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ON OBTAINING LUNAR MISSION LAUNCH OPPORTUNITIES

HOBART SWARTWOOD, JR.

RAE-B MISSION ANALYSIS REPORT NO. 2

FEBRUARY 1972

GSFC — GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND

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ABSTRACT

A general procedure for predicting launch opportunities for the RAE-B lunar orbiter mission is presented. It is shown that knowledge of the Earth-Moon distance and lunar phase and declination are sufficient to determine launch periods consistent with present mission constraints and to approximately predict launch, transfer, and arrival parameters such as park orbit coast time, the possibility of shadows in all phases of the mission, arrival energy, and the amount of sunlit orbit time in the lunar orbit. Constraints on RAE-B include bounds on the spin axis-Sun angle in the translunar trajectory, an upper limit on the arrival energy, and a minimum time for sunlight duration in lunar orbit. Presently, both posigrade and retrograde lunar orbits are being considered for the mission. Launch opportunities and trajectory characteristics for the retrograde orbit have been published previously; similar data for the posigrade orbit are determined and presented in this paper. It is shown that there is very little difference in the launch and transfer characteristics for the two, but the arrival energy for the retrograde approach is consistently higher than for the posigrade. For given arrival conditions and a given launch azimuth there are two launch opportunities per day if out-of-plane engine burns are not allowed. Comparisons are made of relevant parameters for the two cases. The general features of launch, transfer, and arrival parameters are discussed and are shown to be a function of lunar declination, flight time, and launch azimuth.

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ON OBTAINING LUNAR MISSION LAUNCH OPPORTUNITIES

I. INTRODUCTION

Allowable Earth-to-Moon transfer trajectories for the RAE-B mission are subject to mission constraints due to fuel and power system requirements on flight time, arrival energy, shadow duration, and vehicle attitude with respect to the Sun and Earth. The selection of launch dates which satisfy all the constraints can be simplified and computer time minimized if approximate launch periods can first be determined.

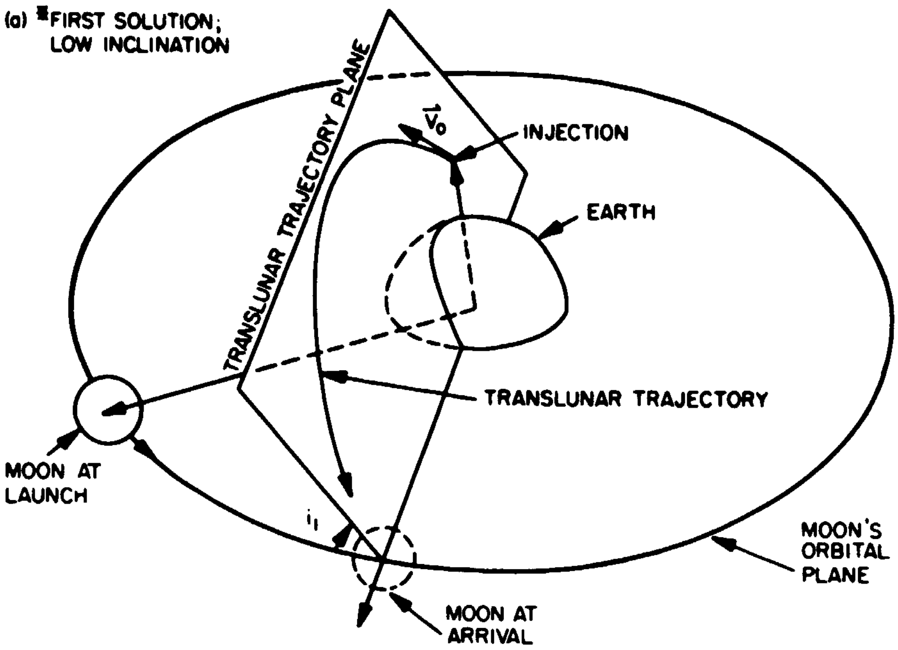
A general procedure is presented which facilitates launch opportunity selection when mission constraints can be directly or indirectly related to such parameters as Earth-Moon distance, lunar phase, and lunar declination. The procedure quite accurately predicts RAE-B lunar launch opportunities from March through Decmeber 1973, and also predicts qualitative features of the park orbit, transfer trajectory, and arrival conditions such as park orbit coast time, shadow possibility and approximate duration, and arrival energy.

The general characteristics of Earth-to-Moon missions in the park orbit phase, translunar phase, and arrival phase will be discussed tutorially. It is shown that for a given launch azimuth there are exactly two launch times per day to reach the Moon without making out-of-plane burns. Further, it is shown that launch and arrival characteristics are a function of launch azimuth, flight time, and the launch time on a given day.

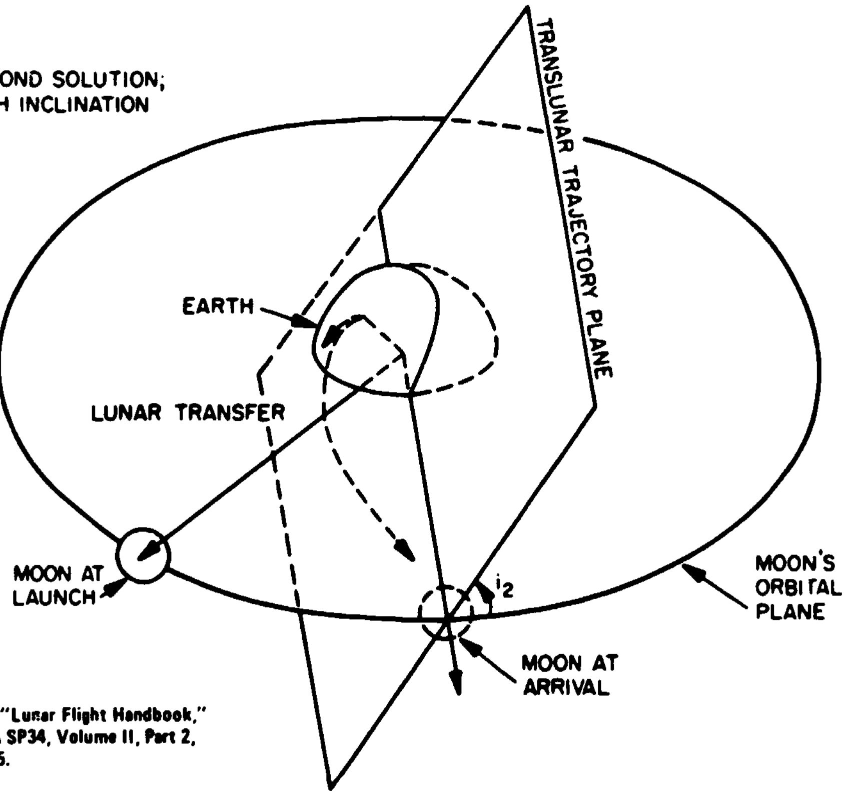
II. LAUNCH TIME

Plane changes are very expensive fuel-wise and therefore it is highly desirable to effect the Earth-to-Moon transfer making no out-of-plane engine burns. Thus it is necessary for the park orbit plane to contain the Moon at arrival of the spacecraft. The inertial orientation of the park orbit plane is a function of launch azimuth and launch time on a given day. For a given launch azimuth, the launch time determines the intersection of the transfer plane and the Moon's orbital plane. This becomes immediately clear when it is realized that the transfer plane must contain the launch site at launch, and the launch site is rotating with the Earth. Therefore, for a given launch azimuth, the problem is reduced to determining a launch time to constrain the park orbit to contain the Moon at arrival. There are two launch times per day when this can be done; see fig. 1. Figure 1(a) shows a favorable launch time for lunar transfer; fig. 1(b) is also acceptable but at a later time on the same day. The inclination of the park orbit planes to the Moon's orbital plane vary with the Moon's

(a) *FIRST SOLUTION;
LOW INCLINATION



(b) SECOND SOLUTION;
HIGH INCLINATION



*From "Lunar Flight Handbook,"
NASA SP34, Volume II, Part 2,
p. V15.

Figure 1. Lunar Transfer Geometry

declination. Figure 2 shows the variation of the inclination for a typical time period. Notice that on a given day there are two solutions for a given launch azimuth as expected. Changing the launch azimuth from 90° increases the inclination of the park orbit. The difference in inclination of the two solutions for a given launch azimuth varies from about 50 degrees when the Moon is coincident with the intersection of the Earth's equator and the Moon's orbital plane, to about 0 degrees when the Moon is at its maximum and minimum declination with respect to the Earth's equator. The two solutions are referred to a "high" and "low" with reference to their inclination.

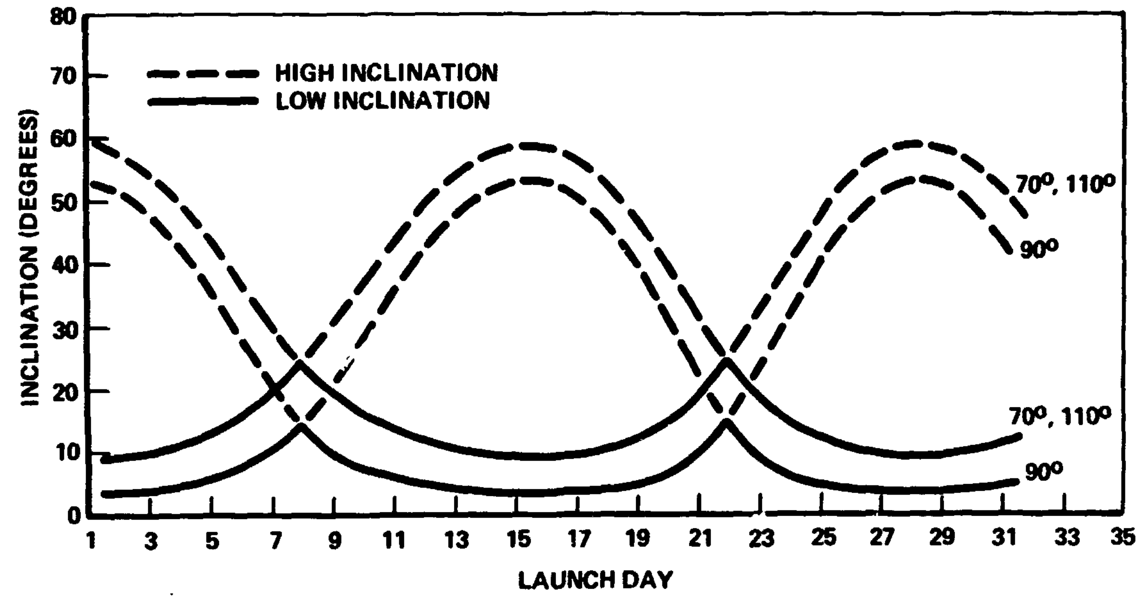


Figure 2. Inclination of Transfer Trajectory to Moon's Orbital Plane vs. Launch Date for Various Launch Azimuths

III. THE RADIO ASTRONOMY EXPLORER-B MISSION

A. General

As our objective is to apply the principles and procedures developed in this paper to RAE-B, it is appropriate to discuss that mission and its peculiarities. RAE-B will be launched during the period March through December 1973 and will be inserted into a circular orbit around the Moon at an altitude of

1100 km. The lunar orbit will be inclined either 116.5 degrees or 63.5 degrees to the lunar equator depending upon whether a retrograde or a posigrade approach is selected. Launch opportunities for the mission are subject to constraints imposed by thermal and electrical power considerations, on-board propulsion system limitations, and spacecraft gravity gradient requirements. In particular it is required that throughout the spin-stablized phases of the mission, the spin axis-Sun angle be maintained between 60 degrees and 120 degrees. Midcourse guidance corrections will be necessary during the translunar trajectory, and as each correction requires some amount of fuel, the mass of the spacecraft at perlune will vary accordingly. However, the retro motor for lunar orbit insertion is a solid propellant motor of fixed total impulse, and therefore the velocity increment which the motor applies varies as the spacecraft's mass. After lunar orbit insertion, the on-board hydrazine system (VCPS) will be used to remove insertion errors and to circularize the orbit. The total velocity increment necessary for orbit insertion and orbit trim, ΔV_c, is currently constrained to be no more than 0.720 km/sec.

The RAE-B lunar orbit is required to remain nearly circular. It has been shown (Ref. 4) that a selenographic inclination of approximately 116.5 degrees for the retrograde orbit, or 63.5 degrees for the posigrade orbit, will minimize growth in eccentricity. Launch opportunities for the retrograde approach have previously been determined (Ref. 5); the posigrade launch opportunities will be determined as a heuristic example. Thus the orbit about the Moon is constrained to be initially inclined 63.5 degrees, circular, and at an altitude of 1100 km.

The orbit about the Moon is subject to perturbations primarily from the Earth and from lunar gravitational anomalies. One effect of these perturbations is to cause a regression of the line of nodes, Ω, the intersection of the orbit plane and the ecliptic plane. Then by adding the rate of rotation of the Moon-Sun line, we obtain the net regression rate with respect to the Moon-Sun line. This regression will cause some part of the orbit to be in lunar shadow for certain values of the node; and at times the entire orbit will be sunlit. For boom deployment and satellite calibration purposes, the initial orbit must be oriented to give a minimum of 50 days in sunlight.

The nominal flight time for the mission is currently 110 hours and the launch azimuth is assumed to be 90 degrees. Under these considerations the arrival energy is essentially minimized and payload capability maximized.

B. Mission Constraints and Lunar Phase and Distance

1. Spin Axis-Sun Angle. Let us now relate the RAE-B mission constraints and requirements to lunar ephemeris data. As discussed previously, the vehicle's spin axis-sun angle must be maintained between 60 degrees and 120 degrees. The spin axis of the vehicle is assumed to be along the velocity vector at injection into the translunar trajectory, so the spin axis-Sun angle is identically the injection velocity-Sun angle. For low energy, long flight time lunar transfer trajectories, the Earth-centered transfer angle is very nearly 180 degrees. Since the nominal flight time for the mission is 110 hours, and since the Moon progresses along its orbit at the rate of about 13 degrees/day, the Moon is approximately 60 degrees from rendezvous at launch. Fig. 3 depicts the situation when the Earth is between Moon and Sun.

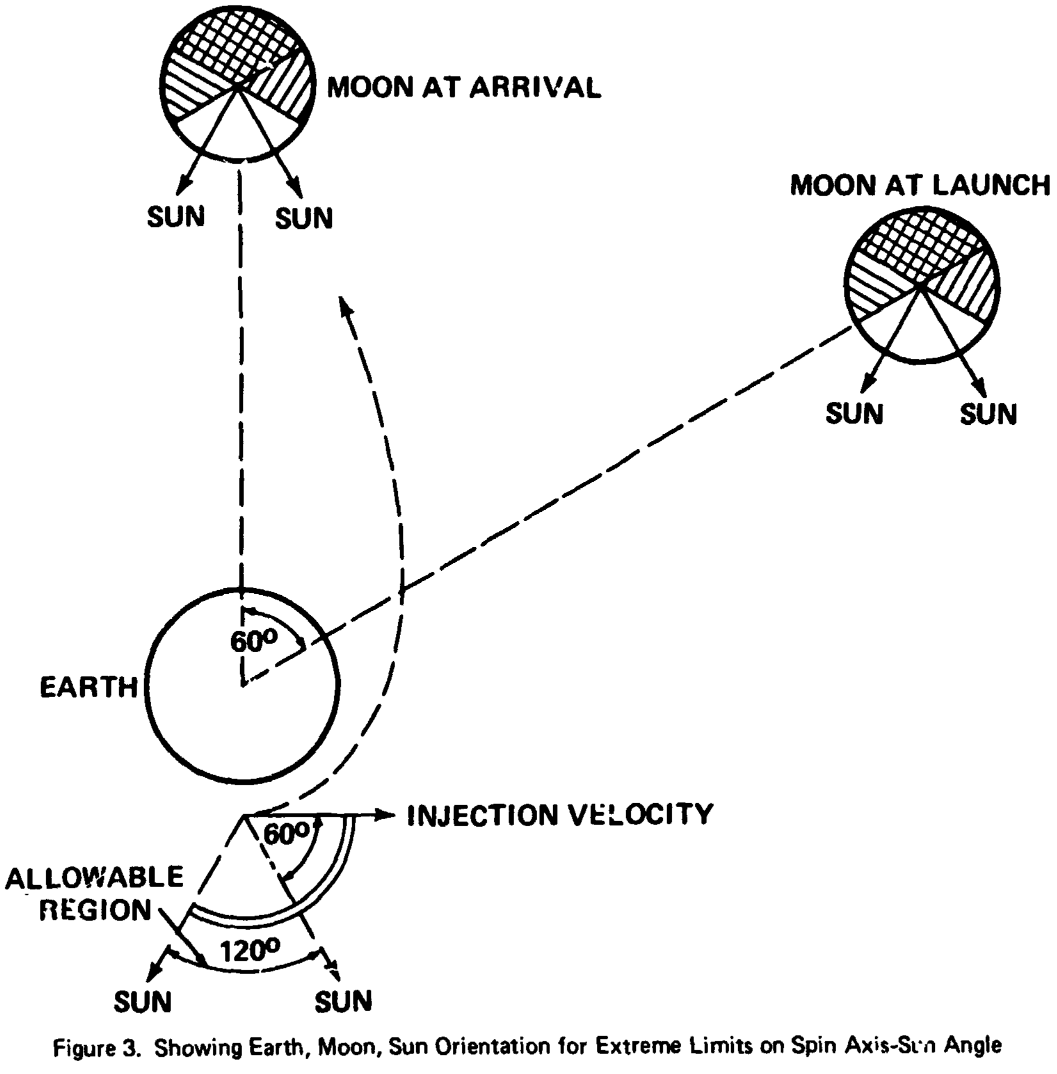


Figure 3. Showing Earth, Moon, Sun Orientation for Extreme Limits on Spin Axis-Sun Angle

The direction of the Sun at its extreme allowed values is also shown; acceptable directions lie within these extremes. Examination of the Earth-Moon-Sun angle at injection for the two extreme directions of the Sun immediately reveals that the angle can be between 90 degrees, when the spin axis-Sun angle is 60°, and 30 degrees, when the spin axis-Sun angle is 120°. The lunar phase correspondingly varies from first quarter to 30 degrees less than full, or approximately 2-1/2 days before full Moon. Clearly, when the Moon is between Earth and Sun, a similar situation occurs and it follows that launch opportunities are between third quarter and about 2-1/2 days before new Moon.

2. Velocity for Orbit Circularization and Trim, ΔV_c. Recall that there is also a constraint on ΔV_c. A constraint on velocity however is essentially a constraint on energy. For a given flight time the arrival energy is principally a function of Earth-Moon distance. This can be seen from the following. Consider the vis viva equation,

$$V^2 = G(m_1 + m_2)\left(\frac{2}{r} - \frac{1}{a}\right), \tag{1}$$

where
G = universal gravitational constant
m_1, m_2 = masses of two mutually attracting bodies,
r = distance between the bodies
a = semi-major axis of the orbit.

For the case of a spacecraft in orbit about the Earth, the mass of the spacecraft is negligible and eq. (1) reduces to

$$V^2 = \mu_E\left(\frac{2}{r} - \frac{1}{a}\right) \tag{2}$$

where

$$\mu_E = Gm_E.$$

For two-body motion the energy of the spacecraft is constant and is given by

$$C_3 = V^2 - \frac{2\mu}{r}. \tag{3}$$

The velocity of the spacecraft with respect to the Moon is given by

$$\vec{V} = \vec{V}_s - \vec{V}_m \tag{4}$$

where $\vec{V}_s$ = velocity of spacecraft with respect to Earth, and $\vec{V}_m$ = velocity of the Moon with respect to the Earth.

Therefore,

$$V^2 = V_S^2 + V_m^2 - 2V_S V_m \cos\theta. \tag{5}$$

For long flight time missions the apogee of the transfer ellipse is roughly at Earth-Moon distance. Applying the vis viva equation at apogee of the Earth-centered transfer ellipse, we get

$$V_S^2 = \mu_E\left(\frac{2}{r_S} - \frac{1}{a_S}\right) = \mu_E\left(\frac{2}{r_m} - \frac{1}{a_S}\right). \tag{6}$$

Similarly, assuming two-body motion of the Earth-Moon system we can apply the vis viva equation to the Moon, but cannot neglect the Moon's mass, so we get

$$V_m^2 = (\mu_E + \mu_m)\left(\frac{2}{r_m} - \frac{1}{a_m}\right). \tag{7}$$

Strictly, r_m is the distance from the Earth-Moon barycenter, but as the barycenter is inside the Earth, r_m is essentially the Earth-Moon distance.

Inserting the latter two equations into eq. (5) we get for the square of the velocity with respect to the Moon,

$$V^2 = \mu_E\left(\frac{2}{r_m} - \frac{1}{a_S}\right) + (\mu_E + \mu_m)\left(\frac{2}{r_m} - \frac{1}{a_m}\right)$$

$$-2\left[\mu_E\left(\frac{2}{r_m} - \frac{1}{a_S}\right)\right]^{1/2}\left[(\mu_E + \mu_m)\left(\frac{2}{r_m} - \frac{1}{a_m}\right)\right]^{1/2}\cos\theta. \tag{8}$$

This function monotonically decreases as r_m gets large, and never reaches a minimum. But as r_m is bounded between approximately 357000 km and 407000 km, then V^2, and thus C_3, and thus ΔV_c, do reach a minimum, when r_m takes on its maximum value, and reaches a maximum when r_m is minimum.

Consider the third term in eq. (8). It involves a cos θ factor, where θ is the angle between the Earth-centered velocity vectors of the vehicle and the Moon. The apogee of the transfer ellipse is at lunar distance and is on the node. The angle θ therefore is very nearly equal to the inclination of the transfer trajectory to the Moon's orbital plane. Referring to fig. 2 we see that this angle varies from about 4 degrees to about 54 degrees for the 90 degree launch azimuth. The larger the angle the smaller the term for a fixed r_m, and since the term is negative, the value of V^2, and thus C_3, and thus ΔV_c, is larger. It follows that the high inclination trajectory requires a larger ΔV_c than the low, execpt when the inclinations are about equal which occurs when the Moon is near its maximum and minimum declination, as seen previously.

In figure 4 we observe exactly how C_3 and ΔV_c vary with date for the two-month period of July and August. As expected the curves are qualitatively very similar; values for the high inclination solution are correspondingly higher than for the low.

Consider now the lunar ephemeris data as presented in figures 5 and 6. Relating fig. 4 with 5 and 6 it is seen that when the Moon is near its minimum declination, around July 13, the ΔV_c's for the high and low inclinations are equal and just about their lowest value, as would be expected since the Earth-Moon distance, $|\vec{R}_{mE}|$, is near its maximum. The Moon reaches its maximum declination on about July 26, and the ΔV_c for the high and low inclinations are again equal, but larger since $|\vec{R}_{mE}|$ is near its minimum. Indeed the general qualitative behavior can be explained from the motion of the Moon.

Launch dates satisfying the constraint that the vehicle remain in sunlight for at least 50 days in lunar orbit can also be related to lunar phase at launch. Let

lunar phase angle = Sun-Moon-Earth angle
λ_a = lunar phase angle at arrival
λ_ℓ = lunar phase at launch
Ω_S = longitude of ascending node with respect to the Moon-Sun line
Ω_E = longitude of ascending node with respect to the Moon-Earth line

Then, at arrival

$$\lambda_a = \Omega_S - \Omega_E, \tag{9}$$

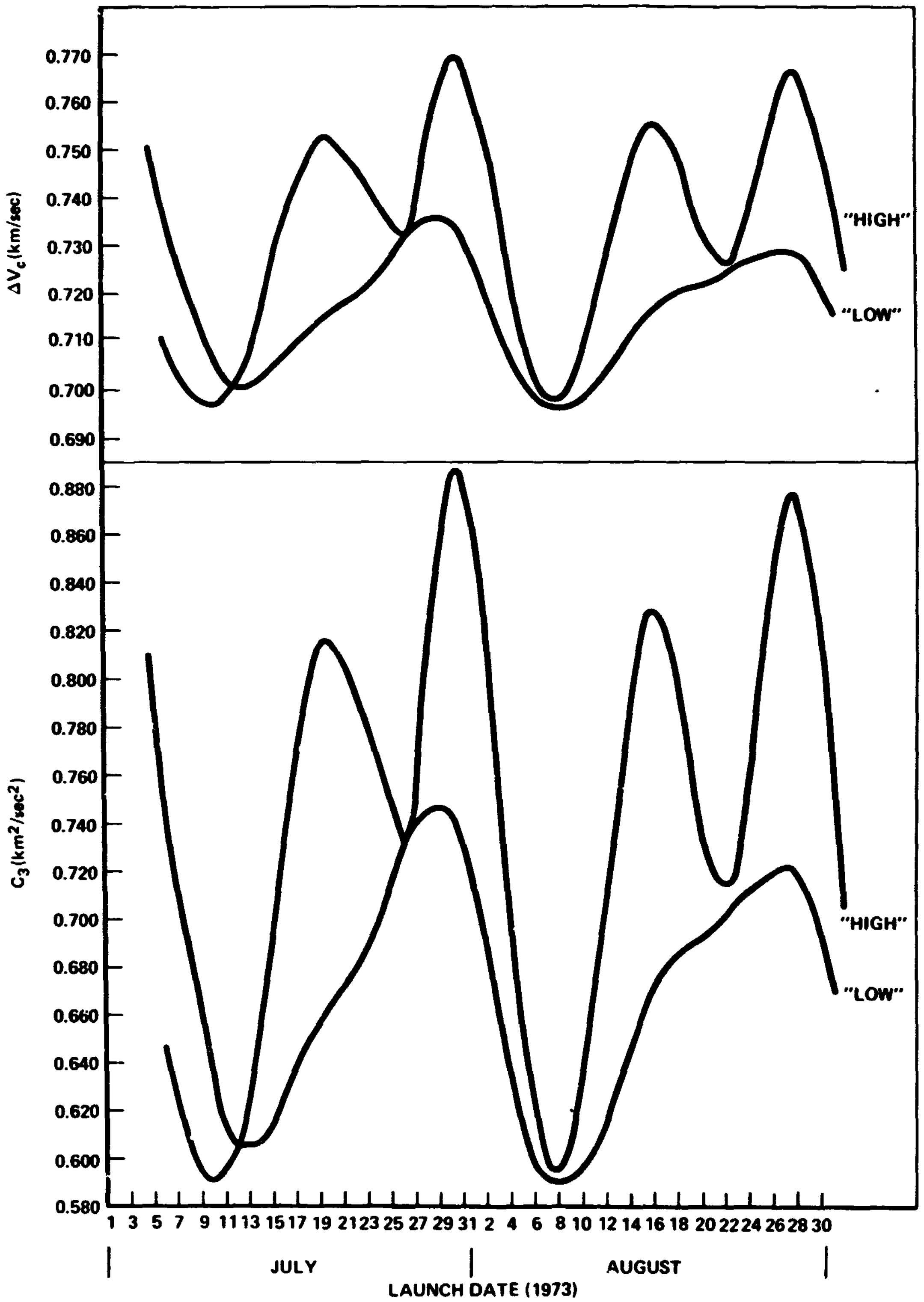


Figure 4. Showing Variation of ΔV_c and C_3 with Launch Date

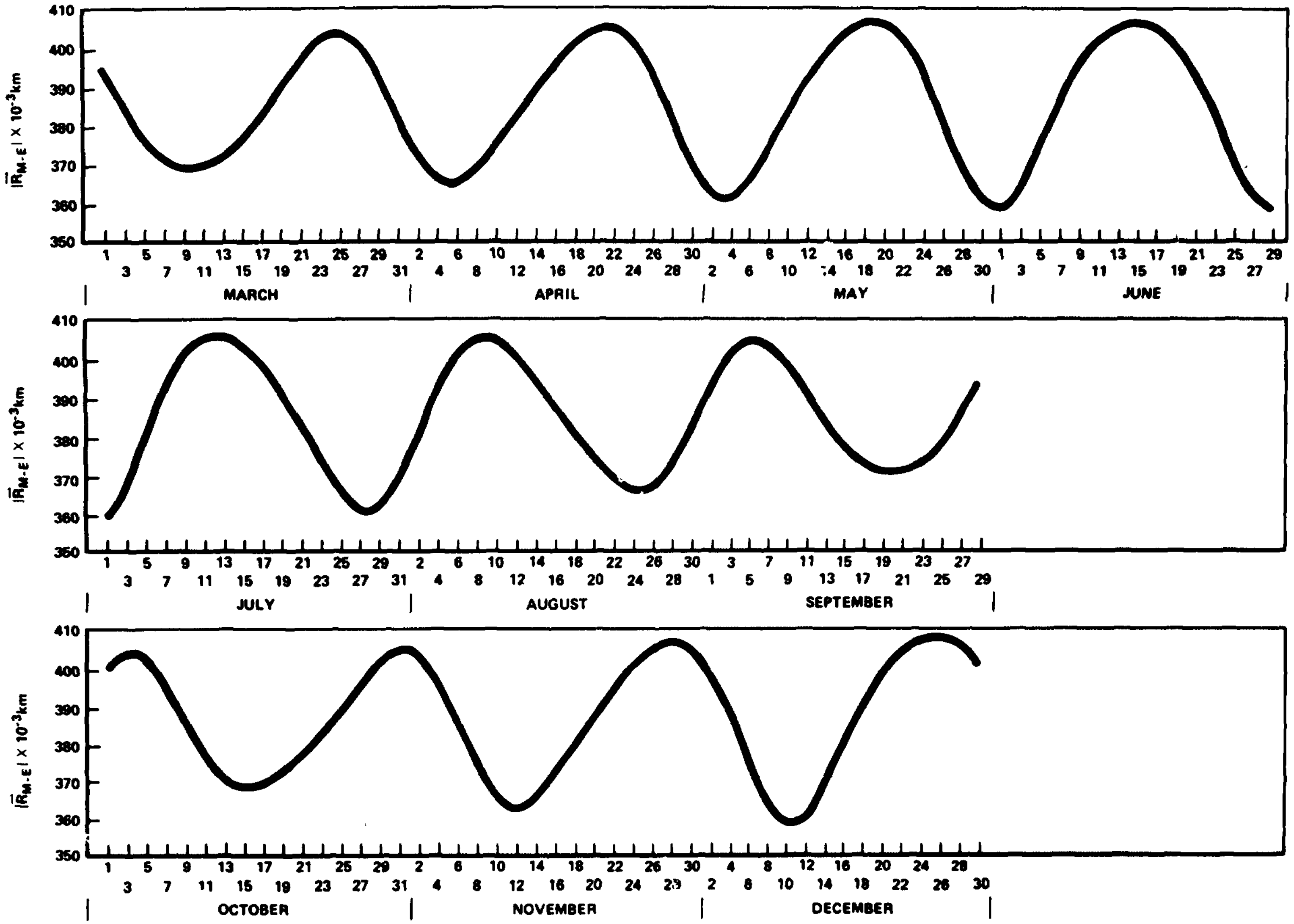


Figure 5. Earth-Moon Distance During RAE-B Launch Period, 1973

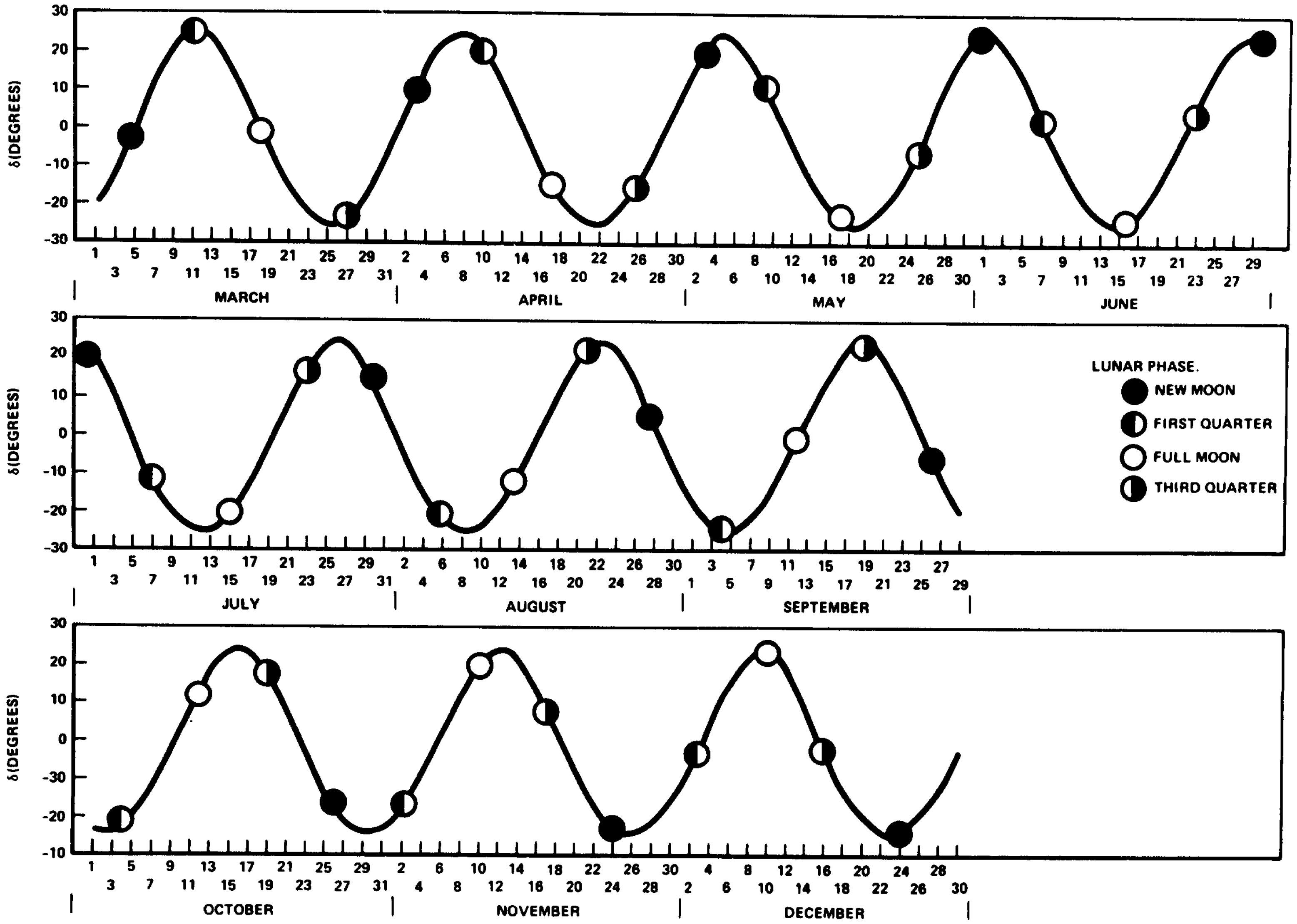


Figure 6. Lunar Declination and Phase During RAE-B Launch Period, 1973

and clearly,

$$\Omega_S = \dot{\Omega}_S \, t. \qquad (10)$$

Therefore, at launch the phase angle is found from

$$\lambda_\ell = \lambda_a - \text{(flight time) (angular rate of Moon)} \qquad (11)$$

$$= \dot{\Omega}_S \, t - \Omega_E - \text{(flight time) (angular rate of Moon)} \qquad (12)$$

For the RAE-B lunar orbit it can be shown that for a shadow-free orbit,

$$45^\circ \leqslant \Omega_S \leqslant 135^\circ \text{ and } 225^\circ \leqslant \Omega_S \leqslant 315^\circ. \qquad (13)$$

Lunar gravitational anomalies and perturbations primarily due to Earth, and the very motion of the Earth-Moon system about the Sun cause Ω_S to change with time, i.e., a regression of the node for a retrograde orbit, or a progression of the node for a posigrade orbit. The major contribution by gravitational anomalies to the nodal rate is due to lunar oblateness, J_2, and the nodal rate is proportional to -cos i. For posigrade orbits,

$$0^\circ \leqslant i \leqslant 90^\circ \Rightarrow \dot{\Omega} < 0; \qquad (14)$$

for retrograde orbits,

$$90^\circ \leqslant i \leqslant 180^\circ \Rightarrow \dot{\Omega} > 0\,. \qquad (15)$$

An exact value of $\dot{\Omega}$ can be obtained by running numerical integration N-body computer programs, and can be shown to be about -0.169 deg/day for the posigrade orbit. Then if the node rate due to the motion of the Earth-Moon system about the Sun is added, approximately -0.986 deg/day, the net nodal rate with respect to the Moon-Sun line, $\dot{\Omega}_S$, is obtained. For the posigrade RAE-B orbit,

$$\dot{\Omega}_S \approx -1.155 \text{ deg/day.}$$

The value of Ω_E can best be found from computer runs, and for RAE-B, it averages about 277°. In order to maximize sunlit orbit time, Ω_S should be about 315°, see fig. 7, since $\dot{\Omega}_S < 0$. Substituting into eq. (9),

$$\begin{aligned} \lambda_a &= \Omega_S - \Omega_E \\ &= 315^\circ - 277^\circ \\ &= 38^\circ \end{aligned}$$

From eq. (11),

$$\lambda_{\ell} = \lambda_a - \text{(flight time) (angular rate of Moon)}$$
$$= 38^\circ - (4.583 \text{ days}) (12.9 \text{ deg/day})$$
$$= -21 \text{ deg.}$$

The minimum acceptable sunlit orbit time is 50 days, and the corresponding change in the node is

$$\Delta\Omega_S = \dot{\Omega}_S \Delta t$$
$$= (-1.155 \text{ deg/day}) (50)$$

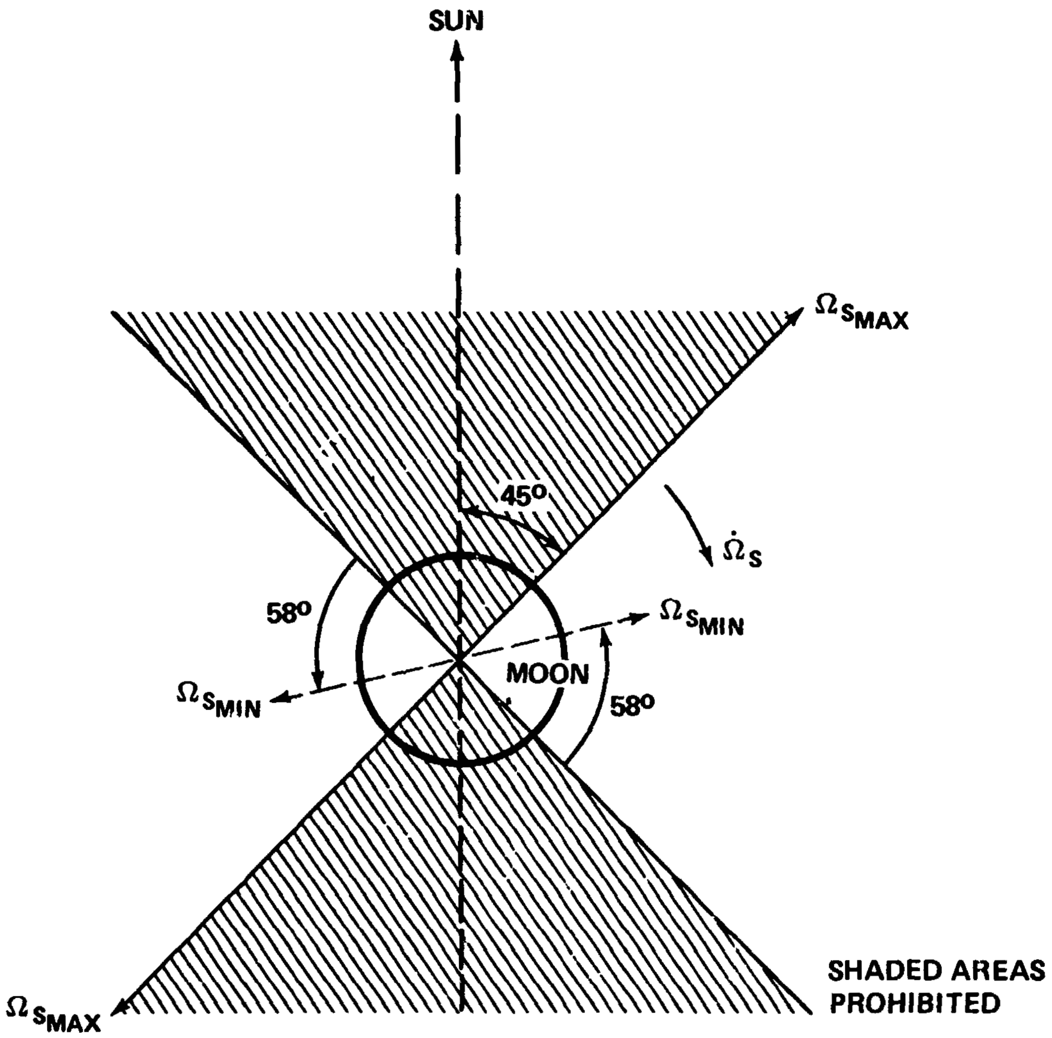


Figure 7. Region of Acceptable and Unacceptable Values for Ω_S

$$\therefore \Delta\Omega_S = -58$$

$$\therefore \quad \Omega_S = 58^\circ + 45^\circ + 180^\circ$$

$$= 283^\circ$$

$$= -77^\circ$$

From eq. (9)

$$\lambda_a = -77^\circ - 277^\circ$$

$$= -354^\circ,$$

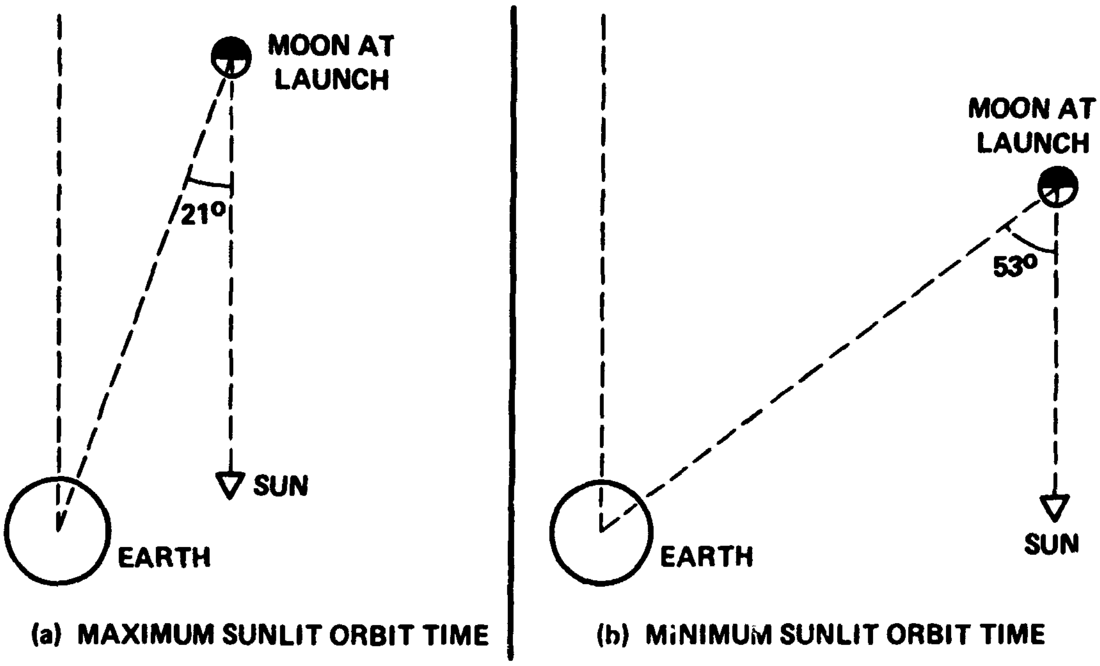
and therefore

$$\lambda_{\ell} = -354^\circ - (4.583 \text{ day}) (-12.9 \text{ deg/day})$$

$$= 307^\circ$$

$$= -53^\circ$$

is the lunar phase angle at launch which just allows adequate sunlit orbit time. Fig. 8 shows the Earth-Moon-Sun geometry for the extreme cases.



(a) MAXIMUM SUNLIT ORBIT TIME

(b) MINIMUM SUNLIT ORBIT TIME

Figure 8. Earth-Moon-Sun Orientation at Launch

Acceptable launch periods are from about 3 days after first quarter to about 2 days before full moon. Considering the bounds on Ω_S, eq. (13), it is apparent that $\Omega_S = 135, 103°$ correspond to maximum and minimum sunlit orbit time also. Symmetrically, launch opportunities lie between 3 days after third quarter and about 2 days before new moon.

IV. SHADOW

Shadows in any phase of the mission prior to at least 50 days after lunar orbit insertion are undesirable from thermal and power system considerations. Certainly, if the park orbit coast time is greater than one half its period, shadow will be inevitable in the park orbit. Even if the park orbit coast time is less than one half its period, shadows may occur depending upon the time of day of the launch.

Shadows in the transfer trajectory can be related to lunar phase at launch. In figure 9 four possible Earth-Moon-Sun orientations are presented. In situation (a) there will be no shadows in the translunar trajectory and are avoidable in the park orbit if the park orbit coast time is sufficiently short; for long coasts there will be a shadow. In (b) shadows are again avoidable in the park orbit, but a distinct possibility exists in the transfer trajectory; whether a shadow is encountered is a function of the distance of the transfer trajectory out of the ecliptic, which is essentially a function of its inclination. In (c) & (d) the vehicle definitely encounters park orbit shadows, or transfer shadows, or both. Situations (a) & (b) occur near full moon, and (c) & (d) occur near new moon. The RAE-B mission prohibits shadows and therefore only launches near the full moon phase are acceptable. Accurate values of shadow duration in the transfer trajectory can be obtained from N-body numerical integration programs.

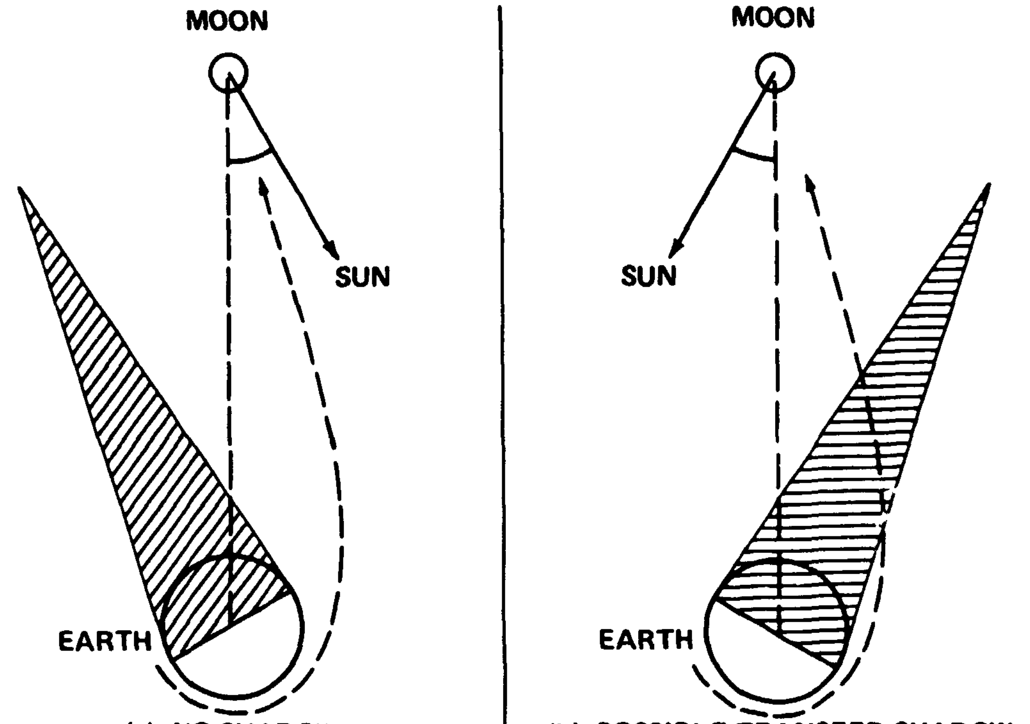
V. LUNAR APPROACH

The lunar approach trajectory is hyperbolic and its inclination is given by

$$i = \cos^{-1} \frac{(\vec{r} \times \vec{v} \cdot \hat{z})}{|\vec{r} \times \vec{v}|} \qquad (16)$$

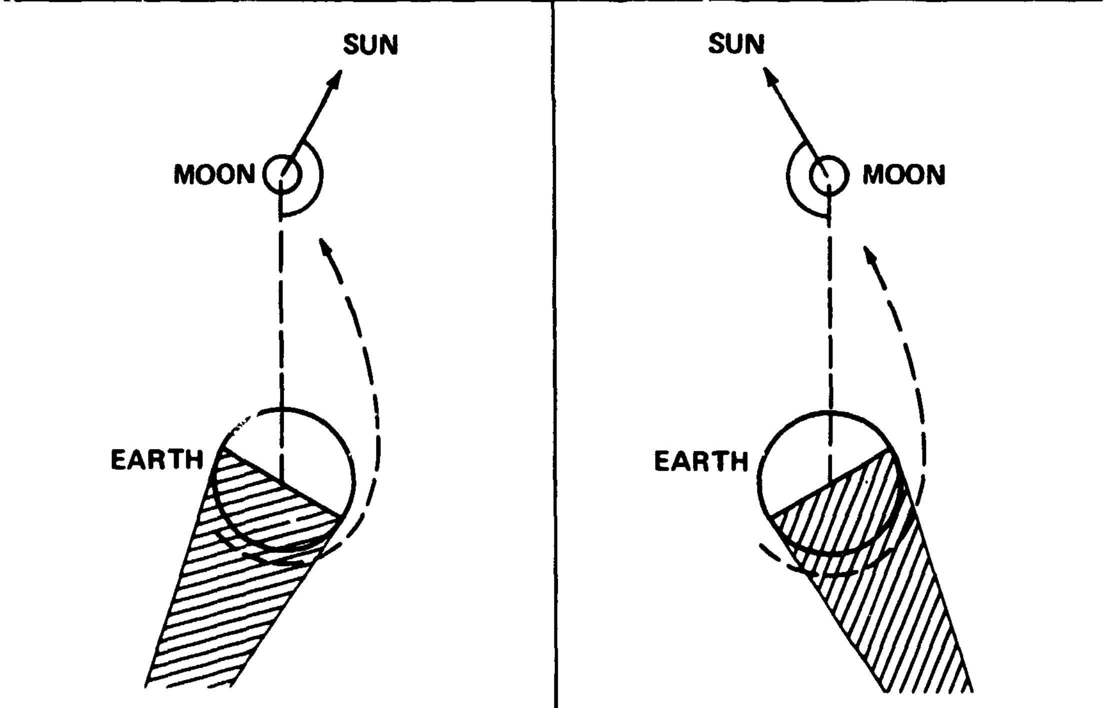
where

$\vec{r}$ = position vector of spacecraft
$\vec{v}$ = velocity vector of spacecraft
$\hat{z}$ = unit normal to fundamental plane.



(a) NO SHADOW

(b) POSSIBLE TRANSFER SHADOW



(c) PARK AND INJECTION SHADOW

(d) PARK AND INJECTION SHADOW

Figure 9. Possible Shadow Situations

But, as the arccosine function is double-valued, two possible approach trajectories will have the required inclination. In figure 10 we see the two possible retrograde approach trajectories for a given inclination. Similarly there are two possible posigrade approach trajectories for a given inclination. The approach trajectories can be referred to as "northwestern", "southwestern", "northeastern", and "southeastern" depending upon whether they are retrograde or posigrade and whether they approach from above or below the Moon's equator.

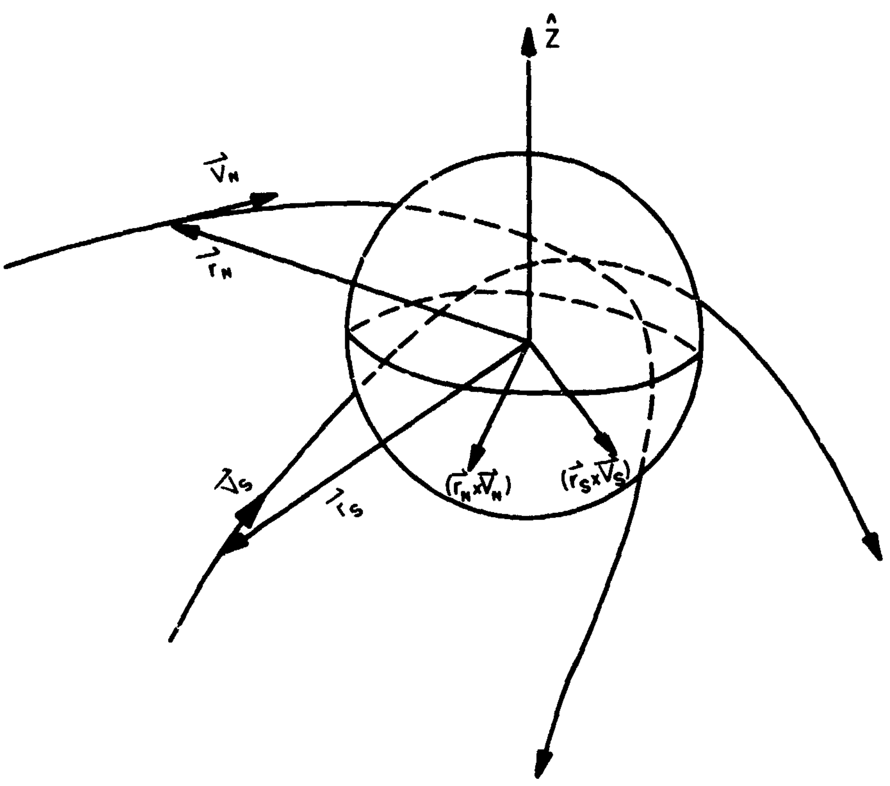


Figure 10. Northern and Southern Retrograde Lunar Approach Trajectories

VI. VARIATION OF TRAJECTORY PARAMETERS DURING THE MONTH

It is interesting to examine the launch, transfer, and arrival parameters as they vary with launch date and correlate them to the lunar ephemeris data. Tables I, II, III, IV give all pertinent data for the month of July for the four possible lunar approaches. For the present consider only Table I, which was derived for the nominal RAE-B flight time, 110 hours, and the northern retrograde approach. The data can be understood and indeed predicted from information already available. Refer to figures 5 and 6, the lunar ephemeris information. During the first part of July, the Moon is on its descending leg. Similarly the vehicle in the low inclination transfer trajectory at arrival will be on its descending leg. The transfer angle is 180° and therefore injection is on the ascending leg. But for a 90° launch azimuth, the park orbit does not reach its minimum declination and begin its ascending leg until the second half of the park orbit, and thus the coast time should be very long. As seen in the table, this is indeed the case.

The Moon is approaching its first quarter phase during early July. As we saw previously the constraint on spin axis-Sun angle prohibits launch during this period. This is verified by the table. The long park orbit coast and the phase of the Moon demand a long shadow, and it does exist. Sunlit orbit time is related to lunar phase at launch; recalling the acceptable lunar phases for launch to satisfy the constraint on sunlit orbit time, it is seen that this launch period is unacceptable. For launches between the first and seventh day of the month, arrival would be between the sixth and twelfth days. The Earth-Moon distance at arrival is fairly great and thus C_3, and ΔV_c, should be acceptable, and they are.

The Moon reaches its minimum declination around July 13. Therefore, for a launch around the eighth or ninth, injection would be near the maximum declination, and the coast time would be very short. There should be no park orbit shadows shortly after first quarter. But as full moon approaches, the Earth's shadow extends toward the Moon and transfer shadows of varying durations occur.

Acceptable launch dates for the spin axis-Sun angle constraint are between first quarter and 2-1/2 days before full moon, or between July 7 and about July 12, as verified in the table. Sunlit orbit time is acceptable from July 9 through July 13. Notice that $|\vec{R}_{M-E}|$ is decreasing and ΔV_c is correspondingly increasing.

Table I

Mission Parameters for Northwestern Approach

Launch Date	Launch Time (GMT)	Park Orbit Coast Time (Sec)	Spin Axis-Sun Angle (Deg)	Shadow			Lunar Phase Angle at Arrival (Deg)	Sunlit Orbit Time (Days)	ΔV for Circular Orbit (km/sec)
				Park (Min)	Injection (Min)	Transfer (Hrs)			
7-01-73	16:12	3816	6	41	0.0	0.0	-102	- 41	0.711
-03-	16:23	4134	20	41	0.0	0.0	- 79	- 74	0.701
-05-	16:48	4388	43	41	0.0	0.0	- 56	-106	0.697
-07-	17:52	4501	65	41	0.0	0.0	- 34	30	0.700
-09-	14:09	239	85	0	0.0	0.0	- 14	51	0.701
-11-	14:37	474	107	0	0.0	11.8	8	78	0.706
-13-	14:49	769	129	0	0.0	2.8	31	104	0.711
-15-	14:55	1089	152	0	0.0	1.3	54	- 23	0.715
-17-	15:03	1420	176	0	0.0	0.8	78	- 49	0.719
-19-	15:19	1739	158	3	31.0	0.0	104	- 76	0.723
-21-	16:21	1930	131	10	20.0	0.0	131	-102	0.731
-23-	13:48	2821	105	16	15.0	0.0	158	27	0.735
-25-	14:09	3173	77	23	9.0	0.0	-174	59	0.735
-27-	14:18	3552	50	30	5.0	0.0	-147	91	0.727
-29-	14:26	3910	23	37	2.0	0.0	-121	- 17	0.714

Table II

Mission Parameters for Southwestern Approach

Launch Date	Launch Time (GMT)	Park Orbit Coast Time (Sec)	Spin Axis-Sun Angle (Deg)	Shadow			Lunar Phase Angle at Arrival (Deg)	Sunlit Orbit Time (Days)	ΔV for Circular Orbit (km/sec)
				Park (Min)	Injection (Min)	Transfer (Hrs)			
7-01-73	15:56	3868	5	41	0.0	0.0	-102	- 40	0.711
-03-	16:04	4196	20	41	0.0	0.0	- 79	- 73	0.701
-05-	16:22	4472	43	41	0.0	0.0	- 56	-104	0.697
-07-	17:21	4606	65	41	0.0	0.0	- 34	26	0.699
-09-	14:35	152	84	0	0.0	0.0	- 14	56	0.701
-11-	14:56	411	106	0	0.0	11.7	8	80	0.706
-13-	15:04	716	129	0	0.0	2.7	31	104	0.711
-15-	15:10	1039	152	0	0.0	1.3	54	- 24	0.715
-17-	15:20	1363	175	0	0.0	0.8	78	- 50	0.718
-19-	15:42	1661	158	3	31.0	0.0	104	- 77	0.724
-21-	16:53	1822	131	10	20.0	0.0	132	1	0.733
-23-	13:23	2904	105	15	14.0	0.0	158	25	0.735
-25-	13:51	3230	77	24	9.0	0.0	-174	57	0.735
-27-	14:03	3603	49	30	5.0	0.0	-147	90	0.727
-29-	14:10	3964	23	37	2.0	0.0	-122	- 17	0.715

Table III

Mission Parameters for Northeastern Approach

Launch Date	Launch Time (GMT)	Park Orbit Coast Time (Sec)	Spin Axis-Sun Angle (Deg)	Shadow			Lunar Phase Angle at Arrival (Deg)	Sunlit Orbit Time (Days)	ΔV for Circular Orbit (km/sec)
				Park (Min)	Injection (Min)	Transfer (Hrs)			
7-01-73	16:11	3814	6	41	0.0	0.0	-102	-28	0.710
-03-	16:22	4132	20	41	0.0	0.0	- 79	-52	0.699
-05-	16:47	4385	42	41	0.0	0.0	- 56	-75	0.695
-07-	17:52	4497	65	41	0.0	0.0	- 34	18	0.698
-09-	14:08	235	84	0	0.0	0.0	- 14	38	0.698
-11-	14:36	471	106	0	0.0	12.0	8	57	0.703
-13-	14:48	766	129	0	0.0	2.8	31	76	0.708
-15-	14:54	1087	152	0	0.0	1.2	54	-16	0.713
-17-	15:02	1419	175	0	0.0	0.8	78	-35	0.716
-19-	15:17	1739	159	3	31.0	0.0	104	-55	0.722
-21-	16:21	1929	131	10	20.0	0.0	131	-77	0.730
-23-	13:48	2817	106	16	14.0	0.0	158	17	0.734
-25-	14:09	3171	77	23	9.0	0.0	-174	41	0.734
-27-	14:17	3551	50	30	5.0	0.0	-147	65	0.727
-29-	14:26	3909	24	37	2.0	0.0	-121	-11	0.713

Table IV

Mission Parameters for Southeastern Approach

Launch Date	Launch Time (GMT)	Park Orbit Coast Time (Sec)	Spin Axis-Sun Angle (Deg)	Shadow			Lunar Phase Angle at Arrival (Deg)	Sunlit Orbit Time (Days)	ΔV for Circular Orbit (km/sec)
				Park (Min)	Injection (Min)	Transfer (Hrs)			
7-01-73	15:55	3867	6	41	0.0	0.0	-102	-29	0.710
-03-	16:03	4195	20	41	0.0	0.0	- 79	-53	0.699
-05-	16:21	4471	42	41	0.0	0.0	- 56	-76	0.695
-07-	17:20	4604	65	41	0.0	0.0	- 34	21	0.696
-09-	14:35	147	84	0	0.0	0.0	- 14	37	0.698
-11-	14:56	407	106	0	0.0	12.1	8	56	0.703
-13-	15:04	713	129	0	0.0	2.8	31	75	0.708
-15-	15:10	1036	152	0	0.0	1.3	54	-15	0.712
-17-	15:19	1360	175	0	0.0	0.8	78	-34	0.716
-19-	15:42	1659	158	3	31.0	0.0	104	-54	0.722
-21-	16:53	1819	131	10	20.0	0.0	132	-74	0.733
-23-	13:23	2901	105	16	14.0	0.0	158	19	0.735
-25-	13:51	3230	77	23	9.0	0.0	-174	41	0.735
-27-	14:02	3602	50	30	5.0	0.0	-147	66	0.727
-29-	14:09	3963	24	37	2.0	0.0	-122	-14	0.714

For arrivals when the Moon is on its ascending leg and near zero declination, injection would be on the descending leg of the park orbit and also near zero declination. Therefore, around July 19, the park orbit coast time would be about 1/4 of a period or about 1100-1200 seconds. Launches between full moon and third quarter do not satisfy sunlit orbit time constraints, $|\vec{R}_{M-E}|$ is near minimum at arrival and ΔV_c is unacceptably large.

For launches between July 19 and July 29 the park orbit coast time is rather long, and for the lunar phase shown, park orbit shadows are unavoidable. $|\vec{R}_{M-E}$ is small and ΔV_c is rather large.

How do the launch and arrival parameters vary as one targets to the southwestern sector of the Moon? Intuitively one would expect little if any difference, as a very small perturbation in the launch parameters should suffice to satisfy the new target parameters. This is immediately verified upon comparison of tables I and II. Similarly, it is expected that only a small perturbation in launch parameters can target a trajectory to the two posigrade, i.e., the northeastern and southeastern approach. The essential difference in the retrograde and posigrade orbits is in their sunlit orbit time, resulting from the difference in nodal regression rates for the two; see tables III and IV.

The preceding tables were derived for the low inclination transfer trajectory; similar information for the high inclination northwestern approach is presented in Table V. The data change so minutely for the other three targetings that it suffices to study any one. The launch time and park orbit coast time are expectedly quite different. A very long park orbit coast in the low inclination transfer is complemented by a very short park orbit coast in the high, and vice versa. The later launch time implies a different spin axis-Sun angle, and the short park orbit implies a short park orbit shadow. There should be very little difference in sunlit orbit time. However, ΔV_c, as discussed before, is much greater than for the low. When arrival occurs when the Moon is near its maximum or minimum declination the inclination of the two transfer trajectories are equal and thus the ΔV_c's are also equal. For arrival at the Moon's minimum declination, July 12, launch would be on July 7, and the ΔV_c's are indeed equal. The same holds for July 21 launches also.

For completeness the effect of varying flight time and launch azimuth is presented in Tables VI, VII, VIII, IX, and X for a typical time frame, July 9 through July 13. Launch and arrival parameters for flight times of 90 hrs., 110 hrs., and 130 hrs. and launch azimuths of 70°, 90° and 110° are compared for each date. The most salient points are that the longer flight times have lower arrival energy; and for a given flight time, launch azimuth affects the inclination of the transfer and thus the ΔV_c. Perhaps the most important

Table V

Mission Parameters for Northwestern Approach, High Inclination Transfer

Launch Date	Launch Time (GMT)	Park Orbit Coast Time (Sec)	Spin Axis-Sun Angle (Deg)	Shadow			Lunar Phase Angle at Arrival (Deg)	Sunlit Orbit Time (Days)	ΔV for Circular Orbit (km/sec)
				Park (Min)	Injection (Min)	Transfer (Hrs)			
7-01-73	4:52	902	53	15	0.0	0.0	-109	- 28	0.751
-03-	7:49	569	48	2	0.0	0.0	- 83	- 53	0.729
-05-	10:38	293	50	0	0.0	0.0	- 59	- 97	0.709
-07-	12:56	154	64	0	0.0	0.0	- 37	23	0.700
-09-	20:00	4390	87	41	0.0	0.0	- 11	60	0.713
-11-	22:44	4130	104	41	0.0	0.0	12	89	0.732
-13-	00:08	3974	109	37	0.0	0.0	24	103	0.740
-15-	2:58	3636	119	26	0.0	0.0	48	- 24	0.751
-17-	5:57	3276	129	11	0.0	0.0	74	- 51	0.750
-19-	9:16	2915	141	3	21.0	0.0	101	- 78	0.740
-21-	12:26	2673	132	10	20.0	0.0	130	-105	0.732
-23-	19:24	1739	101	17	13.0	0.0	161	26	0.750
-25-	23:02	1365	80	23	9.0	0.0	169	59	0.768
-27-	00:44	1170	72	20	7.0	0.0	-155	78	0.769
-30-	3:55	800	59	13	4.0	0.0	-127	- 6	0.753

Table VI

Mission Parameters as a Function of Flight Time and Launch Azimuth for Launch on 7-09-73

7-09-73		Launch Time (GMT)	Park Orbit Coast Time (sec)	Spin Axis Sun Angle (deg)	Shadow Time			Lunar Phase Angle (deg)	Sunlit Orbit Time (days)	ΔV for Circular Orbit (km/sec)
Flight Time	Launch Azimuth (deg)				Park (min)	Injection (min)	Transfer (hrs)			
90 hrs	70	10:12	861	77	0	0	0	-25	56	0.741
	90	13.43	212	77	0	0	0	-24	59	0.737
	110	15:23	5208	79	41	0	0	-22	60	0.741
110 hrs	70	10:47	861	84	0	0	0	-16	48	0.704
	90	14:09	239	85	0	0	0	-14	52	0.701
	110	15.55	5216	86	41	0	0	-13	52	0.704
130 hrs	70	11:14	894	92	0	0	0	- 7	45	0.700
	90	14:27	300	93	0	0	0	- 5	47	0.698
	110	16:21	5255	94	41	0	0	- 4	48	0.701

Table VII

Mission Parameters as a Function of Flight Time and Launch Azimuth for Launch on 7-10-73

7-10-73		Launch Time (GMT)	Park Orbit Coast Time (sec)	Spin Axis Sun Angle (deg)	Shadow Time			Lunar Phase Angle (deg)	Sunlit Orbit Time (days)	ΔV for Circular Orbit (km/sec)
Flight Time	Launch Azimuth (deg)				Park (min)	Injection (min)	Transfer (hrs)			
90 hrs	70	10:48	907	87	0	0	0	-14	70	0.742
	90	14:09	287	88	0	0	0	-13	73	0.739
	110	15:56	5263	90	41	0	0	-11	73	0.742
110 hrs	70	11:15	935	94	0	0	0	- 5	62	0.705
	90	14:26	345	96	0	0	0	- 3	65	0.703
	110	16:21	5296	97	41	0	0	- 2	65	0.706
130 hrs	70	11:36	988	102	0	0	17.5	5	58	0.703
	90	14:39	426	104	0	0	14.7	6	61	0.701
	110	16:40	56	105	0	0	12.0	7	61	0.703

Table VIII

Mission Parameters as a Function of Flight Time and Launch Azimuth for Launch on 7-11-73

7-11-73		Launch Time (GMT)	Park Orbit Coast Time (sec)	Spin Axis Sun Angle (deg)	Shadow Time			Lunar Phase Angle (deg)	Sunlit Orbit Time (days)	ΔV for Circular Orbit (km/sec)
Flight Time	Launch Azimuth (deg)				Park (min)	Injection (min)	Transfer (hrs)			
90 hrs	70	11:15	984	98	0	0	0	- 3	83	0.743
	90	14.25	397	99	0	0	2.9	- 1	86	0.741
	110	16:21	5346	100	41	0	15.0	0	87	0.744
110 hrs	70	11:36	1032	105	0	0	12.5	7	75	0.707
	90	14:37	474	107	0	0	11.4	8	78	0.706
	110	16:40	99	107	0	0	9.3	9	78	0.708
130 hrs	70	11:53	1100	113	0	0	4.1	16	71	0.705
	90	14:46	568	115	0	0	6.0	17	74	0.704
	110	16:56	170	116	0	0	3.7	18	74	0.706

Table XI

Mission Parameters as a Function of Flight Time and Launch Azimuth for Launch on 7-12-73

7-12-73		Launch Time (GMT)	Park Orbit Coast Time (sec)	Spin Axis Sun Angle (deg)	Shadow Time			Lunar Phase Angle (deg)	Sunlit Orbit Time (days)	ΔV for Circular Orbit (km/sec)
Flight Time	Launch Azimuth (deg)				Park (min)	Injection (min)	Transfer (hrs)			
90 hrs	70	11:35	1083	108	0	0	9.5	8	98	0.745
	90	14:35	528	110	0	0	8.9	10	99	0.743
	110	16:39	152	111	0	0	7.5	11	99	0.745
110 hrs	70	11:52	1145	116	0	0	4.0	18	88	0.710
	90	14:44	617	118	0	0	5.0	19	91	0.709
	110	16:55	216	118	0	0	3.5	20	91	0.710
130 hrs	70	12:07	1223	125	0	0	2.5	27	84	0.709
	90	14:51	718	126	0	0	3.1	29	86	0.708
	110	17:10	295	127	0	0	2.3	30	87	0.709

Table X

Mission Parameters as a Function of Flight Time and Launch Azimuth for Launch on 7-13-73

7-13-73		Launch Time (GMT)	Park Orbit Coast Time (sec)	Spin Axis Sun Angle (deg)	Shadow Time			Lunar Phase Angle (deg)	Sunlit Orbit Time (days)	ΔV for Circular Orbit (km/sec)
Flight Time	Launch Azimuth (deg)				Park (min)	Injection (min)	Transfer (hrs)			
90 hrs	70	11:51	1198	120	0	0	3.7	20	- 3	0.746
	90	14:42	673	121	0	0	4.2	21	- 5	0.745
	110	16:54	269	122	0	0	3.3	22	- 6	0.746
110 hrs	70	12:06	1270	127	0	0	2.4	30	- 5	0.712
	90	14:49	769	129	0	0	2.8	31	103	0.711
	110	17:08	342	130	0	0	2.2	32	104	0.713
130 hrs	70	12:19	1355	136	0	0	1.4	39	97	0.712
	90	14:55	875	138	0	0	1.9	40	100	0.711
	110	17:22	428	138	0	0	1.6	41	100	0.712

observation is that for the 90 hr. flight time, the ΔV_c requirement is much greater than for the longer flights and is prohibitively high for the RAE-B mission, at least during this time frame.

VII. LAUNCH OPPORTUNITIES FOR THE POSIGRADE MISSION

The RAE-B mission constraints have been related to the position and phase of the Moon and thus acceptable launch opportunities can be determined given only the lunar ephemeris date. Launch opportunities for the retrograde lunar orbit have been published previously (Ref. 5); posigrade launch opportunities will now be determined utilizing the principles and procedures developed in this paper. The launch period under consideration is from March through December 1973. It is further assumed that:

- **the flight time from translunar injection to closest approach to the Moon is 110 hrs.,**
- **all rocket engine burns are impulsive and in plane,**

- the vehicle's spin axis-Sun angle is coincident with its velocity vector at translunar injection,
- the park orbit is less than one revolution,
- the powered flight ascent from the launch pad to park orbit insertion takes 600 seconds and burns through an Earth-fixed central angle of 21°.

To maximize payload capability the 90 degree launch azimuth is selected. The circular lunar orbit will be inclined 63.5 degrees to the Moon's equator to minimize eccentricity growth, and will be at an altitude of 1100 km. Reviewing the mission constraints and their associated acceptable lunar phases at launch.

Mission Constraint	Lunar Ephemeris Data at Launch
$60^\circ \leq$ spin axis-Sun angle $\leq 120^\circ$	first quarter $\lesssim$ lunar phase $\lesssim$ full moon - 2-1/2 days
	third quarter $\lesssim$ lunar phase $\lesssim$ new moon - 2-1/2 days
sunlit orbit time $\geq$ 50 days	first quarter + 3 days $\lesssim$ lunar phase $\lesssim$ full moon - 2 days
	third quarter + 3 days $\lesssim$ lunar phase $\lesssim$ new moon - 2 days
$\Delta V_c \leq 0.720$ km/sec	$\lvert\vec{R}_{M-E}\rvert$ near maximum
short park orbit coast	Moon on ascending leg
no park orbit shadows	full moon

Consider first the month of March. The Moon is near its new moon phase in the early part of the month and park orbit shadows result; no launch opportunities are available. Near mid month when the Moon is approaching new Moon, it is also on its descending leg; injection into the translunar trajectory would be on the ascending leg of the park orbit with the result that the park orbit would be too long. In the latter part of the month $\lvert\vec{R}_{M-E}\rvert$ is too small and ΔV_c is unacceptably high. There are no launch possibilities then in March 1973.

Now consider April. The early month is prohibited as park shadows exist since the Moon is near new moon. Near mid month the spin axis-Sun angle is within its acceptable bounds. At arrival the Moon will be near its minimum

Table XI

Posigrade Launch Opportunities

Launch Date	Launch Time (GMT)	Park Orbit Coast Time (Sec)	Spin Axis-Sun Angle (Deg)	Shadow			Lunar Phase Angle at Arrival (Deg)	Sunlit Orbit Time (Days)	ΔV for Circular Orbit (km/sec)
				Park (Min)	Injection (Min)	Transfer (Hrs)			
4-14-73	16:04	273	111	0.0	0.0	0.0	19	66	0.714
5-12-73	15:07	210	96	0.0	0.0	0.0	0	50	0.708
-13-	16:16	143	108	0.0	0.0	0.0	12	60	0.703
-14-	17:07	139	119	0.0	0.0	0.0	23	70	0.700
6-11-73	15:41	177	102	0.0	0.0	0.0	4	54	0.700
-12-	16:05	262	113	0.0	0.0	0.0	15	64	0.700
8-09-73	12:58	714	99	0.0	0.0	0.0	1	50	0.708
-10-	13:02	872	110	0.0	0.0	0.0	12	59	0.712
9-08-73	11:20	1139	103	0.0	0.0	0.0	5	51	0.717

Table XII

Celestial Coordinates of Orbit Normal

Launch Date	a	δ
4-14-73	-118.08	7.18
5-12-73	-110.49	4.54
-13-	- 99.56	3.98
-14-	- 88.61	3.31
6-11-73	- 81.05	3.34
-12-	- 70.15	4.22
8-09-73	- 34.85	12.87
-10-	- 24.34	16.99
9-08-73	- 7.05	24.30

declination, and injection would then occur near maximum declination of the park orbit resulting in a very short park orbit coast; and $|R_{M-E}|$ is near its maximum, so ΔV_c should be acceptable. In the latter month there will be park orbit shadow. Thus mid-April should be acceptable. Computer simulation confirms that April 14 is indeed an acceptable launch opportunity.

Continuing in this fashion for each month one can generate possible launch opportunities for the entire year. Of course, a computer program should be used to verify, and quantify, the results. Table XI presents the launch opportunities with all pertinent data for the 1973 posigrade mission.

The plane of the lunar approach trajectory can be oriented inertially by giving the celestial coordinates of the normal to the plane. In a given coordinate system.

$$\alpha = \Omega - 90^\circ$$

$$\delta = -(i - 90^\circ)$$

where

α = right ascension of normal

δ = declination of normal

Ω = right ascension of the ascending node

i = inclination.

Table XII presents this data for each acceptable launch date, in the mean Earth equator and equinox of date coordinate system.

VIII. CONCLUSION

It has been shown that if mission constraints for Earth-to-Moon missions can be related, directly or indirectly, to lunar ephemeris data such as lunar declination, lunar phase, and Earth-Moon distance, then approximate launch dates, and even approximate values for initial launch and arrival parameters, can be determined. Indeed the mission analyst's job can be greatly simplified and considerable computer time saved.

This also allows more insight into the problem of determining launch opportunities for RAE-B.

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