THE VISCOSITY OF MAGMATIC SILICATE LIQUIDS: A MODEL FOR CALCULATION

Yan Bottinga[*] and Daniel F. Weill

Center for Volcanology, University of Oregon, Eugene, Oregon 97403

* Present address: Groupe de Recherches Géochimiques Louis Barrabé, Institut de Physique du Globe, Faculté des Sciences de Paris, 9 Quai St. Bernard, Paris 5, France.

INTRODUCTION

There are very few aspects of igneous petrology which are not in some way closely related to the viscosity of magma. Rate of crystal growth, gravitative settling or rise of solids and bubbles, mode of volcanic eruption, flow differentiation mechanisms, flow characteristics of lavas, mass transfer in magmas under externally imposed pressure gradients or natural convection, rate of cooling of magmatic intrusions and lavas; all of these phenomena are critically dependent upon the viscosity of magmatic liquid or the effective viscosity of a magmatic suspension. Quantitative evaluation of any of the above igneous processes must begin with a knowledge of the viscosity of silicate liquids within the rather wide range of temperature and composition displayed by natural melts. The need for such viscosity data has long been recognized by petrologists, and many attempts have been made to determine effective viscosities from field observations of lava flow rates (Becker, 1897; Palmer, 1927; Nichols, 1939; Krauskopf, 1948; Minakami, 1951; Walker, 1967, among others). Uncertainties related to temperature measurements; size, shape, and roughness of the flow channel; and gas and solid particle content make evaluation of such viscosity determinations very difficult.

Laboratory viscosity measurements on geologically significant liquids date back to the turn of the century. The first determinations were probably made by Doelter (1902), who was able to classify on a qualitative scale the viscosities of various rocks and minerals melted in the laboratory. Much of the early work is summarized by Kittl (1913). The measurements of Kani (1934a, 1934b, 1935), Kozu and Kani (1935), Kani and Hosakawa (1936), Volarovich (1936), and Volarovich and Tolstoi (1936) were valuable contributions towards an understanding of the temperature and composition dependence of silicate viscosity. Bowen (1934) measured the viscosities of molten orthoclase and albite, and included an interesting discussion of the significance of viscosity in petrology and liquid silicate experimental work. More recently, viscosity measurements on molten rocks have been reported by Euler and Winkler (1957), Shaw (1969), Carron (1969), and Murase and McBirney (1970).

Newton's law of viscosity states that the shear stress in liquids undergoing laminar flow is proportional to the local velocity gradient perpendicular to the stress. The proportionality constant in this relation is known as the viscosity coefficient, η. In Newtonian fluids it is independent of the magnitude of the shear stress. The silicate liquids discussed in this paper closely approximate the Newtonian model. Fluid suspensions of solids or bubbles are best described by non-Newtonian models. For a discussion of this problem in geological context the reader is referred to Shaw et al. (1968) and Shaw (1969).

The viscosities of silicate liquids are strongly dependent upon chemical composition. For example, at $1300^{\circ}C$ where most igneous rocks are completely melted, the composition range "basaltic" to "granitic" corresponds to a viscosity range of 10^2 to 10^8 poises. Such extreme variability emphasizes the need for systematizing the composition dependence of viscosity. Without such an approach, much of igneous petrology remains largely qualitative. In discussing the differentiation of the Stillwater magma, Hess (1960) found it necessary to know the viscosity of the magmatic liquid. Taking what he considered to be reasonable estimates of the upper $(3 \times 10^4$ poises) and lower $(3 \times 10^2$ poises) viscosity limits of basalts, Hess was forced to adopt the geometric mean $(3 \times 10^3$ poises) as the best value for the Stillwater magmas. This same value was adopted as the viscosity of the Skaergaard magmas by Wager and Brown (1967). Jaeger (1968) states that: "In all discussions of convection a knowledge of the viscosity of the liquid is vital, and no certain information is available about this." In subsequent calculations, Jaeger (op. cit.) is forced to adopt Hess' figure as typical for magmas.

In petrology it has been difficult to make direct and quantitative use of viscosity data partly because the measurements have been performed on individual molten rocks without a systematic investigation of composition dependence. Fortunately a large number of measurements have been made in simpler, two to five component systems, which make it possible to systematically analyze the composition dependence of viscosity in anhydrous silicate liquids. It is the main purpose of this paper to present the

results of such an analysis in the form of a simple model designed to allow reasonably accurate calculations of viscosity as a function of temperature and composition.

In this paper we treat directly the problem of predicting viscosities of anhydrous silicate liquids. Such viscosity numbers are applicable to many extrusive melts and to nearly dry magmatic liquids in general (e.g., lunar liquids). The fluidizing action of H_2O dissolved in silicate melts is well recognized (Saucier, 1952; Sabatier, 1956; Friedman, et al., 1963; Burnham, 1963; Carron, 1964 and 1969; Shaw, 1965), and it is now possible to predict the effect of H_2O content on viscosity in a semi-quantitative way. Data for hydrous melts are not yet sufficient, however, to warrant direct integration with the more complete set of data on the effect of other melt constituents. We have chosen, therefore, not to incorporate H_2O directly in our model. Rather, it seems to us that the best procedure is to use the present model for calculating viscosities of anhydrous compositions, and, where necessary, to estimate the effect of added H_2O according to the suggestions of Shaw (1965), and Carron (1969). The model presented here can easily be modified to incorporate the effect of H_2O whenever sufficient additional data are accumulated.

INPUT DATA

Selecting an unbiased set of viscosity input data presents certain problems which cannot always be resolved in a purely objective fashion. Fortunately the influence of the selection procedure is minimized when

the total data population is large as is the case for viscosity measure-
ments of silicate liquids. We have used a total of 2,440 observations
which span the temperature range 1100-1800°C and the composition range
35-91 mole % SiO_2. In Table 1 we list the silicate systems, approximate
temperature ranges, and references for the input data selected.

In general, all the measurements selected for this study are reported
to be accurate to within 10% or better, and there is no reason to doubt the
validity of this error limit for the viscosities per se. For purposes
of attempting a correlation between viscosity, temperature and composition,
the reliability is probably not as good as this figure suggests. Accurate
control and measurement of high temperatures are difficult. The same
is true of composition. Bubble formation, volatilization, contamina-
tion of sample by ambient atmosphere and crucible are potential sources of
error in reporting accurate viscosity-composition data. Since viscosity
is strongly dependent on temperature and composition we estimate the
overall uncertainty in the input data to be substantially greater than
10%. The simplest way to gauge the reliability of the data is to directly
compare the results of various investigators wherever this is possible.
This comparison is made in Table 2 and shows that the agreement is good,
but well outside the 10% limit in many cases. Nevertheless, the data of
Table 2, and the selected input data in general, constitute the most
accurate viscosity measurements available, and the scatter is certainly
well within the limits that would be deemed petrologically useful.

A few published viscosity data are not included in Table 1. As a general guide, we have assumed that modern data is more likely to be accurate. In order to avoid a biased set of data, we have followed the practice of either totally accepting or rejecting the data reported in a single investigation rather than selecting only those measurements which closely fitted our model. The data of Saito and Saeki (1965) for the system SiO_2-CaO-Cr_2O_3-FeO-TiO_2 were not included because they are in serious disagreement with the results of Bockris and Lowe (1954) and Kozakevitch (1960) at comparable compositions. We have not included the measurements of Kato and Minowa (1969) in the system SiO_2-Al_2O_3-CaO (with additions of MgO, FeO, MnO, TiO_2, V_2O_5, Cr_2O_3, and P_2O_5) because of insufficient evidence for the control of composition. The data of Saito and Kawai (1951) on SiO_2-Al_2O_3-MgO-CaO, Behrendt and Kootz (1949) on SiO_2-Al_2O_3-MgO-CaO and SiO_2-MgO-CaO-TiO_2, Shil (1961) and Schleier (1958) on SiO_2-Al_2O_3-MgO-CaO were obtained using graphite crucibles at temperatures in excess of 1500° C. Silicate melts are known to attack graphite seriously at elevated temperatures and consequently these data have not been incorporated. The results of Gimmel'farb (1968) in the system SiO_2-Al_2O_3-CaO-FeO were also not selected because of possible contamination problems.
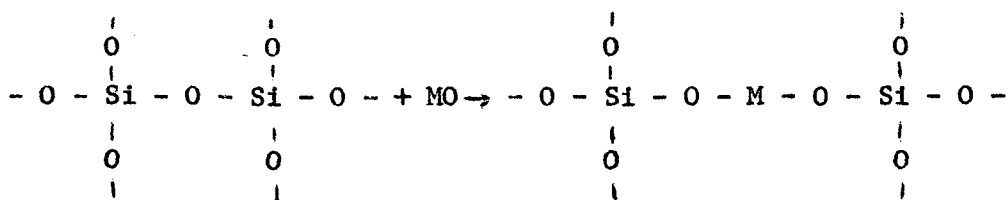
The 2,440 observations which have been selected as input data are a compromise between our evaluation of reliability and a desire to incorporate maximum information on the effect of petrologically significant components. From this point of view the weaknesses of the input are: (1) the effect

of K_2O is known only from the binary K_2O-SiO_2 system, (2) there are few

reliable data on the effect of Fe_2O_3. Additional measurements on the above

and on liquids containing TiO_2, FeO, and MnO would be very desirable.

CHEMICAL DEPENDENCE OF VISCOSITY

General

The dependence of silicate liquid viscosity on bulk chemistry may be

explained qualitatively in terms of the classic concepts of silicate

structure. Liquid SiO_2 is considered to be a non-periodic array of SiO_4

tetrahedra linked together by strong Si-O covalent bonding at all corners.

In such an idealized system, flow must involve the rupture of some of the

Si-O bonds. Consequently, the activation energy and viscosity for pure

SiO_2 flow are predictably high (Hofmaier, 1968). The addition of

metallic oxides to the SiO_2 liquid results in a breakdown of the continuous

Si-O-Si linkage due to the ionic nature of the metallic atoms. Some of

the Si-O bonds are replaced by weaker M-O bonds, and the Si-O bonds adjacent

to cations are weakened because the latter polarize the shared oxygen.

The process may be schematically represented by:

$$
\begin{array}{ccccc}
| & | & & | & | \\
O & O & & O & O \\
| & | & & | & | \\
-O-Si-O-Si-O- & +MO\rightarrow & -O-Si-O-M-O-Si-O- \\
| & | & & | & | \\
O & O & & O & O \\
| & | & & | & | \\
\end{array}
$$

At the orthosilicate composition, $(MO + M_2O)/SiO_2 = 2$, the tetrahedral

linkages are supposedly gone, i.e., there are no silicon bridging oxygens

and the structure consists of independent SiO_4 tetrahedra. The progressive weakening of the bonding of the SiO_4 network by addition of M_2O and MO components has given rise to their description as network modifiers, and it follows that the addition of such network modifiers causes a decrease of viscosity. Other oxide components such as Fe_2O_3 and TiO_2 are also treated as network modifiers since the ionic radii of Fe^{3+} and Ti^{4+} preclude any extensive participation of these cations in the network forming tetrahedra. The dual role of Al_2O_3 is discussed in another section.

Model for Correlation

Regardless of structural details, it is clear from the above generalizations that viscosity-composition variations will be most strongly dependent on the concentration of network forming atoms such as silicon. It also follows that the effect of various network modifiers may be quite specific. Figures 1, 2, and 3 show the variation of the logarithm of viscosity with composition in binary systems of the type M_2O-SiO_2, MO-SiO_2, $MAlO_2$-SiO_2, and MAl_2O_4-SiO_2. In addition to the obvious increase of viscosity with mole fraction of SiO_2, several features of the plots are noteworthy. It is apparent that the effect of the various network modifiers is quite specific, and that a satisfactory quantitative model of viscosity-composition variation must be more discriminating than the usual "network former" and "network modifier" categories. The data for individual systems can be represented by a small number of straight line segments, i.e., the logarithm of viscosity may be satisfactorily expressed

as a linear function of composition over restricted composition intervals. We may generalize this relationship in analytical form:

$$\ln \eta = \sum_i X_i D_i \qquad (1)$$

where X_i is the mole fraction of the ith component, and D_i is a constant associated with component i over a restricted range of composition. Each temperature has a particular set of D_i constants. Our model is essentially based on an extension of these observations to multicomponent systems. The composition ranges have been arbitrarily selected at 35-45, 45-55, 55-65, 65-75, and 75-81 mole % SiO_2, and the D_i constants are determined by minimizing the sum

$$S = \sum_j \left(\ln \eta_{measured} - \sum_i X_i D_i \right)^2_j \qquad (2)$$

where j indexes all measurements within a composition range at a given temperature. The set of D_i constants which minimize S are determined by iteration, starting with initial D_i values that are obtained from linear least squares analyses of the two component systems of Table 1. The method of Marquard (1963), combining the Gauss method and the method of steepest descent, was used to obtain improved D_i values for each subsequent iteration.

Choice of Components

Aluminum plays a dual role in silicate liquid structures. The

addition of Al_2O_3 to SiO_2 liquid rapidly lowers viscosity (Kozakevitch,

1960 and Rossin et al., 1964). In such melts most of the aluminum is

evidently not isomorphous with silicon; it is predominantly in six-fold

coordination with oxygen and acts as a network modifier. The addition

of other monovalent or divalent cation oxides, however, allows aluminum to

enter the tetrahedral structural sites in place of silicon while preserving

local charge balance, and we postulate the existence of $MAlO_2$ and MAl_2O_4

groupings as schematically represented by:

$$
\begin{array}{ccc}
\begin{array}{c}
| \quad | \\
O \quad O \\
| \quad | \\
-O-Si-O-Si-O- \\
| \quad | \\
O \quad O \\
| \quad |
\end{array}
&
+ \quad MAlO_2 \quad =
&
\begin{array}{c}
| \quad | \quad | \\
O \quad O \; M^+ \; O \\
| \quad | \quad | \\
-O-Si-O-Al-O-Si-O- \\
| \quad | \quad | \\
O \quad O \quad O \\
| \quad | \quad |
\end{array}
\end{array}
$$
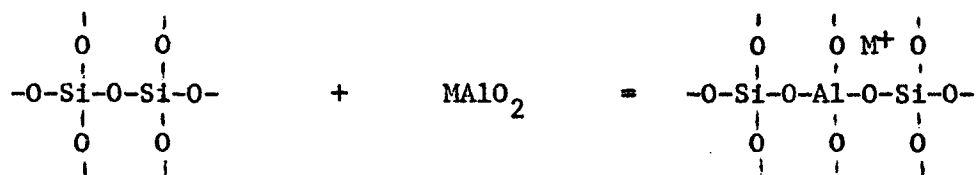
Fig. 4 shows that as Al_2O_3 is substituted for Na_2O or CaO (keeping the

mole % of SiO_2 constant) there is a steep increase of viscosity until a

maximum value is reached in the vicinity of compositions equimolar in

Al_2O_3 and Na_2O or CaO. This increase corresponds to the progressive

incorporation of aluminum in tetrahedral positions until all the associated

cations are used up. The tetrahedral framework is thereby strengthened

and the average size of the flow units is increased until the equimolar

composition is reached. Any additional aluminum is octahedrally coordinated

and does not cause a further increase in viscosity. Similar observations

have been made on the viscosity relations in the system $MgO-Al_2O_3-SiO_2$

(Riebling, 1964). The structural role of aluminum inferred from the viscosity

data is in agreement with determinations of the coordination number of

aluminum in soda aluminosilicate glasses by Day and Rindone (1962) using

an X-ray fluorescence technique. These authors found that the wavelength

of aluminum $K\alpha$ radiation at compositions Al/Na $\leq$ 1 corresponds to aluminum in

tetrahedral coordination whereas Al/Na > 1 compositions showed peak shifts

corresponding to aluminum in sixfold coordination. Thermodynamic analysis

of melts in the SiO_2-$KAlO_2$ and SiO_2-$NaAlO_2$ systems by Førland (1964) also

indicates the existence of $KAlO_2$ and $NaAlO_2$ species.

Most melts of geological interest have compositions such that the

sum of the MO and M_2O oxides exceeds Al_2O_3 on a molar basis, and we

therefore assume that in such melts aluminum will be predominantly in

tetrahedral coordination. In accord with this observation and the preceding

arguments we have chosen to express the chemical composition of magmatic

silicate melts in terms of the following major oxide components: $KAlO_2$,

$NaAlO_2$, $CaAl_2O_4$, $MgAl_2O_4$, SiO_2, TiO_2, FeO, MgO, CaO, Na_2O, and K_2O. In

addition, MnO, SrO, BaO, Li_2O, $BaAl_2O_4$, and $MnAl_2O_4$ may also be minor

components. In calculating the mole fraction of the various components,

Al_2O_3 is first combined with K_2O to form $KAlO_2$. Any excess potassium is

assigned to K_2O. More usually an excess of Al_2O_3 is further combined with

Na_2O, BaO, SrO, CaO, MgO, and MnO in that order until all Al_2O_3 is used

up. The sequence in which the aluminate components are calculated

corresponds to the order of stability suggested by viscosity measurements.

For example, Kani (1935) observed that molten $KAlSi_3O_8$ is more viscous

than molten $NaAlSi_3O_8$. We interpret this to mean that K^+ is favored over

Na^+ when competing for positions associated with tetrahedrally coordinated aluminum. Comparison of viscosities between the systems $Na_2O-Al_2O_3-SiO_2$ and $CaO-Al_2O_3-SiO_2$ are also informative in this regard. For this purpose, the melts to be compared must have equal molar concentrations of SiO_2 and Al_2O_3 and $Al_2O_3 \geq CaO$ or Na_2O. In such melts, differences of viscosity (at equal temperatures) correspond to differences in the relative stabilities of $NaAlO_2$ and $CaAl_2O_4$ groups, and the data indicate greater stability for the former. In binary $MO-SiO_2$ liquids, the replacement of CaO by MgO increases viscosity (e.g., Fig. 2). However, in $MO-Al_2O_3-SiO_2$ systems where molar $Al_2O_3 \geq MO$, the replacement of CaO by MgO decreases the viscosity (e.g., Fig. 3), indicating that calcium aluminate groups are more stable than magnesium aluminate groups. These and similar arguments lead us to calculate the mole fractions of the aluminate components in the order: $KAlO_2$, $NaAlO_2$, $BaAl_2O_4$, $SrAl_2O_4$, $CaAl_2O_4$, and $MgAl_2O_4$. Few data are presently available on the viscosity of silicate liquids containing MnO. Fortunately, the concentration of MnO in natural liquids is usually low, and calculation of $MnAl_2O_4$ is arbitrarily performed last in the sequence.

Previous correlation models

The degree to which silica tetrahedra are directly bonded to each other exerts an important influence on viscosity. In pure silica liquids (O/Si = 2) all oxygens are involved in Si-O-Si "bridges". At the ortho-silicate composition (O/Si = 4) there are only independent SiO_4 tetrahedra, i.e., no bridging oxygens. The variation of the O/Si ratio in silicate

liquids is a measure of the degree to which the SiO_2 tetrahedra are linked together. This line of reasoning led Murase (1962) to propose that the bridge density (b.d. = 4 - O/Si) of silicate melts be used to estimate the composition dependence of viscosity. In Fig. 5-c we have plotted the logarithm of viscosity as a function of bridge density for 123 representative measurements selected from the input data of Table 1. The correlation is very poor primarily because the model neglects the effect of aluminum. As previously pointed out, the role of aluminum in most natural liquid compositions will be that of tetrahedral network former.

More realistic models based on the ratio of oxygen to tetrahedrally coordinated cations (including aluminum) have been proposed by Shaw (1965) and Carron (1969). Figures 5a and 5b show an improved viscosity correlation for these parameters although the scatter may still be several orders of magnitude larger than the stated errors of the measurements. This scatter is mainly due to the non-specific roles assigned to the individual network forming and network modifying atoms in such general models. The quality and quantity of viscosity data now available (Table 1) makes it worthwhile to attempt a more quantitative correlation with composition. Where only limited data are available, as with the effects of H_2O, the generalized composition parameters are useful for semi-quantitative estimates of viscosity as shown by Shaw (1965), and Carron (1969).

RESULTS

Constants

An initial set of D_i constants (equation 1) was determined at $50^\circ C$

intervals between 1200 and $1800^\circ C$ by a least squares fit to the input

data (equation 2). The initial D_{SiO_2} values were then plotted against

reciprocal absolute temperature, and the departures from linearity assumed

to be mainly due to insufficient data in certain composition ranges.

The best straight lines were then drawn by visual inspection, and the

resultant smoothed values of D_{SiO_2} were used to determine new values of

D_i constants for the remaining components by minimizing the sum:

$$S = \sum_j (\ln \eta_{observed} - X_{SiO_2} D_{SiO_2} - \sum_{i \neq SiO_2} X_i D_i)^2_j \qquad (3)$$

The final D_i constants are given in Table 3 along with the number (N) of

input data at each temperature and composition range. We also list the

mean observed viscosity value and the root mean square of the difference

between calculated and observed viscosity for each set. There is generally

a smooth variation of the calculated D_i constants against reciprocal

temperature. In most cases the variation is approximately linear with

respect to reciprocal absolute temperature over an appreciable range,

and linearly extrapolated or interpolated values are denoted by italics

in Table 3. In all cases the actual values calculated by the final

least square analysis (equation 3) have been retained without further

smoothing.

Figure 6 is a frequency diagram of $\Delta \ln \eta = \ln \eta$ (calculated) -

$\ln \eta$ (measured) for 2,440 measurements. The average difference is zero,

and the frequency distribution is quite symmetrical. Of the total number

of comparisons made, 77% lie between $-0.25 < \Delta \ln \eta < 0.25$, and 99% lie

between $-0.75 < \Delta \ln \eta < 0.75$.

Viscosity Calculations

The entries of Table 3 are sufficient in many cases for direct

calculations of viscosity according to equation 1. In other cases lack

of sufficient input data makes it necessary to estimate D_i values for

certain components. This paper is primarily concerned with geological

applications, and we have chosen several compositions representative of

the magmatic range in order to discuss the calculations and the necessary

approximations. The compositions of the representative magma types are

given in weight percent in Table 4 along with the calculated mole

percentages of the components necessary for the viscosity calculations.

The results of the calculations are illustrated in Fig. 7.

Unfortunately, very few data are available on the viscosity of silicate

liquids containing Fe_2O_3. The few measurements which have been made are

all in the low Fe_2O_3 range with poor control of oxidation state, and it

is impossible to obtain accurate figures on the effect of this component.

To a first approximation the available data suggests that equimolar amounts

of Fe^{3+} and Fe^{2+} have roughly the same influence on the viscosity of

silicate melts (Endell and Zauleck, 1950; Johannsen and Brunion, 1959; and

Röntgen et al., 1960). In addition to the lack of experimental data on

the influence of Fe_2O_3, it must also be realized that the actual Fe_2O_3

content of magmas is usually not known to any degree of certainty. Analyses

of solidified rocks often reflect post liquid-state oxidation due to

cooling and/or exposure to oxidizing atmospheres. Accordingly, all Fe^{3+}

is converted to Fe^{2+} for the calculations. The small amount of phosphorus

usually present in magmas is added to silicon (cf. calculations in Table 4).

For present purposes it is convenient to consider as major components

all those present in concentrations greater than 5 mole %. For all major

components it is recommended that only the D_i values actually listed in

Table 3 be used. This will mean that for certain compositions, the

calculations will be possible only over certain restricted temperature

ranges (e.g., see compositions 2-6 of figure 7). In most cases the

temperature range will be sufficient for petrological applications, and

the linear reciprocal temperature dependence may be extrapolated to some

extent provided the system in question remains above the liquidus. The only

possible major component of rock systems for which there are no data in

Table 3 is $KAlO_2$. The measurements of Kani (1935) on molten alkali

feldspar systems indicate that D_{KAlO_2} should be somewhat larger than

D_{NaAlO_2}. When sufficient viscosity data become available for potassic

aluminosilicate systems the D_{KAlO_2} constants can be quantitatively

evaluated. In the meanwhile, we have been forced to use $D_{KAlO_2} = D_{NaAlO_2}$

for purposes of calculation. Fortunately $KAlO_2$ is not a major component of most magma types (only composition number 1 of Table 4).

There is not always available a complete set of constants for the minor (< 5%) components, and the following recommended approximations have been used in the sample calculations. Where values of D_{TiO_2} are not available, we have used the approximation, $D_{TiO_2} = D_{CaO}$. This is consistent with the experimental results of Johannsen and Brunion (1959) on the effect of these two components. No data are available for D_{NaAlO_2} in the composition range $0.35 < X_{SiO_2} < 0.45$. Fortunately most rock systems in this range also tend to have only relatively minor amounts of alkali metals (cf. analyses 5 and 6 of Table 4). In these cases we have used the D_{NaAlO_2} constants for the composition range $0.45 < X_{SiO_2} < 0.55$. Wherever D_i constants are lacking for certain minor MO components, we have used the arithmetic average of the D_{MO} values listed in Table 3. All the above approximations should be valid to within ± 2 (natural logarithm units) or better, and the error associated with each minor component estimate will therefore be less than ± 0.10. The average error of several such estimates is often less.

Figure 7 illustrates the range of viscosities calculated for typical natural silicate liquids. Over 95% of the 65 calculated points fall within ± 0.20 (ln η) of a straight line plot, and we believe that linearly smoothed reciprocal temperature plots are the most valid representations of the calculated viscosities. Our model is based on a quasi-linear variation of the logarithm of viscosity with composition over somewhat

arbitrarily defined composition ranges. This necessarily imposes an artificial discontinuity on the calculated viscosities when the composition is varied from one range to the next. Analysis number 2 (X_{SiO_2} = 0.659, Table 4 and Fig. 7) falls close to the arbitrary division between two composition ranges. In order to illustrate this effect we have calculated the viscosity using D_i constants for the range 0.65 < X_{SiO_2} < 0.75 (curve 2a) and 0.55 < X_{SiO_2} ≷ 0.65 (curve 2b).

Viscosity of molten rock and mineral systems

The proposed model is primarily intended for calculating melt viscosities in chemically complex geological systems. The most direct test of the model is a comparison of calculated and measured viscosities in such systems. Unfortunately there is not an abundance of viscosity data for geological systems, and we have decided to make comparisons with all the measurements known to us rather than attempting to select only superior experimental data. In Table 5 we have compiled most of the published viscosity measurements for molten rock systems, and have compared them to calculated viscosities. In each case the calculated viscosity is obtained from a linear fit (against reciprocal absolute temperature) of individual points calculated at 50° C intervals using the data of Table 3. The compositions of the melts are listed in Table 6. The total range of viscosities covered and the correspondence between measured and calculated values is given in Fig. 8. The general agreement shown in Table 5 and Fig. 8 indicates that the model may certainly be used with confidence for the purpose of obtaining viscosity values for geological calculations.

Considering the experimental data by sets according to author(s) we detect no systematic trends in the differences between measured and calculated viscosities except with the data of Euler and Winkler (1957). The calculated values for this set of compositions tend to be somewhat higher than the measured viscosities. For the 55 data points of these authors given in Table 5, the average deviation, $\Sigma(\ln \eta_c - \ln \eta_m)/55$, is equal to 0.51. For the remaining 54 data points listed in Table 5, the average deviation is very close to zero (-0.06).

We may also compare the measured and calculated viscosities of Table 5 and Fig. 8 for individual rock compositions. The only serious discrepancies occur with an olivine basalt and olivine dolerite (analyses 20 and 21, Euler and Winkler, 1957), a nepheline basalt (analysis 4, Kani, 1934), and an andesite (analysis 8, Volarovich, 1936). There is strong evidence that for the first three of these molten rock systems, the measurements are not self-consistent. In Fig. 9 we have plotted the logarithm of the measured and calculated viscosities against reciprocal absolute temperature, and have shown the activation energy for viscous flow that is indicated from the slope in each case. The measurements of Kani (1934) on the nepheline basalt (Fig. 9a) imply an activation energy of only 15 kcal/mole, well below the range of observed values for all comparable silicate liquids. The measurements of Euler and Winkler (1957) on molten basalt and dolerite (Fig. 9b and c) indicate activation energies which are very much too large since the activation energy for pure SiO_2 liquid is 120 kcal/mole (Rossin et al., 1964). The calculated

viscosities in all three cases indicate activation energies which are consistent with values normally observed for comparable systems.

It is impossible at this stage to attempt any further quantitative evaluation of the accuracy of the viscosity calculations. The differences between measured and calculated values may be due to a large number of factors. First of all, the model itself is only a convenient approximation. Our fit to the data on the simple synthetic systems is by no means perfect, and in some instances the D_i constants are based on relatively few measurements. The measurements themselves are subject to all the errors previously discussed in reference to the reliability of the input data. The problem of accurate composition is perhaps more acute in rock systems. Many of the measurements were performed on molten rocks which were analyzed in separate splits without special precautions taken to insure homogeneity. Most of the rocks contained some H_2O before melting. Alkali metals are subject to volatilization during heating in the presence of H_2O. Crucibles of graphite, alumina and platinum have been used in the experiments. All three materials are known to react with silicate liquids under certain conditions. The undetected presence of crystals in the melt may also seriously affect measured viscosities. The effective viscosity of a liquid-solid suspension may be estimated from the equation due to Roscoe (1952)

$$\eta_e = \eta(1 - 1.35 \ \phi)^{-2.5} \tag{4}$$

where η_e is the effective viscosity of a suspension made up of a liquid of viscosity η and ϕ volume fraction of solid particles (ideally uniform spheres). The equation leads to an estimate of $\eta_e/\eta = 1.2$ when $\phi = 0.05$. The presence of bubbles usually affects viscosity measurements in a similar way. The gas pressure inside the bubble and the surface tension at the gas-liquid interface oppose the deformation caused by shear strain and increase the effective viscosity. An increase of viscosity due to the presence of bubbles is well known from the study of foams made from relatively low viscosity materials. It is perhaps less widely appreciated that bubbles can also effectively lower viscosity if the energy necessary to deform a bubble in a liquid is less than the energy required to induce viscous flow in the bubble-free liquid phase. This phenomenon may be anticipated in highly viscous liquids, and was observed by Murase (1962) who recorded decreasing viscosity of a glassy specimen of the Oshima 1950 lava at 1000° C during vesiculation.

A general evaluation of the over-all accuracy of the existing viscosity-temperature-composition data for geologic systems could be obtained by comparing the results of separate laboratory measurements on identical compositions. Unfortunately, we are not aware of any published duplicate measurements in rock systems which would allow such intercomparisons. However, several independent measurements in feldspar composition liquids have been published. A comparison of these is made in Table 7 where it can be seen that the scatter of measured values is comparable to the differences between calculated and measured viscosities for rock systems noted in Table 5 and Fig. 8.

DISCUSSION

In order to arrive at a valid calculation model it has been necessary to examine a wide spectrum of the literature covering measurement and theory of viscosity in silicate systems. Such a comprehensive review, combined with extensive statistical treatment of much of the presently accumulated data inevitably results in a broad overview of many interesting aspects of the problem of viscous flow in silicate liquids. We present in this section a brief discussion of a few points which we believe to be of interest.

Compositional Dependence of Viscosity

The linearity of the composition dependence of logarithmic viscosity in silicate melts over even restricted composition ranges may appear somewhat surprising, but it is interesting to note that similar relations have long been known in ionic solutions. Jones and Dole (1929) found that the viscosity of such solutions is given by

$$\eta = \eta_o (1 + A\sqrt{C} + BC) \qquad (5)$$

where η_o is the viscosity of the solvent, C is the molar concentration of solute, and A and B are constants for a particular solute. The $A\sqrt{C}$ term is always positive and accounts for the expected increase in viscosity due to the electrostatic attraction between oppositely charged nearest neighbor ions (Falkenhagen and Dole, 1929; Onsager and Fuoss, 1932). It is much

smaller than the BC contribution at all but very dilute concentrations. Values of B coefficients are characteristic for individual ions and can be either positive or negative. Negative viscosity coefficients in aqueous solutions have been explained in terms of the "depolymerization" of water by certain ions (Cox and Wolfenden, 1934). The analogy with network modifying components which have low or negative D_i coefficients in our model is obvious.

In reviewing viscosity data for moderately dilute aqueous electrolyte solutions, Gurney (1953) concluded that the contributions of individual ions to the BC term were approximately additive and independent, i.e.,

$$\eta \approx \eta_0 \left(1 + \sum_i B_i C_i\right) \tag{6}$$

where i indexes the individual solute ions. We note that since $B_i C_i \ll 1$ for all solutions studied, equation 6 can be rewritten as

$$\ln \eta \approx \ln \eta_0 + \sum_i B_i C_i \tag{7}$$

In our model we have proposed that the composition dependence of viscosity for silicate liquids is given by equation 1, which can be rewritten in the form

$$\ln \eta = D_{SiO_2} + \sum_j (D_j - D_{SiO_2}) X_j \tag{8}$$

where j indexes all components other than SiO_2. If we regard SiO_2 as the "solvent" of silicate melts, there is a direct analogy in the composition dependence of viscosity between the two types of liquids.

Temperature Dependence of Viscosity

Almost all of the viscosity measurements we have examined were found to obey closely an Arrhenius type equation, $\eta = A\exp(E/RT)$. This holds true for measured viscosities in molten rock systems as well as measurements in synthetic silicate systems of fewer components. The fit to an Arrhenius equation is good enough in many cases to allow the authors to report the measurements exclusively in terms of the Arrhenius parameters A and E. Out of all the data that were selected for the input set, only those of Liutikov and Tsylev (1963) and Staronka and Choma (1968) could not be fitted to an Arrhenius equation with 10% maximum deviation. The reasons for these exceptions to the rule are not known to us.

Because the input data closely obeys the Arrhenius expression, it is reasonable to expect a linear dependence against reciprocal temperature for the deduced D_i constants. As described in a previous section, this was found to be approximately the case for the initial set of D_{SiO_2} constants, and the final set of D_{SiO_2} constants were constrained to this type of temperature dependence. It is interesting to note then that most of the remaining D_i constants also show a $1/T$ linear dependence. This is not true for all components, and some of the D_i vs. $1/T$ linear plots exhibit kinks. It is not possible to state with certainty whether these

irregularities are fundamental or simply due to the sparse data available for certain components. In any case, constraining the D_{SiO_2} coefficients to a linear 1/T dependence does not appreciably influence the agreement between calculated and measured viscosity (root mean square difference between calculated and measured natural logarithm of viscosity for the entire input data set increased only from 0.239 to 0.246 when D_{SiO_2} was thus constrained). It should also be pointed out that calculated viscosities for all silicate systems which we have considered so far indicate Arrhenius E values which are consistent with the range of values to be expected from estimates based on composition. Calculated viscosities do not show any of the irregularities which are occasionally displayed by the coefficients for the individual components, and some restricted temperature extrapolation via reciprocal temperature plots is probably permissible. These should not extend appreciably below the stable liquidus temperature for any system since there is evidence (see below) that E in the Arrhenius equation becomes temperature sensitive in the metastable liquid range.

Many theories dealing with the liquid state have been proposed to explain the temperature dependence of transport properties such as diffusion, conductivity and viscosity. Some of these theories result in modified forms of the simple Arrhenius equation. For example, reaction rate theory (Glasstone et al., 1941) applied to viscous flow in liquids predicts a relation of the form

$$\eta = K \sqrt{T} \exp(E/RT) \qquad (9)$$

In this equation and in equations 10 and 11, K denotes a constant that is
temperature independent. From our analysis of the data for silicate
liquids we conclude that there is no difference in goodness of fit between
equation 9 and the simpler Arrhenius relation. The existing data are
not sufficiently precise to act as a basis for either retaining or
rejecting the $\sqrt{T}$ term in the pre-exponential.

When glass-forming liquids are cooled metastably below their
equilibrium liquidus temperatures no discontinuity is observed in their
second-order thermodynamic properties such as heat capacity or thermal
expansion coefficient. This temperature range represents a true region of
stability (really metastability with respect to ordered crystalline phases)
for the supercooled liquid. Further cooling to a temperature which is
characteristic for each particular liquid system causes a sharp decrease
in heat capacity and thermal expansion. Such measurements serve to
operationally define the supercooled liquid-glass transition in glass-
forming systems. It is recognized that the glass transition defined in
this way is a rate controlled phenomenon since with reduced cooling rates
it is possible to lower the temperature at which discontinuities are
observed in thermal expansion and heat capacity. In practice then, the
glass transition point, T_g, is somewhat arbitrarily determined by the
coincidence of molecular relaxation times in the supercooled liquid system
and "normal" cooling rates as judged by human standards (for most glass-
forming systems this usually corresponds to a measured viscosity coefficient
of approximately 10^{13} poises at T_g). Based to a large extent on the type

of behavior outlined above which is typical of classical glass-forming systems such as the silicates, the glass transition has often been discussed in purely kinetic terms, and glasses regarded strictly as supercooled liquids which because of their great structural complexity are only very slowly approaching the true equilibrium liquid structural state. This view maintains that only the barrier of time separates glasses from equilibrium supercooled liquids.

It is becoming increasingly clear that a purely kinetic concept of the glassy state cannot be strictly correct. Kauzmann (1948) showed that the equilibrium properties of the true supercooled liquid (i.e., above T_g) when extrapolated to temperatures below T_g quickly lead to impossible thermodynamic results such as negative configurational entropy. This result led Kauzmann to believe that "a non-vitreous stable [really metastable] liquid cannot exist below a certain temperature". Gibbs and DiMarzio (1958) have subsequently shown that this paradox can be resolved by assuming that all supercooled liquids, given sufficient time, will undergo a true thermodynamic second-order transition to a glass. The temperature of this thermodynamic glass transition, T_o (as opposed to the operationally defined kinetic glass transition) corresponds to Kauzmann's "certain temperature" and is always somewhat below T_g. It is, in fact, the lower limit of T_g as would be determined by measurements of heat capacity or thermal expansion in experiments of ever-increasing duration. This theory of glasses differs fundamentally from previous ones in that the glassy state is considered a true thermodynamic equilibrium (albeit metastable) state.

Recent developments of the ideal glass concept (Gibbs and DiMarzio, 1958; Adam and Gibbs, 1965; Angell, 1968a and 1968b, among others) lead to some interesting conclusions regarding the temperature dependence of viscosity in liquids. The entropy of a liquid may be considered in terms of thermal and configurational contributions. The latter vanishes to zero at T_o according to the theory of the ideal glass transition. Since viscous flow can take place only via configurational changes, it follows that the ideal supercooled liquid has infinite viscosity at T_o. Consequently, T_o can be considered as the temperature at which the ideal liquid reaches a configurational ground state, and this characteristic temperature should be the effective temperature "zero point" in equations describing the temperature dependence of viscosity. In keeping with this generalization, the theoretical treatment of Adam and Gibbs (1965) predicts a temperature dependence of the form

$$\eta = K\exp\left(\frac{E}{T\ln T/T_o}\right) \tag{10}$$

As pointed out by Angell (1968a), when T is not far above T_o this is approximately equivalent to the Vogel-Tammann-Fulcher empirical equation (Fulcher, 1925) which successfully describes the temperature dependence of viscosity in many liquids.

$$\eta = K\exp\left(\frac{E}{T - T_o}\right) \tag{11}$$

We have attempted to fit the input data to both equations 10 and 11, but despite the fact that they have an additional parameter over the Arrhenius equation, they did not provide a better fit to the measurements. This may be related to the fact that our set of measurements are valid for "high temperature liquids" (i.e., above the stable liquidus) whereas equations 10 and 11 have been most successful in describing the properties of liquid systems at lower temperatures (see Angell, 1968a for discussion of this point). The above considerations serve to emphasize our caution about extrapolation of calculated viscosities much below the liquidus. This should not prove to be a major obstacle in using the calculations for petrologic purposes since the existence of greatly supercooled liquids in nature is presumably rare.

CONCLUDING REMARKS

We have presented this model for viscosity in the hope that, in spite of its obvious shortcomings, it will allow a more quantitative treatment of many geologic problems involving viscosity. The approach has already proved useful in explaining certain aspects of lunar petrology which can be related to the viscosity of lunar lavas (Weill et al., 1970 and 1971). Another obvious application is that now it is possible to calculate the progressive changes in viscosity of a multicomponent magmatic or synthetic liquid during crystallization. Space does not permit us to do more in this paper than simply present the model in detail. We leave it to others

to develop additional applications and also to further test its validity as additional data are gathered.

The variation of a physical property such as viscosity in temperature-multicomponent space is clearly only a special case of a more general petrologic problem involving other physical and chemical properties of magmatic liquids. It is encouraging to find that even with the compositional complexities of natural systems taken into account, some of these problems lend themselves to relatively simple approximate analytical solutions (cf., density of magmatic liquids in Bottinga and Weill, 1970, and plagiocalse crystallization in Kudo and Weill, 1970). It is a pleasure to think that perhaps additional aspects of igneous petrology will prove capable of being systematized in like fashion.

ACKNOWLEDGMENTS

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Table 1.   Silicate systems, approximate temperature ranges and
references for viscosity input data.

System	T range (°C)	Reference no.
Li_2O-SiO_2	1100-1700	1,2
Na_2O-SiO_2	1100-1750	1,2,3
K_2O-SiO_2	1100-1600	1,2
$CaO-SiO_2$	1400-1800	4,11,12,14,15,18,19
$MgO-SiO_2$	1600-1800	1,15
$FeO-SiO_2$	1200-1450	5,6,16,17
$SrO-SiO_2$	1550-1800	1,15
$BaO-SiO_2$	1500-1800	1,15
$Na_2O-Li_2O-SiO_2$	1100-1150	7
$Na_2O-K_2O-SiO_2$	1100-1400	7
$Na_2O-CaO-SiO_2$	1100-1400	7
$Na_2O-BaO-SiO_2$	1100-1150	7
$Na_2O-FeO-SiO_2$	1300-1500	9
$Na_2O-TiO_2-SiO_2$	1300-1450	9
$CaO-MgO-SiO_2$	1300-1650	14,18,19
$CaO-FeO-SiO_2$	1200-1350	8
$FeO-MnO-SiO_2$	1300-1500	16,17
$FeO-MnO-TiO_2-SiO_2$	1250-1500	16,17
$Al_2O_3-Na_2O-SiO_2$	1100-1750	9,10
$Al_2O_3-CaO-SiO_2$	1150-1800	11,13,14,15,16,17,18,19, 24,
$Al_2O_3-MgO-SiO_2$	1250-1800	14,15,19,20,21,24
$Al_2O_3-SrO-SiO_2$	1500-1800	15
$Al_2O_3-BaO-SiO_2$	1550-1800	15
$Al_2O_3-CaO-MgO-SiO_2$	1250-1650	14,16,17,18,19,22,23,24
$Al_2O_3-CaO-Bao-SiO_2$	1400-1500	16,17
$Al_2O_3-CaO-TiO_2-SiO_2$	1250-1450	24
$Al_2O_3-CaO-FeO-SiO_2$	1200-1450	24
$Al_2O_3-CaO-MgO-BaO-SiO_2$	1300=1650	14,22
$Al_2O_3-CaO-FeO-MnO-SiO_2$	1400	16,17
$Al_2O_3-CaO-MgO-FeO-SiO_2$	1250-1500	25
$Al_2O_3-CaO-FeO-MnO-TiO_2-SiO_2$	1400	16,17

Table 1. Continued

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Table 2. Intercomparisons of viscosity input data.

Composition (mole %)					Viscosity (poises)			Reference no. (see Table 1)
SiO_2	Li_2O	Na_2O	K_2O	FeO	$1200°$	$1300°$	$1400°(C)$	
69.7	30.3				131	54.9	25.6	2 (interpolated)
70.0	30.0				115	55.0	28.2	1
67.4	32.6				83.0	39.8	20.0	2 (interpolated)
67.0	33.0				81.3	38.9	20.0	1
80.5		19.5			940	385	179	2 (interpolated)
80.5		19.5			911	351	151	2 (interpolated)
80.4		19.6			1030	410	182	3
80.0		20.0			–	224	100	1
70.5		29.5			270	112	55	3
70.0		30.0			200	87.1	41.7	1
69.9		30.1			204	81.8	50.0	2 (interpolated)
67.1		32.9			150	64.1	30.3	2 (interpolated)
67.0		33.0			135	60.3	28.2	1
83.3			16.7		2447	861	343	2 (interpolated)
83.1			16.9		2240	832	355	1
67.0			33.0		355	125	50.2	2 (interpolated)
66.6			33.4		390	145	60.3	1
39.2				60.8	5.75	1.6	–	5
39.0				61.0	–	1.5	1.0	6
39.0				61.0	–	1.1	1.0	16

SiO_2	MgO	CaO	SrO	BaO	$1600°$	$1700°$	$1800°$	
50.0	50.0				3.16	1.79	1.18	1
50.0	50.0				3.82	2.26	1.41	15
69.5		30.5			–	13.6	8.5	4
69.5		30.5			–	18.1	10.0	15
48.3		51.7			1.81	1.11	0.75	4
48.3		51.7			1.8	1.1	0.8	12
58.4		41.6			4.68	2.75	1.8	15
58.3		41.7			4.7	2.7	1.8	12

Table 2. (CONTINUED)

SiO₂	Al₂O₃	CaO	MgO				Ref.
49.7		50.3		3.18	1.95	1.30	1
50.0		50.0		3.08	1.87	1.20	15
50.2			49.8	2.80	1.86	1.50	1
50.0			50.0	2.76	1.68	1.07	15

Composition (mole %)				Viscosity (poises)			Reference no. (see Table 1)
SiO₂	Al₂O₃	CaO	MgO	1300°	1400°	1450°	
62.7	9.2	28.0		720	180	120	24
62.7	9.2	28.0		730	214	128	13
53.2	12.6	34.2		247	70.9	42.3	18
53.3	12.4	34.3		250	80	55	24
53.2	12.6	34.2		–	–	42.4	12
56.6	8.3		35.1	–	–	53	21
56.6	8.3		35.1	–	118	68.6	19
59.1	11.6		29.3	–	–	115	21
59.1	11.6		29.3	–	269	148	19
SiO₂	Al₂O₃	CaO	MgO	1350°	1400°	1450°	
55.0		24.5	20.5	27.7	18.2	12.2	19
55.2		23.9	20.9	29.3	15.4	9.2	14
40.8		38.9	20.3	–	4.11	2.89	18
40.7		39.1	20.2	10.4	5.3	4.2	14
SiO₂	Al₂O₃	CaO	MgO	1600°	1700°	1800°	
52.0	23.5		24.5	19.5	9.71	5.17	20
50.0	25.0		25.0	12.5	6.16	5.25	15
SiO₂	Al₂O₃	CaO	MgO	1400°	1450°	1500°	
39.5	5.8	47.5	7.3	6.77	4.60	3.29	23
39.5	5.8	47.5	7.3	5.8	3.1	2.9	22
43.6	5.7	36.3	14.4	10.2	6.9	4.58	18
43.6	5.7	36.3	14.4	8.8	6.3	4.2	22
49.6	5.8	37.2	7.4	16.9	10.9	7.53	18
49.6	5.8	37.2	7.4	21.7	14.9	8.9	22
61.6	12.1	11.0	15.3	440	238	137	19
62.0	12.1	11.3	14.6	390	222	124	14

Table 3. D_i constants for calculation of viscosity according to $\ln n = \Sigma X_i D_i$.

Root mean square of difference = r.m.s.d. = $[(\Sigma(\ln n_{obs} - \ln n_{calc})^2]^{1/2}/N$.

Numbers in italics are linearly extrapolated or interpolated from a

D_i versus reciprocal absolute temperature plot.

T (°C)	1200	1250	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
					— X_{SiO_2} from 0.81 to 0.75 —								
D_{SiO_2}	13.62	12.80	11.96	11.21	10.50	9.80	9.20	8.56	8.03	7.51	6.97	6.52	6.03
D_{TiO_2}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{FeO}	-	4.4	5.1	5.71	6.17	6.85	7.4	7.8	-	-	-	-	-
D_{MnO}	-	-	-4.59	-4.73	-6.41	-4.40	-	-	-	-	-	-	-
D_{MgO}	-0.6	3.83	-1.19	-1.37	-2.23	-2.05	-2.62	-2.7	-3.0	-3.2	-3.4	-3.6	-3.8
D_{CaO}	-2.5	-3.18	-3.38	-1.87	-3.61	-3.47	-5.17	-4.5	-	-	-	-	-
D_{SrO}	-	-	-	-	-	-5.6	-	-	-	-6.2	-5.88	-5.70	-5.25
D_{BaO}	-	-	-	-	-	-	-5.56	-5.19	-5.16	-5.05	-4.76	-4.74	-4.48
D_{Li_2O}	-	-16.5	-15.98	-14.87	-14.50	-13.87	-13.35	-12.47	-12.16	-12.08	-11.31	-10.7	-
D_{Na_2O}	-18.90	-18.65	-17.63	-16.27	-15.41	-13.76	-13.12	-12.3	-	-	-	-	-
D_{K_2O}	-15.19	-14.71	-14.00	-13.51	-12.96	-12.4	-	-	-	-	-	-	-
D_{KAlO_2}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{NaAlO_2}	19.5	18.4	17.3	16.1	15.1	14.24	13.23	12.49	11.56	10.73	10.09	9.28	8.71
$D_{BaAl_2O_4}$	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 3. (continued)

T (°C)	1200	1250	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
							X_{SiO_2} from 0.81 to 0.75						
$D_{CaAl_2O_4}$	2.1	0.79	0.29	-1.25	-1.74	-2.84	-4.09	-4.84	-5.28	-5.64'	-5.79	-6.19	-6.32
$D_{MgAl_2O_4}$	1.5	-0.2	-1.59	-3.47	-4.82	-6.04	-6.63	-7.91	-8.11	-8.25	-8.26	-8.43	-8.44
$D_{MnAl_2O_4}$	-	-	-11.55	-11.78	-11.49	-10.34	-	-	-	-	-	-	-
N	8	15	24	27	34	30	18	7	8	7	8	7	7
avg ln n_{obs}	6.84	8.98	8.49	8.07	7.43	7.93	7.50	7.67	6.98	6.55	5.90	5.52	5.05
r.m.s.d.	0.37	0.31	0.26	0.26	0.24	0.21	0.13	0.03	0.05	0	0	0	0
							X_{SiO_2} from 0.75 to 0.65						
D_{SiO_2}	13.22	12.40	11.55	10.80	10.05	9.40	8.77	8.13	7.60	7.05	6.50	6.03	5.55
D_{TiO_2}	-6.5	-6.61	-5.08	-4.57	-4.39	-4.10	-3.5	-	-	-	-	-	-
D_{FeO}	-3.9	-4.27	-2.41	-1.87	-2.08	-2.18	-1.2	-	-	-	-	-	-
D_{MnO}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{MgO}	-4.3	-3.93	-2.73	-3.43	-3.87	-3.56	-3.11	-2.72	-2.70	-3.19	-2.79	-2.20	-1.27
D_{CaO}	-2.86	-6.92	-6.53	-6.32	-6.24	-6.06	-6.40	-6.0	-4.30	-5.92	-5.96	-5.83	-5.62
D_{SrO}	-	-	-	-	-	-	-4.7	-8.16	-4.58	-4.63	-4.69	-4.62	-4.61
D_{BaO}	-	-	-	-	-	-8.4	-7.89	-7.76	-7.74	-7.31	-7.08	-6.80	-6.52
D_{Li_2O}	-13.92	-13.42	-12.91	-12.32	-11.73	-11.27	-10.94	-10.95	-10.55	-10.1	-	-	-
D_{Na_2O}	-12.96	-12.20	-11.63	-11.37	-10.79	-10.35	-9.93	-11.06	-12.42	-10.29	-9.07	-8.1	-6.15
D_{K_2O}	-10.20	-10.19	-9.70	-9.55	-9.17	-9.0	-	-	-	-	-	-	-
D_{KAlO_2}	-	-	-	-	-	-	-	-	-	-	-	-	-

Table 3. (continued)

T (°C)	1200	1250	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
						X_{SiO_2} from 0.75 to 0.65 —							
D_{NaAlO_2}	3.34	3.13	2.41	4.93	4.90	4.92	4.88	6.07	6.53	6.33	6.15	4.20	6.29
$D_{BaAl_2O_4}$	–	–	–	–	–	–	–	–	–	-1.4	-1.70	-2.03	-2.29
$D_{CaAl_2O_4}$	4.8	5.0	1.45	-0.33	-1.22	-2.55	-3.38	-5.22	-5.20	-5.26	-5.36	-5.52	-5.62
$D_{MgAl_2O_4}$	3.6	1.86	-1.03	-2.40	-3.55	-5.70	-6.42	-6.72	-6.86	-6.88	-6.94	-7.05	-7.11
$D_{MnAl_2O_4}$	–	–	–	–	–	–	–	–	–	–	–	–	–
N	31	63	75	77	81	76	54	23	23	22	24	23	22
avg lnn_{obs}	7.25	8.10	7.25	6.58	5.95	5.53	4.99	4.88	4.51	4.09	3.68	3.32	2.73
r.m.s.d.	0.38	0.33	0.30	0.28	0.26	0.25	0.21	0.14	0.27	0.20	0.14	0.13	0.13
						X_{SiO_2} from 0.65 to 0.55 —							
D_{SiO_2}	12.32	11.52	10.72	9.97	9.25	8.58	7.97	7.35	6.80	6.27	5.70	5.28	4.76
D_{TiO_2}	–	-4.2	-4.32	-4.41	-4.26	-4.16	-4.1	–	–	–	–	–	–
D_{FeO}	-3.09	-5.38	-5.97	-6.19	-6.64	-6.52	-4.58	-7.2	–	–	–	–	–
D_{MnO}	–	–	–	–	-5.41	–	–	–	–	–	–	–	–
D_{MgO}	-3.1	-3.19	-3.65	-4.18	-4.27	-3.78	-3.33	-4.68	-4.02	-3.90	-3.71	-3.57	-3.22
D_{CaO}	-2.57	-4.29	-4.58	-5.28	-5.54	-5.52	-5.52	-5.66	-5.41	-5.36	-5.19	-5.16	-4.88
D_{SrO}	–	–	–	–	–	–	-5.0	-4.93	-4.90	-4.86	-4.67	-4.72	-4.48
D_{BaO}	–	–	–	–	-5.0	-5.0	-4.79	-5.00	-5.00	-4.92	-4.68	-4.56	-4.22
D_{Li_2O}	-10.65	-10.24	-9.80	-9.38	-9.00	-8.43	-8.09	-8.2	–	–	–	–	–

Table 3. (continued)

T (°C)	1200	1250	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
					— X_{SiO_2} from 0.65 to 0.55 —								
D_{Na_2O}	-8.68	-8.47	-8.31	-8.02	-8.00	-7.85	-7.79	-8.61	-8.12	-5.43	-3.70	–	–
D_{K_2O}	–	–	–	–	–	–	–	–	–	–	–	–	–
D_{KAlO_2}	–	–	–	–	–	–	–	–	–	–	–	–	–
D_{NaAlO_2}	5.63	5.31	6.16	5.92	7.48	7.00	6.54	6.50	5.95	5.39	4.92	3.78	3.7
$D_{BaAl_2O_4}$	–	–	–	–	–	–	–	–	–	–	–	–	–
$D_{CaAl_2O_4}$	1.85	2.48	0.73	0.72	-0.22	-1.40	-1.64	-2.91	-3.04	-3.16	-3.21	-3.50	-3.60
$D_{MgAl_2O_4}$	-0.4	-3.50	-1.82	-2.24	-1.71	-3.51	-3.81	-3.62	-4.50	-4.72	-4.80	-4.97	-4.89
$D_{MnAl_2O_4}$	–	–	–	–	–	–	–	–	–	–	–	–	–
N	17	33	46	56	66	82	34	34	33	32	29	28	23
avg lnn_{obs}	6.33	5.86	5.48	4.84	5.80	5.02	4.45	4.55	4.05	3.55	3.18	2.60	1.17
r.m.s.d.	0.20	0.16	0.15	0.16	0.24	0.24	0.24	0.29	0.19	0.19	0.18	0.16	0.18
					— X_{SiO_2} from 0.55 to 0.45 —								
D_{SiO_2}	10.33	9.50	8.70	7.97	7.26	6.60	6.00	5.39	4.87	4.30	3.77	3.35	2.85
D_{TiO_2}	–	–	-1.15	-3.76	-4.05	-2.43	–	–	–	–	–	–	–
D_{FeO}	-3.4	-4.19	-4.82	-5.40	-4.78	-6.48	-5.39	-6.6	–	–	–	–	–
D_{MnO}	–	–	-4.09	-2.25	-4.57	–	–	–	–	–	–	–	–
D_{MgO}	-2.8	-2.72	-3.07	-2.92	-2.88	-2.69	-2.32	-1.76	-2.24	-2.43	-2.13	-2.19	-2.67

Table 3. (continued)

T (°C)	1200	1250	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
					X_{SiO_2} from 0.55 to 0.45								
D_{CaO}	-1.4	-1.43	-1.77	-2.21	-2.58	-2.72	-2.88	-3.06	-3.12	-3.38	-3.18	-3.17	-3.00
D_{SrO}	-	-	-	-	-	-	-2.7	-2.60	-2.57	-2.51	-2.46	-2.48	-2.39
D_{BaO}	-	-	-	-	-	-	-2.8	-2.82	-2.84	-2.76	-2.64	-2.58	-2.39
D_{Li_2O}	-7.77	-7.61	-7.29	-7.00	-6.73	-6.45	-5.95	-6.0	-	-	-	-	-
D_{Na_2O}	-7.37	-7.21	-6.86	-6.84	-6.64	-6.33	-6.11	-3.86	-3.49	0.49	1.13	-	-
D_{K_2O}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{KAlO_2}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{NaAlO_2}	11.15	10.59	9.98	9.59	9.15	8.14	7.58	6.70	6.14	5.44	4.98	4.47	4.08
$D_{BaAl_2O_4}$	-	-	-	-	-	-	-	-	-	-	-	-	-
$D_{CaAl_2O_4}$	7.1	6.09	4.85	3.50	2.74	1.75	0.73	-0.26	-0.50	-1.07	-1.14	-1.28	-1.30
$D_{MgAl_2O_4}$	-	3.5	2.57	2.74	3.46	6.34	2.41	-1.20	-0.65	-0.06	-0.16	0.45	0.91
$D_{MnAl_2O_4}$	-	-	-	-	-	-	-	-	-	-	-	-	-
N	7	35	49	64	97	107	113	52	42	42	33	32	32
avg $\ln\eta_{obs}$	5.08	4.64	4.19	3.55	3.29	3.36	2.84	2.80	2.51	2.01	1.81	1.19	0.90
r.m.s.d.	0.12	0.14	0.16	0.16	0.18	0.25	0.30	0.33	0.21	0.26	0.15	0.15	0.13
					X_{SiO_2} from 0.45 to 0.35								
D_{SiO_2}	5.35	5.02	4.71	4.43	4.17	3.90	3.67	3.41	3.21	3.00	2.80	2.62	2.42
D_{TiO_2}	-2.2	-1.10	-0.80	-1.32	0.87	1.99	1.9	-	-	-	-	-	-

Table 3. (continued)

T (°C)	1200	1250	1300	1350	1400	1450	1500	1550	1600	1650	1700	1750	1800
			—		X_{SiO_2} from 0.45 to 0.35		—						
D_{FeO}	-1.83	-2.51	-2.82	-2.87	-3.20	-3.00	-4.02	-4.0	-	-	-	-	-
D_{MnO}	-3.6	-3.10	-3.01	-3.17	-2.49	-2.13	-2.2	-	-	-	-	-	-
D_{MgO}	2.0	1.53	2.41	0.54	-0.35	-0.75	-1.23	1.48	1.12	1.07	-0.73	-1.04	-1.13
D_{CaO}	0.74	0.74	0.71	0.39	-0.16	0.71	-1.17	-2.48	-3.32	-2.35	-2.59	-2.80	-2.92
D_{SrO}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{BaO}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{Li_2O}	-4.3	-4.26	-4.38	-4.46	-4.57	-4.64	-4.7	-	-	-	-	-	-
D_{Na_2O}	-4.45	-4.69	-4.82	-4.93	-5.00	-5.03	-5.11	-5.2	-	-	-	-	-
D_{K_2O}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{KAlO_2}	-	-	-	-	-	-	-	-	-	-	-	-	-
D_{NaAlO_2}	-	-	-	-	-	-	-	-	-	-	-	-	-
$D_{BaAl_2O_4}$	-	-	-	-	-	-	-	-	-	-	-	-	-
$D_{CaAl_2O_4}$	8.6	7.7	6.71	5.44	5.39	4.31	4.01	5.85	6.5	7.08	6.71	6.36	5.58
$D_{MgAl_2O_4}$	-	-	-	-	-	-	-	3.48	1.26	-2.27	-0.23	0.25	0.13
$D_{MnAl_2O_4}$	-	-	-	-	-	-	-	-	-	-	-	-	-
N	10	25	32	38	63	70	71	24	12	8	8	9	8
avg $\ln n_{obs}$	1.44	2.74	2.31	2.09	2.04	1.57	1.38	2.06	1.77	1.17	0.54	0.28	0.35
r.m.s.d.	0.38	0.25	0.23	0.20	0.23	0.21	0.17	0.70	0.80	0.36	0.10	0.03	0.05

Table 4. Compositions of representative magma types used in sample calculations. Numbers 1, 2, 3, 5 and 6 are granite, diorite, diabase, peridotite and dunite average analyses respectively (Clark, 1967, Table 1-1). Number 4 is an average nepheline basalt (Manson, 1967, Table VI, no. 14).

Weight %	(1)	(2)	(3)	(4)	(5)	(6)
SiO_2	70.18	56.77	50.48	46.0	43.95	40.49
TiO_2	0.39	0.84	1.45	2.6	0.10	0.02
Al_2O_3	14.47	16.67	15.34	15.6	4.82	0.86
Fe_2O_3 (a)	1.57	3.16	3.84	3.5	2.20	2.84
FeO	1.78	4.40	7.78	7.9	6.34	5.54
MnO	0.12	0.13	0.20	0.16	0.19	0.16
MgO	0.88	4.17	5.79	7.4	36.81	46.32
CaO	1.99	6.74	8.94	10.1	3.57	0.70
Na_2O	3.48	3.39	3.07	3.4	0.63	0.10
K_2O	4.11	2.12	0.97	1.7	0.21	0.04
H_2O (b)	0.84	1.36	1.89	1.1	1.08	2.88
P_2O_5 (c)	0.19	0.25	0.25	0.53	0.10	0.05

Mole %						
$KAlO_2$	5.97	3.13	1.43	2.46	0.24	0.04
$NaAlO_2$	7.68	7.61	6.85	7.45	1.10	0.16
$CaAl_2O_4$	2.41	5.99	6.28	5.45	1.88	0.33
$MgAl_2O_4$	0.47	0	0	0	0	0
CaO	0	2.36	4.76	6.80	1.56	0.40
MgO	1.02	7.18	9.95	12.48	49.23	58.69
MnO	0.11	0.12	0.19	0.16	0.15	0.12
FeO	1.83	6.99	10.83	10.45	6.25	5.76
TiO_2	0.38	0.72	1.26	2.21	0.07	0.02
SiO_2	80.13	65.90	58.45	52.54	39.52	34.48

a. All Fe^{+++} is added to Fe^{++}.

b. Calculations for anhydrous liquids only.

c. All P^{5+} is added to Si^{4+}.

Table 5. Comparison of measured and calculated viscosities in molten rock systems. See Table 6 for chemical compositions. X_{SiO_2} = mole fraction of SiO_2 after recalculation to components described in text.

T (°C)	$10^4/T$ ($°K^{-1}$)	measured η		calculated η		composition		reference
		$\ln\eta$	poises	$\ln\eta$	poises	X_{SiO_2}	analysis no. (Table 6)	
1298	6.365	11.70	1.2×10^5	12.22	2.03×10^5	0.7915	23	Carron (1969)
1400	5.977	10.31	3.0×10^4	10.70	4.44×10^4	0.7915	23	
1491	5.669	9.16	9.5×10^3	9.49	13.2×10^3	0.7915	23	
1587	5.376	8.04	3.1×10^3	8.34	4.19×10^3	0.7915	23	
1700	5.068	7.00	1.1×10^3	7.13	1.25×10^3	0.7915	23	
1300	6.357	8.29	3.980×10^3	10.00	22.0×10^3	0.7783	8	Volarovich (1936)
1400	5.977	7.07	1.175×10^3	8.60	5.43×10^3	0.7783	8	
1400	5.977	12.04	17.000×10^4	11.00	5.99×10^4	0.8131	11	"
1260	6.523	>8.29	>4,000	8.46	4,720	0.6899	13	Euler and Winkler (1957)
1300	6.357	7.78	2,400	8.02	3,041	0.6899	13	
1340	6.200	7.22	1,370	7.61	2,020	0.6899	13	
1380	6.050	6.92	1,010	7.21	1,350	0.6899	13	
1415	5.924	6.58	720	6.88	973	0.6899	13	
1455	5.787	6.15	470	6.51	672	0.6899	13	
1498	5.647	5.70	300	6.14	464	0.6899	13	

Table 5. (continued)

T (°C)	$10^4/T$ (°K^{-1})	measured η lnη	measured η poises	calculated η lnη	calculated η poises	composition X_{SiO_2}	composition analysis no. (Table 6)	reference
1260	6.523	8.01	3,000	8.28	3,944	0.6892	14	Euler and Winkler (1957)
1300	6.357	7.50	1,800	7.85	2,570	0.6892	14	
1340	6.200	6.96	1,050	7.44	1,700	0.6892	14	
1380	6.050	6.67	790	7.06	1,160	0.6892	14	
1418	5.914	6.27	530	6.71	821	0.6892	14	
1458	5.777	5.83	340	6.35	572	0.6892	14	
1493	5.663	5.44	230	6.06	428	0.6892	14	
1264	6.506	7.24	1,400	7.78	2,390	0.6545	15	Euler and Winkler (1957)
1302	6.349	6.80	900	7.45	1,720	0.6545	15	
1341	6.196	6.27	530	7.12	1,240	0.6545	15	
1378	6.057	6.09	440	6.82	916	0.6545	15	
1420	5.907	5.70	300	6.49	659	0.6545	15	
1462	5.764	5.14	170	6.19	488	0.6545	15	
1300	6.357	7.51	1,820	7.50	1,810	0.6623	7	Volarovich (1936)
1400	5.977	6.75	851	6.30	545	0.6623	7	
1200	6.789	9.84	18.8x10^3	9.33	11.3x10^3	0.6990	26	Murase and McBirney (1970)
1250	6.566	8.90	7.33x10^3	8.71	6.06x10^3	0.6990	26	
1300	6.357	8.17	3.53x10^3	8.13	3.39x10^3	0.6990	26	

Table 5. (continued)

T (°C)	$10^4/T$ (°K^{-1})	measured η lnη	measured η poises	Calculated η lnη	Calculated η poises	composition X_{SiO_2}	composition analysis no. (Table 6)	reference
1350	6.161	7.43	1.69×10^3	7.59	1.98×10^3	0.6990	26	Volarovich (1936)
1400	5.977	6.85	944	7.08	1,190	0.6990	26	
1450	5.804	6.39	596	6.60	735	0.6990	26	
1300	6.357	4.84	126	5.55	257	0.5936	6	Volarovich (1936)
1400	5.977	4.01	55	4.45	86	0.5936	6	
1300	6.357	5.98	395	5.65	284	0.5860	9	Volarovich (1936)
1400	5.977	4.84	126	4.58	98	0.5860	9	
1250	6.566	6.01	407	6.14	464	0.5844	25	Murase and McBirney (1970)
1300	6.357	5.46	235	5.55	257	0.5844	25	
1350	6.161	4.84	126	5.00	148	0.5844	25	
1400	5.977	4.26	71	4.49	89	0.5844	25	
1450	5.804	3.80	45	4.00	55	0.5844	25	
1250	6.566	7.17	1,300	6.43	620	0.5556	5	Volarovich and Tolstoi (1936)
1300	6.357	6.59	730	5.93	376	0.5556	5	
1350	6.161	5.98	396	5.47	237	0.5556	5	
1400	5.977	5.55	256	5.03	153	0.5556	5	

Table 5. (continued)

T (°C)	$10^4/T$ (°K^{-1})	measured η lnη	measured η poises	calculated η lnη	calculated η poises	composition X_{SiO_2}	composition analysis no. (Table 6)	reference
1250	6.566	6.30	545	6.32	556	0.6039	3	Kani (1936)
1300	6.357	5.56	259	5.68	293	0.6039	3	
1350	6.161	5.21	183	5.07	159	0.6039	3	
1400	5.977	4.93	139	4.51	91	0.6039	3	
1250	6.566	6.49	656	6.38	590	0.5711	1	Kani (1936)
1300	6.357	5.69	296	5.82	337	0.5711	1	
1350	6.161	5.17	176	5.29	198	0.5711	1	
1400	5.977	4.92	137	4.80	122	0.5711	1	
1250	6.566	5.82	337	6.03	416	0.5697	2	Kani (1936)
1300	6.357	5.15	173	5.45	233	0.5697	2	
1350	6.161	4.91	136	4.90	134	0.5697	2	
1400	5.977	4.79	120	4.39	80	0.5697	2	
1200	6.789	5.77	320	6.12	455	0.5543	22	Shaw (1968)
1210	6.743	5.48	240	5.99	399	0.5543	22	
1220	6.698	5.42	225	5.86	351	0.5543	22	
1235	6.631	5.30	200	5.68	293	0.5543	22	
1300	6.357	4.55	95	4.92	137	0.5543	22	

Table 5. (continued)

T (°C)	$10^4/T$ (°K^{-1})	measured η lnn	measured η poises	calculated η lnn	calculated η poises	composition X_{SiO_2}	analysis no. (Table 6)	reference
1225	6.676	5.86	350	6.33	561	0.5670	17	Euler and Winkler (1957)
1262	6.515	5.46	235	5.92	372	0.5670	17	
1300	6.357	5.01	150	5.51	247	0.5670	17	
1340	6.200	4.58	98	5.11	166	0.5670	17	
1380	6.050	4.13	62	4.73	113	0.5670	17	
1418	5.914	3.50	33	4.38	80	0.5670	17	
1462	5.764	3.00	20	3.99	54	0.5670	17	
1230	6.653	5.78	325	6.13	459	0.5689	18	Euler and Winkler (1957)
1264	6.506	5.39	220	5.70	299	0.5689	18	
1305	6.337	4.85	128	5.23	187	0.5689	18	
1342	6.192	4.39	81	4.82	124	0.5689	18	
1383	6.039	3.85	47	4.39	81	0.5689	18	
1421	5.903	3.40	30	4.01	55	0.5689	18	
1468	5.744	3.00	20	3.57	36	0.5689	18	
1224	6.680	6.21	500	6.92	1,012	0.5400	16	Euler and Winkler (1957)
1261	6.519	5.91	370	6.44	626	0.5400	16	
1300	6.357	5.50	245	5.95	384	0.5400	16	
1340	6.200	5.04	155	5.47	237	0.5400	16	
1378	6.057	4.63	103	5.04	154	0.5400	16	

Table 5. (continued)

T	$10^4/T$	measured η		calculated		composition		reference
(°C)	(°K^{-1})	$\ln\eta$	poises	$\ln\eta$	poises	X_{SiO_2}	analysis no. (Table 6)	
1416	5.921	4.19	66	4.63	103	0.5400	16	
1462	5.764	3.58	36	4.16	64	0.5400	16	Euler and Winkler (1957)
1180	6.882	5.80	330	5.94	380	0.5220	19	
1220	6.698	5.27	195	5.43	228	0.5220	19	
1260	6.523	4.66	106	4.93	138	0.5220	19	
1300	6.357	4.13	62	4.46	86	0.5220	19	
1338	6.207	3.58	36	4.04	57	0.5220	19	
1378	6.057	2.89	18	3.62	37	0.5220	19	
1418	5.914	2.30	10	3.22	25	0.5220	19	
1215	6.720	5.70	300	4.95	141	0.4972	20	
1253	6.553	4.70	110	4.53	93	0.4972	20	Euler and Winkler (1957)
1292	6.390	3.50	33	4.11	61	0.4972	20	
1330	6.238	2.40	11	3.72	41	0.4972	20	
1220	6.698	5.30	200	4.99	147	0.5069	21	
1260	6.523	4.01	55	4.52	92	0.5069	21	Euler and Winkler (1957)
1295	6.378	3.14	23	4.13	62	0.5069	21	
1338	6.207	2.30	10	3.68	40	0.5069	21	
1300	6.357	4.87	130	4.53	93	0.5278	10	Volarovich (1936)
1400	5.977	4.09	60	3.50	33	0.5278	10	

Table 5. (continued)

T (°C)	$10^4/T$ (°K^{-1})	measured η $\ln\eta$	measured η poises	calculated η $\ln\eta$	calculated η poises	composition X_{SiO_2}	composition analysis no. (Table 6)	reference
1250	6.566	4.76	117	3.39	30	0.4126	4	Kani (1936)
1300	6.357	4.57	97	3.02	20	0.4126	4	
1350	6.161	4.48	88	2.68	15	0.4126	4	
1400	5.977	4.26	71	2.36	11	0.4126	4	
1395	5.995	1.96	7.1	1.88	6.6	0.3985	24	Murase and
1450	5.804	1.61	5.0	1.34	3.8	0.3985	24	McBirney (1970)

Table 6. Chemical compositions of melts used for the viscosity measurements listed in Table 5. H_2O and other volatiles listed in the original rock analyses are assumed to be driven off during melting and are not included in the table. See Table 5 for references.

Weight %	1	2	3	4	5	6	7	8	9	10	11	12
SiO_2	49.29	49.24	51.8	35.66	47.79	49.03	50.58	63.76	52.4	48.14	73.74	64.10
TiO_2	2.22	1.33	0.60	3.74	1.02	0.57	0.66	0.44	0.79	0.96	0.16	---
Al_2O_3	18.49	16.81	15.00	11.97	17.72	19.25	18.89	17.81	15.28	12.86	13.77	16.47
Fe_2O_3	2.38	6.16	3.68	5.19	5.68	1.56	4.52	5.53	2.14	7.31	0.40	4.33
FeO	6.77	3.60	10.14	9.69	4.39	7.68	6.41	0.59	6.36	10.62	0.78	2.10
MnO	0.22	0.17	0.20	0.30	0.17	0.16	0.24	0.09	0.15	0.08	0.08	---
MgO	6.09	8.02	5.36	8.35	3.78	8.84	1.80	3.72	8.87	6.12	0.08	0.37
CaO	8.14	9.49	9.77	14.39	8.68	9.61	7.76	5.05	9.45	9.18	1.06	1.43
Na_2O	3.93	2.67	1.76	3.65	2.54	0.84	2.29	0.67	2.89	4.18	4.14	6.26
K_2O	1.79	1.09	0.32	1.89	7.46	0.38	0.79	0.66	0.34	0.66	5.13	5.58
P_2O_5	---	0.43	0.31	1.37	---	0.29	0.29	---	---	---	---	---

(continued)

Table 6 (continued)

Weight %	13	14	15	16	17	18	19	20	21	22	23
SiO_2	59.60	60.35	57.09	48.50	51.20	49.69	47.11	46.66	46.65	50.14	72.50
TiO_2	0.94	1.20	0.72	1.72	2.03	2.63	2.41	2.63	1.93	2.63	0.10
Al_2O_3	17.06	16.26	16.92	16.96	14.77	15.43	13.88	12.10	14.04	13.37	13.50
Fe_2O_3	4.58	2.55	6.98	4.58	1.93	5.85	3.26	3.25	1.62	1.21	0.50
FeO	3.35	3.55	---	3.70	8.04	7.69	9.58	7.52	9.96	10.13	1.85
MnO	0.18	0.53	0.12	0.17	0.14	0.45	0.15	0.18	---	0.17	0.10
MgO	1.37	4.51	7.49	4.62	7.21	6.51	9.02	12.33	13.15	8.20	1.30
CaO	6.38	5.89	3.77	8.56	8.62	8.28	9.60	11.16	9.08	10.80	0.90
Na_2O	4.28	2.77	1.85	6.23	3.89	2.36	2.87	2.22	2.15	2.32	4.55
K_2O	2.17	2.07	4.63	4.26	1.43	1.13	1.39	1.70	0.82	0.53	4.15
P_2O_5	0.65	0.25	1.03	0.44	0.37	0.21	0.48	0.56	0.23	0.27	---

Table 6 (continued)

Weight %	24	25	26
SiO_2	43	50.71	60.71
TiO_2	11	1.70	0.92
Al_2O_3	7.7	14.48	17.46
Fe_2O_3	---	4.89	3.45
FeO	21	9.07	2.67
MnO	0.26	0.22	0.10
MgO	6.5	4.68	3.35
CaO	9.0	8.83	5.53
Na_2O	0.40	3.16	4.22
K_2O	0.21	0.77	1.17
P_2O_5	---	0.36	0.16

Table 7. Comparison of measured and calculated viscosities in feldspar liquids

| Composition | T (°C) | Viscosity (poises) | | | | | | |
| | | measured | | | | | | calculated |
		1	2	3	4	5	6	
NaAlSi$_3$O$_8$	1400	113,000	177,500	40,000				115,000
CaAl$_2$Si$_2$O$_8$	1550			107	38	43	37	40

1. Riebling (1966). Composition given as 0.752 SiO$_2$, 0.123 Na$_2$O, 0.125 Al$_2$O$_3$

2. Kozu and Kani (1935). Measurement on CaAl$_2$Si$_2$O$_8$ at 1555 °C.

3. Bowen (1934).

4. McCaffery et al. (1931)

5. Rossin et al. (1964)

6. Hofmaier (1968)

Figure Captions

Figure 1. Composition dependence of viscosity in SiO_2-M_2O systems at 1400 $^{\circ}$C. Data from Bockris et al. (1955), ◯ , ☐ , △ ; Shartsis et al. (1952), ● , ◪ , ▲ ; Heidtkamp and Endell (1936), ■ .

Figure 2. Composition dependence of viscosity in SiO_2-MO systems at 1700 $^{\circ}$C. Data from Bockris and Lowe (1954), ◯ ; Bockris et al. (1955), ☐ , △ , ◇ ; Hofmaier (1968), ● , ■ , ▲ , ● ; Kozakevitch (1960), ◗ .

Figure 3. Composition dependence of viscosity in $MO \cdot Al_2O_3$-SiO_2 and $M_2O \cdot Al_2O_3$-SiO_2 systems at 1700 $^{\circ}$C. Data from Rossin et al. (1964), ◯ ; Hofmaier (1968), ● , ☐ , ✝ , △ ; Riebling (1964), ■ ; and Riebling (1966), ✳ .

Figure 4. Viscosity at constant mole % SiO_2 and temperature in the SiO_2-Al_2O_3-CaO system at 1800 $^{\circ}$C (Rossin et al., 1964 and Kozakevitch, 1960) and the SiO_2-Al_2O_3-Na_2O system at 1500 $^{\circ}$C (Riebling, 1966).

Figure 5. Variation of viscosity with composition parameters R_1 (Shaw, 1965), R_2 (Carron, 1969), and bridge density (Murase, 1962) in silicate liquids at 1500 $^{\circ}$C. The 123 data points are typical of the composition range (mole %) SiO_2 (40-90%), Al_2O_3 (0-20%), remainder MO + M_2O, and were randomly selected from the input data of Table 1. The symbols Q, Gr, Ga, Pe, and Os denote the composition parameters of SiO_2, average granite, gabbro, peridotite, and orthosilicate respectively.

Figure 6. Frequency diagram of difference between calculated and experi-
mentally measured logarithm of viscosity. Measured values taken
from input data of Table 1. Calculated values from equation 1
using constants of Table 3.

Figure 7. Calculated viscosities for liquids of compositions listed in
Table 4. See text for explanation of curves 2a and 2b.

Figure 8. Correlation of calculated and experimentally measured
viscosities in molten rock systems. ■ , Carron (1969); ☐ , Euler
and Winkler (1957); ✚ , Kani (1934); ◯ , Murase and McBirney (1970);
✕ , Shaw (1968); ▲ , Volarovich and Tolstoi (1936); △ ,
Volarovich (1936).

Figure 9. Activation energy of viscous flow (kcal/mole). a. nepheline
basalt, analysis no. 4, Table 6 (Kani, 1934). b. olivine basalt,
analysis no. 20, Table 6 (Euler and Winkler, 1957). c. olivine
dolerite, analysis no. 21, Table 6 (Euler and Winkler, 1957).
Plotted points are measured viscosities, dashed lines are calculated
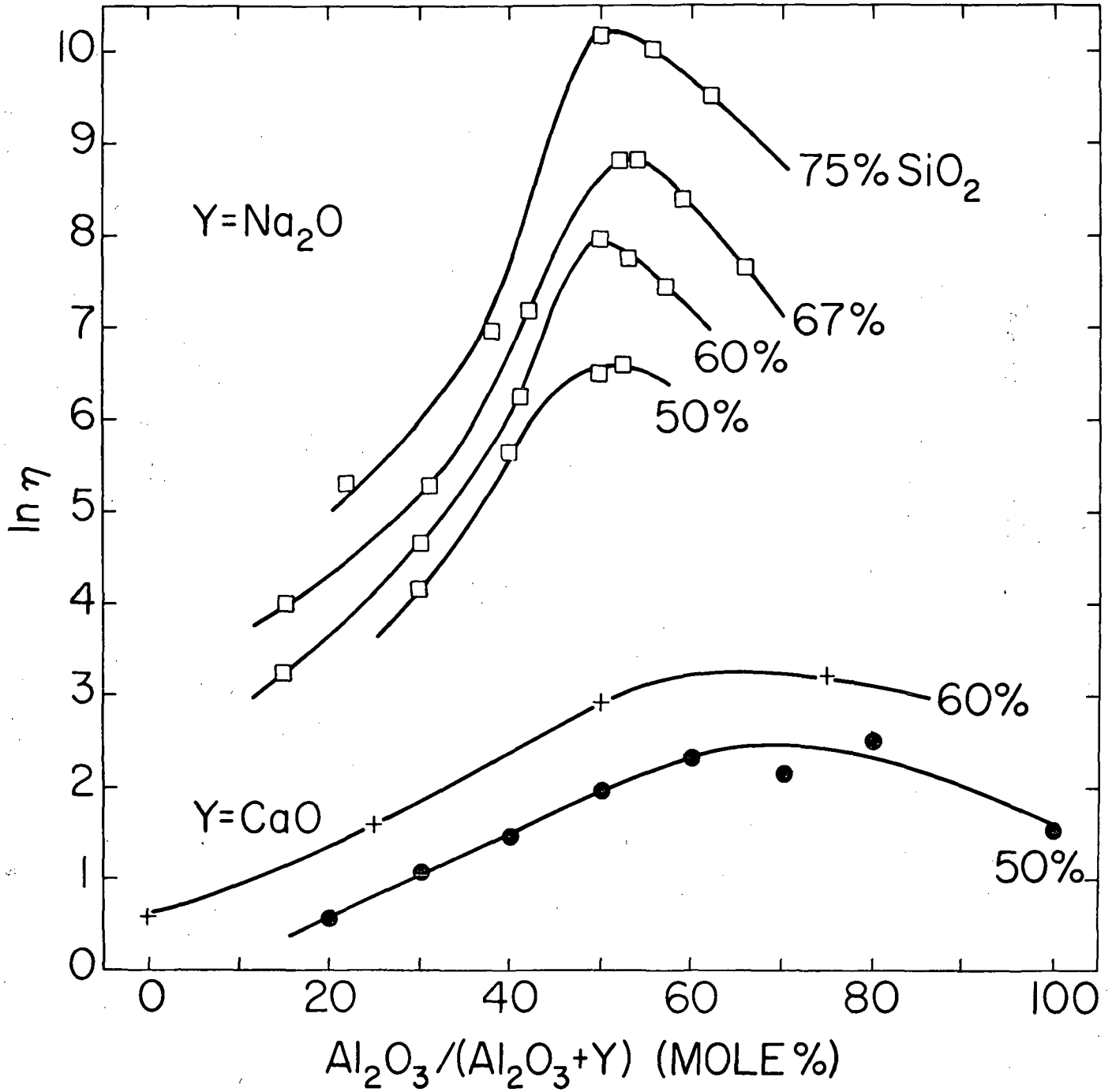viscosities.

Fig. 1

Fig 2.

Fig. 3

Fig. 4

Fig. 5

(a)

$$R_2 = \frac{2(O-2T)}{T} \times 100$$

O = No. OF OXYGENS
T = No. OF Si, Al, Fe^{3+} & P

(b)

$$R_1 = \frac{(Si+Al)}{O} \times 100$$

(c)

b.d. = 4 - (O/Si)

BRIDGE DENSITY
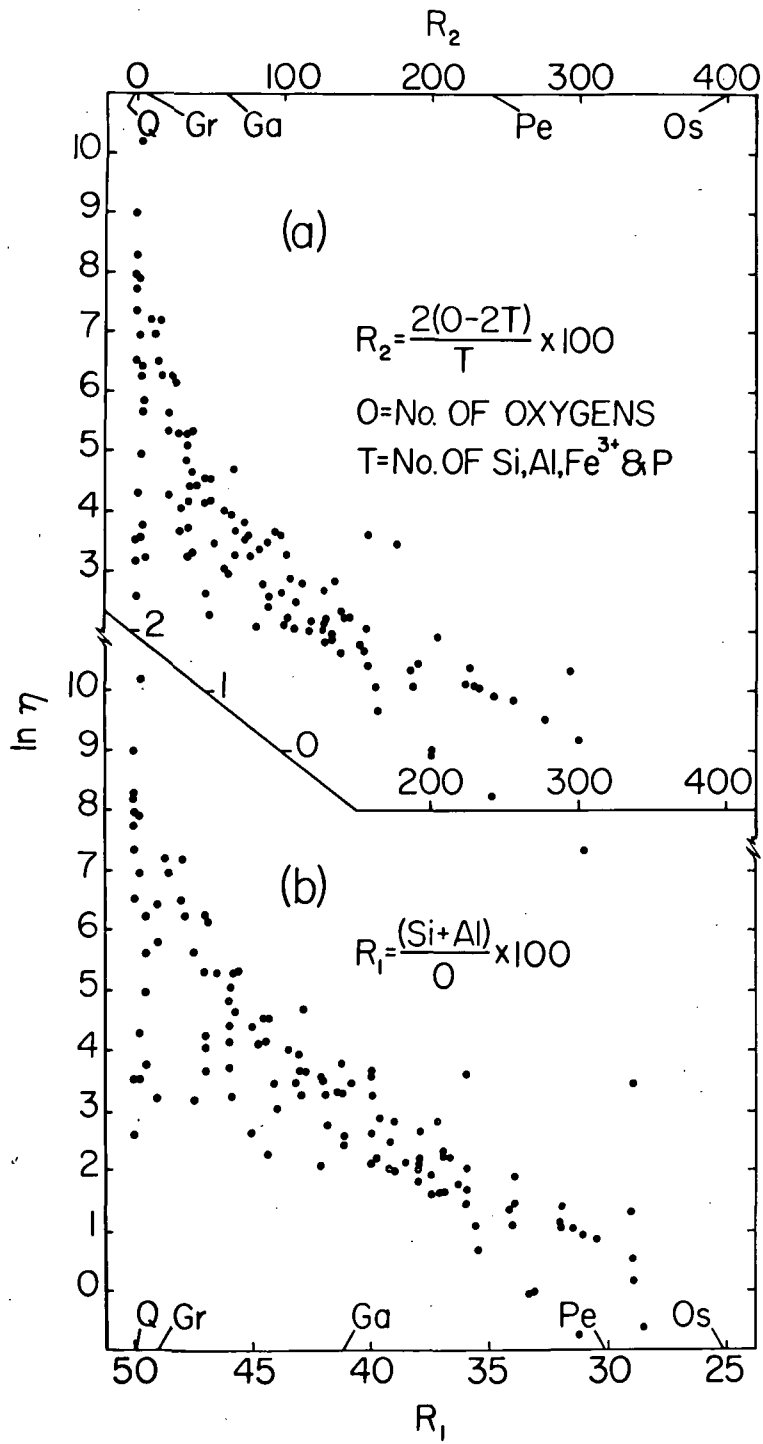
Fig. 6

$\Delta \ln \eta = \ln \eta \, (\text{CALC.}) - \ln \eta \, (\text{MEAS.})$

Fig. 7

Fig. 8.

Fig. 9