. None of these are closer than  $3^\circ$  to  $\theta_p$ . This is simply a selection effect due to nomenclature - orbits closer to  $\theta_p$  have periods longer than 200 years and are termed long period or parabolic.

The bulk of the high velocity comets are between  $3-8^\circ$  away from the parabolic line. Opik (1971) has compared the time required to obtain a given scattering angle with the time estimated for a comet to be destroyed and finds that small angles are more probable for comets with high  $u$ . This is because most encounters with Jupiter are distant with small angular deflections resulting; and also that higher  $u$ 's are less affected by the Jupiter encounter. There are four exceptions, Crommelin,  $13^\circ$ ; Temple-Tuttle,  $11^\circ$ , Stephan-Oterma,  $12^\circ$  and Tuttle,  $24^\circ$ . Possibly these four comets are the result of several deflections; or possibly the result of one larger deflection. Since the number of deflections of a high velocity comet is low statistically and  $\theta$  is small and repeated deflections add up to a random walk, then if at any time  $\theta$  becomes less than  $\theta_p$ , the comet will be ejected from the solar system in a hyperbolic orbit. Therefore, with the limited supply of high velocity comets available, it is not surprising that larger values of  $\theta - \theta_p$  are not observed. (In order to study the statistical properties of the group, the long period orbits which have the same  $u$ 's but smaller values of  $\theta - \theta_p$  should be included.)

The lack of high velocity comets far from the parabolic line is the reason for the lack of retrograde short period orbits. Newton (1893) estimated that 20% of the short period comets should have retrograde orbits if they result from direct capture processes. Recently, Everhart, using numerical procedures, has obtained results indicating 23% of the short period comet orbits should be retrograde. The above analysis indicates that retrograde short period comets ( $a < 20$ .au) occur when high velocity comets are scattered  $10^\circ$  or more from  $\theta_p$ . This combination occurs only for Temple-Tuttle and (just barely) Halley's comet. The reason is that the probability for scattering diminishes as  $u^{-4}$  (Opik, 1968).

### Low Velocity Comets

These are the comets that have been the cause of many unsuccessful attempts to relate the short period comets to parabolic comets, mainly because the encounter velocity is too low to be derivable from parabolic orbits. This has been discussed above, and the emphasis of the present section is to discuss the orbital distributions, having already assumed the low velocity.

The salient feature of the  $u, \theta$  plot of these comets (Fig. 4) is the trend away from the parabolic limit to the upper left hand corner. This trend is caused by observational selection. This is shown by noting that nearly all of these comets fall between the lines of  $\phi = .5$  and  $q = 2.0 \text{ au}$  when  $\phi = 0^\circ$ . (This condition,  $\phi = 0^\circ$ , corresponds to the requirement that the comets lie in the plane of the ecliptic, a condition which is nearly true for the bulk of the low velocity comets.)

The points are well scattered through this space, suggesting that the cometary lifetimes of this group are longer than the time required to randomize these orbits by multiple encounters and/or that the origin of these comets produces a random selection, unlike the high velocity comets. There are several reasons that can be advanced to explain qualitatively why low velocity comets randomize while high velocity comets do not. The maximum scattering angle increases sharply as  $u$  decreases. Then, low velocity orbits have fewer values of  $\theta$  where depletion processes occur. First, for  $u < .6$ ,  $i = 0^\circ$ , there are no perihelia possible with  $q < .5 \text{ au}$ , where the most rapid physical destruction occurs. Second, there is a smaller hyperbolic region as  $u$  decreases. If



$u < .414$ , hyperbolic ejection requires the combined action of Jupiter and Saturn. Third, if a handball effect produces these comets, the initial value of  $\theta$  before encounter with Jupiter may be far from  $\theta_p$  due to a previous encounter with an outer planet so that a large change in  $\theta$  is required to eject the comet.

Further, multiple encounters are frequent for low velocity comets:

Marsden (1970) has remarked that one can say that "Half of the comets of the Jupiter family have passed within half an astronomical unit of Jupiter at some-time during the past half century." And while the comet is at the lower, non-observable values of  $\theta$ , that is, orbiting in the vicinity of Jupiter and Saturn, solar effects are not significant, although the comets may be subject to collisional destruction. Marsden and Sekanina (1971) have estimated that impulsive changes occur in 1 orbit out of 20 for coreless comets which they attributed to collisions with meter-size boulders (perhaps derived from comets with cores). If this and the above assumption of randomization is true, the amount of cometary and meteoric debris in the inner solar system at any time may be dependent of the accumulation of debris in the region of the outer planets.

These results are in agreement with Sitarski's (1968) conclusion that the observed short period comets cannot derive dynamically from the observed long period comets. First, because the short period comets have smaller values of  $u$  than the observed long period comets; and second, because the long period comets near the parabolic line in the  $u$  range of the low velocity short period comets have such high values of perihelia ( $q > 3\text{au}$ ) that they are not observable. However, they must be there - Havnes has shown numerically that the observed field of comets tends to increase in semimajor axis within 100-1000 years. Further, there is ample evidence to support cometary orbital changes due to

Jupiter encounter. For example, P/Oterma existed in the orbit used here only between 1938-1962; it was scattered by Jupiter into this orbit from a larger orbit ( $P=18$  years,  $q=5.62$  au). The small orbit was commensurable with Jupiter in the ratio  $2/3$ ; after 3 of the comet's revolutions and 2 of Jupiter's it was again altered into a larger orbit ( $P=19$  years,  $q=5.4$  au, Kresak, 1965). Bouska (1965) showed that at its next perihelion in 1983 its expected magnitude will be far too low to be observable. Kazimirchak-Polonskaya (1968) has recently summarized the effect of the outer planets on the evolution of cometary orbits for a number of comets and found that in a number of cases Jupiter had captured the comet from an outer planet, Saturn or in one instance, Uranus.

There is one striking exception to the rule that short period comets are not observable with large perihelia: Comet Schwassmann-Wachmann 1. This is a physically hyperactive comet with a perihelion near Jupiter. It would seem to be a large new comet; it is interesting to note that its velocity relative to Jupiter is very low,  $u=.138$ . Its aphelion is between Jupiter and Saturn ( $Q=7.2$  au), analogous to Havnes "primary field" of comets. If this comet is indeed "new" - in the sense of being at its least perihelion - it would seem that capture by Jupiter from an orbit previously established by Saturn is the most plausible mechanism for establishing a low encounter velocity. Non-gravitational forces would seem unlikely to have reduced the relative velocity so much without having depleted the cometary mass: nor could these forces have acted in a time short compared to the intervals between scattering encounters with Jupiter. Orbital stability in this region since the beginning of the solar system followed by a recent orbital change due to encounter seems unlikely, however, Monte Carlo studies of scattering need to be done on this region of the solar system. Also,

commensurabilities may act to prolong orbits beyond statistical expectation, but the significance of commensurabilities cannot be readily evaluated. But in order to explain the observed distribution of comets, orbital stability would have to produce a current field of comets distributed nearly evenly in a large range of  $u$ , which requires an unlikely set of coincidences.

It has been suggested that the short period comets are produced by current volcanism on Jupiter. The physical arguments against this will not be restated here; but in the dynamical context it will be noted that there is not a distinct line of demarkation between short and long period comets.

Everhart (1969) found that the distribution of inclinations predicted from capture of long period comets by Jupiter did not agree with the observed distribution of inclinations of comets with period  $< 21$  yrs and  $q < 2\text{au}$ . The difficulty is that the predicted distribution is nearly flat, that is, there are almost as many high inclinations and retrograde orbits as low inclinations; in contrast to the observed distribution, which has the great bulk of the short period comets below  $30^\circ$ . Because of the predominance of low velocity comets, this is not surprising: retrograde orbits require high velocities, and also, the maximum direct inclination decreases with decreasing  $u$  (Fig. 5). It is possible that the observed distribution of inclinations is still systematically low, even allowing for the unexpectedly low velocities; this requires further calculation.

Everhart also studied the distributions and probabilities resulting from single close encounters of random parabolic comets with the planets. He concluded, "Every calculated distribution is in serious conflict with the corresponding distribution for the known short period comets. These cannot be the immediate or unmodified result of capture by Jupiter." He had sought the origin

of the short period comets in near parabolic comets whose original perihelion distance is close to the orbital radius of Jupiter and whose original inclination is small. These comets have a high capture probability, but Everhart found that most of the captured comets have perihelia near Jupiter after the capture encounter. This region therefore does not contribute any more short period comets at observable distances than comets whose original perihelia are at 1 or 2 au. However, when multiple encounters are considered, the picture changes considerably. During the capture encounter, a small change in  $\theta$  is more probable than a change which is large enough to provide a comet with perihelion less than 2 au. After multiple encounters when the distribution is randomized, a large proportion of the comets which were captured from orbits with small inclinations and perihelia close to Jupiter will have perihelia at observable distances. Since these comets have a high capture probability, they must provide the bulk of the short period comets.

## CONCLUSIONS

The jovicentric encounter velocities of the short period comet orbits have been calculated. It is found that many of the short period comets, for example, P/Halley, resemble long period comets dynamically and should perhaps be reclassified. In the meantime, the short period comets have been divided into two categories, "low velocity" and "high velocity", where "velocity" refers to the magnitude of the jovian encounter velocity,  $u$ . If  $u < 1.$ , the comet is called low velocity; this group is approximately the group often called "Jupiter family."

The large number of short period comets with low velocities confirms previous conclusions that these comets cannot have been derived directly from parabolic orbits by an encounter with Jupiter if Laplace's method is correct. In

fact, by Laplace's method it is impossible for many of these comets, those with  $u < .414$ , to have been emplaced in their observed orbits through this mechanism alone. At the same time, however, the emphasis of the present paper is to seek alternate mechanisms for reducing the Jovian encounter velocity rather than on suggesting an alternate origin of the short period comets. It is suggested that capture by Saturn is a good mechanism for providing low velocity comets later captured by Jupiter or that comets captured by Jupiter are scattered by Saturn with a reduction in  $u$ . Approximately half the time, this handball effect will have the opposite effect and will accelerate the comets out of the solar system; however, these comets cannot be observed. Further, it seems that  $u$ , if originally low, may be altered substantially during sustained encounters with the major planets. This problem of low velocity encounters requires further study with accurate numerical integration, but it may be that the Monte Carlo methods used to estimate solar system lifetimes are in too much error to give reasonable answers for low velocity objects.

This paper is in agreement with Kazimirchak-Polonskaya's (1967) conclusion: "We see that as the problem becomes more complicated and as we approach the real conditions of comet motion, the basic disparities between the results of the capture theory and observational facts are removed. The conflicts with observation have arisen, then, not because the capture theory is false, but from an oversimplified formulation of the problem."

The high velocity comets have  $u$  values equivalent to the parabolic comets; the only difference is values of  $\theta$  that are over  $3^\circ$ , resulting in a shorter semi-major axis. They do not show a random pattern of  $\theta$ , indicating a lifetime in the solar system short compared with time for angular deflection. For statistical

purposes, they should be compared with the long period or parabolic comets.

Because the average angular deflection for high velocity comets is small and because the perihelions of low velocity parabolic comets are high, the conclusion can be drawn that the observed long period comets cannot be expected to produce short period comets, and that the long period comets that can produce short period comets are not observable.

The lack of retrograde short period comets has often been considered to be a proof that short period comets do not derive from long period comets. But, the retrograde short period comets require a high velocity and a large scattering angle, a combination which is not statistically frequent. Kazimirchak-Polonskaya (1967) points out the Schulhof demonstrated this in the last century; therefore, it is long past time that this criticism of the derivation of short period comets from long period comets was discarded.

The demonstration that many of the short period comets have lower velocities than expected from the parabolic derivation theory may have consequences involving the interrelationships of comets, meteors, meteorites, asteroids, and the zodiacal light. Opik's (1966) conclusion that the origin of the meteorites is extinct nuclei of short period comets inside Jupiter's orbit requires reexamination in view of the fact that Jupiter is unable to remove many of the orbits by hyperbolic ejection; rather there may be substantial quantities of comets and cometary debris in the region between Jupiter and Saturn. Roosen (1970) concluded that the gegenschein is due to a circumsolar cloud which increased in density at some distance from the Sun outside the Earth's orbit. He notes that previous authors have suggested particular comets whose perihelia are near the earth's as the source of the zodiacal cloud, but he concludes that this would

be in contradiction to the observation that the Earth's shadow is not visible at the antisolar point. He finds that the radial distribution can be equivalent to that of the known asteroids; but that the radial density can vary widely so long as the spatial intensity increases outside the Earth's orbit. Also he finds that the average inclination of the particles producing the gegenschein must be much greater than that of the known asteroids. Therefore his observations may be compatible with a zodiacal cloud concentrated at 5-9 au and derived from unobserved short period comets.

There are two ways in which cometary decay can put dust into orbits in the regions of between Jupiter and Saturn. Meteor streams arising from the "Jupiter family" comets which penetrate into the inner regions of the solar system may be perturbed by Jupiter back to the cognate orbits with perihelia somewhere in the vicinity of Jupiter and aphelia extending out toward Saturn just as the comets themselves become so perturbed. Second, collisions in the region near Jupiter may be causing debris. Marsden and Sekanina have invoked the mechanism of interplanetary boulders colliding with coreless comets to explain the impulsive changes in the erratic comet orbits. If, as they have postulated, there are meter-sized boulders which strike the erratic comets approximately once in 20 orbits, there must be substantial quantities of smaller sized debris as well.

The outer planet missions provide an opportunity to actively study the distribution of dust as a function of radial distribution from the sun. Particularly helpful would be an instrument capable of measuring the inclination to the ecliptic, in order to obtain an accurate distribution of  $u$  as a function of distance. Also, an instrument capable of determining the proportion of fragmentable or

friable material as a function of distance might be of interest in view of the fact that observers of comets, meteors, meteor streams, meteorites and fireballs have repeatedly found this distinction. With this information, it may be possible to assess the relative importance of asteroids and comets in the origin and maintenance of the interplanetary debris and the meteorites.

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Table I

Comet	$u$	$\cos\theta$	$\theta$	$\phi$	$\theta - \theta_p$
ENCKE	-0.157	***	***	***	***
GRIGG-SKJELLERUP	0.436	-1.135	***	***	***
HONDA-MRKOS-PAJDUSAKOVA	0.643	-0.887	152.5	19.8	89.6
TEMPLE 2	0.186	-2.038	***	***	***
NEUJMIN 2	0.245	-1.542	***	***	***
BRORSEN	0.730	-0.829	146.0	33.0	74.6
TUTTLE-GIACOBINI-KRESAK	0.418	-1.012	***	***	***
TEMPLE-SWIFT	0.407	-0.984	169.6	50.6	***
DEVICO-SWIFT	0.324	-1.088	***	***	***
TEMPLE 1	0.177	-1.722	***	***	***
PONS-WINNECKE	0.569	-0.766	140.0	37.9	86.4
KOPFF	0.383	-0.871	150.5	16.9	***
GIACOBINI-ZINNER	0.733	-0.711	135.3	33.8	63.7
FORBES	0.376	-0.860	149.3	16.5	***
WOLF-HARRINGTON	0.448	-0.773	140.6	50.2	113.6
SCHWASSMANN-WACHMANN 2	0.287	-0.669	132.0	14.2	***
BIELA	0.688	-0.689	133.6	13.6	66.1
DANIEL	0.522	-0.710	135.3	37.9	89.5
WIRTANEN	0.429	-0.758	139.3	35.1	121.2
D'ARREST	0.542	-0.698	134.2	31.5	84.8
PERRINE-MRKOS	0.576	-0.690	133.6	27.4	79.0
AREND-RIGAUX	0.537	-0.698	134.2	30.3	85.8
REINMUTH 2	0.278	-0.971	166.2	***	***
BROOKS 2	0.343	-0.843	147.4	22.2	***
HARRINGTON	0.435	-0.733	137.1	20.6	115.7
HOLMES	0.374	-0.775	140.8	***	***
JOHNSON	0.247	-1.011	***	***	***
FINLAY	0.617	-0.661	131.3	4.6	71.5
BORRELLY	0.660	-0.647	130.3	43.4	65.6
HARRINGTON-ABELL	0.473	-0.651	130.6	35.6	95.7
FAYE	0.500	-0.622	128.5	16.4	87.1
WHIPPLE	0.246	-0.869	150.3	***	***
ASHBROOK-JACKSON	0.320	-0.719	136.0	50.3	***
REINMUTH 1	0.399	-0.624	128.6	20.8	***
AREND	0.558	-0.569	124.7	36.3	72.8
OTERMA	-0.190	***	***	***	***
SCHAUMASSE	0.702	-0.551	123.4	12.8	54.5
WOLF	0.536	-0.505	120.4	54.4	71.9
COMAS SOLA	0.576	-0.496	119.7	19.9	65.1
VAISALA 1	0.679	-0.404	113.8	13.5	47.2
NEUJMIN 3	0.611	-0.350	110.5	5.2	51.3
GALE	0.842	-0.452	116.9	9.8	36.8
TUTTLE	1.182	-0.554	123.6	29.7	23.9
SCHWASSMANN-WACHMANN 1	0.138	0.599	53.2	***	***
NEUJMIN 1	0.915	-0.325	109.0	12.6	24.0
CROMMELIN	1.231	-0.439	116.1	13.2	13.9
TEMPLE-TUTTLE	1.908	-0.824	145.5	9.5	11.6

Table I (Continued)

Comet	u	cos $\theta$	$\theta$	$\phi$	$\theta - \theta_p$
STEPHAN-DTERMA	1.053	-0.266	105.4	13.2	12.4
WESTPHAL	1.281	-0.380	112.3	22.0	7.8
BRORSEM-METCALF	1.371	-0.433	115.7	6.6	6.9
OLBERS	1.322	-0.399	113.5	22.6	7.0
PONS-BROOKS	1.550	-0.551	123.4	23.6	6.4
HALLEY	1.900	-0.763	139.8	6.8	6.3
HERSCHEL-RIGOLLET	1.535	-0.500	120.0	21.2	3.8
GRIGG-MELLISH	1.797	-0.668	131.9	24.7	3.6
WILSON-HARRINGTON	-1.014	***	***	***	***
HELFENZRIEDER	0.580	***	***	***	***
BLANPAIN	0.422	***	***	***	***
DU TOIT 2	0.244	***	***	***	***
BARNARD 1	0.257	***	***	***	***
SCHWASSMANN-WACHMANN-3	0.479	-0.953	162.3	***	***
GRISCHOW	0.499	-0.933	159.0	5.7	117.6
DU TOIT-NEUJMIN-DELPORTE	0.280	***	***	***	***
BROOKS 1	0.336	***	***	***	***
LEXELL	0.621	-0.837	146.8	2.3	86.4
KULIN	-0.178	***	***	***	***
PIGOTT	0.686	-0.778	141.1	***	***
TAYLOR	0.415	-0.829	146.0	51.5	140.6
SPITALER	0.296	***	***	***	***
HARRINGTON-WILSON	0.389	-0.852	148.4	74.5	***
BARNARD 3	0.628	-0.691	133.7	49.3	72.5
GIACOBINI	0.471	-0.736	137.4	24.4	103.1
SCHORR	0.292	-0.938	159.6	44.4	***
HARRINGTON-ABELL	0.472	-0.655	131.0	35.8	96.3
SWIFT-2	0.569	-0.631	129.1	4.4	75.6
SHAJN-SCHALDACH	0.263	-0.868	150.2	38.4	***
DENNING 2	0.643	-0.608	127.4	6.6	65.6
METCALF	0.556	-0.572	124.9	22.9	73.2
JACKSON-NEUJMIN	0.658	-0.512	120.8	16.1	56.3
DENNING 1	0.874	-0.571	124.8	4.9	42.6
SWIFT 1	0.698	-0.498	119.9	11.3	51.3
SLAUGHTER-BURNHAM	0.537	-0.281	106.3	13.7	57.8
VAN BIESBROECK	0.588	-0.267	105.5	9.9	49.3
WILD	0.771	-0.341	109.9	21.4	35.1
PETERS	0.925	-0.420	114.8	25.9	29.3
DU TOIT 1	0.938	-0.398	113.5	14.3	27.1
PERRINEE	1.543	-0.709	135.2	21.9	18.5
PONS-GAMBERT	1.864	-0.751	138.7	18.0	7.1
ROSS	1.721	-0.664	131.6	14.0	6.8
DUBIAGO	1.221	-0.330	109.3	12.3	7.6
DE VICO	1.620	-0.591	126.2	22.4	6.1
VAISALA 2	1.292	-0.356	110.8	20.8	6.2
SWIFT-TUTTLE	1.808	-0.687	133.4	25.0	4.5
BARNARD 2	1.303	-0.346	110.3	15.8	4.7
MELLISH	1.534	-0.502	120.2	6.4	3.9

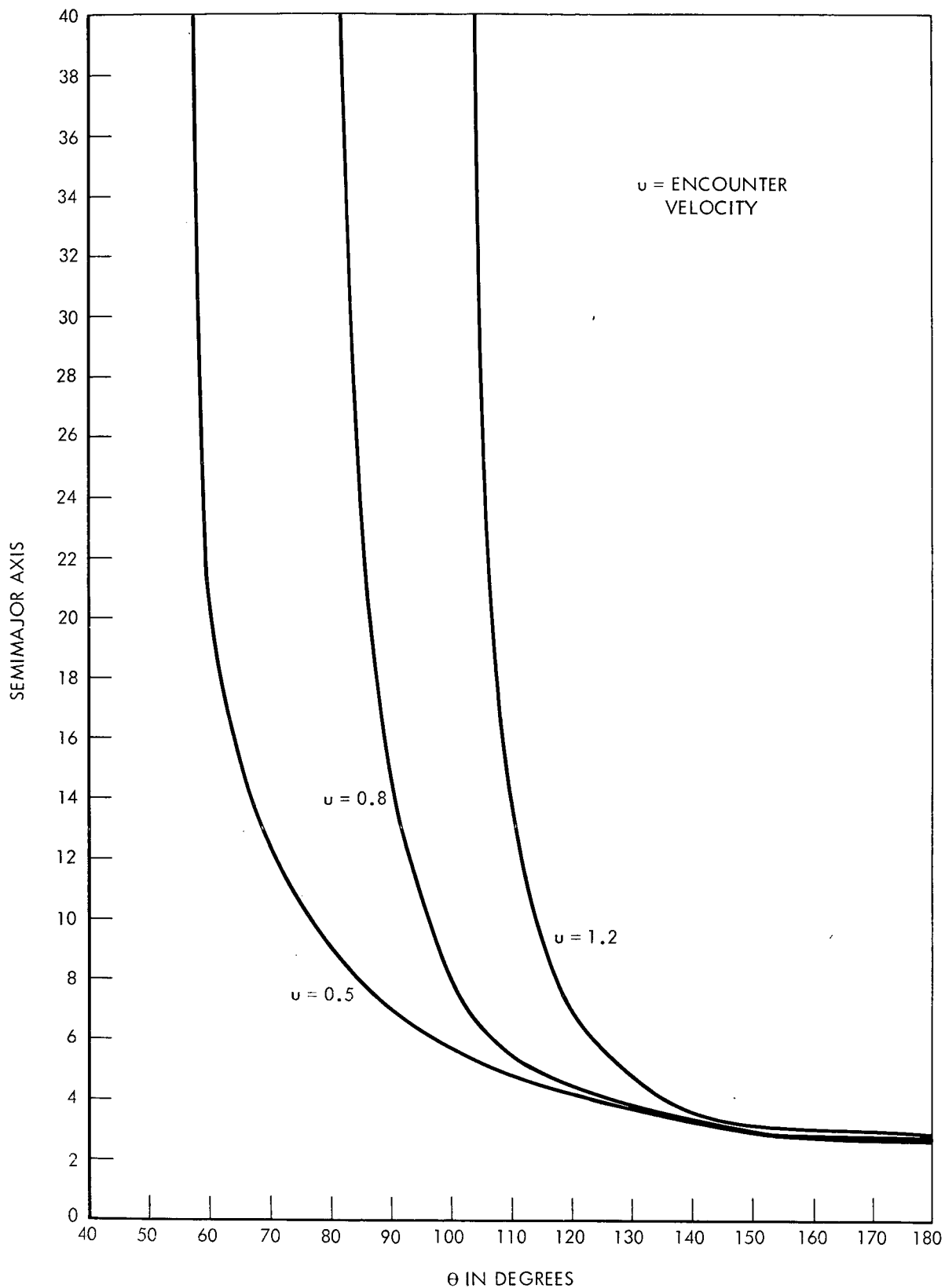


Figure 1. Semimajor Axis vs.  $\theta$

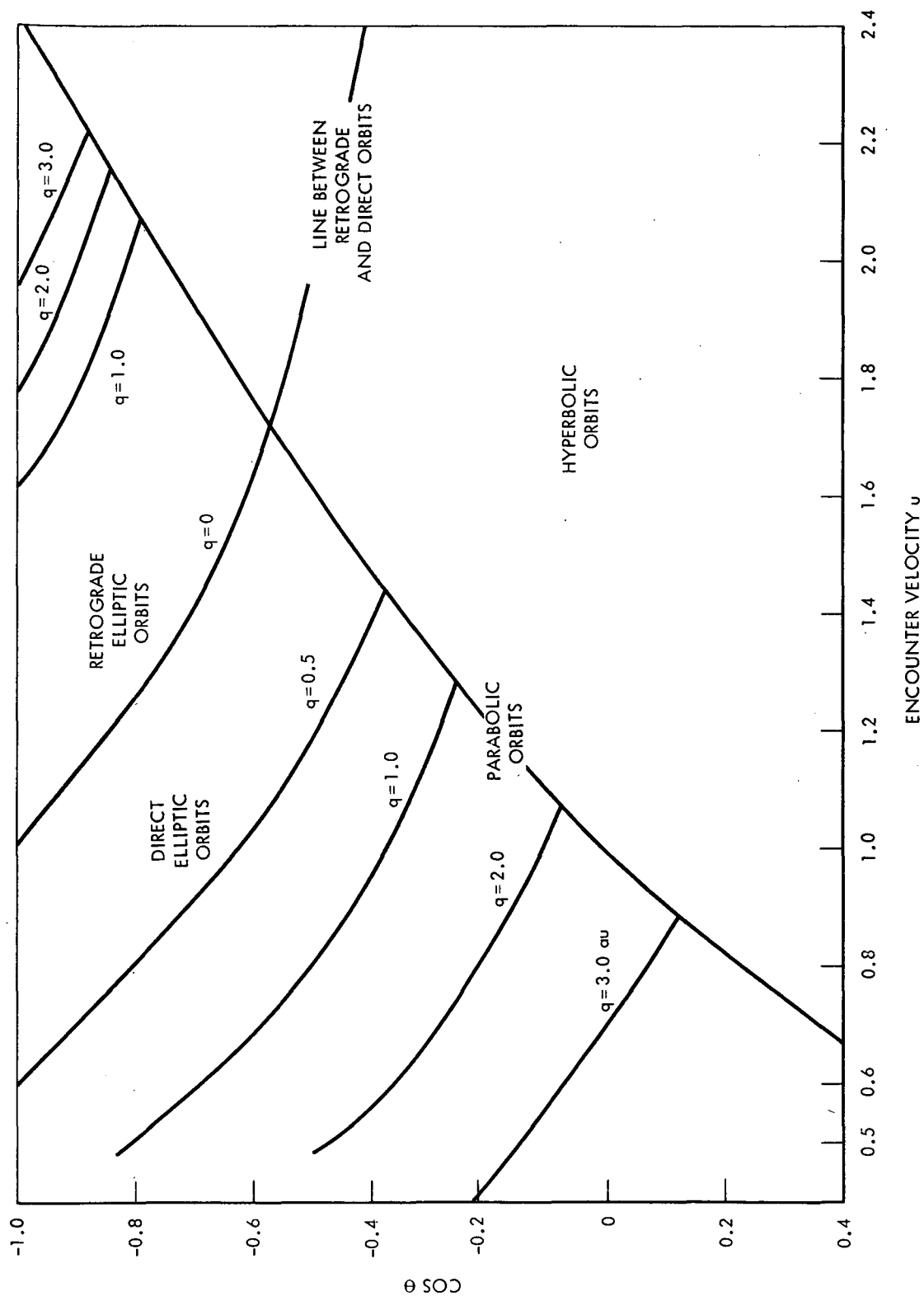


Figure 2a. Location of Orbits when  $\phi = 0^\circ$

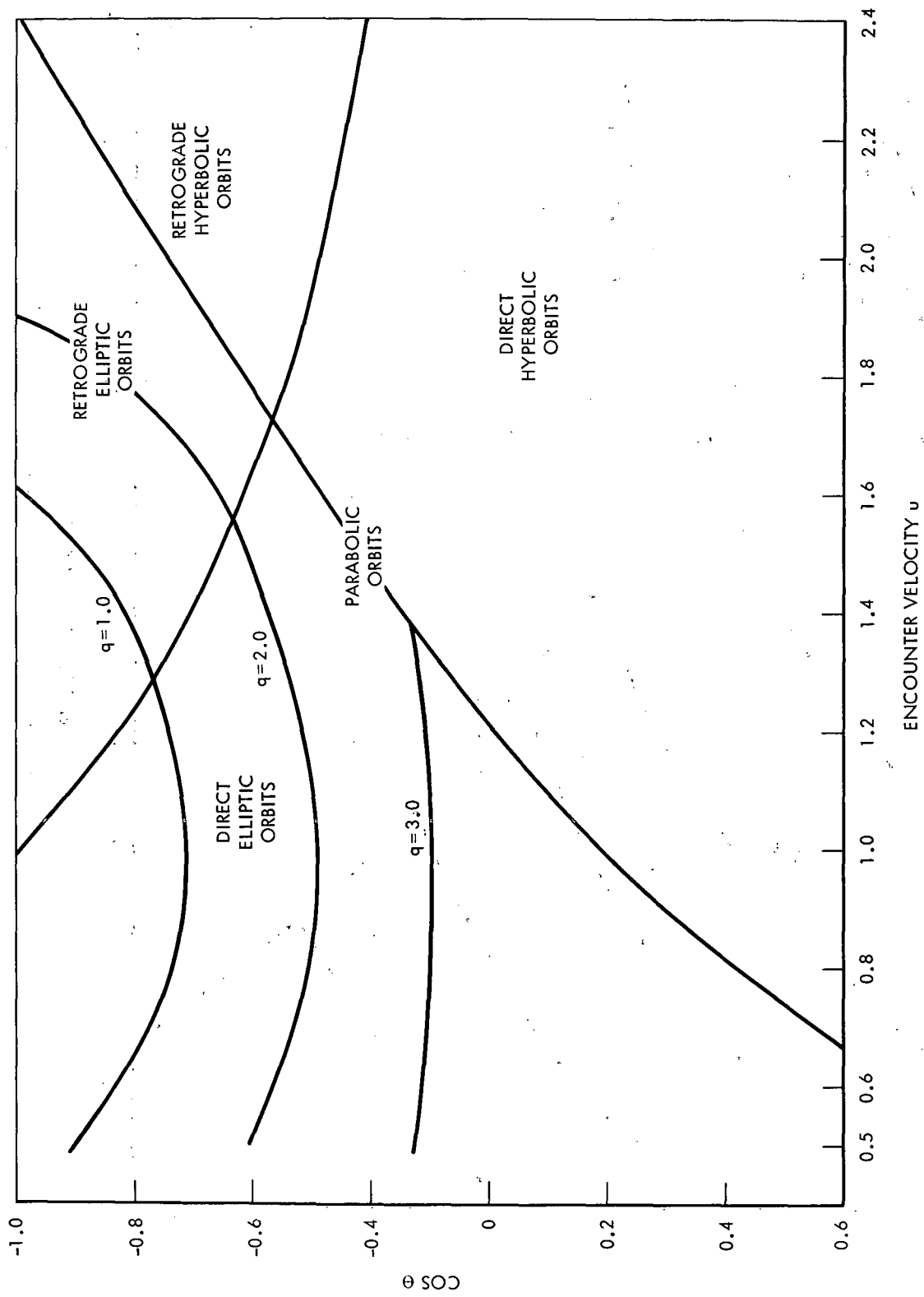


Figure 2b. Location of Orbits when  $\phi = 45^\circ$

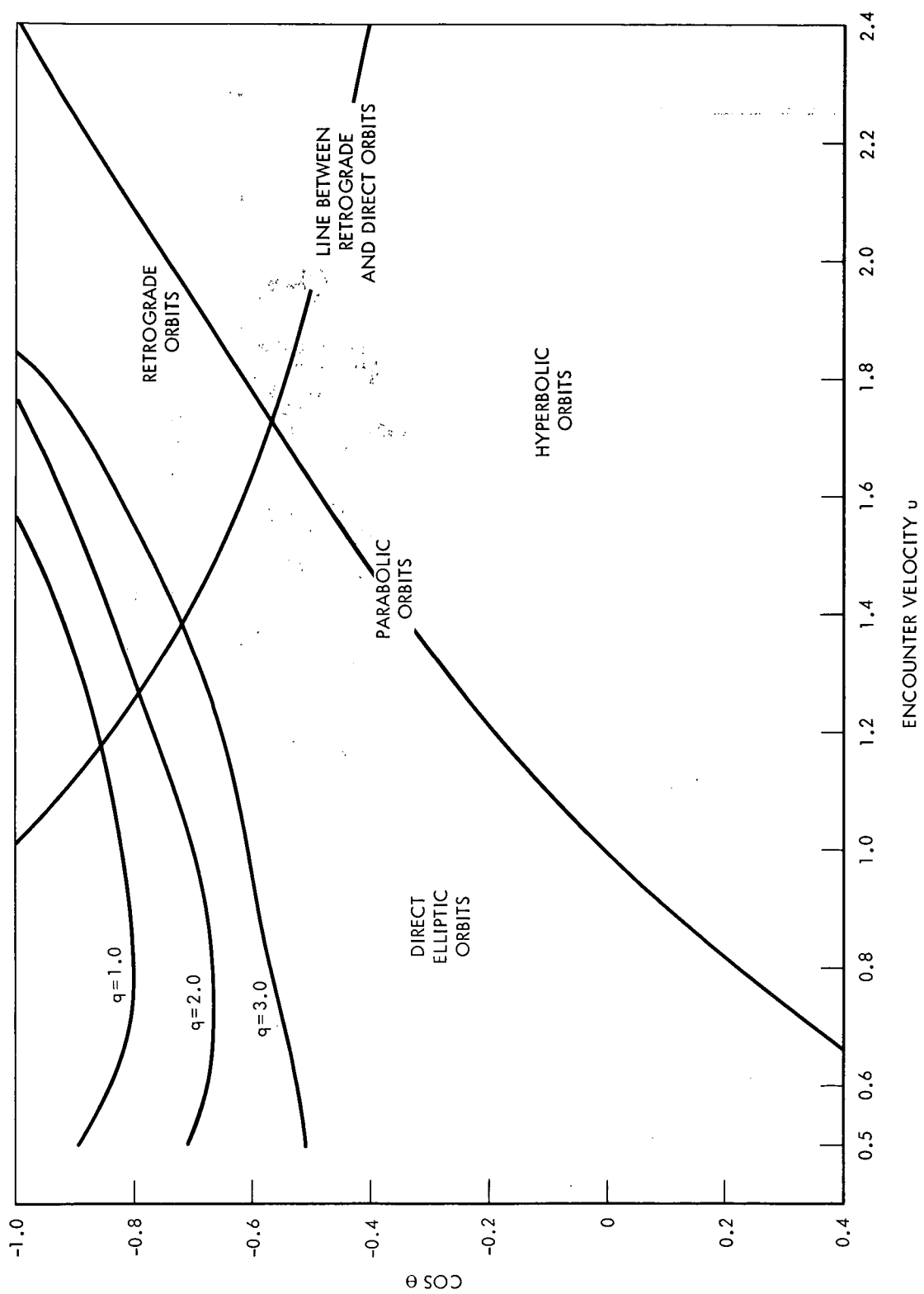


Figure 2c. Location of Orbits when  $\phi = 85^\circ$



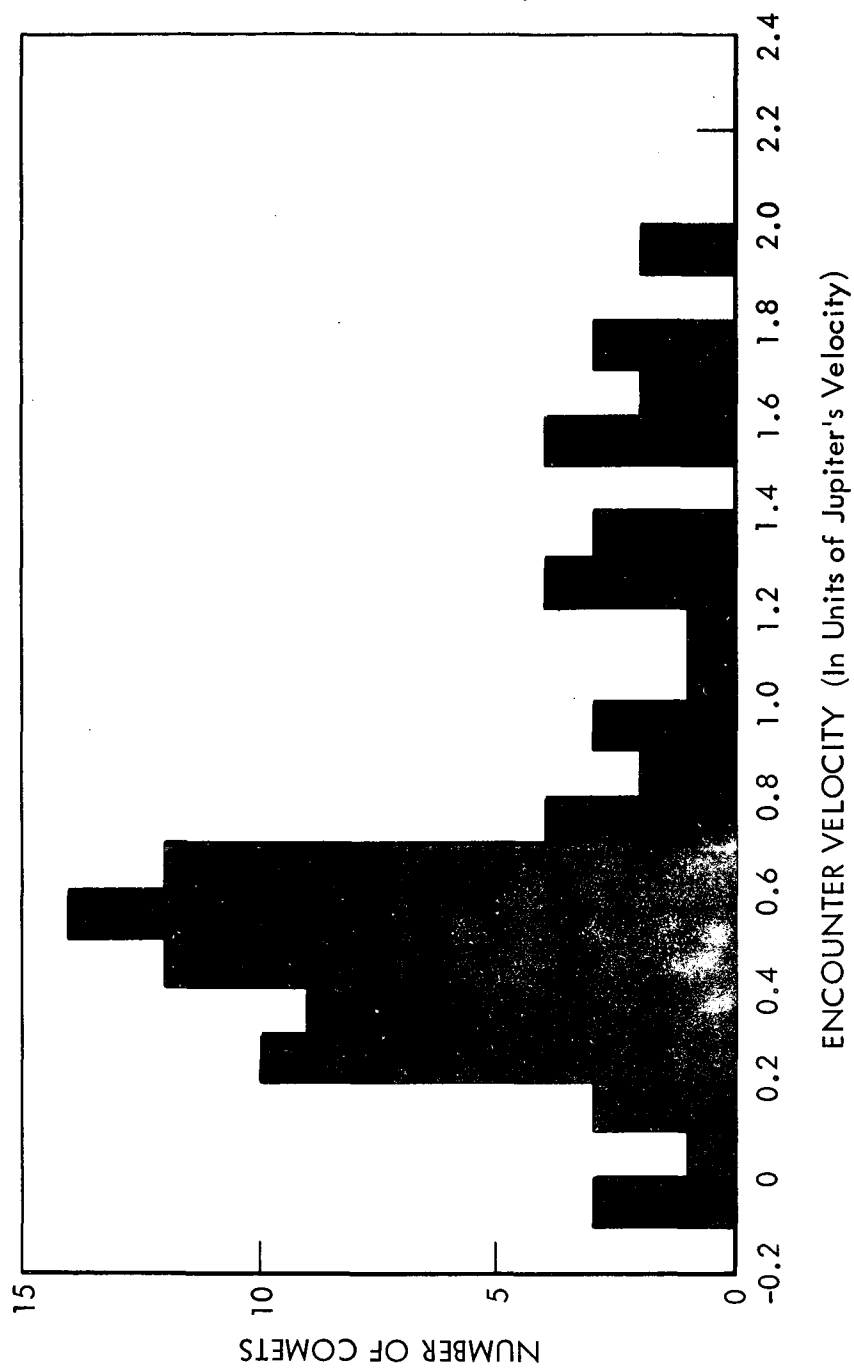


Figure 3. Distribution in u

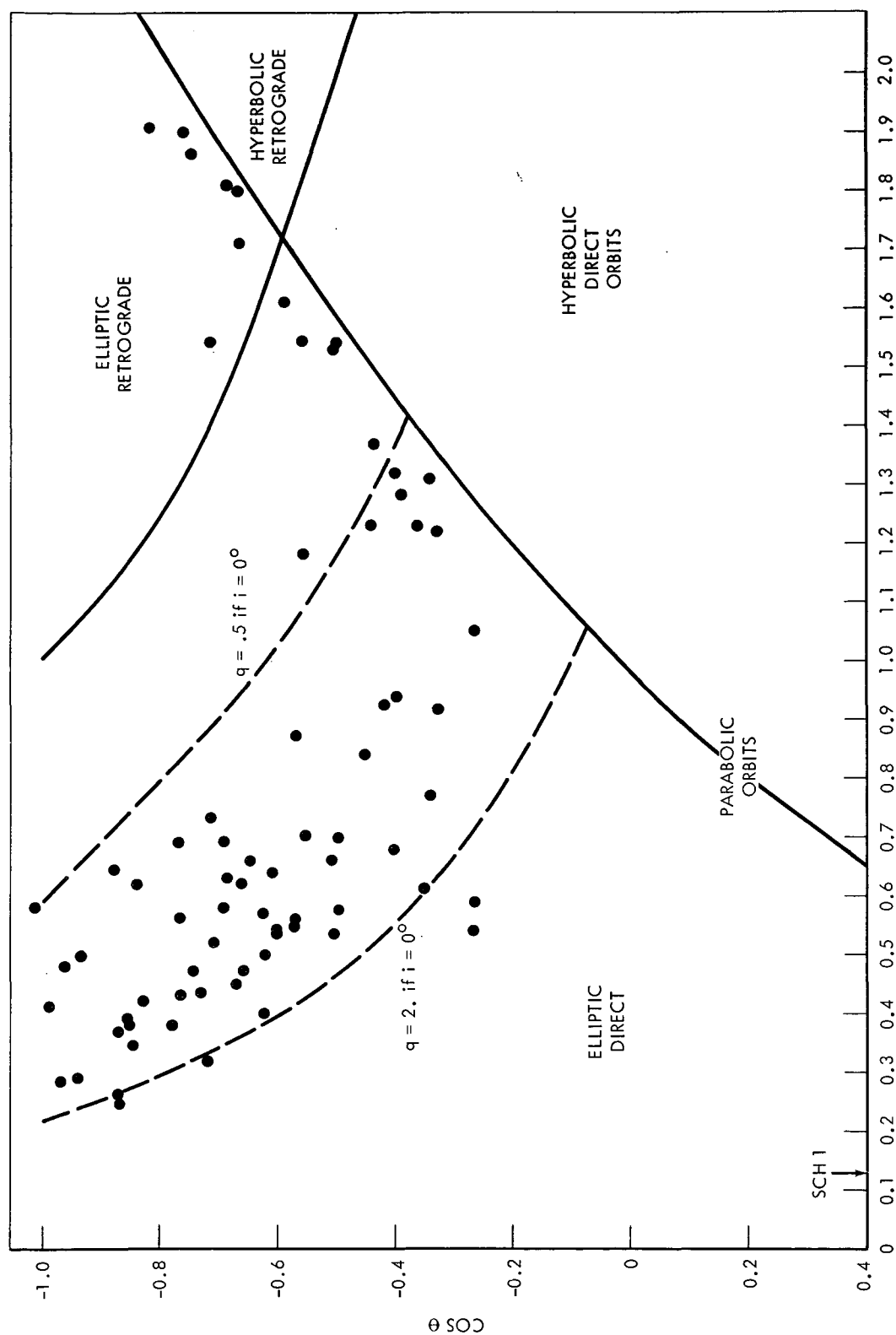


Figure 4. Short Period Comets in  $(u, \cos \theta)$  Space

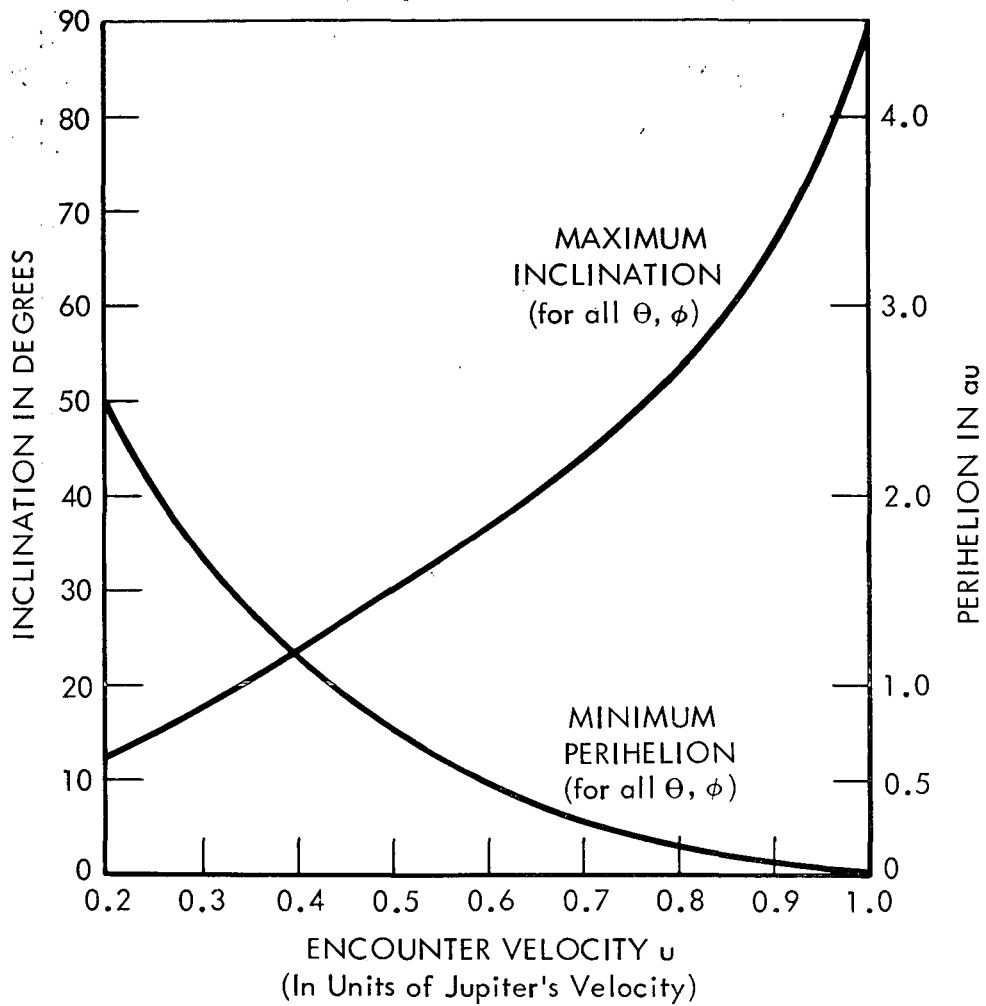


Figure 5. Maximum Inclination and Minimum Perihelion as a Function of  $u$