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N. A. Rynin

INTERPLANETARY FLIGHT AND COMMUNICATION

Volume III, No. 8

Theory of Space Flight

TRANSLATED FROM RUSSIAN

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INTERPLANETARY FLIGHT AND COMMUNICATION

(Mezhplanetnye soobshcheniya)

Volume III, No. 8

THEORY OF SPACE FLIGHT

(Teoriya kosmicheskogo poleta)

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Leningrad 1932

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NOTE ON THE TRANSLATION

The general title of Rynin's series, "Mezhplanetnye Soobshcheniya," is sometimes translated literally as "Interplanetary Communications." However, as has been pointed out by various writers on the history of space science (Willy Ley, Carsbie Adams, and others), the actual meaning is closer to "Interplanetary Travel." The title of the series in the present English translation, "Interplanetary Flight and Communications," is thus a compromise between the two interpretations.

The subjects dealt with in this volume range from the most speculative (the origins of life and the possibility of traveling to other stellar systems) to the most practical (data from rocket experiments).

In general, an attempt has been made to use the terminology current in 1932, the year this collection was published, rather than more modern space jargon. Thus, "entry" or "return" is used instead of "re-entry," "propulsion by reaction" or "reaction aircraft" instead of "jet propulsion" or "jet aircraft," "useful load" instead of "payload," and even, at times, "velocity of gas ejection" instead of "exhaust velocity."

The commentaries on the papers, the biographical notes, and the final sections by Kondratyuk and Rynin were written in Russian. The rest of this volume, with the exception of Goddard's paper, was translated into Russian from French or German. The abridged version of Goddard's paper has been copied from the original paper in English, rather than re-translated.

Notes or additions in square brackets, and footnotes labelled as translator's notes, have been added during the translation from Russian into English. All other notes or additions were made by Rynin during compilation or editing of this volume.

A great number of proofreading errors in the Russian text have been corrected without comment, for the most part mistakes in symbolic notation.

September 1971 Ron Hardin

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FOREWORD

This work is the eighth volume in a series of studies undertaken by the author. The overall title of the series is "Interplanetary Travel," and the seven volumes which have already been published are:

I. "Dreams, Legends, and Early Fantasies," Leningrad, 1928;
II. "Spacecraft in Science Fiction," Leningrad, 1928;
III. "Radiant Energy: Science Fiction and Scientific Projects," Leningrad, 1931;
IV. "Rockets," Leningrad, 1929;
V. "Theory of Rocket Propulsion," Leningrad, 1929;
VI. "Superaviation and Superartillery," Leningrad, 1929;
VII. "K. E. Tsiolkovskii: Life, Writings, and Rockets," Leningrad, 1931.

The ninth and last volume, entitled: "Astronavigation. Annals, Bibliography, and Index," is now in press.

Any comments regarding the volumes which have already appeared, or regarding the sending of these to readers, may be directed to the author at the following address:

Leningrad, Kolomenskaya Ulitsa 37, apt. 25.

Nikolai Alekseevich Rynin

Leningrad, 1 October 1931

INTRODUCTION

This issue, Theory of Space Flight, is a collection of translations of classical publications, mainly by foreign and some Russian authors.

It presents translations from the work of the French scientists Esnault-Pelterie (three papers) and Maurice Roy, the American scientist Goddard, the Germans Oberth, Hohmann, Lorenz, Shershevskii [Scherschevski], Kunz, Pirquet, Debus, and Ley, and the Russians Kondratyuk and Lebedev.

Many of the results included in this issue were also used in the previous publications in the series. A separate issue (No. 7) has been devoted exclusively to the work of K. Tsiolkovskii. Analysis of the work on interplanetary travel included in this issue clearly shows that different people in different countries independently came to the same conclusion, namely that interplanetary travel is feasible but impracticable at this stage because of technical and financial difficulties. These difficulties will certainly be overcome in the future, and man will finally pierce the armor of the atmosphere and the earth's gravitation, escaping into the mysterious and luring abysses of interplanetary space.

ROBERT ESNAULT-PELTERIE

3 FOREWORD TO ESNAULT-PELTERIE'S PAPERS

In all, four works by Esnault-Pelterie dealing with interplanetary travel are known to us. These are:

1. "Considération sur les résultats d'un allégement indéfini des moteurs," 1913;
2. "L'exploration par fusées de la très haute atmosphère et la possibilité des voyages interplanétaires," 1928;
3. "Astronautik und Relativitätstheorie," 1928;
4. "L'Astronautique," 1930.

Translations of the first three of these are presented below. The fourth work, which is Esnault-Pelterie's chef d'oeuvre, was published in 1930 in Paris. In addition to the first three papers, it includes a number of new studies containing some large nomograms. Although it would be extremely useful to publish a translation of this book as well, for financial reasons this was not possible. We cite here just the titles of the chapters of "L'Astronautique":

1. History of the Subject (pp. 17–24);
2. Résumé of Works of Goddard, Oberth, and Hohmann (25–38);
3. Rocket Motion in a Vacuum (39–78);
4. Rocket Motion in Air (79–108);
5. Expansion of Fuel Gas in a Nozzle (109–130);
6. Combustion in a Chamber (131–152);
7. The Use of Rockets (153–168);
8. Interplanetary Travel (169–206);
9. Interest in Interplanetary Studies (207–224);
10. Conclusion (225–248).

N. Rynin.

4 SOME INFORMATION ABOUT ESNAULT-PELTERIE

Robert Esnault-Pelterie (Figure 1) was born in Paris on 8 November 1881. He studied at the Janson De Sailly Lycée until 1898 and then at the Sorbonne. In 1902 he completed his military service. Esnault-Pelterie was active in the field of aviation as early as the year 1900. At first his experiments with an airplane similar to that of Wright were unsuccessful. However, then he began to look for optimum wing shapes and studied wing resistance with the aid of an automobile. On the basis of these data, he designed a monoplane in 1907 and made a successful flight in it in 1908. Then he took up the study of flight engines, as well as some other subjects related to aviation. In 1908 Esnault-Pelterie was awarded a large prize from the French Society of Civil Engineers for his engine. At present he is honorary President of the Board of the French Aircraft Industry, holds the Legion of Honor, and is a Licentiate of the Physical Sciences.

FIGURE 1. Robert Esnault-Pelterie.

In Paris in 1927, on the initiative of the engineer Esnault-Pelterie and the banker André Hirsch, an annual prize of 5,000 francs was set up for the best work on "Astronautics" (a term suggested by J. H. Rosny, President of the Goncourt Academy). This work had to be of a scientific nature and it could deal with subjects such as the following:

Astronomy and ballistics;

Physics: atomic theory, transmutation of elements, electromagnetic interplanetary communication, storage of energy, use of a telescope with
5 a movable base, etc.;

Chemistry: storage of air for respiration in a container, removal of respiration products, preparation and storage of atomic hydrogen, etc.;

Mechanics: construction of interplanetary ships, control and guidance, parachutes, etc.;

Metallurgy: superlight alloys (calcium, lithium, beryllium, etc.);

Physiology: effect of acceleration on organisms.

In 1930, Esnault-Pelterie presented an interesting lecture to the French Institute concerning the possibility of rocket flights around the world in 1 hr 26 min and from Paris to New York in 24 min.

In 1930 he traveled to the USA and, at the request of the American Interplanetary Society, delivered a lecture in New York on the subject of interplanetary travel. In this lecture, Esnault-Pelterie predicted flight into interplanetary space after 25 years and he stressed the need for considerable sums of money (about 2 million dollars) to make such flight possible.

His first work on interplanetary travel was published in a French journal in 1913: Esnault-Pelterie, R. "Considération sur les résultats d'un allégement indéfini des moteurs." — Journal de physique théorique et appliquée. Cinquième série. Tome III, Année 1913, Mars, pag. 218. Paris.

The work of Esnault-Pelterie has been cited at various times by writers in Russian. The following are examples:

Veigelin, K. — Priroda i Lyudi, No. 4. 1914.

Tsiolkovskii, K. Issledovanie mirovykh prostranstv reaktivnymi priborami (The Exploration of Space by Jet Machines), pp. 4—7, Kaluga. 1914.

Novaya Vechernyaya Gazeta, No. 210, 20 November 1925, Leningrad.

Outside the USSR, Esnault-Pelterie's work has been mentioned in:

Gussalli, L. Si può già tentare un viaggio dalla terra alla luna? Milano. 1923.

An article in the journal "Il Secolo" XIX, Genova, Martedi 4 Maggio 1926.

Now let us proceed to the first paper by Esnault-Pelterie. It should be noted that the high value for the fuel weight on a rocket obtained by Esnault-Pelterie is due not to any error in his calculations, as has been suggested by Tsiolkovskii, but rather to his assumption of a very low acceleration of the rocket ($11/10g$), a value which Esnault-Pelterie considered to be safe for a man. This acceleration can, of course, be assumed to be higher.

6 *First Paper*

*CONSIDERATIONS CONCERNING THE RESULTS OF AN INDEFINITE WEIGHT-REDUCTION OF ENGINES**

The ideas presented in this paper were engendered by results obtainable at present from applications of light engines. It would be interesting to find out what could be expected from such engines if their weights were considerably less. In other words, what will be the possibilities if the weight of an engine is decreased indefinitely for a [given] horsepower. Will the resulting progress relate only to the field of aviation, or will new horizons open up? If they exist, what will these new horizons be?

Many writers have used voyages from star to star as the subjects of novels. Even today, star travel is said to be impossible, however, without taking into account the actual physical data which could help solve the problem. The purpose of this work will be to present some such physical data, which have resulted from certain considerations having a bearing on the calculations.

I

The first difficulty encountered by us is the absence of an atmosphere between the stars. Since there is no atmosphere, an airplane could not be used for a flight in outer space, because there would be nothing to hold it up.

Problems of a physiological nature will be considered below. Here let us limit ourselves to the question of whether our knowledge of mechanics is sufficient to create an engine capable of propelling a ship regardless of whether there is any external support.

Although it may seem strange to someone who has not made a study of this subject, such an engine has been known for a long time, namely the rocket (Jules Verne's cannon, which would crush the passengers during
7 launching, cannot be considered "an engine for a spaceship").

It is often said that a rocket moves by virtue of its reaction "on air." The first part of this statement is true, but the second part "on air" is false. A rocket moves just as well in a vacuum, and even better, than in air.

In order to better understand this phenomenon, let us assume that a machine gun is mounted on a trolley which can move without friction along rails parallel to the axis of the weapon. With each firing, the machine gun will move backward according to a familiar law of mechanics. The

* [Considération sur les résultats d'un allégement indéfini des moteurs.]

momentum acquired by the machine gun and its trolley, considered together, will be equal and opposite to that acquired by the projectiles. The air resistance will only serve to reduce the velocities.

In a rocket the role of the bullet is played by the gas produced when the fuel explodes. This gas is ejected from the rocket in a continuous flow. Let us assume that M_0 is the total mass of the rocket at launch, M_1 is its mass at a time P, and dm is the mass of fuel ejected from the rocket during a time dt.

Let us also assume that the fuel efflux takes place at a constant rate relative to the rocket, and that the flow rate of the fuel remains constant and equal to μ. Finally, V is the velocity acquired by the rocket, F is the reaction force, and γ is the acceleration at a time t.

Calculations show that the phenomenon can be described by the formula

$$-MdV=\mu\, dt.v=v.dm. \quad (1)$$

It should be noted that, if the rocket consists completely of fuel (an idea which, though purely abstract, is of some interest), it would burn up completely in a time

$$T=\frac{M_0}{\mu}. \quad (2)$$

The introduction of this limiting time into the formula giving V as a function of t leads to

$$(T-t)\,dV=v.dt,^*$$

from which

$$V=v\log\frac{(T-t)}{T}.$$

8 For $t=T$ we have $v=-\infty$ (assuming that v is positive). This result should not surprise us, since the reaction force remains constant; thus the mass decreases in proportion to the decrease in fuel and, at the limit, goes to zero. The acceleration increases to an infinitely large value. The expression for the path traversed as a function of t is

$$x=-v\left\{T\left[\left(\frac{T-t}{T}\right)\log\frac{T-t}{T}\right]+t\right\}.$$

After all the fuel is used up, the path will be

$$X_T=-v.T.$$

Therefore, leaving aside other questions for the moment, we may conclude that flight in a vacuum is after all not impossible. However, it is not enough just to propel the device, it must be directed as well.

* From (1) we have $MdV=\mu.dt.v$; $-\frac{M}{\mu}dV=v.dt$; but $M=M_0-\mu t$; and thus $\frac{(M_0-\mu t)}{\mu}dV=vdt$; $\left(-\frac{M_0}{\mu}+t\right).dV=vdt$ or $-(T-t)\,dV=vdt'$. However, assuming that the velocities have unlike signs, we obtain the equation given in the text.

In principle, this does not present great difficulties. In order to change the direction of flight, it is sufficient to change the orientation of the engine in such a way that the direction of the reaction force will be at an angle to the flight path. If such a shifting of the engine cannot be made in all directions, then one or two small engines can be used to give complete maneuverability.

II

In order to move a body of known weight away from the center of a star, energy must be expended. Let us consider a mass M at a distance x from the center of a star of radius R. Here γ is taken to be the acceleration of gravity at the surface of this star. In order for the body to traverse a distance dx, an element of work

$$dB = M\gamma \frac{R^2}{x^2} \cdot dx$$

must be performed. This gives a total work done of

$$B = M\gamma . R\left(1 - \frac{R}{x}\right).$$

From this it is clear that, to move a given mass out to infinity, it is necessary to perform an amount of work

$$B = M.\gamma.R$$

or, designating the weight of the body as $P = M\gamma$, the work is

$$B = P.R. \qquad (3)$$

9 Let us consider the weight of the body as a result of universal gravitation, that is, the force acting between the body and the star. Then, designating the mass of the star as U, we obtain

$$P = k.\frac{M.U}{R^2},$$

where k is the gravitational constant. Now, the work required to move the body to infinity will be

$$B = k\frac{M.U}{R^2}.$$

Therefore, if a sufficiently high velocity is imparted to a body leaving the earth's surface, this body will depart to infinity.

For the earth this velocity is 11,280 m/sec. In other words, if a projectile leaves the earth with this velocity, it will never return (provided air resistance is not taken into account). This critical velocity is equal to that acquired by a body falling to the planet from infinity, without any initial velocity.

The law of motion for such a body can be expressed as

$$V^2 = 2g\frac{R^2}{x}.$$

For $x = R$

$$V_R = -\sqrt{2gR} \qquad (1°)$$

and

$$\frac{1}{2} m V^2 = P.R. \qquad (2°)$$

For the earth

$$V_R = 11{,}280 \quad \text{m/sec.}$$

For a body 1 kg in weight and for the earth, we have, from equation (3), $B = 6{,}371{,}103$ kg·m, which is equivalent to 14,970 cal. It should be recalled that 1 kg of a hydrogen-oxygen mixture in an appropriate proportion gives 3,860 cal. For comparison, 1 kg of gunpowder (fulmicotton [cotton powder] and potassium chlorate) gives only 1,420 cal. Therefore, 1 kg of an oxygen-hydrogen mixture gives nearly $^1/_4$ of the energy required to lift 1 kg from the earth to infinity. On the other hand, 1 kg of radium, which yields a total of $2.9 \cdot 10^9$ cal, provides an energy 194,000 times greater than that required. Here, however, we are not yet taking into account the efficiency of a reaction engine. Let us consider a body which recedes from a star with an accelerated motion described by some law. At the moment when its velocity is opposite in sign to, and greater than, the velocity it would have at the same point if it were falling from infinity without any initial velocity, it would be useless to impart to the body any
10 further energy. Its kinetic energy would be sufficient to send it off to infinity.

The law of motion for a body acted upon by a constant force F greater than the weight of the body and directed vertically and centrifugally relative to a star can be expressed by the equation

$$v = \sqrt{2Ax + \frac{2gR^2}{x} - 2R(A + g)}.$$

The body acquires a velocity for which a further expenditure of energy will not be necessary at a distance from the center of the star equal to

$$x = R\left(1 + \frac{g}{A}\right),$$

where $A = \frac{F}{M}$.

Once a body has left the earth under the influence of a force equal to its weight, that is, for

$$A = g,$$

it will attain the critical velocity at a distance from the center equal to two earth radii, or at a height above the earth's surface equal to the terrestrial radius. This tells us that a body can escape completely from a star with the aid of a thrust force less than its weight. If the star has an atmosphere, the body may first fly like an airplane, gradually ascending and increasing its velocity in proportion to the decrease in air density, until it attains the critical velocity.

Let us determine the energy required to take a body from the earth to the moon and back (Figure 2). Such a flight can be divided into three periods:

1. The body is accelerated to the critical velocity required to escape from terrestrial gravitation;

2. The energy expenditure (burning of the rocket) terminates. The body moves under the influence of the velocity attained;

3. At a certain point it turns its lower side toward the moon, the engine begins to operate, and the velocity decreases, so as to slow down to zero at the moment of contact with the lunar surface.

First Period. Let us apply to the body a force

$$F = \frac{11}{10} p$$

with $A = 11/10\, g$, a force which can be applied even assuming the presence of living beings aboard the body. The critical distance will be

$$x = \frac{21}{11} R,$$

12 which corresponds to a height of 5,780,000 m above the earth's surface.

At this moment the velocity will be

$$V = 8180$$

The flight time during this period will be

$$t = 24 \text{ min } 9 \text{ sec.}$$

Second Period. The body continues flying by inertia and is attracted by both the earth and the moon. Let us assume that P is the weight of the body, at the earth's surface, P_l is its weight at the lunar surface, ϱ is the radius of the moon, and $D = x + y$ is the distance between the earth and the moon. Calculations show that

$$V = \sqrt{2\left(g\frac{R^2}{x} + 0.165\, g\frac{\varrho^2}{y} + 0.820 \cdot 10^6\right)}.$$

At the point where the attractions of the earth and moon are equal, the velocity will be

$$V = 20.30 \text{ m/sec.}$$

This is the lowest velocity on the flight path.

With the approach to the lunar surface the velocity will be

$$v = 3060 \text{ m/sec.}$$

The velocity of free fall from infinity to the lunar surface is

$$v = 2370 \text{ m/sec.}$$

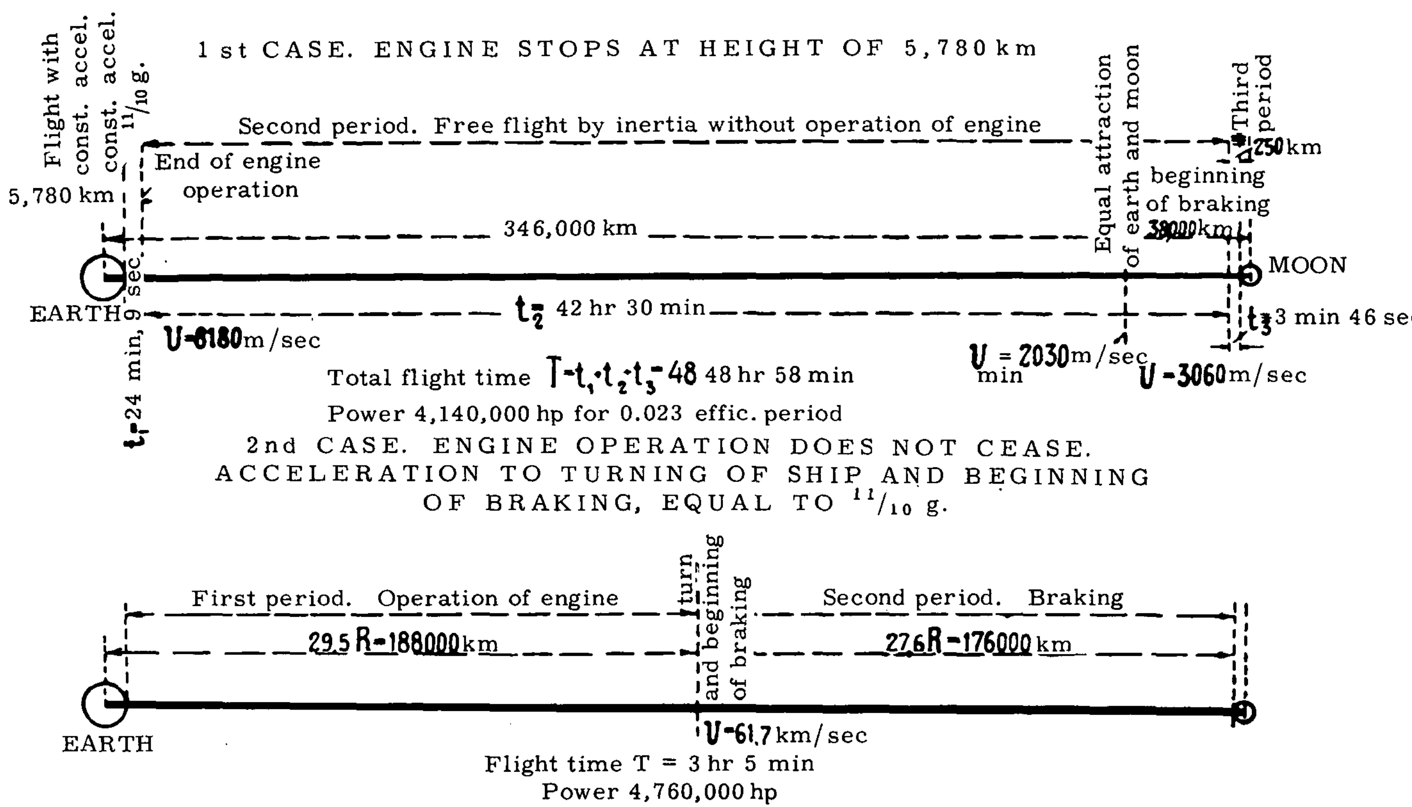

FIGURE 2. Flight from earth to moon.

The time required to traverse the distance of the second period can be determined neglecting the attractive force of the moon, which proves to be negligible. This time will be equal to the time required for the body to fly from the moon to the point where the engine stopped operating:

$$t = 48 \text{ hr } 30 \text{ min}$$

Third Period. Now the motion has to be retarded, once the ship has been turned and the engine started. What will be the law governing this retarded motion? We can compare this motion to a similar motion relative to the earth, taking into account that the attraction of the moon will be much weaker.

Since great accuracy is not necessary, let us limit the acceleration which must be overcome by the engine to a value equal to half the acceleration at the lunar surface, and let us assume that the motion takes place with
13 deceleration under the influence of this fictitious acceleration.

We find that the ship must be turned at the following distance from the lunar surface:

$$d = 250{,}000 \text{ m}.$$

This point is so close to the lunar surface that, since our calculation is not perfectly accurate, the duration of the third period can be assumed equal to the time required for flight of the ship to the moon itself.

The deceleration will last for a time

$$t = 226 \text{ sec} = 3 \text{ min } 46 \text{ sec}$$

Thus the approximate flight time is:

1 period	0 hr 24 min 9 sec
2 period	48 hr 30 min
3 period	0 hr 3 min 46 sec
Total about	48 hr 58 min

For the return flight almost the same amount of time, in the opposite sequence, will be required. It should be noted that the engine will work only during 28 min of the flight to the moon and about the same amount of time during the return flight, provided the braking effect of the earth's atmosphere is not utilized during decent.

Now let us determine the actual minimum power required to carry out the flight, taking into account the engine efficiency. Let us assume that the ship weighs 1,000 kg, of which 300 kg is propellant. If (taking into account that during descent to the earth the retardation is just due to the atmosphere) the engine operates only 27 + 3.5 min, or, with some reserve,

$$35 \text{ min} = 2100 \text{ sec},$$

the fuel consumption per second will be

$$\frac{300}{2100} = 0.143 \text{ kg},$$

so that the velocity of ejection is

$$v = 65{,}300 \text{ m/sec},$$

and so 1 kg of propellant will yield

$$T = 217.2 \cdot 10^6 \text{ kg. m or } 512 \cdot 10^3 \text{ cal.}$$

A mixture of $H_2 + O$ contains only $^1/_{133}$ of this energy and other powerful energy-producing substances contain only $^1/_{360}$ of this energy. On the other
14 hand, 1 kg of radium contains 5,670 times as much energy as this.

The engine power required for our ship will be

$$\frac{300 \cdot 217.2 \cdot 10^6}{2100 \cdot 75} = 414{,}000 \text{ hp.}$$

It should also be noted that the efficiency of a reaction engine is quite low. Actually, in order to take a 1-kg mass from the earth out to infinity, 6,371,103 kg. m of work must be performed on it. We have determined that this work for an engine is $217.2 \cdot 10^6$ kg. m. Therefore, the efficiency will be

$$k = 0.0293.$$

In addition, to impart to the gas an ejection velocity of 65,300 m/sec in a vacuum, the gas must be heated to an improbable temperature of $2.525 \cdot 10^6$ degrees. For flight in air, moreover, the conditions will be even worse, since in addition to an increase of this temperature a greater pressure will also be necessary.

IV

Let us assume that, after the ship has reached the critical velocity, the engine continues to operate and stops working when the velocity is 10 km/sec. Then the times required to reach the planets closest to the earth at their closest approach will be:

for Venus	47 d 20 hr
for Mars	90 d 15 hr.

It should be noted that the energy expenditure for these flights will not be too much higher than the minimum required to overcome terrestrial gravity. Actually, once the ship has reached a sufficient distance away from the earth, it will continue in free flight and the attraction of the earth will slow the flight very little.

Consequently, the main difficulty is to overcome terrestrial gravity, and once this has been accomplished, it will not be especially difficult to get to other planets, be they near or far. Here, naturally, the safety of the passengers during their stay inside the hermetically sealed ship must be guaranteed, and this will be considered below.

In the preceding sections we have suggested only the theoretical possibility of a flight between the earth and the moon. This is a problem in pure mechanics, which does not bring up the question of whether man will actually someday be able to leave our planet and investigate others. This leads us to study the physiological conditions necessary to carry out
15 such a flight.

The successes which have been attained during underwater voyages indicate that it is possible to exist away from the air for a certain period of time. The question of the temperature requires special consideration. It is usually assumed that the temperature in interplanetary space is absolute zero. It is the opinion of the author, however, that this is not the case.

The concept of temperature has meaning only for material bodies and it does not apply to a vacuum (Dewar flasks are evidence of this). If the heat influx to a ship per units time is less than the heat outflow, then its temperature will drop; however, if influx is greater than efflux, the temperature rises. It is possible to construct a ship in such a way that half of its surface will be of polished metal which does not conduct heat from within. The other half of the surface may be, for example, oxidized copper, forming a black surface. If the polished surface is turned toward the sun, the temperature of the ship drops; in the opposite case, the temperature rises.

The foregoing indicates that the problem has, in principle, been solved. However, it should be kept in mind that there is one more difficulty which complicates the practical solution of the problem. Actually, in our example of a flight from the earth to the moon, we have suggested an acceleration of

$$A = \frac{11}{10} g$$

for the time it takes to travel a distance of 5,780 km from the earth's surface. During this entire period the passengers will weigh $^{11}/_{10}$ of their weight on earth. It may be assumed that this should not cause them any special discomfort. However, the feelings experienced by them will be more unusual when the engine ceases operation. Then they will lose their weight and have the feeling that they are falling into a void.

If the organism is not accustomed to such a change, then in the absence of a gravitational field an artificial field should be created, at any rate one equal to the terrestrial field, and then the passengers will retain their terrestrial weights, wherever they may happen to be in outer space. However, the system required for this will call for a large expenditure of energy and makes an already difficult problem even more complex.

Let us consider the formula expressing the law of motion of a body acted upon by a constant force from the moment it leaves the earth. We assume that, until the maximum velocity between the earth and moon is attained, an acceleration equal to $^{11}/_{10}$ of terrestrial gravity is used, and that all the other maneuvers take place with an acceleration equal to the terrestrial acceleration. In addition, we assume that the effect of the lunar attraction, in view of its smallness, can be neglected. Under these conditions calculations show that the vehicle should turn at a distance of 29.5 earth radii from the center of the earth. At this moment, the velocity will be 61,700 m/sec. After this the turned vehicle will be retarded by a force equal to its terrestrial weight.

The time required to reach the moon will be

$$t = 3 \text{ hours } 5 \text{ min.}$$

16

However, in this case the work required for the flight of a 1000 kg space-ship, 300 kg of which is fuel, will be $67.2 \cdot 10^6$ cal per kg of propellant, that is, 131 times more than in the first case. Dynamite provides only 1/47,300 of the required power, while radium provides 433 times as much power. The required power is

$$\frac{857 \cdot 10^{10}}{24{,}000 \cdot 75} = 4{,}760{,}000 \text{ hP.} \qquad \text{(a)}$$

If this means of propulsion is used for a flight to a nearby planet, then we obtain the following maximum velocities and flight times:

	Velocity	Flight time
for Venus	643 km/sec	35 hr 04 min
for Mars	883 km/sec	49 hr 20 min

VI

Although the above velocities are also improbable, still there are celestial bodies which attain velocities of the order of these, for instance, any comet.

Only molecular forces and particle energies can make such flights possible. If we assume that a loaded vehicle weighing 1000 kg includes 400 kg of radium, and that we can obtain energy from it at any time, as desired, then this amount of fuel will suffice for a flight to Venus and back with some fuel left over, and it will be barely enough for a flight to Mars and back (with constant acceleration).

17

Second Paper

*THE EXPLORATION BY ROCKET OF THE UPPER ATMOSPHERE AND THE POSSIBILITY OF INTERPLANETARY TRAVEL**

FROM THE SOVIET TRANSLATOR

This work is a development of the paper written by Esnault-Pelterie back in 1913. In this paper the author presents a number of original conclusions and hypotheses, which other scientists dealing with interplanetary travel have little touched upon or completely ignored. The subjects considered include:

1. Representation of the motion of a rocket in a vacuum without gravity, with the aid of so-called critical curves, and a study of the economics of the motion, that is, the expenditure of a minimum of fuel.
2. Analysis of the optimum rocket shape. The author considers three types of rockets: cylindrical, conical, and exponential (a rocket moving with constant thrust), and gives preference to the latter, especially for a manned flight.
3. For manned flights the author recommends an acceleration differing little from terrestrial (1.1 to $2g$), because of possible danger to an organism as a result of high accelerations.
4. The heating of a vehicle during passage through the atmosphere is given special consideration, as well as the temperatures of vehicles approaching close to the earth, Venus, Mars, and Mercury, on the side toward the sun and on the dark side.
5. With regard to fuel, the author believes that existing types can be used to send rockets into the upper atmosphere, but he feels that flights to the moon or to other planets will be possible only when man possesses atomic energy. For the present, it is advisable to use atomic hydrogen, but the properties of this substance have as yet been little studied.
6. The author considers specious the theory of Arrhenius ("panspermism") that spores can travel from planet to planet. Instead, a hypothesis concerning the appearance of life on a planet is put forward, life being considered as one type of the physicochemical phenomena which continue through all time and go through a gradual evolution of forms from simpler to more complex.
7. In conclusion, the author appeals for more progress in the exciting field of interplanetary travel, by carrying out studies of a number of particular subjects, so as to prepare for the moment when physics provides mankind with the possibility of using atomic energy.

N. Rynin

* L'Exploration par fusées de la très haute atmosphère et la possibilité des voyages interplanétaires.

18

FOREWORD

Dreams of flying from the earth into the limitless starry heavens are as old as mankind itself. In this paper Robert Esnault-Pelterie approaches in a scientific manner a problem which for many centuries has been treated by various writers from a predominantly fantastic viewpoint. Lucian in ancient Greece and Cyrano de Bergerac in 17th-Century France suggested quite fantastic means of overcoming terrestrial gravity. And who does not recall the more recent plans of Jules Verne's projectile, or the curious device of H.G. Wells, whereby the first man reached the moon because the outer shell of the vehicle possessed the mysterious property that formed a screen against the force of gravity? With regard to this field of fantasy, it is appropriate here to mention the little-known novelist Achille Eyraud, who in 1865 suggested a type of rocket or reaction engine for flights away from the earth.

The scientific study of such engines can be traced back to the time just twenty years ago (1907) when Robert Esnault-Pelterie first* took up this subject. His ideas were published later, in 1912, the date of his report to the French Physical Society. Although others have studied this fascinating subject since then, of whom Esnault-Pelterie mentions Dr. Bing and the American professor, Goddard, we can safely say that the author of this paper was the first to tackle the subject in its entirety. Esnault-Pelterie has initiated and greatly developed the scientific study of the flight of living beings into the mysterious reaches of interplanetary space.

Naturally, the problem is still far from being resolved, but the first step has been taken and it is now clear what obstacles have to be overcome in the construction of a rocket which will be able to take us to the heavenly bodies. The day may be coming soon when mankind will have at its disposal atomic energy, and then it will be possible to carry out the ideas expressed in such a brilliant and talented manner by Esnault-Pelterie.

Esnault-Pelterie has already completed a number of elegant scientific endeavors of different types. In particular, he was a pioneer in aviation and he came up with ideas which were often far ahead of his time, and
19 which showed the perspicacity and intuitive ability of the author. Most people know of him as the inventor of the "Manche à balai," that is, the control stick used in aviation. He is also the author of a number of other significant works related to aviation, automotive theory, and, in general, mechanics.

Esnault-Pelterie was the first to suggest a direct method for studying the laws of aerodynamics (1905). In 1906 he constructed a motor-driven model plane, which was a novelty at that time. He suggested testing the strength of an airplane by loading it with sand, and he worked out a new method for measuring the strength of metals.

The following paper was read before the annual general meeting of the French Astronomical Society in 1927. In addition to the formulas and calculations, which are of great interest, Esnault-Pelterie opens up before the reader a number of possibilities such as to inspire man's imagination.

* Here the writer is in error, since the first to present a theory of rocket flight in general, and rocket motion in interplanetary space, was the Russian scientist K. E. Tsiolkovskii (in 1903).

In the hope of seeing the future epic of interplanetary voyages come to pass, let us say along with the poet:

Si nous pouvions franchir ces solitudes mornes;
Si nous pouvions passer les bleus septentrions;
Si nous pouvions atteindre au fond des cieux sans bornes,
Jusqu'à ce qu'à la fin, éperdus, nous voyions,
Comme un navire en mer croît, monte et semble éclore,
Cette petite étoile, atome de phosphore,
Devenir par degrés un monstre de rayons.

V. Hugo

General Ferrier
Member of the Institute

AUTHOR'S NOTE

In October 1927 my friend André Hirsch drew my attention to a number of works dealing with a subject of interest to me. I tried without success to obtain these works in Vienna, where I was obliged to be later. There I found out about the studies by Lorenz (Danzig), published on 7 May 1927 in the "Zeitschrift des Vereins deutscher Ingenieure." This very serious, albeit somewhat short, work included a bibliography which listed, in addition to Goddard's books, which were already known to me, the following new works:

1925. H. Oberth. "Die Rakete zu den Planeten-Räumen";
1925. W. Hohmann. "Die Erreichbarkeit der Himmelskörper";
1925. M. Valier. "Der Vorstoss in den Weltenraum."

I was able to get the first two books on 14 January 1928, although Oberth's work was the 1923 edition and not the 1925 edition. In Hohmann's work I found, with some surprise, a number of subjects which had been studied independently by myself, and in some parts he had progressed even further than I, for instance concerning the braking of flight in the atmosphere, where he speaks of successive circuits of the earth along ellipses. However, Hohmann considers passage through the atmosphere at a height of 75 km with a speed of 11 km/sec, without taking into account the heating up of the vehicle, which will be so considerable as to render it uncontrollable.

With regard to the ratio of initial and final masses of the rocket, the results of Hohmann are identical to mine, and this is very significant. Interestingly enough, he, like myself, carried out calculations up to gas-ejection velocities of 10,000 m/sec. However, he assumed an acceleration of 20 g, which does not provide much of an advantage over an acceleration of 10 g. Hohmann's work deserves a serious study rather than just a mention, and I am very sorry that I was not familiar with it earlier.

Oberth's book is also very thorough and deserving of attention. Some questions related to the effect of acceleration are developed and rocket designs are even presented. Before beginning my own study, I could not help but mention these two works and express my great respect for them.

Finally, I should ask to be excused if I have passed over other works that were unknown to me, since it was not easy to obtain bibliographies on this subject and in fact I then still had not obtained the above book by Valier.

Robert Esnault-Pelterie

21
INTRODUCTION

Mr. President, Ladies and Gentlemen. *

Our president, General Ferrier, on the suggestion of our colleague, André Hirsch, approached me recently about presenting to the members of the society a more detailed report of the subject discussed by me on 15 November 1912 at the French Physical Society. But first, let me mention some works with which I have become familiar since the above date.

Fifteen years ago, I wished to deliver a lecture on the possibility of interplanetary travel, and the difficulties related to it, at a time when aviation had just been born and when expectations had been raised. At that time, for many, perhaps somewhat ill-advised reasons, it seemed to me more prudent to hide the actual purpose of my study under the title: "Considerations Concerning the Results of an Indefinite Weight-Reduction of Engines." Now, however, I am able to make known my ideas under their true title.

The volume of my previous report was reduced so much by the secretary of the Physical Society that my ideas were often barely intelligible to the reader, and this leads me now to present the material in more detail then was possible earlier. My ideas on this subject go back to a much earlier time. Long ago, I was surprised by Jules Verne's error in the novel "From the Earth to the Moon," in which he described his travelers enclosed in a projectile which was to be shot from a cannon 300 meters long. In order to keep his passengers from being crushed by the inertial forces during launching, Verne provided a frame 2 m high at the base of the projectile, which would break up when the cannon was fired. Actually, the effect of such a frame would be equal to just lengthening the cannon from 300 to 302 m, that is, there would be almost no change in the effect of the inertial forces or the danger that the passengers would be flattened.

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Hence I concluded that it was necessary to give a projectile a running start of several kilometers, and this led to the use of a rocket. I myself would not be able to establish just when this idea occurred to me, if it had not, fortunately, been referred to in an old book by Captain Ferber: "From Hill to Hill, from City to City, from Continent to Continent," on page 161 of which he writes:

"In order to go higher, and man would like to go higher, a different principle must be adopted. The principle of the rocket is the most applicable, implying the use of a reaction engine. Man will be sealed up in an enclosure where air for breathing will be produced artificially. Actually, he will no longer ride a flying machine, but rather a controllable projectile. The realization of this idea will not be improbable, as long as the sun provides our planet with reserves of energy. A reduction of the heat at the earth may serve as a stimulus to new progress, since then life on earth will be different. Man will be faced by a serious dilemma; either to return to the age of his ancestors and follow the path of regression, or to proceed to new conquests by human genius.

* Report to general meeting of French Astronomical Society on 8 June 1927.

"The more powerful, more developed, men of the future will have to do this. Some of them will then leave our inhospitable planet, and the triumph of the lighter-than-air craft, which has been born before our eyes, will be realized." *

Ferber has dated the note in his book to 26 July 1908. Therefore, my ideas can be said to go back to sometime during the first half of 1908. I should note that similar ideas were stated at that time by someone else, namely by Dr. André Bing, whom I did not know earlier and who, after my report in 1912, sent me his Belgian patent No. 236377 of 10 June 1911. This patent is entitled: "A Device For Studying the Upper Atmosphere." Dr. Bing also noted that, some years before, he had discussed this problem with one of my colleagues at the Society of French Scientists and Inventors, Edouard Belin, the inventor of the phototelegraph.

Finally, in 1912—1913, Robert Goddard, an American professor at Princeton University, made a number of theoretical calculations. Then, in 1915—1916 at Clark University (Worcester, Mass.), Goddard carried out some experiments with rockets designed to study the upper atmosphere, following the idea expressed so wonderfully by Dr. Bing. Professor Goddard concluded that a projectile with a charge of magnesium powder could be sent to the moon and that the explosion could be seen from the earth with a telescope.

When reading Dr. Bing's patent, the impression is obtained that its author probably did not carry out calculations to check the feasibility
23 of the invention. However, as he wrote me in 1913 and as is quite evident, Dr. Bing simply wished to retain the priority for himself by means of this patent. The patent leads one to conclude, albeit not completely directly, that almost any height can be reached using successive rockets, which would then fall away one after the other when burned out. This was also Professor Goddard's main principle, when he calculated sending a rocket out of the atmosphere with an initial weight 600 times that of the useful load. In other words, for instance for a flight into interplanetary space or to the moon (which is practically the same thing), to send a load of 1 kg, the initial weight must be 600 kg.

The results obtained by Professor Goddard would appear, at first glance, to contradict my results. He assumes it to be possible to send a projectile out into space, while I feel that it is not possible at present to send out a device capable of overcoming terrestrial gravity; first a more powerful energy source, such as radium, must be found, and no such source is as yet at our disposal. However, this contradiction is only apparent and can be explained by the fact that Goddard and I are approaching the problem from different points of view.

He wishes simply to send to the moon a projectile with powder aboard and to determine the moment of the explosion on the moon using a telescope. On the other hand, I consider the problem of sending living beings from celestial body to celestial body and bringing them back to earth. I have shown clearly that it is possible to send a small part of a projectile a given distance, as was indicated by a formula in my 1912 report, and also by a statement following it at the top of page 5 (Section II). However, I was also aware that to do this an enormous initial mass of the projectile

* "Let us recall the persons who developed this idea, mainly Wells, Esnault-Pelterie, Archdeacon, Quinton, and other philosophers" (note quoted from Captain F. Ferber de Rue).

would be necessary. I consider the principle to be impracticable for flights with living creatures. In such a case, as I will show below, the initial mass must be not 600 times, but several thousand times, greater than the final mass, if the passengers are not to be crushed during the launch, as would be the case for Jules Verne's heroes when they are launched from a cannon. They would also be crushed for other reasons to be given below.

These then are the conclusions from my 1912 report which I felt should be presented here in order to prevent any misunderstanding by the reader. The present report deals with the following:

Chapter I. A study of rocket flight in vacuum; the equation of motion; the most economical shape; cylindrical, conical, and exponential rockets; heights and velocities of escape (beginning of free flight); utilization coefficient.

Chapter II. Study of rocket flight in air; equation of motion; equation of air resistance; ballistic coefficient; most economical shape; under known conditions, air resistance does not alter significantly the conditions obtained for flight in a vacuum; temperature of compressed air ahead of rocket;
24 attainable accelerations.

Chapter III. Application of rockets to study of the upper atmosphere and for interplanetary travel; launching to the moon; flight around the moon; conditions depending on exhaust velocity; for what exhaust velocities calculations can be made; possibility of implementation.

Chapter IV. Conditions necessary for transport of living beings; an interplanetary ship; living conditions aboard it; the physiological effects of the absence of acceleration; maneuverability; conditions for implementing it; duration and velocity of a flight to Venus or Mars.

Chapter V. Of what scientific interest are visits to other worlds? What might we find on them? Are they inhabited?

Conclusion.

Chapter I

ROCKET MOTION IN A VACUUM

The study of this simpler problem is very important for the later consideration of the general problem taking air resistance into account. The rocket ascent is divided into two periods: a first, or burning, period with acceleration in flight; and a second period, after burning of all the fuel, when the rocket does not have thrust but flies in its trajectory under the influence of the velocity attained.

For the present let us consider just a rectilinear trajectory toward the zenith, and let us introduce the following notation:

V: rocket speed at given moment
v: absolute velocity of gas ejection
m: effective mass of fuel (for time t_0, $m = m_0$)
p: mass of empty rocket
$M = m + p$: total mass of rocket at given moment
F: reaction force at given moment

Γ: acceleration
dm: element of mass ejected during given time element dt
y: height at given moment
G: acceleration of gravity at given height (at sea level $G = g$)
R: air resistance.

Note. I consider distances, forces, and accelerations directed upward, like the velocity V, to be positive. The quantities v, G, and R will be
25 positive by definition (par essence).

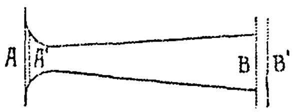

FIGURE 3.

Reaction in a nozzle. Let us assume that a constant regime of gas ejection has been established in a nozzle (Figure 3). At a moment t the nozzle contains a certain gas mass between surfaces A and B; A^1 and B^1 are taken to be the position of this mass at a moment $t + dt$.

The part included between surfaces A^1 and B will be common to the two cases. The part between B and B^1 is the mass dm ejected during a time dt and it equals the mass between A and A^1. The latter has a very low velocity and its momentum is an infinitesimal second-order quantity. The former, on the other hand, acquires the ejection velocity and its momentum vdm will be a first-order quantity.

Since the rest of the gas mass retains its velocity, the theorem of the momentum component gives

$$F.dt = -v.dm \qquad (1)$$

or

$$F = -v.\frac{dm}{dt}. \qquad (2)$$

The acceleration will be

$$\Gamma = -\frac{v}{M}\frac{dm}{dt} = -\frac{v}{M}\frac{dM}{dt}^{*}. \qquad (3)$$

Since dm and dM are negative, Γ will be positive.

Equation of motion. Diagram. In order to present the formulas more clearly, let us consider the absolute values of G and R and let us write the general equation of motion:

$$M\frac{d^2y}{dt^2} = M\Gamma - MG - R. \qquad (4)$$

However, remembering that the motion is in a vacuum, we obtain:

$$M\frac{d^2y}{dt^2} = M\Gamma - MG. \qquad (4\text{ bis})$$

or, on the basis of (3),

$$\frac{d^2y}{dt^2} = -\frac{v}{M}\frac{dM}{dt} - G. \qquad (5)$$

* $-dM$ is the part of the total mass M of the rocket ejected during the time dt; naturally, $dM = dm < 0$.

The motion can be depicted using a diagram on which V is the abscissa and y is the ordinate, the case being limited to positive values of y and V. Then

$$\frac{dy}{dt} = V, \tag{6}$$

from which

$$\frac{d^2y}{dt^2} = \frac{dV}{dt} = V\frac{dV}{dy}, \quad \frac{dM}{dt} = V\frac{dM}{dy}. \tag{7}$$

Now equation (5) can be written as

$$V\frac{dV}{dy} = -\frac{V \cdot v}{M} \cdot \frac{dM}{dy} - G, \tag{8}$$

or

$$-\frac{dM}{M} = \frac{V \cdot dV + G \cdot dy}{Vv}. \tag{9}$$

The critical curve. The critical curve is understood to be a curve representing the motion of the rocket without gas ejection (without thrust). The curve is so called because, in order to reach a given height y, there is no need to continue accelerating the rocket all the way to this height. It is enough to accelerate it to some lower height, corresponding to some point on the critical curve passing through a point at the height y, after which the flight continues by inertia. The equation for the critical curve is obtained from (4 bis), by setting $\Gamma = 0$ in it. Then, from (7) we obtain

$$V\frac{dV}{dy} = -G \tag{10}$$

For small heights (from the following it will be clear which heights)

$$V\frac{dV}{dy} = -g = \text{const.} \tag{10 bis}$$

After integration we obtain

$$V_0^2 - V^2 = 2gy. \tag{10 ter}$$

Equation (10) may be written as

$$VdV + Gdy = 0, \tag{11}$$

and this shows that at all times during flight in a vacuum a projectile of constant mass retains a constant energy. If we designate the total energy of a unit mass as $g\eta$, then we obtain

$$VdV + Gdy = gd\eta. \tag{12}$$

27 The equation for the critical curve in a vacuum, as compared with a variable η, will be

$$d\eta = 0. \tag{13}$$

To obtain the value of $\eta(V, y)$, it is sufficient to integrate (12). Let us denote the radius of the earth as a, and then

$$G = \frac{g}{\left(1 + \frac{y}{a}\right)^2}. \tag{14}$$

Consequently,

$$g\,d\eta = V\,dV + \frac{g\,dy}{\left(1 + \frac{y}{a}\right)^2}, \tag{14 bis}$$

from which

$$g\eta = \frac{V^2}{2} - g\frac{a}{\left(1 + \frac{y}{a}\right)} + \text{const.} \tag{15}$$

Setting $\eta = 0$ for $y = V = 0$, we have

$$\eta = \frac{V^2}{2g} + \frac{y}{1 + \frac{y}{a}}. \tag{16}$$

If y is small enough compared with a, then

$$\eta = \frac{V^2}{2g} + y. \tag{16 bis}$$

The most economical curve. Assuming that the medium does not present any resistance to the flight, we find that for an ascent to even a few hundred kilometers a great energy expenditure is necessary. Therefore, *the main problem is to use the minimum mass of propellant required to* lift the given final mass p to the given height.

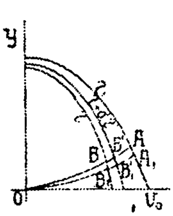

FIGURE 4.

Let us draw the critical curve $V_0AY(\eta_1)$ in Figure 4 passing through the final height, and let us assume that OBA is some curve corresponding to the theories of burning and thrust. Integration of equation (9) gives

$$L\frac{M_0}{p} = \int_{(OBA)} \frac{V\,dV + G\,dy}{Vv}. \tag{17}$$

Since

$$\left(1 + \frac{m_0}{p}\right) = \frac{M_0}{p}$$

28 varies with

$$L\left(\frac{M_0}{p}\right),$$

it is sufficient to find the minimum of the integral on the right side. Let us draw the two critical curves for η and $(\eta + d\eta)$ in such a way that

$$\eta < \eta + d\eta < \eta_1. \tag{18}$$

and the thrust curves intersect at points B and B^1.

From equations (9) and (12) we obtain

$$g d\eta = - Vv \frac{dM}{M}, \tag{19}$$

where dM is negative and g, V, v, and M are positive; $d\eta$ is by definition positive, and the point corresponding to the period of thrust passes successively through all the critical curves on the side of increasing η and does not turn backward.

The differential element of the right side of (17) can be written, according to (12), as

$$\frac{g d\eta}{Vv}. \tag{20}$$

Let us draw a curve $OB_1\ B_1^1 A_1$, below curve OBB^1A in the figure, and let us consider an element $B_1B^1_1$, which, like BB^1 corresponds to the value $d\eta$. Of these two elements, the smallest will be the one for which the product Vv is greater, independently of what pair of elements is chosen.

This compels us to select the maximum value of v as determined by the physicochemical properties of the explosives available to us. If the explosive is selected appropriately, v can be assumed constant.

Because of the shape of the η curves, the second of the two elements BB^1 and $B_1B^1_1$ will correspond to a larger V, and this holds true for all elements of curves OBA and OB_1A_1. Therefore, the second curve will be more useful than the first. Passing to the limit, we see that the most economical curve for fuel consumption is the part OV_0 on the V axis, and for it the ratio M_0/P will be a minimum.

In this case, the burning period will be instantaneous, the acceleration will be infinite, and the projectile will have a lift $dy = 0$, so that formula (17) reduces to

$$L\frac{M_0}{p} = \int_0^{V_0} \frac{dV}{v} = \frac{V_0}{v}, \tag{21}$$

from which

$$\left(\frac{M_0}{p}\right) \min = e^{\frac{V_0}{v}} \tag{22}$$

29 Now if we consider formula (16), after applying it successively to points V_0 and Y of the η_1 = const curve, we obtain

$$\frac{V_0^2}{2g} = \frac{Y}{1 + \frac{Y}{a}}. \tag{23}$$

and (22) becomes

$$\left(\frac{M_0}{P}\right)\min = e^{\frac{1}{v}\sqrt{\frac{2gY}{1+\frac{Y}{a}}}}. \tag{23 bis}$$

If Y is small in comparison with the radius of the earth a, then

$$\left(\frac{M_0}{P}\right)\min = e^{\frac{1}{v}\sqrt{2gY}}. \tag{23 ter}$$

Under the most favorable theoretical conditions, and assuming an exhaust velocity of 2,000 m/sec, in order to overcome the force of terrestrial gravity with a final mass of 1 kg, an initial mass of 269 kg is necessary. This value is considerably lower than that obtained by Goddard for the case of air rather than a vacuum. If we assume v = 2,500 m/sec, this value drops to 88 kg.

However, it should be kept in mind that these figures correspond to strictly abstract conditions. If it were necessary to impart to a finite mass an instantaneous and infinitely large acceleration, the mass would have to be stretched out into a plate without any thickness in order for its quantity per unit area to be 0. Its area then would be infinite, and the limits of the mass would lose their physical meaning; finally, for a flight in the atmosphere we would have to introduce an important condition with regard to reduction of the cross section of the rocket.

Minimum cross section. The above theory indicates the upper, infinite, limit for the ejection of a unit mass. It would be desirable to consider how this cross section, for a unit mass, can be decreased as desired, either infinitely or at least to some lower limit, which (since again this pertains to flight in a vacuum and to a theoretical point of view) will be useful to us later when studying flight in air.

Determination of ejection area. For the expansion of an ideal gas in a nozzle, the ejection velocity is given by the equation

$$v^2 = 2RT_0\frac{\gamma}{\gamma-1}\left[1-\left(\frac{p}{p_0}\right)^{\frac{\gamma-1}{\gamma}}\right], \tag{24}$$

where, if the gas expands to zero pressure, then theoretically we convert all the energy to kinetic energy without any loss to friction.

It should be noted that the pressure at the nozzle exit is not determined
30 by the pressure in the center, where the gas is expanding. Instead, it is determined by the ratio between the orifice cross section and the throat cross section, taking into account the initial temperature and pressure; here I will not discuss all of the theory of the Laval nozzle. However, it follows that in the case of a vacuum, in order to be logical, the area of the orifice has to be infinite, which leads us, as in the previous case, to an absurd condition.

In order to avoid this difficulty, a rocket with very high pressure (1,000 or even 2,000 kg/cm) can be used, so that for a very high degree of expansion (100 or 200) the gas would have a sufficiently high pressure at the nozzle exit (10 or 20 kg/cm). Thus a large part of the energy of the gas would be converted to kinetic energy at that time; theoretically it would be 75% for a degree of expansion of a hundred, and practically, as in Goddard's experiment, it would be 64% for an unspecified degree of expansion.

From this it follows that the cross section of the nozzle throat should be as large as possible, that is, equal to the midship section of the rocket. For the very high pressures under which it will function, this cross section makes it possible to attain a degree of expansion sufficient to turn most of the energy into kinetic energy.

These considerations lead us to draw the following simple theoretical conclusions concerning rockets: the cross section of the nozzle orifice is the gas-ejection section and is equal to the midship section of the rocket. Through this orifice gas in its final expansion stage is ejected at a velocity v. If we assume that ahead of the nozzle there is a reservoir of fuel, then the flow rate of the latter will be proportional to the flow rate of the mass of ejected gas.

Therefore, let us replace the actual rocket by a theoretical one consisting of a solid fuel, in the shape of a surface of revolution. At a given moment, the rocket has a velocity in some direction serving as an axis of this surface and limited at the rear by a surface normal to this velocity. The latter plane surface is the surface of combustion and from it gas is ejected backward at a velocity v. As the fuel flows out, this surface moves into its mass at a rate such that the gas discharge continually corresponds to the exhaust velocity v through the orifice.

This purely theoretical simplification is actually not compatible with the condition of proper utilization of energy, as required by the use of a nozzle. However, it must be demonstrated that it is as completely law-abiding as possible, since later it will greatly simplify the entire discussion. When a cylindrical rocket is discussed, this will indicate that the gas-ejection section remains constant; if the rocket is conical, then the ejection section is proportional to 2/3 of the residual mass; and finally, if a rocket with constant thrust is considered, then the ejection section will be proportional to the residual mass.

Consequently, the ejection section has now been determined. The volume of gas ejected in a time dt will be

$$v.Sdt. \qquad (25)$$

31 For this surface the ejected mass will be

$$v.Sdt = -dM. \qquad (27)$$

and taking (3) into account,

$$\Gamma = \varrho \frac{v^2.S}{M}. \qquad (27)$$

The thrust (reaction) will be

$$F = \varrho . v^2 . S. \qquad (28)$$

Here ϱ and v are determined by the physical properties of the fuel. Therefore, we can specify arbitrarily only the value of $\frac{S}{M}$. For a launching from the earth we have

$$\Gamma_0 \geqslant g \tag{29}$$

that is,

$$S_0 \geqslant \frac{g M_0}{\varrho . v^2}. \tag{30}$$

The right side of this inequality expresses the minimum of the ejection section for an initial lifted mass M_0.

$$\sigma \min = \frac{g M_0}{\varrho . v^2}. \tag{31}$$

Optimum utilization of given section. Let us assume that a certain rocket A has the shape of a surface of revolution about the velocity direction, and that a meridian of this shape is specified. We compare it with a cylindrical rocket C having the same initial and final masses, and with an ejection section which is continually equal to the most effective ejection section of A. Then it will always be true that

$$S_A \leqslant S_c, \tag{32}$$

and, according to (28),

$$F_A \leqslant F_c. \tag{33}$$

This will be the case at any given arbitrary height. The velocity of fuel efflux, and thus the lightening of rocket A, will always be less than, or at any rate equal to, that of rocket C. For the efflux at a given time, the residual mass of A will always be higher than, or at any rate equal to, the residual mass of C. If, however, as is sometimes the case, the height y is taken as the independent variable instead of the time, this condition may not hold true, and then the following two cases are possible.

1. For the same height of ascent the residual mass of A is always great-
32 er than that of C.

For a given random height interval dy, the following elementary quantities of work will be done:

$$F_A\, dy \leqslant F_c\, dy. \tag{34}$$

Since this work is performed to overcome the force of gravity and to impart kinetic energy, therefore for the same heights we have

$$M_A (V_A\, dV_A + G dy) \leqslant M_0 (V_c\, dV_c + G dy). \tag{35}$$

However, since in this case it is always true that

$$M_A \geqslant M_c, \qquad (36)$$

therefore the greater will be

$$V_A dV_A + Gdy \leqslant V_c dV_c + Gdy, \qquad (37)$$

and thus

$$V_A dV_A \leqslant V_c dV_c. \qquad (37\,\text{bis})$$

After taking the sum from 0 to a certain y and extracting the square root, we obtain

$$V_A \leqslant V_c. \qquad (38)$$

However, rocket A has at least at one point a section smaller than that of the other rocket, otherwise the two rockets would be identical. Therefore, it is always true that

$$V_A < V_c. \qquad (38\,\text{bis})$$

The latter inequality also applies to the case when, at some height, one of the rockets has used up all its fuel. According to the foregoing, this will take place for a cylindrical rocket at a height where the other rocket still has fuel reserves.

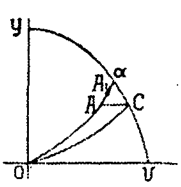

FIGURE 5.

Let us plot the fuel curves of a Vy diagram (Figure 5). Because of inequality (38 bis), curve OC lies below curve OA, but for a height C rocket A will still have energy reserves. Let us assume that these reserves are used up at just the moment when rocket A reaches a height corresponding to the end of burning of C. Then this curve will be parallel to the V axis, but it may not reach the end point of curve C. Actually, if this were the case, then it would be only due to the consumption of more propellant than in the case of C, since the previous considerations show that curve OAC corresponds to a greater fuel expenditure than curve OC.

33 Moreover, an instantaneous expenditure of the residual fuel of A requires an infinitely large section, and curve A cannot bend down along AC. It will continue rising, for instance, to point A_1, where there is even less reason for it to bend and go to point C.

2. If rocket A stays long at heights which differ little from one another, then it may be that, for the portion of fuel used, it will attain a large height with less reserves of fuel than the cylindrical rocket at the same heights.

Let us assume that at each height which rocket A tries to reach more easily than C, we prevent this from happening, by hindering the corresponding consumption of its active mass in such a way that at all heights the following inequality holds true:

$$M_c \leqslant M_A \qquad (38\ \text{ter})$$

In this case the previous sequence of arguments remains in force, although the effect of the action of rocket C is, in the final analysis, reduced.

Conclusion. Let us define the utilization coefficient [efficiency] of the rocket as:

$$U = \frac{P}{M_0}. \qquad (39)$$

Now it can be stated that a cylindrical rocket has a better utilization coefficient than some other rocket with the same maximum section. In other words, it can lift a greater final mass to a given height or it can lift a given final mass to a greater height.

Comparison of cylindrical rockets of like section with one another. Let us consider a cylindrical rocket for which

$$\Sigma > \sigma\ \text{min}. \qquad (40)$$

Here,

$$\Gamma_0 > g \qquad (40')$$

and the rocket is launched and rises according to the familiar law.

Now let us assume that we hinder the launching by adding to the rocket an explosive cylinder of the same section and of a mass m_1, so that

$$M_0 + m_1 = \frac{\varrho \cdot v^2 \Sigma}{g}. \qquad (41)$$

At the moment when this mass m_1 reaches burnout, and the main rocket begins to operate, the latter will already possess a certain velocity and will have reached a certain height. Therefore, the main rocket will attain a greater velocity and a greater height than previously during the consumption of its fuel.

34 Therefore, we either increase the final height or increase the final mass, if burning ceases at the moment where the corresponding point of the second rocket reaches the critical curve of the main rocket.

Conclusion. Of all the cylindrical rockets of like cross section, the rocket with the greatest initial mass lifts a given final mass higher, or lifts a greater mass to a given altitude, but with an attendant reduction of the utilization coefficient

$$\left(\frac{P}{M_0}\right).$$

Critical curve. We have already seen that, in order to reach a given height H, it is enough to continue burning until the moment when point (V, y) gets to a critical curve with a limit $V=0$ and $y=H$. The equation of this curve is obtained from (16) and on the basis of the two above cases we have

$$\frac{V^2}{2g}+\frac{y}{1+\frac{y}{a}}=\frac{H}{1+\frac{H}{a}}, \tag{42}$$

or

$$\frac{V^2}{2g}=\frac{H}{1+\frac{H}{a}}-\frac{y}{1+\frac{y}{a}}. \tag{43}$$

For $H=\infty$ we have

$$\frac{V^2}{2g}=a-\frac{y}{1+\frac{y}{a}}=\frac{a}{1+\frac{y}{a}}. \tag{44}$$

This is the equation for the curve of motion (escape) of a rocket in a vacuum.

Properties of rockets of different shapes. Before solving completely the theoretical problem taking air resistance into account, it would be interesting to determine the limits of the theoretical possibilities, as applied to their actual implementation.

For simplicity, I will assume the rockets to be actually cylindrical, conical, or some other definite shape. The ejection section will be designated as S, and the length of the moment t as l. For a fuel of uniform composition with a density ϱ and a burning rate of v^1, we will have

$$-\frac{dl}{dt}=v^1. \tag{45}$$

In addition, at each moment

$$-\frac{dM}{dt}=\varrho\, v\, S=\varrho'\, v^1\, S, \tag{46}$$

from which

$$-v^1=v\frac{\varrho}{\varrho'}=\text{const.} \tag{47}$$

35 Integrating (45), we obtain

$$l=l_0-v^1 t. \tag{48}$$

For a final initial length l_0 of the rocket, the total burning time will be

$$T=\frac{l_0}{v^1}=\frac{l_0}{v}\frac{\varrho^1}{\varrho}, \tag{49}$$

from which

$$l = v^1 (T - t). \tag{50}$$

Cylindrical rocket. The equation of motion gives

$$M \frac{d^2 y}{dt^2} = \varrho \cdot v^2 S - Mg \frac{1}{\left(1 + \frac{y}{a}\right)}. \tag{51}$$

Integrating (46), we obtain

$$M = M_0 - \varrho v S t, \tag{52}$$

from which, assuming

$$M_0 = \varrho . v S T, \tag{53}$$

and

$$M = \varrho v S (T - t) \tag{54}$$

we obtain

$$\frac{d^2 y}{dt^2} = \frac{v}{T - t} - \frac{g}{\left(1 + \frac{y}{a}\right)^2}. \tag{55}$$

The condition for ascent from the earth

$$\frac{M_0}{S} \leqslant \varrho \frac{v^2}{g} \tag{56}$$

indicates that

$$T \leqslant \frac{v}{g}, \tag{57}$$

and

$$T \max = \frac{V}{g} = \tau. \tag{58}$$

Let us set

$$T' = k\tau = k \frac{v}{g}, \tag{59}$$

where k is arbitrarily a portion of the maximum of the fictitious length or the hypothetical assumed length.

Let us also introduce the variable

$$\lambda = \frac{t}{T} = 1 - \frac{T - t}{T} = 1 - \frac{M}{M_0} = \frac{M_0 - M}{M_0}, \tag{60}$$

which can be represented for a given moment in terms of the ratio of the consumed mass to the initial mass:

$$u = 1 - \lambda = \frac{T-t}{T} = \frac{M}{M_0}, \tag{61}$$

and this gives the ratio of the available mass to the initial mass. At the end of flight this ratio gives the utilization coefficient

$$U = \frac{P}{M_0}. \tag{62}$$

With this notation, equation (55) can be written in the following form:

$$\frac{d^2y}{d\lambda^2} = \frac{v.T}{1-\lambda} - \frac{gT^2}{\left(1+\frac{y}{a}\right)^2}, \tag{63}$$

or, introducing τ,

$$\frac{d^2y}{d\lambda^2} = \frac{kv\tau}{1-\lambda} - \frac{k^2g\tau^2}{\left(1+\frac{y}{a}\right)^2} = \tag{64}$$

$$= \frac{kv\tau}{1-\lambda} - \frac{k^2v\tau}{\left(1+\frac{y}{a}\right)^2}. \tag{65}$$

Finally,

$$\frac{d^2y}{d\lambda^2} = kv\tau\left[\frac{1}{1-\lambda} - \frac{k}{\left(1+\frac{y}{a}\right)^2}\right], \tag{66}$$

and after integration we obtain

$$\frac{dy}{d\lambda} = kv\tau\left[L\frac{1}{1-\lambda} - \frac{k\lambda}{\left(1+\frac{y_i}{a}\right)^2}\right] = VT = k\tau V \tag{67}$$

and

$$y = kv\tau\left[\lambda - \frac{1}{2}\cdot\frac{k\lambda^2}{\left(1+\frac{y_j}{a}\right)^2} - (1-\lambda)L\frac{1}{(1-\lambda)}\right]. \tag{68}$$

Here y_i and y_j designate average values depending on λ. If it is possible to neglect y in comparison with a, these equations give

$$V = v\left[L\frac{1}{1-\lambda} - k\lambda\right]. \tag{69}$$

and

$$y = kv\tau\left[\lambda - \frac{1}{2}k\lambda^2 - (1-\lambda)L\frac{1}{1-\lambda}\right]. \tag{70}$$

If in these formulas we set $t = T$, that is, $\lambda = 1$, then we find that for consumption of all the fuel the velocity will be infinite, but the height attained will be finite. If in (70) we set $\lambda = 1$, and $k = 1$, we obtain this maximum height for a given v. For a velocity

$$v = 2000 \text{ m/sec},$$

37 which is almost the same as that of Goddard, we obtain the burnout heights listed in Table 1.

TABLE 1.

λ \ k	0.01	0.05	0.1	0.25	0.5	1.0
0	0 m	0 m	0 m	0 m	0 m	0 m
0.25	138	666	1,269	2,694	3,795	1,218
0.4	378	1,824	3,486	7,493	10,908	5,506
0.5	620	3,001	5,746	12,454	18,537	11,591
0.7	1,371	6,658	12,816	28,293	44,100	38,250
0.9	2,714	13,241	25,657	57,950	95,258	107,947
0.95	3,244	15,854	30,788	70,071	117,144	142,289
0.99	3,829	18,745	36,491	83,735	142,492	185,076
0.999	4,025	19,718	38,418	88,414	151,395	201,056
0.99999	4,053	19,857	38,695	89,093	151,708	203,498
1	4,057	19,878	38,739	89,195	152,906	203,874

It is clear from this table that a cylindrical rocket, that is, rocket C with constant ejection sections, will not burn at heights above 204 km for an exhaust velocity of 2,000 m/sec. For a utilization coefficient of 1%, it will burn up to 185 km, and for $k = 0.5$ for the same λ it will burn up to 142.5 km.

Velocities V at the end of burning are found from (6). Table 2 gives additional height values.

38 The height of ascent of a rocket as a projectile is obtained from (42);

$$H = \frac{1}{\frac{1}{\frac{V^2}{2g} + \frac{y}{1 + \frac{y}{a}}} - \frac{1}{a}}. \tag{71}$$

The velocity required to overcome terrestrial gravity is 11,180 m/sec. The lower three rows of Table 2 satisfy this condition.

TABLE 2.

λ \ k	0	0.01	0.05	0.1	0.25	0.5	1.0
0	0 m	0 m	0 m	0 m	0 m	0 m	0 m
0.25	575	570	550	525	450	325	75
0.4	1,022	1,014	982	942	822	622	222
60.5	1,386	1,376	1,336	1,286	1,136	886	386
0.7	2,408	2,394	2,338	2,268	2,058	1,708	1,008
0.9	4,605	4,587	4,515	4,425	4,155	3,705	2,805
0.95	5,991	5,972	5,896	5,801	5,516	5,041	4,091
0.99	9,210	9,191	9,111	9,012	8,715	8,220	7,230
0.999	13,816	13,796	13,716	13,616	13,316	13,817	11,818
0.9999	18,421	18,401	18,321	18,221	17,921	17,421	16,421
1.0	∞	∞	∞	∞	∞	∞	∞

Conical rocket. The shape of this rocket is determined by the equation

$$S = S_0 \left(\frac{M}{M_0}\right)^{\frac{2}{3}}. \tag{72}$$

Its total mass is

$$M = \varrho' \frac{Sl}{3}. \tag{73}$$

In a particular case

$$M_0 = \varrho' \frac{S_0 l_0}{3}. \tag{74}$$

The scaling law gives

$$\frac{l}{l_0} = \left(\frac{M}{M_0}\right)^{\frac{1}{3}}. \tag{75}$$

The equation of motion (51) has the same form, but in this case we have

$$M \frac{d^2 y}{dt^2} = \varrho v^2 S_0 \left(\frac{M}{M_0}\right)^{\frac{2}{3}} - Mg \frac{1}{\left(1+\frac{y}{a}\right)^2}. \tag{76}$$

or

$$\frac{d^2 y}{dt^2} = \varrho v^2 \frac{S_0}{M_0} \left(\frac{M_0}{M}\right)^{\frac{1}{3}} - \frac{g}{\left(1+\frac{y}{a}\right)^2}. \tag{77}$$

From (74) and (49) we obtain

$$\frac{S_0}{M_0}=\frac{3}{\varrho' l_0}=\frac{3}{\varrho' v' T_1}, \tag{78}$$

and from (75), (49), and (51), we have

$$\left(\frac{M_0}{M}\right)^{\frac{1}{3}}=\frac{l_0}{l}=\frac{T_1}{T_{1-t}}. \tag{79}$$

Finally, taking (47) and (77) into account,

$$\frac{d^2 y}{dt^2}=\frac{3v}{T_{1-t}}-\frac{g}{\left(1+\frac{y}{a}\right)^2}. \tag{80}$$

39 This equation is identical to (55), except that in it v is replaced by $3v$. Let us call this velocity v_1 the fictitious velocity:

$$v_1=3v. \tag{81}$$

From (80) and (81) we obtain the ascent condition:

$$T_1 \leqslant \frac{3v}{g}=\frac{v_1}{g}. \tag{82}$$

We assume that

$$T_{max}=k\tau_1=k\frac{v_1}{g}, \tag{83}$$

where k has the same meaning as for a cylinder.

Equation (60) now becomes

$$\lambda=\frac{t}{T_1} \tag{84}$$

so that

$$\lambda=1-\frac{l}{l_0}=1-\left(\frac{M}{M_0}\right)^{\frac{1}{3}}. \tag{85}$$

The quantity $(1-\lambda)$ now represents the coefficient of linear utilization but not the mass utilization. The latter will be

$$u=\frac{M}{M_0}=(1-\lambda)^3. \tag{86}$$

Under these conditions we obtain the same integrals as in (66) and in (67), except that v is replaced by $v_1=3v$, that is,

$$V=v_1\left[L\frac{1}{1-\lambda}-\frac{k\lambda}{\left(1+\frac{gt}{a}\right)^2}\right]; \tag{87}$$

$$y = kv_1 \tau_1 \left[\lambda - \frac{1}{2} \frac{k\lambda^2}{\left(1+\frac{y}{a}\right)^2} - (1-\lambda) L \frac{1}{(1-\lambda)} \right]. \tag{88}$$

If y is small in comparison with a, then formulas are obtained which are analogous to those for a cylindrical rocket. For equal λ the velocity of a cone will be three times that of a cylinder and the height will be nine times greater. However, formula (86) shows that the coefficient of mass utilization (u) for a cone is lower than that for a cylinder, that is, the former uses more propellant than the latter.

The theorem following formula (33) and the subsequent result indicate that, for sections which are identical per unit mass, a cylinder is more economical than a cone. A cone and cylinder can also be compared for identical fuel consumptions and the theorem can be modified accordingly.

For the sake of clarity, the subscript $_1$ will be used for all quantities pertaining to a cone, and quantities without this subscript will be used for
40 a cylinder.

Let us compare the velocities and heights attainable with conical and cylindrical rockets for identical mass utilizations. From (61) and (86) it follows that

$$1-\lambda = (1-\lambda_1)^3, \tag{89}$$

and thus

$$\lambda = 3\lambda_1 - 3\lambda_1^2 + \lambda_1^3. \tag{90}$$

By specifying some arbitrary λ_1, we obtain the corresponding λ. For example, for

$$\lambda_1 = 0.5 \tag{91}$$

we have

$$\lambda = 1 - 0.5^3 = 1 - 0.125 = 0.875. \tag{92}$$

In order to obtain the values of V and y for a cone, the same values for a cylinder with $\lambda = 0.5$ must be tripled and multiplied by 9, respectively, and then the values obtained must be recalculated for a cylinder with $\lambda = 0.875$. Thus we obtain Tables 3 and 4.

TABLE 3. Cone with $\lambda = 0.5$

k	0	0.01	0.05	0.1	0.5	1
y in m	0	5,580	27,009	51,714	166,833	104,319
V in m/sec	4,159	4,128	4,008	3,858	2,658	1,158

TABLE 4. Cylinder with $\lambda = 0.875$

k	0	0.01	0.05	0.1	0.5	1
y in m	0	2,492	12,140	23,515	86,370	94,700
V in m/sec	4,159	4,141	4,071	3,984	3,284	2,409

These tables show that the larger kinetic energy of the residual mass of a cylinder compensates and increases the difference in potential energy corresponding to the difference in the heights obtained at burnout. If, for example, the residual mass is 1 kg and $k = 1$, the kinetic-energy excess of a cylinder will be 223,000 kg·m, and its potential-energy efficiency will be about 9600 kg·m. It is clear from (16) that η for a cylinder remains considerably larger than that for a cone under the above conditions of conformability.

Rocket with constant thrust. Previously, we defined such a rocket by the condition

41

$$\frac{S}{M}=\frac{S_0}{M_0}=\text{const.} \tag{100}$$

Such a rocket may be called "exponential" for the following reasons. First let us assume that

$$\frac{S_0\,\varrho\,v^2}{M_0}=\frac{g}{k}=\frac{v}{k\tau}, \tag{101}$$

where $k\tau$ has the same value as for a cylinder, that is,

$$k\tau=k\frac{v}{g}. \tag{59}$$

Now let us write (100) in the form

$$S=\frac{1}{k\varrho\, v\tau}\cdot M, \tag{102}$$

Now, differentiating with respect to t and taking (16) into account, we obtain

$$\frac{dS}{dt}=-\frac{\varrho\, v\, S}{\varrho v\, k\tau}=-\frac{1}{k\tau}S, \tag{103}$$

from which

$$S = S_0 e^{-\frac{t}{k\tau}}$$

and

$$M = M_0 e^{-\frac{t}{k\tau}}. \tag{104}$$

The shape of the rocket will be a surface of revolution about the OZ axis. Let us take x and z to be the coordinates of its meridian.

Then

$$S = \pi \,.\, x^2 \tag{105}$$

and

$$x^2 = x_0^2 e^{-\frac{t}{k\tau}}; \tag{106}$$

$$z = v^1 t; \tag{107}$$

$$x = x_0 e^{\frac{z}{2v^1} : \frac{t}{k\tau} -}. \tag{108}$$

This expression shows that as z goes to infinity, x goes to 0. Thus such a rocket will have infinite length and burning time.

From (49) and (59) we obtain

$$v^1 k\tau = L. \tag{109}$$

42 If we designate l as the actual length of the exponential rocket, then

$$x = x_0 e^{-\frac{1}{2L}} \tag{110}$$

and

$$S = S_0 \cdot e^{-\frac{1}{L}}. \tag{111}$$

Finally, its total mass will be (according to (104))

$$M = M_0 e^{-\frac{1}{L}}. \tag{112}$$

This shows that for such a rocket not only the radius and area of any normal section, but also the residual mass, vary according to a power law as a function of the rocket length, indicating the correctness of the name of this rocket.

Considering (109) and replacing ϱv by $\varrho' v^1$, we can write (101) in the form

$$M_0 = S_0 \varrho' v^1 k\tau = \varrho'.S.L. \tag{113}$$

This relation shows that L represents the length of a cylindrical rocket of the same mass and the same initial cross sections as the exponential rocket being considered.

The equation of motion

$$\frac{d^2y}{dt^2} = \frac{\varrho \cdot v^2 S}{M} - \frac{g}{\left(1+\frac{y}{a}\right)^2} \tag{114}$$

is transformed, taking into account (100) and (101), into

$$\frac{d^2y}{dt^2} = \frac{g}{k} - \frac{g}{\left(1+\frac{y}{a}\right)^2}. \tag{115}$$

The condition of ascent from the earth gives

$$\frac{d^2y}{dt^2} > 0; \tag{116}$$

and for this to be the case it is necessary and sufficient that

$$k < 1 \tag{117}$$

where y varies from 0 to ∞, and the acceleration also varies from some initial value to the limiting value:

$$\frac{g}{k} = \varrho \cdot v^2 \frac{S_0}{M_0}, \tag{118}$$

which constitutes the "thrust acceleration." This is why I have called such a rocket a "rocket with constant thrust," and not a "rocket with constant
43 acceleration." The latter would be correct only for a reduction of the acceleration of gravity and a small value of this acceleration in comparison with the thrust acceleration.

Introducing the velocity V, we can write equation (115) as

$$V\frac{dV}{dy} = \frac{g}{k} - \frac{g}{\left(1+\frac{y}{a}\right)^2}, \tag{119}$$

which gives

$$V^2 = 2g\left(\frac{y}{k} + \frac{a}{1+\frac{y}{a}} - a\right) = 2gy\left(\frac{1}{k} - \frac{1}{1+\frac{y}{a}}\right), \tag{120}$$

and

$$V = \sqrt{2gy\left(\frac{1}{k} - \frac{1}{1+\frac{y}{a}}\right)}. \tag{121}$$

This is the equation of the (V, y) curve for a period of infinitely long burning. The velocity V increases with y, and both these quantities increase without limit.

Critical height at which such a rocket reaches free-flight (or critical) velocity If we eliminate $\frac{V^2}{2g}$ from (120) and (144), we obtain

$$\frac{a}{1+\frac{y}{a}}=\frac{y}{k}-\frac{y}{1+\frac{y}{a}}, \tag{122}$$

or

$$a=\frac{y}{k}, \tag{123}$$

which gives a critical height

$$y_c=ka.$$

Note. Since $k<1$, therefore $y_c<a$.
From (120) and (123) we obtain

$$\frac{V^2}{2g}=\frac{a}{1+k}, \tag{124}$$

from which

$$V_c=\frac{\sqrt{2ga}}{\sqrt{1+k}}. \tag{125}$$

When k varies from

$$0\left(\frac{g}{k}=\infty\right)$$

to

$$1\left(\frac{g}{k}=g\right),$$

44 the velocity V_c decreases from

$$\sqrt{2ga}$$

to

$$\sqrt{ga}.$$

Calculation of time; critical time. Previously we had

$$\frac{dy}{dt}=\sqrt{2g}\sqrt{y\left(\frac{1}{k}-\frac{1}{1+\frac{y}{a}}\right)}, \tag{121}$$

which gives

$$t=\frac{1}{\sqrt{2g}}\int_0^y\frac{dy}{\sqrt{y\left(\frac{1}{k}-\frac{1}{1+\frac{y}{a}}\right)}}. \tag{126)*$$

This is the equation of an elliptic integral. Since it was not possible to solve this integral accurately, I have obtained an approximate solution (see Appendix at end of this paper).

Numerical results for exponential rockets. This rocket is of special interest in that it subjects the rocket components and any living beings which might be aboard, to an almost constant acceleration. I will make acceleration. I will make calculations for three values of the acceleration, for reasons to be given below. Here the following values of y_c, V_c and t_c were obtained:

Γ	k	y_c	V_c	t_c
10 g	0.1	637 km	10,660 m/sec	120 sec
2 g	0.5	3,185	9,133	750
1.1 g	0.91	5,800	8,080	36 min 40 sec

The reciprocals $\frac{M_0}{p}$ of the utilization coefficient are of special interest; they are given in Table 5 for various values of v.

45

TABLE 5.

v m/sec	$\Gamma = 1.1\,g$	$\Gamma = 2\,g$	$\Gamma = 10\,g$
2,000	143,000	1,574	358.5
2,500	13,270	361.3	110.6
3,000	2,700	135.2	50.5
3,500	883	67.1	28.8
4,000	378	39.7	18.9
4,500	196	26.3	13.6
5,000	115	19.1	10.5
6,000	52.2	11.6	7.10
7,000	29.7	8.19	5.37
8,000	19.4	6.30	4.35
9,000	14.0	5.13	3.69
10,000	10.7	4.36	3.24

Chapter II

ROCKET MOTION IN AIR

Let us designate as R the absolute resistance of the air (that is, the force resulting from the opposition of the air to the motion of an object). This quantity increases with V, decreases with the height y, and depends on the way in which the rocket penetrates the air.

The equation of motion can be written as

$$\frac{d^2y}{dt^2} = \frac{S \cdot \varrho \cdot v^2 - R}{M} - \frac{g}{\left(1 + \frac{y}{a}\right)^2}\,. \tag{145}$$

Let us assume that the area of ejection is equal to the "caliber" of the rocket. Then

$$\frac{d^2y}{dt^2}=\frac{S}{M}\left[\varrho v^2-\frac{R}{S}\right]-\frac{g}{1+\frac{y}{a}^2}. \tag{146}$$

1. The condition of minimum cross section is

$$M_0 \leqslant \varrho . \frac{S_0 v^2}{g}. \tag{147}$$

46 2. During the entire burning period, if $\frac{S}{M}$ does not increase, the total acceleration $\frac{d^2y}{dt^2}$ will never be negative. Actually, according to hypothesis,

$$\frac{S_0 . \varrho . v^2}{M_0} - g \geqslant 0,$$

or, also according to hypothesis, $\frac{S}{M}$ does not diminish, so that we obtain the same for

$$\frac{S}{M}\varrho . v^2.$$

Moreover,

$$\frac{g}{\left(1+\frac{y}{a}\right)^2}$$

decreases, when the height increases. Consequently, the difference

$$\frac{S . \varrho . v^2}{M} - \frac{g}{\left(1+\frac{y}{a}\right)^2}$$

increases with height.

In order for the total acceleration

$$\frac{d^2y}{dt^2} = \frac{\varrho . Sv^2 - R}{M} - \frac{g}{\left(1+\frac{y}{a}\right)^2}$$

to become negative, R must increase, but this requires an increase in V, which in turn means that the total acceleration must be $\geqslant 0$.

Corrections. If during the burning period $\frac{M}{S}$ is a constant quantity (exponential rocket) or a nonincreasing function of time, then:

a) the velocity and height will continually increase;

b) it will be true at all times that

$$\frac{R}{S} < \rho \cdot v^2,$$

except for the case when the acceleration is less than 0.

For rocket flight in a vacuum we have seen that, given identical maximum sections, a cylinder is the most economical shape and also that a cylindrical rocket of maximum length could transport the greatest final mass.

For the case of air, we can compare rockets possessing equal capabilities of air penetration. The slowing-down process can be expressed as

$$\frac{R}{M} = \frac{1}{\bar{\omega}} \varphi (V, y) \qquad (147)$$

where $\bar{\omega}$ is the "coefficient of ballistic penetration," and $\varphi (V, y)$ is a function
47 which increases with V, decreases with y, and depends on just these two variables.

We obtain the following results: of two cylindrical rockets with identical ballistic penetrations, the longer of the two, or, what comes to the same thing, the one with greater mass per unit section area, will ascend higher or (for equal heights) lift a greater final mass.

Here it would be sufficient to repeat the earlier discussion for the case of a vacuum. However, so that the question of the velocity loss due to air resistance will not perplex the reader, let us throw more light upon it by first studying the case of two projectiles P and P^1 with equal ballistic penetrations, sent upward at the same moment and from the same height; the initial velocities of the projectiles are V_0 and V_0^1.

Let us assume that

$$V_0 > V_0^1,$$

in which case, regardless of the air resistance, projectile P will reach a greater final height than P^1. Let us consider this case in more detail.

At the end of a very small time interval following the launch, the first projectile will be higher. However, it will slow down more due to the air resistance than P^1 will, so that its velocity loss will be greater. The second projectile will nevertheless fail to catch up with it, since, just when its velocity is equal to that of the first projectile, its retardation due to air resistance will not only be equal to, but will even exceed, that of the first projectile, since it is lower and since the effect of gravity is stronger there. Consequently, the first projectile attains a greater final height.

Now let us return to the case of two rockets, in this instance two identical rockets Φ and Φ^1. Rocket Φ is assumed to have a velocity V_0 at a height y_0, while Φ^1 is still on the ground with zero velocity.

Now both rockets are ignited. It can be assumed that, for equal fuel consumptions, the first rocket will fly ahead of the second. Actually, when the velocity of the second rocket becomes equal to that of the first, the retardation due to air resistance will be the same (if not greater) for the second rocket than for the first. Thus the second rocket will fly in front and will still have an advantage at higher altitudes.

As a result, for flight in air as well as for flight in a vacuum, a cylindrical rocket of greater length will lift a given mass higher or will lift a greater mass to a given height.

Curve of total fuel consumption. For the case of a cylinder we have

$$\frac{R}{S}=\varrho . v^2, \tag{148}$$

48 which, for cylinders with equal $\frac{\overline{\omega}}{S}$, is the equation for a (V,y) curve possessing unusual properties.

Actually, we see that the velocity varies, increasing. Let us assume that the burning of the rocket proceeds to the end, so that M goes to zero. The height of burnout will be limited (and, of course, less than for a vacuum), if the velocity becomes infinite; in such a case the ratio $\frac{R}{S}$ also goes to infinity. However, at a certain time it will be true that

$$\frac{R}{S}>\varrho\cdot v^2$$

and the acceleration becomes negative. Accordingly, the velocity cannot increase above some given limiting value.

On the other hand, toward the end of burning, as M approaches 0, it can be assumed that, if the difference $\varrho . v^2-\frac{R}{S}$ remains finite (that is, greater than some specified small quantity), the velocity increases and exceeds the given limit, which contradicts the first part of this discussion. Thus, the difference

$$\varrho . v^2-\frac{R}{S}$$

goes to zero. The following is therefore true: the equation

$$\frac{R}{S}=\varrho . v^2.$$

is the **curve of total burning**. All the burning curves approach this curve.

For a cylinder, using the same notation as in the case of flight in a vacuum, we obtain

$$\frac{d^2y}{dt^2}=\frac{v-\frac{1}{\varrho v}\cdot\frac{R}{S}}{T-t}-\frac{g}{\left(1+\frac{y}{a}\right)^2}, \tag{149}$$

and assuming

$$t=\lambda T,$$

we obtain

$$\frac{dV}{d\lambda}=\frac{v-\frac{1}{\varrho v}\cdot\frac{R}{S}}{1-\lambda}-\frac{gT}{\left(1+\frac{y}{a}\right)^{2}}. \tag{150}$$

Let us now introduce coefficient k, where

$$T=k\tau=k\frac{V}{g}\ (0\leqslant k\leqslant 1). \tag{151}$$

The formulas of ballistics are used to determine R. The acceleration
49 Γ, expressed in the cgs system, is understood to be the retardation due to the air resistance faced by a projectile of mass p, for some fictitious ogival angle. Therefore,

$$R=p.\Gamma. \tag{152}$$

According to Havre's formula,

$$\Gamma=\Delta_0'\frac{a'^2}{p^1}\sin\gamma\, e^{-hy}F(V)\times 100, \tag{153}$$

where $\Delta_0'=1.208$ (mass in kg of 1 m^3 of air at the earth, according to Havre). a^1 is the diameter (caliber) in m, p^1 is the mass in kg, and $h=10^{-4}$.

In the expressions for e^{-hy} and $F(V)$, quantities y and V are in m and m/sec. Thus we have

$$\frac{R}{S}=\frac{400}{\pi}\left(\frac{p}{p'}\right)\left(\frac{a'}{a}\right)^{2}\Delta_0'\sin\gamma\, e^{-hy}F(V)\begin{cases}S=\frac{\pi a^2}{4}\\ S \text{ in cm}^3\\ a \quad \text{in cm}\end{cases}$$

where

$$a^1=\frac{a}{100}\quad p=1000\,p^1.$$

Consequently,

$$\frac{R}{S}=\frac{40}{\pi}\Delta_0'\sin\gamma . e^{-hy}F(V). \tag{154}$$

Now our equation can be written as

$$\frac{dV}{d\lambda}=\frac{v-\frac{40}{\pi\varrho . v}\Delta_0'\sin\gamma\,\varepsilon^{-hy}F(V)}{1-\lambda}-\frac{gT}{\left(1+\frac{y}{a}\right)^{2}}, \tag{155}$$

where all the units are cgs, except y and V in e^{-hy} and $F(V)$, and the lengths are in meters.

If y, v and V are everywhere expressed in m and m/sec, the equation becomes

$$100\,\frac{dV}{d\lambda}=\frac{100v-\frac{40}{100\,\pi.\varrho.v}\,\Delta_0'\sin\gamma.e^{-hy}F(V)}{1-\lambda}-981\,\frac{T}{\left(1+\frac{y}{v}\right)^2} \qquad (155\text{ bis})$$

or

$$\frac{dV}{d\lambda}=\frac{v-\frac{4}{1000\,\pi.\varrho.v}\,\Delta_0'\sin\gamma\cdot e^{-hy}F(V)}{1-\lambda}-9.81\,\frac{T}{\left(1+\frac{y}{a}\right)^2}, \qquad (155\text{ ter})$$

where ϱ is in cgs units, $\Delta_0'=1.208$ (or some other value in kg/m³ corresponding to $y=0$), and y, a, and V are in m.

For

$$v=2000\,\text{m/sec},\ \varrho=\frac{1}{4000},$$

50 we find the coefficient of $\sin\gamma.e^{-hy}F(V)$ to be

$$\frac{4}{1000\,\pi.\varrho.v}\,\Delta_0'=\frac{4\cdot 1.208}{1000\,\pi.\frac{1}{4000}\cdot 2000}=\frac{8\,.\,1.208.10^{-3}}{\pi}=3.0761\,.\,10^{-3}.$$

Accordingly,

$$\frac{dV}{d\lambda}=\frac{2000-3.0761\,.\,10^{-3}\sin\gamma.e^{-hy}F(V)}{1-\lambda}-9.81\,\frac{T}{\left(1+\frac{y}{a}\right)^2}. \qquad (155_4)$$

As already stated above, a cylindrical rocket is one which has a constant ejection area. A conical rocket, on the other hand, has an ejection area proportional to the power of $^2/_3$ of the residual mass.

However, assuming that in this case the diameter of the projectile remains constant, it would be of no use to derive the corresponding formulas, since it is not clear how the throat area of the nozzle would change, this being essentially the ejection area.

The case is similar for an exponential rocket. Moreover, further discussions of particular types of rockets more or less approximating conical or exponential rockets will be, more precisely, discussions of successive cylindrical or composite rockets (fusées gigognes). Special study of each of these cases is necessary.

AIR RESISTANCE

The foregoing formulas were established using ballistic data. However, function $F(V)$, which enters into the expression for R, has still not been determined. Although the velocities involved here are greater than those in ballistics, still, since only an approximate evaluation of the phenomenon is desired, let us use the usual formula of aviation:

$$R = KSV^2. \tag{156}$$

This formula gives only a first approximation, and later it should be changed to show the variations with pressure, temperature, and humidity, by introducing appropriate coefficients. In addition, it is unfortunately necessary to introduce another, quite arbitrary, coefficient to compare the midship sections (of rockets) with different profiles moving in air.

This method of comparison seems to me to be erroneous, since the aerodynamic properties of a plate moving orthogonally depends on its dimensions and on the shape of its contours. Consequently, the choice of a standard for the square section will be completely arbitrary.

I propose that we always compare the resistance to the penetration of the air by the rocket with the momentum, relative to this rocket, of an air column with the same midship section as the rocket, and with a length equal
51 to its velocity of motion in air. This resistance will be equal to the force produced by a complete cancellation of the momentum relative to the air which would be encountered by the rocket if all its molecules were to move across the plane. Such a determination has the advantage that it is possible to establish the absolute coefficient of penetration relative to the cross section of the rocket.

If such a form were realized, its resistance to motion through the air would be expressed in ordinary aerodynamic units (kg of weight, m, sec), giving

$$R = \frac{a}{g} SV^2, \tag{157}$$

where $g = 9.81$ and a is the weight (in kg) of one m^3 of air at the given point. Since all the units are cgs, this formula may be written as

$$f = aSw^2, \tag{158}$$

where a is the mass in grams of one cm^3 of air at the given point.

In ballistics the exponent of w is assumed to increase with the velocity, reaching almost four for the speed of sound. However, let us retain the formula as given in (158), since it yields more favorable results.

For an arbitrary rocket shape,

$$f = kaSw^2, \tag{159}$$

where $k = 1$ for the standard shape. In accordance with experiments in aerodynamics laboratories, we have

$k = 0.70$ for a plane;
$k = 0.106$ for a sphere.

If, as in our case, the rocket moves at a velocity considerably greater than the average velocity of molecules of the surrounding gas, then we can assume that a perfect vacuum exists astern of it. Thus the entire force f

may be attributed to the compression of gas ahead of the rocket nose. Now it is easy to obtain the average pressure; from (159) we have

$$p_m = \frac{f}{S} = kaw^2. \qquad (160)$$

Let us designate as p the external (overall) pressure, giving a compression ratio of

$$\frac{p_m}{p} = \frac{a}{p}\, kw^2, \qquad (161)$$

where

$$\frac{a}{p} = \frac{1}{pv} = \frac{1}{RT}. \qquad (162)$$

Here R is the perfect-gas constant divided by the molecular mass of the gas, T is the absolute temperature, and a and p are the specific mass and
52 pressure of the air at the given atmospheric point (cgs units).

Thus

$$\frac{p_m}{p} = \frac{k}{R} \cdot \frac{w^2}{T}. \qquad (163)$$

This remarkable expression shows that in a gas at constant temperature the degree of compression depends only on the velocity, being proportional to the square of the latter. The degree of compression is therefore independent of the gas density at the point in question.

To determine the gas temperature ahead of the rocket, we use the expression

$$\frac{T}{T_{amb}} = \left[\frac{k w^2}{RT_{amb}}\right]^{\frac{\gamma-1}{\gamma}} \qquad (164)$$

or

$$T = T_a^{\frac{1}{\gamma}} \cdot \left[\frac{k w^2}{R}\right]^{\frac{1-\gamma}{\gamma}}. \qquad (165)$$

This formula shows that the final temperature increases with an increase in the temperature of the surrounding gas, but less rapidly than the latter does. Moreover, this final temperature does not depend on the pressure of the surrounding air. Consequently, it is incorrect to say that the rocket heats up due to "air friction," as is usually stated in the case of meteors. Friction itself could not exert any appreciable effects, since it is a function of the first power of the velocity, rather than the square. At high velocities the effect of friction will be completely overshadowed by that of the kinetic energy of the air, which is proportional at least to w^2.

The heating is a result of the compression, which is quite sufficient to heat meteors as well. For instance, let us consider a moving body with

$k = 0.1$ (for a projectile of ogival shape k will be somewhat lower, and for a meteor it will be somewhat higher). For $T_a = 250°$ abs, the heating ΔT of the air ahead of the body will be a function of the velocity (see Table 6).

TABLE 6.

u km/sec	1	2	3	5	7	10	50	100
ΔT	24°	159°	266°	445°	595°	754°	2,390°	3,705°

It is clear from the table that for a velocity of 2 km/sec the heating is already great enough to preclude the presence of living beings aboard the rocket. It is true that this heating will not last long, and that the heat capacity of the rocket slows down somewhat the influx of thermal energy. In addition, the rocket will cool off toward the stern, where the air is
53 rarefied and cooler.

Note 1. In aeronautics k is assumed to be much lower. For example, for streamlined bodies k may be as low as 0.03 (using my units of measurement). However, it should be noted that this result is obtained because small jets of the medium converge behind the body and give rise to a propelling force forward. This force reduces the drag, but in itself this resistance does not decrease.

The enormous velocities with which we are dealing here are many times greater than the average molecular velocity of the surrounding gas. Consequently, this weakening of the penetration need not be taken into account, since the small jets of the medium are not able to close the rocket in from behind. On the other hand, it may be that the pointed nose of the rocket, by reducing the relative impact velocity of molecules, will have a marked influence with respect to both the resistance to penetration and the thermal situation, compressing the air. Be that as it may, it is hardly possible that k will be less than 0.5, but nevertheless the temperature will still be quite high.

Note 2. Formula (155) indicates that the temperature T does not depend on the surrounding pressure. Thus it might be concluded that no manned rocket could ever leave the earth without vaporizing. If this were the case, however, meteors would become ignited when they arrived from infinity just to heights of about 120 km. Hence it follows that a certain temperature may be insufficient for heating. Somewhat more heat is required.

Later it will be seen that the energy produced by retardation as a projectile falls toward the earth reaches an appreciable value only at a height of about 120 km, the height at which "falling stars" are observed. Above this level neither energy nor heat are developed. If a meteor or projectile were to become slightly heated there, it would soon radiate into surrounding space a quantity of heat equal to that obtained earlier and would not become heated further.

Experience with meteors convinces us in this respect. Only a cylindrical rocket, which develops its maximum velocity below 200 km, will heat up to a dangerous degree. A conical rocket, with a maximum speed at a level nine times higher (around 1,800 km), will not be exposed to this danger, and the same is true of an exponential rocket, except for an acceleration $\Gamma = 10\,g$ which should be considered unsuitable for other reasons as well.

On the basis of the foregoing considerations, we conclude that the presence of air resistance does not modify considerably the results obtained for motion in a vacuum. This is because a rocket, in contrast to a
54 projectile fired from a gun, develops its maximum velocity gradually rather than rapidly. Except for the two cases cited, this velocity becomes very large only above the dangerous zone at 120 km, where the density and the resistance to penetration are so low that the work required to overcome them is negligible, regardless of the velocity. Consequently, the rocket can serve as a vehicle for flights into interplanetary space.

Chapter III

POSSIBLE ROCKET APPLICATIONS

The study of the upper atmosphere may well be the first application of rockets. Theory shows that the nitrogen content should increase with height; then, at very great heights, this gas should give way to hydrogen. Above the hydrogen zone there is probably an even lighter gas, the basis of the luminous phenomena of the aurora borealis. This gas, as yet chemically unknown and hypothetical, is called geocoronium.

It would be of interest to study the regions of the atmosphere above the heights (up to 30 km) attained by sounding balloons. Any altitude can be reached with the aid of rockets; the only problem will be to obtain a sufficient amount of such a rarefied gas. Physicists, of course, would be quite satisfied to study just a small sample of it.

In 1919 Prof. Goddard proposed another rocket application, namely a "moon shot." His suggestion was to send a pound of magnesia [flash] powder (American "Victor" powder) to the moon and to watch the explosion through a telescope. Calculations show indisputably that such an experiment is theoretically possible. Moreover, the American newspapers soon announced that an appropriate rocket was ready to take off. It is not known to me whether such experiments, worthy of American enterprise, have been carried out. As yet nothing has been heard of their results.

This problem is solvable for certain conditions. As I concluded 15 years ago, and as was assumed later by Goddard, the velocity of gas ejection should not be more than 2,000 m/sec. Table 5 shows that at low accelerations unacceptably high ratios of the initial and final masses are obtained. The highest acceleration assumed by me ($\Gamma = 10\,g$), which is acceptable for recording instruments or for specially constructed photo apparatus, can be attained without any insurmountable difficulties. For a vacuum the mass ratio is 358.5, so that an initial mass of 358.5 kg is required in order to send a final mass of 1 kg into space: Here, however, the initial mass is assumed to be almost all fuel. I say "almost all" because actually the fuel can provide an exhaust velocity considerably higher than 2,000 m/sec; during
55 the calculation of this velocity it was assumed that the rocket was relieved of its inert mass to only a small extent, in comparison with the fuel. In addition, I do not take into account the heating which can occur for an acceleration of 10 g.

Goddard obtained a less favorable ratio for air than I did: 602. However, in the transition from theory to practice, unbelievable difficulties are encountered, even assuming (as did Goddard) that for a moderate assumed velocity the weight of the surrounding gas equals only $^1/_4$ the weight of air. Consequently, for 1 kg of final mass, 43 kg of shell and 558 kg of fuel will be necessary. It must be admitted that I do not have in mind a similar rocket. But Goddard worked with a powder providing 1,238.5 cal/kg, while, as early as 1912, I noted in my brochure that fuels more powerful than this exist. Then I referred to a powder similar to the American powder mentioned previously, but at the same time I also drew attention to a mixture of hydrogen and oxygen in appropriate proportions, which provides 3,860 cal/kg.

For his powder, Goddard obtained an experimental value of

$$v = 2434 \text{ m/sec.}$$

A mixture of H_2 and O, on the other hand, may give about 3,400 m/sec. Here, however, a reservation should be made. For a high degree of expansion the exhaust velocity depends mainly on the initial temperature, while the latter depends in turn on the rapidity with which the combustion products are dissipated. Thus the problem is very complicated. In order to evaluate the results, we must know the combustion reaction for Goddard's powder. If the combustion products are vapors of water and carbon dioxide, then a marked dissociation takes place, especially of the carbon dioxide. If, on the other hand, vapors of water and carbon monoxide are produced, only the first of these will undergo a certain degree of dissociation.

In any case, the dissociation increases so rapidly with temperature that the latter is appreciably reduced. For example, if hydrogen is burned with oxygen in the right proportion, water vapor at a temperature of 5,300 to 5,400°C should be produced, whereas the flame of an oxygen jet will not have a temperature exceeding 2,500°, due to the radiation losses. The limitation of the increase in temperature is also known to be due to dissociation. Because of the foregoing, the $H_2 + O = H_2O$ reaction cannot be expected to give a velocity greater than 3,000 m/sec. However, this constitutes a very considerable improvement over a rocket with an assumed acceleration of $\Gamma = 5\,g$ (the limit for heating). Here the mass ratio will be only 63, which facilitates the construction of the rocket. Moreover, even better results can be obtained. Prof. Langmuir, working at the General Electric Company in America, prepared some atomic hydrogen and used it in a burner according to the reaction $H + H = H_2$. This reaction liberates more heat per molecule than the formation of water vapor does (58°C), and it has the advantage
56 that it reduces the dissociation temperature even more.*

The final molecular mass is $^1/_9$ of that for water, and this reaction would have a great [overall] advantage were it not for the fact that, unfortunately, the enormous specific heat (3.8) cancels out this advantage in part, limiting the theoretical temperature to 9,900°. In the final analysis, the practical results depend on the dissociation of molecular hydrogen into atomic hydrogen. Obviously, if this dissociation is not great at high temperatures, then this means can be used to obtain very high temperatures.

* According to different data, we have: 1) 75 to 80 cal per molecule; 2) 90 cal at constant volume and 3,000°; and 3) 85 cal at constant pressure and the same temperature. I have assumed the lowest value, 75 cal.

In the absence of more precise data at present, let us assume that the velocity may reach 10,000 m/sec, with a theoretical limit of 12,000 m/sec. Then, according to Table 5, we obtain quite acceptable mass ratios, even for $\Gamma = 2\, g$.

However, will it be possible to keep atomic hydrogen in liquid form? Will there not be a danger of explosion? Will it not be easily detonated? Can it be stored conveniently?

I do not have the answers to these questions. However, even if satisfactory answers can be found to them, there is another problem of a special kind, which Goddard did not foresee and which I will now discuss.

In order to overcome the earth's gravity, a rocket must develop a velocity of 8,000 to 11,200 m/sec, depending on the height of flight. This velocity will be equal to that acquired by an object falling to the same place from infinity without any initial velocity.

Lunar gravity is much weaker than terrestrial gravity. At the surface of the moon it is only 0.165 as great. The moon's radius is 0.237 of the earth's radius. At a distance from the lunar center equal to the radius of the earth, the acceleration due to gravity will be only

$$0.165 \cdot 0.273^2 = 0.01229,$$

that is, it will be a little more than $^1/_{100}$ of that at the earth's surface. This figure indicates the ratio of the masses of the moon and earth.

If there is even a slight error in either the launching angle or the velocity at burnout, the rocket will not have the proper trajectory. If the purpose is to hit the moon and if the rocket is aimed well enough, then the terminal velocity is essentially unimportant, provided it is high enough. It should be noted that it will be very difficult to aim the rocket precisely, unless the launching site is so chosen that the moon is in the equatorial zone, where the tangential velocity due to the rotation of the earth is about 463 m/sec. This velocity has to be added to the velocity of the rocket relative to the earth, irrespective of the effect of air currents. All this complicates the
57 aiming of the rocket.

If a zenith launching is carried out at a higher latitude, the smallest excess in the terminal velocity will cause the rocket to bypass the moon and to go off either into infinity or else into a descent on the invisible side of the moon.

In any case the point of impact of a rocket on the moon, even under the most favorable conditions, cannot be specified accurately, and it would be very difficult to observe it with a telescope, as proposed by Goddard. In a letter which I sent to Goddard on 16 June 1920, I pointed out how interesting it would be to send a rocket not to the moon, but rather around it. We see only one side, and as yet not a single person on the earth has been able to view the other side of the moon. It would be of great interest to photograph this invisible side.

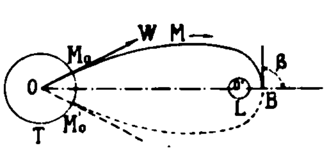

FIGURE 6.

With respect to this, however, certain difficulties arise which I did not foresee in 1920, but which I will try to evaluate here. Let us consider just the symmetrical branches of the trajectory, as shown in Figure 6. We assume that these branches intersect the line between the centers of the earth and moon at a right angle at point B.

Let us use the following notation: M_0 is the point of departure from the earth, α is the angle M_0TL, l is the distance from B to the lunar surface, W_0 is the velocity at launching (I assume that this velocity is attained instantaneously at point M_0), and W_c is the critical velocity of free flight at point A.

In order for the trajectory to pass behind the moon, angle α must be between 1° and 9°. The corresponding values of W_0 will range from 0.99 W_c to 1.0001 W_c, and the values of l will range from zero to infinity. Consequently, if the angle varies by 8° and the velocity varies by 1 %, the distance at which the rocket will circumvent the moon will vary from zero to infinity.

The trajectory will be symmetrical with respect to line TB [or OB] only if angle α and velocity W_0 are calculated very precisely. An error on the high side means that the rocket will not return to the earth, and for an error on the low side it will fall to the moon.

58 These considerations indicate the great difficulty, or even the impossibility, of sending a rocket around the moon, just on the basis of the aiming accuracy and the selection of the departure velocity.

Now let us consider the question of whether the retarding effect of the atmosphere can be used during the return of a rocket to the earth, a question which I discussed back in 1912. The corresponding calculations are as follows.

The velocity of an object falling to earth from infinity with zero initial velocity is

$$V^2 = 2g\frac{a}{a+y}, \tag{190}$$

where α is the radius of the earth, and y is the height. At a height of 200 km, this velocity will be 11,105 m/sec.

The density of the atmosphere can be expressed approximately by the formula

$$H = \zeta L\frac{\mu_0}{\mu}, \tag{191}$$

where μ_0 is the specific mass at the earth, and μ is this quantity at a height H. In the cgs system we have $\zeta = 10^6$ for very great heights. Let us set

$$z = \frac{H}{\zeta}. \tag{192}$$

Then (191) gives

$$\mu = \mu_0 e^{-z}. \tag{193}$$

The acceleration will be

$$F = \frac{d^2H}{dt^2} = \zeta\frac{d^2z}{dt^2}. \tag{194}$$

However, F is the sum of two quantities: one due to gravity,

$$f_1 = -\frac{Mg}{\left(1+\frac{y}{a}\right)^2} \tag{195}$$

and the other due to air resistance,

$$f_2 = k\cdot\mu S w^2 = k\mu_0 S w^2 \cdot e^{-z}\,. \tag{196}$$

The equation of motion is

$$\frac{d^2H}{dt^2} - \frac{k\mu_0 S}{M} w^2 e^{-z} - \frac{g}{\left(1+\frac{y}{a}\right)^2} = 0, \tag{197}$$

where

$$w = \frac{dH}{dt} = \zeta\frac{dz}{dt} \text{ and } \frac{d^2H}{dt^2} = \zeta\frac{d^2z}{dt^2}\,. \tag{198}$$

For simplicity we take

$$\frac{k\cdot\mu_0\cdot S\cdot\zeta^2}{\zeta M} = A \text{ and } \frac{g}{\zeta} = B; \tag{199}$$

59 so that (197) may be written as

$$\frac{d^2z}{dt^2} = A\left(\frac{dz}{dt}\right)^2 e^{-z} + \frac{B}{\left(1+\frac{\zeta}{a}z^2\right)}\,. \tag{200}$$

This equation can be solved, but the solution is complicated. We can see what it represents if we note that the effect of air resistance is nearly imperceptible at heights above 200 km. For this reason, above such heights, I determined the velocity with which the rocket approaches. Whether it falls from the moon or from infinity, this velocity changes very little.

Below 200 km the force of gravity may be assumed to be constant, and equal to 951 in the cgs system. However, for simplicity, it can even be neglected, and the approximation will still be sufficiently accurate.

Let us designate as z_0' the velocity with which the rocket approaches a height of 200 km. Then equation (200), simplified in the manner indicated above, can be integrated easily

$$\frac{d^2H}{dt^2} = A\zeta z_0'^2 e^{-2A\left(e^{-z}-e^{-z_0}\right)-z}. \tag{201}$$

If an ordinary parachute is used in the descent $(K=1)$, $\frac{M}{S} = 2$ kg/m², and the deceleration becomes appreciable only at a height of 150 km, where it will be equal to 1.8 times the acceleration of gravity. Unfortunately, however, it increases rapidly and reaches a maximum at a height of 91.5 km, where it is 229 times the acceleration of gravity. Subsequently it diminishes at the same rate, going to zero at a height of 70 km. Special instruments could survive this maximum deceleration, but it would be fatal to living beings.

In order to avoid this, I assumed that it would be possible to enter the atmosphere at a tangent, thereby utilizing the air density more uniformly. Unfortunately, the braking begins only at a height of 150 km and increases extremely rapidly over a distance of 80 km. If the trajectory follows a tangent at a height of 150 km, then for a flight at 1,340 km with passage of this height at an angle of 12°, ζ will become 4.5 times greater. Calculation shows that the greatest deceleration will be much less than the previous figure, but that it will still be 51 times terrestrial gravity. Assuming a descent angle of 6°, we obtain a value of ζ ten times as great and a deceleration of about 23.4 g.

I do not know how, without taking special measures, an organism can survive such an increase in the force of gravity (similar to that at the surface of the sun), which would make a 75-kg person weight 1,750 kg. Perhaps automatic parachutes of variable area could be used, which would begin to work earlier and then gradually vary their surface area.

However, this would require such an accuracy of the tangential descent that it would be attainable only through a control of the rocket by means of additional bursts. Such bursts could be better used, on the other hand, to
60 slow the rocket down during descent.

Now let us consider the power of the deceleration, relative to a gram of mass of the rocket. We have

$$\frac{d^2 H}{dt^2} w = \frac{d^2 H}{dt^2} \cdot \frac{dH}{dt} = \frac{d^2 H}{dt^2} \zeta \ z^1, \tag{202}$$

from which

$$P = A . \zeta^2 . z_0'^3 e^{-3A(e^{-z} - e^{-z_0}) - z}. \tag{203}$$

This power reaches a maximum at a height of 95 km, that is, only slightly above the height of maximum deceleration. The maximum power is very high, 14.8 kw, or about 20 HP per gram. As noted previously, the retardation is a consequence of the velocity. The air is compressed ahead of the parachute, developing a pressure of 458 kg/m², if the parachute is rated at 2 kg/m² at the ground.

This pressure, corresponding to only 46 g/cm², is very low in absolute value, but compared to the normal pressure that high in the atmosphere it is enormous. The degree of compression can easily be expressed as a function of z. The pressure ahead of the falling object will be

$$\rho_m = \frac{M}{S} \frac{d^2 H}{dt^2} = \frac{M}{S} A . \zeta . z_0'^2 e^{-2A(e^{-z} - e^{-z_0}) - z}. \tag{204}$$

The pressure at a height z, on the other hand, is

$$p = p_0 . e^{-z}. \tag{205}$$

The ratio, or degree, of compression will be

$$\frac{\rho_m}{\rho}=\frac{M}{S\,\rho_0}A\zeta z_0'^2 e^{-2A(e^{-z}-e^{-z_0})}=\frac{k\cdot\mu_0}{\rho_0}w_0^2 e^{-2A(e^{-z}-e^{-z_0})}, \qquad (206)$$

or, when z is very great,

$$\frac{\rho_m}{\rho}=\frac{k\mu_0}{\rho_0}w_0^2. \qquad (207)$$

The temperature of this, instantaneously compressed, gas will be

$$T=\dot{T}_{amb}\left[\frac{k\mu_0}{\rho_0}w_0^2\right]^{\frac{\gamma-1}{\gamma}}. \qquad (208)$$

Calculations show the degree of compression to be 1,950, and if the temperature of the surrounding air is $-50°C$, then the temperature of the compressed gas ahead of the parachute will be 1,730°. There is no need to ask what would happen to it under such conditions.

This calculation gives such a result because the temperature does not vary gradually; instead, it begins with a maximum at infinity for zero deceleration, decreases imperceptibly until the beginning of deceleration,
61 and then takes a sharp upward jump for the maximum developed power.

Therefore, meteors will probably not become ignited above 120 km either, this being the region in which their appearance is noted. At great heights the power developed by them is not great, and the temperature corresponds only to slight heating, a small amount compared to the radiation and not enough to heat the meteor.

This temperature rises only when the energy of deceleration suddenly reaches an enormous level. Calculations show that this occurs at heights of 120 to 130 km, where the power becomes 1.25 to 3.55 kW/g, respectively. This energy is applied just to the forward surface of a parachute, and thus the energy concentration per unit mass will be even higher. It continues down to a height of 80 km, the region where "falling stars" disappear.

Taking account of this significant result, I consider the use of a parachute for braking a rocket in the atmosphere to be impossible; for this purpose it will be necessary to equip the rocket itself with some type of counter-engine. Referring again to the figures given at the end of Chapter I, we see that for the optimum case (according to Table 5) it is necessary to have a propellant supply of

$$3.24^2 = 10.5 \text{ times the useful load,} \qquad (209)$$

or, for $\Gamma=2g$

$$4.36^2 = 19 \text{ times the useful load.} \qquad (210)$$

If the useful load is one ton, then 10.5 or 19 tons of atomic hydrogen will be necessary at launching, even for the limiting case, which is not practicable in reality. However, the problem is not insoluble. It is merely difficult to obtain a practical solution, especially when working with atomic hydrogen, the properties of which are still unknown.

Chapter IV

CONDITIONS FOR TRANSPORTING LIVING BEINGS

INTERPLANETARY SPACESHIPS

I have indicated above that it is possible to construct recording instruments capable of withstanding an acceleration of 10 g. Now, however, the question arises of the limit for living creatures.

I already have some indication of this from practice. In my airplanes I provided the pilots with elastic belts; these could be so adjusted that at the end of their extension they would bear, without being damaged, ten times the weight of the body. Thus this danger can apparently be eliminated. The problem of the heating, however, still remains. It would be more prudent, perhaps, to limit the acceleration to $\Gamma = 2g$. The use of an H + O mixture is inadvisable, so that atomic hydrogen has to be used, but [as noted above] its
62 properties are almost unknown.

Finally, let us assume that the heating problem has also been resolved. There will still be a number of difficulties to be overcome. It will be necessary, for instance, to have a fuel supply sufficient to overcome the earth's gravity, and it will be almost impossible to calculate a circumlunar flight accurately.*

Such a bold venture will involve several probable inconveniences, and it is difficult at present to assess their significance. For instance, at the moment when the thrust ceases, the passengers will experience a sudden transition from an acceleration of **2 g**, which in itself will be burdensome, to the absence of acceleration, which has not yet been experienced either.

CONDITIONS FOR STAYING ABOARD THE SPACESHIP

Provision of air for breathing

With respect to an air supply, our experience with submarines is very useful. Moreover, it may be assumed that an even more successful solution will be possible, especially when we have available the enormous quantities of energy associated with the splitting of atoms, which will provide new possibilities for chemical reactions acting on the atoms themselves. The main goal will be to retain, without any losses, the gaseous mass contained within a spaceship flying in a vacuum. This will be much simpler than it would be to maintain a vacuum inside a spaceship located in a gas under pressure, where enormous losses would be sustained for small quantities of gas penetrating the shell. In our case, however, such losses will pertain only to the mass of gas situated inside the spaceship under pressure.

It should be noted that it might be possible to fill the spaceship with an atmosphere of pure oxygen, so that the pressure could be reduced to $^1/_{10}$ of atmospheric. Then the losses would be even lower.

* [This, of course, was written long before the electronic computer was even imagined (Translator).]

Maintenance of suitable temperature

Temperature exists only where there is matter. Interstellar space, accordingly, is not as icy as is often thought. We are familiar with the low temperatures of the upper layers of our atmosphere. However, as tenuous as these regions are, they still contain some matter. The absolute temperature, on the other hand, can be neither cold nor warm. We know that heat is a sign of molecular motion, and if no molecules are present such motion cannot, of course, exist.

In my previous paper I only had room to describe in a few words the possibility of varying the temperature of a spaceship by blackening one of its surfaces and polishing the other, with one or the other side to be turned toward the sun. Rotation of the spaceship would then produce some average temperature, equal to that which a heat conductor would have in the same place, if its entire surface possessed the same radiating capacity, no matter what its degree. A black body represents a particular case of this. For
63 simplicity, I will call this temperature "the temperature of a black spherical conductor subjected to the effect of solar radiation at the given point." This is the temperature which would correspond to the nonexistent temperature of a vacuum, and we can see that in our regions it differs markedly from absolute zero.

BLACK-BODY TEMPERATURE AND LIMITING TEMPERATURES OF THE SPACESHIP

Let us consider a spherical heat conductor (Figure 7) of diameter D and radiating capacity K. The conductor is exposed to solar radiation in a direction z^0. We assume that ds is a surface element of the sphere determined by angles α, $\alpha + d\alpha$, β, and $\beta + b\beta$. In this case

$$ds = \frac{D}{2}\, d\beta \cdot \frac{D}{2}\, d\beta \cos\alpha, \qquad (220)$$

from which

$$ds = \frac{D^2}{4} \cos\alpha \, d\alpha \, d\beta. \qquad (221)$$

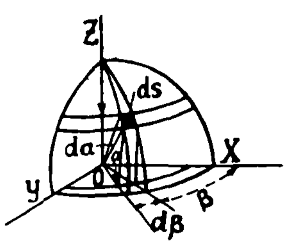

FIGURE 7.

If σ is Stefan's constant, and if θ is the absolute temperature of the sun, then the amount of heat which element ds will absorb if the radiating capacity of the star is unity, for the whole hemisphere, will be

$$dQ = k \cdot \sigma \cdot ds \cdot \theta^4. \qquad (222)$$

The amount of heat reflected or scattered in space will be

$$dQ_D = (1 - k)\, \sigma \cdot ds\theta^4. \qquad (223)$$

We designate as γ the solid angle subtended by the sun as seen from the sphere. Then a surface element absorbs only an amount of heat

$$dq = dQ\frac{\gamma}{2\pi}\sin\alpha = k\cdot\sigma\cdot\theta^4\frac{\gamma}{2\pi}\times\frac{D^2}{4}\sin\alpha\cdot\cos\alpha\cdot d\alpha\cdot d\beta. \qquad (224)$$

Integration over limits from $\alpha = 0$ to $\frac{\pi}{2}$ and from $\beta = 0$ to 2π gives

$$q = k\sigma\theta^4\frac{\gamma}{2\pi}\cdot\frac{D^2}{4}\left(\frac{\sin^2\alpha}{2}\right)_0^{\frac{\pi}{2}}\cdot 2\pi; \qquad (226)$$

$$q = \frac{\pi D^2}{4}\cdot k\cdot\sigma\theta^4\frac{\gamma}{2\pi}. \qquad (227)$$

However, this is the same amount which would be absorbed by a plane
64 disk of diameter D receiving radiation normal to it.

If the absolute temperature of the sphere is T, then its entire surface will radiate a quantity of heat

$$q^1 = \pi D^2 k\cdot\sigma\cdot T^4. \qquad (228)$$

Equilibrium will exist when

$$q^1 = q, \qquad (229)$$

that is, when

$$T^4 = \theta^4\frac{\gamma}{8\pi},$$

or

$$T = \xi\sqrt[4]{\frac{\gamma}{8\pi}}\,\theta. \qquad (230)$$

Vicinity of earth

If the sphere is close to the earth, then from it the sun subtends an angle of about 32', corresponding to a solid angle

$$\gamma = \frac{\pi}{4}\cdot 32'^2 = 804.8'^2 = 0.2235^{\circ 2}. \qquad (231)$$

A solid angle of 8π, on the other hand, equals twice the total surface, or $82{,}506^{\circ 2}$. Consequently,

$$\frac{\gamma}{8\pi} = \frac{0.2235}{82\,506} = 2.709\cdot 10^{-6}. \qquad (232)$$

Assuming $\theta = 6{,}300°$ absolute, we have

$$\theta^4 = 1.5753 \cdot 10^{15} \tag{233}$$

and

$$T^4 = 2.709 \cdot 10^{-6} \cdot 1.5753 \cdot 10^{15} = 4.267 \cdot 10^9, \tag{234}$$

and hence

$$T = 255.6° = -17.4° \text{ C} \tag{235}$$

The foregoing considerations can also be applied to the earth. Assuming an average surface temperature of 15°C or 288° abs., we see that the earth's central fire and the difference in atmospheric absorption between infra-red and visible light account for only 32.4°, that is, about 12%. Therefore, the conditions of our life on the earth's surface depend to a much greater degree on the sun than on the earth itself.

Life would continue on earth even if its inner central heat were used up, provided that the sun continued to shine in the sky. On the other hand, if the sun were gone, life on earth would be impossible on the basis of the internal heat alone.

65

Let us now consider a plane disk of diameter D which is continually oriented normally with respect to the solar radiation and which is backed by an absolute nonconductor of heat. The disk will receive an amount of heat

$$q = \frac{\pi D^2}{4} \cdot k \cdot \sigma \cdot \theta^4 \frac{\gamma}{2\pi} \tag{236}$$

and radiate an amount

$$q_1 = \frac{\pi D^2}{4} k\sigma \cdot \tau_M^4. \tag{237}$$

The condition for equilibrium is

$$\tau^4_M = \theta^4 \frac{\gamma}{2\pi} \tag{238}$$

that is,

$$\tau^4_M = 4T^4. \tag{239}$$

At the distance of the earth this gives

$$\tau_M = \sqrt{2} \cdot T = 361.5° = 88.5 \, C. \tag{240}$$

If the earth always turned the same side toward the sun, and if there was an excess of 32.4°C as a result of the factors indicated above, then at a point on the earth's surface where the sun was always at the zenith the temperature would be of the order of 120°C and the sea would boil there.

Now let us calculate the temperature τ_m on the other side of the earth. Below the surface of the earth, the temperature increases by 1° for each 30°. The average conductivity of the surface rock is $300 \cdot 10^{-5}$ cgs units. Therefore the heat flow per cm² will be

$$q = \frac{300 \cdot 10^{-5}}{3000} = 10^{-6} \text{ cal/g/sec.} \tag{241}$$

Stefan's law gives

$$\tau^4 = \frac{q}{\sigma} = \frac{10^{-6}}{1.39 \cdot 10^{-12}} = 720{,}000 \tag{242}$$

and thus

$$\tau = 29^\circ 13 = -244^\circ \text{C}.$$

In this case the entire atmosphere (with the exception of helium and hydrogen) would be frozen, and no form of life would be possible.

Note. The obtained values of 88° and −244° should correspond approximately to the extreme temperatures on the lunar surface, on the side toward the sun and directly opposite, taking into account the slowness of the rotation of this body.

Next let us assume that half the sphere is covered with a layer of
66 oxidized copper, having a radiating capacity $k = 0.85$. The other half is covered with a layer of polished aluminum ($k' = 0.13$). If the latter surface is turned toward the sun, then it absorbs an amount of heat

$$q = \frac{\pi D^2}{4} k' \sigma \theta^4 \frac{\gamma}{2\pi} \tag{243}$$

and radiates an amount

$$q_1' = \frac{\pi D^2}{4} k' \sigma \cdot T_m^4. \tag{244}$$

The other half will radiate an amount

$$q_2' = \frac{\pi D^2}{2} k\sigma \cdot T_m^4. \tag{245}$$

The condition for equilibrium is

$$q = q_1' + q_2' \tag{246}$$

or

$$T_m^4 = \frac{2k'}{k+k'} \theta^4 \frac{\gamma}{8\pi} = \frac{2k'}{k+k'} T^4, \tag{247}$$

and hence

$$T_m = T \sqrt[4]{\frac{2k'}{k+k'}}. \tag{248}$$

In our case

$$T_m = 0.7178\ T = 183.4° = -89.6°\ C. \tag{249}$$

If the blackened side is turned toward the sun, then we interchange k and k^1 in (248) to obtain

$$T_m = 1.1478\ T = 293.4° = +20.4°\ C. \tag{250}$$

Consequently, near the earth it is easier to bring about cooling than heating.

Vicinity of Venus

Now let us consider what would happen to a spherical spaceship near the planet Venus, which lies at an average distance from the sun equal to 0.72 of that of the earth. The solid angle subtended by the sun as seen from Venus is

$$\gamma_v = \gamma_T \left(\frac{1}{0.72}\right)^2 = 1.3887^2\, \gamma_T. \tag{251}$$

From formula (230) we obtain

$$T_v = \sqrt[2]{1.3887}\,.\, T_T = 1.1787\ T = 301.1° = +28.1°\ C, \tag{252}$$

which is a moderate temperature. However, the temperature due to the internal heat of Venus must be added to this value. Since the internal heat is
67 probably of the same order as that of the earth, the average temperature at the surface of Venus will be aout 60°C; this results in extensive evaporation and the formation of large clouds. This conclusion is in accordance with telescopic observations and, in particular, with albedo measurements.

I should mention in passing that it is quite unlikely that Venus always turns the same side toward the sun. If we multiply the temperature corresponding to the earth's distance by this same factor 1.1787, then we obtain 426° abs = 153°C. As indicated above, another 32° must be added to this value, so that the temperature of a place where the sun is at the zenith will be

$$+185°\ C, \tag{253}$$

a value quite different from the temperatures at the earth.

The temperature at a point diametrically opposite to this one is difficult to determine, since it depends on the heat provided by the internal fire of the planet. Assuming this heat to be twice that of the earth, we obtain a value of 35° = -238°C. Under such conditions almost the entire atmosphere should descend and solidify on this side of the planet. However, observations carried out during a transit of Venus ahead of the solar disk have shown that it possesses an atmosphere which is denser than ours, and in such an atmosphere the refraction will be almost twice as great. On the other hand, the albedo of the planet corresponds to a layer of freshly fallen snow or to clouds, so that Venus is most likely covered by a nearly solid cloud cover. It may be that our earth would have a similar appearance if it were viewed from above.

Such considerations lead us to believe that the days and years on Venus are different from those on earth. In order for the planet to have the appearance which is observed, it must rotate on its axis with a velocity which is at least equal to that of the earth, but which is probably considerably higher.

Returning now to our spaceship, let us determine the minimum and maximum temperatures using formulas (249) and (250), with $\boldsymbol{T}$ replaced by $\boldsymbol{T_v}$. We obtain

$$Tv_m = 216.1° = -56.9° C \text{ and } Tv_M = 345.5° = +72.5° C. \qquad (254)$$

This time the passengers would have no trouble keeping warm, and they would even have to take special measures to keep from being cooked.

Vicinity of Mars

In the vicinity of Mars, which is 1.52 times as far away from the sun as the earth is, analogous calculations give

$$\left.\begin{aligned} T_M &= 207.3° = -\ 65.7° C \\ T_{Mm} &= 148.7_0 = -124.3° C \\ T_{MM} &= 237.8° = -\ 35.2° C \end{aligned}\right\}. \qquad (255)$$

68 In this case the passengers would have to take special measures to keep from freezing. The walls of the spaceship would have to be impervious to heat, and a heating system would have to be provided inside the ship.

I should mention that Mars, with a diameter considerably smaller than that of the earth, should also have less internal heat as well. Living conditions on this planet will depend almost entirely on the solar radiation. The average temperature on Mars will be around 65°C below zero, even taking into account the effect of the atmosphere.

The question also arises of whether the Martian polar ice is not carbon-dioxide snow [dry ice] rather than water. Actually, because of the lower gravity at the surface of Mars, the lighter components of its atmosphere may escape into space, and water vapor is a very light gas. In any case, the atmosphere of this planet is known to be quite thin. If Mars did not rotate, then the side toward the sun would have a temperature of

$$293° = +20° C. \qquad (256)$$

However, it makes one rotation every 24 hr 37 min, and the latest measurements give the following temperature for the sunny side:

$$+7° C. \qquad (257)$$

Vicinity of Mercury

Analogous calculations give the following temperatures in the vicinity of Mercury:

$$T_{merc} = 409^\circ = +136^\circ C$$
$$T_{me\text{-}m} = 239.5^\circ = + 20.5^\circ C. \qquad (258)$$

The spaceship can still fly here, with just its polished side facing the sun. A ship with a shape similar to that of an artillery shell would be able to present a minimum profile to the sun. If certain precautionary measures were taken, it might even be able to approach closer to the sun.

PHYSIOLOGICAL EFFECT OF ABSENCE OF GRAVITY; ARTIFICIAL ACCELERATION

In my 1912 report I indicated some possible physiological effects of the reduction or elimination of the gravitational field which would be experienced by passengers in a spaceship. Here it will be appropriate to consider an error committed by Jules Verne in his novel "From the Earth to the Moon." Verne assumed that, if they survived the launching, the passengers would continually experience a feeling of normal gravity, except when they arrived at the "neutral point," where the attractions of the moon and earth are equal.
69 At this moment they would suddenly float up to the ceiling of the spaceship.

Actually, at the very beginning, during launching, the passengers would be crushed against the floor of the ship, and then, when the projectile emerged from the cannon, they would be flattened against its ceiling, since it would strike the atmosphere with a terrific velocity.

Let us assume, however, that the projectile is not stopped by the resistance of the atmosphere. In this case, after having been killed twice, the passengers would be subjected to the conditions of falling in a vacuum, even though their vessel is traveling at a great velocity. The sensation experienced during falling does not depend on the velocity, but only on the acceleration. When a body is in free flight, that is, when it is not acted upon by any external force, living beings inside of it will have a feeling of falling, regardless of the direction and velocity of fall.

Even without leaving the region of terrestrial gravity, we may be aware of an annoying feeling when we speed up or slow down in an elevator. Breathing is retarded and the feeling is given that, if it were to continue thus, the heart would stop as well. Future interplanetary travelers will find little consolation in the fact that, although their hearts may continue to work, their breathing may cease. During a fall from a great height in the atmosphere (for example, when a parachute does not open for a long time), the sensation of falling cannot long endure, since the fall is rapidly transformed into uniform motion by the air resistance balancing the weight of the body. Although the fall continues, a person will not experience it, since there is no acceleration. Only a strong air current would be felt, and not the falling itself, due to the absence of a gravitational field.

Many of us are familiar with the disagreeable sensations which are experienced at the ends of a ship during prolonged rolling of the sea. It might be useful at this point to consider the reason for the feeling of normal weight, when each molecule of the body is located in a gravitational field; if a molecule does not move, it is just because it is connected to adjacent molecules. To put it simply, the sensation of weight consists in our feeling that the head presses down on the shoulders, the shoulders on the back, the body on the legs, the legs on the feet, and the feet on the ground, which resists this pressure caused by the acceleration of gravity.

If the support of the earth were taken away, then each molecule and each organ composed of these molecules would be free to react to the acceleration of gravity, and they would all begin to move with the same velocity. The interaction of forces inside the body would cease, and, to put it simply, the head would no longer press down on the shoulders, the shoulders on the back, etc., and the legs would not be supported by the earth, which we have assumed to be gone.

There will apparently be a particularly marked effect on the hydrostatic system of the semicircular canals of the ears, which serve to orient the organs of the body and which are directly related to the sympathetic nervous system.

Consequently, the effect of the absence or marked weakening of gravity
70 should be considered very seriously. It is to be expected that persons who suffer from "seasickness" and "airsickness" will also be afflicted with "spacesickness." Back in 1912, in the hope of protecting space travelers from the risk of the absence of gravity, I considered creating a gravitational field using the engine of the spaceship. The passengers would then still have the sensation of normal weight. At that time I was not yet familiar with the work of Einstein, whose principle of general relativity demonstrates the equivalence of fields of gravitation and acceleration.

It is interesting to note that during the transition from travel on the ground to aviation, and then to astronautics,* we pass accordingly from transportation at an arbitrary variable velocity to transportation at a constant velocity, and finally to transportation at a constant acceleration.

I have already mentioned that this motion at constant acceleration requires the expenditure of a great deal more energy than in the case of free flight from the earth. In addition, I have assumed that, once the spaceship has reached a given height, it will be able to continue without any thrust. This moment of transition also presents a danger from the physiological point of view. However, I was unable to suggest the solution to this problem, namely to reduce the acceleration gradually using the engine. In this way an organism gradually becomes accustomed to the transition. This solution will be verifiable, however, only when we have at our disposal atomic engines and interplanetary spaceships, which, unfortunately, is still far in the future.

SPACESHIP MANEUVERABILITY

This subject was touched upon only very briefly in my previous report, because of the limited volume of the paper. However, it is of great interest. A projectile will follow a rectilinear trajectory only if the resultant of all the external forces acting on it has a constant direction and passes through its center of gravity.

* This term was proposed by J.H.Rosny.

In our case the resultant of the thrust and the reaction of the spaceship must at all times pass through the center of gravity of the ship. However, this condition will never be satisfied with mathematical precision, and so the ship must be provided with controls.

My first idea was to equip the ship with a reaction engine, which could be turned to any side with a control stick, as desired by the pilot. In this case, in contrast to airplane controls, the control stick can be moved automatically, with the aid of a pendulum. For example, when the rocket deviates from its
71 path, an electrical contact moves the reaction engine in the desired direction. Naturally, if the thrust force passes outside the center of gravity of the ship, the latter will change position due to the torque, and the trajectory will bend. Such deviations can be carried out at will, by adjusting the electrical contacts of the pendulum in such a way that the equilibrium position of the latter does not correspond to the direction of thrust, parallel to the velocity at a given moment.

In order to keep the ship from rotating about the direction of its flight velocity, it is possible to make use of tangential rockets. If the ship is so constructed that it has screw threads on its surface which impart to it a *similar rotation in the atmosphere during the upward flight, then with the* aid of the above rockets it will be easy to prevent this rotation.

Finally, if no initial rotation takes place, these rockets can be used to turn the ship through some desired angle, a result which can also be accomplished using an internal electric motor with a flywheel having a sufficient moment of inertia. To turn the motor on, the ship begins to rotate in the opposite direction, the angular velocities of the two rotations being inversely proportional to the corresponding moments of inertia. When the motor stops, the rotation of the ship ceases as well. During such an operation, friction between the rotor and the stator does not interfere.

It is difficult at present to predict just what features a motor utilizing the splitting of atoms will have. It may be that other methods for controlling the ship will be necessary, such as the use of several reaction engines, distributed outside the ship's axis of symmetry (for instance, around a circle of given diameter). One of these engines could be made to run faster and the others more slowly, etc.

Whatever the case may be, a spaceship in a vacuum will not be helpless. The laws of mechanics show clearly that a thrust can be imparted to it and that it can be maneuvered, in the same way as vehicles on the ground, in the water, and in the air. The main task will be to develop new powerful sources of thrust.

Let us assume, however, that such sources are available. What energy expenditure will be necessary and possible ? The solution of the first problem depends on the solution of the second. A body falling to a planet from infinity develops some terminal velocity of descent. The general law describing its motion is

$$V^2 = 2g\frac{a^2}{a+y}. \qquad (259)$$

For $y=0$

$$V_a = -\sqrt{2ga}. \qquad (260)$$

For the case of the earth, at the limit we obtain

$$V_{lim} = 11\,180 \text{ m/sec.}$$

This is the velocity which must be imparted to a body, launched toward
72 the zenith, in order for it not to fall back to earth (air resistance not taken into account). As the body disappears to infinity, its velocity gradually decreases, going to zero.

Next let us calculate the work performed in such a case by a 1-kg body. If, in general, the weight of a body at the surface of a planet of radius a is p, then this work will be

$$\tau = P.a. \quad (261)$$

For a terrestrial radius of 6,371 km, the amount of work done by a body with a mass of 1 kg will be

$$\tau_1 = 6\,371\,000 \text{ kg} \cdot \text{m} \quad (262)$$

or 14,940 cal.

We recall that 1 kg of powder (fulmicoton et chlorate de potasse) produces 1,420 large calories, 1 kg of a hydrogen-oxygen mixture in the correct proportion produces 3,860 Cal, 1 kg of atomic hydrogen produces 34,000 Cal (that is eight times as many as the preceding mixture), and 1 kg of radium produces $2.9 \cdot 10^9$ large calories (that is, 85,000 times as many). Finally, according to relativity theory, matter is just a stable form of energy with enormous amounts of the latter stored up in it. Thus, 1 kg of matter may be equivalent to $9.17 \cdot 10^{15}$ kg/m or $21.5 \cdot 10^{12}$ large calories (15 billion times as many calories as the powder mentioned above).

When such energy sources are finally at our disposal, the conditions of travel will be quite different, and the contrast will be reminiscent of that between present-day sleeping cars and the first simple railroad cars.

However, if we had available just an $H_2 + O$ mixture, then I do not see how space flight would be possible, since the acceleration $\Gamma = 10g$ at launching would be dangerous, and during the return there would be the additional danger of burning up in the atmosphere. Even under the best circumstances the mass ratio would be

$$200^2 = 40\,000, \quad (263)$$

which is not feasible.

If we could use atomic hydrogen, on the other hand, then, according to Table 5, a spaceship would be practicable, although there would be difficulty involved. In this case, however, it would be impossible to imagine an unmanned spaceship with automatic instruments; only a spaceship with a pilot would be possible. It would be more prudent perhaps to keep Γ under $2g$, so as to avoid the danger of heating up during the upward flight. In order to provide braking during the descent and control during flight, the mass ratio must be 20 or 25, which is rather difficult to achieve. But perhaps by that time a more suitable material will be available for the structure, such as metallic beryllium.

Now let us imagine such a flight. We assume an acceleration $\Gamma=2g$ and a velocity great enough to overcome terrestrial gravity, namely 9 km/sec. at a height of 3,185 km. The latter height will be attained in 12 min 30 sec.
73 Subsequently the spaceship will fly just under the influence of the acquired velocity. This is the boundary at which a sudden termination of thrust causes the feeling of a loss of weight, together with the physiological phenomena mentioned above. For the time being, I assume that we have safely survived these. Now our ship flies according to the laws of universal gravitation, just like any other celestial body.

The duration of the flight will be shorter or longer, depending on whether the ship passes close to or far from the moon. Half the duration of the flight will naturally be longer than the time it would take to fly to the moon along a straight line. However, a study of the latter case aids us in evaluating a flight along a curved trajectory.

Beginning at the moment when the thrust ceases, the ship slows down in accordance with the law

$$V=\sqrt{2\left(g\frac{a^2}{x}+0.165\,g\frac{\rho^2}{y}+0.820\cdot 10^6\right)}. \tag{264}$$

At the point where the attractions of the moon and earth are equal, this velocity drops to a minimum:

$$V=2030\ \text{m/sec}. \tag{265}$$

During the approach to the lunar surface, it increases to

$$V=3060\ \text{m/sec}. \tag{266}$$

On the other hand, the velocity of free fall to the moon from infinity is

$$V_0=2373\ \text{m/sec}. \tag{267}$$

The time required to traverse the second part of the path can be calculated approximately if we neglect the influence of the moon, which is comparatively insignificant. This time will be equal to that required for free fall along the path from the moon to the point of termination of thrust:

$$t=48^h\,30^m. \tag{268}$$

Consequently, the flight over the first half of the path will take a time

$$12^m+48^h\,30^m=48^h\,42^m. \tag{269}$$

A flight to the moon and back will thus take about $4\,{}^1/_2$ days.

The velocities calculated above seem to be enormous in comparison with those to which we are accustomed, but they are quite modest relative to the velocities of celestial bodies. The maximum velocity at the end of the thrust period will be 33,000 km/hr. In the vicinity of the moon it drops to 7,000 km/hr, which is a very modest value.

On the return path retardation of the ship should begin at the point where the thrust ceased earlier, that is, at a height of 3,200 km. A parachute should be employed only very near the earth (at a height of around 10 km).

74 In spite of the fact that we can now look forward to utilizing the energy of $H + H = H_2$, still we should limit ourselves to just a study of the moon. This, however, constitutes a great step forward, although it involves enormous danger. It should not be forgotten that we assume a successful conversion of atomic hydrogen to liquid form and a retention of it in this state without any danger of explosion. These are things about which nothing is known as yet and which, unfortunately, may well be impossible.

Before we can dream about the future, we must wait until physicists have learned more about atoms and about methods of utilizing them. The methods being used at present are still very primitive and of almost no value, except for the experiments of Rutherford, who has succeeded in splitting some nitrogen atoms. Although this result is in itself quite noteworthy, still we have a long way to go before it will be possible to use atomic energy in any significant amounts.

The nitrogen atom which was disintegrated in this way had a diameter of

$$0.000\,000\,028 \text{ cm}$$

and a mass of

$$0.000\,000\,000\,000\,000\,000\,000\,0233 \text{ g}$$

It is clear from this what a long way we still have to go. As yet it is quite difficult to foresee just how atomic energy will be used. Will an almost unlimited supply of such energy be available in some reservoir, to be used unendingly? Or will this energy be so inaccessible that we will not be able to affect it directly, so that its liberation will require a certain amount of work? I do not know what methods will be used, but I hope nevertheless that one day we will possess these sources of the kinetic energy of minute particles, which possess such colossal velocities close to the speed of light. Although neither the energy of radium nor an energy 10,000 times greater than the energy of that substance is as yet available, still it is to be expected that we will possess such energies in the immediate future.

Let us assume that a spaceship escapes with $\Gamma = 1.1g$ and that it flies for 37 min, after which it acquires the required velocity in a direction straight toward the moon. In this case the velocities will be almost the same as those calculated above. In order to keep the ship from crashing on our satellite, counterbursts of the rockets should be initiated at a height of 250 km above the moon, according to approximate calculations. This will turn the ship so that its bottom is toward the moon (it was stated earlier how this is done). The retardation will last for several minutes, so that the approximate total flight time will be

$$49^h\,11^m.$$

The return to the earth takes place in reverse order, and it will be much easier, since the attraction of the moon is only 0.165 of that of the earth.
75 This means that a ship with a weight of 1,000 kg on the earth weighs only 165 kg on the moon. For the return flight the ship must once again be turned, with a resumption of deceleration, as described above. A parachute is used only very near the earth, when the velocity has been greatly reduced.

Assuming that the engine will operate for only 75 min and that the spaceship weighs 1,000 kg at launching, the fuel needed will weigh 300 kg and the exhaust velocity will be 150,000 m/sec. This indicates that we are still far from our goal, even if atomic hydrogen is used. It should be noted, by the way, that the use of an atomic engine will involve the ejection of gas of a corresponding temperature. However, even for the lightest substance, atomic hydrogen, the exhaust velocity will require an initial temperature of 315,000°, while for other substances it will exceed 2,000,000°.

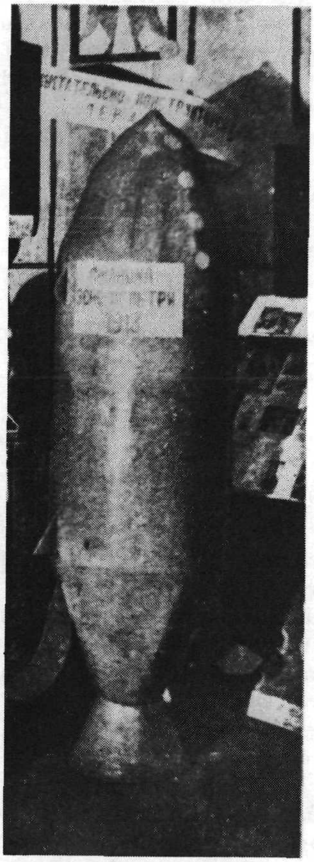
FIGURE 8. Rocket of Esnault-Pelterie on display at interplanetary exhibition in Moscow in 1927 [Figure added by Rynin].

It would be more expedient if an atomic engine could be made to eject electrons or positive ions directly. It is interesting to consider what the power would be in such a case, and we obtain a value of around 450,000 HP. The problem is to construct an engine with this power for a total spaceship weight of 1,000 kg. We assume an engine efficiency of 3%, which is not too bad. The exhaust velocity for ions will be considerably lower than 150,000 m/sec. A 1,000-kg ship will require much less than 300 kg of fuel, but at the same time its efficiency will not be as high.

Let us assume that we start the engine and that, after attaining the critical velocity, we wish to reach and maintain a velocity of 10 km/sec. Our flight is directed toward one of the planets closest to the earth, during its nearest approach to us. The length of such a trip will be:
to Venus: 42,000,000 km in 48 days 14 hr;
to Mars: 78,000,000 km in 90 days 8 hr.

It should be noted that the amount of work required for such a journey will not be much greater than the work done during the departure from the
76 earth. Actually, once the craft is far enough away to be out of the earth's considerable gravitational field, the flight will proceed by inertia alone.

Consequently, the problem is mainly to overcome terrestrial gravity, and once this has been accomplished it will be comparatively easy to reach both distant and nearby planets. However, and this is very important, the spaceship must be hermetically sealed and living conditions aboard it must be suitable throughout the time of the flight. Moreover, conditions must be such that the absence of a gravitational field will not be injurious to the living organisms aboard.

If the organism cannot bear such [gravityless] conditions, on the other hand, it will be necessary to create an artificial gravitational field, by using the engine to produce a constant acceleration. If a field corresponding to that of the earth is set up, then the passengers will not experience any discomfort, no matter where they may be. However, such measures will naturally necessitate the expenditure of an enormous amount of fuel energy, and they will also postpone even further into the future a flight which is already difficult under present conditions.

Let us use the law of motion for a body acted upon by a constant thrust during flight away from the earth. We assume that, until the moment when it develops the maximum velocity between the moon and the earth, the spaceship has an acceleration equal to $^{11}/_{10}$ of terrestrial gravity. All the maneuvers will thus be carried out at this acceleration. The influence of the moon is small enough to be neglected. Under such conditions, calculations show that the craft should be turned when it is 29.5 earth radii away from the earth, when the velocity is 61,700 m/sec. Afterward we begin to decelerate it with a force equal to its terrestrial weight.

The time required to reach the moon is

$$t = 3^h 27^m.$$

In this new case, assuming a 1,000-kg spaceship of which 300 kg is fuel, the work required is $67.2 \cdot 10^6$ cal/kg of propellant, that is, 131 times as much as in the previous case. Dynamite can provide only $^1/_{47,300}$ of this power, while radium gives 43.2 times as much. The power needed is

$$\frac{857 \cdot 10^{10}}{24\,000.75} = 4\,760\,000 \text{ HP}.$$

Let us assume that we use this means to fly to the nearest planets. The durations of these trips and the maximum velocities will be:

for Venus: 42,000,000 km in 35 hr 40 min; 643 km/sec = 2,320,000 km/hr;

77 for Mars: 78,000,000 km in 49 hr 20 min; 885 km/sec = 3,180,000 km/hr.

At first glance, these velocities may seem to be astonishing. However, some celestial bodies, such as Halley's Comet, have similar speeds. Consequently, it is clear that only atoms can provide us with the required forces and velocities.

Note. Figure 8 shows a general view of one of Esnault-Pelterie's rockets. This model was on display at the Interplanetary Travel Exhibition in Moscow in April 1927. It is not known to us what data were used for the construction of this model [note added by Rynin].

Chapter V

INTEREST IN INTERPLANETARY TRAVEL

We should not expect to discover any new elements on our neighboring celestial bodies. Helium, which was detected on the sun when it was still unknown here on earth, was later found on our planet as well, and, from the chemical point of view, the sun does not offer us anything which we do not have in the laboratory. Furthermore, since we are now familiar with the laws of radioactivity, it can be concluded that on bodies having the same origin as the earth the distribution of different elements should be almost the same. Not only is there little hope of discovering new elements, but those which are rare on earth cannot be expected to be more prevalent either.*

Why should we be interested in visiting other celestial bodies ? Similar questions naturally were asked by sceptics, who accompanied them with their customary sarcastic smiles, in times past when steam power, the automobile, and, within my own memory, aviation came on the scene. Perhaps they feel that "this time the situation is somewhat different." Naturally it is "different." However, I will answer these sceptics just as, time and again, they have been answered in the past. Scientific studies which appear to be completely useless have a way of ultimately turning out to be useful in some quite unexpected manner. In addition to such unforseeable advantages, however, interplanetary travel is of very great interest.

ARE OTHER PLANETS INHABITED ?

Life is the subject which interests us the most, since we ourselves are living beings and must compete with other beings for survival. However, we are only familiar with life in its terrestrial forms. If we became acquainted with extraterrestrial forms of life, would this not widen our understanding of life ? Would we not find answers to certain as yet unresolved questions ? Naturally, the answer is yes.

WHAT IS LIFE ?

I think that the following definition is a satisfactory one: "Life is a process by which certain chemical compounds of 'living matter' grow due
78 to the intake of various external chemical compounds." Clearly, the basic principle of life is assimilation; other factors are secondary. The growth and reproduction of cells, which at first glance seem to be very important, are seen, after some reflection, to be instead a result of an equilibrium of osmotic pressures. Prof. Leduc has succeeded in reproducing quite similar phenomena in compounds which could not be considered "living matter," since they did not exhibit actual assimilation** or unlimited reproduction.

The field of "organic chemistry" originally was so named in order to demonstrate the difference between it and the chemistry of minerals. Now, however, this science is just the chemistry of carbon, and, although it is a complex field, it obeys the general laws of chemistry and physical chemistry.

The number of natural organic substances which can be produced in the laboratory has risen considerably since Marcellin Berthelot first used a voltaic arc to convert acetylene into benzene and obtained, with the aid of

* This is not completely true, since the densities of the planets decrease from Mercury outward, which is similar to the situation in a nebula, the center of which is more compressed.

** Their chemical composition was altered slightly by the absorption of water, which caused a volume increase or "growth."

"mineral" carbon and hydrogen, a certain elementary substance of similar nature. He left it to his son, Daniel Berthelot, to carry out an analogous experiment using ultraviolet light. The beginning of life on earth may well have come about as a result of reactions of this type, which occurred under the influence of light at a time when the physical conditions on the earth made these, at present impossible, conditions possible.

Consequently, we are indebted to our sun for originating, as well as maintaining, life. In any case, this explanation sees the phenomena of life as being very unusual, and as being a consequence of unique conditions. These conditions led to the creation of a substance with special properties, and all living beings ultimately developed from this basic substance. From such a point of view, it would seem highly unlikely that the same exceptional conditions could have existed in some other place, and it is not to be expected that future interplanetary travelers will find life on other planets.

The idea that the phenomena of life and those of chemistry and physics are completely different has come to dominate our minds so much that it has seemed necessary to explain the origin of life on earth either as an act of divine will or as an importation from another system, as proposed by Svante Arrhenius. Arrhenius's theory appears to solve the problem, so let me consider it here in some detail.

It is known that when light encounters an obstacle it exerts on it a pressure proportional to the amount of luminous energy per second received at the place. By our standards this pressure is very minute, but for very small particles the ratio of the particle surface to the mass becomes greater
79 and greater, until finally the light pressure exceeds the weight of the particle.

Arrhenius considers plant seeds and spores, which are lifted into the upper atmosphere by air currents, and which, because they weigh very little, can escape into interplanetary space and reach other worlds. In this way, life may be transported into outer space by light. Apart from its poetic attraction, this theory is based on the concept of vitalism, which I have referred to above. According to the vitalistic point of view, life is a unique phenomenon depending on special profound causes, and it develops on its own, having nothing in common with other phenomena. However, if Arrhenius's hypothesis is subjected to mathematical analysis, there are numerous objections to it.

1. Let us consider a spherical particle composed of a white substance which reflects 60% of the incident light; the particle is assumed to be at a height of about 200 km. Calculations show that the solar radiation will exert on a particle a pressure equal to the particle weight for diameters not exceeding 0.00000048 mm. It will be seen below that this size corresponds to molecules which, although large, are not complex (chloroform, benzene). All known seeds which are visible with a microscope have diameters at least 300 times as great, and conditions are not such that they are able to escape from the earth into space.

We do not know of any seeds the size of chloroform molecules, and it appears to me that none could exist. Such a small mass would not include enough atoms to make up an organic substance as complex as protoplasm.

2. If we consider spores 0.0002 mm in diameter, on the other hand, we see that they can ascend into the atmosphere in two ways: by Brownian movement or via air currents. Calculations show that, if the entire earth's surface were covered with such spores to a density of one per mm^2 (which

would amount to $5.1 \cdot 10^{20}$, or 510,000,000,000,000,000,000 spores), the Brownian movement would lift only 34 spores per million to a height of 1 mm. Even though this is an impressive number of spores (17,300,000,000,000, 000), as we go higher the number drops very rapidly. For instance, only one particle will be lifted 4.8 mm, 10^{-24} will be lifted 1 cm, and $10^{-400,000,000}$ will be lifted 200 km.*

Consequently, it cannot be assumed that Brownian movement will lift even one seed into the atmosphere. If we consider particles with diameters $^1/_{300}$ as great (that is, which weigh $^1/_{27,000,000}$ as much), we obtain a distribution with height similar to that of a gas, which is quite natural, since such dimensions are already the dimensions of molecules. However, from the point of view taken by us, this case is of no interest, because it is absolutely impossible for living seeds to be as small as this.

80 Air currents can lift seeds of normal dimensions very high, but the number of seeds diminishes quite rapidly with height. Unfortunately, I do not possess accurate data on the number per cm^3, but the experiments of Pasteur show that their number at the levels of a field, at 850 m in the Jura Mountains and at 2,000 m at Montanvert, and at the shore of a sea covered with ice, will be, respectively, 8.5 and 1. Assuming a power law, we obtain

$$\frac{n}{n_0} = e^{-0.608 \cdot 10^{-3} H}.$$

Thus at 11,000 m we have $\frac{n}{n_0} = 0.00125$ (we will see later why I have chosen this height), and at 200,000 m we have $1.6 \cdot 10^{-53}$. Since the air of a field does not contain many seeds per cm^3, there will definitely not be any at 200 km (perhaps one for the entire earth).

It should be noted that Pasteur indicated the presence of strong winds at lower places, with the exception of Montanvert. The proportion given should actually thus be higher there, so that the number of seeds per unit volume at a height should be even less than the number on a mountain.

The following should also be noted: up to a height of 11,000 m the temperature variation obeys, albeit approximately, the adiabatic law, which means that the air is mixed vertically. Above this level, on the other hand, the temperature remains unchanged, which obviously excludes the possibility of vertical currents. Therefore, even assuming that seeds can be lifted to an altitude of 11,000 m, it would appear to be very unlikely that they could go any higher.

3. Although any one of the foregoing objections rules out the possibility of seeds traveling into space, I would still like to go a bit further and formulate a hypothesis. Let us assume that a seed of $^1/_{1,000}$ the size ascends to a height of 200 km, and that it is subjected to a solar-radiation pressure equal to its weight, which, let me repeat, will not be true. Then the particle falls at an angle of 45° to the vertical. In order for the particle to escape from the earth, its diameter will have to be reduced to $^1/_{1,000}$, or at least to $^1/_{100}$, but in such a case it would be smaller than a molecule, or even than an atom, and repulsion could not take place. Interestingly enough, Arrhenius mentions this difficulty.

* Actually, such a particle will experience only a very slight pressure; its size is not great enough to allow it to reflect or absorb light, and it can only refract it. Later I will mention this fact, which is unfavorable to Arrhenius's hypothesis.

He writes that "if a spore 0.00016 mm in diameter has a charge of $5 \cdot 10^{-1}$ electrostatic units, then a field of 140 V/m^2 is enough to overcome its weight and lift it. Such an electric field is typically observed at the earth's surface in clear weather." However, at a height of 200 km, the atmospheric pressure is only two billionths of the standard value, and above 60 km the air is so tenuous that it stops being a conductor. Thus the latter possibility is also eliminated.

4. Although the barriers to it are insurmountable, still I would like to
81 suppose that a seed has left the earth and travels into space at an ever increasing velocity. Calculations show that, if it departs from the earth under a pressure equal to its weight, the greatest velocity it can attain will be 1,700 km/sec. It will arrive at Mars with a velocity of 1,000 km/sec.

What will happen if a seed enters the atmosphere of a planet at such a speed? To find out, let us make the following comparisons. Let us consider a spore 0.0002 mm in diameter arriving at the earth from some other system. We assume that the solar radiation imparts to it a velocity of only 170 km/sec. Calculations show that it begins to decelerate markedly at a height of 200 km, and that this deceleration reaches a maximum at a height of 167 km, where the force will be equal to 53,000 times the weight of the spore. It will stop completely at a height of 156 km.

In order to give some idea of the force of this retardation, let me just point out that at 171 km it amounts to 6,000 kW, or 92,000 HP, per gram. A particle with a diameter $^1/_{300}$ as great suffers the same fate, but there is no reason to repeat such calculations for it. In the following I will apply a different study method. The air ahead of a moving projectile becomes compressed, and the indicated retardation can be attained for a pressure with an absolute value which is comparatively quite low, but which will be enormous compared to the pressure at the place in question. Moreover, for simple adiabatic compression, air heats up to a temperature of 45,000°. It is easy to imagine what would happen to the projectile under such conditions, even if most of the heat developed were absorbed by the air itself.

We do not know the density of the Martian atmosphere, but it can be shown that, even if it is equivalent just to the terrestrial atmosphere at a height of 100 km (that is, 0.000046 of that at the ground), any spore entering it would burn up. If the Martian atmosphere is even thinner than this, then a spore which does not burn up will probably disintegrate when it strikes the surface.

Consequently, the assumption of fertilization of the earth from Venus, or of Mars from the earth or Venus, has absolutely no basis in fact. In his memoir, Arrhenius did not consider these difficulties.

5. Ultraviolet rays from the sun would inevitably kill any seeds which were not protected by an absorbing atmosphere. Arrhenius considers this and concludes that, in the absence of humidity and oxygen, some seeds would survive. However, the experiments on this do not seem to me to have been exhaustive, and, moreover, ultraviolet light is such a strong sterilizer that a number of corroborating experiments will still be necessary in order to show that it loses its power under certain conditions.

6. Arrhenius does not consider possible the arrival of seeds aboard meteorites, the surfaces of which become baked during the descent. But couldn't seeds be situated inside of deep pores, where the heat cannot reach

them? I think that Arrhenius was correct in rejecting this possibility,
82 since the origin of a bolide is, in any case, catastrophic and any seeds in it would have burned up at the beginning.

Finally, if they were hidden in deep crevices, seeds would inevitably catch fire as a result of the violent retardation, and all life on the surface of the bolide would thus be destroyed.

On the other hand, Arrhenius assumes that seeds wandering about in space may encounter "dust particles 1,000 times larger," on their way to the sun, and that, obeying the laws of attraction, the seeds may adhere to their surfaces and travel with them. I confess, however, that I do not understand how a seed traveling at 1,000 km/sec can collide with a grain of dust without being smashed and becoming dust itself, or, finally, what happens during an encounter between one particle going to the sun and one coming from the sun.

7. If seeds migrate from one celestial system to another, they must exist in outer space for thousands of years at a temperature of absolute zero (-273°C). How will this affect them?

Arrhenius is an optimist. He assumes that the simplest forms, and in particular spores, can stand very low temperatures and furthermore that the rate of chemical reactions decreases with temperature. And anyway, the retarded vital processes of the seeds are slowed down even more, so that for them "three million years at a temperature of −220°C are like a day at 10°C..." He cites as proof experiments at −252°C lasting 20 hr and at −200°C lasting six months. However, the last 20° of cold are more dangerous than the entire first 252°, since there is a real difference between molecular motions which are reduced to $^1/_{12}$ and motions which are halted completely; between eight months and three million years, of course, the difference is even greater.

8. Finally, I would like to offer one more objection, which has apparently not yet been put forward by anyone. Assuming that all the foregoing objections are shown to be unfounded, let us consider what the probability would be that one of the seeds covering the earth's surface with an average density of one seed per mm^2 will succeed in getting to some other world. This probability would seem to me to be zero. But let us assume that the particle has left the earth and is flying through outer space.

I have already shown that it cannot fertilize a planet in our system. Let us suppose that it flies to one of the stars. In order to determine the probability of an encounter with the star, we must find the ratio of the sum of all the solid angles subtended by all the stars to the total angle (that is, to 4π).

We can get an idea of this ratio by comparing the light incident on the earth from all the visible stars to the light of a sun which would replace them. For the stars of a hemisphere this corresponds to a brightness of 92,000 suns. The probability will be of the order of $1.7 \cdot 10^{-13}$ to $4 \cdot 10^{-15}$. However, let us go further and assume that a spore flies to a star during the
83 course of millions of years spent at a temperature of −273°C. The light of the star will gradually slow the spore down and, under the influence of this pressure, it will describe a hyperbola about the star and fly for still more millions of years through the icy reaches of interplanetary space. If the star has cool satellites, an encounter between the seed and one of them is possible, but there will again be little certainty that conditions on the satellite will be suitable for the seed to live.

However, first it will be necessary for the orbital plane of the satellite to pass through the path of the seed, and this is also of low probability. Finally, the satellite itself will have to situated in the path of the seed, another improbable occurrence.

A spore entering the atmosphere of the satellite must not burn up. Thus it has to move obliquely in the orbital plane in the right direction, rather than directly toward the center of the satellite, and the velocity of the spore must equal that of the satellite in its orbit.

The probability that this entire set of circumstances will be realized is equal to the product of the probabilities of each of them, and this is, most likely, about the same as the probability that a brick will be lifted to the second floor of a building by Brownian movement. According to Jean Perrin, we would have to wait for $10^{10^{10}}$ years before one instance of either of these phenomena would occur. Compared with this time, geological periods, and even the lifetime of the solar system, are negligible.

If the number of seeds transported by light to the surface of a planet were great, and if at the same time the number of planets from which seeds could migrate were also great, then very likely there would be some slight possibility that Arrhenius's theory is true. However, as the foregoing shows, the number of such seeds is essentially zero, and Arrhenius's idea of panspermism is highly improbable.

Moreover, according to Arrhenius, two kinds of material exist in nature, living and nonliving, and I, personally, do not believe this. Each phenomenon, considered in one of its characteristic forms, seems to be completely different from another. When Thales of Miletus noted that amber attracts straw when rubbed, he did not doubt but that other substances possess a similar property, though to a different degree.

Even in recent years it was thought that radioactivity is just a property of radium. Now, however, it is assumed that any substance may be radioactive, even if our sensitive instruments cannot detect it. Very different degrees of radioactivity are recognized, from substances 200,000 times as strong as radium to substances with $1/_{3,000,000}$ of the power of uranium.

It is also difficult to differentiate between animals and plants. For instance, some plants (heliotrope, sunflower, or catchfly) are able to close their leaves over a fly which settles on them, pierce it with their sharp pinnas, and eat it. If we consider simpler species, the difference becomes even less.

We recognize differences between things solely on the basis of the theory of probability. Certain probabilities, however, are so great that they
84 correspond in practice to certainty. For instance, I release a pencil that I have been holding in my hand. Will it fall? The kinetic theory of gases replies: it is not definite, but the probability is so great that it can hardly be expressed using the decimal system. Thus I consider the falling of the pencil to be practically certain.

With respect to the beginning of life, I would reason in a similar manner. The probability that such a widespread phenomenon as life had a chance beginning is very low. Consequently, with a low probability of erring, I assume that such a beginning is just as common (exceptionally uncommon) as life itself.

Before the microscope was invented, people assumed a spontaneous genesis of living species, since they could not view them on a small scale.

After its invention, on the other hand, they began to deny this possibility, even though it was still not possible to see all the species. There is a remarkable analogy between living beings and crystals. (Here Esnault-Pelterie devotes a page to this [Rynin]).

The sizes of the most minute particles making up minerals, living beings, and other objects are:

diameter of electron	0.000 000 000 00372 mm
diameter of hydrogen molecule	0.000 000 217 mm.

The formula for the hydrogen molecule is H–H; the oleic acid molecule is one of the largest, with a length of 0.0000022 mm, although even larger molecules exist. The sizes of the smallest bacteria (B. influenzae) range from 0.0002 to 0.0005 mm, that is, they are 100 times greater than the foregoing. The largest bacteria (B. Bütschlii) have the following dimensions: widths from 0.0004 to 0.005 mm and lengths from 0.050 to 0.060 mm. Living cells range from 0.001 to 0.02 mm. The smallest visible bacteria have diameters 100 million times greater than electrons (for comparison, we should note that the diameter of the earth is a mere 6 *million times greater than we are*). *These bacteria, however, are only* 100 times greater in linear size than a large organic molecule.

All the foregoing pertains to the linear dimensions. The corresponding masses (in grams) are:

Electron	0. 000 000 000 000 000 000 000 000 000 9
Hydrogen atom	0.000 000 000 000 000 000 000 001 66
Hydrogen molecule	0.000 000 000 000 000 000 000 003 32
Nitrogen atom	0.000 000 000 000 000 000 000 023 3
Oleic acid molecule	0.000 000 000 000 000 000 000 465
B. influenzae min.	0.000 000 000 000 008
" " max.	0.000 000 000 000 125
B. Bütschlii min.	0.000 000 000 008
" " max.	0.000 000 001 5

This table gives an idea of the difference between the weights of the oleic acid molecule and B. influenzae; the former weighs only $^1/_{20,000,000}$
85 of the latter.

To show the complexity of a molecular cell, I will make a comparison with a nitrogen atom, which weighs 14 times more than a hydrogen atom. This choice was dictated by the fact that the atomic weight of nitrogen is equal to that of the CH_2 group, which is usually the basic element of organic substances. The above-mentioned bacilli may contain the following numbers of atoms or groups of atomic weights, taken as a basis:

Bacillus influenzae	min.	343 000 000
" "	max	5 400 000 000
B. Bütschlii	min	345 000 000 000
" "	max	646 000 000 000 000

If we take only the cell nucleus into account, then this number can be divided by ten. However, even in this case a colossal number of combinations in the groupings of individual elements will be possible, even for 12 varieties.* The latest discoveries indicate that even smaller living

* Here we omit half a page of unessential discussion of the compositions and properties of minute substances [Rynin].

things exist. Finally, if we assume that a further reduction of their dimensions is possible, then their compositions will also have to become simplified, until, at the limit, their properties will approximate purely physicochemical processes. Life continually reproduces itself, once it has originated in physicochemical phenomena; this theory is known as "physicochemical aidiogenesis" (from the Greek ἀΐδιος, meaning "eternal," and γένεσις, meaning "origin").

In this way countless substances originate, perhaps in part under the influence of light. Some of these possess only normal osmotic properties, others have an increased sensitivity. Relative to external interactions, development and further complication occur. Of the trillions of molecules so produced on the earth, almost all decay, but of these there will nevertheless be many from which new species originate, and a new strain of living things. The process is analogous to the transition from bacteria to plants and to the higher animals, and it requires geological periods for its completion.

If this theory is valid, then the existence of living beings on Mars and Venus enters the realm of possibility, and, although the chemical composition of such beings will be the same as on earth, other species may exist, but in essence they will be similar to terrestrial beings, since the principles of their genesis were the same.

To conclude this discussion, I should say a bit more about the means of transporting life from planet to planet. Given the present state of our knowledge, it seems to me that we will be able to visit our neighbors in the solar system only after several centuries. Then we will introduce microbes there and, if these worlds are fertile, the latter will reproduce.

However, the reverse may have already occurred. Could not the Martians themselves have visited us several hundred million years ago? Are we not the descendents of them or of their microbes? I must admit, however, that this explanation seems to me to be very unlikely.

86 *Chapter VI*

CONCLUSION

It is clear from the foregoing that we are still far from achieving interplanetary travel or even flying to the moon. If it were possible to use atomic hydrogen as a fuel, the only remaining problem would be to construct an engine capable of operating at a temperature of at least 6,000°C and with a velocity of gas ejection of about 10 m/sec.

What would be the weight of a manned spaceship with all the necessary equipment (for oxygen recovery, CO_2 absorption, etc.)? How will an organism stand the absence of gravity? Will it not be necessary at all times to create an artificial gravitational field, and what portion of the earth's field should it be equal to? Will this not require an excessive amount of fuel? It may be that, in order to reduce the required reserves, it would be advisable to anesthetize the travelers, for instance, with a mixture of nitrous oxide (protoxyde d'azote) and oxygen, during the entire flight. Interplanetary travel will be realized without any risk, once we have atomic energy at our disposal.

Unfortunately, in spite of remarkable progress in this direction, namely in the study of the structures of the simplest atoms (hydrogen and helium), science has been stopped by the complexity of the lithium atom. What, then, will happen when we study more complex atoms ? It may be that radiated atomic energy, like thermal energy, will be utilized according to principles similar to that of Carnot. However, even in this case the energy will be about 100,000 times greater per unit mass than for atomic hydrogen. If we could use all the energy of matter, which is even 10,000 times greater, new possibilities would present themselves, including the possibility of destroying the world, and ourselves with it.

It is hard to say how many of these hypotheses are realizable. In any case, it is desirable to give every possible support to studies which might promote the advancement of "Astronautics," a term suggested by J. N. Rosny. Moreover, I have suggested to my friend Andre Hirsch that he join with me in establishing an annual prize of the Astronomical Society of France. This annual international prize will be called the REP-Hirsch Prize and it will be awarded for the best original technical work of the year which brings us closer to one of the stages of astronomical knowledge.

In order to coordinate the studies, we have requested the Astronomical Society to set up an Astronautics Commission, whose activities will deal with the following subjects: atomic theory, transmutation of elements, development of an atmosphere suitable for breathing, ultralight alloys, physiological effects of changes in acceleration, equipment for inter-
87 planetary navigation, etc.

In the same way as, prior to the age of aviation, a number of investigators, Col. Charles Renard in particular, showed that flying would be possible with a light motor of a certain weight, so should the Astronomical Society of France promote the elucidation of all subjects related to future flights.

It is necessary to be completely ready for the day when physicists will place at the disposal of mankind the powerful energy whose existence we now foresee, unless some insurmountable difficulty compels man to be an eternal prisoner of the earth.

It is my hope that this study will stimulate other investigators to deal with these questions, and that it will serve as a point of orientation in indicating the most important points remaining to be clarified.

APPENDIX

Let us assume that the curve for the fuel as a function of (V, y), as given by equation (121), is extended until it intersects the critical curve of escape. We next divide y/a into successive intervals. For one of these intervals we have:

y_0 and y_1 are the initial and final values of y,
$\Delta y = y_1 - y_0$ is the amplitude,
V_0 and V_1 are the initial and final velocities,
$V_m = \frac{V_0 + V_1}{2}$ is the mean velocity,
Δt is the duration of the interval.

We can use the approximate formula

$$\Delta t = \frac{\Delta y}{V_m}. \tag{127}$$

For the first interval this formula gives an unsatisfactory result ($y_0 = 0$; y_1), and so for this interval I will use another formula, obtained in the following way:

We assume that the motion is determined by the equation

$$\frac{d^2 y}{dt^2} = J(y). \tag{128}$$

Here

$$y = 0 \text{ and } \frac{dy}{dt} = 0 \text{ for } t = 0. \tag{129}$$

We next expand function $J(y)$ in a series for $y = 0$ and assume that

$$J(0) \neq 0 \; J'(0) \neq 0. \tag{130}$$

88 If we take

$$y = \frac{1}{2} J(y_1) t^2, \tag{131}$$

then for $y_1 = \frac{y}{6}$ we have the following result for terms up to t^6:

$$y = \frac{1}{2} J\left(\frac{y}{6}\right) t^2. \tag{132}$$

Therefore

$$t = \sqrt{\frac{2y}{J\left(\frac{y}{6}\right)}}. \tag{133}$$

However, from (115) we have

$$t = \sqrt{\frac{2g}{\frac{g}{k} - \left(\frac{g}{1 + \frac{y}{6a}}\right)^2}} \tag{134}$$

or

$$t = \sqrt{\frac{y}{\frac{2}{g} \cdot \frac{1}{k} \frac{1}{\left(1 + \frac{y}{6a}\right)^2}}}. \tag{134^1}$$

Second approximate method. On the basis of equation (115), and noting that during the entire burning time $y < a$, that is,

$$\frac{y}{a} < 1, \tag{135}$$

we have

$$\frac{1}{\left(1+\frac{y}{a}\right)^2} = 1 - 2\frac{y}{a} + \ \ldots \tag{136}$$

or

$$\frac{d^2y}{dt^2} = g\left(\frac{1}{k} - 1 + 2\frac{y}{a} - \cdots\right), \tag{137}$$

and we retain only the indicated terms. Let us set $\frac{y}{a} = z$. In this case

$$\frac{d^2z}{dt^2} = \frac{g}{a}\left(\frac{1}{k} - 1 + 2z \cdots\right). \tag{138}$$

This approximation will be better, the lower the value of k.

Equation (138) is a linear second-order equation in z. Its solution, taking into account the initial conditions

$$z_0 = 0; \ \frac{dz}{dt} = 0,$$

will be

$$z = \frac{1}{2}\left(\frac{1}{k} - 1\right)\left[\frac{e^{t\sqrt{\frac{2g}{a}}} + e^{-t\sqrt{\frac{2g}{a}}}}{2} - 1\right]. \tag{139}$$

89 We can simplify (139) by setting

$$u = e^{t\sqrt{\frac{2g}{a}}}. \tag{140}$$

Then (139) becomes

$$z = \frac{1}{2}\left(\frac{1}{k} - 1\right)\left[\frac{u + \frac{1}{u}}{2} - 1\right]; \ y = az, \tag{141}$$

$$\frac{dz}{dt} = \frac{1}{2}\left(\frac{1}{k} - 1\right)\sqrt{\frac{2g}{a}}\left[\frac{u - \frac{1}{u}}{2}\right], \ V = a\frac{dz}{dt}, \tag{142}$$

and

$$L\frac{M_0}{M}=\frac{1}{kt}=\frac{1}{kv}\sqrt{\frac{\alpha g}{2}}\,Lu\,. \tag{143}$$

In order to determine the critical elements, it is sufficient to set

$$z=z_0=k. \tag{144}$$

By this means quite simple expressions are obtained.

91 *Third Paper*

*ASTRONAUTICS AND THE THEORY OF RELATIVITY**

My studies of the subject of space flights have indicated how unfeasible such flights will be as long as we have available only existing, known chemical reactions to provide the required energy. The situation will be otherwise, however, when physicists are able to place atomic energy at our disposal.

New advances in science compel me to consider how the new theory will affect the force of an ordinary action, and I will make use of mathematical analysis for this. Let us consider two systems: the first system (0) with its axes fixed to the observer will not be designated by any index, and the second system, moving relative to the first, will be designated by indexes (1).

The axes are oriented as usual and may coincide at the beginning: $t=t^1=0$; $x=x^1=0$. The velocity of motion of system (1) relative to system (0) is directed along OX and is such that axes OX and OX^1 coincide when extended.

Let us assume that at a moment t a ship moves with a velocity v in the direction of positive x, and that system (1) has an equal and constant velocity. In system (1) let there now be a material point which is at rest at moment t. Its mass is designated as m_0. We apply to it a force which is designated as F^1 in system (1) and which is directed toward positive x and x^1. Then, in system (1) we have

$$m_0\frac{d^2x^1}{dt'^2}=F^1.$$

The equations of the Lorentz transformation are:

$$x^1=\frac{1}{a}(x-vt) \qquad (1)$$

$$y^1=y \qquad (2)$$

$$z^1=z \qquad (3)$$

$$t=\frac{1}{a}\left(t-\frac{vx}{c^2}\right). \qquad (4)$$

* In 1928, in Nos.8–10 of the journal "Die Rakete," a paper by R. Esnault-Pelterie appeared (translated from French by J. Winkler). Later this paper was included in the collection of Esnault-Pelterie's works entitled "L'Astronautique" (Paris, 1930). A translation of this paper follows.

$$x=\frac{1}{a}(x^1+vt^1) \quad (1')$$

$$y=y^1 \quad (2')$$

$$z=z^1 \quad (3')$$

$$t=\frac{1}{a}\left(t^1+\frac{vx^1}{c^2}\right). \quad (4')$$

Here v is the velocity of the body, c is the velocity of light, and

$$a=\sqrt{1-\frac{v^2}{c^2}}.$$

From (1) and (4), we obtain

$$\frac{dx^1}{dt^1}=\frac{\frac{dx}{dt}-v}{1-\frac{v}{c^2}\cdot\frac{dx}{dt}} \quad (5)$$

and

$$\frac{d^2x^1}{dt'^2}=\frac{d}{dt}\left(\frac{\frac{dx}{dt}-v}{1-\frac{v}{c^2}\frac{dx}{dt}}\right)\frac{1}{\frac{1}{a}\left(1-\frac{v}{c^2}\frac{dx}{dt}\right)}=a^3\frac{d^2x}{dt^2}\frac{1}{\left(1-\frac{v}{c^2}\frac{dx}{dt}\right)^3}. \quad (6)$$

If we consider the motion during an infinitesimal time interval dt, immediately following t, then, as an approximation (to infinitesimal quantities),

$$\frac{dx}{dt}=v. \quad (7)$$

However, according to Lorentz's equation,

$$a=\sqrt{1-\frac{v^2}{c^2}}. \quad (8)$$

Therefore (6) becomes

$$\frac{d^2x^1}{dt'^2}=\frac{1}{a^3}\frac{d^2x}{dt^2}. \quad (9)$$

Let us assume that, for physiological reasons, the pilot can tolerate a constant acceleration g (the acceleration of gravity). To simplify the calculations, we suppose that the reaction engine is so regulated that

$$\frac{d^2x^1}{dt'^2}=g=\text{const.} \quad (10)$$

We assume that the ship leaves the earth with an acceleration of $2g$, that is, a person aboard will feel twice as heavy. Then, for (9), we have

$$\frac{d^2x}{dt^2}=a^3g \tag{11}$$

for the interval dt. This indicates that the velocity $\frac{dx}{dt}$ changes very little, and that in system (1) the velocity $\frac{dx^1}{dt^1}$ will be very small.

93 This can always be attained, since dt may be arbitrarily short. Passing to the limit, we see that relation (11) remains valid for any given moment, if system (1) moves with the ship. Since now $\frac{dx^1}{dt^1}=0$, from (4) and (4') we have

$$\frac{dt^1}{dt}=a \text{ and } \frac{dt}{dt^1}=\frac{1}{a}. \tag{12}$$

However, a is a function of t, so that

$$\frac{d}{dt}\left(\frac{1}{a}\frac{dx}{dt}\right)=\frac{1}{a^3}\frac{d^2x}{dt^2}. \tag{13}$$

This holds true for any moment, with acceleration of the system's motion.

In addition, for the given system, with a constant acceleration (g) in this system, we have

$$\frac{d}{dt}\left(\frac{1}{a}\frac{dx}{dt}\right)=g, \quad \frac{dx}{dt}=gat+\text{const.} \tag{14}$$

Assuming $\frac{dx}{dt}=0$ for the initial conditions, we find that for $t=0$ the constant of integration is also 0. Consequently, for the system moving with the rocket, since $\frac{dx}{dt}=v$, we have

$$v=gat. \tag{15}$$

From (8) it follows that

$$t=\frac{1}{g}\sqrt{\frac{1}{\frac{1}{v^2}-\frac{1}{c^2}}} \tag{16}$$

or

$$v=\sqrt{\frac{1}{\frac{1}{g^2t^2}+\frac{1}{c^2}}}=\frac{dx}{dt}. \tag{17}$$

Equation (17) can be rewritten as

$$dx=\sqrt{\frac{1}{\frac{1}{g^2t^2}+\frac{1}{c^2}}}\,dt \tag{18}$$

or

$$dx = \frac{c}{g} \frac{g^2 t}{\sqrt{c^2 + g^2 t^2}} dt. \tag{19}$$

For initial conditions $x=0$ and $t=0$,

$$x = \frac{c}{g} (\sqrt{c^2 + g^2 t^2} - c). \tag{20}$$

94 so that for very large t we have $x = ct$.
Equations (8) and (17) give

$$dt^1 = \frac{c}{\sqrt{c^2 + g^2 t^2}} dt \tag{21}$$

or

$$dt^1 = \frac{c}{g} = \frac{d(gt)}{\sqrt{c^2 + g^2 t^2}}. \tag{22}$$

For initial conditions $t^1 = 0$, we have for $t=0$,

$$t^1 = \frac{c}{g} \log\left(\frac{g}{c} t + \sqrt{1 + \frac{g^2}{c^2} t^2}\right) \tag{23}$$

Proof. Elements of distance in space should have the same values in both systems, so that

$$\partial s^2 = -\partial x^2 + c^2 \partial t^2$$

must be equal to

$$\partial s'^2 = c^2 \partial t'^2 \tag{24}$$

and thus

$$c^2 (\partial t^2 - \partial t'^2) = \partial x^2. \tag{25}$$

However, from (21) we obtain

$$c^2 dt^2 \left(1 - \frac{c^2}{c^2 + g^2 t^2}\right) = dx^2 \tag{26}$$

or

$$dt \frac{cgt}{\sqrt{c^2 + g^2 t^2}} = dx, \tag{27}$$

that is, equation (19) is obtained.

Now let us suppose that the pilots are not familiar with the laws of relativity. They only know that they have an acceleration g =const., and they believe that their motion obeys the law

$$v = gt^1; \quad x = \frac{1}{2} gt'^2. \tag{28}$$

In order to traverse a distance X, the following time would appear to be required:

$$T = \sqrt{\frac{2X}{g}}. \tag{29}$$

For sufficiently long distances, they think that in a time $t^1 = c/g$ (for instance, in a year, or in 354.2 days, for g =981 cgs units) they will attain the speed of light or even exceed it.

Actually, however, according to equation (20), in the system of the observer their time will be

$$t = \sqrt{\frac{X^2}{c^2} + \frac{2X}{g}}, \tag{30}$$

95 that is, in their own system, according to (23),

$$t^1 = \frac{c}{g} \log \left[\frac{g}{c} \sqrt{\frac{X^2}{c^2} + \frac{3X}{g}} + \sqrt{1 + \frac{g^2}{c^2} \left(\frac{X^2}{c^2} + \frac{2X}{g} \right)} \right] \tag{31}$$

or

$$t^1 \neq \frac{c}{g} \log \left[\sqrt{\frac{g^2 X^2}{c^4} + \frac{2gX}{c^2}} + \left(\frac{gX}{c^2} + 1 \right) \right]. \tag{32}$$

Consequently, the ratio of the actual time in their system to the apparent time will be

$$\frac{t^1}{T} = \frac{\log \sqrt{\left(\frac{g^2 X^2}{c^4} + 2 \frac{gX}{c^2} \right) + \left(\frac{2X}{c^2} + 1 \right)}}{\sqrt{2 \frac{gX}{c^2}}}. \tag{33}$$

If X is very large, then

$$\frac{t^1}{T \lim} = \frac{\log \left(\frac{2gX}{c^2} \right)}{\sqrt{\frac{2gX}{c^2}}} \text{ and, at the limit, 0.} \tag{34}$$

On the other hand, if X goes to 0, then

$$\frac{t^1}{T \lim} = \frac{\log \left(\sqrt{\frac{2gX}{c^2}} + 1 \right)}{\sqrt{\frac{2gX}{c^2}}} \text{ and, at the limit, 1.} \tag{35}$$

It is noteworthy that the apparent duration of the voyage in the system of the pilots will be shorter than the duration according to the data of classical mechanics, when the flight speed exceeds the velocity of light; the difference will be greater, the longer the flight is in time and space.

Numerical results

In order to simplify the calculations, we take as a unit of length

$$L=\frac{c^2}{g}=9.18\cdot 10^{17}\text{cm}=918\cdot 10^{10}\text{km}.$$

This length unit has the advantage that it is close in value to the astronomical unit [$1.5\cdot 10^{13}$ cm] and the light year ($9.467\cdot 10^{17}$ cm).

Let us compare the various times required to cover certain distances in the system of an observer, away from whom the pilots are flying. The times involved are:

T (formula (29)), the time which they would measure if they were not familiar with the laws of relativity theory;

t, the time according to their opinion, in a system moving with the observer;

96 t', the time which they assume in the system of their ship.

All these times are reckoned in tropical years (1 year $\doteq 3.1556\cdot 10^7$ sec).

X = L	2 L	5 L	10 L	100 L	1,000 L	10,000 L
T = 1.3675	1.9348	3.0585	4.325	13.675	43.25	136.75
t = 1.674	2.735	5.720	10.59	97.68	968	9,670
t¹ = 1.275	1.7068	2.400	2.992	5.143	7.370	9,600

These figures indicate an amazing gain in time not only in system (0), but in the Euclidean system as well, assuming that the flight velocity exceeds the speed of light. Consequently, when we obtain the velocity in space, we also obtain the velocity in time, but this will be possible only in the future.

WORK AND CONSUMPTION OF MATERIALS

Now let us assume that the system of the observer coincides with the system of the ship, and that the moving system, from the beginning of motion, coincides with some atom (electron or nucleus). Then equation (13) can be used, and the thrust in the system of the ship, for each moment, can be expressed as

$$F.dt=d\left(\frac{m_0}{a}\cdot\frac{dx}{dt}\right). \qquad (13)$$

Let us designate as v the final velocity of a particle during its ejection from the engine, and let us find the work performed by it. Then it will be possible to calculate the thrust imparted by it to the ship at a given moment.

At a time t, a total of ν atoms are assumed to be present between surfaces A and B of the nozzle, along a normal to the direction of motion, these atoms being at rest relative to the engine. Surface A is traversed by particles with a very low velocity. However, at surface B the particles have acquired the exhaust velocity v.

During a time $t+dt$ particles which were earlier between A and B move to surfaces A^1 and B^1, which are infinitely close to A and B. For steady motion the number of atoms between surfaces A^1 and B will be constant, and their momentum will also be constant. However, the number of atoms between A and A^1 will not be equal to the number between B and B^1.

Let us designate their total rest mass as δm_0. The process takes place as if this mass m_0 had, during a time interval δt, zero velocity relative to velocity v. Then the momentum would not obey the law $F=f(t)$. Therefore, it can be calculated using formula (13), assuming F constant and equal to its mean value. Summation over a time t and in the proposition

$$\frac{dx}{dt}=v$$

gives

$$f.\delta t=\frac{\delta m_0}{a}v. \qquad (13a)$$

97 Moreover,

$$\frac{v}{a}=\frac{1}{\sqrt{\frac{1}{v^2}-\frac{1}{c^2}}}. \qquad (36)$$

If we set

$$\frac{\delta m_0}{\delta t}=\mu_0, \qquad (37)$$

then

$$F=\frac{\mu_0}{\sqrt{\frac{1}{v^2}-\frac{1}{c^2}}}. \qquad (38)$$

The radiated energy, according to the classical formula, is

$$P=\mu_0 c^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}-1\right). \qquad (39)$$

This energy is obtained in a time δt.

Next let us determine the energy required to produce a unit of reaction force [thrust]. This will be

$$\frac{P}{F}=c^2\left(\frac{1}{v}-\sqrt{\frac{1}{v^2}-\frac{1}{c^2}}\right). \tag{40}$$

This formula shows that, if the velocity varies from 0 to c for a constant ejected mass μ_0, the force will increase from 0 to ∞ and the quotient $\frac{P}{F}$ will increase from 0 to c.

Reaction due to ejection of energy. According to the classical equation

$$F.dt=\frac{dW}{c},$$

we obtain

$$F=\frac{P}{c};\quad \frac{P}{F}=c.$$

This is the upper limit for the case of energy ejection.

EJECTED MASS REQUIRED TO PRODUCE UNIT FORCE

Ejection of matter. From equation (38) we have

$$\frac{\mu_0}{F}=\sqrt{\frac{1}{v^2}-\frac{1}{c^2}}. \tag{42}$$

"**Ejection of energy.**" The mass of the energy is

$$m=\frac{W}{c^2}. \tag{43}$$

98 Thus we obtain

$$\mu_0=\frac{P}{c^2} \tag{44}$$

and from (41)

$$\frac{\mu_0}{F}=\frac{1}{c}. \tag{45}$$

From this we see that, in the case of matter, as v varies from 0 to c, the amount of ejection required to produce a unit of force varies from ∞ to 0. In the case of the ejection of energy, however, it will be constant at $\frac{1}{c}$, for $v=\frac{c}{\sqrt{2}}$. The latter is considerably less than the velocity of electrons from radioactive substances, and is even lower in comparison with the velocities of α rays.

CONSUMPTION FOR CONSTANT ACCELERATION IN THE SHIP'S SYSTEM

We designate as m_0 the mass of the ship in its own system, that is, its mass at rest. The initial mass of the ship is M_0. The equation for the acceleration constant in the ship's system can be written as

$$\Gamma = \frac{F}{m_0} \tag{46}$$

and, from (38),

$$\Gamma = \frac{\mu_0}{m_0} \cdot \frac{1}{\sqrt{\frac{1}{v^2} - \frac{1}{c^2}}}, \tag{47}$$

from which

$$\mu_0 = m_0 \Gamma \cdot \sqrt{\frac{1}{v^2} - \frac{1}{c^2}}. \tag{48}$$

The mass consumption for the energy obtained will be, from (44) and (39),

$$\mu_1 = \frac{P}{c^2} = \mu_0 \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1 \right), \tag{49}$$

and the total ejected mass will be

$$\frac{-dm_0}{dt} = \mu_0 + \mu_1 = m_0 \Gamma \sqrt{\frac{1}{v^2} - \frac{1}{c^2}} \cdot \left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \right) \tag{50}$$

or

$$\frac{-dm_0}{dt} = m_0 \frac{\Gamma}{v}, \quad \frac{-dm_0}{m_0} = \frac{\Gamma}{v}\, dt, \tag{51}$$

from which, for initial conditions $m_0 = M_0$ and $t = 0$,

$$\frac{m_0}{M_0} = e^{-\frac{\Gamma}{v} t}. \tag{52}$$

99 **Ejection of energy only.** From equations (46) and (45) we have

$$\Gamma = \frac{\mu_1 c}{m_0}, \tag{53}$$

and

$$\frac{-dm_0}{dt} = \mu_1 = \frac{m_0 \Gamma}{c} \tag{54}$$

or

$$-\frac{dm_0}{dt}=\frac{\Gamma}{c}t \tag{55}$$

and finally

$$\frac{m_0}{M_0}=e^{-\frac{\Gamma}{c}t}. \tag{56}$$

Consequently, the consumption of matter during its ejection is always considerably greater than during the ejection of [just] energy, and the two would become equal only if the velocity of ejection of matter were to reach the velocity of light.

Let us calculate the ratio $\frac{m_0}{M_0}$ for this limiting case of consumption, for different distances. Keeping in mind that t in equation (56) denotes the local time of the ship and that it is given as t^1 in equation (32), we obtain, after substitution,

$$\frac{m_0}{M_0}=e^{-\frac{\Gamma}{c}\cdot\frac{c}{\Gamma}\log\left[\sqrt{\frac{\Gamma^2 X^2}{c^4}+2\frac{\Gamma X}{c^2}}+\left(\frac{\Gamma X}{c^2}+1\right)\right]} \tag{57}$$

or

$$\frac{m_0}{M_0}=\frac{1}{\sqrt{\frac{\Gamma X^2}{c^4}+2\frac{\Gamma X}{c^2}}+\left(\frac{\Gamma X}{c^2}+1\right)}. \tag{58}$$

Taking $L=\frac{c^2}{\Gamma}$, as previously, to be the unit of length, we obtain

$$\frac{m_0}{M_0}=\frac{1}{\sqrt{X^2+2X}+X+1}. \tag{59}$$

If, as previously, we assume an acceleration g, then L will be equal to one light year, giving

$\frac{X}{L}=$	0.01	0.02	0.05	0.1	0.2	0.5	1
$\frac{m_0}{M_0}=$	0.868	0.819	0.730	0.642	0.537	0.382	0.218

If the pilot has covered the given distance and wishes to slow down, then in order to cancel the velocity he must turn the ship and carry out all the operations in reverse. For the flight away, the mass ratios will be

$\frac{X}{L}=$	0.02	0.04	0.01	0.22	0.4	1	2
$\frac{m_0}{M_0}=$	0.753	0.671	0.533	0.412	0.288	0.146	0.0718

100 For a round-trip flight, where no new fuel supply is obtained at the end of the flight away, the ratios are

$\frac{X}{L}=$	0.02	0.04	0.1	0.2	0.4	1	2
$\frac{m_0}{M_0}=$	0.567	0.450	0.284	0.170	0.0829	0.213	0.00515

The nearest star, Alpha Centauri, is $4.5\,L$ away, and the distance to Sirius is $10\,L$. Therefore, with respect to these stars, the figures obtained are not very comforting. However, if we consider Neptune, which is $4.905 \cdot 10^{-14}\,L$ from the sun, a flight to it with acceleration over the first half of the path and retardation over the second half requires an amount of fuel equal to $0.0434\,M_0$. However, if the acceleration were constant, the flight time would be 3 days 12 hr, for a velocity of 3,000 mm/sec and a fuel supply of $0.039\,M_0$.

Such considerations induce us to make a study of a case where the ship is accelerated to a certain velocity, but with the special condition that the duration of the trip in the ship's system be as short as possible.

In this case, returning to equations (8), (12), and (17), we have

$$\frac{dt^1}{dt}=a$$

for

$$a^2=\frac{1}{1+\frac{\Gamma^2 t^2}{c^2}}, \tag{60}$$

from which

$$\frac{\Gamma^2 t^2}{c^2}=\frac{1}{a^2}-1. \tag{61}$$

Moreover, taking (52) and (23) into account,

$$\frac{m_0}{M_0}=e^{-\frac{\Gamma}{v}\cdot\frac{c}{\Gamma}\log\left(\frac{\Gamma}{c}t+\sqrt{1-\frac{\Gamma^2}{c^2}t^2}\right)}, \tag{62}$$

$$\frac{m_0}{M_0}=\left[\frac{\Gamma}{c}t+\sqrt{1+\frac{\Gamma^2}{c^2}t^2}\right]^{-\frac{c}{v}} \tag{63}$$

or, from (61),

$$\frac{m_0}{M_0}=\left[\sqrt{\frac{1}{a^2}-1}+\sqrt{1-\frac{1}{a^2}-1}\right]^{-\frac{c}{v}} \tag{64}$$

and finally

$$\frac{m_0}{M_0}=\left[\frac{a}{\sqrt{1-a^2}+1}\right]^{\frac{c}{v}}. \tag{65}$$

For

$$v=0, \quad a=\sqrt{1-\frac{v^2}{c^2}}=1$$

and

$$\frac{m_0}{M_0}=1$$

101 and for

$$v=c, \quad a=0$$

and

$$\frac{m_0}{M_0}=0.$$

It is interesting that this expression does not depend on Γ, that is, no additional expenditures are necessary if the ship flies with an acceleration greater than g. Then the flight gains time as a result of an increased velocity, without increasing the fuel expenditure, naturally provided that the human organism can stand the overload [the g-load].

The numerical results obtained from (65) show, however, that even in the optimum case, when $v=c$, we have

$a=$	0.5	0.2	0.1
$\frac{m_0}{M_0}=$	0.268	0.102	0.050

that is, the ratios are very unsatisfactory.

Consequently, using atomic energy, it will be comparatively easy to reach the limits of the solar system, but visiting other solar systems will not be possible, in view of the enormous distances involved. On the other hand, it is impossible to set limits to human knowledge. Perhaps physiology will present us with a means of prolonging life and rejuvenating an organism, so that this problem will also be capable of solution.

103 *ROBERT GODDARD*

FOREWORD

In 1912–1913 the American Professor Goddard presented a theory of rocket flight and formulated equations describing rocket motion. These equations were published in a paper entitled "A Method of Reaching Extreme Altitudes" (Washington, 1919).

He poses the problem in the following way: "The problem was to determine the minimum initial mass of an ideal rocket necessary, in order that on continuous loss of mass, a final mass of one pound would remain, at any desired altitude."* Here a continuous consumption of the mass, for instance in the form of propellant, is assumed.

First Goddard derives accurate formulas [rigorous solution] and shows that the use of these leads to an insoluble problem in the calculus of variations. Then he presents an approximate calculation method, suitable for practical work.

Robert Goddard published his studies of rocket flight in 1919. He was apparently the first to carry out scientific experiments determining the efficiency of a rocket and optimum rocket construction. Below we present a brief biographical note on Goddard (sent to us by him), the essential part of his paper "A Method of Reaching Extreme Altitudes," and a description of some patents on new types of rockets which were taken out by him.

* [Goddard, R.H. A Method of Reaching Extreme Altitudes, page 1.– Smithsonian Institution, Washington. 1919.]

104 SOME INFORMATION ABOUT R. GODDARD

Robert Goddard (Figure 9) was born in Worcester [Mass.], USA, on 5 October 1882. His parents were Nahum Danford Goddard and Fannie Louise (Hoyt) Goddard. He obtained a B. Sc. degree from the Worcester Polytechnic Institute in 1908, and advanced degrees from Clark University: an M. A. in 1910 and a Ph. D. in physics in 1911. In 1908–1909 he was an instructor in physics at the Worcester Institute, and in 1912–1913 he worked as a research fellow in physics at Princeton University. In 1914–1915 Goddard was an instructor and an honorary member of the Physical Society at Clark University, and in 1915–1919 he served as an assistant professor of physics there. In 1919 he became a professor of physics at Clark, and in 1923 he took over as director of the physics laboratories at this university.

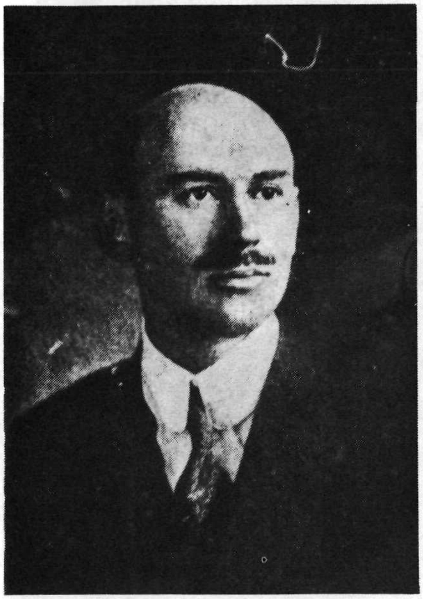

FIGURE 9. R.Goddard.

During the World War in 1918, Goddard served as a research director for the US Signal Corps, at the Worcester Institute, and for the Mount Wilson Observatory. His affiliations include: member, AAAS; member, American Physical Society; member, American Meteorological Society; American Institute of Social Sciences; Sigma Xi, and Sigma Alpha Epsilon. In June 1924 he married Esther Christine Kisk.

In 1929 Goddard began to work for the American War Department, his services having been enlisted by a colonel (Signal Corps of the US Army).* His experiments were financed principally by the D. Guggenheim Foundation.

* [Col. Charles A. Lindbergh]

Goddard's chief scientific studies deal with: electrical conduction in powders, crystal rectifiers, mechanical strengths of dielectrics in a magnetic field, interference colors in clouds, balancing of airplanes, gas production by electrical discharges in vacuum tubes, and a method of reaching high altitudes for research purposes.*

* [The parts of Goddard's paper given by Rynin have here been copied directly from English rather than retranslated from Russian. In Rynin's text only the most essential parts of the paper were presented, and the sections given were rearranged considerably. The order in the Russian book has been followed, and in addition some of the section headings are those of Rynin rather than Goddard. Otherwise, the following represents an abridged version of Goddard's historic work.

Comments interspersed by the Soviet editor or Soviet translator are given in smaller type, to distinguish them from the paper itself. The Soviets have added an extra table, as well as brief descriptions of some of Goddard's studies and four of his rocket patents. The solution of a relevant problem in rocket theory, by the German mathematician Hamel, is also included, at the end of the section on Goddard.]

105 *THEORY OF ROCKET FLIGHT*

DERIVATION OF DIFFERENTIAL EQUATION OF ROCKET MOTION AND APPROXIMATE METHOD FOR ITS SOLUTION

Referring to Figure 10, a mass *H*, weighing one pound, is to be raised as high as possible in a vertical direction by a rocket formed of a cone, *P*, of propellant material, surrounded by a casing *K*. The material *P* is expelled downward with a constant velocity, *c*. It is further supposed that the casing, *K*, drops away continuously as the propellant material *P* burns, so that the base of the rocket always remains plane.

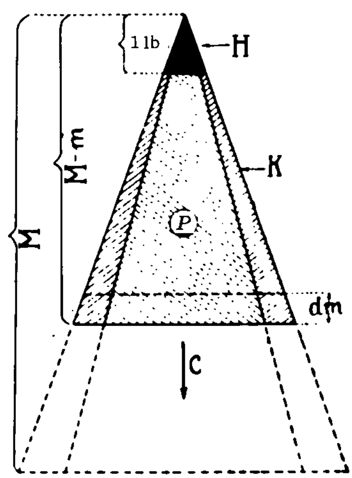

FIGURE 10. Theoretical rocket of Goddard

Let us call

M the initial mass of the rocket;

m the mass that has been ejected up to the time, *t*;

v the velocity of the rocket, at time *t*;

c the velocity of ejection of the mass expelled;

R the force, in absolute units, due to air resistance;

g the acceleration of gravity;

dm the mass expelled during the time *dt* ;

k the constant fraction of the mass *dm* that consists of casing *K*, expelled with zero velocity relative to the remainder of the rocket; and

dv the increment of velocity given the remaining mass of the rocket.

The differential equation for this ideal rocket will be the analytical statement of Newton's Third Law, obtained by equating the momentum at a time *t* to that at the time *t* + *dt*, plus the impulse of the forces of air resistance and gravity,

$$(M-m)v = dm\,(1-k)\,(v-c) + vk\,dm + (M-m-dm)\,(v+dv) + [R+g\,(M-m)]\,dt$$

106 If we neglect terms of the second order, this equation reduces to

$$c\,(1-k)\,dm = (M-m)\,dv + [R+g\,(M-m)]\,dm \qquad (1)$$

A check upon the correctness of this equation may be had from the analytical expression for the Conservation of Energy, obtained by equating the heat energy evolved by the burning of the mass of propellant, $dm\ (1-k)$, to the additional kinetic energy of the system produced by this mass plus the work done against gravity and air resistance during the time dt. The equation thus derived is found to be identical with equation (1).

In the most general case, it will be found that R and g are most simply expressed when in terms of v and s [the flight altitude]. In particular, the quantity R, the air resistance of the rocket at time t, depends not only upon the density of the air and the velocity of the rocket, but also upon the cross section, S, at the time t. The cross section, S, should obviously be as small as possible; and this condition will be satisfied at all times, provided it is the following function of the mass of the rocket $(M-m)$,

$$S = A\ (M-m)^{\frac{2}{3}}, \tag{2}$$

where A is a constant of proportionality. This condition is evidently satisfied by the ideal rocket, Figure 10. Equation (2) expresses the fact that the shape of the rocket apparatus is at all times similar to the shape at the start; or, expressed differently, S must vary as the square of the linear dimensions, whereas the mass $(M-m)$ varies as the cube. Provision that this condition may approximately be fulfilled is contained in the principle of primary and secondary rockets.

The resistance, R, may be taken as independent of the length of the rocket by neglecting "skin friction." For velocities exceeding that of sound this is entirely permissible, provided the cross section is greatest at the head of the apparatus . . .

The quantities R, g, and v, are evidently expressible most simply in terms of the altitude s, provided the cross section S is also so expressed, giving, in place of equation (1)

$$c(1-k)\,dm = (M-m)\,dv + \frac{1}{v(s)}\cdot\ [R(s) + g(s)(M-m)]\ ds \tag{3}$$

The success of the method depends entirely upon the possibility of using an initial mass, M, of explosive material that is not impracticably large. It amounts to the same thing, of course, if we say that the mass ejected up to the time t (i.e., m) must be a minimum, conditions for the existence of a minimum being involved in the integration of the equation of motion.

That a minimum mass, m, exists when a required mass is to be given an assigned upward velocity at a given altitude is evident intuitively from the following consideration: if, at any intermediate altitude, the velocity of ascent be very great, the air resistance R (depending upon the square of the
107 velocity) will also be great. On the other hand, if the velocity of ascent be very small, force will be required to overcome gravity for a long period of time. In both cases the mass necessary to be expelled will be excessively large.

Evidently, then, the velocity of ascent must have some special value at each point of the ascent. In other words it is necessary to determine an unknown function $f(s)$, defined by

$$v = f(s)$$

such that m is a minimum.

It is possible to put $f(s)$ and $\frac{df(s)}{ds}ds$ in place of v and dv, in equation (3), and to obtain m by integration. But in order that m shall be a minimum, δm must be put equal to zero, and the function $f(s)$ determined. The procedure necessary for this determination presents a new and unsolved problem in the Calculus of Variations.*

In order to obtain a solution that will be sufficiently exact to show the possibilities of the method, and will at the same time avoid the difficulties involved in the employment of the rigorous method just described, use may be made of the fact that if we divide the altitude into a large number of parts, let us say, n, we may consider the quantities R, g, and also the acceleration, to be constant over each interval.

If we denote by a the constant acceleration defined by $v=at$ in any interval, we shall have, in place of the equation of motion (3), a linear equation of the first order in m and t, as follows:

$$\frac{dm}{dt}=\frac{(M-m)(a+g)+R}{c(1-k)}. \tag{4}$$

the solution of which, on multiplying and dividing the right number by $(a+g)$ is

$$m=e^{-\frac{a+g}{c(1-k)}t}\cdot\frac{M(a+g)+R}{a+g}\cdot$$
$$\cdot\left[\int e^{\frac{a+g}{c(1-k)}t}\left(\frac{a+g}{c(1-k)}\right)dt+C\right]=e^{\frac{-a+g}{c(1-k)}t}\cdot\frac{M(a+g)+R}{a+g}\cdot\left[e^{\frac{a+g}{c(1-k)}t}+C\right].$$

where C is an arbitrary constant. This constant is at once determined as -1 from the fact that m must equal zero when $t=0$.

We then have

$$m=\left(M+\frac{R}{(a+g)}\right)\left[1-e^{\frac{-a+g}{c(1-k)}t}\right]. \tag{5}$$

This equation applies, of course, to each interval, R, g, and a being considered constant. We may make a further simplification if, for each interval, we determine what initial mass, M, would be required when the final mass in the interval is one pound. The initial mass at the beginning of the first interval, or what may be called the "**total initial mass,**"
108 required to propel the apparatus through the n intervals will then be the product of the n quantities obtained in this way.

If we thus place the final mass $(M-m)$, in any interval equal to unity, we have $M=m+1$ and when this relation is used in equation (5), we have for the mass at the beginning of the interval in question

$$M=\frac{R}{a+g}\left(e^{\frac{a+g}{c(1-k)}t}-1\right)+e^{\frac{a+g}{c(1-k)}t}. \tag{6}$$

* Later this problem was solved by Hamel in Germany, and a translation of his paper is given below [at the end of this selection of Goddard's works].

Now the initial mass that would be required to give the one pound mass the same velocity at the end of the interval, if R ang g had both been zero, is, from (6)

$$M = e^{\frac{at}{c(1-k)}}. \tag{7}$$

The ratio of equation (6) to equation (7) is a measure of the additional mass that is required for overcoming the two resistances, R and g; and when this ratio is least, we know that M is a minimum for the interval in question. The "total initial mass" required to raise one pound to any desired altitude may thus be had as the product of the minimum M's for each interval, obtained in this way.

From equations (6) and (7) we see at once the importance of high efficiency, if the "total initial mass" is to be reduced to a minimum. Consider the exponent of e. The quantities a, g, and t depend upon the particular ascent that is to be made, whereas $c(1-k)$ depends entirely upon the efficiency of the rocket, c being the velocity of expulsion of the gases, and k, the fraction of the entire mass that consists of loading and firing mechanism, and of magazine. In order to see the importance of making $c(1-k)$ as large as possible, suppose that it were decreased tenfold. Then $e^{\frac{a+g}{c(1-k)}t}$ would be raised to the 10th power, in other words, the mass for each interval would be the original value multiplied by itself ten times.

According to Goddard's experiments the velocity of expulsion of gases in an improved rocket can be raised to a value 6 or 7 times greater (2,434 m/sec) than the gas velocity in, for example, a ship rocket (314 m/sec), and thus the mass of the former rocket may be reduced to $\sqrt[7]{\ }$ of the mass of the latter. The mass of propellant has to be as large as possible, relative to the remaining mass of the rocket. In Goddard's experiments with steel rockets, the rocket walls were made very thick, corresponding to the outer shaded section in Figure 10. However, these walls could have a thickness extending out as far as the solid outer lines (k) in the drawing. Goddard assumes that the minimum mass of the casing in his experiments could be raised to 120 grams per gram of powder.

NUMERICAL EXAMPLE OF ROCKET-MOTION CALCULATION

As already explained, this method consists in employing equations (6) and (7) to obtain a minimum M in each interval, where

$M =$ the initial mass, for the interval, when the final mass is one pound, and

$R =$ the air resistance in poundals over the cross section S, at the altitude of the rocket. If we call P the air resistance per unit cross section, we shall have for $R = P.\,S.\,\frac{\varrho}{\varrho_0}$, where ϱ is the density at the altitude of the rocket,

109 and ϱ_0 is the density at sea level.

$a =$ the acceleration in ft per sec^2, taken constant throughout the interval;

$g =$ the acceleration of gravity;

$t =$ the time of ascent through the interval, and
$c(1-k) =$ what will be called the "effective velocity," for the reason that the problem would remain unchanged if the rocket were considered to be composed entirely of propellant material, ejected with the velocity $c(1-k)$. It will be remembered that c actually stands for the true velocity of ejection of the propellant, and k for the fraction of the entire mass that consists of material other than propellant. *The effective velocity is taken constant* throughout any one calculation.

The altitude is divided into intervals short enough to justify the quantities involved in the above equations being taken as constants. The equations are then used to find the minimum value of M for each interval – the mean values of R and g, in the interval, being employed – and the "total initial mass" required to raise a final mass of one pound to a desired altitude is then obtained as the product of these M's.

VALUES OF THE QUANTITIES OCCURRING IN EQUATIONS (6) AND (7)

On the basis of his experiments, Goddard assumes $c = 7{,}500$ ft/sec. Then, for $k = 1/15$, we have $c(1-k) = 7{,}000$ ft/sec. In order to calculate the air resistance, Goddard divides the height of ascent into 7 intervals (Figure 11) and determines coefficient P using the following formula.

The coefficient, for projectiles with pointed heads, becomes

$$P = 0.000\,064\,30\,v^2\left(\frac{v^1}{a}\right)^{3.375} + 480 \text{ (poundals)}, \qquad (8)$$

where v^1 is the velocity with which a wave is propagated in the air immediately in front of the projectile; which equals the velocity of the body when that velocity exceeds the velocity of sound in the undisturbed gas; and a is the velocity of sound in the undisturbed gas.*

Beyond 120,000 ft the density is calculated by the empirical rule which assumes the density to become halved at every increase in altitude of 3.5 miles. A comparison was made between the values obtained in this way and those obtained from the very probable pressures deduced by Wegener, in the following way: the mean density between two levels for which Wegener gives pressures was obtained by multiplying the difference in pressure by 13.6, and dividing by the difference in level in cm. A comparison showed that the densities used in the present calculations beyond 125,000 ft were from three to twentyfold larger than those derived from Wegener's data, so that the values used in the present case were doubtless perfectly safe. Densities beyond 700,000 ft must be negligible. . . .

* Ratios ρ/ρ_0 are shown in Figure 11, and data for the various selected intervals are shown in Table 1 [Rynin].

110 TABLE 1

Interval	Length of interval		Height of upper end of interval above sea level		Mean density in terms of ρ_0	Mean gravity chosen, in terms of gravity at sea level
	feet	meters	feet	meters		
s_1	5,000	1,524	5,000	1,524	0.928	1
s_2	10,000	3,048	15,000	4,572	0.730	1
s_3	10,000	3,048	25,000	7,620	0.520	1
s_4	20,000	6,096	45,000	13,716	0.278	1
s_5	40,000	12,192	85,000	25,908	0.080	1
s_6	40,000	12,192	125,000	38,100	0.015	1
s_6	75,000	22,860	200,000	60,960	0.0026	1
s_6	300,000	91,440	500,000	152,400	0.000025	1
s_9 { a = 150	3,415,000	1,040,900	3,915,000	1,193,300	....	0.839
s_9 { a = 50	8,810,000	2,685,000	9,310,000	2,837,400	....	0.684

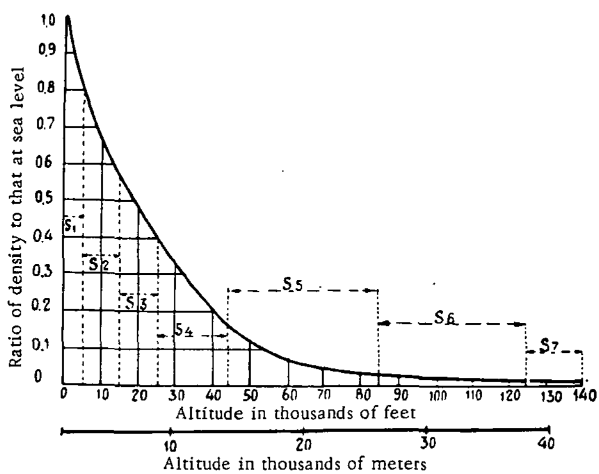

FIGURE 11.

111 CALCULATION OF MINIMUM MASS FOR EACH INTERVAL

Tables 2 and 3 are calculated for a start, respectively, from sea level and from an altitude 15,000 ft, i.e., the beginning of s_3. The procedure in each case is, however, identical.

The process of calculation is as follows: at the beginning of any interval we have the velocity already acquired during the previous intervals, let us say v_0. This velocity is, of course, zero at the beginning of the first interval. Assume any final velocity at random, v_1, for the interval in question.

The value of may be had from the equation

$$v_1 = v_0 + at \tag{9}$$

and t is at once obtained from the relation

TABLE 2.

Interval	V_1, ft/sec	at	t, sec	a	$\frac{at}{c(1-k)}$	$\frac{a+g}{c(1-k)}t$	$e^{\frac{at}{c(1-k)}}$	$e^{\frac{(a+g)t}{c(1-k)}}$	P, poundals per sq. in	R, $\left(P.S\cdot\frac{\varrho}{\varrho_0}\right)$	$\frac{R}{a+g}$	M, lbs	$\frac{M}{e^{\frac{at}{c(1-k)}}}$	$e^{\frac{2(a+g)t}{c(1-k)}}$	M_2, lbs	$e^{\frac{7.28(a+g)t}{c(1-k)}}$	M_R, lbs	$e^{\frac{27.2(a+g)t}{c(1-k)}}$	M_{R_1}, lbs	Time to upper end of interval
S_1	500	500	20.0	25	0.0716	0.1630	1.074	1.176	7.36	6.85	0.120	1.1972	1.113							
	800	800	12.5	64	0.1145	0.1720	1.120	1.186	20.0	18.5	0.193	1.2218	1.092							
•	1,000	1,000	10.0	100	0.143	0.1890	1.153	1.207	31.25	29.0	0.219	1.252	1.086	1.458	1.5584	3.94	4.586	167.3	203.91	10.0 sec
	1,200	1,200	8.34	144	0.172	0.212	1.185	1.235	61.4	57.0	0.323	1.311	1.106							
	1,500	1,500	6.7	226	0.215	0.2475	1.242	1.276	104.6	98.0	0.378	1.380	1.112							
	2,000	2,000	5.0	400	0.287	0.309	1.332	1.362	202.5	188.0	0.436	1.5195	1.138							
S_2	1,100	100	9.54	10.47	0.0143	0.0578	1.014	1.061	153.3	112.1	2.64	1.222	1.206							
•	1,200	200	9.10	22.0	0.0286	0.0704	1.034	1.073	166.6	121.6	2.24	1.237	1.199	1.150	1.4860	1.665	3.155	6.73	20.60	19.1
	1,400	400	8.33	47.9	0.0574	0.0954	1.060	1.100	216.0	158.7	1.97	1.297	1.223							
S_3	1,300	100	8.0	12.5	0.0143	0.0508	1.014	1.052	250.0	130.0	2.925	1.204	1.186							
•	1,400	200	7.7	25.8	0.0286	0.0637	1.034	1.066	262.8	136.9	2.37	1.222	1.182	1.137	1.462	1.589	2.974	5.62	16.52	26.8
	1,600	400	7.15	56.4	0.0574	0.0906	1.060	1.096	294.5	152.6	1.74	1.261	1.191							
S_4	1,500	100	13.8	7.23	0.0143	0.0775	1.014	1.080	339.0	94.3	2.42	1.273	1.255							
•	1,600	200	13.33	15.0	0.0286	0.0898	1.034	1.094	372.0	101.5	2.17	1.297	1.253	1.198	1.626	1.92	3.91	11.33	33.73	40.13
	1,700	300	12.9	23.24	0.0429	0.1022	1.046	1.107	394.0	109.4	1.975	1.319	1.26							
	1,800	400	12.5	33.25	0.0574	0.1170	1.060	1.123	424.0	118.0	1.81	1.346	1.267							
S_5	1,700	100	24.25	4.125	0.0143	0.1258	1.014	1.133	439.0	35.1	0.974	1.262	1.245							
•	1,800	200	23.7	8.45	0.0286	0.1366	1.034	1.146	480.0	38.4	0.951	1.2845	1.242	1.313	1.711	2.694	4.304	40.70	88.45	63.83
	2,000	400	22.24	18.0	0.0574	0.159	1.060	1.173	535.0	42.8	0.854	1.321	1.246							
S_6	1,900	100	21.7	4.62	0.0143	0.1135	1.014	1.12	567.0	8.50	0.232	1.1478	1.13							
•	2,000	200	21.1	9.50	0.0286	0.1255	1.034	1.133	603.0	9.01	0.2175	1.162	1.123	1.280	1.3406	2.488	2.810	29.76	36.02	84.93
	2,200	400	20.0	20.0	0.0574	0.1490	1.060	1.16	669.0	10.02	0.1923	1.1907	1.124							
S_7 a = 150	5,160	3,160	21.0	150	0.4523	0.5452	1.572	1.725	1,878.0	4.84	0.0264	1.7442	1.108	2.97	3.022	52.6	53.96	2.63×10^{6}	2.70×10^{6}	105.93
S_7 a = 50	3,393	1,393	27.8	50	0.199	0.3276	1.218	1.387	1,122.0	3.1	0.0355	1.4007	1.15	1.900	1.9319	10.79	11.13	7.03×10^{3}	7.28×10^{3}	112.73
S_8 a = 150	10,790	5,630	37.5	150	0.804	0.976	2.23	2.65	10,600	0.272	0.00146	2.6524	1.19	7.02	7.0288	1,192.0	1,193.7	2.88×10^{11}	2.88×10^{11}	143.43
S_8 a = 50	6,833	2,840	55.8	50	0.399	0.652	1.49	1.92	4,000	0.0994	0.00121	1.9211	1.293	3.680	3.6832	117.4	117.54	4.67×10^{7}	4.67×10^{7}	168.53
S_9 a = 150	33,790	23,000	153.5	150	3.29	3.89	26.9	48.8	–	–	–	48.8	–	2,380.0	2,380.0	1.906×10^{12}	1.906×10^{12}	5.74×10^{15}	5.74×10^{45}	296.93
S_9 a = 50	30,533	23,700	472.5	50	3.38	4.85	29.13	129.0	–	–	–	129.0	–	16,700.0	16,700.0	1.995×10^{15}	1.995×10^{15}	1.25×10^{57}	1.25×10^{57}	641.03

TABLE 3.

Interval	v_1, ft/sec	at	t, sec	a	$\frac{at}{c(1-k)}$	$\frac{(a+g)t}{c(1-k)}$	$e^{\frac{at}{c(1-k)}}$	$e^{\frac{(a+g)t}{c(1-k)}}$	P, poundals per sq.in	R, $\left(P.S.\frac{\varrho}{\varrho_0}\right)$	$\frac{R}{a+g}$	M, lbs	$\frac{M}{e^{\frac{at}{c(1-k)}}}$	$e^{\frac{2(a+g)t}{c(1-k)}}$	M_g, lbs	$e^{\frac{7.28(a+g)t}{c(1-k)}}$	M_{R_1}, lbs
S_3	500	500	40.0	12.5	0.0715	0.255	1.074	1.29	11.53	5.97	0.134	1.329	1.236				
•	800	800	25.0	32.0	0.1147	0.2277	1.120	1.256	30.7	16.00	0.250	1.300	1.162	1.574	1.718	5.225	6.545
	1,000	1,000	20.0	50.0	0.142	0.235	1.152	1.263	46.7	24.3	0.295	1.341	1.165				
	1,500	1,500	13.4	112.0	0.2145	0.277	1.24	1.318	165.0	83.3	0.570	1.499	1.207				
S_4	900	100	23.7	4.23	0.0143	0.1227	1.013	1.132	95.7	27.7	0.764	1.232	1.216				
•	1,000	200	22.2	9.00	0.0286	0.1305	1.034	1.137	108.8	31.4	0.767	1.242	1.200	1.293	1.518	2.581	3.794
	1,300	500	19.1	26.2	0.0714	0.1645	1.073	1.177	165.0	46.25	0.794	1.318	1.227				
	1,800	1,000	15.4	65.0	0.1430	0.2136	1.152	1.238	305.0	87.90	0.908	1.455	1.263				
S_5	1,100	100	38.1	2.625	0.0124	0.1888	1.013	1.207	150.1	12.0	0.347	1.278	1.261				
	1,200	200	36.5	5.47	0.0286	0.1960	1.03	1.215	170.0	13.55	0.362	1.293	1.255				
•	1,300	300	34.75	8.64	0.0430	0.202	1.044	1.223	195.0	15.65	0.384	1.306	1.250	1.495	1.685	4.32	5.594
	1,400	400	33.3	12.0	0.0571	0.210	1.058	1.233	218.8	17.49	0.397	1.325	1.252				
	1,500	500	32.1	15.60	0.0715	0.2192	1.073	1.245	243.5	19.45	0.520	1.372	1.280				
	2,200	1,000	26.1	21.40	0.1147	0.268	1.12	1.308	417.0	33.4	0.623	1.501	1.340				
S_6	1,600	300	27.7	10.8	0.0430	0.1690	1.045	1.184	343.0	5.16	0.1203	1.206	1.153				
	1,800	500	25.7	19.5	0.0714	0.1890	1.074	1.206	406.0	6.10	0.1186	1.230	1.147				
	1,900	600	25.0	24.0	0.0857	0.201	1.091	1.223	430.0	6.43	0.1150	1.248	1.142				
•	2,000	700	24.2	28.9	0.1002	0.212	1.105	1.234	460.0	6.90	0.1134	1.260	1.140	1.522	1.581	4.66	5.075
	2,100	800	23.6	33.8	0.1142	0.224	1.118	1.249	510.0	7.65	0.1165	1.278	1.142				
	2,200	900	22.8	40.0	0.1285	0.237	1.124	1.266	534.0	8.02	0.1115	1.295	1.151				

$$s = v_0 t + \frac{1}{2} a t^2,$$

i.e.,

$$t = \frac{s}{v_0 + \frac{1}{2} a t}, \qquad (10)$$

whence, of course, a is at once known.

The calculations of $\exp \frac{a+g}{c(1-k)} t$ and $\exp \frac{at}{c(1-k)}$ call for no comment; and R is obtained as P, the mean ordinate between v_0 and v_1, from the curves as already explained, multiplied by S and $\frac{\varrho}{\rho_0}$.

The value of M, the initial mass, for the interval, necessary in order that the final mass in the interval shall be one pound, is then obtained from equation (7); and finally, the ratio of equations (6) to (7) (i.e.,

$$\frac{M}{\exp \frac{at}{c(1-k)}}$$

is calculated. This is the ratio of the initial mass necessary, including losses due to both R and g, to the mass necessary to give the one pound the same velocity, r, without overcoming R and g; and the entire calculation must be repeated until a minimum value of this ratio is obtained – when the corresponding mass, M, will be the minimum mass for the interval in question. Each minimum M is marked in the tables by an asterisk.

This process is carried out for each interval beginning with the first. It should be noticed that, although P and the density are not really constant in any interval, the result obtained by taking the mean of the quantities must nevertheless give results close to the truth, owing to the fact that P increases during the ascent, whereas the density decreases.

116 EXPLANATION OF TABLES 2 AND 3

It should first be explained why no minimum M has been calculated for the intervals s_7 and s_8. Although the minima for the preceding intervals are clearly defined, a trial will show that a minimum M can occur, for s_7 and s_8, only for extremely high velocities v_1; although for s_7, a secondary minimum occurs for $v_1 = 8{,}000$ ft/sec. Even for $v_1 = 30{,}000$ ft/sec the minimum has not yet been attained for this interval, although the acceleration required to produce this velocity is 6,000 ft/sec^2. The reason for this state of affairs is evident at once from the fact that the density ratio, $\frac{\varrho}{\rho_0}$,

is very small for s_7, and also from the fact that a occurs in the denominator of the term containing R in equation (6), so that the large acceleration counterbalances the increase in R.

Thus, in order that the initial mass for s_7 shall be a minimum, the acceleration must become very large, with consequent severe strains in the rocket apparatus and instruments carried by the rocket, to say nothing of the difficulty of firing with sufficient rapidity to produce such large accelerations. It thus becomes advisable to choose a moderate acceleration in s_7 and s_8, and not to assign a velocity v_1 as was done in the preceding intervals. Two accelerations are chosen: 50 ft/sec^2 and 150 ft/sec^2, respectively. The interval s_9, also calculated for assigned accelerations, will be explained in detail below. In all cases, when either one of these accelerations is mentioned in connection with s_8 and s_9, this acceleration will be understood as having been taken also in the preceding intervals, beyond s_6.

In order to see how far the effective velocity, $c\,(1-k)$ may fall short of 7,000 ft/sec and still not render the rocket impracticable, a few additional columns for M are calculated.

In the first of the additional columns, M_2, the effective velocity is taken as 3,500 ft/sec, namely, half that of the preceding calculations. This allows of considerable inefficiency of the apparatus, in a number of ways. For example, the product

$$c\,(1-k)=3500$$

may be given by the same proportionality, k, as before, but with a velocity of ejection of the gases as low as 3,750 ft/sec. On the other hand, the velocity of ejection may be as large as before (i.e., 7,500 ft/sec); and the proportionality, k, increased to 0.533; meaning, of course, that the rocket now consists more of mechanism than of propellant.

The second additional calculations, M_{k_1}, are carried out under the assumption that a reloading mechanism is used, with k as in the original calculations ($k=\frac{1}{15}$), but that the velocity of expulsion of the gases is the mean found by experiment for the Coston ship rockets, namely 1,029.25 ft/sec. In this case the effective velocity is

$$c\,(1-k)=1029.25\left(1-\frac{1}{15}\right)=960\text{ ft/sec.}$$

The third additional calculations, M_{R_2}, are carried out for the case of a rocket built up of Coston rockets in bundles (shown in section in Figure 12), the lowest bundle of which is fired first and then released; after which the bundle above is fired and then released, and so on. For the Coston ship
117 rocket (having a range of a quarter of a mile, with the charge of red fire removed, as already stated) the ratio of the powder charge to the remaining mass of the rocket is found to be closely $^1/_4$. Hence the "effective velocity" in this case is only

$$c\,(1-k)=1029.25\left(1-\frac{4}{5}\right)=257.3\text{ ft/sec.}$$

The M's in the last two cases are calculated only for the accelerations that make M minima for the first case (effective velocity 7,500 ft/sec). Hence in these cases, the M's are not minima, although only in the last two cases is there probably much discrepancy from the actual minima.

The cross-section, throughout any interval, is taken as one square inch except for interval s_9. It will be seen from the table that this is justifiable, as the largest mass in intervals s_1 to s_8 does not differ much from one pound.

CALCULATION OF MINIMUM MASS TO RAISE ONE POUND TO VARIOUS ALTITUDES IN THE ATMOSPHERE

The "total initial masses" required to raise one pound from sea level to the upper end of intervals s_6, s_7 and s_8 are given in Table 4. They are obtained by multiplying together the minimum masses (marked by stars in Table 2), from s_1 up to and including the interval in question, and represent, as already explained, the mass in pounds of a rocket which, starting at sea level, would become reduced to one pound at the altitude given.

The highest altitude attained by the one pound mass is not, however, the upper end of the interval in question, but is a very considerable distance higher. This, of course, follows from the fact that the one pound reaches the upper end of each interval with a considerable velocity, and will continue to rise after propulsion has ceased until this velocity is reduced to zero, by gravity and air resistance.

If we call v_n the velocity with which the pound mass reaches the upper end of the particular interval where propulsion ceases, h the distance beyond which the one pound will rise (the cross section still being one square inch), and p the mean air resistance in poundals over the distance h, we have by the Principle of Work and Energy,

$$h = \frac{v_n^2}{2(g+p)}.$$

The values of p are small, owing to small atmospheric density, being 1.59 poundals for the h beyond s_6; 0.28 beyond s_7 (a = 150). For s_8 the low density makes this quantity negligible.

The altitudes obtained by adding to the interval the corresponding h, are called the "Greatest altitude attained" in Table 4.

Obviously if the start is made at a high elevation, the "total initial mass" required to reach a given height will be less than for a start at sea level, due not only to the fact that the apparatus is not raised through so great a height, but also to the fact that the denser part of the atmosphere is avoided.
120 Table 3 gives minimum masses, M, calculated for a start with zero velocity from the beginning of interval s_3 (i.e., 15,000 ft), the effective velocity being 7,000 ft/sec, as in Table 2.

It happens that the velocity v_1 for minimum M in the interval s_6 of Table 3 is the same as the v_1 for the same interval in Table 2. The calculations that have been made for the intervals beyond s_7 apply therefore to the present case, and the only difference between the two cases is that the masses required to reach s_7 will be greater, for the start at sea level, than for the start at 15,000 ft.

The calculations beginning at 15,000 ft have been carried out in Table 4 for all but the lowest "effective velocity"; and it will be observed that the start from a high elevation becomes important only for the lower "effective velocities."

The most striking as well as the most important conclusion to be drawn from Table 4 is the small "total initial mass" required to raise one pound to very great altitudes when the "effective velocity" is 7,000 ft/sec, the mass for the height of 437 miles (2,310,000 ft) for example, being but 12.33 lbs, starting from sea level. Even for an "effective velocity" of 3,500 ft/sec,

(118)

TABLE 4.

Interval	Altitude of upper end of interval in feet	Greatest altitude attained (feet)	Time (sec) to reach greatest altitude from sea level	Total initial masses (in lbs) Starting from	
				$c(1-k)=7000$	$c(1-k)=3500$
S_6	125,000	184,500	144.13	3.665	12.61
S_7 (a = 50)	200,000	377,500	217.73	5.14	24.36
(a = 150)	200,000	610,000	265.93	6.40	38.10
S_8 (a = 50)	500,000	1,228,000	380.53	9.875	89.60
(a = 150)	500,000	2,310,000	475.23	12.33	267.70
S_9 (a = 50)	9,310,000	∞	∞	1,274.0	1.497×10^6
(a = 150)	3,915,000	∞	∞	602.0	6.370×10^5

(118)

AUXILIARY TABLE. Data for various rockets [added by Rynin]

ft/sec	Small common rocket	Coston ship rocket	Goddard's large steel rocket	Hydrogen + oxygen start 4,572 m	
Initial weight in kg	0.120	0.640	19.1	54.000	19.7
Weight of charge in kg	0.223	0.130	0.082	53.546	19.246
b/a	$1/5$	$1/5$	$1/2{,}800$	$1/1.08$	$1/1.02$
$\frac{a}{a-b}$	$1/0.8$	$1/0.8$	1	$1/119$	$1/43.5$

(119)

for one pound final mass					
sea level			Starting from 15,000 feet		
$c(1-k) = 960$	**$c(1-k) \doteq 257.3$**	**$c(1-k) \doteq 257.3$** k taken = 0	**$c(1-k) = 7000$**	**$c(1-k) = 3500$**	**$c(1-k) = 960$**
2,030.0	7.40×10^{9}	8.63×10^{8}	2.66	6.95	702.0
2.260×10^{4}	5.46×10^{12}	6.08×10^{11}	3.74	13.38	7,820.0
1.096×10^{5}	2.00×10^{15}	2.28×10^{14}	4.65	20.90	37,800.0
2.660×10^{6}	2.55×10^{19}	2.89×10^{18}	7.19	49.30	9.17×10^{5}
1.318×10^{8}	5.77×10^{26}	6.53×10^{25}	8.97	147.30	4.51×10^{7}
5.320×10^{21}	3.21×10^{76}	3.63×10^{75}	926.0	8.22×10^{5}	1.82×10^{21}
2.490×10^{20}	3.32×10^{71}	3.76×10^{70}	438.0	3.51×10^{5}	8.59×10^{19}

(119)

ft/sec	Small common rocket	Coston ship rocket	Goddard's large steel rocket	Hydrogen + oxygen start 4,572 m	
c m/sec	292	1,029	–	2,865	3,627
$c(1-k)$ m/sec ...	58	78.4	–	–	–
Height of ascent ...	meter	149	0.05	∞	∞
Flight range (m) ...	–	402	–	–	–

which allows of considerable inefficiency in the rocket apparatus, the mass is sufficiently moderate to render the method perfectly practicable, for in this case an altitude of over 230 miles from sea level, practically the limit of the earth's atmosphere, requires under 90 lbs; and an altitude of 118 miles, close under the geocoronium sphere, only 38 lbs. For a start at 15,000 ft, the masses are of course, less, namely 49.3 lbs and 20.9 lbs, respectively.

The enormous difference between the total initial masses required for low-efficiency rockets, compared with those for high, may at first appear surprising; but they should be expected from the exponential nature of equations (6) and (7). Thus if the "effective velocity" is reduced from 7,000 ft/sec to half this value, the minimum masses for each interval, neglecting air resistance, will be those for 7,000 ft/sec squared; and including air resistance, still greater. Similarly for an effective velocity of 960 ft/sec which is that for reloading rockets having the same velocity of ejection as Coston ship rockets, the minimum masses will be those for 7,000 ft/sec raised to the 7.28th power; and for bundles or groups of ship rockets, as shown in Figure 12, the minimum masses will be those for 7,000 ft/sec, raised to the 27.2th power. Even when air resistance is entirely neglected in the calculations for the last case, the masses are of much the same magnitude, as shown in Table 4. The large values of the masses M_{R_1} and M_{R_2} simply express the impossibility of employing rockets of low efficiency. Attention may be called to the particular case under M_{R_2} (the groups of ship rockets indicated in Figure 12) in which one pound is raised to the altitude of 1,228,000 feet (232 miles); the "total initial mass" in this case, even neglecting air resistance entirely, is 2.89×10^{18} lbs, or over sixfold greater than the entire mass of the earth.

These large numbers, to be sure, agree with one's first impression as to the probable initial mass of a rocket designed to reach extreme altitudes; but the comparatively small initial masses, possible with high efficiency, are not intuitively evident until one realizes what an enormous reduction is involved in extracting anything as large as the 27th root of a number.

It should be observed that the apparatus is taken as weighing one pound. Strictly speaking, if the recording instruments have a mass of one pound, the entire final mass of the apparatus must be at least three or four pounds. The mass for the recording instruments may be considered as being very
121 small, yet many valuable researches could, of course, be performed with an apparatus weighing no more than this. The entire final apparatus should if possible be designed to weigh not over 3 or 4 lbs at most, unless the efficiency of the apparatus is so high that the "effective velocity," $c\,(1-k)$, is at least in the neighborhood of 7,000 ft/sec. An examination of Table 4 makes very evident the necessity of securing maximum effectiveness of the apparatus before a rocket for such a purpose as meteorological work, for example, is constructed, in order to make the method as inexpensive as possible. It should be remarked, however, that the "total initial mass" will really not be increased in as large a proportion as the final mass if the latter is made greater than one pound by virtue of equation (2).

Before proceeding further it will be well to consider carefully the question of air resistance as dependent upon the cross section of the rocket during flight. It has already been assumed that the cross section, in the calculation of the minimum M for each interval, was one square inch. If we make the apparatus as long, narrow, and compact, as possible, the assumption of a cross section of one square inch for an apparatus weighing one pound will not be unreasonable. A glance at Tables 2 and 3 will show that, for "effective velocities" of 7,000 ft/sec and 3,500 ft/sec, the mass at the beginning of any interval (except s_0) does not greatly exceed one pound – the mass at the end of each interval being one pound – so that the computations are in agreement with this assumption of area of cross section. For the two cases of the adapted Coston rockets, the masses at the beginning of the intervals are much larger; and hence we see that the "total initial masses" in Table 4, large as they are, would have been even larger if a proper value of cross section had been employed.

The important point is, however, that cross-sectional areas of even less than one square inch should have been used. The reason for this is obvious when one remembers that in calculating the "total initial masses," when we multiply minimum masses, M, together we are also multiplying the cross sections in the same ratio. In other words, we are considering numbers of rockets, each of one square inch cross section, grouped together side by side, into a bundle. But such an arrangement would have its cross section proportional to its mass and not to the $^2/_3$d power of its mass, as would be the case if the shape of the rocket apparatus were at all times similar to the shape at the start (as in the ideal rocket, Figure 10). This constant similarity of shape is, as we have seen (equation 2), one of the conditions for a minimum initial mass. Hence the "total initial masses" that have been calculated are really larger than the true minima, which would be obtained only by repeating the calculations, assuming a smaller cross section except in the last few intervals, in which the rocket has become so small that the condition of one-square-inch-per-pound is approximately satisfied.

Before leaving the subject of air resistance, attention should be called to the fact that the velocities (Table 2), do not exceed that for which air resistance has been studied by Mallock until in s_7, for $a = 150$ ft/sec^2, and in s_8, for $a = 50$ ft/sec^2; and furthermore, that the velocities do not become much in excess until the densities have become almost negligible.

CHECK ON APPROXIMATE METHOD OF CALCULATION

A simple calculation, involving only the most elementary formulae instead of equations (6) and (7) will show that the "total initial masses" in Table 4 cannot be far from the truth.

Consider, for simplicity, a rocket of the form shown in Figure 10, and suppose that one-third of the mass of the rocket is fired downward, with a velocity of 7,000 ft/sec at the first shot; one-third of the remaining mass, at the second shot; and so on, for successive shots. From the principle of the Conservation of Momentum it will be evident that the mass that remains is given an additional upward velocity of 3,500 ft/sec after each shot.

Thus, after the fourth shot, the mass that remains is $^{16}/_{81}$, or practically $^1/_5$, of the initial mass, and the velocity is 14,000 ft/sec. This velocity is

sufficient, if we neglect air resistance, to raise the part of the rocket that
122 remains to an altitude of 580 miles (by the familiar relation $v^2 = 2gh$). Although the range would be much reduced if air resistance were considered, it should nevertheless be remembered that the values in Table 4 are calculated for the condition under which air resistance is a minimum.

The above simple case is not realizable in practice because of the large mass of propellant for each shot compared with the total mass, i. e., provision is not made for the mass of the chamber. The result will be the same, however, if smaller charges are fired in rapid succession, as will be evident from a calculation similar to the above, . . . under the assumption of smaller charges for successive shots.

CHECK ON APPROXIMATE METHOD OF CALCULATION, FOR SMALL CHARGES FIRED IN RAPID SUCCESSION

Consider a rocket weighing 10 lbs, having 2 lbs of propelling material, fired two ounces at a time, eight times per second, with a velocity of 6,000 ft/sec – much less than the highest velocity attained in the experiments, either in air or in vacuo.

Let us suppose that, for simplicity, the rocket is directed upward and that each shot takes place instantly (a supposition not far from the truth); the velocity remaining constant between successive shots.

After the first shot, the mass, $9\,^7/_8$ lbs, has an upward velocity v_0 due to the downward velocity of the $^1/_8$ lb expelled. This velocity, v_0, is at once found by the Conservation of Momentum. But it is decreased by gravity until, at the end of $^1/_8$ sec, it is reduced to

$$v_0^1 = v_0 - gt$$

the space passed over during this time being

$$s = v_0 t - \frac{1}{2} g t^2.$$

We have then, $v_0^1 = 71.8$ ft/sec, and $s = 9.23$ ft.

At the beginning of the second interval of $^1/_8$ sec, an additional velocity is given the remaining mass, of 76.8 ft/sec, and the final velocity and space passed over may be found in the same way. By completing the calculations for the remaining intervals we shall have

for time just under $^1/_2$ sec: $v_0^1 =$ 291.1 ft/sec; $s =$ 91.98 ft
" " " " 1 " $v_0^1 =$ 603.8 ft/sec; $s =$ 335.48 ft, and
" " " " 2 " $v_0^1 =$ 1,284.1 ft/sec; $s =$ 1,315.68 ft

These figures compare well with those in Table 2, for s_1. In the present check, air resistance would doubtless be unimportant until the velocity had reached 1,000 ft/sec or so; but the velocity would, even if decreased somewhat by air resistance, compare favorably with that of a projectile fired from a gun.

CALCULATION OF MINIMUM MASS REQUIRED TO RAISE ONE POUND TO AN "INFINITE" ALTITUDE

From the fact that the preceding calculation* leads us to conclude that such an extreme altitude as 2,310,000 ft (over 437 miles) can be reached by the employment of a moderate mass, provided the efficiency is high, it becomes of interest to speculate as to whether or not a velocity as high as
123 the "parabolic" velocity for the earth could be attained by an apparatus of reasonably small initial mass.

Theoretically, a mass projected from the surface of the earth with a velocity of 6.95 miles/sec would, neglecting air resistance, reach an infinite distance, after an infinite time; or, in short, would never return. Such a projection without air resistance, is, of course, impossible. Moreover, the mass would not reach infinity but would come under the gravitational influence of some other heavenly body.

We may; however, consider the following conceivable case: if a rocket apparatus such as has here been discussed were projected to the upper end of interval s_8, either with an acceleration of 50 or 150 ft/sec^2, and this acceleration were maintained to a sufficient distance beyond s_8, until the parabolic velocity were attained, the mass finally remaining would certainly never return.

If we designate as the upper end of s_9 the height at which the velocity of ascent becomes the "parabolic" velocity, it will be evident that this height will be different for the two accelerations chosen, inasmuch as the "parabolic" velocity decreases with increasing distance from the center of the earth.

If we call u = the "parabolic" velocity at a distance H above the surface of the earth;

v_1 = the velocity acquired at the upper end of interval s_8;

s_0 = the height of the upper end of s_8 above sea-level,

we have, taking the radius of the earth as 20,900,000 feet,

$$u = v_1 + at \tag{11}$$

$$H = s_0 + v_1 t + \frac{1}{2} at^2 \tag{12}$$

and also the equation relating "parabolic" velocity to distance from the center of the earth

$$\frac{36\,700}{u} = \sqrt{\frac{20\,900\,000 + H}{20\,900\,000}}. \tag{13}$$

On putting the values of u and H from (11) and (12), in (13), we have

$$\sqrt{20{,}900{,}000}\cdot 36{,}700 = (v_1 + at)\ \sqrt{21{,}400{,}000 + v_1 t + {}^1/_2\, at^2}. \tag{14}$$

* [Here the results in Table 4 are meant.]

Equation (14) is a biquadratic in t, from which t may easily be obtained (by trial and error). The values of t, for the two accelerations chosen, given in Table 2, enables u and the initial masses for s_9, to be at once obtained.

The effect of air resistance in s_9 is negligible, if we accept Wegener's conclusicns, above mentioned, concerning the properties of geocoronium. But even if we use the empirical rule of a fall of density to one-half for every 3.5 miles we shall find the reduction of velocity very small on passing from the upper end of s_8 (500,000 ft) to 1,000,000 ft (beyond which the density is negligible).

124 PROOF THAT THE RETARDATION BETWEEN 500,000 ft AND 1,000,000 ft IS NEGLIGIBLE

The falling-off of velocity, W, due to air resistance, is given by

$$P \cdot \frac{\varrho}{\varrho_0} \cdot s \cdot h = \frac{1}{2} M_0 \cdot W^2$$

where P = the mean air resistance in poundals per square inch between the altitudes 500,000 and 1,000,000 ft from the previously mentioned velocity curves, the pressure being considered as atmospheric; ϱ = the mean density over this distance; s = the mean area of cross section of the apparatus throughout the distance, taken as 25 square inches in view of the average mass, M_0, throughout the interval, and h = the distance traversed: 500,000 ft.

It is thus found that the loss of velocity w is less than 10 ft/sec (for a = 150 ft/sec) even when $\frac{\varrho}{\varrho_0}$ is taken as constant throughout the distance and equal to that at 500,000 ft (i. e., $2.73 \cdot 10^{-9}$).

The "total initial masses," to raise one pound to an "infinite" altitude, for the two accelerations chosen, are given in Table 4. It will be observed that they are astonishingly small, provided the efficiency is high. Thus with an "effective velocity" of 7,000 ft/sec, and an acceleration of 150 ft/sec^2, the "total initial mass," starting at sea level is 602 lbs, and starting from 15,000 ft is 438 lbs. The mass required increases enormously with decreasing efficiency, for, with but half of the former "effective velocity" (3,500 ft/sec) the "total initial mass," even for a start from 15,000 ft, is 351,000 lbs. The masses would obviously be slightly less if the acceleration exceeded 150 ft/sec^2.

Attention is called to the fact that hydrogen and oxygen, combining in atomic proportions, afford the greatest heat per unit mass of all chemical transformations. For this reason, if the calculations are made under the assumption that hydrogen and oxygen are used . . . the velocities would be

9,400 and 11,900 ft/sec; and the total initial masses for a start from 15,000 feet, respectively, 119 pounds and 43.5 pounds.

For comparison with the data on powder rockets, calculated using the formulas of the approximate method and presented in Table 4, an auxiliary table is given with the latter table. The auxiliary table presents Goddard's data on other rockets: a common small rocket, a Coston ship rocket, a large steel rocket used by Goddard in his experiments, and rockets using a hydrogen-oxygen mixture as propellant instead of powder.

TYPES OF ROCKETS AND CORRESPONDING EXPERIMENTS

Goddard's compound rockets

There is no need for a rocket to continue carrying the part of its casing which housed the propellant which has already been consumed. Thus, in order to reduce the amount of fuel required during a flight, Goddard
125 proposed using a compound rocket, the unnecessary parts of which would gradually fall away as the propellant was used up. A rough example of such a compound rocket is shown in Figure 12, it being just made up of bundles of conventional ship rockets (Coston ship rockets). Goddard also gives examples of more improved types: powder rockets for which the total initial mass must be large, because of the height of the flight.

There are, under any circumstances, two possibilities: either the secondaries may be small, so that each time a secondary rocket, or group of secondaries, is discarded, the total mass is not appreciably changed, as indicated schematically in Figure 13; or a series of as large secondaries as possible may be used, Figure 14, in which case the empty casings constitute a considerable fraction of the entire weight at the time the discarding takes place.

In so far as avoiding difficulties of construction are [sic] concerned, the use of a smaller number of larger secondaries is preferable, but they should be long and narrow, as otherwise the air resistance on the nearly empty casings will be greater for the same weight of propellant than would be the case if groups of small secondaries, Figure 13, were used, in as compact an arrangement as possible. It should be explained, also, that if very small secondaries were employed, the metal of the magazines and
126 casings would become a considerable fraction of the entire weight, as the amount of surface enclosing the propellant would then be a maximum.

Possibility of employing Figure 14. A rough calculation shows at once the possibility of using a comparatively small number of large secondaries (or groups), provided, as is, of course, to be expected from dimensional considerations, that the larger any individual rocket, the less, in proportion, need be the ratio of weight of metal to weight of propellant.

Such a calculation can be made by finding the number of secondary rockets, for the case in Figure 14, that would be required for the same total initial mass, other conditions being the same, as for continuous loss of mass with zero relative velocity, which is practically the case in Figure 13.

For the latter, equation (7), in which R and g are neglected, is evidently sufficient for the purpose, for the reason that the form of the expression, so far as $(1-k)$ is concerned, is the same whether or not R and g is [sic] included.

(125)

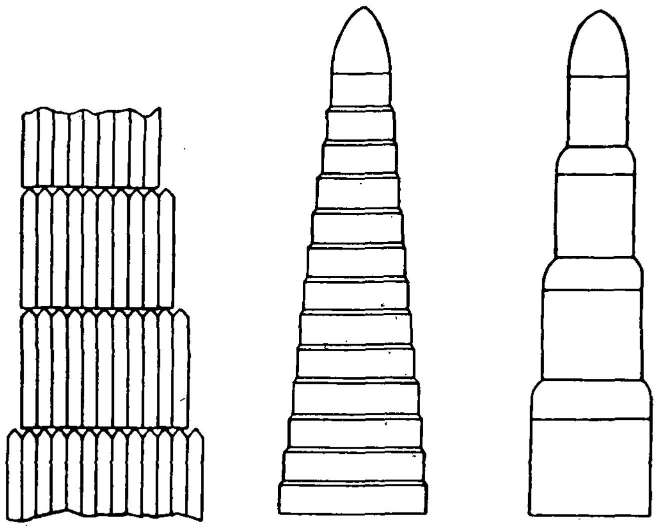

FIGURE 12. FIGURE 13. FIGURE 14.

Goddard's compound rockets.

Let us now find what conditions must hold for Figure 14, in order that the total initial mass shall equal that for Figure 13. Assume, first, that the casings are discarded successively at the end of n equal intervals of time, no mass being discarded except at these times; the velocity of gas ejection being c, as before. The total initial mass is obtained as the product of the initial masses for each interval, from equation (7) with $k=0$, assuming the final mass for each interval is, as before, 1 lb, after first multiplying the initial masses by a greater factor than unity, the excess over unity being the weight h, of the casings which are discarded at the end of the intervals.

If, in Figure 13, we divide the time into n equal intervals in the same way, we shall have, as the condition that the total initial masses are the same in the two cases,

$$M = e^{\frac{a\left(\frac{t}{n}\right)n}{c(1-k)}} = (1+h)^n e^{\frac{a\left(\frac{t}{n}\right)n}{c}}. \quad (15)$$

We obtain, then, on combining (15) with (7),

$$M^k = (1+h)^n,$$

from which

$$n = k \frac{\log M}{\log (1+h)}. \tag{16}$$

Let us assume, for Figure 13 (many small secondary rockets), as well as for Figure 14 (large secondary rockets), that the ratio of mass of metal to mass of propellant is the minimum reasonable amount that can be expected, which may be put tentatively, at least, as $^1/_{14}$ and $^1/_{18}$, respectively.

Two cases will suffice for purpose of illustration: one in which the ratio of initial to final mass is moderately large, e. g., 40, and the other in which
127 the ratio is extreme, e. g., 600.

The numbers of secondaries (or separate groups) for Figure 14, for these two cases, are, from (16), 5 and 9 respectively, n being necessarily an integer.

It is to be understood that the numbers could be made even smaller, although this would necessitate larger total initial masses.

GODDARD'S EXPERIMENTS ON ROCKET EFFICIENCIES

Between 1915 and 1918 Professor Goddard carried out a number of experiments with rockets of different types, so as to determine their efficiencies. The efficiency is here defined as the ratio of the kinetic energy of the gas ejected from the rocket to the thermal energy of the propellant. The experiments were carried out in a vacuum as well as under atmospheric pressure, and powders of various kinds were used. The velocities of gas ejection were also determined.

Types of experimental rockets. Figure 15 shows four kinds of rockets:

a) common rocket with total weight of 120 grams, 23 g of which is a powder charge;

b) large Coston ship rocket, 640 g in weight, including 130 g powder charge;

c) small steel rocket, three models of which were tested: short-nozzle type (9 cm), medium-nozzle type (14.2 cm), and long-nozzle type (19.2 cm);*

d) large steel rocket.

In rockets c and d the nozzles are conical, with a taper of 8°. The powder charge is placed ahead of the nozzle, and its length C can be varied with the aid of a bushing.

The experimental results are given in Table 5, which clearly shows the common rocket to have the lowest
129 efficiency. The efficiency of a ship rocket is a little higher, and that of a steel rocket is much higher (up to 64.53 %).

There are three factors which may influence the efficiency of a rocket in a positive way: 1) the thermodynamic properties of the propellant and the selection of a proper shape and length for the conical nozzle through which the gases are ejected, so as to convert all the energy of gas expansion into kinetic and so as to effect total burning; 2) possible lightening of the rocket by putting maximum propellant weight in a very small volume, with minimum weight of casing and rest of load; 3) use of compound [secondary] rockets, the casings of which would fall away one after the other, as the propellant in them is burned up.

* Goddard does not give the exact lengths of the nozzles in his paper, but since photos with scales indicated were given, we were able to determine the nozzle lengths approximately.

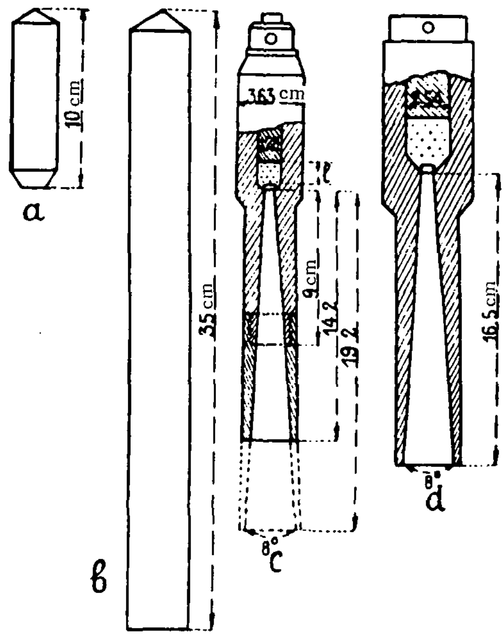

FIGURE 15. Diagrams of Goddard's rockets

USE OF PARACHUTE DURING ROCKET DESCENT

In his "Method of Reaching Extreme Altitudes," Goddard discusses the possibility of lifting recording instruments to the upper atmosphere with the aid of a rocket and indicates that it may be possible to use a parachute to effect a safe, gradual descent of the instruments to the earth.

Some means will, of course, be necessary to check the velocity of the returning instruments. It might not appear, at first sight, that a parachute would be operative at a velocity of 10,000 ft/sec or more; but it should be remembered that this velocity will occur in air of very small density, so that the pressure, or force per unit area of the parachute, would not be excessive, notwithstanding the high velocity of the apparatus . . .

If the parachute is so large that the velocity will be decreased greatly when the denser air is reached, the descent will be so slow that finding of the apparatus will not be so easy as would be the case with a more rapid descent. For this reason, part of the parachute device must be lost automatically when the apparatus has fallen into air of a certain density; or else the parachute must be small enough to facilitate a rapid descent, with additional parachute devices rendered operative as the rocket nears the ground. Such devices are not described in the present paper, but can be of simple and light construction.

(128) TABLE 5. Velocities of gas ejection from rocket (according to Goddard)

Rocket	Propellant	Efficiency	Gas-ejection velocity	
			ft/sec	m/sec
a) Experiments in atmosphere				
Common rocket	powder	1.86 %	957.6	292
Coston ship rocket	powder	2.21	1,029.25	314
Small steel rocket	Du Pont powder	44.73	6,257	1,907
–	"Infallible" powder	41.88	6,832	2,082
–	"Infallible" powder	44.78	7,064	2,154
Large steel rocket	"Infallible" powder	57.25	7,987	2,434
–	Du Pont powder	64.53	7,515	2,290
b) Experiments in a vacuum				
Small steel rocket	Du Pont powder	39.73	5,897	1,797
–	"Infallible" powder	52.93	7,680	2,340
–	–	55.90	7,893	2,405
c) Velocities using hydrogen-oxygen mixture as propellant				
	Hydrogen + oxygen (liquid or solid)		5,500–7,500	

Notes

1) Efficiency defined as ratio of kinetic energy of gases expelled from rocket to thermal energy of propellant.
2) In a vacuum velocities are somewhat higher, for same charge length and powder mass, than in atmosphere. Du Pont powder gives high velocity in vacuum for medium and short nozzles. No difference for long nozzles.
3) "Infallible" powder gives higher velocity in vacuum (up to 22 %).
4) Nozzles of medium length give higher velocities than short or long nozzles.
5) There is reason to believe that the velocities in a vacuum are actually somewhat higher than those shown in the table, due to the inaccuracy of the experiments.

The effectiveness of a parachute of even moderate size, operating in a region where the density is small, may be demonstrated by the following concrete example. Suppose that an apparatus weighing one pound and having a parachute of one square foot area descends from the altitude, 1,228,000 ft. (over 200 miles), and does not encounter any atmospheric resistance until it is level with the upper limit of s_6 (125,000 ft). This condition will not, of course, be that which would actually obtain in practice, for a continually increasing resistance will be experienced as the apparatus descends; but if a sufficient braking action can be shown to exist in the present example, the parachute device will a fortiori be satisfactory in practice.

The velocity acquired by the apparatus in falling freely under the influence of gravity between the two levels is

$$\sqrt{64\cdot 1\cdot 103{,}000} = 8400 \text{ ft/sec}.$$

Now the air resistance in poundals per square inch of section at atmospheric pressure for this velocity is, from the plot of Mallock's formula, 360 · 32 poundals per square inch, making the value of R for the area of the parachute

$$R = 1{,}653{,}000 \text{ poundals/in}^2.$$

But the actual resistance is R, multiplied by the relative density at 125,000 ft, which is approximately 0.01, giving for the resistance,

$$F = 16{,}530 \text{ poundals/in}^2$$

A retarding acceleration must therefore act upon the apparatus, of amount given by

$$a = \frac{F}{M} = \frac{16{,}530}{1} = 16{,}530 \text{ ft/sec}$$

Hence it is safe to say that, long before the apparatus had fallen to the 125,000 ft level, the velocity would have been reduced to, and maintained at, a safe value, with the employment of even a small parachute. This case, it should be noticed, is entirely different from that of a falling meteor; in that the apparatus under discussion falls from rest, at the highest point reached, whereas the meteor enters the earth's atmosphere with an enormous initial velocity.

If it is considered desirable, for any reason, to dispense with a sufficiently large parachute,* the retarding of the apparatus may be accomplished to any degree by having the rocket consist, at its highest point of flight, not merely of instruments plus parachute, but of instruments together with a chamber, and considerable propellant material. Then, after the rocket has descended to some lower level, . . . this propellant material can be ejected, so that the velocity is considerably checked before the apparatus reaches as low an altitude as, say, 5,000 ft . . . But . . . this method can hardly be as satisfactory as the parachute method; for if the "final" mass to be elevated is made a number of pounds, let us say n, the "total initial mass" (which is large even for one pound final mass) will be n fold larger, and the apparatus correspondingly more expensive.**

RECOVERY OF APPARATUS ON RETURN

A point of considerable practical importance is the question of finding the apparatus on its return, and of following it during flight, both of which depend in a large measure upon the time of flight.

Concerning the times of ascent, Table 4 shows that these are remarkably short. For example, a height of over 230 miles is reached in less than $6\,{}^1/_2$ minutes . . .

The short time of ascent and descent is, of course, highly advantageous as regards following the apparatus during ascent, and recovering it on landing.

* [Here the Russian translation varies somewhat from the original, in that it reads: " ... if the parachute descends together with the rocket..."]

** In 1926 a successful descent of an airplane, with a flight weight (including pilot and equipment) of 740 kg, was accomplished using a parachute, at the Naval Air Station in San Diego [California]. The pilot stopped the motor at an altitude of 750 m, and the craft began a descent lasting 1.5 min, at an initial speed of about 11 m/sec. A parachute attached to the airplane immediately opened, and the craft descended to the ground safely, striking it at about 6 m/sec. Although the chassis was damaged in the process, the experiment was considered to be a success.

The path can be followed, by day, by the ejection of smoke at intervals, and at night by flashes. Any distinctive feature, as for example, a long black streamer, could assist in rendering the instruments visible on the return.

PROBABILITY OF COLLISION WITH METEORS

The probability of collision with meteors of "visible" size is negligible. This can be shown by deriving an expression for the probability of collision of of a sphere with particles moving in directions at random, all having constant velocity, the expression being obtained on the assumption that the speed of the sphere is small compared with the speed of the particles.

The probable number of collisions here calculated is the sum of the probable numbers obtained by taking the velocity of the spherical body, and of the meteors, separately equal to zero.

Let v = velocity of the spherical body;

V = velocity of the meteors;

n = the number of meteors per unit volume, which number is, of course, a fraction (mutual collisions between meteors being neglected), and

131 S = the area of cross section of the spherical body.

For $v=0$, the meteors, if any, which strike the sphere during the time t to $t+dt$, will have come from a spherical shell of radii Vt and $V(t+dt)$, neglecting the diameter of the spherical body in comparison with that of the spherical shell. Further, the probable number in any small volume, in this shell, which are so directed as to strike the body, is

$$\frac{S}{4\pi V^2 t^2};$$

being the ratio of the solid angle subtended at the element, by the spherical body, to the whole solid angle, 4π. Hence the probable number of collisions, N, from all directions, between the time t_1 and t_2 is, evidently,

$$N = n \cdot S \cdot V\,(t_2 - t_1).$$

For $V=0$, an expression of the same form is obtained for the probable number of meteors within the space swept out by the spherical body.

If we accept Newton's estimate of the average distance apart of meteors as being 250 miles, we have by considering collision between very small meteors of velocity 30 miles/sec, and a sphere one foot in diameter of velocity one mile/sec, moving over a distance of 220,000 miles, the probability as $1.23 \cdot 10^{-8}$; which is, of course, practically negligible. The value would be slightly greater if the meteors were considered as having a diameter of several centimeters, rather than being particles; but the probability would be less, however, if meteor swarms were avoided.

Attention is called to the fact that, even if meteor swarms were not avoided, the probable number of collisions would be reduced if the direction of motion were substantially that of the swarm.

In general, for any values of v and V, the meteors reaching the spherical body at successive instants come from a spherical surface of increasing radius, Vt, with moving center distant vt in front of the initial position of the spherical body.

It should be explained that when v differs but little from V, the relative velocity of the body and meteors is small enough to be neglected, for meteors on this expanding spherical surface lying outside a certain cone, the vertex of which coincides with the moving center of the spherical body.

NOTE BY N.RYNIN

It is the opinion of Prof.Graff (Hamburg) that the probability of a collision between a spaceship and a meteor is very small, since the number of meteors in a unit volume of space is negligible, equivalent to one gram of mass for a volume of 100 km^3 (Scheiner-Graff: Astrophysik, 1922, S.305–306).

Moreover, in a paper entitled "Kometen und Meteore" (Stuttgart), p.68, K.Meier notes that in the Leonid shower of 1866 the meteors were separated by 110 km, even in the densest part of the shower.

WORKS ATTRIBUTED TO GODDARD

In writings in Russian, as well as in other languages, references have often been made to Goddard's studies. Without vouching for their authenticity, we now present summaries of a few of these references.

132 GODDARD'S MANNED ROCKET

Issue No. 7 of the journal "Ekho" for 1923 (5?) contained a brief description, with an illustration (Figure 16), of a plan for a manned rocket, attributed to Goddard; the rocket was ostensibly designed with a trip to Mars in mind. It was provided with a buffer at the top, so as to mitigate the shock during launching, and the passengers were to ride amidships in a freely rotating sphere; the sphere was to include a cabin and a chamber for observations.

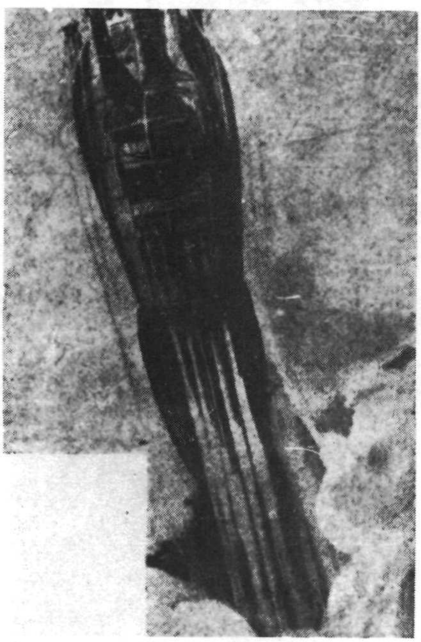

FIGURE 16. Manned rocket attributed to Goddard.

FIGURE 17. Interplanetary radio transmitter attributed to Goddard.

GODDARD'S INTERPLANETARY RADIO TRANSMITTER

In 1925 "Vestnik Znaniya" (No. 8, p. 581) contained a reference to a plan by Goddard for sending radio signals from a rocket to the ground (Figure 17).

According to this plan, the rocket is to be equipped with a radiotelegraphic transmitter, which issues signals automatically during flight. The rocket is described as a steel projectile, about 20 meters in length, divided into two parts by an insulating ring. The upper part serves as an antenna, and the lower part as a counterweight for the radio transmitter. The idea is that, when the rocket approaches the limit of the earth's atmosphere, radio signals will begin to be transmitted automatically. In this way, radio stations on the ground will be able to ascertain directly how radio waves coming in from outside are propagated.

133 Since 1926 no information on Goddard's more recent work has appeared in print. However, he is apparently (according to German technicians) continuing his work, but now for the United States War Department, for which he is constructing rocket torpedo bombs capable of bombarding London, Paris, or Berlin from America. Figure 18 shows an artist's conception of such a rocket.

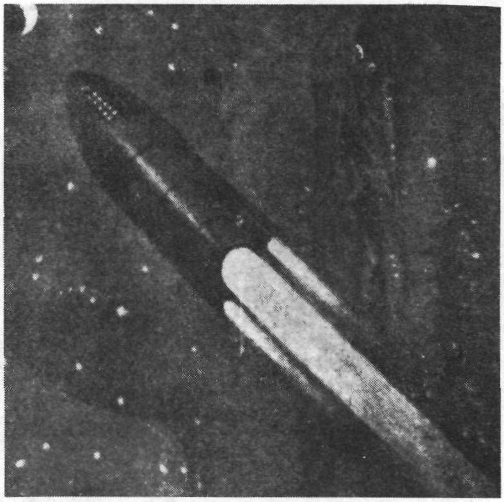

FIGURE 18. Rocket torpedo bomb attributed to Goddard.

GODDARD'S ROCKET

A model of the large compound steel rocket originally proposed by Goddard in 1919 (see Figure 19) was on display at the exhibition of interplanetary apparatus in Moscow in 1927. The propellant for this rocket was alcohol diluted with water, which, as it burned was to lift the rocket to a certain altitude. Then liquid hydrogen in the sphere of an oxygen spray in a second rocket, located in the same chassis, was to burn. After the operation of these two rockets, their casings were to fall away, leaving the forward part of the rocket to fly alone, charged by a smokeless powder (nitrocellulose). Later Goddard turned away from liquid propellants and manned rockets and used dry propellants instead.

(134)

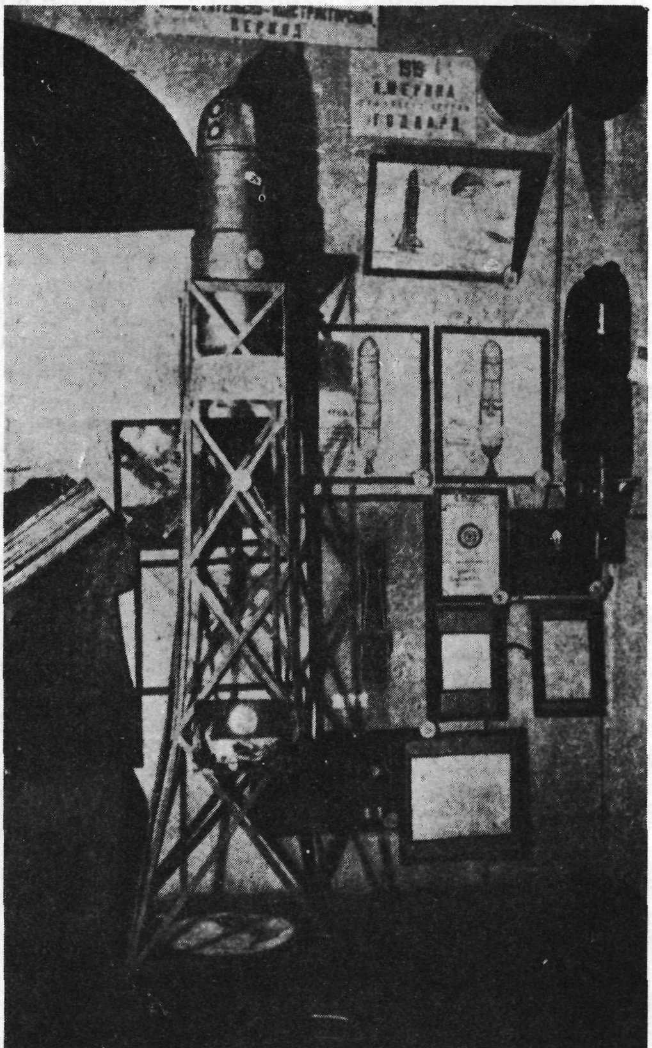

FIGURE 19. Compound passenger rocket attributed to Goddard [Moscow exhibition].

In his most recent experiments Goddard has returned to a liquid propellant (gasoline plus liquid oxygen). His test rockets are cylindrical, with conical heads and with tail groups having four fins.

"GODDARD'S SHOT" ON 17 JULY 1929

Under the above title a description of Goddard's test flight of his rocket was published in the "Bulletin of the American Interplanetary Society." In 1928 Goddard had improved his rocket nozzle and had determined by experiment the composition of a suitable mixture of liquid hydrogen and liquid oxygen. Prior to his main experiment, he carried out a number of preliminary tests at Auburn, Mass. When these produced satisfactory results, Goddard shifted his work to Worcester, where he prepared a rocket equipped with a barometer and a parachute. The experiment known as the "July 17 Shot" was also carried out there.*

Goddard constructed a steel tower 12 m (40 ft) high; rails led from the
134 base of the tower to the top, to aid in lifting the rocket. The latter was
2.74 m (9 ft) long and 0.71 m (28 in) in diameter. The rocket bursts were to take place at intervals, rather than continuously, and each burst could be heard for 3 km. The experiment was a brilliant success. Although the ascent was not high, the parachute brought the casing and the barometer to a safe landing.

The most important consequence of this experiment was financial support
135 for Goddard's work. From 1919 to 1929 the Smithsonian Institution spent
12,000 dollars on the experiments, in addition to the money spent by Goddard himself. Then, in July 1930, D. Guggenheim offered 100,000 dollars to finance the continuation of the experiments. Accordingly, construction was begun on a large rocket provided with stabilizing and descent equipment and with an instrument compartment as well. Ascents to heights from 75 to 300 km are proposed. The experiments are to be carried out near Roswell, New Mexico, where atmospheric conditions are better than at Worcester.

FUEL CONSUMPTION OF THE ROCKET AND HEIGHT OF ITS ASCENT

Professor Goddard has calculated that, under the most favorable conditions, the following amounts of powder are necessary to raise each kg of an empty rocket to the heights indicated:

Powder (kg)	Height of ascent (km)
12.5	55
89	368
167.7	693
802	Beyond range of gravity

* [Actually, the July 17 flight was also made at Auburn (Translator).]

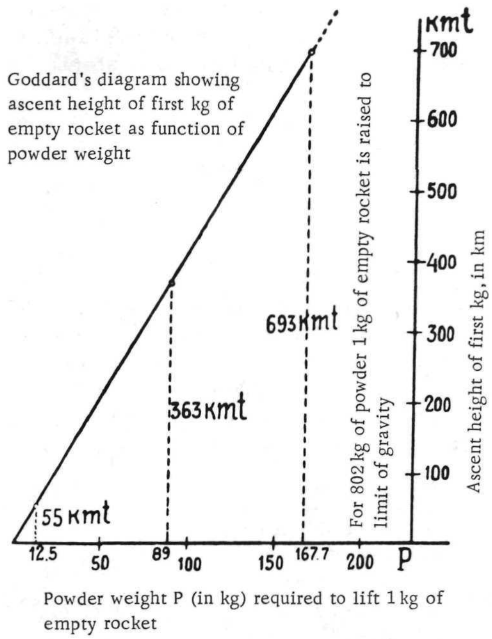

FIGURE 20.

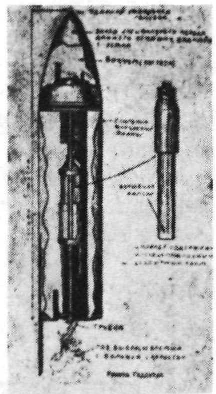

FIGURE 21. Moon-flight rocket attributed to Goddard.

This relationship is expressed graphically in Figure 20. For a velocity of free flight equal to 12,000 m/sec, a velocity of gas ejection of 2,000 m/sec
136 will be $^1/_6$ as great, while a velocity of gas ejection of 1,800 m/sec will be nearly $^1/_7$ as great. Consequently, the initial mass must be either $e^6 = 403.4$ or $e^7 = 1096.5$ times the final mass. Goddard assumes a velocity of gas ejection of 1,900 m/sec and obtains an initial mass 802 times greater than the final mass.

GODDARD'S MOON-FLIGHT ROCKET

One of the newspapers printed a picture of a rocket, supposedly designed by Goddard for a flight to the moon. Upon hitting the moon, this rocket was to produce an explosion visible from the earth. Figure 21 shows a small diagram of this rocket, with an explanation [in Russian].

GODDARD'S PATENTS FOR NEW TYPES OF ROCKETS

Along with his theoretical and experimental studies, Goddard was also responsible for a number of inventions connected with the improvement of ordinary rockets, and he took out several patents on these. Drawings and brief descriptions of these patented rockets are given below, taken by us from American patent publications (United States Letters Patent).

Goddard's compound rocket

(Patent 1102653, 1 Oct. 1913)

The rocket as a whole consists of two parts: a large lower rocket and a small upper rocket (Figures 22 and 26). Each of these has a combustion chamber with propellant and a conical nozzle, the length of which is at least three times its diameter. At its top the lower rocket has a tube into which the upper one is inserted, and, when the lower rocket stops burning, the upper one separates from this tube. In order to stabilize flight, the rocket is made to rotate by means of bursts in curved horizontal ducts located at the head of the rocket.

Goddard's revolver rocket

(Patent 1103303, 15 May 1914)

Rocket bursts are brought about via a successive downward feeding of cartridges to the nozzle. The spent cartridges are removed to a special chamber inside the rocket (Figure 23).

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FIGURE 22.

FIGURE 23.

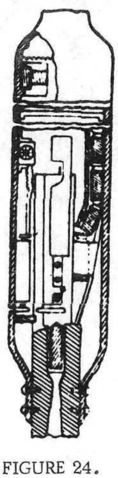

FIGURE 24.

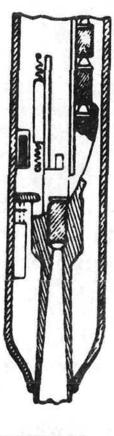

FIGURE 25.

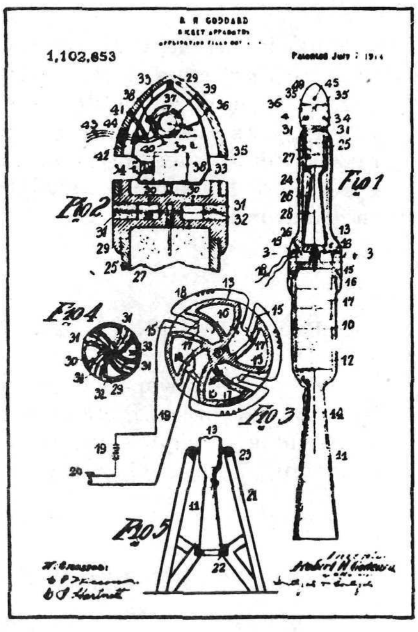

FIGURE 26.

Goddard's rockets.

Goddard's revolver rocket

(Patent 1191299, 8 Nov. 1915)

Rocket bursts are brought about via a successive downward feeding of cartridges to the nozzle. By means of a special mechanism, spent cartridges are removed from the combustion chamber and ejected outward through a special opening (Figure 24).

Goddard's revolver rocket

(Patent 1194496, 23 Dec. 1915)

Cartridges are fed automatically to the nozzle along the rocket, their removal, and opening and closing of the chamber where the cartridges explode, being effected by a spring mechanism. Used cartridges are ejected outward (Figure 25).

Appendix

A PROBLEM IN THE CALCULUS OF VARIATIONS RELATED TO ROCKET THEORY

Georg Hamel (Prof. Math., Berlin-Charlottenburg
Institute of Technology, Germany)

Let us consider a rigid body acted upon by the earth's gravity and the
138 air resistance W. The body, which has an instantaneous (in general variable) mass M, is lifted vertically by the reaction force of a stream of gas (that is, by a rocket). According to Newton's law and the law of conservation of mass, we can write the following differential equation:

$$M\frac{du}{dt}+C\frac{dM}{dt}+W(s,u)+Mg=0 \qquad (1)$$

where   is the path traversed, t is the time, $u=ds/dt$ is the flight velocity, and C is the relative velocity of the exhaust.

In the subsequent calculations we introduce the following simplifications: 1) since the ascent is only to 100 or 200 km, we assume that g is constant; 2) in calculating the air resistance W, the effect of the exhaust stream is neglected; 3) momentum changes inside the rocket, due to variation of the combustion surface (in the case of powder, etc.), are neglected, as well as other similar minute phenomena; and 4) the earth's rotation is neglected.

We introduce the following notation: M_e is the final mass (that is, the mass of the empty rocket), M_a is the initial mass, and $q=(M_a-M_e)/M_e$ is the mass ratio. Our goal will be to determine the minimum value of M_a, that is, the minimum amount of fuel (and thus minimum q), for certain previously specified conditions: M_e, total height of ascent h, initial velocity u_a for $t_a=0, s_a=0$, and C constant (constant relative exhaust velocity).*

This problem was posed by the American Professor R. H. Goddard,** who also made an attempt at solving it, but using a method that is, mathematically, very questionable. Here this problem will be solved using the methods of the calculus of variations.

Taking equation (1) to be a linear differential equation in M, we can integrate it and solve for M_a by substituting in the final (in the sense of the end of the ascent process) values. This gives

$$M_a=e^{-\frac{u_a}{C}}\int_0^{t_e}\frac{1}{C}W(s,u)\cdot e^{\frac{u}{C}+\frac{gt}{C}}\,dt+M_e e^{-\frac{u_a}{C}+\frac{u_e}{C}+\frac{gt_e}{C}}. \qquad (2)$$

* The condition that C be constant was first demonstrated by Tsiolkovskii; it is a fundamental condition.

** A Method of Reaching Extreme Altitudes. Washington, USA. 1919. Publication of the Smithsonian Institution

The values t_e, u_e, and s_e pertain to the end of burning (that is, to the beginning of flight by inertia), and it is clear that $s_e < h$, where h is the total height of ascent. Since I am considering the problem for a rocket alone (without a catapult, so that the initial velocity is zero), we can write $u_a = 0$. However, a consequence of this condition is the absence of a real minimum; there is only a lower limit (boundary), which can be approached to any degree,
139 and it is sufficient to consider that the velocity u increases rapidly from $u_a = 0$ (at $t_a = 0$) to the specified value. In the following, the "minimum" will be understood to refer to this lower limit, which is the actual minimum.

Hence our problem can be stated as

$$M_a = \int_0^{t_e} f(u \cdot s,\ t)\ dt + F(u_e t_e) = \text{min.} \tag{3}$$

with the values s_e and t_e free ($u = ds/dt$). There will then be a relation between u_e and s_e. With the consumption of all the propellant ($M = M_e$), the rocket will ascend due to the kinetic energy developed. This free (motorless) ascent should be used, and it can be expressed by the equation

$$M_e \cdot \frac{du}{dt} + W(s,\ u) + M_e\, g = 0, \tag{4}$$

which is identical to the condition $f + (dF/dt) = 0$. This equation has the form $u\ (du/ds) = f_1\ (u,\ s)$, and it is integrable when the final values $u = 0$ and $s = h$ are substituted in

$$u = \psi\ (s). \tag{5}$$

Because of the low value of the air resistance at great heights (for high h and s), it will be approximately true that

$$u = \sqrt{2g(h - s)}. \tag{6}$$

Consequently, we must still take into account the boundary condition $u_e = \psi\ (s_e)$. This expression can be substituted directly into function F, so that the additive term will have the form $F\,[\psi\ (s_e),\ t_e]$.

However, while seeking a minimum value for M_a, we have no right to substitute the boundary condition into the integral, since it is known that, for the limiting values of the function of the boundary condition $\psi(s_e)$, it is always possible to substitute another value of u_e; the value of the integral can vary from it to any small degree.

Therefore, the integral in equation (3) should become a minimum for certain values of s_e and t_e.

This is an ordinary problem in the calculus of variations, corresponding to Euler's equation and giving regular limiting values without a conjugate point. Thus we have, if the conditions

$$W > 0;\ (\partial W/\partial u) > 0;\ (\partial^2 W/\partial u^2) > 0 \tag{7}$$

are always satisfied, the inequality

$$(d^2f/du^2) > 0, \tag{8}$$

where an equals sign can be used only for $u=0$. Assuming still that
140 $(dW/ds)<0$, we can obtain results which are more and more general.

Consequently, in itself the integral has a marked minimum. Difficulties arise only for variations of s_e and t_e or of s_e and u_e. In this case the following results are obtained:

1) only a single stationary point exists, at $s_e=s_0$, $u_e=u_0$ for which condition $(\partial M_a/\partial t_e)=0$ and $(\partial M_a/\partial s_e)=0$ is satisfied. This point lies on the $u=\psi(s)$ curve, so that it is a final (end) point (condition), in the sense of a Courant natural point.

2) For variation of s_e, u_e along the $u=\psi(s)$ curve, point s_0, u_0 corresponds to a real minimum.

3) For any variation the discriminant of the second-order terms

$$\left[\frac{\partial^2 M}{\partial s^2_e}\cdot\frac{\partial^2 M}{\partial t^2_e}-\left(\frac{\partial^2 M}{\partial s_e\cdot\partial t_e}\right)^2\right]_0=0, \tag{9}$$

so it is possible that pointed regions, in which M_a reaches values lower than at point s_0, u_0, may be as low as the values δs_0, δt_0.

4) However, such a pointed region can reach the $u=\psi(s)$ curve only from outside. But these outer points do not have a physical meaning, like the final values s_e, u_e, since in this case the retardation of the speed of the rocket should occur via a sudden increase in its mass, which is physically impossible. A minimum is guaranteed mathematically by the inequality

$$\frac{dM}{dt}<0. \tag{10}$$

Consequently, a true minimum does exist. For the numerical calculations, we used the following formula for the air resistance:

$$W=C\cdot\delta_0\cdot e^{-\frac{s}{f}}\,U^2 \tag{11}$$

where δ_0 is the air density at sea level and $f=6.666$ km, that is, at a height of 6.666 km the air density is $e=2.71$ times the density at the ground (sea level). Calculations show that $u_0\,(=u_e)$ depends only slightly on C, for C = 1,000 and 2,000 m/sec and, for the corresponding possible values of $C\delta_0$ and M_e, it depends only slightly on these quantities as well. Here, for a total ascent height $h=100$ km, we have u_0 $(=u_e)=$ 1,000 to 1,100 m/sec and $s_0=0.5\ h$. The actual minimum $(M_a)_{min}$ and also lim u, were not calculated. During the calculations Dipl. Eng. Rossmann, assistant to Prof. Cranz, was kind enough to help me, and it was on his suggestion that the rocket problem was dealt with in a report to the mechanics seminar at the Berlin-Charlottenburg Institute of Technology. All the equations proved to be easily integrable.

Note. This paper was published in German in: "Zeitschrift für angewandte Mathematik und Mechanik," Vol.7, Book 6, Nov.–Dec.1927, pp.451–452.

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HERMANN OBERTH

SOME INFORMATION ABOUT OBERTH

Hermann Oberth (Figure 27) was born on 25 June 1894 in Hermannstadt, Transylvania. He completed the gymnasium in 1912 in Schässburg, and for two semesters he studied medicine in Munich. Later Oberth studied physics and astronomy at Klausenburg, Munich, Göttingen, and Heidelberg. During the World War (1914–1915) he served in the infantry and later in the medical corps. In 1923 a study by Oberth entitled "The Rocket Into Interplanetary Space" was first published. At present (since 1925) H. Oberth is a teacher in the town of Medias (Mediasch) in Rumania.

FIGURE 27. H.Oberth.

A second edition of the above-mentioned work (Hermann Oberth, "Die Rakete zu den Planetenräumen") was published in 1925, and our version of it will be given below.

In his work, Oberth presents quite complex mathematical arguments to prove that, given the present state of technology, it is possible to leave the earth with the aid of a rocket. The rocket would later either fall back to earth, begin to revolve about the earth as a satellite, or go off into interplanetary space. In the first case the rocket could be used to study the upper atmosphere, by placing recording instruments aboard it. In the second case, in which the rocket revolves around the earth, it could serve as a station for other rockets passing between it and the earth. Such a rocket could reflect solar radiation
142
to the earth, melt the ice in polar regions, and increase the amount of arable land in the world. However, the author himself admits that the latter plan is still in the realm of fantasy and will only be possible in the distant future. Now, however, Oberth suggests constructing a rocket of the first type and launching it without passengers, although he gives drawings of another rocket to be used for manned flight as well.

A basic feature of Oberth's rocket is that it consists of two, and in some cases even three, individual rockets. As the propellant burns, the individual rockets fall away one by one, the lower one first and then the middle one. During the return descent, the upper rocket also separates, and only the nose of this rocket remains. This nose section includes the parachutes, and the instruments or passenger compartment, which are to descend to the earth. In spite of the fantastic nature, and even the lack of basis, of many of Oberth's suggestions (for example, the use of stabilizers during flight in the vacuum of space, and the advisability of using a parachute), it must still be admitted that his approach to the solution of the problem of rocket flight is of great significance, since it is based on mathematical analysis and physicomechanical laws. Therefore, we present below Oberth's main calculations, together with a description and some drawings of his rocket. Finally, in conclusion, we also quote the author's hopes concerning the possible application of his rocket.*

In 1929 in Berlin the UFA motion-picture company produced a film entitled "The Girl in The Moon" ["Frau im Mond"], the subject of which was a manned rocket flight to the moon and back. Oberth helped work out the technical aspects of this film, by giving advice on rocket construction. Figure 28 shows the rocket ready for launching. In Figure 29 a model of

(143)

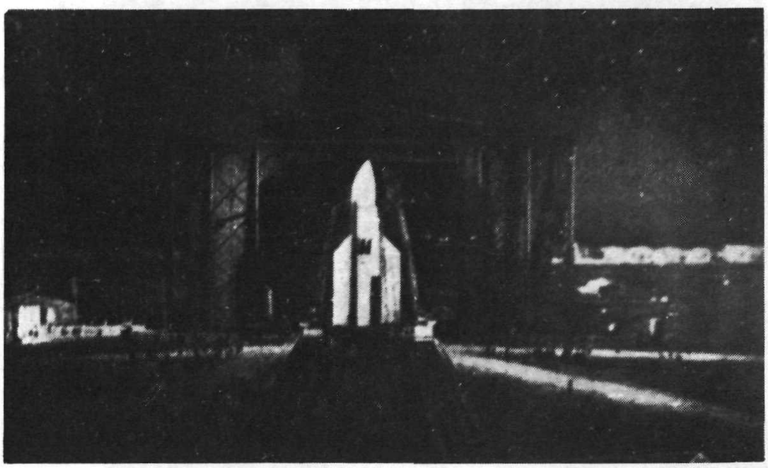

FIGURE 28. Oberth's rocket at launching (from "The Girl in the Moon").

the inside of this rocket is shown, and in Figure 30 the control panel and conditions aboard the rocket during launching are shown. Figure 31 shows a future rocket flight with observers aboard (from "Die Umschau").

After the production of this film in May 1929, the company drew up a contract with Oberth to aid him in his studies of rocket construction. Oberth began carrying out experiments, with the assistance of an engineer named Nebel. At first he tried various kinds of liquid propellants: C_7H_8, C_8H_{18}, and gasoline + O_2; CH_4 + O_2, and gasoline + N_2O_4 Mixtures of gasoline and O_2 and

* The well-known German balloonist, August von Parseval, believes that the flight of a rocket to the moon or to Mars, which in Oberth's opinion will not take place soon, may actually be realized earlier, since technology is developing at a rapid pace.

$CH_4 + O_2$ turned out to be preferable. The experiments were carried out on a small island in Haffe (Greifswalder Oie, near Stettin). The rocket weighed 9.8 kg empty and the propellant weighed about 10 kg.

(143)

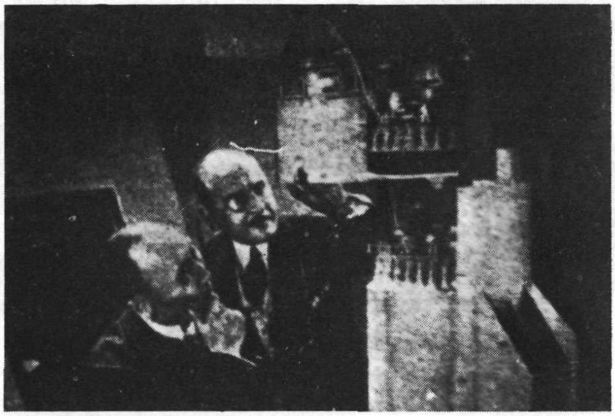

FIGURE 29. Internal construction of Oberth's rocket (from "The Girl in the Moon").

In July 1929 near Berlin the construction of two Oberth liquid-propellant rockets was begun: one of these was wingless and 1.5 m long, and the other had wings and was 1.9 m long. The work was carried out under the direction of Oberth, by A. B. Shershevskii, and a group of young engineers. In October a reaction engine was already op-
143 erating on gasoline and liquefied
144) oxygen (volume ratio 1:3.1). The fuel was supplied at a pressure of 5 to 10 atm, using centrifugal force pumps. Also in October, work was begun on construction of a meteorological rocket, using
144 liquefied methane (CH_4) as a fuel; the latter has a specific weight of 0.46 (for a temperature of −160°C in the liquid state), and was stored in Dewar flasks. An endogenic oxygen compound which was liquid at room temperature was poured into a container along with liquid methane, and the reaction engine operated well. The rockets were made of Elektron. However, late in 1929 Oberth was compelled to discontinue his work, due to a shortage of funds.

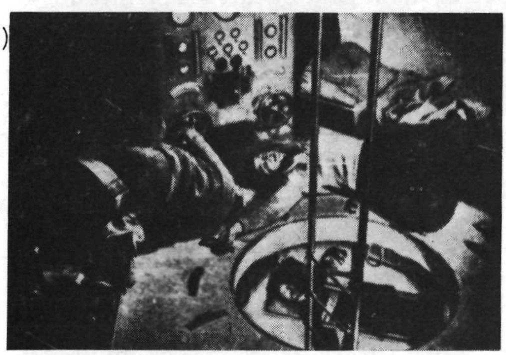

FIGURE 30. Control compartment of Oberth's rocket (from "The Girl in the Moon")

On 21 December 1929 Oberth left for Rumania (Mediasch, or Medias, Hermannstadterstrasse, 9) after having a serious disagreement with the UFA

Company over the contract. UFA had spent, together with a donation, about 27,000 marks on the research and construction work.

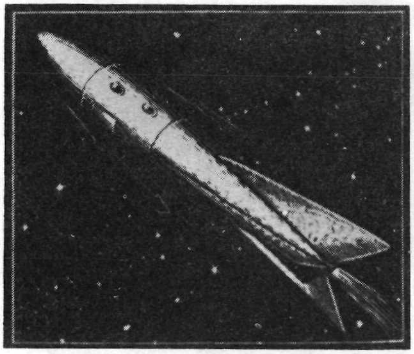

FIGURE 31. Flight of rocket attributed to Oberth.

The contract specified that UFA would, for 100—1 = 99 years, receive $33\frac{1}{2}\%$ of the net profit for all rocket projectiles which would be constructed by Oberth or his representative. Since Oberth was bound by this contract and was required to pay these sums whereever the rockets might be built, he decided to return no sooner than 1 April and to start legal proceedings against UFA to dispute the contract. The work had continued until 20 December 1929. At that time an almost completed rocket lay in the construction department of the Elektron Werke S. G. Farbenindustrie at Bitterfeld. The rocket was 70 to 80% finished. Completed were: the casing, the reaction engine with control panel, the fuel injectors, and the nozzle. Oberth's immediate co-workers were: A. B. Shershevskii, Dipl. Ing. Rudolf Nebel, Dipl. Ing. Max Langgut, and a design engineer named Alfred Krontz.

145 On 20 July 1929 the work had reached a standstill, and Nebel had conducted negotiations with Alfred Frommherr (Berlin), a representative of the Magdeburger Werkzeug und Maschinen Aktien Gesellschaft (MAG) concerning continuation of the work. MAG agreed to provide 20,000 marks to complete the first rocket, but only with very stringent conditions in the contract, namely that MAG would receive a considerable part of the net profits for a long period (about 50 years) and also that the contract with UFA would be broken.

A further condition of the contract required Oberth to plan and demonstrate to Lauff, the director of MAG, a flight of the first rocket to a height of 50 km. For this Oberth required approximately another 4,000 marks. At first he preferred to complete the construction in Mediasch, but later he decided to carry on his work at the Zurich branch of MAG (Switzerland).

Figure 32 shows a general view of a rocket for studying the stratosphere, constructed by Nebel, in cooperation with Professor Oberth, in the town of Tegel. This rocket was 2 m long, and was provided with a recording instrument as well as a parachute, with the aid of which it could make a gradual descent to the earth. The stand for the launching of the rocket is shown in Figure 33, and Figure 34 gives a representation of the proposed parachute descent. This descent will take about an hour. Since parachutes are often carried long distances by the wind, making it difficult to find them, Oberth's rocket is to be provided with a flashing red light, to facilitate observation of the rocket descent.

In 1929 Oberth published a third edition of his book, under the title "Wege zur Raumschiffahrt." In this completely revised work he analyzes three types of problems: physical and engineering problems, structural problems, and problems of rocket applications. Since this entire study has been translated by us under the direction of Gostekhizdat, who will publish it in the near future, we will not discuss its contents here.

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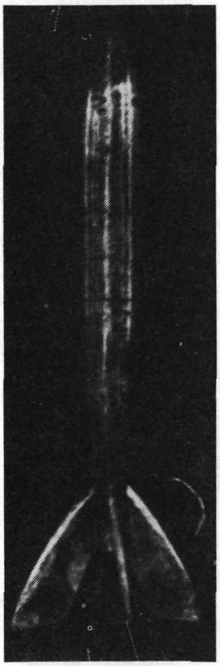

FIGURE 32. Oberth's Mirak rocket.

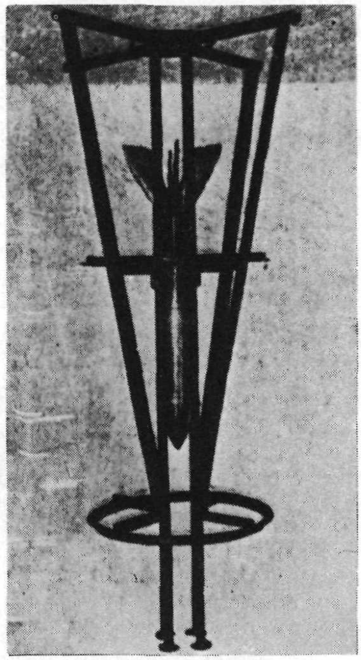

FIGURE 33. Stand for launching of Oberth's rocket.

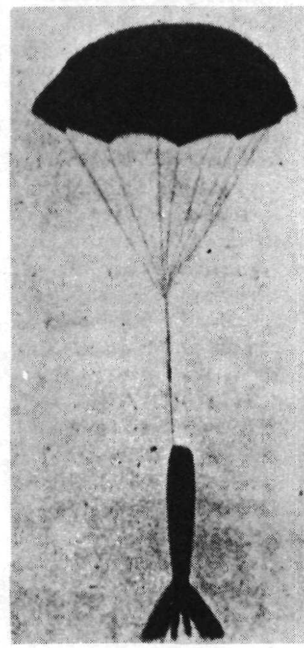

FIGURE 34. Descent of Oberth's rocket by parachute.

In 1929 the latter work won for Oberth the first REP-Hirsch prize, which had been set up in France for the Astronomical Society there. The main reason for awarding the price to Oberth was that he had succeeded in increasing the velocity of gas ejection from a rocket to 4,000 m/sec, by increasing the amount of hydrogen used in the hydrogen-oxygen mixture. Consequently, "only" 24 tons of propellant were needed for each ton of useful load, in order to escape into outer space.

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WORKS OF HERMANN OBERTH

Part I. Theory of Rocket Flight

FUNDAMENTAL EQUATION OF MOTION AND OPTIMUM ROCKET VELOCITY

A longitudinal section of a rocket is shown in Figure 35. When an explosion [burst] occurs inside of it, gases are ejected through the nozzle at the bottom, and the recoil pushes the rocket upward. The following symbols will be used: P is the recoil, dt is the duration of the burst, c is the velocity of gas ejection, and dm is the mass ejected from the rocket.

From the law of momentum, we have

$$P \cdot dt = -c \cdot dm. \tag{1}$$

The fuel consumption over a period of time is found from equation (1) by integration:

$$m_0 - m_1 = \frac{1}{c}\int_{t_0}^{t_1} P\,dt.$$

Now let us introduce the following notation: L is the air resistance to flight of the rocket, G is the rocket weight (force of gravity), $Q = L + G$, v is the flight velocity at a given moment, b is the acceleration $\frac{dv}{dt}$, $R = P - Q$ is the force imparted to the rocket by acceleration b, and m is the mass of the rocket.

Now we can write $R = m \cdot b = m\,\frac{dv}{dt}$. From (1) we have

$$Rdt + Qdt = -c \cdot dm$$

or

$$mdv + Qdt + cdm = 0 \tag{2}$$

148 This is the fundamental differential equation of motion, connecting the mass, velocity, time, fuel consumption, and resistance.

Let us consider the motion of a rocket within the earth's atmosphere. We wish to determine the velocity of motion for which: 1) the momentum mdv determining the flight stays at a specified value, and 2) the fuel consumption dm is a minimum. This velocity will be called the optimum velocity.

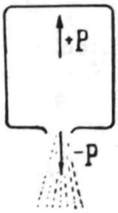

FIGURE 35.

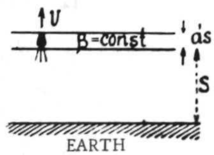

FIGURE 36.

The rocket, located at a height s above the earth (Figure 36), traverses an air layer ds, which is so thin that during the traversal: 1) the air density does not change, 2) the rocket mass m also does not change,* and 3) the momentum increases constantly by an amount mdv.

Then the time required to traverse the layer will be $dt=\frac{ds}{v}$, and from equation (2) we have

$$\frac{mdv}{ds}+\frac{Q}{v}+c\frac{dm}{ds}=0. \qquad (2a)$$

Quantities mdv and ds are assumed to be constant. Differentiating with respect to v gives

$$\frac{\partial\left(\frac{Q}{v}\right)}{\partial v}+\frac{\partial c}{\partial v}\cdot\frac{dm}{ds}+c\,\frac{\partial\left(\frac{dm}{ds}\right)}{dv}=0. \qquad (3)$$

Oberth assumes that the velocity of gas ejection c of his rocket is constant. Thus the second term in (3) goes to zero. The condition that the fuel consumption dm be a minimum gives

$$\frac{\partial\frac{(dm)}{ds}}{\partial v}=\frac{1}{ds}\cdot\frac{\partial\,(dm)}{\partial v}=0,$$

so that equation (3) becomes

$$\frac{\partial\left(\frac{Q}{v}\right)}{\partial v}=0. \qquad (4)$$

However, $Q=L+G$, where G is the force of gravity mg, g being the acceleration of gravity at a height s. For the layer ds we assume constant g, so that the air resistance L is

$$L=F\cdot\beta\cdot\gamma\cdot v^2,$$

where F is the area of the midship section of the rocket, β is the air density,
149 and γ is the air resistance, a function of the rocket shape and the velocity v.

Substitution of these quantities into the expression for Q, gives

$$\frac{Q}{v}=F\beta\cdot\gamma\cdot v+\frac{mg}{v} \qquad (4a)$$

* Here the author makes a contradictory statement. The mass m will not remain constant, but rather will decrease.

and, after substituting into (4) and differentiating, we have

$$\frac{\partial\left(\frac{Q}{v}\right)}{\partial v}=\frac{-mg}{v^2}+F\beta\left(\gamma+v\frac{d\gamma}{dv}\right).$$

When this expression is equal to zero, we obtain the optimum velocity, as determined by the condition

$$\bar{v}^2=\frac{mg}{F\cdot\beta\left(v\cdot\frac{d\gamma}{dv}+\gamma\right)}. \tag{5}$$

Everywhere in the following this optimum velocity will be assumed, and this simplifies all the calculations considerably.

From (5) we have

$$m=\frac{F\cdot\beta}{g}\cdot\bar{v}^2\left(\bar{v}\frac{d\gamma}{dv}+\gamma\right). \tag{5a}$$

Here all the variables are functions of just one independent variable, v. Differentiation gives

$$dm=\frac{F\cdot\beta}{g}\bar{v}^2\cdot\left(\bar{v}\frac{d\gamma}{dv}+\gamma\right)\cdot\left[\frac{d\beta}{\beta}-\frac{dg}{g}+\frac{2d\bar{v}}{v}+\frac{\bar{v}\frac{d^2\gamma}{dv^2}+2\frac{d\gamma}{dv}}{v\cdot\frac{d\gamma}{dv}+\gamma}\,dv\right]. \tag{5b}$$

If the quantity

$$\frac{\bar{v}\frac{d^2\gamma}{dv^2}+2\frac{d\gamma}{dv}}{\bar{v}\frac{d\gamma}{dv}+\gamma}$$

is called z, then from (4a) and (5a) we have

$$Q=F\beta\bar{v}^2\gamma+F\beta\bar{v}^2\left(\bar{v}\frac{d\gamma}{d\bar{v}}+\gamma\right)$$

or

$$Q=F\beta\bar{v}^2\left(\bar{v}\frac{d\gamma}{d\bar{v}}+2\gamma\right). \tag{5c}$$

Dividing (5c) by (5a), we obtain

$$\frac{Q}{mg}=\frac{\bar{v}\frac{d\gamma}{dv}+2\gamma}{\bar{v}\frac{d\gamma}{d\bar{v}}+\gamma}. \tag{5d}$$

This quantity will be called y.

RELATION BETWEEN FLIGHT TIME, MASS, FORCE, DISTANCE, AIR RESISTANCE, AND OPTIMUM FLIGHT VELOCITY

Equation (2), or its other form (2a), relates the mass, flight time, force, distance, fuel consumption, and velocity of a rocket flight. Now let us rewrite this equation, in terms of the optimum flight velocity $(\bar{v})$ rather than the velocity v.

150 When all the terms of equation (2a) are multiplied by $\frac{ds}{m \cdot c}$, we obtain

$$\frac{d\bar{v}}{c} + \frac{Qdt}{m \cdot c} + \frac{dm}{m} = 0. \qquad (6)$$

However, from (5a) and (5b) we have

$$\frac{dm}{m} = \frac{d\beta}{\beta} - \frac{dg}{g} + \frac{2 \cdot d\bar{v}}{v} + z \cdot d\bar{v}.$$

Moreover, from (5d),

$$\frac{Q}{mg} = y; \quad \frac{Q}{m \cdot c} dt = \frac{g}{c} ydt,$$

so that (6) can be rewritten as

$$\frac{d\bar{v}}{c} + \frac{g}{c} \cdot ydt + \frac{d\beta}{\beta} - \frac{dg}{g} + \frac{2d\bar{v}}{\bar{v}} + z \cdot d\bar{v} = 0. \qquad (6a)$$

Next Oberth expresses all the variables of equation (6a) in terms of $\bar{v}$ and t.

The acceleration of gravity g is inversely proportional to the square of the distance from the earth's center. If r is the radius of the earth and s is the flight altitude, then

$$g = 9.81 \cdot \frac{r^2}{(r+s)^2} \text{ m/sec}^2$$

and

$$dg = \frac{-9.81 \cdot 2 r^2 ds}{(r+r)^3} \cdot \frac{-dg}{g} = \frac{2ds}{r+s} = \frac{2vdt}{r}.$$

In the second term of formula (6a), as a first approximation, let us take a value of 9.7 m/sec^2 for g (the average value for $s_0 = 5$ km and $s_1 = 50$ km). This term then becomes

$$\frac{g}{c} ydt = \frac{9.7}{c} ydt.$$

In order to determine the third term of (6a), for convenience Oberth integrates the approximate expression for the air density as a function of height:

$$\beta=\beta_0 e^{-\frac{s-s_0}{H^1}},$$

where e is the base of natural logarithms, and H^1 is a constant.

Differentiation with respect to s, gives

$$\frac{d\beta}{ds}=\beta_0\cdot e^{-\frac{s-s_0}{H^1}}\cdot\left(-\frac{1}{H^1}\right)$$

and thus

$$\frac{d\beta}{\beta}=-\frac{ds}{H^1}=-\frac{\bar{v}dt}{H^1}.$$

Substitution of these expressions into (6a) now gives

$$\frac{d\bar{v}}{c}+\frac{(9.7\,\mathrm{m/sec^2})}{c}\,y dt-\frac{\bar{v}}{H^1}dt+\frac{2\bar{v}dt}{r}+\frac{2}{\bar{v}}dt+z\cdot d\bar{v}=0\,.$$

If we designate

$$\frac{\bar{v}}{H^1}-\frac{2\bar{v}}{r}=\left(\frac{1}{H^1}-\frac{2}{r}\right)\bar{v}$$

151 as $\frac{v}{H}$. then we obtain

$$\frac{dt}{d\bar{v}}=\frac{\frac{1}{c}+\frac{2}{\bar{v}}+z}{\frac{\bar{v}}{H}-\frac{g}{c}y}. \tag{7}$$

Flight time. If we have a velocity $\bar{v}$ greater than 460 m/sec, ballistic experiments indicate that the air resistance (γ) can be assumed constant. Then, from (5d), we obtain

$$y=2;\quad Q=2mg=L+G=L+mg,$$

giving $L=G$; so that equation (7) can be written as

$$\frac{dt}{d\bar{v}}=\frac{\frac{1}{c}+\frac{2}{\bar{v}}}{\frac{\bar{v}}{H}-\frac{2g}{c}}=\frac{H}{c}\cdot\frac{\bar{v}+2c}{\bar{v}\left(\bar{v}-\frac{2gH}{c}\right)}. \tag{7a}$$

Integration then gives

$$(t-t_0)=\left(\frac{c}{g}+\frac{H}{c}\right)\ln\frac{\bar{v}-\frac{2gH}{c}}{\bar{v}_0-\frac{2gH}{c}}-\frac{c}{g}\ln\frac{\bar{v}}{\bar{v}_0}.$$

The acceleration b is found from (7a) to be

$$b=\frac{d\bar{v}}{dt}=\frac{c}{H}\frac{\bar{v}\left(\bar{v}-\frac{2gH}{c}\right)}{\bar{v}+2c}=\frac{\bar{v}\,(\bar{v}c-2gH)}{H\,(\bar{v}+2c)}. \qquad (7c)$$

Determination of mass. If we substitute into (6) the quantity

$$\frac{Qdt}{m\cdot c}=\frac{yg}{c}\,dt$$

and assume, as previously, that $y=2$, then we obtain

$$\frac{d\bar{v}}{c}+2\frac{g}{c}\cdot dt+\frac{dm}{m}=0 \qquad (8)$$

and, after integration

$$\ln\frac{m_0}{m}=\left[\frac{1}{c}\,\bar{v}-\bar{v}_0+2g\,(t-t_0)\,.\right] \qquad (8a)$$

The recoil force P is found from equation (1):

$$Pdt=-c\,dm. \qquad (1)$$

However, from (8) we have

$$dm=-m\left(\frac{d\bar{v}}{c}+\frac{yg}{c}\,dt\right) \qquad (8)$$

so that

$$Pdt=m\,(d\bar{v}+ygdt);$$

or

$$P=m\left(\frac{d\bar{v}}{dt}+yg\right)=mb+ymg;$$

and for $v>460$ m/sec

$$P=m\cdot(b+2g). \qquad (9)$$

152 The height of ascent $ds=\bar{v}\cdot dt$, taking (7a) into account, will be

$$ds=\frac{H}{c}\cdot\frac{\bar{v}+2c}{\bar{v}-\frac{2gH}{c}}\cdot d\bar{v} \qquad (10)$$

for $\bar{v}>460$ m/sec.

Integration then gives

$$s-s_0=\frac{H}{c}(\bar{v}-\bar{v}_0)+2H\left(1+g\frac{H}{c^2}\right)\ln\frac{\bar{v}-\frac{2gH}{c}}{\bar{v}_0-\frac{2gH}{c}}. \qquad (10a)$$

Sample calculation. We assume that $H=6{,}300$m, $v_0=500$ m/sec, $v_1=11{,}000$ m/sec, $c=3{,}000$m/sec, and $g=9.7$ m/sec^2. In this case the terms in formula (10a) will have the following values:

$$\frac{2gH}{c}=40.74\text{m/sec};\ \frac{2gH}{c^2}=0.01358;$$

$$\ln\bar{n}=2.3026\ \lg\ =2.3026\cdot 1.37721=3.17233;$$

$$2+\frac{2gH}{c^2}\ln\bar{n}=2.01358\cdot 3.17233=6.37882;$$

$$\frac{\bar{v}_1-\bar{v}_0}{c}=3.5000;\quad \frac{s_1-s_0}{H}=9.76822;$$

$$s_1-s_0=6300\cdot 9.87822=62232.8:$$

The fuel consumption is found from (8a):

$$\log\frac{m_0}{m_1}=\frac{1}{c}\left[(\bar{v}-\bar{v}_0)\,0.4343+2g(t_1-t_0)\,0.4343\right]$$

where $0.4343=\frac{1}{2.3026}$ is the modulus of the logarithms.

The time interval t_1-t_0 is found from (7b):

$$(t_1-t_0)\,0.4343=\frac{c}{g}\lg\frac{\bar{v}_1-\frac{2gH}{c}}{\bar{v}_0-\frac{2gH}{c}}\cdot\frac{\bar{v}_0}{\bar{v}_1}+\frac{H}{c}\lg\frac{\bar{v}_1-\frac{2gH}{c}}{\bar{v}_0-\frac{2gH}{c}};$$

however,

$$\frac{c}{g}=309.28\text{sec}$$

$$\log\frac{\bar{v}_1-\frac{2gH}{c}}{\bar{v}_0-\frac{2gH}{c}}\cdot\frac{\bar{v}_0}{\bar{v}_1}=0.03530;$$

$$(t_1-t_0)\,0.4343=309.28\cdot 0.03530+2.1\cdot 13772=13.811\text{sec};$$

$$\lg\frac{m_0}{m_1}=(4560.15+267.93)/3000=1.60936;$$

$$\frac{m_0}{m_1}=40.678$$

that is, for the assumptions made, in order to ascend from a height s_0, corresponding to a velocity $\bar{v}_0=500$ m/sec to a height $s_1-s_0=62.23$ km, it will be necessary to use an amount of fuel (or, in general, to lose a mass) equal to
153 nearly $\frac{39}{40}$ of the entire mass of the rocket. The ascent time will be 13.8 sec.

The following air-density ratio corresponds to the height interval $s_1-s_0=62{,}232.8$ m between heights s_1 and s_0:

$$\frac{\beta_0}{\beta_1}=e^{\frac{s_1-s_0}{H}}=e^{\frac{62{,}232.8}{6300}}=19{,}530.$$

If the density β_1 at height s_1 is assumed to be greater, then the flight conditions have to vary. Oberth performs the above calculation and obtains, for a density β_1 which is 60 times greater, the following figures:

$$(t_1 - t_0)\ 0.4343 = 24.309\ \text{c};\ s_0 = 5000\text{m};$$
$$s_1 = 67{,}233\ \text{m};$$
$$\frac{m_0}{m_1} = 47{,}560\ \text{m};$$
$$H = 10{,}759\ \text{m}.$$

Next he demonstrates that, even though the values assumed for the air density, air resistance, and acceleration of gravity g may not be perfectly accurate, the results obtained are still precise enough (deviations not exceeding ± 7 or 8%).

Results. For an ascent from a height $s_0 = 5{,}000$ m to a height $s_1 = 67{,}233$ m, with an initial rocket mass m_0 and an initial velocity $v_0 = 500$ m/sec, we obtain the following figures:

1) The rocket mass at height s_1 is $m_1 = 0.023\, m_0$;
2) The velocity $\bar{v}_1$ at height s_1 is 11,000 m/sec;
3) The ascent time is about 19 sec.

ENGINE AND VELOCITY OF GAS EJECTION

Figure 37 shows a drawing of the propelling (lower) part of a rocket. Liquid oxygen and a liquid fuel are used as a propellant. Oberth assumes his rocket to be compound, that is, consisting of two parts: an upper part and a lower part. Each of these is a separate rocket. When the propellant in the lower rocket is used up, this rocket falls away and the upper one begins to operate. For the upper rocket the fuel used is liquid hydrogen, and for the lower one it is a mixture of water and alcohol. The fuel is mixed with oxygen in the combustion chamber. The gaseous oxygen is heated to 700°C and sprayed
154 into the chamber through the side walls of tubes E (a detail of the tube walls from the side of the combustion chamber is shown separately in Figure 37a). On the outside, from above, these tubes have liquid fuel flowing around them at a pressure of 3 or 4 atm. The group of small tubes E represents the oxygen injector. It has a length of 3 to 5 cm. Below the injector, in the combustion chamber, the mixture is ignited, the motion being retarded somewhat during its free expansion by the throat *Fm*, in order to increase the recoil. From *Fm* the expanding gases escape via the nozzle and move outward through the nozzle orifice *Fd*.

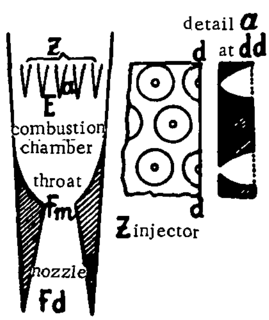

FIGURE 37.

The velocity of gas ejection at any point in the nozzle is determined by Oberth using Zeuner's formula:

$$c = \sqrt{2 \cdot 9.81 \cdot \frac{k}{k-1} p_0\ V_0 \left[1 - \left(\frac{p}{p_0}\right)^{\frac{k-1}{k}}\right]}, \qquad (12)$$

where k is the ratio of the specific heat of the gas at constant pressure to the specific heat of the gas at constant volume, p_0 is the absolute pressure in the combustion chamber in kg/m², p is the absolute pressure in the chamber at a given point in the nozzle, in kg/m², P is taken to be $\geqslant$ the air pressure β, and V_0 is the gas volume in m³. The value of $P_0 V_0$ depends on the gas mixture used.

The velocity W increases with a rise in the pressure P in the combustion chamber, and with a rise in R $(p \cdot v = RT)$, the gas constant, or T (the absolute temperature), and it decreases with an increase in k (for hydrogen $k = 1.4$). The pressure P rarely exceeds 5 atm, for temperatures up to 2,000° abs. Hydrogen has the largest gas constant ($R = 420$), whereas for oxygen $R = 26.5$, for water vapor it is 47, and for air it is 29.26.

According to Zeuner, the nozzle shape is given by the formula

$$\frac{Fd}{Fm} \leq \sqrt{\frac{\frac{k-1}{k+1} \cdot \left(\frac{2}{k+1}\right)^{\frac{2}{k-1}}}{\left(\frac{p}{p_0}\right)^{\frac{2}{k}} - \left(\frac{p}{p_0}\right)^{\frac{k+1}{k}}}}, \tag{13}$$

where F_p is the nozzle section at the given place.

Oberth assumes that the outer section of the nozzle Fd is 705 cm². If k and $\frac{Fd}{Fm}$ are constant, then $\frac{Pd}{P0}$ will also be constant, Pd being the gas pressure at the nozzle exit.

Here, according to (12), the velocity of gas ejection from the nozzle orifice, that is, C_d, will also be constant and independent of the internal gas pressure P_0. However, with an increase in P_0, Pd will also increase, as well as the recoil P and the mass of expelled gas.

The recoil is defined as

$$P = \int\int (p - \beta)\, dF = \int\int p dF - \beta F \cdot$$

155 From this formula it follows that at great heights, where β is zero or nearly zero, the recoil is greater by an amount F.

However, this statement is not completely accurate. The recoil will actually not be so much greater, for the following reasons: 1) with a decrease in β there will be an expansion of the gas beyond the nozzle, so that (p) drops and thus pdF also decreases; 2) the velocity of ejection c at the throat increases; and 3) more gas flows through Fm.

Oberth assumes the lowest value of the exhaust velocity c to be from 1,530 to 1,700 m/sec. As an example, Oberth assumes the propellant mixture to consist of: 46 g ethyl alcohol for 96 g oxygen or 1 g hydrogen for 8 g oxygen.

The amount of heat required to heat H kg of liquid hydrogen to the flash point T_1, determined according to the formula, is $H \cdot 3.4\,(T_1 + 12)$ cal, where $3.4 = c_p$ is the specific heat of the gas at constant pressure.

In order to heat S kg of liquid oxygen to this same temperature, we need

$$S \cdot 0.218\,(T_1 + 144) \text{ cal.}$$

If liquid air is used instead of oxygen, then the nitrogen in the air must also be heated, so that an amount of heat (for N kg of nitrogen)

$$N \cdot 0.244 \cdot (T_1 + 121) \text{ cal.}$$

is required.

In order to calculate the velocity of gas ejection, it is first necessary to know the values of k. For the lower (alcohol) rocket we take $k = 1.30$, and for the upper rocket, which burns hydrogen and water vapor with oxygen, the value of k is determined for various weight ratios of the component gases, as shown in the following table.

Wt. of oxygen / Wt. of hydrogen	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
k	1.400	1.398	1.396	1.394	1.393	1.391	1.389	1.388	1.386	1.385	1.384	1.383

For a diatomic gas (oxygen) $k = 1.406$.

FREE FLIGHT OF ROCKET

After the fuel has been used up, free flight of the rocket in space begins at some velocity v_1. This velocity consists of the rocket's own velocity v plus the tangential velocity w imparted to the rocket by the earth's rotation
156 and the wind. It should be noted that, while the rocket flies through the earth's atmosphere, the air resistance reduces the rocket's free-flight velocity. However, this reduction will be inconsiderable at great heights, and, according to Oberth's calculation for a velocity $\bar{v}_1 = 1{,}000$ m/sec, it will be only 69 m/sec; for $\bar{v}_1 = 10{,}000$ m/sec, this reduction is only 2.2 m/sec,* which is negligible. Oberth also derives some formulas giving the altitude of a rocket for a vertical launch or for a launch at some angle to the earth's surface.

Let us assume that a rocket of mass m_1 at a height h above the center of the earth moves over a distance dh (Figure 38), the acceleration of gravity at height h being g. Then the work done against gravity over this distance will be

$$dA = m_1 \cdot g \cdot dh = m_1 \cdot g_0 \cdot \frac{r^2}{h^2} \cdot dh,$$

* This deceleration is found by Oberth using the formula

$$\int_0^\infty \frac{L}{m_1} dt = \approx \frac{L_1}{m_1} \cdot \frac{H}{\bar{v}_1}, \qquad (14)$$

where L_1 is the air resistance and H has the value indicated previously.

where r, the radius of the earth, is equal to h_0, and g_0 is the acceleration of gravity at the earth's surface.

When the rocket ascends from height h_1 to height h_2, the work done against gravity will be

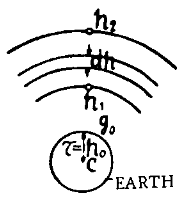

FIGURE 38.

$$A=\int_{h_1}^{h_2} dA = m_1\cdot g_1\cdot h_0^2\left(\frac{1}{h_1}-\frac{1}{h_2}\right). \tag{15}$$

Since such an ascent occurs at the expense of kinetic energy of the rocket, therefore

$$A=\frac{1}{2}m_1\cdot(v_1^2-v_2^2).$$

These two formulas combine to give

$$v_1^2-v_2^2=2\cdot g_0 h_0^2\left(\frac{1}{h_1}-\frac{1}{h_2}\right)=2g_1\cdot h_1^2\left(\frac{1}{h_1}-\frac{1}{h_2}\right). \tag{16}$$

In the derivation of this formula it was assumed that other celestial bodies do not affect the flight of the rocket. For $v_1^2<2\,g_1h_1$, the rocket will describe an ellipse in space, for $v_1^2=2\,g_1h_1$, it will describe a parabola, and for $v_1^2>2\,g_1h_1$, it will describe a hyperbola.

157 According to Kepler's second law, during equal time intervals the areas swept out by the radius vectors of the rocket orbit will be equal (Figure 39). Using this law we can determine the flight altitude:

$$\Delta\,\mathrm{I}=\Delta\,\mathrm{II}.$$

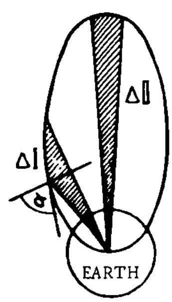

FIGURE 39.

In the figure the sides of triangle $\Delta\,\mathrm{I}$ are: $v_1\cdot dt$; h_1, and $h_1+v_1\,dt\cdot\sin a$, where a is the angle between velocity and the horizontal.

The area of triangle $\Delta\,\mathrm{I}$ is $v_1\cdot h_1\cdot\cos a\cdot\frac{dt}{2}$, the sides of triangle $\Delta\,\mathrm{II}$ are $v_2\,dt$; h_2; and h_2, and the area of triangle $\Delta\,\mathrm{II}$ is $h_2 v_2\frac{dt}{2}$.

If we set these areas equal to one another, we obtain

$$h_1 v_1\cdot\cos a=h_2 v_2$$

or

$$(v_1^2-v_2^2)=v_1^2\left(1-\frac{h^2_1}{h^2_2}\cos^2 a\right),$$

and from (16) it follows that

$$v_1^2\left(1-\left(\frac{h_1^2}{h_2}\right)\cos^2 a\right)=2g_1\,h_1^2\left(\frac{1}{h_1}-\frac{1}{h_2}\right).$$

For an elliptical orbit this equation has two roots, one of which in our case is imaginary (inside the earth or under it) and the other is real, determining the highest point of the ascent. This height is found from an expression given previously.

$$h_2 = h_1 \cdot \frac{g_1 \cdot h_1 + \sqrt{g_1^2 h_1^2 - (2g_1 h_1 - v_1^2)\, v_1^2 \cos^2 a}}{2g_1 h_1 - v_1^2}. \tag{17}$$

If the rocket is launched perpendicularly with respect to the earth's surface, as assumed by Oberth for his rocket, then the foregoing formula becomes

$$h_2 = h_1 \cdot \frac{g_1 h_1 + \sqrt{g_1 h_1 - (2g_1 h_1 - \bar{v}_1^2 - w^2)\, w^2}}{2g_1 h_1 - \bar{v}_1^2 - w^2}, \tag{17a}$$

where

$$v_1^2 = \bar{v}_1^2 + w^2; \quad v_1 \cos a = w.$$

The rocket will not return to the launching point, because of 1) the effect of the wind, 2) the rotation of the earth, and 3) the conditions of the rocket flight.

Actually, let us assume that the rocket is launched vertically from a point a on the earth's surface (Figure 40), the velocity of rotation of the

(158)

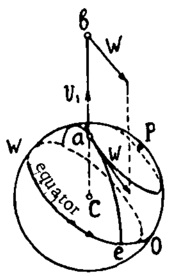

FIGURE 40.

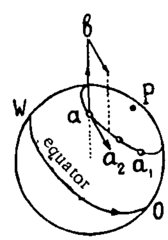

FIGURE 41.

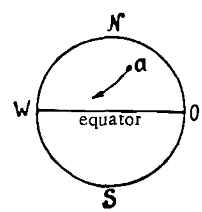

FIGURE 42.

earth (and the point) being w. Moving upward to a height b, the rocket is under the influence of velocities $\bar{v}$ and w, and it travels in the plane of these velocities; the latter plane passes through the earth's center and intersects the earth's surface along one of the great circles. When it returns to earth,
158 the rocket should descend somewhere on this great circle, that is, it will come down at some point located south of the parallel through point a. At the same time, a rocket leaving the earth at point a will have the same angular velocity as the earth (Figure 41), and this quantity will decrease steadily as the rocket ascends. Therefore, when the rocket falls back to earth, it will lag somewhat to the west of the launching point, and it will not land at the shifted launching point a_1, but rather at a point a_2 to the west of it. As a result of the motions depicted in Figures 40 and 41, the rocket will descend somewhere along a curve lying to the south and west of the launching point a (Figure 42).

EFFECT OF ACCELERATION

We measure the weight of a body in terms of its pressure on some supporting object, such as the pan of a balance. This pressure is proportional to the product of the mass of the body times its acceleration. A body located at the earth's surface is acted upon by a gravitational acceleration $g=9.8$ to $9.83\ m/sec^2$. If the support were to be taken away, the body would fall.

Let us call the influence of the acceleration on the body the "effect of the acceleration." According to the law of relativity of motion, such an effect will be observed: 1) when all the molecules of the body experience acceleration but the body itself is at rest (an example is the pressure of a weight on the pan of a balance), and 2) when all the molecules of the body are at rest but the body moves with accelerated motion (an example is the effect of inertia on passengers during sudden accelerations or decelerations of a streetcar). The effect of acceleration is measured in the same units as the acceleration itself (that is, in m/sec^2).

As an example, let us calculate the effect of acceleration when an ivory billiard ball falls onto a marble slab. The relevant data are: height of fall, 20 cm; velocity of descent, $v=2$ m/sec; deformation on impact, $s=1$ mm.

The effect of acceleration will be denoted as a, and the time during which the deformation takes place as t. The formulas used in the calcu-
159 lation are

$$v=at;\quad s=\frac{1}{2}at^2,$$

from which we have

$$t=\frac{v}{a};\quad s=\frac{1}{2}\frac{v^2}{a};\quad a=\frac{v^2}{2s}\ \frac{4}{2\cdot 0.001}=2000\ m/sec^2.$$

Another example of an acceleration effect is the squeezing of the bicycle wheels at the high point of a "devil's wheel," along which a cyclist (or an aviator) moves rapidly, describing a "loop." Here the reason for the effect is the centrifugal acceleration. The result of the effect disappears when the inertia of the body becomes equal to its weight, for instance, when the body is in free flight. If a rocket falls freely toward the earth, persons aboard it will lose their weight and will be suspended freely in the air inside the rocket, liquids will assume a spherical shape and cease pressing against the walls of their containers, etc. On the other hand, when a rocket acquires a considerable acceleration, liquids will press more forcefully against the vessel walls, and this must be kept in mind when testing the strength of the containers, so as to avoid rupturing them.

CONCLUSIONS

Formulas were derived above for the flight time (t_1-t_0) (7a), the acceleration b (7c), the fuel consumption and general reduction of rocket mass $\log\frac{m_0}{m_1}$ (8a), and the specific recoil $\frac{P}{m_0}$ (9). In these equations the velocity of gas ejection c, the initial velocity v_0, the initial height H and the acceleration g can all be assumed to be given and constant. In this case $t_1 - t_0$, b, $\frac{m_0}{m_1}$, and $\frac{P}{m_0}$ are functions of v and may be calculated for different $\bar{v}$. The results of such calculations are given in the table, for $c = 1{,}400$ m/sec and $H = 7{,}200$ m.

(160)

Velocity $\bar{v}$ m/sec	Flight duration $(t - t_0)$ sec	Acceleration b m/sec^2	$\log \frac{m_0}{m}$	$\frac{m_0}{m}$	$\frac{P}{m_0}$	Notation
500	0.0	11.7	0.0000	1.000	31.4	m_0 is mass of loaded rocket
600	7.3	17.0	0.0754	1.190	30.9	
700	11.9	23.3	0.134	1.362	31.4	
800	16.1	30.1	0.191	1.552	31.4	m is mass of rocket in general
900	21.5	37.8	0.240	1.738	33.0	
1000	21.5	40.0	0.286	1.931	34.1	
1200	25.2	64.1	0.371	2.349	35.6	P is recoil
1400	27.7	84.3	0.448	2.803	37.0	
1500	29.0	95.0	0.486	3.062	37.2	
1700	31.2	117.1	0.560	3.631	37.8	
2000	33.6	153.7	0.625	4.217	41.2	
2200	35.0	179.5	0.735	5.434	36.7	
2400	35.9	206.0	0.808	6.427	35.1	
2600	36.5	234.0	0.872	7.446	34.1	
3000	38.2	291.5	1.006	10.139	29.9	
3400	39.3	351.0	1.138	13.74	26.9	
3800	40.3	414.0	1.267	18.49	23.4	
4000	40.7	447.0	1.330	21.38	21.8	

This table can be used to determine the mass ratio $\frac{m_0}{m}$ for any velocity range and for some initial velocity. Let us suppose, for example, that we wish to determine the mass ratio for an initial velocity $v_a = 800$ m/sec and a final velocity $v_b = 3{,}000$ m/sec.

Since

$$\log \frac{m_a}{m_b} = \log m_a - \log m_{500} + \log m_{500} - \log m_b = \log \frac{m_{800}}{m_{3000}} - \log \frac{m_{500}}{m_{800}},$$

therefore, according to the table, $\log \frac{m_a}{m_b} = 1.006 - 0.191 = 0.815$, and $\frac{m_a}{m_b}$ will be 6.5. The flight time will be $38.2 - 16.1 = 22.1$ sec.

160 Composite rocket

Since the ratio $\frac{m_0}{m_1}$ increases very rapidly with an increase in the velocity and flight time, and since, for technological reasons, a limit will soon be reached, Oberth suggests making the rocket composite, with one rocket inside another. Each rocket will have its own engine and propellant, and when the latter burns up the corresponding rocket will fall away, causing a new increase in the ratio $\frac{m_0}{m}$ of the remaining rockets; in this way a high velocity will be attained.

If M_0 and m_0 are the masses of the loaded rockets, and M_1 and m_1, are the masses of the rockets without fuel, then we can substitute the following quantity into equation (8a) instead of $\frac{m_0}{m}$:

$$\frac{m_0}{m}=\frac{M_0+m_0+\mu_0+\quad m_0+\mu_0+\quad \mu_0+}{M_1+m_0-\mu_0+\quad m_1+\mu_0+\quad \mu_1+}.$$

161 This quantity can be made as large as desired, by employing a number of rockets situated one inside the other (Oberth assumes two rockets for his apparatus). Each outer rocket (Figure 43) must be greater than the sum of the remaining rockets, and the last rocket should weigh as little as possible.

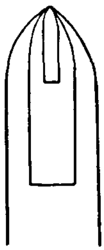

FIGURE 43.

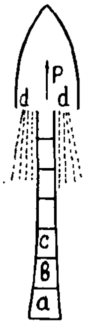

FIGURE 44.

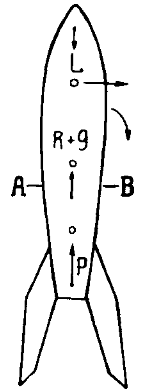

FIGURE 45.

An **excess pressure inside the rocket** is advisable, in that it increases the stress on the rocket walls and the fuel containers, and thereby also increases their resistance to bending, as is the case, for instance, for the fuel tanks of soft dirigibles. Such an excess pressure is particularly advisable when the acceleration of the rocket is to be increased considerably.

According to equation (12), the velocity of gas ejection c for given $\frac{P_d}{P_0}$ and k will be higher, the greater is the product P_0V_0. In turn, the latter product will be greater, the lower the specific weight of the expelled gas and the higher its temperature. The velocity c is highest for hydrogen.

Methods for increasing flight velocity. It is evident from expression (5) that the velocity of the rocket will be higher: 1) the lower the air pressure, and 2) the greater the load per unit area F of the rocket cross section, that is, the greater the ratio $\frac{mg}{F}$.

The latter quantity will be more appreciable when: a) the rocket is quite long, and b) the specific weight of the rocket is considerable. However, if the rocket is long, then measures must be taken to ensure that the force of air resistance will not cause it to break up. To do this, the point of application of the recoil force P could be moved upward (Figure 44), by moving the fuel tanks downward to form a tail section (a, b, c, . . .) and discarding them as they become empty. However, this design has a number of structural shortcomings. On the other hand, the engine could also be placed at
162 the bottom, as was suggested by Oberth for his rocket. Such an arrangement is shown in Figure 45.

If the flight is not perfectly straight, there may be a transverse air pressure on the forward part of the rocket. Then, because of the combination of forces acting on it, the rocket may break up at some section AB. The strength of the rocket can be ensured by maintaining an internal excess pressure and by providing special ribs. During flight through the lower layers of the atmosphere, when the velocity is still low, the air density considerable, and the flight time long, the exhaust velocity c should be increased, so as to give a higher flight velocity and ratio $\frac{m_0}{m_1}$. For his composite rocket, Oberth assumes values of $c = 1{,}530$ to $1{,}700$ m/sec for the lower, alcohol, rocket, and $c = 3{,}800$ to $4{,}250$ m/sec for the upper, hydrogen, rocket. Here the specific weight of the fuel for the first rocket will be eight times as great. If two hydrogen rockets were employed instead, the entire apparatus would have to be five times as long, its volume would have to be 125 times as great, and it would have to be 18 times as heavy.

Advantages of launching the rocket from a great height

Let us assume that a rocket begins its [powered] upward flight from some height where the air density is only $1/n$ of that at the ground. In this case the following conclusions can be drawn: a) the initial velocity will be higher, or, for the same v_0, the load per unit area of rocket cross section will be $1/n$ as great. The fuel consumption will be reduced accordingly. Oberth proposes lifting his rocket to a height of 5,500 m with the aid of two dirigibles (Figure 46), and then launching it on its way from there; b) since the air resistance per square unit of cross-sectional area will be only $1/n$ as great, the excess pressure inside the liquid-fuel tanks need only be $1/n$ as great. Thus the mass of the rocket can be reduced considerably; c) the areas of the nozzle orifices need only be $1/n^2$ as great, and the length of the combustion chamber can be reduced.

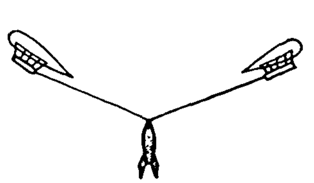

FIGURE 46.

Reasons for making lower rocket run on alcohol and upper rocket on hydrogen

Oberth demonstrates that an alcohol rocket should be used in the lower atmospheric layers, where the air density is high. The velocity of gas ejection (c) and the flight velocity v_x of this rocket will be relatively low. At greater heights, where the air density is low, a hydrogen rocket, for which c and v_x are higher, should be employed.

Oberth gives the following justification for his use of different types of rockets at different heights:

163 The ratio $\frac{m_r}{m_1}$ may be greater, the lower the air pressure at the beginning of the rocket ascent.

If b_r is the weight of the propellant and m_1 is the weight of the empty rocket, then it will be approximately true that $\frac{b_r}{m_1}=\frac{k}{\beta_0}$, where k is a proportionality factor.

The fuel for an alcohol rocket weights q times as much as the fuel for a hydrogen rocket. If we use capital letters for the alcohol rocket and lower-case letters for the hydrogen rocket, then we can write

$$\frac{B_r}{M_1}=q\cdot\frac{b_r}{m_1}$$

Considering the elementary effect of the recoil force, we can also write

$$c\cdot dm+mdv_x=0,$$

where c is the exhaust velocity, dm is the fuel consumption, m is the rocket mass, and dv_x is the velocity increment. Integration gives

$$v_x=c\ln\frac{m_0}{m_1}. \qquad (19)$$

For the two cases in question we have:

for the alcohol rocket,

$$V_x=C\ln\frac{M_1+B_r}{M_1}=C\ln\left(1+\frac{B_r}{M_1}\right)=C\ln\left(1+q\frac{b_r}{m_1}\right);$$

for the hydrogen rocket,

$$v_x=c\ln\left(1+\frac{b_r}{m_1}\right).$$

Since $V_x<v_x$, therefore

$$C\ln\left(1+q\frac{b_r}{m_1}\right)<c\cdot\ln\left(1+\frac{b_r}{m_1}\right)$$

and

$$\frac{\ln\left(1+q\frac{b_r}{m_1}\right)}{\ln\left(1+\frac{b_r}{m_1}\right)}<\frac{c}{C}. \qquad (20)$$

The ratio $\frac{c}{C}$ is constant and equal to about

$$\frac{1530}{4200}=2.3.$$

Let us denote the left side of inequality (20) as f. The limiting values of f will be: at the earth's surface, where β is high but $\frac{b_r}{m_1}$ is low, $f=q$; at infinity, where $\beta=0$,

$$f=\frac{\ln\left(1+q\frac{b_r}{m_1}\right)}{\ln\left(1+\frac{b_r}{m_1}\right)}<\frac{\ln\left(1+\frac{b_r}{m_1}\right)+\ln q}{\ln\left(1+\frac{b_r}{m_1}\right)}=1+\frac{\ln q}{\ln\left(1+\frac{b_r}{m_1}\right)};\ f\lim. =1.$$

164 Therefore, where it is necessary to satisfy inequality (20), a hydrogen rocket is used. This will be the case for heights where $\frac{c}{C}=2.3$ or above (where $f<2.3$ and approaches unity at the limit). Below this, an alcohol rocket is employed, for which $\frac{c}{C}<q$. We have plotted an illustrative diagram (Figure 47) indicating the limits of applicability of the two types of rockets (this drawing does not appear in Oberth's paper).

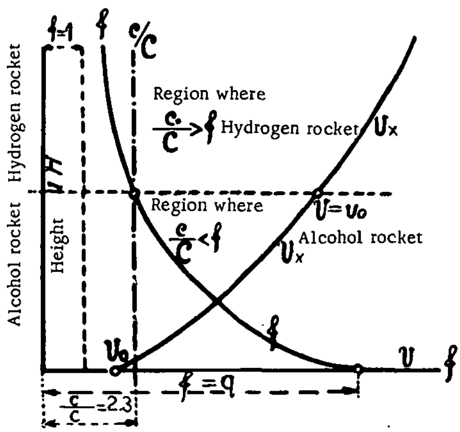

FIGURE 47.

The ratio $\frac{m_0}{m_1}$ of the mass of the loaded rocket to the mass of the empty rocket could be made arbitrarily large in the absence of air resistance and terrestrial gravity.

The altitude of the rocket depends only on velocity v_1 (formulas (16) and (17)), and it will be finite as long as $v_1'^2<2g_1h_1$. When $v_1=\sqrt{2g_1h_1}$, the velocity will be parabolic, and at a height of 70 km above the equator it will equal 11,160 m/sec. In addition, the flight altitude will be influenced by the latitude of the place (according to formula (17a), w is a function of the latitude).

Optimum Flight Direction

According to formula (16), the greatest flight altitude h corresponds to the largest difference $v_1^2 - v_2^2$. Therefore, in order to increase h, it is necessary to raise v_1 and to reduce v_2.

Velocity v_2 will be a minimum if the ellipse (Figure 39) is as elongated as possible, that is, if the initial velocity v_1 is directed along the vertical. On the other hand, if v_1 is to be a maximum, then it must lie in a direction tangent to the earth's surface, since then the velocity of the earth's rotation will be added to the rocket's own velocity. The optimum launching direction will be somewhere between the two above directions and it will be toward the east. If the rocket is to have a parabolic velocity, the launch should be directly eastward (along the tangent).

Part II. Description of "Model B" Rocket

GENERAL REMARKS

Oberth does not give detailed drawings of his apparatus, only a rough sketch of it. In addition, he points out that the actual construction of such
165 a rocket would entail many modifications.

Purpose of apparatus: to study the height, composition, and temperature of the earth's atmosphere, to determine the law of air resistance for different heights and velocities, and also to study the operation of the rocket itself, which Oberth calls the "Model B" rocket.

The apparatus consists of two rockets: an upper, inner, hydrogen rocket (H. R.)* and a lower, outer, alcohol rocket (A. R.).** The composite rocket is 5 m in length, 55.6 cm wide, and it weighs 544 kg, 6.9 kg of which constitute the H. R. In addition, an auxiliary rocket is provided. The question of the rocket material has not been finally decided. This material must operate under tension because of the internal excess pressure. For the A. R. Oberth uses aluminum alloys having a specific weight of 3.0 g/cm^3 and a tensile strength of 30 to 32 kg/cm^2. Because the stress is applied for only a short time (about 1/2 minute), 20 kg/cm^2 can be withstood without rupture. The oxygen tanks are to be made of a copper-lead alloy, the contraction coefficient of which, for cooling to 170 or 180°, is the same as an aluminum alloy. The parts subjected to intense heating are made of copper. The walls of the fuel injector may be made of silver (Silberblech). The H. R. is made of lead, the strength of which at low temperatures is equal to that of steel.

ALCOHOL ROCKET

General data: The ascent begins from a height $s_0 = 7{,}700$ m, since the apparatus is lifted to a height of 5,500 m by means of dirigibles (Figure 46),

* H.R. stands for "hydrogen rocket."

** A.R. stands for "alcohol rocket."

and another 2,200 m is traversed with the aid of the auxiliary rocket, so as to obtain an initial velocity $\bar{v}_0$.

The pressure in the combustion chamber is 16.5 kg/cm² $< p_0 <$ 20 kg/cm².

The propellant is: 341.5 kg of water, to which is added 45.8 kg of alcohol, 1.67 kg of purified alcohol, and 98.8 kg of liquid oxygen or a corresponding amount of liquid air. In the latter case less water will be required.

The combustion temperature is 1,700°C $< T_0 <$ 1,750°C.

The pressure at the nozzle orifice is $P_d = \beta_0 = 0.39$ kg/cm².

The ratio of the nozzle-orifice area to the midship-section area of the rocket is

$$\frac{Fd}{F} = 0.329 \text{ and } \frac{Fd}{Fm} = 5.86;$$

$$\frac{d}{dm} = \sqrt{5.86} = 2.42 \text{ (Figure 37);}$$

$$d = 55.6\sqrt{0.329} = 29.9 \text{ cm}; \quad d_m = \frac{29.9}{2.42}\ 12.35 \text{ cm}.$$

166 The velocity of gas ejection is taken to be $c = 1{,}400$ m/sec.

The container for the mixture of water and alcohol is maintained at an excess pressure of 3 atm.

The space for the H. R. is maintained at this same pressure.

The oxygen containers are maintained at a pressure of $P_0 + 1.5$ atm.

The propelling apparatus has walls 2.35 mm thick, and the walls of the area with the oxygen tanks are 2.8 mm thick.

167 The weights of individual parts of the rocket are shown in the table below.

In the following this ratio will be taken to be 9.

The load on the rocket cross section is 0.225 kg/cm².

The initial velocity is $V_0 = 500$ m/sec.

The velocity after burning of all the propellant is $V_1 =$ 2,800 to 2,900 m/sec.

When the fuel has been used up, the load on the cross section is 0.0232 kg/cm².

The duration of the burning is 36 to 40 sec.

The amount of fuel burned per second is

$$12.01 \text{ kg/sec} < \frac{dm}{dt} < 13.21 \text{ kg/sec}.$$

Mixture composition and burning (Figures 48 and 49). In the upper part of the combustion chamber a row of tubes is provided (in space A), the width at the bottom being 2.5 cm and the width at the top being 3.6 cm. These tubes do not reach to the top of the chamber. There is purified alcohol between these tubes, and this is brought to boiling by the oxygen-rich combustible gas fed in by pump mn in the form of bubbles. The alcohol vapor enters the tubes, into which tubes D penetrate from above, from the bottom of the oxygen container S; tubes D have openings in their side walls (Figure 37a). The pressure in space A is slightly higher than p_0 atm, while in the oxygen container the pressure is $p_0 + 1.5$ atm, so that oxygen is injected through tubes D along thin channels. An igniter G is placed at the ends of the tubes, Weights of parts of alcohol rocket

Item	Weight in kg
A. Parts	
1. Propelling apparatus	16.2
2. Oxygen containers	10.0
3. Pumps	8.0
4. Floats	4.0
5. Upper part (wall thickness 0.4 mm)	6.0
6. Injector	3.0
7. Other	4.0
$M_1 =$	51.2
8. Hydrogen rocket m_0 [parts plus propellant]	6.9
$M_1 + m_0 =$	58.1
B. Propellant of A.R.	
1. Water	341.5
2. Alcohol	45.8
3. Purified alcohol	1.67
4. Liquid oxygen	48.80 *
	437.77

* Incorrect value. According to the text this should be 98.80 kg, giving a total of 487.77 kg, as in the ratio given below by Oberth [Translator].

From the table we can find the ratio of the rocket mass before flight to the mass after the mixture in the A. R. has been burned:

$$\frac{M_0 + m_0}{M_1 + m_1} = \frac{487.77 + 58.1}{58.1} = \frac{545.87}{58.1}{}^{**} = 9.4.$$

** Here Oberth commits an arithmetical error, assuming that $M_0 + m_0 = 544$ kg, giving $M_1 + m_1 = 56.2$ kg [Rynin].

The values and notation given here are inconsistent. According to the text, the combined weight of both rockets with propellant ($M_0 + m_0$) is 544 kg, and the weight of the A. R. propellant ($M_0 + m_1$) is 487.77 kg (the latter weight is given erroneously in the table as 437.77 kg; see first footnote). The weight $M_0 + m_0$ (487.77 kg) after burning of the propellant in the A. R. is thus 544 − 487.77 = 56.2 kg, the figure given in Rynin's note. The ratio in question should then actually be

$$\frac{M_0 + m_0}{M_1 + m_1} = \frac{544}{56.2} = 9.7.$$

If the total weight at launching is assumed to be 545.87 kg, as given in the table, then, using the correct notation in the denominator, we have

$$\frac{M_0 + m_0}{M_1 + m_1} = 9.4.$$

Since the Soviet book is plagued with proofreading errors throughout, it is sometimes difficult to determine the sources of errors and inconsistencies [Translator].

to ignite the fuel mixture. Since considerably more oxygen is injected through the tubes than is needed for ignition, a gas containing 95% oxygen and giving (at 700°C) a pressure up to 20 atm is obtained. This gas continues through tubes *E* to chamber *O*, while a mixture of water and alcohol is added to it along the way, the latter mixture being injected through small openings and then ignited.

Rocket construction (Figure 49). The upper part of the rocket has the form of a cap over the two rockets and it is kept from opening by springs *b* and b^1. When all the propellant in the alcohol rocket has been consumed, the connection between the tip of the rocket and the main part is broken, the tip opens up, dividing into two parts (Figure 50), and the inner hydrogen rocket separates from the alcohol rocket. There is air inside the two halves of the rocket tip (*c*), to prevent these parts from sinking if they land in the water. For a flight velocity of about 3,000 m/sec, the rocket tip will heat up considerably, so that special coolers are necessary (not shown in the drawing). In addition, it will be cooled from inside by vaporizing hydrogen escaping from the nozzle of the inner rocket and rising in the space between the walls of *A* and the rocket wall. The hydrogen then leaves through safety valves *K*.

The diameter of the space inside the alcohol rocket is 30 cm, and the
169 diameter of the hydrogen rocket is 25 cm. Therefore, between the walls of the two rockets there is a gap 2.5 cm thick, which is filled with hydrogen and further divided by the wall of ***A***. The tip of the H. R. is one cm below the tip of the A. R. Buffers *f* are located at different places between the H. R. and the A. R., to protect the H. R. from shocks, which at the very low temperatures involved could rupture it. Space *e* contains a mixture of water and alcohol. A float *g* is also placed in this space, and its purpose will be explained below. This mixture is at a pressure of 3 atm, maintained by pumps ***mn***, which supply hot gas to the double bottom *h*, from which the gas ascends through a number of openings. The pressure is regulated by automatic valves *K*. The mixture of water and alcohol is fed through valve *y* and tubes ***O*** in turn to chambers p_1 and p_2, which are also connected to safety valves *K*, and in addition to tube ***k***, which also feeds the mixture to injector *Z*. Chambers p_1 and p_2 have a double bottom *i*, through the openings of which the gas delivered to them by pumps ***mn*** arrives. Therefore, these chambers also act as pumps.

The operation of valves a_1 is such that, when one of them is full of mixture from ***e***, the other forces this mixture into the injector at a pressure of 20 to 23 atm. The oxygen container *S* is maintained at a pressure of 18 to 21 atm, and the pressure in space *A* is one atm lower. In order to prevent buckling of the bottom of *S*, it is supported by wires leading to the top of this container. The top is ellipsoidal in shape, and, for a circular rocket section, there are depressed places at the two opposite points where the top touches the walls. Valves O_2 are placed there, and liquid alcohol flows through them into injector *Z*. The liquid in chamber p_1 is collected in the middle, at *K*. The vaporizing oxygen will be at a pressure of 21 atm, and it vaporizes because: 1) a hot surface *A* lies under it (Figure 48), and 2) pumps ***mn*** supply hot gas. This gas also contains some water vapor, which, when the oxygen vaporizes, is converted into ice crystals. The crystals will float above the surface of the liquid oxygen and, when it is used up, will be ejected through

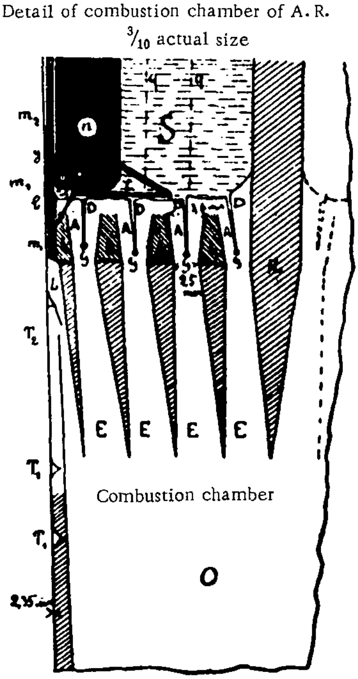

FIGURE 48. Engine of Oberth's rocket.

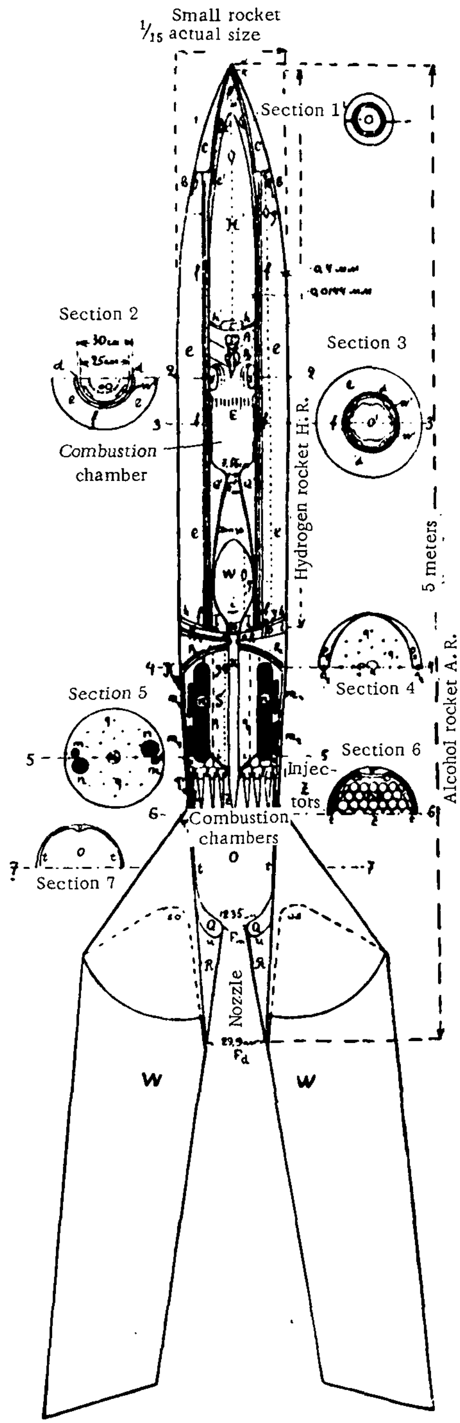

FIGURE 49. Oberth's double rocket.

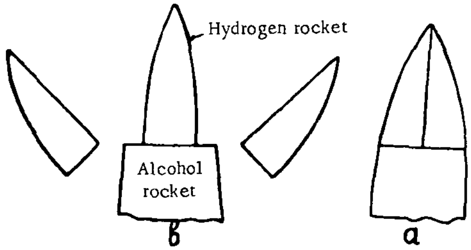

FIGURE 50. Head of rocket.

the wide opening m_1, so as to prevent clogging of the pores of the injector tubes.

The *oxygen container S contains a float, which maintains the proper flow* rates of fuel and oxygen. This float is electrically connected to float g of the alcohol container W, and also to the safety container S, which functions similarly to valves K. If, for example, the level of liquid oxygen drops slowly, the pressure over it will increase and thus more oxygen will be fed to the injector.

The walls of the oxygen container are 2.8 to 3 mm thick. The container W holding the liquid alcohol is joined to injector Z via tube K. Its purpose is: 1) to maintain here the pressure of a certain height, since the effect of p_1 and p_2 does not reach there. The pressure in container W itself is main-
170 tained by pumps mn, which force hot gas into it. The container has a float g *in it, which, in addition to its previously described function, also regulates* the operation of p_1 and p_2. Container W is situated under the nozzle of the H. R. and must be protected from cooling. It is oval in shape. Between W and p_1 there is a space l, in which instruments can be mounted to record the operation of the A. R. They must also be protected from cooling. Electrical instruments and a small dynamo can be placed there as well.

Pumps mn operate as follows: (Figure 51): the small pump m_1 feeds

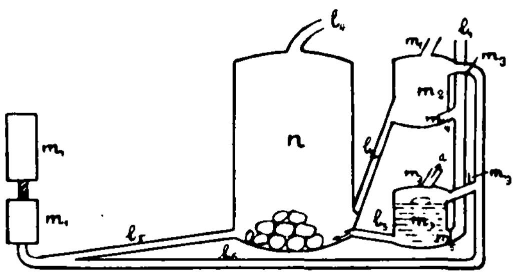

FIGURE 51. The rocket pumps.

alcohol alternately to the two containers m_2 and m_3 and continuously feeds tank n. Containers $m_{2,3}$, like chambers $p_{1,2}$, pump oxygen to n. Some lumps of sodium lie at the bottom of $m_{2,3}$. When valves $m_{4,6}$ or $m_{5,7}$ are open, oxygen flows into containers $m_{2,3}$. When both containers are full of oxygen, these valves close, and alcohol flows into the oxygen through the open valves $m_{8,9}$. Because the sodium is present, intense burning begins and oxygen flows along $l_{2,3}$ to tank n, where an appropriate mixture of oxygen with alcohol takes place. There is also sodium in container n, and it converts all the alcohol and oxygen into vapor, so that a hot, oxygen-rich gas leaves through tube l_4. Tank n is lined on the inside with a refractory material. On the outside, n is surrounded by liquid oxygen. Tube l_4 has valves in it which regulate the influx of gas at h or i.

Note. An electrical igniter can be used instead of the sodium.

Combustion chamber O (Figure 49) does not actually touch the outer

casing; it is separated from it by a thin wall t, which is connected to the casing by a number of spacers. Liquid from the injector circulates in the space between t and the casing. It is converted into vapor in this space and, by cooling wall t, it prevents it from catching fire. From this separating wall the vapor goes out into chamber O through opening L, and, flowing outward, it moves along walls t, shielding them from the hot gas. If an intense vaporization of liquid occurs at the separating wall, thermoelement T^1 begins to operate, so as to lower the temperature. In addition, the separating wall has a wide part (see section 7), where the float is situated, which, when the influx of liquid is too great, rises and shuts off the flow, so that the liquid will not overflow through opening L into the combustion chamber. Partition u at the nozzle throat Fm serves as a separating wall between the two parts Q and R (Figure 49). When all the propellant is used up, pumps mn operate so as to start vaporization of the liquid first in R, and then in Q. If this
171 construction is used, it is not necessary to line the nozzle with a refractory material, and the weight of the rocket will be less. The nozzle itself has either one orifice, if the rocket is small, or a number of them, supplied by a common combustion chamber.

In all, there are four stabilizers, each of these being double. The stabilizer fins can rotate about the x axis. During ascent they stabilize the rocket and regulate the direction of motion, operating partly as controls under the influence of the instruments in L. During descent, they are turned backward, and their resistance slows down the fall.

The search for a rocket which has fallen to earth can be facilitated using the following technique. A small compartment closed on the outside by a door is built into the rocket wall. A rubber balloon with gas in it is placed inside the compartment. The pressure inside the compartment is maintained at 10 atm. When the rocket falls to the ground, a special acid begins to act on the latch of the door, eating through it, and the door opens. The balloon, which has become inflated because of the drop in pressure to 1/10, leaves the chamber and rises to a certain height above the rocket, where it is held by a cord and indicates the landing spot.

Instruments required for the alcohol rocket:

1. Constant-current generator.
2. Gyroscope with electric motor. It is controlled by stabilizers.
3. Acceleration indicator. It may consist of a weight, attached to an electric strip. When the acceleration changes, a pen connected to the weight will trace out a line on a moving strip of paper, giving an indication of the velocity, and thus the flight altitude as well.
4. Floats regulating the levels of alcohol and oxygen. These can also turn on an electric current.
5. A manometer recording the internal pressure.
6. An instrument for measuring the external air pressure. An aneroid may be used for this, or, since the latter can hardly provide reliable readings, a special instrument may be designed. The latter is connected to the acceleration indicator and has an indicator in the form of a roller which can move along a curve to the edge of a strip, the lower horizontal edge of which moves on rollers. The upper edge of the strip is traced out as a curve.

7. The internal pressure, which is higher than the external air resistance L , may push the rocket tip off and thus plates b and b_1 must operate at a rupture, which also measures and serves to take into account the resistance.

8. All the electric currents which can be produced in the rocket by various devices (floats, etc.) will affect electromagnets, and ultimately they will influence the operation of pumps mn and the flight of the rocket.

9. Thermographs. One of these is mounted near the top of the rocket, in order to record the properties of the air.

172 HYDROGEN ROCKET

General remarks

The flight of the hydrogen rocket begins at an altitude $s_1 = 56.2$ km.

The table shows the distribution of weight for the parts of the rocket and for the propellant.

Weights of parts of the hydrogen rocket

Item	Weight in kg
A. Rocket Parts	
1. Weight of hydrogen container and rocket tip	0.033
2. Combustion chamber and injector	0.466
3. Instruments	1.500
4. Pumps, annular container for oxygen	0.500
5. Nozzle and its casing	0.3
6. Stabilizers	0.3
7. Parachute	0.5
m_1	3.6
B. Propellant	
1. Hydrogen	1.36
2. Oxygen	1.94
	3.3

Total weight
$m_0 = 3.6 + 3.3 = 6.9$ kg.

Pressure in combustion chamber $P_0 = 3$ atm.
Temperature $T_0 = 1{,}700°C$.
Diameter of nozzle exit 25 cm.
Diameter of nozzle throat $d_m = 7.55$ cm.
Velocity of gas ejection $c = 3{,}400$ m/sec.
Pressure in hydrogen container 0.24 atm, which for the initial flight altitude of this rocket gives an excess pressure of about 0.12 atm.
The thickness of its walls is 0.0144 mm,

$$\frac{m_0}{m_1} = \frac{6.9}{3.6} = 1.915; \ \log \frac{m_0}{m_1} = 0.2825; \ \ln \frac{m_0}{m_1} = 0.650.$$

173 The flight velocity $V_x = 3400 \cdot 0.650 = 2210$ m/sec.
The acceleration in the first second is $b_0 = 200$ m/sec^2.
The fuel consumption is

$$\frac{dm}{dt} = 6.9 \cdot \frac{200}{3400} = 0.406 \text{ kg/sec.}$$

The gas pressure at the nozzle orifice is $P_d = 0.0196$ atm.
The duration of burning is $\frac{3.30}{0.406} = 8.15$ sec.
When all the propellant has been burned, the rocket velocity will be

$$3000 + 2210 - 64.3 - 7 = 5139 \text{ m/sec.}$$

Here 3,000 is the terminal velocity of the alcohol rocket, 2,210 is the hydrogen rocket's own velocity, 64.3 is the velocity reduction due to terrestrial gravity and air resistance, and 7 is the velocity reduction due to air resistance on the remaining path (after the term 64.3 stops having an effect). For a velocity of 5,139 m/sec, the rocket will ascend to a height of 1,960 km.

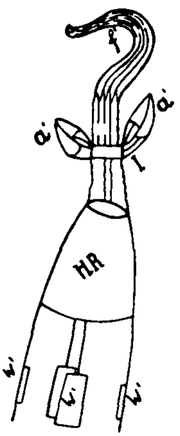

FIGURE 52. Separation of rocket head.

Rocket construction. The tip a^1 of the H. R. (Figure 52) is constructed like the tip of the A. R. It opens during descent, and a parachute f^1 emerges from a place under it. The tip flaps remain connected to the rocket. The inside of the tip is lined with a porous fabric which can be wetted by the water in c^1, the water being sprayed onto the fabric by pump e^1. The other (primed) symbols in Figure 49 designate parts which are analogous to those in the A. R. The oxygen is kept in an annular container S^1, from which it is fed in a vapor form into tubes E^1 under a pressure of 3.1 atm. Hydrogen (H^1) is forced into the space between tubes E^1 by pumps $P^1_{1,2}$, at a pressure of 5 atm. The space inside the annular oxygen container serves as a tank.

Tubes (i^1) carrying hot gas extend into pumps $P^1_{1,2}$. These are provided with special filters, so as to prevent ice crystals from entering injector E^1; the crystals may form as a result of the presence of water vapor in the gases. It should be kept in mind that it is only at 253°C below zero that liquid hydrogen stops evaporating, and that for liquid oxygen the corresponding temperature is 183°C below zero. Therefore, as soon as

the temperature goes above these limits, these substances begin to evaporate. Consequently, ventilators and coolers must be used. In addition, for
174 such low temperatures, the metal walls become so brittle that very likely only lead would be suitable for the wall material. Liquid hydrogen is made to flow around the combustion chamber O^1 and the nozzle. The stabilizers are so constructed that they can be rotated. When the H. R. is still inside the A. R. (section 3), these stabilizers (W^1) are turned and they fit into appropriate depressions in the rocket frame. When the H. R. leaves the A. R., these stabilizers move downward on special hinges and extend below the nozzle, directing the motion of the rocket.

Instruments aboard H. R. 1. Electric battery; 2. gyroscope; 3. acceleration indicator; 4. instruments recording regime of fluids; 5. manometer; 6. thermograph; 7. meter measuring pressures at top (*b* for the A. R.).

(175)

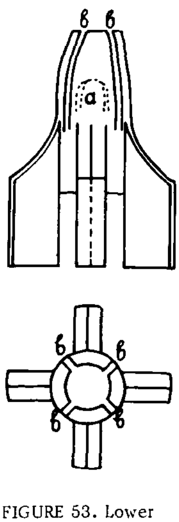

FIGURE 53. Lower rocket.

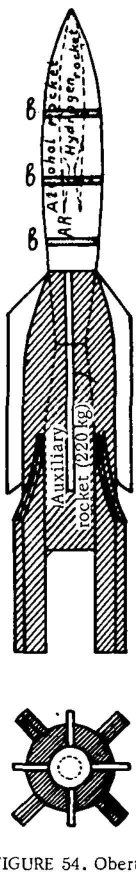

FIGURE 54. Oberth's triple rocket.

PURPOSES OF ROCKET FLIGHT

Using the rocket described above, the following are possible:

1. Determination of the air resistance at great heights and the law of its variation as a function of velocity.

2. Determination of the density and specific weight of air at these heights.

3. Determination of its pressure and temperature.

4. Determination of motions in the upper atmosphere (according to the difference between the calculated and actual landing sites on the earth).

NOTES ON ROCKET FLIGHT

1. The preliminary experiments must include tests of nozzle and injector operation, tests of the efflux of fluids from small openings, etc.

2. The task of the auxiliary rocket (Figure 53) is to lift the above-described composite rocket from a height of 5,550 m to 7,750 m and, after its own propellant is burned up, to impart an initial velocity of 500 m/sec to the main rocket (A. R.). Its weight with propellant is 220 kg and it operates for 8 sec, during which time it imparts an acceleration of 100 m/sec^2 to the A. R. The auxiliary rocket is fitted onto the stabilizers of the A. R., in its slots (*b*), and its oxygen tank (*a*) is located in the nozzle of the A. R. For strength, the A.R. is braced on the outside with rings which drop off at the same time as the auxiliary rocket does. Figure 54 shows schematically the positions of all

three rockets: the hydrogen rocket (dashed lined), the alcohol rocket (solid lines), and the auxiliary rocket (shaded).

3. Pumps $P_{1,2}$ will be of greater significance, the more the rocket weighs.

175 4. The greater the rocket is, the higher will be the weight ratio of the loaded and empty rockets $\left(\frac{m_0}{m_1}\right)$.

5. The composite rocket shown in Figure 49 is quite complex. If very high altitudes do not have to be reached, then by gradually excluding individual parts of it the height can be reduced (300, 250, or 100 km).

WHAT OBERTH CONSIDERS NOVEL ABOUT HIS SUGGESTIONS

1. Use of liquid fuel instead of the solid or powdered fuel proposed up till now. The advantages of a liquid fuel are: a) the velocity can be regulated; and b) a higher ratio $\frac{m_0}{m_1}$ can be obtained, as well as a greater exhaust velocity, so that lighter gases are ejected and, because of the more suitable nozzle shape, the fuel will be used more efficiently.

(176)

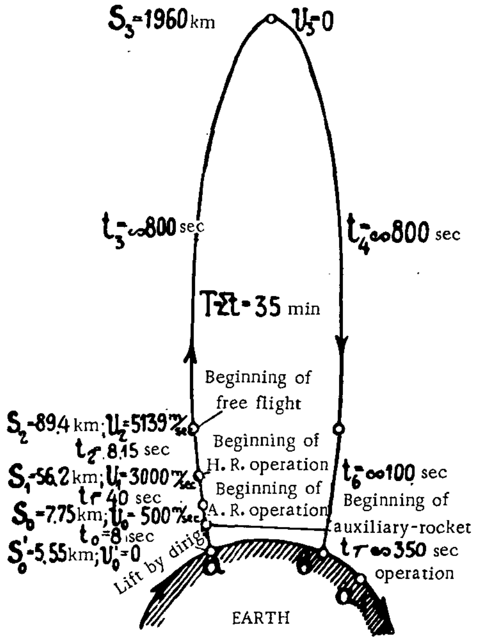

FIGURE 55. Flight path of Oberth's rocket.

2. Separation of the rocket into parts. The advantages are: a) less dead weight is carried into space, and b) the individual component rockets can be constructed in accordance with the tasks to be performed.

3. Velocity regulator, ascent technique, chamber pump, and evaporation by means of bubble injection. Finally, some new formulas (3 to 11) are suggested, and the effect of acceleration is studied.

Figure 55 shows the flight path of Oberth's rocket, plotted by us on the basis of his data. From point (*a*) at sea level the rocket ascent is begun using dirigibles (Figure 46), to a height $S_0^1 = 5.55$ km. Here the rocket separates from the dirigibles and during the course of 8 sec the auxiliary rocket raises it to a height $S_0 = 7.75$ km, where it has developed a velocity $V_0 = 500$ m/sec. At this height the auxiliary rocket falls away and the alcohol rocket begins to operate. The latter raises the apparatus to a height $S_1 = 56.2$ km in 40 sec, where the velocity will be $V_1 = 3{,}000$ m/sec. Here the A. R. falls
176 away and the hydrogen rocket begins to operate, up to a height $S_2 = 89.4$ km, which is attained in 8.15 sec, giving a velocity $V_2 = 5{,}139$ m/sec. Then the propelling portion of the hydrogen rocket falls away and only its upper compartment with the stabilizers remains, continuing to a height $S_3 = 1{,}960$ km. Finally, the rocket describes an ellipse back to earth to point (*b*), which will lie behind (to the west of) the launching point (*a*), which during this time has shifted to some point a_1.

Part III. Thoughts about the Future

EFFECTS OF ABNORMAL ACCELERATIONS ON HUMAN BEINGS

Oberth cites the following examples of abnormal accelerations which have taken place, indicating how these have affected human beings.

1. A fireman jumped from a height of 25 m and landed flat on a canvas, depressing it 1 m. The fireman was not injured, although the acceleration attained was about 240 m/sec^2.

2. A swimmer jumped upright from a height of 8 m into water, without injury. The acceleration attained was about 40 m/sec^2.

3. A swimmer dove backward into water from a height of 2 m gliding
177 over the surface somewhat while lying flat on his back. In this case the skin of the back experienced an acceleration of 200 m/sec^2, the back muscles and kidneys an acceleration of 160 m/sec^2, other parts of the body 80 m/sec^2 and the head and bones 70 m/sec^2.

In general a person can stand a greater acceleration effect if it is directed from head to feet, rather than the reverse. An even greater effect can be endured in a recumbent position or along a tangent.

4. During the war an aviator flying at 60 m/sec executed four loops of a spiral 140 m in diameter without injuring himself; in this case, for 29 sec an acceleration effect of about 51.5 m/sec^2 was experienced.

On the basis of these events and other considerations, Oberth assumes that a person can endure an acceleration of about 51.2 m/sec^2 for 200 to 400 sec. A lower acceleration would not have any harmful physiological effects.

PHYSIOLOGICAL EFFECT OF ABNORMAL ACCELERATION

The organs which sense the effect of acceleration are situated at the vestibule of the internal ear of the human being, where the fluid [lymph] of the ear, the hair cells, and the lime crystals are located. For different positions of the body and motions of it, these crystals press on the corresponding hairs and transmit the sensation to the brain. This sensation of the acceleration effect may be different in different cases. Let us consider some examples.

1. **A carousel.** In a carousel the ceiling, which has seats suspended from its outer ring, rotates. If the radius of the carousel is 4 m, the length of the suspending arms is 2 m, and one rotation is completed in 6.5 sec, then the seats will swing 1.15 m outward and will move around a circle 5.15 m in radius. Here the velocity of the seats will be 5.1 m/sec, and the centrifugal acceleration will be 5 m/sec^2. The acceleration effect reaches 11 m/sec^2 and is inclined 26.6° to the vertical. In spite of this, passengers with their eyes closed can indicate the vertical accurately.

2. **Circular flight with banking of airplane.** In contrast to the preceding case, an aviator experiences a different sensation when he flies in a circle of radius 520 m at a speed of 190 km/hr in a banking aircraft. The earth seems to him to be tipped rather than immovable.

The effect of acceleration is unpleasant during such a circular flight, and it is even more unpleasant when there are slight rises and dips in the motion (tossing of the ship). On the other hand, rapid decelerations have less effect. For example, if an elevator descends at a speed of 1 m/sec, and if it comes to a stop over a distance of 20 cm, the effect of the acceleration will be $2.5+g$ m/sec^2 during $^2/_5$ sec. Greater discomfort will be experienced than in the case of a dive into water, where this effect will be $25+g$ m/sec^2 during the same $^2/_5$ sec. Similarly, these effects will be felt differently depending on whether they are unexpected or known about in ad-
178 vance, whether they are voluntary or involuntary, etc.

An increase in acceleration need not, in Oberth's opinion, cause illness or unpleasant feelings in a passenger. A decrease in acceleration, on the other hand, will cause fear during the first fraction of a second, but this will be less: 1) the more frequently we are subjected to this experience, and 2) the more prepared we are for its occurrence. This feeling of fear gradually disappears, although during the first seconds it will seem to last very long.

When a person begins to fly rockets, he should first make comparatively low flights (50 to 200 km), which will take from 100 to 200 sec. Then, once he has become accustomed to the effect of acceleration, he can ascend higher. For training purposes, and in order to determine the effect of acceleration, Oberth suggests constructing a special large ($r=60$ m) carousel, in one of the wagons of which an experimenter can sit.

Assuming that a person can safely bear an acceleration effect of 40 m/sec^2, which corresponds to a vertical acceleration of $40-g=\sim$ 30 m/sec^2, in order to obtain, for example, a velocity $v_1=900$ m/sec, a time of 300 sec will be necessary. However, for an ideal velocity

$$\left(v=\sqrt{2g_1 h_1}=11.160 \text{ m/sec for } h_1=r+70 \text{ km}\right),$$

which under normal conditions (ascent from a small height) will be reduced by terrestrial gravity by an amount $\int_0^{301} g\,dt = 2{,}400$ m/sec and by the air resistance (200 m/sec), the acceleration effect will not be dangerous to a person.

PASSENGER ROCKET

Figure 56 shows a plan for another passenger rocket, which consists of three parts: an upper part containing the parachute f' and the passenger

(179)

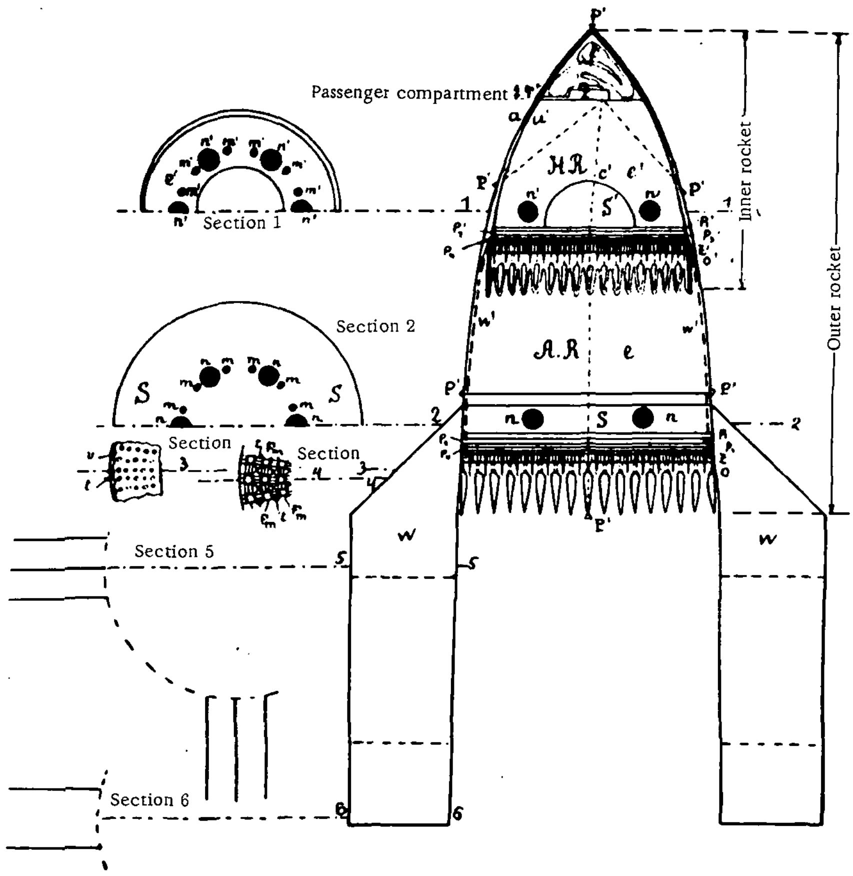

FIGURE 56. Oberth's composite passenger rocket.

compartment *I*, a middle part (the hydrogen rocket, H. R.), the top of which surrounds the upper part, with stabilizers extending downward to the injector of the lower rocket, and a lower alcohol rocket (A. R.), the upper part of which envelops all the parts above it. The A. R. operates first, up to a certain altitude, and then it falls away. The H. R., which has left the A. R., then begins to operate. The passenger compartment with the parachute then separates from the H. R. and completes the rest of the flight, remaining connected to the H. R. just by electrical wires. The parts of the alcohol rocket are indicated by unprimed letters and those of the hydrogen rocket by primed letters. These parts are listed in the following table.

(180)

Alcohol rocket	Hydrogen rocket	Rocket part
a	a'	Tip of rocket
—	f'	Parachute
—	T'	Passage way to passenger area I
e	—	Container for water and alcohol
—	e'	Hydrogen container
S	S'	Oxygen container
—	I	Compartment for passenger and instruments
	P'	Periscopes
m, n	m', n'	Pumps for hot gas
$P_{1,2}$	$P'_{1,2}$	Pump chambers for fuel
$P_{3,4}$	$P'_{3,4}$	Pump chambers for oxygen
F_m	—	Minimum nozzle section
z	z'	Injector
l	l'	Regulating pins*
t	—	Nozzle wall
v	—	Rear channel t and channel regulators
w	w'	Stabilizers
O	O'	Combustion chambers

* These pins project into the nozzle throat and reduce its section, regulating the pressure P_0 in the injector and combustion chamber and making it independent of the recoil P'.

The passenger rocket will be launched at sea, far from any populated places, so that the A. R. will not cause damage when it falls from a height. When container S is empty, the rocket floats on the surface of the water in a tilted position (Figure 57a), and when the container is full of fuel, the rocket is vertical (Figure 57b) but does not sink. The walls of the passenger compartment are from 1.5 to 2.5 cm thick and are made of aluminum.

179 Oberth considers three possible types of emergency during the ascent: 1) failure of pumps, 2) loss of equilibrium, and 3) explosion. He assumes that these emergencies should not represent any danger to a passenger.

1. If the pumps stop working, the apparatus will keep floating on the water.

2. In the event of breakage or incorrect operation of the stabilizers, the pilot can restore equilibrium with the aid of the appropriate pump operation.

3. An explosion in the A. R. will just cause ejection of the H. R., while an explosion in the H. R. will eject the passenger compartment, such explosions being in general of low probability.

Collisions with meteors are difficult to prevent. However, even if a hole is made in compartment *I*, it is fairly easy to seal a small opening* and then to refill the compartment with air.

180 When the rocket falls into the water, it will float. For descent onto land, the parachute should be used.

Instrumentation. For the initial flight direction inclined toward the east, two gyroscopes should be used (with vertical and horizontal axes), which should give a stable trajectory. It would also be useful to have a third gyroscope with its axis perpendicular to the axes of the first two, to control them.

The acceleration should be determined in the directions of the three coordinate axes. The instruments for measuring acceleration are to be connected to the gyroscopes. From the accelerations, the flight velocity is determined, as well as the spatial coordinates of the rocket relative to the center of the earth or the sun. Oberth also gives a plan for constructing such a device, together with a brief theoretical description of its operation in the sphere of terrestrial gravitation.

181

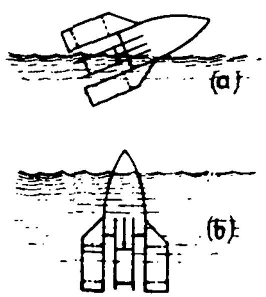

FIGURE 57.

FIGURE 58.

The acceleration effect is measured using a special device (Figure 58). Tube G_1 is immersed in container G_2, but does not reach the bottom. Above the two tubes the volumes of air L_1 and L_2 are such as to maintain the mercury column (shaded) at equilibrium. In the figure d_1 and d_2 indicate the ends of wires leading to an electrical meter; a current of a certain intensity flows in these wires. The wires are attached to floats which rest on the mercury surface (shown in black on the drawing). As the mercury levels fluctuate, the floats sometimes approach and sometimes recede from one another, moving along the amalgam or gold-plated surface of tube Q, thereby increasing or decreasing the resistance to the current flowing in the wires and producing different readings on the electrical meter. The variation of the distance between floats depends on the acceleration effect. When the latter increases (if the device moves upward), the mercury in the upper tube drops and that in the lower tube rises, varying the reading of the electrical meter, which must be calibrated appropriately.

A passenger can determine his position (v and h), by observing the apparent diameter of the earth and its position among the stars. Windows are provided in compartment *I*, so as to make such observations possible.

* Meteors greater than 2 cm in diameter are very rare.

Figure 52 shows the mutual positions of passenger compartment *I*, parachute *f'*, flaps *a'*, *a'*, the hydrogen rocket H. R., and its stabilizers *w'* during free flight (without thrust) in outer space. In this case an acceleration effect is experienced. It is important to provide regulation of the heating and cooling of compartment *I* and the H. R. during insolation, which equals about 2.3 g cal/cm^2. According to the law of heat transfer, a small sphere floating freely in outer space will be heated to 240° above absolute zero, after which equilibrium between the influx and efflux of heat will exist. In order to attain such equilibrium and to maintain a moderate temperature inside compartment *I* (25°C), one side of the rocket should be white and the other side black. By rotating the compartment appropriately relative to the sun, the desired temperature inside of it can be obtained. At considerable distances from the sun, compartment *I* may have the shape of a half cylinder, blackened on its rectangular wall; it can then be turned toward the sun for optimum heat absorption. In addition, the inner surfaces of flaps (*a*) may be mirrors which direct reflected light into *I*.

In order to prevent evaporation of the hydrogen inside the H. R. during free flight, one wall of this rocket should be light in color and turned toward the sun. During descent the compartment should be pulled back into the H. R. In order to provide air for breathing in compartment *I*, containers with liquid oxygen and nitrogen must be available; these substances are converted
182 to gas a little at a time during flight, either under the influence of solar heat or with the aid of artificial heating. The used air is absorbed by kalium causticum.* For extended flights it moves through a black tube to the shady side; there all the harmful admixtures are taken out, leaving only gaseous oxygen and nitrogen, which move through a tube to the sunny side, where they are heated and then returned to compartment *I*. In order to cleanse the black tube of sediments, from time to time it is rotated to the sunny side, separated from compartment *I*, and opened; then the sediments are vaporized and removed.

Compartment *I*, like the rocket itself, is provided with periscopes. Compartment *I* is 2 m in length, with a cross section of 1.1 m. During the ascent and descent a passenger lies on a suspended couch. The rest of the time he can move about freely in the compartment.**

During flight in interplanetary space without gravity, an observer can go out of the rocket through a double door (lock). Then, joined to the rocket by a line, he can ride with it through space (Figure 59). In order to protect himself from the cold, he must wear a space suit; the latter, made on the same principle as a thermos bottle, would keep the heat of the body from passing outward. In addition, the space suit can be black on one side and white on the other, and the black side can be turned toward the sun so as to be heated. Finally, solar radiation can also be reflected toward the observer using mirrors on the rocket.

* [Caustic potash, KOH.]

** Oberth is somewhat vague about the size of compartment *I*. Judging by Figure 56 and the above statements, the height of the chamber should not be less than a person's height, that is, about 2 m. Then, according to the scale of the drawing, the whole rocket must have a height of about 110 m; however, if the length of compartment *I* in Figure 56 is 2 m, the height of the rocket will be about 22 m. Then the height of compartment *I* will be only 50 cm, that is, too small for walking.

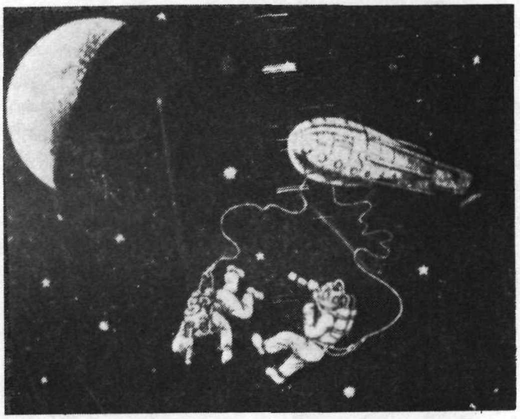

FIGURE 59. Observers outside a rocket.

PROSPECTS FOR THE FUTURE

The cost of a research rocket (Figures 48 and 49) will be about 20,000 marks (gold). It will be possible to make a number of scientific discoveries using such a rocket. A large rocket (Figure 56) capable of carrying a per-
183 son into interplanetary space, on the other hand, will enable man to make many more new and valuable discoveries, such as those on a flight around the moon (at a velocity $v_1 = 11$ km/sec). A passenger rocket will cost about 1,000,000 marks, but it will be good for up to 100 ascents, and on each ascent it will lift a considerable weight into space (see following table).

WEIGHT OF PASSENGER ROCKET

Alcohol	25,000 kg
Hydrogen	4,000 kg
Oxygen, water, etc.	271,000 kg
Total for one passenger	300,000 kg
Rocket weight for two passengers	400,000 kg

A rocket like this can fly around the earth as well as around the moon. Contact between the earth and the rocket can be maintained with the aid of small rockets. If an extended stay in such a rocket of an "observing station" is found to have unpleasant physiological consequences, due to the absence of an acceleration effect, then two rockets can be launched, these being joined by a wire one km long, and the two rockets can be made to revolve about one another.

The following studies will be possible using such an interplanetary station:

1. Determination, using the appropriate instruments, of all the details of the earth's surface.

2. Transmission of light signals or electrical signals to the earth.

3. Warnings to ships concerning icebergs, to their country concerning the approach of an enemy, etc.

4. Transmission of solar heat energy to northern lands with the aid of mirrors. This could melt the eternal ice of the north and transform uninhabited areas into fertile, populated regions. To do this, a network of wires (Figure 60) might be deployed around the rocket, by rotation; a mirror could then be mounted on the wires and tilted as desired with the aid of an electric current, so as to direct the sun's rays either toward the earth (Figure 61b) or away from the earth (Figure 61a). Oberth assumes

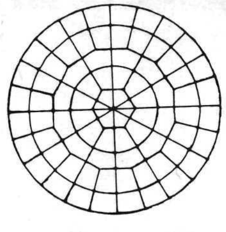

FIGURE 60.

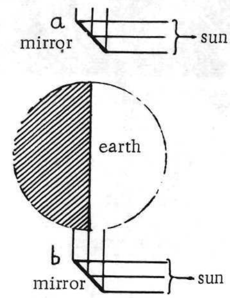

FIGURE 61.

a mirror diameter of 100 km. Sodium may serve as the mirror material (spec. weight of unity with high strength). The thickness of the reflecting
184 layer is taken to be 0.005, for a total mirror weight of 10 g per m^2 or 100 g per hectare, giving a mirror cost of 3,500 marks per hectare. One ascent of a rocket carrying a load of 2,000 kg of sodium comes to 60,000 marks. A mirror 100 km in diameter will be constructed in 15 years, at a cost of 3 billion marks, if 100,000 kg of sodium are sent up each week. Using such a mirror, it would be possible to blow up enemy storehouses, to cause waterspouts and hurricanes, to burn up entire cities, etc.

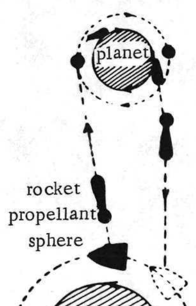

FIGURE 62.

5. If a rocket is considered as an interplanetary station, and if it has sufficient fuel reserves aboard, then other rockets can be sent from it to study other worlds. The energy required to propel these rockets would be incomparably lower than that required to launch them from the earth, since the earth's attraction and the air resistance would be considerably less. A fuel supply, for example, in the form of a sodium (Natriumblech) sphere, could be connected to a powerful rocket and sent off to some other planet. At a specified height above the planet the sphere could be detached and left to revolve around the planet. The rocket itself would then land on the planet to carry out studies, after which it would ascend once again, join the sphere, and fly back. *Figure 62 shows a plan for such an interplanetary voyage.*

OBERTH'S COMMENTS ON THE WORKS OF GODDARD

In 1919 Robert Goddard, an American professor, published a paper entitled "A Method of Reaching Extreme Altitudes" in the Collections of the Smithsonian Institution. In this paper Goddard describes the results of his preliminary experiments, concerning which Oberth could only make theoretical proposals and which complemented the studies of Oberth. For instance, using smokeless nitrocellulose powder and for a funnel-shaped nozzle with an 8° slant relative to the axis, Goddard was able to utilize $64\frac{1}{2}\%$ * of the entire burst energy for the recoil, whereas previous rockets had never been able to utilize more than 2%. In addition, Goddard found that the efficiency of a rocket increases with an increase in the nozzle size, keeping the same ratio of nozzle volume to weight of powder, a result which is explained by the relative difference in the effects of gas friction at the walls of large and
185 small nozzles. Moreover, Goddard took special pains to obtain a smooth surface inside the nozzle. Finally, Goddard showed, on the basis of experiments in a vacuum, that the efficiency of a rocket is greater under such conditions, due to the lack of air resistance.

On the basis of experiments with different propellants, Goddard obtained the following figures:

"Infallible" powder (Hercules Powder Co.): 1,238.5 cal/g released during burning, with a velocity of gas ejection equal to 2.434 km/sec.

"Dupont Pisolen Pulver No. 3": 972.5 cal/g and 2.290 km/sec.

Goddard suggests constructing the propelling apparatus like the muzzle of a gun which has automatic, rapid injection of cartridges one after another into the breech part. Goddard also proposes sending his rocket to the moon, where it would explode upon landing, causing a blast which could be observed from the earth. In conclusion, Oberth states that his work was independent of that of Goddard and that he began his rockets in 1907.

OBERTH'S REPLIES TO CRITICISM OF HIS PLAN. SUPPLEMENT TO SECOND EDITION

1. The container for the liquid oxygen should be made of sheet copper, the hydrogen rocket being made of lead.

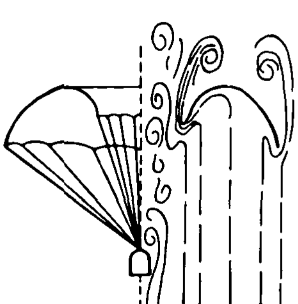

FIGURE 63.

2. The temperature of the exploding gases will be higher than that assumed, so that the results of the burning will be favorable.

3. Critical comments which have been made concerning the possibility of employing a parachute during descent do not constitute a serious objection, in general, to carrying out a safe landing. Rocket bursts can also be used to slow the descent, but this will require making the entire apparatus heavier. The descent can be braked at first using bursts and then a parachute can be used afterward. The latter may be ring-shaped (see Figure 63), in which case the heating will

* Cf. coefficient of energy utilization of fuel in diesel motors (40%) and steam engines (21%).

be less for a rapid descent. Use of the parachute may begin at a height of 7 km. Since a passenger rocket will fly along a second-order curve during descent, rather than come in perpendicular to the earth's surface, its path to the ground will be quite long. Assuming a parabolic path, let us determine the length of the flight path from a height of 7 km to the ground.

In polar coordinates the equation for the parabola is

$$\varrho = \frac{p}{1+\cos\varphi}; \quad \cos\varphi = \frac{p}{\varrho} - 1.$$

186 where ϱ is the radius vector, φ is the angle of deviation, and p is the parabolic parameter: $p = 2r$, where r is the radius of the earth.

For $\varrho = r$ we have cos $\varphi = 1$ and $\varphi = 0°$. For $\varrho = r + h$, where h is the height (7km), we have

$$\cos\varphi = \frac{p}{r+h} - 1 = \frac{\frac{p}{r}}{1+\frac{h}{r}} - 1 = \frac{2}{1+\frac{7}{370}} - 1 =$$

$$= \infty\, 2\,(1 - 0.0011) - 1 = 0.9978;\ \varphi = \pm 3.8°.$$

This gives a path length, for the entire descent, of

$$s = 2\ \ r = 840 \text{ km}.$$

However, it should be noted that during the return of the rocket to the earth the parabolic velocity gradually becomes an elliptic velocity, and then a circular velocity, that is, it decreases, the path of the rocket at approach to the earth being helical. Therefore, the parachute will also have a gradual effect.

4. Stabilizers are actually not needed for the hydrogen rocket, since it operates in a near vacuum. However, they can be used as controls, since the gases escaping from the nozzle will strike them; the gas efflux can be regulated by the pins shown in Figure 48 below the atomizer.

Figure 64 shows the descent of a rocket onto water with the aid of a parachute, and Figure 65 shows a rocket being braked by ejected gases. The effect of the gases facilitates the work of the parachute (figs. from M. Valier).

Some details of the construction of Oberth's rocket for flight to an altitude of 50 km are given in a work entitled "Die Möglichkeiten der Weltraumfahrt," Leipzig, 1928, p. 130. A discussion of the control of a rocket is included in this same work (p. 136 and 216).

OBERTH'S MOST RECENT WORKS

Oberth, H. "Grundprobleme der Raumschiffahrt" (paper in book "Die Möglichkeiten der Weltraumfahrt," Leipzig. 1928).

"Der Raketenantrieb bei Flugzeugen" (1931).

(187)

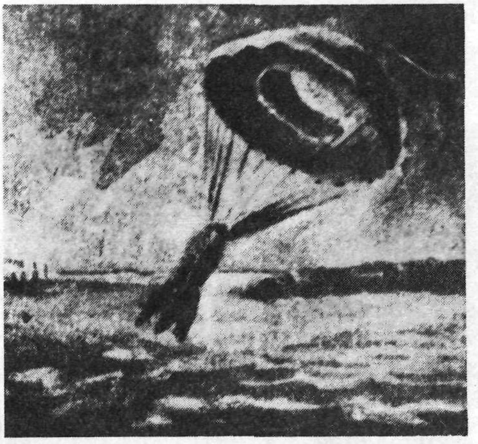

FIGURE 64. Parachute descent of rocket.

(187)

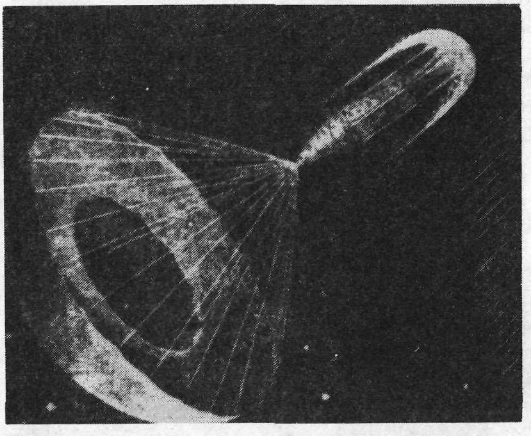

FIGURE 65. Rocket descent with the aid of parachute and backward reaction.

Figure 66 shows a model of one of the rockets attributed to Oberth. It was on display at the Exhibition of Interplanetary Rockets at Moscow in 1927.

Oberth has proposed constructing a recording rocket for heights above 70 km, in which the nozzles for gas ejection are located at the head of the rocket (Figure 67), the propellant being carried aft in the tail section and pumped up to the nozzles.

Note. In "Kosmos" (1925, S. 149) Heinrich Hein presents some calculations for such a rocket. He takes the height of ascent to be 6,400 km. The terminal velocity is found to be 800 km/sec, for a flight duration of 70 min. *If the rocket is launched from the equator along the earth's radius, then due*

187 to the rotation of the earth (at 480 m/sec near the equator) it will land 4,000 km to the west, after describing an ellipse.

(188)

FIGURE 66. Rocket attributed to Oberth.

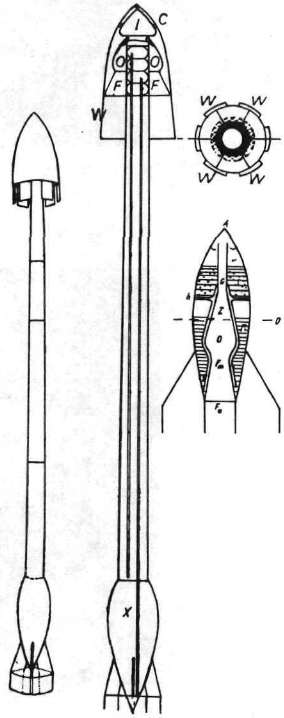

FIGURE 67. Oberth's rocket.

For the flight of a passenger rocket Oberth suggests launching the rocket
188 not vertically, that is, along the earth's radius, but rather at an angle, along a curve called by him a "synergic" curve. Thus the acceleration during the upward flight can be increased, since the effect of the earth's acceleration is essentially paralyzed, the takeoff being nearly parallel to the earth's surface.

WALTER HOHMANN

[SOVIET] TRANSLATOR'S FOREWORD

In 1925 Walter Hohmann, a German engineer, published a work entitled "The Attainability of the Celestial Bodies" ("Die Erreichbarkeit der Himmelskörper"), * in which he studied the conditions of rocket flight into outer space. This study was based on an analysis of the mechanics and mathematics of space flight, and in it the problems of flight trajectories and landings on planets were developed in a particularly interesting manner. A complete translation of this work is given below, preceded by a brief biographical note sent to us by the author.

In 1929, for his work and, in particular, for his idea of a gliding rocket descent to the earth, Hohmann was awarded the second REP-Hirsch prize (France). **

* Later he wrote another paper, entitled "Fahrtrouten, Fahrzeiten, Landungsmöglichkeiten" (included in the book: "Die Möglichkeit der Weltraumfahrt," Leipzig, 1928, p. 177).

** [Actually, the REP-Hirsch prize was not awarded in 1929 (Translator).]

190 BIOGRAPHICAL NOTE ON WALTER HOHMANN

Walter Hohmann (Figure 68) was born at Hardheim am Odenwald, the son of a doctor, on 18 March 1880. His secondary education was obtained at the gymnasium in Würzburg, where he studied from 1891 to 1900. For his higher education, Hohmann attended the Technische Hochschule in Munich (from 1900 to 1904), where he majored in mathematics and theoretical mechanics under Prof. Finsterwalder and Prof. Föppel. Upon completion of his studies at this institution, he worked as a construction engineer (Bauingenieur): in Vienna from 1904 to 1906, in Berlin from 1906 to 1908, in Hannover from 1908 to 1911, in Breslau from 1911 to 1912, and in Essen in 1912.

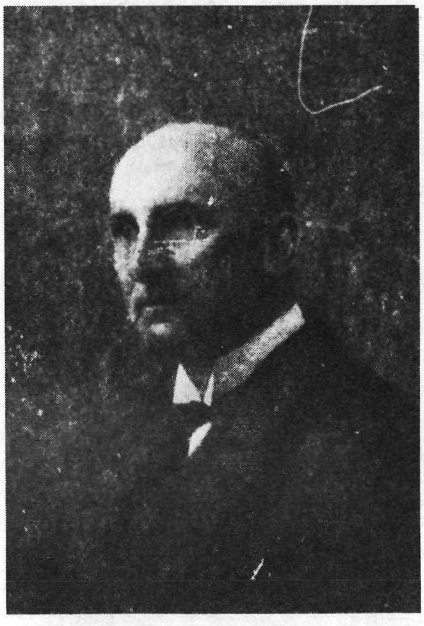

FIGURE 68. W. Hohmann.

His scientific works deal with the statics of structures and with reinforced concrete. The subject of interplanetary travel began to interest Hohmann in 1914, when he started to prepare the work mentioned above; he focuses particular attention on the astronomical and ballistic aspects of the subject. Now let us go on to our translation of Hohmann's book.

191 *THE ATTAINABILITY OF THE CELESTIAL BODIES*

FOREWORD

The present work has the following goals: to evaluate, with the aid of a mathematical study, the difficulties involved in solving the problem of interplanetary travel, and to demonstrate that, with an appropriate development of the technical means already at man's disposal, this problem can be brought to a successful solution.

In his first studies, which were carried out about 10 years ago, the author assumed the upper limit for the gas velocity during a rocket burst to be 2,000 m/sec, the velocity attainable at that time. Consequently, all the calculations were at first carried out for that velocity. However, since then three works on rocket flight have appeared which indicate that the above velocity may be higher. These works are:

Goddard. "A Method of Reaching Extreme Altitudes" (based mainly on Goddard's own experiments);

Oberth. "Die Rakete zu den Planetenräumen" (valuable for its theoretical studies);

Valier. "Der Vorstoss in den Weltraum" (general statement of the problem).

On the basis of these studies, and, in particular, from a comparison of my results with those in Oberth's book, I carried out additional calculations for higher gas velocities during a burst (2,500, 3,000, 4,000, and 5,000 m/sec), taking the above-mentioned velocity of 2,000 m/sec to be a minimum initial value. The results of these calculations turned out to be more promising.

In connection with this, the following should be noted. When comparatively low gas velocities are used, an attempt must be made to eliminate all dead weight (ballast). This led to the idea of portraying the fuel of a rocket in the form of a tower, composed of a solid explosive material, which gradually becomes smaller as its component substance burns up. Such a device would represent an ideal solution, with no dead weight present; however, it is feasible for comparatively low gas velocities. For higher velocities, according to Oberth, the gas must be ejected through a narrow nozzle. But the use of the latter, like the use of a liquid fuel, entails provision of the
192 appropriate containers and casings, making the dead weight more or less considerable, the propulsion of the additional weight being easier, the higher the velocity of gas ejection.

The total rocket weights given in the last two chapters [parts] of this work were determined without taking these dead weights into account, since it was difficult to determine their values without carrying out experiments on the best shapes and materials for the containers and nozzle. The weights G_0, indicated there, represent the lower limit for the optimum fuel.

In my treatment of a number of subjects I am indebted to the works of Oberth and Valier: the influence of high gas velocities, certain further improvements, and, in particular, the possibility of descending to a planet without using a braking ellipse (see end of Part II), as well as problems related to an intersecting ellipse (end of Part V) and heating phenomena during descent.

At times approximate formulas were used in the calculations instead of precise mathematical expressions, for the simple reason that the author is an engineer rather than a mathematician. However, this does not greatly affect the final results obtained.

W. Hohmann

Essen, October 1925

Part I

193 *DEPARTURE FROM THE EARTH*

Let us assume that we are beyond the influence of gravity, aboard a rocket of mass m which is at rest. Now we can impart to the rocket a certain velocity Δv in any direction, provided we eject from it a portion of the mass Δm in the opposite direction, with a velocity c relative to the rocket. Since the center of mass (center of gravity) of system m as a whole remains the same, therefore for a certain time t we have (see Figure 69):

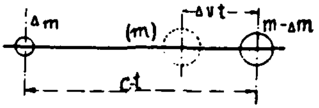

FIGURE 69.

$$\Delta m (c \cdot t - \Delta v \cdot t) = (m - \Delta m) \cdot \Delta v \cdot t,$$

or

$$\frac{m - \Delta m}{\Delta m} = \frac{c - \Delta v}{\Delta v},$$

or

$$\frac{m}{\Delta m} = \frac{c}{\Delta v}, \tag{1}$$

and thus

$$\Delta v = c \cdot \frac{\Delta m}{m}.$$

Consequently, a single ejection of part of the mass Δm at a velocity c imparts to the remaining mass $(m - \Delta m)$ a velocity from the starting point given by

$$\Delta v = c \cdot \frac{\Delta m}{m},$$

directed in the opposite direction relative to Δm. This velocity will endure until a new ejection of mass modifies the motion of the rocket.

If a part of the mass $\frac{dm}{dt}$ is ejected during each second, at a constant rate c, then the remaining mass has an acceleration

$$\frac{dv}{dt} = \frac{c}{m} \cdot \frac{dm}{dt} \tag{1a}$$

with a gradual reduction of the mass m.

194 Let us assume the fuel consumption to be so regulated that at any moment the amount of fuel required per second, $\frac{dm}{dt}$, is proportional to the remaining mass m. Then

$$\frac{dm}{dt} ; m = a = \text{const.}$$

In this case the acceleration will be uniform and independent of the mass:

$$\frac{dv}{dt} = c \cdot a \qquad (1b)$$

as long as the velocity of gas ejection does not change.

The mass consumption obeys the law

$$\frac{dm}{dt} = -am \qquad (1c)$$

(the right side being negative because m decreases with an increase in time). Therefore,

$$\int \frac{dm}{m} = -a \int dt$$

and after integration we have

$$\ln m = -at + C.$$

For initial conditions $t = 0$ and $m = m_0$,

$$\ln m_0 = 0 + C \text{ and } C = \ln m_0;$$

so that

$$\ln m = -at + \ln m_0$$

or

$$\ln \frac{m}{m_0} = -at,$$

or

$$\frac{m}{m_0} = e^{-at} \quad \text{or} \quad \frac{m_0}{m} = e^{at} \qquad (2)$$

and thus after a time t the following mass remains:

$$m = \frac{m}{e^{at}}.$$

If a rocket with an acceleration ca of its own is subjected to a gravitational acceleration g of opposite sign, then the total acceleration will be

$$\frac{dv}{dt} = ca - g.$$

For example, let us assume that a rocket located a distance r from the earth's center moves away from the earth in a radial direction. We designate the acceleration of gravity at the earth's surface as g_0, the radius of the
(195) earth being r_0 (Figure 70). In this case the acceleration of gravity, which is directed opposite to the rocket's own acceleration, will, at a distance r, be equal to *

$$g = g_0 \frac{r_0^2}{r^2} \tag{3}$$

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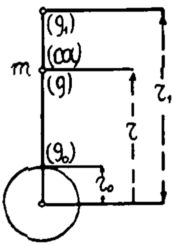

FIGURE 70.

and the total acceleration is

$$\frac{dv}{dt} = ca - g_0 \frac{r_0^2}{r^2}.$$

In addition,

$$\frac{dr}{dt} = v,$$

from which we have

$$\frac{dv}{dr} = \frac{ca - \frac{g_0 r_0^2}{r^2}}{v}; \int v\,dv = \int \left(ca - \frac{g_0 r_0^2}{r^2}\right) dr;$$

and

$$\frac{v^2}{2} = car + \frac{g_0 r_0^2}{r} + C.$$

For initial conditions (at the earth's surface) $r = r_0$ and $v = 0$, we can write

$$0 = car_0 + \frac{g_0 r_0^2}{r_0} + C,$$

which gives

$$C = -car_0 - g_0 r_0 = -r_0 (ca + g_0).$$

Therefore,

$$\frac{v^2}{2} = car + \frac{g_0 r_0^2}{r} - r_0 (ca + g_0) = (r - r_0) \cdot \left(ca - g_0 \frac{r_0}{r}\right) \tag{4}$$

If at a distance r_1, where a maximum velocity v_1 is attained, the rocket's own acceleration ceases, the rocket will behave like a body thrown up vertically with an initial velocity v_1. At a distance

$$r' > r_1$$

* Some remarks concerning the law of gravity are given at the end of Part III.

its velocity will be

$$v' = \frac{dr'}{dt}$$

and the retardation will be

$$\frac{dv'}{dt} = -g_0 \cdot \frac{r_0^2}{r'^2}.$$

From the last two equations, we obtain

$$v'\,dv' = -g_0 r_0^2 \frac{dr'}{r'^2}$$

or

$$\frac{v'^2}{2} = +\frac{g_0 r_0^2}{r'} + C,$$

but since

$$C = \frac{v_1^2}{2} - \frac{g_0 r_0^2}{r_1},$$

therefore

$$\frac{v'^2}{2} = \frac{gr^2}{r'} + \frac{v_1^2}{2} - \frac{g_0 r_0^2}{r_1} \qquad (5)$$

196 If the rocket attains a great enough maximum velocity v_1 at a distance r_1 from the center of attraction, then the cessation of its own acceleration ca will not cause it to fall back under the influence of gravity. In such a case the final velocity $v' = 0$ only for $r' = \infty$.

Then, from equation (5),

$$\frac{v_1^2}{r} = \frac{g_0 r_0^2}{r_1}, \qquad (6)$$

while, from (4),

$$\frac{v_1^2}{2} = car_1 + \frac{g_0 r_0^2}{r_1} - r_0(ca + g_0),$$

which gives

$$car_1 = r_0(ca + g_0),$$

or

$$r_1 = r_0 \frac{ca + g_0}{c} = r_0\left(1 + \frac{g_0}{ca}\right) \qquad (7)$$

and

$$v_1 = \sqrt{\frac{2 g_0 r_0^2}{r_1}} = \sqrt{\frac{2 g_0 r_0}{1 + \frac{g_0}{ca}}}. \qquad (8)$$

The time t_1 required to reach this distance r_1 and this maximum velocity can be found from the relation

$$\frac{dr}{dt} = v$$

as

$$t_1 = \int_{r_0}^{r_1} \frac{dr}{v} = \int_{r_0}^{r_1} \frac{dr}{\sqrt{2car + \frac{2g_0 r_0^2}{r} - 2r_0(ca + g_0)}} .$$

Since it is quite difficult to find this integral, we must give up trying to calculate t_1 for an acceleration of gravity g which varies with the distance. Instead we take some value g_m between g_0 and g_1 which is, for convenience of calculation, not even the average value

$$g_m = \frac{g_0 + g_1}{2},$$

but rather

$$g_m = \frac{2g_0 + g_1}{3},$$

or, returning to equation (3),

$$g_m = \frac{2g_0 + g_0 \frac{r_0^2}{r_1^2}}{3} = \frac{g_0}{3}\left(2 + \frac{r_0^2}{r_1^2}\right).^*$$

197 The flight time is obtained if, instead of the expression for the total acceleration

$$ca - g_0 \frac{r_0^3}{r^2} ,$$

we use the expression

$$\beta = ca - \frac{g_0}{3}\left(2 + \frac{r_0^2}{r_1^2}\right). \qquad (9)$$

Then, in accordance with equations (7) and (8), we have

$$t_1 = \frac{v_1}{\beta} = \frac{v_1}{ca - \frac{g_0}{3}\left(2 + \frac{r_0^2}{r_1^2}\right)} = \frac{\sqrt{\frac{2g_0 r_0}{1 + \frac{g_0}{ca}}}}{ca - \frac{g_0}{3}\left(2 + \frac{1}{\left(1 + \frac{g_0}{ca}\right)^2}\right)} . \qquad (10)$$

* For low values of ac this average value is acceptable. The following expression would be more precise:

$$g_m = \frac{\xi \cdot g_0 + g_1}{\xi + 1},$$

where

$$\xi = \frac{r_0}{2r_0 - r_1},$$

so that for $ac = g_0$ the total acceleration β is correspondingly zero.

Substituting this value of t_1 into equation (2), we obtain

$$\frac{m_1}{m_0}=e^{-at_1} \text{ or } \frac{m_0}{m_1}=e^{at_1}. \qquad (11)$$

which shows the relationship between the mass m_0 at the beginning of the accelerated motion and the mass m_1 at the end of it (after a time t_1).

The difference $m_0 - m_1$ indicates the weight of propellant ejected at a constant rate c during a time t_1, such that the remaining mass m_1 will attain the highest velocity v_1 at a distance r_1.

The mass m_1 represents the useful load which is liberated from the influence of terrestrial gravity. After determining the velocity of gas ejection c, and the rocket acceleration ca, we can, on the basis of practical considerations, find r_1, v_1, t_1, and m_0 from equations (7, 8, 10, and 11). Table 1 shows how the ratio $\frac{m_0}{m_1}$ is affected by differences in the values of c and ca. Here we assume

$$r_0 = 6380 \text{ km and } g_1 = 9.8 \text{ m/sec}^2 = 0.0098 \text{ kg/cm}^2$$

(in rounded-off figures).

It is clear from the table that ca has less effect then c does. Therefore, it should first be attempted to obtain a value of c which is as high as possible, and then to select an acceptable value for the acceleration of the rocket itself. Passengers will experience the latter as an increase in their weight, so that physiological factors will limit this acceleration.

In order to determine the acceptable acceleration, let us note the following: a person jumping from a height $h = 2$ m attains a velocity $v=\sqrt{2hg_0}$ when he hits the earth; at the moment of contact with the ground, he bends his knees and over a distance of about $h_1 = 0.5$ m the velocity drops to zero. Consequently, the deceleration (β) can be found from the formula

$$v=\sqrt{2h'\beta}.$$

199 From these two expressions we have

$$\beta = g_0 \cdot \frac{h}{h_1} = g_0 \cdot \frac{2.0}{0.5} = 4g_0 = \backsim 40 \text{ m/sec}^2 .$$

Naturally, a person will experience this retardation β for only a fraction of a second, whereas for our rocket its own acceleration ca will last for some minutes. Therefore, it is advisable to assume values of ca between 20 and 30 m/sec^2.*

It is somewhat more difficult to satisfy the requirement that the velocity of gas ejection c be a maximum. The highest velocities attained so far by artillery shells have been about 1,000 to 1,500 m/sec, but such velocities, as Table 1 indicates, give values of $\frac{m_0}{m_1}$ which are too high and thus are unsuitable. Consequently, $c = 2{,}000$ m/sec should be taken as a lower limit, which gives, for $ca = 30$ m/sec^2, a ratio $\frac{m_0}{m_1} = 825$.

For these lower values ($ca = 30$ and $c = 2{,}000$) the following calculations were performed. More favorable results for increased values of c are given

* For details concerning the physiological effects of acceleration, see the work by Oberth mentioned earlier.

TABLE 1.

Rocket's own acceleration (m/sec²)	15	20	25	30	40	50	100	200
$r_1 = r_0\left(1 + \frac{g_0}{ca}\right)$ (km)	10,600	9,510	8,860	8,490	7,950	7,640	7,000	6,680
$v_1 = \sqrt{\frac{2g_0 r_0}{1 + \frac{g_0}{ca}}}$ (m/sec)	8,660	9,150	9,470	9,680	10,000	10,200	10,650	10,890
$\beta = ca - \frac{g_0}{3}\left(2 + \frac{r_0^2}{r_1^2}\right)$ (m/sec²)	7.27	12.00	16.76	21.61	32.35	41.18	90.76	190.46
$t_1 = \frac{v_1}{\beta}$ . . . . . . (sec)	1,192	762	565	448	319	248	117	57
Ratio $\frac{m_0}{m_1} = e^{\alpha t_1}$ for velocity of gas ejection: $c = 1{,}000$ (m/sec)	58,700 000	4,160,000	1,545,000	675,000	346,000	240,000	120,300	89,130
$c = 1{,}500$ "	149,000	25,000	12,000	7,750	4,950	3,840	2,400	2,000
$c = 2{,}000$ "	7,570	2,010	1,160	825	587	495	347	299
$c = 2{,}500$ "	1,270	438	282	216	164	143	108	95.5
$c = 3{,}000$ "	388	159	110	88	70	62	49	44.7
$c = 4{,}000$ "	87.3	44.8	34.1	28.7	24.2	22.2	18.7	17.2
$c = 5{,}000$ "	35.7	20.9	16.7	14.6	12.8	11.9	10.4	9.8
$c = 10{,}000$ "	6.0	4.6	4.1	3.8	3.6	3.5	3.2	3.1

in appropriate places, in the form of a comparison just with the calculation results.

At the beginning of the upward flight (at launch) the amount of gas ejected per second is determined from equation (1c):

$$\frac{dm_0}{dt}=am_0,$$

but

$$a=\frac{ca}{c}=\frac{30\,\mathrm{m/sec}^2}{2000\,\mathrm{m/sec}}=\frac{0.015}{\mathrm{sec}}.$$

and

$$m_0=825\,m_1.$$

Therefore,

$$\frac{dm_0}{dt}=0.015\cdot 825\cdot m_1=12.4\,m_1.$$

Consequently, during the first part of the upward flight, the mass consumed per second will constitute a considerable portion of the remaining useful load.

If the bursts are made to be similar to the firing of a cannon, then in this case it will be necessary to carry a large dead weight, which will increase the initial mass m_0 of the rocket correspondingly. In order to avoid this, we distribute the mass of the fuel m_0-m_1 similarly to that in an ordinary [ship] rocket, so that the combustion products will be expelled into the vacuum of space with a velocity c. Let us assume that the consumption of the fuel mass per second corresponds to the rocket cross section and to the available residual mass of the rocket; then we can assume that each section is proportional to the overlying mass, and the shape of the fuel will be similar to that of a tower with the same resistance to compression (Figure 71).

200 The mass expelled per second through some cross section F is found using equation (1c) and Figure 71:

$$\frac{dm}{dt}=am=F\cdot\frac{dh}{dt}\cdot\frac{\gamma'}{g_0},$$

where g_0 is the acceleration of gravity, and γ' is the specific weight of the tower material, reduced to the value at the earth's surface.

Moreover,

$$\frac{dh}{dt}=\frac{am}{F}\cdot\frac{g_0}{\gamma'},$$

but, since

$$\frac{m}{F}=\frac{m_1}{F_1}=\frac{m_0}{F_0}, \tag{12}$$

therefore

$$dh=\frac{am_1}{F_1}\cdot\frac{g_0}{\gamma'}\,dt$$

and

$$h=\frac{am_1}{F_1}\cdot\frac{g_0}{\gamma'}\cdot\int_0^{t_1}dt=\frac{am_1}{F_1}\cdot\frac{g_0}{\gamma'}\cdot t_1.$$

If we designate $G_1 = m_1 g_0$ as the weight of the residual mass of the rocket relative to the earth's surface, then

$$h = \frac{at_1}{\gamma'} \cdot \frac{G_1}{F_1}, \tag{12a}$$

and, from equation (12),

$$F_0 = \frac{m_0}{m_1} \cdot F_1.$$

For example, assuming that the weight to be lifted G_1 is two tons, for a specific weight of the fuel $\gamma' = 1.5$ tons/m^2, we have in the given case

$$(ca = 30\ \text{m/sec}^2;\ c = 2000\ \text{m/sec}\ ;\ a = 0.015\ \text{sec}^{-1};\ t_1 = 448\ \text{sec};\ \frac{m_0}{m_1} = 825)$$

the following values:

$$h = \frac{0.015 \cdot 448}{1.5} \cdot \frac{2.0}{F_1} = \frac{8.96}{F_1}\ \text{and}\ F_0 = 825\, F_1.$$

If the area of the upper section of the tower is $F_1 = 0.332$ m^2, which corresponds to a circle 0.65 m in diameter, then we obtain

$$F_0 = 825 \cdot 0.332 = 273\ \text{m}^2,\ \text{for a diameter of 18.7 m}$$

and

$$h = \frac{8.96}{0.332} = 27\,\text{m (Figure 72).}$$

The resistance of the material to compression will be, if the rocket's own acceleration $ca = 30$ m/sec^2 (instead of the normal $g = 9.8$ m/sec^2),

$$\sigma = \frac{ca}{g} \cdot \frac{G_1}{F} = \frac{30}{9.8}\ \frac{2\ \text{tons}}{0.332\ m^2} = 18.5\ \text{tons/m}^2 = 1.85\ \text{kg/sec}^2$$

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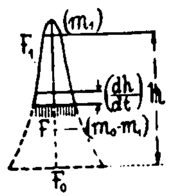


FIGURE 71.

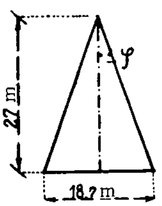


FIGURE 72. Hohmann's rocket

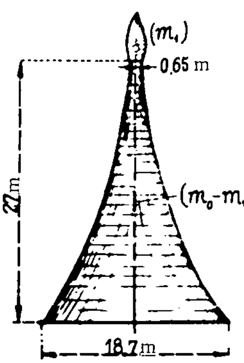


FIGURE 73.

The search for a material of the required strength, which would at the same time yield an ejection velocity c and the corresponding energy, represents a problem in explosives engineering.

We have not yet taken into account the air resistance. Although the rocket shape described above (Figure 72) is suitable for overcoming the air resistance and although high velocities occur only at considerable heights, where the atmosphere is either very tenuous or else nonexistent, still the effect of the dense lower layers of air must be evaluated, if only approximately.

According to Lossel, the resistance W of air with a specific weight γ, through which a body with a midship section F moves with a velocity perpendicular to F, is

$$W=\frac{\gamma\cdot v^2}{g}\cdot F\cdot\psi,$$

(cf. equation (14) in Part II). Here g is the acceleration of gravity, and ψ is a coefficient depending on the shape of the body (for a plane moving in a direction perpendicular to itself, $\psi=1$).

The retardation occurring as a result of this will be

$$\Delta\beta=\frac{W}{m}=\frac{\gamma v^2}{g}\cdot\frac{F}{m}\psi.$$

For the given case, equation (12) gives

$$\frac{F}{m}=\frac{F_1}{m_1}=\frac{0.332}{2000/10}=\frac{1}{600}\ \frac{\mathrm{m}^3}{\mathrm{kg/sec}^2}.$$

202 For a conical tower (Figure 73) we have

$$\psi=\sin^2\varphi=\infty\left(\frac{18.7}{2.27}\right)^2=0.12.$$

Therefore,

$$\Delta\beta=\frac{\gamma v^2}{g}\cdot\frac{0.12}{600}=\frac{\gamma v^2}{g}\cdot\frac{1}{5000} \qquad (13)$$

For the given limits we can take $g=10$ m/sec, and equation (4) gives

$$v^2=2(r-r_0)\left(c\alpha-g_0\frac{r_0}{r}\right),$$

the values for γ being given in Table 3 of Part II. Table 2 shows the results of calculations of $\frac{\gamma v^2}{g}$ kg/m^2 for various distances r.

203 At heights greater than 50 km above the earth's surface, for the velocities attained there, the effect of air resistance according to equation (13) is negligible. Let us consider a more unfavorable case, when, for heights from 0 to 50 km, the average value is

$$\frac{\gamma v^2}{g}=12\,000\ \mathrm{kg/m}^2$$

Then, according to equation (13), the average retardation will be

$$\Delta\beta=\frac{12\,000}{5000}=2.4\ \mathrm{m/sec}^2$$

202) TABLE 2.

r km	$(r-r_0)$ km	$\left(ca-g_0\frac{r_0}{r_2}\right)$ km/sec	v^2 km²/sec²	γ kg/m³ from Tab.3	$\frac{\gamma v^2}{g}$ kg/m²
6,380	0	0.02020	0.00	1.30	0
6,381	1	0.02020	0.04	1.15	4,600
6,382	2	0.02020	0.08	1.00	8,000
6,383	3	0.02020	0.122	0.90	11,000
6,384	4	0.02020	0.162	0.80	13,000
6,385	5	0.02020	0.202	0.70	14,200
6,386	6	0.02020	0.243	0.62	15,100
6,388	8	0.02021	0.323	0.48	15,500
6,390	10	0.02021	0.404	0.375	15,200
6,395	15	0.02022	0.606	0.215	13,000
6,400	20	0.02023	0.810	0.105	8,500
6,410	30	0.02024	1.214	0.0283	3,440
6,420	40	0.02026	1.620	0.0074	1,200
6,430	50	0.02027	2.028	0.00187	370
6,440	60	0.02028	2.434	0.00045	110
6,460	80	0.02032	3.250	0.000023	7.5
6,480	100	0.02035	4.070	0.000001	6.4

and thus at heights below 50 km, instead of $ca = 30$ m/sec², we will have an actual acceleration of

$$ca - \Delta\beta = 30 - 2.4 = 27.6 \text{ m/sec}^2$$

For $r = 6{,}430$ km or $r - r_0 = 50$ km, we have from equation (4)

$$\frac{v^2}{2} = 50\left(0.0276 - 0.0098\frac{6380}{6430}\right) = 0.895 \text{ km}^2/\text{sec}^2$$

instead of

$$50\left(0.03 - 0.0098\frac{6380}{6430}\right) = 1.014 \text{ km}^2/\text{sec}^2,$$

and

$$v = \sqrt{2.0.895} = 1.340 \text{ km/sec}$$

instead of

$$\sqrt{2.1.014} = 1.425 \text{ km/sec}$$

and a corresponding flight time

$$t' = \frac{1340}{27.6 - \frac{9.8}{3}\left(2 + \frac{6380^2}{6430^2}\right)} = 75 \text{ sec}$$

instead of

$$\frac{1425}{30 - \frac{9.8}{3}\left(2 + \frac{6380^2}{6430^2}\right)} = 70.3 \text{ sec.}$$

In all, the time difference $\Delta t = 4.7$ sec.

In addition, the final velocity will be less by an amount

$$\Delta v' = 1.425 - 1.340 = 0.085 \text{ km/sec},$$

and thus the rocket's own acceleration must be prolonged by a time

$$\Delta t' = \frac{\Delta v'}{\beta'} = \frac{85}{30 - 9.8 \cdot \frac{6380^2}{6490^2}} = 3.5 \text{ sec}.$$

The length of the bursts will be, instead of the value $t_1 = 448$ sec given in the table,

$$t_1' = 448 + 4.7 + 3.5 = 456 \text{ sec},$$

204 and thus

$$at_1' = 0.015 \cdot 456 = 6.84$$

giving a mass ratio

$$\frac{m_0}{m_1} = e^{at_1'} = 933 \text{ instead of } 825.$$

The result will be somewhat better if we simply increase the rocket's own acceleration by an amount $\Delta\beta = 2.4$ m/sec^2 throughout the first 50 km. Then the total duration of the gas ejection will remain the same as that without air resistance, that is, 448 sec; the first 70.3 sec of this will correspond to an acceleration $ac = 32.4$ m/sec^2 for $a = \frac{32.4}{2{,}000} = 0.0162$. The other 377.7 sec will correspond to $ac = 30$ m/sec^2 for $a = 0.015$, giving a ratio of

$$\frac{m_0}{m_1} = e^{\Sigma at} = e^{0.0162 \cdot 70.3 + 0.015 \cdot 377.7} = 898.$$

The table below indicates the effect of the air resistance, for other values of ac and c, on the ratio

$$\frac{m_0}{m_1} = e^{\frac{ac}{c} t_1'}.$$

	$ac = 30$ m/sec^2 ($t_1' = 456$ inst. of 448 sec)	$ac = 100$ m/sec^2 ($t_1' = 123$ inst. of 117 sec)	$ac = 200$ m/sec^2 ($t_1' = 64$ inst. of 57 sec)
$c = 2{,}000$ m/sec	933 inst. of 825	468 inst. of 347	602 inst. of 299
$c = 2{,}500$ "	235 " 216	138 " 108	166 " 95.5
$c = 3{,}000$ "	95 " 88	60 " 49	71 " 44.7
$c = 4{,}000$ "	30 " 28	22 " 18.7	25 " 17.2
$c = 5{,}000$ "	15 " 14.6	12 " 10.4	13 " 9.8

It is clear from the table that, with an increase in the rocket's own acceleration ac, the effect of air resistance increases very much. Consequently, a high value of ac resulting from a high velocity may prove to be less advantageous than a low ac.

The foregoing ideas, according to which a body is propelled by prolonged bursts overcoming the force of gravity, are not new. In "Around the Moon" ["Autour de la Lune"], Jules Verne presented similar ideas, when he described a means of reducing the velocity of a cannon ball with the aid of rockets. Also, in the novel "On Two Planets" ["Auf Zwei Planeten"], Kurd Lasswitz describes the use of particle ejection at the velocity of light, which involves a very small reduction of the weight of the vehicle.

205 The recent studies by Goddard, Oberth, and Valier were mentioned in the Foreword. Back in 1890 Hermann Ganswindt, the famous pioneer aeronaut, demonstrated that it was possible to construct a rocket airplane; the studies of the Russian scientist, Tsiolkovskii, date back to the same period. Finally, we should note that Newton, in his lectures on the principle of recoil, pointed out the possibility of utilizing this principle for flight in a vacuum.

Part II

RETURN TO EARTH

Let us consider a rocket falling from a great distance away from the center of attraction (see Part I and Figure 70), this distance being between r_1 and r_0. The velocity is to be reduced from v_1 to zero. This will take the same amount of time t_1 as previously (equation 10) and a consumption of fuel $\frac{dm}{dt}$, ejected in the direction of motion. For the ascent and the return to earth, the flight time will thus be doubled. The ratio between the initial and final masses will be

$$\frac{m_0'}{m_1} = e^{at_1 \cdot 2},$$

that is, it will not be twice as great, but rather proportional to the second power of the values of $\frac{m_0}{m_1}$, given in Table 1. For instance, for $ac = 30$ m/sec^2 and $c = 2,000$ m/sec,

$$\frac{m_0^1}{m_1} = 825^2 = 680,625.$$

Using this method of retardation and for the gas velocity c assumed, the mass ratio is very unfavorable. Therefore, it is advisable to seek some other method of descent, for example, one making use of the braking effect of the earth's atmosphere.

According to Lossel, the air resistance to a body moving through the atmosphere is

$$W = w \cdot F \cdot \varphi = \gamma \cdot \frac{v^2}{g} \cdot F \cdot \varphi_1, \qquad (14)$$

where v is the velocity of the body at a given moment, g is the acceleration of gravity, γ is the specific weight of air, w is the pressure per unit area perpendicular to the direction of motion, F is the cross-sectional area (of the body) perpendicular to the direction of motion, and φ is a coefficient depending on the shape of the body (for a plane surface $\psi = 1$, and for a convex hemisphere $\psi = 0.5$).

206 We assume that the air pressure, which is p_0 at the earth's surface and zero at some height h, varies according to the following law (Figure 74):

$$p = p_0\left(\frac{y}{h}\right)^n \tag{15}$$

Then the drop in pressure for a height variation dy will be

$$\frac{dp}{dy} = \frac{np_0}{h^n} y^{n-1}.$$

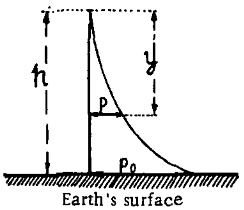


FIGURE 74.

However, we also have

$$dp = \gamma dy \quad \text{or} \quad \frac{dp}{dy} = \gamma.$$

Therefore,

$$\gamma = \frac{np_0}{h^n} y^{n-1} \tag{16}$$

At the earth's surface $y = h$, and $p = p_0$, so that

$$\gamma = \frac{np_0}{h}$$

and

$$n = \frac{\gamma_0}{p_0} h \tag{17}$$

and from equation (16)

$$\gamma = \frac{\gamma_0}{p_0} h \cdot \frac{p_0}{h^n} y^{n-1} = \gamma_0 \left(\frac{y}{h}\right)^{n-1}. \tag{16a}$$

We assume:

$$\gamma_0 = 1.293 \text{ kg/m}^3$$

$$p_0 = 0.76 \text{ m} \cdot 13{,}600 \text{ kg/m}^3 = 10{,}330 \text{ kg/m}^2 \text{ (weight of mercury column).}$$

Then

$$\frac{\gamma_0}{p_0} = \frac{0.293 \text{ kg/m}^3}{10330 \text{ kg/m}^2} = \frac{1}{8000 \text{ m}} = \frac{1}{8 \text{ km}} \tag{17a}$$

According to sounding-balloon observations, the atmospheric pressure at a height $h - y = 10$ km is about 210 mm Hg, so that

$$\frac{p}{p_0} = \frac{210}{760} = \sim \frac{1}{3.6}.$$

This value can also be obtained independently of equation (15), for h from 100 to 1,000 km. Observations of falling meteors, as well as theoretical considerations, indicate that the height of the atmosphere is at least $h = 400$ km (see, for example, Trabert, "Lehrbuch der kosmischer Physik," p. 304). This value will be used in the following. Then, from equations (17) and (17a), we have

$$n = \frac{400}{8} = 50; \; n - 1 = 49.$$

(208 209)

TABLE 3.

$h-y$ km	y km	$\gamma = 1.293\left(\frac{y}{h}\right)^{49}$ kg/m³	$h-y$ km	y km	$\gamma = 1.293\left(\frac{y}{h}\right)^{49}$ kg/m³	$h-y$ km	y km	$\gamma = 1.293\left(\frac{y}{h}\right)^{49}$ kg/m³	$h-y$ km	y km	$\gamma = 1.293\left(\frac{y}{h}\right)^{49}$ kg/m³
0	400	1.3	25	375	0.055	70	330	0.0001025	150	250	0.00000000013
1	399	1.15	30	370	0.0283	75	325	0.0000497	200	200	0.00000000000000023
2	398	1.00	35	365	0.01464	80	320	0.0000230	400	0	0.00000000000000000000
3	397	0.90	40	360	0.0074	85	315	0.0000106	—	—	—
4	396	0.80	45	355	0.00376	90	310	0.0000049	—	—	—
5	395	0.70	50	350	0.00187	95	305	0.0000022	—	—	—
10	390	0.375	55	345	0.000915	100	300	0.00000098	—	—	—
15	385	0.205	60	340	0.000448	105	295	0.000000423	—	—	—
20	380	0.105	65	335	0.000217	110	290	0.000000185	—	—	—

207 The values of γ were calculated for various heights $h-y$, and these are given in Table 3.

If a body approaches from outer space to a distance 400 km from the earth's surface, or a distance $r = 6{,}780$ km from the earth's center, and if it moves under the influence of gravity, then equation (6) gives a velocity

$$v = \sqrt{2g_0 \cdot \frac{r_0^2}{r}} = \sqrt{2 \cdot 0.0098 \cdot \frac{6380^2}{6780}} = 10.9 \text{ km/sec}$$

Clearly, for a **radial** descent, this velocity cannot be reduced to zero without damaging the rocket itself and harming the passengers. However, the duration of the braking process can be reduced considerably if the body enters the atmosphere **tangentially**.

For a body approaching the earth from far away and moving solely under the influence of terrestrial gravity, the trajectory will be nearly parabolic, with the focus at the earth's center, provided that the body does not descend radially. Then, at any distance r, the velocity will be (see Figure 70)

$$v = \sqrt{2g_0 \frac{r_0^2}{r}}.$$

For passage right near the earth's surface, the tangential velocity will be

$$v_{max} = \sqrt{2g_0 r_0} = \sqrt{2 \cdot 0.0098 \cdot 6380} = 11.2 \text{ km/sec}.$$

At the limit of atmosphere the tangential velocity will be

$$v = \sqrt{2 \cdot 0.0098 \cdot \frac{6380^2}{6780}} = 10.9 \text{ km/sec}.$$

Thus, within the atmosphere this velocity will be about

$$v' = 11.1 \text{ km/sec},$$

and we can also take this to be the average velocity for the entry of a body into the atmosphere. In order to determine the layers of air in which it is advisable to carry out retardation, the air resistance $w=\frac{\gamma v^2}{g}$ was calculated for different heights and for a flat surface (1 m^2) moving perpendicular to its plane at a velocity of 11.1 km/sec. The results, in kg/m^2, are shown in Table 4.

209 TABLE 4.

$h-y$ km	y km	r km	$g=g_0\frac{r_0^2}{r^2}$ m/sec^2	$\gamma=\gamma_0\left(\frac{y}{h}\right)^{49}$ kg/m^3	$w=\gamma\frac{v^2}{g}$ kg/m^2
400	0	6,780	8.69	0.000000000000000000	0.000000000
200	200	6,580	9.21	0.0000000000000023	0.00000003
150	250	6,530	9.36	0.00000000013	0.0017
110	290	6,490	9.48	0.000000185	2.4
105	295	6,485	9.50	0.000000423	5.5
100	300	6,480	9.51	0.00000098	12.7
95	305	6,475	9.53	0.0000022	28.5
90	310	6,470	9.54	0.0000049	63.4
85	315	6,465	9.56	0.0000106	137
80	320	6,460	9.57	0.0000230	297
75	325	6,455	9.59	0.0000497	640
70	330	6,450	9.60	0.0001025	1,320
65	335	6,445	9.62	0.000217	2,780
60	340	6,440	9.63	0.000448	5,720
55	345	6,435	9.65	0.000915	11,800
50	350	6,430	9.66	0.001870	23,900

Air layers lying above 100 km were not taken into account when calculating the braking effect and for the flight velocities taken. Our rocket now, in contrast to the conditions at departure from the earth (considered at the end of Part I), will not profit from a reduction of the air resistance to its low mass m_1. Instead, this resistance must be used to the best advantage by selecting the optimum rocket shape.

Here the situation is similar to that for an airplane, which, in the lower
208 layers of the atmosphere, for $g=9.8$ m/sec^2, $\gamma=1.3$ kg/m^3, and a velocity of 50 m/sec, will face a normal drag of

$$w=\frac{\gamma v^2}{g}=\frac{1.3\cdot 50^2}{9.8}=330 \text{ kg/m}^2$$

According to Table 4, this drag corresponds to a height from 75 to 100 km above the earth's surface (Figure 75).

The rocket will be assumed to enter the earth's atmosphere in such a way that the vertex of the parabolic path lies at a height of 75 km above the earth's surface, or at a distance

$$r_0=6380+75=6455 \text{ km}$$

from the earth's center, taken to be the focus of the parabola.

210 The path length between heights of 75 and 100 km, which is the distance over which braking takes place, is found from Figure 75. From the equation

for the parabola, we have

$$\frac{r_0}{r'} = \cos^2 \alpha',$$

from which

$$\cos \alpha' = \sqrt{\frac{r_0}{r}} = \sqrt{\frac{6455}{6480}} = 0.998\,075$$

and

$$\alpha' = 3°34'; \;\; 2\alpha' = 7°8'.$$

(208)

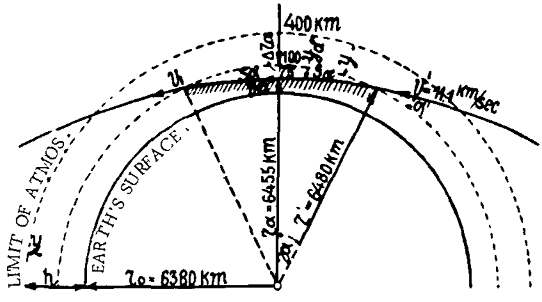


FIGURE 75.

In addition, as a good approximation, we have

$$S_a = r' \cdot \sin 2\alpha' = 6480 \cdot 0.12428 = 805 \text{ km}$$

and thus the length of the braking path between the 75-km and 100-km levels is

$$2\, s_a = 1610 \text{ km}.$$

Here, as a first approximation, we assume that the path does not vary as a result of the slowing down (the effect of retardation is given special consideration at the end of Part II).

Over a path s_a the retardation β of the mass m_1 of the rocket, because of the air resistance w, will have a variable value:

$$\beta = \frac{w}{m_1},$$

or (from equations (14) and (16a) with $g = \sim g_0$)

$$\frac{dv}{dt} = -\frac{\gamma_0 F \psi}{g_0 m_1} \cdot v^3 \cdot \left(\frac{y}{h}\right)^{49}.$$

In addition,

$$\frac{ds}{dt} = v$$

and, approximately,

$$\frac{ds}{dy}=\frac{s}{\Delta r_a}=\frac{s_a}{r'-r_a}.$$

Consequently,

$$\frac{dv}{dy}=\frac{dv}{dt}\cdot\frac{dt}{ds}\cdot\frac{ds}{dy}=-\frac{\gamma_0 F\varphi}{g_0 m_1}\cdot\frac{s_a}{\Delta r_a}\cdot v\left(\frac{y}{h}\right)^{49}$$

or

$$\frac{dv}{v}=-\frac{\gamma_0 F\cdot\varphi}{g_0 m_1}\cdot\frac{s_a}{\Delta r_a}\left(\frac{y}{h}\right)^{49}\cdot dy;$$

and

$$\ln v=-\frac{\gamma_0\ F\ \varphi.}{50\cdot g_0 m_1}\cdot\frac{s_a}{\Delta r_a}\cdot\frac{y^{50}}{h^{49}}+C.$$

If the vehicle enters the braking path at $y=y'$, we have

$$\ln v'=-\frac{\gamma_0 F\cdot\varphi}{50 g_0 m_1}\cdot\frac{s_a}{\Delta s_a}\cdot\frac{y'^{50}}{h^{49}}+C.$$

211 In the middle of the braking path, at $y=y_a$, we have

$$\ln v=-\frac{\gamma_0 . F\ \varphi}{50 g_0 m_1}\cdot\frac{s_a}{\Delta r_a}\cdot\frac{y_a{}^{50}}{h^{49}}+C.$$

Therefore, during the traversal of the first half s_a of the braking path,

$$\ln v'-\ln v_a=\ln\frac{v'}{v_a}=\frac{\gamma_0 . F.\varphi}{50 g_0 m_1}\cdot\frac{s_a}{\Delta r_a}\cdot h\left[\left(\frac{y_a}{h}\right)^{50}-\left(\frac{y'}{h}\right)^{50}\right]. \qquad (18)$$

When we put in the numerical values, we obtain:

$$\gamma_0=1.3\text{kg/m}^3; \Delta r_a=r'-r_a=100-75=25\text{ km}$$

$$s_a=805\text{km};\ \frac{s_a}{\Delta r_a}=\frac{805}{25}=32.2;$$

$$h=400\text{ km}=400{,}000\text{ m};\ g_a=325\text{km};\ y'=300\text{ km}$$

Moreover, as previously, $g_0 m_1$ = the weight G_1, of the rocket relative to the earth's surface) which equals 2,000 kg, and $F\cdot\varphi$, the area corresponding to an open parachute 2.8 m in diameter, oriented perpendicular to the flight direction, is 6.1 m^2. Then, the highest value of the retardation at a height of 75 km will be

$$\beta_{max}=\frac{w}{m_1}\cdot F\cdot\varphi=\frac{140}{200}\cdot 6.1=19.5\text{ m/sec}^2,$$

and the velocity v_a at the vertex of the parabola is found from the equation

$$\ln\frac{v'}{v_a}=\frac{1.3.6.1}{50\ 2000}\cdot 32.2\cdot 400{,}000\cdot\left[\left(\frac{325}{400}\right)^{50}-\left(\frac{300}{400}\right)^{50}\right]=0.031$$

or

$$\frac{v'}{v_a}=e^{0.031}=1.032$$

or

$$v_a=\frac{v'}{1032}.$$

In a similar manner, we can calculate the exit velocity of the rocket from the second half s_a of the braking path:

$$v_1 = \frac{v_a}{1.032} = \frac{v'}{1.032^2} = \frac{11.1}{1.032^2} = 10.4 \text{ km/sec}.$$

As a consequence of the velocity reduction, the shape of the flight path will change, so that the rocket describes an ellipse instead of the parabola it has followed up to that time. While traversing the ellipse, the rocket again follows a braking path, entering the latter at a velocity $v_1 = 10.4$ km/sec. Due to the shortness of the braking path, the arc of the ellipse will differ little from the parabola, so that the length of the new braking path may also be taken to be $2s_a = 2(805) = 1{,}610$ km.

After traversal of this distance, the new exit velocity from it will be

$$v_2 = \frac{v_1}{1.032^2} = \frac{v'}{1.032^4} = \frac{11.1}{1.032^4} = 9.8 \text{ km/sec}.$$

212 As a result of this new velocity reduction, the body moves along a new reduced ellipse instead of the previous one. Accordingly, a new braking in the atmosphere occurs along this ellipse, with an entry velocity $v_2 =$ 9.8 km/sec. Let us again assume the length of the braking path to be $2s_a =$ 1,610 km, although actually it will be somewhat greater and the retardation will be more marked. Then,

$$v_3 = \frac{11.1}{1.032^6} = 9.2 \text{ km/sec}$$

and also

$$v_4 = \frac{11.1}{1.032^8} = 8.6 \text{ km/sec}$$

and

$$v_5 = \frac{11.1}{1.032^{10}} = 8.1 \text{ km/sec}.$$

Finally, after one more such elliptical path, with braking over half the distance s_a, the velocity at the vertex will be

$$v_a = \frac{v_5}{1.032} = \frac{11.1}{1.032^{11}} = 7.85 \text{ km/sec},$$

but this is precisely the velocity

$$\sqrt{g_a r_a} = \sqrt{g_0 \frac{r_0^2}{r_a^2} r_a} = \sqrt{g_0 \frac{r_0^2}{r_a}} = \sqrt{0.0098 \cdot \frac{6380^2}{6455}} = 7.85 \text{ km/sec},$$

for which a body at a distance $r_a = 6{,}455$ km from the earth's center (or at a height of 75 km above the earth's surface) will describe a circle around the earth, when the air resistance is not taken into account. In this case the vehicle will remain in the earth's atmosphere and the subsequent descent will be similar to the gliding approach of an airplane.

In order to determine how long it will take for the rocket to traverse different ellipses, it is sufficient to make the calculations for just one of them (Figure 76). If a body mass m is located a distance r from the earth's

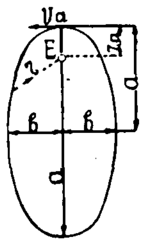


FIGURE 76.

center E, it will experience an attraction

$$P = -\frac{\mu \cdot m}{r^2}.$$

At the earth's surface, where $r = r_0$ the attractive force will equal the weight mg_0, of the body:

$$mg_0 = \frac{\mu \cdot m}{r_0^2},$$

so that

$$\mu = g_0 r_0^2 = 0.0098 \cdot 6380^2 = 4{,}000{,}000 \text{ km}^3/\text{sec}^2.$$

If the body in Figure 76 is a very small (or very great) distance r_a from the center of attraction, then the velocity will be $v_a \perp r_a$, and the body will describe an ellipse with semiaxes

213

$$a = \frac{\mu}{\frac{2\mu}{r_a} - v_a^2}; \quad b = \frac{v_a r_a}{\sqrt{\frac{2\mu}{r_a} - v_a^2}}$$

(for derivation, see end of Part III).

Assuming the error to be small, we take the velocities v_1, v_2, etc., of exit from the braking path to be at the vertex, where $r_a = 6{,}455$ km; then, rounding off, we have

$$\frac{2\mu}{r_a} = \frac{800\,000}{6455} = 124$$

and, for $v_1 = 10.4$ km/sec,

$$a_1 = \frac{400\,000}{124 - 10.4^2} = 25{,}000 \text{ km}$$

$$b_1 = \frac{10.4 \cdot 6455}{\sqrt{124 - 10.4^2}} = 16{,}800 \text{ km};$$

for $v_2 = 9.8$ km/sec,

$$a_2 = \frac{400\,000}{124 - 9.8^2} = 14{,}300 \text{ km}$$

$$b_2 = \frac{9.8 \cdot 6455}{\sqrt{124 - 9.8^2}} = 11{,}950 \text{ km};$$

for $v_3 = 9.2$ km/sec,

$$a_3 = \frac{400\,000}{124 - 9.2^2} = 10{,}250 \text{ km}$$

$$b_3 = \frac{9.2 \; 6455}{\sqrt{124 - 9.2^2}} = 9500 \text{ km};$$

for $v_4 = 8.6$ km/sec,

$$a_4 = \frac{400\,000}{124 - 8.6^2} = 8000 \text{ km}$$

$$b_4 = \frac{8.6 \cdot 6455}{\sqrt{124 - 8.6^2}} = 7850 \text{ km};$$

and, for $v_5 = 8.1$ km/sec,

$$a_5 = \frac{400\,000}{124 - 8.1^2} = 6900 \text{ km}$$

$$b_5 = \frac{8.1 \cdot 6455}{\sqrt{124 - 8.1^2}} = 6860 \text{ km}.$$

The duration of the flight along each ellipse is calculated on the basis of the law of equal areas (equation (39) at end of Part III):

$$\frac{dF}{dt} = \text{const.} = \frac{v_a r_a}{2};$$

$$dF = \frac{v_a r_a}{2} \cdot dt;$$

$$F = \frac{v_a r_a}{2} \cdot t = ab\pi.$$

Consequently,

$$t = \frac{2\ ab\cdot a}{v_a r_a} \tag{18a}$$

214 Accordingly, to traverse all five ellipses (Figure 77), the following time will be necessary:

$$t_1 = \frac{2.25\,000 \cdot 16\,800.\pi}{10.4 \cdot 6455} = 39{,}300 \text{ sec} = 10.9 \text{ hr},$$

$$t_2 = \frac{2.14\,300 \cdot 11\,950.\pi}{9.8 \cdot 6455} = 16{,}900 \text{ sec} = 4.7 \text{ hr},$$

$$t_3 = \frac{2.10\,250 \cdot 9500.\pi}{9.2 \cdot 6455} = 10{,}300 \text{ sec} = 2.9 \text{ hr},$$

$$t_4 = \frac{2\ 8000 \cdot 7850.\pi}{8.6\ 6455} = 7100 \text{ sec} = 2.0 \text{ hr},$$

$$t_5 = \frac{2.6900 \cdot 6860.\pi}{8.1 \cdot 6455} = 5700 \text{ sec} = 1.6 \text{ hr}.$$

Giving a total time $t_a = 79{,}300$ sec $= \sim 22.1$ hr.

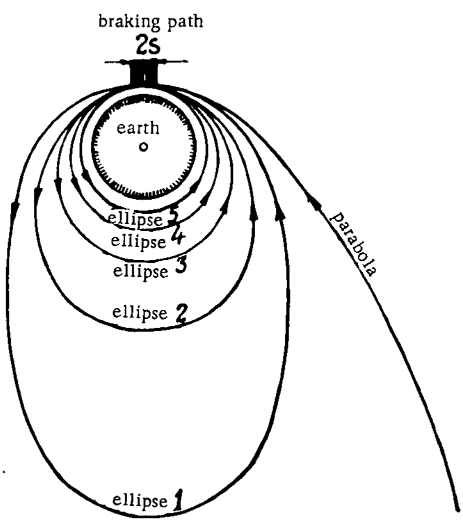


FIGURE 77. Descent of Hohmann rocket

The gliding flight which begins after this may be represented as follows: it starts at a height $h-y_a=$ 75 km, with a tangential velocity $v_a=7.85$ km/sec. At this velocity the centrifugal acceleration $z_a=\frac{v_a^2}{r_a}$ equals the acceleration of gravity g_a, since $v_a^2=g_a r_a$ (see page 203). Due to the prolonged retardation β, the air resistance will cause a reduction in the velocity v and the centrifugal acceleration

$$z=\frac{v^2}{r},$$

so that the acceleration of gravity remains almost unchanged. In addition, an ever-increasing radial retardation ϱ must act upon the rocket, as well as the tangential retardation β, in order to compensate the preponderance of the acceleration of gravity g over the centrifugal acceleration z, that is,

$$\varrho=g-z=g\left(1-\frac{z}{g}\right).$$

or, since $z=\frac{v^2}{r}$ for the given region between heights of 0 and 75 km as well, it is accurate enough to take $g=\frac{v_a^2}{r}$, and

$$\varrho=g\left(1-\frac{v^2}{v_a^2}\right) \qquad (19)$$

215 The radial retardation can be obtained owing to the effect of the air resistance on the supporting surface F_0, which must be turned so that it is inclined to its original horizontal position, with the aid of the altitude controls, and this inclination must gradually become greater and greater (see Figure 79):

$$\varrho=\frac{w}{m}\cdot F_0\cdot\sin^2\alpha\cdot\cos\alpha \qquad (20)$$

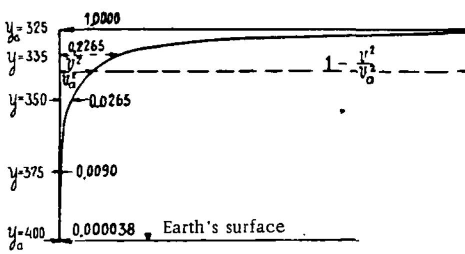


FIGURE 78.

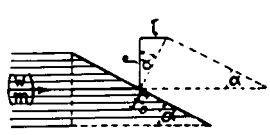


FIGURE 79.

However, the resulting tangential resistance $v=\varrho\tan\alpha$ can be neglected in comparison with the high retardation β along the path. In order to ensure that the height regulation will always be easy to carry out, the drag w cannot exceed that at the beginning of the glide. Therefore, from equations (14) and (16a), we have

$$w=\frac{\gamma_0}{g_0}v^2\left(\frac{y}{h}\right)^{49}=\frac{\gamma_0}{g_0}v_a^2\left(\frac{y_a}{h}\right)^{49};$$

that is, the flight must proceed in such a way that always

$$\frac{v^2}{v_a^2}=\frac{\left(\frac{y_a}{h}\right)^{49}}{\left(\frac{y}{h}\right)^{49}}=\left(\frac{y_a}{y}\right)^{49} \tag{21}$$

In other words, a different flight velocity should correspond to each height. Figure 78 shows how each height y has its own ratio $\frac{v^2}{v_a^2}$. This same diagram also shows the difference $1-\frac{v^2}{v_a^2}$, which (according to equation (19)) expresses the increment of the radial acceleration ϱ in units of $1/g$. Moreover, when it attains a certain velocity v, the rocket traverses a path s with a constant retardation $\beta=\beta_a$:

$$s=\frac{v_a^2-v^2}{2\beta_a}=\frac{v_a^2}{2\beta_a}\left(1-\frac{v^2}{v_a^2}\right)=\frac{v_a^2}{2\beta_a}\left[1-\left(\frac{y_a}{y}\right)^{49}\right]. \tag{22}$$

Formula (22) indicates that the path s is expressed with the aid of Figure 78, via segments of length $1-\frac{v^2}{v_a^2}$, in units of $1/\frac{v_a^2}{2}$. It is evident from the figure

216 that, if the retardation β remains constant, then favorable flight at the beginning of the glide, may turn into a fall at the end. Consequently, the value of β should remain constant only until the inclination of the flight begins to intersect the horizontal more sharply.

According to equation (22), this inclination can be expressed as

$$\frac{ds}{dy}=\frac{v_a^2}{2\beta_a}\cdot 49\cdot\frac{y_a^{49}}{y^{50}}=\frac{v_a^2}{2\beta_a}\cdot\frac{49}{y_a}\left(\frac{y_a}{y}\right)^{50},$$

so that

$$\left(\frac{y}{y_a}\right)^{50}=\frac{49}{y_a}\cdot\frac{v_a^2}{2\beta_a}\cdot\frac{dy}{ds}. \tag{23}$$

At a height $h-y_a=75$ km, or for $y_a=325$ km, with a velocity

$$v_a=7.85\ \text{km/sec}$$

and a retarding surface $F=6.1$ m^2, the retardation will be

$$\beta_a=\frac{w}{m_1}\cdot F=\frac{\gamma_0}{g_0 m_1}v_a^2\left(\frac{y_a}{h}\right)^{49}\cdot F=\frac{1.3}{2000}\cdot 7850^2\cdot 6.1\left(\frac{325}{400}\right)^{49}=9.3\ \text{m/sec}^2=$$
$$=0.0093\ \text{km/sec}^2.$$

Assuming that this retardation is retained down to an inclination $\frac{dy}{ds}=\frac{1}{m}$, we can find the altitude to which the rocket descends from equation (23):

$$\left(\frac{y_b}{y_a}\right)^{50}=\frac{49}{325}\cdot\frac{7.85^2}{2\cdot 0.0093}\cdot\frac{1}{10}=50,$$

or

$$y_b=y_a\cdot 50^{\frac{1}{50}}=325\cdot 1.0814=352\ \text{km},$$

which corresponds to a height

$$h-y_b=400-352=48\,\text{km}$$

above the earth's surface. The corresponding velocity v_b is found from equation (21):

$$\frac{v_b^2}{v_a^2}=\left(\frac{y_a}{y_b}\right)^{49}=\left(\frac{y_a}{y_b}\right)^{50}\cdot\frac{y_b}{y_a}=\frac{1.0814}{50}=0.02163,$$

or

$$v_b=v_a\sqrt{0.02163}=7.85\cdot 0.147=1.15\,\text{km/sec}.$$

The path traversed will be (according to equation (22))

$$s_b=\frac{v_a}{2\beta_a}\left(1-\frac{v_b^2}{v_a^2}\right)=\frac{7.85^2}{2\cdot 0.0093}(1-0.02163)=3250\,\text{km}.$$

The flight time is

$$t_b=\frac{v_a-v_b}{\beta_a}=\frac{7850-1150}{9.3}=720\,\text{sec},$$

217 and the radial retardation, which must be found at this point, is given by equation (19):

$$\varrho_b=g\left(1-\frac{v_b^2}{v_a^2}\right)=g(1-0.02163)=0.97837\,g,$$

that is, it is almost equal to the total acceleration of gravity. It can be obtained with the aid of the supporting surface F_0, determined using equation (20):

$$\varrho=\frac{w}{m}\cdot F_0\cdot\sin^2\alpha\cdot\cos\alpha=\backsim g,$$

where w has the value

$$w=\frac{\gamma_0}{g_0}\cdot v_a^2\left(\frac{y_a}{h}\right)^{49}=\frac{1.3}{9.8}\cdot 7850^2\left(\frac{325}{400}\right)^{49}=310\,\text{kg/m}^2.$$

Therefore,

$$F_0\sin^2\alpha\cdot\cos\alpha=\frac{m_1 g}{w}=\backsim\frac{2000}{310}6.5\,\text{m}^2.$$

Since the quantity $\tau=\varrho\tan\alpha$ in the relation for β_a is not large, angle α should be as small as possible, at any rate, max $\alpha=20°$, that is

$$\max\tau=0.364\cdot 9.8=3.56\,\text{m/sec}^2;$$
$$\beta_a=9\,\text{m/sec}^2$$

and

$$F_0=\frac{6.5}{0.342^2\cdot 0.940}=59\text{m}^2(5\text{m}\times 12\text{m}).$$

From the foregoing it follows that, from a height $h-y=75$ km down to a height of 48 km above the earth's surface, over a path length $s_b=3{,}250$ km, with a constant retarding area $F=6.1$ m^2 and a constant supporting area $F_0=59$ m^2, the angle (α) of inclination (intersection) of the supporting surface to the horizontal must increase from 0° to 20°. Thus, for an invariable air resistance $w=310$ kg/m^2, the velocity will drop from $v_a=7{,}850$ m/sec to $V_b=1{,}150$ m/sec, and the radial retardation (ϱ) will rise from 0 to a value equaling the acceleration of gravity (see Figure 80, from A to B).

Beginning from a height $h - y_b = 48$ km, in order to avoid a rapid fall, the retardation of the motion must be decreased; moreover, so as not to use a retarding surface F, in the form of a parachute, a supporting surface F_0, which gives a component $\tau = 3.56$ m/sec$^2 = 0.00356$ km/sec^2 should be used for braking, since it will retard the motion further. However, this value of τ should not be used up to the end either, since, if it is, a short flight may be followed by a sharp descent (fall); therefore, for constant ϱ (equal to the acceleration of gravity), the retardation during flight should be reduced more and more, for instance, for a movement from position B further beyond D (Figure 80), shifting the supporting surface F_0 into a horizontal position.
218

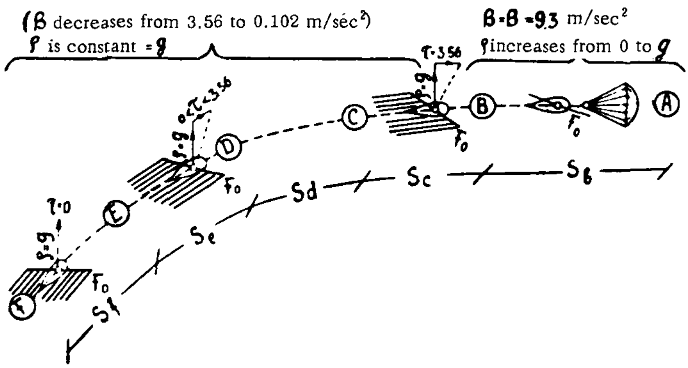
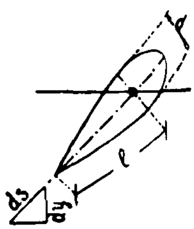


FIGURE 80.



JRE 81.

For each point on the trajectory we have the relation

$$-\beta \cdot ds = d\left(\frac{v^2}{2}\right),$$

or, since

$$v^2 = v_a^2\left(\frac{y_a}{y}\right)^{49}$$

we obtain

$$-\beta ds = \frac{v_a^2}{2} d\left(\frac{y_a}{y}\right)^{49} = -\frac{v_a^2}{2}\cdot\frac{49}{y_a}\left(\frac{y_a}{y}\right)^{50}\cdot dy,$$

or

$$\frac{ds}{dy} = \frac{v_a^2}{2\beta}\cdot\frac{49}{y_a}\left(\frac{y_a}{y}\right)^{50}, \qquad (24)$$

where β is a variable.

Let us assume that the glide near the earth is at an angle of 45°; then, for $y = y_0 = 400$ km, we have

$$\frac{dy}{ds} = \frac{1}{\sqrt{2}} \text{ (Figure 81)}$$

and a final value of

$$\beta\min=\frac{v_a^2}{2}\cdot\frac{49}{y_a}\cdot\left(\frac{y_a}{y_0}\right)^{50}\cdot\frac{dy}{ds}=\frac{7{,}85^2}{2}\cdot\frac{49}{325}\cdot\left(\frac{325}{400}\right)^{50}\cdot\frac{1}{\sqrt{2}}=0.000102\ \text{km/sec}^2=$$
$$=0.102\ \text{m/sec}^2.$$

At the end of the path (F in Figure 80) the tangential component τ of the air resistance at a wing is zero. Thus the retardation β_{min} at this point is due only to the shape of the rocket itself (Figure 81), and it will be

$$\beta_{\min}=\frac{w}{m_1}\cdot\frac{d^2\pi}{4}\cdot\left(\frac{d}{2l}\right)^2,$$

from which we obtain

$$l=\frac{d^2}{4}\sqrt{\frac{w\cdot\pi}{m_1\cdot\beta_{\min}}}.$$

219 We substitute the numerical values

$w=310$ kg/m^2 (the value taken, with some margin);

$$m_1=\frac{2000\ \text{kg}}{9{\cdot}8\ \text{m/sec}^2}=\backsim 200\ \frac{\text{kg/sec}^2}{\text{m}};$$

$d=1.5$ (least allowable diameter of rocket);

$$l=\frac{1{,}5^2}{4}\sqrt{\frac{310\cdot\pi}{200\cdot 0.102}}=3.88\ \text{m}.$$

At the end of the remaining path the flight velocity is found from the relation

$$\frac{v^2}{v_a^2}=\left(\frac{325}{400}\right)^{49},$$

which gives

$$v=v_a\left(\frac{325}{400}\right)^{49/2}=7850\cdot 0.062=48.5\ \text{m/sec},$$

and the resistance will be

$$w=\frac{\gamma_0}{g_0}\cdot v^2=\frac{1.3}{4.8}\cdot 48.5^2=310\ \text{kg/m}^2,$$

which makes it possible to descend without difficulty.

To simplify the calculation, we assume that β varies from 3.56 to 0.102 m/sec^2 in jumps rather than continuously. The jumps occur in the four regions $B-C$, $C-D$, $D-E$, and $E-F$ (Figure 80), the values in these regions being $\beta_c=3.5$ m/sec^2, $\beta_d=1.0$ m/sec^2, $\beta_e=0.2$ m/sec^2, and $\beta_f=0.102$ m/sec^2. The inclinations of the path are taken to be

$$\frac{dy}{ds}=\frac{1}{6},\ \frac{1}{3},\ \frac{1}{2}\ \text{and}\ \frac{1}{\sqrt{2}}.$$

Then, for the end of each part, we have:

For segment $B-C$, according to equation (24),

$$\frac{ds}{dy}=\frac{v_a^2}{2\beta_c}\cdot\frac{49}{y_a}\cdot\left(\frac{y_a}{y_c}\right)^{50}$$

or

$$\left(\frac{y_c}{y_a}\right)^{50} = \frac{v_a^2}{2\beta_c} \cdot \frac{49}{y_a} \cdot \frac{dy}{ds} = \frac{7.85^2}{2 \cdot 0.0035} \cdot \frac{49}{325} \cdot \frac{1}{6} = 222;$$

so that

$$y_c = y_a \cdot 222^{\frac{1}{50}} = 325 \cdot 1.114 = 362 \text{ km}$$
$$h - y_c = 38 \text{ km}.$$

In addition, from equation (21),

$$\frac{v_c^2}{v_a^2} = \left(\frac{y_a}{y_c}\right)^{49} = \frac{1.114}{222} = 0.00502;$$
$$v_c = v_a \sqrt{0.00502} = 7\ 85 \cdot 0.0706 = 0.555 \text{ km/sec},$$

and from (22),

$$s_c = \frac{v_b^2 - v_c^2}{2\beta_c} = \frac{1.15^2 - 0.555^2}{2\ 0.0035} = 146 \text{ km}$$

220 so that

$$t_c = \frac{v_b - v_c}{\beta_c} = \frac{1150 - 555}{3.5} = 170 \text{ sec}.$$

For segment $C - D$,

$$\left(\frac{y_d}{y_a}\right)^{50} = \frac{v_a^2}{2\beta_d} \cdot \frac{49}{y_a} \cdot \frac{dy}{ds} = \frac{7.85^2}{2 \cdot 0.001} \cdot \frac{49}{325} \cdot \frac{1}{3} = 1550;$$
$$y_d = y_a \cdot 1550^{\frac{1}{50}} = 325 \cdot 1.158 = 377 \text{ km} \quad h - y_d = 23 \text{ km}$$
$$\frac{v_d^2}{v_a^2} = \left(\frac{y_a}{y_d}\right)^{49} = \frac{1.158}{1550} = 0.00075;$$
$$v_d = 7.85 \sqrt{0.00075} = 0.215 \text{ km/sec}$$
$$s_d = \frac{v_c^2 - v_d^2}{2\beta_d} = \frac{0.555^2 - 0.215^2}{2 \cdot 0.001} = 131 \text{ km}$$
$$t_d = \frac{v_c - v_d}{\beta_d} = \frac{555 - 215}{1} = 340 \text{ sec}.$$

For segment $D - E$,

$$\left(\frac{y_e}{y_a}\right)^{50} = \frac{v_a^2}{2\beta_e} \cdot \frac{49}{y_a} \cdot \frac{dy}{ds} = \frac{7.85^2}{2\ .\ 0{,}0002} \cdot \frac{49}{325} \cdot \frac{1}{2} = 11{,}600;$$
$$y_e = y_a \cdot 11\ 600^{\frac{1}{50}} = 325 \cdot 1.206 = 392 \text{ km} \quad h - y_e = 8 \text{ km}$$
$$\frac{v_e^2}{v_a^2} = \left(\frac{y_a}{y_e}\right)^{49} = \frac{1.206}{11600} = 0.000104;$$
$$v_e = 7.85 \cdot \sqrt{0.000104} = 0.080 \text{ km/sec}$$
$$s_e = \frac{v_d^2 - v_e^2}{2\beta_e} = \frac{0.215^2 - 0.080^2}{2 \cdot 0.0002} = 99 \text{ km}$$
$$t_e = \frac{v_d - v_e}{\beta_e} = \frac{215 - 80}{0.2} = 675 \text{ sec}.$$

For segment $E-F$,

$$y=400\,\text{km}\quad h-y=0;\; v_f=49\,\text{m/sec}$$

$$s_f=\frac{v_e^2-v_f^2}{2\beta_f}=\frac{0.080^2-0.049^2}{2\cdot 0.0001}=20\,\text{km}$$

$$t_f=\frac{v_e-v_f}{\beta_f}=\frac{80-49}{0.1}=310\,\text{sec}.$$

The length of the entire gliding flight will be

$$s_{b-f}=3250+146+131+99+20=3646\ \text{km}$$

and its duration will be

$$t_{b-f}=720+170+340+675+310=2215\ \text{sec}=37\ \text{min}.$$

The total duration of the descent [flight], from the original launching into the atmosphere to the landing on the earth, will be about

$$79{,}300+2200=81{,}500\ \text{sec}\ =\sim 22.6\ \text{hr}.$$

221 When determining the braking ellipses, it was assumed that, at the point of tangency where the parabola approaches the first ellipse, the remaining ellipses begin immediately, without a gradual transition from one to another. Actually, the braking action takes place gradually, rather than all at once, all along the length of each ellipse, and the path of the rocket will be helical instead of elliptical. Along its path, the rocket will encounter lower, and thus denser, air layers presenting greater resistance, so that the retardation will be correspondingly greater than that assumed above. As a result, it is desirable to evaluate the shape of the exit ellipse, as well as the inclination and shortening of its axis.

In order to determine the pattern of the possible change in flight conditions, in the following the first ellipse after the parabola (Figure 77) will be replaced by a spiral. For this purpose, in Figure 75 the angle $4\alpha'=14°16'$, within which the parabola cuts through the air layers, is divided into six parts equal to $\Delta\varphi = 2°22\tfrac{2}{3}'$ each, within which the length of the corresponding segment of the spiral is about $\Delta s=\frac{1610}{6}=\sim 270$ km. If necessary to the left of the angle (Figure 77), we can assume more such angles. Let us suppose that, at the points of contact with adjacent paths Δs, the retardation occurs in jumps, corresponding to an instantaneous velocity reduction $\Delta v=\frac{\beta\cdot\Delta s}{v}$, where v denotes the final velocity in the preceding part of the path, and β is found with the aid of Table 4, according to the formula

$$\beta=\frac{w}{m_1}\cdot F\cdot\left(\frac{v}{v'}\right)^2.$$

If the exact value of w is not given in the table, it can be obtained by rectilinear interpolation, which gives a result somewhat higher than the true value. For the initial point of each branch of an ellipse, the quantities r_1, v_1 and a_1 are assumed to be given and are obtained through Δv, as results of the study of the preceding ellipse.

In addition, we can use the equations

$$a=\frac{\mu}{\frac{2\mu}{r_1}-v_1^2};$$

$$b=\frac{v_1^2\quad r_1^2\cdot\cos^2 a_1}{\frac{2\mu}{r_1}-v_1^2};\quad \mu=g_0\cdot r_0^2$$

Region	0	I	II	III	IV	V	VI	VII
r_1 km	← parabolic path →	6,480	6,466	6,457	6,454	6,456	6,462	6, 472
v_1 km/sec		11.09	11.00	10.66	10.20	9.80	9.60	9.57
α_1		3°34'	2°22⅔'	1°17'	> 0°1' < 0°16'	> 0°55' < 0°59'	1°45'	2°30'
$a = \dfrac{\mu}{\dfrac{2\mu}{r_1} - v_1^2}$		< ∞	148,030	38,987	20,080	14,347	12,641	12,486
$b^2 = \dfrac{(v_1 r_1 \cos \alpha_1)^2}{\dfrac{2\mu}{r_1} - v_1^2}$		< ∞	$1,869.1 \cdot 10^6$	$461.56 \cdot 10^6$	$217,525 \cdot 10^6$	$143.57 \cdot 10^6$	$121.5 \cdot 10^6$	$119.5 \cdot 10^6$
$\epsilon = \sqrt{a^2 - b^2}$		—	141,580	32,534	13,627	7,894	6,188	6,033.8
φ_1 from $\cos \varphi_1 = \dfrac{b^2/r_1 - a}{\epsilon}$		7°8'	5°4'	2°50'	0°	2°30'	5°15'	7°41'
$\Delta\varphi$		2°22⅔'	2°22⅔'	2°22⅔'	2°22⅔'	2°22⅔'	2°22⅔'	2°22⅔'
$\varphi_2 = \varphi_1 \pm \Delta\varphi$		4°45⅓'	2°41⅓'	0°27⅓'	2°22⅔'	4°52⅔'	7°37⅔'	10°2⅔'
$r_2 = \dfrac{b_2}{a + e \cos \varphi_2}$	6,480	6,466	6,457	6,454	6,456	6,462	6,472	6, 485
$v_2 = \sqrt{\dfrac{2\mu}{r_2} - \left(\dfrac{2\mu}{r_1} - v_1^2\right)}$	11.10	11.10	11.01	10.663	10.198	9.794	9.5905	—
α_2 from $\cos \alpha_2 = \cos \alpha_1 \cdot \dfrac{r_1 v_1}{r_2 v_2}$	3°34'	2°22⅔'	1°17'	>0°0' < 0°16'	>0°55' < 0°59'	1°45'	2°30'	—
$\beta = \dfrac{w}{m_1} \cdot F \cdot \left(\dfrac{v_2}{v}\right)^2$	0.00038	0.00412	0.014	0.018	0.016	0.0067	0.00064	—
Δs	270	270	270	270	270	270	270	—
$\Delta v = \dfrac{\beta s}{v_2}$	0.01	0.10	0.35	0.46	0.40	0.19	0.018	—
$v_2 - \Delta v$	~ 11.09	~ 11.00	~ 10.66	~ 10.20	~ 9.80	~ 9.60	~ 9.57	—

(see equations (45) and (46), in relation to the law of equal areas), and

$$\cos\varphi_1 = \frac{\frac{b^2}{r_1} - a}{\sqrt{a^2 - b^2}}$$

(see the equation for an ellipse).

From these equations we obtain the angle φ_1 between the incoming path and the major axis of the ellipse in question; moreover, since $\Delta\varphi = 2°22\frac{2}{3}'$, we also obtain the angle $\varphi_2 = \varphi_1 \mp \Delta\varphi$ between the final path and the major axis a, as well as, finally, the corresponding values for the final point of the branch of the ellipse:

$$r_2 = \frac{b^2}{a + \sqrt{a^2 - b^2}\cos\varphi_2}$$

(see equation of ellipse),

$$v_2 = \sqrt{\frac{2\mu}{r^2} - \left(\frac{2\mu}{r_1} - v_1^2\right)}$$

(see equation (41)), and

$$\cos\alpha_2 = \cos\alpha_1 \cdot \frac{r_1 v_1}{r_2 v_2}$$

(see law of equal areas and equation (39)), etc., until a distance $r > 6{,}480$ km is reached. The calculations carried out are presented here.

223 For purposes of comparison, the elements of the braking paths for elliptical and helical trajectories are given below.

Limits		0–I	I–II	II–III	III–IV	IV–V	V–VI	VI–VII	VII–VIII
Parabola and first braking ellipse	r	6,480	6,466	6,458	6,455	6,457.5	6,464.5	6,476.3	–
	v	11.10	11.11	11.12	10.40	10.40	10.39	10.38	–
	α	3°34'	2°22⅔'	1°11⅓'	0°0'	1°0'	2°1'	3°2'	–
Transition spiral	r	6,480	6,466	6,457	6,454	6,456	6,462	6,472	6,485
	v	11.10	11.00	10.66	10.20	9.80	9.60	9.57	–
	α	3°34'	2°22⅔'	1°17'	>0° <0°16'	> 0°55' <0°59'	1°45'	2°30'	–

The escape ellipse obtained, with a =12,486 km instead of 25,000 km and with $b = \sqrt{119{,}500{,}000} = 10{,}931$ km instead of 16,800 km, is considerably smaller than the calculated first braking ellipse; the two major axes differ from one another by an angle of 7°41' – 7°8' = 33'. The nearest point of escape from the earth will lie at a distance

$$r_a = \frac{b^2}{a + e} = \frac{119{,}500{,}000}{12\,486 + 6033} = 6452.7 \text{ km instead of } 6455 \text{ km}.$$

Thus, in reality, it may be possible to limit ourselves to two braking ellipses, instead of the five ellipses mentioned above, and then to pass directly to a circular trajectory. This will be particularly advisable if the braking surface F is increased somewhat.

In conclusion, it should be determined whether or not it is possible to pass directly to a circular orbit without completing any elliptical orbits, during the escape of a rocket into a retarding air envelope. This would be possible, of course, only with the use of altitude controls. The latter will not present any difficulty, since in any case such controls will be needed in the subsequent gliding flight.

For our first calculation, which is the most unsuitable case from the point of view of the effect of the retardation, we assume that the rocket reaches the vertex of the parabola at $r_a = 6{,}455$ km, with a reduced (due to the air resistance) velocity of about $v_a = \frac{11.1}{1.032} = 10.75$ km/sec. If, under these conditions, the vehicle must then describe a circular trajectory, it will have to be subjected to a centripetal acceleration

$$z_a = \frac{v_a^2}{r_a} = \frac{10\,750^2}{6{,}455{,}000} = 17.9 \text{ m/sec}^2,$$

instead of the acceleration of gravity

$$g_a = 9.8 \left(\frac{6380}{6455}\right)^2 = 9.6 \text{ m/sec}^2$$

at that point.

Consequently, an additional radial acceleration

$$\varrho = z_a - g_a = 8.3 \text{ m/sec}^2$$

224 is necessary, which should be obtained using the effect of air resistance on the supporting surface F_0 (which is needed anyway), the latter surface being inclined at an angle α to the horizontal, as shown in Figure 82. The angle is selected so that

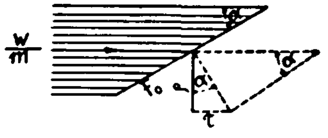


FIGURE 82.

$$\varrho = \frac{w}{m} \cdot F_0 \cdot \sin^2 \alpha \cdot \cos \alpha.$$

As the flight velocity v is gradually reduced, the radial acceleration ϱ will have to be made smaller as well, and this can be accomplished by an appropriate reduction of angle α. For $v_a = 10.75$ km/sec and $r_a = 6{,}455$ km, and with the same supporting-surface area $F_0 = 59$ m^2 as is required for gliding, we have:

a rocket mass

$$m = \sim \frac{2000}{10 \text{ m/sec}^2} = 200 \ \frac{\text{kg/sec}^2}{\text{m}}$$

a resistance

$$w = 640 \cdot \left(\frac{10.75}{11.10}\right)^2 = 600 \text{ kg/m}^2,$$

and

$$\frac{w}{m} \cdot F_0 = \frac{600 \text{ kg/m}^2}{200 \frac{\text{kg/sec}^2}{\text{m}}} ; 59 \text{ m}^2 = 177 \text{ m/sec}^2.$$

In addition, for a circular trajectory,

$$\sin^2 \alpha \cdot \cos \alpha = \frac{\varrho}{\frac{w}{m} \cdot F_0} = \frac{8.8}{177} 0.047 \text{ m}$$

$$\alpha = \sim 12^2/_3{}^\circ.$$

Angle α will decrease gradually, reaching 0° at the transition to a free circular [circling] velocity of 7.85 km/sec. The greatest retardation, at a height of 75 km and at $v_{max} = 11.1$ km/sec, for a parachute area $F = 6.1$ m^2, was earlier found to be

$$\beta_{max} = 0.0193 \text{ km/sec}^2.$$

During the forced circling motion at this height (75 km), the retardation for an instantaneous velocity v will be

$$\beta = \frac{dv}{dt} = -v^2 \frac{\beta_{max}}{v^2_{max}} = -v^2 \cdot k, \left(\text{where } k = \frac{0.0193}{11.1^2}\right),$$

and also

$$\frac{ds}{dt} = v;$$

so that

$$\frac{dv}{ds} = -vk;$$

$$kds = -\frac{dv}{v};$$

$$-ks = \ln v + C.$$

225 At the vertex of the parabola, for $s = 0$,

$$0 = \ln v_a + C; \; C = -\ln v_a;$$

and thus

$$-ks = \ln v - \ln v_a = \ln \frac{v}{v_a},$$

or

$$s = \frac{1}{k} \ln \frac{v}{v_a}.$$

Therefore, at the end of the forced flight, and at the beginning of the free flight in a circle, that is, for $v = 7.85$ km/sec, the rocket moves away from the vertex of the parabola along a path

$$\max s = \frac{11{,}1^2}{0.0193} \ln \frac{1075}{785} 6400 \cdot (6.98\,008 - 6.66\,568) = 2000 \text{ km}.$$

The time required to traverse this distance can be found from the relations:

$$\frac{dv}{dt} = -v^2 k;$$

$$kdt = -\frac{dv}{v^2}$$

$$kt = +\frac{1}{v} + C.$$

For $t = 0$, that is, at the parabola vertex,

$$0 = \frac{1}{v_a} + C; \; C = -\frac{1}{v_a}.$$

Therefore,

$$kt = \frac{1}{v} - \frac{1}{v_a}$$

and

$$t = \frac{1}{k}\left(\frac{1}{v} - \frac{1}{v_a}\right) = \frac{1}{\beta_{max}}\left(\frac{v^2_{max}}{v} - \frac{v^2_{max}}{v_a}\right);$$

$$t = \frac{1}{0.0193}\left(\frac{11.10^2}{7.85} = \frac{11.10^2}{10.75}\right) = \frac{15.7 - 11.5}{0.0193} = 218 \text{ sec} \quad = 3.63 \text{ min.}$$

Consequently, from the moment the rocket passes through the vertex of the parabola until the end of the glide, the following time elapses:

$$218 + 2200 = \sim 2400 \text{ sec} = 40 \text{ min.}$$

A descent to the earth without flying along braking ellipses is thus quite possible. The case is somewhat different during the forced circular flight, when the passengers aboard the vehicle will be pushed toward the upper part of the craft due to the centrifugal force, and when they will fly with their heads or their backs downward, which may make maneuvering difficult. The pilot has to be careful not to fall prematurely into the denser layers of the atmosphere, which (Figure 78) may cause the rocket to fall. On the other hand, if he flies higher than he should, then in the worst case he will have to leave the earth's atmosphere and describe a greater or smaller ellipse, giving him an opportunity to choose a more favorable descent.

226 The ignition of bolides and meteors in the atmosphere represents an apparent contradiction to the method of descent described above. Such phenomena might lead us to conclude that a body entering the earth's atmosphere from outer space must be subjected to extreme heating, due to the air resistance. However, it must be kept in mind that such meteors possess velocities which are considerably higher than that of our rocket. The latter, as we have assumed, is acted upon just by terrestrial gravity, and it possesses, like the earth itself, a motion about the sun at a velocity of 30 km/sec. Meteors, on the other hand, at distances from the sun equal to the radius of the earth's orbit, attain velocities of about 42 km/sec relative to the sun, because of the attraction of the latter. Therefore, if they are flying toward the earth, then for a meteor velocity of about 30 m/sec these bodies will have velocities relative to the earth of $42 + 30 = 72$ km/sec instead of the 11.1 km/sec possessed by our rocket. Since the air resistance is proportional to the square of the velocity, therefore in the most unfavorable direction a falling meteor will encounter a resistance $\left(\frac{72}{11}\right)^2 = 43$ times as great as that encountered by the rocket.

However, it must not be forgotten that for a reduction of the velocity from

$$v' = 11{,}100 \text{ m/sec to } v = 0,$$

an energy $\frac{mv'^2}{2} - 0$ is released. Assuming, as before, that the mass

$$m = \frac{2000 \text{ kg}}{10 \text{ m/sec}^2} = 200 \frac{\text{kg/sec}^2}{\text{m}}$$

we obtain

$$\frac{mv'^2}{2} = \frac{200}{2} \cdot 11{,}100^2 = 12{,}300{,}000{,}000 \ \mu g.$$

This energy must be converted into either turbulence of the air, heat, or both of these together. So far, considerations of descent to the earth have tacitly assumed the former, that is, conversion of the energy to air motion. The other extreme case, a complete conversion to heat, gives the following results.

Assuming a mechanical equivalent of heat equal to $\frac{1}{427}$, we obtain the number of calories liberated during descent:

$$Q = \frac{12{,}300{,}000{,}000}{427} = 24{,}800{,}000 \text{ W. E. (Wärmeeinheiten = units of heat).}$$

For the previous assumption of as rapid a retardation as possible, the parachute used will be greatly heated and will burn up. Therefore, it is necessary to make use of a series of parachutes with appropriate shapes, one after the other, when passing through the retardation region (Figure 80). This continues until, finally, the transition to gliding is made, at point B, where the velocity has dropped to only 1,150 m/sec and where there is no longer any danger of heating.

227 In order to reduce the ignition hazard, the braking should be so planned that the heated surfaces have enough time to transmit heat outward via radiation. In general, the energy produced during retardation from a velocity v' to a velocity v will be

$$E = \frac{mv'^2}{2} - \frac{mv^2}{2};$$

and the energy increment per second is

$$\frac{dE}{dt} = mv\frac{dv}{dt}.$$

This corresponds to an influx of heat per second of

$$\frac{dQ}{dt} = \frac{mv}{427} \cdot \frac{dv}{dt}.$$

If the permissible per-second heat influx $\frac{dQ}{dt}$ is known, then the deceleration during braking for the moment when the velocity is v must be no greater than

$$\frac{dv}{dt} = \frac{dQ}{dt} \cdot \frac{427}{mv}.$$

The permissible influx of heat per second must be compensated, if possible, by a heat efflux via conduction and radiation. Assuming the surface of the rocket to be corrugated, we can take the influx per second to be $500\,\frac{\text{W.E.}}{\text{sec}}$, so that for $m = 200\,\frac{\text{kg/sec}^2}{\text{m}}$, we have

$$\frac{dv}{dt} = \frac{500 \cdot 427}{200 \cdot v} = \frac{1000}{v}; \; (v \text{ in m/sec}).$$

The retardations for various values of v will be:

for $v = 10000$ m/sec: $\frac{dv}{dt} = \frac{1000}{10000} = 0.1$ m/sec^2,

„ $v = 5000$ „ $\frac{dv}{dt} = \frac{1000}{5000} = 0.2$ „

„ $v = 1000$ „ $\frac{dv}{dt} = \frac{1000}{1000} = 1.0$ „

„ $v = 100$ „ $\frac{dv}{dt} = \frac{1000}{100} = 10.0$ „

Such small retardations can almost be obtained without using a parachute, since the air resistance to a body's motion and the drag on the wings of a vehicle will be sufficient to provide a slight braking effect.

The total distance s covered during descent is obtained from the relations

$$\left.\begin{aligned}\frac{dv}{dt} &= \frac{10}{\cdot}\\ \frac{ds}{dt} &= v\end{aligned}\right\} \frac{dv}{ds} = \frac{1000}{v^2}$$

228

$$ds = \frac{v^2\,dv}{1000};\quad s = \frac{1}{1000}\int_0^{11100} v^2\,ds^2 =$$

$\frac{11100^3}{3\cdot 1000} = 410{,}700{,}000$ m $= 410{,}700$ km = about 10 circumferences of the earth.

In this case, during the forced circular motion, the following distances must be covered:

- for $v = 11{,}100$ to $7{,}850$ m/sec, a distance of $\frac{11{,}100^3 - 7850^3}{3\cdot 1000} = 249{,}450{,}000$ m = =about 6 circuits of the earth;

for 7,850 to 4,000 m/sec, $\frac{7850^3 - 4000^3}{3\cdot 1000} = 139{,}920{,}000$ m = about 3.5 circuits;

and for $v = 4{,}000$ to 0 m/sec, $\frac{4000^3}{3\cdot 1000} = 21{,}330{,}000$ m = about 0.5 circuit.

All the foregoing would be true, if we could assume that all the energy of retardation is converted to heat.

The truth lies somewhere between the two extremes. In any case, with regard to a descent to the earth, the following factors must be taken into account:

1. Since the braking does not have to be great, a comparatively small parachute can be employed.

2. The parachute should cause as much air turbulence as possible, which means that it must have an appropriate shape (conditions 1 and 2 will be satisfied best if, as suggested by Valier, the parachute consists of a row of cones situated along a common axis, at large distances from one another and with their vertexes pointing forward).

3. Since ignition of the parachute may occur, additional spare parachutes (cones) should be carried.

4. The rocket should be provided with metal fins for cooling, as well as with wings. The operation of the latter at very high velocities and in a tenuous atmosphere still remains to be studied.

Part III

FREE COASTING IN SPACE

Two portions of an interplanetary flight have been considered in Parts I and II: upward flight away from the earth, until the velocity is great enough to preclude backward falling, and descent to the earth, from the moment of entry into the earth's atmosphere. Now let us consider whether it will actually be possible, after leaving the earth, to so direct the flight that a return to the earth can be made along a desired (for instance, tangential) trajectory.

After its own acceleration terminates, a rocket will move away from the
229 earth in a radial direction if, for the sake of simplicity, the lateral velocity is neglected. The latter originates as a result of the earth's rotation (at the equator it amounts to around 463 m/sec). The rocket ascends or "falls at a steadily diminishing velocity" into space, and its passengers, with the sudden disappearance of the sensation of weight, will probably at first be concerned about the feeling of falling. However, once they become somewhat used to it, they will probably have a pleasant sensation of being suspended in space.

In order for the flight speed at infinity to actually be zero, the rocket must attain a corresponding maximum velocity v_1 at the distance r_1 where the rocket's own acceleration ceases. However, this velocity will still be influenced by the air resistance, which was not determined perfectly accurately in the previous calculations.

In any case let us assume that, at some distance r_2 from the center of the earth (this distance can be found by direct measurements over certain time intervals), the flight velocity is v_2'. At a distance r from the center of the earth the retardation will be

$$\frac{dv}{dt} = -g_0 \cdot \frac{r_0^2}{r^2}$$

and the velocity is

$$\frac{dr}{dt} = v;$$

so that we have

$$\frac{dv}{dr} = -\frac{g_0 r_0^2}{r^2 v},$$

or

$$v\,dv = -g_0 \cdot r_0^2 \cdot \frac{dr}{r^2},$$

from which

$$\frac{v^2}{2} = +\frac{g_0 r_0^2}{r} + C;$$

and at a distance r_2

$$\frac{v_2'^2}{2} = \frac{g_0 r_0^2}{r_2} + C.$$

Consequently,

$$\frac{v_2'^2 - v^2}{2} = \frac{g_0 r_0^2}{r_2} - \frac{g_0 r_0^2}{r} \qquad (25)$$

The height r_3', at which the velocity $v = 0$, is found from the relation

$$\frac{v_3'^2}{2} = \frac{g_0 r_0^2}{r_2} - \frac{g_0 r_0^2}{r_3'} = g_0 r_0^2 \left(\frac{1}{r_2} - \frac{1}{r_3'}\right); \qquad (25a)$$

$$r_3' = \frac{2 g_0 r_0^2}{\frac{2 g_0 r_0^2}{r_2} - v_3'^2} \qquad (26)$$

If the height of ascent should be r_3 rather than r_3', then at r_2, instead of the velocity v_2' determined from equation (25a), the velocity will have to be

$$v_2 = \sqrt{2 g_0 r_0^2 \left(\frac{1}{r_2} - \frac{1}{r_3}\right)} = \sqrt{2 g_0 r_0^3 \frac{r_3 - r_2}{r_2 r_3}} \qquad (27)$$

Thus the given velocity v_2' has to be varied by an amount

$$\Delta v_2 = v_2 - v_2'.$$

230 This can be accomplished by a controlling [correcting] burst of mass Δm, with a velocity of ejection c, thereby reducing the previous mass m of the rocket.

From equation (1) we have

$$\frac{\Delta m}{m} = \frac{\Delta v_2}{c}.$$

The signs will be plus or minus, depending on whether v is directed backward or forward.

Let us assume, for example, that at a distance $r_2 = 40{,}000$ km the given velocity is

$$v_2' = 4.46 \text{ km/sec},$$

(for which the height of the flight $r_3' = \infty$), and that we wish to reach a distance $r_3 = 800{,}000$ km (twice the distance from the earth to the moon).

Then, from equation (27), for

$$2g_0 \cdot r_0^2 = 2 \cdot 0.0098 \cdot 6380^2 = 800{,}000 \text{ km}^3/\text{sec}^2,$$

we must have

$$v_2 = \sqrt{2g_0 \cdot r_0^2 \frac{r_3 - r_2}{r_2 r_3}} = \sqrt{800{,}000 \cdot \frac{800{,}000 - 40{,}000}{40{,}000 \cdot 800{,}000}} = 4.35 \text{ km/sec}$$

from which

$$\Delta v_2 = v_2 - v_2' = 4.35 - 4.46 = -0.11 \text{ km/sec},$$

and for a velocity of gas ejection $c = 1.0$ km/sec

$$\frac{\Delta m}{m} = \frac{0.11}{1.0} = 0.11;$$

that is, about $1/9$ of the original mass must be burned, in a forward direction and with an [exhaust] velocity of 1,000 m/sec. The result obtained will be better, the earlier the burst is carried out.

Until it attains the desired height, r_3, the rocket, if left to itself, will again fall back radially to the earth. However, if the condition stated in Part II, namely a tangential approach to the earth's atmosphere, is satisfied, then the rocket must have a certain tangential velocity v_3 at the instant when the radial velocity becomes zero, that is, at a distance r_3 (Figure 83). Then the return path will not be parabolic, as was the case in Part II, but rather a very elongated ellipse, the semimajor axis of which will be

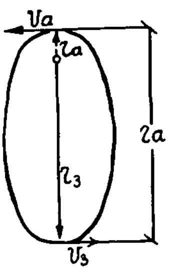


FIGURE 83.

$$a = \frac{r_3 + r_a}{2}.$$

On the other hand, on the basis of the law of gravity (see equation (45) at the end of this part), we have

$$a = \frac{g_0 r_0^2}{\frac{2g_0 \cdot r_0^2}{r_3} - v_3^2}.$$

Therefore,

$$\frac{g_0 \cdot r_0^2}{\frac{2g_0 \cdot r_0^2}{r_3} - v_3^2} = \frac{r_3 + r_a}{2},$$

231 and thus

$$v_3^2 = \frac{2g_0 \cdot r_0^2}{r_3} - \frac{2g_0 \cdot r_0^2}{r_3 + r_a} = 2g_0 \cdot r_0^2 : \frac{r_a}{r_3 (r_3 + r_a)},$$

or

$$v_3 = \sqrt{2g_0 \cdot r_0^2 \frac{r_a}{r_3 (r_3 + r_a)}}. \qquad (28)$$

Similarly,

$$v_a^2 = 2g_0 \cdot r_0^2 \cdot \frac{r_3}{r_a (r_3 + r_a)} = v_3^2 \cdot \frac{r_3^2}{r_a^2},$$

or

$$v_a = v_3 \cdot \frac{r_3}{r_a},$$

for example, for $r_3 = 800{,}000$ km, $r_a = 6{,}455$ km, and $g_0 r_0^2 = 400{,}000$, we obtain

$$v_3 = \sqrt{800{,}000 \cdot \frac{6455}{800\,000\,(806\,455)}} = 0.09 \text{ km/sec} \approx 90 \text{ m/sec}.$$

The tangential velocity can once again be obtained by burning some propellant, the relative mass of which is

$$\frac{\Delta m}{m} = \frac{0.09 - 0.00}{1.0} = 0.09,$$

that is, about $1/11$ of the mass of the rocket must be burned, with a velocity of gas ejection of 1,000 m/sec and in a direction perpendicular to the previous trajectory.

Then the velocity v_a near the earth, at a distance r_a from it, will be

$$v_a = 0.09 \cdot \frac{800\,000}{6455} - 11.1 \text{ km/sec},$$

that is, it is nearly the same as the velocity assumed earlier for a parabolic path.

Since the velocities and distances measured during flight will not be free of error, the correctness of the trajectory will have to be checked during the subsequent flight, and this can be done as follows (Figure 84).

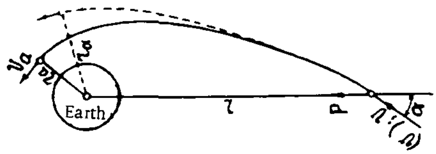


FIGURE 84.

Let us assume that measurements carried out at a distance r gave a velocity v' and a flight direction (angle α) which will send the rocket to the earth over a distance r_a', which is undesirable since the rocket should
232 actually appear at a distance r_a. Then, relations should exist between r_a, r, α, and the required velocities v_1 and v (see end of this part).

1. According to the law of gravity,

$$P = -g_0 \cdot r_0^2 \cdot \frac{m}{r^2}.$$

2. According to the universal laws of work,

$$\int P dr = -g_0 r_0^2 m \cdot \int \frac{dr}{r^2} = \frac{mv^2}{2} - \frac{mv_a^2}{2}$$

or

$$\frac{g_0 r_0^2}{r} + C = \frac{v^2}{2} - \frac{v_a^2}{2}.$$

For $r = r_a$

$$\frac{g_0 r_0^2}{r_a} + C = 0.$$

Consequently,

$$\frac{g_0 r_0^2}{r} - \frac{g_0 r_0^2}{r_a} = \frac{v^2}{2} - \frac{v_a^2}{2},$$

or

$$v_a^2 = v^2 + 2g_0 r_0^2 \left(\frac{1}{r_a} - \frac{1}{r}\right)$$

3. According to the law of equal areas,

$$v \cdot r \cdot \sin\alpha = v_a \cdot r_a,$$

or

$$v_a^2 = \frac{v^2 \cdot r^2 \sin^2\alpha}{r_a^2};$$

therefore, it should be true that

$$v^2 \left(\frac{r^2}{r_a^2} \sin^2\alpha - 1\right) = 2g_0 r_0^2 \left(\frac{1}{r_a} - \frac{1}{r}\right) \qquad (29)$$

or

$$v^2 = \frac{2g_0 r_0^2}{r^2 \sin^2\alpha - r_a^2} \cdot r_a \cdot \frac{r - r_a}{r}$$

and

$$v = \sqrt{\frac{2g_0\, r_0^2}{r^2 \sin^2 a - r_a^2}\, r_a \frac{r - r_a}{r}} \tag{30}$$

instead of v'.

Let us assume, for instance, that at a distance $r_4 = 400{,}000$ km the velocity is

$$v_4' = 1.415 \text{ km/sec},$$

in a direction lying at an angle $a_4 = 7°50'$ (both these values correspond to a parabola with a perigee at $r_a' = 7{,}500$ km), in which case

$$\frac{r_4^2 \sin^2 a_4}{r_a} = \frac{400{,}000^2 \cdot 0.137^2}{6455} = 465{,}000 \text{ km}.$$

In order to reach a point lying a distance $r_a = 6{,}455$ km from the earth, equation (30) gives

$$v_4 = \sqrt{\frac{2g_0\, r_0^2}{r_4^2 \sin^2 a_4 - r_a^2} \cdot r_a \frac{r_4 - r_a}{r_4}} = \sqrt{\frac{800{,}000}{465{,}000 - 6455} \cdot \frac{400{,}000 - 6455}{40{,}000}} = 1.31 \text{ km/sec}.$$

233 Therefore,

$$\Delta v_4 = v_4 - v_4' = 1.310 - 1.415 = 0.105 \text{ km/sec},$$

and the flight direction must be corrected by burning an amount of fuel

$$\frac{\Delta m}{m} = \frac{\Delta v_4}{C} = \frac{0.105}{1.0} = 0.105,$$

that is, about $\frac{1}{9.5}$ of the former mass of the rocket, the burst being directed ahead.

With the aid of equation (29), the effect of the earth's rotation can be established, a factor which has been neglected so far. It imparts to an ascending rocket an initial velocity v_u, which at the equator is

$$\frac{400{,}000}{86{,}400} = 0.463 \text{ km/sec},$$

and at a latitude of 50° is about $0.463 \cos 50° = \backsim 0.3$ km/sec.

As a result of this, it turns out that, when the rocket's own acceleration ceases at a distance r_1 and a velocity v_1 has been attained, the motion of the rocket is not exactly radial, but makes an angle a_1, with the radius r_1, so that

$$\sin a_1 = \frac{v_u}{v_1} \text{ (Figure 85)}.$$

For the values assumed previously, $r_1 = 8{,}490$ km and $v_1 = 9.68$ km/sec, the subsequent flight will be along a parabola, which passes very close to the center of the earth (about 8 km).

At a distance $r_2 = 40{,}000$ km, the flight velocity along the parabola will be

$$v_2' = \sqrt{\frac{2g_0 \cdot r_0^2}{r_2}} = 4.46 \text{ km/sec},$$

and, according to the law of equal areas,

$$v_3 r_2 \sin a_2 = v_1 r_1 \sin a_1 .$$

Therefore,

$$\sin a_2 = \sin a_1 \cdot \frac{v_1 r_1}{v_2 r_2} = \frac{u r_1}{v_2 r_2} = \frac{0.3 \cdot 8490}{4.46 \cdot 40\,000} = 0.0143.$$

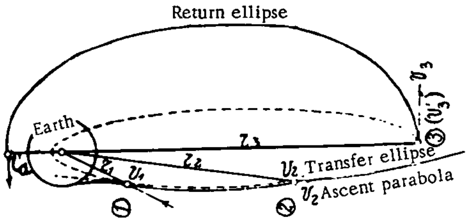


FIGURE 85.

234 Now let us assume that the velocity is reduced from $v_2' = 4.46$ to $v_2 = 4.35$ km/sec, by means of a correcting burst with $c = 1$ km/sec and $\frac{\Delta m}{m} = 0.11$. Then the rocket will fly along a transfer ellipse, the apogee and perigee of which will lie at distances which can be found from equation (29):

$$\frac{v_2^2 r_2^2 \cdot \sin^2 a_2}{r_3^2} - v_2^2 = \frac{2 g_0 r_0^2}{r_3} - \frac{2 g_0 r_0^2}{r^2};$$

$$r_3^2 \left(\frac{2 g_0 r_0^2}{r_2} - v_2^2\right) - r_3 \cdot 2 g_0 r_0^2 = - v_2^2 r_2^2 \sin^2 a_2;$$

$$\frac{\max}{\min} r_3 = \frac{g_0 r_0^2}{\frac{2 g_0 r_0^2}{r_2} - v_2^2} \cdot \left[1 \pm \sqrt{1 - \left(\frac{v_2 r_2 \sin a_2}{g_0 r_0^2}\right)^2 \left(\frac{g_0 r_0^2}{r_2} - v_2^2\right)}\,\right].$$

Accordingly, we have

$$\frac{\max}{\min} r_3 = \frac{400{,}000}{\frac{800{,}000}{40{,}000} - 4.35^2} \cdot \left[1 \pm \sqrt{1 - \left(\frac{4{,}35 \cdot 40{,}000 \; 0.0143^2}{400{,}000}\right) \left(\frac{800{,}000}{40{,}000} - 4.35^2\right)}\,\right]$$

$$\frac{\max}{\min} r_3 = 370{,}500\,(1 \pm 0.99\,999);$$

thus the point on the transfer ellipse which is closest to the earth's center will lie about 4 km from it, that is, it is practically at the center. The farthest distance away, on the other hand, will be 741,000 km from it, that is, at a distance nearly equal to the previous height of ascent. However, now at this distance $r_3 = 741{,}000$ km the velocity will not be zero, but rather, according to the law of equal areas,

$$v_3 = \frac{v_2 r_2 \sin a_2}{r_3} = \frac{4{,}35 \cdot 40{,}000 \cdot 0.0143}{741{,}000} = 0.0034 \text{ km/sec} = 3.4 \text{ m/sec},$$

directed along the tangent.

For transfer to some desired backward elliptical path, the following velocity, found from equation (28), must be taken instead of the previous value $v_3 = 0.09$ km/sec:

$$v_3 = \sqrt{2 \cdot g_0 \cdot r_0^2 \frac{r_a}{r_3(r_3 + r_a)}} = \sqrt{800{,}000 \cdot \frac{6455}{74{,}100 \cdot 747{,}455}} =$$
$$= 0.0964 \text{ km/sec} = 96.4 \text{ m/sec}$$

so that

$$\Delta v = 96.4 - 3.4 = 93 \text{ m/sec}$$

and

$$\frac{\Delta m}{m} = \frac{\Delta v}{c} = 0.093 = \sim \frac{1}{10.8},$$

235 instead of the value of 1/11 obtained earlier; consequently, the earth's rotation does not exert any special effect.

The study of the subsequent trajectory between ascent and descent does not present any special difficulty. Let us assume that, in order to achieve the desired velocity variation, we execute one burst of the rocket (as was assumed earlier), denoting the mass of the rocket before the burst as m_0 and the mass after the burst as m_1. Then, from equation (1),

$$\frac{\Delta m}{m} = \frac{m_0 - m_1}{m_0} = \frac{\Delta v}{c}$$

or

$$\frac{m_0}{m_1} = \frac{1}{1 - \frac{\Delta v}{c}}. \qquad (31)$$

However, the rocket should be protected from the effect of an instantaneous burst and, in addition, it is desirable to reduce the amount of material burned during the burst. Therefore, a series of successive weak bursts will be preferable to a single strong burst. Then the general pattern of the bursts will be close to that given for the fuel consumption in Part I, so that

$$\frac{dm}{m} = \frac{dv}{c}$$

or, in general,

$$\ln m = \frac{v}{c} + C.$$

If at the beginning of the velocity change the mass is m_0 and the velocity v_0, while at the end they are m_1 and v_1, then

$$\ln m_0 = \frac{v_0}{c} + C$$

$$\ln m_1 = \frac{v_1}{c} + C.$$

Accordingly,

$$\ln \frac{m_0}{m_1} = \frac{v_0 - v_1}{c} = \frac{\Delta v}{c}$$

and

$$\frac{m_0}{m_1} = e^{\frac{\Delta v}{c}} \; . \qquad (32)$$

Since here a mass decrease occurs rather than an increase, the sign of Δv will be determined by the direction of the gas ejection. For small values of $\frac{\Delta v}{c}$ the results obtained using equations (31) and (32) will differ little from each other. For large values, on the other hand, a series of bursts turns out to be more suitable than a single burst. For instance, for

$$\frac{\Delta v}{c} = 0.1$$

236 we obtain

$$\frac{1}{1-0.1} = 1.11$$

and

$$e^{0.1} = 1.105,$$

for

$$\frac{\Delta v}{c} = 0.5$$

we obtain

$$\frac{1}{1-0.5} = 2.0$$

and

$$e^{0.5} = 1.65,$$

for

$$\frac{\Delta v}{c} = 0.9$$

we obtain

$$\frac{1}{1-0.9} = 10.0$$

and

$$e^{0.9} = 2.46,$$

and for

$$\frac{\Delta v}{c} = 1.0$$

we obtain

$$\frac{1}{1-1} = \infty$$

and

$$e^{1.0} = 2.72.$$

When determining the duration of the free flight [coasting] and the amount of time between the termination of the rocket's own acceleration and the first entry into the earth's atmosphere, the comparatively insignificant effect of the earth's rotation can be neglected, and r_2 can be assumed to be the same as r_1. Accordingly, the flight time can be divided into two parts:

I. A time t_I, from the end of the rocket's own acceleration at $r_1 = 8,490$ km to the beginning of the ellipse on the return trajectory at $r_3 = 800,000$ km.

II. A time t_{II}, during which the rocket flies along the return ellipse from apogee at $r_3 = 800,000$ km to perigee at $r_a = 6,455$ km.

Time t_I will be identical to the time required for a body without any initial velocity to fall from a height $r_3 = 800,000$ km to a height $r_1 = 8,490$ km. Here, for any distance r, the velocity v is found from equation (27):

$$v = \sqrt{2g_0 r_0^2 \frac{r_3 - r}{rr_3}}$$

or, since

$$v = -\frac{dr}{dt}$$

$$-\frac{dr}{dt} = \sqrt{\frac{2g_0 r_0^2}{r_3}} \cdot \sqrt{\frac{r_3 - r}{r}}$$

237 and

$$-\sqrt{\frac{2g_0 r_0^2}{r_3}} \cdot t = \int \frac{\sqrt{r}\,.\,dr}{\sqrt{r_3 - r}} + C;$$

$$-\sqrt{\frac{2g_0 r_0^2}{r_3}} \cdot t = -\sqrt{r(r_3 - r)} + r_3 \arcsin\sqrt{\frac{r}{r_3}} + C;$$

therefore, for $r = r_3$,

$$0 = 0 + r_3 \frac{\pi}{r} + C.$$

Consequently, we have

$$\sqrt{\frac{2g_0 r_0^2}{r_3}} \cdot t = \sqrt{r(r_3 - r)} + r_3\left(\frac{\pi}{2} \arcsin\sqrt{\frac{r}{r_3}}\right).$$

and for $r = r_1$

$$t_1 = \sqrt{\frac{r_3}{2g_0 r_0^2}}\left[\sqrt{r_1(r_3 - r_1)} + r_3\left(\frac{\pi}{2} - \arcsin\sqrt{\frac{r_1}{r_3}}\right)\right].$$

Since r_3 is very large compared with r_1, we can write

$$\arcsin\sqrt{\frac{r_1}{r_2}} = \sqrt{\frac{r_1}{r_3}}$$

and

$$t_1 = \backsim \sqrt{\frac{r_3}{2g_0 r_0^2}}\left[\sqrt{r_1(r_3 - r_1)} + r_3\left(\frac{\pi}{2}\sqrt{\frac{r_1}{r_3}}\right)\right].$$

Thus

$$t_1 = \sqrt{\frac{800\,000}{800\,000}}\left[\sqrt{8490\,(800,000 - 8490)} + 800,000\left(\frac{3,1416}{2} - \sqrt{\frac{8490}{800,000}}\right)\right] =$$

$$= 1 \cdot [81,900 + 1,174,400] = 1,256,300 \text{ sec} = \backsim 349 \text{ hr}.$$

The time t_{II} required to cover half the arc of the ellipse is found from the law of equal areas (see equation (18 a)):

$$t_{II} = \frac{ab\pi}{v_3 r_3},$$

where

$$a = \frac{r_3 + r_a}{2} = \frac{800\,000 + 6455}{2} = 403{,}227 \text{ km}$$

and

$$b = \frac{v_3 r_3}{\sqrt{\frac{2g_0 r_0^2}{r_3} - v_3^2}} = \frac{0.09 . 800\,000}{\sqrt{\frac{800{,}000}{800{,}000} - 0.09^2}} = 72{,}400 \text{ km}.$$

Therefore,

$$t_{II} = \frac{403{,}227 \cdot 72{,}400\,\pi}{0.09 \cdot 800{,}000} = 1{,}272{,}000 \text{ sec} = 354 \text{ hr}.$$

The total time of the free flight will be

$$t_I + t_{II} = 349 + 354 = 703 \text{ hr} = \backsim 29^1/_3 \text{ days},$$

and the duration of the entire flight, including ascent and descent, will be

$$703 + 22.6 = 726.6 \text{ hr} = \backsim 30^1/_5 \text{ days},$$

that is, about one month.

238 The foregoing considerations enable us to carry out at this point a more precise determination of the value $G_1 = 2$ tons, assumed previously for the rocket weight. This weight will include:

a) passengers with jackets, etc.,
b) solid and liquid food supplies,
c) supply of fuel for heating,
d) supply of oxygen for breathing and combustion,
e) containers for above-mentioned food,
f) equipment for heating, supplying air, removing waste, and making measurements and observations,
g) equipment required for gliding flight: supporting and retarding surfaces, altitude controls, equipment in rocket nose, and corresponding fittings,
h) the rocket casing itself, and the supply of rocket fuel for the controlling bursts, together with equipment.

Now let us evaluate each of these quantitatively.

a) Two persons with clothing and personal effects will weigh $2 \cdot 100 = 200$ kg.

b) Food for a person for one day weighs about 4 kg, giving $2 \cdot 30 \cdot 4 = 240$ kg for two persons for a month.

c) Since the rocket loses heat in outer space via radiation rather than conduction, the loss will be no greater than that for a thermos bottle (a container with an evacuated space in it), being of a similar amount and form, and for a bright surface this loss will be very small. If, in addition, a partly or completely blackened surface is turned toward the sun, then it will absorb solar heat to such an extent that the interior of the rocket may have a temperature high enough to make other heating methods unnecessary.

Just to be on the safe side, let us assume that the rocket loses heat via conduction rather than radiation. The heat loss per hour will then be $V = t.f\varphi$ where t is the difference between the internal and external temperatures, f is the area of the dividing surface, and φ is a coefficient depending on the properties of this surface and representing the amount of heat (in calories) passing through 1 m^2 of the surface for a temperature difference of 1°C (1 W. E. is the amount of heat required to heat 1 kg of water by 1°C).

If the walls of the rocket are covered with a good insulator, which is at the same time as light as possible (a turflike mass), then we may obtain $\varphi = 0.5$. The rocket surface f should be as small as possible; of all the bodies of a given volume, the sphere has the least area. Since the previous considerations indicated a minimum rocket dimension of about 1.5 m (see Figure 81), its volume must be at least 4.5 m^3, in order to accomodate two persons and the necessary equipment. Therefore, instead of a sphere, an ellipsoid of revolution is more advisable, the latter having a diameter of 1.6 m, a length of 3.4 m, an internal volume of 4.55 m^3, and an outer surface 14.45 m^2 in area.

The internal temperature is assumed to be + 10°C. We assume that the side turned toward the sun is heated to + 70°C and that the opposite side is
239 at − 270°C. Thus the average external temperature will be − 100°C, with a difference of 110°C between inside and outside. The heat loss per hour will be $V = 110 \cdot 14.45 \cdot 0.5 = 800$ W. E. and for a day it will be $24 \cdot 800 = 19{,}900$ W. E. These losses must be compensated by burning some kind of fuel. Kerosene gives the greatest amount of heat (11,000 W. E. for 1 kg), and for one day 1.7 kg of it will be necessary. For reasons to be explained in (d) below, we assume a kerosene consumption of 2 kg per day. Then, for 30 days we have $30 \cdot 2 = 60$ kg.

d) Since 2.7 kg of oxygen are needed for the combustion of 1 kg of kerosene, the oxygen consumption per day will be $2 \cdot 2.7 = 5.4$ kg. In addition, about 0.6 kg of oxygen per day are needed for the breathing of one person, or 1.2 kg for two persons. Thus the daily consumption of oxygen for heating and respiration will be $5.4 + 1.2 = 6.6$ kg, giving $30 \cdot 6.6 = 200$ kg per month.

The oxygen will be carried in liquid form, in containers from which the air has been evacuated. If it were transported in the form of compressed air, then the walls of the containers would have to be very thick, and thus very heavy, because of the enormous pressures involved. Liquid oxygen has a temperature of about − 190°C. To convert 1 kg of liquid oxygen to gaseous form, 500 W. E. are necessary; to heat the gas from −190° to + 10°, for a specific heat of 0.27, another $0.27 \cdot 200 = 54$ W. E. /kg are needed. Thus, in all, a daily consumption of 6.6 kg of oxygen is required, or $6.6 \cdot 554 = 3{,}560$ W. E. /day. Along with this, $\frac{3560}{11000} = 0.3$ kg of kerosene are needed.

Therefore, we have to add 0.3 kg of kerosene to the amount obtained in (c), bringing the total consumption to 2 kg.

e) Let us assume that the containers for the liquid oxygen weigh 0.4 of their contents, while the containers for food and kerosene weigh 0.2 of their contents. This gives a total container weight of

$$200 \cdot 0.4 + (240 + 60) \cdot 0.2 = 140 \text{ kg}.$$

f) We assume that the kerosene stove, the ventilating and waste-removal equipment, and the instruments for time, angle, and distance measurements and other observations, all together weigh 200 kg.

g) The various surfaces on the rocket have the following areas: braking surfaces $F = 6$ m^2; supporting surfaces $F_0 = 59$ m^2; control surfaces (for altitude and rotations) = 5 m^2; and the nose section of the rocket, which is so constructed that it can be detached from the rocket in order to reduce the weight and the amount of radiated heat, and which has a conical surface with a base diameter of 1.6 m and a generatrix 4 m long, has an area of $1.6\ \pi \frac{4.0}{2} =$ $= 10$ m^2. The total area is thus $6 + 59 + 5 + 10 = 80$ m^2 and, at 6 kg/m^2, a weight of 240 kg.*

h) According to (c) above, the outer surface of the rocket has an area of 14.45 m^2. Its weight, including the insulating layer, is taken to be 50 kg/m^2, giving a total of $14.45 \cdot 50 = 780$ kg.

i) The correcting-burst equipment weighs 200 kg.

Thus the total weight of the rocket, without the charge [rocket fuel], is 2,260 kg.**

240 Let us assume that, during the flight, three correcting bursts of the rocket are executed, with a consumption of $1/10$ of the mass. Then, taking into account the steady use of food and fuel, we obtain an initial weight $G_1 = 2{,}260 \cdot 1.1^3 = 3{,}000$ kg, giving a rocket-fuel weight of $3{,}000 - 2{,}260 = 740$ kg.

By the beginning of the coasting part of the flight, the supplies of [rocket] fuel, food, kerosene, and oxygen will have disappeared, leaving just a weight

$$G_1' = 3000 - 740 - 240 - 60 - 200 = 3000 - 1240 = 1760 \text{ kg.}$$

Consequently, the final weight during descent turns out to be even less than that assumed in Part II (2 tons). The initial weight, however, is 1.5 times greater than that in Part I. Therefore, 1.5 times as much propellant will be needed, in comparison with the amount assumed in Part I for the period when the rocket has its own acceleration; thus the linear dimensions of the vehicle shown in Figure 72 must be $\sqrt[3]{1.5}$ times greater. If the effect of air resistance during ascent is taken into account, which (according to the data at the end of Part I) corresponds to an increase in initial mass of 933/825, the linear dimensions of the tower in Figure 72 will have to be increased by a factor of

$$\sqrt[3]{1.5 \cdot \frac{933}{825}} = \sqrt[3]{1.69} = 1.192$$

For $c = 2{,}000$ m/sec and $ac = 30$ m/sec^2, we have:

height of tower	$27 \cdot 1.192 = 32$ m;
lower diameter	$18.7 \cdot 1.192 = 22$ m;
upper diameter	$0.65 \cdot 1.192 = 0.77$ m.

The total weight at the beginning of the ascent will then be

$$G_0 = G_1 \frac{m_0}{m_1} = 3 \cdot 933 = 2799 \text{ tons [sic].}$$

* Here Hohmann makes an error in multiplying: 80.6 = 480 [Rynin].

** [The corrected weight is 2,500 kg.]

In order to keep the weight down, we have assumed that changes in flight direction are effected solely by means of a correcting burst of the rocket. Thus some means will have to be provided for turning the rocket, so as to point the correcting burst in the proper direction. This can be accomplished by moving the masses inside the rocket the other way; for instance, passengers hanging onto special handrails will be able to move along the walls of the compartment. Let us assume that living beings of mass m_e, situated at an average distance x_e from the center of gravity of the rocket, move at an angular velocity w_e. At the same time, inert masses m_t, located an average distance x_t from the center of gravity, move with an opposing angular velocity w_t (Figure 86). Then, according to the law which states that the static momentum (Σmv) of the entire system must be zero, we have

$$\Sigma mv \cdot x = 0;$$

241 or, since $v = x\omega$,

$$\Sigma m\omega x^2 = 0$$

or

$$m_t \cdot \omega_t \cdot x_t^2 = m_e \cdot \omega_e \cdot x_e^2,$$

so that

$$\frac{\omega_t}{\omega_e} = \frac{m_e \cdot x_e^2}{m_t\, x_t^2} \qquad (33)$$

that is, the angular velocities are inversely proportional to the moments of inertia of the masses. If the passengers weigh 140 kg, then in an adverse case (at the beginning of free flight) the remaining mass will weigh 3,000–140 = 2,860 kg, and, according to Figure 86, we obtain

$$\frac{\omega_t}{\omega_e} = \frac{140 \cdot 0.5^2}{2860 \cdot 1.2^2} = \sim \frac{1}{120}.$$

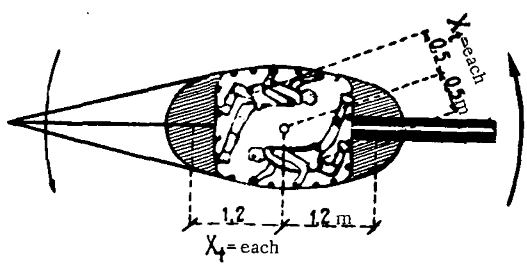


FIGURE 86.

Thus, in order to cause the rocket to negotiate one [complete] turn, the passengers have to crawl around the walls a total of 120 times; for a half turn they must go around 60 times, for a quarter turn, 30 times, etc. Such climbing exercises will provide a sensation of gravity for the arms and legs, and this will even constitute a pleasant diversion during a prolonged state of weightlessness.

If the passengers move around the center of gravity with a velocity of 0.5 m/sec, then it will take them $\frac{1.0 \cdot \pi}{0.5} = 6$ sec to go around once, and to move the vehicle through a quarter turn it will take $30 \cdot 6 = 180$ sec. At a distance $r_1 = 40{,}000$ km from the earth's center, where the first correcting burst is necessary, the flight speed will be about 4.46 km/sec, and, during the time the passengers are climbing around the vehicle, the latter will traverse a distance of $4.46 \cdot 180 = 800$ km. Therefore, the turn must be begun 800 km ahead of the point where the velocity is to be varied by an amount Δv_2 and where the rocket has to rotate its nozzle forward or backward (depending on the sign of Δv_2).

Compared to the distance from the earth (40,000 km), a distance of 800 km is not very great. To turn the rocket during descent, for a correct positioning of the supporting surfaces at the beginning of coasting, rotation of the ellipsoid about its major axis can be carried out more rapidly, since in this case the inert mass of the rocket will be closer to the axis of rotation.

242 In conclusion, let us consider some laws and some results related to the motion of a body under the influence of gravity. These laws have already been applied in the foregoing, and they will frequently be used below.

1. **Observational data:** the planets follow approximately circular trajectories around the sun.

2. If a body of mass m, moving with a velocity v, describes a circular trajectory of radius r, then it is acted upon by a "centripetal" acceleration $\frac{dv_r}{dt}$, directed toward the center of the circle (see Figure 87). For a very small time interval Δt, the components of the path traversed will be

$$\Delta x = v \cdot \Delta t,$$

from which we have

$$\Delta t = \frac{\Delta x}{v}$$

and

$$\Delta y = \frac{dv_r}{dt} \cdot \frac{(\Delta t)^2}{2} = \frac{dv_r}{dt} \cdot \frac{(\Delta x)^2}{2v^2}.$$

In addition, from the similarity of the right triangles containing $\Delta \varphi$, we can write

$$\Delta y = \frac{\Delta x}{2} \cdot \frac{\Delta x}{r} = \frac{(\Delta x)^2}{2r}.$$

A comparison of the two expressions shows that

$$\frac{dv_r}{dt} = \frac{v^2}{r},$$

or, if the centripetal acceleration is due to a central force P, we have

$$P = -m\frac{v^2}{r} \tag{34}$$

(negative if P is directed opposite to r, that is, inward).

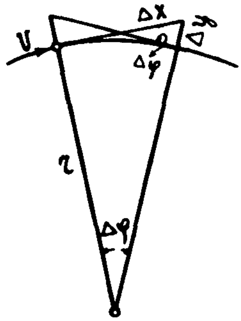


FIGURE 87.

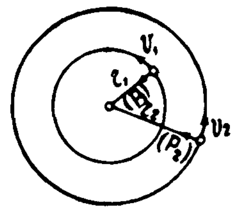


FIGURE 88.

3. **Observational data**: the squares of the revolution times T_1 and T_2 of two planets vary as the cubes of their distances r_1 and r_2 from the sun (Figure 88):

$$\frac{T_1^2}{T_2^2}=\frac{r_1^3}{r_2^3}.$$

If v_1 and v_2 are the corresponding velocities of the planets, then

$$T_1=\frac{2r_1\pi}{v_1} \quad \text{and} \quad T_2=\frac{2r_2\cdot\pi}{v_2}$$

243 and

$$\frac{r_1^2}{v_1^2}\cdot\frac{v_2^2}{r_2^2}=\frac{r_1^3}{r_2^3},$$

or

$$\frac{v_2^2}{v_1^3}=\frac{r_1}{r_2} \tag{35}$$

4. It follows from (34) and (35) that

$$\frac{P_1}{P_2}=\frac{\frac{m_1 v_1^2}{r_1}}{\frac{m_2\cdot v_2^2}{r_2}}=\frac{m_1 v_1^2 r_2}{m_2 v_2^2 r_1}=\frac{m_1 r_2^2}{m_2 r_1^2}$$

and thus

$$\left.\begin{aligned} P_1&=-\mu\cdot\frac{m_1}{r_1^2}\\ P_2&=-\mu\cdot\frac{m_2}{r_2^2}\end{aligned}\right\}$$ (negative because P is toward the center, whereas r is measured outward from the center)

or, as a **universal law of gravity,**

$$P=-\mu\frac{m}{r^2}, \tag{36}$$

where μ has a different value for each center of attraction. This quantity will be determined below.

5. If **the sun** is the center of attraction, μ can be found from the following data: the distance from the earth to the sun is, on the average, $r_e =$ $= 149,000,000$ km, the time for one revolution around the sun is $T_e = 365$ days, and the average velocity is

$$v_e = \frac{2r_e \cdot \pi}{T_e} = \frac{2 \cdot 149{,}000{,}000\pi}{365 \cdot 86{,}400} = 29.7\ \text{km/sec.}$$

From equations (34) and (36), we now have

$$-P = m_e \cdot \frac{v_e^2}{r_e} = \mu \cdot \frac{m_e}{r_e^2},$$

or

$$\mu = v_e^2 \cdot r_e = (29.7\ \text{km/sec})^2 \cdot 149{,}000{,}000\ \text{km}$$
$$\mu = 132{,}000{,}000{,}000\ \text{km}^3/\text{sec}^2 \tag{37}$$

6. For **the earth** as the center of attraction, μ is obtained as follows: the distance of the moon from the earth $r_m = 392,000$ km, the time for one revolution about the earth is 28 days, and the velocity is

$$v_m = \frac{2r_m \cdot \pi}{T_m} = \frac{2 \cdot 392{,}000\,\pi}{28 \cdot 86{,}400} = 1.01\ \text{km/sec},$$

so that

$$\mu = v_m^2 \cdot r_m = 1.01^2 \cdot 392{,}000 = 400{,}000\ \frac{\text{km}^3}{\text{sec}^2}.$$

7. At the **earth's surface** $r_0 = 3,680$ m, and the terrestrial attraction is found from equation (36) as

$$P_0 = \frac{\mu \cdot m}{r_0^2} = \frac{400{,}000}{6380^2}\, m$$

and the central acceleration

$$g_0 = \frac{\mu}{r_0^2} = \frac{400{,}000}{6380^2} = 0.0098\ \text{km/sec}^2 = 9.8\ \text{m/sec}^2,$$

which is also the observed acceleration for the free fall of bodies. If we assume that g_0 is known, then

$$\mu = g_0 \cdot r_0^2 = 0.0098 \cdot 6380^2 = 400{,}000\ \frac{\text{km}^3}{\text{sec}^2}.$$

244 8. **Law of equal areas.** For every central motion, that is, for the motion of a material point acted upon by a force P directed toward a center which is at rest, the following things will be true: at a distance r_1 the velocity v_1 varies in direction and magnitude because of the effect of the central acceleration caused by force P. The new velocity v_2 is obtained as the diagonal of the velocity parallelogram. The area swept out by line r_1 per unit time will be (Figure 89), for a velocity v_1,

$$\frac{dF_1}{dt} = \frac{r_1 v_1 \sin \varphi_1}{2},$$

and for a velocity v_2,

$$\frac{dF_2}{dt}=\frac{r_1 v_1 \sin\varphi_1}{2}.$$

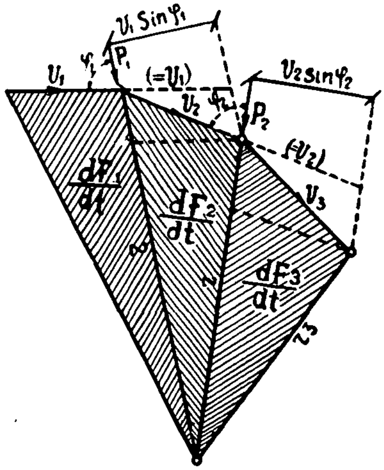


FIGURE 89.

In a similar manner, for the distance r_2 and the velocity v_2, we determine v_3 as the diagonal of the velocity parallelogram including the velocity due to the effect of the central acceleration from force P_2. The area swept out per unit time by line r will be

for velocity v_2, $\frac{dF_2}{dt}=\frac{r_2 v_2 \sin\varphi_2}{2}$,

for velocity v_3, $\frac{dF_3}{dt}=\frac{r_2 v_2 \sin\varphi_2}{2}$;

and thus, from the foregoing, we can write

$$\frac{dF_1}{dt}=\frac{dF_2}{dt}=\frac{dF_3}{dt}=\mathrm{const} \tag{39}$$

that is, during equal time intervals the radius vector sweeps out equal areas.

9. **Law of work.** At each point along the flight path the force P (Figure 90) can be divided into two components, X and Y, of constant direction:

$$X=m\frac{dv_x}{dt};\qquad Y=m\frac{dv_y}{dt};$$

where

$$\frac{dx}{dt}=v_x;\qquad \frac{dy}{dt}=v_y.$$

From this we have

$$Xd_x = mv_x\,dv_x; \quad Yd_y = mv_y\,dv_y;$$

$$\int Xdx = \frac{mv_x^2}{2} - \frac{mv_{ax}^2}{2}; \quad \int Yd_y = \frac{mv_y^2}{2} - \frac{mv_{ay}^2}{2};$$

or, since

$$v^2 = v_x^2 + v_y^2,$$

therefore, between two points at which the velocities are v_a and v,

$$\int Xdx + \int Ydy = \frac{mv^2}{2} - \frac{mv_a^2}{2}.$$

Moreover, from Figure 90:

$$\left.\begin{matrix} x = P\cos\xi; & dx = ds.\cos\zeta \\ y = P\sin\xi; & dy = ds.\sin\zeta \end{matrix}\right\} ds = \frac{dr}{\cos\varphi}.$$

Consequently,

$$\int P(\cos\xi\cos\zeta + \sin\xi.\sin\zeta)\frac{dr}{\cos\varphi} = \frac{mv^2}{2} - \frac{mv_a^2}{2},$$

245 or, since

$$\cos\xi\cos\zeta + \sin\xi\sin\zeta = \cos(\xi - \zeta) = \cos\varphi$$

we obtain

$$\int Pdr = \frac{mv^2}{2} - \frac{mv_a^2}{2} \qquad (40)$$

10. **Application to any motion under influence of gravity**. Figure 91 shows: z, the center of attraction; v_a, the velocity of a body at r_a, when it is closest to the center; and v, the velocity of the body at any distance r. The components of this velocity are: $\frac{dr}{dt}$ along r, and $r\cdot\frac{d\varphi}{dt}$ perpendicular to r.

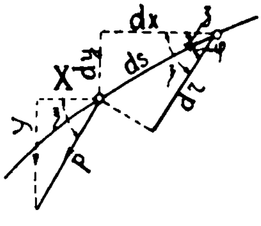


FIGURE 90.

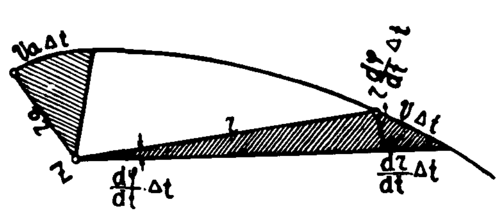


FIGURE 91.

Then, according to equation (36) for the law of gravity,

$$P = -\frac{\mu.m}{r^2}$$

and, according to equation (40) for the law of work,

$$\int P dr = -\mu m \int \frac{dr}{r^2} = \frac{mv^2}{2} - \frac{mv_a^2}{2}$$

or

$$+\frac{\mu}{r} + C = \frac{v^2}{2} - \frac{v_a^2}{2}.$$

For $r = r_a$,

$$\frac{\mu}{r_a} + C = 0.$$

Therefore,

$$\frac{\mu}{r} - \frac{\mu}{r_a} = \frac{v^2}{2} - \frac{v_a^2}{2},$$

or

$$v^2 = v_a^2 + \frac{2\mu}{r} - \frac{2\mu}{r_a}. \qquad (41)$$

From the law of equal areas (39),

$$\frac{v_a \cdot \Delta t \cdot r_a}{2} = \left(r + \frac{dr}{dt} \cdot \frac{t}{\Delta t}\right) \cdot \frac{r}{2} \cdot \frac{d\varphi}{dt} \cdot \Delta t;$$

and thus

$$\frac{d\varphi}{dt} = \frac{v_a r_a}{r^2 + r\frac{dr}{dt}\Delta t},$$

or, for $\Delta t = dt = 0$,

$$\frac{d\varphi}{dt} = \frac{v_a r_a}{r^2}. \qquad (42)$$

Pythagoras's theorem gives

$$(v\,\Delta t)^2 = \left(\frac{dr}{dr} \cdot \Delta t\right)^2 + \left(r\frac{d\varphi}{dt} \cdot \Delta t\right)^2$$

246 or

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\varphi}{dt}\right)^2 = \left(\frac{dr}{dt}\right)^2 + \frac{v_a^2 r_a^2}{r^2}$$

and, taking equation (41) into account,

$$\left(\frac{dr}{dt}\right)^2 = v_a^2 + \frac{2\mu}{r} - \frac{2\mu}{r_a} - \frac{v_a^2 r_a^2}{r^2};$$

moreover, from (42),

$$\left(\frac{d\varphi}{dt}\right)^2 = \frac{v_a^2 r_a^2}{r^4};$$

and so

$$\left(\frac{dr}{d\varphi}\right)^2=\frac{r^4}{v_a^2 r_a^2}\left(v_a^2-\frac{2\mu}{r_a}+\frac{2\mu}{r}-\frac{v_a^2 r_a^2}{r^2}\right),$$

or

$$\frac{dr}{d\varphi}=r\sqrt{\frac{v_a^2-\frac{2\mu}{r_a}}{v_a^2 r_a^2}\cdot r^2+\frac{2\mu}{v_a^2 r_a^2}r-1} \tag{43}$$

11. The **ellipse equation** (Figure 92) is

$$r=\frac{b^2}{a+e\cos\varphi},$$

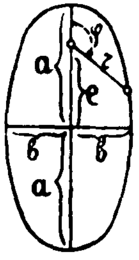


FIGURE 92.

where

$$e^2=a^2-b^2$$

or

$$a^2-e^2=b^2;$$

$$\frac{dr}{d\varphi}=\frac{b^2 e\sin\varphi}{(a+e\cos\varphi)^2},$$

but

$$\frac{b^2}{(a+e\cos\varphi)^2}=\frac{r^2}{b^2}$$

and

$$e\sin\varphi=\sqrt{e^2-e^2\cos^2\varphi}.$$

In addition,

$$e^2\cos^2\varphi=\left(\frac{b^2}{r}-a\right)^2=\frac{b^4}{r^2}-\frac{2ab^2}{r}+a^2,$$

so that

$$e\sin\varphi=\sqrt{e^2-a^2+\frac{2ab^2}{r}-\frac{b^4}{r^2}}=\sqrt{-b^2+\frac{2ab^2}{r}-\frac{b^4}{r^2}};$$

and so

$$\frac{dr}{d\mu}=\frac{r^2}{b^2}\sqrt{-b^2+\frac{2ab^2}{r}-\frac{b^4}{r^2}},$$

or

$$\frac{dr}{d\varphi}=r\sqrt{-\frac{1}{b^2}r^2+\frac{2a}{b^2}r-1} \tag{44}$$

12. A **comparison of equations (43) and (44)** indicates that a body moving under the influence of gravity describes an ellipse, for which

$$-\frac{1}{b^2}=\frac{v_a^2-\frac{2\mu}{r_a}}{v_a^2 r_a^2}$$

and

$$\frac{2a}{b^2}=\frac{2\mu}{v_a^2 r_a^2},$$

247 so that

$$a=\frac{\mu}{\frac{2\mu}{r_a}-v_a^2}; \tag{45}$$

moreover,

$$b^2=a\,\frac{v_a^2 r_a^2}{\mu}=\frac{v_a^2 r_a^2}{\frac{2\mu}{r_a}-v_a^2};$$

consequently,

$$b=v_a r_a\sqrt{\frac{a}{\mu}}=\frac{v_a r_a}{\sqrt{\frac{2\mu}{r_a}-v_a^2}}. \tag{46}$$

In addition,

$$e^2=a^2-b^2=a^2-a\,\frac{v_a^2 r_a^2}{\mu};$$

and, if we add on

$$0=+2ar_a-2ar_a,$$

we obtain

$$e^2=a^2-2ar_a+\frac{r_a^2 a}{\mu}\left(\frac{2\mu}{r_a}-v_a^2\right),$$

or, since

$$\frac{1}{\mu}\left(\frac{2\mu}{r_a}-v_a^2\right)=\frac{1}{a},$$

therefore,

$$e^2=a^2-2ar_a+r_a^2=(a-r_a)^2,$$

or

$$e=\pm(a-r_a),$$

that is, the focus of the ellipse (Figure 92) coincides with the center of attraction (Figure 91).

13. As long as

$$\frac{2\mu}{r_a}-v_a^2>0,$$

the value of a will stay positive and b will be real, that is, the path remains elliptical.

If

$$\frac{2\mu}{r_2}-v_a^2=0,$$

then

$$a = \infty \quad \text{and} \quad b = \infty,$$

that is, the path is parabolic.

If

$$\frac{2\mu}{r_0} - v_a^2 < 0,$$

then a is negative and b is imaginary, that is, the path is **hyperbolic**.

If $a = r_a$, then

$$r_a = \frac{\mu \cdot r_a}{\frac{2\mu}{r_0} - v_a^2},$$

or

$$2 - v_a^2 r_a = \mu;$$

so that

$$v_a^2 = \frac{\mu}{r_a},$$

that is, the path is **circular**.

248 14. The **flight time** for any ellipse is found from equation (39), the law of equal areas:

$$\frac{dF}{dt} = \text{const.} = \frac{v_a r_a}{2},$$

$$F = \frac{v_a r_a}{2} \cdot t = ab\pi,$$

which gives

$$t = \frac{2ab\pi}{v_a r_a}. \qquad (47)$$

When we substitute into (47) the following expression from (46),

$$b = v_a r_a \sqrt{\frac{a}{\mu}},$$

we obtain finally

$$t = 2a\pi \sqrt{\frac{a}{\mu}} = 2\pi \sqrt{\frac{a^3}{\mu}} \qquad (48)$$

Part IV

FLIGHT ORBITS AROUND OTHER CELESTIAL BODIES

A flight around the moon, in order to study its unknown other side, will diffe little from the free flight of a rocket considered in Part III, provided the ship does not come so close to the moon that the lunar attraction (which is $\frac{1}{80}$ of that of the earth at the same distance) has an effect. During 30 days of rocket flight, the moon describes an almost complete circle about the earth, so that here, strictly speaking, it is a case of intersection of the paths of the rocket and the moon, rather than a flight around the moon. In Figure 93, E is the earth, M is the moon, and F is the rocket, with identical subscripts indicating corresponding positions of the rocket and the moon. The closest approach to the moon will equal about half the greatest distance of the rocket from the earth, and the maximum relative lunar attraction will be about $\frac{4}{80}=\frac{1}{20}$ of that of the earth at that same time. Its further effect will not be considered here.

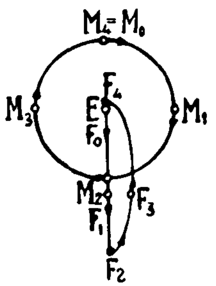


FIGURE 93.

In the previous considerations only the earth's attraction was taken into account. The attraction of the sun was ignored, since the rocket was assumed to take part in the earth's motion around the sun at a velocity of 30 km/sec. Strictly speaking, however, this will be the case only at an instant when the rocket is at rest relative to the earth, that is, when the maximum distance r_3 from the earth is reached; not only that, but this rest point will have to lie
249 on the earth's orbit, that is, it will have to be at the same distance from the sun as the earth is. It was assumed that the rocket leaves the earth along a tangent to the orbit of the latter, with a velocity of 10 km/sec relative to the earth. Thus, the velocity relative to the sun will be $30+10=40$ or $30-10=20$ km/sec, depending on whether the rocket flies with or against the orbital motion of the earth. In the latter case the trajectory of the rocket will be steeper than the earth's orbit, and in the former case it will be less steep.

Since the velocity of the rocket relative to the earth diminishes rapidly, due to the earth's attraction, while the times of the ascents considered so far only amounted to about 15 days (that is, about $\frac{1}{24}$ of the time for the earth to revolve around the sun), the path of the rocket during the given time interval can hardly deviate much from the earth's orbit. If, on the other hand, the ascent takes place radially with respect to the earth's orbit, then at the moment the maximum height r_3 is reached, even though the rocket speed relative to the sun equals that of the earth, the rocket's distance from the sun will be greater or less than the earth's distance, depending on whether the rocket moves away from the sun or toward it. In the latter case, because of the solar attraction, the rocket's path will be more curved than the earth's, while in the former case it will be less curved.

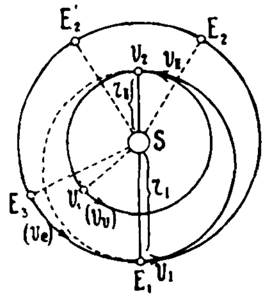
However, as along as the ascent is only to 800,000 km, as calculated previously, the distance will not be great enough, in comparison with the earth-sun distance of 150,000,000 km, to cause much solar influence on the path of the rocket, and it will be immaterial in which direction the ascent from the earth is carried out. It is even advisable to ascend toward the sun, so as to facilitate measurements of distances and velocities, the earth being visible as a brightly illuminated disk. A height of ascent $r_3 = 800{,}000$ km in such a direction will, in the following, be considered an initial value for which this distance can be neglected in comparison with the distance from the sun.

Let us assume that, at this distance r_3, the tangential velocity v_3 is not 0.09 km/sec, as was taken to be the value in Part III (Figure 83), but rather about 3 km/sec. Then, under the influence of the earth's gravity, the path of the rocket will not be elliptical but instead a very gently sloping hyperbola, since

$$\frac{2\mu}{r_3}-v_3^2=\frac{2\cdot 400{,}000}{800{,}000}-3^2=-8.$$

The rocket will move along this trajectory with an almost constant velocity and at an ever-increasing distance from the sphere of terrestrial

(250) gravitation. Then, ultimately, it will move just under the influence of solar gravitation, like an independent comet. At the beginning of this trajectory the tangential velocity of the rocket relative to the sun will be $v_1 = 29.7 \pm 3.0 = 32.7$ or 26.7 km/sec, depending on whether the rocket flies with or against the earth's motion in its orbit. In either case the rocket will describe an ellipse about the sun, outside the earth's orbit in the former case and inside it in the latter.



250 FIGURE 94.

Let us assume that the rocket describes an ellipse inside the earth's orbit, and that it touches the latter orbit at a point situated a distance r from the sun. We further assume that the rocket flies *tangentially to the orbit of another planet at a point at a* distance r_{II} (Figure 94). The semimajor axis of the ellipse will be

$$a=\frac{r_1+r_{II}}{2};$$

however, from (45),

$$a=\frac{\mu}{\frac{2\mu}{r_1}-v_1^2};$$

accordingly,

$$\frac{2\mu}{r_1}-v_1^2=\frac{2\mu}{r_1+r_{II}};$$

and thus

$$v_1^2=\frac{2\mu}{r_1+r_{II}}\cdot\frac{r_{II}}{r_1}$$

or

$$v_1=\sqrt{\frac{2\mu}{r_1+r_{II}}\cdot\frac{r_{II}}{r_1}}. \tag{49}$$

The mean distance of the earth from the sun $r_1 = 149{,}000{,}000$ km, whereas for Venus, for example, $r_{II} = 108{,}000{,}000$ km. For the sun, according to equation (37), $\mu = 132{,}000{,}000{,}000\ \text{km}^3/\text{sec}^2$. Therefore, for the line of flight to pass close to Venus, we must have

$$v_1=\sqrt{\frac{264{,}000}{257}\cdot\frac{108}{149}}=27.3\ \text{km/sec}.$$

Assuming a velocity $v_e = 29.7$ km/sec for the earth, we obtain the required difference between the velocities of the rocket and the earth for attainment of the height of ascent by the rocket:

$$\Delta v_1 = v_1 - v_e = 27.3 - 29.7 = -2.4 \text{ m/sec}.$$

This velocity change can be effected by means of a correcting burst in the tangential direction, the mass consumed in the burst being

$$\Delta m = m \frac{\Delta v_1}{c},$$

where m is the rocket mass before the burst and c is the velocity of gas ejection. However, here the value of c assumed in Part III (1 km/sec) for the correcting burst is no longer suitable, and, in addition, a single powerful rocket burst would be dangerous for both the vehicle and the passengers. In the given case, a series of bursts should be used, as discussed in Part I, with a gas velocity

$$c = 2 \text{ km/sec}.$$

The ratio between the total rocket mass before and after the bursts is given by equation (32):

$$\frac{m_0}{m_\cdot} = e^{\left(\frac{\Delta v}{c}\right)};$$

251 here it must be kept in mind that, during a rocket flight to a planet, deviations from the flight path are possible. Thus, to be certain, a correction factor of about $\nu = 1.1$ should be introduced.* Therefore,

$$\left(\frac{m_0}{m_1}\right)_1 = \nu \cdot e^{\frac{\Delta v_1}{c}} = 1.1 \cdot e^{\frac{2.4}{2.0}} = 1.1 \cdot e^{1.20} = 3.65$$

and the ejection should be in the direction of the earth's motion, that is, forward.

* These path deviations can be eliminated by ejecting a mass $\frac{dm}{dt} = -am$ (see equation (1c)), in the direction of the perturbing planet and equivalent to the perturbing gravitational acceleration g. Thus, at a distance x from the planet, equations (1a) and (2) give

$$\frac{dv}{dt} = ca = g = g_0 \frac{r_0^2}{x^2} = \frac{m_0}{m} = e^{at}.$$

For instance, for the assumed initial distance from the earth, $x = 800{,}000$ km, $g_0 = 9.8$ m/sec^2 and $r_0 = 6{,}380$ km:

$$ca = 9.8 \cdot \frac{6380^2}{800{,}000^2} = \frac{1}{16{,}000} \text{ m/sec}^2$$

After a day, or 8,640 sec, for $c = 2{,}000$ m/sec, we have

$$at = \frac{ca}{c} \cdot t = \frac{86{,}400}{16{,}000 \cdot 2000} = 0.0270;$$

For a distance $x = 800{,}000$ km from Venus, when $g_0 = 8.7$ and $r_0 = 6{,}090$.

$$ca = 8.7 \cdot \frac{6090^2}{800{,}000^2} = \frac{1}{20{,}000} \text{ m/sec}^2$$

(continued on next page)

The duration of the flight over half the arc of the ellipse is found from equation (48) for

$$a = \frac{r_{I} + r_{II}}{2} = 128{,}500{,}000 \text{ km}:$$

$$T_1 = \pi \sqrt{\frac{a^3}{\mu}} = \pi \sqrt{\frac{128{,}500{,}000^3}{132{,}000{,}000{,}000}} = 12{,}600{,}000 \text{ sec} = 146 \text{ days}.$$

252 The earth moves along its orbit around the sun with an angular velocity $\frac{360°}{365 \text{ dys.}} = 0.987°$ per day; for Venus this velocity is $\frac{360}{224 \text{ dys.}} = 1.607°$ per day.

In 146 days the earth traverses an arc of $146 \cdot 0.987 = 144°$, while Venus covers $146 \cdot 1.607 = 234.5°$. In order to fly from the earth to Venus (to a point 800,000 km from its center, on the side toward the sun), the launch

(251) (Footnote continued)

and

$$at = \frac{86{,}400}{20{,}000 \cdot 2000} = 0.0216.$$

For a distance $x = 800{,}000$ km from Mars, when $g_0 = 3.7$ and $r_0 = 3{,}392$,

$$\varepsilon a = 3.7 \cdot \frac{3393^2}{800{,}000^2} = \frac{1}{150{,}000}$$

and

$$at = \frac{86{,}400}{150{,}000 \cdot 2000} = 0.00288.$$

With each succeeding day, x will be greater and the daily increment at will be less.

(252) The following table gives the calculated values of t for various distances x and for the first five days of flight from different planets.

Day	Earth		Venus		Mars	
	x km	at	x km	at	x km	at
0	800,000	0.0270	800,000	0.0216	800,000	0.0029
1	850,000	0.0240	850,000	0.0191	900,000	0.0023
2	900,000	0.0213	900,000	0.0170	1,000,000	0.0018
3	1,000,000	0.0173	1,000,000	0.0138	1,200,000	0.0013
4	1,000,000	0.0143	1,200,000	0.0096	1,400,000	0.0009
5	1,200,000	0.0120	1,400,000	0.0070	1,700,000	0.0006
Total	$\Sigma at = 0.1159$		$\Sigma at = 0.0881$		$\Sigma at = 0.0098$	

After five days we will have

$$\nu = \frac{m_0}{m_1} = e^{\Sigma at}$$

for the earth,

$$\nu = e^{0.116} = 1.123;$$

for Venus,

$$\nu = e^{0.088} = 1.093;$$

for Mars,

$$\nu = e^{0.01} = 1.01.$$

The value $\nu = 1.1$ given above for the correction factor is only a rough average of this quantity. An accurate calculation for each distance from the planet gives a more favorable value. It need not be determined for each second; it is sufficient to calculate it each day, either once or several times.

from the earth must take place when Venus is situated 234.5−180 = 54.5° behind the earth, reckoning according to the direction of motion of the planets (points V_1 and E_1 in Figure 94), and 146 days after this the earth will be 36° behind Venus (points V_2 and E_2 in Figure 94).

If the rocket continues along this path, then after another 146 days it will cover the dotted half of the ellipse and reach the initial point of the flight. Then the earth will be another 36° away, that is, a total of 72° (point E_3 in Figure 94). The flight must be continued in order to reach the earth. Here, there are two possible ways to carry out the return to the earth:

First alternative. If the dotted part of the ellipse [in the figure] is actually to lead back to the earth, then when the rocket leaves Venus (V_2) the earth must be 36° ahead of Venus (that is, at point E_2^1), rather than 36° behind it (at E_2). Thus the rocket must remain in the vicinity of Venus until a favorable juxtaposition of the two planets occurs, that is, until Venus moves to
253 a position 36° behind the earth. Since Venus moves more rapidly than the earth (it covers 1.607−0.987 = 0.62° more per day), therefore, in order for it to move from a position 36° ahead of the earth to a position 36° behind the earth, it will have to traverse an arc of 360−72 = 288°, which corresponds to $\frac{288}{0.62}$ = 464 earth days. During all this time, the rocket must remain in orbit around Venus. To accomplish this, its velocity must be reduced by an amount Δv_{II}, corresponding to the prolonged effect of Venus's attraction; this will be analogous to the situation earlier, when the velocity decreased by an amount Δv_I under the influence of the earth's attraction. Venus (V_2 in Figure 94) will be overtaken for a rocket velocity

$$v_{II} = v_I \cdot \frac{r_I}{r_{II}} = 27.3 \cdot \frac{149}{108} = 37.6 \text{ km/sec.}$$

However, at this time the velocity of Venus is

$$v_v = \frac{2 \cdot 108\,000\,000}{224 \cdot 86400} \cdot \pi = 35.1 \text{ km/sec.}$$

In order to arrive at zero rocket velocity relative to Venus, the speed of the rocket must be reduced by 37.6−35.1 = 2.5 km/sec.

If the rocket is to revolve about Venus along a circle of radius a, then the duration of one revolution will be, from equation (48),

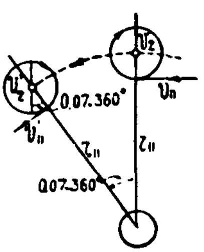


FIGURE 95.

$$t = 2\pi\sqrt{\frac{a^3}{\mu}}.$$

In order to calculate the exact position of the rocket for the farthest departure from Venus, we must note the following during the determination of t: in 464 earth days Venus goes around the sun a total of $\frac{464}{224}$ = 2.07 = = 2 + 0.07 times. Thus, at the moment when the rocket leaves it, Venus will be 0.07 of a revolution away from its position at the beginning of the departure process (Figure 95). Since the velocity of the rocket when it enters (v_{II}) as well as when it leaves (v_{II}') the circling orbit (around Venus), must be ⊥ to the sun-Venus radius, therefore the difference of 0.07 of a revolution for the moment of rocket departure from Venus can be obtained from Figure 95.

The total number of circuits must thus be 3.93, 4.93, 5.93, etc. For example, for 5.93,

$$t=\frac{464}{5.93}=78.2 \text{ days } =6{,}750{,}000 \text{ sec}.$$

To simplify the calculations, we assume the mass of the earth to be the same as that of Venus (according to observations of perturbations of comet motion, Venus has a mass equal to 0.82 of that of the earth). Consequently,
254 for Venus we can also take a value [for μ] of 400,000 km^3/sec^2. Accordingly, a will be

$$a=\sqrt[3]{\mu\left(\frac{t}{2\pi}\right)^2}=\sqrt[3]{400{,}000\left(\frac{6{,}750{,}000}{2\pi}\right)^2}=773{,}000 \text{ km}$$

and the velocity of the circling motion of the rocket will be

$$v_3=\frac{2a\pi}{t}=\frac{2\cdot 773{,}000\,\pi}{6{,}750{,}000}=0.72 \text{ km/sec}.$$

The desired circling motion around Venus will come about if, at the moment when the rocket passes through point V_1 (Figure 94), its relative velocity is not zero but rather 0.72 km/sec. Thus the required velocity reduction will be

$$\Delta v_{II}=37.6-35.1-0.72=1.8 \text{ km/sec},$$

rather than 2.5 km/sec. For this a mass

$$\left(\frac{m_0}{m_1}\right)_{II}=\nu\cdot e^{\frac{\Delta v_{II}}{c}}=1.1\cdot e^{\frac{1.8}{2.0}}=1.1\cdot e^{0.9}=2.65,$$

will have to be consumed, the direction of the burst being ahead of the rocket.

After the 465 earth days needed for 5.93 circuits of Venus, an ejection in the opposite direction of a mass given by the ratio $\left(\frac{m_0}{m_1}\right)_{II}=2.65$ will enable the rocket to overcome the attraction of Venus, and follow an elliptical trajectory, so that 146 days later it will once again approach the earth. At the moment of perigee, at a distance $r_3=800{,}000$ km from the earth's center, another correcting burst will be necessary to make the rocket's velocity relative to the earth equal to $v_3=0.09$ km/sec (see Part II), for which the descent will begin. Since at this moment the rocket velocity $v_1=27.3$ km/sec and the earth's velocity $v_e=29.7$ km/sec, the velocity increment will have to be

$$\Delta v_I'=29.7-27.3-0.09=2.3 \text{ km/sec}$$

and the burst will have to be directed backward, that is, opposite to the direction of flight.

The amount of mass ejected is found from the expression

$$\left(\frac{m_0}{m_1}\right)_I=\nu\cdot e^{\frac{2.3}{2.0}}=1.1\cdot e^{1.15}=3.47.$$

The duration of the entire journey, including ascent and descent (30 days), will be

$$30+146+464+146=786 \text{ earth days} = 2.15 \text{ years},$$

255 If m_1 is the mass of the returning vehicle and m_0 was the mass at the beginning of the flight (including propellant), then, disregarding the small mass variation due to consumption of food by the passengers, etc., it will be approximately true that

$$\frac{m_0}{m_1}=933\cdot 3.65\cdot 2.65^2\cdot 3.47=83{,}000.$$

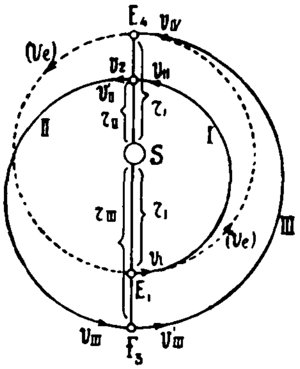


FIGURE 96.

Second alternative. In this case the rocket (Figure 96) is to fly from point V_2 to the earth via an indirect path. It will follow an external trajectory through point F_3 until it returns to the earth at E_4. A very rapid return can be effected in only 1.5 earth years after departure from the earth at point E_1. The distance from the sun (r_{III}) of point F_3 must be such that the rocket can fly from point E_1 via V_2 and F_3 to point E_4 in 1.5 years (547.5 earth days). This time will be the sum of the three times T_1, T_2 and T_3 of the flight along the three semiellipses I, II, and III, the semimajor axes of which are:

$$a_1=\frac{r_I+r_{II}}{2}=128{,}500{,}000 \text{ km};$$

$$a_2=\frac{r_{II}+r_{III}}{2};$$

$$a_3=\frac{r_{III}+r_I}{2}.$$

From the last two expressions we obtain

$$a_3-a_2=\frac{r_I-r_{II}}{2}=\frac{149{,}000{,}000-108{,}000{,}000}{2}=20{,}500{,}000 \text{ km}.$$

In addition,

$$T_3+T_2=T-T_1=547.5-146=401.5 \text{ days}$$

or, from equation (48), for half the arc of the ellipse,

$$\pi\sqrt{\frac{a_3^3}{\mu}}+\pi\sqrt{\frac{a_2^3}{\mu}}=401.5 \text{ days}=34{,}700{,}000 \text{ sec},$$

or

$$\sqrt{a_3^3}+\sqrt{a_2^3}=\frac{34{,}700{,}000}{\pi}\cdot\sqrt{\mu}=\frac{34{,}700{,}000}{\pi}\sqrt{132{,}000{,}000{,}000}.$$

Therefore,

$$\sqrt{a_3^3}+\sqrt{a_2^3}=4{,}010{,}000{,}000{,}000,$$

$$a_3-a_2=20{,}500{,}000.$$

and thus

$$a_3 = 169{,}000{,}000 \text{ km and } a_2 = 148{,}500{,}000 \text{ km};$$

256 from the equation

$$a_2 = \frac{r_{II} + r_{III}}{2}$$

we have

$$r_{III} = 2a_2 - r_{II} = 297{,}000{,}000 - 108{,}000{,}000 = 189{,}000{,}000 \text{ km}.$$

The rocket leaves the earth at E_1 with a velocity $v_I = 27.3$ km/sec and arrives at V_2 with a velocity

$$v_{II} = v_I \cdot \frac{r_1}{r_{II}} = 27{,}3 \cdot \frac{149}{108} = 37.6 \text{ km/sec}.$$

In order to reach F_3 with the required speed, the rocket must leave V_2 with a velocity given by equation (49):

$$v'_{II} = \sqrt{\frac{2\mu}{r_{II} + r_{III}} \cdot \frac{r_{III}}{r_{II}}} = \sqrt{\frac{264{,}000}{297} \cdot \frac{189}{108}} = 39.4 \text{ km/sec}.$$

Then the velocity of approach to F_3 will be

$$v_{III} = v'_{II} \frac{r_{II}}{r_{III}} = 39.4 \cdot \frac{108}{189}\ 22.5 \text{ km/sec}.$$

In order to reach E_4, the rocket must leave F_3 with a velocity

$$v_{III} = \sqrt{\frac{2\mu}{r_{III} + r_{II}} \cdot \frac{r_1}{r_{III}}} = \sqrt{\frac{264{,}000}{338} \cdot \frac{149}{189}} = 24.8 \text{ km/sec},$$

and it will approach E_4 with a velocity

$$v_{IV} = v'_{III} \cdot \frac{r_{III}}{r_1} = 24.8 \cdot \frac{189}{149} = 31.5 \text{ km/sec},$$

whereas the velocity of the earth is

$$v_e = 29.7 \text{ km/sec}.$$

Consequently, over the entire flight, the following velocity variations are necessary:

at launching from E_1,

$$\Delta v_I = 27.3 - 29.7 = -2.4 \text{ km/sec};$$

at approach to V_2,

$$\Delta v_{II} = 39.4 - 37.6 = +1.8 \text{ km/sec};$$

at approach to F_3,

$$\Delta v_{III} = 24.8 - 22.5 = +2.3 \text{ km/sec};$$

257 and at approach to E_4,

$$\Delta v_{IV} = 29.7 - 31.5 + 0.09 = 1.7 \text{ km/sec (including descent).}$$

In order to produce these velocity changes, the following masses must be ejected at a velocity $c = 2$ km/sec:

$$\left(\frac{m_0}{m_1}\right)_I = \nu \cdot e^{\frac{2.4}{2.0}} = 1.1 \cdot e^{1.20} = 3.65$$

$$\left(\frac{m_0}{m_1}\right)_{II} = \nu \cdot e^{\frac{1.8}{2.0}} = 1.1 \cdot e^{0.90} = 2.71$$

$$\left(\frac{m_0}{m_1}\right)_{III} = \nu \cdot e^{\frac{2.3}{2.0}} = 1.1 \cdot e^{1.15} = 3.47$$

$$\left(\frac{m_0}{m_1}\right)_{IV} = \nu \cdot e^{\frac{1.7}{2.0}} = 1.1 \cdot e^{0.85} = 2.57$$

The bursts at E_1 and E_4 are directed forward, while those at V_2 and F_3 are backward (relative to direction of flight).

On the basis of these data, we find, as before, that

$$\frac{m_0}{m_1} = 933 \cdot 3.65 \cdot 2.71 \cdot 3.47 \cdot 2.57 = 82{,}000.$$

The total duration of the flight, including ascent and descent, will be

$$30.5 + 547.5 = 578 \text{ earth days} = 1.58 \text{ years.}$$

The fuel requirements for the two alternatives considered above will be nearly the same. In the second case, however, the duration of the flight is shorter, while in the first case a longer time is spent in the vicinity of Venus.

The situation will be similar for a flight to Mars. However, here a more precise determination of its position at the moment of launching is necessary, since the eccentricity of the Martian orbit is much greater than in the case of the earth and Venus (the aphelion of Mars is about 248,000,000 km and the perihelion is about 205,000,000 km). In Figure 96, when the rocket reaches position F_3, it will be $r_{III} = 189{,}000{,}000$ km from the sun, which is almost equal to the perihelion distance of Mars (205,000,000 km), the difference being only 16,000,000 km. If the flight is coordinated with an opposition of the earth, Venus, and Mars, the most favorable ratios of r_{II} and r_{III} being selected, it will be possible to reduce this different to around $\frac{16}{2} =$ $= 8$ million km, from Venus as well as from Mars, and to complete the flight in $1\frac{1}{2}$ years. This 580-day journey will last almost 20 times as long as the 30-day journey described in Part III.

The masses determined earlier, and designated as b), c), d), and e) in Part III, depend on the duration of the flight. Consequently, they will be
258 20 times as great. Masses a), f), g), and i) are independent of the flight duration and will retain their original values. Finally, mass h) depends on the space required for the load, and it will have to be three times the previous value.

Since, in addition to the increase in space, the heat-emission surface will also have to be greater, it will be necessary to provide better insulation as well. Thus the initial mass of the rocket (without rocket fuel) will be

(240 + 60 + 200 + 140)·20 =	12,800 kg
+ 200 + 200 + 240 + 200 + 740 =	1,580 kg
+ 780·3 =	2,340 kg
Total..	16,720 kg = 16,72 tons

The duration of the flight between E_1 and V_2 will be $T_1 = 146$ days, and between V_1 and F_3 it will be

$$T_2 = T_1 \sqrt{\frac{a_2^3}{a_1^3}} = 146 \sqrt{\frac{148.5^3}{128.5^3}} = 181 \text{ days.}$$

Between F_3 and E_4 we have

$$T_3 = T_1 \sqrt{\frac{a_3^3}{a_1^3}} = 146 \sqrt{\frac{169.0^3}{128.5^3}} = 220 \text{ days.}$$

The consumption of the 12.8 tons will be distributed as follows:

15-day ascent to	$E_1: 12.8 \cdot \frac{15}{578}$	= 0.33 ton
flight between E_1 and	$V_2: 12.8 \cdot \frac{146}{578}$	= 3.20 "
flight between V_2 and	$F_3: 12.8 \cdot \frac{181}{578}$	= 3.95 "
flight between F_3 and	$E_4: 12.8 \cdot \frac{220}{578}$	= 4.80 "
total between ascent and E_4:		12.28 tons

At the approach to E_4 the total rocket weight will be 16.72−12.28 = 4.44 tons.
Immediately prior to reaching E_4, the total weight will be
4.44·2.57 = 11.40 tons.
After reaching F_3: 11.40 + 4.80 = 16.20 tons.
Immediately before reaching F_3: 16.20·3.47 = 56.30 tons.
After reaching V_2: 56.30 + 3.95 = 60.25 tons.
Immediately before reaching V_2: 60.25·2.71 = 163.00 tons.
Upon reaching E_1: 163.00 + 3.20 = 166.20 tons.
Immediately before reaching E_1: 166.20·3.65 = 606.67.
After rocket's own acceleration ceases: 606.67 + 0.33 = 607 tons.
At launching $G_0 = 607 \cdot 933 = 567{,}000$ tons, or, in short,

$$G_0 = [\{[(4.44 \cdot 2.57 + 4.8) \cdot 3.47 + 3.95] \cdot 2.71 + 3.2\} \cdot 3.65 + 0.33] \cdot 9.33 = 567{,}000 \text{ tons.}$$

259 The great quantity of equipment which has to be carried aboard the rocket necessitates an increase in the rocket's own acceleration during ascent as well. Moreover, a velocity variation during flight with so much ballast aboard (about 607−17 = 590 tons), as well as the transportation of this ballast, will make maneuvering quite difficult. The effect which the velocity of gas ejection c has on the variation of the weight G is shown by the following figures (the rocket's own acceleration ac is assumed to be 30 m/sec² for all c):

$$c = 2 \text{ km/sec}: \quad G_0 = [\{[(4.44 \cdot 2.57 + 4.8) \cdot 3.47 + 3.95] \cdot 2.71 + 3.2\} \cdot 3.65 + 0.33] \cdot 933 = 567{,}000 \text{ tons.}$$

$c = 2.5$ km/sec: $G_0 = [\{[(4.44 \cdot 2.17 + 4.8) \cdot 2.77 + 3.95] \cdot 2.27 + 3.2\} \cdot 2.87 + 0.33] \cdot 235 = 69{,}500$ tons.

$c = 3$ km/sec: $G_0 = [\{[(4.44 \cdot 1.95 + 4.8) \cdot 2.38 + 3.95] \cdot 2.00 + 3.2\} \cdot 95 = 17{,}600$ tons.

$c = 4$ km/sec: $G_0 = [\{[(4.44 \cdot 1.69 + 4.8) \cdot 1.98 + 3.95] \cdot 1.73 + 3.2\} \cdot 2.00 + 0.33] \cdot 30 = 3{,}150$ tons.

$c = 5$ km/sec: $G_0 = [\{[(4.44 \cdot 1.55 + 4.8) \cdot 1.75 + 3.95] \cdot 1.57 + 3.2\} \cdot 1.78 + 0.33] \cdot 15 = 1{,}130$ tons.

259 *Part V*

LANDING ON OTHER CELESTIAL BODIES

Of the planets closest to the earth, Venus is the most suitable for landing, since in all probability it possesses an atmosphere similar to that of the earth. For this reason, and also because the gravitational attraction of Venus can be assumed to be almost the same as that of the earth, a landing on Venus should be similar to a landing on the earth (the latter has been described in Parts II and III). In this case, a rocket at a distance $r_3 = 800{,}000$ km from the center of Venus will have to develop a tangential velocity $v_3 = 0.09$ km/sec (see Figure 83).* The flight up to this point proceeds like the flight from E_1 to V_2 in Figure 94.

The rocket approaches V_2 with a velocity $v_{II} = 37.6$ km/sec, whereas the velocity of Venus in its orbit $v_v = 35.1$ km/sec, giving a relative velocity of $37.6 - 35.1 = 2.5$ km/sec. In order to reduce this to 0.09 km/sec, a velocity decrease $\Delta v_{II} = 2.4$ km/sec is necessary, for which the ejection of a mass given by the ratio

$$\left(\frac{m_0}{m_1}\right)_{II} = \nu \cdot e^{\frac{\Delta v_{II}}{c}} = 1.1 \cdot e^{\frac{2.4}{2.0}} = 1.1 : e^{1.2} = 3.65$$

is required, whereas for point E_1, as before, we have

$$\left(\frac{m_0}{m_1}\right)_{I} = 3.65.$$

260 The duration of the flight will be:

Ascent to E_1	15 days
Cometlike flight from E_1 to V_2	146 "
Descent at V_2	15 "
Total	176 days

that is, it is six times as long as the 30-day flight described in Part III. The weights previously denoted as b), c), d), and e) will be six times as great; those

(259)

* Compare with the mass of Venus [0.82 times the mass of the earth] quoted earlier. Moreover, because of the great height and high density of Venus's atmosphere, a landing on Venus will be easier than an earth landing.

denoted as a), f), g), and i) will be the same as before, and weight h), the weight of the rocket casing, will be twice as great. Consequently, the initial weight (without propellant) will be:

$$(240+60+200+140)\cdot 6 = 3{,}680$$
$$+\ 200+200+240+200+740 = 1{,}580$$
$$+\ 780\cdot 2 = 1{,}560$$

Total . . . 7,000 kg = 7.0 tons

The supplies required will be, as before,

Between ascent and E_1	0.3 ton
Between E_1 and V_2	3.2 tons
Total between ascent and V_2	3.5 tons

Therefore, at the approach to Venus, a weight of 7.0–3.5 = 3.5 tons will remain. The total weights for ascents from the earth at various velocities will be:

for $c = 2$ km/sec $G_0 = [(3.5\cdot 3.65 + 3.2)\cdot 3.65 + 0.3]\cdot 933 = 54{,}800$ tons
" $c = 2.5$ " $G_0 = [(3.5\cdot 2.87 + 3.2)\cdot 2.87 + 0.3]\cdot 235 = 8{,}800$ "
" $c = 3$ " $G_0 = [(3.5\cdot 2.45 + 3.2)\cdot 2.45 + 0.3]\cdot 95 = 2{,}800$ "
" $c = 4$ " $G_0 = [(3.5\cdot 2.00 + 3.2)\cdot 2.00 + 0.3]\cdot 30 = 620$ "
" $c = 5$ " $G_0 = [(3.5\cdot 1.78 + 3.2)\cdot 1.78 + 0.3]\cdot 15 = 260$ "

For an independent return from Venus to the earth, a similar weight will be required at ascent. However, if the fuel for the return flight has to be carried along from the earth, then the weight of vehicle plus fuel, for the ascent from the earth, will be at least:

for $c = 2$ km/sec $54{,}800\cdot 3.65^2\cdot 933 = 670{,}000{,}000$ tons
" $c = 2.5$ " $8{,}800\cdot 2.87^2\cdot 235 = 17{,}000{,}000$ "
" $c = 3$ " $2{,}800\cdot 2.45^2\cdot 95 = 1{,}600{,}000$ "
" $c = 4$ " $620\cdot 2.00^2\cdot 30 = 74{,}000$ "
" $c = 5$ " $260\cdot 1.78^2\cdot 15 = 12{,}400$ "

If the rocket lands on Venus, it may also be assumed that the mass required for the return flight can be obtained from raw materials available on the planet, with the aid of simple equipment.

A landing on Mars will have to be carried out somewhat differently from
261 one on the earth or Venus, due to the probable absence of a dense atmosphere. Moreover, in this case a much greater braking of the rocket will be necessary, using the techniques described in Part I. The radius of Mars $r_0 = 3{,}373$ km, and the acceleration of gravity at its surface, obtained from observations of the motion of the Martian satellites, is $g_0 = 3.7\ \text{m/sec}^2 = 0.0037\ \text{km/sec}^2$.

Assuming the same value as previously for the rocket's own acceleration, that is, $c\alpha = 0.03$ km/sec^2, and a velocity of gas ejection $c = 2.0$ km/sec, we obtain

$$\alpha = \frac{c\alpha}{c} = \frac{0.03}{2.0} = \frac{0.015}{\text{sec}}.$$

Thus the distance r_1 from the center of Mars, where the rocket's own acceleration begins, can be found from equation (7):

$$r_1 = r_0\left(1 + \frac{g_0}{ca}\right) = 3392\left(1 + \frac{0.0037}{0.03}\right) = 3800 \text{ km}.$$

The velocity of a rocket approaching a distance r_1 from infinity will be, according to equation (8),

$$v_1 = \sqrt{\frac{2g_0 r_0^2}{r_1}} = \sqrt{\frac{2 \quad 0.0037 \cdot 3392^2}{3800}} = 4.70 \text{ km/sec}.$$

Then, equation (9) gives the deceleration during the braking period:

$$\beta = ca \cdot \frac{g_0}{3}\left(2 + \frac{r_0^3}{r_1^2}\right) = 0.03 - \frac{0.0037}{3}\left(2 + \frac{3392^2}{3800^2}\right) = 0.02655 \text{ km/sec}^2,$$

and equation (10) gives the braking time:

$$t_1 = \frac{v_1}{\beta} = \frac{4{,}70}{0.02655} = 177 \text{ sec}.$$

According to equation (11), the mass ratio is

$$\frac{m_0}{m_1} = e^{\alpha t_1} = e^{0.015.177} = e^{2.66} = 14.3.$$

If $r_1 = 149{,}000{,}000$ km is the distance of the earth from the sun, and $r_2 = 205{,}000{,}000$ km is the distance of Mars from the sun, then during its ascent from the earth the rocket must develop a tangential velocity given by equation (49) as

$$v_1 = \sqrt{\frac{264{,}000}{354} \cdot \frac{205}{149}} = 32.0 \text{ km/sec},$$

whereas the earth's velocity is only 29.7 km/sec.

At the approach to Mars, on the other hand, the velocity of the rocket will be

$$v_{II} = 32.0\,\frac{149}{205} = 23.2 \text{ km/sec},$$

whereas the velocity of Mars at perihelion is 26.5 km/sec. Consequently, the velocity variations will be:
at departure from the earth,

262 $$\Delta v_I = 32.0 - 29.7 = 2.3 \text{ km/sec},$$

with

$$\left(\frac{m_0}{m_1}\right)_I = \nu \cdot e^{\frac{2.3}{2.0}} = 1.1 \cdot e^{1.15} = 3.47;$$

and, prior to landing on Mars,

$$\Delta v_{II} = 26.5 - 23.2 = 3.3 \text{ km/sec},$$

with

$$\left(\frac{m_0}{m_1}\right)_{II} = \nu \cdot e^{\frac{3.3}{2.0}} = 1.1 \cdot e^{1.65} = 5.73.$$

The duration of the flight will be:

for the ascent from the earth, 15 days;
for the cometlike flight from earth to Mars, $\pi \cdot \sqrt{\frac{a^3}{\mu}}$,

with

$$a = \frac{r_I + r_{II}}{2} = 177{,}000{,}000$$

and

$$\mu = 132{,}000{,}000{,}000 \frac{\text{km}^3}{\text{sec}^2},$$

so that we have

$$\pi \sqrt{\frac{177{,}000{,}000^3}{132{,}000{,}000{,}000}} = 20{,}350{,}000 \text{ sec} = 235 \text{ days;}$$

for the descent to Mars, about 15 days;

thus we obtain a total of 265 days, or nearly nine times as long as the 30-day flight in Part III.

Now let us calculate, as for the flight to Venus, the initial weight of the rocket without the propellant:

$$\frac{9}{6} \cdot 3680 + 1580 + 1560 = 5790 + 3140 = 8930 \text{ kg} = 9 \text{ tons.}$$

The fuel required to get to Mars (which weighs about 5.8 tons) will be consumed as follows:

$\frac{15}{265} \cdot 15.8 = 0.3$ ton for ascent from the earth,
$\frac{235}{265} \cdot 5.8 = 5.2$ tons for cometlike flight from earth to Mars,
0.3 ton for descent to Mars
(9.0–5.8 = 3.2 tons will remain at arrival on Mars).

Thus the total weight at the beginning of flight will be:

for $c = 2$ km/sec, $G_0 = \{[(3.2 \cdot 14.3 + 0.3) \cdot 5.73 + 5.2] \cdot 3.47 + 0.3\} \cdot 933 = 875{,}000$ tons
" $c = 2.5$ km/sec, $G_0 = \{[(3.2 \cdot 8.3 + 0.3) \cdot 4.13 + 5.2] \cdot 2.77 + 0.3\} \cdot 235 = 76{,}500$ tons
263 " $c = 3$ km/sec, $G_0 = \{[(3.2 \cdot 5.9 + 0.3) \cdot 3.32 + 5.2] \cdot 2.38 + 0.3\} \cdot 95 = 15{,}600$ tons
" $c = 4$ km/sec, $G_0 = \{[(3.2 \cdot 3.8 + 0.3) \cdot 2.51 + 5.2] \cdot 1.98 + 0.3\} \cdot 30 = 2{,}200$ tons
" $c = 5$ km/sec, $G_0 = \{[(3.2 \cdot 2.9 + 0.3) \cdot 2.14 + 5.2] \cdot 1.75 + 0.3\} \cdot 15 = 690$ tons

These results are much less favorable than those for a flight to Venus, which has a denser atmosphere. The situation is much better, however, for the return flight from Mars to the earth, once again assuming that fuel can be procured on Mars from the raw materials available there. In such a case, the dense atmosphere of the earth can be utilized in the descent, so that, instead of the previous weights multiplied by 933, etc., we have considerably lower values:

lower values:

for $c = 2$ km/sec, $G_0 = \{[(3.2+0.3)\cdot 3.47+5.2]\cdot 5.73+0.3\}\cdot 14.3 = 1{,}430$ tons
" $c = 2.5$ " $G_0 = \{[(3.2+0.3)\cdot 2.77+5.2]\cdot 4.13+0.3\}\cdot 8.3 = 515$ "
" $c = 3$ " $G_0 = \{[(3.2+0.3)\cdot 2.38+5.2]\cdot 3.32+0.3\}\cdot 5.9 = 265$ "
" $c = 4$ " $G_0 = \{[(3.2+0.3)\cdot 1.98+5.2]\cdot 2.51+0.3\}\cdot 3.8 = 118$ "
" $c = 5$ " $G_0 = \{[(3.2+0.3)\cdot 1.75+5.2]\cdot 2.14+0.3\}\cdot 2.9 = 71$ "

A landing on the moon will be similar to one on Mars. As in the case of Mars, for a lunar landing we first introduce the values $r = 1{,}740$ km and $g_0 = 0.0016$ km/sec^2. Since the density of the moon is less than that of the earth, we have:

$$g_0\ 0.0098\,\frac{1740}{6380}:$$

$$ac = 0.03\ \text{km/sec}^2\ c = 2.0\ \text{km/sec};\ a = \frac{0.015}{\text{sec}};$$

$$r_1 = 1740\left(1+\frac{0.0016}{0.03}\right) = 1830\ \text{km};$$

$$v_1 = \sqrt{\frac{2\cdot 0.0016\cdot 1740^2}{1830}} = 2.30\ \text{km/sec};$$

$$\beta = \backsim 0.03 - \frac{0.0016}{3}\left(2+\frac{1740^2}{1830^2}\right) = 0.0284\ \text{km/sec}^2;$$

$$t_1 = \frac{v_1}{\beta} = \frac{2.30}{0.0284} = 81\ \text{sec};$$

$$\frac{m_0}{m_1} = e^{at_1} = e^{0.015\cdot 81} = e^{1.22} = 3.40.$$

In this case the duration of the flight is no more than half that of the flight assumed in Part III for twice the distance between the earth and the moon. Moreover, a correspondingly smaller store of provisions will have to be carried. Thus the weight of the vehicle (less propellant) will be about
264 2.6 tons instead of 3.0 tons. Consequently, the initial weights for an earth-moon flight will be:

for $c = 2$ km/sec, $G_0 = 2.6\cdot 3.4\cdot 933 = 8{,}250$ tons
" $c = 2.5$ " $G_0 = 2.6\cdot 2.64\cdot 235 = 1{,}610$ "
" $c = 3$ " $G_0 = 2.6\cdot 2.25\cdot 95 = 555$ "
" $c = 4$ " $G_0 = 2.6\cdot 1.85\cdot 30 = 144$ "
" $c = 5$ " $G_0 = 2.6\cdot 1.64\cdot 15 = 64$ "

The weights at the ascent for the return flight (moon to earth) are:

for $c = 2$ km/sec, $G_0 = 2.6\cdot 3.4 = 8.9$ tons
" $c = 2.5$ " $G_0 = 2.6\cdot 2.64 = 6.9$ "
" $c = 3$ " $G_0 = 2.6\cdot 2.25 = 5.9$ "
" $c = 4$ " $G_0 = 2.6\cdot 1.85 = 4.8$ "
" $c = 5$ " $G_0 = 2.6\cdot 1.64 = 4.3$ "

However, if propellant is carried from the earth for the round-trip flight, the weights at ascent from the earth will be:

for $c = 2$ km/sec, $G_0 = 2.6\cdot 3.4^2\cdot 933 = 28{,}000$ tons
" $c = 2.5$ " $G_0 = 2.6\cdot 2.64^2\cdot 235 = 4{,}250$ "
" $c = 3$ " $G_0 = 2.6\cdot 2.25^2\cdot 95 = 1{,}250$ "
" $c = 4$ " $G_0 = 2.6\cdot 1.85^2\cdot 30 = 890$ "
" $c = 5$ " $G_0 = 2.6\cdot 1.64^2\cdot 15 = 700$ "

The comparative ease with which the moon can be reached, together with the low relative fuel consumption, $\frac{m_0}{m_1} = 4.0$, for the ascent from the lunar surface, suggests using the moon as a station for more distant flights. A precondition for this will be the presence on the moon of the materials required to produce an explosive mixture, and in addition a suitable factory will have to be constructed on the moon. In order to investigate this possibility, a ship will first have to be sent to the moon with enough propellant to complete the round trip on its own. For this we must take $c = 2$ km/sec and $G_0 = 28{,}000$ tons, which does not present any insurmountable difficulties. For a successful result, further flights to the moon will require only 8,250 tons, and return flights only 8.9 tons. However, for flights from the moon to other planets, instead of the ascent ratio of $\frac{m_0}{m_1} = 933$ for the earth, the ratio will be only $\frac{m_0}{m_1} = 3.4$ (from lunar surface), etc. Finally, the landing can be made on the earth, under more favorable conditions, rather than on the moon.

265 The following weights are obtained for the flights indicated:

a) Round trip from moon to Venus, Mars, and earth (without landing on Venus or Mars):

for $c = 2$	km/sec;	$G_0 = \frac{3.4}{933} \cdot 567{,}000 =$	2,070 tons
" $c = 2.5$	"	$G_0 = \frac{2.64}{235} \cdot 69{,}500 =$	780 "
" $c = 3$	"	$G_0 = \frac{2.25}{95} \cdot 17{,}600 =$	417 "
" $c = 4.$	"	$G_0 = \frac{1.85}{30} \cdot 3{,}150 =$	194 "
" $c = 5$	"	$G_0 = \frac{1.64}{15} \cdot 1{,}130 =$	124 "

b) Flight from moon to Mars with landing, but without supplies for return trip:

for $c = 2$	km/sec;	$G_0 = \frac{3.4}{933} \cdot 875{,}000 =$	3,190 tons
" $c = 2.5$	"	$G_0 = \frac{2.64}{235} \cdot 76{,}500 =$	860 "
" $c = 3$	"	$G_0 = \frac{2.25}{95} \cdot 15{,}600 =$	370 "
" $c = 4$	"	$G_0 = \frac{1.85}{30} \cdot 2{,}200 =$	136 "
" $c = 5$	"	$G_0 = \frac{1.64}{15} \cdot 690 =$	76 "

c) Flight from moon to Venus with landing, but without supplies for return trip:

for $c = 2$	km/sec;	$G_0 = \frac{3.4}{933} \cdot 54{,}800 =$	200 tons
" $c = 2.5$	"	$G_0 = \frac{2.64}{235} \cdot 8{,}800 =$	99 "
" $c = 3$	"	$G_0 = \frac{2.25}{95} \cdot 2{,}800 =$	67 "
" $c = 4$	"	$G_0 = \frac{1.85}{30} \cdot 620 =$	38 "
" $c = 5$	"	$G_0 = \frac{1.64}{15} \cdot 260 =$	29 "

d) Exploratory flight to Mars, with landing and with supplies for return trip; mass ratio for ascent from Mars taken to be $\frac{m_0}{m_1} = 14.3$, allowing 5.8 tons of necessary provisions (food, etc.) for return trip:

for $c = 2$ km/sec; $G_0 = 14.3 \cdot \frac{9+5.8}{9} = 75{,}000$ tons

" $c = 2.5$ " $G_0 = 8.3 \cdot \frac{9+5.8}{9} = 11{,}800$ "

" $c = 3$ " $G_0 = 370 \cdot 5.9 \cdot \frac{9+5.8}{9} = 3{,}600$ "

266 " $c = 4$ " $G_0 = 136 \cdot 3.8 \cdot \frac{9+5.8}{9} = 850$ "

" $c = 5$ " $G_0 = 76 \cdot 2.9 \cdot \frac{9+5.8}{9} = 360$ "

e) Flight to Venus with landing, under same conditions:

for $c = 2$ km/sec; $G_0 = 200 \cdot 933 \cdot \frac{7+3.9}{7} = 290{,}000$ tons

" $c = 2.5$ " $G_0 = 99 \cdot 235 \cdot \frac{7+3.9}{7} = 36{,}300$ "

" $c = 3$ " $G_0 = 67 \cdot 95 \cdot \frac{7+3.9}{7} = 9{,}900$ "

" $c = 4$ " $G_0 = 38 \cdot 30 \cdot \frac{7+3.9}{7} = 1{,}780$ "

" $c = 5$ " $G_0 = 29 \cdot 15 \cdot \frac{7+3.9}{7} = 680$ "

It is much more difficult to carry out the return flight in case (e) than in case (d). However, in spite of this, and even taking into account that an independent return flight from Venus (with almost the same weight requirement G as for a direct flight from the earth to Venus) must be carried out with a high exhaust velocity c, nevertheless the probability of finding an atmosphere there, and living conditions similar to those on earth, is so great, while the difficulties involved in a flight to Venus are so minor (assuming a stop at the moon first), that it will be more advisable to begin our studies of the planets with Venus rather than Mars, and to leave the latter just as a subject of research.

During all ascents from the lunar surface, the moon's velocity around the earth must be carefully taken into account, similarly the earth's orbital velocity for ascents from the earth (see Figure 85); this effect will not be studied below. For simplicity, earlier we considered only those ellipses joining planets which were tangent to the planetary orbits, so that only a variation of the magnitude of the velocity was necessary, and not the direction. Naturally, these tangential ellipses will also represent the **optimum paths**. However, it would be good if there were other ellipses intersecting the planetary orbits, but which give shorter paths. Thus the opposite case should also be studied, namely the case where the direction of the velocity changes but not the magnitude. The desired ellipse may intersect both planetary orbits with velocities equal to the respective velocities of the planets. Using the notation in Figure 97, we obtain the following expressions for the joining ellipse, according to equation (41):

$$1.\quad v_a^2 - \frac{2\mu}{r_a} = v_1^2 - \frac{2\mu}{r_1};$$

$$2.\quad v_a^2 - \frac{2\mu}{r_a} = v_2^2 - \frac{2\mu}{r_2}.$$

For circling trajectories r_1 and r_2, equation (37) gives

$$v_1^2 = \frac{\mu}{r_1},$$

$$v_2^2 = \frac{\mu}{r_2}.$$

267 Therefore it should be true that

$$1.\ v_a^2 - \frac{2\mu}{r_a} = \frac{\mu}{r_1} - \frac{2\mu}{r_1},$$

$$2.\ v_a^2 - \frac{2\mu}{r_a} = \frac{\mu}{r_2} - \frac{2\mu}{r_2},$$

or

$$1.\ \frac{2\mu}{r_a} - v_a^2 = \frac{\mu}{r_1},$$

$$2.\ \frac{2\mu}{r_a} - v_a^2 = \frac{\mu}{r_2}.$$

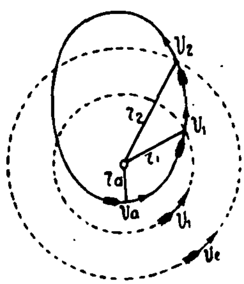


FIGURE 97.

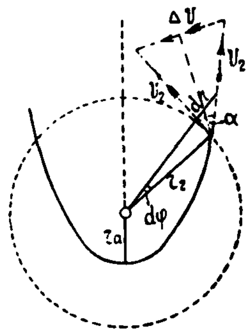


FIGURE 98.

These two equations contradict each other. Consequently, the condition that the rocket cross both planetary orbits, with velocities corresponding to those of the planets, cannot in general be satisfied. Now let us make the condition that the rocket cross only one planetary orbit, for instance, at radial distance r_2, the velocity of the rocket at the intersection point being equal to the orbital velocity of the planet. In this case we are left with only one equation,

$$\frac{2\mu}{r_a} - V_a^2 = \frac{\mu}{r_2},$$

from which we obtain, after an appropriate choice of r_a,

$$v_a^2 = \frac{2\mu}{r_a} \cdot \frac{\mu}{r_2}.$$

Moreover, from equation (45),

$$a = \frac{\mu}{\frac{2}{r_a} - v_a^2} = \frac{\mu}{\frac{\mu}{r_2}} = r_2$$

and from equation (46)

$$b=\frac{v_a r_a}{\sqrt{\frac{2\mu}{r_a}-v_a^2}}=\frac{v_a r_a}{\frac{\mu}{\sqrt{\frac{v}{r_2}}}}=r_a\sqrt{\frac{2r_2}{r_a}-1},$$

268 that is, each ellipse whose semimajor axis (a) equals the radius r_2 of the circular planetary orbit will, at its point of intersection with this orbit, give a velocity equal to the velocity of the planet. The angle at the intersection between the ellipse and the orbit, equal to the angle between the tangents (Figure 98), is found from the expression

$$\tan\alpha=\frac{dr}{r_2\,d\varphi}=\frac{1}{r_2}\cdot\frac{dr}{d\varphi},$$

and from equation (43), for $r=r_2$,

$$\tan\alpha=\sqrt{\frac{v_a^2-\frac{2\mu}{r_a}}{v_a^2 r_a^2}\cdot r_2^2+\frac{2\mu}{v_a^2\cdot r_a^2}\cdot r_2-1},$$

or, since in this case

$$v_a^2-\frac{2\mu}{r_a}=-\frac{\mu}{r_2},$$

therefore,

$$\tan\alpha=\sqrt{-\frac{\mu r_2}{v_a^2 r_a^2}+\frac{2\mu r_2}{v_a^2 r_a^2}-1}=\sqrt{\frac{\mu r_2}{v_a^2 r_a^2}-1}.$$

Of all the different possible connecting ellipses with semimajor axes $a=r_2$, only those which are at the same time tangent to the planetary orbit of radius r_1 should be considered in more detail, since for these ellipses a variation of the magnitude of the velocity is sufficient, whereas for the others the direction must be varied as well.

For this we take

$$r_a=r_1.$$

Then

$$v_a^2=\frac{2\mu}{r_1}-\frac{\mu}{r_2}=\mu\frac{2r_2-r_1}{r_1 r_2},$$

and

$$\tan\alpha=\sqrt{\frac{\mu r_2}{r_1^2\mu\cdot\frac{2r_2+r_1}{r_1 r_2}}-1}=\sqrt{\frac{r_2^2}{r_1(2r_2-r_1)}-1}.$$

or

$$\tan\alpha=\sqrt{\frac{r_2^2-2r_1 r_2+r_1^2}{r_1(2r_2-r_1)}}=\sqrt{\frac{(r_2-r_1)^2}{r(2r_2-r_1)}}.$$

At the place of intersection, for transition from one path to the other, a variation in direction will be necessary without a variation in the magnitude of v_2. Thus we will have to have a velocity component perpendicular to the bisector of the intersection angle α and having a magnitude

$$\Delta v=2\cdot v_2\cdot\sin\frac{\alpha}{2}\text{ (Figure 98).}$$

Let us assume, for example, that the connecting ellipse is tangent to the earth's orbit and intersects the orbit of Venus. Then

$$r_1 = 149{,}000{,}000 \text{ km},$$
$$r_2 = 108{,}000{,}000 \text{ km},$$
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$$v_2 = 35.1 \text{ km/sec}.$$

$$\tan a = \sqrt{\frac{(108-149)^2}{149 \cdot (216-149)}} = \frac{41}{\sqrt{149 \cdot 67}} = 0.41.$$

$$a = \backsim 22^1/_4{}^\circ; \quad \Delta v = r \cdot 35.1 \cdot \sin 11^1/_8{}^\circ = 13.5 \text{ km/sec}.$$

If the ellipse is tangent to Venus's orbit and intersects the orbit of the earth, then

$$r_1 = 108{,}000{,}000 \text{ km},$$
$$r_2 = 149{,}000{,}000 \text{ km}$$
$$v_2 = 29.7 \text{ km/sec}$$

$$\tan a = \sqrt{\frac{(149-108)^2}{108 \cdot (298-108)}} = \frac{41}{\sqrt{108 \cdot 190}} = 0.286;$$
$$a = \backsim 16^\circ; \quad \Delta v = 2 \cdot 29{,}7 \cdot \sin 8^\circ = 8.3 \text{ km/sec}.$$

If the ellipse is tangent to the earth's orbit and intersects the orbit of Mars, then

$$\left.\begin{aligned} r_1 &= 149{,}000{,}000 \text{ km} \\ r_2 &= 205{,}000{,}000 \text{ ''} \\ v_2 &= 26.5 \text{ km/sec} \end{aligned}\right\} \quad \text{assuming a circular orbit;}$$

$$\tan a = \sqrt{\frac{(205-149)^2}{149\,(410-149)}} = \frac{56}{\sqrt{149 \cdot 261}} = 0.284;$$
$$a = \backsim 16^\circ; \quad \Delta v = 2 \cdot 26.5 \cdot \sin 8^\circ = 7.4 \text{ km/sec}$$

If the ellipse is tangent to the orbit of Mars and intersects the earth's orbit, then

$$r_1 = 205{,}000{,}000 \text{ km},$$
$$r_2 = 149{,}000{,}000 \text{ ''}$$
$$v_2 = 29.7 \text{ km/sec}$$

$$\tan a = \sqrt{\frac{(149-205)^2}{205\,(298-205)}} = \frac{56}{\sqrt{205 \cdot 93}} = 0.405\,.$$
$$a = \backsim 22^\circ; \quad \Delta v = 2 \cdot 29.7 \sin 11^\circ = 11.4 \text{ km/sec}$$

It is evident from the foregoing that the velocity components will in all cases be considerably greater than for ellipses tangent to both planetary orbits. Even in the most favorable case (tangent to the earth's orbit and intersecting that of Mars), for $\Delta v = 7.4$ km/sec (instead of the $\Delta v_{11} = 3.3$ km/sec obtained earlier for Mars), the mass-consumption ratios $\frac{m_0}{m_1} = \nu_e^{\frac{\Delta v}{c}}$ are as follows:

for $c = 2$ km/sec; $\frac{m_0}{m_1} = 1.1 \cdot e^{\frac{7.4}{2.0}} = 14.5$ instead of 5.73;

" $c = 2.5$ " $\frac{m_0}{m_1} = 1.1 \cdot e^{\frac{7.4}{2.5}} = 21.4$ " 4.13;

270 " $c = 3$ km/sec; $\frac{m_0}{m_1} = 1.1 \cdot e^{\frac{7.4}{3.0}} = 14.1$ instead of 3.32;

" $c =$ " $\frac{m_0}{m_1} = 1.1 \cdot e^{\frac{7.4}{4.0}} = 7.05$ " 2.51;

" $c = 5$ " $\frac{m_0}{m_1} = 1.1 \cdot e^{\frac{7.4}{5.0}} = 4.85$ " 2.14.

Consequently, for transfer to an ellipse tangent to the orbit of one planet and intersecting the orbit of another, the required velocity variation Δv_1 will be greater than for an ellipse tangent to both trajectories, since in the latter case the curvature of the path varies less. The foregoing results make it clear than an ellipse tangent to the orbits of both planets gives the optimum trajectory for the rocket.

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HANS LORENZ

The journal "Zeitschrift des Vereins Deutscher Ingenieure" for 7 May 1927 (No. 19) included a paper by H. Lorenz, entitled "Die Möglichkeit der Weltraumfahrt." In this work the author outlined in very clear and concise mathematical form the conditions of flight in outer space, either by shooting a projectile from a cannon or on the basis of the rocket principle. Lorenz does not consider the important problem of the resistance of the atmosphere to the flight of a projectile, and, in addition, the historical outline of related works given at the beginning of the paper is incomplete (the studies of Tsiolkovskii, Esnault-Pelterie, etc., are not mentioned). Lorenz's paper, however, is still of considerable interest. Moreover, in 1928 another paper by him on this same subject was published, and this is also given below.

First Paper. THE POSSIBILITY OF SPACE TRAVEL

Once the light engine had been developed and the centuries-old dream of flying through the air had come true, man's hopes went even further, and some daring minds made visits to other celestial bodies the subject of their studies. The first stimulus to thoughts concerning such flights was provided by the novels of Jules Verne, who described a flight around the moon by some persons riding a projectile which had been fired from a cannon on the earth. Another novelist, Kurd Lasswitz, who was at the same time a physicist and a philosopher, described a space flight in a "rocket" ship, in his novel "On Two Planets" ["Auf Zwei Planeten"]. This flight principle (that of the rocket) forms the basis for the recent mathematical and mechanical
272 studies of Goddard,* Oberth,** and Hohmann,† who have even suggested techniques for the practical implementation of such flights. A number of general plans and projects, such as those of Valier†† have also been proposed, interest has been shown by wide circles of people, and a Society for Space Travel [Verein für Raumschiffahrt] has even been founded.

* Goddard, R. H. A Method of Reaching Extreme Altitudes. — Smithsonian Institute, Washington. 1919.
** Oberth, H. Die Rakete zu den Planetenräumen, 2nd ed.—R. Oldenbourg, München und Berlin. 1925.
† Hohmann, W. Die Erreichbarkeit der Himmelskörper. — R. Oldenbourg, München und Berlin. 1925.
†† Valier, M. Der Vorstoss in den Weltenraum. — R. Oldenbourg, München und Berlin. 1925.

The situation being thus, a careful, sober assessment of the possibility of carrying out a space flight is necessary, from the point of view of mechanics. This will bring up such problems as lifting a body to a given distance from the earth, and even completely beyond the field of the earth's gravity, propelling a body in space, and, finally, returning to the earth, taking the resistance of the atmosphere into account.

Our goal will be just to study the possibility of escaping from the earth into airless space, using the means available to us at present. Since the earth itself moves through space along its orbit, and rotates about its axis as well, the point of departure of a projectile will already have a velocity component in the direction of flight, equal to some value v_0.

Let us consider two masses m_1 and m_2 with a common initial velocity v_0. As a result of the forces acting on them, these masses attain final absolute velocities v_1 and v_2. The momentum equation will then be

$$m_1 v_1 + m_2 v_2 - (m_1 + m_2) v_0 = 0 \tag{1}$$

and the equation of work will be

$$L = \frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2 - \frac{m_1 + m_2}{2} v_0^2. \tag{2}$$

From these two equations we obtain

$$L = \frac{m_1}{2} v_1^2 + \frac{m_2}{2} v_2^2 - v_0 (m_1 v_1 + m_2 v_2) + \frac{m_1 + m_2}{2} v_0^2.$$

or

$$L = \frac{m_1}{2} (v_1 - v_0)^2 + \frac{m_2}{2} (v_2 - v_0)^2. \tag{2a}$$

Equations (2) and (2a) indicate that the same amount of work must be expended to modify either the absolute or the relative motion. Equation (1) can be rewritten as

$$m_1 (v_1 - v_0) + m_2 (v_2 - v_0) = 0. \tag{1a}$$

Then, from (2a), by eliminating $(v_2 - v_0)$, we obtain

$$L = \frac{m_1}{2}\left(1 + \frac{m_1}{m_2}\right)(v_1 - v_0)^2. \tag{2b}$$

For $m_2 = \infty$ and $v_2 = v_0$, equation (2b) gives a work

$$L = \frac{m_1}{2} (v_1 - v_0)^2. \tag{2c}$$

273 This corresponds to lifting from the earth's surface a mass which is in practice infinitely great, in comparison with that of a projectile, and which is thus not susceptible to work.

If a projectile is situated between two celestial bodies with masses m_1 and m_2, at a distance r from the first and a distance r_0-r from the second (Figure 99), an acceleration

(276)

$$q=k\frac{m_1}{r^2}-k\frac{m_2}{(r_0-r)^2}, \tag{3}$$

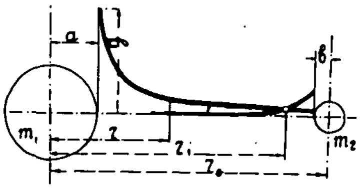

FIGURE 99.

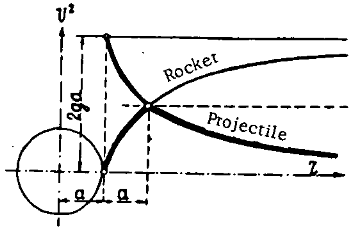

FIGURE 100.

with respect to m_1 will be imparted to it, where k is the gravitational constant according to Gauss. If g is the acceleration at the surface of body m_1, which has a radius a, then k is given by the equation

$$km_1=ga^2, \tag{4}$$

and instead of equation (3) we have

$$q=ga^2\left[\frac{1}{r^2}-\frac{m_2}{m_1}\frac{1}{(r_0-r)^2}\right]. \tag{3a}$$

This quantity goes to zero for a point at a distance r_1 given by the condition

$$\frac{r_0-r_1}{r_1}=\sqrt{\frac{m_2}{m_1}};\quad \frac{1}{r_1}=\frac{1}{r_0}\left(1+\sqrt{\frac{m_2}{m_1}}\right). \tag{3b}$$

The work required to lift a mass m from the surface of body m_1 to a distance r is found from equation (3a):

$$\begin{gathered}L=m\int_a^r q\,dr=mga^2\int_a^r\left(\frac{1}{r^2}-\frac{m_2}{m_1}\frac{1}{(r_0-r)^2}\right)dr,\\ L=mga^2\left[\frac{1}{a}-\frac{1}{r}+\frac{m_2}{m_1}\left(\frac{1}{r_0-a}-\frac{1}{r_0-r}\right)\right].\end{gathered} \tag{5}$$

Setting $r=r_1$ in this equation, we obtain from it (taking (3) into account), the work expended in lifting the mass to the neutral point:

$$L_1=mga\left[1-\frac{a}{r_0}\left(1+\sqrt{\frac{m_2}{m_1}}\right)+\frac{m_2}{m_1}\left(\frac{a}{r_0-a}-\frac{a}{r_0}-\frac{a}{r_0}\sqrt{\frac{m_1}{m_2}}\right)\right].$$

However, since $a << r_0$, therefore, with sufficient accuracy, we have

$$L_1 = mga\left[1 - \frac{a}{r_0}\left(1 + 2\sqrt{\frac{m_2}{m_1}} - \frac{a}{r_0}\frac{m_2}{m_1}\right)\right], \tag{5a}$$

and, setting $r_0 - r = b$, we obtain the work required to lift the mass to the surface of a body m_2 with a radius b:

$$L_2 = mga\left[\left(1 - \frac{a}{r_0 - b} + \frac{m_2}{m_1}\right)\left(\frac{a}{r_0 - a} - \frac{a}{b}\right)\right]$$

274 or, since $b << r_0$,

$$L_2 = mga\left[1 - \frac{a}{r_0}\left(1 - \frac{m_2}{m_1}\right) - \frac{m_2}{m_1}\frac{a}{b} - \frac{a^2}{r_0^2}\left(\frac{b}{a} - \frac{m_2}{m_1}\right)\right] \tag{5b}$$

and, finally, for $r = r_0 = \infty$, we obtain from (5) the total work required to remove a mass m from the sphere of attraction of m_1:

$$L_0 = mga. \tag{5c}$$

In the particular case of the earth and the moon,

$$r_0/a = 63;\ b/a = 0.27;\ m_1/m_2 = 80;\ \sqrt{m_1}/\sqrt{m_2} = 9;$$

and, neglecting the small quantities $\frac{a^2}{r_0^2}$, $\frac{a}{r_0}$, and $\frac{m_2}{m_1}$, we can write

$$L_1 = L_0\left(1 - \frac{1}{51.5}\right);\ L_2 = L_0\left(1 - \frac{1}{16.1}\right). \tag{6}$$

These two expressions indicate that the lunar attraction reduces the work needed to lift a projectile from the earth to the neutral region (at a distance $r_1 = \frac{9}{10} r_0$) by about 2%, and to the moon's surface by 6%. This saving is so inconsiderable that it may be neglected when calculating the expenditure of work, especially when celestial bodies which lie essentially beyond the earth's sphere of gravity are to be reached.

In all such cases the work required to lift the body is found from equation (5c), as $L_0 = mga$, and the corresponding variation in kinetic energy will be

$$2ga = W_0^2 - W^2. \tag{7}$$

At infinity $W = 0$ and

$$W_0 = \sqrt{2ga} = 11{,}180 \text{ m/sec}, \tag{7a}$$

this being the velocity necessary in order to overcome terrestrial gravity and escape from the earth (without an atmosphere). Let us assume that such a velocity is to be obtained by shooting an object from a cannon, for which an explosive charge must be provided. Let us denote as h the charge energy to be converted into mechanical work, relative to a unit of weight. This will be none other than the height of ascent in meters which is attained by a unit weight of this substance by means of its own energy. At firing, a projectile of mass m_0 leaves a cannon with a velocity W_0. If we call the projectile mass m and the average velocity $\frac{W_0}{2}$, we obtain an average kinetic energy $\frac{m W_0^2}{2}$, and the equation of work will be

$$mgh = \frac{m W_0^2}{6} + \frac{m_0 W_0^2}{2} \tag{8}$$

275 or, taking equation (7a) into account,

$$\frac{m_0}{m} = \frac{h}{a} - \frac{1}{3} \tag{8a}$$

Since the mass ratio must be positive, therefore

$$h > \frac{a}{3} \tag{8b}$$

that is, the free ascent of the propellant must be more than three times the earth's radius.

Table 1 gives some figures for the two most powerful explosives, namely nitroglycerine and guncotton (Schiesswolle); data are also given for two ideal explosives: hydrogen plus oxygen, and carbon plus oxygen.

TABLE 1.

Propellant	Q, WE/kg	h_0, km	h, km	w, m/sec
H_2+O	3550	1520	1010	4430
$C+O_2$	2930	1250	835	4048
Nitroglycerine	1580	670	446	2950
Guncotton	1100	460	306	2450

In the table, Q is the number of calories, and h_0 is the corresponding amount (coefficient) of work. According to ballistic experiments, a value of only $h = \frac{2}{3} h_0$ should be taken as the permissible height of ascent, since the gases will carry off an amount of heat up to at least $\frac{1}{3} h_0$. For all the substances in the table, $h < \frac{a}{3}$. Consequently, at present there is not a single propellant available which could impart the required velocity to the projectile being fired. Thus it is of no use to carry out further studies of the effect of projectile acceleration on the cannon length or studies of the effect of air resistance.

Therefore, let us now go on to consider rocket action instead. In a rocket the motive force is provided by the recoil of the gases given off by the explosive substance [propellant]. Let us denote the relative velocity of these gases as w, and the varying velocity of the variable (because of the gas efflux) mass m relative to the earth as v. Once again, h is the actual height of ascent of the propellant. Then,

$$w^2 = 2gh. \tag{9}$$

276 The quantities entering into this formula are evaluated in Table I for different propellants.

In addition to the recoil, ejection of a mass of gas at a rate w per unit time also occurs, and this imparts to the total mass m an acceleration capable of overcoming the acceleration of gravity. If the mass of the exploding gas is dm, then

$$w\frac{dm}{dt} = -m\left(\frac{dv}{dt} + g\frac{a^2}{r^2}\right), \tag{10}$$

but since $dr = vdt$, therefore

$$wv\frac{dm}{m} = -vdv + ga^2 d\left(\frac{1}{r}\right) \tag{10a}$$

and, adding and subtracting on the basis of

$$\frac{w^2 dm}{2} = gh\,dm,$$

we obtain

$$-ghdm = mvdv - \frac{dm}{2}[(v-w)^2 - v^2] - mga^2 d\left(\frac{1}{r}\right). \tag{10b}$$

This is none other than an energy equation, in which the left-hand term expresses the formation of mechanical energy of a gas particle dm, which serves to vary the kinetic energy of particle dm itself, as well as of the mass m, and finally to perform the work of ascent (last term on right).

Three variables, m, v, and r, enter into equations (10a) and (10b). The total mass m continually diminishes during the burning, while velocity v increases. In addition, denoting the (as yet unknown) initial velocity as v_0, and the total initial mass as m_0, we have

$$\frac{m}{m_1} = e^{\frac{-v}{v_0}}, \quad \frac{dm}{m} = -\frac{dv}{v_0}. \tag{11}$$

Therefore, equation (10a) becomes

$$\left(1 - \frac{w}{v_0}\right)vdv = ga^2 d\left(\frac{1}{r}\right). \tag{12}$$

277 Integration of (12) for initial conditions $v = 0$ and $r = a$ at the earth's surface gives

$$v^2 = \frac{2ga^2 v_0}{w - v_0}\left(\frac{1}{a} - \frac{1}{r}\right) \tag{12a}$$

For $r=\infty$ we obtain a final velocity

$$v_1^2=\frac{2gav_0}{w-v_0}$$

or

$$v_0=\frac{wv_1^2}{v_1^2+2ga}. \tag{12b}$$

From equation (11) we have

$$\frac{m_0}{m}=e^{\frac{a}{w}\left(1+\frac{2ga}{v_i^2}\right)} \tag{13}$$

and for $r=\infty$, that is, for $v=v_1$,

$$\frac{m_0}{m}=e^{\frac{1}{w}\left(v_1+\frac{2ga}{v_i}\right)} \tag{13a}$$

This expression will be a minimum for

$$v_1^2=2ga$$

or, according to equation (12b),

$$v_0=\frac{w}{2}. \tag{13b}$$

Consequently, the minimum will be

$$\frac{m_0}{m_1}=e^{\frac{2v_1}{w}}=e^{\frac{2\sqrt{2ga}}{w}}=e^{2\sqrt{\frac{a}{h}}}. \tag{13c}$$

In general, however, from (12b) and (13),

$$\frac{m_0}{m}=e^{\frac{2v}{w}},\ v^2=2ga^2\left(\frac{1}{a}-\frac{1}{r}\right). \tag{14}$$

Thus the acceleration in the flight path will be

$$\frac{dv}{dt}=v\,\frac{dv}{dr}=g\,\frac{a^2}{r^2}$$

and the total acceleration produced by the gas recoil will be

$$q=\frac{dv}{dt}+g\,\frac{a^2}{r^2}=2g\,\frac{a^2}{r^2}. \tag{15}$$

For the earth's surface ($a=r$) this corresponds to twice the acceleration of gravity, which passengers in a reclining position can withstand. For the propellants in Table 1, equation (13a) gives the following figures (Table 2).

TABLE 2.

Propellant	$\frac{a}{h}$	$2\sqrt{\frac{a}{h}}$	$\frac{m_0}{m_1}$
H_2+O	6.37	5.05	156
$C+O_2$	7.63	5.53	252
Nitroglycerine	14.28	7.56	1920
Guncotton	20.82	9.10	8900

The table shows that, even in the best case and without taking air resistance into account, only a very small fraction of the initial mass of the rocket will be able to escape from the gravity of the earth. Therefore, the rocket flight will not be successful.

The flight time from the earth's surface to some given distance r is found from (14), taking into account that $dr=vdt$:

$$dt\sqrt{2ga}=dr\sqrt{\frac{r}{r-a}}\,. \qquad (15a)$$

Integrating for $t=0$ and $r=a$, we obtain

$$t=\sqrt{\frac{a}{2g}}\left[\frac{r}{a}\sqrt{1-\frac{a}{r}}-ln\left(\sqrt{\frac{r}{a}}-\sqrt{\frac{r}{a}-1}\right)\right] \qquad (15b)$$

where

$$\sqrt{\frac{a}{2g}}=570\text{ sec.}$$

For the distance ratios

$$\frac{r}{a}=1,\ 2,\ 3,\ 4,\ 10,\ 25,\ 50,\ 63$$

(distance of the moon), the flight times will be

$$t=0{,}21'55'',\ 34'10'',\ 45'25'',\ 1\text{hr}\,47'20'',\ 4\text{hr}\,15',\ 8\text{hr}\,15',\ 10\text{hr}\,21'.$$

If we could be satisfied with a lower flight velocity, then, according to Oberth, the fuel consumption could be reduced, giving a more favorable ratio $\frac{m_0}{m}$. When the burning stops, the rocket should fly like a projectile from a cannon. Thus we have a combination of the two means of propulsion described above, firing of a projectile and recoil (Figure 100). Burning of the propellant in the rocket should cease only when, at a distance r_2, a sufficiently high projectile velocity is attained, since otherwise it will not be possible to overcome gravity.

For a radial projectile velocity we have $v=\frac{dr}{dt}$ and

279 $$\frac{dv}{dt}=-g\frac{a^2}{r^2},\quad v\,dv=ga^2\,d\left(\frac{1}{r}\right).$$

If the initial velocity (at the earth's surface) is $v_0=\sqrt{2ga}$, then

$$(v^2-v_0^2)=2ga^2\left(\frac{1}{r_1}-\frac{1}{a}\right),\quad v^2=2g\frac{a^2}{r_1}. \qquad (16)$$

Let us substitute this into (14), in order to obtain the distance where the burning should stop:

$$r_1=2a \qquad (16a)$$

that is, a distance equal to twice the earth's radius.

Here

$$v^2=ga;\quad v=7900\ \text{m/sec}. \qquad (16b)$$

Substituting this expression for v into (14) and keeping in mind that

$$w=\sqrt{2gh},$$

we obtain

$$\frac{m_0}{m_1}=e^{\sqrt{2\frac{a}{h}}}. \qquad (17)$$

A comparison of this result with (13a) shows that, by stopping the burning, the mass ratio can be reduced by the following amount:

$$1\ \sqrt{2}=0.7\ \text{time}.$$

Table 3 shows the values of $\frac{m_0}{m_1}$ in this case for the different propellants.

TABLE 3.

Propellant	$\frac{m_0}{m_1}$
H_2+O	34
$C+O_2$	48
Nitroglycerine	199
Guncotton	582

These figures indicate the impossibility of a rocket flight, without even taking into account the extremely low velocity (less than 8,000 m/sec and going to zero at infinity).

We define the efficiency during the rocket ascent as the ratio of the work performed, $m_1 g\left(a+\frac{v_1^2}{2g}\right)$, to the work of the propellant which is converted into a gas, $(m_0-m_1)\,gh$, that is:

$$\eta=\frac{m_1}{m_0-m_1}\cdot\frac{a}{h}\left(1+\frac{v_1^2}{2ga}\right). \tag{18}$$

Then, for a rocket with continual gas ejection, when

$$v_1^2=2ga$$

and for cessation of burning, when $v_1^2=0$, we have

$$\eta=\frac{2a}{\left(\frac{m_0}{m_1}-1\right)h},\quad \eta''=\frac{a}{\left(\frac{m_0}{m_2}-1\right)h}. \tag{18a}$$

Values of these efficiencies are given in Table 4, the last column of which shows the mass ratios $\frac{a}{h}+1$, corresponding to an efficiency $\eta''=1$ of the rocket at cessation of burning and for conversion of all the energy of the propellant into work of ascent.

TABLE 4.

Propellant	η'	η''	$\frac{a}{h}+1$
H_2+O	0.082	0.193	7.37
$C+O_2$	0.061	0.162	8.63
Nitroglycerine	0.015	0.072	15.28
Guncotton	0.0047	0.036	26.82

Here we have still not taken into account the mass of propellant which will be needed to brake the rocket during its return to the earth. This mass will be about the same as that required for the ascent, which has been calculated above. However, the total mass ratio of the rocket, for both ascent and descent, will be the product of the two ratios, and this leads to improbable numbers. The foregoing study does not pertain to flights in the upper atmosphere, since its composition, density, and effect on the flight are still unknown.

Note. On p. 143 of the journal "Die Rakete" for 1927, there is a review by Oberth of Lorenz's paper; this review indicates more favorable prospects for rocket flights.

Second Paper. *THE FEASIBILITY OF SPACE TRAVEL*

The "Jahrbuch der Wissenschaftlichen Gesellschaft für Luftfahrt" for 1928 included a paper by Hans Lorenz entitled "Die Ausführbarkeit der Weltraumfahrt," a translation of which is presented below. Although this paper repeats the conclusions arrived at in the first work, it also includes some interesting new observations as well.

THE ENGINE

The intensive development of aeronautical engineering, based on the use of light, powerful motors, has given rise to new ideas on sending a passenger vehicle into outer space. Such a vehicle will have to overcome terrestrial gravity and reach other celestial bodies. The solution of this problem, as novelists like Jules Verne and Kurd Lasswitz have indicated, is based on a dynamic principle. The vehicle must overcome gravity, but wings and a propeller will no longer be of any use. An engine is needed which operates on fuels other than those used in internal-combustion engines, since the necessary oxygen will not be available in outer space, and in the atmosphere, at heights from 30 to 50 km, the amount of oxygen will be insufficient. Therefore, a propellant is needed which already contains oxygen in it, and as a result its weight per unit energy will be greater. In ballistics the most powerful propellants are assumed to be nitroglycerine and guncotton (collodium). To these we might add the detonating gas and the mixture of carbon and oxygen used in mining. Table I gives the heat energy Q per unit weight, the height of ascent h_0, and the quantity $h = \frac{2}{3} h_0$, which is

TABLE 1.

Propellant	Q, cal/kg	h_0, km	h, km	w, m/sec	$\frac{a}{h}+1$
H_2+O	3550	1570	1010	4430	7.37
$C+O_2$	2930	1250	835	4048	8.63
Nitroglycerine	1580	670	446	2950	15.28
Guncotton	1100	460	304	2456	21.82

282 the part of the height h_0 utilized in ballistics, the other third being lost, since some of the heat is carried off by the combustion products. The last column but one of the table gives the exhaust velocity $w=\sqrt{2gh}$.

ENERGY CONSUMPTION

Let us determine the energy consumption during the motion of a rocket in outer space, proceeding just from the condition of overcoming gravity. The acceleration of gravity at the earth's surface is g and the earth's radius is a. At distances $r>a$ from the earth's center, the acceleration will be

$$q=-g\frac{a^2}{r^2}. \tag{1}$$

The work done in lifting a mass m will be

$$L=mga^2\left(\frac{1}{a}-\frac{1}{r}\right). \tag{2}$$

For $r=\infty$ we obtain the limiting value $L_0=mga$.

For a flight to another planet, this work will be reduced due to the attraction of the latter. Thus, beginning at the neutral point on the line joining the centers of the two planets, there will no longer be any energy expenditure. For the moon, which has a mass equal to about 1/80 of that of the earth, this point lies 0.9 of the way to the moon. Therefore, the energy saving up to this point will be only 0.02 L_0, and all the way to the moon it will be 0.06 L_0, an amount so negligible that it need not be taken into account; the energy expenditure thus may be assumed to be L_0, for flights to the moon and to other celestial bodies as well. *

The most favorable energy expenditure will be when the energy is used only for ascent. If m_0 is the initial mass of the ship, and m when it is loaded with fuel, the fuel mass will be m_0-m. The energy formula will then be

$$(m_0-m)g\cdot h=L_0=mga$$

for an efficiency $\eta=1$. The minimum of the mass ratio is

$$\frac{m_0}{m}=\frac{a}{h}+1. \tag{3}$$

Some values of this quantity are given in the last column of Table 1. Here it is assumed that only the vehicle itself is lifted, but not the parts containing propellant. The latter is assumed to deliver all its energy during the launching at the earth's surface, which will be possible only if the vehicle is fired like a cannon.

* See preceding paper: "The Possibility of Space Travel."

If we do not take into account the resistance of the atmosphere to a body passing through it, then the firing of a projectile will impart to the latter a minimum energy corresponding to a velocity

$$v_0 = \sqrt{2ga} = 11.180 \text{ m/sec.}$$

Since this takes place in the tube in which the fuel comes in contact with the base of the projectile, the average velocity of the fuel particles will be $\frac{v_0}{2}$, and the weight increase will be

$$(m_0 - m) \cdot \frac{v_0^2}{6}.$$

For total conversion of the fuel energy into kinetic energy (Wucht), we obtain

$$(m_0 - m)\left(gh - \frac{mv_0^2}{6}\right) = \frac{mv_0^2}{2} \tag{4}$$

but since

$$v_0^2 = 2ga,$$

therefore

$$\frac{m_0}{m} = \frac{3h + 2a}{3h - a}. \tag{5}$$

This ratio will remain positive until $3h > a$, that is, until the free height of ascent of the fuel becomes greater than 1/3 of the earth's radius.

According to Table 1, even detonating gas will not satisfy this condition. Therefore, at present no propellant exists which is capable of imparting to a body the minimum velocity required for a flight into space, even assuming a vacuum. Consequently, it is useless to seek the optimum accelerations, or to determine the cannon length or the effect of the air, which will constitute a barrier to a projectile leaving a cannon at a planetary velocity. If, for such a firing, the velocity of the projectile at the earth is v_0, then its velocity v at a distance r from the earth's center is given by formula (1). For $q = dv/dt$ and $vdt = dr$, it can be found from the relation

$$v^2 = 2g\frac{a^2}{r} \tag{6}$$

that is, at an infinitely large distance the velocity is zero. Thus the kinetic energy of a space projectile will vary in inverse proportion to its distance from the earth's center (Figure 101).

LIMITING VALUES OF MASS RATIOS FOR SPACE FLIGHT

Since the firing of a projectile from a cannon does not make flight into space possible, let us consider the use of the principle of reaction, that is,
284 the flight of a rocket. In the case of a projectile we could already speak of a possible limiting case, when burning of the charge leads to the mass ratios for ascent of equation (3) (see last column of Table 1). These represent the lower limiting values, and they are very high in comparison with those for land, water, and air transports, amounting to from 6 to 20 times the limit, even without taking into account auxiliary mechanisms and control devices, assuming a full load. In addition, the weight of the passengers must be counted, as well as that of the food products, air supply, instruments, shielding devices, etc. A more favorable mass ratio can be obtained if we assume that a continuous consumption of a fuel $h.g.dm$ goes just to lift an instantaneous total mass m. Then we have the simple relation

$$-hgdm = mg\frac{a^2}{r^2}dr = -mga^2 d\left(\frac{1}{r}\right).$$

After integration over limits from $r=a$ to $r=\infty$, we obtain

$$\lg\frac{m_0}{m} = \frac{a}{h}; \quad \frac{m_0}{m} = e^{\frac{a}{h}} \tag{7}$$

and an efficiency

$$\eta = \frac{ma}{(m_0 - m)h}. \tag{7a}$$

The values calculated using these formulas are given in Table 2. The mass ratios obtained are so great, and the efficiencies so small, that the feasibility of such a device is out of the question. Here we have the upper limit for the mass ratio.

TABLE 2.

Propellant	$\frac{a}{h}$	$\frac{m_0}{m}$	η
H_2+O	6.37	584	0.011
$C+O_2$	7.63	2060	0.003
Nitroglycerine	14.28	$\sim 1.6.10^{6}$	$7.2.10^{-6}$
Guncotton	20.82	$\sim 11.10^{7}$	$1.1.10^{-8}$

ROCKET FLIGHT WITH THRUST

A vehicle propelled by reaction must leave the earth with a certain acceleration (starting from a state of rest). Let us assume for simplicity that the ascent is vertical, so that the reaction [thrust] serves both to

285 impart an acceleration to the total mass m and to overcome gravity. Keeping in mind that the thrust per unit time will be $w\frac{dm}{dt}$, we obtain

$$w\frac{dm}{dt}=-m\left(\frac{dv}{dt}+g\frac{a^2}{r^2}\right) \tag{8}$$

where v is the flight velocity. For $dr=vdt$,

$$w\cdot v\frac{dm}{m}=-\left[vdv-ga^2\,d\left(\frac{1}{r}\right)\right]. \tag{8a}$$

But since

$$\frac{w^2}{2}dm-gh\cdot dm=0,$$

therefore

$$-ghdm=mvdv-mga^2\,d\left(\frac{1}{r}\right)-\frac{dm}{2}\left[(v-w)^2-v^2\right]. \tag{8b}$$

Consequently, here we have an energy equation, the left side of which is the energy developed during ejection of the combustion products, this being the energy which lifts the rocket.

This formula contains three variables: m, v, and r. Thus certain assumptions have to be made in order to arrive at a solution. For example, the regime of the fuel consumption can be established, that is, the ratio $\frac{dm}{dt}$ can be given some specified value. The acceleration limits, as dictated by the possible danger to a passenger, can also be determined (maximum of $2g$). It is not possible, using the calculus of variations, to determine from equation (8a) the function $v=f(r)$ for which the ratio $\frac{m_0}{m}$ would be an absolute minimum. Therefore, we assume the flight to be such that the acceleration is equal to n^2 times the acceleration of gravity at a distance r, that is,

$$\frac{dv}{dt}=v\frac{dv}{dr}=n^2 g\frac{a^2}{r^2}. \tag{9}$$

At launching $v=0$ and $r=a$, so that

$$v^2=2n^2 ga^2\left(\frac{1}{a}-\frac{1}{r}\right) \tag{9a}$$

and from (8a),

$$w\frac{dm}{m}=-(n^2+1)g\frac{a^2}{r^2}\frac{dr}{v}=\left(n+\frac{1}{n}\right)\cdot\frac{gad\left(\frac{1}{r}\right)}{\sqrt{2g\left(\frac{1}{a}-\frac{1}{r}\right)}}. \tag{9b}$$

For $n=1$, we have

$$\left(n+\frac{1}{n}\right)_{min}=2.$$

From (9a) and (9b), we obtain

$$v^2=2ga^2\left(\frac{1}{a}-\frac{1}{r}\right) \qquad (10)$$

and

$$w\frac{dm}{m}=-2dv;\ lg\frac{m_0}{m}=2\frac{v}{w}. \qquad (11)$$

286 The variation in rocket weight [kinetic energy] according to (10) has been plotted in Figure 101, where it is compared with the case of a projectile from a cannon. The two curves intersect at the point

$$r_1=2a \text{ for } v_1{}^2=ga;\ v_1=7900 \text{ m/sec}. \qquad (10a)$$

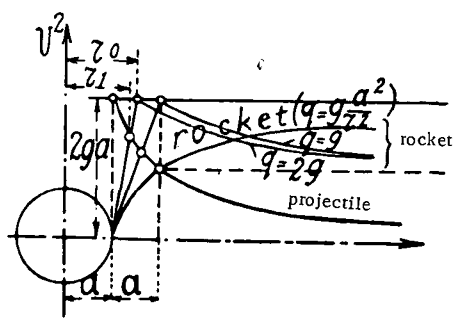

FIGURE 101.

For $r=\infty$ the weight of a rocket moving with a prolonged acceleration will be, for the limiting velocity

$$v_0{}^2=2ga, \qquad (10b)$$

the same as the theoretical initial weight of a projectile from a cannon.
For $w^2=2gh$, we have from (11)

$$\lg\frac{m_0}{m}=2\sqrt{\frac{a}{h}} \qquad (11a)$$

The burning of a mass m_0-m develops an energy $(m_0-m)\,gh$, which is transmitted to the remaining mass of the rocket m. At a distance r the latter performs a quantity of work

$$mga^2\left(\frac{1}{a}-\frac{1}{r}\right)+m\frac{v^2}{2}=mv^2.$$

Here the efficiency will be

$$\eta=\frac{mv^2}{(m_0-m)gh} \tag{12}$$

and, at the limit,

$$\eta=\frac{2ma}{(m_0-m)h}\,. \tag{12a}$$

The data given in Table 3 were calculated using this formula (enormous mass ratios and low efficiencies).

287 TABLE 3.

Propellant	$2\sqrt{a/h}$	m_0/m	η
H_2+O	5.05	156	0.082
$C+O_2$	5.53	252	0.061
Nitroglycerine	7.56	1920	0.015
Guncotton	9.10	8900	0.005

The time of ascent, from the launching point to a distance r, is determined from (10):

$$dt=\frac{dr}{v}=\frac{dr}{\sqrt{2ga}}\sqrt{\frac{r}{r-a}}\,. \tag{13}$$

For $r=0$ and $r=a$, the time is

$$t=\sqrt{\frac{a}{2g}}\left[\frac{r}{a}\sqrt{1-\frac{a}{r}}-lg\left(\sqrt{\frac{r}{a}}-\sqrt{\frac{r}{a}-1}\right)\right], \tag{13a}$$

where

$$\sqrt{\frac{a}{2g}}=570 \text{ sec.}$$

Table 4 gives some data for various distances.

TABLE 4.

$\frac{r}{a}=$	1	2	4	25	50	63 (distance of moon)
$t=$	0	21'55"	45'25"	4 hr 15'	8 hr 15'	10 hr 21'
$t'=$	0	21'55"	54 hr 40'	13 hr 16'	37 hr 32'	52 hr 52'

ROCKET FLIGHT WITH INTERMITTENT THRUST

The two weight curves corresponding to equations (6) and (10) intersect at $r_1 = 2a$, where an apparent cessation of thrust occurs and the rest of the flight is slowed down. For a numerical evaluation of this case, it is sufficient to apply the formulas of the last section, taking (10a) into account and using the velocity v_1 corresponding to $r_1 = 2a$. Then, from (11) and (12),

$$w \lg \frac{m_0}{m} = \frac{2v_1}{w} = \sqrt{2\,\frac{a}{h}} \tag{11b}$$

$$\eta = \frac{ma}{(m_0 - m)h} \tag{12b}$$

288 The values given in Table 5 were calculated using these formulas.

TABLE 5.

Propellant	$2a/h$	m_0/m	η
$H_2 + O$	3.57	34	0.193
$C + O_2$	3.91	48	0.162
Nitroglycerine	5.35	199	0.072
Guncotton	6.43	582	0.036

The duration of the flight to $\frac{r}{a} = 2$ is determined from (13a), which taking (6) into account, gives

$$dt = \frac{dr}{v} = \sqrt{\frac{r}{2ga}} \cdot dr;$$
$$t_1 = \frac{1}{3}\sqrt{\frac{2a}{g}}\left[\left(\frac{r}{a}\right)^{\frac{3}{2}} - 2^{\frac{3}{2}}\right] \tag{13b}$$

and for $t' = t_1 + 2'55''$ we obtain the values given in the second row of Table 4.

ROCKET FLIGHT WITH UNIFORM ACCELERATION

The laws of rocket motion presented above give relatively low values of $\frac{m_0}{m}$, irrespective of the dependence on certain other factors, the choice of which may reduce these ratios.

Equation (8) can be rewritten for radial motion:

$$w \lg \frac{m_0}{m} = v + g a^2 \int_a^r \frac{dr}{r^2 v} \, \frac{x}{dx} \, . \tag{14}$$

Here the second term indicates the effect of gravity. Let us assume that the flight proceeds with a constant acceleration q. Then, in accordance with the rectilinear ascent, the curve of the upward flight will be (Figure 101)

$$q = \frac{dv}{dt} = \frac{v\,dv}{dr} = \frac{d}{dr}\left(\frac{v^2}{e}\right); \qquad v^2 = 2q(r-a) \tag{15}$$

and, from (14),

$$w \lg \frac{m_0}{m} = v + g a^2 \int_a^r \frac{dr}{r^2 \sqrt{2q(r-a)}} \tag{14a}$$

289 so that, after integrating and substituting $r - a = a \tan^2 \varphi$, we obtain

$$w \lg \frac{m_0}{m} = v + g \sqrt{\frac{a}{2q}} \left(\arctan \sqrt{\frac{r}{a} - 1} + \frac{a}{r} \sqrt{\frac{r}{a} - 1} \right) \tag{16}$$

The work performed will be

$$L = m g a^2 \left(\frac{1}{a} - \frac{1}{r} \right) = m g a \left(1 - \frac{a}{r} \right)$$

for a kinetic energy [Wucht] of

$$\frac{m v^2}{2} .$$

The efficiency in this case is

$$\eta = \frac{m \left[a \left(1 - \frac{a}{r} \right) + \frac{v^2}{2g} \right]}{(m_0 - m) h} . \tag{17}$$

At our limit, for an acceleration $q = \infty$ and a launching velocity $v_0^2 = 2ga$ at $r = a$, we have

$$\lg \frac{m_0}{m} = \frac{v}{w} = \sqrt{\frac{a}{h}}; \quad \eta = \frac{m a}{(m_0 - m) h} . \tag{17a}$$

Some values calculated using these formulas are shown in Table 6.

In spite of the relatively efficient use of the propellant energy, the limiting values of the mass ratio $\frac{m_0}{m}$ are still considerably higher than for an ideal shot with $q = 1$ (Table 1).

TABLE 6.

Propellant	$\frac{v_0}{w} = \sqrt{a/h}$	$\frac{m_0}{m}$	η
H_2+O	2.53	12.5	0.556
$C+O_2$	2.77	15.8	0.517
Nitroglycerine	3.79	44.3	0.331
Guncotton	4.56	95.9	0.228

UNIFORM ACCELERATION TO SHOT VELOCITY

Two cases are possible for a finite acceleration. First let us assume that such an acceleration acts only until the shot velocity v_0 is reached, which corresponds to the upper horizontal asymptote in Figure 101. This asymptote originates at a point corresponding to r_0, the straight part of the
290 line for the kinetic-energy variation.

We can write the equations

$$v_0^2 = 2q(r_0 - a) = 2ga;\ \frac{r_0}{a} = 1 + \frac{g}{q}. \tag{18}$$

Taking into account (16), (17), and the relation $w^2 = 2qh$, we obtain

$$\lg\frac{m_0}{m} = \sqrt{\frac{a}{h}}\left[1 + \frac{1}{2}\sqrt{\frac{r_0}{a} - 1}\left(\operatorname{arctan}\sqrt{\frac{r_0}{a} - 1} + \frac{a}{r_0}\sqrt{\frac{r_0}{a} - 1}\right)\right] \tag{16b}$$

$$\eta = \frac{m\left(2 - \frac{a}{r_0}\right)a}{(m_0 - m)h}. \tag{17b}$$

The data in Table 7 were calculated using these formulas.

TABLE 7.

$q/g =$	1	1.5	2	3	4	η
$r_0/a =$	2	5/3	3/2	4/3	5/4	—
H_2+O	63.2	41.8	33.0	25.0	21.5	0.154 — 0.372
$C+O_2$	93.3	59.4	45.8	33.9	28.7	0.124 — 0.332
Nitroglycerine	506	272	191	126	100	0.043 — 0.174
Guncotton.	1800	853	555	337	257	0.017 — 0.098

This table shows that the ratios of the masses and efficiencies are more favorable in this case, in comparison with the ratios for the reduced accelerations of Tables 3 and 5, although more energy will be expended by the rocket.

Let us assume that the acceleration is terminated when the velocity v_1 is reached. This corresponds to a distance r_1, in the case of a projectile, or to the point of intersection between the ascending kinetic-energy line and the descending kinetic-energy curve (hyperbola). Then

$$v_1^2 = 2q(r_1 - a) = 2g\frac{a^2}{r_1} \qquad (20)$$

$$\frac{r_1}{a}\left(\frac{r_1}{a} - 1\right) = \frac{g}{q}.$$

Taking (16) and (17) into account, we have

$$\lg\frac{m_0}{m} = \frac{1}{2}\sqrt{\frac{a}{h}\cdot\frac{a}{r_1}}\left[1 + \frac{r_1}{a} + \frac{r_1}{a}\sqrt{\frac{r_1}{a} - 1}\ \arctan\sqrt{\frac{r_1}{a} - 1}\right] \qquad (16c)$$

$$\eta = \frac{ma}{(m_0 - m)h}. \qquad (17c)$$

291 The data in Table 8 were obtained using these formulas, and they are found to be closest to the values in Table 6. However, it should be kept in mind that accelerations of $q/g > 2q$ will probably not be possible, so that only the figures in the first two columns of Tables 7 and 8 will be meaningful.

TABLE 8.

$q/g =$	1	1.5	2	3	4	r
$r_1/a =$.	1.618	1.458	1.366	1.264	1.207	—
$v_1/v_0 =$.	0.785	0.827	0.855	0.888	0.910	—
$H_2 + O$	23.7	17.4	15.1	13.3	12.6	0.280 – 0.548
$C + O_2$	31.9	22.8	19.4	16.9	16.0	0.247 – 0.508
Nitroglycerine	116	72.9	58.7	48.5	45.0	0.125 – 0.327
Guncotton.	306	175	135	107	97.6	0.068 – 0.215

The duration of the flight may be found from (9) as

$$t = \frac{v_1}{q} + \int_{r_1}^{r}\frac{dr}{v} = \frac{v_1}{q} + \frac{1}{3}\sqrt{\frac{2a}{g}}\left[\left(\frac{r}{a}\right)^{\frac{3}{2}} - \left(\frac{r_1}{a}\right)^{\frac{3}{2}}\right]. \qquad (21)$$

Since the variation of kinetic energy for such motion differs from that in case 6 [sic] only in its first part, before r_1, therefore the flight durations will differ very little from the values of t' in the second line of Table 4, and there is no point in recalculating them.

Now let us assume that a reaction-propelled vehicle moves at an angle ϑ to the radial direction. The radial and tangential components are w_r, w_t, v_r, and v_t. In this case

$$\frac{w_r}{w} = \frac{v_r}{v} = \cos\vartheta;\ \frac{w_t}{w} = \frac{v_t}{v} = \sin\vartheta\,. \tag{22}$$

The equations of motion for the two directions are

$$\left.\begin{aligned} w_r\frac{dm}{dt} &= -m\left(\frac{dv_r}{dt} - \frac{v_t^2}{r} + g\frac{a^2}{r^2}\right)\\ w_t\frac{dm}{dt} &= -m\left(\frac{dv_t}{dt} + \frac{v_r v_t}{r}\right). \end{aligned}\right\} \tag{23}$$

Setting $dr = v_r dt$ or $v_t dt$, we add these equations to obtain

$$(w_r v_r + w_t v_t)\frac{dm}{m} = -\left(v_r dv_r + v_t dv_t + g\frac{a^2}{r^2}dr\right)$$

292 and, taking (22) into account, together with (8a), we have

$$wv\frac{dm}{m} = m - \left(vdv + g\frac{a^2}{r^2}dr\right). \tag{23a}$$

For uniform acceleration over a path $ds = vdt$, we can write

$$\frac{dv}{dt} = q;\ vdv = \frac{qdr}{\cos\vartheta} = qds$$

or, after integration,

$$v^2 = \frac{2q(r-a)}{\cos\vartheta}\,. \tag{24}$$

Therefore, instead of (14a), for constant ϑ, we have

$$w\lg\frac{m_0}{m} = v + ga^2\sqrt{\cos\vartheta}\int\frac{dr}{r^2\sqrt{2q(r-a)}} \tag{23b}$$

and, according to the equations

$$rd\varphi = dr\,\mathrm{tg}\,\vartheta;\ \varphi = \mathrm{tg}\,\vartheta\lg\frac{r}{a}\,. \tag{25}$$

The flight path will be a logarithmic spiral (Figure 102). In order to determine the mass ratios, we use the formulas derived above, but with $q' = q/\cos\vartheta$ instead of q. For the same distances r from the earth and the same final velocities, formulas (17b) and (17c) for the efficiencies will remain unchanged, since q does not enter into these formulas directly, but only through the mass ratio $\frac{m_0}{m}$. If, as an example, $\vartheta = 75°30'$, corresponding to an angle of 14°30' to the horizontal, then cos $\vartheta = 0.25$ and $q' = 4q$. For a uniform acceleration $q = g$, and for a radial acceleration of the earth $g_1 = ga^2 / r^2$, the values in the last columns of Tables 7 and 8 must be used, instead of those in the first column, which gives a considerable reduction of the mass ratio and a higher efficiency, in the case of an oblique launching.

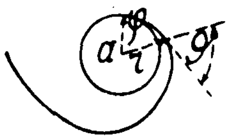

FIGURE 102.

For uniform acceleration the flight duration is found from the relations:

$$q_r = q\cos\vartheta;\ dt' = \frac{dr}{v_1} = \frac{dr}{v\cos\vartheta}.$$

The duration will be longer than for a radial flight, given the same final velocities and lengths of flight with respect to $\frac{1}{\cos\vartheta}$.

At the same time, for free flight in space, they will not depend on ϑ. For
293 $\vartheta = 90°$, cos $\vartheta = 0$, and then, for $q = \infty$, the second term in equation (23a) disappears. Thus we have the limiting case and equation (17a). Launching the rocket at an angle is especially important in the case of a return to the earth.

FLIGHT IN OUTER SPACE AND RETURN TO EARTH

In a case studied above it was assumed that a velocity $v_0 = 11{,}180$ m/sec was imparted to a rocket, this speed being sufficient to begin a flight into outer space and around some other celestial body, such as the moon. To control the flight, it was then necessary to make some lateral correcting bursts of the rocket, which requires additional propellant.

The vehicle will heat up considerably during a descent to another planet or to the earth, when it enters an atmosphere at a space velocity. Consequently, the flight speed will have to be reduced. For example, a reverse thrust may be employed, which will require new masses of propellant and an increase in the ratio $\frac{m_0}{m}$. Even if we ignore the propellant needed to visit another planet and to guide the rocket, and if we consider just the descent to the earth, the increase in mass will still be equal to $\frac{m_0}{m}$ squared, even for the optimum ratio of the quantities in the last column of Tables 7 and 8. If $H_2 + 0$ or $C + 0_2$ is used as the propellant (Table 6), the figures will be even higher. For nitroglycerine and guncotton the values obtained are quite fantastic. This difficulty will not be circumvented by the composite [step] rockets proposed by Professor Oberth, the parts of which will fall away one by one, leaving only a vehicle of mass m at the end of the space flight.

Let us denote the masses which fall away gradually as

$$(m_0 - m_n), \ldots (m_3 - m_2), \ldots (m_1 - m),$$

the initial mass being m_0 and the final mass being m. Then,

$$m+(m_1-m)+(m_2-m_1)+(m_3-m_2)+\ldots+(m_0-m_n)=m_0.$$

the velocity increments being $v_n-v_{n+1}\ldots$ and, in the ideal case (17a),

$$\lg\frac{m_0}{m_n}=\frac{v_n}{w};\quad \lg\frac{m_2}{m_1}=\frac{v_1-v_2}{w};\quad \lg\frac{m_1}{m}=\frac{v-v_1}{w},$$

so that, for a final velocity

$$v=v_n+(v_{n+1}-v_n)\ldots+(v_1-v_2)$$

we have

$$\frac{m_0}{m_n}\cdot\frac{m_n}{m_{n+1}}\cdot\frac{m_{n+1}}{m_{n+2}}\ldots=\frac{m_0}{m}.$$

294 Consequently, we obtain the ratio of initial and final masses in the same form as earlier, but with the addition of the useless weight of the casings of the intermediate rockets.

Therefore, the basic reasons for the incredible difficulties involved in carrying out a rocket flight into space are: improper utilization of the thermal effect during the chemical reaction between oxygen and propellant, the tremendous mass of propellant required, the lack of the proper light but strong materials for the rocket itself, etc.

DISCUSSION OF REPORT

Ing. M. Schrenck. The use of rockets will be of benefit only at space velocities. Rocket flights in the atmosphere are not advisable, if other means are available. Let us consider whether it would be possible to use a rocket on a racing plane in order to attain high velocities. The results of the corresponding calculations are shown in Figure 103. The velocity of gas ejection is taken to be 1,000 m/sec. If a rocket were employed, it turns out that a new speed record could be set over a short distance.

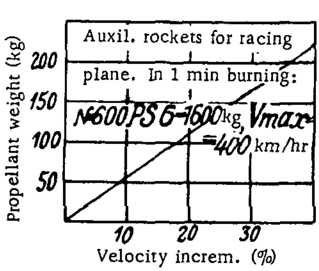

FIGURE 103.

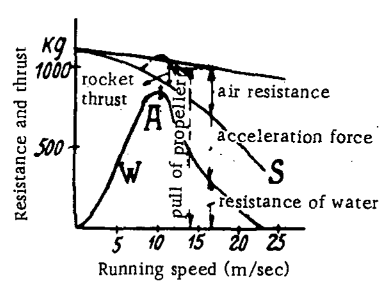

FIGURE 104.

It is to be expected that rockets will be used to send recording instruments to great heights. For a mass ratio of 0.3, for example, a height of 24 km can be attained at a speed of 200 m/sec, or a height of 36 km at a speed of 300 m/sec, without taking air resistance into account. The latter will have an effect, but it will be less for large rockets than for small ones.

Dr. Kölzer. It was mentioned in the report that the composition of the atmosphere up to 50 km is such that the velocity of propagation of sound in it may be considered constant up to that height. According to the latest studies, on the other hand, the following sound velocities have been obtained for different heights: 330 m/sec at the ground, 290–295 m/sec at 10 to
295 35 km, and 330 m/sec again at 50 km. This increase is due not to a rising hydrogen content but rather to the temperature variation. In addition, I have been informed of experiments with rocket-borne meteorographs up to a height of 1000 m. The instruments stood up well under an acceleration of 50 m/sec^2.

Prof. Pröll. An essential difference between a rocket and an aircraft is that the former develops its velocity rapidly with a large energy expenditure, while the latter develops speed slowly with a low energy loss. When a seaplane is taking off, it may be that the pull of the propeller is insufficient to lift the plane off the water. For instance, in Figure 104, curve w represents the variation in the resistance of the water, while curve S shows the pull of the propeller less the air resistance. The ordinates of the area between the two curves give the force of the acceleration imparted. Point A shows the beginning of the critical moment for the takeoff. It may be delayed for a long time, until the velocity increases so much that the pontoons are pulled out of the water. If at this moment an additional thrust is supplied by a rocket (dotted line), the upward flight will proceed much more rapidly.

Note. The remarks of Oberth, who also took part in the discussion, have essentially already been given above in his paper.

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A. SHERSHEVSKII

An article entitled "The Spaceship," by A. B. Shershevskii, was published in the journal "Flugsport" in 1927 (p. 386). In it the author first gave a brief history of the subject and then went on to develop a theory of the flight of an interplanetary rocketship. A translation of this work [which was written in German] is presented below, following a short biography of A. Shershevskii [Alexander Boris Scherschevsky].

LIFE OF A. SHERSHEVSKII

Aleksandr Borisovich Shershevskii (Figure 105) was born on 22 October 1894 in Leningrad. He obtained his secondary education at the private Shtemberg Realschule. In 1913 he was admitted to the Mechanics Department of the Leningrad Polytechnic Institute. There he studied mechanical engineering, shipbuilding, and aeronautical engineering under the following professors (listed alphabetically): (the late) A. P. Boklevskii, J. de Bottesatte (now in the USA), D. N. Zeiliger (now at Kazan State University), A. Ioffe (Leningrad), N. A. Rynin (Leningrad), (the late) V. A. Slesarev, van der Vleet (now in Prague), (the late) A. A. Fridman, (the late) V. I. Yarkovskii, and others.

FIGURE 105.

In the spring of 1915 Shershevskii volunteered for the aviation division of the Aero-Club, where he completed courses in aircraft motors and pilot training. In the summer of 1916 he was released from this program because of poor eyesight. In 1916 and part of 1917 (a total of six months) Shershevskii worked at the Lebedev aircraft factory in Leningrad, Novaya Derevnya (construction practice, manufacture, and assembly). In 1919 he went to Berlin, Germany. There he continued his studies, as an auditor at Berlin University (Physics and Mathematics Department of the Philosophy Faculty) and the Tech-

298 nische Hochschule, under the following professors (listed alphabetically): Bieber-Bieberbach (pure mathematics), A. Einstein (relativity), R. von Mises (pure and applied math.), M. Plank (physics), H. Reissner (statics), R. Fuchs (aerodynamics), and G. Hamel (mechanics). In 1925 he worked at the patent department of the Rohrbach aircraft factory (all-metal airplanes and airships). From 1924 to 1926, under the direction of Major Tschudi, the president of the German Aero-Club, Shershevskii prepared the Russian section of a seven-language international dictionary. He contributed to a number of aviation journals (Z. F. M., Flugsport, Luftfahrt, Illustrierte Flug-Woche, Jungflieger, Die Rakete, Zeits. für angewandte Mathem. u. Mechanik, Vestnik Vozdushnogo Flota, and others). In 1928 a popular science book by Shershevskii was published by the C. I. E. Volckmann Publishing Company in Berlin-Charlottenburg: "Die Rakete für Fahrt und Flug, Eine allgemeinverständliche Einführung in das Raketenproblem" ("The Rocket for Travel and Flight, A Popular Introduction to the Rocket Problem"). He worked with Professor Oberth. At present he is participating in the work of the Deutsche Versuchsanstalt für Luftfahrt in Berlin-Adlershof. In the near future he intends to complete a study of long-range rockets (Zum Variatsionsproblem der Fernrakete) and a study of the development of shapes and sizes of animals and mechanisms, moving in a liquid or gaseous medium or in a vacuum (spaceships).

Shershevskii was interested in aeronautics almost from his very childhood, and while in school he organized a model-aircraft club. He constructed models himself, and from 1911 to 1914 he contributed to the journals "Vestnik Vozdukhoplavaniya" and "Aerozhizn'" (Leningrad society). As early as 1912–1913 he carried out tests with tailless aircraft (which are only now beginning to be developed at the Research Division of the Rhön-Rositten Gesellschaft at Wasserkuppe, Rhön, Germany, by Ing. A. Lippisch, Fr. Stamer, and F. Wenck); the results of these tests have not been published. Shershevskii became interested in rockets and interplanetary travel while reading Tsiolkovskii's classic work "Exploration of Planetary Space with Jet Machines" (Vestnik Vozdukhoplavaniya, Leningrad, 1911–1913).

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THEORY OF THE INTERPLANETARY ROCKET SHIP

FIRING OF PROJECTILE FROM A CANNON OR CENTRIFUGAL MACHINE

First let us consider the case of a projectile fired into outer space by a cannon with an ordinary charge or by an electric (solenoid) cannon, and also the case of a projectile fired from a centrifugal machine. Both cases are unfeasible, for the following reasons: a) insufficient strength of materials, b) technical impracticability, c) excessive acceleration forces, of the order of $10^9 g$ (where $g = 9.81$ m/sec^2), making it impossible to use instruments or to take passengers, and d) enormous air resistance.

If the length of the cannon is L, the projectile velocity at firing is v, the projectile acceleration is b, the acceleration of gravity at the earth (constant) is g, and the vertical height of ascent is h, then we have

$$L = v^2/2(b-g) \tag{1}$$

and

$$b = (v^2 + 2gL)/2L. \tag{2}$$

The effect of acceleration, that is, the apparent heaviness in the cannon projectile, will be

$$\underline{b} = b/g = (h/L) + 1. \tag{3}$$

For a cannon 300 m long and an ascent height of 300 km, we have a muzzle velocity of 2,450 m/sec and an overload [excessive acceleration force] of 1,001 g. However, in order to overcome gravity, that is, to escape into outer space at zero final velocity, an initial velocity of

$$v = \sqrt{2gr} \tag{4}$$

is required. If r is the radius of the earth, we obtain

$$v_\infty = 11{,}180 \text{ km/sec} \tag{5}$$

Actually, to ascend to a height h where the velocity is still v_r, we need an initial velocity

$$v = \sqrt{v_r^2 + \frac{2grh}{r+h}}. \qquad (6)$$

300 If $v_r = 0$, then

$$v = \sqrt{\frac{2grh}{r+h}}. \qquad (7)$$

Setting $h = \infty$, we obtain

$$v_\infty = \sqrt{2gh}. \qquad (8)$$

However, if we wish to retain a certain velocity at infinity, then

$$v_{\infty r} = \sqrt{v_r^2 + 2gr} \qquad (9)$$

where $v_{\infty r} > v_\infty$. The latter expression plays an important role in the theory of a reaction-propelled ship.

In contrast to the latter velocity, the initial velocity of a projectile at firing is very high. Substituting the quantiy

$$v = \sqrt{2gr},$$

into equation (1), we obtain

$$L = (\sqrt{2gr})^2 / 2(b-g) = gr/(b-g). \qquad (10)$$

A brief computation indicates that an enormous overload, of the order of $10^9\,g$, will be produced, without even taking the air resistance into account. This will be the case for a centrifugal machine as well as for a cannon. In addition, it will not be possible to guide the projectile either.

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THE REACTION-PROPELLED SPACESHIP

Let us define a reaction-propelled spaceship as a large passenger rocket which can be controlled. The propellant used must have a very high reaction energy. Liquid H and O may serve as such a substance (Table 1).

FLIGHT OF A REACTION-PROPELLED SHIP IN AIRLESS AND GRAVITY-FREE SPACE

The theory of motion of a reaction-propelled ship is based on the following two assumptions: 1) the relative velocity of gas ejection stays constant, and 2) this ejection [exhaust] is optimum, that is, the points of application of the external forces, and the center of intertia of the mass, lie on the vector of the resultant forces of the reaction.

300) TABLE 1.

	Heat conducted, kcal per kg	Exhaust velocity v_a in km/sec
Burning in oxygen-free space		
Propellant: H and O. Combustion product: water vapor.	3.200	5.18
" H and O. Product: water	3.736	5.60
" H and O. " ice	3.816	5.65
" C_6H_6 and O_2. " H_2O and CO_2	2.370	4.45
Burning in oxygen-rich atmosphere		
Fuel H_2. Product: H_2O	28.780	15.52
" C_6H_6	10.000	9.6

The following symbols will be used: M is the total mass of the ship, V is its velocity, m_r is the mass of the empty ship, m_{aa} is the mass of propellant at the beginning of flight, and m_a is the mass of propellant left at a given moment.

Then, at any time,

$$M = m_r + m_a$$

For $t=0$,

$$m_a = m_{aa}$$

So that for $t=0$,

$$M = m_r + m_{aa} \qquad (11)$$

Let us denote the mass ratio m_{aa}/m_r as q, and the velocity of gas ejection as v_a. Then, from the law of conservation of momentum,

$$(m_r + m_a)\, dV = -v_a\, dm_a. \qquad (14)^*$$

The integral equation is

$$\int \frac{dV}{v_a} = -\int \frac{dm_a}{m_r + m_a} + C \qquad (15)$$

which gives

$$\frac{V}{v_a} = -\lg(m_r + m_a) + C. \qquad (16)$$

For $t=0$; $m_a = m_{aa}$ and $V=0$, so that

$$C = +\lg(m_r + m_{aa}) \qquad (17)$$

* Here we omit the elementary derivations of (14) (formulas (12) and (13)).

and

$$\frac{V}{v_a}=\lg\left(\frac{m_r+m_{aa}}{m_r+m_a}\right). \qquad (18)$$

The highest velocity is for $m_a=0$, when

$$V_{max}=v_a\lg\left(1+\frac{m_{aa}}{m_r}\right). \qquad (19)$$

302 TABLE 2.

$\frac{m_{aa}}{m_r}=q$	V_{max} in m/sec		W_{ex}, %
	for $v_a=5000$ m/sec	for $v_a=4000$ m/sec	
0	0	0	0
0.1	472.5	378	8.87
0.2	910.0	728	16.55
0.3	1,310	1,048	22.9
0.4	1,680	1,344	28.2
0.5	2,025	1,620	32.8
0.6	2,345	1,876	36.7
0.7	2,645	2,116	40.0
0.8	2,930	2,344	42.9
0.9	3,210	2,568	45.8
1.0	3,465	2,772	48.0
1.5	4,575	3,660	55.8
2.0	5,490	4,392	60.3
3.0	6,900	5,520	63.5
4.0	6,045	6,436	64.7
5.0	8,960	7,168	64.1
6.0	9,730	7,784	63.0
7.0	10,375	8,316	61.7
8.0	10,985	8,788	60.5
9.0	11,515	9,212	58.9
10.0	11,990	9,592	57.6
15.0	13,865	11,092	51.2
20.0	15,220	12,176	46.3
30.0	17,170	13,736	39.3
50.0	22,400	17,920	31.0
100.0	26,280	21,040	21.0
193.0	30,038	24,032	14.4
∞	∞	∞	0

303 Let us denote as W_{en} the energy efficiency of a rocket ship in a gravity-free medium. It will be equal to the ratio of the energy developed by the rocket ship to the propellant energy:

$$W_{en} = \frac{m_r}{m_{aa}}\left[\lg\left(1 + \frac{m_{aa}}{m_r}\right)\right] \quad (19)$$

An essentially simple, but somewhat tedious in execution, calculation indicates a maximum energy efficiency of 64.7%, for a mass ratio

$$q_{opt} = 3.997 = \sim 4 \quad (20)$$

For $q=0$, $W_{en}=0$, according to equation (19). For $q \Rightarrow \infty$ it will also be true that $W_{en}=0$. The relevant data are listed in Table 2* and plotted in Figure 106.

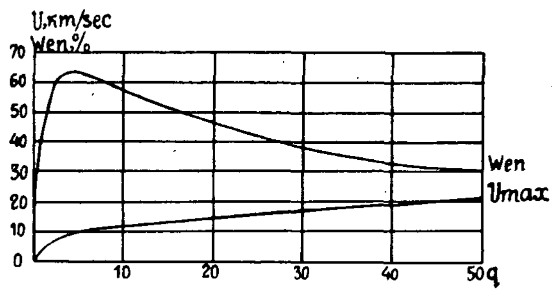

FIGURE 106.

The flight velocity is also a function of q (equations (18) and (12)). The flight in question corresponds to gravity-free space, that is, 1) between suns of the Milky Way, 2) at small planetoids having low gravitational accelerations, and 3) at distances from celestial bodies which are about equal to their radii. An additional calculation shows that, for motion in a medium with constant gravity, such as motion within the sphere of the earth's attraction, the formulas remain the same, except that a multiplying factor called the "acceleration term" must be introduced.

FLIGHT IN A VACUUM WITH CONSTANT GRAVITY (TERRESTRIAL)

In the case of terrestrial gravity we have, instead of equations (12) and (19),

$$V_{max_0} = \underset{\text{chem.}}{v_a}\underset{\text{b.t.}}{\left(1 - \frac{g}{b}\right)}\underset{\text{tech.}}{\lg\left(1 + \frac{m_{aa}}{m_r}\right)} \quad (21)$$

* $V_a = 5{,}000$ m/sec for pure hydrogen and oxygen, and $V_a = 4{,}000$ m/sec for a hydrocarbon fuel and oxygen or for endogenic oxygen compounds.

$$W_{en_0}=\frac{m_r}{m_{aa}}\overset{\text{tech.}}{\left[\lg\left(1+\frac{m_{aa}}{m_r}\right)\right]}\overset{\text{b.t.}}{\left(1-\frac{g}{b}\right)} \tag{22}$$

and

$$W_{\text{dyn}}=\left(1-\frac{g}{b}\right). \tag{23}$$

Here W_{dyn} is the dynamic efficiency, and b is the acceleration of the reaction-propelled ship (see Note 1 below). In these formulas, chem. refers to the chemical factor, b.t. to the biological-terrestrial factor, and tech. to the technological factor. The chemical factor depends on the type of propellant, the environment (air or vacuum), and the mixture, and it has an effect on the exhaust velocity. The technological factor, that is, the mass ratio q, affects the strength and the construction of the large, light containers, which have to withstand accelerations of both signs (see Note 2 below). Finally, the biological-terrestrial factor can be divided into the ship's own acceleration and the earth's acceleration. The former must not exceed a limiting value corresponding to danger to a person ($b=5g$). The latter is a characteristic of our planet. Assuming $b=5g$, we obtain

$$\left.\begin{aligned} V_0&=0.8\ V \\ W_{cn_0}&=W_{\text{dyn}}\cdot W_{en}=0.8\ W_{en} \end{aligned}\right\} \tag{28}$$

Note 1. Equations (21) and (22) were derived as follows: The burning time of a given mass of propellant is independent of the presence of an attracting body [such as the earth] and is

$$t=v/(b-g), \tag{24}$$

where v is the ship velocity attained by consuming a certain mass of propellant in t sec. Since we have assumed that the vectors of (b) and (g) lie along the same line in opposite directions (Figure 107 below), therefore the quantity $b-g$ represents the relative acceleration of the ship. The acceleration effect (apparent weight) will be

$$\underline{b}=b/g. \tag{25}$$

For motion by inertia (without the action of external forces like acceleration or air resistance) $\underline{b}=0$.

If t_s is the time it takes for the whole supply of propellant to burn up, and V_{max} is the corresponding maximum velocity, then

$$t_s=V_{max}/b; \tag{26}$$

and, from equation (24),

$$(V_{max}=v\cdot[b/(b-g)]. \tag{27}$$

305 Equation (27) together with equation (12) gives equation (21). Similar considerations lead to equation (22).*

Note 2. According to Tsiolkovskii, it is possible to have a ratio q of 25 or even 35.

For $W_{dyn}=0.8$, the values in Table 2 will be correspondingly lower. It follows from equations (12), (19), (21), and (22) and the table that the velocity will become infinite as q increases. In addition, for constant q, velocity V will be constant as well, that is, the flight velocity does not depend on the absolute weight of the ship. In general, the flight velocity, like the maximum speed attained, will not depend on the length of burning either. If $b=g$, then equation (21) gives zero velocity within the sphere of gravity (terrestrial, where $g=9.81$ m/sec^2), irrespective of the amount of propellant burned. An increase in the dynamic efficiency leads to a lower exhaust velocity and, which is even more important, a lower ratio q. Accordingly, the ship can be made stronger (higher structure weight).

A person can withstand (according to Tsiolkovskii) accelerations of $5g$ or more, provided he is immersed in a container of liquid. For an instantaneous burst, $b=\infty$, $\frac{g}{b}=0$, and the dynamic efficiency is 1 (100%). Here the velocity with gravity will be the same as that without gravity.

Table 2 showed that an increase in the exhaust velocity v_a gives higher final velocities, while for equal velocities q is lower. Moreover, W_{en} also increases with an increase in v_a. Thus a propellant with a high velocity of gas ejection must be used, one which would at least give an efficiency W_{en} of 65%, or 50 to 60% and a low q.

By interpolation, we find that for $b=5g$, $W_{dyn}=0.8$ (80%), and with H + O as propellant, upon departure from the earth we have $q \sim 18$, while for the optimum case ($b=4g$), $W_{dyn}=0.75$ (75%) and $q=20.5$. A further reduction of q is possible by employing a catapult.

Calculations show that, if a catapult is used, the mass ratio will be

$$q_k=\frac{m_{aa}}{m_r}=1+e^{\frac{V_{max}-v_k}{v_a}}, \qquad (29)$$

where v_k is the initial velocity attained with the catapult. Table 3 was

306 prepared using equation (29).

Descending into the earth's atmosphere at a speed of 12 km/sec represents a difficult problem. This problem can be solved in two ways: 1) by means of a reverse reaction [thrust], or 2) by utilization of air resistance (both ways may be used together, of course).

* Apparently, to agree with (27), equations (22) and (21) should have a multiplying factor

$$\left(1-\frac{g}{b}\right), \text{ rather than } \left(\frac{b}{b-g}\right).$$

(Note by N. R.)

TABLE 3.

V_{max}, km/sec	8	11	17
$v_k = 5$ km/sec			
$V_{max} - v_k =$	3	6	12
$q_k =$	0.8	2.31	10.0
$q =$	4	8	20
$v_k = 4$ km/sec			
$V_{max} - v_k =$	4	7	13
$q_k =$	1.24	3.08	12.0
$q =$	4	8	30
$v_k = 3$ km/sec			
$V_{max} - v_k =$	5	8	14
$q_k =$	1.72	4	15
$q =$	4	8	30

Calculations show that for a normal launching a descent using a reverse thrust is unfeasible, since, even in the optimum case,

$$(W_{dyn} = 0.8), \quad q_1 = 323.^*$$

However, the situation will be improved if a catapult is used. Different authors propose different methods. Oberth, Valier, and Goddard suggest reverse thrust and a parachute, and Tsander and Tsiolkovskii suggest aero-
307 dynamic descent (winged rocket).

In the foregoing, a vertical ascent was assumed. However, a rocket can also be launched at an angle and fly horizontally. For flight at a great height

(306)

* For a ship to have zero velocity when it reaches the earth, if a reverse thrust is to be used, the mass ratio must be

$$q_1 = (1 + q)^2 \qquad (30)$$

For low ascents $q \leq 0.5$ and, from (30), we have $q_1 = \sim 2q$. For ascent from the earth and descent to another planet, the ratio is

$$q_2 = (1 + q)(1 + q_3) - 1 \qquad (31)$$

where q_3 is analogous to q_1 for the other planet. Thus, to visit a planet and return to earth, the ratio will be

$$q_4 = (1 + q)^2 (1 + q_3)^2 - 1 \qquad (32)$$

with a velocity $V=\sqrt{(r+h)\cdot g}=8{,}000$ m/sec* (where r is the earth's radius and h is the flight altitude), the centrifugal acceleration will be equal to that of the earth (Figure 107), and the weight of the ship will be cancelled. Without taking air resistance into account, we find that horizontal flight (and launching along a tangent to the earth) will be much more satisfactory than vertical launching, since in this case the dynamic efficiency will not be

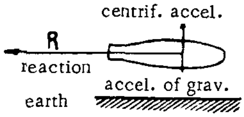

FIGURE 107.

$$W_{dyn}=1-\frac{g}{b}, \qquad (33)$$

but rather

$$(W_{dyn})=\left[1-\left(\frac{g}{b}\right)^2\right].$$

For instance, for $b=5g$, we have $(W_{dyn})_w=0.96$ (96%). **

An analysis of the conditions of an oblique launching gives a dynamic efficiency which is even more favorable. †

AIR RESISTANCE ††

The effect of the air resistance is not clearly defined in our problem. However, as will be shown in the following, it does not represent an Achilles' heel, since all the studies which have been made so far show that during the flight of a spaceship the air resistance does not play as great a role as might be thought at first glance. During ascent and descent it can be used to provide a lifting force. ‡ Experts on external ballistics are coming around to this same opinion. ‡‡

* For $h=0$, this velocity will be $v_\infty/\sqrt{2g}$.

** Actually, if $R=\sqrt{b^2 g^2}$ is the horizontal acceleration of the ship, the energy in t sec will be $\frac{b^2 g^2}{2g}\cdot t^2$ corresponding to a force of $\frac{b^2 t^2}{2g}$. Dividing one of these by the other, we obtain (33).

† Here

$$W_{dyn}=\frac{R_1}{b}\left(\frac{R_1}{b}-\frac{g}{b}\cos\alpha\right).$$

where α is the angle between the resultant forces and the vertical, and R_1 is the acceleration of the ship along its oblique path.

†† Cf. the paper by Prof. Ludwig Hopf: "Über Modellregeln und Dimensionsbetrachtungen" in "Naturwissenschaften," 8 Jahrg., Heft vom Januar 1920, SS. 81–85.

‡ Differentiation between the concepts of "air resistance" and "lifting force" is based on the fact that the latter, as a more serious analysis shows, constitutes a force which is, sui generis, independent of the resistance of the medium. Cf. the studies of: Bjerknes (father and son), Kutta, Zhukovskii, and Prandtl.

‡‡ Cf. the studies of Becker, Cranz, Eberhardt, Krupp, Mach, Roschdestwensky, Rothe, and Siacci.

308 For velocities greater than the speed of sound, Tsiolkovskii uses the ordinary velocity-squared law to determine the resistance:

$$W = f(v)^2 \qquad (34)$$

and obtains a formula for the work done against air resistance.* This formula indicates that, during the ascent of a 10-ton rocket (H + O), only about 1/4,000 of the entire work of ascent is performed against the resistance. For an oblique ascent, of course, it will be greater. However, if the ascent path is inclined 10° to the horizontal, this work will still only be about 1% of the entire work of ascent (an oblique ascent is, in general, more satisfactory).

Oberth also assumes a square law on the basis of ballistic data, changing only the resistance coefficient C_w. For $v \leqslant 300$ m/sec, c_w is constant.** However, as v approaches the speed of sound, this quantity increases rapidly, and at $v = 425$ m/sec it reaches a maximum (about 2.6 times the value for $v <$ the velocity of sound), after which it approaches asymptotically a value equal to 1.3 to 1.5 times that for a velocity < the velocity of sound. The increase in c_w for $v = 300$ to 400 m/sec is quite easy to explain: the compression of the air ahead of the nose decreases for $v < c$ (the velocity of sound), because of air runoff to the side. When $v > c$, only flows to the side are possible. As a result of the compression of the air, the pressure will be proportional to the square of the velocity, both for $v < c$ and for $v > c$. Behind the moving body rarefaction occurs, and for $v < c$ this produces an inflow which is also proportional to the velocity squared. When $v = c$ a situation begins for which, at the limit, a perfect vacuum is approached; it is not possible to compress air and it is not possible to increase more rapidly than c.

Therefore, at high velocities the inflow at the rear of the rocket decreases, and the quantity

$$c_w = (\text{pressure} + \text{inflow})/(F \cdot q) \qquad (35)$$

goes to the limit

$$c_w = \text{pressure}/(F \cdot q). \qquad (36)$$

* For the work against air resistance, Tsiolkovskii gives the equation

$$A_w = \frac{F(b - \sin a \cdot g)\gamma \cdot h^2 \; c_w}{g \cdot \sin^2 a}.$$

where F is the midship-section area, a is the inclination of the trajectory to the horizontal, γ is the specific weight of air at sea level, h is the height of ascent, and c_w is the resistance coefficient. For a vertical ascent

$$a = 90^\circ; \quad \sin a = 1;$$

and

$$A_w = F \cdot (b - g) \cdot \gamma \cdot h^2 \cdot c_w \cdot g.$$

** c is the velocity of sound.

Here F is the area of the midship section, and q is the static pressure,
309 equal to $\varrho / v^2/2$, where ϱ is the air density. For a spaceship there is no inflow, since the space behind the ship is full of the ejected gases. Tsiolkovskii* assumes that the air resistance at high velocities ($v > c$) is expressed better as a power series and may be limited by the term $a_3 v^3$.

The lift forces at high velocities have been investigated even less. The following quotation from Prandtl gives some indication of this. ** ... "My calculations are based on the conditions of flow around flat profiles with low lift. It turns out that, for such a profile and for flow in a compressible fluid, the pressure distribution will be the same as in an incompressible fluid for some other profile, whose transverse size exceeds the size of the former profile in the ratio

$$\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\text{"}$$

It follows from this that, close to the velocity of sound, separation of the flow occurs much more readily than at low velocities (see Figure 108).

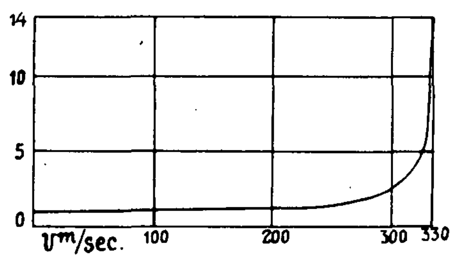

FIGURE 108.

Thus for $v < c$ we may assume during our calculations that the flow is two-dimensional. †

It is very difficult to carry out experiments with models at these velocities. The most suitable type of experiment consists in catapulting a body into a pressurized channel filled with water, glycerine, or some other liquid (this method was suggested by the author, by Tsiolkovskii, and by Oberth). At the Göttingen laboratory such a setup has been constructed for $v < c$; $v = c$ and $v > c$.

* Letter of 11 May 1927.

** Letter of 15 December 1926.

† Cf. Albert Betz. Einführing in die Theorie der Flugzeug Tragflüge. Die Naturwissenschaften, 6 Jahrg. NoNo.38 u. 39, SS.557–552 [sic] and 573–578.

The materials used to construct a spaceship must be suitable for the conditions to be met with during flight; in particular, they must be able to
310 withstand extended periods at temperatures ranging from absolute zero to 2,500 or 3,000°C and pressures from 30 to 50 atm. The operating conditions in the combustion chamber and in the nozzle will be especially severe. Let us consider separately passenger ships and ships without passengers. Goddard proposed plans for the latter, including a plan for an explosion to light up a dark part of the moon, the explosion being observed through a telescope. Similar projects have been suggested by von Hoefft, Oberth, and Tsiolkovskii, who envisioned the construction of a recording rocket carrying automatic instruments. In addition, Oberth and Tsiolkovskii also proposed manned rockets.

Rockets can also be divided into solid-fuel (powder) rockets, as proposed by Goddard for a small ship (without passengers), and liquid-fuel rockets, as proposed by the other investigators. The latter rockets have either a single combustion chamber and a single nozzle (Tsiolkovskii and Tsander) or several of these (Oberth). Oberth suggests a stage rocket, sections of which will fall away as portions of the propellant are used up, the passenger cabin then descending by parachute. The problem of the winged ship has still not been solved, since the advisability of such a ship has not been proven by calculations and the author of this plan (Tsander in Moscow) has not published his studies. The designs of Oberth, Oberth and Valier, and Tsiolkovskii are in general similar, and the frames of their ships will experience bending stresses, as in the case of soft dirigibles (Parseval), due to the internal pressure.

311 Tsiolkovskii's rocket (Figure 109) has a tapered steel frame with double walls having a vacuum in the space between them (as in a Thermos). Large

(310)

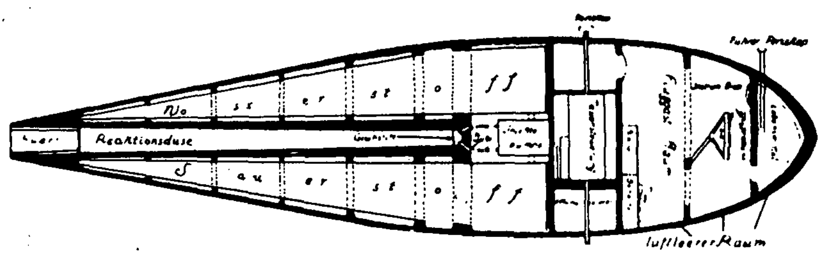

FIGURE 109.

fuel tanks are arranged around a single large central combustion chamber and a conical nozzle (angle of opening only 8 to 10°). Propellant at a temperature of absolute zero is injected into the combustion chamber by pumps (according to Tsiolkovskii, the pumps are very simple) and ignited by an electric spark.

The ship is guided either with the aid of controls located in the gas flow or by shifting masses so as to change the position of the center of gravity. The masses are moved using an electrical servomotor. The steering controls are regulated with the aid of a periscope, which receives directing rays from the sun or stars and transmits them to solenoids.

Oberth's double rocket is shown schematically in Figure 110. Propellant enters the combustion chamber via several injectors, after which it

(310)

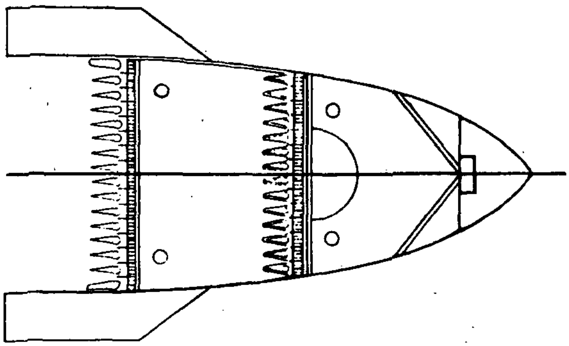

FIGURE 110.

passes through Laval-type nozzles into space. The lower rocket uses alcohol, water, and oxygen, and the upper rocket uses pure H and O. Oberth suggests using an aluminum alloy (sp. wt. of 3, tensile strength of 30 to 32 kg/mm^2) for the frame of the alcohol rocket, copper and lead for the oxygen tanks, and lead, copper, and soft iron for the H — O rocket. The rocket is guided with the aid of fins and regulated combusion. Non-manned,
312 rockets are controlled automatically.

The Oberth-Valier design shown in Figure 111 is another version of the Oberth rocket. The chambers are located amidships, around a sternpost

(311)

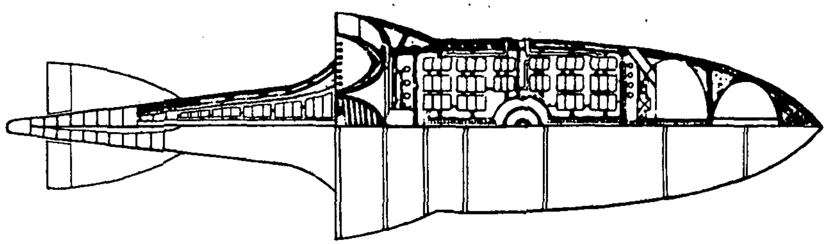

FIGURE 111.

with controls, and they take up about 80% of the midship-section area. From the nose to the stern of the rocket, we see: a detachable nose with a parachute, and inside the nose two lenticular passenger compartments with a central passageway; large fuel tanks, eight combustion chambers, a control rod, more fuel tanks, and, finally, the tail section. The combustion chamber contains a system of tubes (Figure 112) for supplying propellant, honeycomb nozzles, and cooling tubes, which protect the chamber casing.

(311)

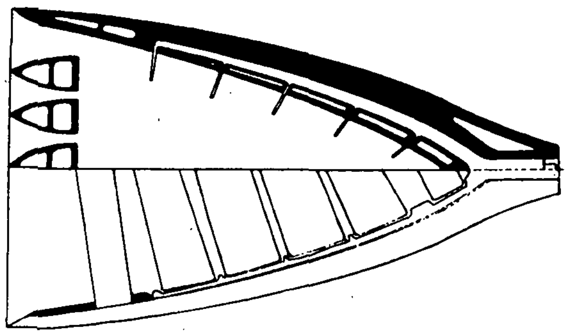

FIGURE 112.

CONCLUDING REMARKS

At present the only means of flying into outer space is with the aid of a reaction-propelled device. The physical and psychological aspects of the problem are such that a practical implementation must now be considered. The development of this subject still has a very short history. In principle, a reaction-propelled ship is feasible and its dynamic theory is known, but problems of air resistance, materials, and construction have not yet been solved completely. The main types of rockets (those of Oberth, Goddard, and Tsiolkovskii) have been worked out in considerable detail, and their practical implementation depends only on funds. It is imperative that reaction-propelled ships be built, in order to allow man to solve a number of scientific problems.

APPENDIX

The following information is relevant to the foregoing paper:

1. Dr. Franz von Hoefft (Vienna) is at present constructing the first rocket for research using automatic equipment. Its length is 1.2 m, with a diameter of 0.3 m, giving a ratio $\frac{L}{D}=4$; in addition, $M=30$ kg, $m_{aa}=22$ kg, $m_r=8$ kg, $q=2.75$, the ascent height $h=100$ km, and the propellant is H+O. A parachute will be used in the descent.

2. Tsiolkovskii is carrying out some preliminary experiments with models. The results will probably be published in 1928.

3. Dr. Ing. Rossmann, Prof. Cranz's assistant, gave a lecture on reaction-propelled ships, at the Charlottenburg Polytechnic Institute (Berlin), in which he cited the studies of Goddard, Oberth, and Tsiolkovskii. However, his theory of air resistance gives rise to certain objections. Prof. H. Reissner and G. Hamel are working on the integration of the equations of motion of a ship, taking air resistance into account.

4. Professor Oberth has written me (29 December 1926) that "The idea of testing models with the aid of a catapult seems to me to be very good.
313 Your results for small experimental models moving through dense air, although not completely applicable to large machines moving in a tenuous atmosphere, are better than nothing." In addition, Oberth maintains that the theory and construction of a reaction-propelled ship (rocket) are simpler than for a reaction-propelled aircraft.

5. The problem of the "reaction-propelled aircraft" is being pursued further. Its theory is being developed by me for Valier.

6. The plan for reaction motors to be used on a Junkers monoplane (J 24) and a new light plane (a combination of 20-power Klemm-Daimler airplanes) is unfeasible. Each velocity corresponds to a certain optimum plan view and profile wing configuration, and for velocities greater than the speed of sound the relevant studies have not yet been made.

7. Considerable research carried out by R. Goddard and R. Lademann (Berlin) still remains to be published.

315 *JULIUS KUNTZ*

In the journal "Die Rakete" for 15 Jan. 1928 Engineer Julius Kuntz presents some typical solutions for some of the simplest problems related to rocket flight. As a starting point for his calculations, he assumes that the rocket reaches a height of 1,600 km above sea level and attains a space velocity [escape velocity] of 10,000 m/sec, sufficient to fly beyond the neutral layer between the earth and the moon.

PROBLEMS IN THE THEORY OF ROCKET FLIGHT TO THE MOON

Problem 1. What must be the acceleration γ at a height $S = 1{,}600$ km, in order to impart to a mass m a velocity $v = 10{,}000$ m/sec ?
Solution:

$$\gamma = \frac{V^2}{2S} = \frac{10^8}{2 \cdot 1.6 \cdot 10^6} = 31.25 \text{ m/sec}^2.$$

Problem 2. How long will it take to develop this acceleration ?
Solution:

$$t = \frac{v}{\gamma} = \frac{10\,000}{31.25} = 320 \text{ sec}.$$

Problem 3. What force P is required in order to impart to a mass m an acceleration of 31.25 m/sec² ?
Solution:

$$P = m \cdot \gamma = 31.25\, m.$$

If

$$m = \frac{1000}{9.81},$$

then

$$P = \frac{1000}{9.81} \cdot 31 \cdot 25 = 3185.5 \text{ kg}$$

Problem 4. What force is necessary in order to overcome gravity if a rocket weighing 1,000 kg is to ascend from sea level to a height of 1,600 km ?

Solution. The weight of the rocket at sea level is 1,000 kg. The weight of the rocket at a height of 1,600 kg is 0.64% of 1,000 kg, or 640 kg. Assuming on the average, and with a little to spare, that this weight is 1,000 kg, and taking into account the result of Problem 3, we obtain a total force of

$$3185.5 + 1000 = 4185.4 \text{ kg}$$

Note. Kuntz neglects air resistance, assuming it to be small

Problem 5. Where is the neutral zone of the attractions of the earth and moon situated?

Solution. Let us denote the distances of this point from the centers of the earth and moon, respectively, as R and r. We also assume that the average distance between the earth and the moon is $R+r=384{,}000$ km, and that the mass M_1 of the moon is $^1/_{81}$ of the mass M of the earth. Then,

$$\frac{kM}{R^2}=\frac{kM_1}{r^2}=\frac{kM}{81r^2}$$

$$\frac{R^2}{r^2}=\frac{M}{M_1}=81;\quad R=9r;\quad r=38{,}400 \text{ km};$$

thus $R=345{,}600$ km (here k is the gravitational constant).

Problem 6. What will be the velocity of a mass attracted by the earth and the moon: a) at the neutral point N, and b) when it hits the moon, if it has a velocity of 10,000 m/sec at 1,600 km above sea level (Figure 113)?

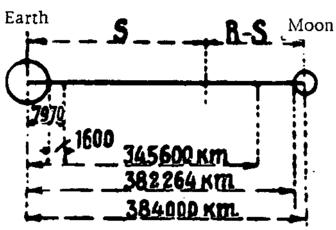

FIGURE 113.

Solution. Let us denote the distance from the earth to the moon as $R = 384 \cdot 10^8$ cm, the mass of the earth as $M = 6{,}064 \cdot 10^{24}$ g, the mass of the moon as $M_1 = \frac{M}{81}$, the gravitational constant as $k = 66 \cdot 10^{-9}$, and the distance of a moving point C from the earth's center at any moment as S. Here the acceleration of terrestrial gravity is taken to be negative and that of lunar gravity to be positive. The total acceleration will be

$$\frac{d^2S}{dt^2}=\frac{-kM}{S^2}+\frac{kM}{81(R-S)^2}.$$

In order to solve this differential equation, we assume that

$$\frac{dS}{dt}=P;\ \frac{d^2S}{dt^2}=\frac{dP}{dt}=\frac{P\cdot dP}{dS}=-kM\left[\frac{1}{S^2}-\frac{1}{81(R-S)^2}\right].$$

Integration gives

$$\frac{P^2}{2}=-KM\left[-\frac{1}{S}-\frac{1}{81(R-S)}\right]+C$$

and, finally, a velocity

$$P=\frac{dS}{dt}=\sqrt{2kM\frac{81R-80S}{81S(R-S)}+C}.$$

317 **Determination of constant C.** Constant C is found from the condition that for $S = 7{,}970 \cdot 10^6$ cm

$$\frac{dS}{dt}=10{,}000 \text{ m/sec}=10^6 \text{ cm/sec}$$

Thus

$$10^{12}=2KM\frac{81\cdot 384\cdot 10^3-80\cdot 7970\cdot 10^5}{81\cdot 7970\cdot 10^5\cdot(384\cdot 10^8-7970\cdot 10^5)}+C$$

so that

$$C=-0.406\cdot 10^{10}.$$

The velocity is therefore

$$\frac{dS}{dt}=\sqrt{2kM\frac{81\cdot R-80S}{81S(R-S)}-0.406\cdot 10^{10}}$$

a) **Velocity at neutral point.** In this case

$$S=345.6\cdot 10^8 \text{ cm} \quad R-S=38.4\cdot 10^8 \text{ cm}$$

$$\frac{dS}{dt}=\sqrt{2kM\frac{81\cdot 384\cdot 10^8-80\cdot 345.6\cdot 10^8}{81\cdot 345.6\cdot 10^8\cdot 38.4\cdot 10^8}-0.406\cdot 10^{10}}=$$
$$=1.473\cdot 10^5 \text{ cm/sec} =1473 \text{ m/sec}.$$

b) **Velocity at impact on moon:**

$$S=382.264\cdot 10^8 \text{(see figure)} R-S=1.736\cdot 10^8 \text{ cm}$$

$$\frac{dS}{dt}=\sqrt{2kM\frac{81\cdot 384\cdot 10^8-80\cdot 382.264\cdot 10^8}{81\cdot 1.736\cdot 10^8\cdot 382.264\cdot 10^8}-0.406\cdot 10^{10}}=$$
$$=2.713\cdot 10^5 \text{ cm/sec}=2713 \text{ m/sec}.$$

assuming that the moon has no atmosphere.

Problem 7. Determination of flight time for mass m: a) over distance from point 1,600 km above sea level to neutral point, and b) from there to lunar surface.

Solution. Starting from the formula for the velocity, we obtain

$$\frac{dt}{dS}=\frac{1}{\sqrt{2kM\frac{81R-80S}{81S(R-S)}+C}},$$

which gives a time

$$t = \int \frac{dS}{\sqrt{2KM\frac{81R - 80S}{81S(R-S)} + C}}.$$

318 The solution of this integral is complex. It is simpler to obtain an approximate solution (accurate to $^1/_2\%$) with the aid of a velocity curve (Figure 114). For each interval along the ordinate axis we assume that the velocity is constant and equal to the average over the interval. We then divide the corresponding distance by this velocity and obtain the time required to traverse the interval. The sum of these times gives the total flight time:

	hr	min	sec
a) Flight time from point 1,600 km above sea level to neutral point, 155,830 sec ...	43	17	10
b) Flight time from neutral point to lunar surface, 22,546 sec	6	15	46
c) Flight time from earth's surface to point 1,600 km above sea level (see Prob. 2), 320 sec	–	5	20
Total............. 178,696 sec	49	38	16

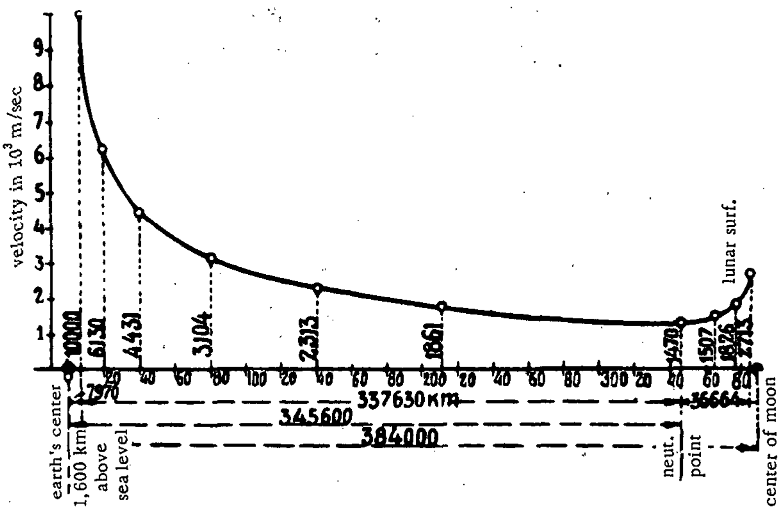

FIGURE 114.

If the body has a velocity of 1,000 m/sec at the neutral point, then its velocity as it approaches the moon will be 2,493 m/sec, and its maximum velocity will be 9,944 m/sec. If its velocity is zero at the neutral point, the velocity at the moon will be 2,284 m/sec, for a maximum velocity of 9,892 m/sec.

All the foregoing problems have been solved assuming that the mass begins its motion over a terrestrial pole. For launching from some other point the earth's velocity of rotation must be taken into account; then calculations show that the maximum velocity will vary slightly.

319 *GUIDO VON PIRQUET*

Guido von Pirquet was born at his family castle, Schloss Hirschstetten, in 1890. He began his studies at a Realschule and then continued at the Technische Hochschule in Vienna (Engineering department) and in Graz. Von Pirquet has made independent studies of astronomy and other scientific subjects, and he has served on the testing committee for inventions and as secretary of the Society for High-Altitude Exploration [Gesellschaft für Höhenforschung] in Vienna. He has written a number of works on interplanetary travel.

FIGURE 115. G. von Pirquet.

321 *K. DEBUS*

Karl Debus was born at Lustadt (Rheinpfalz) on 10 Sept. 1891. He attended gymnasium at Bad Dürkheim, Speyer, and Ludwigshafen (Rhein), and then continued his education in Munich and Würzburg. From 1915 to 1918 he participated in the World War, and during recent years he has written for newspapers and journals, particularly on the subject of the earth as a body in space.

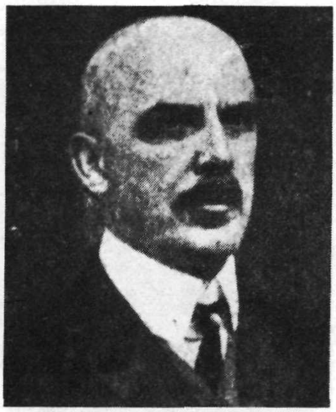

FIGURE 116. K. Debus.

323 *W. LEY*

Willy Ley was born in Berlin on 2 Oct. 1906. He attended a Realschule there but did not take the final examinations, because of illness. Ley worked in a bank until 1926 and then studied literature. Next he made studies of biology and astronomy. In 1926 he published a work entitled "Space Flight" ["Die Fahrt ins Weltall"], and he has also written a number of papers on paleontology, astronomy, and rocket flight. *

FIGURE 117. W. Ley.

325 *MAURICE ROY*

*PROPULSION BY REACTION**

For any propulsion system originating in a fluid, it can be said that a reaction is produced. The reaction causing the motion (the recoil) can be obtained either by means of a mechanical engine operating in this fluid medium or else by ejecting backward a certain momentum from the moving system.

An example of the first technique is the classical motor-propeller, and an example of the second is the common rocket. Both of these devices are thus, in a certain sense, reaction engines. However, according to generally accepted terminology, the name "reaction engine" is applied predominantly to exhaust engines (or to engines ejecting a stream of fluid into the surrounding medium). The most well-known examples of such devices are the rockets used in fireworks displays and the hydraulic tourniquet.

In this report I intend to discuss only engines of this type. It should be noted that a report has already been presented here on this same subject, and that it was followed by a very interesting discussion. I would like now to consider one extremely important point which was brought up during this discussion.

Although the rocket was invented very long ago, still it has always greatly excited the imagination of inventors. Rockets have been considered not only with respect to interplanetary travel, for which they represent the only possible means of locomotion, but also for flights in air, and recent experiments have indicated the feasibility of this. According to designs proposed by inventors, a rocket engine can be constructed either as an ordinary explosive rocket or it can operate on a liquid fuel, causing expulsion of the exhaust
326 gases from the rocket.

In the latter case the air required for combustion is taken from the surrounding atmosphere by the rocket. More or less considerable extra supplies of air may sometimes be added to this amount.

However, if the reaction experienced by the rocket is not applied directly to set some system in motion, but instead the rocket is mounted on the end of a rotating rod in such a way that a peripheral reaction is obtained, then an actual gas tourniquet is produced. The latter is a device which can be used to operate any mechanical engine. This principle, a favorite principle of many gas-turbine inventors, is also encountered in the device known as the

* Maurice Roy, a mining engineer, is a professor at the National School of Transportation (Paris). This report was presented to the French Aeronautical Society at its meeting on 29 Jan.1930. The translation was taken from "La Technique Aéronautique" for 15 Jan.1930.

reaction propeller, which has been developed by some inventors. In this device the propeller is actuated by the exhausts of several rockets mounted at the ends of the blades and oriented in appropriate directions.

There is a great similarity between all these engine systems, regardless of how different they might seem at first glance. The classical motor-propeller system can also be included in this category. Actually, all these engines are based on the combustion of an explosive substance or explosive mixture. The result of this combustion may be the ejection of a stream of gas through a fixed or adjustable aperture, the direct reaction to which may provide a useful propulsive effect, or, if desired, it may be a mechanical effect on a shaft, so as to turn a propeller and provide propulsion.

These systems can be compared, once we have established some basis for comparison. Let us consider here just the efficiencies of the engines. However, we must start by giving a precise definition of this efficiency and formulating an expression for it.

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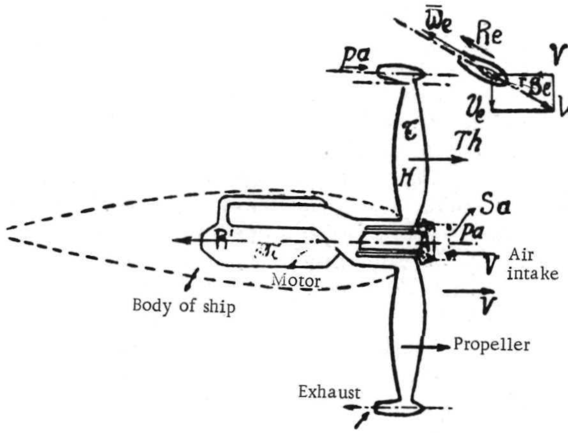

FIGURE 118.

A general diagram of the systems considered by us is presented in Figure 118. Air from the surrounding atmosphere enters the apparatus through fixed axial aperture ***A***, which points forward. Supplies of fuel are carried along on board. As they progress through the apparatus, the air and fuel undergo certain physical and chemical transformations, the main ones being compression, combustion, and explosion. These thermodynamic transformations occur partly in the heat engine (motor) ***M*** and partly in the rotary device or turbine ***T***.

Turbine ***T*** is connected to motor ***M*** and can, as needed, be actuated by the latter, or, conversely, cause it to operate (in the latter case motor ***M*** actually functions as a receiver). Turbine ***T*** ejects gas into the atmosphere through apertures directed backward, perpendicular to the absolute trajectory, which is helical. Turbine ***T*** actuates propeller ***H***; it may even be combined with the latter, as in the device shown in Figure 118.

327 Obviously, this general diagram covers various particular applications, such as all the systems considered above and the classical motor-propeller system. In order to obtain the latter, it is necessary just to reduce the role of turbine ***T*** to that of a simple transmission mechanism, which transfers the motion from motor ***M*** to the propeller. Then this part of the apparatus will be stopped automatically by the motor gear box and, since the ejection of gases is from a fixed aperture, the aircraft motor will once again assume its normal configuration.

In order to obtain a rocket engine, we need only make turbine ***T*** stationary and completely eliminate propeller ***H***. Then the work done by motor ***M*** will not be transmitted outward, and the ejection of gases will take place, as follows logically, after the gases have left the motor, in the rear part of the assembly, as is the case in an ordinary rocket.

In order to obtain an explosive [solid-fuel] rocket which does not take in air from the surroundings, it is sufficient just to eliminate aperture ***A***. A pure reaction propeller can be obtained merely by eliminating motor ***M***. Then the compression will occur in the hollow (reamed out) blades of the propeller, and the combustion will take place in a combustion chamber located at the top of the blade and the gas-intake tube of the rocket, the direct reaction of which actuates the propeller.

In addition, it is evident that the foregoing plan makes it possible to construct many other types of engines, representing a whole succession of combinations of the given elements, a succession whose extreme cases have just been considered by us as particular examples.

How shall we define the overall efficiency of each of these systems of traction engines? First of all, let us define, somewhat arbitrarily but as logically as possible, the useful effect of the traction forces, and secondly let us determine the fuel consumption for which this useful action is accomplished.

If the motion of an assembly being towed [through the air] is carried out in an ideal manner, considering the simplified case in which no traction engine is necessary, the aerodynamic resistance [drag] will be equal to some value R and the power required to propel the object at a velocity V will be equal to RV.

When a traction engine operates, the pulling force produced by it at a velocity V counteracts the actual drag of the towed assembly, while the presence of the traction engine and its work affects the latter. If this actual pulling force is R', then we have

$$R = R'(1 + \varepsilon).$$

Coefficient ε, which is usually a small, positive quantity, indicates, for the regime being considered by us, the overall effect of the traction engine on the resistance which is to be overcome.

328 The power of the motion produced will be $R'V$, but let us take the quantity RV to be a measure of the useful power, independently of which system we choose for the traction engine. In order to produce a useful power $R'V$, the traction engine must consume m kg/sec of fuel, the calorific power of which is L.*

* Here the units are assumed to be so chosen that they are consistent with one another and, in particular, that quantities of heat or work are expressed in the same units. The kilogram is used as the unit of mass.

The net efficiency of the engine will be

$$\frac{R' V}{mL}.$$

This ratio can be separated into two parts, in order to show more clearly the role of the thermodynamic transformation undergone by the air and the fuel.

This transformation is generally characterized by the so-called thermal efficiency of the process. The latter quantity can be defined as the ratio between two quantities: 1) the effective (useful) work which would result from the same transformation if it were to take place for the same heat exchange with the surroundings and for the same passive resistance in a conventional stationary engine, and 2) the calorific power.

In view of this, let us define the net efficiency of the traction engine as the product of its thermal efficiency and the quantity which we have called the efficiency of the engine. Thus we have

$$\frac{R' V}{mL} = \eta_{th} \times \eta_p,$$

or, according to our definition,

$$\eta_p = \frac{R' V}{m \cdot \eta_{th} \cdot L}.$$

Next let us consider the overall efficiency. This will be equal to the ratio between the useful action, measured conventionally in terms of the quantity $RV = R' V(1-\varepsilon)$, and the fuel consumption required for this useful action.

The most common measure of the consumption is the calorific power (mL) of the weight of consumed fuel. I will retain this arbitrary criterion, but at the same time I will indicate below that it leads us to a conclusion
329 which, at first glance, seems to be paradoxical.

We can now write

$$\eta_g = \frac{RV}{mL} = (1-\varepsilon)\frac{R' V}{mL} = (1-\varepsilon) \times \eta_{th} \times \eta_p.$$

This formula, which is a consequence of the definitions made by us, has the theoretical advantage that it sets off clearly the following three factors, which are each of completely different character:

1. The effect (ε) of the traction engine on the resistance being overcome.
2. The nature (η_{th}) of the thermodynamic transformation undergone by the active substances (air and fuel) as they pass through the apparatus.
3. The efficiency η_p of the engine.

Now let us consider how we can calculate the overall efficiency of one of the assemblies described by the general plan in Figure 118.

The pulling force R' can be found by applying the theorems for the momentum components in the direction of transfer V to the assembly and to the substances (air and fuel) in it, during the course of some period of its motion (operation), which is assumed to be periodic. Here the following quantities must be taken into account: the resistance of a stationary ship body [airframe], the pressure or thrust of the propeller blades, the resistance of the rocket casings, and, finally, the impulse of the pressures exerted upon the intake and exhaust apertures, together with the momentum lost ahead of the motor and recovered behind it.

With the aid of certain considerations which are quite complex but easy to follow, it is not difficult to show that, in general, the drag on the rocket casings (hollow objects with openings in them) can be neglected, and that the
330 resistance of the rocket bodies themselves, which to some degree include the casings and the fuselage of its shaft (hub), can be included in the pull of the propeller.

On the other hand, application of the law of angular momenta about the propeller axis for the system described above gives us a second ratio, and from it we can find the power consumed by the resistance of the aerodynamic force couple of the propeller resistance. This power is related to the usable power of the pull of the above-mentioned propeller by the efficiency η_{th} of this propeller, determined in the usual way, which has now become a universal characteristic of air screws.

The equations formulated in this way also introduce the mechanical power transferred by motor M to turbine T, which in theory is identical to propeller H. It is convenient to consider this power to be a certain fraction h of the effective work of the thermodynamic transformation of the active substances consumed. Thus the quantities h, m, η_{th}, and L will appear in the formula.

In order to determine the relative velocity of gas ejection, which is a very important unknown, we must find another (third) ratio from the law of conservation of energy. This is done by applying this law under the same conditions as were assumed when applying the theorems mentioned by us above.

It will not be necessary to consider these calculations in detail, since they do not present any difficulty. Certain assumptions have to be made, but these are not of any particular significance, so that it will be sufficient just to give the following formulas obtained as a result of the calculations:

$$\eta_g = (1-\varepsilon)\,\eta_{th}\,\eta_p$$

$$\eta_p = h\eta_h + (1-h)\,\eta_f$$

$$(1-h)\,\eta_f = \frac{1}{q}\left\{(1+\eta_h \operatorname{tg}^2\beta_e)\left(\sqrt{a\,[1+2\,(1-h)\,q\cos^2\beta_e]}-a\right)+(q-1)\right\}.$$

In these expressions, η_{th}, h, and η_h have the meanings indicated by us above.

Parameter α represents the ratio $\frac{a+1}{a}$, where α is the weight of the air picked up by the apparatus in the time required to consume 1 kg of fuel.

Angle β_e is the angle between the final velocity (resultant velocity) of the rocket and its velocity of transfer. When the rocket is mounted at the end of a blade, tg βe_e is a functional parameter $\frac{U_e}{V}$ of the propeller.

Finally, parameter q is defined as $q = \frac{\eta_{th} L}{a V^2}$. This parameter is of very great importance, as will be shown below.

The equation given above can be simplified considerably if parameter α is quite large in comparison with unity. This will be the case for all
331 motors operating on liquid fuel, provided there is a small air surplus. Then we can set α equal to unity, giving the following relation:

$$(1-h)\eta_f = \{(1+\eta_h \operatorname{tg}^2\beta_e)(\sqrt{1+2(1-h)q\cos^2\beta_e}-1)\}.^*$$

Let us use this equation to make a direct comparison of different assemblies characterized by the same value of q and having propellers with equal efficiencies. Such assemblies will differ from one another only by the amount h of thermodynamic work performed by the engine and transmitted either to the propeller or by means of the action $\frac{U}{V}$ of the propeller.

For $h=1$ we have a conventional motor-propeller assembly. If $h=0$ we have a pure reaction-type rocket, comprising an engine. Figures 119 and 120 show some curves for the variations of h, η_{th}, $(1-h)\eta_f$, and finally η_p, as h varies from 0 to 1 for different values of $\frac{U}{V}$, with $q=50$, the latter value being taken as a starting point for the analysis.

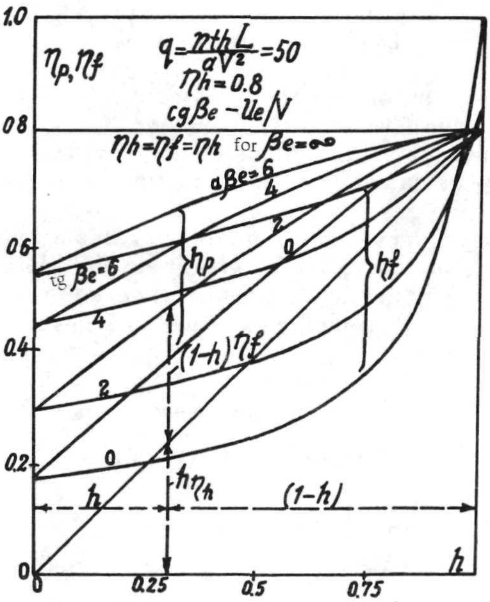

FIGURE 119.

* [This result is not consistent with the previous equation, apparently due to a proofreading mistake (Trans.).]

332 This value is obtained when

$$a = 20$$
$$L = 11{,}000 \text{ km/kg}$$
$$\eta_{th} = 0.30$$
$$V = 117 \text{ m/sec} = 420 \text{ km/hr}.$$

These conditions correspond to an aircraft motor of very high quality and a high-speed airplane.

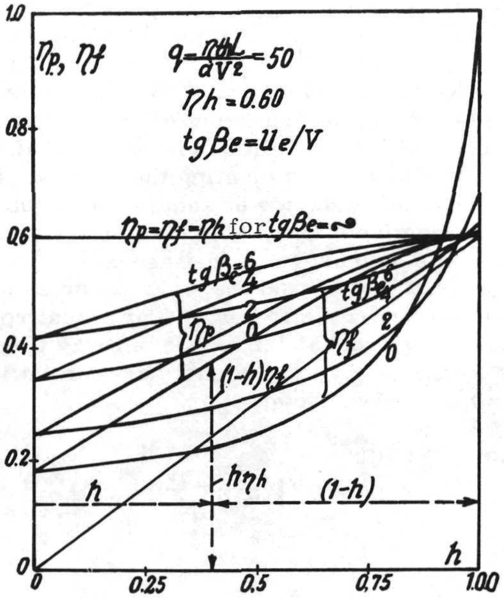

FIGURE 120.

A study of the curves for η_p is particularly instructive. The following becomes clear from these curves: if, as in the case considered by us, q and η_{th} are assumed to be constant, then it will be best if h is as far from zero, and as close to unity, as possible.

A number of other examples could be quoted here to demonstrate that, for all the values of q which are of interest in aviation and which are attainable at present, the result just derived by us remains valid.

333 This leads us to conclude that the pure reaction-type propeller is completely unsuitable, in comparison with the classical motor-propeller assembly, for the conditions assumed by us, namely for

$$\frac{\eta_{th} L}{a V^2} = q$$

to be constant, and for equal thermal efficiencies.

In order to avoid such an unfavorable result, it must be possible to operate with a reaction propeller, with very low values of q, or with more suitable values of the thermal efficiency η_{th} (the quantity η_g will increase with η_{th}, in spite of the fact that q, which also increases, thereby reduces η_p).

Moreover, it is quite evident that a pure reaction engine will not satisfy similar conditions. Actually, such a propeller may provide a compression which is much lower than that of a conventional aircraft motor. On the other hand, such a propeller cannot operate with a highly diluted fuel mixture, since the compression channels inside the propeller blades are very narrow.

I should mention in passing that, in all probability, it would be possible to obtain an overall efficiency which is a little higher, in comparison with the classical motor-propeller assembly, if a propeller with a partial reaction were used. The same result could certainly be achieved if it were possible to raise the thermal efficiency of the entire assembly as a whole, without at the same time reducing the efficiency of the engine itself too much. To do this, it would be necessary to expel the exhaust gases of the motor through tubes located at the top of the propeller blades, or, in other words, to use the propeller itself as a device for ejecting the exhaust gases. The possibility of constructing such a device at some time should by no means be ruled out, and this alternative might well serve as a starting point for some very interesting studies (at least from the theoretical point of view).

Now let us consider another case, one which is much more tempting to inventors, namely the direct-reaction engine, or actual rocket. Before considering the liquid-fuel rocket, let us take a look at the explosive rocket. For the latter, the formulas given previously are simplified considerably, since $h=0$ and $a=0$. Thus we have

$$\eta_f = V\sqrt{\frac{2}{\eta_{th} L}} = \frac{2V}{w}$$

334 and

$$\eta_g = (1-\varepsilon)\,\eta_{th}\,\eta_f = (1-\varepsilon)\,V\sqrt{\frac{2\eta_{th}}{L}}.$$

These formulas can be derived directly, on the basis of the following considerations. The usable part $m\eta_{th}\cdot L$ of the fuel energy is converted into the relative kinetic energy $m\frac{w^2}{2}$. The exhaust reaction has a value mw, while the power generated by this reaction, (that is, the useful power) will be, if we neglect the internal work of the engine ($\varepsilon=0$),

$$mwV = \frac{2V}{w}\cdot m\frac{w^2}{2} = \frac{2V}{w}\cdot m\eta_{th} L.$$

This gives an overall efficiency

$$\eta_g = \frac{2V}{w}\eta_{th} = \eta_{th} \times \eta_f.$$

Thus we find that $\eta_f = \frac{2V}{w}$, and, since $w^2 = 2\eta_{th} L$, therefore

$$\eta_f = V\sqrt{\frac{2}{\eta_{th} L}}$$

and

$$\eta_g = V\sqrt{\frac{2\eta_{th}}{L}}.$$

The efficiencies given by these formulas increase to infinity along with V. However, if the concept of the efficiency is to have meaning, it cannot exceed unity. This paradox, though, is only an apparent one, and it is easily explained. We arrived at it, as I pointed out at the beginning of this paper, only because we took the calorific power mL to be a measure of the power consumption during the motion, a fact which sometimes is not accorded enough attention. The quantity mL, of course, only represents a portion of this consumption. Actually, the absolute energy theoretically available in a unit mass of fuel (taking this unit mass to be 1 kg) will be $L + \frac{V^2}{2}$ (the calorific power plus the absolute kinetic energy), rather than just L. If we keep this in mind while calculating the denominator of the overall efficiency, then the expression obtained for the latter will be somewhat different, namely (assuming once again, for simplicity, that $s = 0$)

$$\eta_g = \frac{mwV}{mL + m\frac{V^2}{2}}.$$

335 For $mw = 2m\eta_{th} L$ we obtain, eliminating w,

$$\eta_g = V\frac{\sqrt{2\eta_{th} L}}{L + \frac{V^2}{L}}.^*$$

In this form, η_g no longer goes to infinity with V.

If we assume that η_{th} is constant, then η_g will reach a maximum value of $\sqrt{\eta_{th}}$ for $V = \sqrt{2\eta_{th} L}$, that is, a value which will always be less than unity.

But how can the values of the overall efficiency η_g be calculated? The formulas given above make it possible to find these easily, simply by calculating η_{th} and L.

The thermal efficiency of an explosive [solid-fuel] rocket is a function of the pressure caused by the combustion and also of the quality of the construction of the explosion tubes. Calculations indicate that, even for the most favorable conditions with respect to the resistance (behavior) of the walls of the combustion chamber and the tubes, this efficiency can hardly be more than 45 to 50%.

The value of L will be much lower using explosives, in comparison with the values for any of the liquid fuels available at present, since 1 kg of explosive material contains, in addition to the fuel substance itself (atoms of CH. . .), an oxidizer (O_2). Thus

for oil, L = 10,000 to 11,000 km/kg;

for black powder, L = 650 km/kg;

for a colloidal powder or B powder, L =1,200 km/kg.

* [This result does not follow from the previous expression (Translator).]

The values of the overall efficiency of ***B*** powder under various conditions are given below.

	$\eta_{th}=0.40$	$\eta_{th}=0.60$
$V=40$ m/sec (144 km/hr)	$\eta_g=0.016$	$\eta_g=0.019$
$V=120$ m/sec (432 km/hr).	$\eta_g=0.048$	$\eta_g=0.058$
$V=200$ m/sec (720 km/hr)	$\eta_g=0.080$	$\eta_g=0.098$

However, in practice, for velocities of 700 km/hr, overall efficiencies of even 8% cannot be counted upon, whereas motors and propellers used at present frequently have overall efficiencies of 15 to 22%.

It is easy to calculate the initial velocity of transfer at which a rocket
336 becomes more efficient than an ordinary motor-propeller assembly. This velocity will range from 1,200 to 1,600 km/hr, depending on whether black powder or colloidal powder is used.

In addition to its low thrust efficiency, a solid-fuel rocket has another disadvantage which affects its velocity. This is the high propellant weight, which is a function of the low efficiency just referred to and also of the low calorific power. Because of these very serious shortcomings, the solid-fuel rocket cannot be of any interest to us as a device for pulling aircraft. It can be used for such purposes only at speeds of 1,000 to 1,500 km/hr or above.

At this point it should be mentioned that rocket-engine studies bring out a number of problems in interior ballistics which are extremely interesting from the technical point of view. These problems were worked out during the past War by certain French scientists. Foremost among the names of these scientists is that of a famous former president of our Society, Auguste Rateau.

Since the solid-fuel rocket cannot at present be used for flights in the air, let us go on to consider rockets operating on liquid fuel. Such rockets may be thought of as internal-combustion engines with greatly curtailed explosions, so as to ensure that the work of the gases in the motor exactly compensates the work performed in the preliminary compression of the carburated mixture or air required for combustion. The efflux of the gases at high pressure is regulated by appropriate tubes, which convert this efflux into a true exhaust, the direct reaction to which produces thrust.

The general equations given above by us are also applicable to this case. We need only set $h=0$ and $\mathrm{tg}\,\beta=0$, and then, provided there is a high enough surplus of air in the fuel mixture, α can be equated to unity. In this way we obtain the following very simple formulas:

$$\eta_g=(1-\varepsilon)\,\eta_{th}\,\eta_f$$

$$\eta_f=\frac{1}{q}\left[\sqrt{1+2q}-1\right].$$

Efficiency η_f continually increases, approaching unity as q decreases and goes to zero. This efficiency is a function just of parameter $q=\frac{\eta_{th}L}{aV^2}$.

Now let us return to the expression for η_g, into which the thermal efficiency η_{th} enters twice. It turns out that, in order to raise η_g, we must increase η_{th},
337 α, or V.

As always, a direct-reaction engine becomes of greater interest, the higher the thrust velocity obtained. The increase in the quantity α is of great significance, this quantity being the weight of air required by the engine for 1 kg of fuel. This subject, which is directly related to the improvement of rockets with the aid of ejectors (trompes), was dealt with during the scientific discussion which I mentioned above.

In addition, it is easy to show that an increase in the fuel flow rate (throughput) of an engine constitutes a definite advantage. In the calculation of this flow rate, or fuel consumption, the ratio between the mass of fuel and the mass of air will not be taken into account. Here our hypothesis that $a=1$ is perfectly valid, provided the dilution is sufficient.

Let us assume that the states of the liquids and their flow rates are the same at the intake and the exhaust of the engine. The thrust (if we do not take coefficient ε into account) will be equal to the increase in momentum relative to the mass (ma) of liquid, that is, as the latter passes through the engine, it will be $ma(w-V)$. The useful power will then be $[ma(w-V)V]$.

The power consumed will be mL. The variation of the relative kinetic energy $ma\frac{w^2-V^2}{2}$ of the liquid leaving the engine depends on the usable portion $(m\eta_{th}L)$ of the consumed power:

$$\frac{ma(w^2-V^2)}{2}=m\eta_{th}L \qquad (1)$$

Thus the overall efficiency will be given directly (ε being omitted) by the ratio

$$\eta_g=\frac{ma(w-V)V}{mL}=\frac{(w-V)V}{w^2-V^2}\cdot 2\eta_{th}=\eta_{th}\times\frac{2V}{V+w}.$$

If η_{th} is assumed to be constant, then in order to increase η_g we must decrease w and thus, according to equation (1), increase α. In other words, a high consumption of liquid at a low velocity is more favorable than low consumption at a high velocity. At the limit, for infinite α, we would have $w=0$, $\eta_f=1$, and $\eta_g=\eta_{th}$, which also determines the upper limit of the efficiency.

For this conclusion to be valid, the quantity η_{th} must not vary when the fuel consumption is increased. In relation to this, the theory of ejectors has not yet been worked out sufficiently to be considered completely reliable. However, since the time allotted for this report is limited, I cannot state here all the reasons leading me to believe that it will not be possible to increase the consumption of liquid by means of a more or less suitable mounting of
338 ejectors, without at the same time reducing, to some degree at least, the thermal efficiency of the overall transformation of the active substances (that is, the air for combustion and the air for ignition captured by the ejectors).

However, it must not be concluded that ejectors are thus of no interest whatsoever to us. It is enough that the reduction of the effect of η_{th} brought about by them is less significant than the main advantage of using ejectors, namely that they enable a higher consumption. Moreover, this subject has not yet been studied with the aid of systematic experiments. The formulas given above are clearly similar to the classical approximation formulas providing a general expression for the efficiency of a propeller. In fact, it is not difficult to show than these formulas are identical.

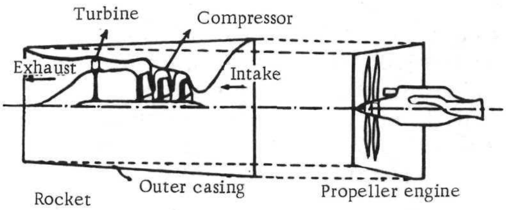

FIGURE 121.

Actually, if we consider a propeller to be an engine operating only in a bounded medium, and if we neglect the rotational energy of this medium, the efficiency of the propeller, as a function of the recoil velocity V, will be

$$\eta_h = \frac{2V}{2V+v} = \frac{2V}{V+(V+v)},$$

and the overall efficiency of a motor-propeller assembly will be

$$\eta_g = \eta_{th} \times \frac{2V}{V+(V+v)}.$$

In order for the efficiency of an ejector (trompe) rocket to be the same as that of a motor-propeller assembly, it is clear that the thermal efficiencies and the sums $V+v=w$ must be the same for both of these, that is, the relative air consumptions of the two devices must be equal.

Figure 121 shows an ejector (trompe) rocket with an efficiency equal to that of a motor-propeller assembly. It is clear from the figure that in this case, when it can hardly be assumed that an ejector (trompe) system has the same high thermal efficiency as a high-quality motor, the ejector (trompe)
339 engine will no longer possess its characteristic features of simplicity and small size.

However, there is no reason to conclude that direct-reaction engines are in general devoid of interest. They might prove to be applicable in certain instances which are as yet unforeseen, such as for towing special-purpose mines or aircraft at very high velocities (of the order of 1,000 km/hr). Accordingly, the experimental study of these engines is quite justified, the more so since it is very probable that the rapidity of motion of a propeller would be curtailed at such high speeds, approaching the velocity of sound in air, due to the considerable reduction of the propeller efficiency.

Now, to conclude this somewhat sketchy report, let me summarize the conclusions which I have arrived at, conclusions that, in all probability, will not be surprising to my audience. These conclusions consist simply in the following: the best type of engine for an aircraft is a combination of a thermal motor (internal-combustion motor) and an air screw, that is, the very type which has been used since the inception of aviation and which made possible the first flights. Reaction engines can compete with these only for flights at very high speeds, which are at present either unattainable or unfeasible in practice.

Accordingly, may those who are working to perfect the thermal motor (internal-combustion engine) and the air screw be encouraged by this statement, and may they continue along that same path of endeavor which has led mankind to such brilliant successes in the field of aviation. They can rest assured that the prospects for the further development of these engines remain the same, and that they are not yet threatened by any serious competition.

341 *YU. V. KONDRATYUK*

In 1929 a book by Yu. Kondratyuk entitled "The Conquest of Interplanetary Space" ["Zavoevanie mezhplanetnykh prostranstv"] was published in Novosibirsk. The book was edited by Prof. V. P. Vetchinkin and published by the author.

Referring those who are interested to the book itself, we present here just its table of contents:

1. Rocket Data; Basic Characteristics.
2. Formula for Load.
3. Exhaust Velocity. Chemicals.
4. Combustion Process; Construction of Combustion Chamber and Exhaust Pipe.
5. Proportional Dead Weights.
6. Types of Trajectories and Required Rocket Velocities.
7. Maximum Acceleration.
8. Effect of Atmosphere on Rocket at Launching.
9. Reduction to Zero of Velocity of Return by Atmospheric Resistance.
10. Interplanetary Station and Rocket-Artillery Facility.
11. Rocket Guidance; Measuring and Control Equipment.
12. Overall Prospects.
13. Experiments and Research.

All the techniques used by the author for presentation, notation, and calculation are quite original. The following ideas and conclusions in this work are novel:

1. The suggestion is made to burn various substances (lithium, boron, aluminum, silicon, magnesium) in ozone rather than in oxygen, so as to increase the heat of combustion. In particular, Kondratyuk suggests burning oil in methane, silicon hydride, boron hydride, acetylene, or hydrogen.

2. The heating of the rocket nose is studied, taking into account both the adiabatic compression of the air and the radiation of the rocket surface and the heated air itself.

At our request, Yu. Kondratyuk has sent his picture and some brief biographical information, which we present below.

342 DEAR NIKOLAI ALEKSEEVICH!

On the assumption that the strictly personal aspects of my life are not what interest you, let me try to give a fairly full account of just those things which are related to my studies of the theory of interplanetary travel.

My mind was first oriented toward thoughts of conquering outer space, or rather toward grandiose and extraordinary projects in general, by something which impressed me greatly when I read it in my youth: Kellermann's gifted industrial poem [novel] "The Tunnel."

At that time my scientific and technical baggage consisted of the following: an uncompleted secondary education plus some rather unsystematic additions made independently in the fields of higher mathematics, physics, and the general theoretical fundamentals of technology, with a tendency toward the development of inventions and toward independent research, more than toward a detailed study of what had already been found and discovered.

My "inventions" include: a water turbine of the Pelton-wheel type, using hydraulic devices considered by me to be unique, instead of millwheels; an automobile with treads for travel on soft, sandy ground; springless centrifugal springs; pneumatic springs; an automobile for travel on rough terrain; a vacuum pump of special construction; a barometer; a long-running clock; a high-power ac electric machine; a mercury-vapor turbine; and many other things, some of which are technologically quite impractical, some of which were already known, and some of which are new and deserve further development and realization.

In mathematics, I have made detailed studies of the axioms of geometry (especially the postulate of parallels), and I have "discovered" the basic formulas of the theory of finite differences, as well as some undeveloped further generalizations of this theory and analysis, together with many less significant things which are almost synonymous with previously made discoveries.

In chemistry and engineering, I have offered some basic elementary ideas. In physics, I have consistently tried to disprove the second law of thermodynamics (it is significant that I have this in common with K. E. Tsiolkovskii) and even in philosophy I have made attempts to construct logical systems, finished, along with 99% of my interest in philosophy, by the "discovery" of the difficult to perceive principle of determinism.

343 The impression made on me by Kellermann's novel, "The Tunnel," was such that, immediately after reading it, I took it upon myself to work out, almost simultaneously, as far as my powers enabled me, two themes: 1) the digging of a deep shaft in order to study the interior of the earth and utilize the heat of the earth's core, and 2) travel away from the earth. Curiously enough, the science-fantasy novels by Jules Verne and H. G. Wells, which I had read earlier and which dealt with this very subject of inter-

planetary travel, had not made any special impression on me. This was apparently because these novels were written with less talent and with less clarity than the novel of Kellermann, and also because for me they were clearly unreliable from a scientific point of view.

The idea of digging a deep shaft very soon ran up against the obstacle of the impossibility of my carrying out the corresponding experimental work, once the fundamentals of certain preliminary alternatives had been worked out. The subject of interplanetary flight, on the other hand, turned out to be more satisfactory, admitting of significant theoretical studies, and it occupied me for a long time, during the course of which I returned to it repeatedly, until I reached a point beyond which further productive work was impossible without the corresponding experimentation.

FIGURE 122. Yu. Kondratyuk.

The first period of the study took over half a year, and it consisted in finding out almost all the fundamental things about rocket flight which had been published, but without a more detailed development and often without exact mathematical arguments. At that time Chapter V and VIII of the subsequently published work were not even projected, while Chapters IV and IX were just worked out in principle; due to my meager knowledge of chemistry, in Chapter VII only an oxygen-hydrogen propellant had been considered.

The basic subject matter of the work of that period comprised: derivation of the fundamental rocket formula (formula (4)), finding the optimum trajectory (Chapter VI), and certain general conclusions from other chapters.

Having decided upon the subject of flight in interplanetary space, I turned immediately to the rocket method, where "rocket" is used in the general
344 sense of the word, according to the definition given by me in Chapter I.
Here the artillery rocket is rejected as clearly being too bulky and, what is the main thing, not being capable of a return to the earth, so that its use would be senseless. Before deriving the fundamental formula, I made approximate calculations for several mechanical alternatives. The most recent and best of these was a rapidly turning cylinder, with a steel cable wound around it. The cable would be unwound by inertia on one side, imparting to the cylinder an opposite acceleration. Having obtained, naturally, an improbably tremendous value for the required weight of the rocket ("n"), I proceeded to combinations of rocket-artillery alternatives: a cannon firing a projectile which in turn becomes a cannon firing a projectile, etc. However, once again I obtained a tremendous size for the first cannon.

Next I turned the muzzle of the second cannon (that is, the first projectile) backward, making it a permanent part of the rocket. I then had it fire smaller projectiles in a backward direction, that is, I increased the active mass of the charge at the expense of the dead weights. However, I again obtained a tremendous value for the mass of the rocket cannon. It turned out, though, that the more I increased the mass of the active part of the charge at the expense of the dead weights (projectiles), the more satisfactory were the formulas for the mass of this rocket.

From there it was not difficult to pass logically to a pure thermochemical rocket, which may be considered to be a cannon continually firing blank charges. After this, the fundamental rocket formula (4) was derived, but, because of the simplifying assumptions made by me during the initial calculations, which were subsequently forgotten and overlooked, for some time "2" rather than "1" was used as the basis for this formula. Thus the results obtained were extremely encouraging, due to this error.

However, shortly thereafter, I also determined the principles of optimum utilization of the rocket reaction: imparting the acceleration at the lowest point of the trajectory. After correcting the error lying at the basis of formula (4), I obtained a less favorable value for "n" (the ratio of the rocket mass to the useful load). This value was "n" = 55, without taking into account the unavoidable losses in efficiency and the presence of proportional dead weights.

This value of 55 was already quite alarming, but the subject in question was so fascinating that, deceiving myself somewhat, I assumed this figure to be acceptable until, ultimately, I found antidotes to this "55" in the form of a physicomathematical basis for the possibility of a satisfactory descent to the earth due to the resistance of the atmosphere, and then in the development of an initial velocity by artificial means, by setting up an interplanetary station with a rocket-firing facility.

Another question, vaguely troubling for a long time, involved the very high force of reaction required for the first pure rocket version of the launching. This force was no less than twice the force of gravity. Later I stopped worrying about this, after finding out that aircraft wings could be utilized with advantage during the ascent, thereby reducing the minimum acceptable reaction force severalfold.

Finally, my last serious worry was the meteor hazard. Only a few days
345 ago, after receiving from Ya. I. Perel'man his book entitled "Interplanetary Travel," I learned that foreign [non-Soviet] investigators who have studied this subject mathematically have arrived at favorable results.

By 1917 I had achieved the first positive results in my work. At that time I had no idea that I was not the first and only investigator in this field. Therefore, for a certain period I "rested on my laurels," awaiting an opportunity to begin experiments, which would lead to a realization of the inventions. At that time the work was conducted in strictest secrecy, since, taking consequences of man's emergence into interplanetary space, I naively assumed then that it was sufficient to publish the basic principles, and that someone possessing the means would immediately implement the first interplanetary flight.

In 1918, in a back number of "Niva," I accidentally came across a note about Tsiolkovskii's rocket. However, for a long time I was unable to obtain the issue of "Vestnik Vozdukhoplavaniya" referred to in this note.

This reference, together with those which I subsequently encountered in the periodical literature with regard to foreign studies, provided an incentive for a further more accurate and detailed development of the theory of flight, so as to pass from general physical principles to a consideration of the technical possibility of actual applications.

Going back to the work several times, after breaks during which I was a tutor, a woodchopper, and a mechanic, I succeeded in 1925 to bring it to almost its present form. All the chapters were provided with a more solid mathematical basis, a quite comprehensive selection of chemicals was made, Chapter VIII (on the atmospheric resistance at launching) was developed, the possibility of a safe gliding descent was verified by calculations, and some other, less important, additions were made.

In 1925, when the work was already approaching completion, and when I had finally succeeded in getting the "Vestnik Vozdukhoplavaniya" for 1911 with part of Tsiolkovskii's work in it, although I was somewhat disappointed to learn that the basic conclusions drawn by me had been anticipated, still I was pleased to find that not only did my study repeat the earlier one, albeit using different methods, but also that it made some new, important contributions to the theory of flight.

The main difference between the techniques used in my calculations and those of Tsiolkovskii is that in very many cases Tsiolkovskii starts from the work, while I everywhere start from the velocities and accelerations. Since the work done by the forces in a rocket problem depends on many conditions and in addition can have very different effects, whereas the accelerations, and thus the velocities as well, are much more definite, I consider the velocity method to be easier and more productive.

In 1925 I received a review [of the unpublished book] from Prof. V. P. Vetchinkin which surprised me greatly by its high opinion of my work, since previous to this I had traditionally not been inclined to expect anything good from "professors." Thus from day to day I awaited the publication of my book. However, the typical, benign procrastination of Glavnauka* and
346 Giz** ensued: consideration, reconsideration, appropriation of funds, withdrawal of these funds, until two and a half years had gone by. Fortunately, by this time I had advanced from being a mechanic to being an instrument maker and designer, so that it became possible to acquire the wherewithall to publish the book myself in Novosibirsk, otherwise it is hard to tell when my work would have appeared. Glavnauka not only withdrew the small amount of money appropriated by it earlier for publication, but it even ceased its organizational aid (leaving me to publish at my own expense at some printer suitable for scientific publications). I did not wish to have the work printed in a journal, since I did not see any possibility of shortening it and I did not see how the whole work could be published in a journal.

In 1927, on the suggestion of V. P. Vetchinkin, I modified the system of notation and, in part, the terminology, so as to make it more conventional and intelligible. At the same time I inserted the derivation of formula (4), not included by me earlier, and I corrected an error in formula (6), for the effect of the mass of the proportional dead weights. Prof. Vetchinkin drew my attention to the great importance of working out the design of the "burner" (the exhaust pipe), as a result of which I wrote and inserted Chapter IV.

Further fruitful research on interplanetary flight by purely theoretical methods is obviously impossible, for me at any rate. Experimental studies must now be made. I expect to get the time and money for these from inventions in various fields, in particular from my type of work at present

* Glavnoe upravlenie nauchnymi muzeinymi i nauchno-khudozhestvennymi uchrezhdeniyami (The Central Scientific Board).

** Gosudarstvennoe knigoizdatel'stvo (The State Publishing House).

in the field of elevator design. So far I have had some initial successes, in view of the recent acceptance of my new type of elevator bucket and bucket conveyors, which have already found themselves a place, in competition with a type that has remained unchanged for a long time.

Incidentally, I am sending on to you a curious, classic review by a certain scientist, indicating that some diehards are still around who will, with dogged persistence, find fault with the idea of interplanetary travel, or any new idea for that matter, until the time when regular trips into outer space will have been established and the cold countries will have been heated by sunlight redirected onto thousands of versts of the earth.

1 May 1929

Respectfully yours,
Yur. Kondratyuk

347 *P.N. LEBEDEV*

As the idea of interplanetary travel developed, different persons proposed plans for propelling spacecraft by means of the pressure of light rays. In order to obtain some idea of the magnitude of this pressure, we present here a brief account of the results of the experimental and theoretical works of P. N. Lebedev, whose studies of this subject are now taken to be classical.


FIGURE 123. P. Lebedev.

The Russian physicist Petr Nikolaevich Lebedev was born in Moscow in 1866. He obtained his primary education at the Petropavlovsk school, and then at the Khainovskii Realschule. Upon graduation from the latter in 1884, he entered the Moscow Institute of Technology (the former Imperial Institute of Technology), where he was a student for two years. Interested in physics, he traveled to Germany, where he worked under the direction of some well-known scientists and earned a German Ph. D. for his studies.

Lebedev's first work, "On the Repulsive Force of Radiating Bodies," was presented in Germany (Strassburg) on 30 (18) July 1891. Returning to Moscow, he continued his work in physics under the direction of Professor A. Stoletov. Here one of his first projects was a study of short electromagnetic waves, and a determination of the conditions under which they cause repulsion and attraction. For this work he received a Ph. D. in physics. In 1900 Lebedev became a professor of physics at Moscow University, where he continued until 1911, when he transferred to the A. L. Shanyavskii University. He passed away on 14 March 1912.

348 Lebedev's chief work was his study of light pressure on solids and gases, the latter in connection with the problem of the origin of comet tails. Referring those who are interested in the details of this experimental study to Lebedev's work entitled "The Pressure of Light,"* we present here just the main conclusions arrived at by Lebedev on the basis of his research.

* Lebedev, P.N. Davlenie sveta (The Pressure of Light).– Klassiki Estestvoznaniya, Book 4, Gos. Izd., Moskva. 1922.

349 *DETERMINATION OF LIGHT PRESSURE ON SOLIDS AND GASES*

SOLIDS

1. An incident light ray exerts pressure on reflecting, as well as absorbing, surfaces.

2. The light pressure is directly proportional to the energy of the incident ray and is independent of the color.

3. The observed light pressures are quantitatively equal to the Maxwell-Bartoliev pressures of radiant energy, and they are given by the formula

$$P=\frac{E}{V};$$

where P is the pressure force, E is the energy incident upon an absorbing body per unit time, and V is the velocity of the ray in the medium in which the body is situated. If we take Langley's value of 3 g cal for the amount of heat (C) delivered in 1 min to an area 1 cm^2 in cross section by a pencil of rays from the sun (the so-called "solar" constant), for a mechanical equivalent of heat $B = 425$ g meters, the energy E of a ray incident upon 1 cm^2 in 1 sec will be

$$E=\frac{C}{60}B=\frac{3}{60}:425=21 \text{ g meters.}$$

Assuming a velocity of light $V = 3 \cdot 10^8$ m/sec, we find from formula (1) the pressure P_0 produced by a pencil of solar rays 1 cm^2 in cross section upon an absorbing body located the same distance from the sun as the earth:

$$P_0=\frac{E}{V}=\frac{2}{3}\cdot 10^{-7} \text{ g}$$

Note: the pressure on 1 m^2 will be $^3/_5$ of a milligram, or, in absolute units,

$$P_0=6\cdot 10^{-5} \text{ dyne.} \qquad (2)$$

350 If we assume that:

the earth-sun distance $\varrho = 15 \cdot 10^{12}$ cm;
the earth's orbital velocity $\gamma = 3 \cdot 10^6$ cm/sec,

then the solar acceleration A at the distance of the earth will be

$$A=\frac{v^2}{\varrho}=0.6\ \text{cm/sec}^2$$

Therefore, at a distance ϱ the sun attracts a 1-gram mass with a force A:

$$A=0.6\ \text{dyne}. \tag{3}$$

The effect exerted by the sun on a body revolving about it consists, first of all, of the Newtonian attraction, and, secondly, of the repulsive forces of radiation. Let us assume that a spherical body at a distance ϱ from the sun absorbs all the solar energy incident upon it, and that it then radiates this energy uniformly in all directions. If the radius of the body is r cm and its density is δ, then we can calculate the force G with which it is attracted by the sun and the force H with which it is repelled by it:

$$G=\frac{4}{3}\pi r^3\delta A$$
$$H=\pi r^2 P_0.$$

From this it is easy to calculate the resultant force F with which the sun attracts the given body, and to express it as a fraction of the Newtonian attractive force:

$$F=\frac{G-H}{G}=1-\frac{H}{G}=1-\frac{3}{4}\frac{P_0}{Ar\delta} \tag{4}$$

For a given body this force F will be a characteristic constant which is independent of the distance from the sun, since quantities P_0 and A both depend on this distance to the same degree. Substituting the numerical values of P_0 and A from (2) and (3) into equation (4), we obtain

$$F=1-\frac{10^{-4}}{r\delta}. \tag{5}$$

From this expression it is clear that, for all bodies for which $\delta>1$ and $r<1$ meter, the deviations from Newton's law are so small that they are imperceptible, even for very careful observations.

The smaller we assume the radius of the body to be, the more important
351 will be the repulsive force of the sun. In comet tails, which consist mainly of gaseous hydrocarbons, we have to do with individual molecules, having radii $r<10^{-8}$ cm and densities $\delta<10$. Thus the repulsion of these tails will be many times greater than their attraction.

The problem of the repulsive force of the sun, which we have just considered, can also be solved for the more general case in which, instead of the sun, we have a body of radius R and density Δ which radiates an amount of heat Q from 1 cm^2 of its area in 1 sec. We can pass to this general case, on the basis of the results obtained for the sun, if we remember that the sun's radius

$$R_0=7\cdot 10^{10}\ \text{cm},$$

its density

$$\Delta_0 = 1.4$$

and the radiation of 1 cm^2 of its surface in 1 sec

$$Q = 2000 \text{ g cal.*}$$

If S is the ratio of the repulsive force of the radiation to its Newtonian attractive force, it is clear that S will be directly proportional to Q, and inversely proportional to both Δ and R.**

For the sun, this quantity S_0 is given by formula (5) as

$$S_0 = \frac{10^{-4}}{r\delta}.$$

For any other body we have

$$S = S_0 \frac{Q}{Q_0} \cdot \frac{\Delta_0}{\Delta} \cdot \frac{R_0}{R}, \tag{6}$$

or, replacing S_0, Δ_0, Q_0, and R_0 by their previously given values, we obtain

$$S = \frac{10^{-4}}{r\delta} \cdot \frac{Q}{2000} \cdot \frac{1.4}{\Delta} \cdot \frac{7 \cdot 10^{10}}{R} = 5 \cdot \frac{Q}{r \cdot \delta \cdot R \cdot \Delta} \cdot 10^3 \tag{7}$$

The resultant K of the attractive and repulsive forces for this body will then be

$$K = 1 - S = 1 - 5\frac{Q}{r \cdot \delta \cdot R \cdot \Delta} \cdot 10^3 \tag{8}$$

352 For a black body at 0° C, Christiansen found that in 1 sec 1 cm^2 of its surface radiates an amount Q', equal to

$$Q' = (1.21 \cdot 10^{-12})(273^4) = 0.0037 \text{ g cal.}$$

Consequently, the force K' with which a spherical perfect black body in outer space, with a radius R cm, a density Δ, and a temperature of 0° C, will attract another spherical perfect black body with a radius r cm and a density δ will be approximately

$$K' = 1 - \frac{20}{r \cdot \delta \cdot R \cdot \Delta} \tag{9}$$

indicating that two spherical bodies with temperatures of about 0°C, densities $\Delta = \delta = 10$, and radii $R = r = 10$ mm will neither attract nor repel each other. Dust particles, on the other hand, which have radii not

* If we assume that at the distance of the earth from the sun $\varrho = 15 \cdot 10^{12}$ cm, 3 g cal are incident upon 1 cm^2 per min, or 0.05 g cal per sec, then 1 cm^2 of the sun's surface, at a distance $R_0 = 7 \cdot 10^{12}$ cm from the center, will radiate

$$Q_0 = 0.05\left(\frac{\varrho}{R_0}\right)^2 = 2000 \text{ g cal/sec.}$$

** Since the attractive force of the mass is proportional to R^3, and the repulsive force of the radiation is proportional to R^2.

exceeding one thousandth of a mm, will be repelled at 0°C in outer space by a force of an order about a million times greater than the order of the force of Newtonian attraction.

GASES

Lebedev arrived at the following conclusions as a result of his experiments with gases:

1. The existence of a light pressure on gases has been established experimentally.
2. The magnitudes of this pressure are directly proportional to the energy of the light beam and the absorption coefficient of the gas.
3. Within the limits of error of the observations and calculations, the relation given by Fitzgerald agrees quantitatively with the observations.

Therefore, the force P with which light presses on a gas layer is

$$P = a \cdot \frac{E}{V} \tag{6}$$

where α is the absorption coefficient of the gas layer for radiant energy, E is the amount of this energy incident upon the gas layer in 1 sec, and V is the velocity of propagation of light.

The table shows the values of pressure P obtained with formula (6) for various gases, expressed in millionths of a dyne per cm^2.

Gas	α	P
0.5 methane + 0.5 H_2	0.0057–0.0071	0.66–0.98
0.5 propane + 0.5 H_2	0.0175–0.0200	1.89–2.10
0.5 butane + 0.5 H_2	0.0172–0.0189	2.06–3.03
0.1 butane + 0.9 H_2	0.0063–0.0072	0.87–0.97
0.5 ethylene + 0.5 H_2	0.0068–0.0075	0.73–1.04
0.5 acetylene + 0.5 H_2	0.0063–0.0080	0.77–1.00
0.5 carbon dioxide + 0.5 H_2	0.0055–0.0072	0.69–0.92

353
N. A. RYNIN

EFFECT OF ACCELERATION ON ANIMALS

In two of the previous volumes of this series on interplanetary travel (on p. 144 of "Superaviation and Superartillery," Leningrad, 1929, and in "Theory of Propulsion by Reaction," Leningrad, 1929) we have already presented a detailed summary of experiments, made in various countries, on the effect of acceleration on living organisms.

However, it seemed to us that these experiments had by no means gone into all the aspects of this problem. Consequently, in order to obtain a comprehensive study of the acceleration effect, several additional studies had to be carried out.

Before making tests on humans, we decided to carry out some on several types of lower animals, and to utilize the results of these to set up more suitable experiments with a human being. Two centrifugal machines were constructed; the first, 1 m in radius, had speeds of up to 300 rpm, and the second, a centrifuge 0.32 m in radius, had speeds of up to 2,800 rpm.

The test animals were placed in special compartments in these machines. The animals (flies, beetles, cockroaches, fish (carp), frogs, mice, rats, pigeons, siskins, crows, rabbits, and cats) were observed for effects upon them of the centrifugal acceleration developed during rotation. The experiments, conducted during the spring and summer of 1930, provided material which will be useful in designing a large centrifugal machine for future projected experiments with a human being. In addition to the rotation, the excess loads [due to acceleration] were also determined for the dropping of fresh hens' eggs onto sand.

It should be noted that two factors exerted an effect during the experiments with test animals: a centrifugal force causing excess weight (excess load) of the animal, and rotation. For tests of short duration, the effect of the centrifugal force predominated, sometimes even causing traumatic disturbances. For prolonged tests the effect of rotation was greater, causing a disturbance of coordination and of the feeling of balance.

A full report on these tests is given in Issue No. 1 of the Bulletin of the Institute of the Civil Air Force (Leningrad). Here we present just the
354 most important findings. But first we should mention that the following people took part in the physiological observations, under the direction of Professor A. A. Likhachev: Drs. M. M. Likhachev, V. M. Karasik, A. M. Vasil'ev, and A. A. Sergeev.

CONCLUSIONS

1. **Insects** (dung beetles, German cockroaches, black cockroaches, common house flies, and horseflies) can endure excess loads of up to 2,500 times for as long as 1 min, without any injurious effects.

2. **Fish** (carp) with weights of up to 20 grams in water can endure, with only slight disturbance, the effects of excess loads of up to 2,200 times, for 1 min (1').

3. **Frogs** weighing from 23 grams to 65 grams, in water and out of water, can safely endure excess loads of up to 23 times, without any disturbance, for times up to 5'.

For excess loads of up to 2,200 times during times up to 1', moderate disturbance of motion is observed, with a return to normal after 30 min.

4. **Birds** (siskins, pigeons, and crows):

a) Siskins (weighing 11.4 grams) can safely endure excess loads of 39 times for about 2'. If the same loads are prolonged to 5', disruption of coordination is observed.

b) Pigeons (weighing 275 grams) can endure excess loads of 28 times for 2', with slight disturbance of coordination, but for excess loads of 23 times over 4' the disturbance of coordination is greater.

c) Crows (weighing 380 grams) can endure excess loads of 23 times for 4'50" with only a slight disturbance of coordination.

5. **Mice and rats:**

a) Mice (white, weighing 17 grams). Reactions to excess loads and their durations were as follows:

Excess load	Duration	Effect
12	2'	normal
48	2'	moderate disturbance
58	2'	marked disturbance
58	5'	death

b) Gray rats (weighing 45 grams):

Excess load	Duration	Effect
30	2'	moderate disturbance
25	3'	marked disturbance

6. **Rabbits** (weighing from 1,520 to 2,600 grams):

	Excess load	Duration	Effect
	10	2'	slight disturbance
	16–28	2', duration of test 1'55"	moderate disturbance
355	23	2', duration of entire test 6'25"	marked disturbance
	10	6', duration of test 11'15"	death

7. **Cats** (weighing from 3,250 to 3,729 grams):

Excess load	Duration	Duration of entire test	Effect
10	2"	4'10"	normal
28	2"	1'55"	slight disturbance
28	2"	4'30"	marked disturbance

8. **Raw eggs** (weighing from 38 to 55 grams) can endure without breaking (in water and without water) excess loads of:

39 times, for 30"
30 times, for 1'
280 times, for 0.01" (dropping)

Small cracks without disturbance of the contents are produced for excess loads of:

700 times, for 5"	with thick end outside disk in sand
100 times, for 0.01"	dropping into sand in jar with water
48 times, for 1"	in water

Breakage is observed for excess loads of:

48 times, for 1"	without water
300 times, for 0.01"	dropping into sand
700 times, for 5"	in sand with tip outside disk

The following general conclusions can be drawn:

1. The larger and the heavier the animals, the harder it is for them to withstand excess loads. Some examples are:

Mice endured	58
Birds endured	39
Rabbits endured	28
Cats endured	28 [times for 2']

2. The duration of the excess load has a marked influence on its effect. Whereas this is less true for frogs and birds, for mice, rats, rabbits, and cats the duration has a great effect. Some examples are:

Frogs can endure an excess load of	23 times for 5'
	2,200 times for 1'
Birds " " " " " "	39 times for 5'
Mice " " " " " "	58 times for 2'
Mice die for an excess load of	58 times for 5'
Rats can endure an excess load of	25 times for 3'
Rabbits can endure an excess load of	28 times for 2'
Rabbits die for an excess load of	10 times for 6'
Cats can endure an excess load of	28 times for 2'

356 During prolonged tests the rotation factor had the greatest significance.

3. Insects, fish, and frogs can endure prolonged excess loads of from 2,200 to 2,500 times.

4. The endurance of the animals is greatly influenced by the way they are placed in the compartment, that is, by the uniformity of the pressure of their bodies against the outer wall of the container. For instance, the immersion of the carp and frogs in water resulted in a general increase in their capacity to resist, while eggs resisted better in ordinary water than out of water, better in salt water then in fresh water, and even better in sand. Mice with cotton padding around them resisted better than mice without the padding.

5. The tests with frogs indicated that apparently the same centrifugal force may have different effects on them, depending on whether the force is produced by an increased number of revolutions and a small radius, or vice versa. However, this conclusion has not yet been verified with other animals, particularly with respect to the size of the animal and the radius of rotation.

Therefore, for different animals, the following excess loads may be considered to be completely tolerable:

	Excess-load limit and duration (' is min and " is sec)
German cockroaches, dung beetles	$\frac{2,532}{1'}$
Black cockroaches, horseflies, house flies	$\frac{2,200}{1'10''}$
Carp	$\frac{28}{1'}$
Frogs	$\frac{48}{2'}$
Siskins	$\frac{38.9}{2'}$; $\frac{30.7}{2'}$
White mice	$\frac{12}{2'}$
Rabbits	$\frac{10}{2'}$
Cats	$\frac{10}{2'}$

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