N72-22902

NASA TECHNICAL MEMORANDUM

NASA TM X-64651

GRAVITY GRADIENT TORQUE PROFILES OVER AN ORBIT FOR ARBITRARY MODULAR SPACE STATION CONFIGURATIONS IN THE Y-POP AND INERTIAL HOLD MODES

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March 22, 1972

NASA

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama

TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO.	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
NASA TM X-64651		
4. TITLE AND SUBTITLE Gravity Gradient Torque Profiles Over an Orbit for Arbitrary Modular Space Station Configurations in Y-POP and Inertial Hold Modes		5. REPORT DATE
		6. PERFORMING ORGANIZATION CODE
7. AUTHOR(S) David J. McGill*		8. PERFORMING ORGANIZATION REPORT #
9. PERFORMING ORGANIZATION NAME AND ADDRESS George C. Marshall Space Flight Center Marshall Space Flight Center, Alabama 35812		10. WORK UNIT NO.
		11. CONTRACT OR GRANT NO.
		13. TYPE OF REPORT & PERIOD COVERED Technical Memorandum
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D.C. 20546		
		14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES Prepared by Astrionics Laboratory, Science and Engineering.
*Associate professor of engineering science and mechanics, Georgia Institute of Technology, and NASA/ASEE Summer Faculty Fellow, MSFC Astrionics Laboratory, 1971.

16. ABSTRACT

For several proposed configurations of the modular space station currently under study by NASA, the gravity gradient torque distribution is examined for both Y-POP and inertial hold orbits. The necessary equations are derived to allow for solar panel gimbaling, and a procedure is established which will allow any desired space station configuration to be built up. Results indicated that solar panel gimbaling can be a significant factor in the torque distribution in Y-POP orbits. In addition, for the two configurations that were plotted, bias torque magnitudes varied widely in inertial hold modes.

17. KEY WORDS	18. DISTRIBUTION STATEMENT
Gravity gradient torque, Space station, Solar panels, Inertial hold, Gimbal angles, Bias torque, Y-POP, Cyclic torque	Jack Thompson Unclassified - Unlimited

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	30	$3.00

MSFC - Form 3292 (May 1969)

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DEFINITION OF SYMBOLS

Symbol	Definition
A_i	Component of $\vec{k}''$ in solar reference frame
B', B''	Reference frames (Fig. 2)
$\vec{I}$, $\vec{J}$, $\vec{K}$	Unit vectors in principal directions
I_1, I_2, I_3	Principal moments of inertia
$\vec{i}$, $\vec{j}$, $\vec{k}$	Unit vectors in T reference frame (Fig. 3)
$\vec{i}_R$, $\vec{j}_R$, $\vec{k}_R$	Unit vectors in solar reference system
$\vec{i}''$, $\vec{j}''$, $\vec{k}''$	Unit vectors along symmetry axes of solar panels (Fig. 2)
I_{ij}	Inertia tensor component
$K_1, \ldots K_{10}$	Constants defined in Table 1
$\vec{M}_c$	Gravity gradient torque vector (N-m)
SP	Reference frame (Fig. 2)
T	Reference frame (Fig. 3)
U_1, U_2, U_3	Components of $\vec{k}_R$ in orbital frame (Fig. 4)
V	Reference frame (Fig. 1)
X_o, Y_o, Z_o	Coordinates in orbital frame (Fig. 1)
X', Y', Z'	Coordinates in B' frame
X'', Y'', Z''	Coordinates in B'' frame
X_R, Y_R, Z_R	Coordinates in solar reference frame
X_{SP}, Y_{SP}, Z_{SP}	Coordinates in SP frame

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
X_v, Y_v, Z_v	Coordinates in V-frame
Γ_y	Angle (Fig. 1)
δ_1, δ_2	Gimbal angles (Fig. 2)
η_x, η_y	Angles (Fig. 1)
θ	Orbital angle (Fig. 1)
Λ	Lead angle, $\theta - \eta_y$
λ_y, λ_z	Angles (Fig. 1)
τ	Orbital period (s)
ϕ_z	Earth inclination angle (Fig. 1)
ψ	Angle used in equation (18)
ω	Orbital speed (rad/s)

NASA TECHNICAL MEMORANDUM X- 64651

GRAVITY GRADIENT TORQUE PROFILES OVER AN ORBIT FOR ARBITRARY MODULAR SPACE STATION CONFIGURATIONS IN THE Y-POP AND INERTIAL HOLD MODES

INTRODUCTION

The modular space station is currently receiving a great deal of study by NASA[1, 2] and its control is obviously a topic of considerable importance.

A major contributor to the control problem is the gravity gradient torque, which is considerable for a large orbiting body and which, for that matter, can cause instability of even very small satellites if they are improperly oriented [1, 2].

Prevention of the continual acquisition of angular momentum caused by such torques clearly requires some advance knowledge of the magnitudes of the torques to be controlled. The objective of this report is to examine the gravity gradient torque distribution acting on an arbitrary configuration of the modular space station over any orbit. This is done for both the Y-POP configuration (in which the space station moves with its longitudinal X-body axis always directed along the velocity tangent and the Z-body axis along local vertical) and for the station frozen into and remaining in any inertial hold configuration during the above-mentioned Y-POP orbit.

The procedure used and described here allows for solar panel gimbaling and for varying the values of the spatial and orbital parameters λ_y, λ_z, Γ_y, and orbiting angle θ. Any Initial Space Station (ISS) configuration may be built up, and with modifications the procedure can be made to accommodate the Growth Space Station (GSS), as well as other orbital modes.

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1. Space Station First Performance Review. MSFC-DPD-235/DR NO. MA-02, MDC G2279, McDonnell-Douglas Astronautics Co., April 1971.

2. Space Station Mass Property Status Report. MSFC-DPD-235/DR NO. SE-07, MDC G2313 McDonnell-Douglas Astronautics Co., June 1971.

It is shown by examples that solar panel gimbaling can have a significant effect on gravity gradient torque distribution and hence on momentum requirements. The examples also show wide variations in bias torque for various stations examined.

OUTLINE OF THE GENERAL PROCEDURE

Before the details of this study are discussed, the general procedure that was followed is outlined below for the convenience of the reader:

1. Give or vary the angles Γ_y, ϕ_z, λ_y, λ_z, and θ of Figure 1.

2. Compute the quantities (U_1, U_2, U_3) from equations (21) through (23).

3. Compute η_x and η_y (Fig. 1) from equations (15) and (16).

4. Compute the gimbal angles δ_1 and δ_2 (Fig. 2) which the solar panels must turn through to face the sun [equations (30) and (33)].

5. Transform the solar panel inertia tensor to vehicle coordinates (Fig. 2).

6. Given the configuration of the station, build it up, determine its mass center C, and compute its composite inertia tensor with respect to C. Perform this in two ways as a check.

7. If the vehicle is in inertial hold, transform the inertial properties to the orbiting coordinate system T (Fig. 3).

8. Compute the principal moments of inertia and their directions, using a standard library routine.

9. Compute the gravity gradient torques acting on the vehicle.

10. Increment θ (orbiting angle) and λ_y (to account for orbital regression) and repeat steps 7 and 9 for inertial hold and all but step 7 for Y-POP. Continue for a complete orbit, and repeat the entire procedure for various configurations and parameters of interest. (In the Y-POP mode, only the solar panels' changing contribution to the inertia tensor is recomputed in step 6.)

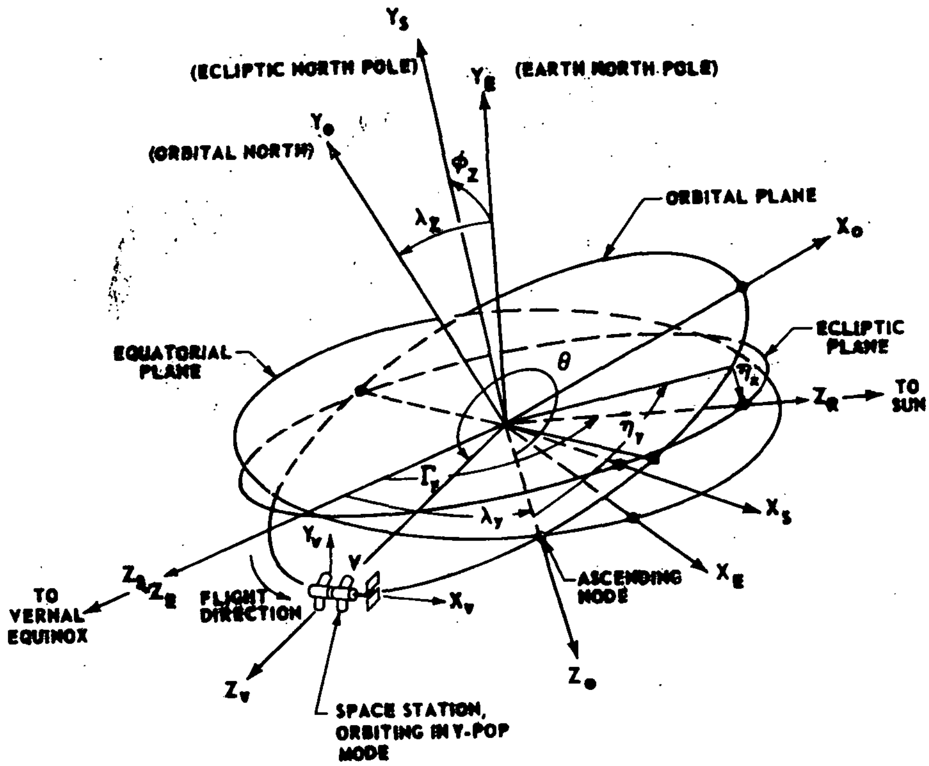

Figure 1. Coordinate systems and parameters.

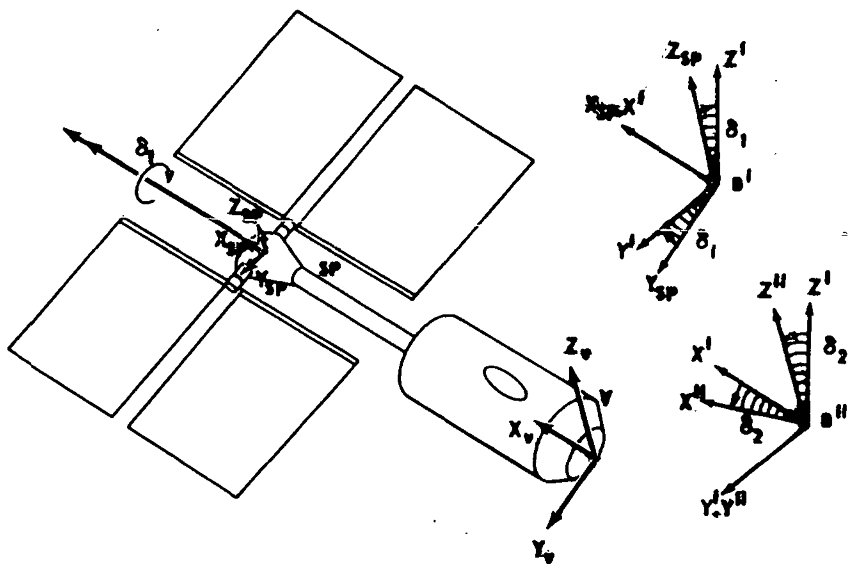

Figure 2. Solar panel gimbal angles.

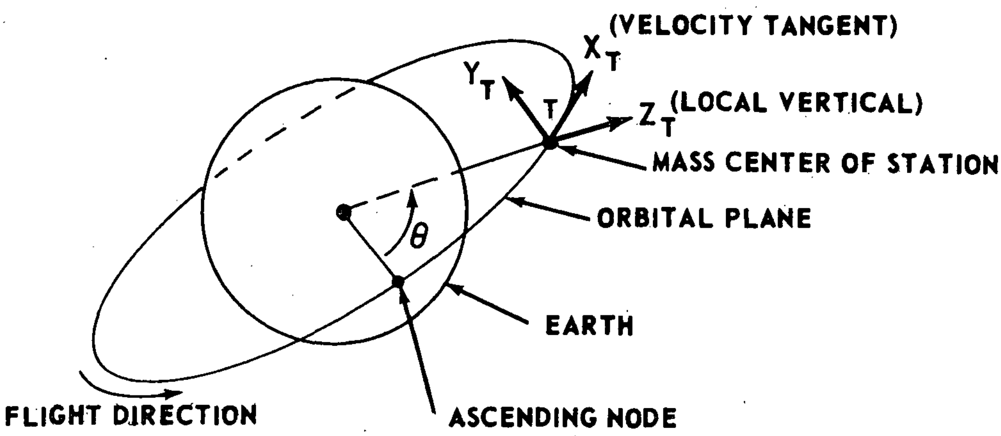

Figure 3. Orbiting coordinates.

VARYING INERTIAL PROPERTIES OF THE SOLAR PANELS

This section describes the gimbaling of the solar panels necessary in order that they always face the sun. Once these angles are computed, they are used to transform the solar panel inertia tensor to the vehicle axes.

The two gimbal angles for the panels are shown in Figure 2. The first angle is a rotation about X_{SP} of amount δ_1. Note that (X_{SP}, Y_{SP}, Z_{SP}) are axes parallel to the vehicle axes (X_V, Y_V, Z_V), passing through the center of mass of the panels. This first rotation brings the panel axes (frame SP) into the primed axes (frame B′) shown at the upper right in Figure 2. The second rotation is about the Y′ axis, through the angle δ_2, bringing the solar panel symmetry axes into the (X″, Y″, Z″) positions (frame B″), shown at the lower right of Figure 2.

Since the inertia tensor with respect to reference frame B″ is known, we can thus transform these properties back through B′ to SP and finally to the V-frame, where all the inertias are being collected.

The condition which must be fulfilled here is that unit vector $\vec{k}''$ (in the Z″ direction) is always pointing toward the sun; i.e., it is equal to $\vec{k}_R$, which is defined to be a unit vector in the Z_R direction. This then gives two additional equations stating that the components of $\vec{k}''$ in the X_R and Y_R directions be zero at all times. From these equations we will arrive at a solution for the gimbal angles δ_1 and δ_2, as will be shown shortly.

We are thus ultimately seeking a transformation matrix $[M_{B''R}]$ which will express the unit vectors of B" in terms of those of the solar reference frame (X_R, Y_R, Z_R), which is labeled R . In steps, we have

$$[M_{B''R}] = [M_{B''O}][M_{OR}] \quad , \tag{1}$$

in which subscript O refers to the orbital plane frame (Fig. 1), having coordinates (X_o, Y_o, Z_o).

Let us first compute matrix $[M_{B''O}]$. Clearly, we have

$$[M_{B''O}] = [M_{B''B'}]\,[M_{B'V}][M_{VO}] \tag{2}$$

and inspection shows that

$$[M_{B''B'}] = \begin{bmatrix} \cos\delta_2 & 0 & -\sin\delta_2 \\ 0 & 1 & 0 \\ \sin\delta_2 & 0 & \cos\delta_2 \end{bmatrix} \quad , \tag{3}$$

$$[M_{B'V}] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\delta_1 & \sin\delta_1 \\ 0 & -\sin\delta_1 & \cos\delta_1 \end{bmatrix} \quad , \tag{4}$$

and

$$[M_{VO}] = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \quad , \tag{5}$$

in which θ is the orbiting angle shown in Figure 1, which locates the space station in the orbital plane with respect to the ascending node.

Substituting equations (3) through (5) into equation (2) yields

$$[M_{B''O}] = \left[\begin{array}{c|c|c} C_{\delta_2} C_{\theta} - S_{\delta_2} C_{\delta_1} S_{\theta} & S_{\delta_1} S_{\delta_2} & -C_{\delta_2} S_{\theta} - S_{\delta_2} C_{\delta_1} C_{\theta} \\ \hline S_{\delta_1} S_{\theta} & C_{\delta_1} & S_{\delta_1} C_{\theta} \\ \hline S_{\delta_2} C_{\theta} + C_{\delta_1} C_{\delta_2} S_{\theta} & -S_{\delta_1} C_{\delta_2} & -S_{\delta_2} S_{\theta} + C_{\delta_1} C_{\delta_2} C_{\theta} \end{array}\right] \qquad (6)$$

in which C_{δ_2} means $\cos \delta_2$, S_{θ} means $\sin \theta$, etc.

The other matrix factor of equation (1) is $[M_{OR}]$. Rotating in the orbital plane about Y_o through the angle η_y, and then rotating through η_x about the rotated position of the axis which began coincident with X_o brings the Z-orbital axis into alignment with the solar reference axis Z_R. Therefore,

$$[M_{OR}] = \begin{bmatrix} C_{\eta_y} & S_{\eta_x} S_{\eta_y} & S_{\eta_y} C_{\eta_x} \\ 0 & C_{\eta_x} & -S_{\eta_x} \\ -S_{\eta_y} & C_{\eta_y} S_{\eta_x} & C_{\eta_x} C_{\eta_y} \end{bmatrix} . \qquad (7)$$

Substituting equations (6) and (7) into equation (1) yields the desired result for $[M_{B''R}]$. We need only the third row here which, written in equation form, is

$$\vec{k''} = A_1 \vec{i_R} + A_2 \vec{j_R} + A_3 \vec{k_R} \quad , \qquad (8)$$

in which the A_i are functions of δ_1, δ_2, η_x, η_y, and θ. If $\vec{k''}$ is to equal $\vec{k_R}$, it follows that $A_1 = A_2 = 0$ and $A_3 = 1$. Written out, these three equations respectively become as follows:

$$S_{\delta_2} C_\theta C_{\eta_y} + C_{\delta_1} C_{\delta_2} S_\theta C_{\eta_y} + S_{\delta_2} S_\theta S_{\eta_y} - S_{\eta_y} C_{\delta_1} C_{\delta_2} C_\theta = 0 \quad , \tag{9}$$

$$S_{\delta_2} C_\theta S_{\eta_x} S_{\eta_y} + C_{\delta_1} C_{\delta_2} S_\theta S_{\eta_x} S_{\eta_y} - S_{\delta_1} C_{\delta_2} C_{\eta_x} - S_{\delta_2} S_\theta C_{\eta_y} S_{\eta_x}$$

$$+ C_{\delta_1} C_{\delta_2} C_\theta C_{\eta_y} S_{\eta_x} = 0 \quad , \tag{10}$$

and

$$S_{\delta_2} C_\theta S_{\eta_y} C_{\eta_x} + C_{\delta_1} C_{\delta_2} S_\theta S_{\eta_y} C_{\eta_x} + S_{\delta_1} C_{\delta_2} S_{\eta_x} - S_{\delta_2} S_\theta C_{\eta_x} C_{\eta_y}$$

$$+ C_{\delta_1} C_{\delta_2} C_\theta C_{\eta_x} C_{\eta_y} = 1 \quad . \tag{11}$$

Even though equations (9) through (11) are unwieldy, they can be solved for δ_1 and δ_2 once we know η_x and η_y in terms of the parameters Γ_y, ϕ_z, λ_y, and λ_z. The value of θ will be incremented over an orbit but will remain constant through each incremental step, so it will be known whenever equations (9) through (11) need to be solved.

Now consider Figure 4. Letting (U_1, U_2, U_3) be the projections of $\vec{k}_R$ onto the axes of the orbital reference frame, it follows that

$$U_1 = \sin \eta_y \cos \eta_x \quad , \tag{12}$$

$$U_2 = -\sin \eta_x \quad , \tag{13}$$

and

$$U_3 = \cos \eta_y \cos \eta_x \quad . \tag{14}$$

Inverting these yields

$$\eta_x = \tan^{-1}\left[\frac{-U_2}{(U_1^2 + U_3^2)^{1/2}}\right] \tag{15}$$

and

$$\eta_y = \tan^{-1}\left[\frac{U_1}{U_3}\right] \quad . \tag{16}$$

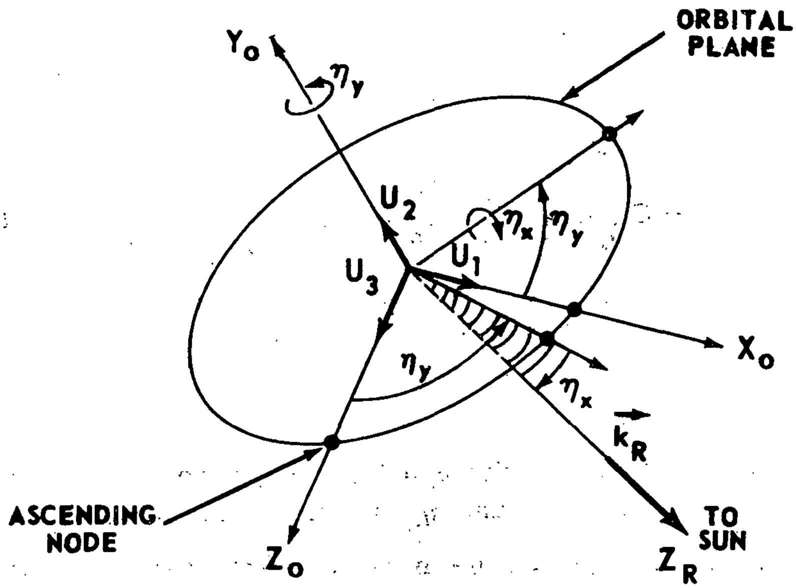

Figure 4. Relation between (η_x, η_y) and (U_1, U_2, U_3)

It is seen from equations (15) and (16) that it remains only to solve for U_1, U_2, and U_3 in terms of Γ_y, ϕ_z, λ_y, and λ_z to be able to solve equations (9) through (11). We do this via the transformation

$$[M_{RO}] = [M_{RS}][M_{SE}][M_{EO}] \quad , \tag{17}$$

in which S refers to the ecliptic and E to the equatorial plane, as depicted in Figure 1.

The matrices of equation (17) which must be multiplied are

$$[M_{RS}] = \begin{bmatrix} C_\psi C_{\Gamma_y} & S_\psi & -C_\psi S_{\Gamma_y} \\ -S_\psi C_{\Gamma_y} & C_\psi & S_\psi S_{\Gamma_y} \\ S_{\Gamma_y} & 0 & C_{\Gamma_y} \end{bmatrix} , \tag{18}$$

$$[M_{SE}] = \begin{bmatrix} C_{\phi_z} & S_{\phi_z} & 0 \\ -S_{\phi_z} & C_{\phi_z} & 0 \\ 0 & 0 & 1 \end{bmatrix} , \tag{19}$$

and

$$[M_{EO}] = \begin{bmatrix} C_{\lambda_y} C_{\lambda_z} & -C_{\lambda_y} S_{\lambda_z} & S_{\lambda_y} \\ S_{\lambda_z} & C_{\lambda_z} & 0 \\ -S_{\lambda_y} C_{\lambda_z} & S_{\lambda_y} S_{\lambda_z} & C_{\lambda_y} \end{bmatrix} . \tag{20}$$

In equation (18), ψ is the angle about Z_R that must be rotated to bring the X-axis into the orbital plane where X_R is defined. This angle is of no consequence, however, as we need only the third row of $[M_{RO}]$. After multiplying, then, the required components of $\vec{k}_R$ become

$$\vec{k}_R \Big|_{(X_o\text{ - component})} = U_1 = S_{\Gamma_y} C_{\phi_z} C_{\lambda_y} C_{\lambda_z} + S_{\Gamma_y} S_{\phi_z} S_{\lambda_z} - C_{\Gamma_y} S_{\lambda_y} C_{\lambda_z} \tag{21}$$

$$\vec{k}_R\Big|_{(Y_o\text{ - component})} = U_2 = -S_{\Gamma_y} C_{\phi_z} C_{\lambda_y} S_{\lambda_z} + S_{\Gamma_y} S_{\phi_z} C_{\lambda_z} + C_{\Gamma_y} S_{\lambda_y} S_{\lambda_z} \tag{22}$$

and

$$\vec{k}_R\Big|_{(Z_o\text{ - component})} = U_3 = S_{\Gamma_y} C_{\phi_z} S_{\lambda_y} + C_{\Gamma_y} C_{\lambda_y} \quad . \tag{23}$$

Thus we may determine: (1) the U_i from equations (21) through (23), (2) η_x and η_y from equations (15) and (16), and (3) δ_1 and δ_2 from equations (9) through (11). This last step goes as follows: Equations (9) through (11) may be represented by (with $S_2 = \sin \delta_2$, etc.):

$$S_2K_1 + C_1C_2K_2 = 0 \quad , \tag{24}$$

$$S_2K_3 + C_1C_2K_4 + S_1C_2K_5 = 0 \quad , \tag{25}$$

and

$$S_2K_6 + C_1C_2K_7 + S_1C_2K_8 = 1 \quad , \tag{26}$$

in which the K_i are constants for any position being examined, listed in Table 1 for completeness. In Table 1, Λ is $\theta - \eta_y$, which is the lead angle of the space station past the sunline's projection into the orbital plane.

Equation (24) yields

$$S_2 = -C_1C_2K_9 \quad , \tag{27}$$

and substituting equation (27) into equation (25) yields

$$C_1C_2K_{10} + S_1C_2K_5 = 0 \tag{28}$$

TABLE 1. CONSTANTS USED IN EQUATIONS (24) THROUGH (29).

i	K_i	
1	C_Λ	
2	S_Λ	
3	$-S_{\eta_x} S_\Lambda$	
4	$S_{\eta_x} C_\Lambda$	
5	$-C_{\eta_x}$	
6	$-C_{\eta_x} S_\Lambda$	
7	$C_{\eta_x} C_\Lambda$	
8	S_{η_x}	
9	T_Λ	[used in equation (27)]
10	S_{η_x}/C_Λ	[used in equation (28)]

so that, if $C_2 \neq 0$,

$$\delta_1 = \tan^{-1}\left[\frac{-K_{10}}{K_5}\right] \quad , \tag{29}$$

which turns out to be, when the constants are evaluated,

$$\delta_1 = \tan^{-1}\left[\frac{\tan \eta_x}{\cos \Lambda}\right] \pm \pi \quad . \tag{30}$$

If $\cos \delta_2 = 0$, it can be seen from equations (24) through (26) that gimbal angle δ_1 drops out of the equations. We are then left with the following resulting trivial cases (after evaluating various possibilities):

$$\eta_x = 0, \eta_y = \theta - \frac{\pi}{2}, \underline{\text{and}} \ \delta_2 = -\frac{\pi}{2} \tag{31}$$

or

$$\eta_x = 0, \eta_y = \theta - \frac{3\pi}{2}, \underline{\text{and}} \ \delta_2 = \frac{\pi}{2} \quad . \tag{32}$$

Returning to the nontrivial case, equation (24) yields, with equation (30),

$$\delta_2 = \tan^{-1}(-\tan \Lambda \cos \delta_1) \pm \pi \quad . \tag{33}$$

There are thus four pairs (δ_1, δ_2) given by equations (30) and (33), which satisfy equations (24) and (25). However, only two of these pairs also satisfy equation (26), the other two answers representing solar panels pointing away from the sun.

The results of an investigation of the proper quadrants which should be used to avoid more than 180-deg traverse in either direction are shown in Table 2.

TABLE 2. GIMBAL ANGLE QUADRANTS

	For η_x between 0 and $\pi/2$	For η_x between 0 and $-\pi/2$
Λ Quadrant	(δ_1, δ_2) Quadrants	(δ_1, δ_2) Quadrants
1	(1, 4)	(4, 4)
2	(2, 4)	(3, 4)
3	(2, 1)	(3, 1)
4	(1, 1)	(4, 1)

It is now a simple matter to transform the inertial properties from the B" frame to the SP frame by the standard tensor transformation

$$I_{ij}^{SP} = \sum_{k=1}^{3} \left[\sum_{\ell=1}^{3} a_{ki} a_{\ell j} I_{k\ell}'' \right] , \qquad (34)$$

in which a_{pq} is the direction cosine of the angle between the X_p'' axis and the X_q^{SP} axis. The computer program performs the transformation given in equation (34).

INERTIA TENSOR FOR THE SPACE STATION

The computation of the gravity gradient torque depends upon a knowledge of the inertial properties of the body referred to its mass center. Therefore, this section briefly describes how the inertia tensor is computed for the space station.

The last section described how inertial properties of the solar panel could be obtained with respect to axes through its center of mass parallel to the vehicle axes. This can be done even more easily for those modules whose inertial properties are known with respect to axes parallel to those of the vehicle; these are the modules attached to ports numbered 1 or 4 in Figure 5.

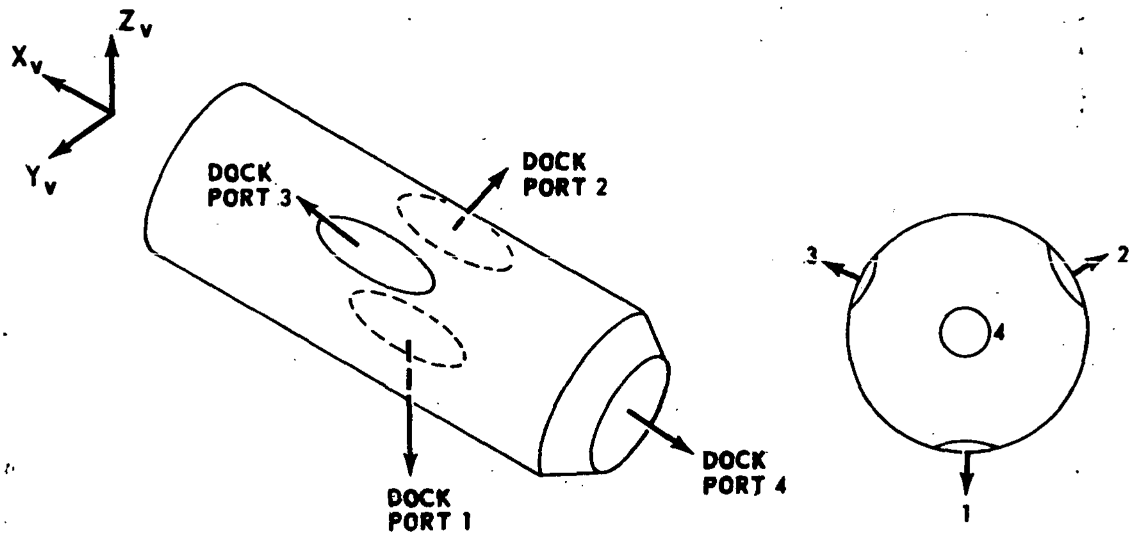

Figure 5. Dock port labeling convention.

For the modules attached to docking ports 2 or 3, one lateral axis is considered pointed parallel to X_v. Then, Mohr's circle is utilized to transform the inertial properties, known with respect to the other two axes, to axes through the mass center of the module parallel to Y_v and Z_v. This is done within the computer program; no transformations need be performed externally.

At this stage, we then have the inertial properties of panels and all modules with respect to their individual centers of mass. One by one, these are then transferred to the origin of the vehicle coordinate frame V by standard transfer theorems, such as

$$I_{\overline{X}\overline{X}} = I_{X_c X_c} + md^2 \quad , \tag{35}$$

$$I_{\overline{X}\overline{Z}} = I_{X_c Z_c} - mac, \text{ etc.} \quad , \tag{36}$$

in which $(\overline{X}, \overline{Y}, \overline{Z})$ are axes parallel to the mass center axes (X_c, Y_c, Z_c); m is the body's mass; d is the distance between $\overline{X}$ and X_c, and a and c are the X_c and Z_c coordinates of the origin of $(\overline{X}, \overline{Y}, \overline{Z})$ with respect to (X_c, Y_c, Z_c). In this application, $(\overline{X}, \overline{Y}, \overline{Z}) = (X_v, Y_v, Z_v)$.

At the same time that the inertias are being transferred, sums are being kept of the systems' mass and of the mass moments with respect to (X_v, Y_v, Z_v).

When all modules have been covered in this way, the mass center of the station is then determined simply by dividing the mass moments by the total mass. Then a transfer of the composite inertial properties from the V origin to the mass center of the station yields the inertias needed for the gravity gradient computations.

Before we proceed to these computations, however, the inertia tensor is checked by going back and transferring each panel and module's inertial properties directly to the now-known mass center. This will furnish a check not only on the inertia tensor but also on the mass center position calculations. This follows from the fact that if the mass center is incorrectly computed, the original inertia transfer from the V origin to this erroneous mass center will be incorrect because the formula used therein requires correct mass center coordinates.

If the station is orbiting in the Y-POP mode, the solar panel inertial properties are transformed and transferred in the manner just described at each station of the orbit. However, the modules' inertial properties are computed only once since they are fixed with respect to the vehicle.

When an inertial hold mode is called for, the inertial properties of the entire station are then frozen with respect to the vehicle axes. However, these combined properties must be transformed to the T-frame, or orbiting frame, of Figure 3, because it is with respect to T that gravity gradient torque computations become rather simple, as described in a later section. Hence, the computer is taught to perform this inertia transformation, which depends upon the orbiting angle θ .

PRINCIPAL MOMENTS OF INERTIA AND CORRESPONDING PRINCIPAL DIRECTIONS

This part of the procedure is concerned with determining principal moments of inertia which, along with their corresponding principal directions, are required for determining gravity gradient torques.

Beginning with an inertia tensor now computed with respect to axes through the space station's mass center, the computer program then calls on the UNIVAC 1108 MATH-PACK subroutine JACMX [3]. This subroutine computes the eigenvalues of the matrix of inertial properties; further, the corresponding eigenvectors are precisely the direction cosines (between the orbiting coordinates of Figure 3 and the principal axes) required in the gravity gradient torque computations to follow. It is noted that in the inertial hold mode, the inertia tensor must be transformed, to be in accordance with this procedure, to the orbiting coordinates; however, the Y-POP mode's tensor is already computed with respect to the orbiting T reference frame.

The method used by JACMX is a modified Jacobian method in which the matrix is diagonalized. This is accomplished by using elementary orthogonal transformations to annihilate successive off-diagonal elements. The matrix of eigenvectors is the product of all the transformation matrices.

GRAVITY GRADIENT TORQUE

It is well known that the gravity gradient torque acting on a rigid body is solely a function of its mass distribution, once its position in space is known. One commonly used form for this torque [4] is

$$\vec{M}_c = 3\omega^2\left[(\vec{j}\cdot\vec{I})(\vec{k}\cdot\vec{I})(I_3 - I_1) + (\vec{j}\cdot\vec{J})(\vec{k}\cdot\vec{J})(I_3 - I_2)\right]\vec{i}$$
$$- 3\omega^2\left[(\vec{i}\cdot\vec{I})(\vec{k}\cdot\vec{I})(I_3 - I_1) + (\vec{i}\cdot\vec{J})(\vec{k}\cdot\vec{J})(I_3 - I_2)\right]\vec{j} \quad , \tag{37}$$

in which, in the nomenclature of the present work,

ω = orbital speed of the space station for a circular orbit, which is 2π over the orbital period τ ;

$(\vec{i}, \vec{j}, \vec{k})$ = unit vectors in the T-frame of Figure 3, which are the same as those of frame V for the Y-POP mode;

$(\vec{I}, \vec{J}, \vec{K})$ = unit vectors in the principal directions;

(I_1, I_2, I_3) = principal moments of inertia with respect to the mass center, about $(\vec{I}, \vec{J}, \vec{K})$, respectively;

$\vec{M}_c$ = gravity gradient torque vector with respect to the body's mass center.

Nothing more need be said here except that the computer is programed to automatically compute the eight required dot products in equation (37) as soon as the eigenvalues (principal moments of inertia) and eigenvectors (principal directions) are found via the library program described in the preceding section.

After performing the arithmetic required in equation (37), the computer program yields the two gravity gradient torque components and their resultant.

Following the printing of these results, the program then increments θ and repeats the procedure until an orbit of Y-POP or inertial hold has been completed. The results obtained for several examples will now be presented and discussed.

RESULTS, CONCLUSIONS, AND RECOMMENDATIONS

The first computer run made was for a space station consisting only of a power module and solar panels. The angles were set simply to $\phi_z = \lambda_z = \Gamma_y = \lambda_y = 0$, thereby placing everything in the same plane for convenient checking.

The program was then debugged successfully by checking the masses, inertial properties, and gimbal angles over Y-POP and inertial hold orbits. Then the parameters were changed to $\phi_z = 23.5$ deg , $\lambda_z = 35$ deg , $\Gamma_y = 90$ deg , and $\lambda_y = 0$, and the results for the same station indicated a rise in gravity torque magnitude from zero at $\theta = 0$ to as much as 0.19 N-m in Y-POP and 0.28 N-m in inertial hold. The Y-POP change is caused solely by solar panel gimbaling, showing its importance for symmetric stations.

The space stations sketched in Figure 6 were then run for two reasons: (1) to further insure that the procedure and the entire program work correctly, and (2) to investigate such effects as solar panel gimbaling and asymmetry of the configuration in station buildups of practical importance.

The magnitudes of the gravity gradient torque on No. 12 for Y-POP and inertial hold modes are shown in Figure 7, along with the orbital parameters used and other data of interest. It is immediately observed again from the Y-POP curve that solar panel gimbaling has a definite effect on the torque. This follows from the observation that the station in the Y-POP mode would feel the gravity torque value at $\theta = 0$ throughout the orbit if the panels were not allowed to gimbal. However, with gimbaling, there is a spread in gravity torque magnitude of about 10 percent over the entire orbit of station No. 12. It is observed that the gimbaling has an even greater effect on station No. 18 (Fig. 8), the spread in torque being about ±15 percent over its orbit. Note from Figure 9 that the largest changes in Y-POP torque correspond closely to the largest changes in the gimbal angles, as expected. Hence, we conclude that solar panel gimbaling is a significant factor to be considered in sizing control moment gyros to meet momentum requirements.

Figures 7 and 8 also indicate interesting results for the inertial hold modes for these two stations. (Note that both curves indicate the periodicity with respect to π that was expected. The torque should indeed be periodic, with twice orbital frequency.) A very important result that can be seen in these figures is that there is about a threefold increase in the bias, or noncyclic, portion of the torque when going from station No. 12 to station No. 18. A run made for an even more unsymmetrical station (No. 47, not shown) showed an even higher bias torque than No. 18, having peak-to-peak magnitudes of 0.15 to 11.52 N-m. A run was also made in which stations Nos. 12 and 18 were locked into inertial hold modes at another orbital angle, with similar results. Thus, a reasonable conclusion seems to be that each proposed asymmetric station configuration should be separately examined and that "average geometry" cases should be avoided.

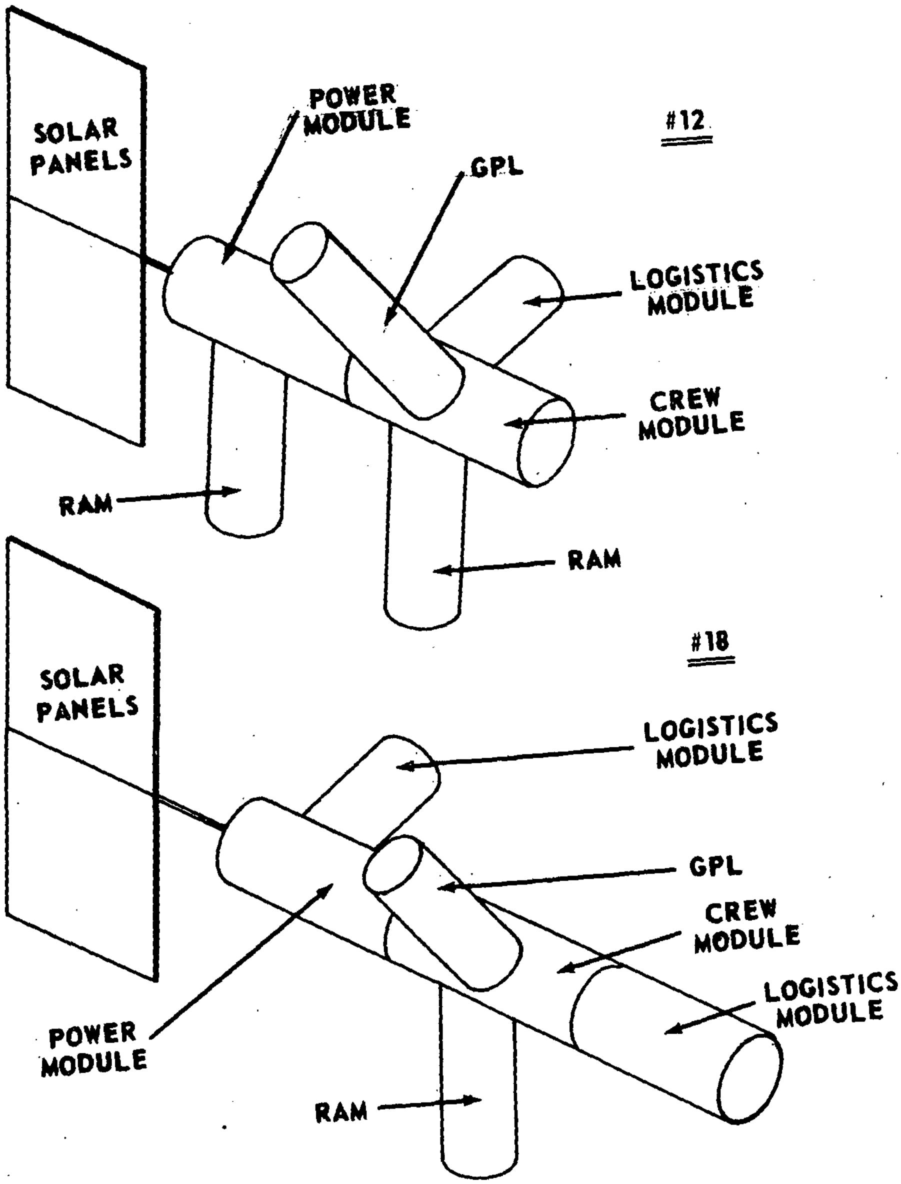

Figure 6. ISS configurations studied.

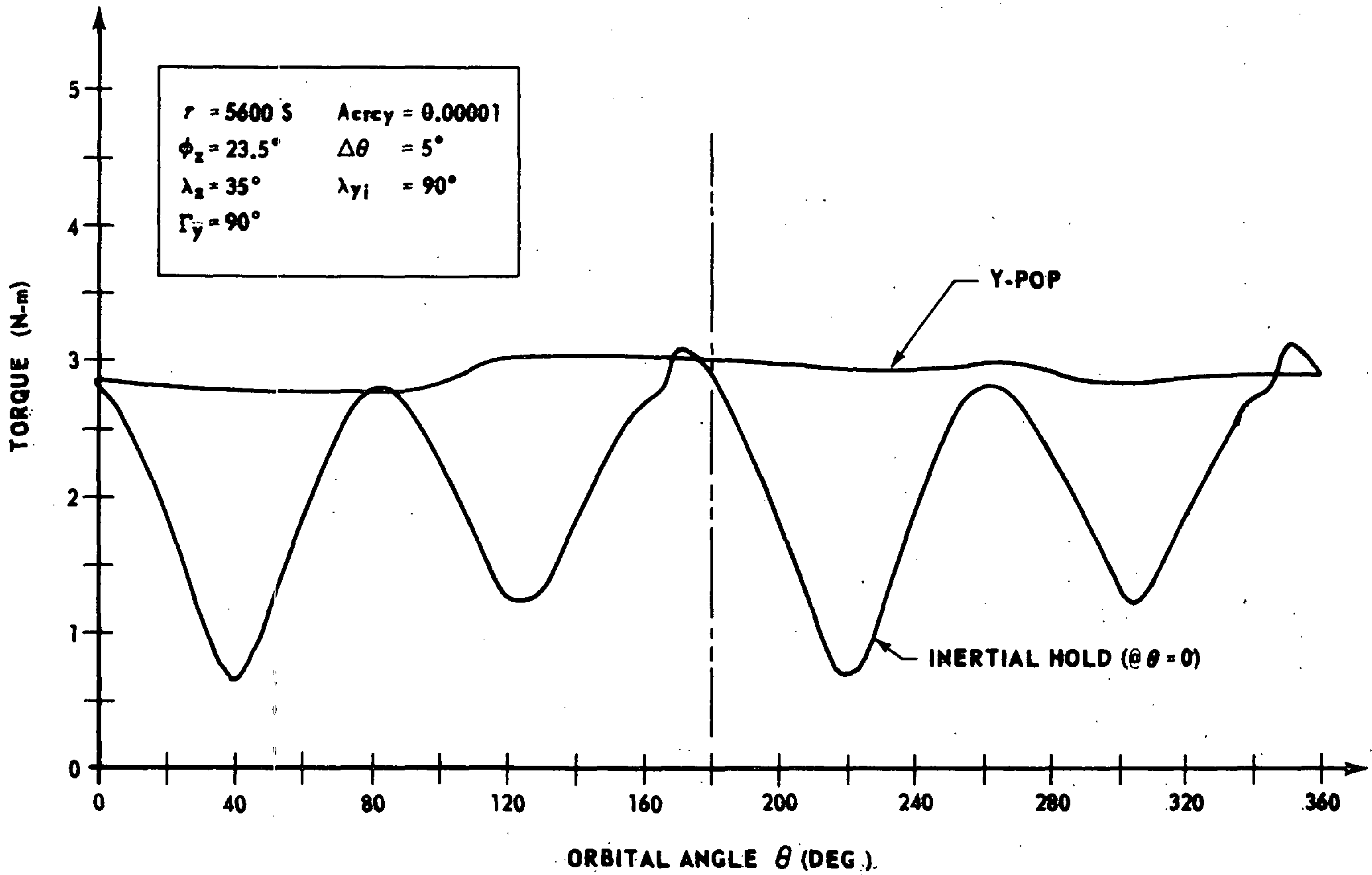

Figure 7. Gravity gradient torques on No. 12.

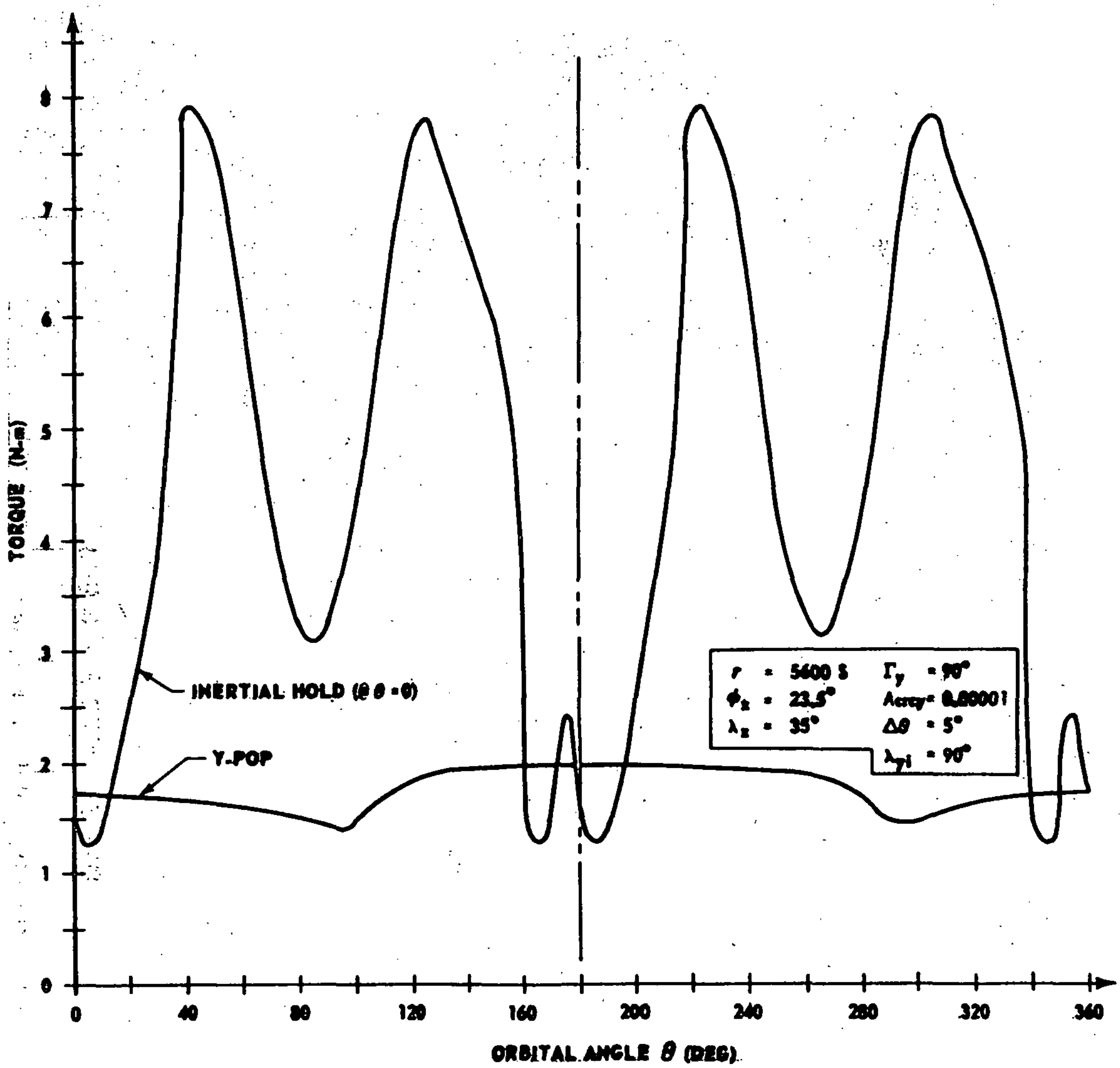

Figure 8. Gravity gradient torques on No. 18.

Some recommendations for extensions of this work are now tabulated. In future work, we wish to include:

1. The capability to trim the axes

2. The computation of angular momentum requirements

3. The ability to feather the solar panels so as to minimize angular momentum buildup during loss of sunlight

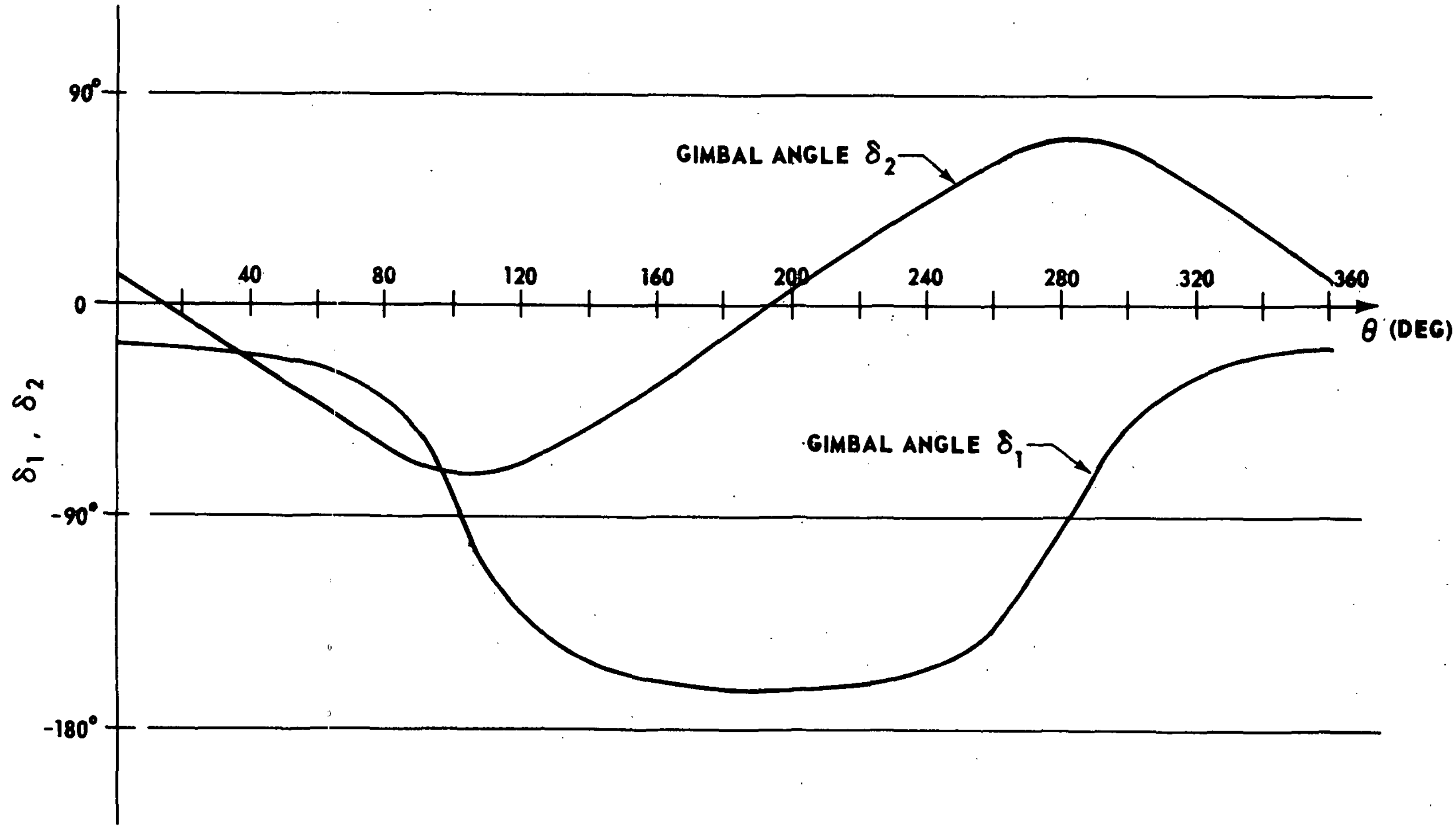

Figure 9. Gimbal angles for orbit of Figures 7 and 8.

4. The forming of the GSS station by adding a second power module and solar panels

5. The capability to directly feed in the beta angle (η_x) instead of the parameters used by the writer; also to feed in a precalculated inertia tensor or to feed out the internally computed inertial properties and/or gimbal angles for other uses

6. The writing of the differential equations of rigid body motion of the station (possibly including aerodynamic torques) and their examination for stability.

Most importantly, we wish to be able to rotate the station through any angle at any point in its orbit and have it remain fixed in that position for a given time, during which torque and momentum would be computed.

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APPROVAL

GRAVITY GRADIENT TORQUE PROFILES OVER AN ORBIT FOR ARBITRARY MODULAR SPACE STATION CONFIGURATIONS IN THE Y-POP AND INERTIAL HOLD MODES

By David J. McGill

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

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