FLUID FLOW ANALYSIS OF A HOT-CORE HYPersonic-WIND-TUNNEL NOZZLE CONCEPT

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SUMMARY

A hypersonic-wind-tunnel nozzle concept which incorporates a hot-core flow surrounded by an annular flow of cold air offers a promising technique for maximizing the model size while minimizing the power required to heat the test core. This capability becomes especially important when providing the true-temperature duplication needed for hypersonic propulsion testing. Several two-dimensional wind-tunnel nozzle configurations that are designed according to this concept are analyzed by using recently developed analytical techniques for prediction of the boundary-layer growth and the mixing between the hot and cold coaxial supersonic airflows. The analyses indicate that introduction of the cold annular flow near the throat results in an unacceptable test core for the nozzle size and stagnation conditions considered because of both mixing and condensation effects. Use of a half-nozzle with a ramp on the flat portion does not appear promising because of the thick boundary layer associated with the extra length. However, the analyses indicate that if the cold annular flow is introduced at the exit of a full two-dimensional nozzle, an acceptable test core will be produced. Predictions of the mixing between the hot and cold supersonic streams for this configuration show that mixing effects from the cold flow do not appreciably penetrate into the hot core for the large downstream distances of interest.

INTRODUCTION

Use of a hot-core wind-tunnel nozzle concept offers a promising technique for maximizing the size model that can be tested in a heated-flow wind tunnel with a minimum power required to heat the flow. This capability is especially important in hypersonic air-breathing propulsion testing where true-temperature duplication of flight conditions is needed and where, for complete engines, there is a minimum-size model that can be realistically tested. A large test section with a high mass flow and a high energy content of the test air is thus required. In free-jet testing in a hypersonic engine facility, the engine cross section might typically be about 25 percent of the test-section cross-sectional
area. The hot-core concept should increase this percentage by a factor of 2 or 3. The basic concept consists of a coaxial flow with a hot center core surrounded by an unheated annular stream. Most of the hot core is ingested by the engine, and the unheated exterior flow alleviates tunnel choking and shock-interference effects.

Although the heated-core supersonic-stream concept has been studied previously (refs. 1 and 2), the analytical methods for evaluating the flows have vastly improved in recent years. The present analytical investigation was conducted to evaluate the hot-core concept for a nozzle design to be used in a small-hypersonic-engine aerothermo-dynamic research facility. A two-dimensional flow is considered since scramjet engines with a rectangular cross section are currently of special interest. The analysis includes nozzle-wall contouring, condensation effects, viscous mixing of the hot and cold streams, and boundary-layer growth on the nozzle walls. Wall-temperature effect on boundary-layer growth is also analyzed. Mach 6.6 flight simulation is considered, with a Mach 6 condition existing at the engine entrance. The forward fuselage of the airplane is assumed to decrease the local Mach number to 6 at the engine inlet while the stagnation-enthalpy conditions are those for flight at Mach 6.6. All the calculations are for a stagnation temperature of 2200 K and a stagnation pressure of \(30.4 \times 10^5\) N/m\(^2\). The nozzle size considered in this analysis is a center core 0.305 meter by 0.279 meter and a surrounding unheated flow that is 0.127 meter thick on three sides. Including the calculated boundary-layer displacement thickness, the power in the airstream of the hot central core is 6.3 megawatts. If the total airstream were heated, the power in the stream would exceed 20 megawatts. These power considerations make further effort to develop the hot-core concept attractive.

Several nozzle configurations are analyzed. These configurations include a half two-dimensional nozzle (one surface contoured) with a ramp on the bottom surface and a full two-dimensional nozzle (two surfaces contoured). Mixing analyses are performed for the cold annular flow introduced both at the nozzle throat, since the previous studies (refs. 1 and 2) of hot-core flows used this configuration, and at the nozzle exit.

With the introduction of the cold annular flow at the nozzle throat, condensation effects become important and are analyzed along with effects of supercooled walls to inhibit boundary-layer growth. When annular flow is injected at the exit, condensation effects need not be considered, and the mixing analysis indicates that this exit injection location is especially attractive.

**SYMBOLS**

\(H\) \hspace{1cm} enthalpy

\(k\) \hspace{1cm} constant

2
M  Mach number
NPr  Prandtl number
NSc  Schmidt number
p  pressure
Rθ  Reynolds number based on momentum thickness
r*  one-half the throat height
T  temperature
u  velocity
x  horizontal distance along axis
y  vertical distance
z  mixing-zone height
δ*  boundary-layer displacement thickness
η  transformed y-coordinate
μ  viscosity
ξ  transformed x-coordinate
ρ  density
(ρε)_t  eddy viscosity

Subscripts:
o  stagnation conditions for hot-core flow
w  wall conditions
free stream for hot-core flow

Superscript:

* throat conditions for hot-core flow

METHODS OF CALCULATION

Nozzle Contouring

The numerical technique for determining the contours for a parallel-flow nozzle used the method of characteristics for a rotational gas mixture whose stagnation enthalpy is allowed to vary normal to the streamlines (refs. 3 and 4). Although chemistry effects are minor for the relatively low stagnation temperatures of this investigation, the analysis considered frozen chemistry and neglected effects of diffusion. This is a reasonable approximation since quasi-one-dimensional calculations have shown that the chemical composition of air is essentially frozen very early in the nozzle expansion (ref. 5). Properties for the chemical species of air as functions of temperature, as needed for this method, are presented in reference 6. The initial uniform profiles for starting the nozzle contouring program are calculated by using the nonequilibrium techniques of reference 5 to determine sonic conditions.

Mixing Analysis

A viscous method-of-characteristics solution was used to analyze the mixing between the hot core and the cold supersonic streams. This method is based on a technique which considers pressure gradients both along and normal to streamlines (ref. 7). The effect of transport properties is assumed to be a function only of gradients normal to the streamlines. The approach has been applied to the analyses of viscous flow problems in the presence of walls in reference 8. The systems of equations for this analysis are presented in reference 9, and a FORTRAN program which calculates the nonuniform supersonic flow with diffusive and dissipative effects within a two-dimensional nozzle is also discussed in reference 9.

In the original program the turbulent viscosity was assumed to be constant throughout the flow field and was arbitrarily set equal to 1000 times the laminar value at the entrance section. For a more realistic second approach, the program was modified by replacing the constant viscosity term with an eddy-viscosity expression which varied in both the axial or horizontal (x) and vertical (y) directions.

The eddy-viscosity model used for this mixing analysis was developed in reference 10 and is given by the equation
\[(\rho e)_t = kz\rho u_\infty\]  
(1)

where \((\rho e)_t\) is the eddy viscosity, \(\rho\) is the density, \(u_\infty\) is the free-stream velocity for the hot core, and \(k\) is an empirical constant which is equal to 0.01 for the conditions of this analysis. The height of the mixing zone \(z\) is defined as a vertical distance between two arbitrarily defined velocities in the velocity profile, as specified in reference 10.

The initial values of viscosity therefore are determined by the initial profile input to the program, and the viscosity changes as the profile changes downstream. This eddy-viscosity model was satisfactorily used in reference 10 to correlate air-air mixing data.

**Boundary-Layer Analysis**

Solutions of nonsimilar laminar and turbulent boundary-layer equations including multicomponent reacting gases and transverse curvature effects were used in the boundary-layer flow analysis. These solutions (ref. 11) use a modification of the Levy-Lees transformation of the equations of motion to the \((\xi,\eta)\) coordinate plane, with the conservation equations integrated across boundary-layer strips. Derivatives in the normal direction are expressed by Taylor series truncated to reflect a cubic approximation, and streamwise derivatives are expressed in a finite-difference form. The resultant set of equations is solved by a general Newton-Raphson iteration. Reference 11 presents the system of equations describing this fluid flow in a boundary layer, and a FORTRAN program which presents the boundary-layer integral matrix procedure is given in reference 12. This analysis assumes local chemical equilibrium conditions.

The turbulent analysis employs a variable-mixing-length model for eddy viscosity near the wall and global parameters of the flow in the outer portion of the boundary layer. For turbulent flow, the time-averaged equations of motion are solved by using the eddy-viscosity expression to determine the Reynolds stress term, along with constant turbulent Prandtl and Schmidt numbers in the energy and species conservation equations. The program uses the laminar equations until a preassigned transition point is reached; then turbulent equations are used. The boundary-layer solutions are started at the entrance of the converging portion of the nozzle, and the step size along the wall is varied throughout the nozzle depending on the pressure gradient at each particular point. The inputs required for computation were \(p_0, H_0, T_\infty,\) turbulent \(Np_T\) and \(Nsc, R_\theta,\) the pressure distribution along the wall, and the initial chemical composition.

**RESULTS AND DISCUSSION**

The two wind-tunnel nozzle concepts analyzed are shown in figures 1 and 2. Both designs include a heated core in a two-dimensional supersonic stream. The annular cold
air is introduced at the throat in the arrangement shown in figure 1 and at the test section for the nozzle shown in figure 2. Another difference between concepts is the use of a half-nozzle design in figure 1, with a ramp placed on the bottom flat surface near the exit, and the use of a full two-dimensional nozzle without ramp in figure 2. The consideration of the half-nozzle with its ramp was intended to provide more testing flexibility since interchangeable ramps, which simulate the forward fuselage of a hypersonic aircraft ahead of the engine, could be used. A Mach number of 6.6 was considered for the half-nozzle concept, with the ramp decelerating the flow to Mach 6 at the exit. The full two-dimensional nozzle of figure 2 expanded the flow directly to Mach 6. A design study indicated no appreciable difference in allowable engine size with or without the ramp as long as the final Mach numbers were identical.

**Throat Injection of Cold Air**

**Mixing analysis.**—The coordinates for the half-nozzle with throat injection were generated from the method of reference 4 by assuming constant initial profiles of temperature, pressure, and velocity. The reference conditions correspond to a Mach 6.6 airstream with $p_0 = 30.4 \times 10^5 \text{N/m}^2$ and $T_0 = 2200 \text{K}$. The initial profiles were set equal to the sonic throat conditions as determined from the nonequilibrium flow analysis of reference 5. The initial profiles shown in figure 3(a) along with the contour coordinates are then used as input to the analysis of reference 9. These initial profiles consist of a central core of hot air surrounded by an outer layer of cold air and are nondimensionalized by dividing the values of temperature and velocity by reference exit values based on hot-flow center-line conditions also computed from the analysis of reference 5. The initial profiles were expanded through the designed nozzle by using the viscous method-of-characteristics technique with frozen chemistry. Both Prandtl and Lewis numbers were assumed equal to unity.

Profiles at the exit plane are shown in figure 3(b) for the eddy-viscosity model given by equation (1), with the eddy viscosity varying in both the x- and y-directions. These results are compared with the exit profiles of figure 3(c) for the turbulent viscosity assumed constant and equal to 500 times the laminar value at the throat. For both of these cases, the mixing between the hot and cold flows is significant and the resulting exit profiles are of poor quality for engine aerothermodynamic testing.

**Condensation analysis.**—A nozzle which has the cold annular flow introduced at the throat (fig. 1) will encounter condensation problems in the unheated annular flow. For cold-flow stagnation conditions of $p_0 = 30.4 \times 10^5 \text{N/m}^2$ and $T_0 = 297 \text{K}$, the air would begin to condense when Mach 4 conditions are reached in the nozzle expansion. This effect is shown in figure 4 where saturation curves for both air and two isentropes for expanding air are shown on a pressure-temperature diagram. The air saturation and
Supersaturation curves shown in this figure were taken from the data of references 13 and 14. Heating the cold annular air to 500 K would avoid the condensation problem as shown by the 500 K isentrope in figure 4; however, this procedure incurs additional expense (2 megawatts of power would be required for a 12.7-cm-thick annular flow at the test section).

In order to visualize the propagation of condensation disturbances in the cold annular flow, figure 5 shows a sketch of the half-nozzle with the paths of the disturbances as they would occur if they traveled along Mach waves and originated in the cold annular flow downstream of the Mach 4 region. The curvature of these disturbance paths was determined by adjusting the slopes of each line to the varying Mach angle as it passed through the streamlines calculated by the mixing analysis of reference 9. Streamlines are not shown in this figure. In reality, a large number of infinitesimal disturbances will propagate through the flow and quickly penetrate to the ramp and affect the entire test stream.

Test-Section Injection of Cold Air

Because of the expected condensation effects and the distortion of the flow profiles due to mixing between the hot and cold flows, the nozzle design of figure 1 does not appear promising and the design of figure 2 appears capable of producing better flow profiles although some of the testing versatility might be lost. When the cold flow is introduced at the nozzle exit as in figure 2, the Mach number of the cold annular flow can be less than 4 to avoid condensation, and the stagnation pressure of the cold flow can be chosen so that the static pressures at the interface between hot and cold flows are identical. The analysis of the suitability of this nozzle concept is therefore based on these conditions.

Boundary-layer analysis.- Pressure-ratio distributions computed along the contoured wall and along the center line of the sidewall of a full two-dimensional Mach 6 nozzle by using the technique of reference 4 are shown in figure 6. These pressure distributions are used as input for the boundary-layer analysis of references 11 and 12 along with \( p_0 \), \( H_0 \), \( N_{Pr} \), \( N_{Sc} \), \( T_w \), and the transition Reynolds number based on momentum thickness. The momentum-thickness Reynolds number was computed along the center line of the sidewall of this nozzle and is presented in figure 7. The transition from laminar to turbulent flow is indicated to be in the range of \( R_\theta \) between 600 and 1000 for the Mach numbers and wall-to-total-temperature ratios of this nozzle (refs. 15 and 16); the flow would therefore become turbulent just downstream of the throat. For the present analysis, transition is assumed to occur at \( R_\theta = 600 \); however, because of the steep gradient of this quantity in the transition region of the nozzle (fig. 7), this choice has only minor effects on the boundary-layer growth.
The computed boundary-layer displacement thickness ($\delta^*$) along the contoured wall and along the center line of the sidewall of this full two-dimensional Mach 6 nozzle is shown in figure 8. Displacement-thickness corrections are of approximately the same magnitude for the contoured wall and the sidewall. The differences between the displacement-thickness distributions of figure 8 are a direct result of the pressure distributions used as input, which are shown in figure 6.

Exit profiles of Mach number and velocity ratio for the full two-dimensional Mach 6 nozzle are shown in figure 9. The magnitude of the boundary-layer displacement thickness at the exit is also shown. A hot-core flow of approximately one-half the total test-section height remains uninfluenced by the boundary layer.

Exit profiles were also computed for the full two-dimensional nozzle with a wall temperature of 100 K. This wall supercooling effect was investigated to determine if liquid nitrogen or similar cryogenic liquid could be used as a wall coolant to decrease the wall boundary-layer thickness. Results are shown in figure 10, and no noticeable improvement in the profiles are predicted when using this value of $T_w$ because the difference between the high stream temperature (2200 K) and the wall temperature when reduced from 290 K to 100 K is not significant.

Exit profiles were also computed for a two-dimensional half-nozzle by using the boundary-layer analysis. The results are shown in figure 11. Although the construction of a half-nozzle would be simpler, the length is twice that of the full nozzle for the same size test section and the profiles of the full nozzle (fig. 9) are superior.

Mixing analysis.- Results of the mixing calculations for the full Mach 6 nozzle with injection at the exit, as obtained by using the method of reference 9, are shown in figure 12. In this configuration (fig. 2), the cold air ($T_o = 298$ K; $p_o = 2 \times 10^5$ N/m$^2$) has been expanded to Mach 3.5 and is introduced at the Mach 6 nozzle exit so that the static pressures of the hot and cold stream are matched at this location. The mixing analysis is started at the nozzle exit by introducing the initial profiles based on the results of the boundary-layer analysis in the hot core and smoothed to fit the cold-air nozzle exit conditions as shown by the exit profile in figure 12. The hot-core flow is assumed to continue downstream at constant area for this analysis.

The exit profile is then allowed to mix by using the viscous method-of-characteristics technique (ref. 9) to obtain the profile shown in figure 12 at 0.61 meter downstream of the exit. This profile indicates insignificant mixing of the cold air into the hot core. In fact, the mixing that does occur extends primarily into the cold flow. At the exit the Mach number on the wall is zero, but the analysis program could not accept an initial profile with so steep a gradient. The initial profile used in figure 12 is thought to be reasonable since viscous forces will accelerate the wall flow downstream of the exit.
Because only the mixing region was of interest, the boundary layer on the outer portion of the cold-flow nozzle was not considered.

Comparison of Nozzle Concepts

Momentum-thickness Reynolds number calculations indicate that the boundary layer will become turbulent just downstream of the throat for all nozzle configurations of the present investigation. The half-nozzle concept with throat injection did not provide an acceptable test core. When injection of the cold flow was not considered, the extra length of a half-nozzle caused a thicker boundary layer, and the flow profiles of figure 11 for a half-nozzle are not as desirable as the flow profiles shown in figure 9 for the full two-dimensional nozzle. When mixing was considered, the injection of the cold annular flow at the throat resulted in even poorer profiles, and the profiles of figure 3 indicate that the cold flow strongly influences the hot-core flow. This effect is especially true for the eddy-viscosity model of reference 10 which uses an eddy-viscosity expression that varies in both the axial and vertical direction.

In contrast, introduction of the cold flow at the nozzle exit appears promising, and for large downstream distances there is little influence of the mixing region on the test core (fig. 12). The full nozzle with annular flow injection at the exit thus appears to provide a technique for alleviating tunnel choking effects and increasing allowable model test size. The present study does not consider any boundary-layer cross-flow effects which tend to thicken the boundary layer in the center of the sidewalls of two-dimensional nozzles (this effect is usually overcome by decreasing the maximum nozzle expansion angle). With the exception of this possible effect, the analyses indicating the suitability of this hot-core nozzle concept with exit injection appear reasonably comprehensive and reliable.

CONCLUDING REMARKS

Several two-dimensional hypersonic-wind-tunnel nozzle configurations, which consist of a hot core surrounded by a cold annular flow, have been examined by using recently developed analytical techniques. The summation of effects of boundary-layer growth, viscous mixing, and flow condensation leads to the conclusion that introduction of the cold annular flow at the nozzle throat region is unacceptable for the nozzle size and stagnation conditions considered. Also use of a two-dimensional concept, with interchangeable ramps on the flat surface to provide versatility for simulating different Mach numbers at the exit, does not appear promising for the stagnation conditions considered since the extra length required for a half-nozzle concept would lead to an undesirably thick boundary layer and too small a test core. Conversely, if a full two-dimensional nozzle is used and the
supersonic annular flow is injected at the nozzle exit, acceptable flow will occur in the hot test core. Predictions of the mixing between the hot and cold supersonic streams show that mixing effects from the cold stream do not appreciably penetrate into the hot core for large downstream distances.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., April 17, 1972.
REFERENCES


Figure 1.- Mach 6.6 nozzle concept (nozzle with ramp).
Figure 2.- Mach 6 exit-injection nozzle concept. (Contouring not scaled.)
Figure 3.- Initial and exit profiles for Mach 6.6 nozzle obtained by using viscous mixing analysis of hot and cold streams.
Figure 4.- Pressure-temperature curves showing saturation and supersaturation relationship and isentropic expansion curves for two stagnation conditions.
Figure 5.- Condensation disturbances originating from the cold flow in the Mach 6.6 nozzle.
Figure 6.- Pressure-ratio distribution along the contoured wall and on the center line of the sidewall of a full two-dimensional Mach 6 nozzle.
Figure 7.- Reynolds number based on momentum thickness along center line of full two-dimensional Mach 6 nozzle.
Figure 8.- Turbulent $\delta^*$ along the walls of the full two-dimensional Mach 6 nozzle; $T_w = 290$ K.
Figure 9.- Exit profiles of Mach number and velocity ratio for a full two-dimensional Mach 6 nozzle. Turbulent-boundary-layer calculations; $T_w = 290$ K.
Figure 10.- Exit profiles of Mach number and velocity ratio for a full two-dimensional Mach 6 nozzle. Turbulent-boundary-layer calculations; $T_w = 100$ K.
Figure 11.- Exit profiles of Mach number and velocity ratio for a two-dimensional half-nozzle at Mach 6. Turbulent-boundary-layer calculations; $T_w = 290$ K.
Figure 12.- Profiles showing mixing of hot Mach 6 and cold Mach 3.5 streams by using viscous mixing analysis with variable-eddy-viscosity model ($k = 0.01$).
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