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# SIMPLIFIED INTERPLANETARY GUIDANCE PROCEDURES USING ONBOARD OPTICAL MEASUREMENTS

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0133683

1. Report No. NASA TN D-6752	2. Government Accession No.	3. Recipient's Catalog No.	
4. Title and Subtitle SIMPLIFIED INTERPLANETARY GUIDANCE PROCEDURES USING ONBOARD OPTICAL MEASUREMENTS		5. Report Date May 1972	6. Performing Organization Code
7. Author(s) Harold A. Hamer and Katherine G. Johnson		8. Performing Organization Report No. L-7630	
9. Performing Organization Name and Address NASA Langley Research Center Hampton, Va. 23365		10. Work Unit No. 110-06-05-06	11. Contract or Grant No.
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, D.C. 20546		13. Type of Report and Period Covered Technical Note	
15. Supplementary Notes		14. Sponsoring Agency Code	
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17. Key Words (Suggested by Author(s)) Interplanetary trajectories Onboard navigation guidance Interplanetary approach guidance		18. Distribution Statement Unclassified - Unlimited	
19. Security Classif. (of this report) Unclassified	20. Security Classif. (of this page) Unclassified	21. No. of Pages 28	22. Price* \$3.00

# SIMPLIFIED INTERPLANETARY GUIDANCE PROCEDURES USING ONBOARD OPTICAL MEASUREMENTS

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## SUMMARY

Simplified guidance procedures are developed which are based on preflight determination of the characteristics of trajectories perturbed about the nominal. The results are devoted principally to planetary approach guidance; however, some considerations for midcourse guidance are included. The methods are studied for an Earth-to-Mars trajectory but would be applicable to Grand Tour and other types of missions.

Generally requiring only a single onboard optical angular measurement, the approach procedure predicts guidance corrections for the control of periapsis radius as well as orbital plane orientation. An error analysis was performed with an assumed  $1\sigma$  error of 10 seconds of arc in the optical measurement and a  $1\sigma$  velocity-cut-off error of 0.1 m/sec. The analysis showed that when the usual type of midcourse guidance is applied, the periapsis radius at Mars can be controlled to a  $1\sigma$  accuracy of about 20 km if the approach guidance is executed 1/2 day before periapsis passage. Also, the orbital plane orientation can be determined to a  $1\sigma$  accuracy of  $0.05^\circ$ . If the guidance is performed at the Martian sphere of influence (2.2 days before periapsis), the periapsis-radius error doubles because of the increased effects of measurement error and maneuvering error.

The effect of using another type of midcourse-guidance procedure was also investigated. This procedure differs in that the guidance correction and ensuing velocity errors are assumed to be approximately normal to the flight path. The analysis showed that this technique resulted in higher approach-guidance accuracy in controlling periapsis radius. However, the fuel requirement was increased by an order of magnitude and the accuracy of controlling the plane orientation was decreased.

## INTRODUCTION

In space missions, the navigation and guidance are normally accomplished by automatic procedures which employ Earth-based radar measurements. The inclusion of procedures which utilize onboard measurements is a desirable feature for interplanetary flight because planet ephemeris errors can lead to unacceptable errors in the trajectory position relative to the planet when only Earth-based measurements are used. This posi-

tion error is most significant during the phase of the mission when the spacecraft is within the sphere of influence of the planet. Consequently, this paper concentrates on the approach-guidance problem.

Over the years, a number of studies have been made to develop onboard guidance procedures for controlling the approach to a celestial body. (For example, see refs. 1 to 7.) In general, these methods require several types of measurements and the measurements must be repeated a number of times inasmuch as the guidance correction is ordinarily based on statistical filtering techniques. The methods may also require more than one guidance maneuver.

The approach procedure presented in this paper requires, in general, only a single onboard star-to-body angular measurement to determine the guidance correction. This measurement, made at a preselected time, is used to determine the position deviation from the nominal in a particular direction. When used in conjunction with some approximations derived from two-body theory, a knowledge of this deviation is sufficient for controlling the periapsis magnitude by using a single guidance maneuver. The method is applied to the approach phase of a Martian trajectory. An analysis is included which shows that for reasonable accuracy, the deviation must be in a specified direction with respect to the nominal trajectory. Also, the direction of the guidance-correction vector is always parallel to the nominal orbital plane and perpendicular to the nominal velocity vector, which is essentially optimum. The accuracy characteristics of the method are examined by means of a Monte Carlo error analysis. The analysis includes the effects of measurement error, thrust-cut-off error, ephemeris error, and the approximation error caused by two-body assumptions and nonlinearity effects.

Results are also included which show that the orbital plane orientation can be readily determined from an onboard measurement. Although plane changes are small at Mars, they can become important in a Grand Tour mission.

Because of less restrictive midcourse-guidance accuracy requirements, Earth-based measurements will ordinarily suffice for this phase in most interplanetary missions. Brief results, however, are presented which could apply to Grand Tour missions, wherein midcourse guidance might necessarily be based on measurements relative to other planets.

## SYMBOLS

- A            angle formed at spacecraft by lines of sight to centers of celestial bodies
- B            angle formed at Earth center by line to spacecraft and line to Moon or Sun center

C	angle formed at Moon or Sun center by line to spacecraft and line to Earth center
D	position deviation in certain direction
E	position ephemeris error in direction of Sun
R	radius of Mars
r	range to Mars center
$r_e$	range to Earth center
$r_{em}$	distance between Earth and Moon centers
$r_{es}$	distance between Earth and Sun centers
$r_m$	range to Moon center
$r_p$	periapsis radius at Mars
$r_s$	range to Sun center
$T_p$	time to nominal periapsis time (time of approach-guidance measurement)
V	spacecraft areocentric velocity (Mars centered)
$V_p$	periapsis velocity at Mars
x,y,z	position coordinates in Cartesian axis system in which X-axis is toward the vernal equinox, XY-plane is parallel to earth equatorial plane, and Z-axis is in direction of north celestial pole
$\beta$	angle between $\overline{\Delta V}_{pc}$ and $\overline{V}$ (see appendix)
$\gamma$	flight-path angle
$\Delta i$	difference in inclination angle of actual and nominal orbital planes

$\Delta r$	difference between actual and nominal range to Mars center
$\Delta r_e$	difference between actual and nominal range to Earth center
$\Delta r_p$	difference between actual and nominal periapsis radius
$\Delta V$	approach-guidance velocity required to correct periapsis radius
$\Delta V_e$	difference between actual and nominal spacecraft geocentric velocity
$\Delta V_{pc}$	guidance velocity required for orbital plane change
$\Delta \theta$	angle between desired deviation direction and measurement star direction
$\delta$	angle between deviation vector and orbital plane
$\epsilon$	eccentricity of orbit
$\theta$	angle between deviation vector and body center
$\theta_m$	angle between measurement star and body center
$\lambda$	angle between $\overline{\Delta V}$ and $\overline{V}$
$\mu$	product of universal gravitational constant and mass of Mars
$\sigma$	standard deviation or root-mean-square value of error
$\phi$	angle between $\overline{V}_1$ and $\overline{V}_2$ (see appendix)

Subscripts:

A,B,C	angles in triangle formed by vehicle, Earth, and Moon (fig. 1)
a	actual value
D	position deviation
i	inclination angle

n	nominal value
R	radius of Mars
r,e	range to Earth center
r,p	periapsis radius at Mars
$\alpha$	half-angle subtended by Mars
1,2	value immediately before and after guidance correction, respectively (see appendix)

Notation:

$| |$  absolute value

A bar over a symbol indicates a vector.

## MIDCOURSE GUIDANCE

The main purpose of this paper is to present results of applying a simplified onboard procedure for interplanetary approach guidance. Such a procedure corrects for errors incurred at midcourse; hence, a brief discussion of midcourse guidance is included. Only onboard midcourse guidance is considered here, inasmuch as this type of guidance was assumed for the approach guidance analysis.

Previous studies of interplanetary midcourse guidance with onboard measurements (for example, refs. 8 and 9) have generally applied to manned flight wherein numerous observations over an extensive portion of the trajectory and, perhaps, several midcourse corrections are required. Results discussed herein correspond to the simplified midcourse-guidance procedure developed in reference 10. This method, based on pre-flight determination of the characteristics of trajectories perturbed about the nominal, requires only one position fix, from which is derived the standard fixed-point-of-arrival guidance correction. The simplicity of the measurements makes the method adaptable for flights such as the Grand Tour mission, where the midcourse-position fix with respect to other planets may be required. As is shown, the errors at the aim point may be relatively large. However, these errors can be corrected if the aim point is selected near the sphere of influence of the target planet.

## Range Determination

In reference 10, the position vector of a body relative to the vehicle (position fix) is determined by three star-to-body angles and an onboard range measurement determined also from angular measurements. In reference 10 the error in the range measurement is shown to be the dominant factor affecting the accuracy of the position fix and hence the accuracy of the midcourse guidance. Because of the importance of this measurement, the pertinent aspects of onboard optical determination of range are presented in this section.

A basic method of determining the range to Earth from onboard midcourse optical measurements taken in interplanetary space is illustrated in figures 1 and 2. The quantities  $r_{em}$  and  $r_{es}$  are known. (The method illustrated in fig. 1 can be applied to any planet if the position of one of its moons is known with reasonable accuracy.)

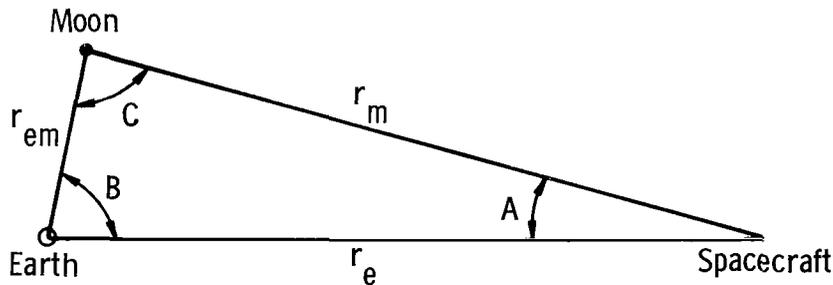


Figure 1.- Vehicle-Earth-Moon system.

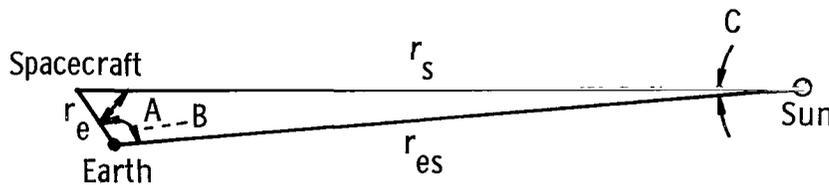


Figure 2.- Vehicle-Earth-Sun system.

For a Mars trajectory at a time 5 days from the Earth, typical values of the angles  $A$ ,  $B$ , and  $C$  in figure 2 are  $56.63^\circ$ ,  $122.91^\circ$ , and  $0.46^\circ$ , respectively. In figure 1, the maximum value of the angle  $A$  at 5 days is approximately  $16^\circ$ .

Two angles of the triangle are measured and the sine law is applied to solve for the range  $r_e$ . For example, from figure 1

$$r_e = \frac{\sin C}{\sin A} r_{em}$$

or

$$r_e = \frac{\sin(A + B)}{\sin A} r_{em}$$

The angle  $A$  can be measured directly, and the angle  $B$  (angle  $C$ ) may be determined by (1) measuring the declination and right ascension of the Earth (Moon) or (2) measuring two star-to-Earth (star-to-Moon) angles. (See ref. 10.) In determining the angles  $B$  and  $C$  in this manner, their nominal values must be known.

#### Range-Determination Accuracy

In the vehicle-Earth-Moon system (fig. 1) the range-error equation, assuming measurement of angle  $B$ , is obtained from the total differential

$$dr_e = \frac{\partial r_e}{\partial A} dA + \frac{\partial r_e}{\partial B} dB + \frac{\partial r_e}{\partial r_{em}} dr_{em}$$

If the measurements are made at a selected nominal time, the third term may be neglected because of the certainty of  $r_{em}$ ; hence,

$$dr_e = \frac{\partial r_e}{\partial A} dA + \frac{\partial r_e}{\partial B} dB$$

For the region in which  $r_e$  varies linearly with  $A$  and  $B$ ,

$$\Delta r_e = \frac{\partial r_e}{\partial A} \Delta A + \frac{\partial r_e}{\partial B} \Delta B$$

or

$$\Delta r_e = -\frac{r_m}{\sin A} \Delta A + r_{em}(\cos B \cot A - \sin B)\Delta B$$

If  $\Delta A$  and  $\Delta B$  are assumed to be random uncorrelated errors in the angular measurements, then

$$(\sigma_{r,e})^2 = \left(\frac{r_m}{\sin A} \sigma_A\right)^2 + \left[r_{em}(\cos B \cot A - \sin B)\sigma_B\right]^2 \quad (1)$$

For  $120^\circ > B > 60^\circ$ ,

$$\sigma_{r,e} \approx \frac{r_m}{\sin A} \sigma_A$$

This approximation can be made because of the combined effect of the relatively small value of  $r_{em}$  and the trigonometric values of the angles.

Similarly, the range-error equation, if angle  $C$  is measured, is

$$(\sigma_{r,e})^2 = (r_e \cot A \sigma_A)^2 + (r_e \cot C \sigma_C)^2 \quad (2)$$

For  $120^\circ > B > 60^\circ$ ,

$$\sigma_{r,e} \approx r_e \cot A \sigma_A$$

Figure 3 shows the variation of range error  $\sigma_{r,e}$  with the angle  $B$  when star-to-Earth and star-to-Moon sightings are used, as calculated from equations (1) and (2). Each curve involves two angular measurements,  $A$  and either  $B$  or  $C$ . Angle  $B$  is determined by the relative position of the Moon. Obviously, measurements should not be taken when the Moon lies near the line of sight to the Earth. The data in figure 3 correspond to a spacecraft position 5 days from Earth ( $r_e \approx 1\,460\,000$  km). The  $1\sigma$  errors in the angular measurements were assumed to be 10 seconds of arc; that is, the error in angle  $A$  would be 10 seconds of arc but the error in angle  $B$  or  $C$  would be greater inasmuch as two star-to-body measurements are required to determine these angles. From an example in reference 10, an error of 10 seconds of arc in each of these two measurements produced component errors in  $B$  or  $C$  of about 14 seconds of arc or a total  $1\sigma$  error of about 20 seconds of arc.

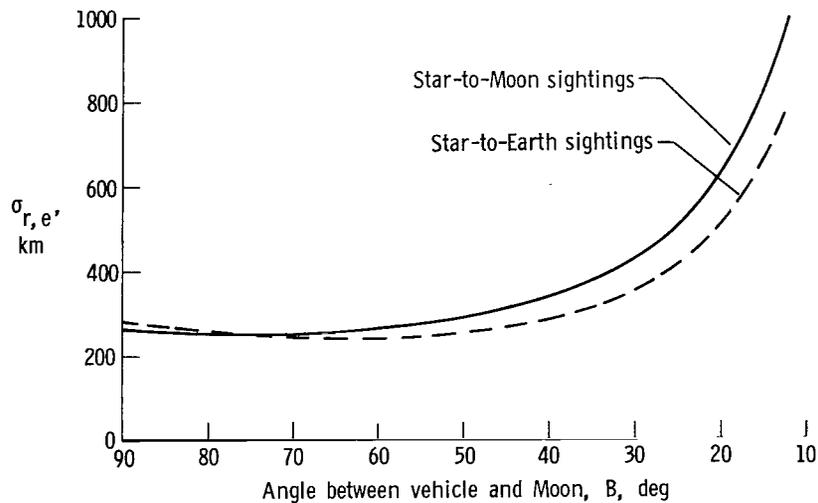


Figure 3.- Range-determination error from angular measurements in vehicle-Earth-Moon system with  $1\sigma$  errors of 10 seconds of arc.

Equations were determined for the vehicle-Earth-Sun system (fig. 2); however, because of the geometry involved, the range errors were prohibitive. For example, at 5 days from the Earth, the  $1\sigma$  range error  $\sigma_{r,e}$  is approximately 18 000 to 20 000 km, depending on whether the Sun or the Earth is used for the star-to-body angular sightings. In sighting to the Sun, the error is attributed to the small value of the angle  $C$ , whereas in sighting to the Earth, the error is caused by the large distance to the Sun. The effect of the uncertainty in the value of the astronomical unit is negligible.

### Guidance-Velocity Requirement

The data in figure 4 illustrate the midcourse velocity requirement. The results in the figure are the velocity deviations 5 days from Earth due to all combinations of errors

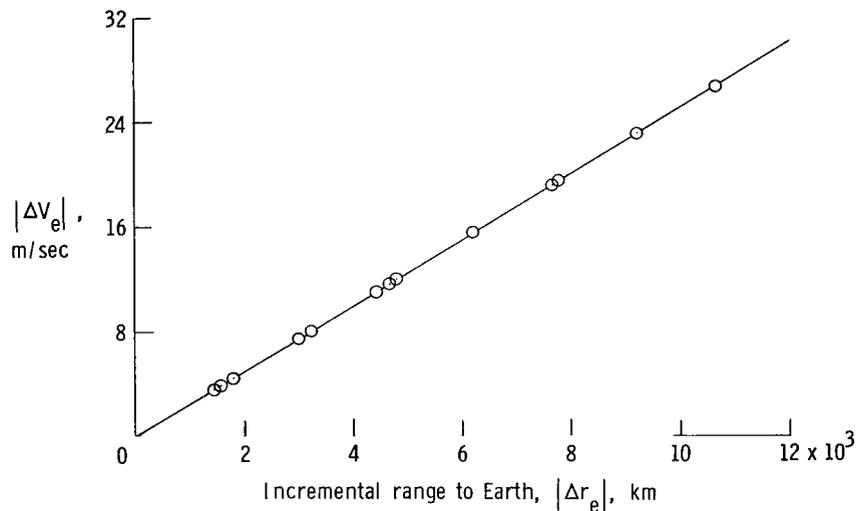


Figure 4.- Perturbed-trajectory characteristics at 5 days from Earth.

of  $\pm 5$  m/sec applied in one or more of the injection-velocity components. The velocity data are shown as a function of the range deviation inasmuch as this quantity is the main contributor to the midcourse correction requirement. The magnitude of the midcourse velocity correction would be slightly higher than the velocity deviation to account for the distance lost (or gained) at the time of first midcourse. The data of figure 4 are used subsequently to estimate the magnitude of the  $\Delta V$  error associated with a first midcourse guidance correction predicted from onboard measurements.

### APPROACH GUIDANCE

The simplified onboard procedure for interplanetary approach guidance is applied to the approach phase of an Earth-to-Mars mission. The approach region is considered

as that part of the trajectory lying within the Martian sphere of influence. The guidance procedure was developed in reference 11 for control of lunar approach trajectories. Characteristics of the method are different when applied to interplanetary approach because of geometry, spacecraft speed, and distance to the target body. The approach guidance results are shown primarily for the case where the final midcourse maneuver is performed relatively close to Mars.

### General Procedure

A schematic sketch of the approach-guidance geometry is presented in figure 5. A nominal trajectory with a periapsis altitude of 1000 km ( $r_p = 4388$  km) was chosen. The

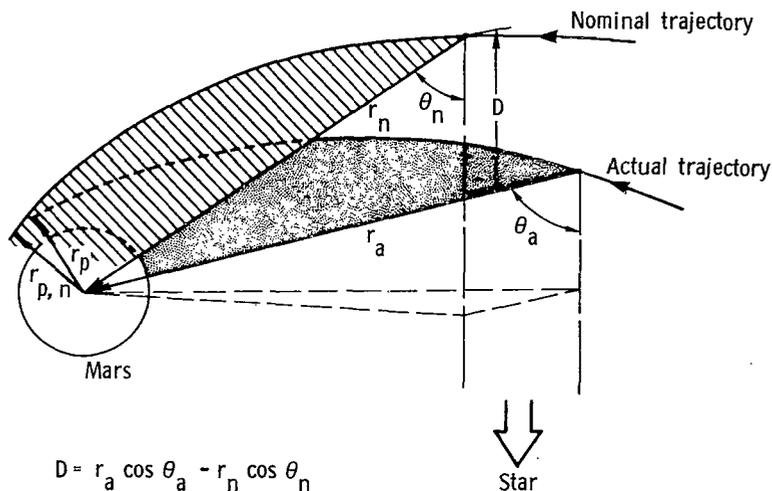


Figure 5.- Approach-guidance geometry.

objective is to control the periapsis magnitude of the perturbed (actual) trajectory to the nominal value. Provision can also be made to control the orientation of the orbital plane to the nominal as discussed in the appendix. The plane changes are shown to be relatively small at Mars; however, such changes can become important in a Grand Tour mission. The measured value of the deviation  $D$  can be employed to determine the magnitude of the guidance correction required to control both periapsis radius and orbital plane orientation.

The equation given in figure 5 shows that  $D$  is determined from nominal and measured values of the angle  $\theta$  and the range. Range can be determined optically by measuring the angle subtended by Mars or by other angular measurements. The direction selected for  $D$  has a large effect on the guidance accuracy. The effect of range measurement error on  $D$  can be minimized by choosing a deviation direction perpendicular

to the nominal range vector; that is,  $\theta_n = 90^\circ$ . For this value of  $\theta_n$ , the deviation can be calculated with the relationship

$$D \approx r_n \cos \theta_a$$

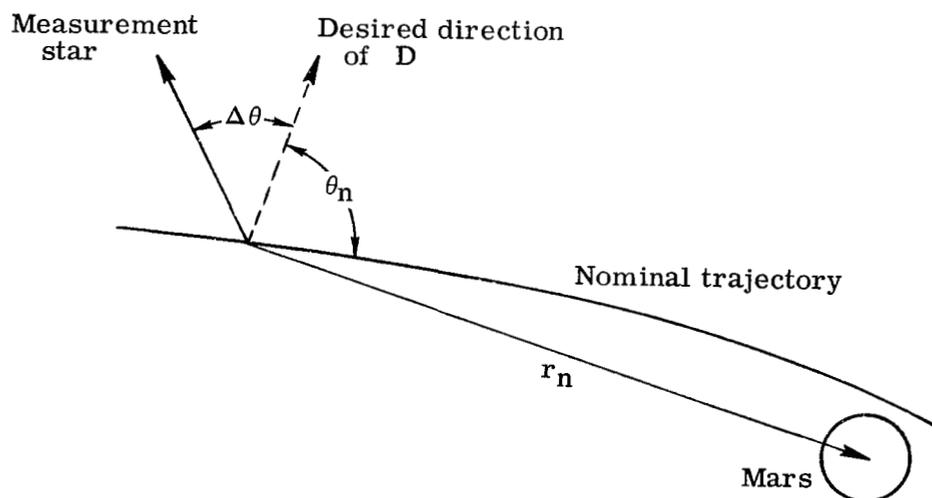
Thus, the error associated with the range measurement is eliminated. As is shown later, this direction is usually best. Under certain conditions, however, another direction could prove superior because of reduced scatter error or increased measurement sensitivity.

Where the final midcourse maneuver is performed far from the Martian sphere of influence, a range measurement is required.

In figure 5, the deviation  $D$  is shown in the direction of the measurement star; as stated, the direction of  $D$  is most important. In practice, if no suitable measurement star lies in the desired direction of  $D$ , another star may be substituted in the following manner. Choose a star which lies in the plane containing  $r_n$  and the desired direction. By adding (or subtracting)  $\Delta\theta$ , the difference between the nominal values (see sketch), the value of  $\theta_a$  can be obtained from

$$\theta_a = \theta_m - \Delta\theta$$

where  $\theta_m$  is the angle between the measurement star and body center. When the chosen star is not in this plane, the direction of the desired star must be rotated about  $r_n$  into the plane of the measurement star. The conversion of the angular measurement and the calculation of  $D$  would then be based on this new direction.



### Guidance-Velocity Requirement

The guidance velocity required to change the periapsis radius to the nominal value is

$$\Delta V = \frac{V \left[ r^2 \cos \gamma \cos(\gamma + \lambda) - r_{p,n}^2 \cos \lambda \right]}{r_{p,n}^2 - r^2 \cos^2(\gamma + \lambda)}$$

$$\pm \frac{r_{p,n} \left\{ \left( r^2 - r_{p,n}^2 \right) V^2 \sin^2 \lambda + \left[ r^2 \cos^2(\gamma + \lambda) - r_{p,n}^2 \right] \left[ \frac{2\mu}{r_{p,n}} - \frac{2\mu}{r} \right] \right\}^{1/2}}{r_{p,n}^2 - r^2 \cos^2(\gamma + \lambda)} \quad (3)$$

This equation was derived in reference 11 from two-body relations. The values derived with the alternate signs of the second term correspond to correcting to either side of Mars. The lesser magnitude of  $\Delta V$  would ordinarily be chosen to assure that the proper direction around Mars was achieved. The  $\Delta V$  required per meter of periapsis radius is shown in figure 6 as a function of distance and time to Mars. (To a good

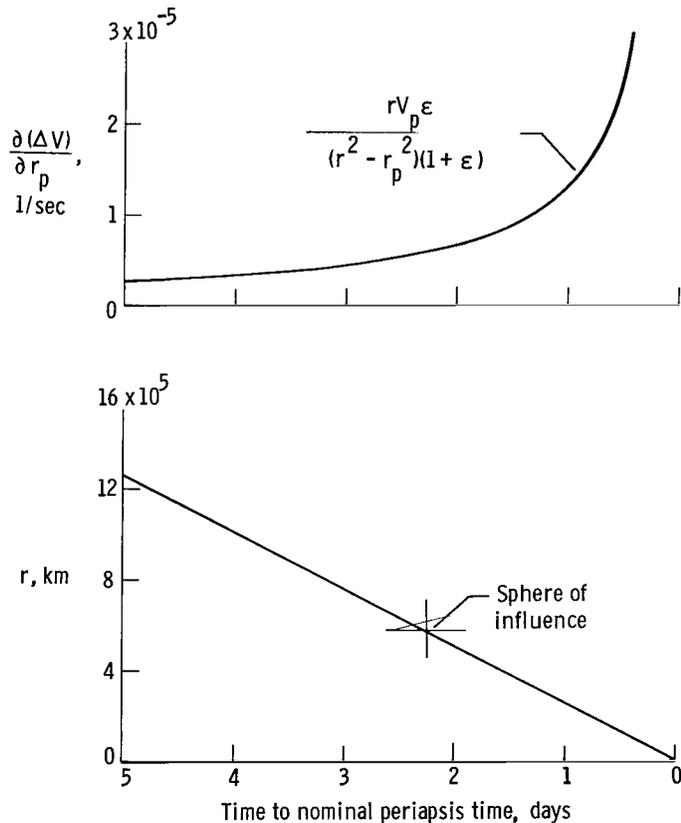


Figure 6.- Approach-guidance velocity requirement in relation to range and time to periapsis.

approximation,  $\Delta V$  is linearly related to periapsis radius.) The curve, computed by a simplified equation (ref. 11) by using nominal values and  $\lambda = 90^\circ$ , illustrates the rapid increase in the velocity requirement as Mars is approached.

In equation (3) the flight-path angle  $\gamma$  has the major influence on the magnitude of  $\Delta V$  for a given guidance pointing angle  $\lambda$  and guidance maneuver time. It will be shown in the following section that the value of  $\gamma$  is highly correlated with the deviation taken in certain directions. A measurement to a star is used to determine  $D$ , which in turn predicts the  $\Delta V$  requirement.

### Results With Close Midcourse Maneuver

General considerations. - For the data presented in figures 7 to 13, an assumed final midcourse correction was performed close to Mars. The time selected was 10 days before arrival at Mars. A Monte Carlo procedure was employed to simulate midcourse guidance errors due to onboard measurement errors resulting from the type of onboard measurements previously discussed. The onboard measurement-error analysis of reference 10 was used to determine the error distributions. (Although these distributions pertain to lunar missions, it was assumed that they would apply to interplanetary flight because of the similar measurements used.) The resulting guidance pointing error distributions had  $1\sigma$  in-plane and  $1\sigma$  out-of-plane errors of  $1.8^\circ$  and  $0.7^\circ$ , respectively. The  $1\sigma$  error in  $\Delta V$  was 2 m/sec. The effect of guidance maneuvering error was considered negligible. Except where noted, the midcourse errors were essentially along the flight path, which is the general case for most midcourse procedures. (For example, see ref. 11.)

Examples of the variation of  $\gamma$  with  $D$  are shown in figures 7 and 8. (The lines in these, as in all figures, were faired through the data points.) Each data point represents the condition at  $T_p = 1$  day on a perturbed trajectory resulting from the errors in the final midcourse correction. The deviation  $D$  was determined by using the relationship

$$D = l(x_a - x_n) + m(y_a - y_n) + n(z_a - z_n) \quad (4)$$

where  $l$ ,  $m$ , and  $n$  are the known direction cosines of the deviation vector. Equation (4) is equivalent to the equation in figure 5. In figure 7, the deviation lies in the nominal orbital plane and is perpendicular to the nominal radius vector, whereas in figure 8, its direction has been changed  $2^\circ$  in the nominal orbital plane.

The scatter of the data such as shown in figures 7 and 8 produces error in the guidance procedure. It is important that the amount of scatter be minimized by selecting the most effective direction for  $D$ . Comparison of figures 7 and 8 shows that this direction

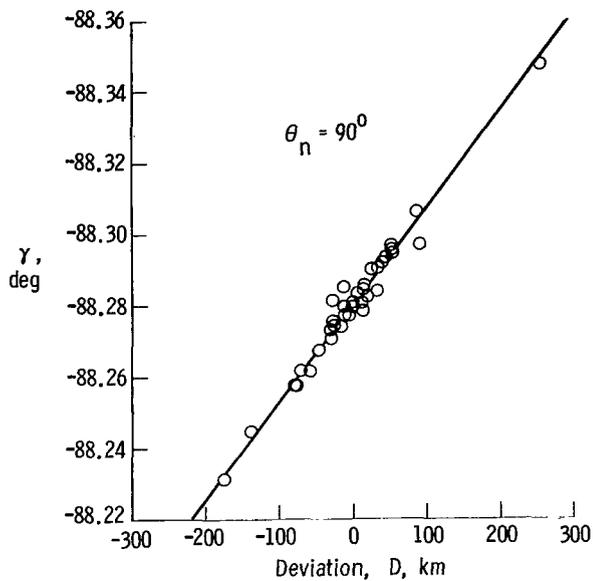


Figure 7.- Example of good correlation between  $\gamma$  and  $D$ .  $\delta_n = 0^\circ$ ;  $T_p = 1$  day.

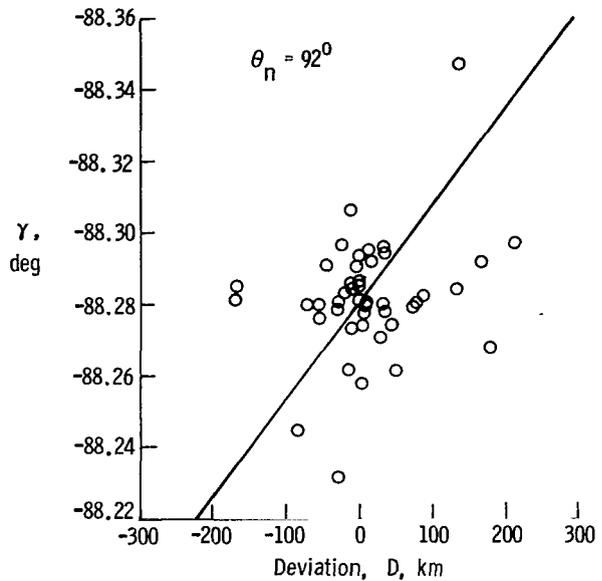


Figure 8.- Example of poor correlation between  $\gamma$  and  $D$ .  $\delta_n = 0^\circ$ ;  $T_p = 1$  day.

should be approximately perpendicular to  $r_p$ . The acceptable range for  $\theta_n$  is approximately  $89.5^\circ$  to  $90.5^\circ$ . Varying the out-of-plane direction by  $10^\circ$  does not greatly affect scatter error; however, the scatter error tends to increase for angles much greater than  $10^\circ$ .

The Sun can be used as the measurement star if it is reasonably close to the orbital plane of the spacecraft. The assumption that the Sun is an inertially fixed point in space contributes a maximum error in  $D$  of 1 km, which is negligible. Sighting on the Sun rather than on a star could prove far superior because of ease of acquisition.

Figures 9 and 10 are presented mainly to indicate the amount of periapsis-radius error caused by approximation (scatter) error. The distance between each point and the line represents the amount of error in determining periapsis radius for the corresponding trajectories. The  $1\sigma$  values in the figures show that somewhat higher accuracy is achieved with a deviation which is  $10^\circ$  out of the nominal orbital plane of the spacecraft. The values of the deviation  $D$  remain essentially unchanged for a given direction at any time within the sphere of influence of the planet.

Uncorrected values of  $\Delta r_p$  in figures 9 and 10 are less than 200 km. These relatively small values are due to the fact that the errors in the final midcourse velocity correction were generally in the direction of the trajectory motion relative to Mars.

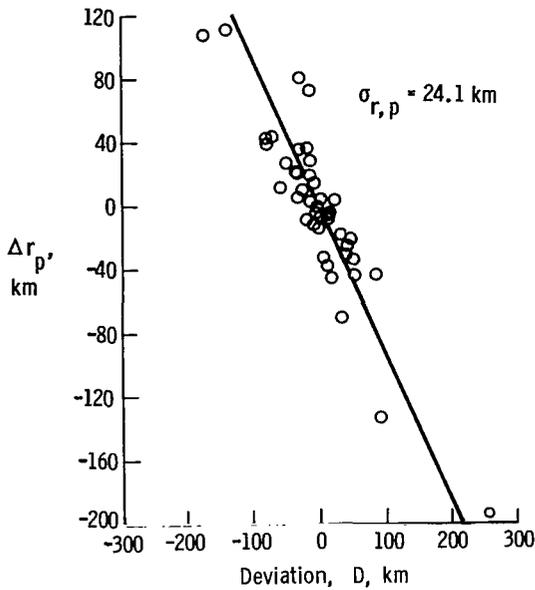


Figure 9.- Accuracy of determining periapsis radius for  $\theta_n = 90^\circ$ ,  $\delta_n = 0^\circ$ , and  $T_p = 1$  day.

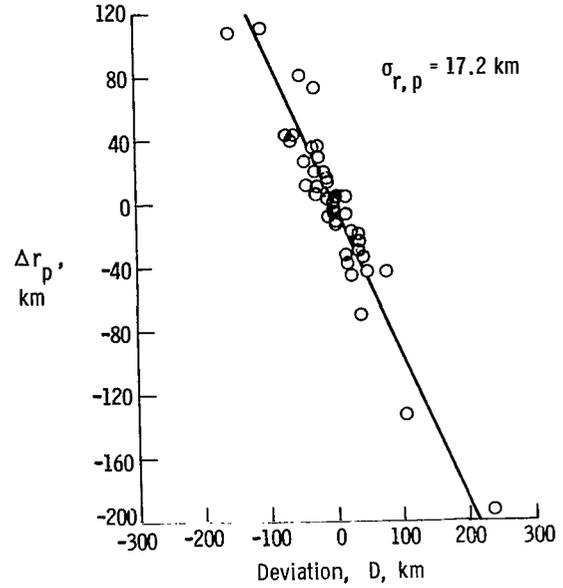


Figure 10.- Accuracy of determining periapsis radius for  $\theta_n = 90^\circ$ ,  $\delta_n = 10^\circ$ , and  $T_p = 1$  day.

To determine the guidance correction, only the variation of approach-guidance correction  $\Delta V$  with deviation  $D$  (fig. 11) is required. This variation is calculated with equations (3) and (4). In all cases, the  $\Delta V$  vector is parallel to the nominal orbital

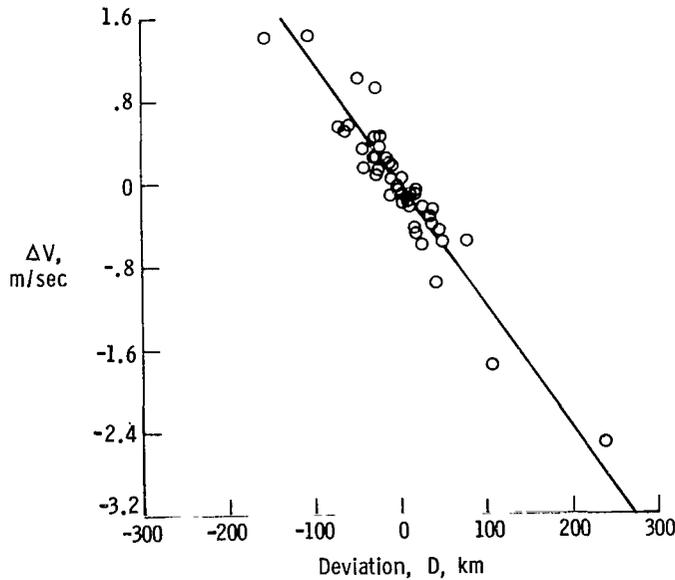


Figure 11.- Guidance velocity as a function of deviation.  $\theta_n = 90^\circ$ ;  $\delta_n = 10^\circ$ ;  $T_p = 1$  day.

plane and perpendicular to the nominal velocity vector, which is essentially optimum for the fuel requirements. Both the deviation and the  $\Delta V$  values in figure 11 pertain to the same time,  $T_p = 1$  day. The guidance maneuver can be performed any time after the measurement through the use of the data given in figure 6. Note that a delay of 1/2 day doubles the fuel requirement.

Effect of different midcourse procedure. - The preceding discussion has dealt with midcourse guidance corrections which produce an error pattern along the flight path. It is conceivable that midcourse errors in a perpendicular direction could be generated by some other midcourse guidance method. The effect of such an error pattern was studied. The data in figures 12 and 13 represent approach conditions for perturbed trajectories in which the final midcourse errors were directed essentially perpendicular to  $\bar{V}$ . The midcourse-guidance error distributions were assumed to be the same as those used to derive figures 7 to 11.

Note in figure 12 that the range of  $\Delta r_p$  has increased by an order of magnitude from that of figures 9 and 10 where the midcourse guidance errors were along  $\bar{V}$ , but the scatter has been drastically reduced. In addition, the data in figure 12 show that a wide range of deviation directions is available for calculating  $D$ . The set of data shown for  $\delta_n = 54.73^\circ$  represents a measurement in the direction of the Sun. No increase in

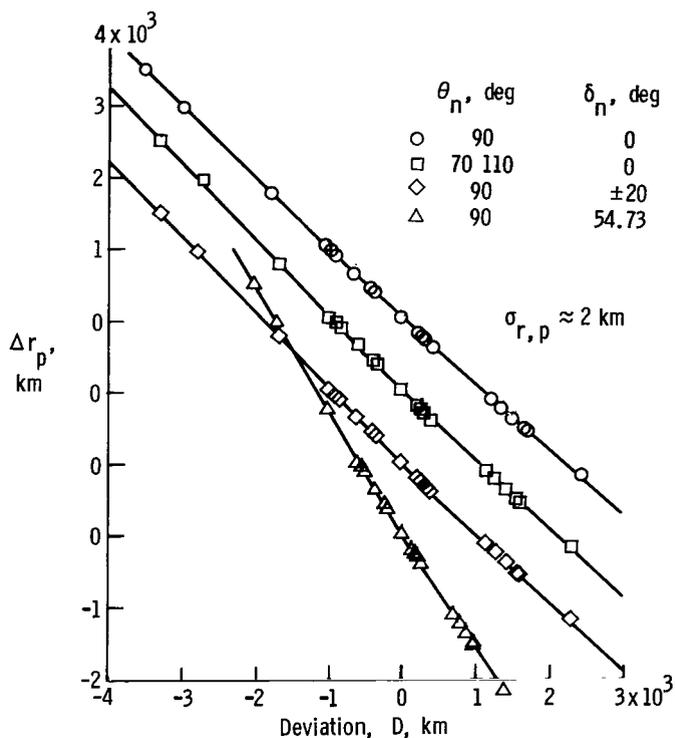


Figure 12.- Accuracy of determining periapsis radius with final midcourse errors approximately perpendicular to flight path. (Note staggered vertical scale.)

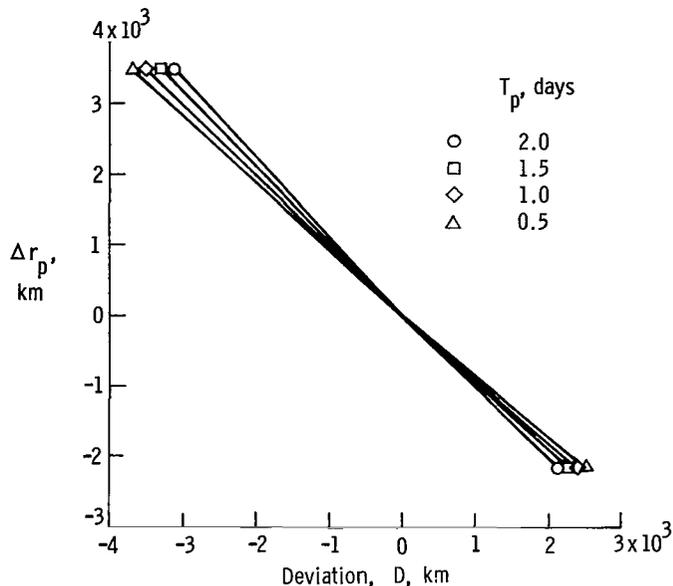


Figure 13.- Effect of measurement time on deviation.  
 $\theta_n = 90^\circ$ ;  $\delta_n = 0^\circ$ .

the scatter error occurs because of this large out-of-plane angle. However, the sensitivity of  $D$  has been reduced, which, in turn, will increase the effect of measurement error on the guidance accuracy.

In figure 13 it is seen that the sensitivity of  $D$  is not greatly affected by the measurement time. As will be shown, the ratio  $\partial r_p / \partial D$  is important in the error analysis. This ratio varies from about 1.06 to 0.90 when midcourse errors are perpendicular to  $\bar{V}$  and the ratio remained constant at approximately 0.93 for all values of  $T_p$  when midcourse errors were along  $\bar{V}$  (for example, figs. 9 and 10).

#### Results Without Close Midcourse Maneuver

For the data presented in figure 14, a first midcourse maneuver at a time 5 days from Earth was assumed to be the only guidance correction prior to approach guidance. As shown, this procedure leads to very large periapsis-radius errors which must be corrected by an approach-guidance maneuver. Normally, the use of only one midcourse maneuver causes excessive fuel requirements. At the sphere of influence the  $1\sigma$  value of  $\Delta V$  required for correcting the  $r_p$  dispersions in figure 14 would be about 120 m/sec. The fuel requirements could be lowered by applying the correction at an earlier time. (See fig. 6.) Even though the trajectory would lie outside the sphere of influence at this time, the two-body approximation used for equation (3) would still be adequate for the time period shown in figure 6.

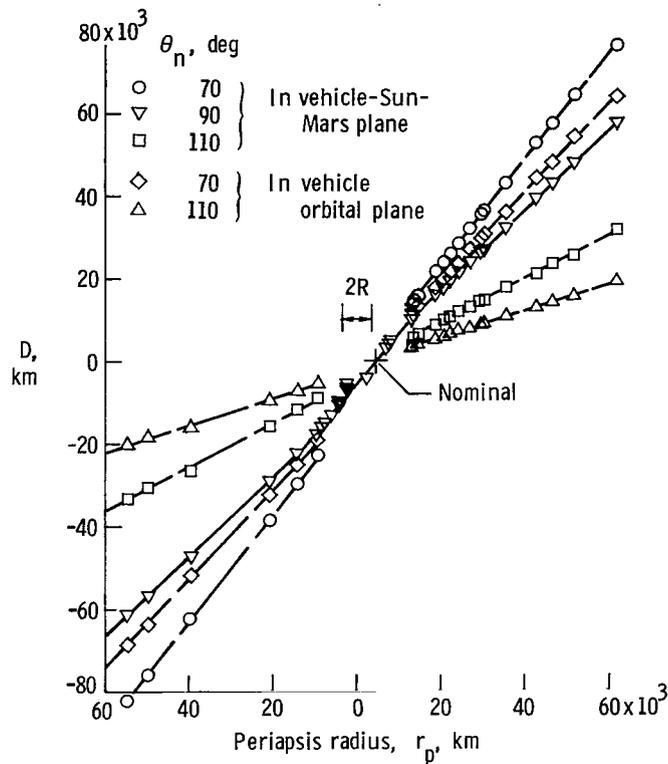


Figure 14.- Deviations at Martian sphere of influence ( $T_p = 2.2$  days) resulting from first midcourse perturbations.

A single midcourse correction might be justified for a Grand Tour mission where midcourse maneuvers between outer planets are based on measurements with respect to a planet other than the Earth. Here the highly accurate Earth-based radar measurements normally required for the more refined second midcourse correction would not be available.

As in previous figures, each data point in figure 14 is the result of the trajectory having been perturbed at midcourse with a Monte Carlo procedure. The same midcourse error distributions as previously discussed were used, except that a  $1\sigma$  error of 1 m/sec in the  $\Delta V$  magnitude was assumed. This value was based on data of figures 3 and 4. The minimum range-determination error from onboard measurements is shown to be about 250 km. This error is the main contributor to the midcourse guidance error and figure 4 shows that this amount of error would lead to a  $\Delta V$  error of roughly 1 m/sec.

The data in figure 14, with little scatter for the cases shown, indicate a wide range of usable deviation directions. The angle between the two planes referred to in the figure is about  $55^\circ$ . The small effect of direction on scatter error is attributed to the highly elongated shape of the position-error ellipsoid at the Martian sphere of influence. This

ellipsoid was determined by analyzing the errors in the perturbed trajectories at this point, which had resulted from the Monte Carlo midcourse-guidance errors. The  $1\sigma$  values of the three axes of the error ellipsoid are 265 000 km, 3880 km, and 550 km, which illustrates that most of the error is concentrated along the major axis. This axis lies about  $9^\circ$  out of the vehicle-Sun-Mars plane about midway between the  $90^\circ$  angle formed by the radius vectors to the Sun and Mars. The inherent advantage of choosing  $\theta_n \approx 90^\circ$  is discussed in the following section.

As aforementioned, the Sun can be assumed a fixed point in inertial space and used as the measurement star. This assumption causes a maximum error in the deviation  $D$  of only 10 km and, for  $\theta_n = 90^\circ$  (fig. 14), an error of about 10 km in determining  $r_p$ .

### APPROACH-GUIDANCE ACCURACY

In this section, the errors associated with the approach-guidance procedure are defined and analyzed and their effect on the accuracy of controlling periapsis radius is determined. The errors are summarized in table I for two times; the time  $T_p = 2.2$  days is at the Martian sphere of influence. In addition, the effect of ephemeris error is shown in figure 15. The results in table I apply to a deviation direction taken perpendicular to the nominal range vector ( $\theta_n = 90^\circ$ ). The guidance maneuver was assumed to be performed at  $T_p$ .

TABLE I. - APPROACH-GUIDANCE ACCURACY CHARACTERISTICS

Type of error	$\sigma_{r,p}$ , km, for -	
	$T_p = 2.2$ days (a)	$T_p = 0.5$ day (a)
Measurement	26.0 (29.5)	6.2 ( 6.0)
Scatter	17.2 ( 2.0)	17.2 ( 2.0)
Velocity cut-off	17.0	4.0
Total	35.5 (34.1)	18.7 ( 7.5)

<sup>a</sup> Results in parentheses correspond to final midcourse-guidance errors in a direction approximately perpendicular to the flight path.

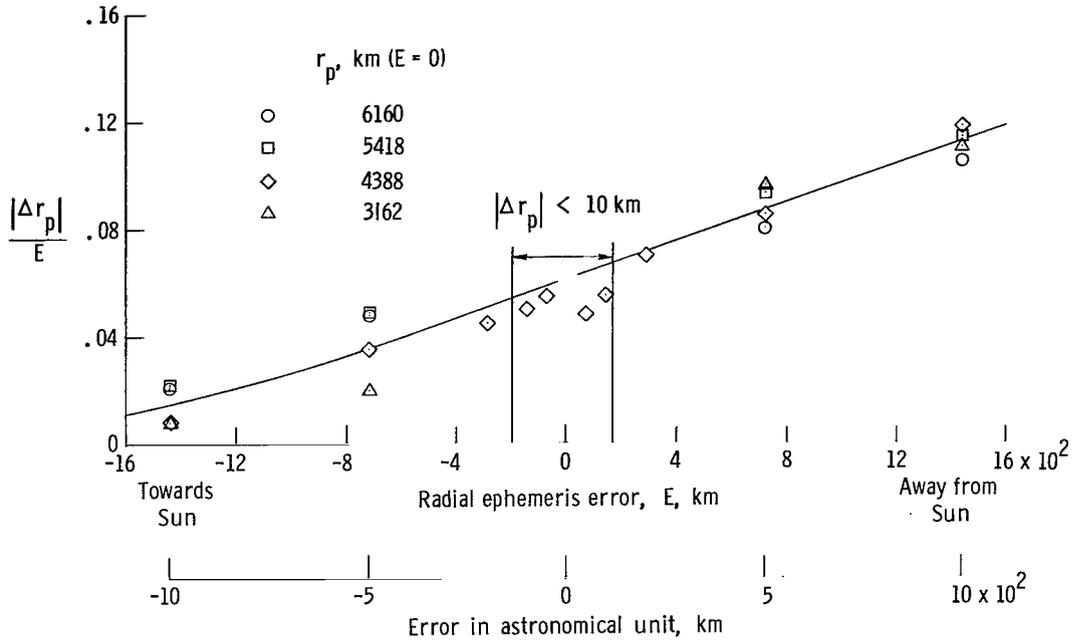


Figure 15.- Effect of ephemeris error on approach-guidance accuracy.  $T_p = 1$  day.

The results in the table apply to the close midcourse case; that is, a final midcourse correction was employed several days prior to the approach guidance. The values are given for midcourse guidance errors both along and normal to  $\bar{V}$ . The value for the scatter error was obtained from figures 10 and 12. This error is caused by the approximation in predicting the guidance correction from a value of the deviation  $D$ . As shown, the scatter error is very small for the case where the midcourse guidance errors are normal to  $\bar{V}$ . This, in turn, reduces considerably the total error for  $T_p = 0.5$  day. The total error was determined by statistically combining the effects of the various types of errors.

The  $1\sigma$  periapsis-radius error due to maneuvering error (velocity cut-off) is shown for a typical  $1\sigma$  value of 0.1 m/sec. The periapsis error was determined by multiplying the reciprocal of the velocity requirement (fig. 6) by  $0.1 \times 10^{-3}$ . The results in table I correspond to a guidance pointing angle  $\lambda = 90^\circ$ . By sacrificing the fuel requirement, which is relatively small (fig. 11), and by applying the approach  $\Delta V$  vector at  $\lambda = 0^\circ$ , the effect of the maneuvering error can be reduced significantly, provided that the maneuver pointing error is small.

The effect of measurement error on the guidance accuracy can be determined from the following equations:

$$\sigma_D = \left[ (\cos \theta \sigma_r)^2 + (r \sin \theta \sigma_\theta)^2 \right]^{1/2} \quad (5)$$

and

$$\sigma_{r,p} = \frac{\partial r_p}{\partial D} \sigma_D \quad (6)$$

where values for  $\partial r_p / \partial D$  are obtained from plots such as those given in figures 10 and 12. Error equations for the onboard optical measurements were developed in reference 11; equation (5) corresponds to uncorrelated errors in the measurement of range  $r$  and the angle  $\theta$ .

In equation (5), nominal values for  $r$  and  $\theta$  are used in calculating the error  $\sigma_D$ . For values of  $\theta_n \approx 90^\circ$ , it is seen that

$$\sigma_D \approx r \sigma_\theta \quad (7)$$

Equation (7), along with equation (6), was used to calculate the results shown in table I. When the midcourse errors were along  $\bar{V}$ , the ratio of  $\partial r_p$  to  $\partial D$  varied slightly with time  $T_p$ . (See fig. 13.) A typical  $1\sigma$  value of 10 seconds of arc was assumed for the angular measurement accuracy.

For  $\theta_n \neq 90^\circ$ , the effect of range-measurement error  $\sigma_r$  becomes important, as shown by equation (5). If range is measured by the half-angle subtended by Mars  $\alpha$ , then according to reference 11

$$\sigma_r = \left[ \frac{r^2}{R^2} (\sigma_R^2 + r^2 \sigma_\alpha^2) \right]^{1/2}$$

where  $\sigma_R$  is the uncertainty in the value for the radius of Mars and  $\sigma_\alpha$  is constant with time. As an example, at  $T_p = 0.5$  day and with  $\theta_n = 85^\circ$ , the error in periapsis radius  $\sigma_{r,p}$  due to measurement error is 28 km. (Typical values of 5 km and 10 seconds of arc, respectively, were assumed for  $\sigma_R$  and  $\sigma_\alpha$ .) This error is due almost entirely to error in the range measurement and would be much higher at greater distances from Mars. Thus, if  $\theta_n$  is much greater (or less) than  $90^\circ$ , some optical method other than the subtended-angle method would be highly desirable for measuring range. One point of import is that the  $1\sigma$  value of the incremental range  $\Delta r$  for the Monte Carlo perturbed trajectories may be well below the range-measurement accuracy. For example, the data for the close final midcourse maneuvers with guidance errors perpendicular to the flight path showed a  $1\sigma$  value of  $\Delta r$  less than 100 km at the Martian sphere of influence. In this case, then, regardless of the value used for  $\theta_n$ , approximating range with its nominal value would be adequate.

## Effect of Ephemeris Error

The effect of Martian ephemeris error on the approach-guidance accuracy was examined (fig. 15). Ephemeris error affects the accuracy because the guidance measurements are referenced to a nominal trajectory which, in turn, is based on a certain location of Mars. This type of error was investigated only for position error in the radial direction; error in the direction of the motion of Mars would have a negligible effect on  $r_p$ . In figure 15, it is seen that the approach-guidance procedure compensates, to a large extent, for presence of ephemeris error. The data are shown for  $T_p = 1$  day but are representative of results for most times within the sphere of influence. The results, shown as the ratio of  $|\Delta r_p|$  to the ephemeris error  $E$ , were obtained by using several trajectories with different nominal values of  $r_p$ . Despite apparent differences in the results between the cases, the trend of the data indicates that increasing the error toward the Sun (which, in effect, moves Mars away from the nominal trajectory) does not increase the guidance error appreciably. The region shown in the figure for  $|\Delta r_p| < 10$  km corresponds to the uncertainty ( $3\sigma$ ) in the Mars ephemeris which is in the neighborhood of 200 km (ref. 12). Adding the 10-km error, statistically, to the data of table I does not appreciably change the results in most cases. The excessive ephemeris errors included in figure 15 are indicative of results which might be expected for other planets.

## Orbital-Plane Orientation Errors

Results in the appendix show that the orbital-plane orientation errors can be controlled to high accuracy if deviations are measured in a certain direction. The acceptable ranges for these measurements, as well as for those used to control periapsis radius, are summarized in table II. As previously noted, the direction of the measurement star need not fall within these ranges.

TABLE II. - APPROXIMATE ALLOWABLE RANGES FOR  $\theta_n$  AND  $\delta_n$

Direction of final midcourse velocity vector	Periapsis radius to be controlled		Orbital plane orientation to be controlled	
	$\theta_n$ , deg	$\delta_n$ , deg	$\theta_n$ , deg	$\delta_n$ , deg
Approximately along flight path	89.5 to 90.5	$\pm 10$	89.8 to 90.2	88 to 90 and -88 to -90
Approximately perpendicular to flight path	70 to 110	$\pm 60$	88 to 92	89.8 to 90 and -89.8 to -90

## CONCLUDING REMARKS

A procedure has been developed for interplanetary approach guidance by using onboard optical measurements. The method is based on characteristics of perturbed trajectories. The procedure requires, in general, only a single star-to-body measurement to determine the position deviation from the nominal in a particular direction. An error analysis showed that the procedure was feasible for controlling the periapsis radius of Martian trajectories; periapsis radius can be controlled to a  $1\sigma$  accuracy of about 35 km if the maneuver is performed at the Martian sphere of influence. The accuracy is substantially increased if the guidance is performed closer to Mars.

Some preliminary results on midcourse guidance for interplanetary flight showed that midcourse procedures based on onboard measurements may be applicable to Grand Tour missions.

The important results concerning the approach-guidance method are as follows:

The method can be applied anywhere within the Martian sphere of influence which extends some 570 000 km.

It has been shown that the guidance method compensates, to a large degree, for the effect of ephemeris error.

The Sun may be assumed to be a fixed point in inertial space and used as the measurement star, with little degradation of guidance accuracy.

Directions that may be selected for the deviation depend on the type of procedure used for final midcourse guidance. A direction lying perpendicular to the nominal radius vector and reasonably close to the orbital plane of the spacecraft, within  $\pm 10^\circ$ , generally provides maximum accuracy for controlling periapsis radius.

The type of final midcourse procedure has a pronounced effect on the approach accuracy, especially if the approach guidance is performed relatively close to Mars.

Langley Research Center,  
National Aeronautics and Space Administration,  
Hampton, Va., April 13, 1972.

## APPENDIX

### CONTROL OF ORBITAL PLANE ORIENTATION

The approach-plane orientation errors at Mars are minor; however, if such errors at the outer planets are not corrected for Grand Tour missions, they could lead to extreme errors at the next planet. The purpose of this appendix is to show that these orbital-plane errors can be determined from onboard measurements. Results in figures 16 and 17 indicate that the error in the inclination angle can be predicted by a measurement of a deviation in a direction perpendicular to the nominal orbital plane. In figure 17, where the final

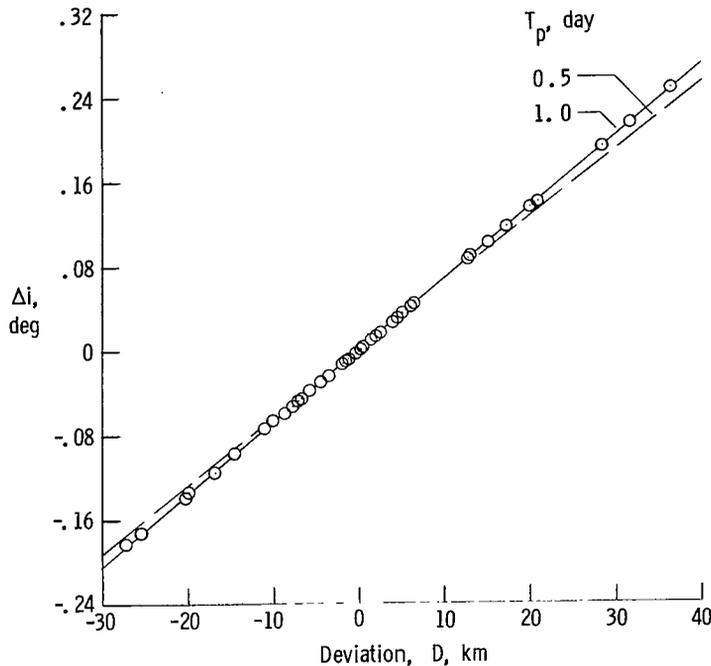


Figure 16.- Variation of  $\Delta i$  with  $D$  for  $\theta_n = 90^\circ$  and  $\delta_n = 90^\circ$ .

midcourse errors were perpendicular to the flight path, there is a noticeable scatter error with a  $1\sigma$  value of about  $0.03^\circ$ . As in the case shown in the main text for predicting  $r_p$  from deviations taken near the orbital plane, the deviation direction for determining inclination angle may be rotated in a plane perpendicular to the orbital plane without affecting the scatter error as long as  $\theta_n \approx 90^\circ$ . The allowable rotation, however, is only about  $\pm 2^\circ$ .

The change in the location of the ascending node can be determined with the same degree of accuracy shown in figures 16 and 17, since in general

$$\Lambda_N = \Lambda_\infty - \sin^{-1}(\tan \zeta_\infty \cot i)$$

APPENDIX - Continued

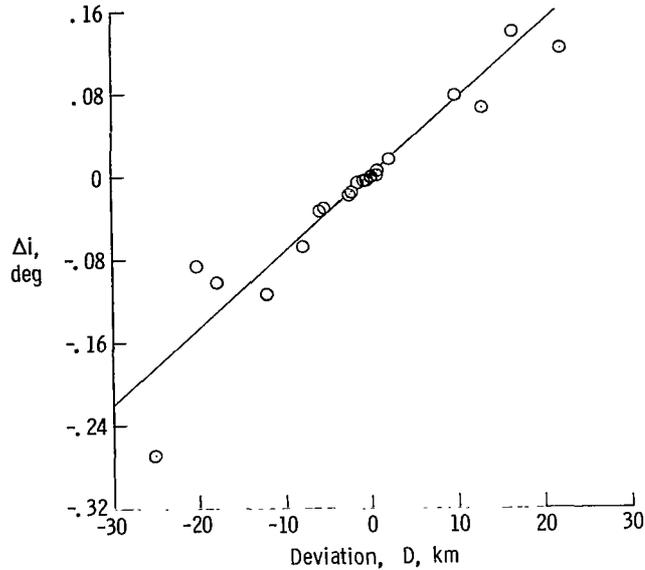


Figure 17.- Variation of  $\Delta i$  with  $D$  for final midcourse errors approximately perpendicular to flight path.  
 $\theta_n = 90^\circ$ ;  $\delta_n = 90^\circ$ ;  $T_p = 1$  day.

or

$$d\Lambda_N = \left(1 - \tan^2 \xi_\infty \cot^2 i\right)^{-1/2} \csc^2 i \, di$$

where

- $\xi_\infty$  latitude of  $\bar{V}$  at Martian sphere of influence
- $\Lambda_\infty$  longitude of  $\bar{V}$  at Martian sphere of influence
- $\Lambda_N$  longitude of ascending node

The change in  $\Lambda_N$  was found to be approximately  $0.67 \Delta i$  in the present cases.

In figure 16 the variation of  $\Delta i$  with  $D$  is not greatly affected by  $T_p$ . The scatter (approximation) error remains negligible at all values of  $T_p$ , except when  $T_p \approx 0$ . The guidance maneuver to correct the plane orientation (derived from the changes in the inclination and the ascending node) should therefore be made close to the planet to take advantage of the smaller effect of measurement error. As shown in figure 18, the guidance-velocity requirement is essentially independent of distance from the planet.

APPENDIX - Continued

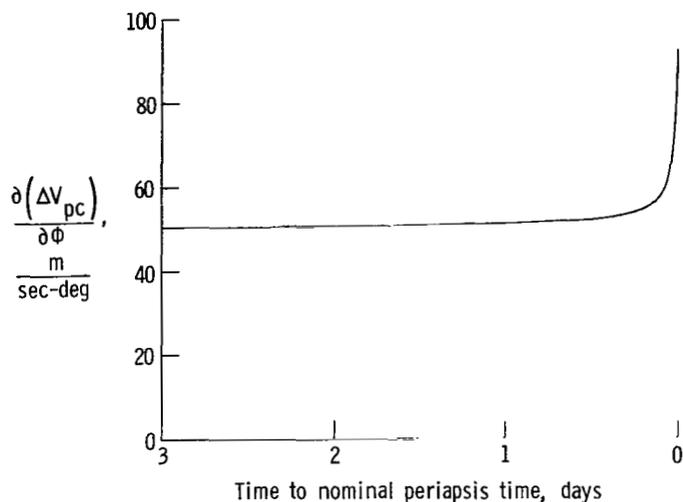
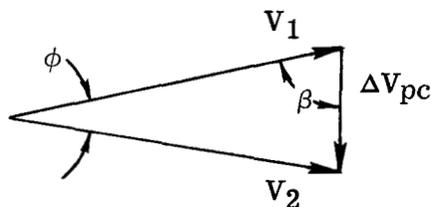


Figure 18.- Guidance-velocity requirement for changing the orbital plane.

The equation for  $\Delta V_{pc}$  was derived from the sketch



as

$$\Delta V_{pc} = 2V_1 \sin \frac{\phi}{2} \approx V_1 \sin \phi$$

where

$$\beta = \sin^{-1} \left( \frac{\sin \phi}{2 \sin \frac{\phi}{2}} \right) \approx 90^\circ$$

These equations correspond to the case where the magnitude  $V_1 = V_2$ . (See ref. 1.)

When the final midcourse errors are perpendicular to the flight path (fig. 17),  $T_p = 0.5$  day is about the closest practical time at which to perform the plane-change

## APPENDIX – Concluded

guidance. At this time, the effect of the measurement error is roughly the same as the  $0.03^\circ$  ( $1\sigma$ ) effect of the scatter error. For example,

$$\sigma_i = \frac{\partial i}{\partial D} \sigma_D = 0.0073 \sigma_D$$

where  $\sigma_D$  at  $T_p = 0.5$  day is approximately 6 km; hence,  $\sigma_i \approx 0.043^\circ$ .

## REFERENCES

1. Friedlander, Alan L.; and Harry, David P., III: An Exploratory Statistical Analysis of a Planet Approach-Phase Guidance Scheme Using Angular Measurements With Significant Error. NASA TN D-471, 1960.
2. Harry, David P., III; and Friedlander, Alan L.: An Analysis of Errors and Requirements of an Optical Guidance Technique for Approaches to Atmospheric Entry With Interplanetary Vehicles. NASA TR R-102, 1961.
3. Hartwell, J. Graham; Danby, J. M. A.; and Jones, Arthur L.: Planetary Approach Navigation. Navigation, vol. 11, no. 2, 1964, pp. 125-144.
4. Taylor, Arthur J.; and Wagner, John T.: On-Board Approach Guidance for a Planet-Orbiter. J. Spacecraft Rockets, vol. 3, no. 12, Dec. 1966, pp. 1731-1737.
5. Seaman, Lewis T.; and Brown, Harold E.: Planetary Approach Guidance for a Martian Mission. Transactions of 1967 National Symposium on "Saturn V/Apollo and Beyond," Vol. III, Steve S. Hu, ed., Amer. Astronaut. Soc., c.1967.
6. Murtagh, Thomas B.: Planetary Probe Guidance Accuracy Influence Factors for Conjunction-Class Missions. NASA TN D-4852, 1969.
7. Duxbury, Thomas C.: A Spacecraft-Based Navigation Instrument for Outer Planet Missions. J. Spacecraft Rockets, vol. 7, no. 8, Aug. 1970, pp. 928-933.
8. White, John S.; Callas, George P.; and Cicolani, Luigi S.: Application of Statistical Filter Theory to the Interplanetary Navigation and Guidance Problem. NASA TN D-2697, 1965.
9. Lowes, Flora B.; and Murtagh, Thomas B.: Navigation and Guidance Systems Performance for Three Typical Manned Interplanetary Missions. NASA TN D-4629, 1968.
10. Hamer, Harold A.; Johnson, Katherine G.; and Blackshear, W. Thomas: Midcourse-Guidance Procedure With Single Position Fix Obtained From Onboard Optical Measurements. NASA TN D-4246, 1967.
11. Hamer, Harold A.; and Johnson, Katherine G.: An Approach-Guidance Method Using a Single Onboard Optical Measurement. NASA TN D-5963, 1970.
12. Anon.: The Deep Space Network for the Period January 1 to February 28, 1967. Space Programs Sum. No. 37-44, Vol. III (Contract No. NAS 7-100), Jet Propulsion Lab., California Inst. Technol., Mar. 31, 1967.



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