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CROSS FLOW IN A STARVED EHD CONTACT

by

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March 1972

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CROSS FLOW IN A STARVED EHD CONTACT

ABSTRACT

It is suggested that fluid film thickness in a starved EHD contact is determined by the rate at which lubricant is forced from the Hertz zone normal to the rolling direction. A simple model allows calculation of this flow rate, together with related quantities of interest to the designer.

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NOMENCLATURE

Symbol	Units	Definition
A, B, C, D		constants
E	in.	pitch diameter
F		$\left[\frac{1}{2}\ln\frac{(K+H)^2}{(K^2-KH+H^2)}+\sqrt{3}\tan^{-1}\left(\frac{2H-K}{K\sqrt{3}}\right)\right]$
H		h/h_o
K		$(-Q_{in}/Q_{out})^{1/3}$
L	in.	half length of Hertz zone
N	lbf	normal load carried by contact
P	lbf/in.^{3}	mean Hertz pressure
$Q_{out\ (in)}$	in.^{3}/s	flow out of (into) Hertz volume
T	s	real time
V	in.^{3}	Hertz half volume
Z		number of balls
d	in.	ball diameter
f		ratio between contact time and real time
h	in.	film thickness
t	s	contact time
w	in.	Hertz width
α	in.^{2}/lbf	pressure viscosity coefficient
μ	poise	viscosity
θ	rad	contact angle
Subscripts		
o		refers to conditions at t = 0
p		refers to conditions prior to t = 0
e		refers to conditions at t = ∞

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CROSS FLOW IN A STARVED EHD CONTACT

INTRODUCTION

The fundamental elastohydrodynamic problem is to calculate lubricant film thickness in a contact where elastic deformations have the same order of magnitude as the film thickness. Various solutions are available, falling into two types: a Grubin or "inlet" analysis, which solves Reynolds equation in the inlet up to the edge of the Hertz zone (1,2,3,4); and a "complete" analysis, involving simultaneous solution of the Reynolds and Hertz equations over the entire contact (5). In general any Grubin analysis ignores flows within the Hertz zone, and the available complete solutions are limited by mathematical complexity to a consideration of one-dimensional flow in the direction of rolling. Moreover, these solutions all suppose that film thickness is determined by parameters of load, speed, viscosity, geometry, and elasticity. The tacit assumption is made that thickness is not a function of the amount of lubricant, which is presumed to be present in excess at the contact. Attempts have been made to impose the effects of side leakage (e. g., 2) and oil supply limitation (e. g., 4) on the classical solutions. However, the modifications seem artificial and do not change the underlying theoretical model.

Considerable experience using ball bearings on the spin axis of precision gyroscopes has shown that successful operation can be obtained although the bearings are starved in an EHD sense. Starvation is defined as any operating condition such that an increase in oil available to the contact will result in an increase in film thickness (6). In this paper, starved EHD behavior is considered in several situations: when no new oil enters the contact, and when a step change in one of the rolling parameters is introduced at an equilibrium contact. Arguing that the important oil flow within the Hertz zone is across the rolling direction, one can formulate a simple theory which explains quantitatively some aspects of starved-ball-bearing behavior, e. g., the "oil jag."

OIL JAGS

An oil jag in a gyro spin-axis bearing is detected as an abrupt increase in driving torque demand, followed by a decay lasting over many seconds back to the original torque level (7,8). If the gyroscope is being accelerated perpendicular to its spin axis during the jag (spin axis and output axis horizontal), a simultaneous and one-one excursion, positive or negative depending upon which bearing of the pair is involved, is observed in its torque-to-balance signal (Fig. 1). This indicates an axial shift of the gyroscope CG.

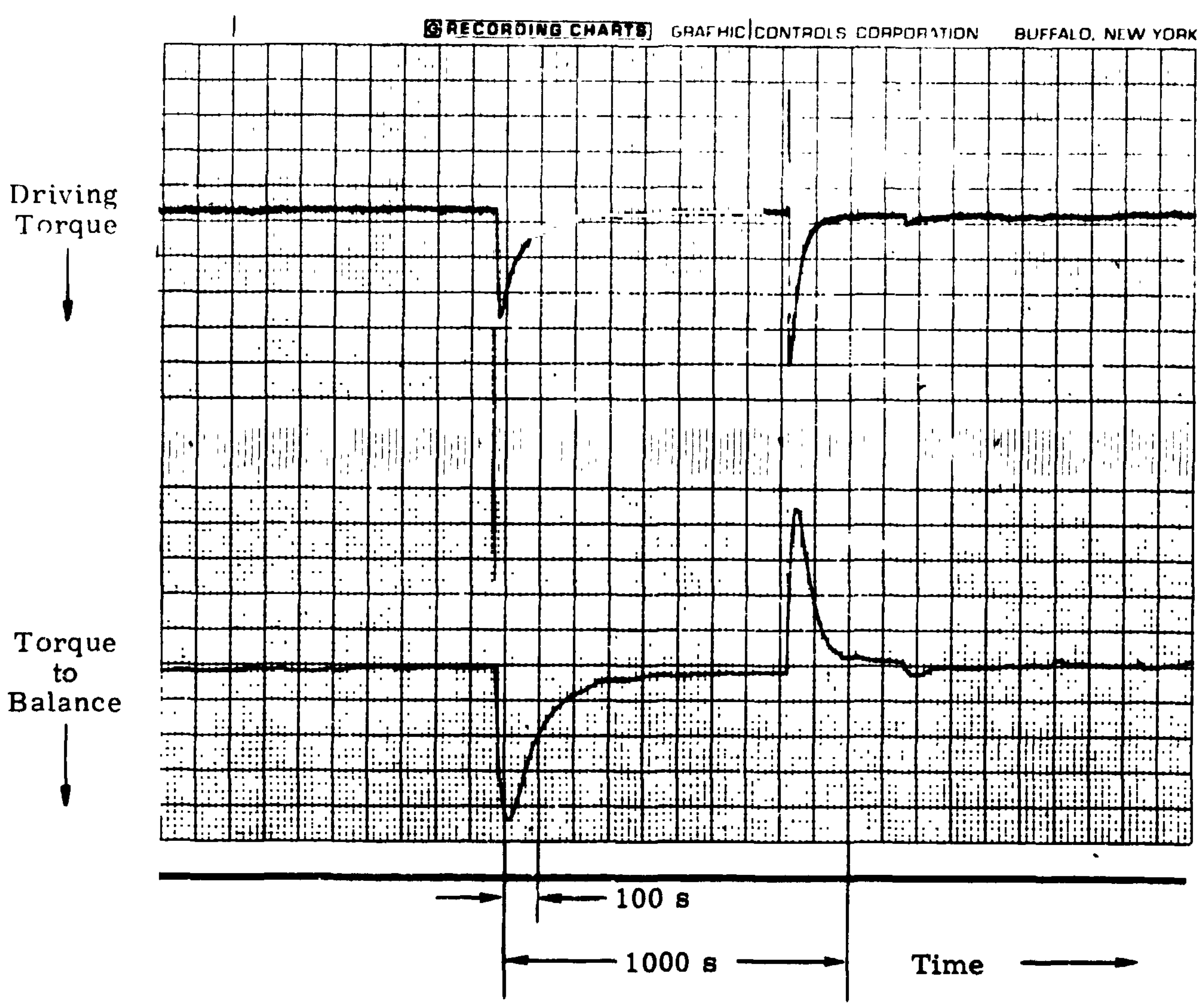

Fig. 1. **Disturbances in driving torque and torque to balance during oil jag.**

Suppose, in one ball retainer, that oil flows in the centrifugal field to its OD, and accumulates in a droplet. The drop will grow to a cricital size determined by its surface tension and the field, whence it will fly off into the ball track. Energy required to roll the ball through the deepened oil is reflected in the driving torque excursion, and an axial shift of the gyro CG due to the increased film thickness in one bearing produces the torque-to-balance excursion (Fig. 2).

Various other jag observations are explained by this model; stroboscopic observation has confirmed oil droplets on the retainer OD, together with a torque excursion upon their disappearance; artificial jags have been produced with a "jag-gun" which throws a known amount of oil into the ball track on demand. Thus the model appears correct, and associates a known increase in film thickness with a known increment in the amount of oil available to the contact. Further, it gives the time required to reduce the film back to its original thickness.

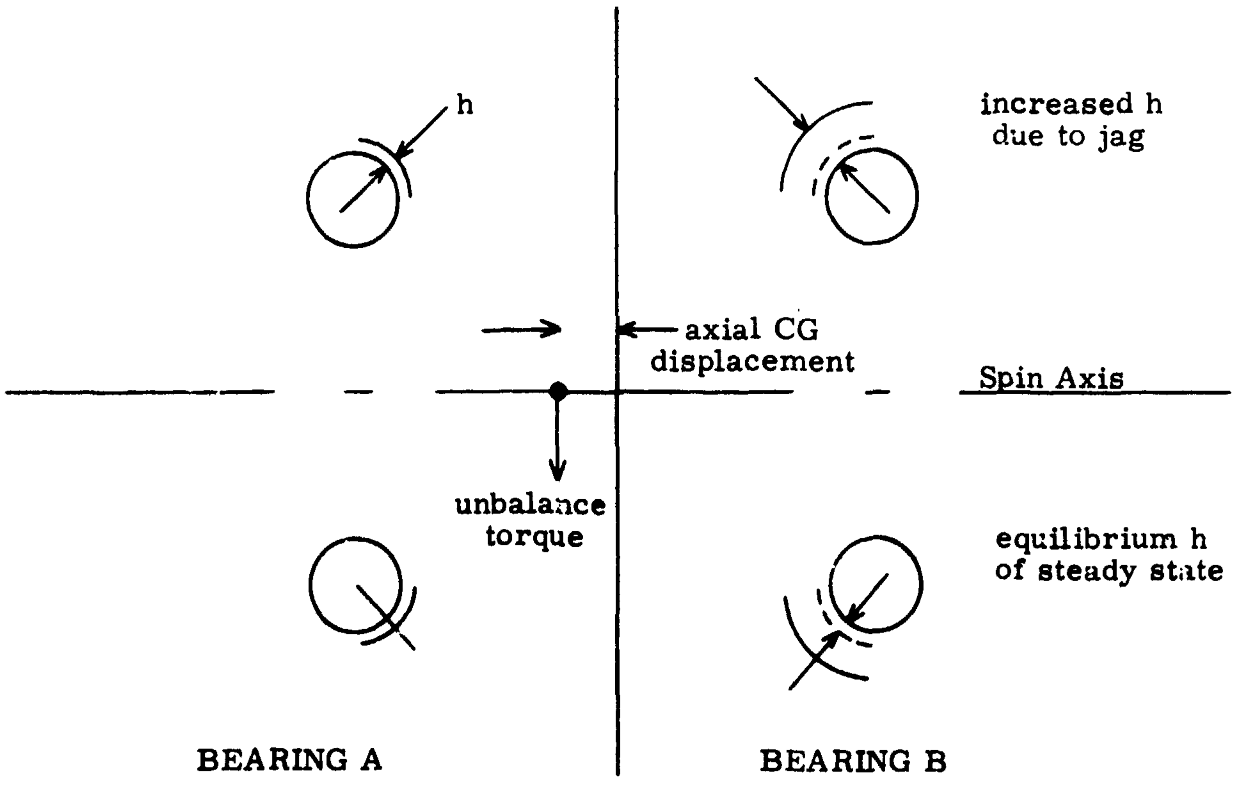

Fig. 2. Oil-jag-mechanism geometry.

FLOW MODEL FOR A STARVED CONTACT

In a starved bearing, the oil on the track is about as thick outside the contact as in it. Stroboscopic observation of interference fringes confirms that films so thin flow very slowly in the centrifugal fields due to bearing rotation. The working oil is thus stationary with respect to the track outside the contact, and is unloaded most of the time. The oil momentarily in the contact may flow in the direction of rolling and also across the direction of rolling under the Hertz pressure acting on it. Its viscosity will be greatly increased while it is under this pressure. Flow in the rolling direction cannot alter the average film thickness since it removes no oil from the track. Only transverse oil flow can change the film thickness. The long-term decays in thickness observed with oil jags thus imply that transverse oil flow out of the Hertz zone should be considered in a starved EHD contact.

To estimate this outflow, approximate the volume of oil in each half of the Hertz zone at any time as (Fig. 3)

$$V = L\,w\,h \qquad [1]$$

where L and w are the Hertz dimensions of the contact, and h is the film thickness. Calculate the oil flow out each end of this box due to a pressure drop equal to the mean Hertz pressure, P, of an oil of viscosity $\mu = \mu(P)$ as

$$Q = \frac{P\,h^3\,w}{12\,\mu\,L} \qquad [2]$$

Flow in the roll direction may be neglected as it must average to zero.

$$dV = L\,w\,dh \doteq -\,Q\,dt \qquad [3]$$

$$\frac{12\,\mu\,L^2}{P} \int \frac{dh}{h^3} = -\int dt + D \qquad [4]$$

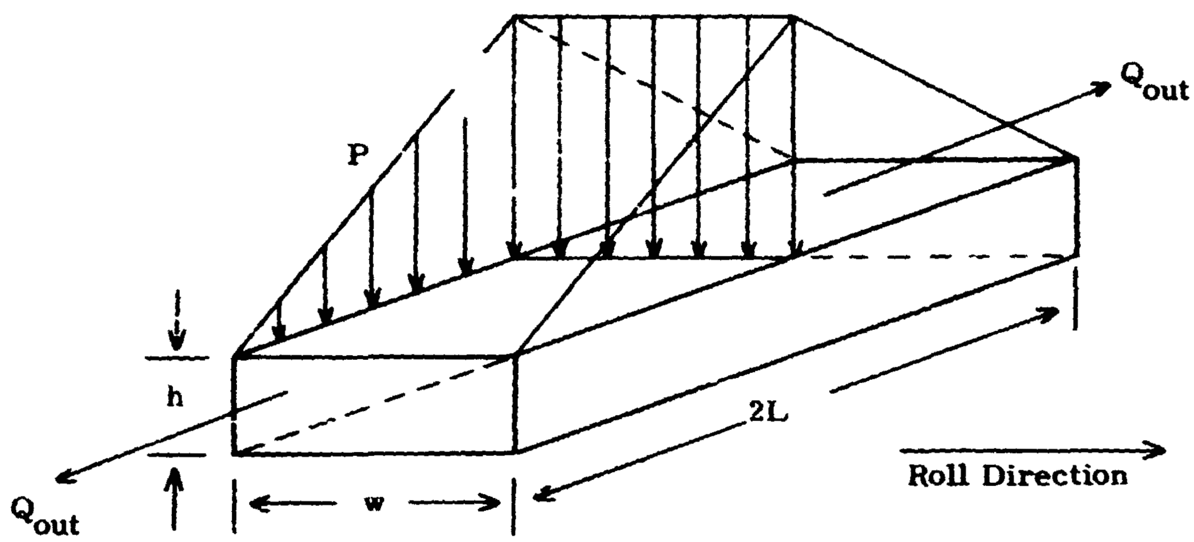

Fig. 3. Oil-film geometry in Hertz zone.

where t is the time an element in the race track is in the contact. It is a fraction f, of real time T, given by the proportion between Zw, the total contact width, and π(E + d cos B), the outer-race-track circumference. Thus,

$$t = fT, \quad f = Zw/(E + d \cos \theta)\pi \qquad [5]$$

written for an outer-race contact in a bearing having Z balls of diameter d, pitch diameter E, and contact angle B.

Finally,

$$\frac{h}{h_o} = \left[\frac{h_o^2 \, Pf}{6 \mu L^2} T + 1 \right]^{-1/2} \qquad [6]$$

which relates the decay of a film of original thickness h_o to real time T in a bearing where no new oil enters the wear track.

This calculation has been carried out for the R3-size bearing (Fig. 4) which produced the jags shown in Fig. 1. The results for two running conditions are plotted in Fig. 5. It takes about three hours of real-time running to pump half the oil out of a contact initially 20 x 10^{-6} inches thick under 150,000 lbf/in.^{2}. When the contact is 4 x 10^{-6} inches thick, it takes about 100 hours to pump out half the remaining

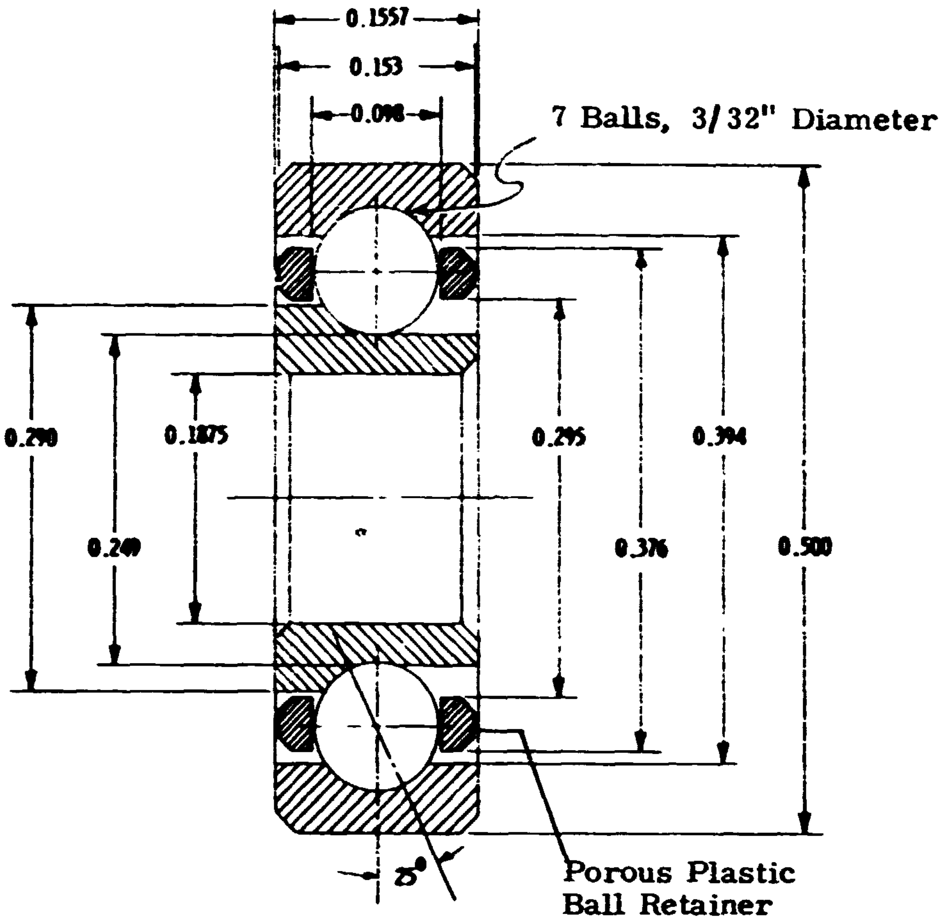

Fig. 4. R3 ball bearing.

oil. These numbers are not out of line with life experienced in retainerless bearings which have no provision for the recirculation of oil.

If the decay is recalculated for a Hertz stress of 100,000 lbf/in.^{2}, there is a decrease in the life of the contact. This is because of the very great decrease in the viscosity of the lubricant with pressure, from 10^6 to 10^4 poise in this case (9).

THE STARVED EQUILIBRIUM CONTACT

Since it is possible to run starved gyro bearings essentially indefinitely (10), there must be a flow of lubricant into the wear track to balance the outflow calculated in the preceeding section. It has been confirmed experimentally (11) that oil circulates from a reservoir in the ball retainer to the track and back, although the

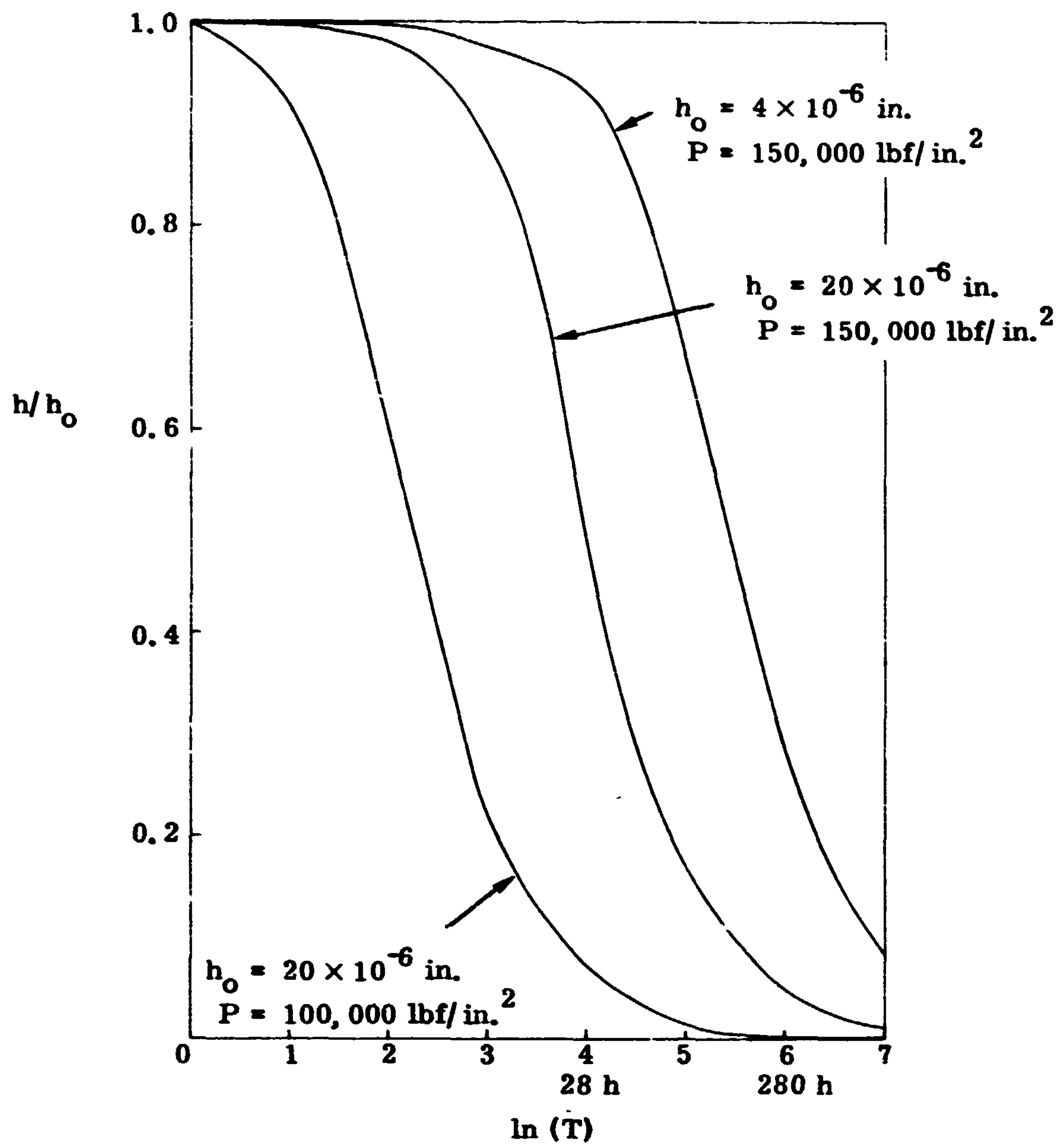

Fig. 5. Film decay without inflow.

details of the process have not been explored. At equilibrium, therefore, there is an inflow equal to the transverse outflow, and an equilibrium film thickness. If equilibrium is disturbed by a change in some running condition, the transient response in film thickness at the contact may be calculated.

In the following analysis, a step change is assumed to occur at $t = 0$; subscript p refers to equilibrium conditions prior to the change ($t < 0$); subscript o to conditions at the change ($t = 0$); and subscript e to final equilibrium ($t = \infty$).

Thus, at an equilibrium contact for $t < 0$

$$Q_{in\ p} = Q_{out\ p} = \frac{P_p w_p h_p^3}{12\,\mu_p L_p} = Q_{in} \tag{7}$$

It is supposed that the inflow to the contact, Q_{in}, is constant throughout the transient, since it is the result of long-term averaging processes in the retainer, but that the outflow steps at $t = 0$ since it depends directly on contact conditions. For $t \geq 0$

$$\frac{dV}{dt} = \frac{d}{dt}\left[L_o w_o h(t)\right] = Q_{in} - (Q_{out})t \tag{8}$$

$$(Q_{out})t = \frac{P_o w_o h^3(t)}{12\,\mu_o L_o} \tag{9}$$

so that

$$12\,\mu_o L_o^2 w_o \int_{h_o}^{h} \frac{dh}{12\,\mu_o L_o Q_{in} - P_o w_o h^3} = \int_0^t dt \tag{10}$$

Equation [10] may be integrated to get

$$\frac{L_o w_o h_o}{3Q_{in}} K\,[F(H,K) - F(1,K)] = t \tag{11}$$

where

$$K^3 = -\frac{Q_{in}}{Q_{out_o}}, \quad H = h/h_o$$

and

$$F(H,K) = \left[\frac{1}{2}\ln\frac{(K+H)^2}{(K^2 - KH + H^2)} + \sqrt{3}\,\tan^{-1}\left(\frac{2H - K}{K\sqrt{3}}\right)\right] \tag{12}$$

F(H,K) has been plotted in Fig. 6 for relevant values of H and K.

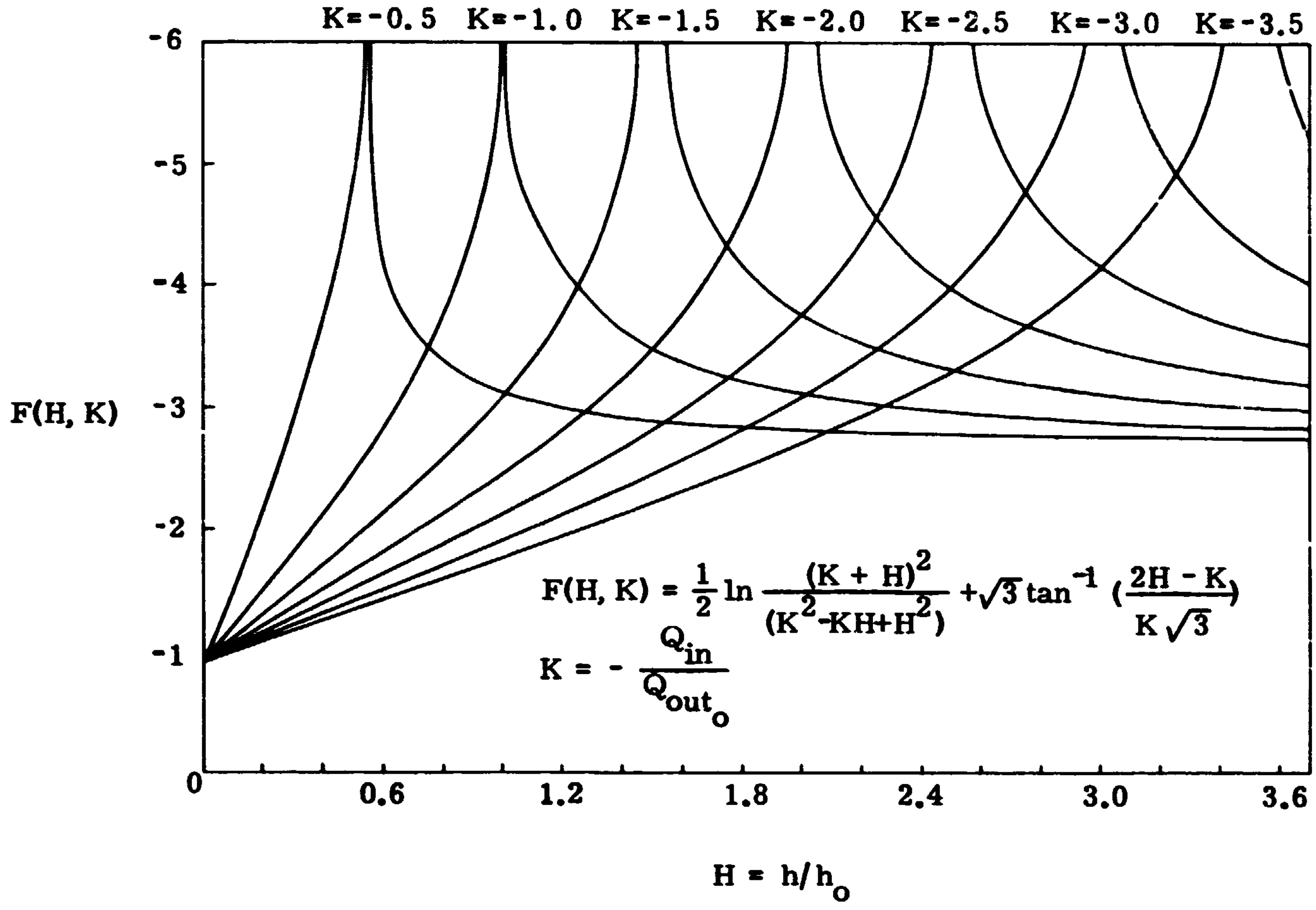

Fig. 6. F(H,K) versus H.

FILM-THICKNESS STEP

An oil jag may be simulated as a step increase in film thickness from h_p to h_o, all other running parameters remaining constant. Then, from Eq. [12]

$$\frac{h_p}{h_o} = H_p = \left| K \right| < 1 \qquad [13]$$

Suppose H_p = - 0.5, meaning that the bearing jags to a new thickness twice the prior equilibrium value. Equation [11] becomes

$$- 0.5\left(\frac{L_o w_o h_o}{Q_{in}}\right) [F(H, -0.5) - F(1, -0.5)] = t \qquad [14]$$

Boundary conditions are satisfied since at $t = 0$

$$F(H_o, -0.5) = F(1, -0.5),\ H_o = 1 \text{ and } h = h_o \qquad [15]$$

and at $t = \infty$, $F(H_e, -0.5) = -\infty$ occurring (Fig. 6) when $H_e = -K$ or

$$0.5 = H_e = h_e/h_o,\ n_e = 1/2\ h_o \qquad [16]$$

Thus, for this case Eq. [11] describes a time decay from the stepped film thickness at $t = 0$ back to the prior equilibrium thickness at $t = \infty$.

As a numerical example, suppose an R3 bearing (F = 0.01) is running on an equilibrium film $h_p = 20 \times 10^{-6}$ inches thick, with a Hertz pressure $P_p = 10^5$ lbf/in.^{2}, and oil viscosity $\mu = 1.1 \times 10^4$ poise. At $t = 0$ the bearing jags to $h_o = 24 \times 10^{-6}$ inches, giving a K value of $-\frac{20}{24} = -0.833$. Using Fig. 6 and Eq. [11], the decay shown in Fig. 7 is calculated. It may be compared with jags experienced in similar bearings (Fig. 1).

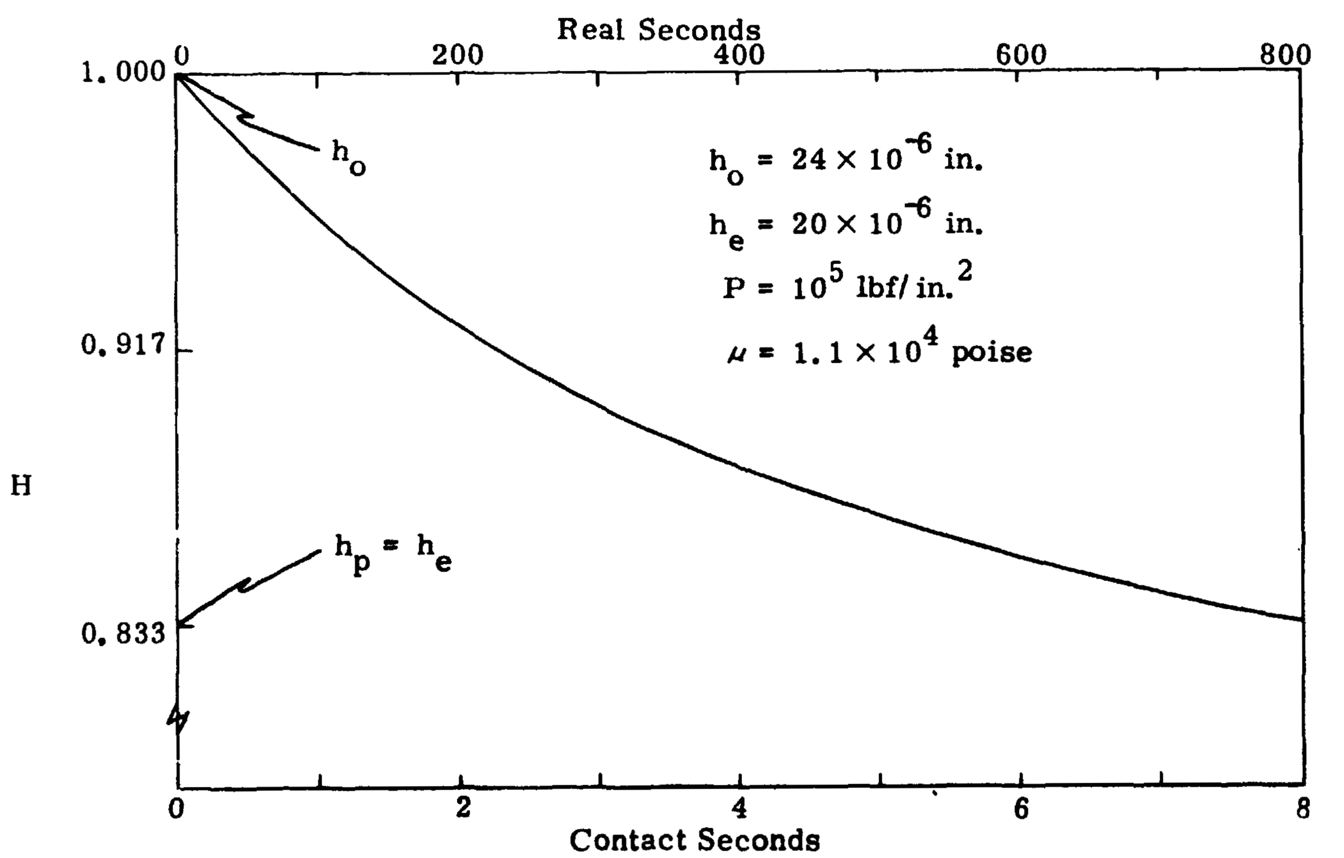

Fig. 7. Film decay after jag.

LOAD STEP

Suppose that at $t = 0$ the load carried at a starved EHD contact steps from N_p to N_o. From elasticity theory

$$(P, w, L) = (A, B, C)N^{1/3} \qquad [17]$$

where A, B, and C are constants which may be calculated according to the particular bearing under consideration.

Assume an exponential dependence of viscosity on pressure, as is done in Ref. (4):

$$\mu_o = \mu_p e^{\alpha(P_o - P_p)} = \mu_p e^{A\alpha(N_o^{1/3} - N_p^{1/3})} \qquad [18]$$

Thus, at the load step, the Hertz pressure, the Hertz dimensions, and the viscosity all change to new values. However, the thickness does not step, $h_p = h_o$. Its time dependence for $t > 0$ is given by Eq. [11]. From Eq. [12]

$$K^3 = -\frac{Q_{in}}{Q_{out_o}} = -\frac{P_p w_p L_o \mu_o}{P_o w_o L_p \mu_p} \qquad [19]$$

or, in terms of load

$$K^3 = -\left(\frac{N_p}{N_o}\right)^{1/3}\left[e^{A\alpha(N_o^{1/3} - N_p^{1/3})}\right] \qquad [20]$$

The exponential part of K ensures that $|K| > 1$, meaning that H values will be found on the left branch of the F curves in Fig. 6. To calculate the final equilibrium film thickness, note that at $t = \infty$, $-F(H,K) = \infty$. From Fig. 6

$$-K = H_e, \quad h_e = h_o |K| \qquad [21]$$

h_e may also be calculated by observing that at $t = \infty$, $Q_{out_e} = Q_{in}$. Substitution gives the same result for h_e.

For this case, Eq. [11] thus describes a build-up in film thickness with time after a step increase in load (Fig. 8).

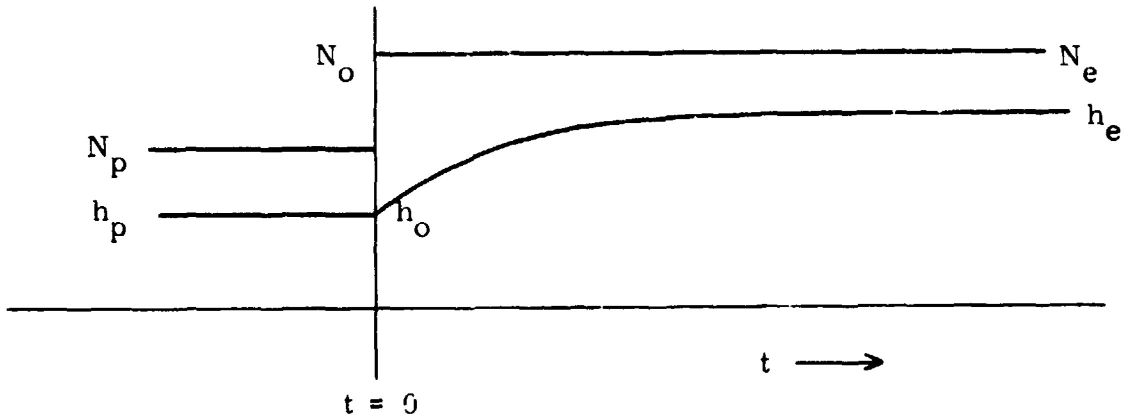

Fig. 8. Film thickness history after load step.

DISCUSSION

This analysis indicates that to calculate the EHD film thickness in a starved contact, it is necessary to know the rate at which lubricant is supplied to the contact. In a conventional instrument bearing, oil is stored in the ball retainer and metered out by unknown processes at unknown rates at the different ball pockets. The direct calculation of film thickness in these bearings is therefore not feasible.

Testing of retainerless bearings which incorporate a rational lubrication supply system has been started. The oil supplies for these bearings have been specifically designed to provide well-defined inflows. As such information becomes available, thickness predictions from the cross-flow analysis can be tested.

The model for cross flow is crude but is believed to reflect some reality. Calculation of life and jag decay seems adequate, and special experiments to check some other predictions are under consideration. Predictions include independence of jag decay period with speed, and increase in decay period with stress. Depending on the outcome of such experiments, this model can be improved or scrapped.

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