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INVESTIGATION OF FACTORS AFFECTING THE
HEATER WIRE METHOD OF CALIBRATING
FINE WIRE THERMOCOUPLES

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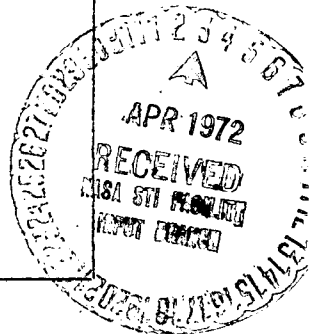
FINAL REPORT

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INVESTIGATION OF FACTORS AFFECTING THE
HEATER WIRE METHOD OF CALIBRATING
FINE WIRE THERMOCOUPLES

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ABSTRACT

An analytical investigation was made of a transient method
of calibrating fine wire thermocouples. The system consisted
of a 10 mil dia. standard thermocouple (Pt, Pt-13% Rh) and an
0.8 mil dia. chromel-alumel thermocouple attached to a 20 mil dia.
electrically heated platinum wire. The calibration procedure
consisted of electrically heating the wire to approximately $2500^{\circ}F$
within about a seven-second period in an environment approximating
atmospheric conditions at 120,000 feet. Rapid periodic readout of
the standard and fine wire thermocouple signals permitted a
comparison of the two temperature indications. An analysis was
performed which indicated that the temperature distortion at the
heater wire produced by the thermocouple junctions appears to be
of negligible magnitude. Consequently, the calibration technique
appears to be basically sound, although several practical changes
which appear desirable are presented and discussed. It appears
that some additonal investigation is warranted to evaluate
radiation effects and transient response characteristics.

INTRODUCTION

The most common method of measuring the temperature of a wall is to attach a small beaded thermocouple to the surface. In the present application it was desired to make such surfaces temperature measurements with very fine wire thermocouples, on the order of 0.001" diameter. The actual application contemplates the measurement of the transient surface temperatures of re-entry spacecraft, such that the temperatures experienced range up to $2500^{O}F$, while the environmental conditions of pressure and gas compositions are those of the atmosphere at re-entry at about 120,000 feet.

In order to insure the accuracy of thermal sensors utilized by the re-entry spacecraft vehicles it was desired to investigate the performance characteristics and accuracy of the fine wire thermocouple to be used. To accomplish this a test procedure was devised in which a 1-mil fine wire chromel-alumel thermocouple, together with a 10-mil platinum-platinum-13% rhodium reference thermocouple, was attached to the center of a 20-mil platinum-13% rhodium electrically heated wire. The two fine wire thermocouple leads and one of the leads of the reference thermocouple formed a single junction as shown in Figure 1. The test assembly was located within a vacuum chamber that permitted operation under vacuum conditions in various gaseous environments, as indicated in Figure 2.

The calibration procedure consisted of rapidly heating the central wire to about $2500^{O}F$ within about a seven-second period

and comparing the output of the two thermocouples. A transient testing procedure was considered essential since extended periods of operation at elevated temperature in an oxidizing atmosphere result in deterioration and damage of the thermocouple materials. Because a transient technique of calibration was required it became imperative to perform an evaluation of the validity and accuracy of the results obtainable from such a technique. Specifically, the goals were to:

1. Perform an analytical investigation of the heat transfer behavior and characteristics of the thermocouple heater wire junction.

2. Estimate the errors to be expected.

3. Present recommendations for improving the accuracy of the technique.

The following sections will describe the efforts made toward the preceding goals.

APPARATUS AND PROCEDURE

The essential features of the transient calibration technique were presented in the previous section. Only additional supplementary information will be presented here.

The thermocouple junction and associated apparatus are presented in Figures 1 and 2. The test procedure consisted simply of applying a pre-set current to the heater wire and obtaining simultaneous readings of the standard and test thermocouples at reading rates up to 40 measurements per second over a seven-second interval.

ANALYSIS

One of the first elements to consider in investigating the accuracy of any temperature measurement is the possible disturbance of the temperature field being measured produced by the measuring device itself. A second factor to consider here, since the calibration procedure is transient in character, is the response of the thermocouple to the rapid temperature variation experienced by the heater wire.

Concerning the former factor, the possibility exists that the temperature field in the heater wire may be distorted appreciably in the region of the standard and fine-wire thermocouple junctions. Such a complex system of intersecting cylinders does not appear to lend itself readily to analytic solution. Instead, it appears

initially that the governing partial differential equations, together with their initial- and boundary-conditions, which could yield temperature-time-space data for the junction, is most amenable to solution by finite-difference approximations and numerical techniques. Initial efforts indicated that obtaining solutions by numerical techniques involved considerable computational difficulties (which are not insurmountable if time is not a factor). These efforts are described in Appendix I. It was decided instead to obtain closed form analytical solutions of the temperature field for a simplified junction approximating the actual one.

The simplified junction considered was that of a 0.010" dia. platinum wire attached to the 0.020" dia. heater wire. Since the cross-sectional area of the reference junction leads are over 100 times larger than the fine wire thermocouple leads, it was believed that disturbance of the temperature field due to the reference lead would be much larger than that due to the test lead. Consequently, the presence of the test thermocouple leads was not considered initially for purposes of analytical tractability.

The system considered, then, is illustrated in Figure 3. The governing equations for the two domain systems are:

Heater Wire:

$$\frac{1}{\alpha_2} \frac{\partial \theta_2}{\partial \tau} = \frac{\partial \theta_2}{\partial r^2} + \frac{1}{r} \frac{\partial \theta_2}{\partial r} + \frac{u'''}{k_2}$$

(1)

$$I.C. : \quad \theta_2 (r, 0) = 0$$
$$B.C. : \quad \theta_2 (0, \tau) \text{ is FINITE}$$
$$-k_2 \frac{\partial \theta_2}{\partial r}(R, \tau) = k_2 \theta_2$$

Reference Thermocouple:

$$\frac{1}{\alpha_1}\frac{\partial \theta_1}{\partial \tau} = \frac{\partial^2 \theta}{\partial x^2} - m^2 \theta_1 \tag{2}$$

$$I.C. : \quad \theta_1 (x, 0) = 0$$

$$B.C. : \quad \theta_1 (0, \tau) = \theta_2 (R, \tau)$$

$$\theta_1 (\infty, \tau) = 0$$

where

$$\theta = T - T_\infty$$

$$m = \sqrt{\frac{hP}{kA}}$$

With reference to equations (1) and (2) several observations should be made. Heat flow in the heater wire is assumed to be one-dimensional in the radial direction. This is a good assumption for wires of large L/d. Heat flow in the reference thermocouple is also assumed one-dimensional in the axial direction. Since the thermocouple has only one-fourth the cross-sectional area as the heater wire and behaves as a fin attached to the heater wire this assumption is believed justified. The implicit assumption is made that the thermocouple heater wire junction is flat, which is a reasonable approximation due to the relative sizes of the wires.

The solutions to equations (1) and (2) are developed in Appendix II and are:

Heater Wire Surface Temperature:

$$T_R = T_\infty + \frac{u''' \alpha \tau}{k} + \frac{u''' R}{2k} - \frac{2 u''' Bi'}{k} \sum_{m=1}^{\infty} \frac{e^{-\alpha \lambda_m^2 \tau}}{\lambda_m^2 [Bi'^2 + \lambda_m^2 R^2]} \tag{3}$$

or in dimensionless form,

$$\frac{T_R - T_\infty}{\frac{u'''R^2}{k}} = F_0 + \frac{1}{2Bi} - 2Bi \sum_{n=1}^{\infty} \frac{e^{-\alpha \lambda_n^2 \tau}}{(\lambda_n R)^2 [Bi^2 + (\lambda_n R)^2]} \qquad (4)$$

Reference Thermocouple Temperature:

$$\frac{T - T_\infty}{T_R - T_\infty} = e^{-m x} \qquad (5)$$

The solutions (3) and (5) were evaluated at 0.5 second intervals for a seven-second period and are listed in Figure 4. For a given instant of time the heater wire surface temperature served as one of the boundary conditions for the thermocouple (i.e., $T_1(0,\tau) = T_2(R,\tau)$). The temperature distribution in the thermocouple was evaluated from (5), which actually describes the steady-state distribution in a fin. Consequently, this procedure contains an implied assumption that the temperature throughout the fin reacts instantaneously to changes in the heater wire surface temperature.

It is conceivable, however, that under actual test conditions thermocouple conduction, convection and radiation losses may drain sufficient energy from the heater wire so as to result in a lower base temperature than calculated from (4) and (5). Practically speaking, the temperature measured by the thermocouple would be lower than the undisturbed surface temperature of the heater wire. In order to estimate the possible difference between the theoretically determined temperature and the actual temperature

the following precedure was followed. As noted in Figure 4, the temperature experienced by the thermocouple node in contact with the heater wire after 3.5 seconds (or midway through the test) was 1317°F. Using this temperature and the steady-state temperature gradient in the thermocouple at that time, the convective, conductive and radiation heat losses from the node were evaluated, in addition to the stored energy within the node. The effect of these heat losses upon the temperature distribution in the thermocouple lead was then determined for a seven-second test period. The modified temperature distribution in the test for a seven-second period are listed in Figure 5. Comparing the temperature distributions of Figures 4 and 5, it is seen that modification for conduction and radiation losses produce very little effect on the junction node temperature--a very important occurrence that will be discussed more thoroughly in a later section.

Since the temperature experienced in these calibration tests range from 70 to 2500°F, accurate solutions to equations (1) and (2) require accurate thermophysical properties data and heat transfer coefficients over the entire temperature range. The thermophysical properties employed are presented in Appendix III. In general, the temperature dependence of all thermophysical properties was taken into account in obtaining transient temperature histories from equations (1) and (2).

The free convection heat transfer coefficient was initially evaluated from correlations for laminar and turbulent free convection from horizontal and vertical cylinders (Equations 7-18, 7-21, 7-29, 7-30 from [1])for the heater wire and thermocouple respectively. The coefficients so evaluated are not accurate, however, for the small wire sizes and high temperatures experienced here. A more accurate correlation is that of McAdams [2], in which data are well correlated over a range of Grashof numbers from 10^{-5} to 10^7 for fluids as different as air and glycerin, for cylinders as small as 0.003 cm in diameter. Further information regarding convective heat transfer coefficients is presented in Appendix III.

DISCUSSION

Temperature Distributions in Junction Region

As indicated earlier, the analytically determined junction node temperature of the 0.020" reference wire is influenced very little by the average heat losses occurring at a temperature of about $1300^\circ F$ for the entire seven-second test period. This occurrence appears to imply that the temperature within the heater wire in the junction region is not changed appreciably by the presence of the reference thermocouple. The results of Figure 5 indicate that the conduction, convection and radiation losses from the reference junction node produce a temperature change of

only a small fraction of a degree over a seven-second period.
Viewed in a different perspective, the energy generated within
the heater wire within a 3.5 second period is of such a magnitude
as to raise the temperature of the thermocouple approximately
1245^OF. In comparison the temperature change occurring as a
result of the maximum heat losses within the reference junction
node acting over twice that length of time produces only a very
small temperature change. Consequently, the energy "drained"
from the heater wire by the reference thermocouple is of such
comparatively small magnitude that the temperature distribution
within the heater wire is not disturbed significantly by the
presence of the reference thermocouple leads. If this is true,
then the disturbance produced by the pressure of the fine-wire
thermocouple leads is even less significant, since they offer a
conduction path less than 1/100 times as large of that of the
reference beads. Other possible sources of error, however, must
be evaluated, one of which is the error associated with the precise
nature of attachment to the heater surface.

Method of Attachment

The temperature gradient in the attached thermocouples, as
exemplified by the temperatures tabulated in Figure 4, is quite
steep close to the heater wire. This occurrence is related to
another type of error that may exist in thermocouple measurements,
namely that associated with the presence of a third material

forming the thermocouple bead or providing the means of attachment to a surface. Generally the possible existence of a temperature difference across a thermocouple bead or substrate material is neglected in theoretical models of thermocouple systems. That is, most analyses deal with the case where the thermocouple bead is very small. Such simplifications are not always justified, as indicated from several investigations. Suh and Tsai [3], for example, found that in a typical measurement of a burning solid propellant the temperature difference across the thermocouple bead is of the order of 70 to 100°C, for a thermocouple of ½ mil diameter.

Errors associated with thermocouple response characteristics also may be influenced by the presence of substrate or bead material. Such errors are discussed in [4], where tests were conducted on thermocouples surrounded with silver epoxy, plain epoxy, no epoxy, solder and glue. Not surprisingly, various response characteristics were observed. Perhaps of even greater importance relative to the present investigation is the following statement by Parker [4]: "In addition to the experimental work, several analytical heat transfer solutions were made with the use of a computer and compared with the experimental data. In general, there was poor agreement because it was extremely difficult to determine very small thicknesses between the thermocouple and substrate accurately."

In the calibration technique investigated here, very high temperature levels and large temperature gradients near the junction region are experienced, in addition to rapid temporal variations. Consequently the preceding comments are quite relevant. In general, any uncertainty in the precise location of the thermocouple junction with respect to the heater wire surface, whether produced by the presence of substrate material or a welding process, may result in significant error. If a bead is formed at the junction, obviously the uncertainty associated with response characteristics may be cause for considerable error.

Nature of Thermocouple Junction

The most common type of thermocouple junction employed is the bead junction, wherein the temperature indicated is a function of the temperatures where the wires leave the bead. Practically, this translates into making the bead small and flattened, being in intimate contact with the surface whose temperature is being measured.

In the present application the separated open junction is employed [5]. That is, each wire is separately joined to the surface, which must be an electrical conductor. This type of junction, which is actually two series junctions, delivers an output that is a weighted mean value of the two individual junction temperatures:

$$\mathcal{E}_o = e_m + (\mathcal{k}' - \mathcal{k}'') \frac{(T' - T'')}{2} \tag{6}$$

From equation (6), the difference between the output and mean emf may be minimized by reducing the temperature difference, T'-T". This type of junction has been shown to be more accurate than the bead junction if this condition is adequately fulfilled [6,7]. In the present case, due to symmetry, the reference leads should be at essentially the same temperature, if they are truly surface welded and no substrate material (or differences in substrate material) is present at the junctions.

It appears, however, that placement of the fine wire junctions is not established as precisely. In particular, it appears quite possible that one junction may be totally in contact with the heater wire surface immediately adjacent to the reference junction, while the second may be in partial contact with the reference thermocouple wire. Thus, since T' ≠ T" in both cases, the indicated temperatures of the two thermocouples would always differ, even if all other errors were either eliminated or negligible. A more judicious placement of the fine wire thermocouple leads appears warranted.

Response Characteristics of Thermocouples

Both analytical and experimental techniques have been developed in the literature to describe the transient response characteristics of thermocouples, [8,9]. In the present application the accuracy of the calibration technique appears strongly dependent upon the comparative response characteristics and may be of greater relevance than errors due to conduction and radiation losses per se. In the present calibration procedure the possibility exists that the test thermocouple responds more quickly to the heater wire surface temperature than the relatively more massive reference thermocouple. Thus, though a difference between the indicated temperatures of the two thermocouples may be observed, it is conceivable that the temperature of the fine wire thermocouple may be more accurate than that of the reference thermocouple which was initially intended to provide the "true" surface temperature.

Generally, Laplace transform techniques are used to analyze the response characteristics of thermocouples. The indicated temperature and actual temperature are related by the transfer function as

$$G_T(s) = \frac{\bar{T}_M}{\bar{T}_A} \tag{7}$$

where $G_T(s)$ = thermocouple transfer function

$\bar{T}_M$ = Laplace transform of the measured temperature

$\bar{T}_A$ = Laplace transform of the actual or calculated temperature

For a thermocouple welded to the external surface of an electrically heated test-section $G_T(s)$ is of the form [9]

$$G_T(s) = \frac{a}{s + c} \tag{8}$$

where a and c depend upon the thermocouple installation and may differ for each thermocouple. It is possible to experimentally determine these values for particular thermocouples by imposing a known temperature change (ramp change, for example) upon the test section; from the known values of the indicated and actual (calculated) temperatures, together with their Laplace transforms, it is possible to calculate the values of a and c in (2), thus determining $G_T(s)$. Once the transfer function has been established, it becomes possible to calculate the true or actual temperatures from the indicated or measured thermocouple temperatures from the relation

$$\overline{T}_M = \frac{\overline{T}_A}{G_T(s)} \tag{9}$$

The present investigation has been concerned primarily with the transient temperature distributions in the heater wire and thermocouple junction region, and the related inaccuracies due to conduction and radiation losses. It is this writer's judgment that additional time should be devoted to investigation of the response characteristics of the thermocouple assembly before the accuracy of the calibration technique may be precisely and quantitatively evaluated.

Radiation Effects

If radiation heat <u>losses</u> from the reference thermocouple and heater wire are taken into consideration, equations (1) and (2) are modified as follows

Heater Wire:

$$\frac{\partial^2 \theta_2}{\partial \lambda^2} + \frac{1}{\lambda} \frac{\partial \theta_2}{\partial \lambda} + \frac{u'''}{k_2} = \frac{1}{\alpha_2} \frac{\partial \theta_2}{\partial \tau} \tag{10}$$

$$I.C. : \quad \theta_2 (\eta, 0) = 0$$
$$B.C. : \quad \theta_2 (0, \tau) \text{ is finite}$$
$$or \quad \frac{\partial \theta_2 (0, \tau)}{\partial \lambda} = 0$$

$$-k_2 \frac{\partial \theta_2 (R, \tau)}{\partial \lambda} = k_2 \theta_2 (R, \tau) + \epsilon \sigma \theta_2^4$$

Thermocouple:

$$\frac{\partial^2 \theta_1}{\partial x^2} - m^2 \theta_1 - \frac{\epsilon P}{k_1 A_1} \sigma \theta_1^4 = \frac{1}{\alpha_1} \frac{\partial \theta_1}{\partial \tau} \tag{11}$$

$$I.C. : \quad \theta_1 (x, 0) = 0$$
$$B.C. : \quad \theta_1 (0, \tau) = \theta_2 (R, \tau)$$
$$\theta_1 (\infty, \tau) = 0$$

It is seen that equations (10) and (11) are both non-linear, (10) by virtue of the radiation boundary condition, and (11) by virtue of the non-linear radiation term in the governing equation itself. Solution of this system of non-linear partial differential equations is a considerable level of difficulty higher than that required in the solution of (1) and (2), and must be done numerically.

Even if (10) and (11) were to be solved, however, a factor which is not taken into account by these equations is the radiation exchange between the heater wire and thermocouple lead(s). Inclusion of this closer approximation to reality involves an additional considerable increase in level of difficulty in solving the governing system of equations. In view of these inadequacies of analytical techniques it appears simpler, and advisable in this writer's opinion to evaluate radiation effects by suitably devised experiments.

Deterioration of Thermocouple Performance at High Temperatures

Initial attempts to calibrate fine wire thermocouples at temperatures above $1220^{\circ}F$ proved difficult since a period of about three minutes was required to obtain each set of temperature measurements. The oxidation occurring at these high temperatures produced deterioration of the thermocouples, thereby producing the need for the rapid transient calibration procedures being investigated here. Although the time at which the thermocouples are at high temperatures is short, the possible influence of surface chemical reactions upon thermocouple accuracy should be considered.

Reference [10] discusses at some length catalytic activity in platinum group temperature sensors in connection with possible measurement errors. As in several other investigations the authors proposed the use of non-catalytic coatings on thermocouples

for most reacting gas temperature measurements.   It appears
advisable in the present transient calibration procedure to
also employ such coatings. In addition to preventing or
minimizing chemical decomposition of the thermocouple wires,
the radiation heat interactions between the heater wire,
thermocouple wires and environment, which are quite difficult
to accurately determine analytically, would be greatly reduced
due to the lowered surface temperature of the coating. The
convective heat losses from the thermocouple leads would also be
somewhat reduced for the same reason. Addition of coating
material, however, could affect the relative response character-
istics of the two thermocouples. The ultimate effect upon the
accuracy of the calibration technique produced by the addition
of coatings appears to be primarily dependent upon the two
phenonmena--reduced radiation heat transfer losses and transient
response characteristics (although conduction losses and
convective heat transfer with the environment must also be
considered). Again, the need for investigation of transient
response characteristics is indicated. Perhaps it would be
possible to sufficiently minimize radiation losses from both
thermocouples with a minimum coating thickness; further addition of
coating material to the fine wire thermocouple might then be
made in such an amount as to match the response characteristics
of both thermocouples, resulting in an accurate calibration
technique over the entire range of temperatures.

As to the particular type(s) of coating that should be employed, several substances have been employed as high temperature coatings in the recent literature, among which are quartz, silica, tetraethyl lead, molten borax, magnesia, oxidized chromium (Cr_2Cl_3), NBS Ceramic Coating A-418, and others [11]. The particular coating employed must of course be capable of withstanding the temperature range to be experienced, should have a low thermal conductivity to (a) minimize conduction losses, (b) to decrease the surface temperature and (c) to minimize aging effects caused by adsorption and desorption processes and exposure to a variety of atmospheres, and should of course be readily depositable upon the thermocouples. A more extensive discussion of thermocouple coatings and a compilation of pertinent literature may be found in [10].

CONCLUSIONS

Although a detailed mapping of the temperature-time-space field within the heater wire thermocouple junction region has not been presented herein the analysis performed indicated that the temperature depression caused by the reference thermocouple leads being attached to the heater wire is negligibly small throughout the test. Consequently, the test thermocouple leads, having a cross-sectional area 1/100 times as large as the reference thermocouple leads, will have essentially no effect upon lowering the heater wire temperature.

Errors associated with the presence of substrate or bead material may be quite significant, due to the existence of a large temperature difference across the material or due to changes in response characteristics. Uncertainties in the placement of the fine wire junctions may also result.in error, indicating a more judicious placement of these junctions appears warranted.

Errors due to a difference in response characteristics of the reference and test thermocouples may be more significant than that due to disturbance of the heater wire temperature produced by the presence (attachment) of the thermocouple wires to it. It is possible to experimentally determine the transfer function of each thermocouple. Additional experimental and theoretical investigation of the comparative transient response characteristics of the thermocouples also appears warranted.

The addition of high temperature chemically inert coating materials to the thermocouples appears desirable principally in order to minimize radiation heat transfer interactions at high temperature levels and to minimize aging effects produced by exposure of bare thermocouple wires to various atmospheres.

RECOMMENDATIONS

For reasons presented herein it is recommended that the fine wire junctions not be placed in the vicinity of the reference

junction(s). Since the reference thermocouple apparently disturbs the heater wire temperature very little, the fine wire leads need not be far removed. A distance of one inch should be more than ample. Both leads in each system should be welded to the surface diametrically opposite each other. Since the heater wire is seven inches in length it appears that conduction losses to the terminals should not influence the wire temperature near its center, where the two thermocouple systems will be located (more precisely, each thermocouple system will be located 2½ inches from the voltage terminals). The temperature depression in the heater wire produced by conduction losses to the voltage terminals may be easily calculated using expressions found in [12,13,14].

Both sets of thermocouple wires should be coated with chemically inert high temperature coating materials to minimize radiation heat losses and to minimize deterioration and aging effects of the thermocouple wires. Addition of such materials, however, influences the transient response of thermocouple systems. Whether or not the thermocouples are coated the comparative transient response characteristics of the thermocouple systems should and must be analyzed before a quantitative evaluation of the transient calibration technique may be realized. In fact, the choice of a particular coating material may be a strong function of its influence upon the transient response character- istics of the thermocouple systems.

The preceding recommendations have been analytically based or oriented. However, it appears essential to this writer that

an experimental program planned co-currently with the previously
recommended activities be conducted. A wealth of information
should be obtainable from a modest experimental program. For
example, the transient response characteristics of the thermocouple
systems with different coatings and coating thicknesses, indicated
by analyses, could be experimentally evaluated.

Likewise, the radiation heat transfer characteristics, which
are rather difficult to establish quantitatively and accurately
(due to mathematical complexity and lack of radiation properties
data), could also be investigated experimentally. In addition, the
comparative effects of radiation and convection losses could be
established by performing tests under varying degrees of vacuum.
Also, aging and deterioration effects may be possible by judicious
testing in various gaseous enviornments at varying degrees of
vacuum.
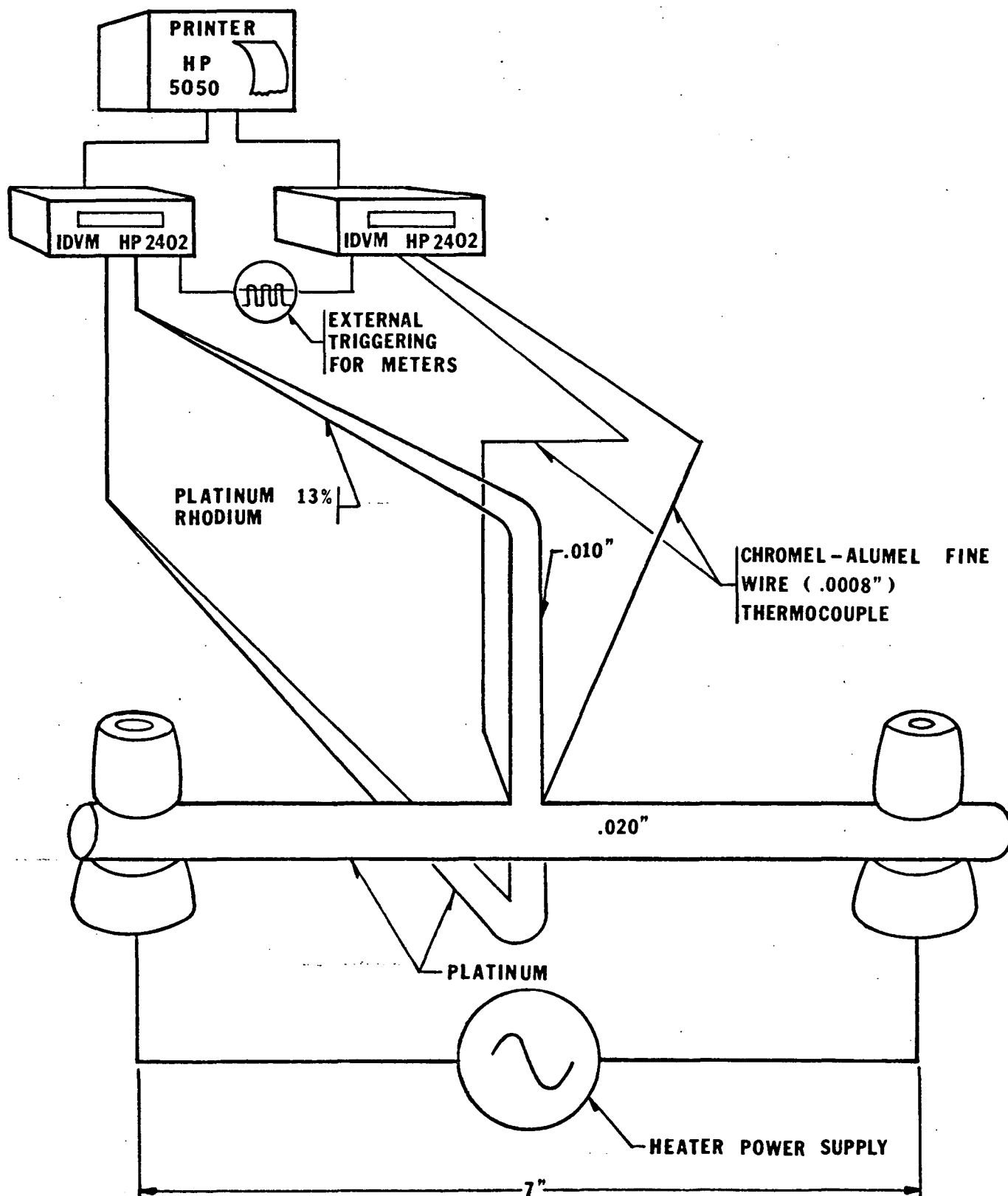
Figure 1.   Schematic of heater wire-thermocouple
junction and instrumentation.

TWO CHANNEL RECORDER

HEATER WIRE

VACUUM CHAMBER

VACUUM PUMP

HEATER POWER SUPPLY
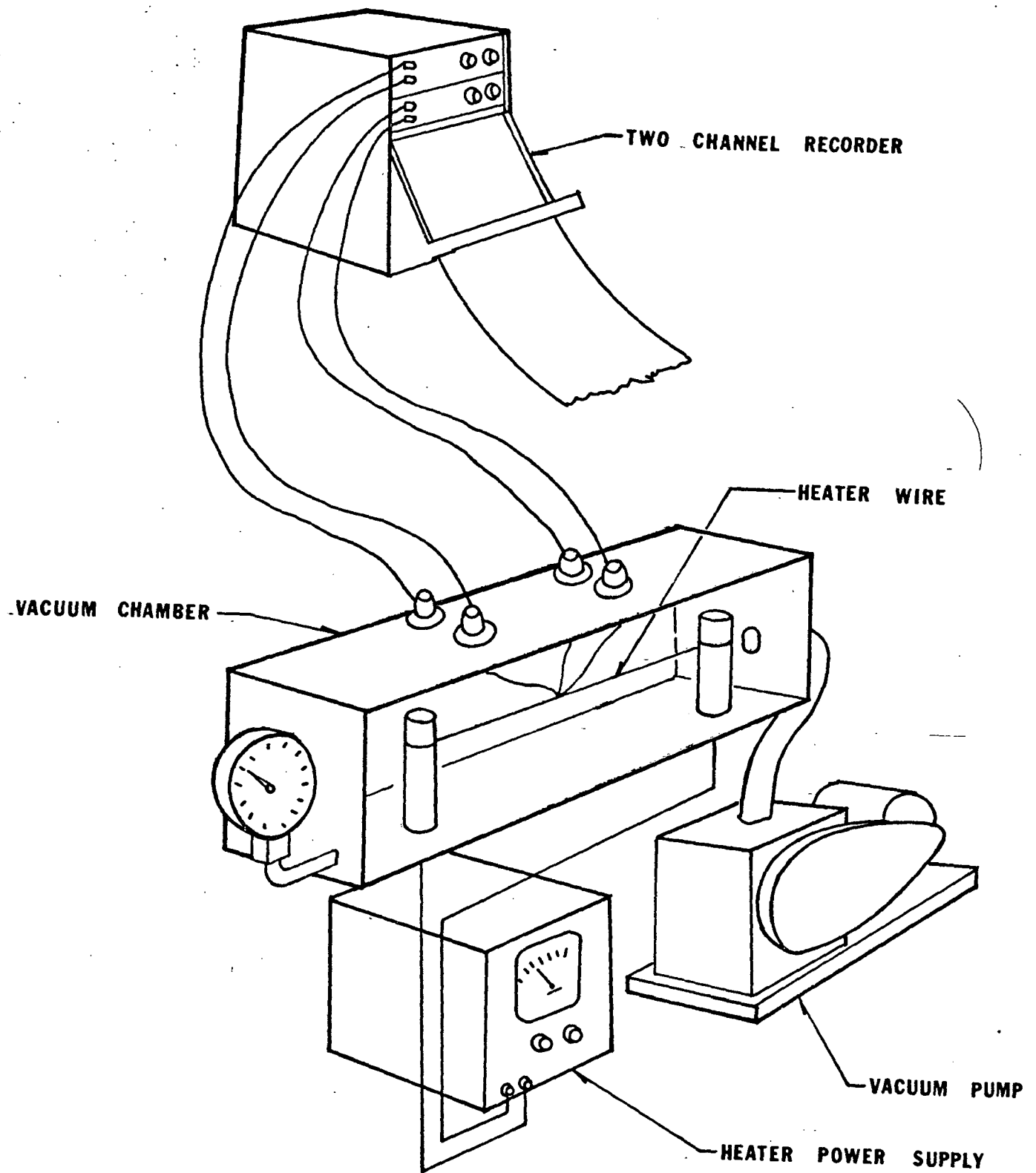
Figure 2. Schematic of experimental apparatus.

PT-13% RHODIUM THERMOCOUPLE
LEAD (0.010" DIA.)

PLATINUM   HEATER   WIRE   (0.020" DIA.)
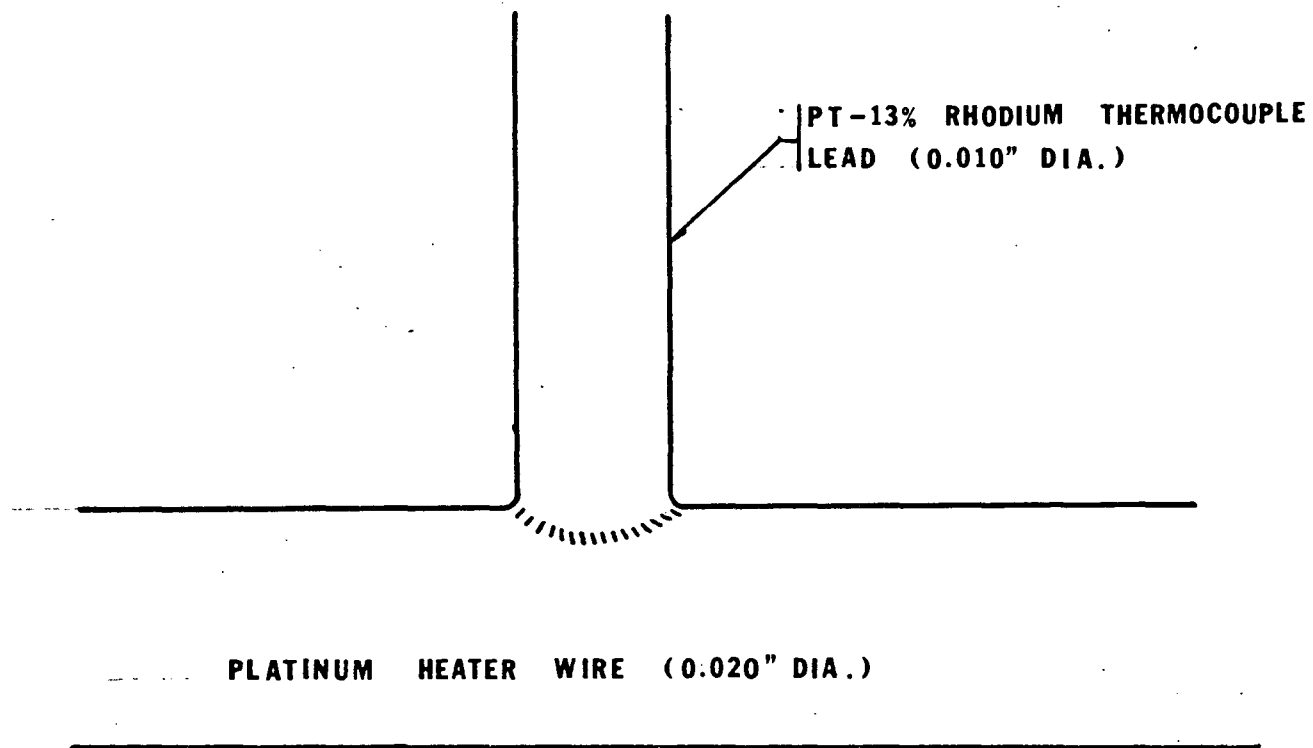
Figure 3. Simplified analytical model of heater
           wire-thermocouple junction.

TIME IS =   0.00

0.6999989E 02
0.6999995E 02
0.6999998E 02
0.7000000E 02
0.7000000E 02
0.7000000E 02

0.7000000E 02
0.7000000E 02
0.7000000E 02
0.7000000E 02
0.7000001E 02
0.7000001E 02
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0.7000001E 02
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0.7000001E 02
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0.7000001E 02
0.7000001E 02
0.7000001E 02

TIME IS =   1.00

0.4266911E 03
0.2235260E 03
0.1360802E 03
0.9844210E 02
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0.7526916E 02
0.7226794E 02
0.7097616E 02
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0.7027127E 02
0.7011676E 02
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0.7000932E 02
0.7000401E 02
0.7000173E 02
0.7000074E 02
0.7000032E 02
0.7000015E 02
0.7000006E 02
0.7000003E 02
0.7000001E 02
0.7000001E 02
0.7000001E 02
0.7000001E 02

Figure 4.   Temperature distribution in thermocouple wire for
             seven-second heating period.

```
TIME IS =        2.00        TIME IS =        3.00        TIME IS =        4.00
0.7833804E 03                0.1140069E 04                0.1496759E 04
0.3770513E 03                0.5305766E 03                0.6841019E 03
0.2221602E 03                0.2682402E 03                0.3343201E 03
0.1268840E 03                0.1553260E 03                0.1837680E 03
0.9448391E 02                0.1067258E 03                0.1189677E 03
0.8053831E 02                0.8580744E 02                0.9107658E 02
0.7453587E 02                0.7680380E 02                0.7907173E 02
0.7195233E 02                0.7292848E 02                0.7390463E 02
0.7094031E 02                0.7126048E 02                0.7168063E 02
0.7036169E 02                0.7054254E 02                0.7072337E 02
0.7015568E 02                0.7023352E 02                0.7031135E 02
0.7006701E 02                0.7010050E 02                0.7013401E 02
0.7002885E 02                0.7004327E 02                0.7005769E 02
0.7001242E 02                0.7001863E 02                0.7002484E 02
0.7000535E 02                0.7000802E 02                0.7001069E 02
0.7000230E 02                0.7000346E 02                0.7000460E 02
0.7000099E 02                0.7000149E 02                0.7000198E 02
0.7000042E 02                0.7000064E 02                0.7000085E 02
0.7000019E 02                0.7000029E 02                0.7000038E 02
0.7000009E 02                0.7000012E 02                0.7000016E 02
0.7000004E 02                0.7000006E 02                0.7000007E 02
0.7000001E 02                0.7000003E 02                0.7000003E 02
0.7000001E 02                0.7000001E 02                0.7000001E 02
0.7000001E 02                0.7000001E 02                0.7000001E 02
0.7000001E 02                                             0.7000001E 02


TIME IS =        2.50        TIME IS =        3.50        TIME IS =        4.50
0.9617252E 03                0.1318414E 04                0.1675104E 04
0.4538140E 03                0.6073393E 03                0.7608646E 03
0.2352002E 03                0.3012802E 03                0.3673601E 03
0.1411050E 03                0.1695470E 03                0.1979890E 03
0.1006048E 03                0.1128468E 03                0.1250887E 03
0.8317288E 02                0.8844201E 02                0.9371111E 02
0.7566984E 02                0.7793777E 02                0.8020570E 02
0.7244039E 02                0.7341656E 02                0.7439271E 02
0.7105039E 02                0.7147055E 02                0.7189071E 02
0.7045211E 02                0.7063295E 02                0.7081379E 02
0.7019459E 02                0.7027244E 02                0.7035028E 02
0.7008377E 02                0.7011726E 02                0.7015077E 02
0.7003605E 02                0.7005047E 02                0.7006489E 02
0.7001551E 02                0.7002172E 02                0.7002793E 02
0.7000668E 02                0.7000935E 02                0.7001202E 02
0.7000288E 02                0.7000402E 02                0.7000513E 02
0.7000125E 02                0.7000173E 02                0.7000222E 02
0.7000053E 02                0.7000074E 02                0.7000096E 02
0.7000024E 02                0.7000033E 02                0.7000042E 02
0.7000010E 02                0.7000015E 02                0.7000018E 02
0.7000004E 02                0.7000006E 02                0.7000009E 02
0.7000003E 02                0.7000003E 02                0.7000004E 02
0.7000001E 02                0.7000001E 02                0.7000001E 02
0.7000001E 02                0.7000001E 02                0.7000001E 02
                             0.7000001E 02                0.7000001E 02
```

Figure 4. Continued.

TIME IS =   5.00

```
0.1853448E 04
0.8376273E 03
0.4004002E 03
0.2122100E 03
0.1312096E 03
0.9634573E 02
0.8133967E 02
0.7488079E 02
0.7210078E 02
0.7090422E 02
0.7038919E 02
0.7016752E 02
0.7007211E 02
0.7003103E 02
0.7001336E 02
0.7000575E 02
0.7000248E 02
0.7000106E 02
0.7000047E 02
0.7000019E 02
0.7000009E 02
0.7000004E 02
0.7000003E 02
0.7000001E 02
0.7000001E 02
```

TIME IS =   6.00

```
0.2210138E 04
0.9911525E 03
0.4664801E 03
0.2406519E 03
0.1434519E 03
0.1016148E 03
0.8360760E 02
0.7585694E 02
0.7252093E 02
0.7108506E 02
0.7046704E 02
0.7020101E 02
0.7008653E 02
0.7003724E 02
0.7001603E 02
0.7000691E 02
0.7000297E 02
0.7000128E 02
0.7000055E 02
0.7000024E 02
0.7000010E 02
0.7000004E 02
```

TIME IS =   7.00

```
0.2566827E 04
0.1144677E 04
0.5325601E 03
0.2690939E 03
0.1556935E 03
0.1068940E 03
0.8587553E 02
0.7683311E 02
0.7294110E 02
0.7125589E 02
0.7054487E 02
0.7023452E 02
0.7010095E 02
0.7004345E 02
0.7001870E 02
0.7000805E 02
0.7000347E 02
0.7000149E 02
0.7000065E 02
0.7000029E 02
0.7000012E 02
0.7000006E 02
0.7000004E 02
0.7000001E 02
0.7000001E 02
```

TIME IS =   5.50

```
0.2031793E 04
0.9142900E 03
0.4334401E 03
0.2264310E 03
0.1373306E 03
0.9898030E 02
0.8247363E 02
0.7536894E 02
0.7231086E 02
0.7099464E 02
0.7042811E 02
0.7018428E 02
0.7007931E 02
0.7003414E 02
0.7001469E 02
0.7000633E 02
0.7000273E 02
0.7000117E 02
0.7000051E 02
0.7000022E 02
0.7000010E 02
0.7000004E 02
0.7000003E 02
0.7000001E 02
0.7000001E 02
0.7000001E 02
```

TIME IS =   6.50

```
0.2388482E 04
0.1067915E 04
0.4995200E 03
0.2548729E 03
0.1495725E 03
0.1042494E 03
0.8474156E 02
0.7634503E 02
0.7273101E 02
0.7117547E 02
0.7050535E 02
0.7021777E 02
0.7009373E 02
0.7004034E 02
0.7001737E 02
0.7000747E 02
0.7000321E 02
0.7000138E 02
0.7000061E 02
0.7000025E 02
0.7000012E 02
0.7000006E 02
0.7000003E 02
0.7000001E 02
0.7000001E 02
```

Figure 4.   Continued.

```
0.1317325E 04        0.1317319E 04
0.17341475 03        0.1734195E 03
0.7857620E 02        0.7857415E 02
0.7071080E 02        0.7071089E 02
0.7005894E 02        0.7005894E 02                    0.1317319E 04
0.7000489E 02        0.7000489E 02                    0.1734147E 03
0.7000041E 02        0.7000041E 02                    0.7857412E 02
0.7000004E 02        0.7000004E 02                    0.7071089E 02
0.7000001E 02        0.7000001E 02                    0.7005894E 02
0.7000001E 02        0.7000001E 02                    0.7000489E 02
0.7000001E 02        0.7000001E 02                    0.7000041E 02
0.7000001E 02        0.7000001E 02                    0.7000004E 02
0.7000001E 02        0.7000001E 02                    0.7000001E 02
0.7000001E 02        0.7000001E 02                    0.7000001E 02
                                                      0.7000001E 02
                                                      0.7000001E 02
                                                      0.7000001E 02
0.1317929E 04        0.1217317E 04                    0.7000001E 02
0.1734195E 03        0.1734150E 03                    0.7000001E 02
0.7857410E 02        0.7857414E 02
0.7071059E 02        0.7071089E 02
0.7005894E 02        0.7005894E 02
0.7000489E 02        0.7000489E 02
0.7000041E 02        0.7000041E 02
0.7000004E 02        0.7000004E 02
0.7000001E 02        0.7000001E 02
0.7000001E 02        0.7000001E 02                    0.1317312E 04
0.7000001E 02        0.7000001E 02                    0.1734145E 03
0.7000001E 02        0.7000001E 02                    0.7857411E 02
0.7000001E 02        0.7000001E 02                    0.7071089E 02
0.7000001E 02        0.7000001E 02                    0.7005894E 02
0.7000001E 02        0.7000001E 02                    0.7000489E 02
                                                      0.7000041E 02
                                                      0.7000004E 02
                                                      0.7000001E 02
0.1317321E 04        0.1317315E 04                    0.7000001E 02
0.17341535 03        0.1734148E 03                    0.7000001E 02
0.7857417E 02        0.7857412E 02                    0.7000001E 02
0.7071089E 02        0.7071089E 02                    0.7000001E 02
0.7005894E 02        0.7005894E 02                    0.7000001E 02
0.7000489E 02        0.7000489E 02                    0.7000001E 02
0.7000041E 02        0.7000041E 02                    0.7000001E 02
0.7000004E 02        0.7000004E 02
0.7000001E 02        0.7000001E 02
0.7000001E 02        0.7000001E 02
0.7000001E 02        0.7000001E 02
0.7000001E 02        0.7000001E 02
0.7000001E 02        0.7000001E 02
0.7000001E 02        0.7000001E 02
```

Figure 5. Modified temperature distribution in thermocouple wire due to conduction and radiation losses.

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NOMENCLATURE

A	cross-sectional area of thermocouple
b', b''	slope of temperature-emf relation for two different materials, i.e. $e = a_o + b T$
Bi	B iot number, $h L/h$
e_m	thermocouple output if both junctions were at the mean temperature, $(T' + T'')/2$
e_o	thermocouple output if junction of wire 1 and surface is at T' and that of wire 2 is at T''
h	convective heat transfer coefficient
k	thermal conductivity
m	$m = (hP/kA)^{\frac{1}{2}}$
P	perimeter of thermocouple cross-section
r	radial coordinate
R	radius of thermocouple
s	Laplace transform variable
T	temperature
T_R	analytically evaluated surface temperature of thermocouple
T_∞	environment temperature
T', T''	temperatures of thermocouple wire-surface junctions in an open junction thermocouple
u'''	internal heat generation rate per unit volume
x	axial coordinate
α	thermal diffusivity
λ_n	eigenvalue
θ	temperature difference, $T-T_\infty$
τ	time

Subscripts

1 reference thermocouple lead wire

2 heater wire

APPENDIX A.

Numerical Solution of Temperature Field of Junction Region

Most of the material presented in this section is of a negative nature. That is, the approaches described here did not lead to the desired temperature-time-space information for the junction regions and are presented here for the sake of completeness.

Initially it appeared that the most direct method of determining the temperature history within the geometrically complex junction region, with non-linear (radiative) boundary conditions, and with temperature dependent thermophysical properties, was by means of numerical techniques. This contention remains valid. However, sufficient practical and even mundane difficulties were encountered in attempting computer solution of the governing equation as to make such solutions unattainable within the time limitations set by the terms of this study. Some of the problems encountered will be described here.

A cursory inspection of the geometry of the junction seems to indicate description of a nodal system in terms of cylindrical coordinates. It quickly becomes apparent that such a system is not workable because of extremely complex "communication" of nodes at the intersection of the thermocouples and heater wire. Consequently, a cartesian coordinate system was chosen in which the circular cross-section of the wires was approximated by a number of small square areas.

A system was chosen that compromised the numerical complexities of a very finely graded network and the approximate temperature distribution obtainable by using a very coarse network. A total of 155 equaions was developed describing the nodal heat balances and the boundary conditions. The equations were written in implicit form. This is necessary since the stability criteria derived considering the small wire sizes involved dictated solution of the 155 equations in time increments of roughly 10^{-5} seconds. For the entire test period of seven seconds, then, a prohibitive amount of computations was required. The implicit method on the other hand is not restricted by the size of the time increment, but does involve the <u>simultaneous</u> solution of the 155 equations.

A computer program using the subroutine SIMEQ from the Langley ACD library was developed to solve the system of equations. Due to inherent inaccuracies associated with the method of solution used in the subroutine it was not possible to obtain accurate nodal point temperature histories. Consequently a Gauss-Seidel iteration scheme, which has proved to be an accurate method of solution for even larger systems of equations, was employed. However, unexpectedly, problems of stability, associated with a parameter appearing in the governing equation, arose. At this point it was believed that time did not permit further investigation of the numerical approach. Thus it was decided to apply analytical techniques to an idealized model of the system to obtain results that would

provide some means for evaluating the accuracy of the transient
calibration technique.

APPENDIX B.

Analytical Solution of Heater Wire and Reference Thermocouple Temperatures

The question which the following analysis attempts to answer is "Does the attachment of the reference thermocouple lead to the electrically heated wire provide a large enough path for heat leakage as to significantly depress the heater wire temperature over what the surface temperature would be without the thermocouple lead attached?" The simplified system, in which the presence and the effects of the fine wire thermocouple leads are neglected, is shown in Figure 3. The governing equations and boundary conditions are repeated here for convenience. It was assumed that the heater wire was infinite in both directions, and thus the temperature gradient in the axial direction was assumed zero. Also one-dimensional heat flow, in the axial direction, was assumed to exist in the smaller diameter reference wire. The governing equations are:

Heater Wire:

$$\frac{1}{\alpha_2} \frac{\partial \theta_2}{\partial \tau} = \frac{\partial \theta_2}{\partial r^2} + \frac{\partial \theta_2}{\partial r} + \frac{u'''}{k_2} \qquad (B\text{-}1)$$

I.C.: $\quad \theta_2(r,0) = 0$ (B-1a)

B.C. $\quad \theta_2(0,\tau)$ is finite or

$$\frac{\partial \theta_2}{\partial r}(0,\tau) = 0 \qquad (B\text{-}1b)$$

$$-k_2 \frac{\partial \theta_2}{\partial r}(R,\tau) = h_2\,\theta_2 \qquad (B\text{-}1c)$$

Reference Thermocouple Lead:

$$\frac{1}{\alpha_1} \frac{\partial \theta_1}{\partial \tau} = \frac{\partial^2 \theta}{\partial x^2} - m^2 \theta_1 \tag{B-2}$$

$$\text{I.C.:} \quad \theta_1 (x,0) = 0 \tag{B-2a}$$

$$\theta_1 (0,\tau) = \theta_2 (R,\tau) \tag{B-2b}$$

$$\theta_1 (\infty,\tau) = 0 \tag{B-2c}$$

To solve equation (B-1) and its boundary conditions the Laplace transform technique is conveniently employed. Taking the Laplace transform of (B-1),

$$\frac{p}{a} \bar{\theta} - \frac{\theta(0)}{a} = \frac{\partial^2 \bar{\theta}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{\theta}}{\partial r} + \frac{u'''}{k\ p} \tag{B-3}$$

where $\quad L(\theta) = \bar{\theta}$

$$L(\frac{\partial \theta}{\partial \tau}) = p\ \bar{\theta} - \theta(0)$$

$$\theta(0) = 0 \qquad \text{from (B-1a)}$$

Letting $P/a = q^2$, and rearranging (B-3),

$$r^2 \frac{d^2 \bar{\theta}}{dr^2} + r \frac{d\bar{\theta}}{dr} - q^2 r^2 \bar{\theta} = \frac{-u'''}{k\ p} r^2 \tag{B-4}$$

The solution to (B-4) may be written as $\bar{\theta} = \bar{\theta}_c + \bar{\theta}_p$.
The complementary solution   is given by

$$\bar{\theta}_c = C_1\ I_0\ (qr) + C_2\ K_0\ (qr) \tag{B-5}$$

From (B-1b), θ_c is finite at $r = 0$, $C_2 = 0$. Thus

$$\bar{\theta}_c = C_1 I_0(qr) \qquad \text{(B-6)}$$

To find the particular solution, let $\theta_p = Ar^2 + Br + C$. Substituting into (B-4), one finds

$$A = 0, \quad B = 0, \quad C = \frac{u'''}{kq^2p} = \frac{a}{kp^2} u''' \qquad \text{(B-7)}$$

Combining the particular and complementary solutions,

$$\bar{\theta} = C_1 I_0(qr) + \frac{a}{kp^2} u''' \qquad \text{(B-8)}$$

From (B-1c)

$$-k \frac{\partial \theta}{\partial r} = h\theta$$

or

$$-k \frac{\partial \bar{\theta}}{\partial r} = h \bar{\theta} \qquad @ \; r = R$$

$$\text{(B-9)}$$

Applying the transformed boundary condition to (B-8), one obtains

$$C_1 = \frac{-\dfrac{h\,a\,u'''}{k\,p^2}}{k\,q\,I_1(qR) + h\,I_0(qR)} \qquad \text{(B-10)}$$

Substituting (B-10) into (B-9) and rearranging,

$$\frac{\bar{\theta}}{\dfrac{u'''a}{k}} = \frac{1}{p^2} - \frac{h\,I_0(qr)}{p^2\left[k\,I_0(qR) + k\,q\,I_1(qR)\right]} \qquad \text{(B-11)}$$

One may now determine θ by taking the inverse Laplace transform of (B-11),

$$\frac{\theta}{\frac{u'''a}{k}} = \tau - \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{e^{\tau z} k I_0[\lambda(z/a)^{1/2}]\,dz}{z^2\left[k I_0\left\{R(\frac{z}{a})^{1/2}\right\} + k(\frac{z}{a})^{1/2} + I_1\left\{(\frac{z}{a})^{1/2}R\right\}\right]} \tag{B-12}$$

The integral of (B-12) is seen to have a second order pole at z = 0 and a simple pole

$$h\,I_0\{(\tfrac{z}{A})^{1/2}R\} + k(\tfrac{z}{A})^{1/2}\,I_1\{(\tfrac{z}{A})^{1/2}R\} = 0 \tag{B-13}$$

Letting $z = -a\lambda_n^2$,, one obtains

$$\frac{hR}{k}\,J_0(\lambda_n R) - \lambda_n R\,J_1(\lambda_n R) = 0 \tag{B-14}$$

which is a multiple value equation solved by many eigenvalues of λ_n.

Now one may find the residues of the integral of (B-12). Thus

$$\text{Res}_{\substack{\text{at } z = 0}} f(z) = \lim_{z \to 0} \frac{d}{dz}\left[\frac{h\,I_0(qr)\,e^{\tau z}}{h\,I_0(qr) + hq\,I_1(qr)}\right] \tag{B-15}$$

where $z = P = q^2 a$.

Now $\dfrac{df}{dz} = \dfrac{df}{dq}\dfrac{dq}{dz}$

and $\dfrac{dq}{dz} = \dfrac{1}{\sqrt{a}} \cdot \dfrac{1}{2\sqrt{z}} = \dfrac{1}{2aq}$

So

$$\operatorname*{Res}_{@\ q=0} f(z) = \lim_{q \to 0} \frac{1}{2aq} \frac{d}{dq} \left[\frac{h I_0(qR) e^{\tau a q^2}}{h I_0(qR) + k q I_1(qR)} \right] \qquad \text{(B-16)}$$

and differentiating,

$$\operatorname*{Res}_{@\ q=0} f(z) = \lim_{q \to 0} \frac{1}{2aq} \left[\{ h I_0(qR) + k q I_1(qR) \} \right.$$
$$\{ h r I_1(qr) e^{\tau a q^2} + 2 q \tau h a I_0(qr) e^{\tau a q^2} \} -$$
$$\left. h I_0(qr) e^{\tau a q^2} \{ R h I_1(qR) + q k R I_0(qR) \} \right] \Big/$$
$$\left[h I_0(qR) + k q I_1(qR) \right]^2 \qquad \text{(B-17)}$$

Now as q → 0, $\qquad I_0(qr) = I_0(qR) = 1$ $\qquad\qquad$ (B-18)
$$I_1(qr) = I_1(qR) = 0$$

Thus

$$\operatorname*{Res}_{q=0} f(z) = \lim_{q \to 0} \frac{1}{2aq} \left[\frac{-h(k q R^2)}{h^2} \right] \qquad \text{(B-19)}$$

$$\operatorname*{Res}_{q=0} f(z) = -\frac{kR}{2a\,h}$$

Now, to find the residue at $z = -a\lambda_n^2$, for simplicity let

$$P = h I_0(qr) e^{-a\lambda_n^2 \tau}$$
$$Q = h I_0(qR) + k q I_1(qR)$$

Then

$$\operatorname*{Res}_{z=-a\lambda_n^2} f(z) = \lim_{z \to -a\lambda_n^2} \frac{P(z)}{P^2 \left[\frac{dQ}{dz}\right]} = \frac{P(-a\lambda_n^2)}{a^2 \lambda_n^4 \left[Q'(-\lambda_n^2) \right]} \qquad \text{(B-20)}$$

$$\operatorname*{Res}_{z=-a\lambda_n^2} f(z) = \frac{h I_0(i\lambda_n r) e^{-\lambda_n^2 \tau}}{a^2 \lambda_n^4 \left[R h I_1(qR) + R k q I_0(qR) \right] \frac{1}{2qa}}$$
$$= \frac{2 h J_0(\lambda_n r) e^{-a\lambda_n^2 \tau}}{a R \lambda_n^3 \left[h J_1(\lambda_n R) + \lambda_n J_0(\lambda_n R) \right]} \qquad \text{(B-21)}$$

But

$$\lambda_n R J_1 (\lambda_n R) = \frac{hR}{k} J_0 (\lambda_n R)$$

and letting

$$Bi = \frac{hR}{k},$$

$$\operatorname{Res}_{z=-a\lambda_n^2} f(z) = \frac{2 Bi J_0 (\lambda_n \Lambda) e^{-a\lambda_n^2 \tau}}{a \lambda_n^2 [Bi^2 + \lambda_n^2 R^2] J_0 (\lambda_n R)} \qquad (B-22)$$

And finally,

$$\frac{\theta}{\frac{u''' a}{k}} = \tau + \frac{k R}{2 a k} - \sum_{n=1}^{\infty} \frac{2 Bi J_0 (\lambda_n \Lambda) e^{-a\lambda_n^2 \tau}}{a \lambda_n^2 [Bi^2 + \lambda_n^2 R^2] J_0 (\lambda_n R)} \qquad (B-23)$$

In terms of temperature, T, at any position within the heater wire,

$$T = T_\infty + \frac{u''' a \tau}{k} + \frac{u''' R}{2 k}$$
$$- \frac{u''' a}{k} \sum_{n=1}^{\infty} \frac{2 Bi J_0 (\lambda_n \Lambda) e^{-a\lambda_n^2 \tau}}{a \lambda_n^2 [Bi^2 + \lambda_n^2 R^2] J_0 (\lambda_n R)} \qquad (B-24)$$

For the surface temperature of the heater wire, i.e. $T_R = T(R)$,

$$T_R = T_\infty + \frac{u''' \tau}{\rho c_p} + \frac{u''' R}{2 k} - \frac{2 u''' Bi}{k} \sum_{n=1}^{\infty} \frac{e^{-\alpha \lambda_n^2 \tau}}{\lambda_n^2 [Bi^2 + \lambda_n^2 R^2]} \qquad (B-25)$$

In dimensionless form,

$$\frac{T_R - T_\infty}{\frac{u''' R^2}{k}} = Fo + \frac{1}{2 Bi} - 2 Bi \sum_{n=1}^{\infty} \frac{e^{-\alpha \lambda_n^2 \tau}}{(\lambda_n R)^2 [Bi^2 + (\lambda_n R)^2]} \qquad (B-26)$$

For the solutions (B-23-26), the values $\lambda_n R$ are zeroes of $J_o(\lambda_n R) = 0$ and are

$$\lambda_1 R = 2.4048$$

$$\lambda_2 R = 5.5201$$

$$\lambda_3 R = 8.6537$$

$$\lambda_4 R = 11.7915$$

$$\lambda_5 R = 14.9309$$

Treating the reference thermocouple lead as an infinitely long fin of circular cross-section, the governing equation and its boundary conditions are:

$$\frac{\partial^2 \theta_1}{\partial x^2} - m^2 \theta_1 = \frac{1}{\alpha} \frac{\partial \theta_1}{\partial \tau} \tag{B-27}$$

where $\theta_1 = T_1 - T_\infty$

I.C.: $\theta_1(x,0) = 0$

$\theta_1(0,\tau) = \theta_2(R,\tau)$

$\theta_1(\infty,\tau) = 0$

Assuming a product solution, $\theta_1 = X(x)\mathcal{T}(\tau)$, one obtains the ordinary differential equations

$$\mathcal{T}' - \gamma^2 \alpha \tau = 0 \tag{B-28}$$

where $\gamma^2 = (\beta^2 - m^2)$, β^2 is a separation constant and

$m^2 = hP/kA$,

and $X'' - m^2 X = 0$ \tag{B-29}

whose solutions are, respectively,

$$\mathcal{T}(\tau) = e^{\alpha \gamma^2 \tau} \tag{B-30}$$

$$\mathcal{X}(x) = Ae^{mx} + Be^{-mx} \tag{B-31}$$

Since θ_1, approaches zero for large values of X, $A = 0$.
Thus,

$$\theta_1 = B e^{-mx} e^{\alpha \gamma^2 \tau} = B e^{\alpha \gamma^2 \tau - mx} \tag{B-32}$$

Now at $x = 0$, $\theta = \theta_0$, so that

$$\theta_0 = Be^{\alpha \gamma^2 \tau} \tag{B-33}$$

But θ_0 is also given by (B-23) evaluated at $r = R$, i.e. $\theta_2(R)$,
or

$$\theta_0(0,\tau) = Be^{\alpha \gamma^2 \tau} = \theta_2(R,\tau)$$

Substituting into (B-32),

$$\theta_1(x,\tau) = \theta_2(R,\tau)e^{-mx}$$

where $m = \sqrt{\dfrac{hp}{kA}}$

Thus

$$T(x,\tau) = T_\infty + \left[T_2(R,\tau) - T_\infty \right] e^{-mx} \tag{B-34}$$

As indicated in the body of the report, this solution contains the tacit assumption that the temperature distribution within the reference lead reacts instantaneously to changes in surface temperature of the heater wire. This is obviously not possible

physically. However, for purposes of estimating heat losses caused by the presence of the thermocouple, this idealization is not unreasonable.

APPENDIX C.

PROPERTIES

For solids experiencing large temperature variations the heat flow and temperature distributions within those solids can be significantly influenced by a temperature dependent thermal conductivity (to a lesser degree the same holds for density and specific heat). Likewise, for a solid subjected to radiation and natural convection boundary conditions, both the radiative characteristics of the surface and the free convection heat transfer coefficient will in general be temperature dependent. Quite often, thermophysical properties data, radiation surface characteristics and accurate convection correlations are not always available. This will become evident in the following sections.

Thermophysical Properties

Thermophysical properties for chromel-P and alumel were obtained from [16] for the temperature range from about 200 to 1000°F. Thermal conductivity values are shown graphically in figure C-1. For temperatures beyond 1000°F extrapolated values were employed by using the straight line equations indicated below and also shown in figure C-1.

$$Alumel: \quad k = 9.83 \ (10^{-3})T + 14.58 \qquad Btu/hr\text{-}ft\text{-}°F$$
$$(T \ in \ °F)$$
$$\rho = 537 \ lb_m/ft^3$$
$$c_p = 0.125 \ Btu/lb_m\text{-}°F$$

Chromel-P: $k = 5.96 \ (10^{-3})T + 9.76$ $Btu/hr\text{-}ft\text{-}^{\circ}F$

$\rho = 545 \ lb_m/ft^3$

$c_p = 0.107 \ Btu/lb_m\text{-}^{\circ}F$

For platinum and platinum-rhodium alloys property values may be found in [17] . For the system of units used herein the following expressions describe the property variations, as recommended in [17]:

Platinum: $k = 41.137 - 0.1387(10^{-2})T$ $Btu/hr\text{-}ft\text{-}^{\circ}F$
$(T \ in \ ^{\circ}F)$

$\rho = 1340 \ lb_m/ft^3$

$c_p = 0.371 \ (10^{-5})T + 0.0311$ $Btu/hr\text{=}ft\text{-}^{\circ}F$
$(T \ in \ ^{\circ}F)$

$\alpha = 0.864 - 0.2913(10^{-4})T$ ft^2/hr
$(T \ in \ ^{\circ}F)$

Platinum-10% Rhodium:

$k = 41.3 \ [1 + 1.11(10^{-4})(T\text{-}64.4)]$ $Btu/hr\text{-}ft\text{-}^{\circ}F$
$(T \ in \ ^{\circ}F)$

$\rho = 1266 \ lb/ft^3$

$c_p = 0.0383 \ Btu/lb_m\text{-}^{\circ}F$

$\alpha = 0.853 \ [1 + 1.11(10^{-4})(T\text{-}64.4)]$ ft^2/hr
$(T \ in \ ^{\circ}F)$

Since properties were not available for Pt-13% Rhodium, the foregoing properties for Pt-10% rhodium were used.

Emissivity

Total and spectral emissivities of platinum may be found in [17]. Emissivity, as understood therein, referred to the ratio of energy radiated hemispherically per unit area per unit time by a surface to that radiated by a black-body at the same temperature.

Expressions describing the total emissivity of platinum are:

$$\epsilon = 0.828 \ (10^{-4}) \ T \qquad\qquad 240 < T < 1690$$

$$\epsilon = 0.0837 + 0.333(10^{-4})T \qquad 1690 < T < 2890$$
$$(T \text{ in } {}^{\circ}F)$$

No similar data was available for platinum-rhodium alloys. In the absence of such data the above expressions could likely be used without appreciable error.

No data describing radiative surface characteristics of chromel-alumel materials appears available in the literature.

Resistivity

Should further computer solutions to the governing heat transfer equations be required in which variations in electrical resistivity with temperature of the heater wire be taken into account in order to simulate the actual calibration test, such values may be found in tabulated or equation form in [17].

For temperatures up to $1500^{\circ}C$, an equation relating the resistivity of platinum to temperature is:

$$R_t = R_o \ [1 + 3.9788(10^{-3})T - 5.88(10^{-7})T] \ \text{ohms/cmf}$$
$$(T \text{ in } {}^{\circ}C)$$

Free Convection Heat Transfer Coefficient

Generalized correlations for free convection heat transfer coefficients involve the property groupings of the Prandtl and Grashof numbers of the surrounding fluid. These properties are mostoften evaluated at the film temperature, i.e. the average of

the heated surface temperature and the temperature of the ambient

fluid. Consequently, large variations of surface temperature

affect the film properties upon which the heat transfer coefficient

is based. In the present case, since the magnitude of the heat

transfer coefficient obviously influences the temperatures experi-

enced by the heater-wire-thermocouple assembly, the properties

of the surrounding medium must be evaluated throughout the transient

heating period.

For air the properties c_p, k, ρ and the property grouping

$g\beta\rho^2/\mu^2$ are presented in Figures C-2 to C-5. Also shown in each

figure are straight line approximations to various segments to the

curves for use in computerized calculations.

Another method for calculating properties as temperature

levels change is to employ the subroutine shown in Figure C-6.

This subroutine, when called upon to calculate a property at a

particular temperature searches the computer memory to find the

two property values at the temperatures immediately bracketing

the temperature in question. A linear interpolation between the

two temperatures is then made to compute the unknown property

value. Since many temperature-property values may be initially

read into the computer memory this straight line approximation

is quite accurate. With this subroutine it is thus not necessary

to use equations of the type shown in Figures 2-5.

Another alternative for calculating the heat transfer coefficient as a function of the changing temperature is to use the applicable correlation (p. 176, [2], in this instance) to calculate 'h' for the different wire sizes at several temperatures. Plotting these values, as in Figure C-7, one may either employ the equations of the curves drawn through the calculated points or make use of the subroutine previously described.
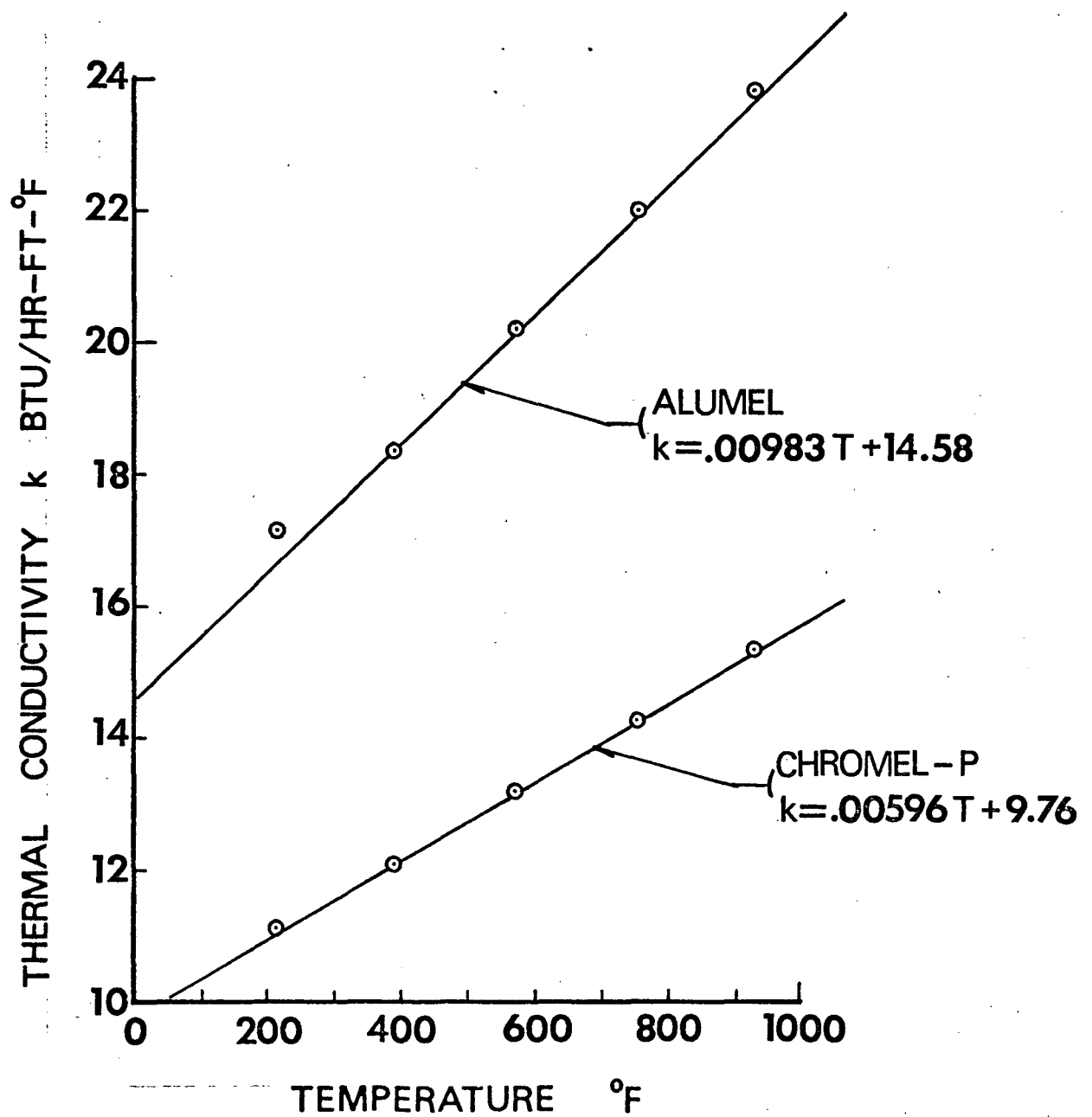
Figure C-1.   Thermal conductivity of alumel and
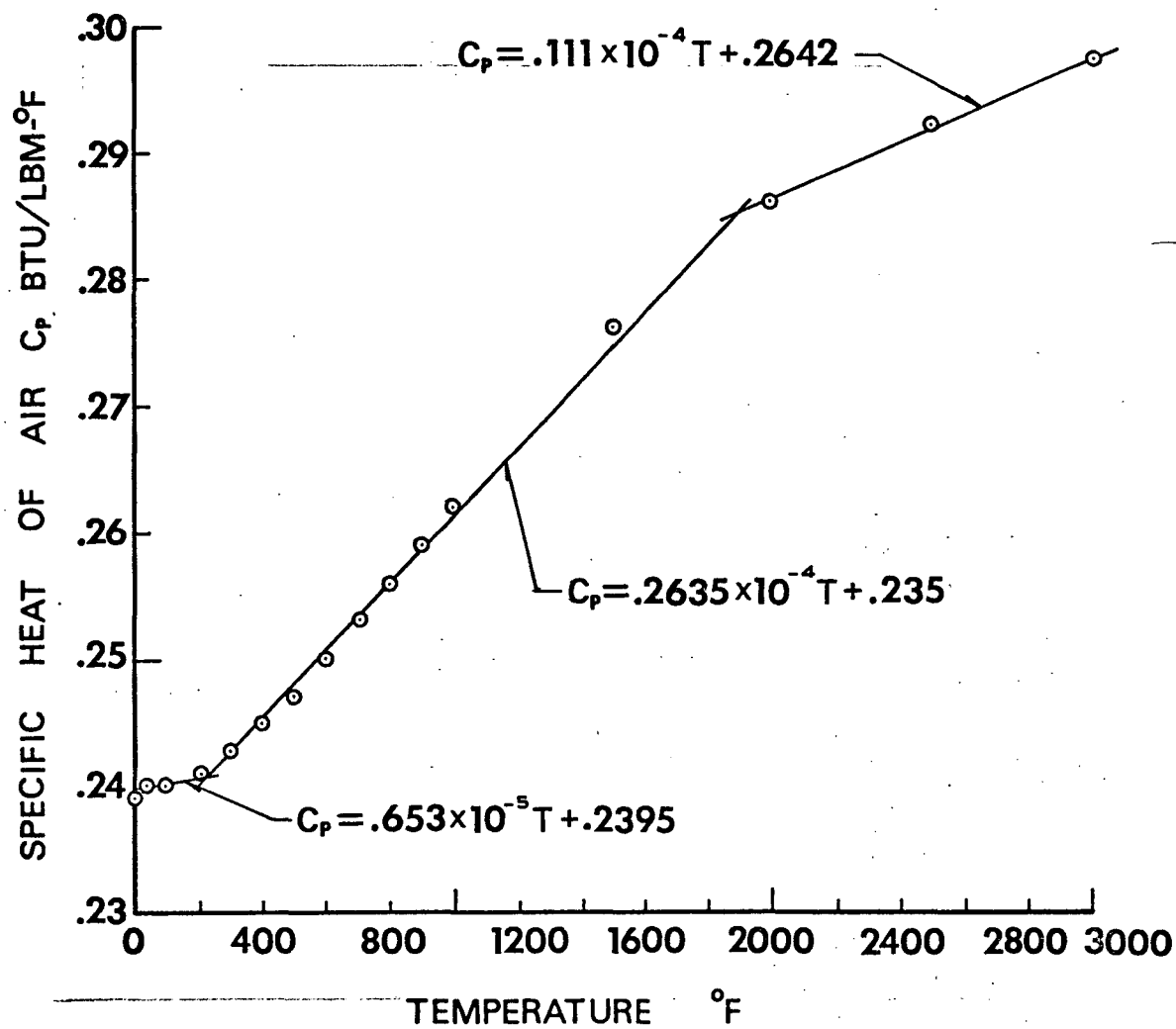   chromel-P as a function of temperature.

The graph shows Specific Heat of Air C_p (BTU/LBM-°F) on the vertical axis versus Temperature °F on the horizontal axis, with the following equations labeled:

$$C_p = .111 \times 10^{-4}\,T + .2642$$

$$C_p = .2635 \times 10^{-4}\,T + .235$$

$$C_p = .653 \times 10^{-5}\,T + .2395$$

Figure C-2.   Specific heat of air at atmospheric pressure as a function of temperature.
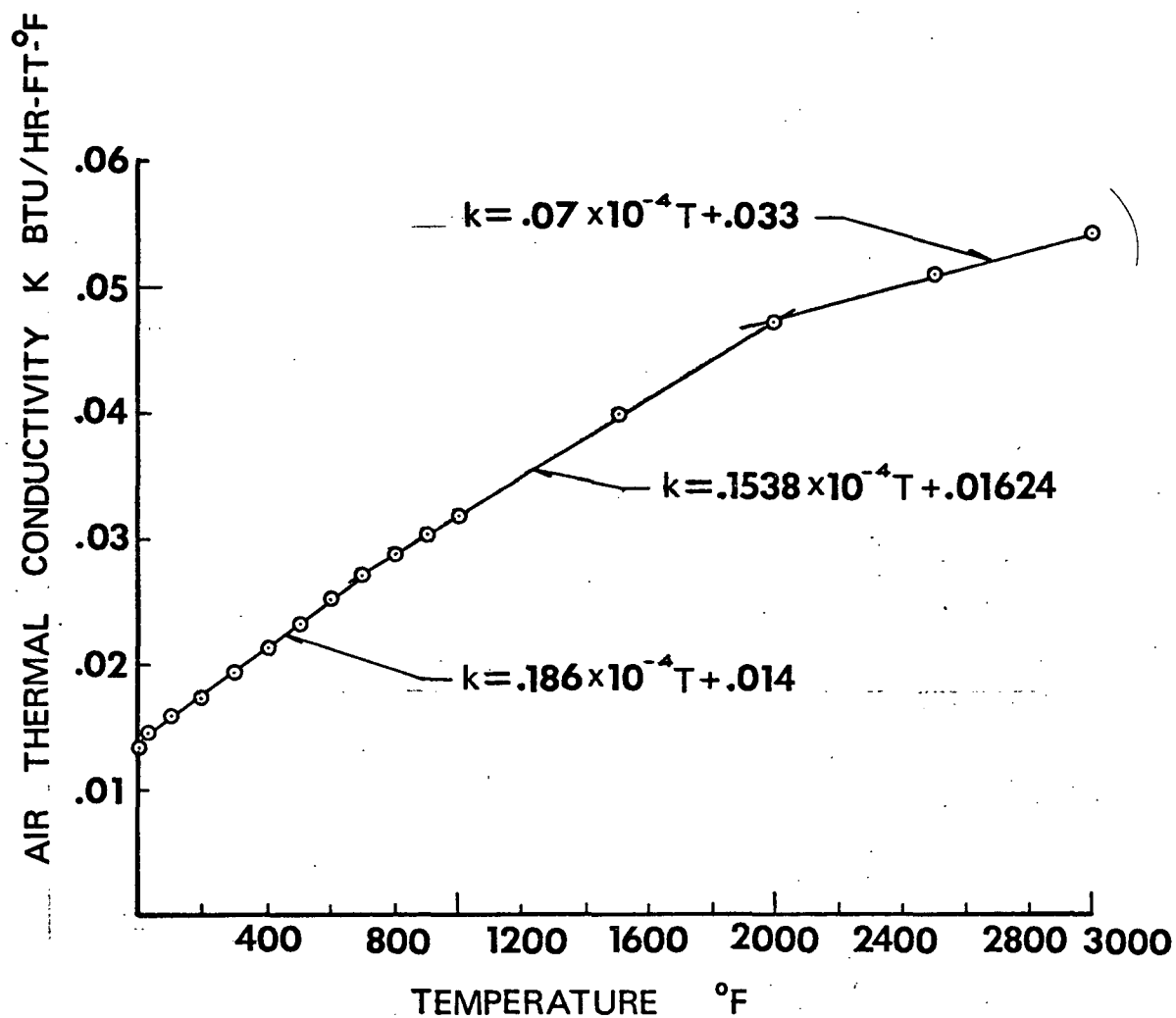
Figure C-3. Thermal conductivity of air at atmospheric pressure as a function of temperature.
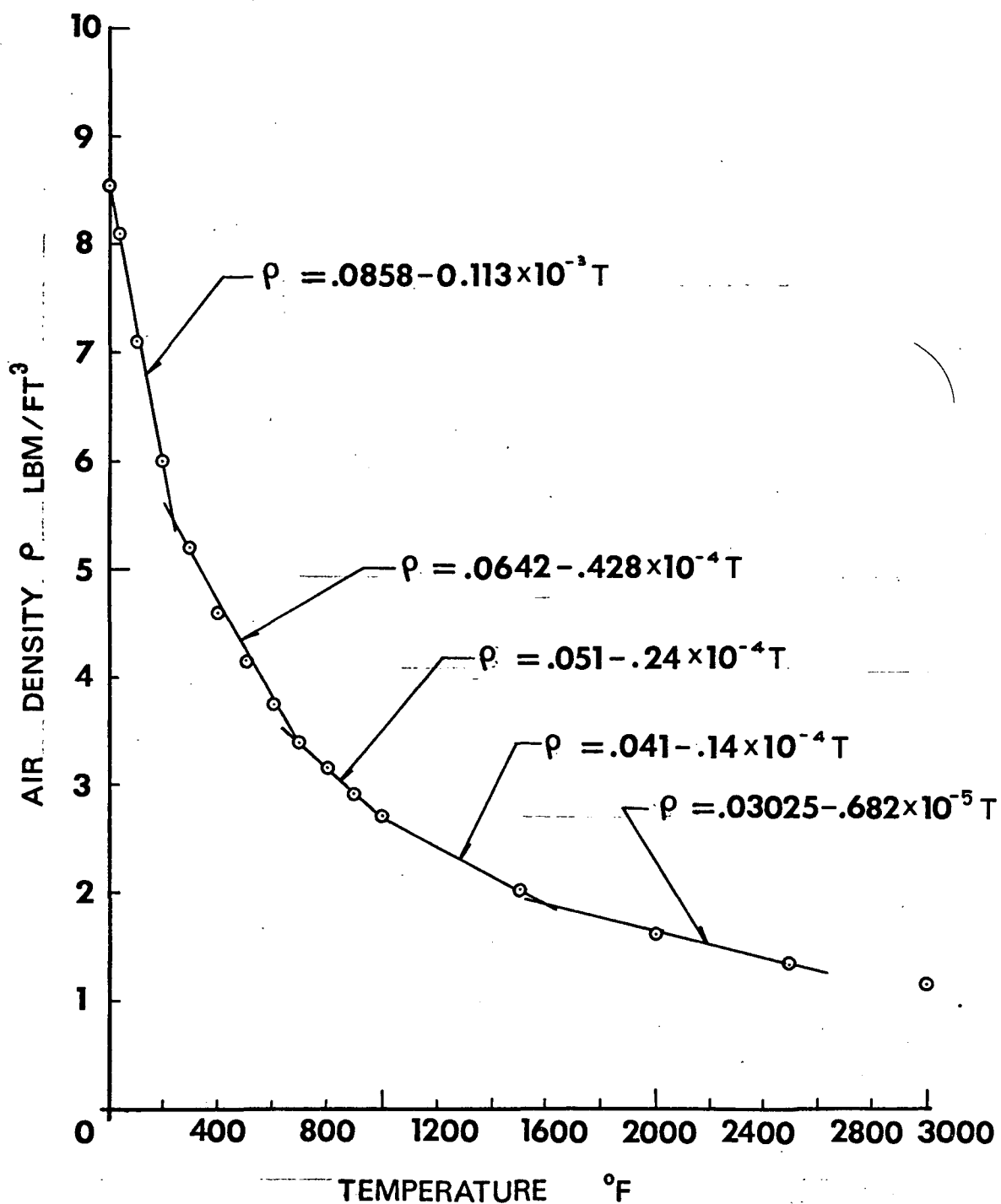
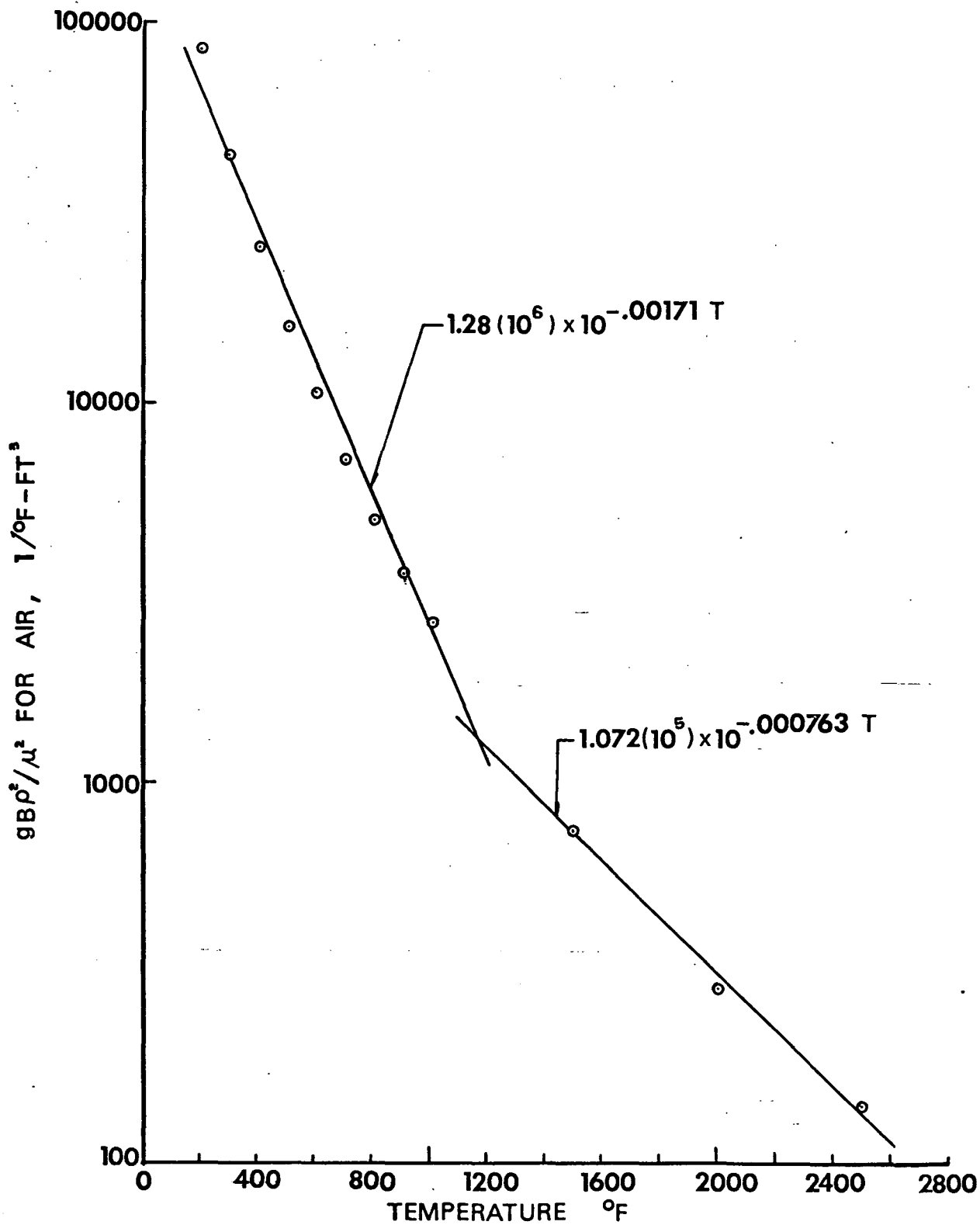Figure C-4. Density of air as a function of temperature.

Figure C-5. Property group of air, $g\beta\rho^2/\mu^2$, as a function of temperature.

$-1.28(10^6) \times 10^{-.00171\,T}$

$-1.072(10^5) \times 10^{-.000763\,T}$

$g\beta\rho^2/\mu^2$ FOR AIR, $1/^{\circ}F-FT^3$

TEMPERATURE $^{\circ}F$

```
$IBFTC VALUE

      SUBROUTINE VALUE(X,G,IG,XA,GA)

      DIMENSION G(1),X(1)

1     FORMAT(1X,63HINPUT VALUE OUTSIDE RANGE OF KNOWN VALUES/1X,12HINPUT
     2 VALUE=,E15.8,9H,10KLOW VALUE=,E15.8,9H,12HUPPER VALUE=,E15.8)

      XX1=XA-X(1)

      IF(XX1.RE.0.0)GO TO 100

      GA=G(1)

      GO TO 102

100   DO 106 K=2,IG

      XX2=XA-X(K)

      IF(XX2*XX1)106,1,GO,106

104   I=K

      IF(XX2.EQ.0.0.OR.XX1.EQ.0.0)GO TO 106

      GO TO 102

105   XX1=XX2

      WRITE(6,1) XA,X(1),X(IG)

      STOP

106   GA=G(I)

      GO TO 102

10?   SLPE=(G(I)-G(I-1))/(X(I)-X(I-1))

      GINSEY=G(I)-SLPE*X(I)

      GA=SLPE*XA+GINSPT

132   RETURN

      END
```
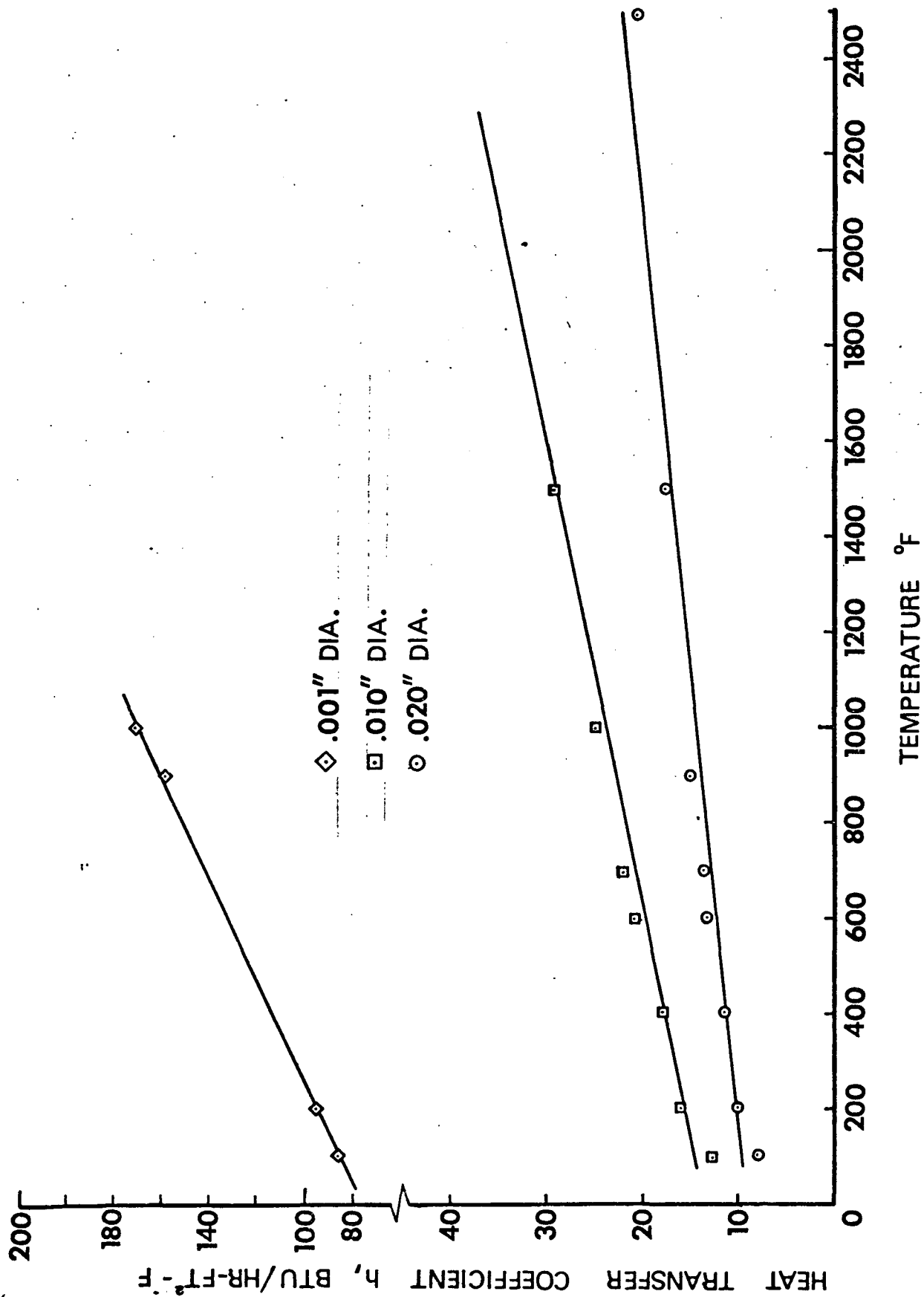
Figure C-6. Subroutine VALUE

Figure C-7. Free convection heat transfer coefficient for horizontal wires in air at atmospheric pressure as a function of temperature.