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EMPIRICAL ANALYTIC TRANSFORMATIONS BETWEEN GEOGRAPHIC AND CORRECTED GEOMAGNETIC COORDINATES

Prepared for:

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
GEORGE C. MARSHALL SPACE FLIGHT CENTER
Aero-Astrodynamics Laboratory

UNDER CONTRACT NO. NAS8-20082

(NASA-CR-123592) EMPIRICAL ANALYTIC TRANSFORMATIONS BETWEEN GEOGRAPHIC AND CORRECTED GEOMAGNETIC COORDINATES R.H. Comfort (Northrop Corp.) Jul. 1970 58 p CSCL 08B

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N72-20369

G3/13 23897

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57p

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FOREWORD

This document describes a method for transforming coordinates, developed for use in studies of high latitude energetics. The work was conducted by the Electro-Mechanical Division, Northrop Corporation, Huntsville, Alabama, for the National Aeronautics and Space Administration, George C. Marshall Space Flight Center, Aero-Astrodynamics Laboratory, under contract NAS8-20082, Appendix A-1, schedule order 32. This program was under the direction of the Space Environment Branch, with Mr. R. E. Smith as NASA/MSFC Technical Coordinator.

ABSTRACT

Based upon a mathematical model of contours of constant corrected geomagnetic latitude in a polar projection of geographic coordinates, analytic equations are developed for converting geographic coordinates to corrected geomagnetic coordinates and vice versa. The equations have been programmed for use on a small computer. Values generated by these programs are compared with published values; accuracies should be sufficient for many studies of high latitude geophysical phenomena. This treatment is restricted to the northern hemisphere.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF SYMBOLS

a	Semi-major axis of ellipse
b	Semi-minor axis of ellipse
c	Dummy constant of proportionality
CGC	Corrected geomagnetic coordinates
GC	Geographic coordinates
$\hat{i}$	Unit cartesian vector
$\hat{j}$	Unit cartesian vector
r	Polar coordinate representing colatitude (CGC)
r'	Intermediate polar coordinate representing colatitude
r''	Polar coordinate representing colatitude (GC)
$\hat{r}$	Unit polar radial vector
$\vec{r}$	Polar vector (CGC)
$\vec{r}'$	Intermediate polar vector
R	Transformation parameter: Latitudinal distance between the geographic and corrected geomagnetic poles in degrees geographic latitude
R_e	Radius of the earth
α	Transformation parameter: The value of ϕ for which the 70° latitude GC circle and the 70° latitude CGC model ellipse intersect when centered on a common origin
β	Transformation parameter: Angle between reference direction of CGC system and major axis of elliptic latitude contour
γ	Angle in transformation geometry
Γ	Intermediate longitudinal polar angle
Γ'	Intermediate longitudinal polar angle
Δ	Transformation parameter: Angle between the reference directions of the GC and CGC systems
$\Delta\chi$	Incremental change in χ, χ representing any of the variables listed here
ε	Transformation parameter: Eccentricity of model ellipse
θ	Polar coordinate representing longitude (CGC)
θ'	Intermediate polar coordinate representing longitude
θ''	Polar coordinate representing longitude (GC)
Θ	Geographic latitude

LIST OF SYMBOLS (Concluded)

Θ_c	Corrected geomagnetic latitude
μ	Proportionality constant between arc length (distance on surface of earth) and geographic latitudinal displacement along a geographic meridian
π	3.14159...
ϕ	Polar coordinate referenced to major axis of model ellipse
ϕ'	Intermediate polar angle referenced to major axis of model ellipse
$\hat{\phi}$	Unit polar angular vector
Φ	Geographic longitude
Φ_c	Corrected geomagnetic longitude
ψ	Angle in transformation geometry

Section I

INTRODUCTION

The purpose of this memorandum is to document the derivation and programming of a set of empirical analytic transformations between geographic coordinates and corrected geomagnetic coordinates. The need for such transformations arises in investigations of the interactions between high latitude geophysical phenomena (e.g., aurorae, polar magnetic substorms, etc.) and the neutral atmosphere. Because high latitude phenomena are controlled by the geomagnetic field, studies of these phenomena employ geomagnetic coordinates. For the neutral atmosphere, on the other hand, dynamic effects associated with the rotation of the earth are more important; hence, geographic coordinates based on the earth's axis of rotation, are the natural frame of reference for such investigations. In order to employ the results of studies in both of these areas together on a large scale, transformations between the coordinate systems are required.

Over the years, several systems of geomagnetic coordinates have been employed, those based on the centered and eccentric dipole approximations of the geomagnetic field being most prominent. It has recently become apparent that the centered dipole approximation is inadequate for quantitative studies. Hultqvist (ref. 1) made a detailed study of the geomagnetic field lines using the first five terms in the spherical harmonic expansion of the field. Based on these results Hakura (ref. 2) has developed a system of geomagnetic coordinates and prepared tables and maps relating them to geographic coordinates. He has designated these "corrected geomagnetic coordinates"; this terminology is followed here. A number of investigators have employed corrected geomagnetic coordinates in detailed morphological studies of auroral and geomagnetic activity (e.g., refs. 3, 4, 5). Consequently, this system is considered to be the most useful for relating such phenomena to effects in the neutral atmosphere.

Hakura's (ref. 2) corrected geomagnetic coordinates are obtained from tabulated data prepared by Hultqvist (ref. 1); and tabulated data are the result of his computations. Direct use of Hakura's coordinates on a computer would require extensive tables relating corrected geomagnetic coordinates to geographic coordinates and an interpolation routine to provide the necessary coordinate transformations. As an alternative to such a procedure, a set of empirical analytic equations is developed to provide a relatively fast and simple means of accomplishing these transformations at a small loss in accuracy. This is accomplished by formulating a mathematical model of contours of constant corrected geomagnetic latitude in the geographic coordinate system. Based on this model, analytic equations are obtained relating the two coordinate systems. Details of the model and the derivation of these equations are contained in Section II. In Section III, computer programs are described and errors arising from the transformations are discussed. Section IV contains a summation. Computer programs and tables relating the two coordinate systems are presented in the appendixes.

Section II

TRANSFORMATION EQUATIONS

2.1. METHOD

The approach taken is intuitive, suggested by the resemblance of the contours of constant corrected geomagnetic latitude, when drawn on an equidistant polar projection of the geographic coordinate (GC) system, to a family of ellipses (see Figure 2-1). Since these contours are circles in the corrected geomagnetic coordinate (CGC) system, one step in the conversion of CGC to GC is the transformation of circles to ellipses. The remaining operations are the usual translation of the origin and rotation of the coordinate frame to the established reference direction.

In the conversion of GC to CGC, it has not been feasible to invert the above transformation equations. Rather a similar procedure has been followed in reverse sequence, making use of the same parameters and some of the same equations.

All calculations are carried out in polar coordinates, r, ϕ, in which r represents colatitude. Where constant parameters are concerned, only the northern hemisphere is considered.

2.2 CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

As previously indicated, circles of constant latitude in the CGC system transform to ellipses in the GC system. Figure 2-2 shows the family of ellipses which are taken to approximate the CGC latitude contours in Figure 2-1. These ellipses are scaled (same eccentricity) from the 70 degree ellipse.

Let r, ϕ represent coordinates of the polar coordinate system shown in Figure 2-2. The equation of an ellipse in polar coordinates is

$$(r')^2 = \frac{a^2\, b^2}{a^2 \sin^2 \phi + b^2 \cos^2 \phi} \qquad (1)$$

where a and b are the semi-axes of the ellipse.

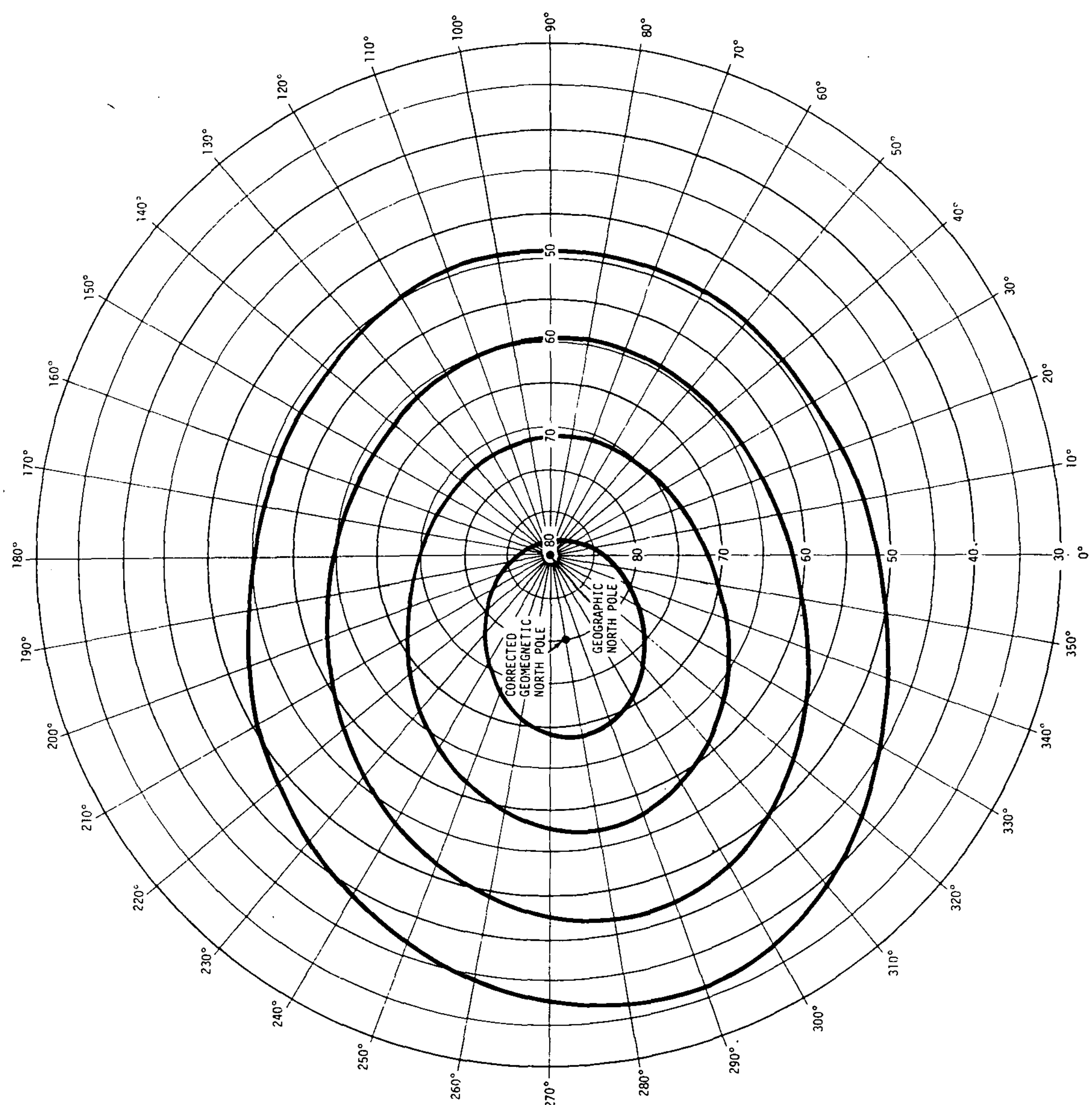

Figure 2-1. CONTOURS OF CONSTANT CORRECTED GEOMAGNETIC LATITUDE IN A NORTH POLAR PROJECTION OF THE GEOGRAPHIC COORDINATE SYSTEM. (PREPARED FROM TABLES IN REFERENCE 2)

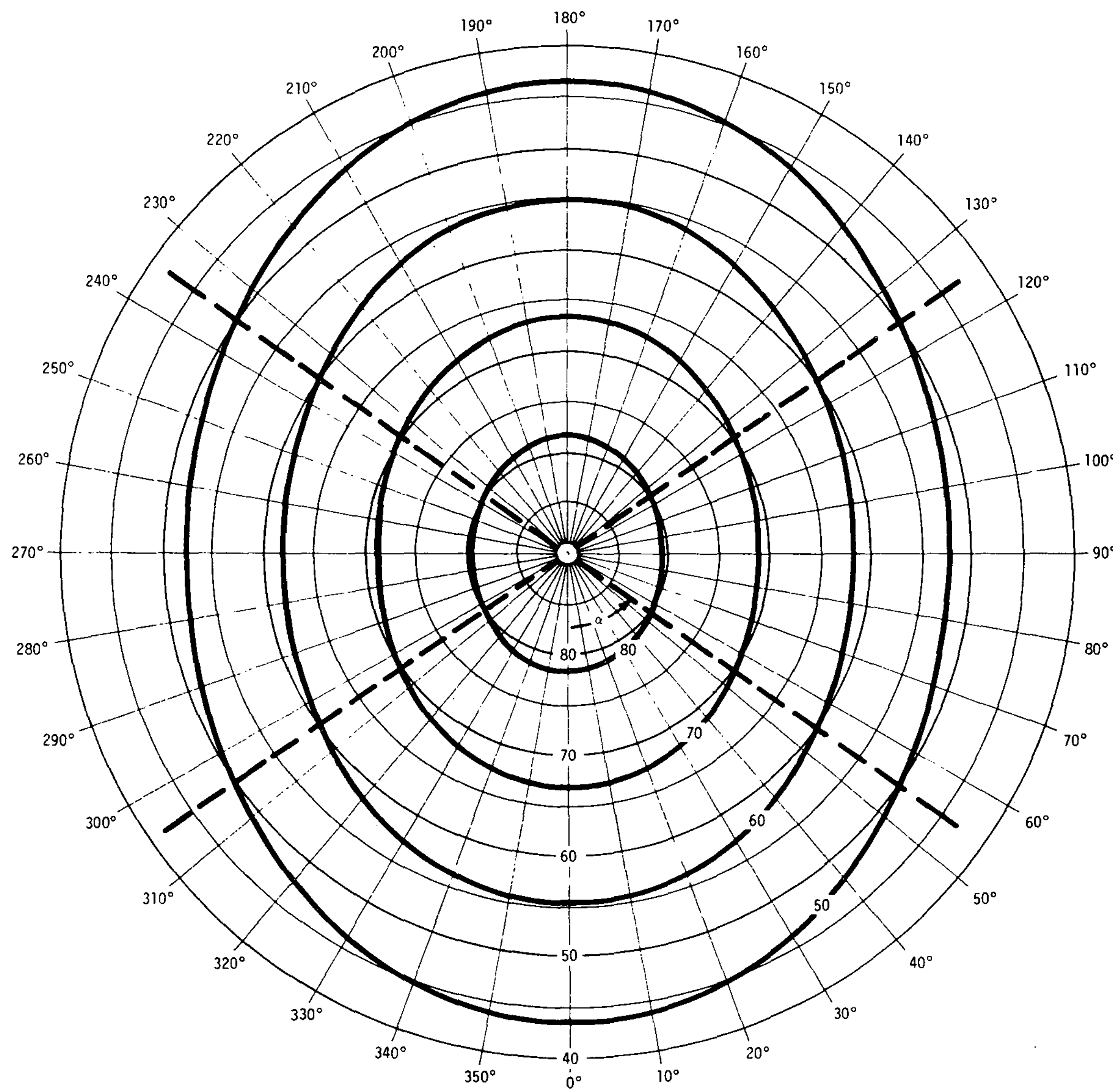

Figure 2-2. MODEL CONTOURS OF CONSTANT CORRECTED GEOMAGNETIC LATITUDE IN A POLAR PROJECTION, CENTERED ON THE CORRECTED GEOMAGNETIC NORTH POLE AND DIRECTION REFERENCED TO THE MAJOR AXIS

Consider a circle of constant latitude, represented by colatitude r. The equation of this circle is

$$r = \text{constant} \tag{2}$$

Equation (2) can be put into the form of equation (1) by writing

$$r^2 = \frac{a^2 b^2}{a^2 \sin^2 \alpha + b^2 \cos^2 \alpha} \tag{3}$$

where α is the (constant) value of ϕ for which the circle (r) and the ellipse (r') intersect (see Figure 2-2). Taking the ratio of equations (1) and (3) gives

$$\left(\frac{r'}{r}\right)^2 = \frac{a^2 \sin^2 \alpha + b^2 \cos^2 \alpha}{a^2 \sin^2 \phi + b^2 \cos^2 \phi} \tag{4}$$

The right side of equation (4) can be rewritten in terms of the eccentricity, which is defined as

$$\varepsilon \equiv \sqrt{1-b^2/a^2}$$

With this parameter it is easy to put equation (4) into the form

$$\left(\frac{r'}{r}\right)^2 = \frac{1 - \varepsilon^2 \cos^2 \alpha}{1 - \varepsilon^2 \cos^2 \phi}$$

Hence circles transform to ellipses by the equation

$$r' = r\left[\frac{1 - \varepsilon^2 \cos^2 \alpha}{1 - \varepsilon^2 \cos^2 \phi}\right]^{1/2} \tag{5}$$

Now a corresponding coordinate ϕ' must also be determined (it was found empirically that merely setting $\phi' = \phi$ gave unacceptable results). A heuristic means of picturing the alteration of the ϕ coordinate (and one which leads to acceptable results) is to imagine the circle of constant latitude being distorted into an ellipse by displacements (of points on the perimeter) which are normal to the circle and outward at the major axis and inward at the minor axis. For each quadrant a constant (small) displacement vector is established by the vector sum of the components at the adjacement semi-axes. The ϕ component of this displacement vector then gives the required modification for each value of ϕ in that quadrant.

Explicitly, let the polar coordinates r, ϕ specify the vector $\vec{r}$, i.e.

$$\vec{r} = \vec{r}(r,\phi) \tag{6}$$

Then $\vec{r}'$ is represented as

$$\vec{r}' = \vec{r}'\,(r + \Delta r,\ \phi + \Delta\phi) \tag{7}$$

The displacement vector is taken to be

$$\Delta\vec{r} = \vec{r}' - \vec{r} \tag{8}$$

where $\Delta\vec{r}$ is treated as having small magnitude. Now $\Delta\vec{r}$ may be resolved along the major and minor axes as

$$\Delta\vec{r} = \hat{i}\,\Delta a \cos\phi + \hat{j}\,\Delta b \sin\phi \tag{9}$$

where $\hat{i}$ and $\hat{j}$ are unit vectors along the major and minor axes respectively and Δa and Δb are the corresponding (empirically known) displacements along these directions (opposite signs on opposite sides of the circle). In terms of polar coordinates, $\Delta\vec{r}$ may be written

$$\Delta\vec{r} = \Delta r\,\hat{r} + r'\,\Delta\phi\,\hat{\phi} \tag{10}$$

where $\hat{r}$ and $\hat{\phi}$ are the unit vectors in the r and ϕ directions, with Δr and $r'\Delta\phi$ the corresponding displacements. It will be noted that r' is used instead of r in equation (10). If Δr were sufficiently small this distinction would not matter. In fact, Δr is not that small and r' gives slightly better results. The unit vectors in equations (9) and (10) are related by

$$\begin{aligned} \hat{r} &= \hat{i}\cos\phi + \hat{j}\sin\phi \\ \hat{\phi} &= -\hat{i}\sin\phi + \hat{j}\cos\phi \end{aligned} \tag{11}$$

These may be inverted to give

$$\hat{i} = \hat{r} \cos\phi - \hat{\phi} \sin\phi$$
$$\hat{j} = \hat{r} \sin\phi + \hat{\phi} \cos\phi \tag{12}$$

Placing these results into equation (9) and substituting that result into equation (10) gives

$$\Delta a \cos^2\phi \; \hat{r} - \frac{\Delta a}{2} \sin 2\phi \; \hat{\phi} + \Delta b \sin^2\phi \; \hat{r} + \frac{\Delta b}{2} \sin 2\phi \; \hat{\phi}$$
$$= \Delta r \; \hat{r} + r' \, \Delta\phi \; \hat{\phi} \tag{13}$$

Since only $\Delta\phi$ is required, only $\hat{\phi}$ components need be considered; hence

$$\Delta\phi = \frac{1}{r'} \left[\frac{\Delta b}{2} - \frac{\Delta a}{2} \right] \sin 2\phi \tag{14}$$

Because of the scaling which is used in generating the ellipses, both Δa and Δb are proportional to r; therefore

$$\Delta b - \Delta a = cr$$

With this substitution equation (14) becomes

$$\Delta\phi = \frac{c}{2} \left(\frac{r}{r'} \right) \sin 2\phi$$

Taking the ratio $\frac{r}{r'}$ from equation (5) gives

$$\Delta\phi = - .13 \left[\frac{1 - \varepsilon^2 \cos^2 \phi}{1 - \varepsilon^2 \cos^2 \alpha} \right]^{1/2} \sin 2\phi \tag{15}$$

where - .13 has been empirically determined. The coordinate ϕ' corresponding to r' is therefore given by

$$\phi' = \phi + \Delta\phi$$
$$= \phi - .13 \left[\frac{1 - \varepsilon^2 \cos^2 \phi}{1 - \varepsilon^2 \cos^2 \alpha} \right]^{1/2} \sin 2\phi \tag{16}$$

In the above treatment the reference axis for ϕ was taken to be along the major axis of the ellipse. In corrected geomagnetic coordinates, the reference axis (zero meridian) is displaced from this direction by an angle β. If the corrected geomagnetic latitude and longitude Θ_c, Φ_c are related to polar coordinates r, θ by

$$\Theta_c = 90° - r \tag{17}$$

$$\Phi_c = \theta \tag{18}$$

then the above transformation may be summarized as

$$r' = r \left[\frac{1 - \varepsilon^2 \cos^2 \alpha}{1 - \varepsilon^2 \cos^2 \Gamma} \right]^{1/2} \tag{19}$$

$$\begin{aligned} \theta' &= \theta + \Delta\theta \\ &= \theta - .13 \left[\frac{1 - \varepsilon^2 \cos^2 \Gamma}{1 - \varepsilon^2 \cos^2 \alpha} \right]^{1/2} \sin 2\Gamma \end{aligned} \tag{20}$$

where

$$\Gamma \equiv \theta - \beta \tag{21}$$

corresponds to ϕ in the above treatment.

The origin must now be shifted from the corrected geomagnetic north pole to the north geographic pole and a new reference direction corresponding to the GC system employed. The relevant geometry is shown in Figure 2-3. From the standard relationships in a plane triangle

$$(r'')^2 = R^2 + r'^2 - 2Rr' \cos \Gamma' \tag{22}$$

where

$$\Gamma' \equiv \theta' - \beta \tag{23}$$

and R is the latitudinal distance between the poles in degrees geographic latitude. Another relation gives

$$\frac{r'}{\sin \gamma} = \frac{R}{\sin \psi} \tag{24}$$

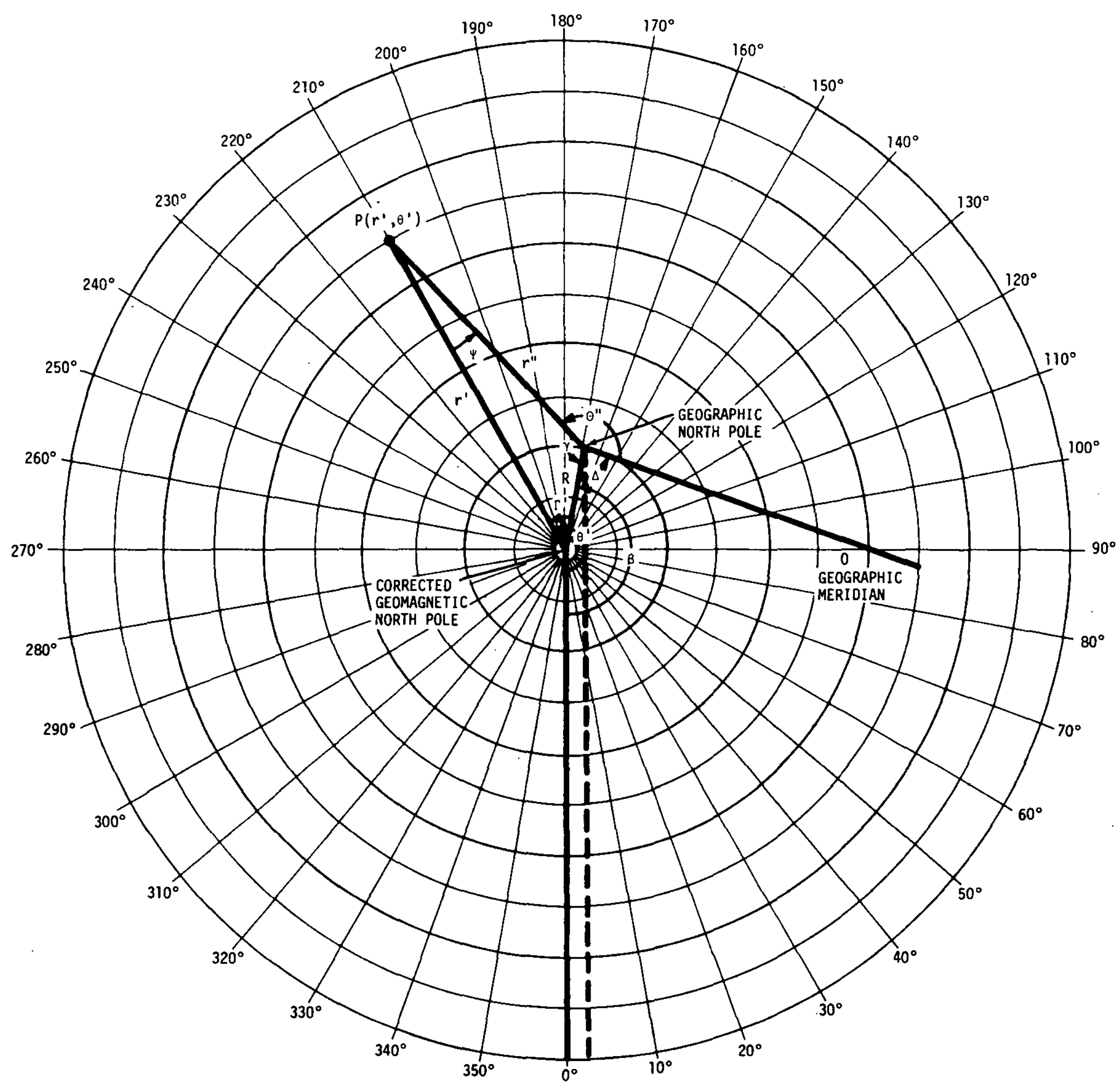

Figure 2-3. GEOMETRIC RELATIONSHIPS IN CHANGING POINT AND DIRECTION OF REFERENCE FROM THOSE OF THE CORRECTED GEOMAGNETIC COORDINATE SYSTEM TO THOSE OF THE GEOGRAPHIC COORDINATE SYSTEM

where the angles are defined by Figure 2-3. From the geometry in Figure 2-3, it is clear that

$$\psi = \pi - (\Gamma' + \gamma) \tag{25}$$

$$\theta'' = \pi + \beta - (\Delta + \gamma) \tag{26}$$

where β and Δ are known constant angles. Substituting equation (25) into (24) gives

$$\frac{r'}{R} = \frac{\sin \gamma}{\sin [\pi - (\Gamma' + \gamma)]} \tag{27}$$

With an expansion of the denominator on the right and rearrangement of terms, equation (27) may be put into the form

$$\tan \gamma = \frac{\sin \Gamma'}{R/r' - \cos \Gamma'} \tag{28}$$

so that

$$\gamma = \text{ARCTAN} \left[\frac{\sin \Gamma'}{R/r' - \cos \Gamma'} \right] \tag{29}$$

Substituting this result into equation (26) gives

$$\theta'' = \pi + \beta - \Delta - \text{ARCTAN} \left[\frac{\sin \Gamma'}{R/r' - \cos \Gamma'} \right] \tag{30}$$

In summary, geographic latitude Θ and longitude Φ are given in terms of corrected geomagnetic coordinates Θ_c, Φ_c by the following equations:

$$\Theta = 90° - r'' \tag{31}$$

$$\Phi = \theta'' \tag{32}$$

$$r'' = [R^2 + r'^2 - 2r' R \cos \Gamma']^{1/2} \tag{22}$$

$$\theta'' = \pi + \beta - \Delta - \text{ARCTAN} \left[\frac{\sin \Gamma'}{R/r' - \cos \Gamma'} \right] \tag{30}$$

$$r' = r \left[\frac{1 - \varepsilon^2 \cos^2 \alpha}{1 - \varepsilon^2 \cos^2 \Gamma} \right]^{1/2} \quad (19)$$

$$\Gamma' = \theta' - \beta \quad (23)$$

$$\Gamma = \theta - \beta \quad (21)$$

$$\theta' = \theta - .13 \left[\frac{1 - \varepsilon^2 \cos^2 \Gamma}{1 - \varepsilon^2 \cos^2 \alpha} \right]^{1/2} \sin 2\Gamma \quad (20)$$

$$r = 90° - \Theta_c \quad (17)$$

$$\theta = \Phi_c \quad (18)$$

where α, β, Δ, ε, and R are empirically determined constants whose values are listed in Table 2-1.

Table 2-1. VALUES OF TRANSFORMATION PARAMETERS

PARAMETER	VALUE
α	55°
β	170°
Δ	70°
ε	.5913
R	9.5°

2.3 GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

Although it has not been feasible to invert the preceding equations, the same parameters are used in a similar manner to obtain the inverse transformation. The pertinent geometry is given in Figure 2-4, where geographic latitude and longitude Θ, Φ are related to the variables shown by equations (31) and (32).

Translating the origin from the north geographic pole to the corrected geomagnetic north pole gives

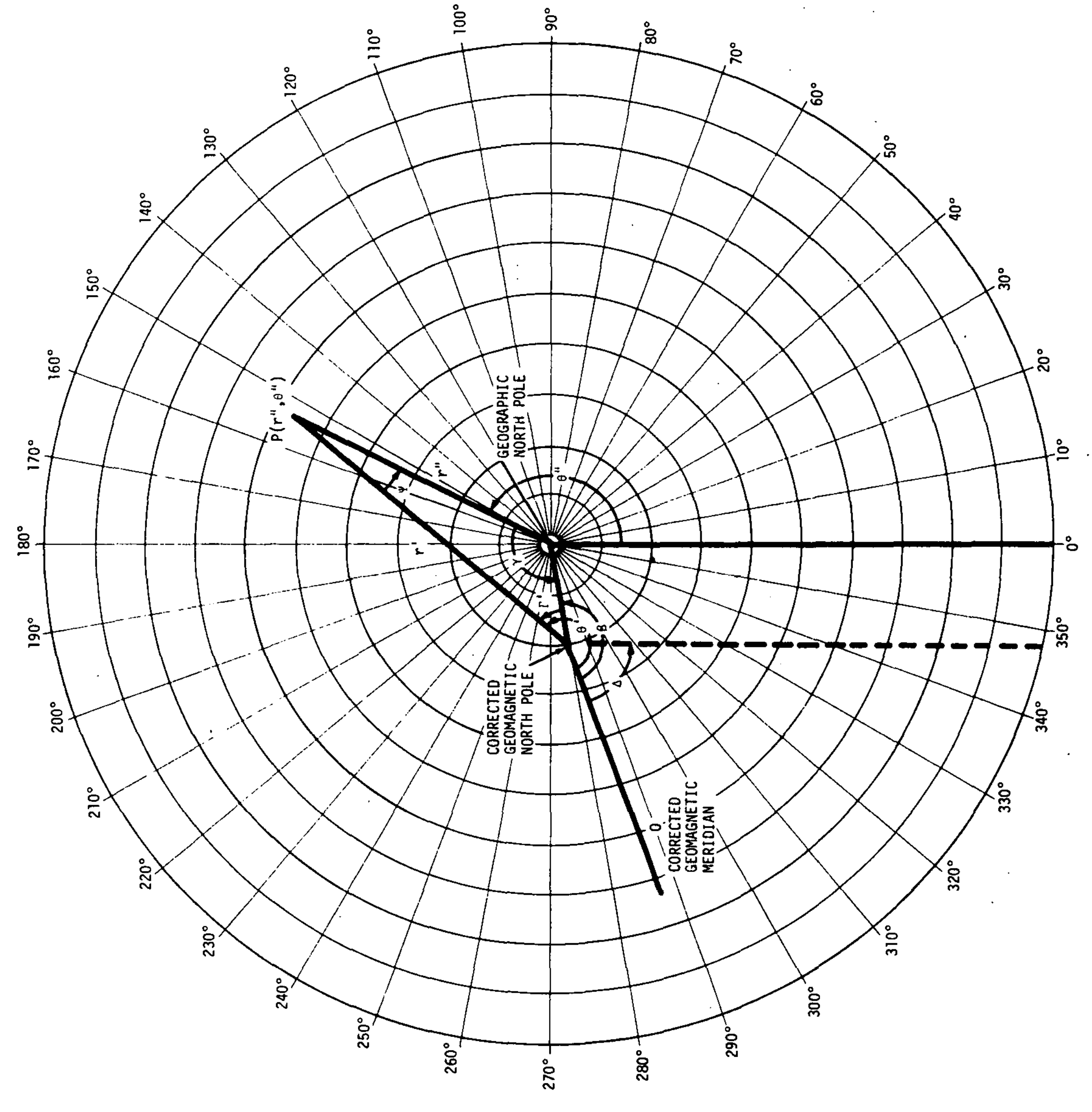

Figure 2-4. GEOMETRIC RELATIONSHIPS IN CHANGING POINT AND DIRECTION OF REFERENCE FROM THOSE OF THE GEOGRAPHIC COORDINATE SYSTEM TO THOSE OF THE CORRECTED GEOMAGNETIC COORDINATE SYSTEM

$$(r')^2 = (r'')^2 + R^2 - 2r'' R \cos \gamma \tag{33}$$

From Figure 2-4

$$\gamma = 2\pi - \theta'' - \Delta - (\pi - \beta)$$

$$= \pi - \theta'' + \beta - \Delta \tag{34}$$

With equation (34), equation (33) may be rewritten

$$r' = [(r'')^2 + R^2 + 2r'' R \cos (\theta'' - \beta + \Delta)]^{1/2} \tag{35}$$

Also from Figure 2-4 and the trigonometry of plane triangles

$$\frac{r''}{\sin \Gamma} = \frac{R}{\sin \psi} \tag{36}$$

where $$\psi = \pi - (\Gamma + \gamma) \tag{37}$$

Substituting equation (34) into (37) gives

$$\psi = \theta'' - \beta + \Delta - \Gamma \tag{38}$$

hence

$$\frac{r''}{R} = \frac{\sin \Gamma}{\sin (\theta'' - \beta + \Delta - \Gamma)}$$

The denominator on the right may be expanded and the equation solved for Γ to obtain

$$\Gamma = \text{ARCTAN} \left[\frac{\sin (\theta'' - \beta + \Delta)}{R/r'' + \cos (\theta'' - \beta + \Delta)} \right] \tag{39}$$

The reference direction of the coordinate system must now be aligned with the zero meridian of the CGC system. This is accomplished through (see Figure 2-4)

$$\theta' = \Gamma + \beta \tag{40}$$

Finally the coordinate system must be distorted so that ellipses become circles. The results of the previous subsection may be used. Combining equations (35) and (19) gives

$$r = \left\{[(r'')^2 + R^2 + 2r'' R \cos(\theta'' - \beta + \Delta)] \left[\frac{1 - \varepsilon^2 \cos^2 \Gamma}{1 - \varepsilon^2 \cos^2 \alpha}\right]\right\}^{1/2} \tag{41}$$

Similarly, combining equations (40) and (20) gives

$$\theta = \Gamma + \beta + .13 \left[\frac{1 - \varepsilon^2 \cos^2 \Gamma}{1 - \varepsilon^2 \cos^2 \alpha}\right]^{1/2} \sin 2\Gamma \tag{42}$$

Summarizing, the conversion of geographic to corrected geomagnetic coordinates may be effected per the following equations:

$$\Theta_c = 90° - r \tag{17}$$

$$\Phi_c = \theta \tag{18}$$

$$r = \left\{[(r'')^2 + R^2 + 2r'' R \cos(\theta'' - \beta + \Delta)] \left[\frac{1 - \varepsilon^2 \cos^2 \Gamma}{1 - \varepsilon^2 \cos^2 \alpha}\right]\right\}^{1/2} \tag{41}$$

$$\theta = \Gamma + \beta + .13 \left[\frac{1 - \varepsilon^2 \cos^2 \Gamma}{1 - \varepsilon^2 \cos^2 \alpha}\right]^{1/2} \sin 2\Gamma \tag{42}$$

$$\Gamma = \text{ARCTAN} \left[\frac{\sin(\theta'' - \beta + \Delta)}{R/r'' + \cos(\theta'' - \beta + \Delta)}\right] \tag{39}$$

$$r'' = 90° - \Theta \tag{31}$$

$$\theta'' = \Phi \tag{32}$$

where R, α, β, Δ, ε are known empirical constants whose values are listed in Table 2-1.

Section III

RESULTS

3.1 COMPUTER PROGRAMS

Equations for transforming corrected geomagnetic coordinates to geographic coordinates have been programmed in Fortran for use on the IBM 1130 computer. This program, designated MC2GC, together with computed tables is presented in Appendix A. A similar program, designated GC2MC, has been prepared for transforming geographic to corrected geomagnetic coordinates. Appendix B contains this program and computed tables. Computer time required for each program to produce the tables shown, including compilation time, was less than 6 minutes, determined primarily by the speed of the printer.

Both programs have been developed with an input option which permits input coordinates to be either read in or generated internally. The latter is useful for synoptic studies, while the former is useful for fixed or instaneous points of interest, such as geophysical stations, satellite passes, or boundaries of the auroral oval. Tables presented in the appendixes have been produced through the synoptic option, covering latitudes $\geq$ 50 degrees at 5-degree intervals and all longitudes at 10-degree intervals.

Minor program modification will permit changes of coverage and intervals. In addition to a switch variable designating the input option, required input includes the transformation parameters listed in Table 2-1, and, unless the synoptic option is designated, the input coordinates together with the number of coordinates pairs for a given run.

3.2 ERRORS

It is useful to establish a common unit in which all errors are to be expressed, so that deviations in both latitude and longitude for both transformations have the same relative significance. For these purposes, it is convenient to take a degree of geographic latitude as such a unit. This unit is proportional to distance on the surface of the (reference sphere) earth; that is, along a geographic meridian, arc length is given by

$$S = \mu \, \Delta\Theta \Big|_{\Phi = \text{constant}} \qquad (43)$$

where

$$\mu = \frac{2\pi}{360°} R_e \tag{44}$$

with R_e the radius of the earth and $\Delta\Theta$ given in degrees. In units of μ then, distance on the surface of the earth (arc length) is given by

$$S = \Delta\Theta \Big|_{\Phi = \text{constant}} \tag{45}$$

Similarly it is easy to see that arc length along a circle of constant geographic latitude is given by

$$S = \mu \cos\Theta \, \Delta\Phi \Big|_{\Theta = \text{constant}}$$

So in units of μ,

$$S = \cos\Theta \, \Delta\Phi \Big|_{\Theta = \text{constant}} \equiv (\Delta\Phi)' \tag{46}$$

When $\Delta\Phi$ and $\Delta\Theta$ are expressed in degrees, both $(\Delta\Phi)'$ and $\Delta\Theta$ represent distances in the same way; they may therefore be compared with one another.

The CGC system has an additional consideration. Since translations and rotations preserve distances, they introduce no alterations into equations (45) and (46). However, this is not true for the transformations of circles to ellipses and vice versa. In the CGC system arc length along a meridian of longitude is still proportional to a change in latitude, as in equation (45), but the constant of proportionality varies with longitude. This variation is just that expressed by equation (19). Hence, from equations (17), (18), (19), (21), and (45)

$$S = \Delta\Theta_c \sqrt{\frac{1 - \varepsilon^2 \cos^2 (\Phi_c - \beta)}{1 - \varepsilon^2 \cos^2 \alpha}} \Bigg|_{\Phi_c = \text{constant}} \equiv (\Delta\theta_c)' \tag{47}$$

where μ is taken as the unit of length. From equation (46)

$$S = \cos \Theta_c \, \Delta\Phi_c \Big|_{\Theta_c = \text{constant}} \equiv (\Delta\Phi_c)' \tag{48}$$

where it is assumed that the variation of $\Delta\theta$ in equation (20) over the interval of $\Delta\Phi_c$ is negligible. The deviations $\Delta\Theta$, $(\Delta\Phi)'$, $(\Delta\Theta_c)'$, and $(\Delta\Phi_c)'$, as defined by equations (45) - (48), are all proportional to distance on the earth in the same way, and on this basis may be compared with one another.

The corrected geomagnetic coordinates of Hakura (ref. 2) are taken as the standard for comparison. Errors are defined as deviations of present results from those in that reference. The extensive tables presented there are used as reference values with coordinate conversions in both directions being checked against them.

For the transformation of geographic to corrected geomagnetic coordinates, comparison between the tables in Appendix B and the tables in reference 2 is direct and convenient. Maximum positive and negative deviations are computed according to

$$\Delta X \equiv \text{Max } (X_{\text{computed}} - X_{\text{Hakura}}) \tag{49}$$

Table 3-1 contains maximum positive and negative deviations in corrected geomagnetic latitude $(\Delta\Theta_c)'$ for each given geographic latitude shown. Also shown is the longitude at which the maximum deviation occurs and the standard values of the corrected geomagnetic coordinates at that point. Similar information for $(\Delta\Phi_c)'$, obtained in the same manner, is presented in Table 3-2.

From Tables 3-1 and 3-2 some general conslusions can be drawn. Errors appear to increase with decreasing latitude, so that the latitude limits for which the transformations are useful depend upon the inaccuracies which can be tolerated. Corrected geomagnetic latitude and longitude errors are comparable for given geographic latitude, although the maximum deviations for both do not

Table 3-1. MAXIMUM DEVIATIONS OF COMPUTED VALUES OF CORRECTED GEOMAGNETIC LATITUDE FROM STANDARD VALUES ALONG CURVES OF CONSTANT GEOGRAPHIC LATITUDE

Θ	$(\Delta\Theta_c)'$*	Φ	Θ_c**	Φ_c**
50°	+ 2.1°	140°	43.9°	210.0°
	- .3°	330°	54.9°	56.0°
55°	+ 1.7°	150°	48.7°	217.8°
	- .4°	330°	59.8°	58.3°
60°	+ 1.5°	150°	53.8°	216.7°
	- .2°	330°	64.4°	60.9°
65°	+ 1.1°	160°	59.1°	222.3°
	- .4°	290°	76.6°	11.0°
70°	+ .9°	160°	64.2°	219.3°
	- .4°	270°	80.9°	327.6°
75°	+ .7°	170°	69.8°	221.0°
	- .3°	280°	85.7°	353.1°
80°	+ .5°	170°	74.6°	212.7°
	- .1°	60°	73.9°	144.8°
85°	+ .4°	190°	79.9°	202.6°

* *Defined by equations (47) and (49)*

** *Standard values - from Hakura (ref. 2)*

Table 3-2. MAXIMUM DEVIATIONS OF COMPUTED VALUES OF CORRECTED GEOMAGNETIC LONGITUDE FROM STANDARD VALUES ALONG CURVES OF CONSTANT GEOGRAPHIC LATITUDE

Θ	$(\Delta\Phi_c)'$*	Φ	Θ_c**	Φ_c**
50°	+ 1.4°	330°	54.9°	56.0°
	- 2.2°	230°	53.9°	285.3°
55°	+ .9°	330°	59.8°	58.3°
	- 1.5°	230°	59.0°	282.9°
60°	+ .7°	330°	64.4°	60.9°
	- 1.0°	230°	63.9°	279.8°
65°	+ .4°	320°	71.2°	54.0°
	- 1.1°	130°	59.3°	198.6°
70°	+ .3°	200°	67.9°	246.4°
	- 1.0°	130°	64.1°	196.8°
75°	+ .5°	290°	84.9°	26.8°
	- .7°	130°	68.8°	194.1°
80°	+ .5°	230°	81.3°	239.9°
	- .4°	130°	73.3°	190.0°
85°	+ .2°	240°	83.7°	203.3°
	- .1°	130°	77.7°	183.0°

* *Defined by equations (48) and (49)*

** *Standard values - from Hakura (ref. 2)* necessarily occur in the same direction (longitude). Not indicated in the tables but apparent in the comparison is the fact that within 1 - 2 degrees latitude of the geomagnetic pole (CGC system) longitude errors become large in terms of degrees, although not in terms of distance as expressed by $(\Delta\Phi_c)'$ due to the cos Θ_c factor. In essence, the longitude conversion breaks down near the pole; but since the latitude conversion holds well in this region, problems of physical location are not very serious.

For the conversion of corrected geomagnetic coordinates to geographic coordinates, the tables of Hakura (ref. 2) are not convenient for comparison. Nevertheless, they can be used because they do establish a correspondence between coordinates in the two systems. In order to check the accuracy of this transformation, corrected geomagnetic coordinates of points along contours of constant geographic latitude were taken from reference 2 and read into program MC2GC. The computed geographic coordinates were then compared with those from the reference, to which the input corrected geomagnetic coordinates corresponded. Tables 3-3 and 3-4, for errors in geographic latitude and longitude respectively, were prepared in this way, according to the notation of equation (49). The correct values of all coordinates at the point where the maximum deviation occurs are also included. It should be noted that to be strictly analogous with Tables 3-1 and 3-2, contours of constant corrected geomagnetic latitude should have been used in determining the maximum deviations. Since this was impractical, given the tables of reference 2, trends in Tables 3-3 and 3-4 are not apparent, unlike Tables 3-1 and 3-2. For present purposes, it is sufficient to note that errors for this transformation are comparable in magnitude to those of the previous one.

To supplement Table 3-3, Figure 3-1 has been prepared. This figure compares computed contours of constant corrected geomagnetic latitude (solid line) with those of Hakura (ref. 2) (dashed line) in the geographic coordinate system. Since geomagnetic latitude is generally the coordinate of importance when the geomagnetic field is important, corrected geomagnetic longitude (computed) is indicated only at the intersection with the (computed) latitude contours. Figure 3-1 clearly shows the "distribution of error" in corrected geomagnetic latitude arising from the transformation. From a comparison of Tables 3-3 and 3-4 it can be seen that longitudinal errors are not necessarily distributed in the same manner, particularly at higher latitudes.

Table 3-3. MAXIMUM DEVIATIONS OF COMPUTED VALUES OF GEOGRAPHIC LATITUDE FROM STANDARD VALUES ALONG CURVES OF CONSTANT GEOGRAPHIC LATITUDE *

Θ	$\Delta\Theta$	Φ	Θ_c**	Φ_c**
40.0°	+ .5°	310.0°	50.7°	30.9°
	- 2.6°	265.0°	51.4°	328.2°
50.0°	+ .8°	315.0°	58.8°	40.3°
	- .9°	215.0°	50.5°	271.5°
60.0°	+ 1.1°	45.0°	55.5°	119.5°
	- .8°	175.0°	54.5°	235.6°
70.0°	+ .9°	50.0°	65.1°	128.2°
	- .4°	190.0°	66.5°	239.6°
80.0°	+ .5°	270.0°	88.3°	256.3°
	- .2°	125.0°	73.2°	186.9°

* *Only those longitude regions where* $\Theta_c \geq 50.0°$ *have been considered.*

** *From Hakura (ref. 2)*

Table 3-4. MAXIMUM DEVIATIONS OF COMPUTED VALUES OF GEOGRAPHIC LONGITUDE FROM STANDARD VALUES ALONG CURVES OF CONSTANT GEOGRAPHIC LATITUDE *

Θ	$(\Delta\Phi)'$ *	Φ	Θ_c**	Φ_c**
40.0°	+ 1.0°	260.0°	50.4°	322.0
	- 1.5°	310.0°	50.7°	30.9
50.0°	+ 1.6°	225.0°	52.8°	280.5
	- 1.1°	340.0°	52.3°	65.1
60.0°	+ .6°	240.0°	66.3°	290.2
	- 1.2	20.0	56.4°	99.8
70.0°	+ .6°	110.0°	64.5°	179.9
	- .7°	25.0°	66.1°	109.8
80.0°	+ .2°	310.0°	84.4°	90.8
	- .4°	210.0°	78.4°	231.5

* *Only those longitude regions where* $\Theta_c \geq 50.0°$ *have been considered.*

** *From Hakura (ref. 2)*

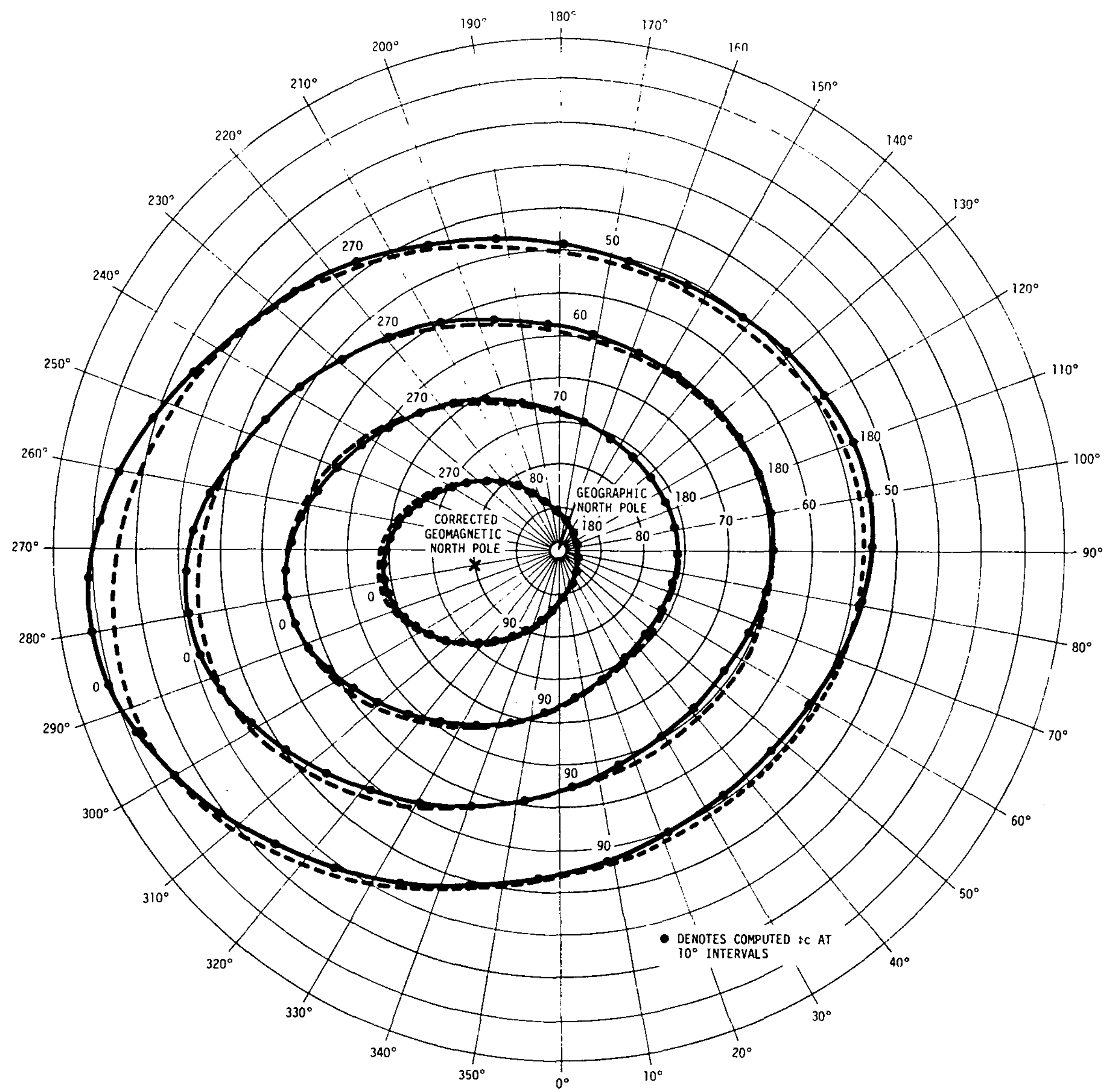

Figure 3-1. COMPARISON OF COMPUTED (SOLID LINE) CONTOURS OF CONSTANT CORRECTED GEOMAGNETIC LATITUDE WITH THOSE OF HAKURA (REF. 2) (DASHED LINE) IN A POLAR PROJECTION OF GEOGRAPHIC COORDINATES. (COMPUTED COORDINATES FROM APPENDIX A)

In the course of determining the errors inherent in the transformations, it has been found that the transformations are only approximately inverses of one another. When geographic coordinates are converted to corrected geomagnetic and back again, deviations of up to 1.2 degrees latitude occur at lower latitudes. As with the other errors noted above, these errors decrease with increasing latitude. Some caution should therefore be exercised in iterative operations involving multiple conversions of the coordinates.

3.3 DISCUSSION

The corrected geomagnetic coordinates computed by Hakura (ref. 2) are based on the geomagnetic field for epoch 1945.0. Gustafsson (ref. 6) recently examined the effect of using the geomagnetic field for epoch 1965.0. He found maximum displacements of dipole field lines of 60 km and 70 km perpendicular to and along the geomagnetic meridians respectively at $\Theta_c = 67$ degrees. This corresponds to .5 degree - .6 degree geographic latitude in the context of subsection 3.2 (see equation (44) and (45)). Hence, it appears that any changes in the coordinate system resulting from use of more recent geomagnetic field data will be of the same order as the errors resulting form the transformations presented here. It is anticipated that such changes can be adequately accommodated by suitable modification of the transformation parameters in Table 2-1.

A basic difference between corrected geomagnetic coordinates and eccentric dipole coordinates, aside from the locations of the respective poles, can be seen by comparing the polar projection maps of Hakura (ref. 2) with those of Cole (ref. 7). Contours of constant eccentric dipole latitude in the latter are circular, whereas similar contours in the former are elliptic. This difference can be seen clearly in Figure 2-2. In terms of the present treatment, the transformation of ellipses to circles and vice versa is the primary difference between eccentric dipole coordinates and corrected geomagnetic coordinates in the approximation presented here.

Gustafsson (ref. 3) has compared eccentric dipole coordinates with corrected geomagnetic coordinates and found the difference in latitude to be generally less than $\pm$ 2 degrees for stations used in his study ($\Theta_c \geq 61$ degrees). From Figure 2-2, however, it appears that over certain longitude ranges deviations of twice this magnitude might obtain. Thus for high latitude ($\Theta_c > 50$ degrees) investigations in the northern hemisphere, the transformations presented here will likely yield better results than eccentric dipole coordinates.

For the southern hemisphere, eccentric dipole coordinates remain a useful approximation. Examination of the contours of constant corrected geomagnetic latitude for the southern hemisphere shows that to the extent that they are elliptic, they are less eccentric (more nearly circular) than the corresponding contours for the northern hemisphere. Since, as was noted above, the major difference between eccentric dipole coordinates and corrected geomagnetic coordinates as approximated here is the ellipicity of the latitude contours, improvements resulting from extension of the present treatment to the southern hemisphere might be slight, if extant. The principal advantages of such an extension would then be uniformity of method for transforming coordinates in both hemispheres and the convenience of simple coordinate conversion in both directions.

Empirical determinations in this report have been based on the 70 degree corrected geomagnetic latitude contour; similarly all scalings have been made from this contour. This CGC latitude was selected in order to minimize error in the vicinity of the auroral oval and because it was intermediate in the high latitude region. Since many investigations of high latitude phenomena require statistical treatment, uncertainties of 1 - 2-degrees latitude are not uncommon, as, for example, in the determination of the boundaries of the auroral oval (ref. 5). Hence, the errors determined in subsection 3.2 should not cause serious problems if the transformations presented here were employed in such studies. The simplicity and flexibility of these transformations should make them particularly useful in model studies, where great accuracy is not required, but where it is desirable to maintain the basic geometry on a large scale. For high precision, however, a more elaborate system such as used by Hakura (ref. 2) for preparing his tables or one based on even more terms in the harmonic expansion of the geomagnetic field may be required.

Section IV

SUMMATION

A mathematical model has been formulated for northern hemisphere contours of constant corrected geomagnetic latitude in a polar projection of the geographic coordinate system. Based on this model, analytic equations have been developed for transforming geographic to corrected geomagnetic coordinates and vice versa. These equations have been programmed for a small computer; programs and computed tables have been presented.

From comparisons of computed coordinates and the coordinates on which this treatment was based, it has been determined that errors generally increase with decreasing latitude. At 50-degrees latitude maximum deviations are 2-degrees - 2.5-degrees latitude, improving with increasing latitude. The transformations are only approximately inverses of one another, so that caution must be exercised in iterative coordinate conversions. These transformations should be sufficiently accurate for use in many investigations of high latitude geophysical phenomena.

The principal advantages of the transformations presented here are: (1) they are more accurate than the dipole approximations; (2) they are fast and easily computed on a small computer; and (3) coordinates may be converted in both directions between geographic and corrected geomagnetic coordinates, while many transformations are limited to the geographic to geomagnetic conversion.

Section V

REFERENCES

1. Hultqvist, B., "The Geomagnetic Field Lines In Higher Approximation", Arkiv För Geofysik, Vol. 3, 1958, pp. 63-77

2. Hakura, Y., "Tables and Maps of Geomagnetic Coordinates Corrected by the Higher Order Spherical Harmonic Terms", Rep. Ionos. Space Res. Japan, Vol. 19, 1965, pp. 121-157

3. Gustafsson, G., "Auroral Orientation Curves and the Auroral Oval I", Tellus, Vol. 21, 1969, pp. 852-859

4. Sandford, B. P., "Variations of Auroral Emissions with Time Magnetic Activity and the Solar Cycle", J. Atmosph. Terr. Phys., Vol. 30, 1968, pp. 1921-1942

5. Feldstein, Y. I. and Starkov, G. V., "Dynamics of Auroral Belt and Polar Geomagnetic Disturbances", Planet. Space Sci., Vol. 15, 1967, pp. 209-229

6. Gustafsson, G., "Numerical Evaluation of the Auroral Orientation Curves II", Tellus, Vol. 21, 1969, pp. 861-866

7. Cole, K. D., "Eccentric Dipole Coordinates", Austral. J. Phys., Vol. 16, 1963, pp. 423-429

Appendix A

PROGRAM FOR TRANSFORMING CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

This appendix contains the computer program MC2GC, which transforms corrected geomagnetic coordinates (ref. 2) to geographic coordinates. Basic equations for this program are developed in subsection 2.2. Characteristics of the program are discussed in subsection 3.1. Also presented are tables of computed values of geographic coordinates obtained through the input option for internal generation of corrected geomagnetic coordinates.

Variables and parameters in the program are related to those in Section II as follows:

GMLT = Θ_c (corrected geomagnetic latitude)

GMLG = Φ_c (corrected geomagnetic longitude)

GGLT = Θ (geographic latitude)

GGLG = Φ (geographic longitude)

DELC = Δ

RC = R

ALPHC = α

BETC = β

EPC = ε

```
LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0001        0001        0000

V2 M07   ACTUAL  8K  CONFIG  8K

// FOR
*IOCS(1132PRINTER,CARD)
*ONE WORD INTEGERS
*LIST SOURCE PROGRAM
C MC2GC TRANSFORMS CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC
C COORDINATES BY MEANS OF AN EMPIRICAL, ANALYTIC SET OF EQUATIONS.
C VALID ONLY FOR HIGH LATITUDES IN THE NORTHERN HEMISPHERE.
C
C DEFINE FUNCTIONS TO BE USED
      AMAG(X,Y,Z) = SQRT(X*X + Y*Y + 2.0*X*Y*COS(Z))
      RAD(X) = .0174533*X
      RADI(X)=57.2957795*X
      ELPS(X,Y,Z) = SQRT((1.0 - X*X*COS(Y)*COS(Y))/(1.0 - X*X*COS(Z)*COS
     1(Z)))
C READ TRANSFORMATION PARAMETERS
      READ (2,77) DELC, RC, BETC, ALPHC, EPC
      WRITE(3,80)
      WRITE(3,88) DELC,RC,BETC,ALPHC,EPC
      DELC=RAD(DELC)
      RC=RAD(RC)
      BETC=RAD(BETC)
      ALPHC=RAD(ALPHC)
C INPUT OPTION DESIGNATED BY SWITCH VARIABLE IO
      READ (2,78) IO
      IF (IO) 11,21,11
C OPTION TO READ IN CORRECTED GEOMAGNETIC COORDINATES (N IS THE NUMBER
C OF LATITUDE-LONGITUDE PAIRS)
   11 READ (2,79) N
      WRITE (3,81)
      WRITE (3,82)
      DO 19 K = 1,N
      READ (2,76) GMLT, GMLG
      GO TO 71
   15 WRITE (3,83) GMLT, GMLG, GGLT, GGLG
   19 CONTINUE
      CALL EXIT
C OPTION TO GENERATE CORRECTED GEOMAGNETIC COORDINATES FOR SYNOPTIC
C STUDIES
   21 GMLT=45.0
      DO 30 I=1,8
      GMLT=GMLT+5.0
      WRITE(3,81)
      WRITE(3,84)
      WRITE(3,85) GMLT
      WRITE(3,86)
      GMLG=-10.0
      DO 29 J=1,36
      GMLG=GMLG+10.0
      GO TO 71
   25 WRITE (3,87) GMLG,GGLT,GGLG
```

```
   29 CONTINUE
   30 CONTINUE
      CALL EXIT
C APPLICATION OF TRANSFORMATION EQUATIONS
   71 R2=RAD(90.0-GMLT)
      T2=RAD(GMLG)
      T1=T2-BETC
      T=T1-.13*ELPS(EPC,T1,ALPHC)*SIN(2.0*T1)
      R=R2*ELPS(EPC,ALPHC,T1)
      S=SIN(T)
      C=RC/R-COS(T)
      T2=ATNR(S,C)
      IF(T2) 72,73,73
   72 T2=T2+6.2831853
   73 T2=3.1415927+BETC-DELC-T2
      IF(T2) 74,75,75
   74 T2=T2+6.2831853
   75 R2=AMAG(-RC,R,T)
      GGLT=90.0-RADI(R2)
      GGLG=RADI(T2)
      IF (IO) 15,25,15
C INPUT-OUTUT FORMATS
   76 FORMAT (20X,2F10.5)
   77 FORMAT(5F10.5)
   78 FORMAT(I2)
   79 FORMAT(I3)
   80 FORMAT (1H0,8X,4HDELC,17X,2HRC,17X,4HBETC,16X,5HALPHC,16X,3HEPC)
   81 FORMAT (1H1,26X,77HTRANSFORMATION OF CORRECTED GEOMAGNETIC COORDIN
     1ATES TO GEOGRAPHIC COORDINATES)
   82 FORMAT (1H0,13X,20HGEOMAGNETIC LATITUDE,5X,21HGEOMAGNETIC LONGITUD
     1E,5X,19HGEOGRAPHIC LATITUDE,6X,20HGEOGRAPHIC LONGITUDE)
   83 FORMAT(1H ,13X,4(4X,F10.3,11X))
   84 FORMAT (1H0,50X,22HGEOMAGNETIC LATITUDE =)
   85 FORMAT(1H+,73X,F10.3)
   86 FORMAT (1H0,25X,21HGEOMAGNETIC LONGITUDE,5X,19HGEOGRAPHIC LATITUDE
     1,5X,20HGEOGRAPHIC LONGITUDE)
   87 FORMAT(1H ,25X,3(4X,F10.3,11X))
   88 FORMAT (1H0,5(5X,F10.5,5X))
      END

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 IOCS

CORE REQUIREMENTS FOR
 COMMON       0  VARIABLES      62  PROGRAM      730

*END OF COMPILATION

// XEQ
```

DELC	RC	BETC	ALPHC	EPC
70.00001	9.50000	170.00003	55.00000	0.59130

TRANSFORMATION OF CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

GEOMAGNETIC LATITUDE = 50.000

GEOMAGNETIC LONGITUDE	GEOGRAPHIC LATITUDE	GEOGRAPHIC LONGITUDE
0.000	34.286	286.472
10.000	35.537	293.051
20.000	37.376	299.881
30.000	39.562	307.152
40.000	41.910	315.075
50.000	44.310	323.834
60.000	46.699	333.545
70.000	49.021	344.234
80.000	51.190	355.830
90.000	53.076	8.157
100.000	54.528	20.934
110.000	55.414	33.802
120.000	55.677	46.375
130.000	55.365	58.348
140.000	54.648	69.587
150.000	53.791	80.142
160.000	53.103	90.199
170.000	52.839	100.000
180.000	53.103	109.800
190.000	53.791	119.857
200.000	54.648	130.412
210.000	55.365	141.651
220.000	55.677	153.624
230.000	55.414	166.197
240.000	54.528	179.065
250.000	53.076	191.842
260.000	51.190	204.169
270.000	49.021	215.764
280.000	46.699	226.454
290.000	44.310	236.165
300.000	41.910	244.924
310.000	39.562	252.847
320.000	37.376	260.118
330.000	35.537	266.948
340.000	34.286	273.527
350.000	33.839	280.000

TRANSFORMATION OF CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

GEOMAGNETIC LATITUDE = 55.000

GEOMAGNETIC LONGITUDE	GEOGRAPHIC LATITUDE	GEOGRAPHIC LONGITUDE
0.000	40.070	286.319
10.000	41.187	292.736
20.000	42.835	299.390
30.000	44.807	306.465
40.000	46.944	314.167
50.000	49.156	322.680
60.000	51.394	332.127
70.000	53.611	342.550
80.000	55.731	353.905
90.000	57.631	6.055
100.000	59.165	18.767
110.000	60.197	31.713
120.000	60.656	44.508
130.000	60.571	56.811
140.000	60.085	68.435
150.000	59.431	79.386
160.000	58.884	89.827
170.000	58.672	100.000
180.000	58.884	110.172
190.000	59.431	120.613
200.000	60.085	131.564
210.000	60.571	143.188
220.000	60.656	155.492
230.000	60.197	168.286
240.000	59.165	181.232
250.000	57.631	193.944
260.000	55.731	206.094
270.000	53.611	217.449
280.000	51.394	227.872
290.000	49.156	237.319
300.000	46.944	245.832
310.000	44.807	253.534
320.000	42.835	260.609
330.000	41.187	267.263
340.000	40.070	273.680
350.000	39.672	280.000

TRANSFORMATION OF CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

GEOMAGNETIC LATITUDE = 60.000

GEOMAGNETIC LONGITUDE	GEOGRAPHIC LATITUDE	GEOGRAPHIC LONGITUDE
0.000	45.854	286.125
10.000	46.836	292.339
20.000	48.291	298.771
30.000	50.044	305.598
40.000	51.965	313.019
50.000	53.982	321.216
60.000	56.059	330.318
70.000	58.161	340.382
80.000	60.222	351.395
90.000	62.129	3.271
100.000	63.742	15.839
110.000	64.925	28.830
120.000	65.591	41.881
130.000	65.747	54.617
140.000	65.505	66.774
150.000	65.064	78.288
160.000	64.664	89.285
170.000	64.504	100.000
180.000	64.664	110.714
190.000	65.064	121.711
200.000	65.505	133.226
210.000	65.747	145.382
220.000	65.591	158.118
230.000	64.925	171.169
240.000	63.742	184.160
250.000	62.129	196.728
260.000	60.222	208.604
270.000	58.161	219.617
280.000	56.059	229.681
290.000	53.982	238.783
300.000	51.965	246.980
310.000	50.044	254.401
320.000	48.291	261.228
330.000	46.836	267.660
340.000	45.854	273.874
350.000	45.504	280.000

TRANSFORMATION OF CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

GEOMAGNETIC LATITUDE = 65.000

GEOMAGNETIC LONGITUDE	GEOGRAPHIC LATITUDE	GEOGRAPHIC LONGITUDE
0.000	51.637	285.872
10.000	52.481	291.822
20.000	53.739	297.966
30.000	55.268	304.469
40.000	56.965	311.522
50.000	58.776	319.301
60.000	60.680	327.936
70.000	62.651	337.498
80.000	64.635	348.004
90.000	66.536	359.428
100.000	68.223	11.691
110.000	69.560	24.626
120.000	70.449	37.938
130.000	70.868	51.239
140.000	70.892	64.171
150.000	70.682	76.551
160.000	70.440	88.422
170.000	70.337	100.000
180.000	70.440	111.577
190.000	70.682	123.448
200.000	70.892	135.828
210.000	70.868	148.760
220.000	70.449	162.061
230.000	69.560	175.373
240.000	68.223	188.308
250.000	66.536	200.571
260.000	64.635	211.995
270.000	62.651	222.501
280.000	60.680	232.063
290.000	58.776	240.698
300.000	56.965	248.477
310.000	55.268	255.530
320.000	53.739	262.033
330.000	52.481	268.177
340.000	51.637	274.127
350.000	51.337	280.000

TRANSFORMATION OF CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

GEOMAGNETIC LATITUDE = 70.000

GEOMAGNETIC LONGITUDE	GEOGRAPHIC LATITUDE	GEOGRAPHIC LONGITUDE
0.000	57.419	285.531
10.000	58.123	291.123
20.000	59.178	296.876
30.000	60.475	302.942
40.000	61.935	309.494
50.000	63.523	316.696
60.000	65.230	324.671
70.000	67.043	333.495
80.000	68.923	343.209
90.000	70.789	353.849
100.000	72.531	5.450
110.000	74.022	18.019
120.000	75.154	31.446
130.000	75.874	45.434
140.000	76.209	59.545
150.000	76.268	73.395
160.000	76.208	86.838
170.000	76.169	100.000
180.000	76.208	113.161
190.000	76.268	126.604
200.000	76.209	140.454
210.000	75.874	154.565
220.000	75.154	168.553
230.000	74.022	181.980
240.000	72.531	194.549
250.000	70.789	206.150
260.000	68.923	216.790
270.000	67.043	226.504
280.000	65.230	235.328
290.000	63.523	243.303
300.000	61.935	250.505
310.000	60.475	257.057
320.000	59.178	263.123
330.000	58.123	268.876
340.000	57.419	274.469
350.000	57.169	280.000

TRANSFORMATION OF CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

GEOMAGNETIC LATITUDE = 75.000

GEOMAGNETIC LONGITUDE	GEOGRAPHIC LATITUDE	GEOGRAPHIC LONGITUDE
0.000	63.199	285.041
10.000	63.758	290.122
20.000	64.602	295.321
30.000	65.651	300.763
40.000	66.854	306.598
50.000	68.192	312.965
60.000	69.665	319.961
70.000	71.272	327.644
80.000	72.989	336.050
90.000	74.759	345.240
100.000	76.500	355.349
110.000	78.112	6.608
120.000	79.499	19.294
130.000	80.582	33.577
140.000	81.326	49.312
150.000	81.755	65.982
160.000	81.950	82.986
170.000	82.002	100.000
180.000	81.950	117.013
190.000	81.755	134.017
200.000	81.326	150.687
210.000	80.582	166.422
220.000	79.499	180.706
230.000	78.112	193.391
240.000	76.500	204.650
250.000	74.759	214.759
260.000	72.989	223.949
270.000	71.272	232.355
280.000	69.665	240.038
290.000	68.192	247.034
300.000	66.854	253.401
310.000	65.651	259.236
320.000	64.602	264.678
330.000	63.758	269.877
340.000	63.199	274.958
350.000	63.002	280.000

TRANSFORMATION OF CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

GEOMAGNETIC LATITUDE = 80.000

GEOMAGNETIC LONGITUDE	GEOGRAPHIC LATITUDE	GEOGRAPHIC LONGITUDE
0.000	68.976	284.283
10.000	69.380	288.575
20.000	69.996	292.924
30.000	70.773	297.415
40.000	71.683	302.159
50.000	72.718	307.242
60.000	73.887	312.710
70.000	75.195	318.549
80.000	76.630	324.719
90.000	78.159	331.185
100.000	79.731	337.989
110.000	81.293	345.318
120.000	82.797	353.579
130.000	84.208	3.503
140.000	85.498	16.336
150.000	86.626	34.308
160.000	87.485	61.367
170.000	87.834	100.000
180.000	87.485	138.632
190.000	86.626	165.691
200.000	85.498	183.663
210.000	84.208	196.496
220.000	82.797	206.420
230.000	81.293	214.681
240.000	79.731	222.010
250.000	78.159	228.814
260.000	76.630	235.280
270.000	75.195	241.450
280.000	73.887	247.289
290.000	72.718	252.757
300.000	71.683	257.840
310.000	70.773	262.584
320.000	69.996	267.076
330.000	69.380	271.424
340.000	68.976	275.716
350.000	68.834	280.000

TRANSFORMATION OF CORRECTED GEOMAGNETIC COORDINATES TO GEOGRAPHIC COORDINATES

GEOMAGNETIC LATITUDE = 85.000

GEOMAGNETIC LONGITUDE	GEOGRAPHIC LATITUDE	GEOGRAPHIC LONGITUDE
0.000	74.747	282.950
10.000	74.976	285.873
20.000	75.330	288.771
30.000	75.783	291.676
40.000	76.324	294.631
50.000	76.953	297.648
60.000	77.676	300.687
70.000	78.495	303.639
80.000	79.399	306.341
90.000	80.363	308.600
100.000	81.352	310.228
110.000	82.333	311.058
120.000	83.276	310.916
130.000	84.163	309.537
140.000	84.973	306.424
150.000	85.662	300.760
160.000	86.151	291.770
170.000	86.332	280.000
180.000	86.151	268.229
190.000	85.662	259.239
200.000	84.973	253.576
210.000	84.163	250.462
220.000	83.276	249.083
230.000	82.333	248.941
240.000	81.352	249.771
250.000	80.363	251.399
260.000	79.399	253.659
270.000	78.495	256.360
280.000	77.676	259.312
290.000	76.953	262.351
300.000	76.324	265.368
310.000	75.783	268.323
320.000	75.330	271.228
330.000	74.976	274.126
340.000	74.747	277.049
350.000	74.667	280.000

Appendix B

PROGRAM FOR TRANSFORMING GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

This appendix contains the computer program GC2MC, which transforms geographic coordinates to corrected geomagnetic coordinates (ref. 2). Basic equations for this program are developed in subsection 2.3. Characteristics of the program are discussed in subsection 3.1. Also presented are tables of computed values of corrected geomagnetic coordinates obtained through the input option for internal generation of geographic coordinates.

Variables and parameters in the program are related to those in Section II as follows:

GMLT = Θ_c (corrected geomagnetic latitude)

GMLG - Φ_c (corrected geomagnetic longitude)

GGLT = Θ (geographic latitude)

GGLG = Φ (geographic longitude)

DELC = Δ

RC = R

ALPHC = α

BETC = β

EPC = ε

```
LOG DRIVE   CART SPEC   CART AVAIL  PHY DRIVE
  0000        0001        0001        0000

V2 M07   ACTUAL  8K  CONFIG  8K

// FOR
*IOCS(1132PRINTER,CARD)
*ONE WORD INTEGERS
*LIST SOURCE PROGRAM
C GC2MC TRANSFORMS GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC
C COORDINATES BY MEANS OF AN EMPIRICAL, ANALYTIC SET OF EQUATIONS. VALID
C ONLY FOR HIGH LATITUDES IN THE NORTHERN HEMISPHERE.
C ONLY FOR HIGH LATITUDES IN THE NORTHERN HEMISPHERE.
C DEFINE FUNCTIONS TO BE USED
      AMAG(X,Y,Z) = SQRT(X*X + Y*Y + 2.0*X*Y*COS(Z))
      RAD(X) = .0174533*X
      RADI(X)=57.2957795*X
      ELPS(X,Y,Z) = SQRT((1.0 - X*X*COS(Y)*COS(Y))/(1.0 - X*X*COS(Z)*COS
     1(Z)))
C READ TRANSFORMATION PARAMETERS
      READ (2,77) DELC, RC, BETC, ALPHC, EPC
      WRITE(3,80)
      WRITE(3,88) DELC,RC,BETC,ALPHC,EPC
      DELC=RAD(DELC)
      RC=RAD(RC)
      BETC=RAD(BETC)
      ALPHC=RAD(ALPHC)
C INPUT OPTION DESIGNATED BY SWITCH VARIABLE IO
      READ (2,78) IO
      IF (IO) 11,21,11
C OPTION TO READ IN GEOGRAPHIC COORDINATES ( N IS NUMBER OF LATITUDE-
C LONGITUDE PAIRS)
   11 READ (2,79) N
      WRITE (3,81)
      WRITE (3,82)
      DO 19 K = 1,N
      READ (2,77) GGLT, GGLG
      GO TO 71
   15 WRITE (3,83) GMLT, GMLG, GGLT, GGLG
   19 CONTINUE
      CALL EXIT
C OPTION TO GENERATE GEOGRAPHIC COORDINATES FOR SYNOPTIC STUDIES
   21 GGLT = 45.0
      DO 30 I=1,8
      GGLT=GGLT + 5.0
      WRITE(3,81)
      WRITE(3,84)
      WRITE(3,85) GGLT
      WRITE(3,86)
      GGLG=-10.0
      DO 29 J=1,36
      GGLG=GGLG+10.0
      GO TO 71
   25 WRITE(3,87)GGLG,GMLT,GMLG
   29 CONTINUE
```

```
   30 CONTINUE
      CALL EXIT
C APPLICATION OF TRANSFORMATION EQUATIONS
   71 R1=RAD(90.0 - GGLT)
      T1=RAD(GGLG)
      T1=T1-BETC+DELC
      S=SIN(T1)
      C=RC/R1+COS(T1)
      T=ATNR(S,C)
      T3=T+.13*ELPS(EPC,T,ALPHC)*SIN(2.0*T)
      IF(T3) 73,75,75
   73 T3=T3+6.2831853
   75 R=AMAG(RC,R1,T1)*ELPS(EPC,T,ALPHC)
      GMLT=90.0-RADI(R)
      GMLG=RADI(T3+BETC)
      IF (GMLG-360.0) 62,62,61
   61 GMLG=GMLG-360.0
   62 IF (IO) 15,25,15
C INPUT-OUTUT FORMATS
   77 FORMAT(5F10.5)
   78 FORMAT(I2)
   79 FORMAT(I3)
   80 FORMAT (1H0,8X,4HDELC,17X,2HRC,17X,4HBETC,16X,5HALPHC,16X,3HEPC)
   81 FORMAT (1H1,26X,77HTRANSFORMATION OF GEOGRAPHIC COORDINATES TO COR
     1RECTED GEOMAGNETIC COORDINATES)
   82 FORMAT (1H0,13X,19HGEOGRAPHIC LATITUDE,6X,20HGEOGRAPHIC LONGITUDE,
     15X,20HGEOMAGNETIC LATITUDE,5X,21HGEOMAGNETIC LONGITUDE)
   83 FORMAT(1H ,13X,4(4X,F10.3,11X))
   84 FORMAT (1H0,50X,21HGEOGRAPHIC LATITUDE = )
   85 FORMAT(1H+,73X,F10.3)
   86 FORMAT (1H0,25X,20HGEOGRAPHIC LONGITUDE,5X,20HGEOMAGNETIC LATITUDE
     1,5X,21HGEOMAGNETIC LONGITUDE)
   87 FORMAT(1H ,25X,3(4X,F10.3,11X))
   88 FORMAT (1H0,5(5X,F10.5,5X))
      END

FEATURES SUPPORTED
 ONE WORD INTEGERS
 IOCS

CORE REQUIREMENTS FOR
 COMMON        0  VARIABLES     62  PROGRAM     722

 END OF COMPILATION

// XEQ
```

DELC	RC	BETC	ALPHC	EPC
70.00001	9.50000	170.00003	55.00000	0.59130

TRANSFORMATION OF GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

GEOGRAPHIC LATITUDE = 50.000

GEOGRAPHIC LONGITUDE	GEOMAGNETIC LATITUDE	GEOMAGNETIC LONGITUDE
0.000	48.068	82.687
10.000	46.706	89.833
20.000	45.816	97.178
30.000	45.375	104.936
40.000	45.328	113.197
50.000	45.591	121.950
60.000	46.054	131.122
70.000	46.596	140.609
80.000	47.093	150.306
90.000	47.441	160.126
100.000	47.565	169.999
110.000	47.441	179.873
120.000	47.093	189.693
130.000	46.596	199.390
140.000	46.054	208.877
150.000	45.591	218.048
160.000	45.328	226.802
170.000	45.375	235.062
180.000	45.816	242.821
190.000	46.706	250.166
200.000	48.068	257.312
210.000	49.883	264.610
220.000	52.092	272.531
230.000	54.586	281.591
240.000	57.205	292.229
250.000	59.726	304.658
260.000	61.870	318.750
270.000	63.331	334.064
280.000	63.853	350.000
290.000	63.331	5.935
300.000	61.870	21.249
310.000	59.726	35.341
320.000	57.205	47.770
330.000	54.586	58.408
340.000	52.092	67.468
350.000	49.883	75.389

TRANSFORMATION OF GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

GEOGRAPHIC LATITUDE = 55.000

GEOGRAPHIC LONGITUDE	GEOMAGNETIC LATITUDE	GEOMAGNETIC LONGITUDE
0.000	53.242	84.115
10.000	51.913	91.230
20.000	51.002	98.556
30.000	50.489	106.277
40.000	50.327	114.462
50.000	50.443	123.090
60.000	50.745	132.089
70.000	51.131	141.365
80.000	51.497	150.824
90.000	51.758	160.388
100.000	51.851	169.999
110.000	51.758	179.610
120.000	51.497	189.175
130.000	51.131	198.634
140.000	50.745	207.910
150.000	50.443	216.909
160.000	50.327	225.537
170.000	50.489	233.722
180.000	51.002	241.443
190.000	51.913	248.769
200.000	53.242	255.884
210.000	54.974	263.114
220.000	57.057	270.922
230.000	59.398	279.857
240.000	61.854	290.439
250.000	64.225	302.983
260.000	66.252	317.435
270.000	67.641	333.340
280.000	68.139	350.000
290.000	67.641	6.659
300.000	66.252	22.564
310.000	64.225	37.016
320.000	61.854	49.560
330.000	59.398	60.142
340.000	57.057	69.077
350.000	54.974	76.885

TRANSFORMATION OF GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

GEOGRAPHIC LATITUDE = 60.000

GEOGRAPHIC LONGITUDE	GEOMAGNETIC LATITUDE	GEOMAGNETIC LONGITUDE
0.000	58.379	86.018
10.000	57.086	93.086
20.000	56.157	100.372
30.000	55.576	108.027
40.000	55.303	116.095
50.000	55.277	124.547
60.000	55.423	133.317
70.000	55.657	142.319
80.000	55.897	151.475
90.000	56.073	160.718
100.000	56.138	169.999
110.000	56.073	179.281
120.000	55.897	188.524
130.000	55.657	197.680
140.000	55.423	206.682
150.000	55.277	215.451
160.000	55.303	223.904
170.000	55.576	231.972
180.000	56.157	239.626
190.000	57.086	246.913
200.000	58.379	253.981
210.000	60.026	261.118
220.000	61.982	268.768
230.000	64.171	277.508
240.000	66.467	287.961
250.000	68.696	300.604
260.000	70.617	315.522
270.000	71.946	332.269
280.000	72.426	350.000
290.000	71.946	7.730
300.000	70.617	24.477
310.000	68.696	39.396
320.000	66.467	52.038
330.000	64.171	62.491
340.000	61.982	71.231
350.000	60.025	78.881

TRANSFORMATION OF GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

GEOGRAPHIC LATITUDE = 65.000

GEOGRAPHIC LONGITUDE	GEOMAGNETIC LATITUDE	GEOMAGNETIC LONGITUDE
0.000	63.457	88.679
10.000	62.205	95.665
20.000	61.264	102.869
30.000	60.622	110.400
40.000	60.244	118.279
50.000	60.084	126.474
60.000	60.082	134.924
70.000	60.172	143.560
80.000	60.292	152.317
90.000	60.388	161.143
100.000	60.424	169.999
110.000	60.388	178.856
120.000	60.292	187.682
130.000	60.172	196.439
140.000	60.082	205.075
150.000	60.084	213.525
160.000	60.244	221.720
170.000	60.622	229.599
180.000	61.264	237.130
190.000	62.205	244.334
200.000	63.457	251.320
210.000	65.014	258.324
220.000	66.842	265.743
230.000	68.877	274.164
240.000	71.019	284.330
250.000	73.116	296.976
260.000	74.950	312.488
270.000	76.241	330.524
280.000	76.712	350.000
290.000	76.241	9.475
300.000	74.950	27.511
310.000	73.116	43.023
320.000	71.019	55.669
330.000	68.877	65.835
340.000	66.842	74.256
350.000	65.014	81.675

TRANSFORMATION OF GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

GEOGRAPHIC LATITUDE = 70.000

GEOGRAPHIC LONGITUDE	GEOMAGNETIC LATITUDE	GEOMAGNETIC LONGITUDE
0.000	68.434	92.650
10.000	67.233	99.471
20.000	66.292	106.492
30.000	65.600	113.778
40.000	65.131	121.335
50.000	64.849	129.129
60.000	64.711	137.114
70.000	64.669	145.236
80.000	64.678	153.448
90.000	64.700	161.712
100.000	64.710	169.999
110.000	64.700	178.287
120.000	64.678	186.551
130.000	64.669	194.763
140.000	64.711	202.885
150.000	64.849	210.870
160.000	65.131	218.664
170.000	65.600	226.221
180.000	66.292	233.507
190.000	67.233	240.528
200.000	68.434	247.349
210.000	69.892	254.134
220.000	71.583	261.196
230.000	73.461	269.071
240.000	75.451	278.589
250.000	77.432	290.870
260.000	79.213	306.998
270.000	80.511	327.185
280.000	80.998	350.000
290.000	80.511	12.814
300.000	79.213	33.001
310.000	77.432	49.129
320.000	75.451	61.410
330.000	73.461	70.928
340.000	71.583	78.803
350.000	69.892	85.865

TRANSFORMATION OF GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

GEOGRAPHIC LATITUDE = 75.000

GEOGRAPHIC LONGITUDE	GEOMAGNETIC LATITUDE	GEOMAGNETIC LONGITUDE
0.000	73.221	99.145
10.000	72.095	105.557
20.000	71.178	112.134
30.000	70.460	118.907
40.000	69.923	125.871
50.000	69.543	133.000
60.000	69.291	140.263
70.000	69.136	147.623
80.000	69.050	155.049
90.000	69.009	162.515
100.000	68.997	169.999
110.000	69.009	177.484
120.000	69.050	184.950
130.000	69.136	192.376
140.000	69.291	199.736
150.000	69.543	206.999
160.000	69.923	214.128
170.000	70.460	221.092
180.000	71.178	227.865
190.000	72.095	234.442
200.000	73.221	240.854
210.000	74.555	247.185
220.000	76.085	253.619
230.000	77.784	260.511
240.000	79.609	268.552
250.000	81.484	279.090
260.000	83.272	294.620
270.000	84.699	318.375
280.000	85.285	350.000
290.000	84.699	21.624
300.000	83.272	45.379
310.000	81.484	60.909
320.000	79.609	71.447
330.000	77.784	79.488
340.000	76.085	86.380
350.000	74.555	92.814

TRANSFORMATION OF GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

GEOGRAPHIC LATITUDE = 80.000

GEOGRAPHIC LONGITUDE	GEOMAGNETIC LATITUDE	GEOMAGNETIC LONGITUDE
0.000	77.595	111.085
10.000	76.608	116.309
20.000	75.774	121.736
30.000	75.085	127.355
40.000	74.531	133.145
50.000	74.101	139.080
60.000	73.779	145.133
70.000	73.549	151.276
80.000	73.397	157.483
90.000	73.311	163.731
100.000	73.283	169.999
110.000	73.311	176.268
120.000	73.397	182.516
130.000	73.549	188.723
140.000	73.779	194.866
150.000	74.101	200.919
160.000	74.531	206.854
170.000	75.085	212.644
180.000	75.774	218.263
190.000	76.608	223.690
200.000	77.595	228.914
210.000	78.733	233.936
220.000	80.018	238.776
230.000	81.441	243.484
240.000	82.987	248.162
250.000	84.636	253.040
260.000	86.363	258.750
270.000	88.130	268.303
280.000	89.571	350.000
290.000	88.130	71.696
300.000	86.363	81.249
310.000	84.636	86.959
320.000	82.987	91.837
330.000	81.441	96.516
340.000	80.018	101.223
350.000	78.733	106.063

TRANSFORMATION OF GEOGRAPHIC COORDINATES TO CORRECTED GEOMAGNETIC COORDINATES

GEOGRAPHIC LATITUDE = 85.000

GEOGRAPHIC LONGITUDE	GEOMAGNETIC LATITUDE	GEOMAGNETIC LONGITUDE
0.000	80.936	134.449
10.000	80.275	136.679
20.000	79.684	139.408
30.000	79.169	142.528
40.000	78.730	145.956
50.000	78.365	149.629
60.000	78.072	153.491
70.000	77.849	157.499
80.000	77.693	161.610
90.000	77.600	165.788
100.000	77.569	169.999
110.000	77.600	174.211
120.000	77.693	178.389
130.000	77.849	182.500
140.000	78.072	186.507
150.000	78.365	190.370
160.000	78.730	194.043
170.000	79.169	197.471
180.000	79.684	200.591
190.000	80.275	203.320
200.000	80.936	205.550
210.000	81.660	207.135
220.000	82.437	207.864
230.000	83.246	207.428
240.000	84.058	205.372
250.000	84.832	201.060
260.000	85.500	193.760
270.000	85.970	183.104
280.000	86.142	169.999
290.000	85.970	156.895
300.000	85.500	146.239
310.000	84.832	138.938
320.000	84.058	134.627
330.000	83.246	132.571
340.000	82.437	132.135
350.000	81.660	132.864