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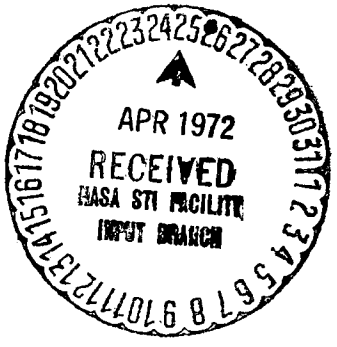
PRESENT-VALUE ANALYSIS: A SYSTEMS APPROACH TO PUBLIC DECISIONMAKING FOR COST EFFECTIVENESS

By Theodore T. Herbert
Program Development

August 20, 1971



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16. ABSTRACT

Present value analysis is an analytical tool which furnishes inputs for the decision-making process. Decisionmakers within Governmental agencies and Congress must somehow evaluate competing (and sometimes conflicting) proposals which seek funding and implementation. These evaluations are typically done on a basis of estimated costs, which consider only the direct, out-of-pocket charges to be disbursed, as they impact a particular organizational unit.

Present-value analysis can be an effective decisionmaking tool by enabling the formal evaluation of the effects of competing proposals on efficient national resource utilization. A project's costs are not only its direct disbursements, but its social costs as well. How much does it cost to have those funds diverted from their use and economic benefit by the private sector to the public project?

While obviously not the only effective decisionmaking input, comparisons of competing projects' social costs allow decisionmakers to expand their decision bases by quantifying the projects' impacts upon the economy and the efficient utilization of the country's limited national resources.

A conceptual model is established for the choosing of the appropriate discount rate to be used in evaluation decisions through the technique.

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Government decisionmakers have been concerned with the problem of how to appropriately evaluate all alternative solutions to a problem. This question assumes even more importance in view of the fact that, from a national point of view, the resources available (manpower, money, machines, and technology) to solve existing problems are limited.

National resources must then be allocated among the various governmental programs. Each of these programs is (either explicitly or implicitly) assigned a priority based on the program's contribution to the nation's goals. High-priority programs are usually fully funded, while lower-priority program funds are eliminated or cut from what was requested.

Each program's administrator is ultimately responsible for maximizing his program's contribution to national goals while minimizing drain on national resources. This is done by determining and recommending the most cost-effective project in attaining a particular goal (solving a particular problem) which supports the administrator's overall mission. A program typically consists of a number of such projects. Should one or more of these supporting projects be less effective than planned, the effectiveness of the overall program is impaired.

A program that does not contribute sufficiently suffers a decrease in funding. The administrator then must operate with reduced funds and still maintain his capability to achieve his objectives as effectively and efficiently as possible.

Before seeking funding, then, an important input to the administrator is the evaluation of competing projects from the point of view of each project's impact on the economy, the private sector (taxpaying individuals and organizations, from which the funding is obtained), mission effectiveness, and efficient use of funds. A project that is (from an administrative, scientific, or engineering viewpoint) sounder or more sophisticated than the other projects with which it is competing could be disapproved; its greater costs, for instance, might not be accompanied by benefits sufficiently greater than those arising from its "cheaper" competitors to justify the extra expenditures.

An analytical tool known as "present-value analysis" is available. The present-value (PV) technique allows the analyst to consider the impact

of the project under analysis on allocation of national resources and alternative uses of the funds required by the project. It further allows the direct comparison of projects using the single criterion of present dollars.

The use of this analytical tool yields an added dimension of information to the decisionmaker which, when added to other analyses, makes possible the selection of the candidate project which fulfills its mission with the least possible expenditure and least possible social cost for a given level of effectiveness.

One major reason that PV analysis has been regarded skeptically by some is the very inclusion of national resource utilization in the analysis. In addition, the human tendency is to analyze a problem primarily as it directly affects the analyst and his organization and subordinate or ignore the impacts on others (even if these others are the Nation as a whole). When these "external" impacts are factored into the analysis as independent variables, some may regard the dependent variable (the PV input to the decisionmaking process) as largely irrelevant.

It is, of course, extremely valuable to formally consider questions of national resource allocation in public decisionmaking. Subjective perception to the contrary, the Nation's resources are not unlimited and must consciously be used as effectively and efficiently as possible. PV analysis allows these issues to be formally addressed.

The following treatment is intended to furnish a conceptual introduction to an element of cost analysis and cost effectiveness. This subsector is known as PV analysis and can furnish an added analytical dimension to the decisionmaker.

Specific illustrations are furnished in Section V on practical applications of the technique. The cost analyst and/or decisionmaker can use this volume as a resource, conceptual overview and introduction, and operational manual.

II. OVERVIEW OF PRESENT-VALUE ANALYSIS

A. The Decisionmaker's Quandary

Typically, the decisionmaker (within agency or Congress) must decide among several different solutions to a problem. If, for example, the problem consists of placing payloads in earth orbit, the decisionmaker might have to decide among several feasible launch vehicles. Any of these vehicles might have the capability of fulfilling the mission (placing the payloads in orbit). Since the candidate vehicles have different characteristics and are at differing

stages of development or operation, some common denominator is sought for comparing the alternatives. The common denominator for a future space program is usually that of the dollars involved in each vehicle's fulfilling the mission. Given the equal capability of alternatives to fulfill the mission, the project having the least costs associated with it is typically considered as being the best of the candidates.

Unfortunately for the decisionmaker, however, the criteria upon which he bases his choice are more complex than is immediately obvious. How does one, for example, equate projects which have different estimated time spans in which the mission is fulfilled? Or projects whose patterns of funding requirements differ drastically? Or projects whose risks of failure are significantly different?

B. A Question of Which Level of Cost to Use

Compounding the problem is the problem of which costs the decision-maker must consider in his analysis. One's pragmatic answer might well be that costs are costs and the total dollars required for each project should be the decision criterion.

The question might then be restated: Costs to whom? The decision-maker should, of course, be concerned with the dollar impacts of each candidate project upon his unit, whether it be an office or an agency. He must also remember that his unit does not exist in a vacuum, independent of other units. The actions taken within or by any unit affect the surrounding units. This phenomenon is akin to the dropping of a pebble into a still pond; the resulting ripples extend over a much larger area than that first affected by the pebble.

In a similar fashion, a unit's commitment to spend X dollars to solve a problem at the unit's level may have far-reaching consequences. The organization (of which the unit in question is but one part) probably operates with a specified budget which must be allocated among its units. Unplanned expenditures would seriously impair the ability of the entire organization to function effectively by decreasing the funds available to meet overall organizational goals. This process could be extended indefinitely. Ultimately, one could trace the impact from a rather small public (governmental) unit to the entire private sector. The unit's commitment was to spend not only its funds, but those of the ultimate source of those funds — the private sector, taxpayers.

When the private sector must furnish the project funds, the project should be compared to the uses of the funds if they were not appropriated. The assumption (and rationale) is that a public project should not be funded if it costs more than its private usage, or if the public benefits from the

project are less than the benefits which would result from its use in the private sector.

Since the impact of the decision is ultimately broader than its immediate effect upon the operating unit, the decisionmaker must take recognition of the different types and levels of costs that are involved. The traditional costs (direct expenditures; out-of-pocket costs) are important for their impact upon the unit and the organization to which it belongs. The "social opportunity" cost cannot be ignored; the use to which the private sector would have put those funds is just as important an input to the decisionmaker. Efficient national resource utilization must be an element considered in each public-sector investment decision.

C. Summary of the Project Evaluation Problems Facing The Decisionmaker

The projects under consideration may be as follows:

1. Characterized by different patterns of dollar flow.

a. Projects may require different lengths of time to achieve the mission.

b. Projects may be characterized by different timing of dollar flows; e.g., some may require relatively early large expenditures while others may require large expenditures relatively late in the project life.

2. Vying for limited funds which must be allocated among the missions.

3. Vying for limited funds which may be more efficiently utilized if left in the private sector; i.e., not appropriated through taxes.

4. Contribute to goals of less importance to the Nation as a whole than the goals of other projects. These other projects may be either in the public or private sector.

D. PV (Discounting) Analysis

The PV technique for evaluating alternative projects is an extension of other analytical methods. As inputs to the decisionmaking process , cost analysis methodologies are oriented strictly toward determining and comparing costs within a closed system. Using these typical tools, for example, the cost analyst will determine funding projections, total and unit costs, and cost per unit of effectiveness for alternative projects. Each of these measurements

provides pieces of the picture which compares the projects under consideration, investigates objectives attained, and other impacts as they directly relate only to a subsystem within the entire socio-politico-economic system known as the United States of America.

These analytical exercises allow then a subsystem (e.g., a branch, department, agency, etc.) to attain its goals with optimum effectiveness. While the effects of a subsystem's choice of action upon the entire system may be referred to, an analytical tool has not heretofore been available to objectively consider and quantify the impact of a particular choice of action upon the entire system — in this case, the Nation as a whole.

1. Cash Prize Example. To introduce the PV concept, consider which one of the following prizes you would choose to receive, if you were given the opportunity by winning a contest.

1. $ 100 immediately.
2. $ 100 one year from now.
3. $ 55 now and $ 55 one year from now.
4. $ 25 now and $ 25 at the end of each of the next 4 years.
5. $ 200 ten years from now.[1]

It is likely that different persons would make different choices, but among the factors which would enter into the decisions of most people would be their current need or desire for money, expectations about inflation, and investment opportunities available for earning a return on money. These are the primary factors which result in money having what is known as "time value." The value of a dollar depends upon the time it is received. A dollar today is worth more than a dollar next year, and that dollar in turn is worth more than a dollar two years hence. Today's dollar exceeds the value of next year's dollar by at least the interest that could be earned on the dollar if it were put in a savings account for one year. Thus, if a savings bank pays 5 percent interest, a dollar today is worth at least as much as $ 1.05 a year from now.

Because of this time value of money, we cannot directly compare dollar amounts received at different points in time. Such dollars

1. This list of options is taken from "The Time Value of Money: Interest Formulas and Computations," by Jerome D. Wiest, p. 1, Humble Oil Company, Humble Marketing Education-Investment Analysis, undated.

are different from each other, just as apples and avocados are different. We cannot directly add or subtract dollars received at several different times any more than we can add apples and avocados, unless we find an equivalence measure — a way of converting dollars at one point of time to their equivalent value at another point of time.

In the example first cited, you have the problem of comparing alternate prizes in which the dollars are received over various periods of time. You can easily eliminate choice 2, since 1 is for the same amount but is available a year earlier. Even though the dollar amounts are the same, prize 1 has greater time value than prize 2, since the former could be invested and earn interest over the year. Prizes 3, 4, and 5 are not as easily compared. Prize 5 is largest in actual dollars ($ 200), but 10 years is a long time to wait for it. You may well prefer $ 100 today. Prize 4 totals $ 125 and exceeds prize 3's $ 110, but the former spreads over a longer time span. All three of these exceed the amount of prize 1, but the latter has the largest sum available immediately.

Somehow you must find a way of making the different sums more directly comparable. For example, consider prize 3: if you could change the $ 55 received a year from now to an equivalent value at the present time, then you could add it to the $ 55 received now and see if the total is worth more than $ 100 of prize 1. Similarly, if you could find the equivalent in today's dollars of $ 25 per year for 4 years, or of $ 200 ten years from now, you could compare prizes 4 and 5 with prize 1 (and with each other). Since a dollar in the future isn't worth as much as a dollar today, in some way you must reduce future dollars to find their present equivalent. The amount of reduction depends upon the rate of return we are able to obtain in the best investment available ("best" in the sense of highest expected return on investment, all risks considered). The higher the returns, the more valuable are present dollars as compared to future dollars.[2]

Table 1 shows an approach to the problem. The time span for the decision impact is based upon the year in which the last direct cash payment of the options may be received.

All alternatives are then compared over the time span paced by the cash receipt of $ 200 in the 10th year. Assuming a 5-percent bank interest rate, one may compute the total cash impact of each option. Thus, for Option 1

2. ibid., pp. 1 and 2.

TABLE 1. COMPARISON OF TOTAL INCOMES FROM OPTIONS

Option	Cash Flow in Year											Total Direct Cash	Total Interest[a]	Total Income
	0	1	2	3	4	5	6	7	8	9	10			
1	+100.00	-	-	-	-	-	-	-	-	-	-	100.00		162.89
interest	-	5.00	5.25	5.51	5.79	6.08	6.38	6.70	7.04	7.39	7.76		62.89	
2	-	100.00	-	-		-	-	-	-	-	-	100.00		155.13
interest	-	-	5.00	5.25	5.51	5.79	6.08	6.38	6.70	7.04	7.39		55.13	
3	+55.00	55.00	-	-	-	-	-	-	-	-	-	110.00		174.90
interest	-	2.75	5.64	5.92	6.21	6.52	6.85	7.20	7.56	7.93	8.33		64.90	
4	+25.00	25.00	25.00	25.00	25.00	-	-	-	-	-	-	125.00		185.10
interest	-	1.25	2.56	3.94	5.39	6.91	7.26	7.63	8.01	8.41	8.83		60.10	
5	-	-	-	-	-	-	-	-	-	-	200.00	200.00		200.00
interest	-	-	-	-	-	-	-	-	-	-	-		-	

a. "Total interest" figures may not be exactly the sum of the individual interest entries because of rounding-off of numbers.

($ 100 immediately), in the year after receipt of the $ 100 one earns $ 5 interest.[3] Leaving that interest in the account, the next year one would receive $ 5.25 interest (5-percent interest on both the $ 100 principal and the $ 5 accrued interest). In similar fashion, the interest earned each year is computed. At the end of the time frame, the total interest accrued under Option 1 ($ 62.89) is added to the direct cash prize of $ 100 for a total cash impact of $ 162.89.

The total cash impacts of each of the other options are derived by following the same technique. The option with the largest total cash impact figure would probably be the one the contest winner should choose, all other things being equal. Thus, Option 5 ($ 200 ten years from now) yields the greatest cash impact; from an economic and rational point of view, that option should be chosen.

The point may be made that many contest winners would choose an option other than that demonstrated to be economically optimum; immediate gratification of desires and needs may be deemed more important than long-run economic optimization.

It must be remembered, however, that the calculations above did not attempt to make the decision for the contest winner; only the winner can decide. Analysis can only furnish objective inputs, upon which the decision-maker may base his choice. The extent to which these inputs are relied upon is not a function of the analysis.

One should notice that, in the analysis above, the interest available from each option plays an important part. Whether the winner elects to accrue that interest is, again, beside the point. The accrual of interest is available; the actual choice to exercise the accrual is outside the scope of the analysis and recommendation.

2. Cash Payment Example. The principles behind the contest example are as readily applicable to costs (outgo) as they are to revenues (income). Table 2 shows the same problem with one fundamental change: instead of receiving cash in specified years, one must now decide which option to choose in paying out cash. One may elect to pay one of the following:

3. This example assumes the prize to be awarded at the beginning of the year with interest earned at the beginning of the next year.

TABLE 2. COMPARISON OF NET OUTLAY FROM OPTIONS

Option	Cash Flow in Year											Total Cash Outlay	Total Interest Income[a]	Net Outlay
	0	1	2	3	4	5	6	7	8	9	10			
1	-100.00	-	-	-	-	-	-	-	-	-	-	100.00		100.00
interest	-	-	-	-	-	-	-	-	-	-	-		0	
2	-	-100.00	-	-	-	-	-	-	-	-	-	100.00		92.24
interest	5.00	0.25	0.26	0.28	0.29	0.30	0.32	0.34	0.35	0.37	-		7.76	
3	-55.00	-55.00	-	-	-	-	-	-	-	-	-	100.00		105.73
interest	2.75	0.14	0.14	0.15	0.16	0.17	0.18	0.18	0.19	0.20	-		4.27	
4	-25.00	-25.00	-25.00	-25.00	-25.00	-	-	-	-	-	-	125.00		106.51
interest	5.00	4.00	2.95	1.85	0.69	0.72	0.76	0.80	0.84	0.88	-		18.49	
5	-	-	-	-	-	-	-	-	-	-	-200.00	200.00		74.22
interest	10.00	10.50	11.03	11.58	12.16	12.76	13.40	14.07	14.77	15.51	-		125.78	

a. Figures may not exactly equal the actual sums from their corresponding interest rows because of rounding-off of numbers.

1. $ 100 immediately.
2. $ 100 one year from now.
3. $ 155 now and $ 55 one year from now.
4. $ 25 now and $ 25 at the end of each of the next 4 years.
5. $ 200 ten years from now.

Which choice should be made?

In the cash prize example, the choice between dissimilar amounts of money was complicated by the dissimilar timing of their receipts. There, the solution was pointed to by considering the total of the cash received plus the amount this cash would earn upon investment. Because of the effect of time upon earnings from investment, an option whose absolute direct cash value was less than another's might be chosen. The investment of the former's cash could conceivably accrue to an extent such that the total of its cash plus earnings would exceed that of the other option.

The present expenditure example in a similar fashion considers, in addition to direct cash outlay, the effect of the timing of the outlay. In Option 1 (Table 2), one pays $ 100 immediately; no investment opportunity exists for the payor with that $ 100. In Option 2, however, by delaying payment of the $ 100 for 1 year, one may invest that amount (at the assumed 5-percent rate) until the $ 100 is due. In the first year, a return of $ 5 can be realized on the investment. Withdrawing the $ 100 to pay the debt still leaves the $ 5 available to accrue interest on itself over the decision period. Interest is drawn on the original return (left to accumulate more interest) until a total of $ 7.76 is accrued. Then, the out-of-pocket impact of Option 2 is $ 92.24 (the $ 100 disbursement minus the $ 7.76 interest earned). One might rationally consider this to be a superior alternative to Option 1, whose direct cash impact was that of an outlay of $ 100 rather than a net outlay of only $ 92.24.

Similarly, with Option 3 one pays $ 55 immediately but the remaining $ 55 (the deferred payment) may be invested until it is due. Once again the return from this investment is left to accrue interest on itself after the second payment is withdrawn. Option 3 requires a direct cash outlay of $ 110 while affording the opportunity to return $ 4.27 as interest. Then, this option costs only $ 105.73. When compared to the preceding alternatives, however, Option 3 seems inferior as the others "cost" still less while satisfying the debt.

A more complicated problem, Option 4, follows the same principles. The alternative of paying $ 125 in five payments costs only $ 106.51 when the interim returns from the dollars earmarked for future expenditure are considered.

Option 5 allows the deferral of payment for 10 years, but $ 200 must then be paid. Investing that $ 200 in the meantime and allowing the interest to accrue, however, has a great impact upon the decision. Despite the relatively great size of the required payment, interest of $ 125.78 accrues so that the option actually costs only $ 74.22. This net outlay is by far the least of any option, suggesting its superiority, other things being equal.

3. Time Value of Money. These examples show that (to paraphrase) all dollars are not created equal. The timing of their receipt (or disbursement) directly affects their value. A dollar due in the future has less value to the creditor than one due immediately. Conversely, a dollar due in the future has more value to the debtor than one due immediately. The reason may simply be stated as the use to which the amount may be put by the parties affected.

A debtor may invest the amount owed until it is due, thereby lessening the out-of-pocket impact of the disbursement. This requires, however, that the creditor be forced to sacrifice the return he could have made had this amount been due immediately. The value of present and future dollars, accordingly, varies.

4. Present Values and Social Costs. These examples have illustrated one perspective from which to consider evaluation decisions. The concept has been established that, indeed, the timing of money is important; a further step may be taken to simplify and systematize evaluation decisions while reflecting the impact of the time-value-of-money concept.

The public decisionmaker is rightly concerned with alternative projects as they (for example) demonstrate peak funding requirements, total funding demanded, etc. These items are relatively easy to quantify.

What is less explicitly an input into the decisionmaking process is the ability and willingness of the Congress (and the people) to make the investment in one of the projects which will fulfill the mission.

An analogy may be readily drawn. The creditor who was trying to decide in which manner to pay a debt is similar to Congress. Congress

(representing the people) tries to evaluate projects and programs in the light of the best way to achieve national goals with limited resources and with minimum negative consequences upon the Nation as a whole.

The program administrator obviously should subject his alternatives to the same types of criteria. In so doing, he explicitly recognizes the social costs associated with his mission. Costs are considered from the point of view of the society rather than from largely parochial viewpoints.

One way in which this social view of costs may be incorporated into the analysis is by way of extending the cost example of Table 2. Instead of investing funds in a bank, the private sector (individuals and organizations who must pay through taxes the debt called for by acceptance of a project) invests its idle funds in the economic system. The economic system contains many individual investment opportunities (such as banks, stocks and bonds, capital equipment, etc.). Suffice it to say that the private sector's idle funds are typically put to work in ways so as to generate returns (interest) from these investments.

Should one know the timing and amount of disbursements required by alternative projects, the problem of minimizing the required investment may be addressed. If a taxpayer is required to pay out $ 1 immediately, he is deprived of that dollar, its use, and what it could have bought.

If, on the other hand, he is required to pay $ 1 a year from now, he may invest 95.2 cents now, earn 4.8 cents (assuming the 5-percent interest rate), and thereby pay the dollar next year. The deferral of the $ 1 payment by 1 year saves the taxpayer 4.8 cents.

Deferring payment of that dollar another year enables the taxpayer to invest 90.7 cents now and accumulate 9.3 cents of interest over the next 2 years. The first year he earns 5-percent on the 90.7 cents (4.5 cents interest), and the second year he earns 5-percent on 95.2 cents (by leaving the first year's interest to accrue further interest) or 4.8 cents interest. Deferring payment of the dollar 2 years saves the taxpayer 9.3 cents.

Table 3 illustrates the use of the concept by applying it to the cost example of Table 2. The contest analogy will be dropped in favor of the private-sector analogy.

Under Option 1, the private sector must pay $ 100 in taxes immediately. Taxpayers may be thought of as investing $ 100 for no time

TABLE 3. EFFECT OF COST OPTIONS ON AMOUNT OF IMMEDIATE INVESTMENT REQUIRED

Option	Type of Cash Flow	Cash Flow in Year											Total Disbursement	Total Investment	Total Interest[a]
		0	1	2	3	4	5	6	7	8	9	10			
1	Disbursement	100											100	100[b]	-
	Investment	-													
	Interest[c]	-													
2	Disbursement		100										100	95.24	4.76
	Investment	95.24													
	Interest	4.76	-												
3	Disbursement	55	55										110	107.38[b]	2.62
	Investment	52.38													
	Interest	2.62	-												
4	Disbursement	25	25	25	25	25							125	113.65[b]	11.35
	Investment	88.65													
	Interest	4.43	3.40	2.32	1.19	-									
5	Disbursement											200	200	122.78	77.22
	Investment	122.78													
	Interest	6.14	6.45	6.77	7.11	7.46	7.84	8.23	8.64	9.07	9.52	-			

a. Figures may not exactly equal the actual sums of their corresponding interest rows because of rounding-off of numbers.

b. Reflects the investment of the disbursement required in year 0 (an investment with zero duration and zero interest) so that all numbers in this column may be directly comparable.

c. Interest rate of 5 percent.

(and no interest), then making the required payment. This option costs (out-of-pocket) $ 100. No savings through interim investment are possible.

Option 2 allows a 1-year interim investment (by deferring the $ 100 payment); $ 95.24 is the balance immediately required to obtain $ 100 1 year from now. Interest of $ 4.76 is earned, so that the present value of $ 100 a year from now is $ 95.24.

The third option combines the characteristics of the first two. An immediate disbursement (of $ 55) and one a year from now are required. The private sector can invest $ 107.38 and immediately withdraw $ 55 for the first payment; $ 52.38 is left to earn $ 2.62 interest, whereupon that $ 55 is withdrawn to make the second payment.

Similarly, investing $ 113.65 allows the disbursement of four annual payments of $ 25 each, commencing immediately, in Option 4. The immediate withdrawal of $ 25 leaves $ 88.65 to earn interest. In the first year, $ 4.43 is earned, giving a total of $ 93.08, and the second payment is made from this, leaving $ 68.08 invested. In the second year, $ 3.40 is earned, giving a total balance of $ 71.48. The third disbursement leaves $ 46.48 available to draw interest of $ 2.32 during the third year, giving a total of $ 48.80. Continuing this process leaves the final disbursement exhausting the investment account.

For Option 5, investing $ 122.78 over 10 years accrues $ 77.22 interest. The total of these sums is sufficient to make the $ 200 disbursement in the 10th year.

The investments referred to above are the aggregated day-to-day routine investment of the private sector of the economy. Given equal tax appropriations due at different points in time, for mutually-exclusive projects, the private sector can better afford the later taxes because of the longer private usage of the funds; the out-of-pocket impact of the appropriation is thereby diminished, resulting in a (in effect) cheaper project even though actual costs may not differ between projects.

From this, one may deduce the appropriateness of the term "present value" as it is applied to the residual of the required disbursement over the interim return earned. The value, at the present, of a future disbursement is the amount required for immediate investment such that the sum of the investment and its accrued interest just equals the amount of the required disbursement when due.

This PV concept is readily applied to evaluation decisions. The project with the lowest PV is the project which requires the private sector to pay the lowest net amount of money. This net amount derives from the private sector's ability to invest this amount in inself, earn returns, accumulate compounded interest, then pay the principal and accrued interest when the disbursement is scheduled.

E. Strengths of the Present-Value Technique

Remembering that the PV technique functions as one tool in the decisionmaker's analytical kit, the strengths of the technique may be summarized as follows:

1. Allows alternative projects to be directly compared in terms of equivalent dollars (dollars today, with the complicating effect unravelled of dollars at different points in time).

2. Reduces differing characteristics of projects to their lowest common denominator — today's dollars.

3. Consciously considers the impacts of the proposed projects upon the Nation's economy.

4. Consciously considers the impacts of the proposed projects upon the private sector from which the funding ultimately derives.

5. Provides a framework for summarizing and presenting the results of analyses pertinent to the problem.

6. Furnishes decision emphases which differ appropriately with hierarchial level of the decisionmaker (e.g., the cost analyst can make relatively low-level decisions on the promise of further efforts on a project; an agency administrator can choose among different approaches to a problem, involving millions of dollars, based on their impacts on national resources).

7. Allows future cost estimates less and less immediate impact on the decision, the further away in time the estimates are. Since the validity of cost estimates decreases as the time span over which they are made increases, this lessened impact on the decision by less credible estimates is appropriate.

8. Forces the analyst/decisionmaker to carefully establish objectives, performance criteria, specific schedules, cost and forecasting data, and items pertinent to the decision itself; i.e., excluding from the analysis irrelevant data.

F. Weaknesses of the Present-Value Technique

A phenomenon typically occurs when a new, not-well-understood technique is referenced. Characteristics are attributed to the technique and its end-product results which range far from the true nature of the tool. Typical reactions range from considering it to be the ultimate, all-encompassing device to dismissing it as totally irrelevant. The true nature of any analytical advance is somewhere in between. Familiarity with the technique, its objectives, and its constraints and limitations allows intelligent usage of the technique. Some of the important weaknesses of the PV analysis are as follows:

1. Fairly difficult concept to understand and/or accept, especially when the analyst/decisionmaker has been trained to deal with real costs (i.e., those costs which pertain directly and only to the specific problem addressed rather than considering the wider impacts of the solution).

2. Contributing to 1. above is the invisibility of the costs considered in the PV analysis. Although these costs physically exist, the specific decisions which gave rise to them may be masked by the complexity of the economy.

3. Modifies a common concept dollar into a slightly different measure.

4. Is not and cannot be used as the single decision input; PV analysis is one of a range of analytical tools, each of which focuses primarily upon a specific element of the problem.

5. Is only as valid an analytical device as its inputs (forecast costs and schedules) are sound.

III. THE DISCOUNT RATE AND ITS EFFECT

Table 3 illustrates the amounts required now to yield the disbursements associated with each option under consideration. This immediate investment

accrues a return (interest) until the disbursement date; the interest and the investment principal are then withdrawn to make the scheduled payment, leaving a balance of zero.

One might well ask the impact of the interest rate at which the principal earns a return. Can the choice of the rate affect the decision? How is the rate determined? What influences the appriopriate rate to use?

A. Discount Rate, Return, and Investment

Tables 3 through 7 show the effect that the choice of interest rate has on the investments needed now (present values) to accumulate to the total required by the several options; the differences arise from the fact that the interest rate chosen determines the return for any year. When larger returns (from larger interest rates) are allowed to accrue further interest, a relatively small initial investment can quickly snowball.

The potential impact of changes in the discount rate on the present value is seen by considering the equation which gives the present value of a future disbursement. The discount rate is i, n is the number of years hence that the disbursement is scheduled, and $ represents the amount of the disbursement.

$$PV = (\$)\left[\frac{1}{(1+i)^{n}}\right].$$

An inverse relationship exists between the present value and discount rate, which is magnified by the number of years hence the disbursement is scheduled as an exponent. The rate chosen can have a powerful influence on the analysis under consideration.

Direct comparisons of the initial investments required for the options at differing interest rates are given in Table 8. For any given option (except the first) the change in the immediate investment required is dramatic. For Option 4, for example, with the low rate of return of 3-percent for interim investment opportunities, $ 117.93 is needed to fulfill the disbursement schedule. As the rate increases, less and less is needed in year 0; at 20-percent, only $ 89.72 is required now to fulfill the disbursement schedule.

TABLE 4. EFFECT OF DISCOUNT RATE ON AMOUNT OF IMMEDIATE INVESTMENT REQUIRED (r OF 3 PERCENT)

Option	Type of Cash Flow	Cash Flow in Year											Total Disbursement	Total Investment	Total Interest[a]
		0	1	2	3	4	5	6	7	8	9	10			
1	Disbursement	100											100	100[b]	
	Investment	-													
	Interest	-													
2	Disbursement		100										100	97.09	2.91
	Investment	97.09													
	Interest	2.91	-												
3	Disbursement	55	55										110	108.40[b]	1.60
	Investment	53.40													
	Interest	1.60													
4	Disbursement	25	25	25	25	25							125	117.93[b]	7.07
	Investment	92.93													
	Interest	2.79	2.12	1.44	0.73	-									
5	Disbursement											200	200	148.82	51.18
	Investment	148.82													
	Interest	4.46	4.60	4.74	4.88	5.02	5.18	5.33	5.49	5.66	5.83	-			

a. Figures may not exactly equal the actual sums of their corresponding interest rows because of rounding-off of numbers.

b. This figure reflects the investment of the disbursement required in Year 0 (an investment with zero duration and zero interest) so that all numbers in this column may be directly comparable.

TABLE 5. EFFECT OF DISCOUNT RATE ON AMOUNT OF IMMEDIATE INVESTMENT REQUIRED (r OF 10 PERCENT)

Option	Type of Cash Flow	Cash Flow in Year											Total Disbursement	Total Investment	Total Interest[a]
		0	1	2	3	4	5	6	7	8	9	10			
1	Disbursement	100											100	100[b]	-
	Investment	-													
	Interest	-													
2	Disbursement		100										100	90.91	9.09
	Investment	90.91													
	Interest	9.09	-												
3	Disbursement	55	55										110	105.00[b]	5.00
	Investment	50.00													
	Interest	5.00	-												
4	Disbursement	25	25	25	25	25							125	104.25[b]	20.75
	Investment	79.25													
	Interest	7.92	6.22	4.34	2.27	-									
5	Disbursement											200	200	77.10	122.90
	Investment	77.10													
	Interest	7.71	8.48	9.33	10.26	11.29	12.42	13.66	15.02	16.53	18.18	-			

a. Figures may not exactly equal the actual sums of their corresponding interest rows because of rounding-off of numbers.

b. This figure reflects the investment of the disbursement required in Year 0 (an investment with zero duration and zero interest) so that all numbers in this column may be directly comparable.

TABLE 6. EFFECT OF DISCOUNT RATE ON AMOUNT OF IMMEDIATE INVESTMENT REQUIRED (r OF 15 PERCENT)

Option	Type of Cash Flow	Cash Flow in Year											Total Disbursement	Total Investment	Total Interest[a]
		0	1	2	3	4	5	6	7	8	9	10			
1	Disbursement	100											100	100[b]	
	Investment	-													
	Interest	-													
2	Disbursement		100										100	86.96	13.04
	Investment	86.96													
	Interest	13.04	-												
3	Disbursement	55	55										110	102.83[b]	7.17
	Investment	47.83													
	Interest	7.17	-												
4	Disbursement	25	25	25	25	25							125	96.38[b]	28.62
	Investment	71.38													
	Interest	10.71	8.56	6.10	3.26	-									
5	Disbursement											200	200	49.44	150.56
	Investment	49.44													
	Interest	7.42	8.53	9.81	11.28	12.97	14.92	17.15	19.73	22.69	26.09	-			

a. Figures may not exactly equal the actual sums of their corresponding interest rows because of rounding-off of numbers.

b. This figure reflects the investment of the disbursement required in year 0 (an investment with zero duration and zero interest) so that all numbers in this column may be directly comparable.

TABLE 7. EFFECT OF DISCOUNT RATE ON AMOUNT OF IMMEDIATE INVESTMENT REQUIRED (r OF 20 PERCENT)

Option	Type of Cash Flow	Cash Flow in Year											Total Disbursement	Total Investment	Total Interest[a]
		0	1	2	3	4	5	6	7	8	9	10			
1	Disbursement	100											100	100[b]	
	Investment	-													
	Interest	-													
2	Disbursement		100										100	83.33	16.67
	Investment	83.33													
	Interest	16.67	-												
3	Disbursement	55	55										110	100.83[b]	9.17
	Investment	45.83													
	Interest	9.17	-												
4	Disbursement	25	25	25	25	25							125	89.72[b]	35.28
	Investment	64.72													
	Interest	12.94	10.53	7.64	4.17	-									
5	Disbursement											200	200	32.30	167.70
	Investment	32.30													
	Interest	6.46	7.75	9.30	11.16	13.40	16.07	19.29	23.15	27.78	33.33	-			

a. Figures may not exactly equal the actual sums of their corresponding interest rows because of rounding-off of numbers.

b. This figure reflects the investment of the disbursement required in year 0 (an investment with zero duration and zero interest) so that all numbers in this coulum may be directly comparable.

TABLE 8. COMPARISON OF IMMEDIATE INVESTMENTS REQUIRED FOR OPTIONS, AT VARYING INTEREST RATES

Option Number	Disbursement Required	Investment Required, Discount Rate (%) of:					
		0	3	5	10	15	20
1	100	100	100	100	100	100	100
2	100	100	97.09	95.24	90.91	86.96	83.33
3	110	110	108.40	107.38	105.00	102.83	100.83
4	125	125	117.93	113.65	104.25	96.38	89.72
5	200	200	148.82	122.78	77.10	49.44	32.30

What practical effect does the choice of the interest rate make? Consider the differences in preference for the options at the different interest rates. At any given interest rate, the option with the lowest required immediate investment (present value) is preferred, with the others being ranked in order by their present values. Table 9 highlights these changes in preferences.

TABLE 9. CHANGES IN PREFERENCE RANKINGS FOR OPTIONS AT DIFFERENT INTEREST RATES

Option Number	Order of Preference at Interest Rate (%) of:					
	0	3	5	10	15	20
1	1.5(Tie)	2	2	3	4	4
2	1.5(Tie)	1	1	2	2	2
3	3	3	3	5	5	5
4	4	4	4	4	3	3
5	5	5	5	1	1	1

For this set of alternatives to the problem, then, the decisionmaker would order his preferences differently (except for the 3 and 5-percent rates) — and sometimes drastically differently — according to the interest rate chosen.

Figure 1 may help illustrate this phenomenon. At any particular number of years hence, the present value of $ 1 changes with the varying interest rates. The compounding of interest at different rates accounts for this. This is a rather straightforward change in present values by interest rates. When different amounts of money are thrown into the calculation, the prior established pattern can change drastically. Further complicate the process by placing the disbursed amounts at different points in time, and extremely complicated relationships derive.

Since the interest or discount rate can have such a profound effect on the decision, the nature and determination of the rate might well be considered.

B. What is the Interest Rate?

The interest (or discount) rate is the variable which equates a future expenditure with a present investment. The accrued interest plus the face value of the investment equal the amount to be disbursed when it is due. Since $ 95.24 can be invested at 5 percent to allow the withdrawal of all interest and principal next year so that a scheduled disbursement of $ 100 may be met, $ 95.24 is the present value of $ 100, with a 5-percent interest rate for 1 year.

The following is a list of items which the interest rate reflects:

1. Investment opportunities given up by the restricting of alternatives to be considered. Before the disbursement, an interim investment opportunity is assumed at a particular rate of interest; the project costs to be incurred are then directly measured against these returns.

2. Risks and uncertainties inherent in the projects under consideration. The rate of return demanded by the private sector increases with the level of risk, which reflects a decrease in investment willing to be made (builds in a risk-allowance or probabilistically weights returns for an average expected return from that risk). In the public sector, risk attaches to mission fulfillment and/or specific scheduled disbursements.

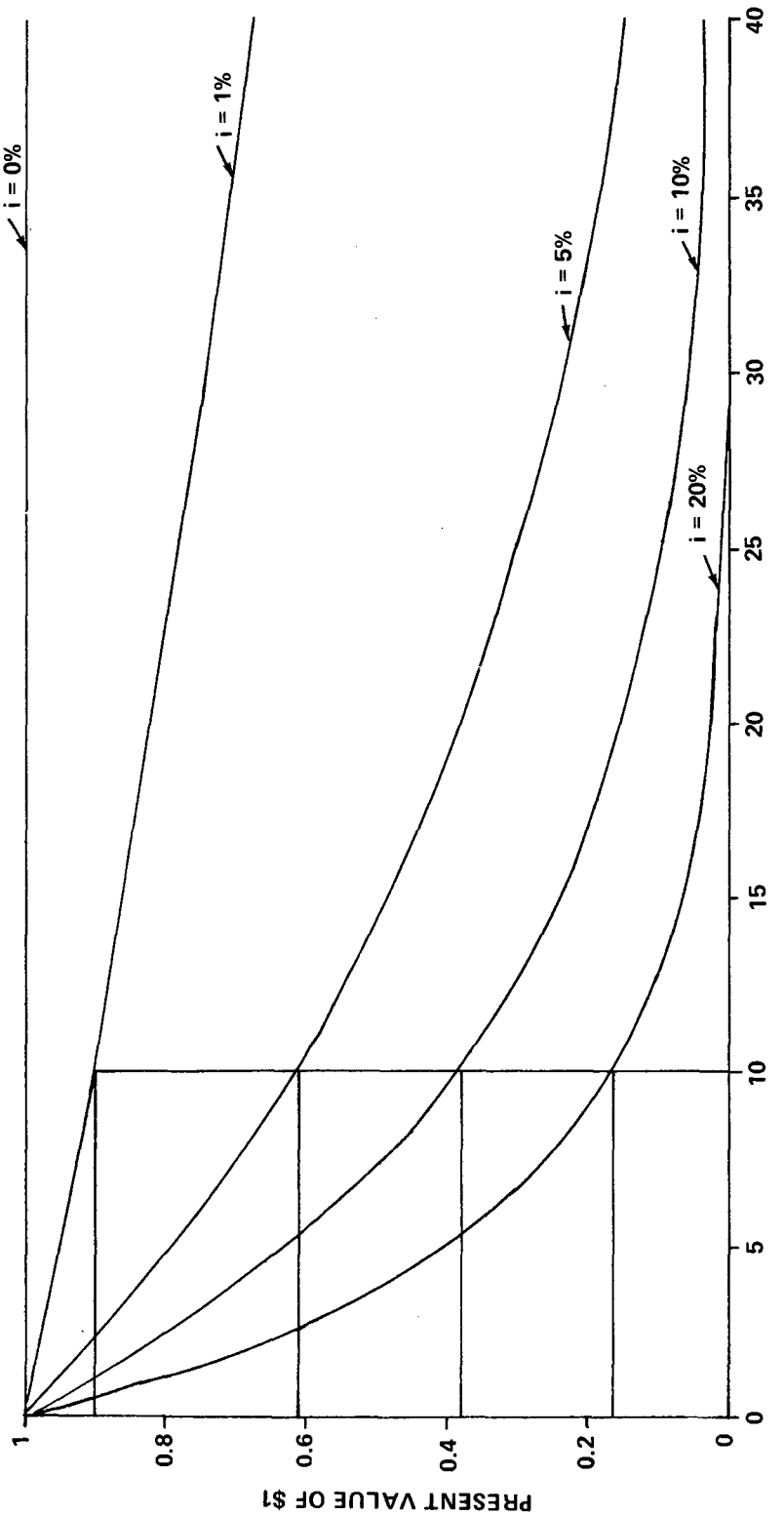

Figure 1. Changes in the present value of $ 1 for different interest rates over time.

High risk here means a low interest rate to penalize riskier alternatives by forcing their present values of costs to be higher than comparable less risky projects.

3. Priorities assigned to various missions. When the fulfillment of certain goals is ranked by preference or degree of contribution to overall objectives, this preference pattern can be built into the analysis of those alternatives. A low interest rate means (all other things remaining equal) a relatively high present value of costs associated with the project; this can be taken to reflect a relatively low priority for alternatives being implemented. A high interest rate means a relatively low present value of the costs associated with the project; this can be taken to reflect a relatively high priority for that alternative's being implemented.

C. What Interest Rate Should be Used?

A matter of practical importance is the determination of the interest rate to be used in evaluating alternative projects. However, since the interest rate is a reflection of investment opportunities, risks, uncertainties, and priorities, finding the correct interest rate is a difficult problem.

No equation exists which tells the analyst/decisionmaker the correct interest rate to be used. The rate must be subjectively chosen, based on considerations discussed above and on any other pertinent characteristics of the specific alternatives under consideration, as well as the mission.

The typical analyst, of course, rarely faces the problem of choosing an interest rate. Usually the rate is determined for him.

Circular Number A-94 (Bureau of the Budget, dated June 26, 1969) requires discounting to be applied to all evaluations whose subsequent decision for adoption is expected to commit the Government to a series of measurable costs extending over 3 or more years and in cases of measurable benefits or outputs that extend 3 or more years beyond the inception date.

The minimum rate to be used was then tied to the Water Resources Council's rate which is related to the current yield on Government bonds. (For FY 1970, this rate was 4 7/8 percent.)

A rationale for establishing a typical range of rates between 5 and 10 percent for Governmental cost analyses may be based on findings that returns on investments in the private sector may range from about 5 percent (Government bonds or bank interest) up to about 13 percent (rate of return on capital expenditures; e.g., plant and equipment). This would constitute a base from which to adjust the discount rate before inclusion of the other factors mentioned in the preceding section.

Sobin investigated returns to corporations and individuals; a weighted average of these rates yielded a 6.8-percent interest rate — a figure that appears both plausible and realistic.[4] It should be emphasized that this base figure can constitute a reasonable beginning to determining the appropriate interest rate for a particular decision process. Returns have probably changed in the private sector since Sobin's data were extracted; hence, the 6.8-percent figure might well be adjusted to reflect current returns.

The establishment of the Water Resources Council rate of 4 7/8 percent as a minimum figure for governmental use may certainly be open to serious question, especially in view of the recognition that the interest rate appropriate to a particular decision is the resultant of far more than the return to the private sector.

Consider a worst-case example. In a depression similar to the one of the late 1920's and early 1930's, a project may be proposed to Congress. The proposal might be to reclaim desert land in New Mexico for a total cost of $ 1 billion, using new and untried reclamation techniques. Impact on GNP or employment by either the project or its result would probably be minimal.

Although the project would probably be rejected out-of-hand by Congress, trying to systematize, quantify, and make objective, the decisionmaking process is important. Using a 4 7/8-percent figure (the minimum allowed) would, in this case, seriously overstate the present value of this proposal. Certainly, 2 or 3 percent might be more appropriate when consideration is made of the state of the economy, risks and uncertainties involved, and current national goals and priorities.

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4. B. Sobin, "Some Interest Rate Aspects of Weapons Systems Investment Policy," Arlington, Va.: Institute for Defense Analyses, Weapons Systems Evaluation Division, Research Paper P-171, February 1965, p. 62.

By contrast, President Kennedy's 1961 establishment of a high national priority on the moon effort by NASA could well have been reflected by an interest rate associated with this program of perhaps 15 percent. The decision must be examined against the backdrop at that time of the state of the economy, risks, uncertainties (balancing technological uncertainties against the risks involved in not achieving at least space equality with the USSR), and national goals and priorities. All of these elements were determining factors in what amounted to a mandate by the people, culminating in the implementation of the people's will by a formal high-priority investment decision by the President.

Thus, while the vast majority of governmental interest rates would probably lie between 5 and 10 percent anyway, exceptional cases will occur which would call for rates outside this range. Most proposals calling for interest rates below the currently established minimum would never get to the decision stage; however, those that do should not have their present values overstated by the artificial floor established.

If one is to determine the interest rate to be used, the following general steps may be helpful:

1. Use as a baseline the current Water Resources Council rate (approximately 5 percent).

2. Consider the current state of the economy (high private investment opportunities pushes the discount rate up).

3. Subjectively evaluate the risks and uncertainties in the overall mission and its competing projects. Relatively high risks and uncertainties push the rate down; low risks and uncertainties push the rate up.

4. Consider national goals and priorities. If the mission is supported by a national mandate, the discount rate rises. Lack of support, interest, or contribution to national goals implies a decline in the discount rate to penalize the mission with high present value of costs.

5. Examine the overall pattern thus subjectively established. Choose a rate that seems to reflect in one number the findings of the other steps.

6. Perform sensitivities about this number to ensure that significant changes do not occur in the analysis because of the choice of the discount rate.

IV. COSTS TO BE INCLUDED IN PRESENT-VALUE ANALYSES

The cost analyst's job is often difficult in the area that would seem to be of minor effort — determining the relevant costs to be included in the analysis.

How are inherited assets accounted for? What types of allocation of support costs should be made? How much amortization is appropriate? These questions are typical of general issues that are important in generating valid analyses.

Almost all questions of which cost elements should be included in the analytical evaluation of alternative projects may be answered with one principle: Consider only those costs which would occur as a direct result of a decision to implement the project. An extension of this principle is to consider only those costs which require an outlay of cash.

Facilities that exist before the choice of a project, and which would be used by the project, are not included in the analysis. No disbursement (cost) is incurred for these facilities by the decision (assuming rent or other actual and direct charge is not levied upon the project by an outside agency which would require an outlay of funds). In deciding among mutually exclusive alternatives, the analyst must remember the responsibility of the decisionmaker. He tries to maximize benefits while minimizing outlay. Including artificial charges (i.e., charges not representative of the actual outlays required) confuses the issue and may invalidate the analysis.

By the same token, amortization (the spreading of initial costs over time) is not included in the analysis. Depreciation, too, is merely a bookkeeping charge and represents no outlay, and thereby is irrelevant to the decision.

V. USING THE PRESENT-VALUE TECHNIQUE

For one to manually compute the present value of a future disbursement is quite tedious. The task is rendered quite simple with the use of tables. Appendix A consists of two such tables.

A. Calculating Present Values

Table A-1 contains present-value factors. Each entry represents the present value of $ 1 at a particular discount rate and to be disbursed n years in the future. The present value of $ 1, then, to be paid 1 year hence, and with an interim investment opportunity of 5 percent, is $ 0.9524. Checking this, 5-percent interest on $ 0.9524 for 1 year is $ 0.0476; the principal ($ 0.9524) plus the interest ($ 0.0476) just equals the $ 1 scheduled to be disbursed.

To use the table for amounts other than $ 1 the amount is simply multiplied by the present value factor (PVF). Thus, the present value of $ 100 to be paid out 1 year hence, and with an interim investment opportunity of 5 percent, is $ 100 × 0.9524 or $ 95.24. Checking this, 5-percent interest on $ 95.24 for 1 year is $ 4.76; the principle ($ 95.24) plus the interest ($ 4.76) equals the $ 100 scheduled to be disbursed.

A useful characteristic of the present-value technique is its additive and subtractive property. The present value of $ 200 for 1 year at 5 percent is twice the present value of $ 100 for 1 year at 5 percent:

$$\text{PV of } \$200 = (200)\ (\text{PVF}) = (2)\ (100)\ (\text{PVF}) = 2\ [(100)\ (\text{PVF})] = 2\ [\text{PV of } \$100]\ .$$

Similarly, the present value of, say, $ 100 is equal to the present value of, say, $ 250 minus the present value of $ 150:[5]

$$\begin{aligned}(\text{PV of } \$250) - (\text{PV of } \$150) &= (250)\ (\text{PVF}) - (150)\ (\text{PVF}) \\ &= (250-150)\ (\text{PVF}) = (100)\ (\text{PVF}) \\ &= (\text{PV of } \$100)\ .\end{aligned}$$

While the calculation of this present-value figure allows the derivation of a useful decision data point, converting the present value to an equivalent annual stream of disbursements may be helpful to the decisionmaker.

5. In this example, each of the disbursements is due in the same year and has equal discount rates.

B. CALCULATING EQUIVALENT ANNUAL VALUES

A figure conceptually easier for the analyst and the decisionmaker to understand than the present value is the equivalent annual disbursement.

The equivalent annual value (EAV) takes the present value of a future disbursement and spreads it over the years from the decision date to the disbursement date. The EAV is an equal annual stream of payments which accrues interest on its declining balance. On the date scheduled for the actual disbursement, the balance reaches zero with the final equal annual payment. Table A-2 contains the Equivalent Annual Factors (EAFs) for computing EAV.

EAV allows comparisons among alternatives in terms of annual costs, a relatively common measure. Even with the complication of the present - value concept inherent within the EAV technique, it is rather easy to understand. The EAV of a $ 1 disbursement 5 years from now at 5 percent is as follows:

1. Find the PV of the $ 1 to be paid 5 years hence, with an interim investment opportunity of 5 percent:

PV of $ 1 = $ 1 × PVF = 0.7835 × $ 1 = $ 0.7835.

2. Find the EAV of the present value of the $ 1 disbursement for 5 years at the 5-percent interest rate:

EAV of $ 0.7835 = $ 0.7835 × EAF = $ 0.7835 × 0.23097 = $ 0.18095.

Thus the equivalent annual value of $ 1 to be paid 5 years hence at 5 percent is 18.1 cents. Checking this gives the following:

Year	Balance	+	Interest	-	Payment
1	0.7835		0.0392		0.18095
2	0.6417		0.0321		0.18095
3	0.4929		0.0246		0.18095
4	0.3366		0.0168		0.18095
5	0.1724		0.0086		0.18095
6	0[6]				

6. Error due to rounding.

C. Calculating PV and EAV of a Series of Future Disbursements

1. Determine the present value of a series of five annual payments of $ 25, beginning 1 year from now with 10 percent interest:

PV of $ 25 (year 1) = $ 25 × 0.9091 = $ 22.73
PV of $ 25 (year 2) = $ 25 × 0.8264 = 20.66
PV of $ 25 (year 3) = $ 25 × 0.7513 = 18.78
PV of $ 25 (year 4) = $ 25 × 0.6830 = 17.08
PV of $ 25 (year 5) = $ 25 × 0.6209 = 15.52
PV of the 5-year series = $ 94.77

2. Determine the equivalent annual value of a series of five annual payments of $ 25, beginning 1 year from now with 10 percent interest:

EAF for n = 5, 10 percent = 0.26380
EAV = EAF × PV = 0.26380 × $ 94.77
= $ 25.00

D. Calculating PV and EAV of a Future Series of Disbursements

1. Determine the present value of a future series of annual disbursements with 4 percent interest, as indicated below:

Year	0	3	9	10
Disbursement	$ 100	$ 25	$ 450	$ 300

PV of $ 100 (year 0) = $ 100 × 1.00 = $ 100
PV of $ 25 (year 3) = $ 25 × 0.8890 = 22.23
PV of $ 450 (year 9) = $ 450 × 0.7026 = 316.17
PV of $ 300 (year 10) = $ 300 × 0.6756 = 202.68
PV of the future series = $ 641.08

2. Determine the equivalent annual value of a future series of annual disbursements with 4 percent interest, as above:

EAF for n = 10, 4 percent = 0.12329
EAV = EAF × PV = 0.12329 × $ 641.08 = $ 79.04

APPENDIX A

PRESENT-VALUE AND EQUIVALENT-ANNUAL-VALUE FACTOR TABLES

TABLE A-1. PRESENT-VALUE FACTORS

(To find the present value of a future disbursement of $ 1, discounted at r percent for n years)

$$PVF = \frac{1}{(1+i)^n}$$

n	1%	1.5%	2%	3%	4%	5%	6%	7%	8%	10%	12%	15%	20%
1	0.9901	0.9852	0.9804	0.9709	0.9615	0.9524	0.9434	0.9346	0.9259	0.9091	0.8929	0.8696	0.8333
2	0.9803	0.9707	0.9612	0.9426	0.9246	0.9070	0.8900	0.8734	0.8573	0.8264	0.7972	0.7561	0.6944
3	0.9706	0.9563	0.9423	0.9151	0.8890	0.8638	0.8396	0.8163	0.7938	0.7513	0.7118	0.6575	0.5787
4	0.9610	0.9422	0.9238	0.8885	0.8548	0.8227	0.7921	0.7629	0 7350	0.6830	0.6355	0.5718	0.4823
5	0.9515	0.9283	0.9057	0.8626	0.8219	0.7835	0.7473	0.7130	0.6806	0.6209	0.5674	0.4972	0.4019
6	0.9420	0.9145	0.8880	0.8375	0.7903	0.7462	0.7050	0.6663	0.6302	0.5645	0.5066	0.4323	0.3349
7	0.9327	0.9010	0.8706	0.8131	0.7599	0.7107	0.6651	0.6227	0.5835	0.5132	0.4523	0.3759	0.2791
8	0.9235	0.8877	0.8535	0.7894	0.7307	0.6768	0.6274	0.5820	0.5403	0.4665	0.4039	0.3269	0.2326
9	0.9143	0.8746	0.8368	0.7664	0.7026	0.6446	0.5919	0.5439	0.5002	0.4241	0.3606	0.2843	0.1938
10	0.9053	0.8617	0.8203	0.7441	0.6756	0.6139	0.5584	0.5083	0.4632	0.3855	0.3220	0.2472	0.1615
11	0.8963	0.8489	0.8043	0.7224	0.6496	0.5847	0.5268	0.4751	0.4289	0.3505	0.2875	0.2149	0.1346
12	0.8874	0.8364	0.7885	0.7014	0.6246	0.5568	0.4970	0.4440	0.3971	0.3186	0.2567	0.1869	0.1122
13	0.8787	0.8240	0.7730	0.6810	0.6006	0.5303	0.4688	0.4150	0.3677	0.2897	0.2292	0.1625	0.0935
14	0.8700	0.8118	0.7579	0.6611	0.5775	0.5051	0.4423	0.3878	0.3405	0.2633	0.2046	0.1413	0.0779
15	0.8613	0.7999	0.7430	0.6419	0.5553	0.4810	0.4173	0.3624	0.3152	0.2394	0.1827	0.1229	0.0649
16	0.8528	0.7880	0.7284	0.6232	0.5339	0.4581	0.3936	0.3387	0.2919	0.2176	0.1631	0.1069	0.0541
17	0.8444	0.7764	0.7142	0.6050	0.5134	0.4363	0.3714	0.3166	0.2703	0.1978	0.1456	0.0929	0.0451
18	0.8360	0.7649	0.7002	0.5874	0.4936	0.4155	0.3503	0.2959	0.2502	0.1799	0.1300	0.0808	0.0376
19	0.8277	0.7536	0.6864	0.5703	0.4746	0.3957	0.3305	0.2765	0.2317	0.1635	0.1161	0.0703	0.0313
20	0.8195	0.7425	0.6730	0.5537	0.4564	0.3769	0.3118	0.2584	0.2145	0.1486	0.1037	0.0611	0.0261
21	0.8114	0.7315	0.6598	0.5375	0.4388	0.3589	0.2942	0.2415	0.1987	0.1351	0.0926	0.0531	0.0217
22	0.8034	0.7207	0.6468	0.5219	0.4220	0.3418	0.2775	0.2257	0.1839	0.1228	0.0826	0.0462	0.0181
23	0.7954	0.7100	0.6342	0.5067	0.4057	0.3256	0.2618	0.2109	0.1703	0.1117	0.0738	0.0402	0.0151
24	0.7876	0.6995	0.6217	0.4919	0.3901	0.3101	0.2470	0.1971	0.1577	0.1015	0.0659	0.0349	0.0126
25	0.7798	0.6892	0.6095	0.4776	0.3751	0.2953	0.2330	0.1842	0.1460	0.0923	0.0588	0.0304	0.0105
26	0.7720	0.6790	0.5976	0.4637	0.3607	0.2812	0.2198	0.1722	0.1352	0.0839	0.0525	0.0264	0.0087
27	0.7644	0.6690	0.5859	0.4502	0.3468	0.2678	0.2074	0.1609	0.1252	0.0763	0.0469	0.0230	0.0073
28	0.7568	0.6591	0.5744	0.4371	0.3335	0.2551	0.1956	0.1504	0.1159	0.0693	0.0419	0.0200	0.0061
29	0.7493	0.6494	0.5631	0.4243	0.3207	0.2429	0.1846	0.1406	0.1073	0.0630	0.0374	0.0174	0.0051
30	0.7419	0.6398	0.5521	0.4120	0.3083	0.2314	0.1741	0.1314	0.0994	0.0573	0.0334	0.0151	0.0042
31	0.7346	0.6303	0.5412	0.4000	0.2965	0.2204	0.1643	0.1228	0.0920	0.0521	0.0298	0.0131	0.0035
32	0.7273	0.6210	0.5306	0.3883	0.2851	0.2099	0.1550	0.1147	0.0852	0.0474	0.0266	0.0114	0.0029
33	0.7201	0.6118	0.5202	0.3770	0.2741	0.1999	0.1462	0.1072	0.0789	0.0431	0.0238	0.0099	0.0024
34	0.7130	0.6028	0.5100	0.3660	0.2636	0.1904	0.1379	0.1002	0.0730	0.0391	0.0212	0.0086	0.0020
35	0.7059	0.5939	0.5000	0.3554	0.2534	0.1813	0.1301	0.0937	0.0676	0.0356	0.0189	0.0075	0.0017
40	0.6717	0.5513	0.4529	0.3066	0.2083	0.1420	0.0972	0.0668	0.0460	0.0221	0.0107	0.0037	0.0007
45	0.6391	0.5117	0.4102	0.2644	0.1712	0.1113	0.0727	0.0476	0.0313	0.0137	0.0061	0.0019	0.0003
50	0.6080	0.4750	0.3715	0.2281	0.1407	0.0872	0.0543	0.0339	0.0213	0.0085	0.0035	0.0009	0.0001
55	0.5785	0.4409	0.3365	0.1968	0.1157	0.0683	0.0406	0.0242	0.0145	0.0053			
60	0.5504	0.4093	0.3048	0.1697	0.0951	0.0535	0.0303	0.0173	0.0099	0.0033			
65	0.5237	0.3799	0.2761	0.1464	0.0781	0.0419	0.0227	0.0123	0.0067	0.0020			
70	0.4983	0.3527	0.2500	0.1263	0.0642	0.0329	0.0169	0.0088	0.0046	0.0013			
75	0.4741	0.3274	0.2265	0.1089	0.0528	0.0258	0.0126	0.0063	0.0031	0.0008			
80	0.4511	0.3039	0.2051	0.0940	0.0434	0.0202	0.0095	0.0045	0.0021	0.0005			
85	0.4292	0.2821	0.1858	0.0811	0.0357	0.0158	0.0071	0.0032	0.0014	0.0003			
90	0.4084	0.2619	0.1683	0.0699	0.0293	0.0124	0.0053	0.0023	0.0010	0.0002			
95	0.3886	0.2431	0.1524	0.0603	0.0241	0.0097	0.0039	0.0016	0.0007	0.0001			
100	0.3697	0.2256	0.1380	0.0520	0.0198	0.0076	0.0029	0.0012	0.0005	0.0001			

TABLE A-2. EQUIVALENT ANNUAL FACTORS
(To find the series of annual equal disbursements, at r percent, equivalent to $ 1 to be disbursed n years hence)

$$EAF = \frac{i(1+i)^n}{(1+i)^n - 1}$$

n	1%	1.5 %	2%	3%	4%	5%	6%	7%	8%	10%	12%	15%	20%
1	1.01000	1.01500	1.02000	1.03000	1.04000	1.05000	1.06000	1.07000	1.08000	1.10000	1.12000	1.15000	1.20000
2	0.50751	0.51128	0.51505	0.52261	0.53020	0.53780	0.54555	0.55309	0.56077	0.57619	0.59170	0.61512	0.65455
3	0.34002	0.34338	0.34675	0.35353	0.36035	0.36721	0.37411	0.38105	0.38803	0.40211	0.41635	0.43798	0.47473
4	0.25628	0.25944	0.26262	0.26903	0.27549	0.28201	0.28859	0.29523	0.30192	0.31547	0.32923	0.35027	0.38629
5	0.20604	0.20909	0.21216	0.21835	0.22463	0.23097	0.23740	0.24389	0.25046	0.26380	0.27741	0.29832	0.33438
6	0.17255	0.17553	0.17853	0.18460	0.19076	0.19702	0.20336	0.20980	0.21632	0.22961	0.24323	0.26424	0.30071
7	0.14863	0.15156	0.15451	0.16051	0.16661	0.17282	0.17914	0.18555	0.19207	0.20541	0.21912	0.24036	0.27742
8	0.13069	0.13358	0.13651	0.14246	0.14853	0.15472	0.16104	0.16747	0.17401	0.18744	0.20130	0.22285	0.26061
9	0.11674	0.11961	0.12252	0.12843	0.13449	0.14069	0.14702	0.15349	0.16008	0.17364	0.18768	0.20957	0.24808
10	0.10558	0.10843	0.11133	0.11723	0.12329	0.12950	0.13587	0.14238	0.14903	0.16275	0.17698	0.19925	0.23852
11	0.09645	0.09929	0.10218	0.10808	0.11415	0.12039	0.12679	0.13336	0.14008	0.15396	0.16842	0.19107	0.23110
12	0.08885	0.09168	0.09456	0.10046	0.10655	0.11283	0.11928	0.12590	0.13270	0.14676	0.16144	0.18448	0.22526
13	0.08241	0.08524	0.08812	0.09403	0.10014	0.10646	0.11296	0.11965	0.12652	0.14078	0.15568	0.17911	0.22062
14	0.07690	0.07972	0.08260	0.08853	0.09467	0.10102	0.10758	0.11434	0.12130	0.13575	0.15087	0.17469	0.21689
15	0.07212	0.07494	0.07783	0.08377	0.08994	0.09634	0.10296	0.10979	0.11683	0.13147	0.14682	0.17102	0.21388
16	0.06794	0.07077	0.07365	0.07961	0.08582	0.09227	0.09895	0.10586	0.11298	0.12782	0.14339	0.16795	0.21144
17	0.06426	0.06708	0.06997	0.07595	0.08220	0.08870	0.09544	0.10243	0.10963	0.12466	0.14046	0.16537	0.20944
18	0.06098	0.06381	0.06670	0.07271	0.07899	0.08555	0.09236	0.09941	0.10670	0.12193	0.13794	0.16319	0.20781
19	0.05805	0.06088	0.06378	0.06981	0.07614	0.08275	0.08962	0.09675	0.10413	0.11955	0.13576	0.16134	0.20646
20	0.05542	0.05825	0.06116	0.06722	0.07358	0.08024	0.08718	0.09439	0.10185	0.11746	0.13388	0.15976	0.20536
21	0.05303	0.05587	0.05878	0.06487	0.07128	0.07800	0.08500	0.09229	0.09983	0.11562	0.13224	0.15842	0.20444
22	0.05086	0.05370	0.05663	0.06275	0.06920	0.07597	0.08305	0.09041	0.09803	0.11401	0.13081	0.15727	0.20369
23	0.04889	0.05173	0.05467	0.06081	0.06731	0.07414	0.08128	0.08871	0.09642	0.11257	0.12956	0.15628	0.20307
24	0.04707	0.04992	0.05287	0.05905	0.06559	0.07247	0.07968	0.08719	0.09498	0.11130	0.12846	0.15543	0.20255
25	0.04541	0.04826	0.05122	0.05743	0.06401	0.07095	0.07823	0.08581	0.09368	0.11017	0.12750	0.15470	0.20212
26	0.04387	0.04673	0.04970	0.05594	0.06257	0.06956	0.07690	0.08456	0.09251	0.10916	0.12665	0.15407	0.20176
27	0.04245	0.04532	0.04829	0.05456	0.06124	0.06829	0.07570	0.08343	0.09145	0.10826	0.12590	0.15353	0.20147
28	0.04112	0.04400	0.04699	0.05329	0.06001	0.06712	0.07459	0.08239	0.09049	0.10745	0.12524	0.15306	0.20122
29	0.03990	0.04278	0.04578	0.05211	0.05888	0.06605	0.07358	0.08145	0.08962	0.10673	0.12466	0.15265	0.20102
30	0.03875	0.04164	0.04465	0.05102	0.05783	0.06505	0.07265	0.08059	0.08883	0.10608	0.12414	0.15230	0.20085
31	0.03768	0.04057	0.04360	0.05000	0.05686	0.06413	0.07179	0.07980	0.08811	0.10550	0.12369	0.15200	0.20070
32	0.03667	0.03958	0.04261	0.04905	0.05595	0.06328	0.07100	0.07907	0.08745	0.10497	0.12328	0.15173	0.20059
33	0.03573	0.03864	0.04169	0.04816	0.05510	0.06249	0.07027	0.07841	0.08685	0.10450	0.12292	0.15150	0.20049
34	0.03484	0.03776	0.04082	0.04732	0.05431	0.06176	0.06960	0.07780	0.08630	0.10407	0.12260	0.15131	0.20041
35	0.03400	0.03693	0.04000	0.04654	0.05358	0.06107	0.06897	0.07723	0.08580	0.10369	0.12232	0.15113	0.20034
40	0.03046	0.03343	0.03656	0.04326	0.05052	0.05828	0.06646	0.07501	0.08386	0.10226	0.12130	0.15056	0.20014
45	0.02771	0.03072	0.03391	0.04079	0.04826	0.05626	0.06470	0.07350	0.08259	0.10139	0.12074	0.15028	0.20005
50	0.02551	0.02857	0.03182	0.03887	0.04655	0.05478	0.06344	0.07246	0.08174	0.10086	0.12042	0.15014	0.20002
55	0.02373	0.02683	0.03014	0.03735	0.04523	0.05367	0.06254	0.07174	0.08118	0.10053	0.12000[a]	0.15000[a]	0.20000[a]
60	0.02224	0.02539	0.02877	0.03613	0.04420	0.05283	0.06188	0.07123	0.08080	0.10033			
65	0.02100	0.02419	0.02763	0.03515	0.04339	0.05219	0.06139	0.07087	0.08054	0.10020			
70	0.01993	0.02317	0.02667	0.03434	0.04275	0.05170	0.06103	0.07062	0.08037	0.10013			
75	0.01902	0.02230	0.02586	0.03367	0.04223	0.05132	0.06077	0.07044	0.08025	0.10008			
80	0.01822	0.02155	0.02516	0.03311	0.04181	0.05103	0.06057	0.07031	0.08017	0.10005			
85	0.01752	0.02089	0.02456	0.03265	0.04148	0.05080	0.06043	0.07022	0.08012	0.10003			
90	0.01690	0.02032	0.02405	0.03226	0.04121	0.05063	0.06032	0.07016	0.08008	0.10002			
95	0.01636	0.01982	0.02360	0.03193	0.04099	0.05049	0.06024	0.07011	0.08005	0.10001			
100	0.01587	0.01937	0.02320	0.03165	0.04081	0.05038	0.06018	0.07008	0.08004	0.10001			

a. $n = \infty$

APPENDIX B

COMPUTER PROGRAM TO CALCULATE PRESENT VALUE AND EQUIVALENT ANNUAL VALUES

Computer Program Listing

```
△ASSIGN S=MTO,SI=CR,BO=MT1,LO=LP.
△REWIND MT1.
△ FORTRAN BO, LO.
=      1          DIMENSION COST[50],PV[12,50],PVC[12,50],ANM[12],RCT[12]
=      2          DIMENSION EQAN[12]
=      3          READ 11,K,[PCT[I],I=1,12]
=      4       11 FORMAT[I2,12F6.3]
=      5        7 READ 4,M,[ANM[J],J=1,12]
=      6        4 FORMAT[I2,12A6]
=      7          PRINT 12,[ANM[J],J=1,12]
=      8       12 FORMAT[1H1,///10X,12A6]
=      9          N=M/10+1
=     10          DO 13 J=1,N
=     11          JJ=[J-1]*10+1
=     12       13 READ 1,[COST[I],I=JJ,JJ+9]
=     13        1 FORMAT[10F8.0]
=     14        6 FORMAT[//1X2HYR6X4HCOST4X3[7X5HPV[I=F4.3,12H]  COM PV[I=F4.3,1H]6X
=     15         1]]
=     16          DO 2 I=1,K
=     17          PV[I,1]=COST[1]*[1./[1.+PCT[I]]]
=     18          PVC[I,1]=PV[I,1]
=     19          DO 2 J=2,M
=     20          PV[I,J]=COST[J]*[1./[1.+PCT[I] ]**J]
=     21          PVC[I,J]=PVC[I,J-1]+PV[I,J]
=     22        2 CONTINUE
=     23          DO 14 J=1,K
=     24       14 EQAN[J]=[PCT[J]*[1+PCT[J]]**M/[[1+PCT[J]]**M-1]]*PVC[J,M]
=     25          DO 15 I=1,4
=     26          N=I
=     27          IF[K-[I*3]]16,16,15
=     28       15 CONTINUE
=     29       16 DO 8 I=1,N
=     30          L=[I-1]*3+1
=     31          PRINT 6,[PCT[NI],PCT[NI],NI=L,L+2]
=     32          COSTSM=0.
=     33          DO 3 J=1,M
=     34          COSTSM=COSTSM+COST[J]
=     35        3 PRINT 5,J,COST[J],[PV[JJ,J],PVC[JJ,J],JJ=L,L+2]
=     36        5 FORMAT[1XI2F10.2,5X3[2F15.2,7X]]
=     37          PRINT 9,COSTSM
=     38        8 PRINT 10,[EQAN[J],J=L,L+2]
=     39        9 FORMAT[1X,F12.2]
=     40       10 FORMAT[//1X17HEQUIVALENT ANNUAL 3[15XF15.2,7X]]
=     41          GO TO 7
=     42          STOP
=     43          END

PROGRAM ALLOCATION

  00014 COST      00160 PV        02440 PVC       04720 ANM
  04750 PCT       05000 EQAN      05030 K         05031 I
  05032 M         05033 J         05034 N         05035 JJ

  05036 L         05037 NI        05040 COSTSM
THE END
```

PV And EAV Program Documentation

Card 1 - Input the number of percents that will be used up to and including 12 (Format I2). Then input in a format of 12F 6.3, the percents to be used.

Example: To use the percents of 0.05, 0.10, and 0.15 (input).

```
3  050  100  150
```

Card 2 - Input the number of years to discount in the first two columns (Format I2) up to 50 years and the title of the case up to Column 74.

```
12      BASELINE OPTIMIZED
```

Cards 3 and Following - Input in a format 1 of 8.0, the dollar value of each year.

NOTE: If number of years is evenly divisible by 10 (10, 20, etc.), a blank card must be placed at end of last data card.

99. 109.

80. 100. 681. 1482. 2003. 2121.

IBM 5081

Additional cases may be run beginning with new title card. Percentages will remain unchanged. An example is as follows:

484. 573. 656. 736. 764. 764.

242 688. 1016. 1174. 1189. 1172. 1209. 904. 557. 434.

16 STAGE AND ONE HALF (A)

IBM 5081

Sample PV And EAV Computer Program Output

YR	COST	PV[I=.175]	CUM PV[I=.175]	PV[I=.200]	CUM PV[I=.200]
1	.00	.00	.00	.00	.00
2	.00	.00	.00	.00	.00
3	55.00	33.90	33.90	31.83	31.83
4	220.00	115.42	149.32	106.10	137.92
5	649.00	289.77	439.09	260.82	398.74
6	2141.00	813.56	1252.65	717.02	1115.76
7	3891.00	1258.33	2510.99	1085.91	2201.67
8	4606.30	1267.80	3778.78	1071.28	3272.94
9	3958.10	927.14	4705.92	767.11	4040.05
10	3856.60	768.82	5474.75	622.86	4662.91
11	3812.80	646.89	6121.63	513.16	5176.07
12	4230.40	610.84	6732.47	474.47	5650.54
13	4401.60	540.90	7273.37	411.39	6061.93
14	4756.90	497.50	7770.87	370.50	6432.43
15	4544.50	404.50	8175.37	294.96	6727.39
16	4211.30	319.01	8494.39	227.78	6955.17
17	3900.80	251.48	8745.87	175.82	7130.99
18	4056.80	222.59	8968.46	152.38	7283.37
19	3271.80	152.78	9121.24	102.41	7385.78
20	1839.10	73.09	9194.33	47.97	7433.75
21	3242.15	109.66	9303.99	70.47	7504.22
22	3242.15	93.33	9397.31	58.73	7562.95
23	3242.15	79.43	9476.74	48.94	7611.89
24	3242.15	67.60	9544.33	40.78	7652.68
25	3242.15	57.53	9601.86	33.99	7686.66
26	3242.15	48.96	9650.82	28.32	7714.98
27	3242.15	41.67	9692.49	23.60	7738.59
28	3242.15	35.46	9727.96	19.67	7758.25
29	3242.15	30.18	9758.14	16.39	7774.64
30	3242.15	25.69	9783.82	13.66	7788.30
31	3242.15	21.86	9805.68	11.38	7799.68
32	3242.15	18.60	9824.29	9.48	7809.17
33	3242.15	15.83	9840.12	7.90	7817.07
34	3242.15	13.48	9853.60	6.59	7823.66
35	3242.15	11.47	9865.07	5.49	7829.15
36	3242.15	9.76	9874.83	4.57	7833.72
37	3242.15	8.31	9883.13	3.81	7837.53
38	3242.15	7.07	9890.20	3.18	7840.71
39	3242.15	6.02	9896.22	2.65	7843.36
40	3242.15	5.12	9901.34	2.21	7845.56
41	3242.15	4.36	9905.70	1.84	7847.40
42	3242.15	3.71	9909.41	1.53	7848.93
43	3242.15	3.16	9912.56	1.28	7850.21
44	3242.15	2.69	9915.25	1.06	7851.27
45	3242.15	2.29	9917.54	.89	7852.16
46	3242.15	1.95	9919.48	.74	7852.90
47	3242.15	1.66	9921.14	.62	7853.52
48	3242.15	1.41	9922.55	.51	7854.03
49	3242.15	1.20	9923.75	.43	7854.46
50	3242.15	1.02	9924.77	.36	7854.81
	155667.62				
EQUIVALENT ANNUAL		1737.38			1571.14

Sample PV And EAV Computer Program Output (Concluded)

YR	COST	PV[I=.175]	CUM PV[I=.175]	PV[I=.200]	CUM PV[I=.200]
1	.00	.00	.00	.00	.00
2	.00	.00	.00	.00	.00
3	55.00	33.90	33.90	31.83	31.83
4	220.00	115.42	149.32	106.10	137.92
5	649.00	289.77	439.09	260.82	398.74
6	2141.00	813.56	1252.65	717.02	1115.76
7	3891.00	1258.33	2510.99	1085.91	2201.67
8	4606.30	1267.80	3778.78	1071.28	3272.94
9	3958.10	927.14	4705.92	767.11	4040.05
10	3856.00	768.70	5474.63	622.77	4662.82
11	3812.80	646.89	6121.51	513.16	5175.97
12	4230.40	610.84	6732.35	474.47	5650.44
13	4401.60	540.90	7273.25	411.39	6061.83
14	4656.90	487.04	7760.30	362.71	6424.54
15	4544.50	404.50	8164.80	294.96	6719.50
16	4211.30	319.01	8483.81	227.78	6947.28
17	3900.80	251.48	8735.29	175.82	7123.11
18	4056.80	222.59	8957.88	152.38	7275.48
19	3271.80	152.78	9110.66	102.41	7377.89
20	1839.10	73.09	9183.75	47.97	7425.87
21	3242.15	109.66	9293.41	70.47	7496.34
22	3242.15	93.33	9386.73	58.73	7555.07
23	3242.15	79.43	9466.16	48.94	7604.01
24	3242.15	67.60	9533.76	40.78	7644.79
25	3242.15	57.53	9591.28	33.99	7678.78
26	3242.15	48.96	9640.25	28.32	7707.10
27	3242.15	41.67	9681.91	23.60	7730.70
28	3242.15	35.46	9717.38	19.67	7750.37
29	3242.15	30.18	9747.56	16.39	7766.76
30	3242.15	25.69	9773.24	13.66	7780.42
31	3242.15	21.86	9795.11	11.38	7791.80
32	3242.15	18.60	9813.71	9.48	7801.28
33	3242.15	15.83	9829.54	7.90	7809.19
34	3242.15	13.48	9843.02	6.59	7815.77
35	3242.15	11.47	9854.49	5.49	7821.26
36	3242.15	9.76	9864.25	4.57	7825.84
37	3242.15	8.31	9872.55	3.81	7829.65
38	3242.15	7.07	9879.62	3.18	7832.83
39	3242.15	6.02	9885.64	2.65	7835.47
40	3242.15	5.12	9890.76	2.21	7837.68
41	3242.15	4.36	9895.12	1.84	7839.52
42	3242.15	3.71	9898.83	1.53	7841.05
43	3242.15	3.16	9901.99	1.28	7842.33
44	3242.15	2.69	9904.67	1.06	7843.39
45	3242.15	2.29	9906.96	.89	7844.28
46	3242.15	1.95	9908.90	.74	7845.01
47	3242.15	1.66	9910.56	.62	7845.63
48	3242.15	1.41	9911.97	.51	7846.14
49	3242.15	1.20	9913.17	.43	7846.57
50	3242.15	1.02	9914.19	.36	7846.93
	155567.02				
EQUIVALENT ANNUAL			1735.53		1569.56

APPROVAL

PRESENT-VALUE ANALYSIS: A SYSTEMS APPROACH TO PUBLIC DECISIONMAKING FOR COST EFFECTIVENESS

By Theodore T. Herbert

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This document has also been reviewed and approved for technical accuracy.

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