23A SKYLAB Memo #1-70

Massachusetts Institute of Technology
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23A SKYLAB Memo #1-70



TO: Distribution

FROM: R. Phillips

DATE: August 7, 1970

SUBJECT: Polynomial Filter Estimation of Range and Range Rate for Terminal Rendezvous.

References:

1. TRW Memo 5522.7-70-85, J.B. Clifford.
2. PCR SLO32, M.C. Contella.

Contents:



(NASA-CR-126400) POLYNOMIAL FILTER ESTIMATION OF RANGE AND RANGE RATE FOR TERMINAL RENDEZVOUS R. Philips (Massachusetts Inst. of Tech.) 7 Aug. 1970 36 p CSCL 22A G3/30
N72-23886
Unclas 21812

## INTRODUCTION

A study of a polynomial filter to compute range rate information from the CSM VHF range data was made. This memo concentrates on the performance of the filter during the terminal phase of the rendezvous. The filter as described in the TRW Memo[1] was incorporated into a simulation of the manual terminal rendezvous. Two modifications to the filter were also made and tested. As specified in PCR SLO32[2] the range rate should be computed to an accuracy of 1 f/s. Private communication with NASA indicated they would like to operate the filter from MCC2 until within 200' of the passive vehicle. For the manual terminal rendezvous scheme assumed, the desired accuracies were achieved for practically the entire period provided the vehicles were on an intercept trajectory. The exceptions were short periods of time following each braking maneuver when the estimated range rate was initially in error by the magnitude of the burn. Astronaut action (or rather inaction) in ignoring the display for a short period of time could null the effect of this deviation. Alternatively the polynomial state (the range rate particularly) could be "updated" to the effect of the burn. With this modification the range rate display is not subject to the short perturbation following the braking maneuvers. Using the range and range rate information thus provided the simulated manned rendezvous was quite successful.

## THE POLYNOMIAL FILTER

The filter as described in the TRW Memo[1] was programmed to include the ability to estimate an additional range derivative, "$\dddot{r}$". The following values for variables were used:

| | | |
|---|---|---|
| $\sigma_{VHF}$ | = 30 ft. | |
| $\Delta q$ | = 60.76 ft. | range quantum |
| $1/f$ | = $\delta t$ = .2 sec. | data sampling interval |
| $\Delta t$ | = 5 sec. | data request interval |

To start the filter the range and range rate were computed from two measurements taken about 5 seconds apart. The filter error covariance matrix was initially diagonal with elements:

$$E_{11} = \sigma^2_{VHF} + \frac{1}{12} \min \left( \Delta_q^2, \ (\dot{r}_{est}/f)^2 \right)$$

$$E_{22} = 2 E_{11}/(\Delta t)^2$$

$E_{33}$ = matched to actual $\ddot{r}$ value

$E_{44}$ = matched to actual $\dddot{r}$ value

Since the values of $\ddot{r}_{est}$ and $\dddot{r}_{est}$ were assumed to be zero in the original state the covariance matrix elements corresponding to those components were set to approximately the values of $\ddot{r}$ and $\dddot{r}$ squared for the particular trajectory. This will not be possible during a mission since the actual trajectory will not be known. The values might be set "a priori" from simulation studies or the complete state and covariance matrix might be computed from four or more "initial" measurements.

## THE TERMINAL RENDEZVOUS

The terminal rendezvous was simulated by a series of burns down the relative range vector (braking) and normal to the range vector ( $\omega_{LOS}$ corrections).

The braking schedule was:

| R (ft.) | R (f/s) | Δt (sec) | ΔV (f/s) |
|---|---|---|---|
| 6000 | 30 | | |
| | | 100 | 3 (nominal) |
| 3000 | 20 | | |
| | | 75 | 10 |
| 1500 | 10 | | |
| | | 90 | 10 |
| 600 | 5 | | |
| | | 120 | 5 |
| 0 | 0 | | |

Several restrictions were made on $\omega_{LOS}$ corrections:

1. Unmeasurable below .3 mr/s
2. Measured to an accuracy of .3 mr/sec ( $1\sigma$ )
3. No correction made if ΔV is less than the values in the following schedule

| R (ft.) | ΔV (f/s) | $\omega_{LOS\ min}$ (mr/s) |
|---|---|---|
| $r > 3000$ | 7 | 2.3 |
| $3000 > r > 1500$ | 5 | 3.3 |
| $1500 > r > 600$ | 3 | 5.0 |
| $600 > r > 0$ | .5 | .3 |

This last restriction was to simulate the astronaut's desire to let the "LOS" profile follow its nominal non-zero path. Only if the "LOS" rate appeared quite large would any corrective maneuver be made before 600 ft.

Under no conditions were burns of less than .5 f/s made. The following restrictions were made on the timing of the burns:

1. No braking burn followed another braking burn by less than 45 sec.
2. No $\omega_{LOS}$ burn followed another $\omega_{LOS}$ burn by less than 30 sec. unless $R < 200'$; then a wait time of 15 sec. was applied.
3. No burn followed any burn by less than 15 seconds.
4. An acceleration of 1 $f/s^2$ was assumed for thrusting.

The burns were applied impulsively but no range data was taken during the assumed thrust time (#4 above). The burn had a magnitude accuracy of $1\sigma$ - 1 f/s (rectangular dist.) and a pointing accuracy of $1\sigma$ - 1 mr. (gaussian dist.) about each of the three axes.

## RANGE DERIVATIVE PROFILES

In terms of more conventional variables the range rate and the next two time derivatives can be written:

$$\dot{r} = \hat{r} \cdot \vec{v}$$

$$\ddot{r} = a_r + v_\perp^2 / r$$

$$\dddot{r} = j_r + 3/r \left( v_\perp a_\perp - (v_r / r) v_\perp^2 \right)$$

with the following definitions:

$\vec{r} = \vec{r}_{pass} - \vec{r}_{act}$

$\vec{v} = d/dt\ (\vec{r})$

$\vec{a} = d/dt\ (\vec{v})$

$\vec{j} = d/dt\ (\vec{a})$

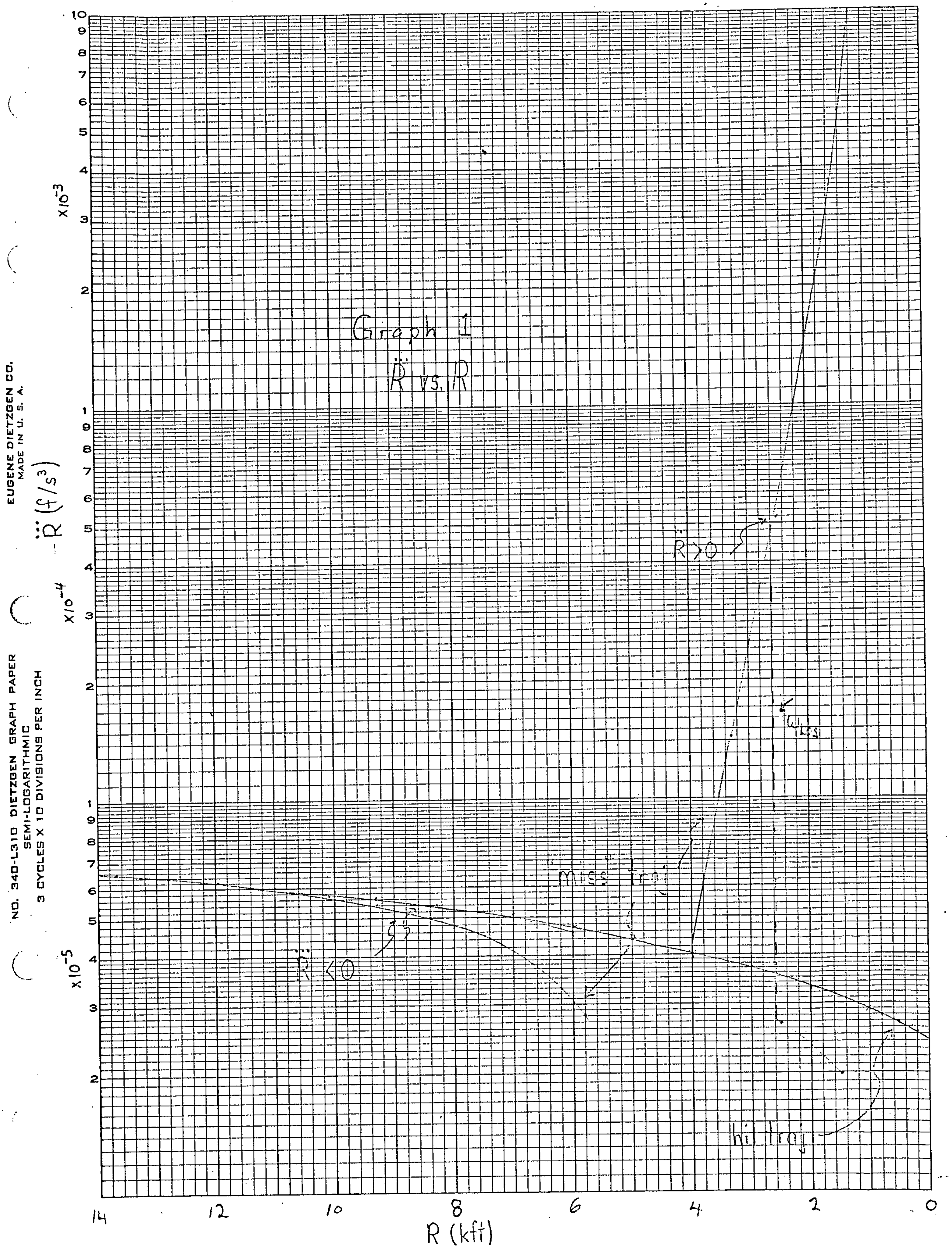
$\wedge$ = denotes unit

$r$ along $\vec{r}$

$\perp$ normal to $\vec{r}$

The dependence of $\ddot{r}$ and $\dddot{r}$ on $v_\perp$ is important. If $v_\perp$ is small $\ddot{r}$ and $\dddot{r}$ will be small; if $v_\perp$ is changing then $\ddot{r}$ and $\dddot{r}$ will be changing. If the two vehicles are not on a "hit" trajectory $v_\perp$ will certainly be non-zero and will grow to the full value of the relative velocity as the two vehicles pass each other. If he wishes to rendezvous the astronaut will of course cancel the $v_\perp$ component of his relative velocity with "$\omega_{LOS}$" (line-of-sight rate) burns as he closes with the target vehicle.

Graphs 1, 2 and 3 of $\ddot{r}$ vs. r, $\dddot{r}$ vs. r, and $v_\perp$ vs. r show the behavior of these quantities for a "hit" trajectory and a "miss" trajectory (of about 400 ft). Also shown is the change in $\ddot{r}$, $\dddot{r}$, and $v_\perp$ due to an $\omega_{LOS}$ burn.



Graph 1
$\ddot{R}$ vs. $R$
$\dddot{R}$ (f/s$^3$)
x10$^{-3}$
x10$^{-4}$
x10$^{-5}$
$\ddot{R} > 0$
$\ddot{R} < 0$
$\omega_{LS}$
miss traj
hit traj
R (kft)
14 12 10 8 6 4 2 0
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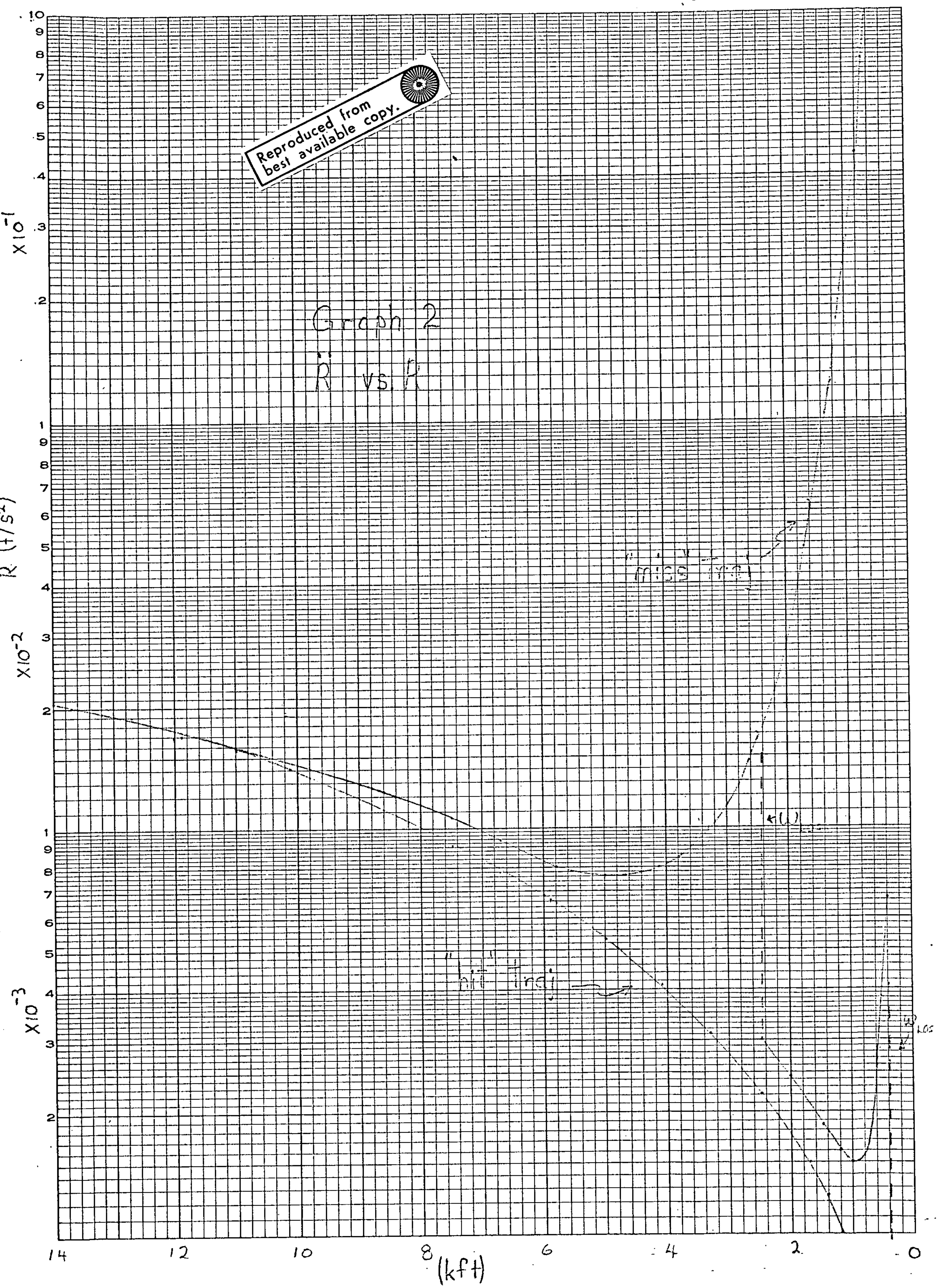
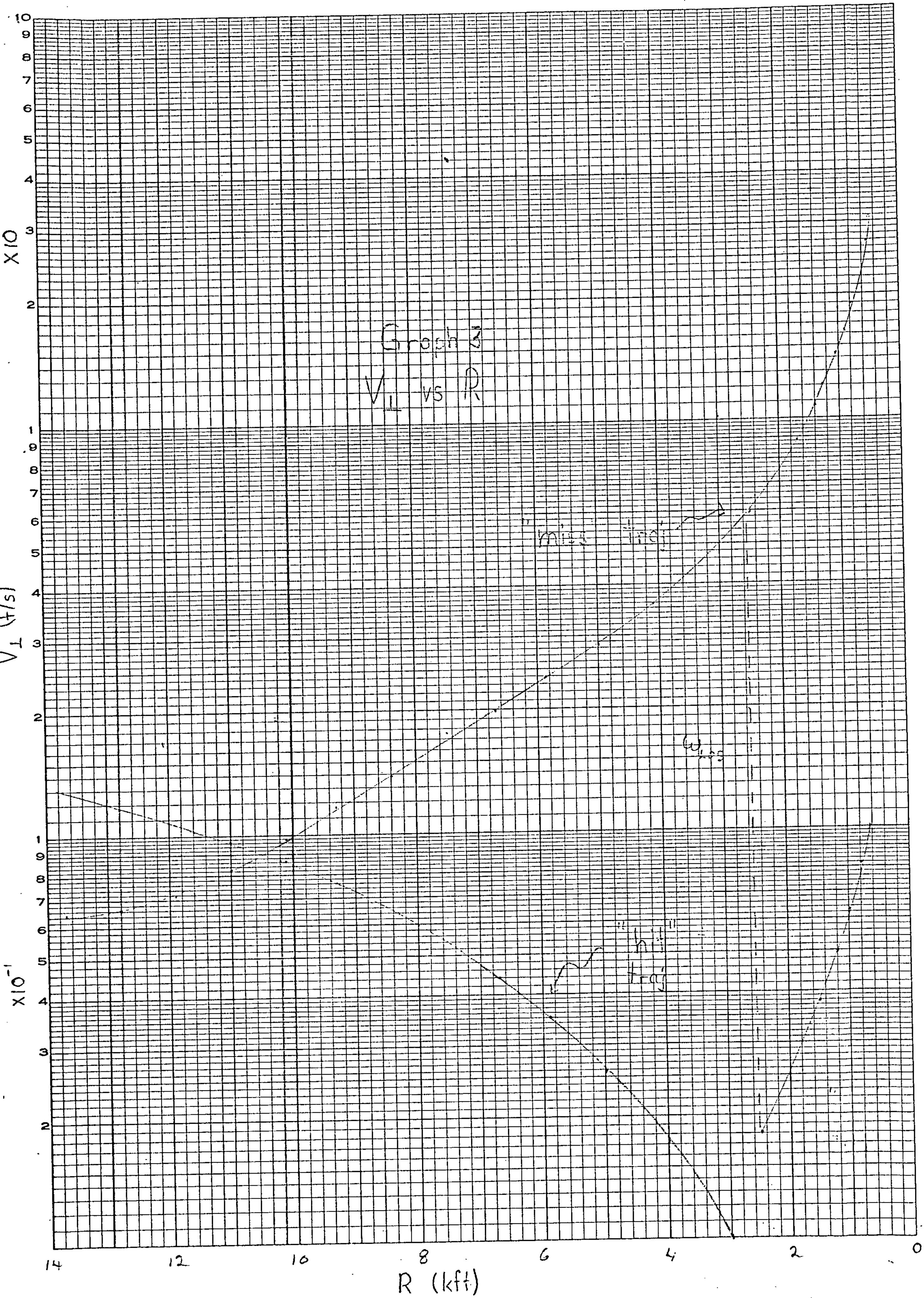

Graph 2
R̈ vs. R
R̈ (f/s²)
X10⁻¹
X10⁻²
X10⁻³
"miss" traj
"hit" traj
ω
ω LOS
14
12
10
8
6
4
2
0
(kft)


Graph 3
V⊥ vs R
"miss traj"
"hit traj"
X10
X10⁻¹
V⊥ (f/s)
R (kft)
14
12
10
8
6
4
2
0

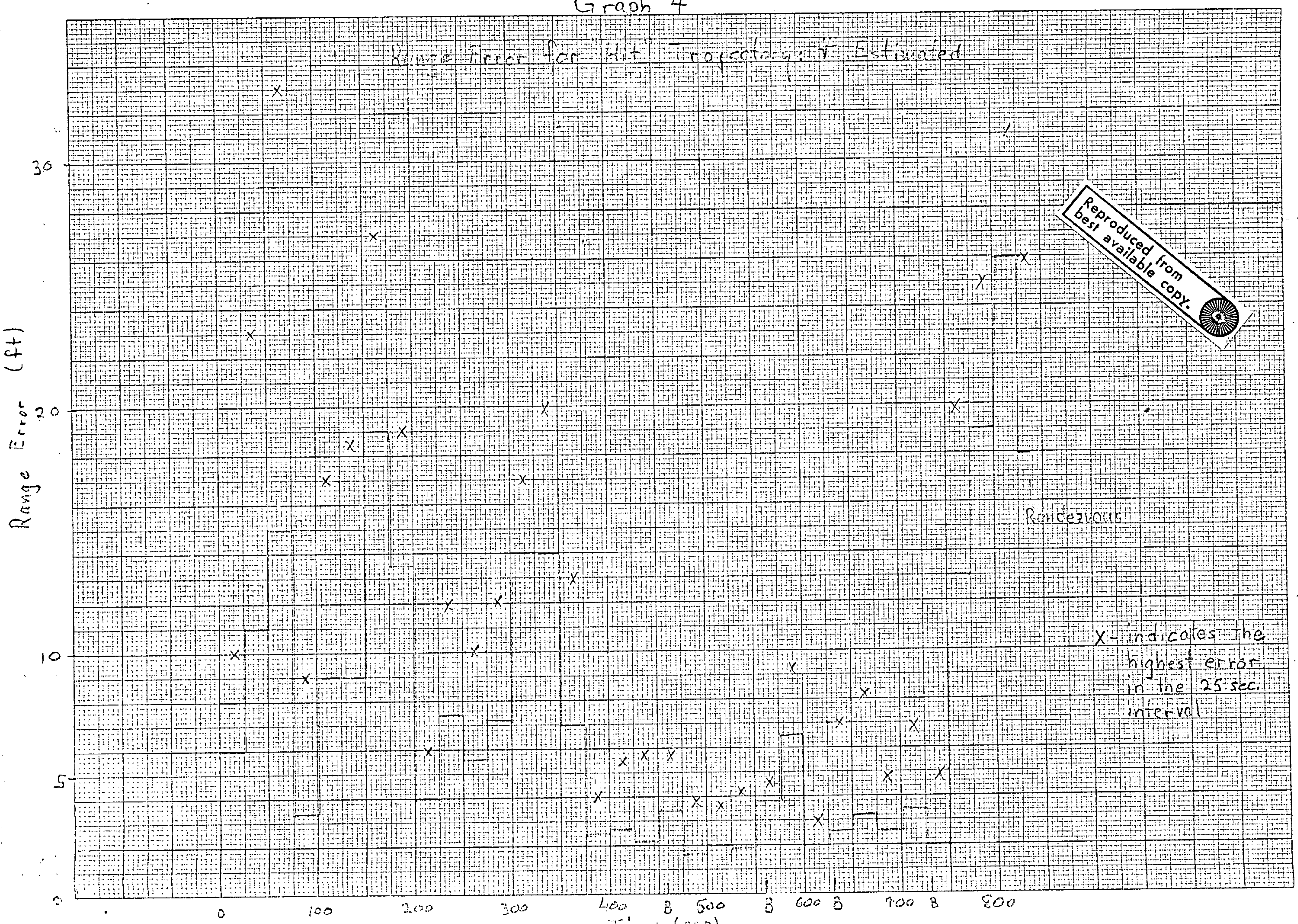
The question that the terminal rendezvous simulation described earlier must answer is whether the polynomial filter can provide accurate range rate information for a trajectory which is not initially on a "hit" but is driven to a rendezvous by correcting $v_\perp$. That the filter will not work unless $v_\perp$ is driven to zero is demonstrated in Appendix 1: "Filter Failure for Miss Trajectories".

## TRIAL SIMULATION RESULTS

Two different trajectories were used for the simulations, one which would have resulted in a hit had not errors in the braking maneuvers caused very slight misses; the other would have resulted in about a 400 ft. miss had no corrective burns been made.

Initially only one run using the hit trajectory was made. In this run "$\dddot{r}$" was estimated and the polynomial state was updated for braking burns. The polynomial filter was initiated 32 mins. after TPI at a range of 22,400 ft. and range rate of -41.8 f/s. Not until after the fourth braking gate was an $\omega_{LOS}$ burn applied. The error in estimated range and range rate is shown in Graphs 4 and 5. Except near the braking gates where the error at each measurement point will be given, only the maximum value and average value for each 25 sec. interval will be shown. Shortly after 1 minute the computed range rate achieves the desired accuracy of 1 f/s. Near the end of the rendezvous the estimate of $\dot{r}$ is beginning to deviate as the error in $\dddot{r}$ is slowly integrating to give an error in $\dot{r}$. By the time of the $\omega_{LOS}$ maneuver, $v_\perp$ has grown to 1.3 f/s and $\dddot{r}$ has grown to $1 \times 10^{-4}$ f/s$^3$. Although the $\omega_{LOS}$ maneuver reduced $\dddot{r}$ to $-4 \times 10^{-6}$ the error in estimating $\dddot{r}$ had risen to $1.6 \times 10^{-4}$ f/s$^3$. The effect being slowly integrated into $\ddot{r}$, $\dot{r}$ and finally r. Before proceeding to the "miss" trajectories it is instructive to consider the behavior of the third derivative and its estimate. Graph 6 shows the actual and estimated values for $\dddot{r}$. The "expected" deviations from the covariance matrix are also shown as bars centered on the estimated value of $\dddot{r}$. After about 3 minutes of marking the estimate begins to converge to the actual value of $\dddot{r}$ and the corresponding covariance matrix element shrinks rapidly so that after another 3 minutes the filter has converged to what it predicts the (constant) value of $\dddot{r}$ to be: $-6 \times 10^{-5}$ f/s$^3$. Unfortunately $\dddot{r}$ is not constant and the filter is unable to follow any further changes in $\dddot{r}$ (the gain having become so low). The same kind of behavior is expected if the polynomial filter is truncated at $\ddot{r}$; in fact, looking ahead to Graph 17 we see the filter converge faster but then deviate by more than 1 f/s before the first braking gate.

# Graph 4


Range Error for "Hit" Trajectory: V Estimated
Range Error (ft)
30
20
10
5
0
0
100
200
300
400
B
500
B
600
B
700
B
800
Rendezvous
X - indicates the highest error in the 25 sec. interval

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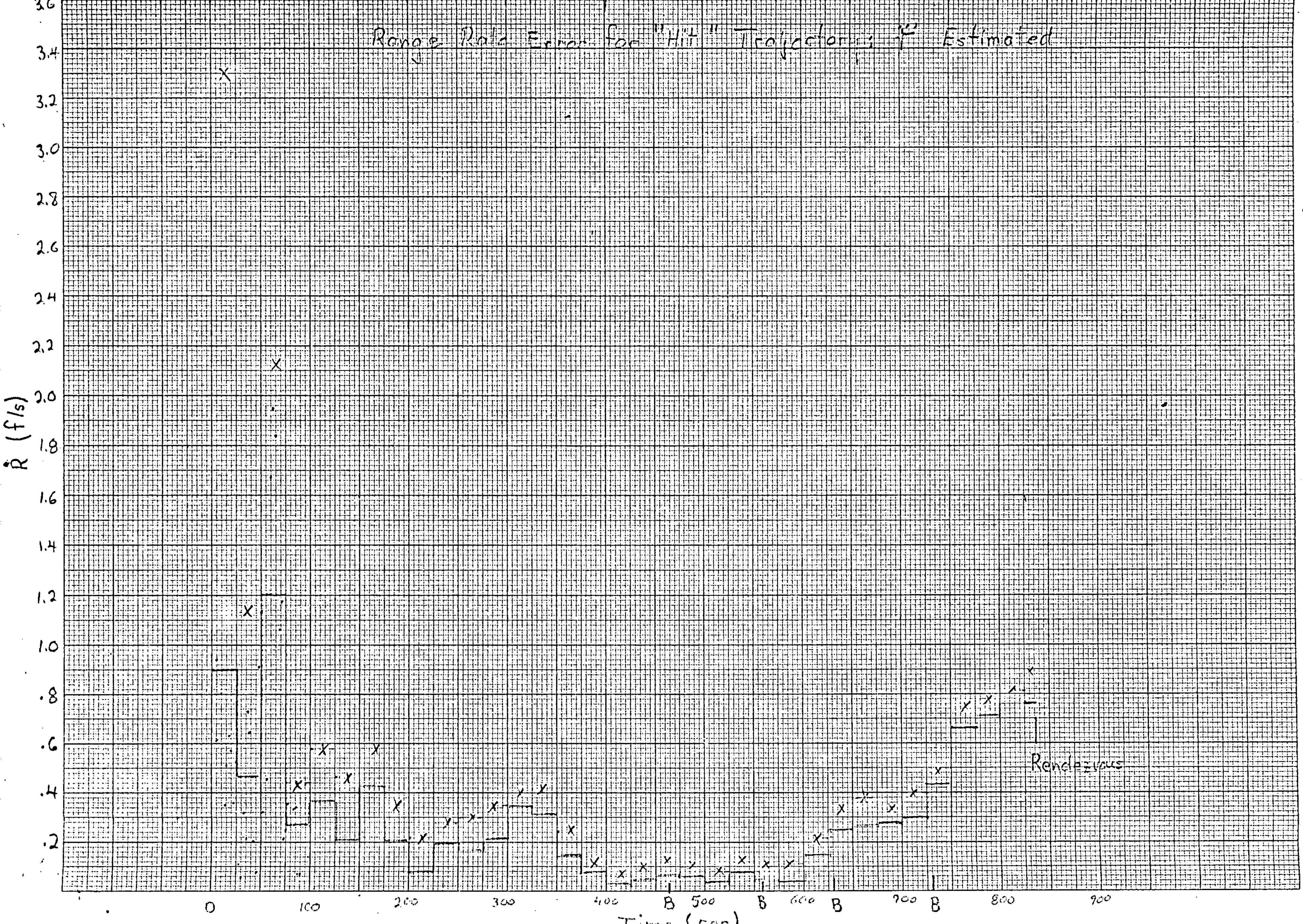
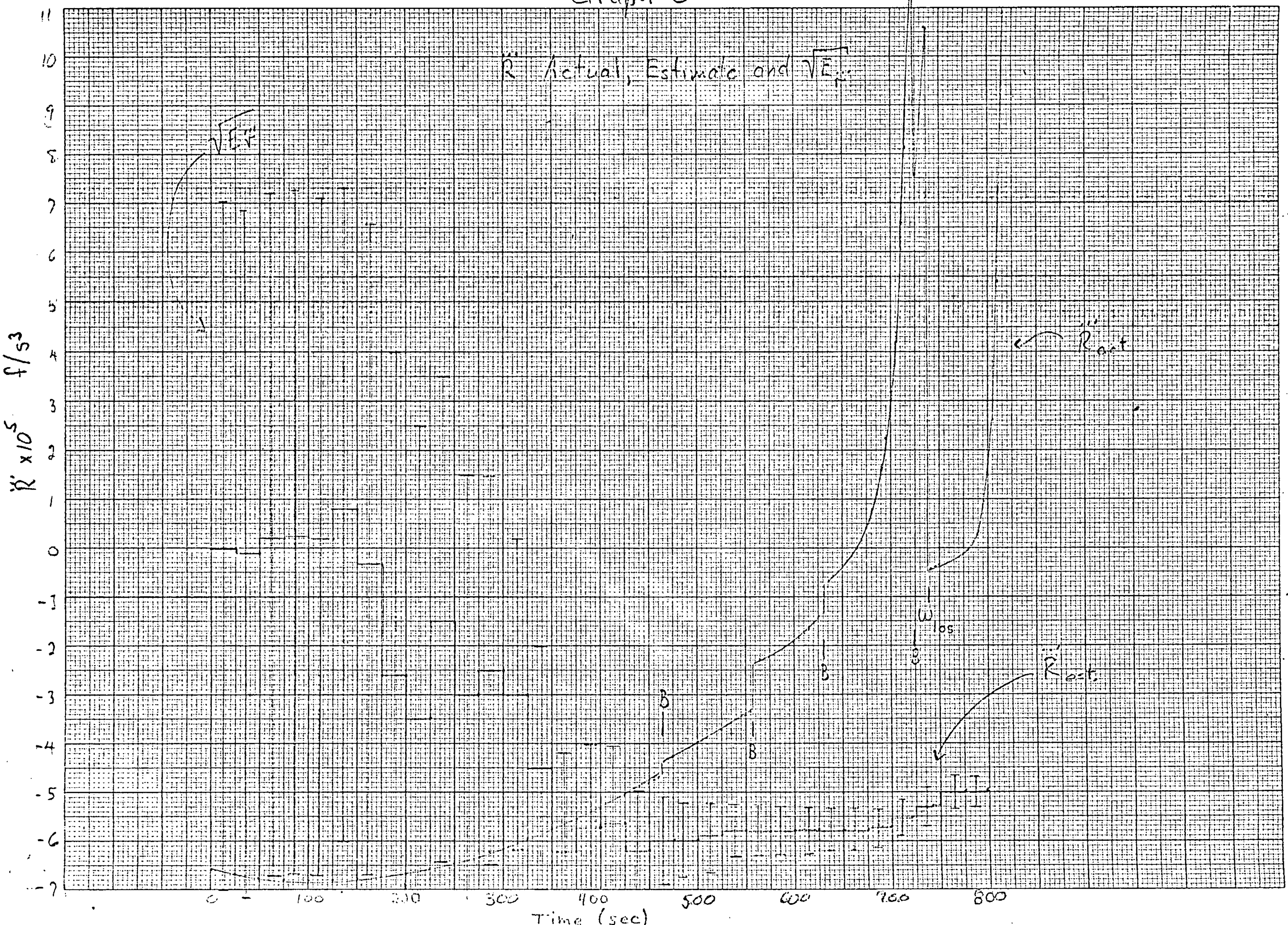
Graph 5



Graph 6


$\ddot{R}$ Actual, Estimate and $\sqrt{E_{\ddot{r}}}$
$\sqrt{E_{\ddot{r}}}$
$\ddot{R}_{act}$
$\ddot{R}_{est}$
$\omega_{los}$
B
B
B
$\ddot{R} \times 10^5$ f/s³
Time (sec)
0
100
200
300
400
500
600
700
800
11
10
9
8
7
6
5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-7

Returning to the behavior of $\dddot{r}$ in Graph 6 we might wonder whether, if the astronaut held $\omega_{LOS}$ down, a constant estimate of zero for $\dddot{r}$ might not be more effective. That is, if $\dddot{r}$ is close enough to zero a filter truncated after $\ddot{r}$ might be better than a filter truncated after $\dddot{r}$.

Before continuing note that the state was updated with the value of the burn in the previous runs. A run was made in which no account was taken of the braking burns in the filter. The estimate of $\dot{r}$ was unable to follow the steps in the actual $\dot{r}$ so that the "astronaut" overestimated his closing rate, did too much braking and "stalled out". The details of the run are shown in Table 1.

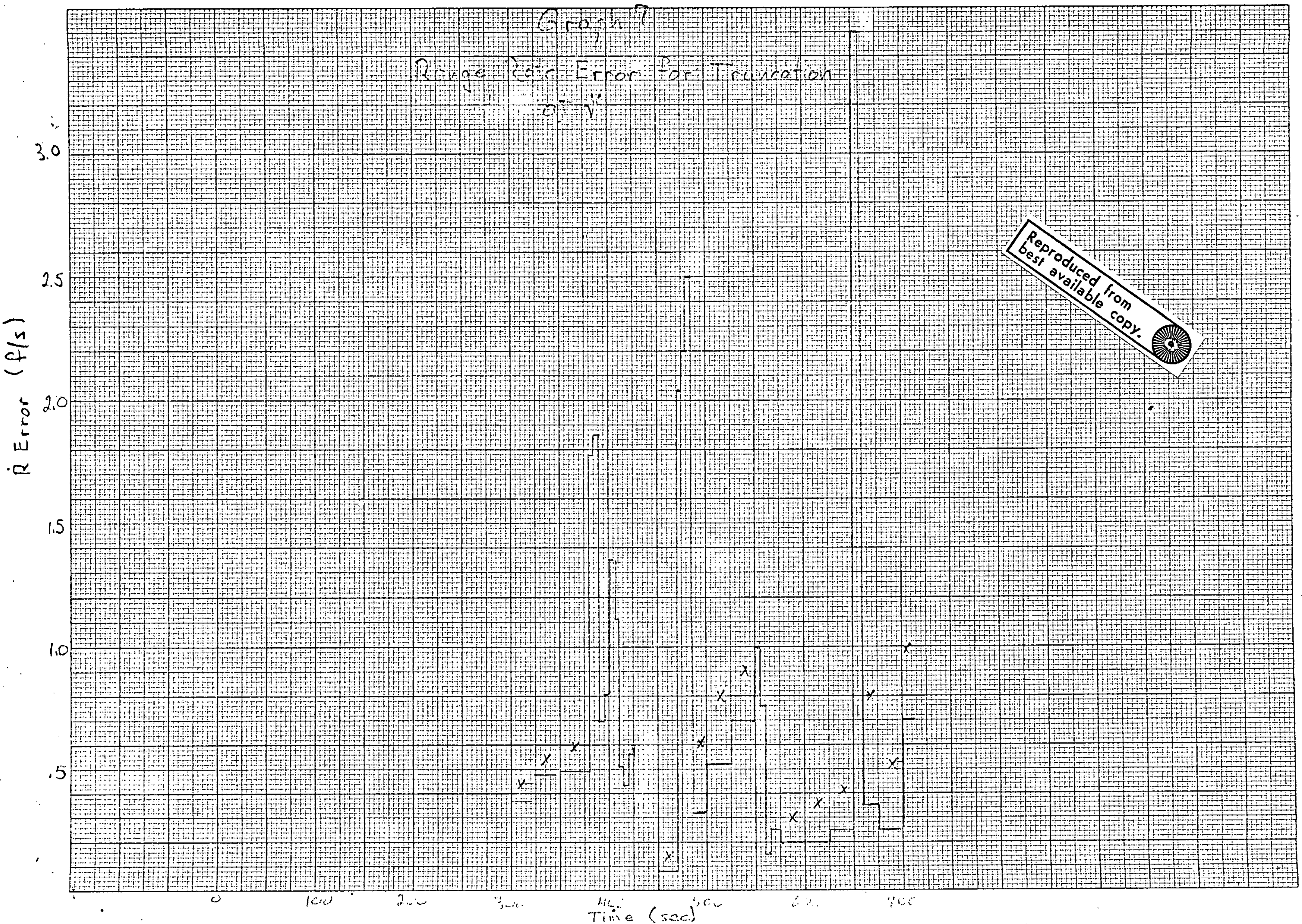
TABLE 1

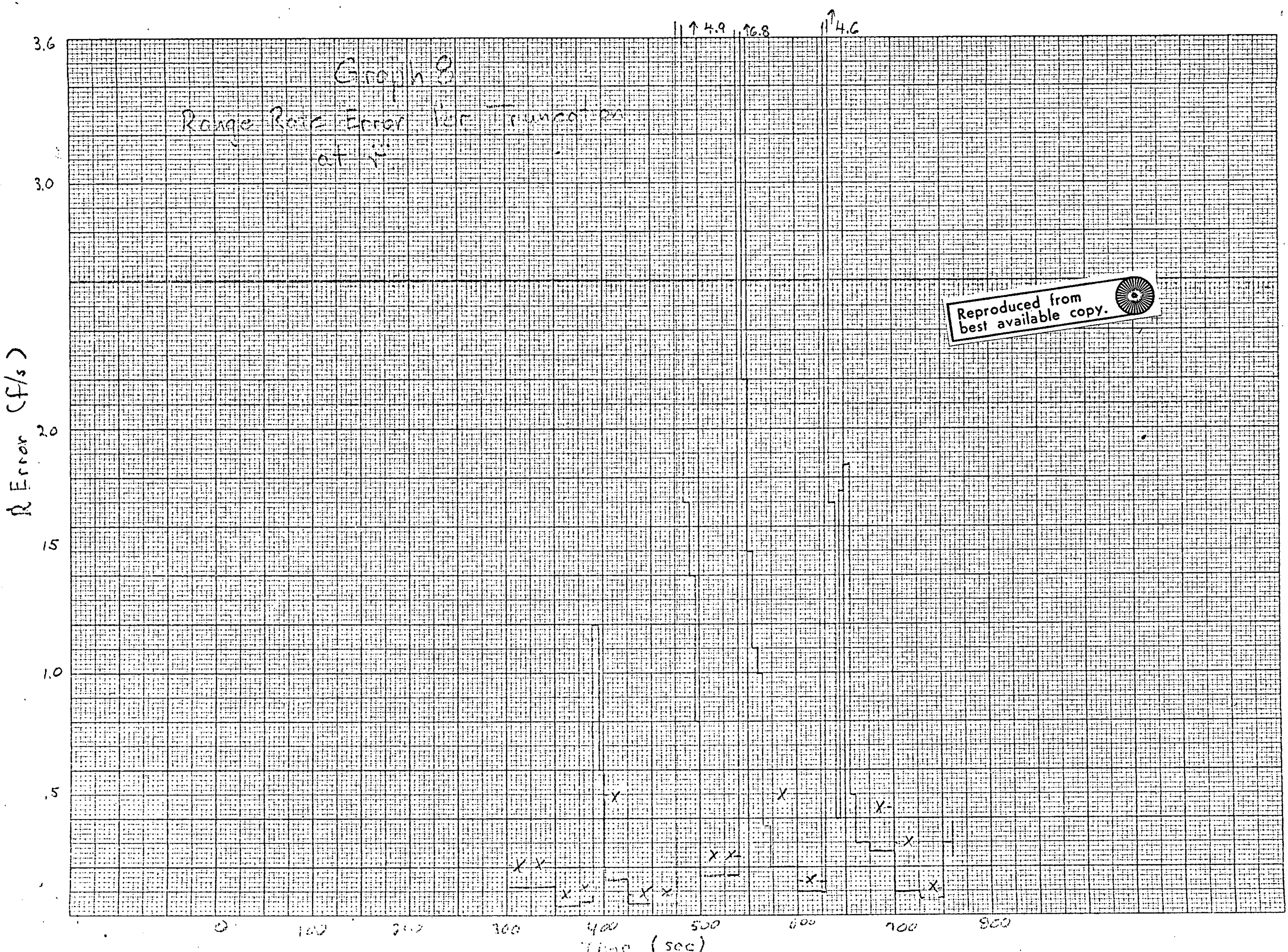
| $R_{est}$ | $\dot{R}_{est}$ | $R_{act}$ | $\dot{R}_{act}$ (bef) | $\dot{R}_{act}$ (aft) |
|---|---|---|---|---|
| 5862 | -33.1 | 5851 | -33.6 | -30.5 |
| 4407 | -32.0 | 4464 | -30.2 | -28.1 |
| 2946 | -30.0 | 3061 | -27.7 | -17.6 |
| 1879 | -27.0 | 2262 | -17.3 | -10.3 |
| 1130 | -22.0 | 1763 | -10.1 | + 1.8 |
| | | | | (positive $\dot{r}$) |

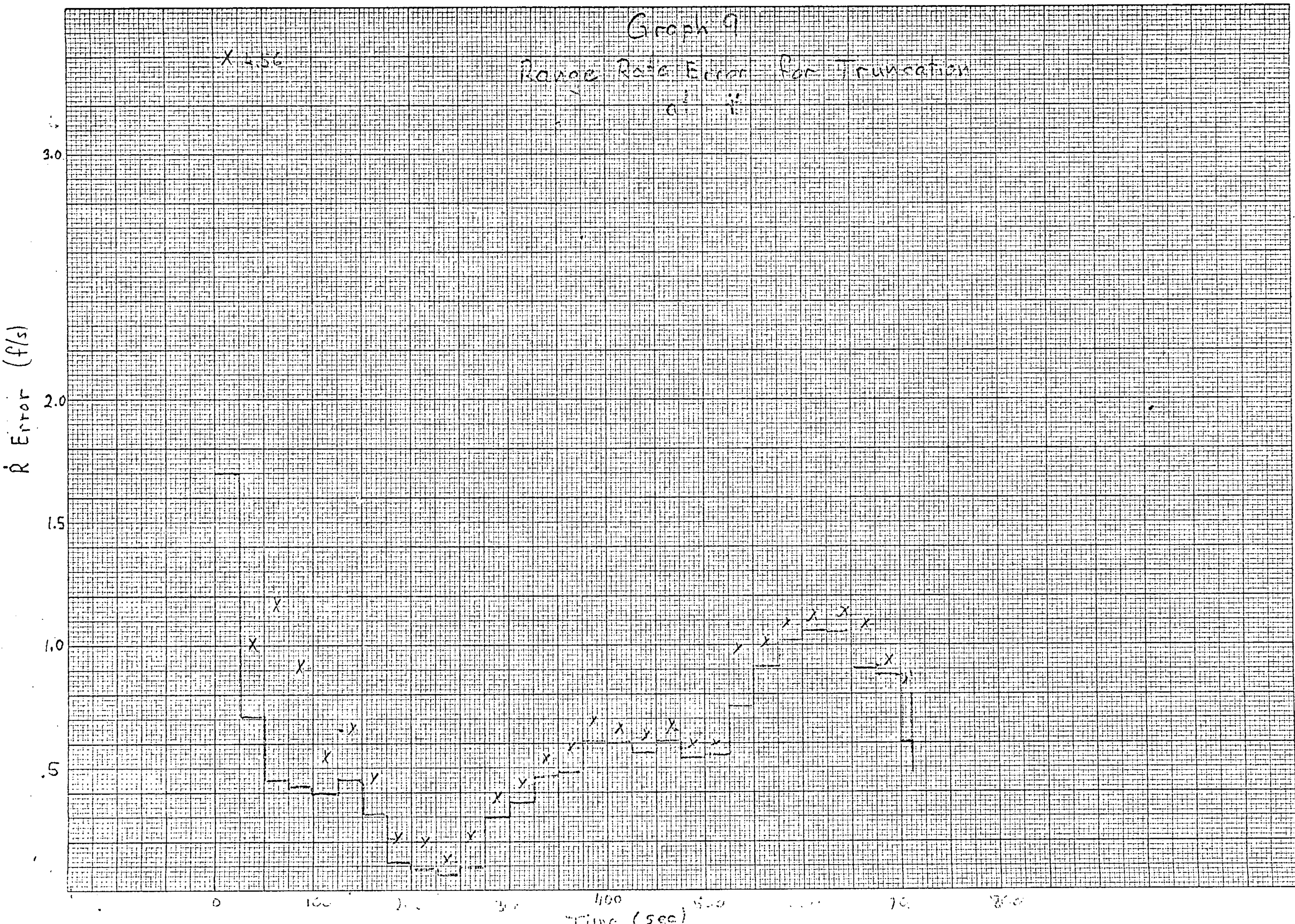
Braking Burns for "No-Update" Run

Obviously the filter must be "notified" in some way of the braking burns.

Continuing, we look at simulations made with the "miss" trajectory. In these simulations the $\dot{r}$ element of the filter covariance matrix, $E_{\dot{r}}$, was degraded by $(\Delta v)^2$ rather than the state being updated by $\Delta v$. Graphs 7 and 8 show the errors in $\dot{r}$ for polynomial filters truncated after $\ddot{r}$ and $\dddot{r}$ respectively. Including the third derivative does not greatly improve the estimation of $\dot{r}$. To eliminate the large transient error which occurs at a braking maneuver the state ($\dot{r}$) was "updated" by the amount, $|\vec{\Delta v}|$, of the burn in the next two simulations. Graphs 9 and 10 show the error in $\dot{r}$ for the same two filters as before (est. $\ddot{r}$, est. $\dddot{r}$). Near the rendezvous time ($\sim$550 sec. or $\sim$1300 ft.) the estimate of $\dot{r}$ for the $\ddot{r}$ filter is in error by about 1.1 f/s. The "gain" in the various filters at this time is a clue to the reason for the large error in $\dot{r}$. Table 2 below shows $\sqrt{E_{\dot{r}}}$ and $\sqrt{E_{\ddot{r}}}$ (corresponding to $\dot{r}$ and $\ddot{r}$) for the four simulations at the time in question (550 - 700 sec).


Graph 7
Range Rate Error for Truncation
3.0
2.5
2.0
1.5
1.0
.5
Ṙ Error (f/s)
0
100
200
300
400
500
600
700
Time (sec)


Graph 8
Range Rate Error
at
↑4.9
↑6.8
↑4.6
3.6
3.0
2.0
15
1.0
.5
0
100
200
300
400
500
600
700
800
(sec)
Error (f/s)


Graph 9
Range Rate Error for Truncation
X 4.56
3.0
2.0
1.5
1.0
.5
Ṙ Error (f/s)
0
100
400
Time (sec)

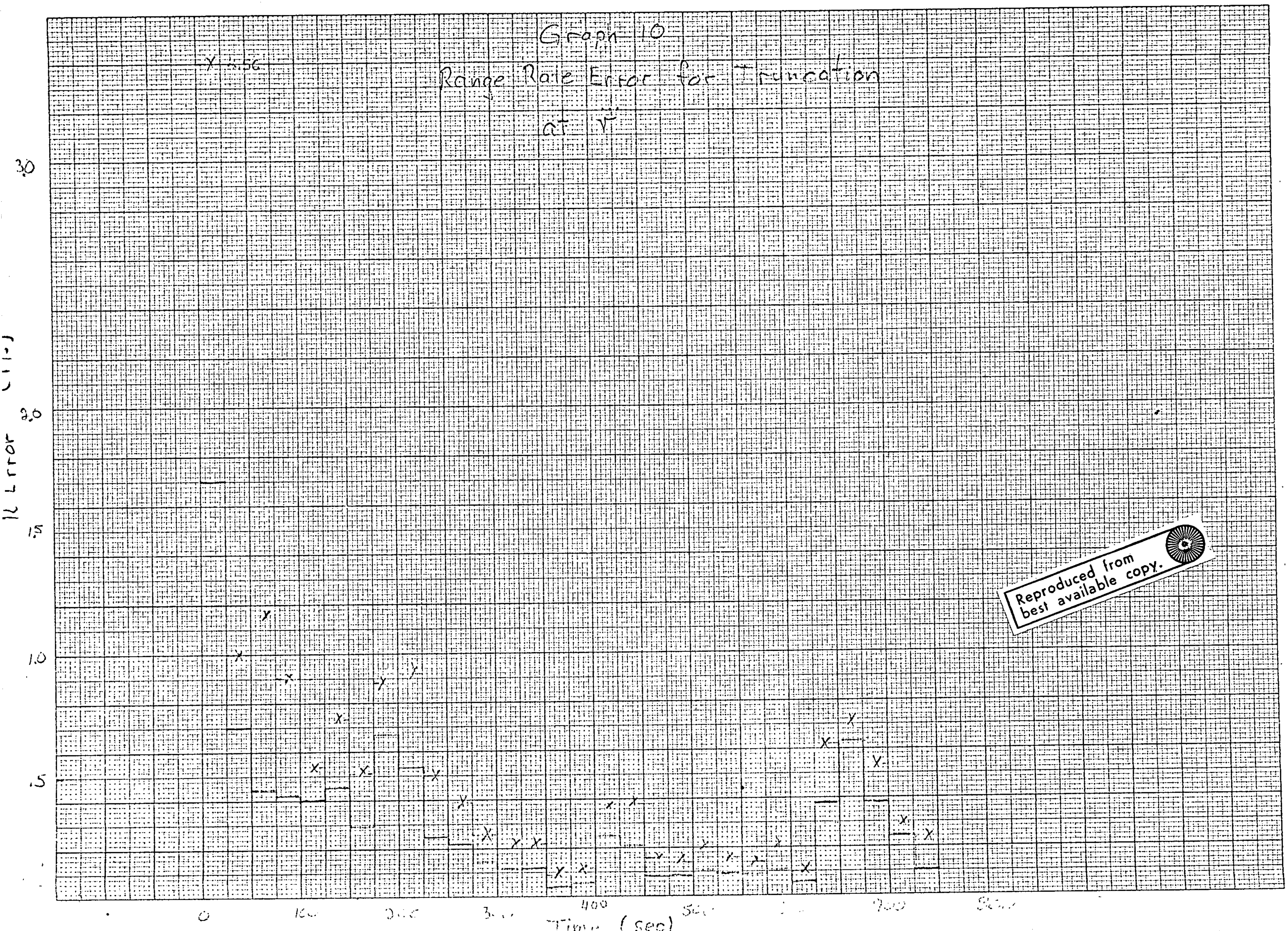

Graph 10
Range Rate Error for Truncation
at $\dot{r}$
.56
30
20
15
10
.5
0
400
Time (sec)

TABLE 2

| | $\sqrt{E_{\dot{r}}}$ | $\sqrt{E_{\ddot{r}}}$ |
|---|---|---|
| $\dddot{r}$ w deg | .33 - 39* | .0067 - .0089 |
| $\dddot{r}$ w upd | .21 - 20 | .0074 - .0020 |
| $\ddot{r}$ w deg | .66 - .39 f/s* | .00059 - .00059 |
| $\ddot{r}$ w upd | .17 - .15 f/s | .00044 - .00043 |

*Exclusive of short transient at braking phase

Covariance Matrix Values

The "gain" added by degrading the $\dot{r}$ element of the filter covariance matrix helps overcome the error in the estimate due to truncation of the polynomial series.

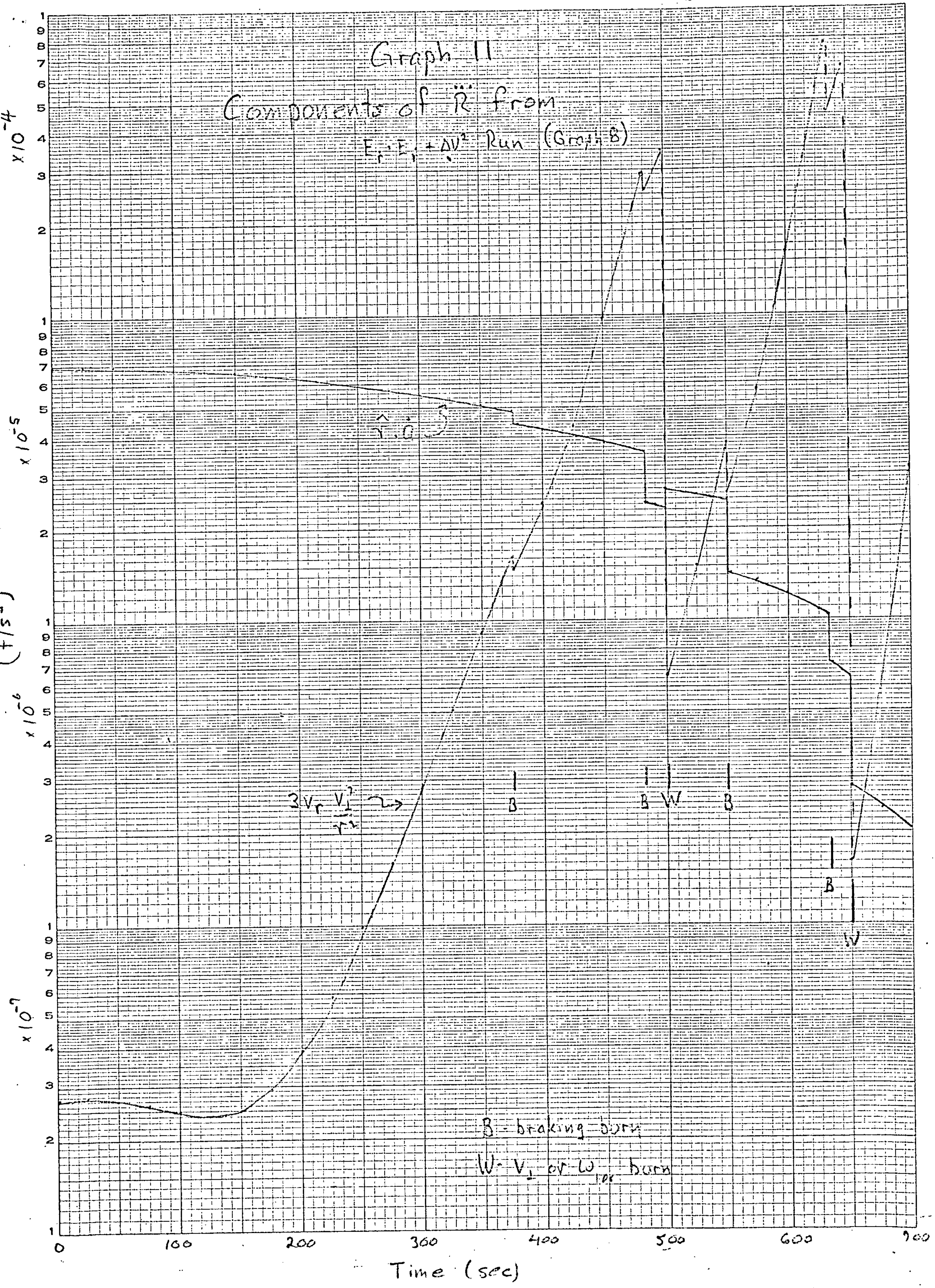
Realizing this a number of other ways of accounting for $\Delta v$ maneuvers were tried with the idea of eliminating the transient error in $\dot{r}$ when the state was not updated and providing additional gain to compensate for the truncation of the series at $\ddot{r}$. Various combinations of:

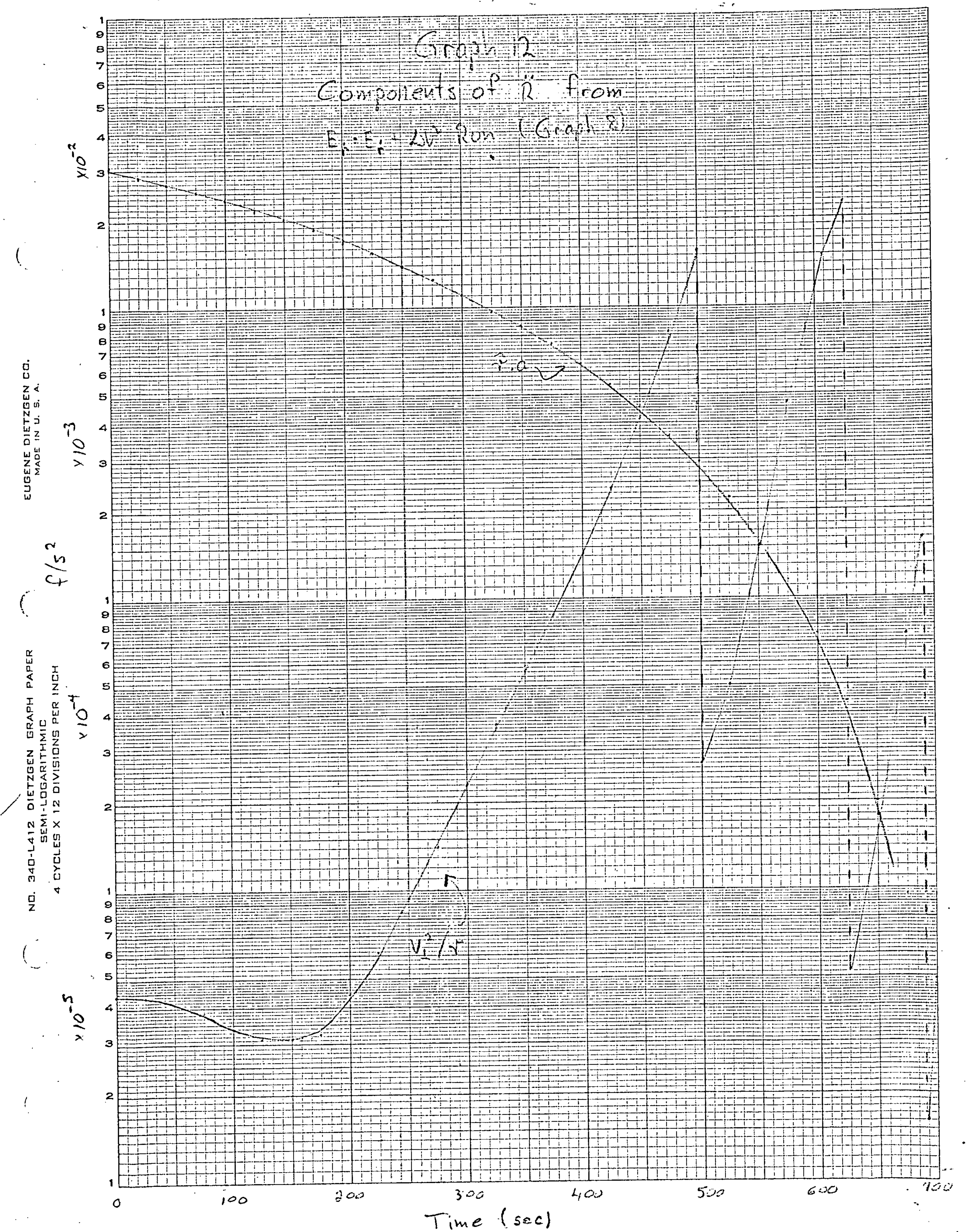
1. update state for brake
2. degrade $E_{\dot{r}}$ for brake
3. update state for $\omega_{LOS}$
4. degrade $E_{\ddot{r}}$ for $\omega_{LOS}$

were used in several simulations as shown in Table 3. Some of these trials are worthy of additional comment. Recalling that the estimate of $\dddot{r}$ was not able to follow the rapidly changing value of $\dddot{r}$ we should expect the same to be true of $\ddot{r}$. Graphs 11 and 12 show the behavior of the components of $\dddot{r}$ and $\ddot{r}$. The component of $\ddot{r}$, $\hat{r} \cdot \vec{a}$, goes rapidly to zero as rendezvous nears leaving only $v_{\perp}^2/r$ which must be zeroed by the astronaut. In run A5 updating the state for an $\omega_{LOS}$ burn,

$$\ddot{r}_{est}' = \ddot{r}_{est} - \Delta v_{\perp}^2 / r_{est}$$

simply preserves the large error that existed before the burn. At the first $\omega_{LOS}$ correction these numbers are:


Graph 11
Components of R̈ from
Er·E1+ΔV² Run (Graph 8)
r̂·â
3Vr V⊥²/r²
B
B W
B
B
W
B - braking burn
W - V⊥ or ω burn
x10^-4
x10^-5
x10^-6
x10^-7
(f/s²)
0
100
200
300
400
500
600
700
Time (sec)



Graph 12
Components of $\ddot{R}$ from
$E_i : E_f + \Delta V$ Run (Graph 8)
$\ddot{r}_0$
$V_1^2/r$
$f/s^2$
$\times 10^{-2}$
$\times 10^{-3}$
$\times 10^{-4}$
$\times 10^{-5}$
0
100
200
300
400
500
600
700
Time (sec)
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| before $\Delta v_\perp$ | after $\Delta v_\perp$ |
|---|---|
| $\ddot{r}_{est} = .014$ | $\ddot{r}_{est}' = -.0027$ |
| $\ddot{r}_{act} = .020$ | $\ddot{r}_{act}' = .0030$ |
| $\Delta = -.006$ | $\Delta = -.006$ |

Graph 13 shows the behavior of $\dot{r}$ for this method of updating the filter. The new error in $\ddot{r}$ is not resolved because the gain in $\ddot{r}$ is too small. If, instead of updating $\ddot{r}_{est}$, it is simply set equal to zero at $\omega_{LOS}$ burns, run A6, we have the following values and differences and the behavior of $\dot{r}_{est}$ shown in Graph 14.

| before $\Delta v_\perp$ | after $\Delta v_\perp$ |
|---|---|
| $\ddot{r}_{est} = .014$ | $\ddot{r}_{est}' = 0$ |
| $\ddot{r}_{act} = .020$ | $\ddot{r}_{act} = .003$ |
| $\Delta = -.006$ | $\Delta = -.003$ |

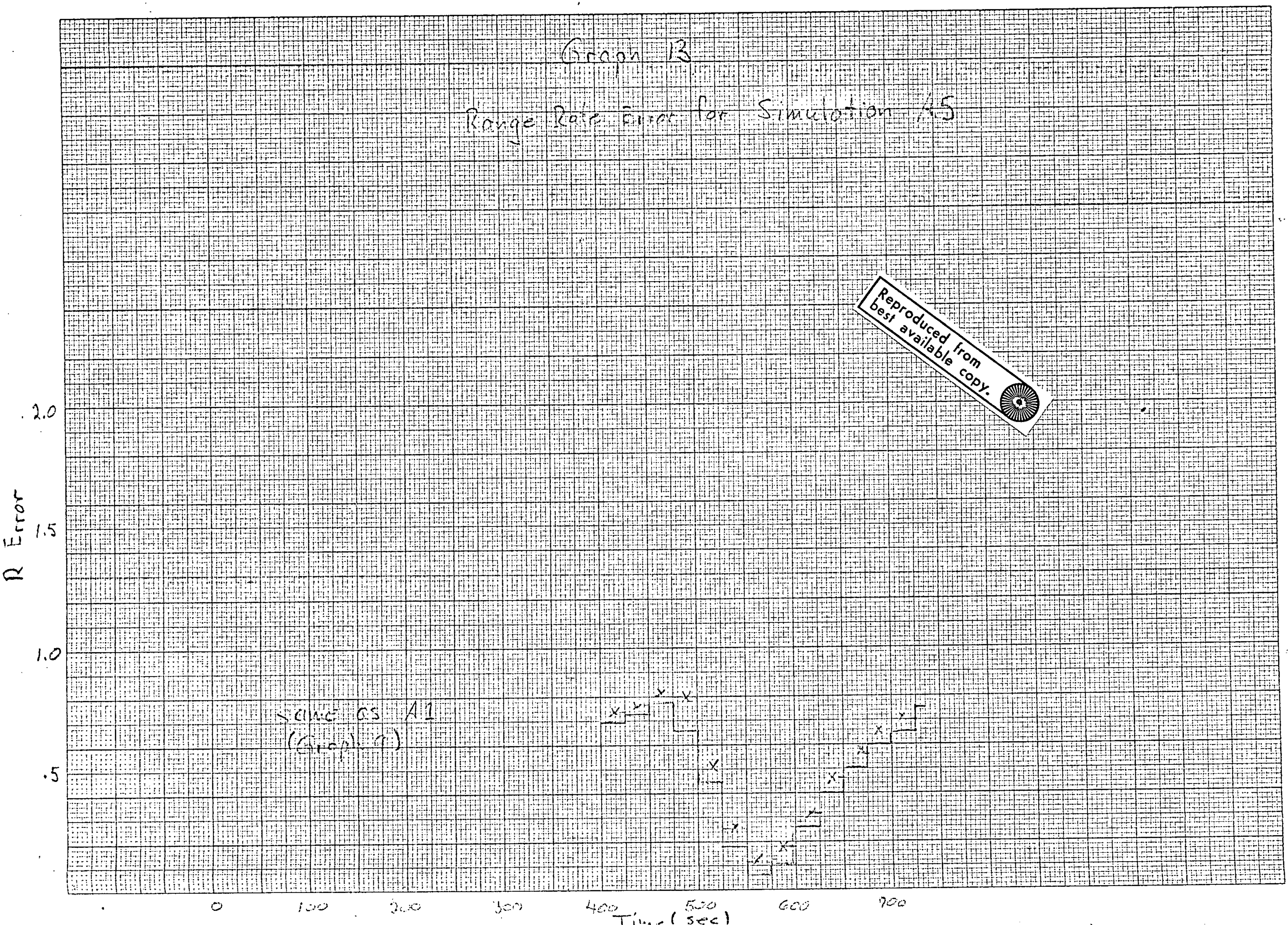
The filter gain in $\ddot{r}$ is still small but that the now smaller $e_{\ddot{r}}$ is not resolved has less effect on the $\dot{r}$ error.

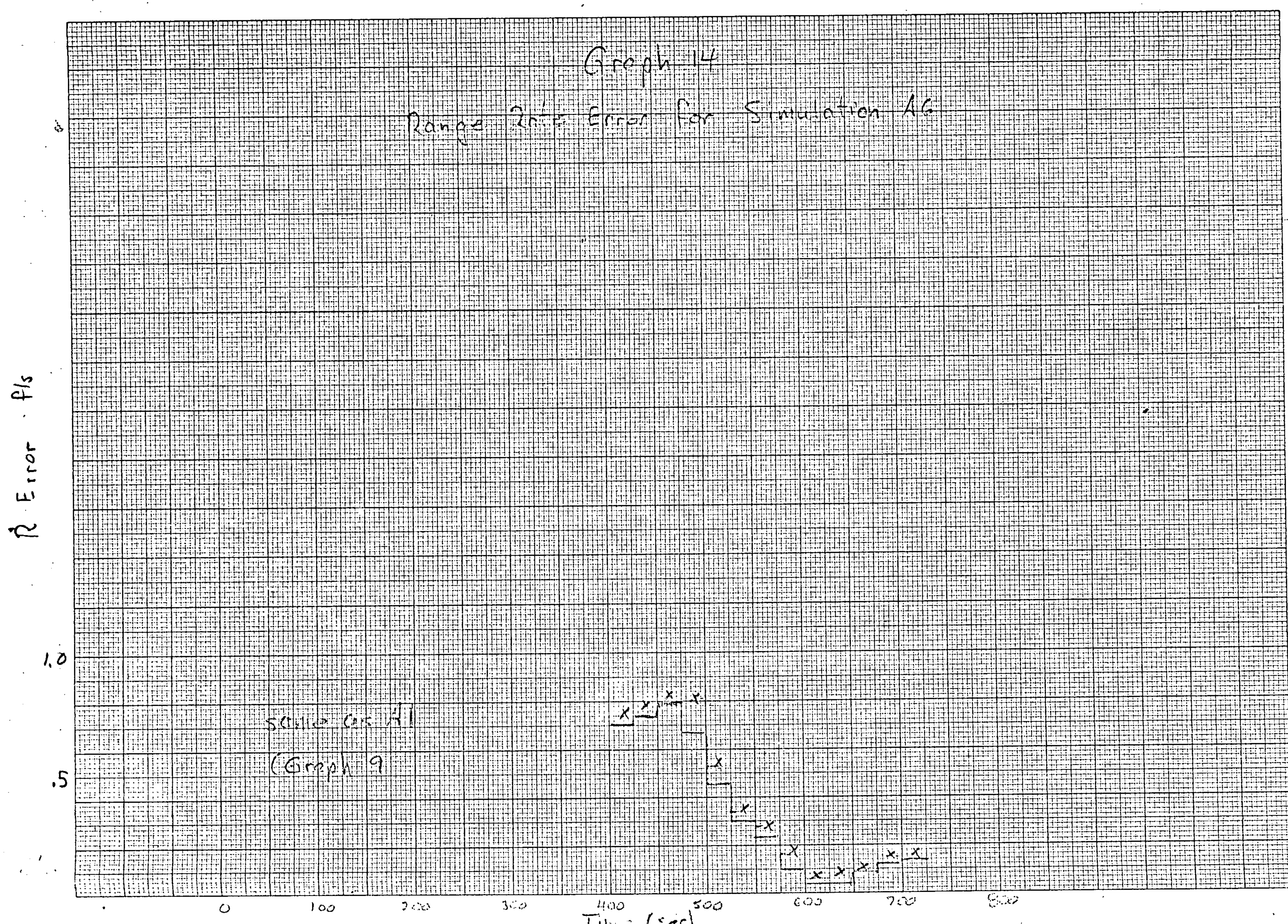
Increasing the gain in $\ddot{r}$ after the first $\omega_{LOS}$ correction should allow the filter to converge to the new value. The value of $\hat{r} \cdot \vec{a}$ is quite small after the rest of the $\omega_{LOS}$ corrections so that reconvergence is not as important. $E_{\ddot{r}}$ was increased in this way for run A7. Results are not significantly different from those of run A6.

In the next two runs B4 and B4a, the $\dot{r}$ element of the filter covariance was degraded by a fixed value at each braking burn; the state was not updated. Results were very good as long as the short "transient" error after each burn is considered tolerable. Graph 15 shows the identical error for these two runs.

The mode of operation used in runs B4 is good because it is logically simple and because it gives accurate estimated $\dot{r}$ values. If chosen as a standard, runs with variations in other parameters can be compared to it. These runs are also listed in Table 3.

As is reasonable the errors in the run are not sensitive to the value of the reinitialized $\dot{r}$ element (runs B4 vs. B4a).


Graph B
Range Rate Error for Simulation A5
Same as A1
(Graph 9)
R Error
2.0
1.5
1.0
.5
0
100
200
300
400
500
600
700
Time (sec)


Graph 14
Range Rate Error for Simulation A6
Ṙ Error ft/s
1.0
.5
same as A1
(Graph 9
0
100
200
300
400
500
600
700
800
Time (sec)

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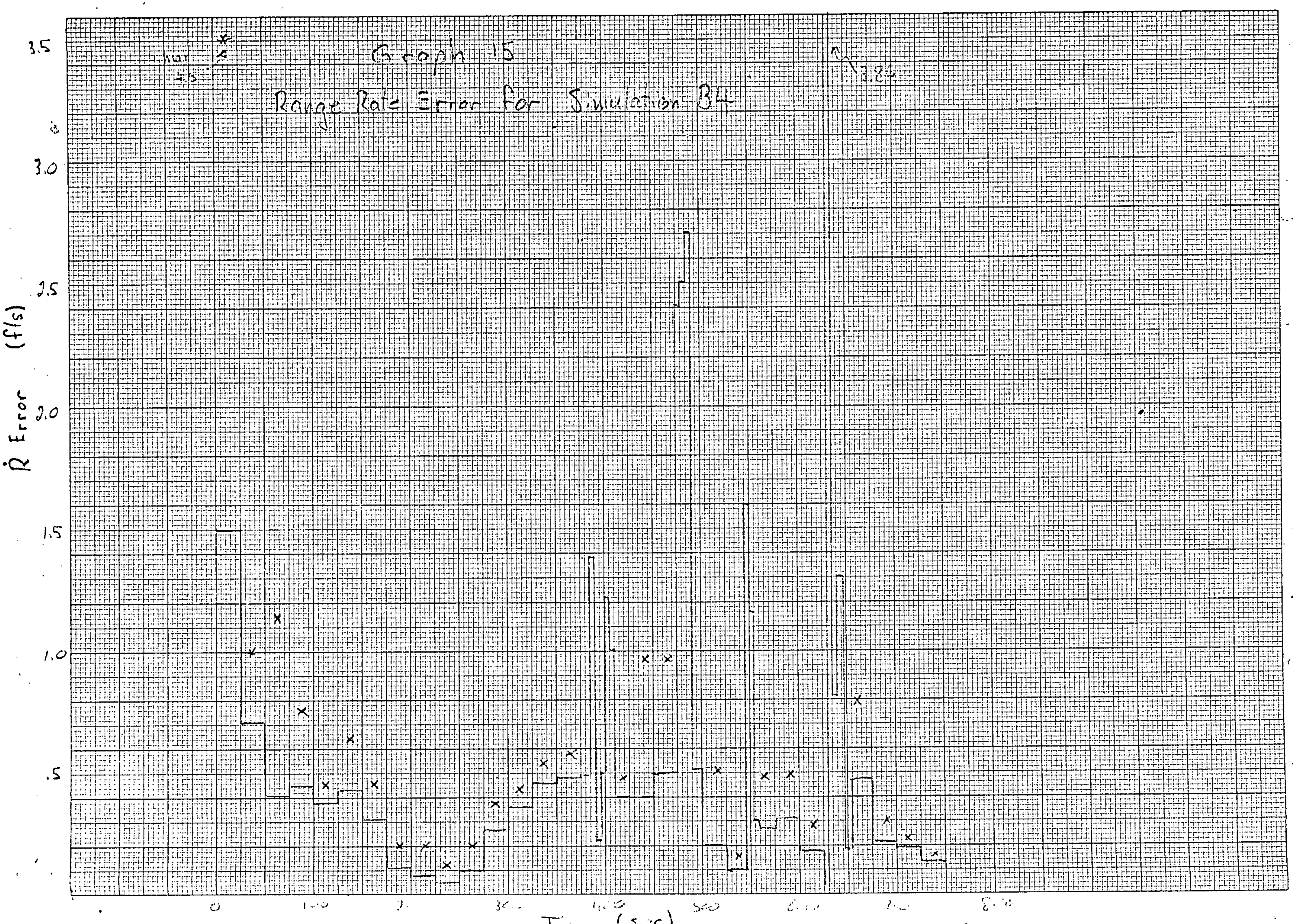


TABLE 3 Trial Simulations

| RUN | DESCRIPTION | RESULTS |
|---|---|---|
| A1 | $\dot{r}_e = \dot{r}_e + \Delta v_b$ | given previously (Graphs 9 and 10) |
| B1 | $E_{\dot{r}} = E_{\dot{r}} + (\Delta v_b)^2$ | given previously (Graphs 7 and 8) |
| A2 | $\dot{r}_e = \dot{r}_e + \Delta v_b$<br>$E_{\ddot{r}} = E_{\ddot{r}} + (\Delta v_{\perp}^2/r)^2$ | slightly worse than A1; too much gain in $\ddot{r}$ |
| B2 | $E_{\dot{r}} = E_{\dot{r}} + (\Delta v_b)^2$<br>$E_{\ddot{r}} = E_{\ddot{r}} + (\Delta v_{\perp}^2/r)^2$ | same performance as B1 |
| A3 | $\dot{r}_e = \dot{r}_e + \Delta v_b$<br>$E_{\dot{r}} = E_{\dot{r}} + (\Delta v_b)^2$ | large error transient as in B1; not enough gain for changing $\ddot{r}$ |
| A4 | $\dot{r}_e = \dot{r}_e + \Delta v_b$<br>$E_{\dot{r}} = E_{\dot{r}} + (\Delta v_b)^2$<br>$E_{\ddot{r}} = E_{\ddot{r}} + (\Delta v_{\perp}^2/r)^2$ | same error transient as A3, additional gain lets estimated $\ddot{r}$ follow true $\ddot{r}$ before error integrates into $\dot{r}$ |

TABLE 3 Trial Simulations (Cont.)

| RUN | DESCRIPTION | RESULTS |
|---|---|---|
| A5 | $\dot{r}_e = \dot{r}_e + \Delta v_b$<br>$\ddot{r}_e = \ddot{r}_e - \Delta v_{\perp}^2 / r$ | not enough gain in $\ddot{r}$ to allow estimate to follow true $\ddot{r}$ (Graph 13) |
| A6 | $\dot{r}_e = \dot{r}_e + \Delta v_b$<br>$\ddot{r}_e = 0$ at $\Delta v$ | good $\dot{r}$ estimate near rendezvous, not so good after first brake (Graph 14) |
| A7 | $\dot{r}_e = \dot{r}_e + \Delta v_b$<br>$\ddot{r}_e = 0$ at $\Delta v_{\perp}$<br>$E_{\dot{r}} = 10^{-6}$ at $\Delta v_{\perp}$ | same as above |
| B3 | $E_{\dot{r}} = E_{\dot{r}} + (10)^2$<br>$\ddot{r}_e = 0$ at $\Delta v_{\perp}$<br>$E_{\ddot{r}} = 1/4 \times 10^{-6}$ | good results $e_{\dot{r}} < 1$ f/s after 1 min from initiation<br>$< 1$ f/s 30 sec. after brake burns |
| B4 | $E_{\dot{r}} = E_{\dot{r}} + (10)^2$<br>$\ddot{r}_e = 0$ at $\Delta v_{\perp}$ | same as B3, degrading $E_{\dot{r}}$ maintains adequate gain in $\ddot{r}$ element so that degrading it was unnecessary (Graph 15) |
| B4a | $E_{\dot{r}} = E_{\dot{r}} + (7)^2$<br>$\ddot{r}_e = 0$ at $\Delta v_{\perp}$ | same as above (Graph 15) |

TABLE 3 Trial Simulations (Cont.)

| RUN | DESCRIPTION | RESULTS |
|---|---|---|
| B4b | same as B4 no range cell sampling | degraded results; (Graph 16) |
| B4c | $E_{\ddot{r}} = 5.8$ initially to correspond to 3 msm'ts initially | larger initial errors, same as B4 after about 1 1/2 to 2 min. |
| B4d | degrade $E_{\ddot{r}}$ by $(1/10\,\dddot{r}\,\Delta t)^2$ at each **extrap.** | in those runs where $\ddot{r}$ gain becomes too low before being boosted by $E_{\ddot{r}}$ degradation this allows $\ddot{r}_e$ to follow true r (Graphs 17, 18 and 19) |
| A1a, b | same as A1 but degrade $E_{\ddot{r}}$ | Excellent results before and during braking makes $\dot{r} \rightarrow 0$ at $v_{\perp}$ unnecessary |
| B1a, b | same as B1 but degrade $E_{\ddot{r}}$ | same as above except for transient at braking burn |

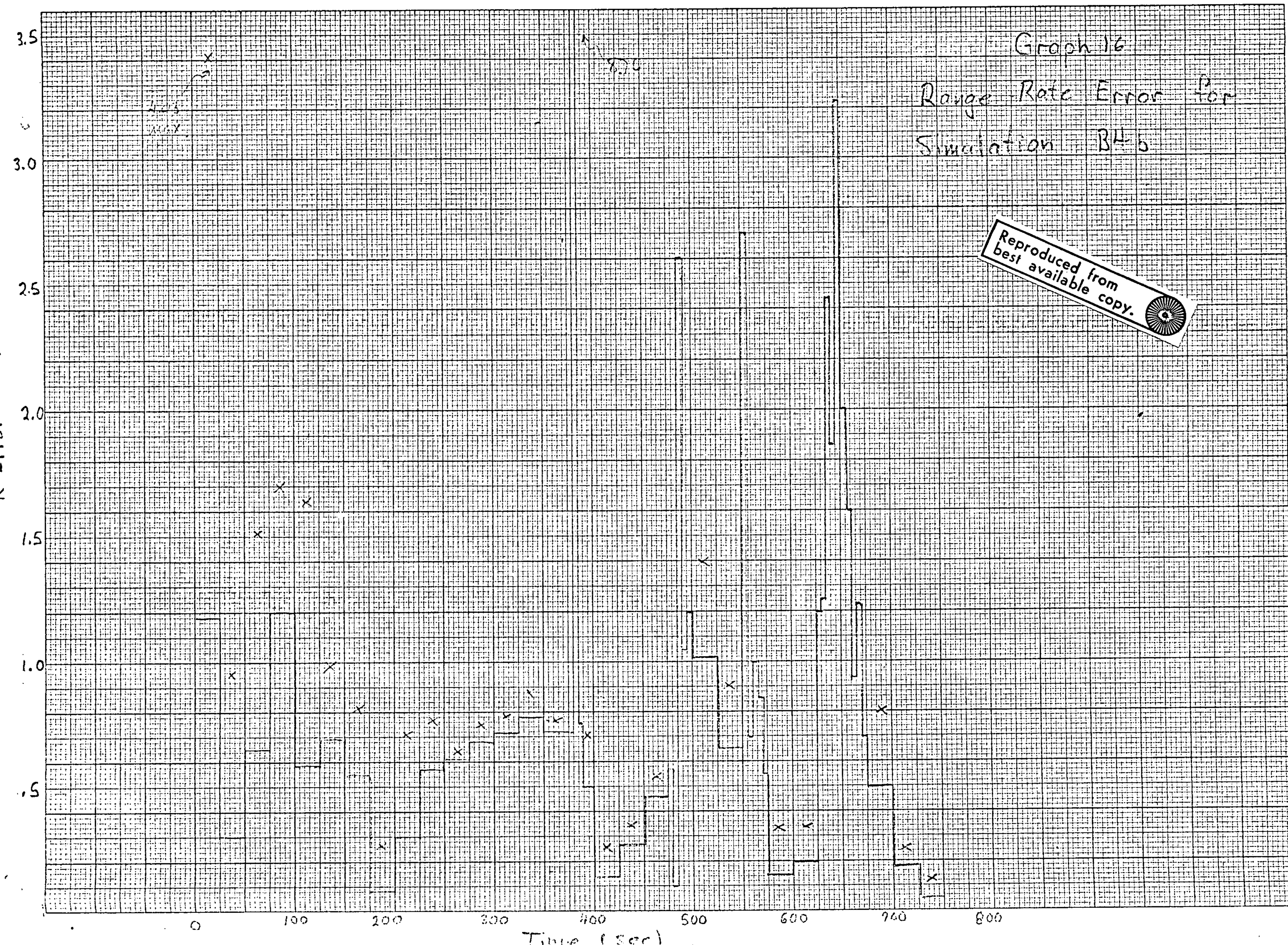
If no range sampling is done results are somewhat degraded after the 20 f/s braking gate. Approximately 40% of the time after this gate the error is greater than 1 f/s, Graph 16. Nonetheless the estimated range rate in each case converged to less than 1 f/s error prior to all braking gates and at rendezvous.

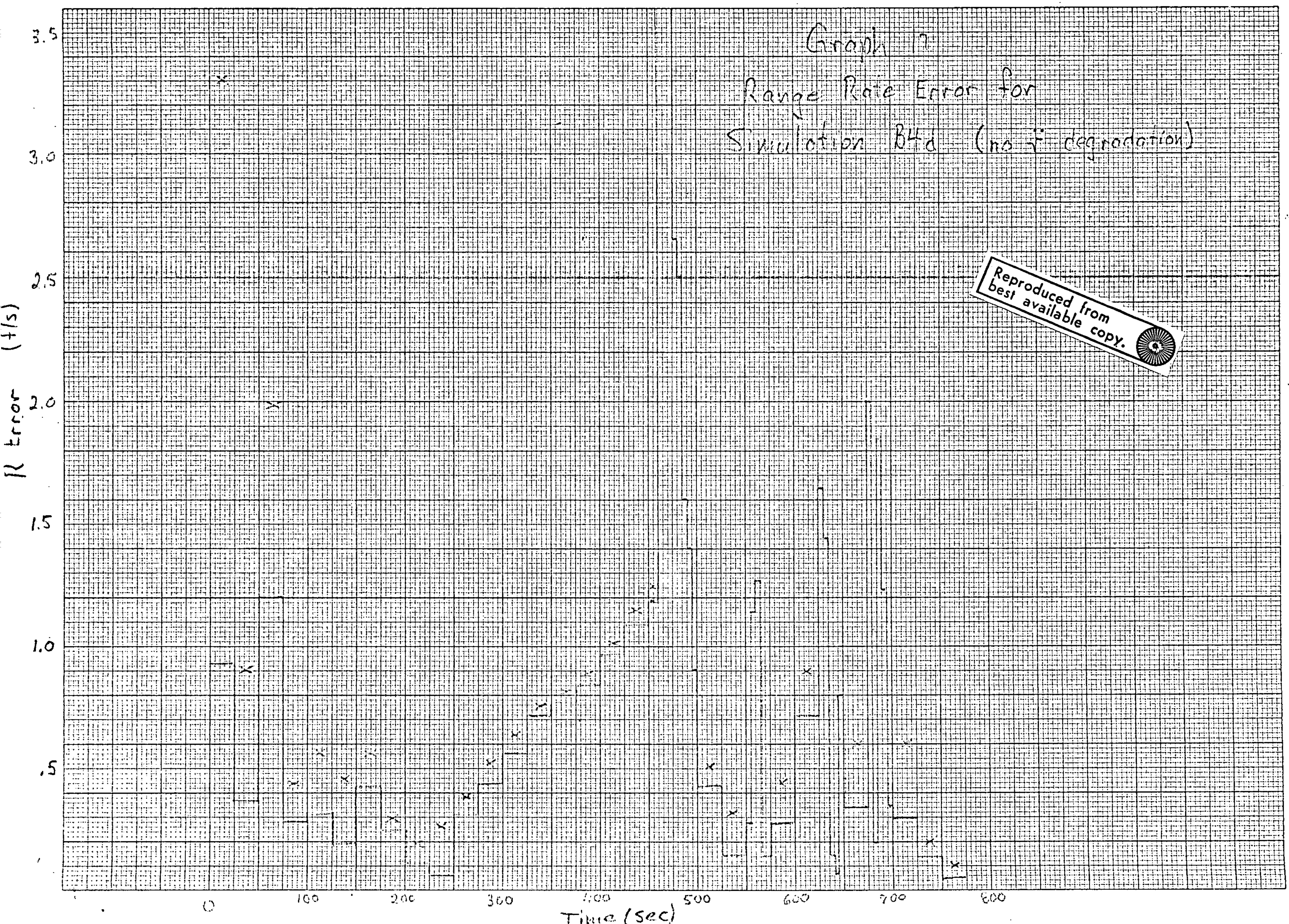
It is possible to make three measurements and determine a complete initial state and error matrix. The value of the $\ddot{r}$ term in the covariance matrix for 5 sec. intervals between measurements would be 5.8 f/s$^2$. Using this value and an initial value of zero for $\ddot{r}_{est}$ yielded results which were the same as in the previous runs after 1 1/2 to 2 minutes of operation, although they were initially worse.

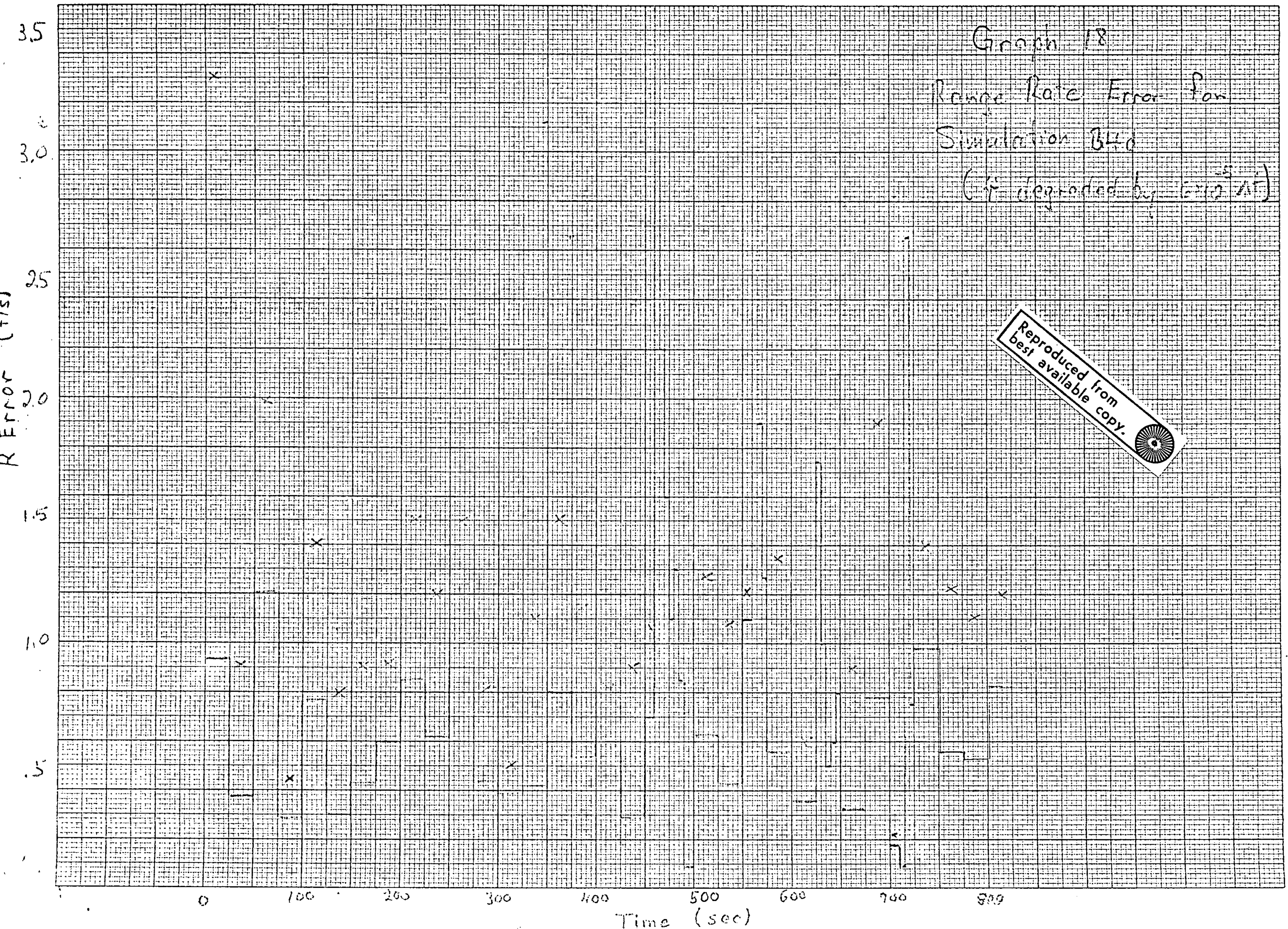
The "hit" trajectory used for the study in Appendix 1 emphasized the weakness of truncating the filter at $\ddot{r}$. Shortly before the first braking gate the error in $\dot{r}_{est}$ had crept slightly over 1 f/s. After the braking and degradation of $E_{\dot{r}}$ (and thus $E_{\ddot{r}}$) the error was resolved and stayed below 1 f/s for the rest of the run. The "hit" trajectory started at a slightly greater distance from rendezvous than the other trajectory resulting in a slightly larger initial $\ddot{r}$ which was quickly estimated by the filter but which then decreased as the two vehicles closed leaving the estimate at too high a value. If the gain of the $\ddot{r}$ term of the covariance matrix were increased the filter might be able to follow the non-constant $\ddot{r}$. The "origin" of the change in $\ddot{r}$ is the non-zero value of $\dddot{r}$. Therefore rather than estimate $\dddot{r}$, the filter element $E_{\ddot{r}}$ was degraded by $(\overline{\dddot{r}}\ \Delta t)^2$ at each extrapolation. The value of $\overline{\dddot{r}}$ for the hit trajectory was used for $\overline{\dddot{r}}$ $(-6 \times 10^{-5})$. The gain thus introduced was too great and $\ddot{r}_{est}$ was noisy, occàssionally jumping to values in error by more than 1 f/s. Another run using the value $(-6 \times 10^{-6}\ \Delta t)^2$ performed beautifully. Graphs 17 and 18, and 19 show the performance of the filter with no $E_{\ddot{r}}$ degradation, and with factors of $(6 \times 10^{-5}\ \Delta t)$ and $(6 \times 10^{-6}\ \Delta t)$.

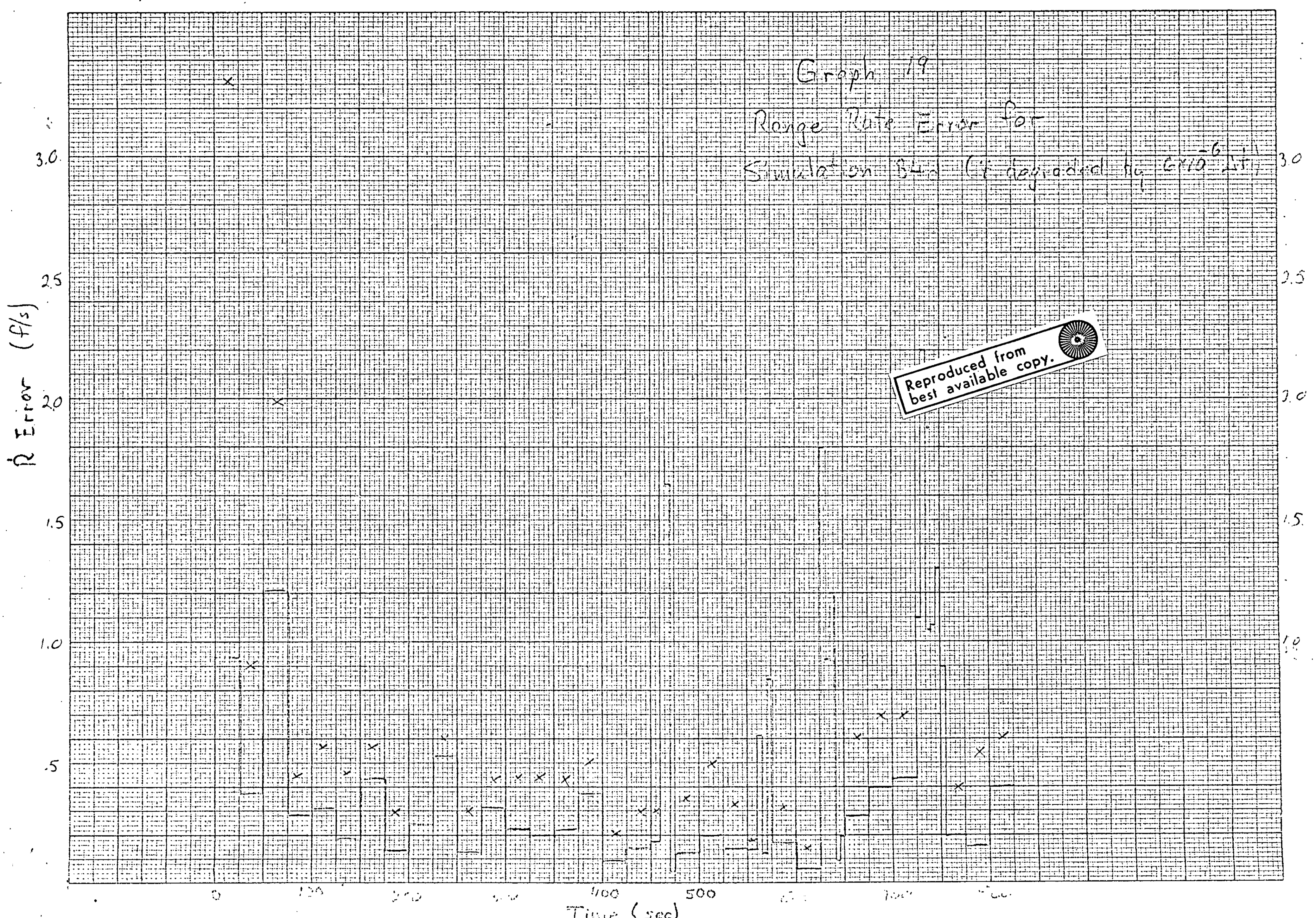
Using the value of $\left(6 \times 10^{-6}\ \Delta t\right)$ runs A1 and B1 were repeated with both the "hit" and "miss" trajectories, A1a, A1b, B1a, B1b. Results were, as expected, quite similar to those presented in Graphs 17 and 19. Prior to the braking the estimate of $\dot{r}$ was momentarily quite accurate then began to stray in the original runs A1 and B1. When the $E_{\ddot{r}}$ degradation was added the solution for $\dot{r}$ was never as accurate but neither did it began to deviate as non-zero $\dddot{r}$ integrated into $\ddot{r}$ and finally into $\dot{r}$.

Based on these runs it is possible to suggest several alternative modes of operation for the polynomial filter.


Graph 16
Range Rate Error for
Simulation B4 b
3.5
3.0
2.5
2.0
1.5
1.0
.5
Ṙ Error (f/s)
0
100
200
300
400
500
600
700
800
Time (sec)


Graph
Range Rate Error for
Simulation B4d (no f degradation)
Reproduced from best available copy.
R Error (f/s)
3.5
3.0
2.5
2.0
1.5
1.0
.5
0
100
200
300
400
500
600
700
800
Time (sec)


Graph 18
Range Rate Error for
Simulation B4d
Ṙ Error (f/s)
3.5
3.0
2.5
2.0
1.5
1.0
.5
0
100
200
300
400
500
600
700
800
Time (sec)


Graph 19
Range Rate Error for
3.0
2.5
2.0
1.5
1.0
.5
0
400
500
Time (sec)
Ṙ Error (f/s)
Reproduced from
best available copy.

## POSSIBLE MODES OF OPERATION

The trial runs of the preceding section have yielded several viable alternative variations in the polynomial filter. Deciding between them may finally rest on either Monte Carlo studies and / or the actual size and performance of the program in the "AGC" simulations. It is possible to list the alternatives and comment on their good and bad features and state what further data is needed for each. The last few simulations listed in Table 3 give the best performance. The alternatives they offer are given in Table 4.

A choice must be made between 1 and 2. On the basis of programming difficulty alternative 2 would be best. The simulations run for the parameter study covered a large range; it is doubtful that a Monte Carlo study beginning with a realistic deviation matrix at MCC2 would reveal any braking maneuvers which were radically different from the nominal.

Option 4 is superior to 3 in that it is effective in reducing $\dot{r}$ error due to truncation of the series throughout the run. Option 3 has no effect until the first $v_\perp$ burn. A value for K must be found from Monte Carlo or parameter type studies however.

To initialize the filter two alternatives are possible. Option 5 will work for any trajectory since r, $\dot{r}$, and $\ddot{r}$ are all measured. The values of the elements of the covariance matrix are also determined. Some extra programming is involved. The alternative, Option 6, gives a slightly quicker solution to $\dot{r}$, provided the value of $\overline{\ddot{r}}$ (ensemble a v) is smaller than the deviation in $\ddot{r}$ obtained by the making 3 measurements in Option 5. The runs made so far indicate that this will be the case. The range cell sampling technique seems to give a quite significant improvement especially at low $\dot{r}$.

## CONCLUSIONS

The simulations in this study indicate that:

1. The effect of braking can be incorporated by adding a constant to the $\dot{r}$ element of the covariance matrix or $\Delta$v to $\dot{r}_{est}$.

2. By adding some portion of the error introduced into $\ddot{r}$ by the truncation of $\ddot{r}$ at each state extrapolation the error in $\dot{r}$ due to that truncation is held under 1 f/s.

TABLE 4 Options for Filter Operation

| Option | Positive Features | Negative Features |
|---|---|---|
| $r_e = r_e + \Delta v_b$ | no transient error at braking burns | programming necessary to measure and incorporate $\Delta v$ applied |
| $E_{\dot{r}} = E_{\dot{r}} + K^2$ | no value needed for $\Delta v$ (although it is necessary to "recognize" a braking maneuver) | "K" must be determined by simulations a radically large brake could cause trouble lengthy programming |
| $\dot{r} = 0$ at $v_\perp$ burns | prevents "truncation drift" | must "recognize" $v_\perp$ burns, works only after $v_\perp$ burns begin |
| $E_{\ddot{r}} = E_{\ddot{r}} + (K\ \Delta t)^2$ | prevents "truncation drift" works throughout run | K must be determined by simulations |
| Initialize with 3 "r" measurements | works for any trajectory | extra programming |
| A priori initial $\ddot{r}$ and $E_{\ddot{r}}$ | if $\ddot{r}$ sufficiently small : quicker solution | a radically different trajectory may give trouble |
| Range cell sampling | reduces errors significantly | extra programming |

3. An a priori value for $\ddot{r}$ and $E_{\ddot{r}}$ is satisfactory by initializaing the problem.

4. Range cell sampling reduces the errors in $\dot{r}$ enough to make it worthwhile.

5. Estimating $\dddot{r}$ unnecessary.

Some systematic study of the value needed in degrading $\ddot{r}$ (#2 above) will be made as will a study to determine the values of $\ddot{r}$ encountered (#3 above).

A change in either the data request rate (5 sec. for this study) or the sampling rate (.2 sec) could alter the options chosen. Additional options or programming should not be necessary however.

# APPENDIX I

## FILTER FAILURE FOR MISS TRAJECTORIES

If the two vehicles are not on intercept trajectories and if the astronaut does not take corrective action, $\omega_{LOS}$ corrections, the estimate of $\dot{r}$ will eventually be completely inaccurate. Restrictions in the manual-terminal simulation do not allow $\omega_{LOS}$ corrections before 600 ft. unless $\omega_{LOS}$ is quite large. These restrictions suffice to prevent any $\omega_{LOS}$ corrections if the vehicles are on an intercept trajectory. In spite of the large $\omega_{LOS}$ (and therefore $\ddot{r}$) allowed before corrective action is taken the filter resolves r correctly.

It is interesting to see how much higher than these limits $\omega_{LOS}$ and the $v_{\perp}$ go when the filter fails on a miss trajectory. Miss trajectories were generated by adding a velocity deviation to the active vehicle at MCC2. Table I. 1 shows the deviation vector, the miss distance, the range at which the error in $\dot{r}$ becomes greater than 1 f/s, and the $\omega_{LOS}$ and $v_{\perp}$ needed for correction at that range. The negative sign on the error in $\dot{r}$ at the 3 rd and 4 th brakes indicates the displayed closing rate is higher than the actual closing rate. For all these runs $\omega_{LOS}$ and $v_{\perp}$ are above minimum levels for astronaut action at the time of filter failure.

The range at which the filter failed varies from about 1. 1 to 2. 5 times the miss distance. When these runs were repeated with $\omega_{LOS}$ corrections rendezvous was achieved. For the undeviated "hit" trajectory the error in $\dot{r}$ drifted to slightly greater than 1 f/s before the first braking gate. The solution to this problem was found in trial B4d of the section on trial simulations.

TABLE I.1 Miss Trajectories

| Velocity Deviation | Miss Dist. | Dist. @ $e_{\dot{r}} > 1$ f/s | $\omega_{LOS}$ @ $e_{\dot{r}} > 1$ f/s | $v_{\perp}$ @ $e_{\dot{r}} > 1$ f/s | (6000) 1st br | (3000) 2nd br | (1500) 3rd br | 4th br (600) |
|---|---|---|---|---|---|---|---|---|
| (0, 0, .1) | 406 ft | 436 ft. | 13.8 mr/s | 6 f/s | < 1 f/s | < 1 f/s | < 1 f/s | 1 f/s |
| (0, 0, .2) | 577 | 655 | 11.6 mr/s | 7.6 f/s | < 1 | < 1 | < 1 | -1.92 f/s |
| (0, 0, .3) | 680 | 1030 | 7.5 mr/s | 7.7 f/s | < 1 | < 1 | < 1 | -5.2 |
| (0, 0, .5) | 991 | 1446 | 6.5 mr/s | " 9.2 | < 1 | < 1 | < 1 | miss > 600ft. $e_{\dot{r}} > 1$ following 3rd br |
| (0, 0, .1) | 1420 | 2033 | 7.0 | 13.1 | < 1 | < 1 | -4.45 | miss > 600 ft. |
| (.1, 0, 0) | 543 | 614 | 13.0 | 8 | < 1 | < 1 | < 1 | -1.4 |
| (.2, 0, 0) | 799 | 1300 | 5.8 | 7.5 | < 1 | < 1 | < 1 | miss 600 ft $e_{\dot{r}} > 1$ following 3rd br |
| (.3, 0, 0) | 1087 | 1360 | 8.1 | 11.0 | < 1 | < 1 | < 1 | " " |
| (.5, 0, 0) | 1430 | 2050 | 6.2 | 12.7 | < 1 | < 1 | -4.12 | miss > 600 ft. |
| | No braking for these | | | | | | | |
| (0, 0, 1) | 1250 | 2000 | 10 | 20.0 | | | | |
| (0, 0, 2) | 2590 | 5570 | 3.3 | 17.7 | | | | |
| (0. 0. 3) | 3980 | 7700 | 2.75 | 21.3 | | | | |
| (1, 0, 0) | 1540 | 3920 | 3.8 | 14.8 | | | | |