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THREE-METER TELESCOPE STUDY FINAL REPORT

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SECTION 1

SUMMARY

The system design concept of the Large Space Telescope program has become sufficiently well definitized for formulation of an optical design configuration that can now be optimized based on a closely constrained set of ground rules. This work, performed under NASA Contract NAS 5-21540 by The Perkin-Elmer Corporation, accomplishes the goals of first defining methods for evaluating the theoretical optical performance of axisymmetric, centrally obscured telescopes based upon the intended astronomy research usage, then proceeds through a series of design parameter variations to determine the optimum telescope configuration. The design optimum requires very fast primary mirrors, so the study also examines the current state of the art in fabricating large, fast primary mirrors. The conclusion is that a 3-meter primary mirror having a focal ratio as low as $f/2$ is feasible using currently established techniques. An improved theory for predicting the effects of misalignment of the primary and secondary mirrors is presented, and tolerances for given levels of optical performance are determined. A tradeoff analysis shows the achievable levels of performance and the design values needed to obtain these levels.

SECTION 2

INTRODUCTION AND STATEMENT OF PROBLEM

2.1 OBJECTIVE OF STUDY

The NASA Goddard Space Flight Center has accomplished a considerable amount of preliminary design leading toward an operational Large Space Telescope late in this decade. These preliminary designs are based on the launch vehicles which are expected to be available in the late 1970's. Figures 2-1 and 2-2 illustrate these design concepts. Succinctly stated, the primary problem to which this study is directed is that of optimizing the optical design of a telescope consistent with the space envelope of these concepts.

The criteria for the design optimization will be extracted from a combination of scientific requirements and Large Space Telescope performance characteristics that will allow the widest and most useful realization of its research potential. In Section 3.3, the current problems in Astronomy and Astrophysics are surveyed and the important properties of the phenomena to be observed with the telescope are categorized. The developing technology of image processing is examined for possible applications and design guidance. From these categories, three measures of performance or figures of merit are derived. Each figure of merit is then examined from two points of view; that of the optical designer, whose aim is to extract the best optical performance within the given parametric constraints, and secondly, from the operational point of view where dimensional instabilities such as thermally induced misalignment, defocus, and mirror deformations are of primary concern. Significantly, slight changes in the definitions of the figures of merit occur when examined from these two view points. This discussion is contained in Section 3.6.

The next step is to use the figure of merit definition to obtain design trends. A correlation or functional relationship between each figure

LARGE SPACE TELESCOPE STRUCTURAL COMPOSITE

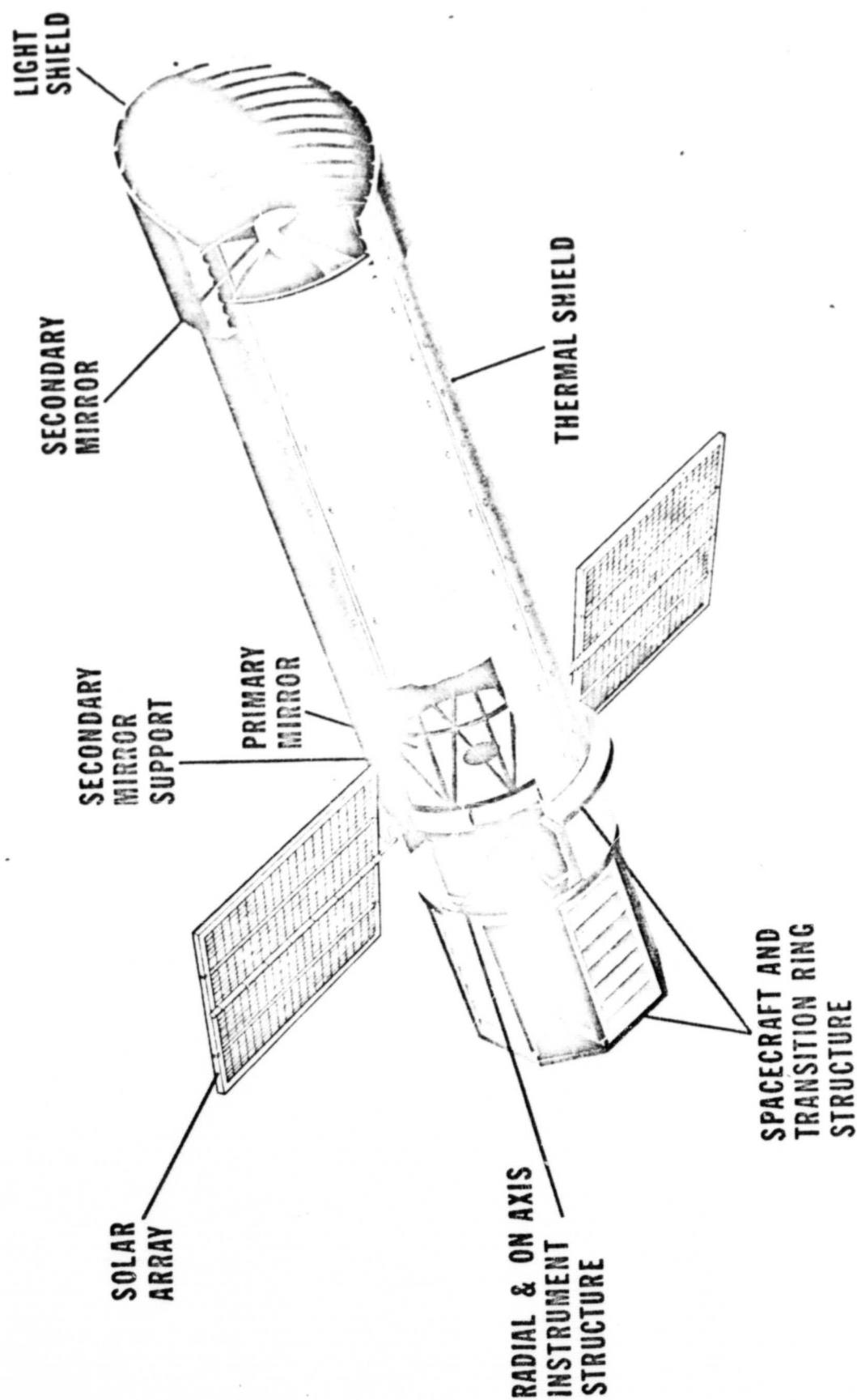


Figure 2-1. Large Space Telescope Structural Composite

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16. Abstract A means is presented whereby the effect of various changes in the most important parameters of a three-meter aperture space astronomy telescope can be evaluated to determine design trends and to optimize the optical design configuration. Methods are defined for evaluating the theoretical optical performance of axisymmetric, centrally obscured telescopes based upon the intended astronomy research usage. A series of design parameter variations is presented to determine the optimum telescope configuration. The design optimum requires very fast primary mirrors, so the study also examines the current state of the art in fabricating large, fast primary mirrors. The conclusion is that a 3-meter primary mirror having a focal ratio as low as f/2 is feasible using currently established techniques. An improved theory for predicting the effects of misalignment of the primary and secondary mirrors is presented, and tolerances for given levels of optical performance are determined. A tradeoff analysis shows the achievable levels of performance and the design values needed to obtain these levels.					
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LIST OF SYMBOLS AND ABBREVIATIONS BY SECTIONS

SECTION 3

ϵ	=	ratio of diameter of the secondary to the diameter of the primary
D_s	=	the diameter of the secondary mirror
D_p	=	the diameter of the primary mirror
A	=	telescope collecting area
I	=	intensity at the center of the diffraction pattern produced by an <u>obscured</u> aperture
I_o	=	intensity at the center of the diffraction pattern produced by an <u>unobscured</u> aperture
S	=	Strehl ratio
$I(o, o)$	=	intensity with no aberrations, no central obscuration
$I(\phi, \epsilon)$	=	intensity with aberrations and central obscuration
$i(p)$	=	intensity at peak of image

NRE	=	normalized relative energy equal to energy of one resolution element divided by 0.84 of total theoretical energy
OTF	=	optical transfer function
$S_i(k_x, k_y)$	=	Fourier transform of the image scene
$S_o(k_x, k_y)$	=	Fourier transform of the object scene
$H(k_x, k_y)$	=	optical system OTF
k_x, k_y	=	variables denoting optical spatial frequency in the X- and Y- directions
S_r	=	Fourier transform of restored image scene
F	=	Linear filter for the spatial frequency spectrum
D	=	desired system OTF
MTF	=	modulation transfer function
ϕ	=	aberration function
A_{nm}	=	aberration weighting coefficient
R_n^m	=	orthogonal Zernike circle polynomials radial component
ϵ	=	central obscuration ratio
ρ	=	radial coordinate of aberration wave front

A	=	polar coordinate of aberration wave front
n, n', l, l', m	=	indices of Zernike circle polynomials
E	=	variance of the aberration function
S	=	modified Strehl ratio
k	=	$2\pi/\lambda$
λ	=	wavelength
$\overline{\phi_{HF}^2}$	=	single term combining mean square of high spatial frequency figure errors of the system
A_{nm}	=	new (balanced) and old (unbalanced) aberration coefficients
A'_{nm}	=	
$\overline{\phi^2}$	=	mean square aberration
ϕ'	=	compensated aberration
$\odot$	=	symbol denoting function is normalized to the obscured aperture
ϵ_{max}	=	maximum possible central obscuration
$*$	=	denotes convolution
RER	=	relative edge response
$s(x)$	=	step function

$e(x)$	=	edge function (image of edge)
$l(x)$	=	line spread function
$T(k)$	=	system transfer function
$p(x,y)$	=	system point spread function
$e'(x)$	=	slope of edge function
j	=	$\sqrt{-1}$
u	=	variable of integration
k_r	=	spatial frequency in radial direction
ϵ	=	obscuration ratio (diameter)

SECTION 4

S	=	modified Strehl ratio
ϵ_s	=	obscuration due to secondary mirror diameter
ϵ_b	=	obscuration due to baffling
N	=	final system $f/\#$ (focal ratio)
ϵ_H	=	obscuration due to hole in primary mirror
N_p	=	primary mirror focal ratio

SECTION 5

W	=	departure from closest reference sphere
W_g	=	third order departure from Gaussian reference sphere
Y_t	=	image height
N	=	$f/\#$ of parabola
M	=	magnification
W_{rms}	=	root mean square departure from closest reference sphere
B	=	distance from primary to Cassegrain image surface
f_p	=	focal length of the primary
Y_t	=	lateral displacement of focal points
σ_t	=	sensitivity coefficient for tilt
σ_d	=	sensitivity coefficient for decenter
ϵ	=	ratio of secondary mirror dia. to primary mirror dia.
$\Delta\phi$	=	rms wavelength aberration
λ	=	wavelength of light

SECTION 6

T	=	manufacturing time
D	=	mirror diameter
N	=	relative aperture (or focal ratio)
$f(Q)$	=	function of mirror accuracy (rms surface error)
C	=	proportionality constant
δ_i	=	optical path difference (OPD) error
RMS (worst case)	=	RMS value obtained if OPD error always contributed to increase the absolute value of the OPD at each point

SECTION 7

C	=	a constant
L_G	=	length of Gregorian
L_C	=	length of Cassegrain

SECTION 1

SUMMARY

The system design concept of the Large Space Telescope program has become sufficiently well definitized for formulation of an optical design configuration that can now be optimized based on a closely constrained set of ground rules. This work, performed under NASA Contract NAS 5-21540 by The Perkin-Elmer Corporation, accomplishes the goals of first defining methods for evaluating the theoretical optical performance of axisymmetric, centrally obscured telescopes based upon the intended astronomy research usage, then proceeds through a series of design parameter variations to determine the optimum telescope configuration. The design optimum requires very fast primary mirrors, so the study also examines the current state of the art in fabricating large, fast primary mirrors. [REDACTED]

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SECTION 2

INTRODUCTION AND STATEMENT OF PROBLEM

2.1 OBJECTIVE OF STUDY

The NASA Goddard Space Flight Center has accomplished a considerable amount of preliminary design leading toward an operational Large Space Telescope late in this decade. These preliminary designs are based on the launch vehicles which are expected to be available in the late 1970's. Figures 2-1 and 2-2 illustrate these design concepts. Succinctly stated,

[REDACTED]

[REDACTED]

[REDACTED] s.

The criteria for the design optimization will be extracted from a combination of scientific requirements and Large Space Telescope performance characteristics that will allow the widest and most useful realization of its research potential. In Section 3.3, the current problems in Astronomy and Astrophysics are surveyed and the important properties of the phenomena to be observed with the telescope are categorized. The developing technology of image processing is examined for possible applications and design guidance. From these categories, three measures of performance or figures of merit are derived. Each figure of merit is then examined from two points of view; that of the optical designer, whose aim is to extract the best optical performance within the given parametric constraints, and secondly, from the operational point of view where dimensional instabilities such as thermally induced misalignment, defocus, and mirror deformations are of primary concern. Significantly, slight changes in the definitions of the figures of merit occur when examined from these two view points. This discussion is contained in Section 3.6.

The next step is to use the figure of merit definition to obtain design trends. A correlation or functional relationship between each figure

LARGE SPACE TELESCOPE STRUCTURAL COMPOSITE

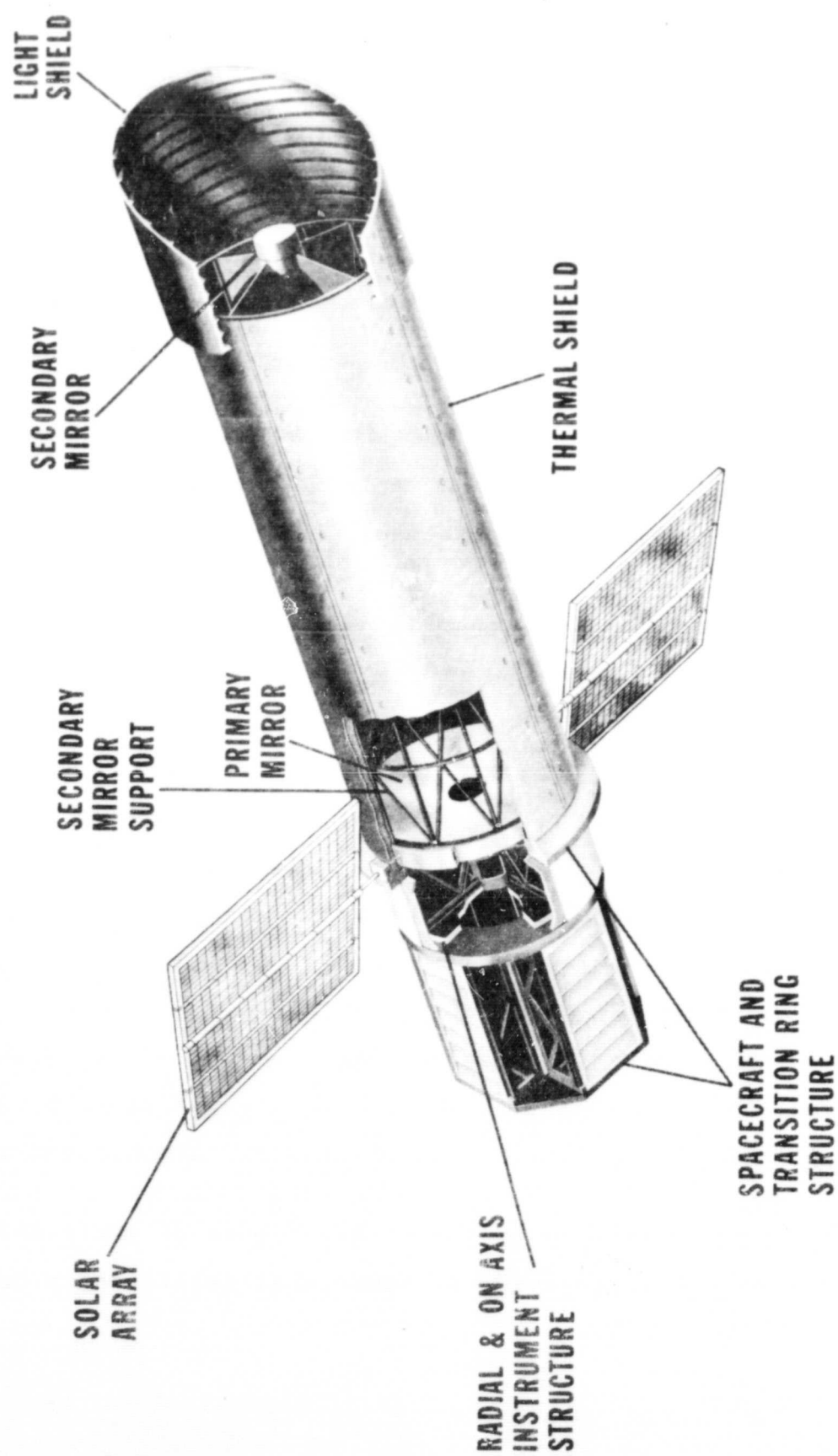


Figure 2-1. Large Space Telescope Structural Composite

3 METER LARGE SPACE TELESCOPE STRUCTURAL CONCEPT

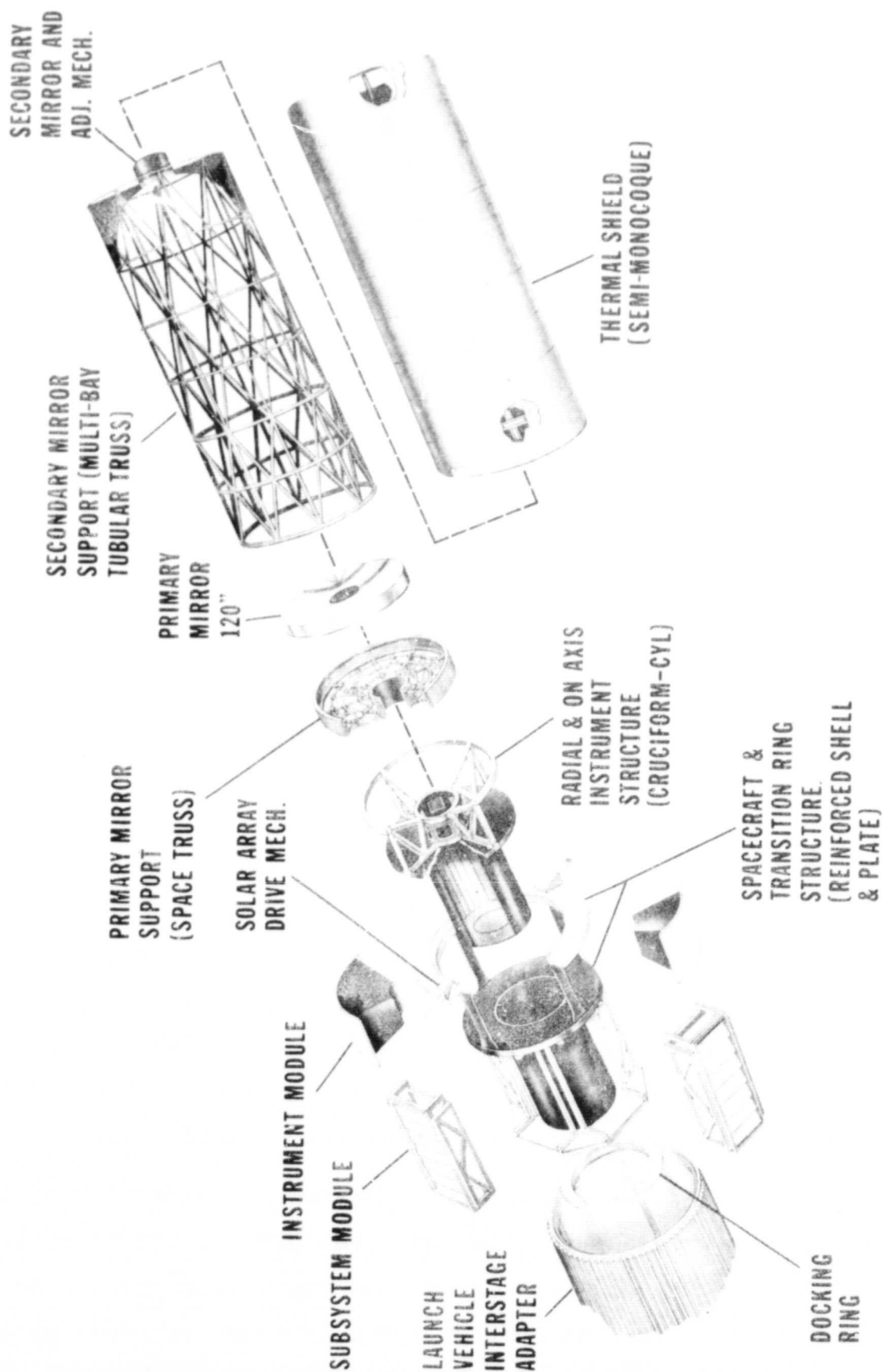


Figure 2-2. Large Space Telescope Structural Concept

of merit definition and design variables, such as obscuration due to secondary mirror and light baffles, and final system focal ratio was then derived. In addition, the operational variables such as misalignment were also related to the appropriate criteria or figures of merit. All of these equations were programmed on an electronic computer, and then the basic parameters were varied within the constraints of the ground rules. A description of the way in which this was done and the results obtained are contained in Section 4.

From the variation in optical performance as the design parameters were varied, a clear picture evolves; namely, that the performance of an axisymmetric two-mirror optical system improves as system magnification is transferred from the primary to the secondary. This also results in shorter overall lengths. The driving influence is the central obscuration caused by the secondary mirror and baffle, which must be minimized for each particular mirror separation and for each type of optical system considered. In order to reduce the central obscuration, the focal ratio of the primary mirror must be made smaller. This leads to tighter requirements in two other areas: manufacture and operation. Therefore, the optical design trend must be moderated by consideration of the relative difficulties and cost of manufacture, and the expected environment and operational degradations which must be accommodated by the optical system. In Section 6 the recent trends in relative cost as faster mirrors have been manufactured is examined, as well as the current state of the art in large optics fabrication. Extrapolations to mirror diameters of three meters indicate a practical limit on the primary mirror focal ratio, and this information is used in the tradeoff study.

Section 5 on "Alignment Discussion" advances new ways of treating telescope misalignments and suggests ways in which the problem can be reduced to a minimum. Appendix A, which is a reprint of the paper given by Mr. Abe Offner at the NASA-sponsored workshop held at MSFC on April 29 - May 1, 1969, gives the basis for analysis of telescope misalignment and also shows that the crucial quantity to be controlled in telescope alignment is the separation of the geometric foci of the primary and secondary mirrors. When this separation is reduced to zero, then the misalignment caused by tilt will be compatible with the expected mechanical and thermal strains, as shown by Mr. Offner's analysis. The equations derived in the paper allow calculation of the wavefront

errors caused by defocus and misalignment, and these wavefront errors can in turn be evaluated using the figure of merit definitions which are used in the study. In this way optical performance can be maximized in the face of misalignment by taking advantage of the trend to faster primary mirror focal ratios indicated by the performance calculations.

Both the effects of central obscuration and the trend in manufacturing are inputs to the tradeoff analysis discussion. Another element in the trade-off decision is the possibility of sensing wavefront errors which cause various types of imperfections in the image produced by the telescope. Recent advances have been made in both sensing and analyzing the resultant wavefront error in terms of the component aberrations. As a result, some aberrations, such as defocus or misalignment can be singled out and appropriate in-orbit corrections made. Application of these techniques shows considerable promise in reducing the amount of performance which must be sacrificed in order to provide safe accommodation of the various operational degradations. These techniques have been partially proven in other work which Perkin-Elmer has accomplished, and development of the necessary components for spaceborne wavefront analyzer mechanization is being contemplated by NASA. The performance potential of the system is discussed in Section 8.

An auxiliary but nonetheless important topic included in the Primary Mirror discussion is the deleterious effect of the light scatter upon faint object detection. Limits on the amount of scattered light which can be tolerated in the detection of 29th and 30th magnitude stars are calculated for several different assumed conditions. The value of these limits is unexpectedly severe in some cases, suggesting that some experimental work is probably in order to determine actual scatter coefficients.

The interrelationship of the various parts of the study under discussion is shown graphically in Figure 2-3, which gives an overall view of the procedures followed in the study.

2.2 GROUND RULES:

The contract is explicit in the dimensional constraints which must be observed in the configuration study. The volume available is cylindrical with a diameter of 3 meters and a length of 8.85 meters. These constraints

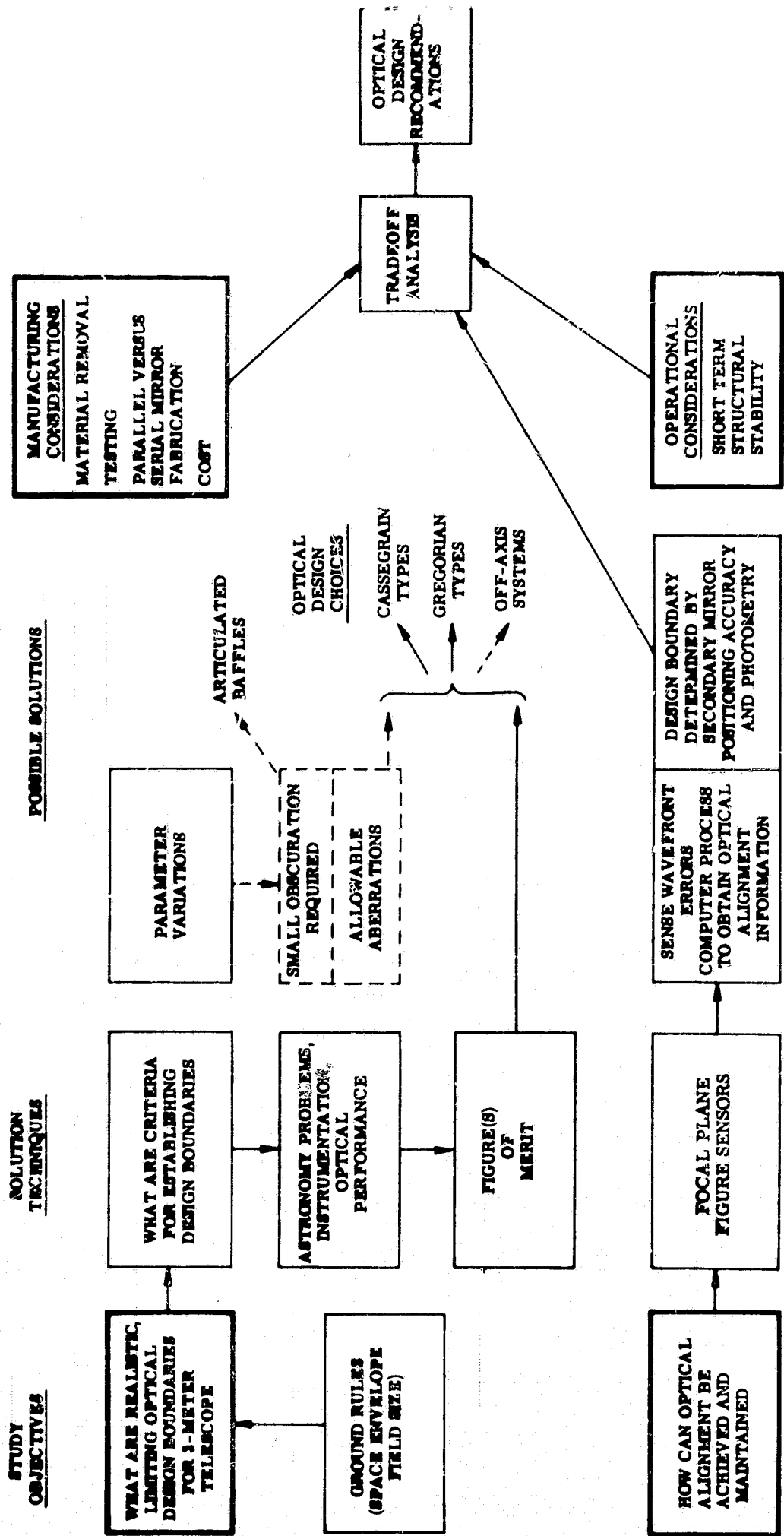


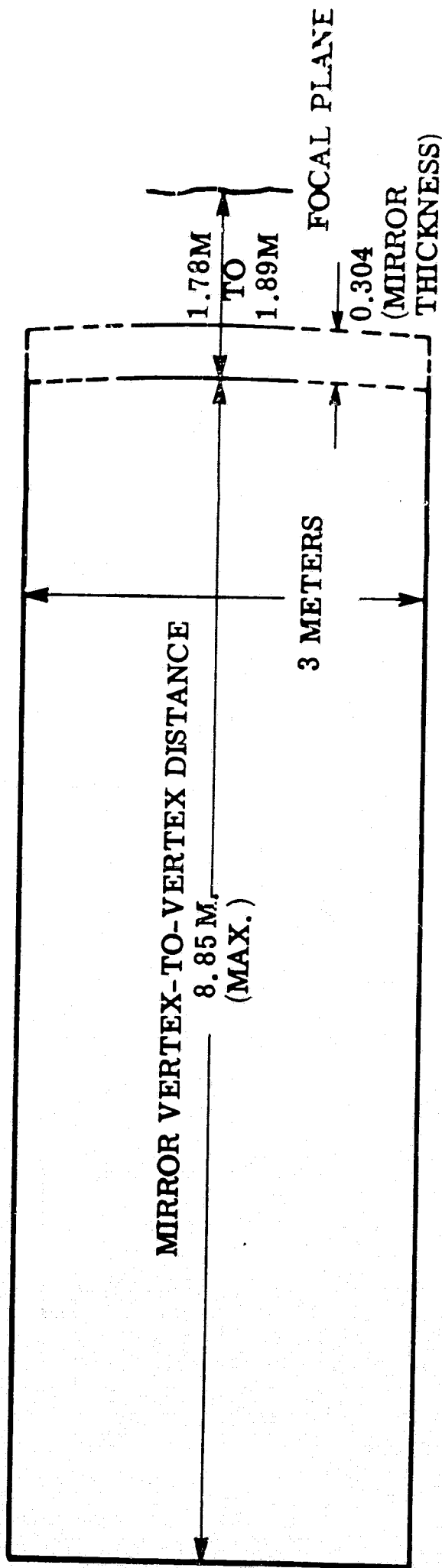
Figure 2-3. Study Plan

are summarized in Figure 2-4. Note that some parameters, such as the diameter of the primary mirror and the angular size of the tracking field, are invariant, while others are stated as maxima (e.g. primary to secondary mirror separation) and are given permissible ranges of variation (e.g. distance from primary mirror vertex to tracking field image plane).

Although, as implied by the various figures of merit examined, it would be desirable to eliminate the central obscuration entirely by going to an off-axis optical system, this violates the ground rules and serious consideration was not to be given to such systems. Therefore, the parametric design variations were limited to axisymmetric two-mirror systems.

The ground rules further permitted investigation of two types of optical system to be selected by Perkin-Elmer. Since the telescope configuration traditionally selected for space usage is the Cassegrain (or its non-conic section refinement, the Ritchey-Chrétien), this is one of the two telescope forms selected by Perkin-Elmer for the investigation. Since the prime variable in the system described by the ground rule constraints is the position of the primary focus shared between the primary and secondary mirrors, it seemed only logical to exercise this variable to the maximum extent practical. In the Cassegrainian systems, this point is in object space, behind the secondary mirror and away from the primary mirror. Another class of telescope designs has the primary focus between the primary mirror and the secondary mirror, as in the Gregorian form. Thus in order to extend the design parameter variation over as broad a range as possible, the Gregorian type of system was selected as the second optical form to be considered in the study. While Gregorian systems are usually considered too long, new developments in technology and an analysis of the manufacturing tradeoff between the two types of systems lead to rather interesting conclusions, as we shall see. The Gregorian type of telescope also has a high performance version which uses non-conic section mirrors equivalent to the Ritchey-Chrétien. We shall term such equivalent telescope design as "Gregorian Aplanats", of which the Schwarzschild telescope is an example. Since the techniques used in the design of Ritchey-Chrétiens and Gregorian Aplanats* can be applied to a wide variety of first order optical designs, and the main objective of the study is to optimize the design to fit the space envelope, consideration relative to this level of optical design refinement was given only cursory attention.

* An Aplanatic Optical System is one which is free of spherical aberration and coma.



FIELD OF VIEW = 30 ARC MINUTES

TRACKING IMAGE QUALITY SHALL PERMIT ACCURACY BETTER THAN 1/10 OF ON-AXIS SPOT SIZE WHEN GUIDING ON STAR AT EDGE OF FIELD

Figure 2-4. 3-Meter Telescope Ground Rules

SECTION 3

OPTIMIZATION TECHNIQUES

In this chapter, the basic techniques for evaluating and optimizing the design of the Large Space Telescope will be developed. This will be done by examining the most current science objectives for the LST and abstracting from these research objectives the important instrumental functions and intrinsic optical measures of performance. In this connection, the rapidly developing technology of image processing will be examined for application to the LST research function to determine whether new design criteria are indicated.

Having formulated the principles upon which an optimization technique can be based, rigorous optical theory is applied to derive working expressions for the criteria or figures of merit which are then used in the balance of the study to optimize the optical design. Three separate criteria are identified, each of which has an area of application. However, the trend of each figure of merit is similar for one of the important design variables (central obscuration), and so only one of the three is selected for most of the design optimization work.

3.1 SCIENTIFIC REQUIREMENTS

Through the publications of the National Academy of Science¹, the astronomy Missions Board², and others, it is possible to identify and categorize the leading observational problems for which the Large Space Telescope should be optimized. A survey of the referenced publications has been made, supplemented by visits to prominent astronomers. A tabular summation of the research functions, the optical performance characteristics required, and instrumentation has been made and is presented in Table 3-1. It is our intent to distill from this compilation the important performance characteristics for a three-meter telescope which can be quantified and used to evaluate and judge various design options.

The most frequently mentioned performance characteristic listed in Table 3-1 is "Wide Spectral Range", which is an obvious requirement, being fundamental to the reasons for going into space for astronomical observations in the first place. The requirement is best met through all-reflective optical systems

TABLE 3-1^a
RESEARCH FUNCTIONS OF THE LARGE SPACE TELESCOPE

1. <u>EXTRA GALACTIC RESEARCH</u>			
Expansion of Universe			
Measurement of Hubble Constant			
a. Red Shift	High Angular Resolution, Wide Spectral Range	Spectrophotometers	
b. Luminosity of Distant Galaxies	High Angular Resolution, Wide Spectral Range	Wide Slit Spectrophotometers	
c. Measurement of Galactic Diameters	High Angular Resolution, Large Field	Cameras	
Density of Matter in the Universe			
Search for Missing Matter			
a. Faint Galaxies	Wide Spectral Range, High Angular Resolution, Large Field	Wide Band Spectrophotometer	PMT Image Tubes Film
b. Interstellar Diffuse Gas	Wide Spectral Range,	Spectrograph	
Curvature of Space	High Angular Resolution, Wide Spectral Range		
Structure and Evolution of Galaxies			
Classification of Galaxies			
Study of Galactic Nuclei	High Angular Resolution, Large Field	Cameras, TV	Film, Image Tubes Cryogenically Re- frigerated UV and IR Detectors
High Resolution Study of Quasars	High Angular Resolution, Wide Spectral Range	Photometer	
Search for Faint Quasi-stellar Objects	High Angular Resolution, Large Field, Wide Spectral Range		

TABLE 3-1

RESEARCH FUNCTIONS OF THE LARGE SPACE TELESCOPE (Continued)

2. SOLAR SYSTEM

Planets			
Physiography of Planets with Little or no Atmosphere			
a.	Photometric and Polarimetric	High Angular Resolution, Large Field	Photometers
Planets with Atmosphere			
a.	Structure and Behavior of Clouds	High Angular Resolution, Wide Spectral Range	Cameras
b.	Absorbing and Scattering of Planetary Atmosphere	High Angular Resolution, Wide Spectral Range	Photometers
Diameters and Photometric Properties of Minor Bodies in Solar System		High Angular Resolution,	
Spectral Analysis of Planet			
a.	Detection and Distribution of Atmospheric Constituents over the Disc of Mars and Venus	High Angular Resolution, Wide Spectral Range, Large Field	Spectroscopy
b.	Temperature Profiles of Mars and Venus	High Angular Resolution, Wide Spectral Range	Wide Range UV and IR Detectors
c.	Energy Balance of Planets	High Angular Resolution, Wide Spectral Range	Image Tubes, PMT
d.	Composition of Major Planets	High Angular Resolution, Wide Spectral Range	Film, Plates
Comets			
Nucleus Structure		High Angular Resolution, Wide Spectral Range	Cameras, Filters
Gas Flow From Nucleus to the Tail		Wide Spectral Range	Spectroscopy
Thermal State		Wide Spectral Range	Spectrophotometers, Radiometer
			Image Tube, PMT
			IR Detectors

TABLE 3-1
RESEARCH FUNCTIONS OF THE LARGE SPACE TELESCOPE (Continued)

3. <u>INTERSTELLAR MATTER</u>	
UV Spectroscopy	
Density of Intergalactic Medium	
Investigation of Galactic Corona	
Structure of Gas in the Galactic Disc	
Structure of Expanding Gas Shells	
Structure of Prestellar Systems such as Orion Nebula	
Measurements of Extinction by Interstellar Grains (900Å to 3000Å)	
IR Spectroscopy	
Infrared Nebulae and Point Sources	
Interstellar Extinction	
Thermal Radiation of Interstellar Grains	
High Angular Resolution, Wide Spectral Range	
	UV Spectrophotometer
	Image Tubes PMT Film
	IR Spectrophotometer Radiometer
	IR Detectors

TABLE 3-1

RESEARCH FUNCTIONS OF THE LARGE SPACE TELESCOPE (Continued)

4. STARS

Measurement of Stellar Masses

Spectroscopic Binaries

Astrometric Binaries

Eclipsing Binaries

Stellar Luminosities

Measurement of Stellar Parallax

Measurement of Stellar Extinction

Study of Radiant Flux

Stellar Spectra

Abundances of Element

Stellar Envelopes

Stellar Emissions

Stellar Evolution

Faint Stars

High Angular Resolution and
Wide Spectral Range

High Angular Resolution
Wide Spectral Range

High Angular Resolution,
Wide Spectral Range

High Angular Resolution, Wide Spectral
Range, Large Field

TV Camera

Camera

Spectroscope

TV Camera

Photometer

UV Spectrophotometer
Camera

TV Camera
Camera

Image Tubes
PMT
Film, Plates

Image Tube
PMT
Film, Plates

designed to have the highest possible optical efficiencies at the wavelengths of interest. Optical efficiency is largely a function of the number of reflections in the optical system and the reflectivity at each surface. The achievement of Wide Spectral Range is auxiliary to the primary objective of this particular study, and, except to constrain the design of two-mirror systems, is not given further consideration. The study does, however, examine the problem of scattered light and set upper limits for a number of situations described in the referenced publication.

The remainder of the performance characteristics are closely related to the research instruments and detectors. These instruments are grouped and the salient characteristics of each are shown in Figure 3-1. The three most important characteristics of the groupings are: 1) luminous energy density in an image over the area defined by either an entrance slit or resolution element, 2) the intensity at the peak of the diffraction pattern, and 3) the sharpness of or precision with which the position of the edge of an image can be defined and measured. To optimize the telescope design, one wishes to maximize the amount of energy which passes through the slit, resolution element, or other defining aperture in the instrument, and a figure of merit which is a measure of the energy density over the instrument-defined resolution area would be useful in evaluating a telescope's performance in this particular mode of operation. Another mode of operation which is also of great importance is that of imaging faint point objects. Here, maximizing the intensity at the peak of the diffraction image is of paramount importance, and again, a mathematical way of forecasting this would be of use in evaluating various candidate telescope designs. Finally, and of prime importance in the area of post-exposure image processing, a mathematical expression which is a measure of the steepness or the gradient of the image at an edge such as that of a filament or a gas cloud would be another attribute which is of importance to astronomy and which would be of aid in evaluating and optimizing the telescope design.

There are already some measures of optical performance which can be applied in this study, and which are directly related to the performance characteristics identified above. Most directly applicable to the maximization of the intensity at the peak of the image of a point object is the Strehl Definition which, following O'Neill³, is defined as: "the ratio of the light intensity at the maximum of the diffraction pattern to that of the same instrument without aberrations". The definition is illustrated graphically in

CHARACTERISTICS OF RESEARCH PROBLEMS

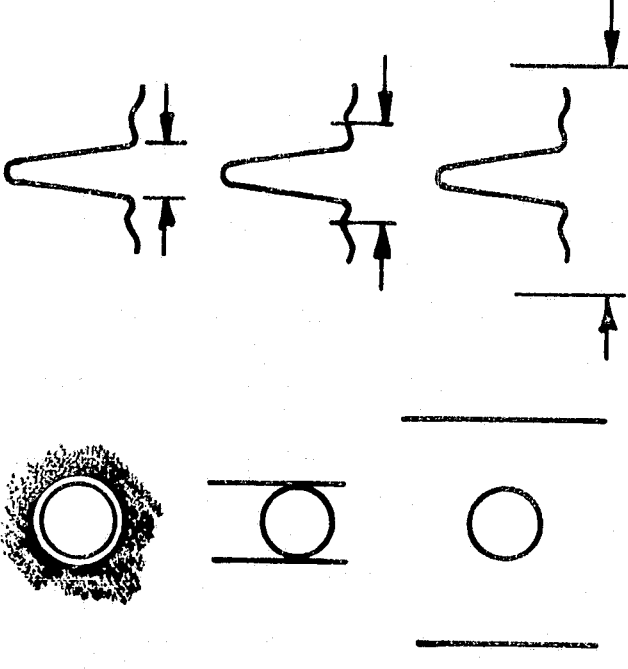
RESEARCH METHOD	IMPORTANT CHARACTERISTIC
PHOTOMETRY SLIT SPECTROSCOPY WIDE SPLIT SPECTROSCOPY	 ENERGY DENSITY IN DEFINED AREA (NORMALIZED RELATIVE ENERGY)
SURVEY FAINT OBJECTS	 INTENSITY AT CENTER OF IMAGE (STREHL RATIO)
ASTROMETRY PHOTOMETRY OF EXTENDED OBJECTS RESOLUTION OF DETAIL	 EDGE SHARPNESS (RELATIVE EDGE RESPONSE)

Figure 3-1. Research Instruments and Characteristics

Figure 3-2, showing typical diffraction patterns for an instrument with, and then without, aberrations. We shall develop a modification of this definition which is valuable in the assessment of specific design variables such as the amount of central obscuration. This derivation is performed in Section 3.6.

In many treatments of the effects of central obscuration, it is not unusual to dismiss the effect by noting that for a secondary mirror diameter equal to 30 to 40 percent of the diameter of the primary mirror, the reduction in collector area is only 9 or 16 percent, and that losing this amount of the light is negligible. It should be borne in mind that there is also a redistribution of the light in the image which, for many applications, is much more serious than the mere reduction in the amount of light collected.

If we define ϵ to mean the ratio of diameter of the secondary, D_s , to that of the primary mirror, D_p , then the telescope collecting area, A , is given by

$$A = \frac{\pi}{4} D_p^2 (1 - \epsilon^2) \quad (3-1)$$

from which the above statements about the amount of light lost can be verified. If we calculate the amount of intensity lost at the center of the image, we find⁴ that the following relationship describes the situation:

$$I = I_0 (1 - \epsilon^2)^2 = I_0 (1 - 2\epsilon^2 + \epsilon^4) \quad (3-2)$$

where I_0 is the intensity at the center of the diffraction pattern produced by an unobscured aperture. Figure 3-3 shows the profile through the images of a point source (such as a star), produced by telescopes of various central obscuration. Note that as the central obscuration is increased, the intensity at the peak of the image decreases while the intensity of the diffraction rings increases. This is because the central obscuration diffracts light from the center of the image into these locations. The exact nature of the redistribution is given in Reference⁴, pp. 416, 417.

If we now plot equation (3-2) above, we obtain the curve shown in Figure 3-4. Note that for the case initially discussed of a 30 or 40 percent obscuration, the reduction in intensity at the peak of the image is 17 and 29 percent respectively, rather than 9 and 16 percent. The significance of the effect of central obscuration is therefore not to be dismissed lightly.

$$S = \frac{I \text{ (Intensity with Aberrations)}}{I \text{ (Intensity with no Aberrations)}}$$

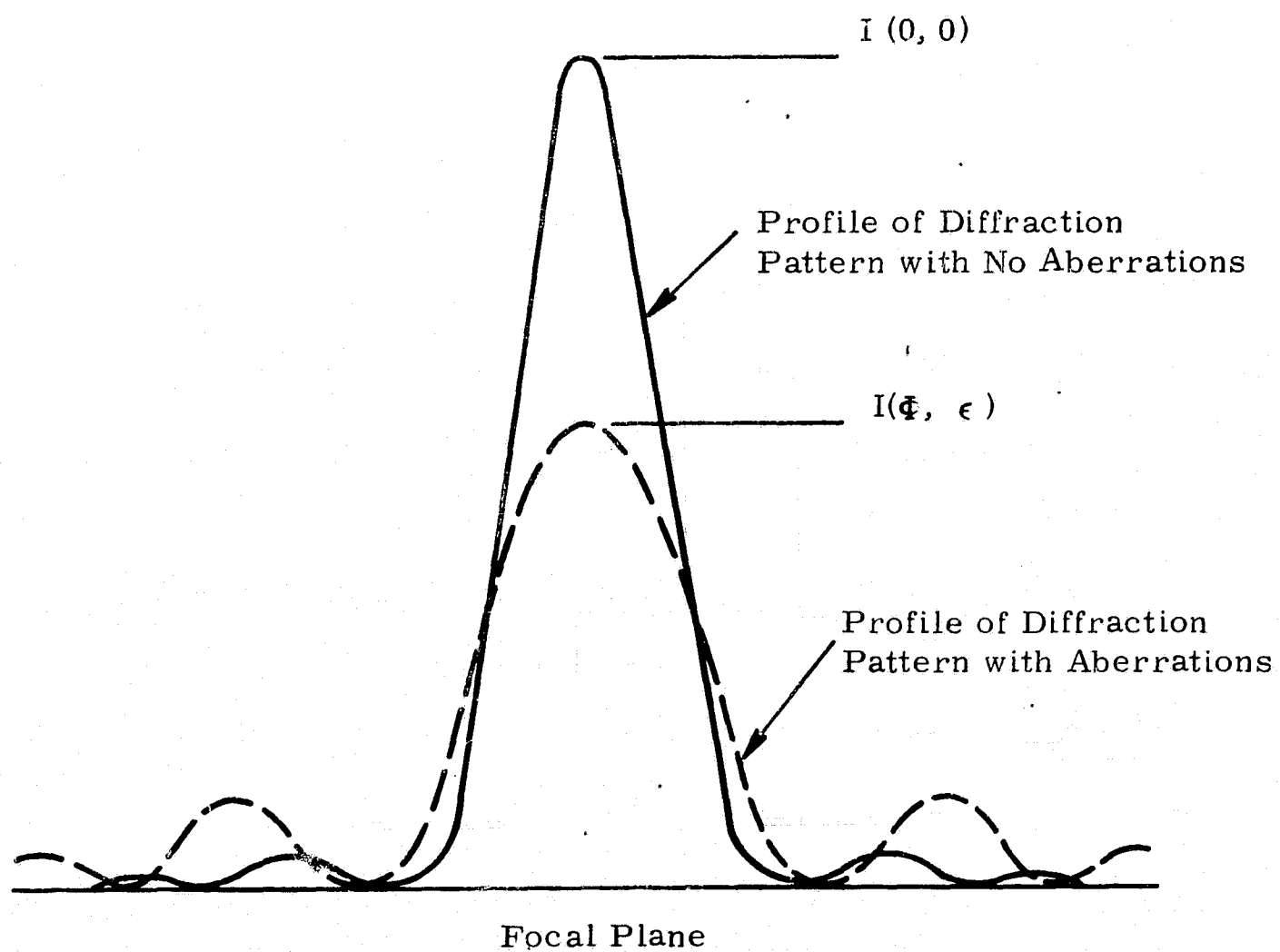


Figure 3-2. Definition of Strehl Ratio

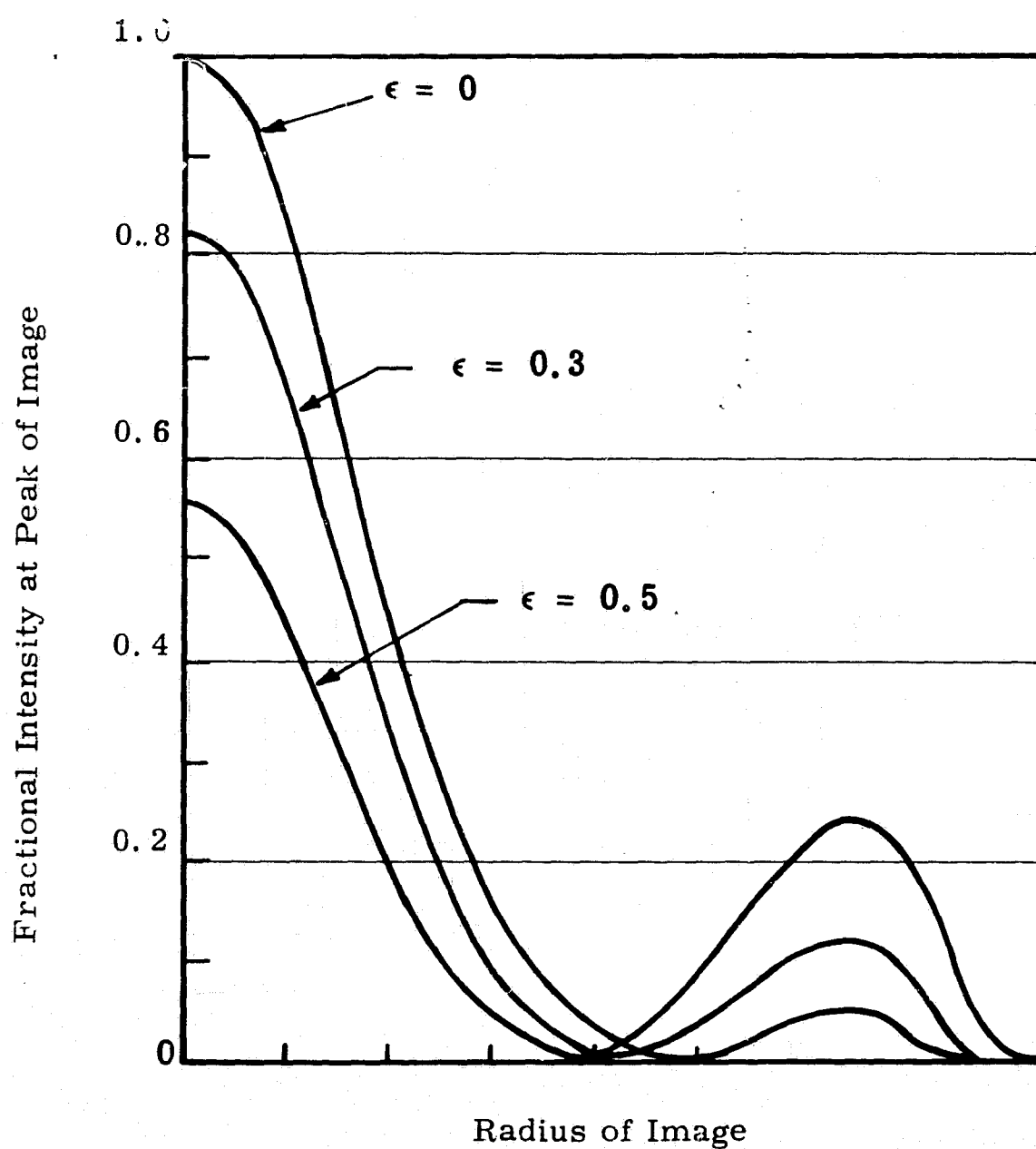


Figure 3-3. Energy Distribution in the Airy Disc as Function of the Obscuration Ratio

$$i(p) = [1 - \epsilon^2]^2$$

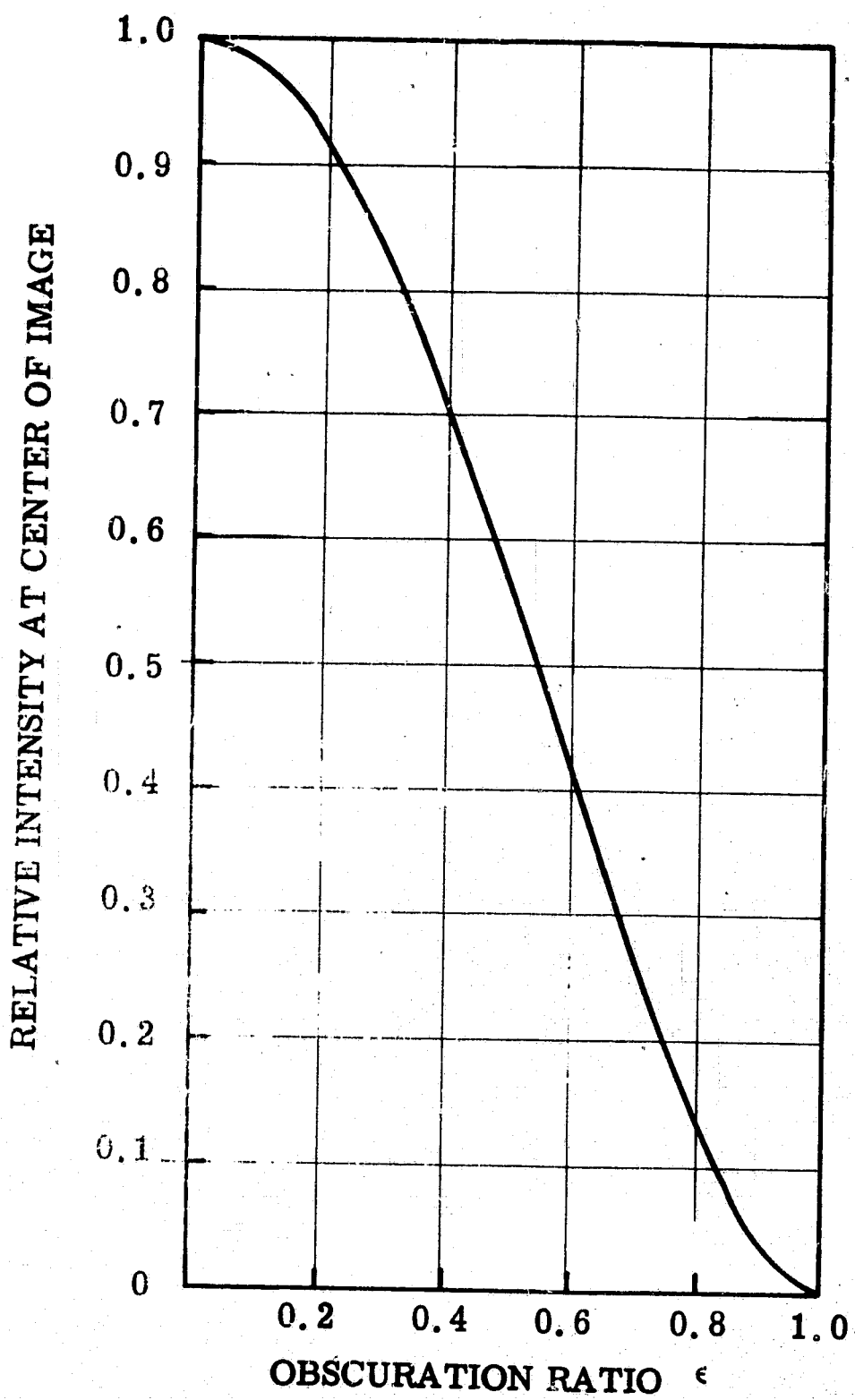


Figure 3-4. Effect of Central Obscuration

The concept of maximizing the energy density over a fixed resolution area is somewhat less commonly known. This concept is suggested by A. Offner in the paper repeated in Appendix A. Offner suggests a measure of performance which he calls the "Normalized Relative Energy" (or NRE for short). The definition of normalized relative energy is the amount of energy falling upon an area defined by the diameter of the first dark ring of the diffraction pattern relative to that produced by an unobscured, perfect optical system, normalized to unity (that is, divided by 0.84, since 84 percent of the energy in an unaberrated diffraction image falls within the first dark ring of the diffraction pattern). This concept is illustrated graphically in Figure 3-5. The advantage of this criterion is that the effects of obscuration as well as aberration, figure and the like can be taken into account. Also, effects of small changes in the profile of the image within the confines of the area defined by the first dark ring of the diffraction pattern will be given less weight than would be the case with the Strehl definition. Since the concept of normalized relative energy is based upon a fixed detector area, its applicability to instruments with small defining apertures (such as spectrographs and photometers) is obvious. A rigorous derivation of the normalized relative energy is presented in Appendix B, and as in the case of the modified Strehl definition, the results are given in the comparison contained in Section 3.7.4.

A third viewpoint from which the scientific data output of the telescope should be viewed is that of post exposure processing by digital computer techniques. The basis for image processing will now be discussed, and later, criteria for design optimization indicated by this new technology will be derived.

$$\text{NRE} = \frac{\text{Energy on Resolution Element}}{0.84 \times \text{Total Theoretical Energy}}$$

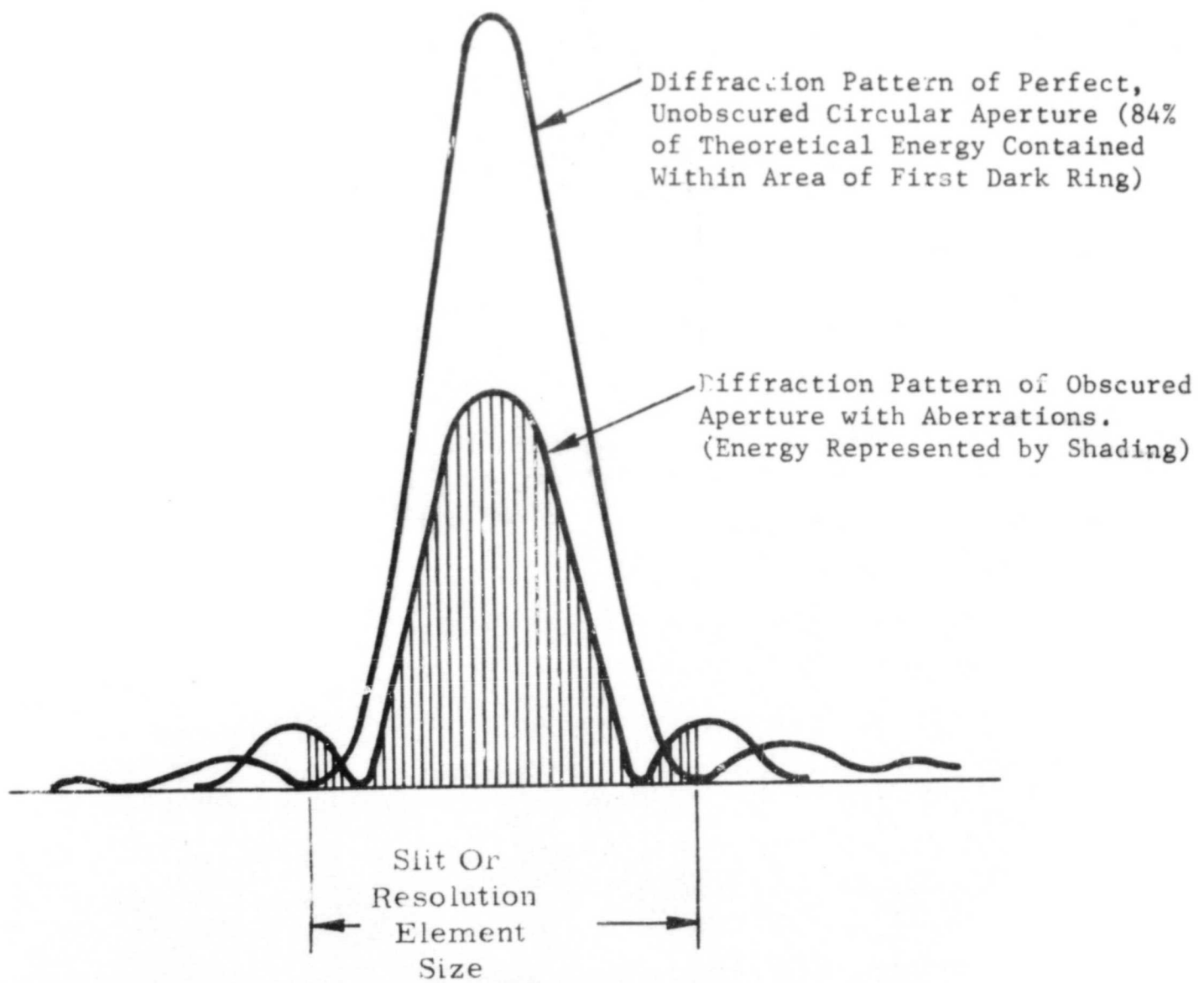


Figure 3-5. Definition of Normalized Relative Energy

3.2 IMPACT AND LIMITATIONS OF IMAGE PROCESSING

3.2.1 Introduction

This section discusses the potential role of image processing as a basic LST system element, by indicating both the uses and limitations of this technique.

Image processing can be used to achieve either of two fundamental objectives, defined below:

Image Restoration - For this application, image processing attempts to take image data suffering from degradations and process that data such that the resultant image is as true as a representation of the object as possible. The resultant image thus contains the maximum possible amount of recoverable object information.

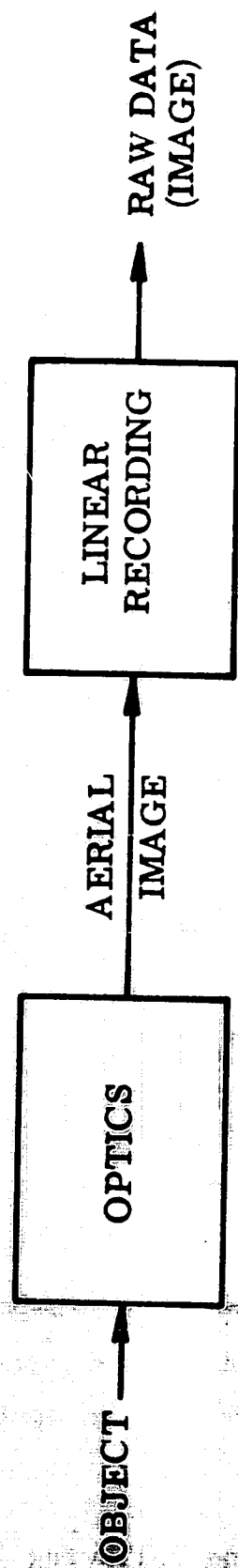
Image Enhancement - For this application, the objective is to take an image containing a fixed amount of recoverable information, and make that information more easily recoverable by changing the presentation of the data - i.e., change the appearance of the image, by various methods (such as an increase in contrast) to make that data more presentable to an observer. A priori knowledge about the object (such as symmetry properties) are often also used in image enhancement.

Thus the processing of an image motion degraded low contrast image to remove the effects of the motion represents restoration, while an artificial increase in the contrast of that image represents enhancement.

Figure 3-6 indicates two basic modes for operation of the LST; for the first mode, shown at the top of Figure 3-5, the prime focus image is relayed to an image recording device, which then records the aerial image with the greatest possible fidelity. This "raw" data representing the recorded image is then transmitted to the ground, where an image is reconstructed, and the data viewed by human observers.

For the second mode, the prime focus image is relayed to an instrument which then optically analyzes the image with the purpose of estimating the value of one or more parameters characterizing the nature of the object. The

● RECORDING MODE



● ANALYSIS MODE

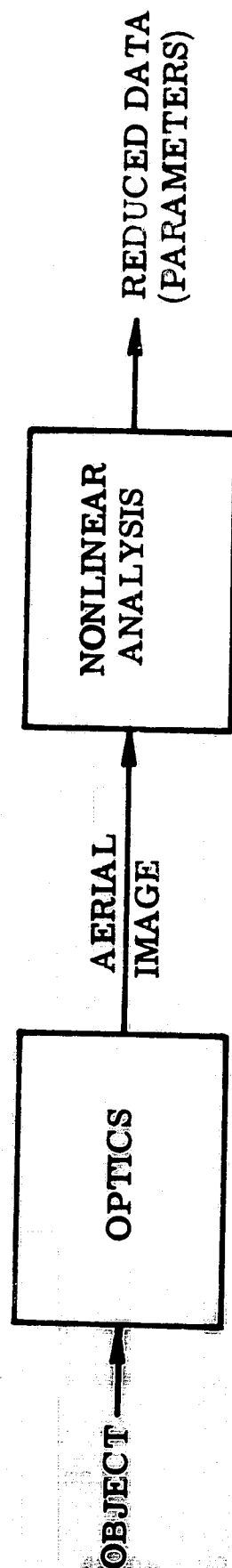


Figure 3-6. Modes of LST Operation

estimates of the parameters of interest are then transmitted to the ground; such parameters include the total flux of dim objects, stellar diameters, spectral characteristics, etc.

Of these two modes, only the first represents a potential use of image processing function, since image processing requires "raw" unreduced data on the intensity distribution of the object.

3.3 IMAGE PROCESSING OF LST IMAGES

The first image processing function mentioned above, restoration, is the function that would most likely be used for LST images, since the prime function of the LST is to probe the unknown; thus one wishes to obtain images which accurately represent the unknown objects being observed. The restoration would be performed to counter various system degradations, as outlined in paragraph 3.4..

The second function mentioned above, enhancement, would entail some processing of images for special applications, in which some a priori information of the object would be assumed (one example might be special processing to aid the separation of a double star); the discussion which follows is restricted to the first function, restoration.

Mathematically, it is convenient to describe the effect of LST system components on the imagery obtained by using Optical Transfer Functions (OTF's). Thus the imaging characteristics of the LST can be written as

$$S_i(k_x, k_y) = H(k_x, k_y) S_o(k_x, k_y) \quad (3-3)$$

where $S_i(k_x, k_y)$ and $S_o(k_x, k_y)$ are the Fourier transforms, respectively, of the image and the object scene, and $H(k_x, k_y)$ is the optical system Optical Transfer Function (OTF). The variables k_x and k_y denote the spatial frequency coordinates in the X- and Y-directions.

Image restoration can be performed by the following linear filtering operation:

$$S_r(k_x, k_y) = F(k_x, k_y) S_i(k_x, k_y) \quad (3-4)$$

Here we have filtered the image, in the frequency domain, with a filter having a transfer function $F(k_x, k_y)$. If $F(k_x, k_y)$ is chosen as the ratio of a desired or ideal OTF $D(k_x, k_y)$ and the actual OTF $H(k_x, k_y)$, i.e.,

$$F(k_x, k_y) = \frac{D(k_x, k_y)}{H(k_x, k_y)} \quad (3-5)$$

then the filter will remove the low-pass effects of the optical system, producing an image that would result from an ideal diffraction-limited system. The result of this image processing is then the modification of the OTF such that it assumes a more desirable form after processing. The resulting image is usually clearer, having been "deblurred".

Such operations have been successfully performed on astronomical imagery, notable the imagery obtained from Stratoscope II (processed by Princeton University).

Any degradation of the system that manifests itself as a lowering of the system transfer function can in principle be compensated for in the image processing function as outlined above; some of the more obvious ones are listed below.

1. Effect of obscuration ratio,
2. MTF of image recording device (vidicon, etc.),
3. differential image motion due to the Bradley effect, and
4. aberrations of the optics (both residual design aberrations, and aberrations resulting from misalignments of the optics).

Figure 3-7 illustrates how some of these can combine to yield a system transfer function significantly lower than the transfer function of the diffraction-limited aperture that represents the design goal of the LST; as long as the degradations do not lower the system MTF to the point where noise overwhelms the recorded data, restoration via digital image processing is a practical, proven technique.

As mentioned above, noise represents the limiting factor in any image processing operation, due to the fact that the processing function boosts

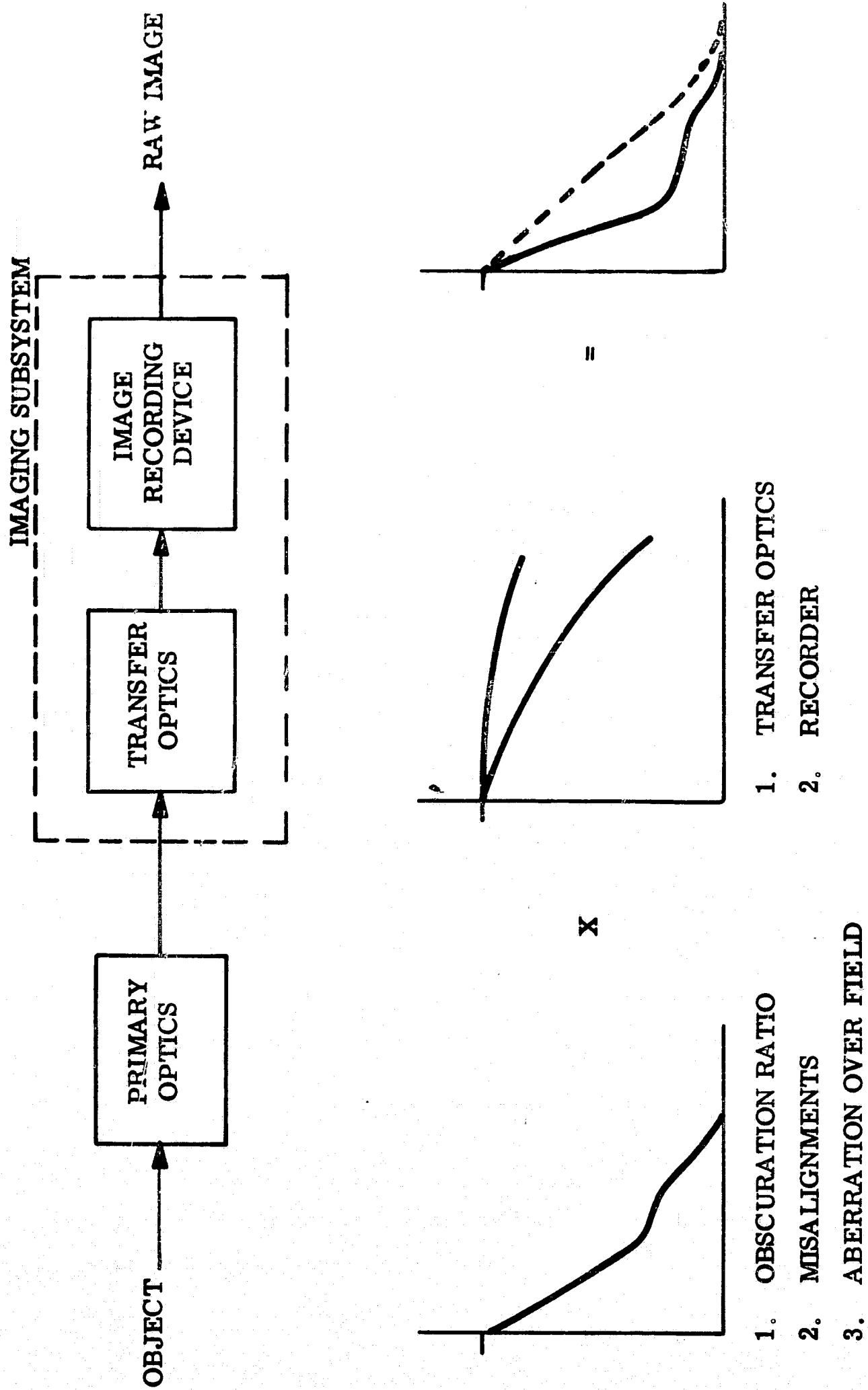


Figure 3-7. LST as Imaging System

the noise frequency components, as well as the image frequency components.

The problem of noise in an image can be reduced by the application of several restoration techniques. The simplest in concept is to average multiple exposures of the same object scene. This procedure will reduce the noise standard deviation by the square root of the number of exposures averaged together, but requires accurate registration of the multiple images (this technique has been successfully applied to Stratoscope II images of Uranus). If only one image of a scene is available, however, more sophisticated techniques must be applied. These include linear (Weiner) filtering, and nonlinear filtering, which is a statistical technique based on a priori knowledge of the statistical properties of the object and noise.

3.4 IMPLICATIONS OF IMAGE PROCESSING ON THE LST CONCEPT

In view of the above discussions, there are two identifiable implications of image processing on the LST concept, as follows:

1. Assuming that the system signal to noise ratio is adequate, images can be obtained through a system whose net system transfer function (after processing) is that of a diffraction-limited system having a zero obscuration ratio, and

2. Since the difficulty of achieving high optical performance in an astronomical telescope increases as the field coverage requirements increases, the possibility is afforded by image processing that this difficulty can be reduced by processing the field images to recover from less well corrected images the same information that would be contained in well corrected field images.

A survey of current accomplishments is contained in Appendix C.

3.5 DESIGN AND OPERATIONAL CONSIDERATIONS

The use of the previously discussed measures of optical performance will fall into two areas: first, they can be used to optimize the optical design within the given space constraints, and secondly, they can be used to evaluate various operational effects, such as thermally induced misalignment, and to determine the sensitivity to such effects of a particular design. In

the first case, prominence is to be given to the design variables that are under consideration, while the second, the operational variables must be explicit in the formulation of the figure of merit. An often used technique for tolerancing optical systems is to apply the Marechal criterion* which relates the relative intensity at the peak of a diffraction image to the rms wavefront aberrations. The wavefront aberrations can be related to both operational and design variables, as will be shown later in this report. The Marechal criteria is usually expressed as

$$i(p) \cong 1 - \left(\frac{2\pi}{\lambda}\right)^2 (\Delta\Phi)^2 \quad (3-6)$$

where $i(p)$ is the peak intensity, λ is the image wavelength of the light, and $\Delta\Phi$ is the rms wavefront aberration.

Where it is possible to combine into a single figure of merit both the design and operationally oriented variables, it becomes possible to determine the relative value of executing a given design option in terms of already known degrading influences. For example, in the previous discussion, the effect of central obscuration upon the peak intensity of the diffraction image was described. By making this relationship explicit, and combining with the conventional Strehl definition (which shows explicitly the effect of uncorrected design aberrations), the relative gains to be achieved by diminishing the central obscuration as opposed to additional compensation for optical aberrations (by adding corrector lenses or field flatteners for example) can be evaluated and decisions reached which are based on quantitative information. One way of reducing the central obscuration is to remove the light baffles when the telescope is being used to view a dark area of the sky. The relative increase in performance can be calculated and then weighed against other design refinements which will undoubtedly be proposed. In this way, those with the highest pay-off in terms of performance can be selected. Our objective, then, is to derive (where possible) a single measure of performance incorporating both design and operational variables.

*Page 469, Reference 4

Equation (3-6) is plotted in Figure 3-8. It is quite interesting to compare Figures 3-4 and 3-8. Note that equivalent reductions in the intensity at the peak of an image are caused by wavefront aberrations and by central obscuration. Figure 3-9 illustrates the effects each causes on the image profile, where the wavefront error is induced by misalignment. For example, a central obscuration of 0.325 appears to cause the same reduction in intensity as a wavefront aberration of 0.0715λ (rms). The question is how do these effects combine: are they additive or multiplicative? This question is answered in the next section, as we shall see.

3.6 TECHNIQUES TO BE USED IN STUDY

In this section the theoretical basis for a figure of merit intended to maximize the intensity at the center of the images will be derived, starting with a definition of the Strehl ratio which includes the effect of the central obscuration as well as the aberrations. An important aspect of the figure of merit is that when the various optical aberrations are considered individually, the amount tolerable for a given value of the Strehl ratio is not constant, but decreases as obscuration increases in the case of some aberrations (coma, astigmatism) and actually increases in the case of others (third order spherical aberration). This result modifies the relationship between misalignment and primary mirror focal ratio. (See Appendix A.)

3.6.1 Method of Attack

In the classical methods of geometric analysis and optimization of an optical system, the optical wavefront emerging from the exit pupil of the optical system is compared to an ideal spherical wavefront converging to the Gaussian focal point. The difference in the optical path between the actual wavefront and the reference sphere at each point on the wavefront is determined and is expressed mathematically as a power series having as variables the location of the Gaussian focal point, and the polar coordinates of the ray in the exit pupil. The coefficients of the terms in the power series can be related to focal point shifts, focal plane curvature, and to other aspects of the optical design. The magnitude of the aberrations (e.g., spherical, coma, astigmatism) are the numerical value of the coefficients of certain

$$i(p) = \left[1 - \frac{4\pi^2}{\lambda^2} (\Delta\phi)^2 \right]$$

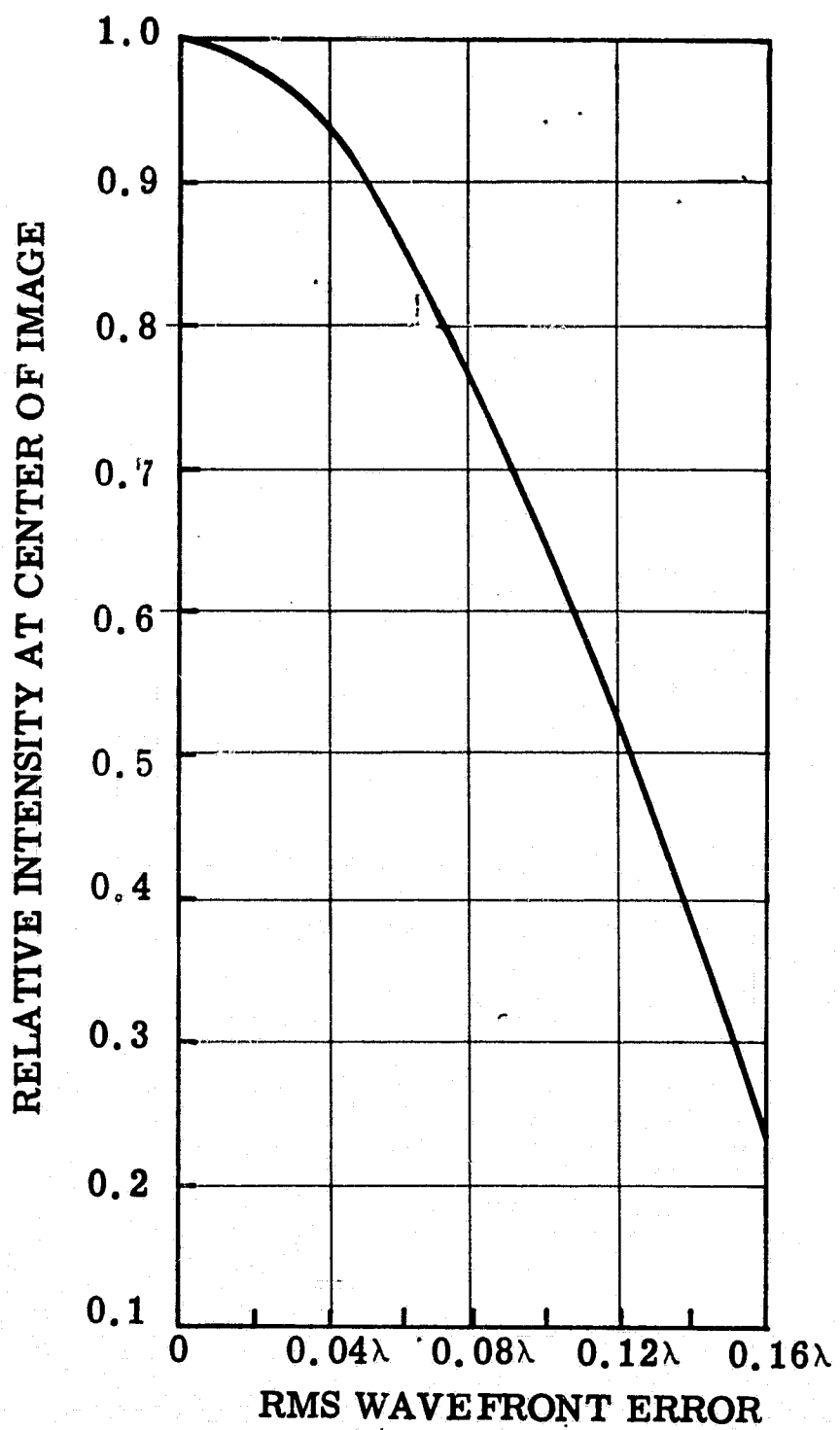
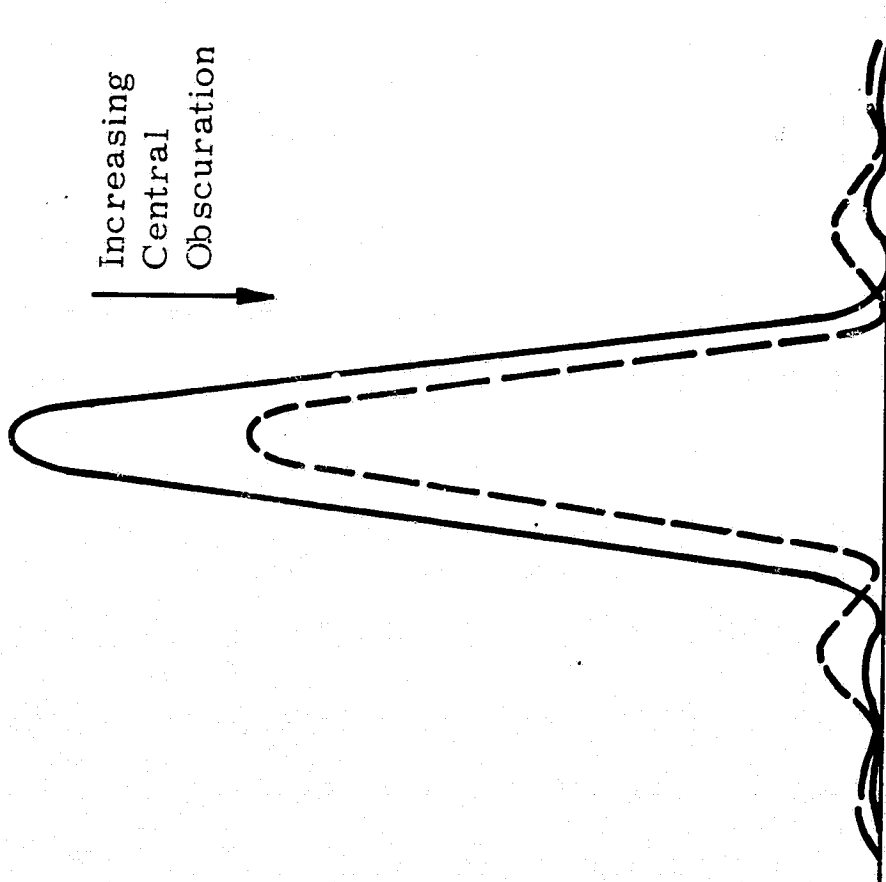


Figure 3-8. Effect of Wavefront Tolerance

DESIGN

EFFECT OF INCREASING
CENTRAL OBSCURATION

Increasing
Central
Obscuration
↓



OPERATIONAL

EFFECT OF
MISALIGNMENT

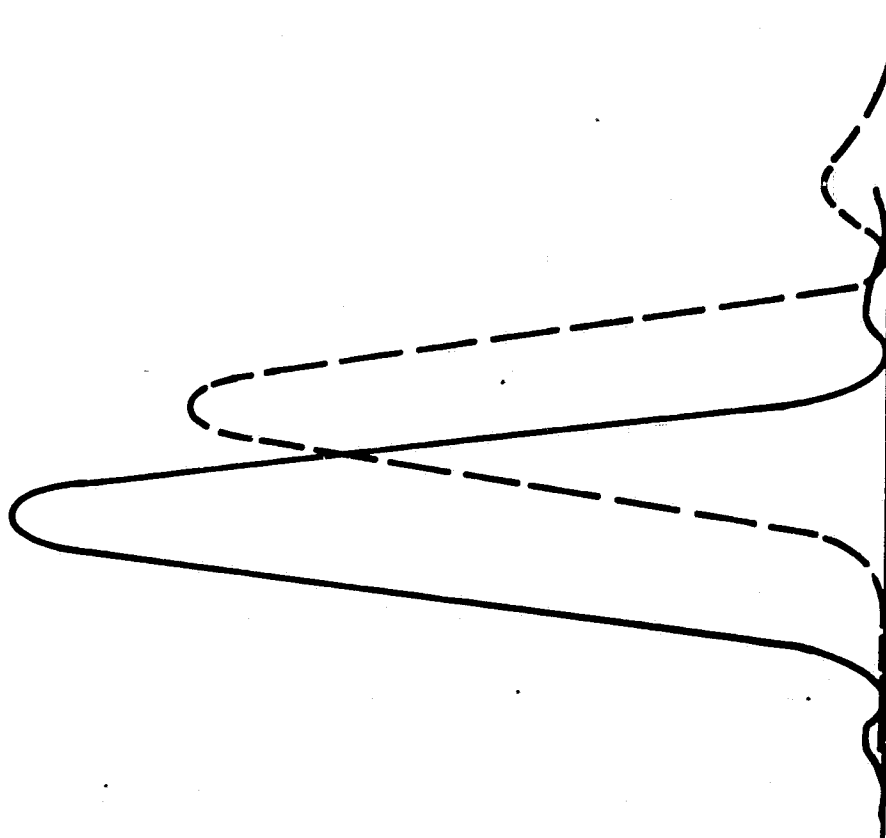


Figure 3-9. Examples of Types of Degradations

terms in the series expansion. The Seidel third-order aberrations of classical geometric optical design are the coefficients of a particular five terms in the series.

As shown in O'Neill's book³, the effects of some of the low order terms of the series expansion (typically coma, astigmatism) representing the wavefront errors can be offset by adjusting the optical design so that certain lower order terms of the series compensate for those of higher order. This process is typically called "balancing the aberrations". O'Neill runs through several examples of optimization. The result of his optimization process is a particular relationship between the coefficients of the various powers of the variables in the series expansion of the wavefront error.

An alternate way of representing the wavefront error has been worked out by Zernike in the form of a set of complete, orthogonal polynomials for a unit circle⁵. O'Neill³ points out that each of these polynomials infers the results of the optimization described above in which controlled amounts of lower order aberrations are used to balance out higher order aberrations. Thus, the Zernike polynomials are a particularly powerful way of representing the optical path errors in that the representation leads to ways of decoupling and optimizing operational and design variables, such as alignment and central obscuration.

While it would be interesting and informative to illustrate here the use and power of the Zernike polynomials, this topic is covered in Chapter 4 of O'Neill³, and Section 9.2 and Appendix VII of Born and Wolf⁴. The interested reader is referred to these books for additional explanation and detail.

Now, in order to utilize the Zernike polynomial representation of the wavefront in the performance analysis of a centrally obscured telescope, it is necessary to re-derive the polynomials, taking into account the central obscuration, since the original polynomials are no longer orthogonal if applied to a wavefront with a central obscuration. This has previously been done at Perkin-Elmer and the results are briefly described in the pages that follow.

Secondly, the Zernike representation must be related to a figure-of-merit criterion that treats the aberrations over a centrally obscured pupil.

The figure-of-merit chosen is the Strehl ratio, which is defined as "the ratio of the light intensity at the maximum of the diffraction pattern (produced by an actual instrument) to that of the same instrument without aberrations"³ or central obscuration. Since we are interested in investigating the effect of central obscuration, the definition we shall adopt for this study will be the above ratio of diffraction pattern intensities, where the denominator of the ratio is the diffraction pattern intensity produced without aberrations and without the central obscuration* and will be called "modified Strehl ratio".

Finally, the maximum tolerable aberrations which achieve a given Strehl ratio with an optical system having a central obscuration will be derived, and the results for typical aberrations as a function of central obscuration ratio are plotted.

3.6.2 Derivation

Following the procedure in Born and Wolf,⁴ we write the aberration functions for a given field position as

$$\Phi = A_{00} + \frac{1}{\sqrt{2}} \sum_{n=2}^{\infty} A_{n0} R_n^0(\rho) + \sum_{n=1}^{\infty} \sum_{m=1}^n A_{nm} R_n^m(\rho) \cos m\theta \quad (3-7)$$

where

A_{nm} are weighting coefficients which are implicitly a function of field position

R_n^m is the orthogonal Zernike circle polynomials radial component.

Since we are concerned with the effect of centrally obscured apertures, we will generate polynomials that are orthogonal over a unit circle

*Note that an implicit assumption in these investigations is that exposure time is maintained constant.

with a central obscuration of radius ϵ . That is,

$$\int_0^{2\pi} d\theta \int_{\epsilon}^1 \rho d\rho \left[V_n^{\ell}(\rho, \theta) \right] V_{n'}^{\ell'}(\rho, \theta) / \int_0^{2\pi} d\theta \int_{\epsilon}^1 \rho d\rho = \frac{1}{n+1} \delta_{\ell\ell'} \delta_{nn'} \quad (3-8)$$

$$\int_{\epsilon}^1 R_n^{\ell}(\rho, \epsilon) R_{n'}^{\ell'}(\rho, \epsilon) \rho d\rho = \frac{(1-\epsilon^2)}{2(n+1)} \delta_{nn'}$$

Note that our generalized Zernike circle polynomials are a function of both ρ and ϵ . The generalized polynomials are listed in Table 3-2.

TABLE 3-2

ZERNIKE CIRCLE POLYNOMIALS FOR
UNIT CIRCULAR APERTURE WITH OBSCURATION OF RADIUS ϵ^{**}

$$R_2^0 = \left[2\rho^2 - (1+\epsilon^2) \right] / (1 - \epsilon^2)$$

$$R_4^0 = \left[6\rho^4 - 6(1+\epsilon^2)\rho^2 + (1 + 4\epsilon^2 + \epsilon^4) \right] / (1-\epsilon^2)^2$$

$$R_1^1 = \rho / (1+\epsilon^2)^{1/2}$$

$$R_3^1 = \left[3(1+\epsilon^2)\rho^3 - 2(1+\epsilon^2 + \epsilon^4)\rho \right] / \sqrt{(1-\epsilon^2)^2 (1+5\epsilon^2 - 5\epsilon^4 + \epsilon^6)}$$

$$R_2^2 = \rho^2 / \sqrt{1+\epsilon^2 + \epsilon^4}$$

*where $V_n^{\ell}(\rho, \theta) = R_n^{\ell}(\rho, \epsilon) e^{i\ell\theta}$, the Zernike circle polynomial

**This table was derived by J. Patterson under the direction of Dr. R.E. Hufnagel⁶. See also Appendix E.

The variance of the aberration function, E, will now be computed.

$$E = \overline{\Phi^2} - (\overline{\Phi})^2 \quad (3-9)$$

By definition

$$\begin{aligned} \overline{\Phi} &= \frac{\int_0^{2\pi} d\theta \int_{\epsilon}^1 \Phi \rho d\rho}{\int_0^{2\pi} d\theta \int_{\epsilon}^1 \rho d\rho} \\ &= \frac{\int_0^{2\pi} d\theta \int_{\epsilon}^1 \Phi \rho d\rho}{\pi(1 - \epsilon^2)} \end{aligned} \quad (3-10)$$

Substituting (3-7) into (3-10)

$$\begin{aligned} \overline{\Phi} &= \frac{1}{\pi(1 - \epsilon^2)} \int_0^{2\pi} d\theta \int_{\epsilon}^1 \rho d\rho \left\{ A_{00} + \frac{1}{\sqrt{2}} \sum_{n=2}^{\infty} A_{n0} R_n^0(\rho, \epsilon) + \sum_{n=1}^{\infty} \sum_{m=1}^n A_{nm} R_n^m(\rho, \epsilon) \cos m\theta \right\} \\ &= A_{00} \end{aligned} \quad (3-11)$$

since all terms containing generalized polynomials $R_n^m(\rho, \epsilon)$ for n or $m \neq 0$ average to zero (i.e., orthogonal to constant term).

Also, by definition

$$\begin{aligned} \overline{\Phi^2} &= \frac{1}{\pi} \frac{1}{(1 - \epsilon^2)} \int_0^{2\pi} d\theta \int_{\epsilon}^1 \Phi^2 \rho d\rho \\ &= \frac{1}{\pi} \frac{1}{(1 - \epsilon^2)} \int_0^{2\pi} d\theta \int_{\epsilon}^1 \rho d\rho \left[A_{00}^2 + \frac{1}{2} \sum_{n=2}^{\infty} A_{n0}^2 [R_n^0(\rho, \epsilon)]^2 \right. \\ &\quad \left. + \sum_{n=1}^{\infty} \sum_{m=1}^n A_{nm}^2 [R_n^m(\rho, \epsilon)]^2 \cos^2 m\theta \right. \\ &\quad \left. + [\text{cross product terms}] \right] \\ &= A_{00}^2 + \frac{1}{2} \sum_{n=1}^{\infty} \sum_{m=0}^n A_{nm}^2 / (1+n) \end{aligned} \quad (3-12)$$

As previously stated, we will reference all image intensity measurements to the image formed by an unobscured aperture (i.e., lower limit of integration in the denominator is zero). This will allow us to obtain directly the reduction in the modified Strehl ratio as the central obscuration is introduced. The modified Strehl ratio is thus defined,

$$\begin{aligned}
 S &= \left| \int_0^{2\pi} d\theta \int_{\epsilon}^1 \rho d\rho e^{ik\Phi} \right|^2 / \left| \int_0^{2\pi} d\theta \int_0^1 \rho d\rho \right|^2 \\
 &\cong \frac{1}{\pi^2} \left| \int_0^{2\pi} d\theta \int_{\epsilon}^1 \rho d\rho \left[1 + ik\Phi + \frac{1}{2}(ik\Phi)^2 + \dots \right] \right|^2 \\
 &= (1-\epsilon^2)^2 \left\{ \left(1 - \frac{1}{2} k^2 \overline{\Phi^2} \right)^2 + k^2 \overline{\Phi^2} \right\} \\
 &\cong (1-\epsilon^2)^2 \left\{ 1 - k^2 \overline{\Phi^2} + k^2 \overline{\Phi^2} \right\} \\
 S &= (1 - \epsilon^2)^2 \left\{ 1 - \left(\frac{2\pi}{\lambda} \right)^2 (\Delta\Phi)^2 \right\} \tag{3-13}
 \end{aligned}$$

where the "mean square deformation" is

$$(\Delta\Phi)^2 = \overline{\Phi^2} - \frac{2}{\Phi}$$

The modified Strehl ratio can be expressed in terms of the aberration weighing coefficients by substituting (3-11) and (3-12) into (3-13)

$$S = (1 - \epsilon^2)^2 \left\{ 1 - k^2 / 2 \sum_{n=1}^{\infty} \sum_{m=0}^n A_{nm}^2 / (1 + n) \right\} \tag{3-14}$$

In practice, optical aberrations are usually considered in two classes: 1) high spatial frequency terms usually resulting from fabrication errors (e.g., concentric rings resulting when the mirror is hand finished); and 2) low spatial frequency terms resulting from misalignments, defocus and bending of the mirror. The high frequency terms (n and/or m large) are usually grouped together under the heading - mean square figure error of the system. We combine these errors into a single term which we define as $\overline{\Phi}_{HF}^2$. If we only consider the $n \leq N$ aberration as significant, (3-14) is rewritten

$$S = (1 - \epsilon^2)^2 \left\{ 1 - \frac{k^2}{2} \sum_{n=1}^N \sum_{m=0}^n A_{nm}^2 / (n+1) + k^2 \overline{\Phi}_{HF}^2 \right\} \quad (3-15)$$

We now consider the possibility of "balancing" a given low spatial frequency aberration with its lower order companion aberration(s). To be more specific, suppose the system suffers from a given single aberration term

$$\Phi = A'_{nm} \rho^n \cos^m \theta \quad (3-16)$$

Now we insert weighted lower order aberrations such that the new wave aberration Φ' , which is measured relative to its best fitting sphere, will give the minimum possible **wavefront** error for the conditions assumed.

$$\Phi' = A'_{nm} \rho^n \cos^m \theta + \sum_{p < n} \sum_{q < p} A'_{pq} \rho^p \cos^q \theta \quad (3-17)$$

In terms of the generalized circle polynomials

$$\Phi' = \epsilon_{nm} A_{nm} R_n^m(\rho, \epsilon) \cos m\theta + \sum_{p < n} \sum_{q < p} R_p^q(\rho, \epsilon) \cos q\theta \quad (3-18)$$

where

ϵ_{nm} is $1/\sqrt{2}$, when m is, 0, n is not equal to 0
is 1 otherwise.

Note A_{nm} and A'_{nm} are the new (balanced) and old (unbalanced) weighting coefficients, respectively.

Noting (3-14), the Strehl condition may be maximized by minimizing the mean square aberration Φ^2 defined in (3-12). This infers that all A_{pq}^2 for $p < n$, $q < p$, must be identically zero and that the Strehl ratio is optimized in terms of the new weighting coefficient A_{nm} (3-18).

$$S = (1 - \epsilon^2)^2 \left\{ 1 - \frac{k^2}{2} \frac{A_{nm}^2}{n+1} \right\} \quad (3-19)$$

The compensated aberration thus becomes

$$\Phi' = \epsilon_{nm} A_{nm} R_n^m(\rho, \epsilon) \cos m \theta \quad (3-20)$$

The relation between A_{nm} and A'_{nm} is easily found by noting the coefficient of highest power of ρ for $R_n^m(\rho, \epsilon)$ as found in Table 3-2. We have taken as an illustrative example the same case considered in Born and Wolf, Second (revised) Edition, p. 467 and 471 (Reference 7).

Suppose the system suffers from third order spherical aberration $\Phi = A'_{40} \rho^4$ and we may introduce required compensations of focus $A'_{20} \rho^2$. We seek the values of these weighting coefficients which make the intensity (and thus the Strehl ratio) maximum. Thus the original aberration $A'_{40} \rho^4$ is mapped into the optimized $A_{40} \rho^4$. We have

$$A'_{40} \rho^4 \rightarrow A_{40} \rho^4 \quad (3-21)$$

In terms of the aberration, Φ , (3-20), and Table 3-2, we have

$$\begin{aligned} \Phi &= \frac{1}{\sqrt{2}} A_{40} R_4^0(\rho, \epsilon) \\ &= \frac{A_{40}}{\sqrt{2}} \left\{ 6 \rho^4 - 6(1 + \epsilon^2) \rho^2 + (1 + 4\epsilon^2 + \epsilon^4) \right\} / (1 - \epsilon^2)^2 \end{aligned} \quad (3-22)$$

The coefficients of ρ^2 and the constant term give the amount of defocus and phase shift (usually not of importance) required to optimize the Strehl ratio. The contribution to the aberration function of the compensated image is obtained by noting the coefficient of the ρ^4 term

$$A'_{40} = \frac{6}{\sqrt{2}} \frac{1}{(1 - \epsilon^2)^2} A_{40} \quad (3-23)$$

Setting the modified Strehl ratio equal to 0.8 (usually considered diffraction limited), from (3-19), we have

$$S = 0.8 = (1 - \epsilon^2)^2 \left\{ 1 - \frac{k^2}{2} A_{40}^2 / 5 \right\} \quad (3-24)$$

Substituting from (3-23) and rearranging (3-24)

$$\left(1 - \frac{0.8}{(1 - \epsilon^2)^2} \right) \frac{5\lambda^2}{2\pi^2} = A_{40}^2 = \left(\frac{\sqrt{2}}{6} \right)^2 (1 - \epsilon^2)^4 (A'_{40})^2 \quad (3-25)$$

which infers, after straight algebra

$$\frac{A'_{40}}{\lambda} \leq \frac{3\sqrt{5}}{\pi(1 - \epsilon^2)^2} \left[1 - \frac{0.8}{(1 - \epsilon^2)^2} \right]^{1/2} \quad (3-26)$$

This gives the amount of third order spherical aberration that can be tolerated if the central obscuration is considered as an aberration in the calculation of the Strehl ratio.

3.6.3 Comparison With Conventional Strehl Ratio

If we go back to (3-13) and redefine the denominator ρ integration limit such that the central obscuration is not considered as an aberration in the computation of the Strehl ratio, the weighting coefficient becomes

$$\left. \frac{A'_{40}}{\lambda} \right|_{\odot} \leq \frac{3}{\pi(1 - \epsilon^2)^2} \quad (3-27)$$

Note this infers that the acceptable peak-peak aberration deviation from the best fitting reference sphere is increased as one increases the central obscuration. These results form a better fit between the new spherical reference and the annular wavefront, in the limit ($\epsilon \rightarrow 1$) any wavefront will exactly fit a reference sphere.

Similar treatments for the cases where the central obscuration is considered as an error component yield the tolerance for compensated coma

$$A'_{31}/\lambda \leq \frac{3\sqrt{2}}{\pi} \sqrt{1 - \frac{S}{(1 - \epsilon^2)^2}} \frac{1 + \epsilon^2}{1 - \epsilon^2} \frac{1}{(1 + 5\epsilon^2 - 5\epsilon^4 + \epsilon^6)^{1/2}} \quad (3-28)$$

and compensated astigmatism

$$A'_{22}/\lambda \leq \frac{1}{\pi} \left[\frac{3}{2} \left(1 - \frac{S}{(1 - \epsilon^2)^2} \right) \left(\frac{1}{1 + \epsilon^2 + \epsilon^4} \right) \right]^{1/2} \quad (3-29)$$

These functions are plotted in Figures 3-10, 3-11, 3-12 and 3-13. The curves are accompanied by an adjacent 0 to denote that the function is normalized to the unobscured aperture.

Likewise, when we compare the obscured aperture with aberrations with that same obscured aperture with no aberrations, the Strehl ratio for compensated coma becomes

$$A'_{31}/\lambda \Big|_{\odot} \leq \frac{3(1 - S)^{1/2}}{\pi} \frac{1 + \epsilon^2}{1 - \epsilon^2} \frac{1}{(1 + 5\epsilon^2 - 5\epsilon^4 + \epsilon^6)^{1/2}} \quad (3-30)$$

and compensated astigmatism is

$$A'_{22}/\lambda \Big|_{\odot} \leq \frac{1}{\pi} \left[\frac{3}{2} (1 - S) \left(\frac{1}{1 + \epsilon^2 + \epsilon^4} \right) \right]^{1/2} \quad (3-31)$$

The functions are also plotted in Figures 3-10, 3-11, 3-12 and 3-13. The curves are accompanied by an adjacent symbol $\odot$ to denote that the function is normalized to the obscured aperture.

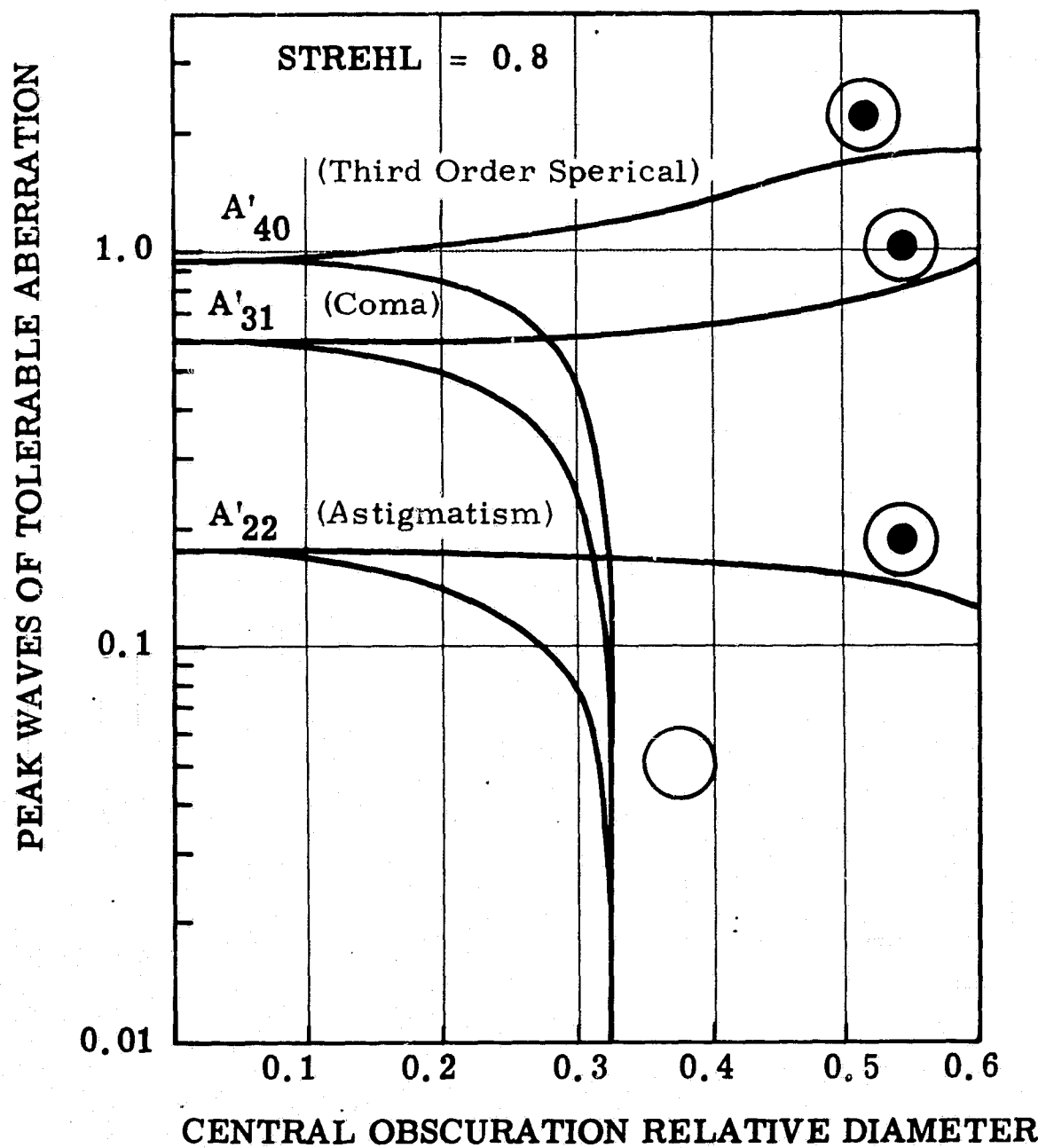


Figure 3-10. Aberration Tolerances, Strehl Ratio 0.8

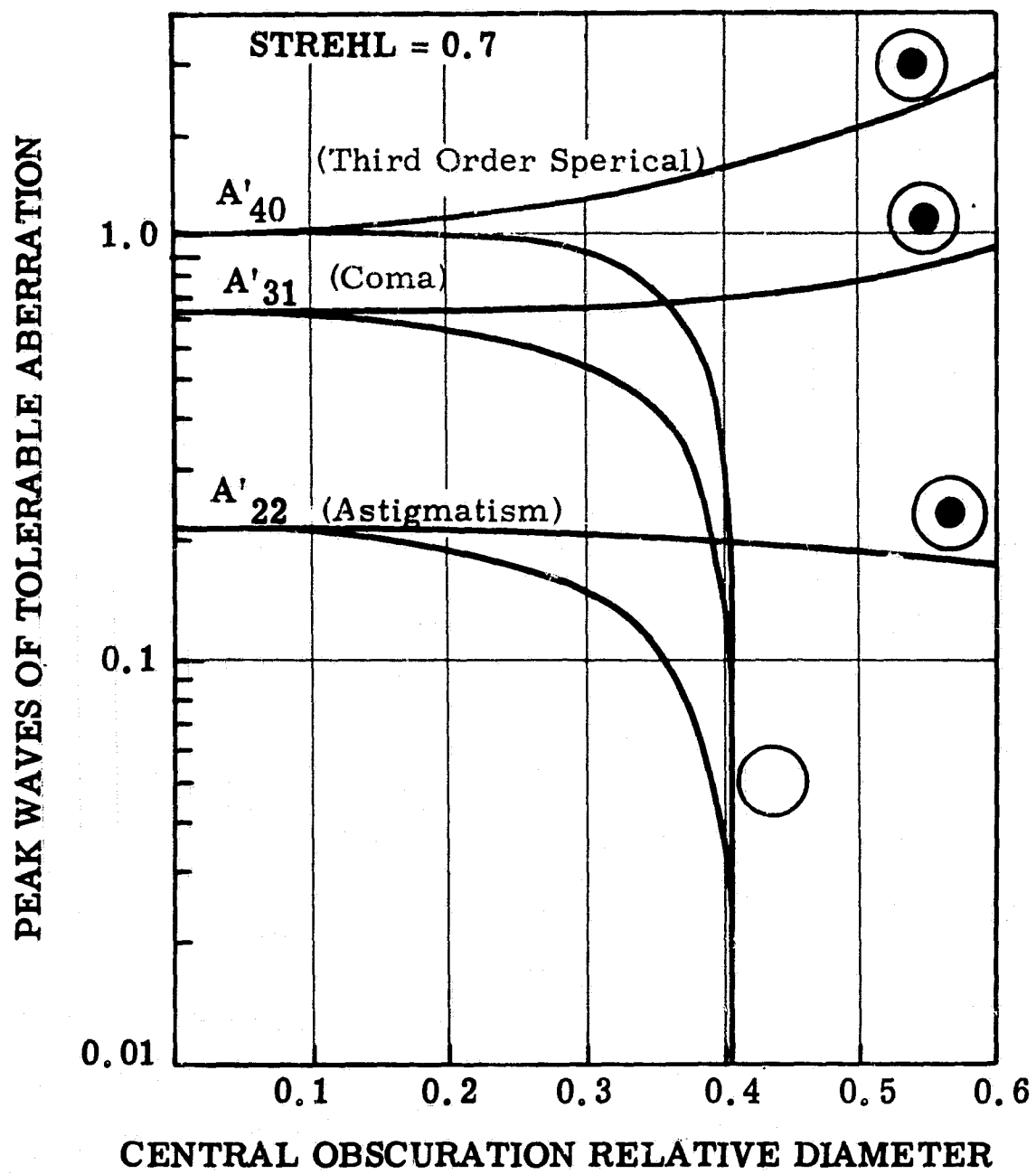


Figure 3-11. Aberration Tolerances, Strehl Ratio 0:7

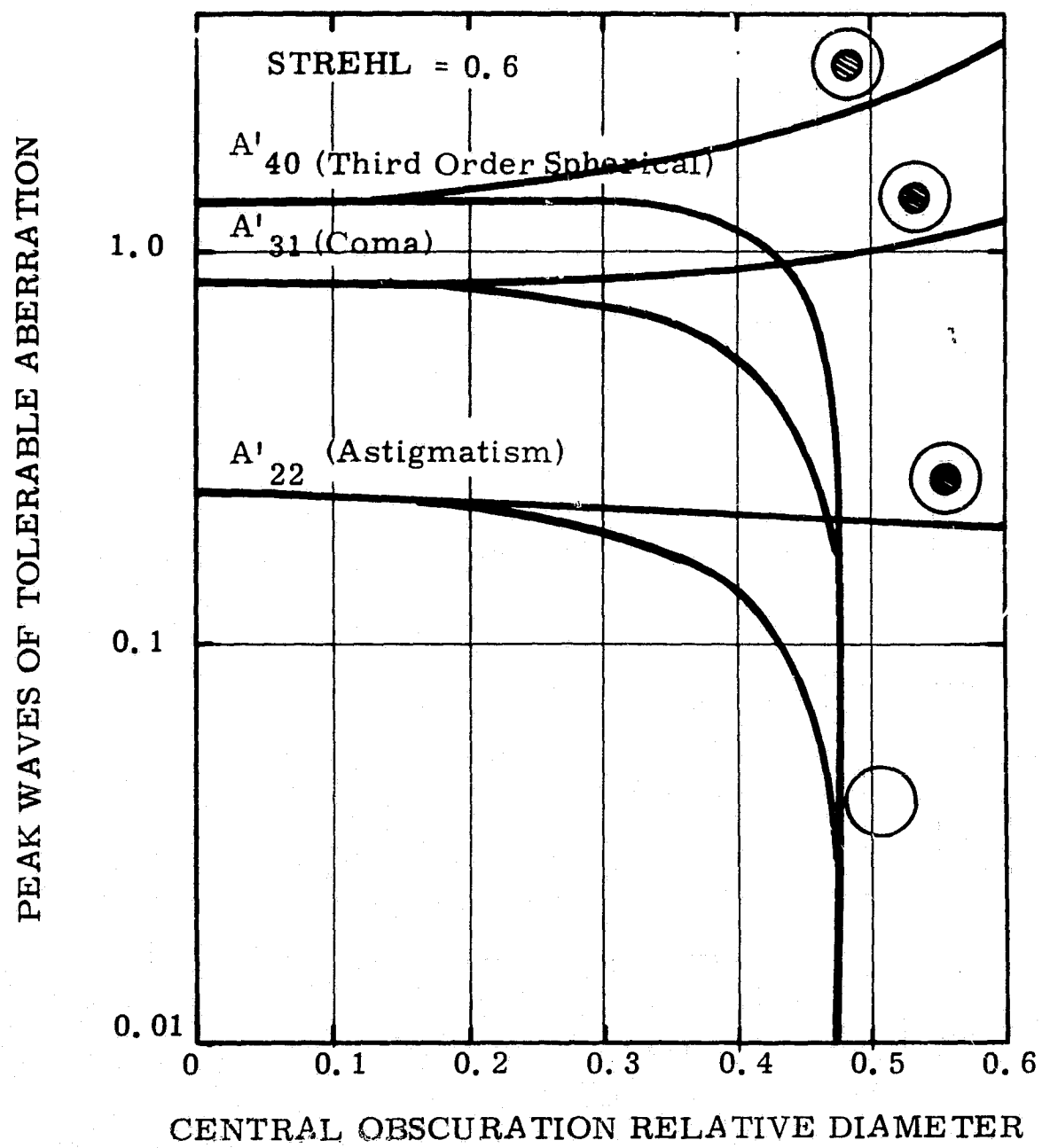


Figure 3-12. Aberration Tolerances, Strehl Ratio 0.6

PEAK WAVES OF TOLERABLE ABERRATION

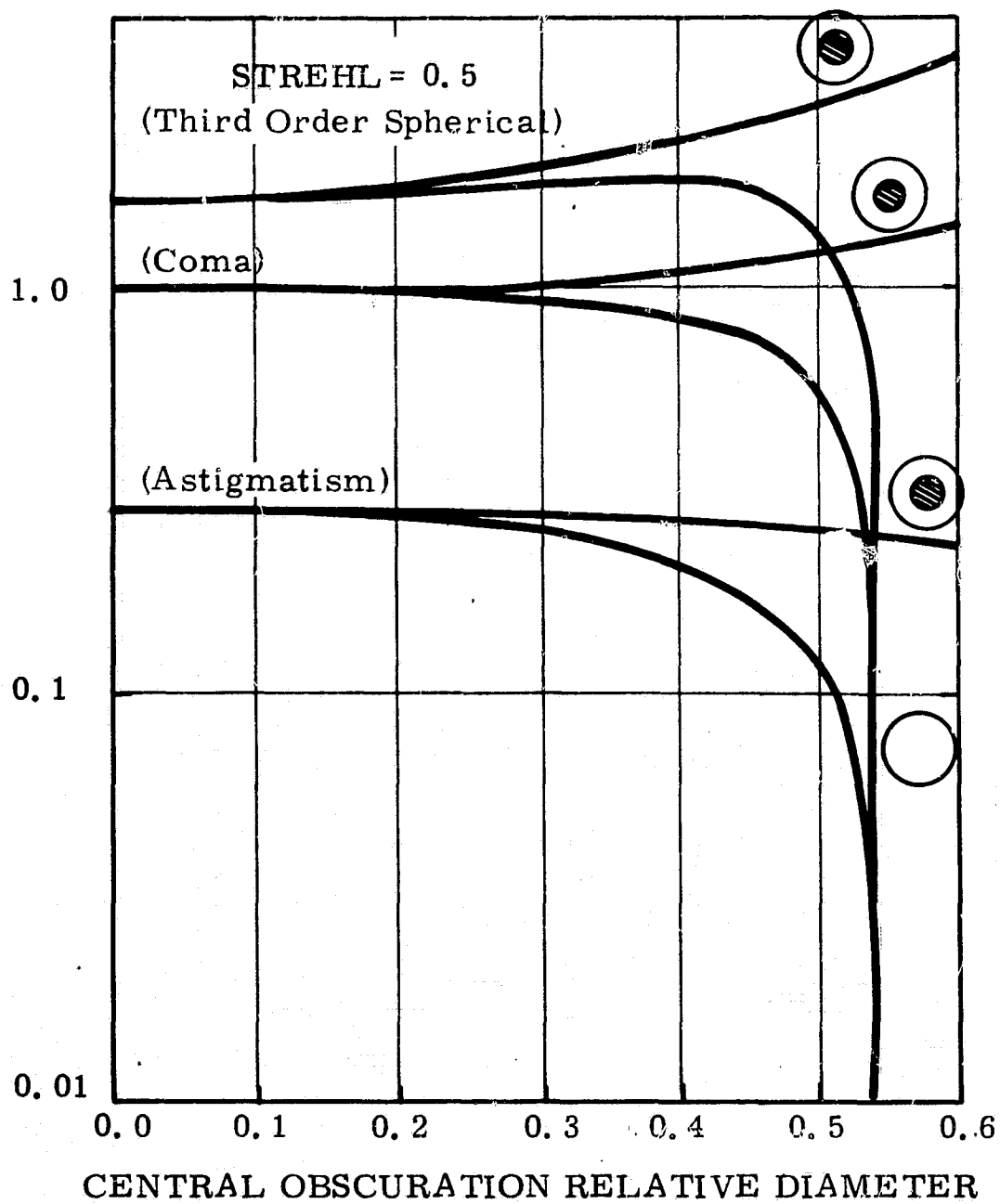


Figure 3-13. Aberration Tolerances, Strehl Ratio 0.5

Study of these curves shows that distinctly different trends for the design tolerance for the variation of the design tolerance with central obscuration result, depending upon the basis for normalization of the Strehl ratio. One basis (inclusion of central obscuration) should be used in design optimization, and the other to determine manufacturing and operational tolerances. The conclusions and numerical values to be derived from the above work are discussed in the next paragraph.

3.6.4 Conclusion

The interpretation of Figures 3-10 through 3-13 is strongly influenced by one's interpretation of the Strehl ratio definition. Recalling the introductory comments of this section, the Strehl ratio is defined as "the ratio of the light intensity at the maximum of the diffraction pattern (produced by an actual instrument) to that of the same instrument without aberrations". In many instrument designs utilizing obscured apertures, the effect of the central obscuration on system performance has been tacitly accepted. Thus the reference for the Strehl definition in this case is the obscured aperture. The data for the curves of Figures 3-10 through 3-13 denoted by the symbol $\odot$ were derived via this definition and give most encouraging results. In general, we may say that an obscured optical system can tolerate increasing amounts of third-order spherical aberration and coma as one increases the diameter of the central obscuration if the above definition of the Strehl ratio is strictly used. Likewise, the same system can tolerate a slightly reduced amount of astigmatism as the obscuration is increased.

Let us now change the "ground rules" for defining the performance of the system "without aberrations". The maximum diameter of the LST will be determined by the launch vehicle quite independent of the optical design. The basis for computation of the Strehl ratio should be the performance of an unobscured clear aperture of maximum permissible diameter. In other words, we

are now considering the central obscuration as an "aberration" in evaluating the performance of a system. At least in the early parametric tradeoff studies for the LST, this is a meaningful procedure. The curves derived using the unobscured aperture as the basis all asymptotically approach the maximum possible central obscuration $\epsilon_{\max}$ for a given Strehl ratio S defined by

$$\epsilon_{\max}^2 = 1 - \sqrt{S}$$

The curves, denoted by the symbol 0, are relatively flat to $\approx 2\epsilon_{\max}/3$, which infers that tolerancing of the positional and/or figure errors of the LST are conventional as long as the central obscuration $\sim < 2\epsilon_{\max}/3$. Since the Strehl ratio of 0.8 is usually considered diffraction limited (which infers $\epsilon_{\max} = 0.325$), the maximum obscuration of the "reasonably toleranced" diffraction-limited system is slightly more than 0.2 and at $\epsilon = 0.3$, the allowable tolerances are reduced by 30 percent.

To state the conclusion in another way, the historic way of tolerancing an optical system (following Marechal) is not much affected by central obscuration until an obscuration ratio (ϵ) of 0.2 is reached at which point the optical aberration tolerances must be tightened in order to achieve the same level of performance. Above $\epsilon = 0.32$, it is no longer possible to achieve the usually desired level of performance.

3.7 RELATIVE EDGE RESPONSE AS A FIGURE OF MERIT

3.7.1 Introduction

Classically, figures of merit have been used to measure quantitatively the performance of optical systems in terms of the system point spread function. The Strehl ratio, which describes the peak intensity of the spread function, is one such figure of merit; the Normalized Relative Energy (NRE), which measures the relative amount of energy within the first minimum of the Airy disk, is another. These criteria are quite useful for most LST applications, since photometers, spectrometers and star trackers usually work directly on star images, which are themselves point spread functions.

However, for applications where the LST is to acquire images of extended objects, the use of point spread function based criteria are not as directly applicable. The following paragraph describes another figure of merit which is more directly keyed to the nature of extended images.

3.7.2 Edge Response as a Figure of Merit

From previous work in the area of image processing of aerial images, and other extended images such as those obtained from electron microscopes, it has been found that the evaluation of such images is based primarily on the effect of the system on the images of lines and edges, since these are far more prevalent in extended imagery than are points. For example, it has been found that the image of an edge should be as sharp as possible, but without ringing, or oscillations. Mathematically, the edge response should be monotonic, and have as large a derivative as possible at the position of the geometrical edge. If ringing does occur, the resultant image contains artifacts, or false ghost edge images, which can mask real objects at the same position, and in general are found to be undesirable to human observers.

It has also been found that if the system transfer function is ~~itself~~ monotonic and does not contain mid-frequency values significantly greater than the value of the MTF of a diffraction-limited aperture, then the system's edge response will be monotonic.

Thus the slope of the edge response of the system is a logical figure of merit to be applied to any imaging system, such as the LST when operating in the imaging mode. Using this figure of merit, it is of interest to investigate

the effect of this figure of merit on the LST system design. This is done in the following section.

3.7.3 Evaluation of the Relative Edge Response (RER)

Equation (3-32), below, describes the image of an edge $e(x)$ obtained through a system having a line spread function $l(x)$

$$e(x) = l(x) * s(x) \quad (3-32)$$

[* denotes convolution]

where

$s(x)$ = step function

$$= \begin{cases} 1 & x > 0 \\ 0 & x < 0 \end{cases}$$

But the line spread function can be expressed in terms of the system point spread function as follows;

$$l(x) = \int_{-\infty}^{\infty} p(x, y) dy \quad (3-33)$$

Thus (3-32) becomes

$$e(x) = \int_0^{\infty} du \int_{-\infty}^{\infty} p(u + x, y) dy \quad (3-34)$$

We now express the system point spread function in terms of the system transfer function $T(k)$ as follows;

$$p(x, y) = \iint T(k_x, k_y) e^{-j(k_x x + k_y y)} dk_x dk_y \quad (3-35)$$

Substituting (3-35) into (3-34) yields the formidable expression given below for the image of the edge described in (3-32):

$$e(x) = \int_0^{\infty} du \int_{-\infty}^{\infty} dy \iint T(k_x, k_y) e^{-j[k_x(u + x) + k_y y]} dk_x dk_y \quad (3-36)$$

Now we find the slope of the edge function described above by taking the derivative of the edge, as follows;

$$e'(x) = \frac{d}{dx} [e(x)] \quad (3-37)$$

Now we can obtain the Relative Edge Response (RER) by evaluating (3-37) at $x=0$, which is the position of the geometrical edge. Thus we have:

$$\text{RER} = e'(x) \Big|_{x=0} \quad (3-38)$$

After some manipulation, the following expression is derived for the RER:

$$\text{RER} = \int_{-\infty}^{\infty} T(k_x, 0) dk_x \quad (3-39)$$

As shown above, the RER is found simply by taking the one-dimension integral of the system transfer function in the spatial frequency direction normal to the edge whose response is of interest. If an average edge response is desired, where the average is to be taken over all possible edge orientations, the (3-39) becomes, for the system transfer function expressed in cylindrical coordinates:

$$\text{RER} = \frac{1}{2\pi} \int_0^{2\pi} \int_0^{\infty} T(k_r, \phi) dk_r d\phi \quad (3-40)$$

Thus the RER can be expressed simply in terms of the system transfer function, in a similar fashion to the Strehl ratio. In fact, the similarity of the RER to the Strehl ratio becomes obvious, by comparing the RER, above, to the Strehl ratio, given in (3-41), below.

$$\text{Strehl} = \iint T(k_r, \phi) k_r dk_r d\phi \quad (3-41)$$

The relationship between the two is more than coincidental since it can be shown that the RER is analogous to the Strehl ratio for line spread function; that is, the RER describes the peak intensity of the line spread function.

Figure 3-14 illustrates the relative values of the RER, the Strehl ratio and the Normalized Relative Energy (NRE) derived in Appendix B for diffraction limited systems having various obscuration ratios. The fact that the RER values are higher than the values for the Strehl for all values of the obscuration ratio stems from the fact that the expression for the RER shown in (3-40) does not contain the factor k_r found in (3-41), which causes the Strehl to weight the higher spatial frequencies more than does the RER. Of the three

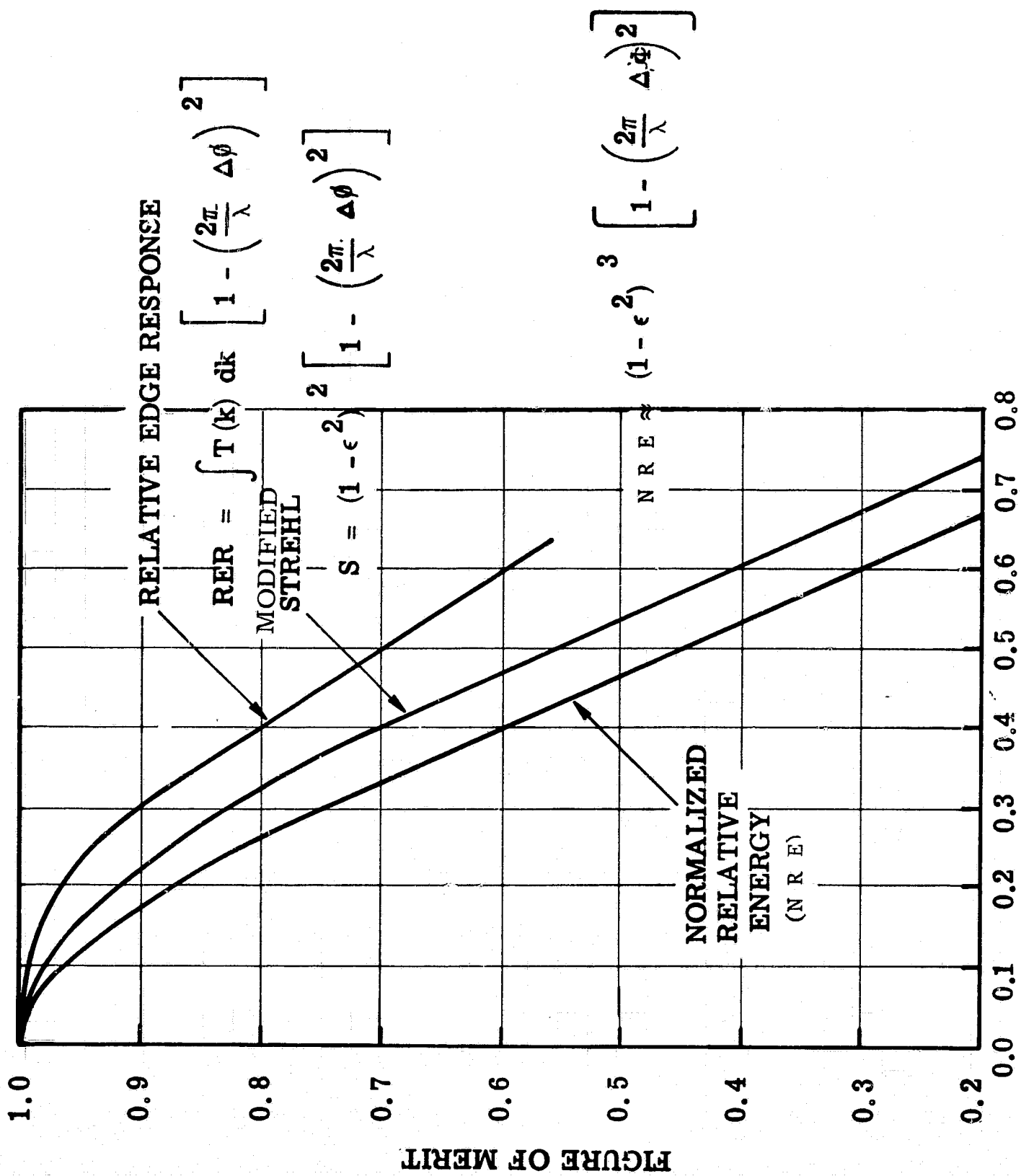


Figure 3-14. Variation of Figures of Merit with Central Obscuration

measures of performance, it is interesting to note that the NRE is most affected by the central obscuration. Since this figure of merit is closely tied to the area of a single resolution element, the addition of another degree (from second power to third power) in the $(1 - \epsilon^2)$ term is perhaps not too surprising as the effect of diffraction around the edges of the smaller central obscuration would be to diffract light onto a larger area.

3.8 TECHNIQUE TO BE EMPLOYED IN STUDY

We have looked at the optical performance of an astronomy telescope from three different viewpoints and have been able to derive three figures of merit which have very similar relationships to the wavefront aberrations, but each of which weight the effect of central obscuration with more or less severity but the falloff in performance as central obscuration is increased is similar in the three cases. Therefore, the design trend which would occur using any one of the three to determine the effect of central obscuration would be similar, as would, of course, the effects of the various wavefront aberrating influences.

One effect of central obscuration has not been taken into account in the figures of merit, and that is the reduction in the diameter of the first dark ring as central obscuration is reduced. This is shown in Figure 3-3, which shows the profile of the diffraction image of a point source for various obscuration ratios. Note that the intensity of the rings surrounding the central core of the image increases with central obscuration even though the radius and intensity of the central core decrease. It is probably that the size of the resolution element or spectrometer slit would be designed to match the diameter of the central core, since the central obscuration and hence the dark ring diameter would certainly be known well before the detectors would have to be specified. Therefore, a somewhat different relationship for the reference area of a normalized relative energy figure of merit would have to be used. Further, the light diffracted into the ring structure should be taken into account as it effects other images and the signal-to-noise obtained at other resolution elements.

Because of these unknowns, and because of the similarity in the trend with central obscuration, the technique selected for general use in the balance of the study is the modified Strehl ratio. Figure 3-15 restates this definition.

DESIGN: STREHL RATIO, MODIFIED TO INCLUDE EFFECT OF CENTRAL OBSCURATION
 DEFINED AS RATIO OF LIGHT INTENSITY AT PEAK OF DIFFRACTION IMAGE WITH
 OBSCURATION AND ABERRATIONS TO IMAGE PRODUCED BY SAME APERTURE
 WITH NO OBSCURATION OR ABERRATIONS

$$i(P) = (1 - \epsilon^2)^2 \left\{ 1 - \left(\frac{2\pi}{\lambda} \right)^2 (\Delta\Phi)^2 \right\}$$

ϵ IS OBSCURATION RATIO (DIAMETERS)

$\Delta\Phi$ IS WAVEFRONT ABERRATION (RMS)

$i(P)$ IS INTENSITY RATIO AT PEAK OF DIFFRACTION IMAGE

Figure 3-15. Optimization Figure of Merit

SECTION 4

OBSCURATION DISCUSSION

In order to determine the effects of the central obscuration, we shall use the modified Strehl ratio which has been previously discussed. In the modified definition, the ratio consists of the value at the peak of the diffraction image produced by a telescope with aberrations and with central obscuration, to the peak intensity in the diffraction image produced by an unaberrated, unobscured circular aperture. The technique then employed is to construct the geometry for the telescope in which the location of the focal point shared between the primary and secondary mirrors and the distance between the mirrors are the variables, and by locating key ray intercepts, pupil locations, etc., the size of the secondary mirror, the size of the hole in the primary mirror, and the size of the baffles required to shield the focal plane from direct light are all determined. The input variables are the speed or focal ratio of the primary mirror and the spacing between the mirrors. Once the size of the central obscuration is determined for a given set of parameters, the value of the Strehl ratio can be calculated using the previously derived equation. Since no aberrations of the optical wavefront were considered in this discussion, the equation for the Strehl ratio reduces to

$$S = (1 - \epsilon^2)^2$$

The ground rules imposed by the statement of work limit the telescope design in the following parameters: the diameter of the primary mirror is to be three meters, the primary to secondary mirror separation is to be equal to or less than 8.7 meters, and the telescope final focal plane is to be 1.8 meters behind the primary mirror front surface. The parameters available as variables in the optimization process include: the telescope configuration (Cassegrain, Gregorian, or other), the primary to secondary spacing, the primary mirror focal length, and the portion of the image plane to be baffled for direct incoming light. An analysis was performed for the Cassegrain and Gregorian configurations to graphically demonstrate the effects of varying the

case was the resulting value of central obscuration. The amount of central obscuration has been shown to produce an upper limit to the attainable Strehl ratio and therefore it will be desirable to minimize this while keeping within the constraints imposed by all other system evaluation criteria.

There are three basic items which determine the central obscuration present in a given system. These are: the diameter of the secondary mirror, the diameter of the hole in the primary mirror required to pass the desired field of view, and the baffling required to provide the desired baffling over the final image plane. Each of these obscurations can be analyzed independently and the obscuration that will govern in a given case will be the largest of the three sources.

There are two particular cases which were noted. The first was where the central obscuration is a minimum without regard to field baffling. This is obtained when the secondary mirror diameter is equal to the primary mirror center hole diameter. The second case is the situation using the minimum baffling required to provide a fully baffled full field with respect to direct incoming stray light. It was determined by the analysis that the imposition of the baffling requirements increased the diameter of the central obscuration by approximately 80 percent for the Cassegrain configuration and by approximately 60 percent for the Gregorian configuration over the minimum achievable in the unbaffled configuration (for the maximum intravertex distance).

The analyses performed are not exact, but are approximations using shallow mirror representations. Particular items neglected are the sagitta for the primary and secondary mirrors, the effects of image plane curvature and the effects due to mirror figure corrections made to compensate for off-axis aberrations. In order to perform a check on the accuracy of the results obtained from the telescope configuration analyses, an exact ray trace was performed for a worst case configuration.* The configuration selected was a Gregorian telescope with primary to secondary mirror spacing of six meters.

* Primary Mirror Focal Ratio of $f/1.72$.

The paths of the rim rays was determined and their intersections with the transverse planes located at the edges of the baffles determined. The maximum distance from the axis of the telescope that any of the rays intersects this plane is then compared with the value obtained from the prior analysis.

The results of the exact ray trace yield a radius for the secondary mirror baffle of $8\frac{1}{2}$ millimeters larger than was determined by the prior analysis. This corresponds to a 2.2% increase. As this diameter of baffle would produce a fully baffled field in excess of the desired 30 arc-minutes, the actual optimum baffle would be slightly shorter and smaller in diameter, for an estimated increase on the order of 1%. The radius of the primary mirror baffle determined by the exact ray trace is only 0.2 millimeter larger than determined by the prior analysis. It is therefore concluded that the geometrical analysis yields a sufficiently close result for the purpose of this study. A summary of the analyses performed for each configuration will now be presented.

The first parameter investigated was the diameter of the secondary mirror. The diameter is defined by the area of intersection of the plane tangent to the vertex of the secondary mirror and the cone containing all the light being imaged by the primary mirror. The outer surface of the cone is identified by that ray which originates at the rim of the primary mirror and passes through the appropriate edge of the image field at the prime focus. This is shown in Figure 4-1. It is noted that the diameter of the secondary mirror increases as the prime focus is moved away from the mirror.

The second obscuration investigated was that due to the hole required in the center of the primary mirror. This analysis required several additional constructions. These were the image of the primary mirror as seen in the secondary mirror (the exit pupil), and the lateral size of the field of view in the final image plane. These are shown in Figures 4-2 and 4-3. The intersection with the primary mirror of the line joining the edge of the exit pupil with the appropriate edge of the final image field of view defines the aperture required in the primary mirror to pass the full field of view unvignetted. The location and size of the exit pupil and the size of the final focus field were computed by using the shallow mirror equations. The same approximations used in the secondary mirror analysis were used here.

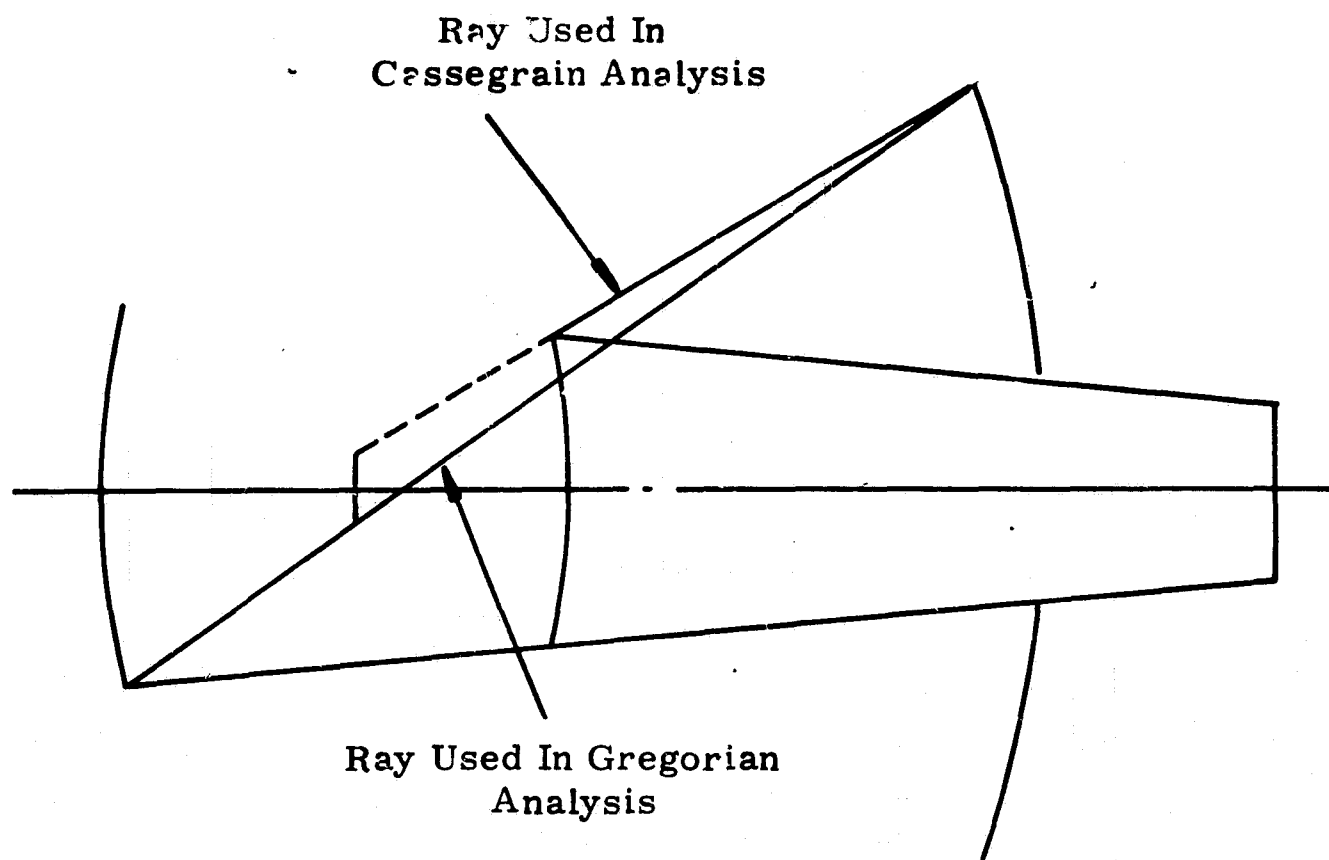


Figure 4-1. Secondary Mirror Diameter

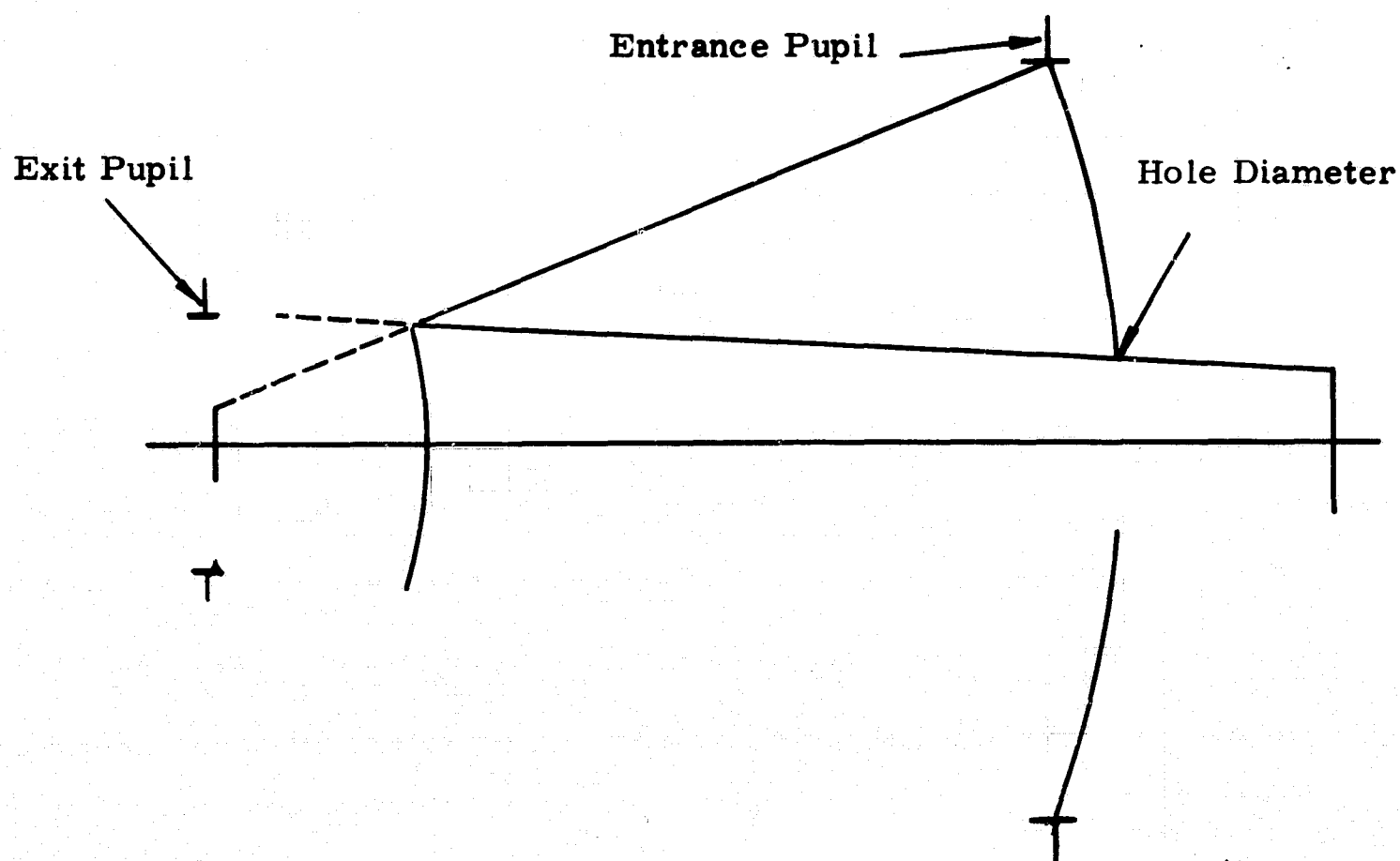


Figure 4-2. Cassegrain Primary Mirror Hole

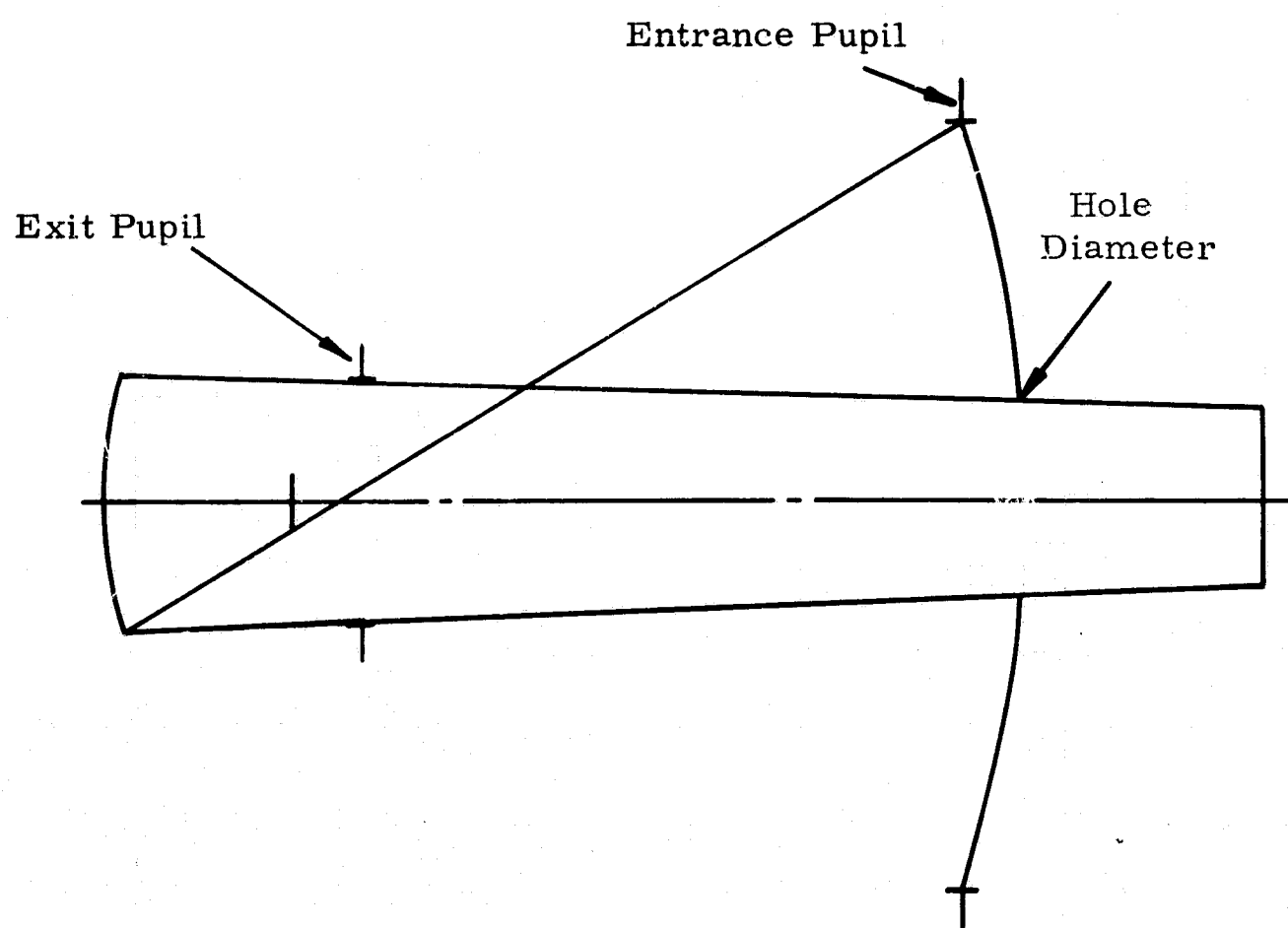


Figure 4-3. Gregorian Primary Mirror Hole

The behavior of the system is such that the diameter of the hole required in the primary mirror increases as the prime focus approaches the secondary mirror. It also occurs that the diameter of the required hole again slowly increases as the prime focus moves far away from the secondary mirror, but in the practical range of values this does not affect the results of the analysis.

As the actual obscuration achieved will be the greater of the two above sources, the minimum achievable will be when these sources are equal. This sets an absolute minimum to the central obscuration unless the ground rules are relaxed, for example, by reducing field of view or permitting vignetting for off-axis images.

The third limitation on central obscuration is the baffling required to fully baffle the image plane over the full field of view against direct incoming stray light. Two baffles are introduced into the analysis to accomplish this. The first baffle extends from the secondary mirror toward the primary and the second baffle extends from the primary toward the secondary. The baffle arrangements are shown in Figures 4-4 and 4-5. The diameter of the baffle extending from the secondary mirror is a variable and defines the amount of central obscuration. This baffle extends back toward the primary mirror, parallel to the telescope axis, until it intercepts the rim ray coming from the edge of the primary mirror and headed for the appropriate edge of the field of view at the prime focus. The baffle emanating from the primary mirror is, in general, smaller in diameter than the secondary mirror baffle and extends out to the point where the ray reflected from the primary mirror at the radial distance of the central obscuration and headed for the on-axis point in the prime focus image intercepts the ray traveling from the appropriate edge of the exit pupil to the appropriate edge of the field of view. This configuration gives a minimum central obscuration for the on-axis image, subject to the parallel requirement that the rim rays of the images at the field edge are not vignetted. It does occur, however, that for off-axis images the amount of central obscuration increases due to vignetting by the baffles since they are, in general, not located at a pupil of the optical system.

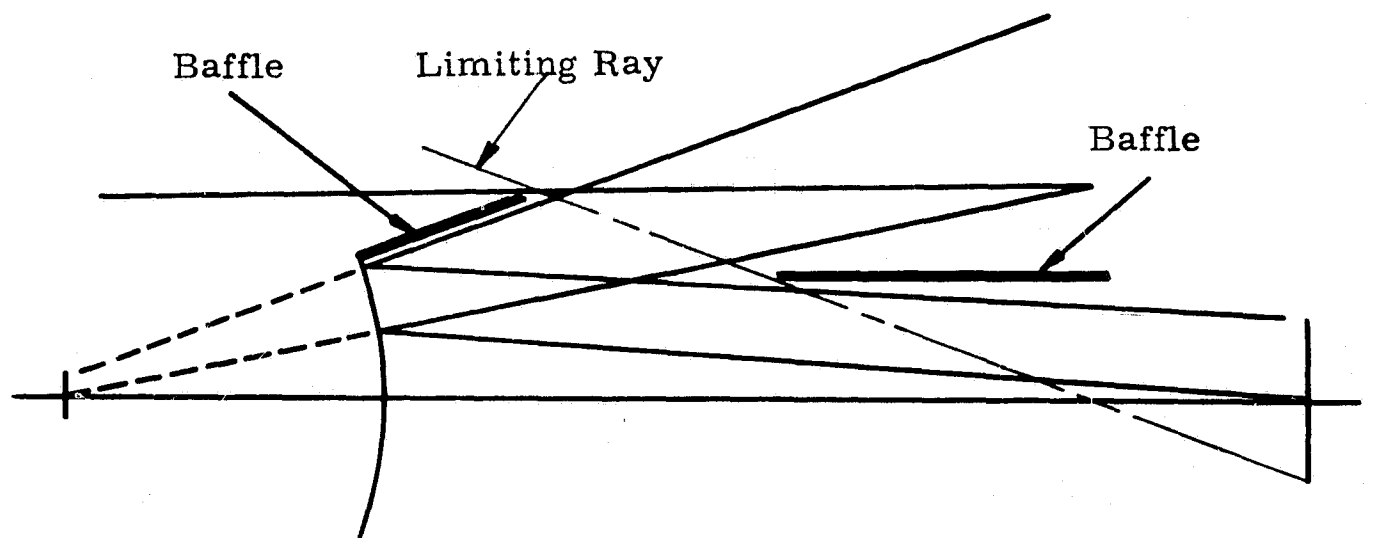


Figure 4-4. Cassegrain Baffling

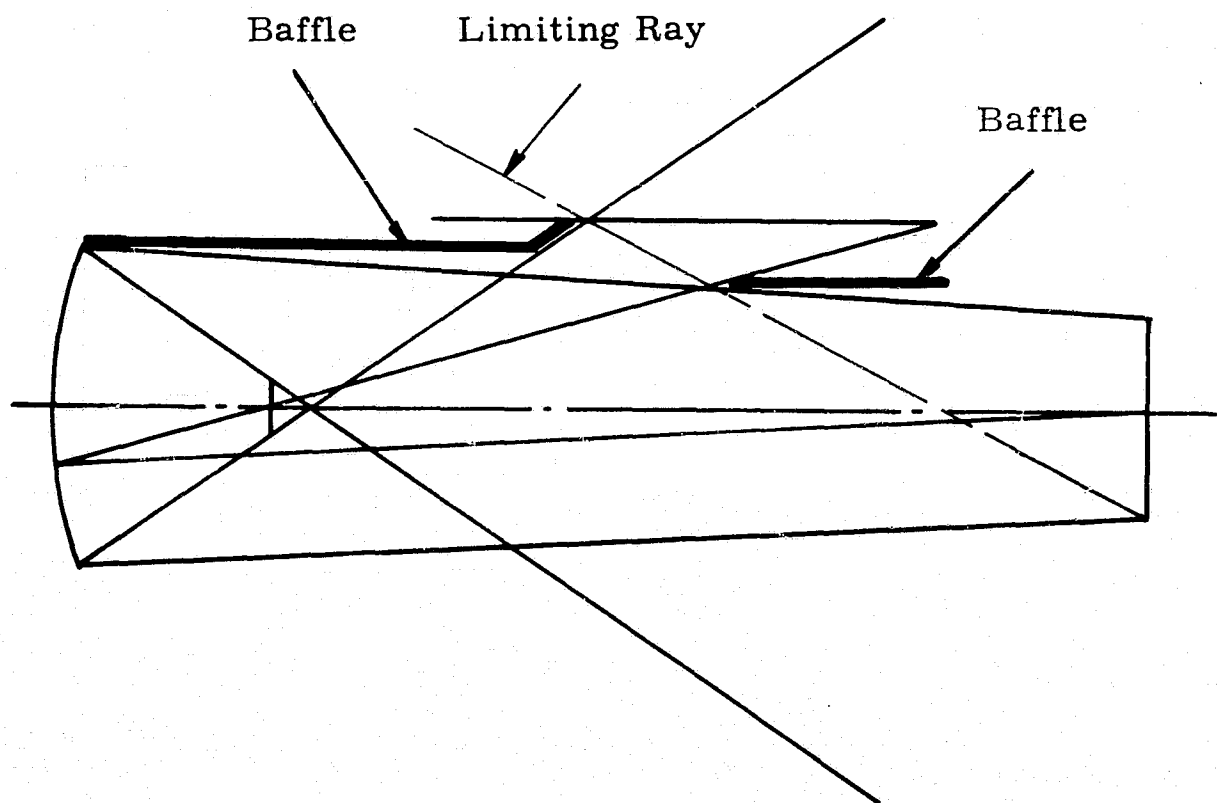


Figure 4-5. Gregorian Baffling

Detailed calculations were performed using the results of these analyses for the two values of mirror vertex separation, 8.7 meters and 6 meters. The results of these calculations are presented in Figures 4-6 and 4-7. Plotted in each figure as a function of primary mirror F number are: ϵ_s ; obscuration due to secondary mirror diameter, ϵ_h ; obscuration due to the hole in the primary mirror, ϵ_b ; obscuration due to baffling, N; the final system F number, and the Strehl ratio resulting from the minimum achievable central obscuration. The solid portion of the Strehl curve is associated with the baffled case and the dashed portion of the curve is associated with the unbaffled case.

Inspection of the two figures reveals several particular relationships. The first is that only a small change in the maximum achievable Strehl ratio is obtained by changing the mirror vertex separation. The second relationship observed is that the maximum achievable Strehl ratio appears to be higher for the Gregorian configuration than it is for the Cassegrain configuration. This gives a measure of preference to the Gregorian configuration with its faster primary mirror to be balanced against other considerations. The third relationship observed is that for the primary mirror focal ratio that gives the minimum central obscuration in the unbaffled case, we are also very near to the minimum central obscuration achievable for the baffled case. This gives rise to the desirability of incorporating controlled or articulated baffles to allow the telescope configuration to be modified to take advantage of dark sky conditions subject to the requirements of particular experiments.

4.1 ARTICULATED BAFFLE

One simple straightforward technique to articulate the secondary mirror baffle will be described.

The scheme in Figure 4-8 depicts the design. The baffle is attached to an arm which is congruent to one of the secondary spider legs. The arm is attached to the telescope via a lower pivot point about which it rotates. It is held in position at either end of its travel by a positive latching mechanism. The current idea for this latch is a magnetic type. The arm is driven to either of its two positions by a cable system whose drive mechanism is housed on the spider supported secondary structure. A motor

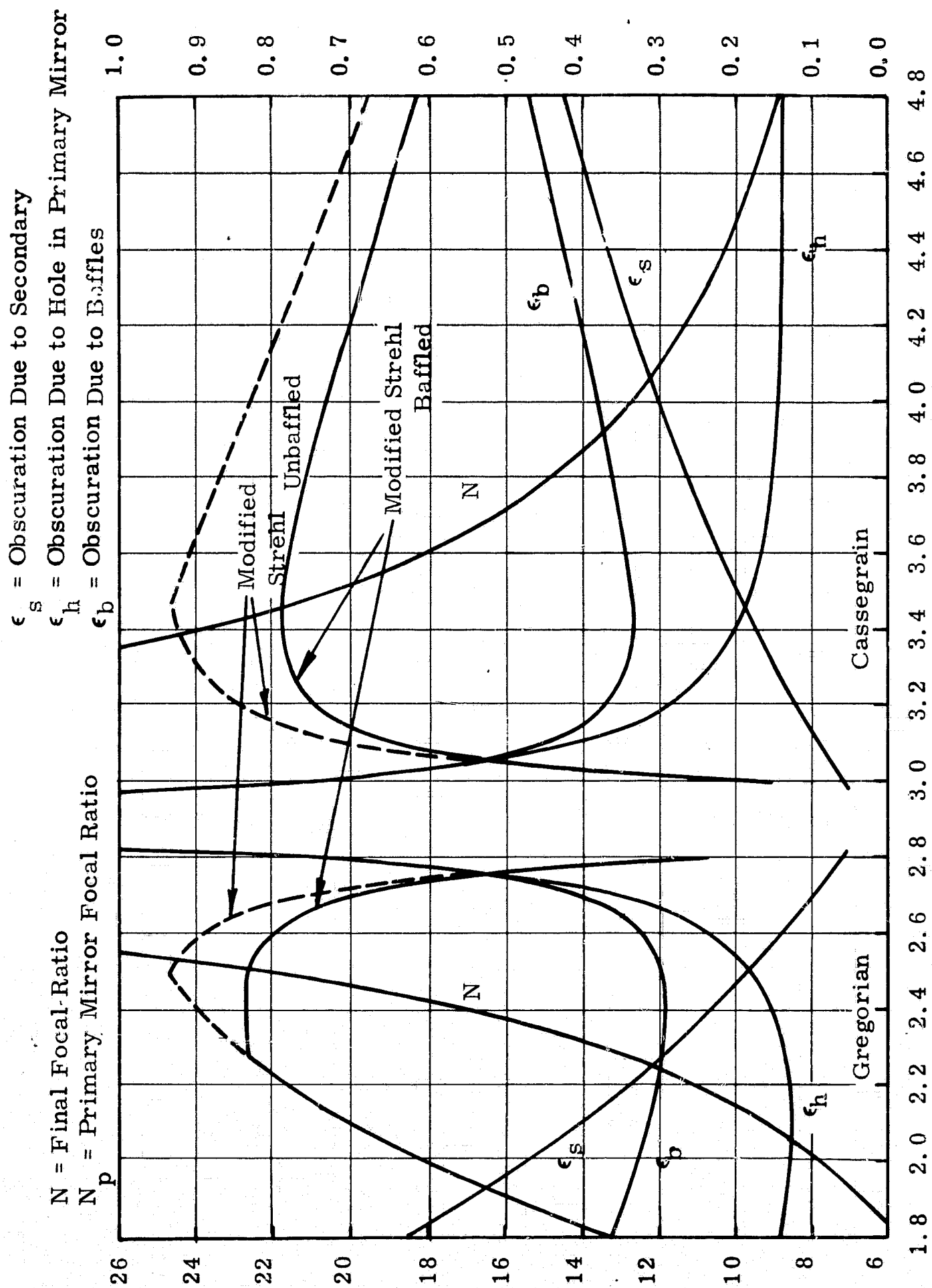
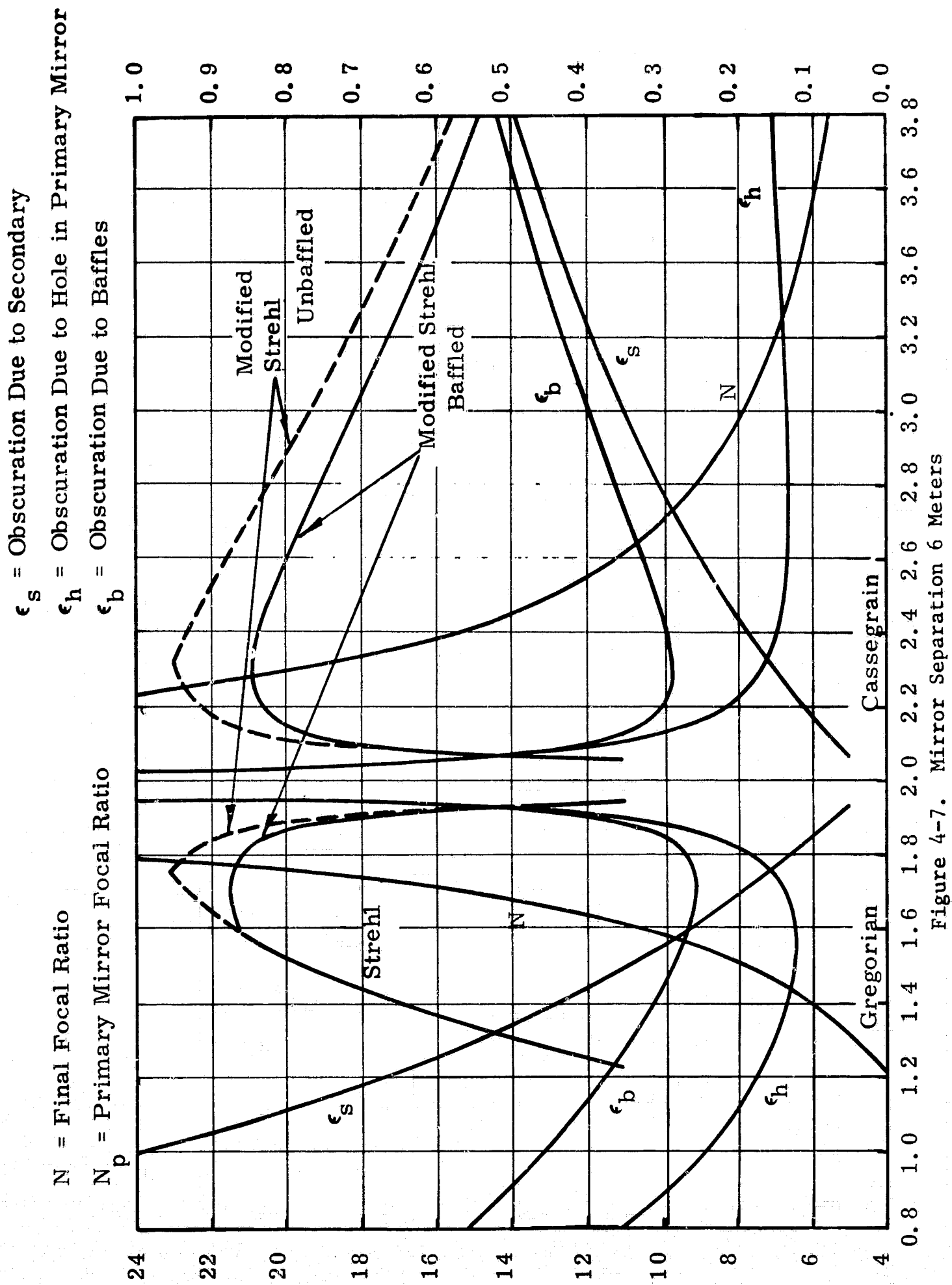


Figure 4-6 Mirror Separation 8.7 Meters



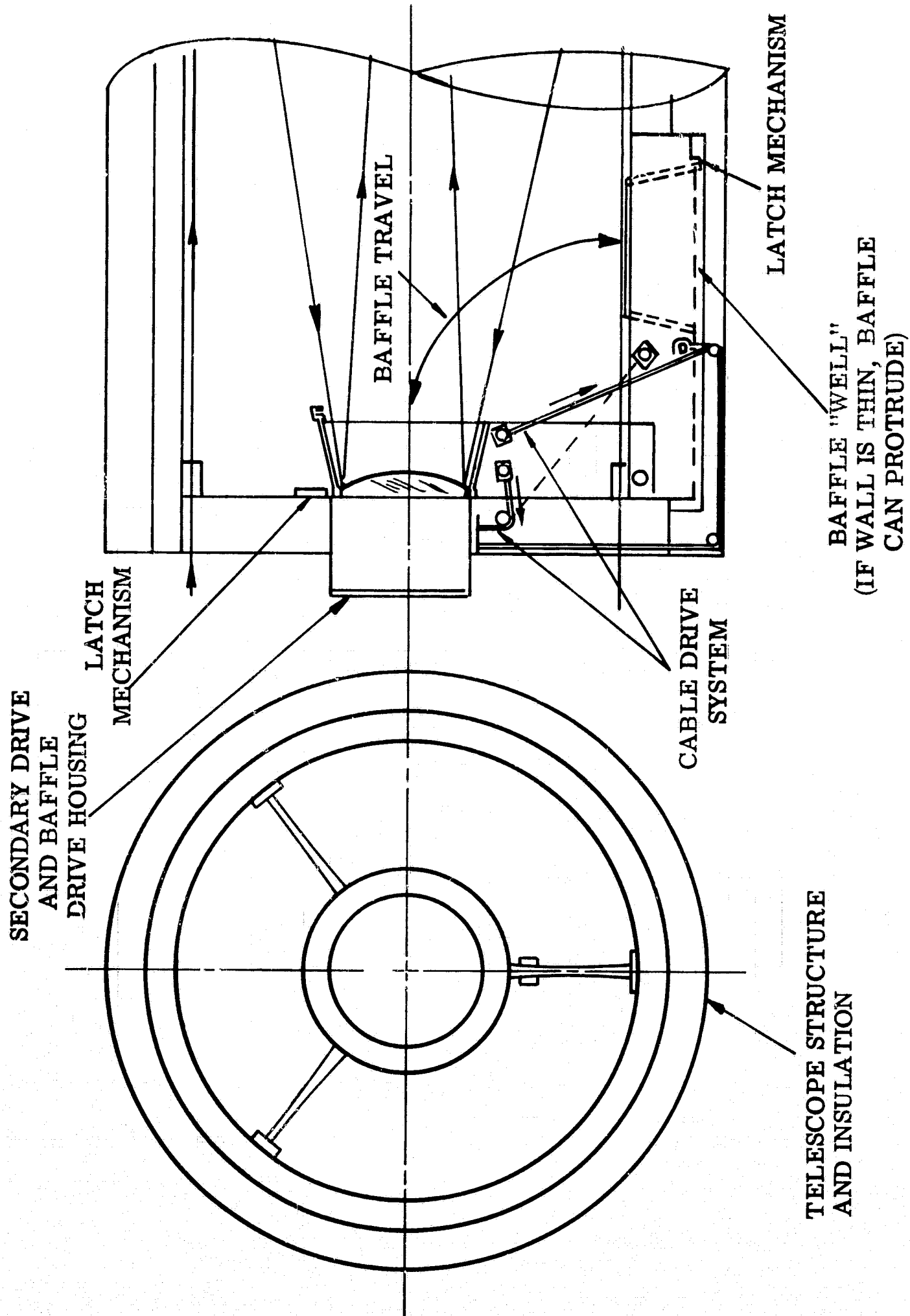
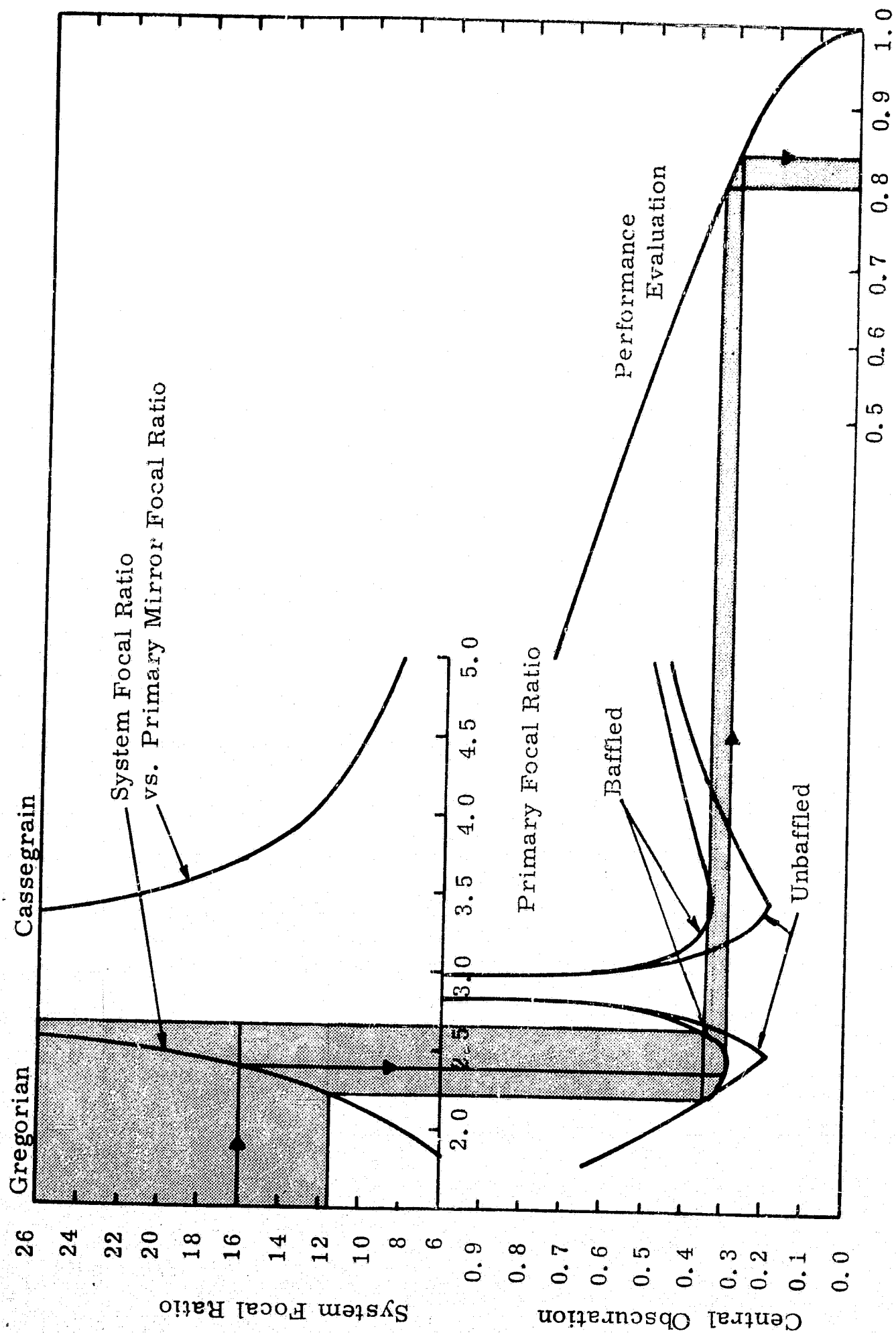


Figure 4-8. Articulated Baffle

driven drum upon which the cable winds and unwinds maintains the proper tension in the system. This system is always able to produce a resulting moment around the pivot point.

The possible gain in performance shown by the difference between the solid and dashed lines in Figures 4-6 and 4-7 make such a mechanization very attractive when compared to alternate means for gaining a comparable increment in performance.

It is observed that for a given mirror separation there is a single well defined value for the primary mirror focal ratio that gives the minimum attainable central obscuration. This in turn defines the secondary mirror diameter and focal length, the minimum attainable central obscuration and the final system F number. If it is desired to utilize some other primary mirror focal length, these results will give a direct indication of the minimum additional image degradation to be expected due to the increased central obscuration. Circumstances that might promote such a desire are image field of view size restrictions or alignment sensing system restrictions. The tradeoff of these considerations will be covered in a later section. The direct relationship between the telescope dimensional parameters and the Strehl ratio in the final image is presented in an additional form in Figures 4-9 and 4-10. Both figures are drawn for the case of a 3-meter aperture telescope with a 30-arc-minute field of view. Figure 4-9 presents the results for an 8.7-meter separation of the primary and secondary mirror and Figure 4-10 presents the results for a 6-meter separation. There are three choices to be made when using either of the figures. The first is what final system focal ratio, N , is desired. With this selection performed, a choice is made with respect to telescope configuration; Gregorian or Cassegrain. The choice of system focal ratio and telescope configuration uniquely determine the primary mirror focal ratio, N_p . The last decision is with respect to baffling. The telescope can be configured for minimum central obscuration allowing some stray light to reach the final image plane, or with additional baffling to avoid direct paths to the focal plane. As a representative baffled case, the situation for minimum obscuration consistent with a fully baffled field of view was analyzed. In the figures a choice is made between the unbaffled and baffled configuration. This choice, together with the primary mirror focal



STREHL

Figure 4-9. Mirror Separation 8.7 Meters

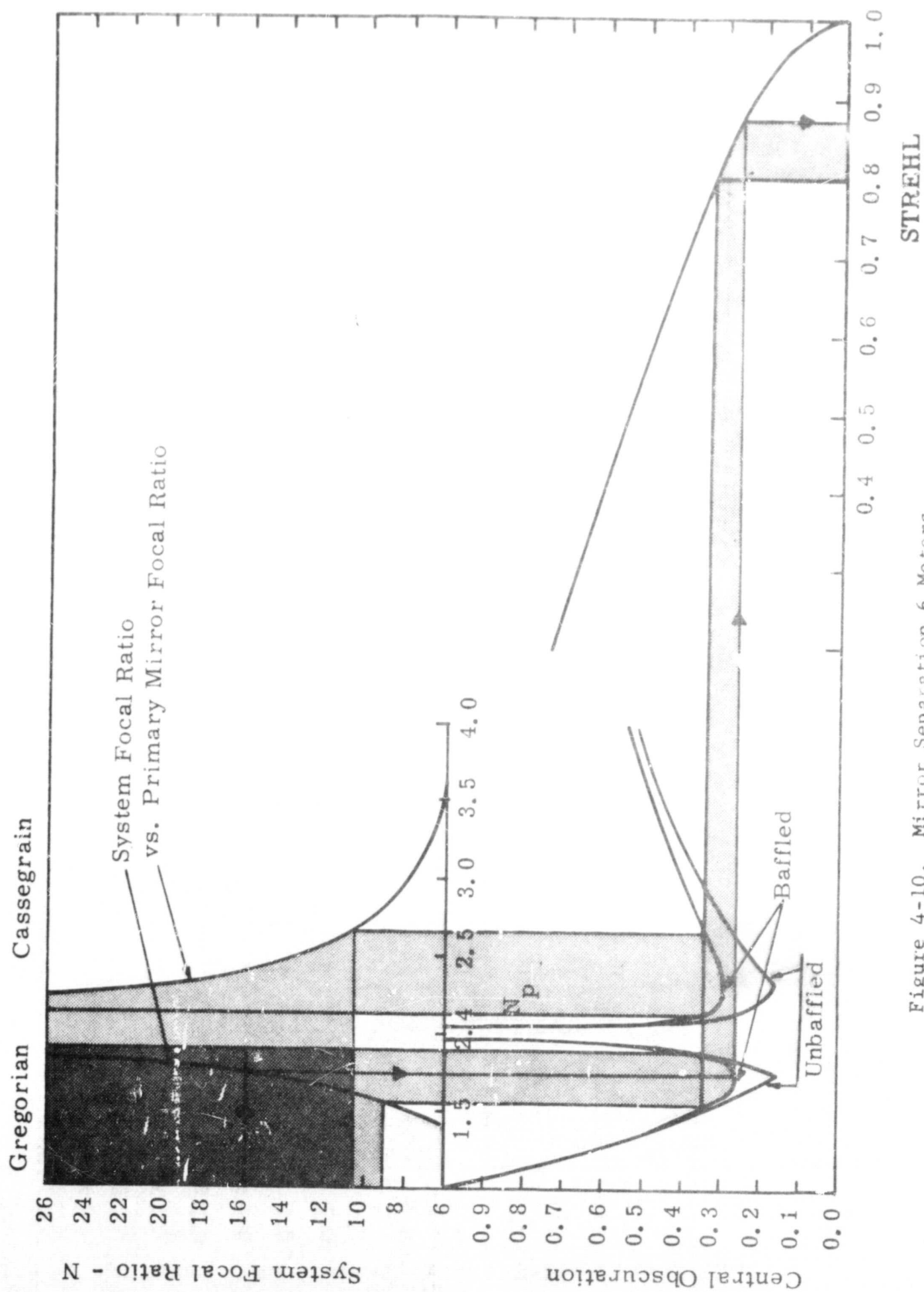


Figure 4-10. Mirror Separation 6 Meters

ratio, determined by the previous choices, determines the minimum amount of central obscuration that is achievable. The value in turn determines the reduction in the Strehl ratio that will be caused in an otherwise perfect optical system.

An example of the flow of parameters is shown on the figures. For Figure 4-9 (mirror separation = 8.7M) a choice of final system focal ratio was made at $f/16$. The selection of the Gregorian configuration gives a primary mirror focal ratio of $f/2.38$. The choice of the baffled configuration next yields a minimum central obscuration of 29 percent of the entrance aperture diameter. This value of central obscuration in turn creates a reduction in the Strehl ratio to a value of 0.83.

The variation of Strehl ratio as a function of final system focal ratio is presented in Figures 4-11 and 4-12. The variation of the maximum achievable Strehl ratio as a function of mirror separation is presented in Figure 4-13. It is noted that the value of the Strehl ratio increases slowly with decreasing mirror separation. A plot of the final system focal ratio produced in the configuration for maximum Strehl ratio versus mirror separation is presented in Figure 4-14. There is observed to be a strong decrease in the final system focal ratio with decreasing mirror spacing.

Finally, if we locate the primary mirror focal ratio corresponding to the maximum achievable Strehl ratio for each mirror separation, we obtain Figure 4-15. A surprising result occurs; namely, that the Gregorian and Cassegrain forms give essentially the same Strehl values for a particular primary mirror focal ratio. Thus, the trend shows that faster primary mirrors yield higher performance.

By determining the final $f/\text{No.}$ (N) associated with the primary mirror $f/\text{No.}$, this result can be plotted on the same coordinates used by Itek to their study.⁸ This result is shown as an overlay in Figure 4-16. Note that the maximum achievable performance lies to the left of the boundary of this diagram. This boundary was set in Ref. 8 to represent the limitations of a particular alignment sensing system. Newer techniques being pioneered at Perkin-Elmer show promise of moving this boundary so that higher performance levels can be achieved.

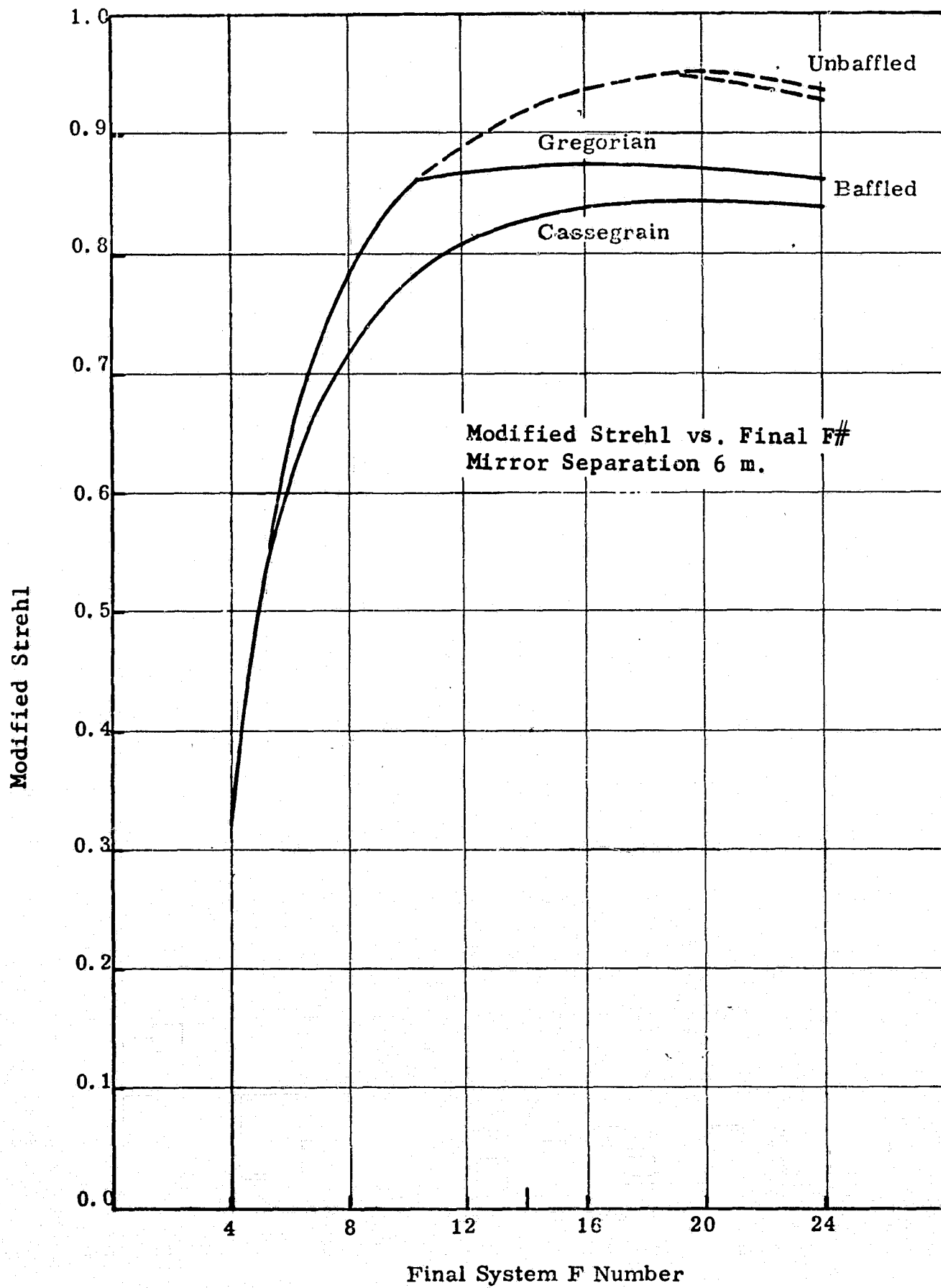


Figure 4-11. Modified Strehl vs. Final F/No., Mirror Separation 6m

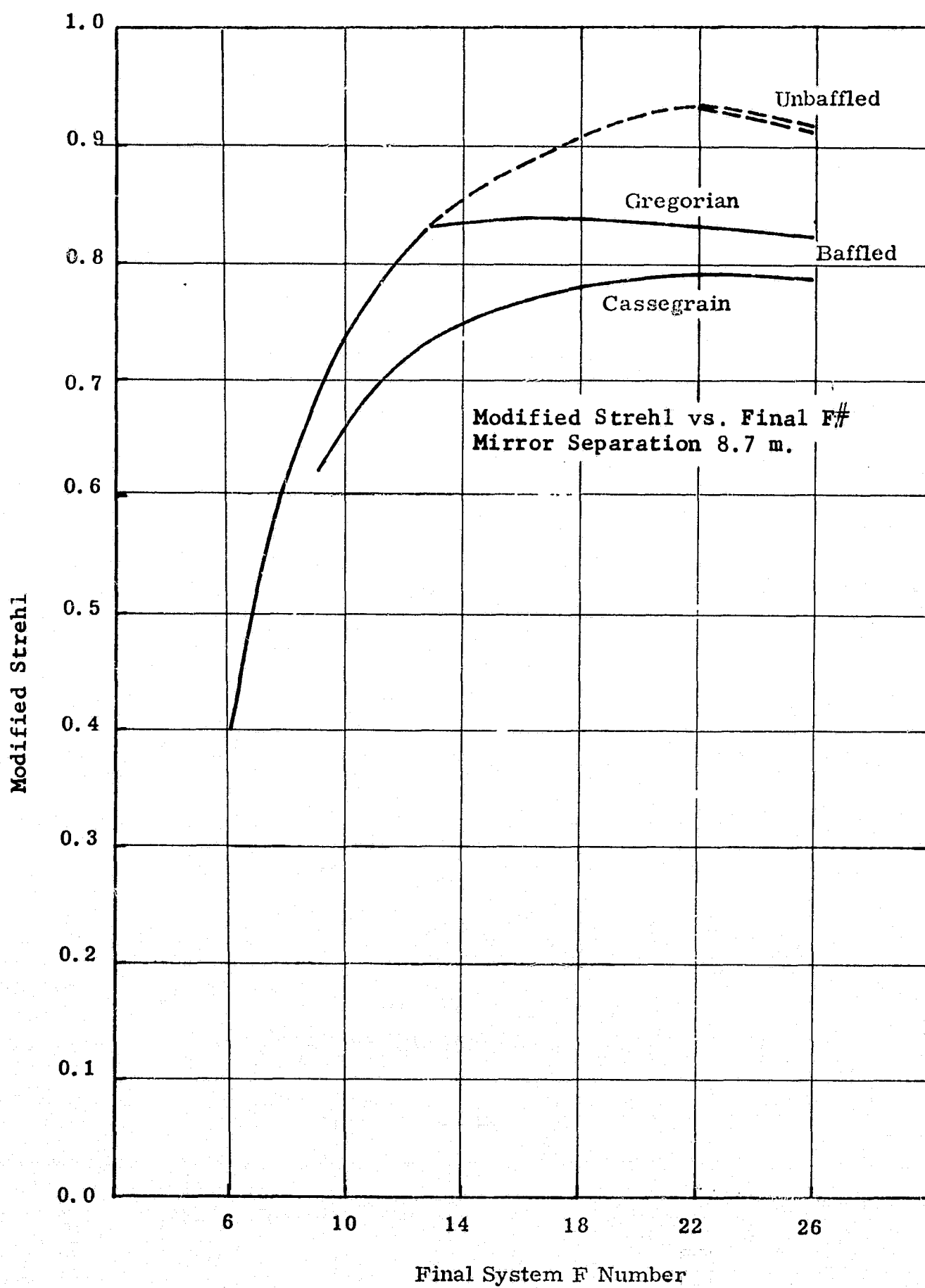


Figure 4-12. Modified Strehl vs. Final F/No., Mirror Separation 8.7 m

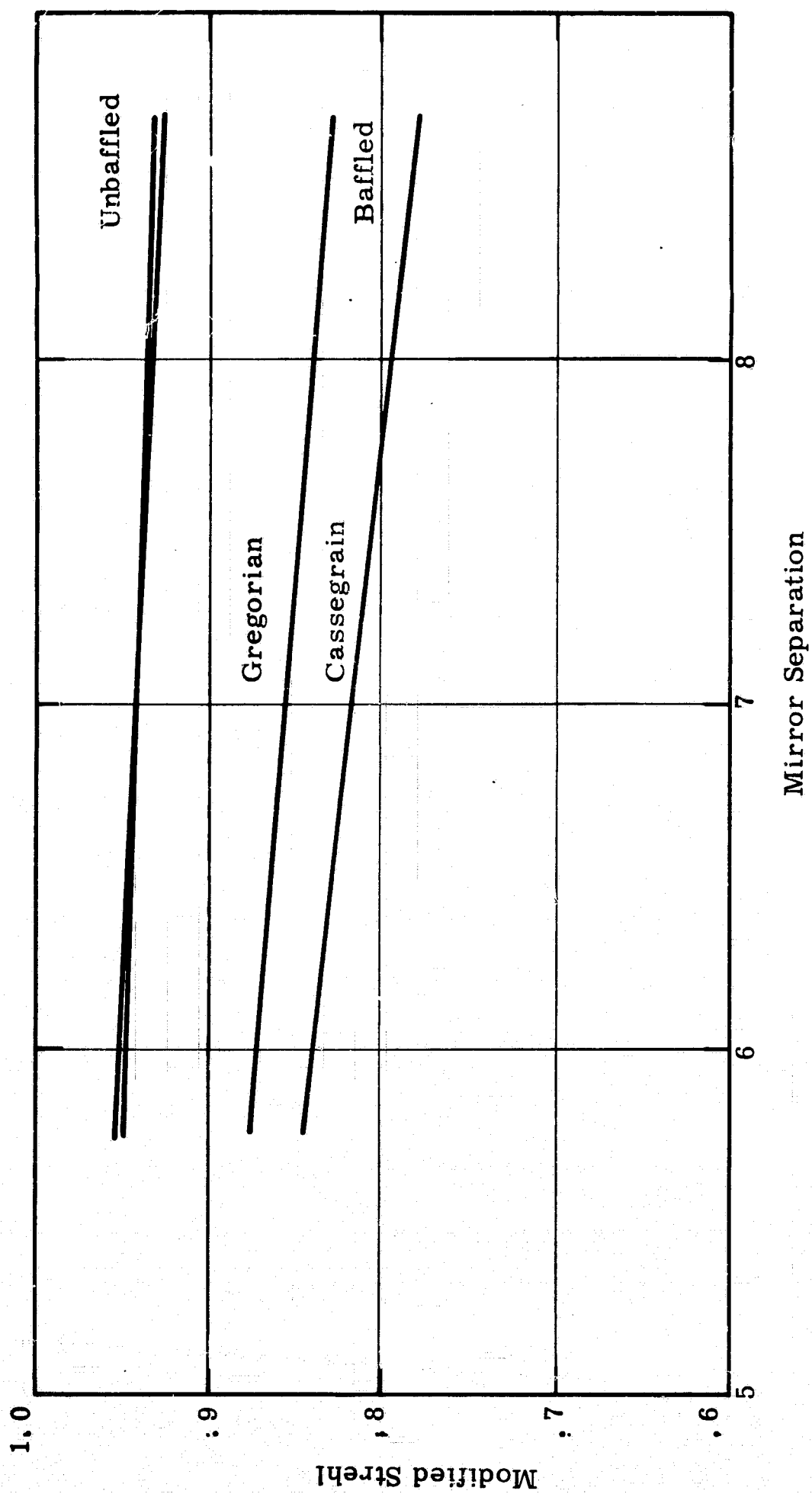


Figure 4-13. Maximum Achievable Modified Strehl Ratio vs. Mirror Separation

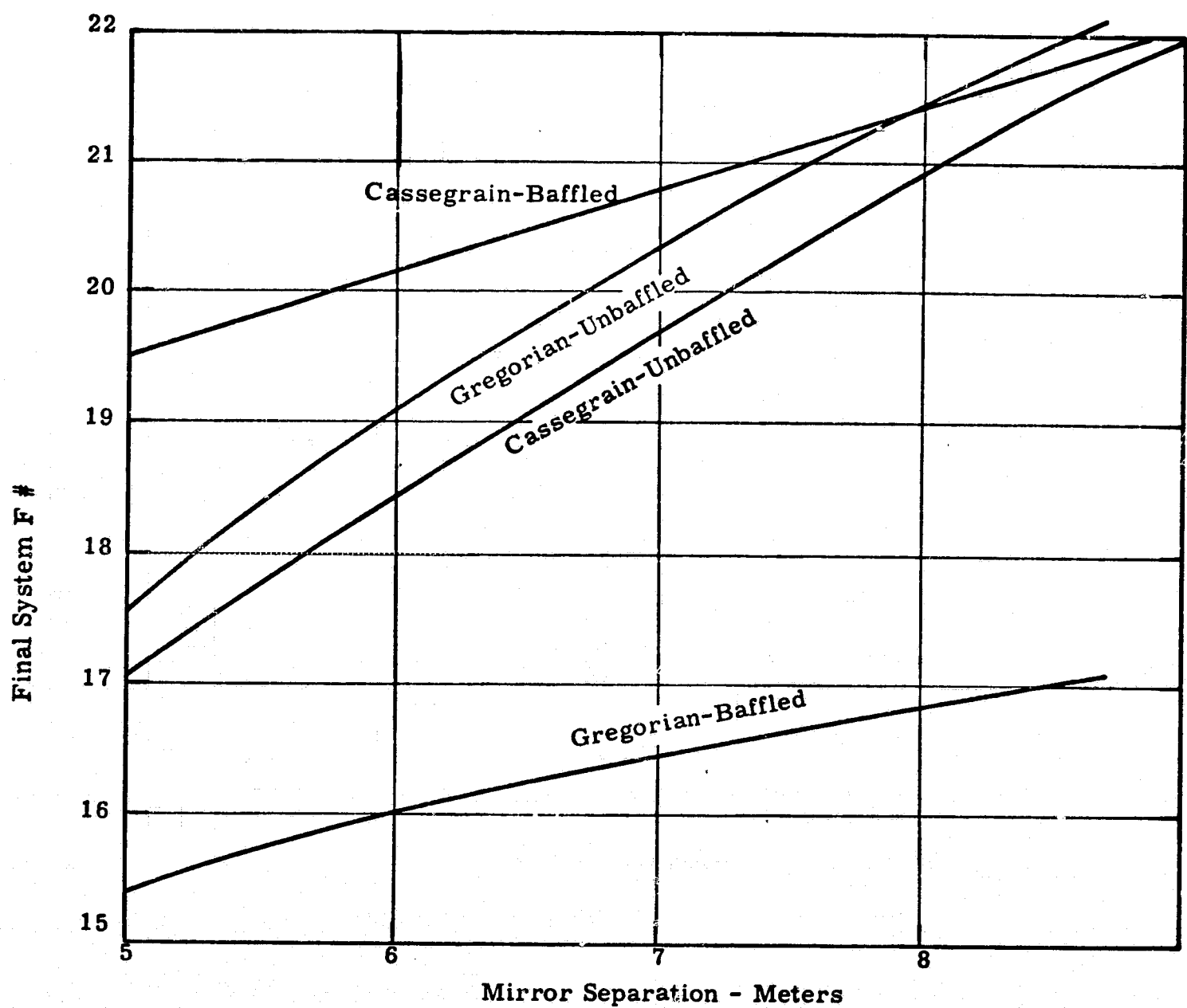


Figure 4-14. Final Focal Ratio vs. Mirror Separation for Maximum Optical Performance (Modified Strehl)

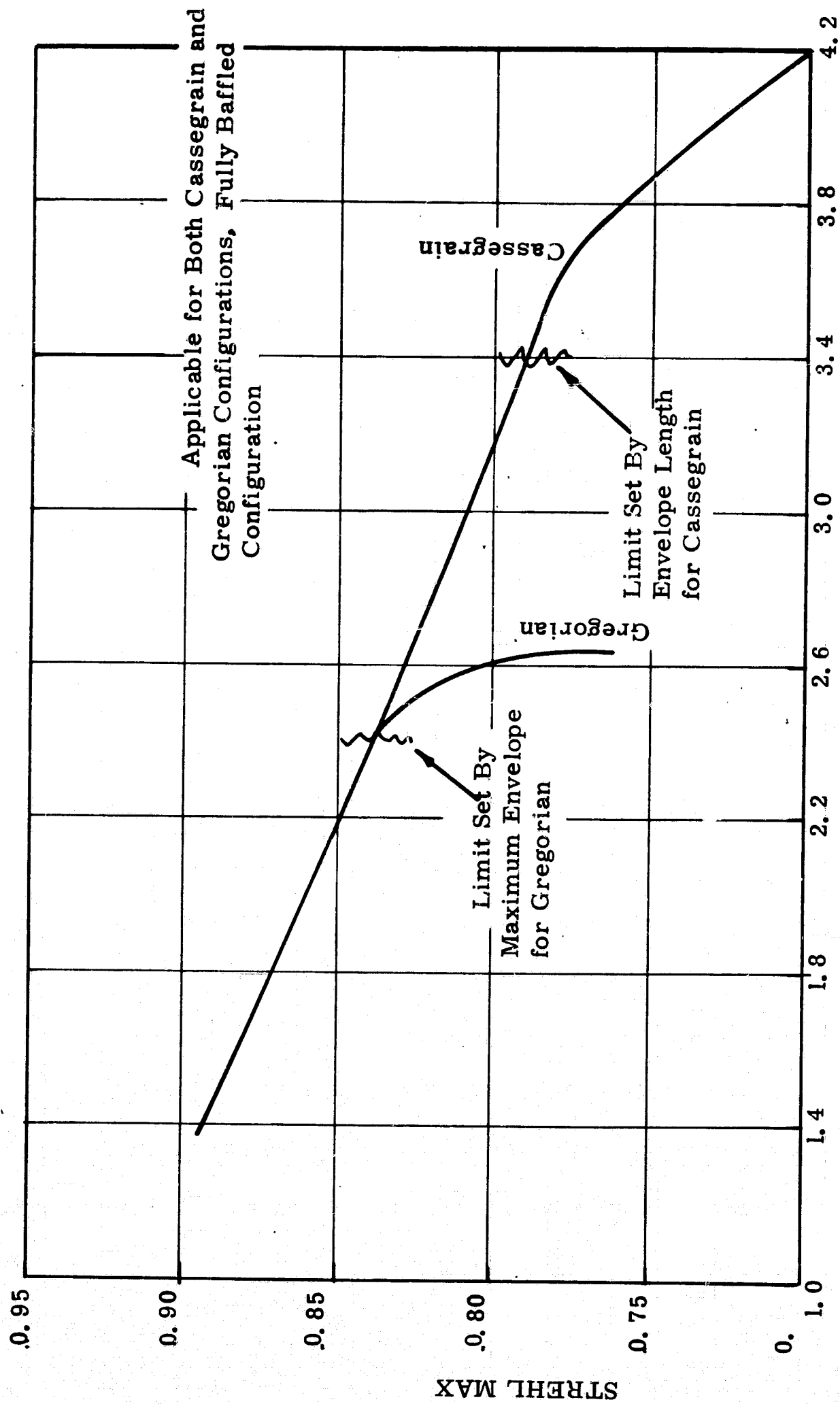


Figure 4-15. Maximum Modified Strehl Value vs. Primary Mirror Focal Ratio

MSR = Modified Strehl Ratio

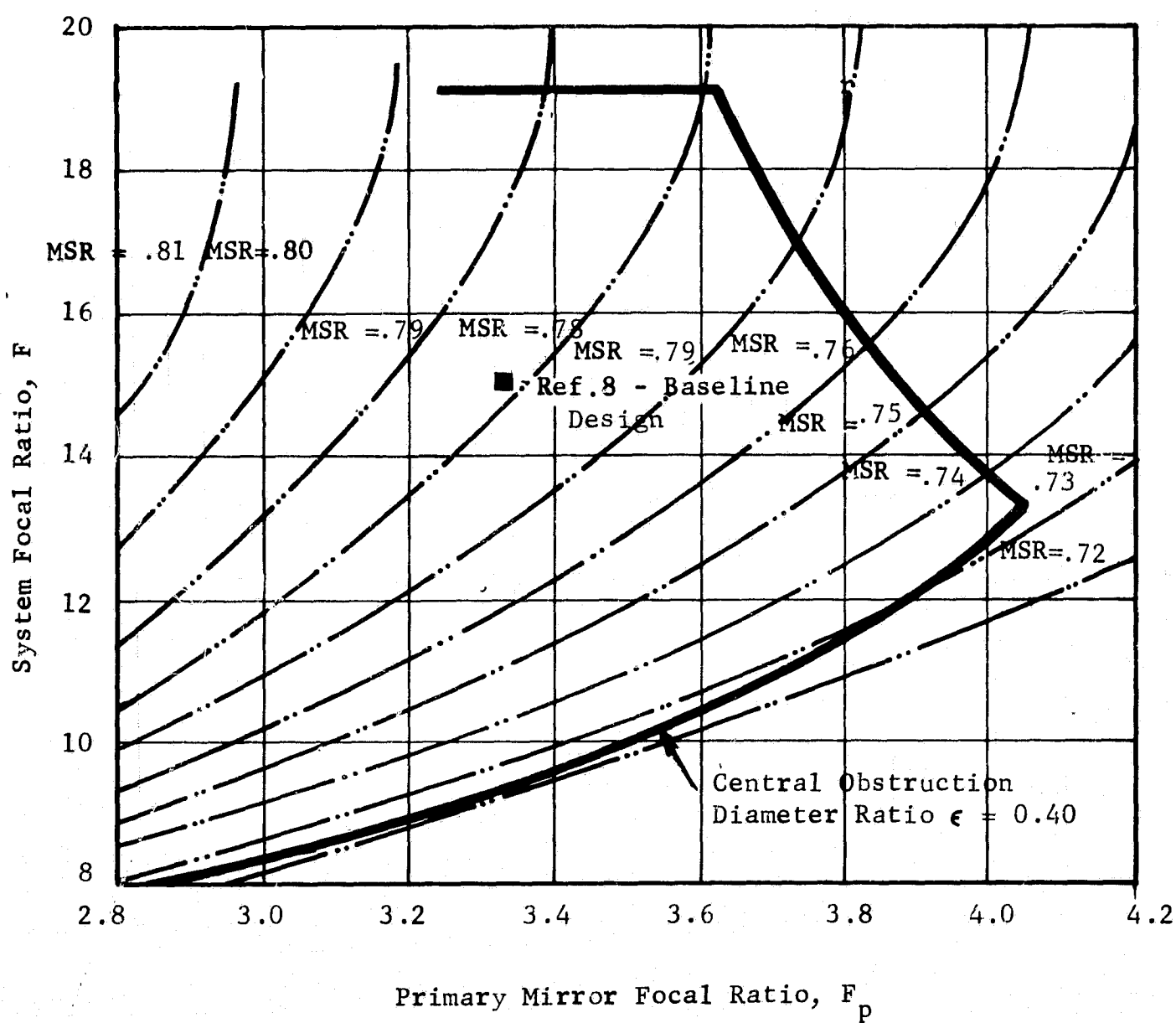


Figure 4-16. Comparison of Ref. 8 Provisional Optical Design Boundary Curves with Modified Strehl Performance Criteria

4.2 CONCLUSIONS

The preceding analysis shows that for any given mirror separation and the fixed location for the focal plane given by the ground rules it is possible to find an optimum primary mirror focal ratio and secondary mirror magnification which will maximize the modified Strehl ratio we have adopted as a design figure of merit. Further, if the process is repeated for various intravertex separations of the mirrors, a trend is developed which shows that the shorter systems having faster primary mirrors (and secondary mirrors with higher magnification) will have increasingly better performance. This trend holds for both the baffled and unbaffled cases. Finally, if the primary mirror speed is considered the variable and the optimum sought for each primary mirror speed, a remarkable trend develops, showing that performance increases as the speed of the primary mirror is increased, and that the performance of the Gregorian and Cassegrain systems are essentially the same for a given primary mirror focal ratio. Numerically, a primary mirror speed of $f/4.2$ yields a Strehl value of 0.75 while a system having an $f/1.7$ primary mirror has a Strehl value of 0.88 (with the trend between these points being nearly linear). (In all of the above cases, the fully baffled configuration was used.)

The prime conclusion to be drawn from this analysis is that the primary mirror in a 3-meter telescope should be as fast as possible, consistent with manufacturing and operational constraints, and the choice between the Cassegrain and Gregorian forms should be based on other considerations than performance. This subject will be discussed further in Section 7.

SECTION 5

ALIGNMENT CONSIDERATIONS

The basic requirements in alignment of a two-mirror telescope are best understood through an appreciation of the interaction of the appropriate mathematical reference points associated with the optical mirrors, and the images formed by these mirrors. The insights afforded by a basic understanding will permit relaxation of some tolerances previously considered necessary.

To the geometrician working with the conic curves that are commonly used in two mirror systems, the word "focus" refers to one or two uniquely defined points in space which help to define the surface. For example, an ellipsoid is, to the geometrician, the surface or locus of all points for which the sum of the distances from two reference points is a constant.

To the optical engineer, the focus means the point to which a wavefront converges (or the point from which a wavefront appears to be diverging, in the case of a virtual image). This focus can occur at any of a large number of points in space, and is a function of the distance and angle from the image-forming mirror (and its axis) to the object. (If the wavefront happens not to be spherical, a poor image will be formed, but the "best" image location is still defined by a point in space.) When considering a scene or an extended object, the optical engineer refers to the focal plane or surface which may (and, in general, does) have curvature. And, of course, when astigmatism is present, he refers to the sagittal and tangential focal surfaces.

For a single image forming mirror, it is axiomatic that the best imagery is obtained when the geometrician's focal point coincides with the optical engineer's best focus. For example, if a perfectly plane, uniform wavefront is reflected by an ideal paraboloid whose axis is perpendicular to the wavefront, the resultant image will be perfect in the sense that it will be free of spherical aberration, coma, and astigmatism. If a wavefront is caused by an extended object, then the above statement will be true only for that point in the image plane which coincides with the geometric focus. Images at other points in the focal plane will suffer to some degree from the usual off-axis aberrations, such as coma.

In the classical two-mirror forms of telescope (Cassegrain and Gregorian), the appropriate geometrical reference points, or foci, must be coincident between the primary and secondary mirrors in order for optimum performance to be achieved. The reasons for this can be appreciated when one realizes that the image formed by the parabolic primary mirror at its geometric focal point will be free of aberrations, and this image is the object for the secondary mirror. If the image produced by the primary mirror is located at the geometric focal point of the secondary mirror, then an aberration-free image will be produced at the far focal point of the secondary mirror, which is at the output focal plane.

Misalignment of a two-mirror telescopic system of the type we have seen considering can be caused by a) decentration of the secondary mirror relative to the primary, and b) tilt of the secondary relative to the primary. Since the secondary mirror will relay perfectly the image produced at its geometric focal point by the primary, the tilt misalignment is relatively unimportant, provided the geometric focal points of the primary and secondary mirrors are caused to remain coincident. The main effect of tilt misalignment under the confocal condition described above is that a slightly different part of the secondary mirror is illuminated by the light which passes through (or, in the Cassegrain, would pass through) the near focal point of the secondary mirror. In spite of the tilt between the axes of the primary and secondary mirrors, an aberration-free image will still be produced at the far focal point of the secondary mirror.

The effects of decenter misalignments of the primary and secondary mirrors have been determined by A. Offner (see Appendix A). The image defects introduced by non-coincidence of the geometric focal points of the primary and secondary mirrors manifest themselves as defocus and coma, with defocus occurring when there is an axial separation of the focal points, and coma occurring when there is a lateral separation of the focal points. The introduction of coma is easily understood when one recognizes that the primary off-axis image defect produced by a parabola is coma. Since the secondary mirror will relay and magnify perfectly an off-axis image located at its geometrical focal point, it is not surprising that the effect of lateral misalignment is to introduce coma. Conversely, aberrations will be added to the "perfect" image produced

by the primary mirror at its geometric focal point, since this will be in the off-axis field of the secondary mirror.

We shall now apply the results of Mr. Offner's derivation to the results obtained in the technology study prepared under Contract NASw-1925 by Itek Corporation, and then use them to derive tolerances for alignment under the ground rules of this study.

5.1 SUPPLEMENTARY NOTE TO "SOME OPTICAL SYSTEMS FOR A SPACEBORNE TELESCOPE"*

In reference 8, entitled "Technology Study for a Large Orbiting Telescope", a table is included in which the sensitivities of Cassegrain and Ritchey-Cretien telescopes to mirror tilt and decentralization has been determined by tracing large numbers of rays through 19 configurations of each of these types of systems. In Appendix A, a single formula (A 14) has been given which supplies the same information based on third order analysis. In this discussion the results obtained by using third order analysis are compared with these results based on ray trace analysis.

The third order departure W_g of a wave from the Gaussian reference sphere by a parabola due to coma at an image height Y_t is given by

$$W_g = Y_t / 32N^3 \quad (5-1)$$

where N is the $f/\#$ of the parabola. (See Reference 9.) Since the coma of a Cassegrain is the same as that of a parabola of the same $f/\#$, the contribution of the secondary hyperboidal mirror of magnification m is given by

$$W_g = mY_t / 32(mN_p)^3 - Y_t / 32N_p^3$$

$$W_g = \frac{m(1-m^2)Y_t}{32N} \quad (5-2)$$

where $N = mN_p$ is the f/number of the Cassegrain (the quantity W used in equation (A13) is the departure of a wave front from the closest reference sphere. In the case of coma, $W = W_g/2$). (See Glossary for definition of various W 's.)

*See Appendix A.

The Maréchal tolerance for coma is reached when $W_g = 0.60\lambda$ (Reference 4). That is to say, $W_g = 0.60\lambda$ corresponds to $Wrms = \lambda/14$ where $Wrms$ is the root mean square departure from the closest reference sphere. Hence equation (a) may be put in the form

$$Wrms = \frac{m(1-m^2)Y_t}{268.8 N_p^3} = \frac{(1-m^2)Y_t}{268.8 m^2 N_p^3} \quad (5-3)$$

For $Wrms$ in wavelength units

$$Wrms = \frac{3.7 \times 10^{-3}}{\lambda N_p^3} \cdot \frac{(1-m^2)Y_t}{m^2} \quad (5-4)$$

For $\bar{Y}_t$ in units of 0.001 inch and $\bar{\lambda}$ in units of micrometers, the units used in the Itek report,

$$Wrms = \frac{0.094}{\bar{\lambda} N_p^3} \frac{(1-m^2)}{m^2 \bar{Y}_t} \quad (5-5)$$

The quantity σ_d is then given by

$$\sigma_d = \frac{0.094}{\bar{\lambda} N_p^3} \frac{(1-m^2)}{m^2} \quad (5-6)$$

For distance from primary to the Cassegrain image surface, B , and f_p , the focal length of the primary, the lateral displacement Y_t of the focus of the secondary mirror from that of the primary mirror corresponding to a tilt of the secondary mirror about its pole of β arc-minutes is given by

$$Y_t = 0.00029 \frac{(f_p + B)\beta}{m+1} \quad (5-7)$$

With this substitution for Y_t in (5-4), it becomes

$$Wrms = 1.07 \times 10^{-6} \frac{(m^2-1)(f_p+B)\beta}{m^2(m+1)\lambda N_p^3} \quad (5-8)$$

For $(\bar{f}_p + \bar{B})$ in inches and $\bar{\lambda}$ in micrometers, (5-8) becomes

$$W_{rms} = \frac{0.027(m^2 - 1)(\bar{f}_p + \bar{B})\beta}{m^2(m+1)\bar{\lambda} N_p^3} \quad (5-9)$$

$$\sigma_t = \frac{0.027(m^2 - 1)(f_p + B)}{m^2(m+1)\bar{\lambda} N_p^3} \quad (5-10)$$

The results obtained by use of (5-6) and (5-10) are compared with the results obtained by ray tracing in Table 5-1 of reference 8.

The excellent agreement of both σ_t and σ_d with the results of ray tracing indicate that the single equation (5-4) is adequate to compute alignment tolerances in Cassegrain (and Cassegrain type) system. The same equation also holds for Gregorian type systems.

We may now use the above relationships to determine the effects of misalignment indicated by the modified Strehl figure of merit that we have been using.

m and N may be expressed as function of N_p as follows:

$$N = m N_p$$

$$m = \frac{2.9 - N_p}{3.5} \quad \text{for Gregorian} \quad (5-11)$$

$$m = \frac{N_p - 2.9}{3.5} \quad \text{for Cassegrain} \quad (5-12)$$

using the maximum mirror separation given in the ground rules. Substituting (5-5) and (5-7a) into (5-5) gives

$$W_{rms} = \left\{ 0.094 \frac{1}{N_p^3} \left(1 - \frac{(2.9 - N_p)^2}{3.5^2} \right) Y_t \right\}^2 \quad (5-13)$$

where N_p is the primary mirror focal ratio, Y_t is the lateral separation of the focii, and the numerical values are based on those given for the ground rules of the study. It is interesting to note that the equation holds for

TABLE 5-1

CLASSICAL CASSEGRAIN

$\frac{N}{P}$	N	<u>Results from</u> <u>Reference 8</u>		<u>Equation (10)</u>		<u>Equation (6)</u>	
		σ_t	σ_d	σ_t		σ_d	
1.5	3.5	1.12	0.072	1.18		0.076	
1.5	5.0	0.97	0.081	1.01		0.084	
1.5	9.0	0.64	0.086	0.67		0.090	
1.5	15.0	0.41	0.088	0.43		0.092	
1.5	30.0	0.22	0.089	0.23		0.093	
2.5	3.5	0.32	0.010	0.33		0.010	
2.5	5.0	0.40	0.015	0.40		0.015	
2.5	9.0	0.32	0.018	0.32		0.018	
2.5	15.0	0.22	0.019	0.22		0.019	
2.5	30.0	0.12	0.019	0.12		0.020	
3.5	3.5	0	0	0		0	
3.5	5.0	0.17	0.0037	0.17		0.0037	
3.5	9.0	0.19	0.0061	0.20		0.0060	
3.5	15.0	0.14	0.0068	0.14		0.0069	
3.5	30.0	0.081	0.0071	0.082		0.0072	

both the Cassegrain as well as the Gregorian telescope forms. This is because, for a fixed mirror separation, the value for secondary mirror magnification changes sign as N_p is varied from values less than 2.9 (Gregorian) to values greater than 2.9 (Cassegrain), but the magnification always appears as a squared quantity, and the change in sign effect is lost. In the curves which follow, the full variation in primary mirror focal ratio is examined.

To relate the rms wavefront errors to the modified Strehl definition we have been using, we can make the substitution $W_{rms} = \frac{2\pi\Delta\phi}{\lambda}$ where W_{rms} is in wavelength units into the defining equation of Figure 3-15. This gives us the working equation

$$\delta = (1-\epsilon^2)^2 \left[1 - \left\{ \frac{0.094}{\lambda N_p^3} \left(1 - \frac{(2.9-N_p)^2}{3.5^2} \right) Y_t \right\}^2 \right] \quad (5-14)$$

By assigning various Strehl values to the left side of the above equation, the boundary for maximum lateral separation of the foci may be investigated as a function of the primary mirror focal ratio. Figures 5-1 and 5-2 are such contour plots. As would be expected, the allowable misalignment is much greater for the slower-primary mirror Cassegrain systems. What is significant is that a Strehl value of 0.8 is achievable for the fast primary (f/2.4) Gregorian with a misalignment of as much as 60 micrometers, which is double the value given in reference 8, and an order of magnitude larger than the one-orbit values that have been calculated by NASA.

Of even more significance is the data plotted in Figure 5-3, which shows the variation in the modified Strehl values as a function of misalignment for particular primary mirror focal ratios. In the upper set of curves, the performance variation based on the usual Strehl definition (ignoring the effect of central obscuration) is plotted. In the lower set, the effect of central obscuration is included. Note that in the lower set of curves, the performance of an f/2.4 system is always better than an f/3.0 system until the amount of misalignment reaches almost 120 microns (when the effect of central obscuration is included in the performance measure). With any active means for adjusting the position of the secondary mirror, the movement between adjustments is almost assuredly smaller than 120 microns. In Figure 5-4, the same relations are plotted for the 8.7 meter mirror separation case. As in the 6 meter separation case, the performance is better for the faster primary mirrors, but more misalignment can be tolerated.

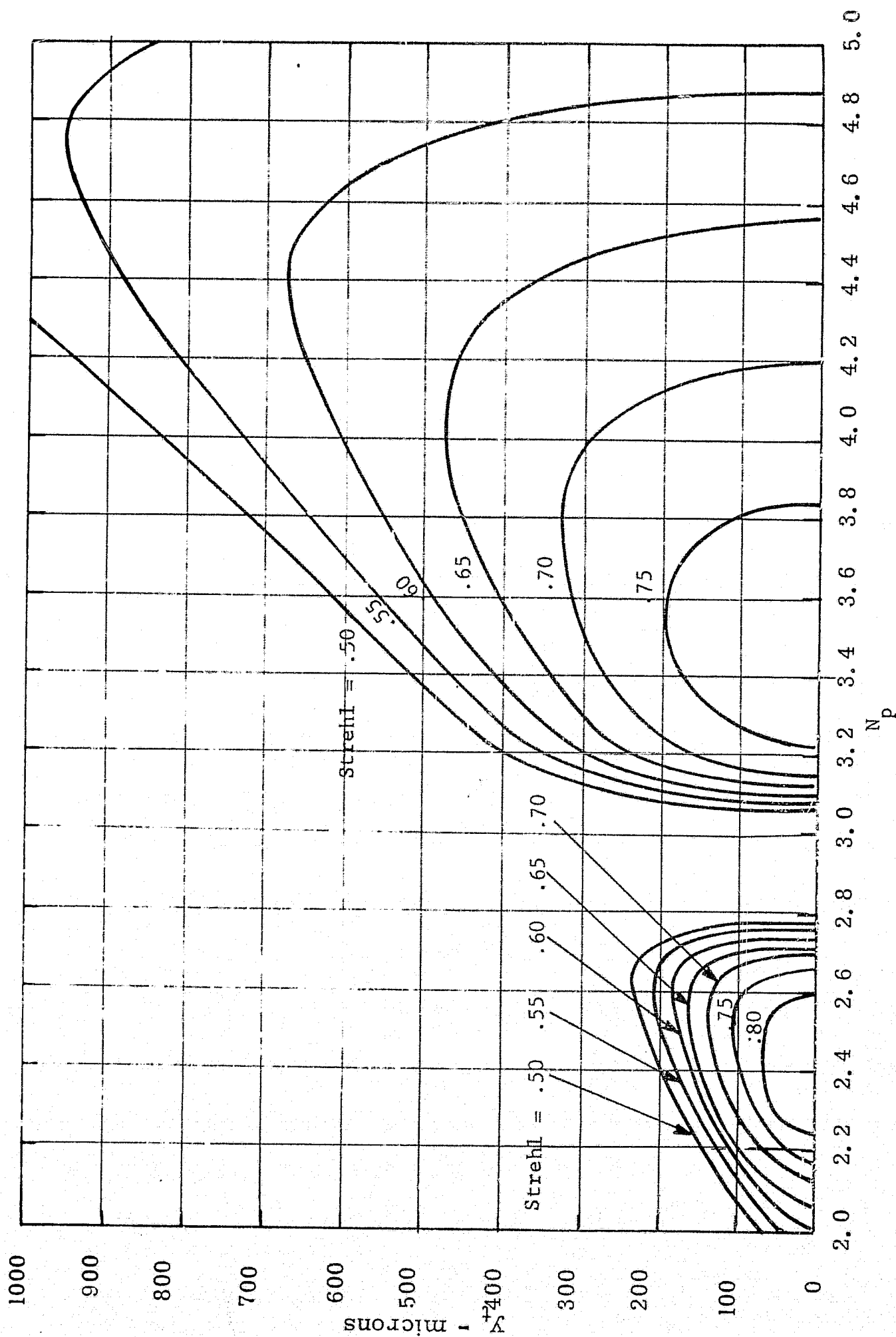


Figure 5-1. Alignment Tolerances vs Primary F Number As a Function of Allowable Strehl
Mirror Separation = 8.7 Meters; Mirror Figure Error = 0; Fully Baffled
Obscuration

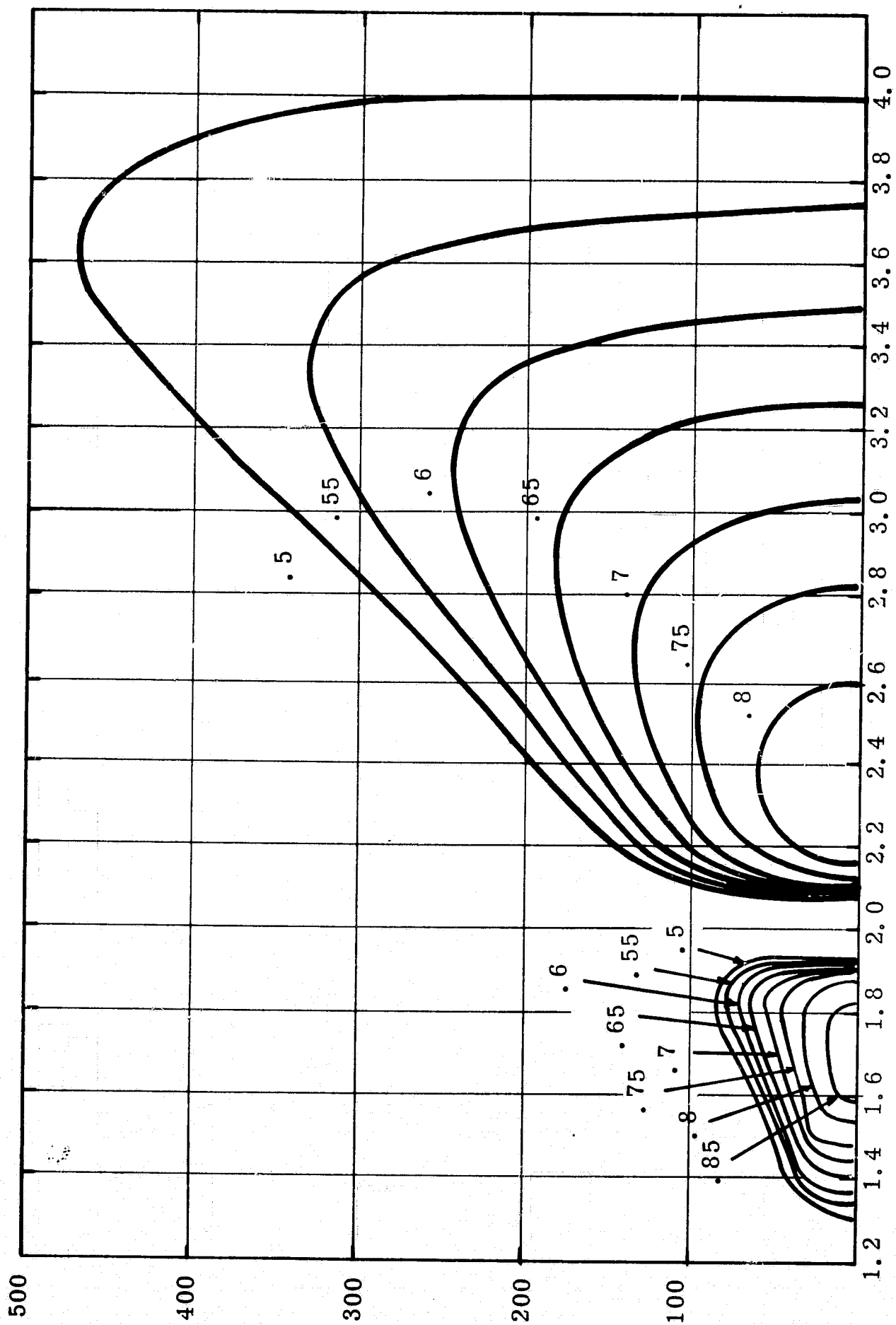


Figure 5-2. Alignment Tolerances vs. Primary F Number as a Function of Allowable Strehl; Mirror Separation = 6 Meters

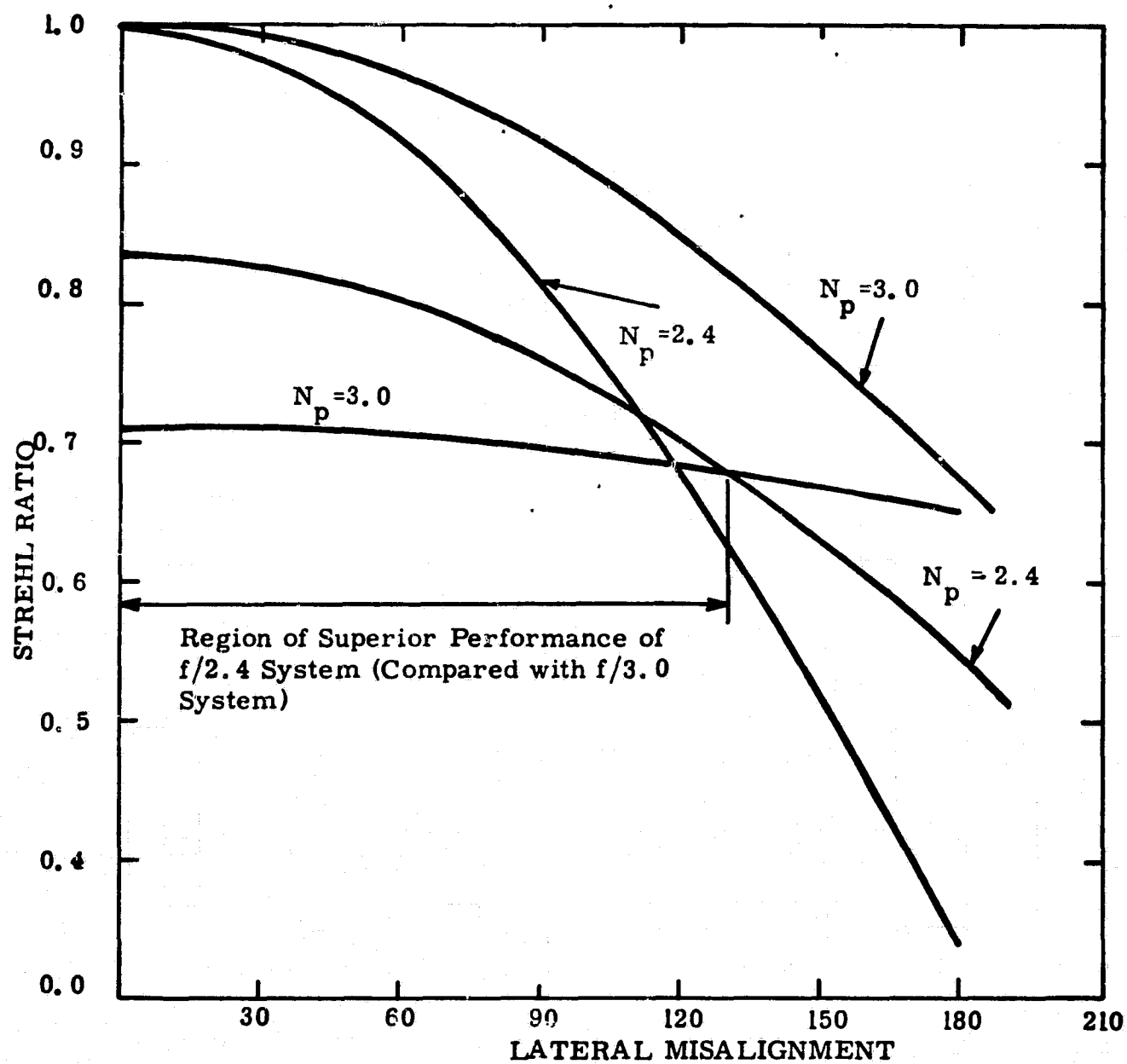


Figure 5-3. Strehl Ratio vs Lateral Misalignment for 6 Meter Mirror Separation

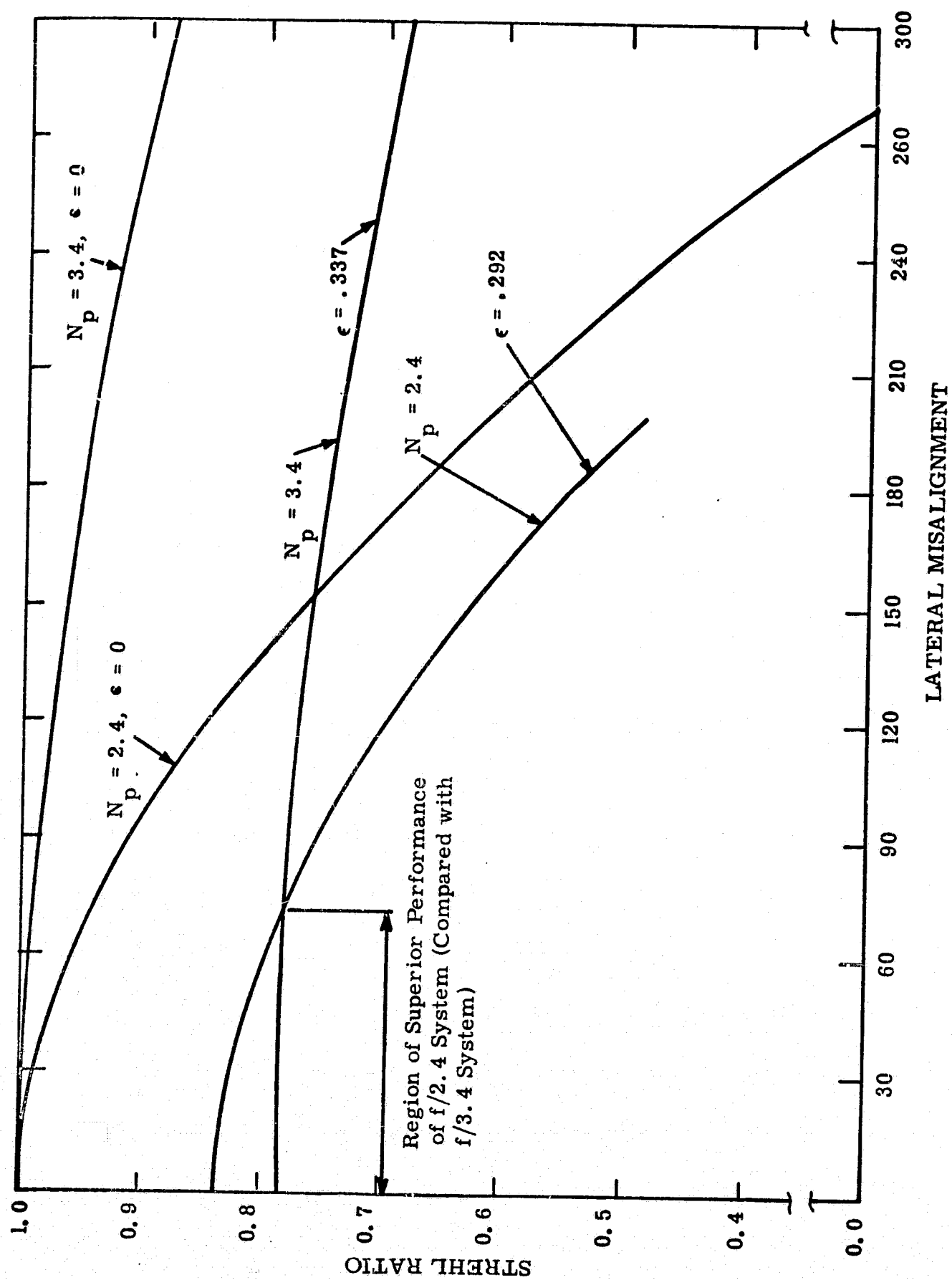


Figure 5-4. Strehl Ratio vs Lateral Misalignment for 8.7 Meter Mirror Separation and Various Primary Mirror Focal Ratios (N_p).

5.2 ADJUSTABLE SECONDARY

The device shown in Figure 5-5 is one possible scheme to position the telescope's secondary mirror. It is essentially a three axis translation mechanism fully contained within a circular housing behind the mirror. Each axis is independent of the other and each has integral with it an actuating mechanism. The present concept for the mechanism is a peristaltic piezoelectric actuator. This type of device is capable of motions in the order of $1/2$ to $1/3$ of a microinch ($1/40$ to $1/60\lambda$), and its range of travel is several inches.

The figure shows that each axis is independently capable of supporting a moment since each carriage consists of a double set of guide rails. The bearings in the system should be of low friction type. Steel in teflon-Delrin type bushings appear favorable since this combination produces a low coefficient of friction (in the order of $\mu = 0.04$) and a good working fit without undue mechanism play.

The design shows that each of the actuators, which are capable of producing forces in the neighborhood of 2 to 3 pounds, are tied into the carriages via a magnetic coupling. This coupling permits both pushing and pulling and in return exerts no excessive sideward loads.

The expected travel of this mechanism could be up to one-inch on each axis and the expected resolution is $1/2$ to $1/3$ of a microinch on any axis ($1/80$ to $1/120$ of a micron).

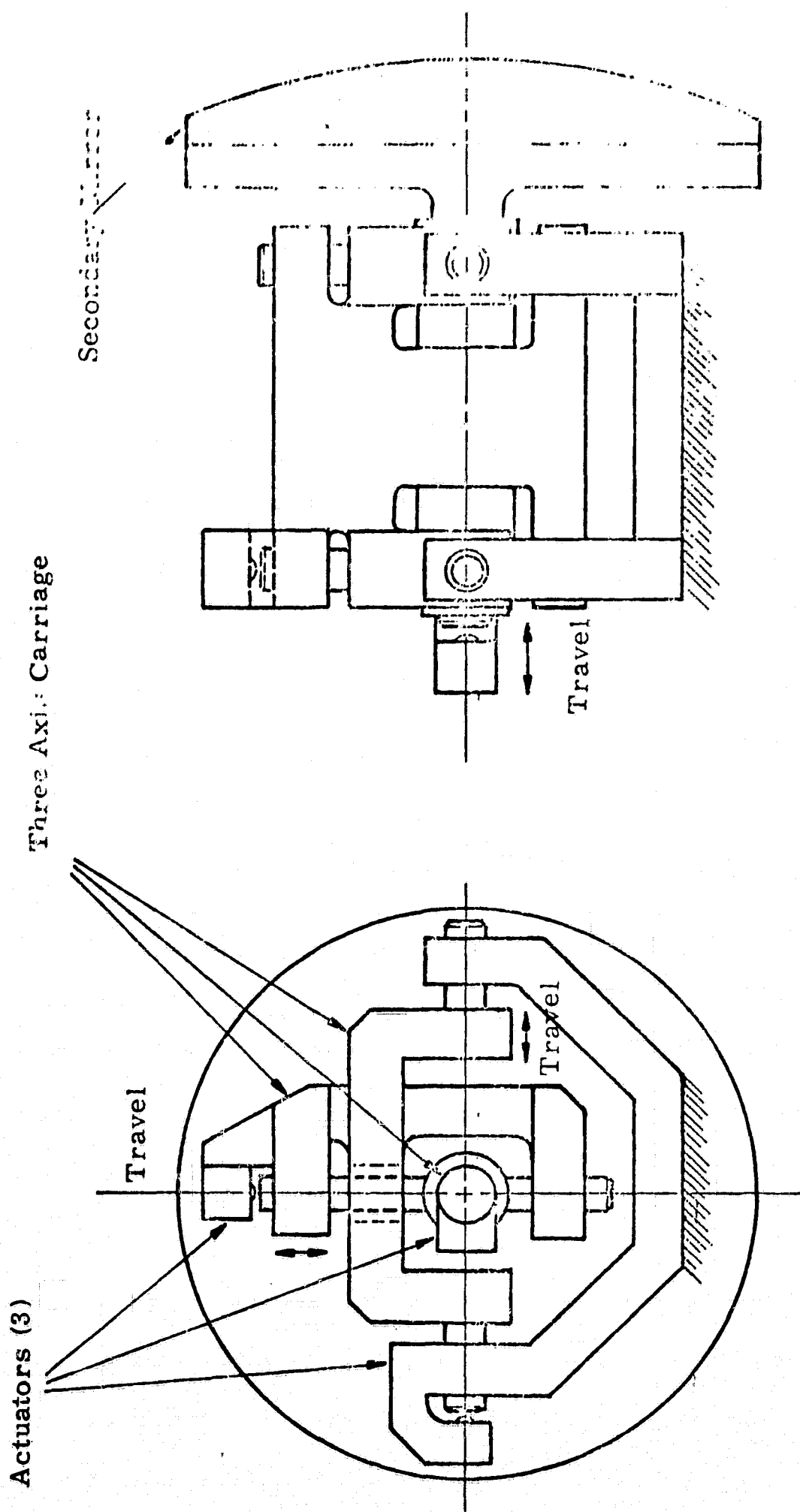


Figure 5-5. Adjustable Secondary

SECTION 6

PRIMARY MIRROR DISCUSSION

Some of the criteria governing the selection of the focal ratio of the primary mirror for the LST are discussed in Sections 3 and 4. These criteria are based chiefly upon the optical performance considerations of a system with perfect mirrors. However, the problems in manufacturing a primary mirror of excellent quality and with a smooth, low-scatter surface multiply rapidly as its size and speed increase. Thus, even in light of recent advances in the state of the art in optical fabrication, achievement of diffraction-limited performance and low-scatter surface in a high-speed 3-meter primary mirror will call for considerable investment in figuring and polishing effort largely because of the number of polishing and measuring cycles to achieve a desired level. Another factor, therefore, in the selection of the speed of the primary mirror is the allocation of time devoted to the task of figuring and polishing. Even so the funding required will represent only a small fraction of the total cost of the LST program because the efforts of relatively few people are required to improve the mirror. The foregoing discussion endeavors to show that in light of the new techniques, the obtainable improvement in performance per increment of cost is such as to warrant investment in the highest quality primary mirror, and justifies selection of the highest performance system as the correct design, consistent with overall program scheduling and the many mission requirements of the ultimate instrument.

In order to develop a formula for extrapolating the available shop experience into the realm of fabrication of the 3-meter mirror the actual fabrication data of a number of large mirrors made by Perkin-Elmer have been compiled and analyzed. The resulting empirical relation which has been fitted to these data gives the mirror fabrication time as function of its diameter, focal ratio and quality:

$$T = C D^{1.5} N^{-1.09} f(Q)$$

where

D is mirror diameter

N is the focal ratio of the mirror

$f(Q)$ is the rms surface error.

and

C is a proportionality constant.

In the above equation the fabrication time includes the necessary metrology support hours. Needless to say that the above formula is not an exact relation, but rather an "order of magnitude" indicator. An assumption was made in the derivation that the physical characteristics of the mirror material does not vary with the size of the blank and that the mount for the mirror during the polishing process is stable and provides nearly uniform support for the mirror during polishing and testing.

Figure 6-1 shows plots of aperture versus fabrication time obtained using the above relation.

The above equation is next used to plot the fabrication time trend versus mirror speed ($f/No.$), which is shown in Figure 6-2. The boxed point shown in this diagram is for a 48-inch $f/2$ parabola and is based on actual experience in fabricating such a mirror recently at Perkin-Elmer. The triangular point on the 88-inch curve is also based on recent Perkin-Elmer experience in fabricating mirrors for ground-based observatories. The curve marked "120 Inch" is the extrapolation based on this and the previous history shown in Figure 6-1. Note that the slope becomes extremely high in the vicinity of $f/1.5$ which probably marks a practical limit for today's technology.

Some of the recent advances in the technology of large mirror fabrication, testing, mirror tolerancing and measurement, will now be discussed since they will have a bearing upon the final selection of the speed of the primary mirror and the optical quality (including scattering characteristics) of the optical system.

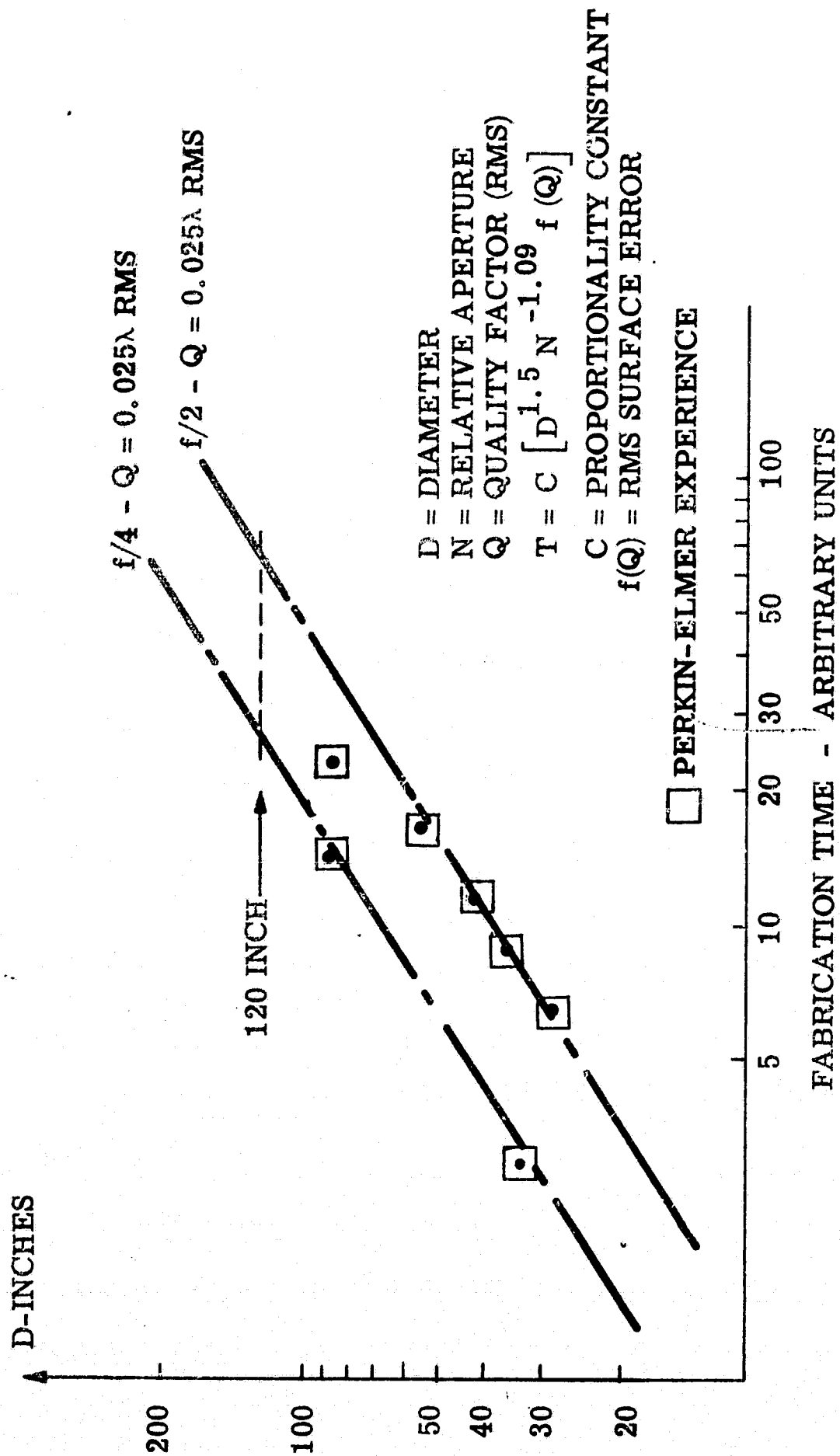


Figure 6-1. Aperture Versus Fabrication Time for $\lambda/40$ RMS Surface

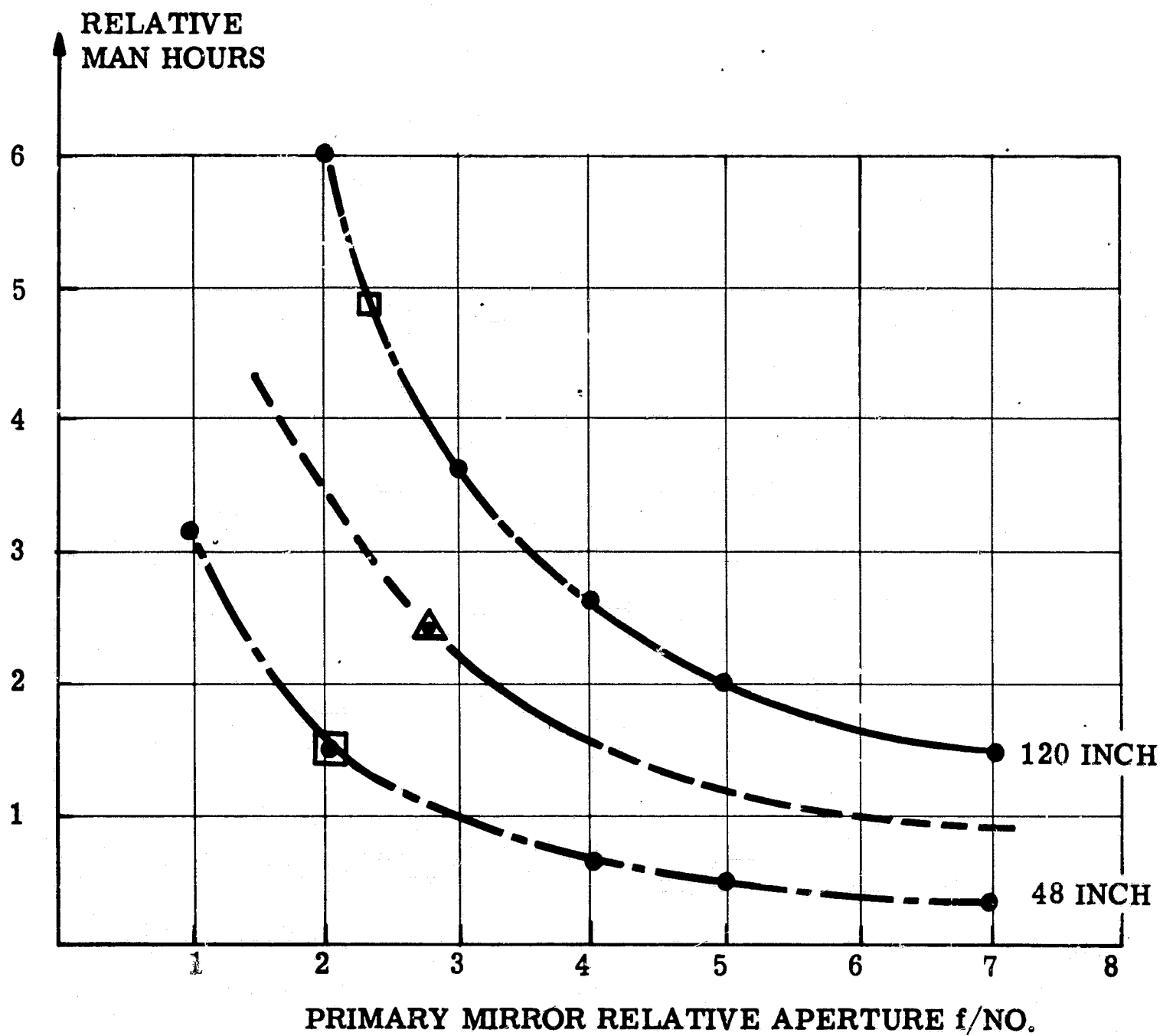


Figure 6-2. Relative Aperture Versus Fabrication Time

6.1 PRIMARY MIRROR TOLERANCING

Recent advances¹⁰ in describing wavefront variations may be applied to the tolerancing of optical surfaces. This new method is similar to the Zernike Polynomials described in Section 3. Over the past three years the method has been used for determining aberrations of an optical system from interferometric data. For this purpose a program which fits a 23-term polynomial by a least-squares method to the OPD data was developed. This method allows subsequent calculation of system performance and diagnosis of manufacturing and assembly errors. The aberration functions used are identified in Appendix E.

A more meaningful method by which the primary mirror can be specified is made possible with the aid of these functions. With this given set of functions the optical designer can recommend the maximum range of primary surface variation for each aberration, taking into account the sensitivity of subsequent optics and the detectors to particular aberrations.

Analysis of the wavefront into its component aberration provides the information needed to determine when the primary is within the specified range. The computed set of functions can then be fed back to optical design so that a complementary set can then be determined for final figuring the secondary or for subsequent correction in the relay optics. These corrections can then be achieved using one of several proprietary figuring methods developed at Perkin-Elmer. One of these methods uses a computer-controlled polishing device currently employed for figuring off-axis paraboloids up to 30 inches.

In any event, the above approach provides a method by which the primary need not be figured to a paraboloid in the absolute sense, but to a surface that is consistent with the ultimate system performance.

Interferometric analysis programs capable of performing the above computations with a set of modified Zernike polynomials are available and have been used for rapid production analysis of interferograms for the past three years

at Perkin-Elmer. This method is described in Reference 10. Briefly, it consists of the following steps:

- a. Negatives of the interferograms are scanned with a system consisting of a precision CRT (0.002 spot) driven by a digital computer and analog subsystem.
- b. A pulse of light is imaged through a Perkin-Elmer 1:1 transfer lens onto the film plane. The light pulse is modulated by the transparency and collected onto a photomultiplier. The resulting voltage is digitized and stored in the computer memory.
- c. Additional equipment includes a graphics terminal with light pen which allows rapid operator interaction with the data.

For the analysis of interferograms the operator loads the transparency into the lens system and starts the execution of the computer program. A fine raster scan is performed using the full raster resolution capability of the cathode ray tube. The locations of the maximum transmission points along each scan are saved in memory and groups of successive scans are averaged together. The averaged fringe positions along 80 scans are displayed and saved if proper. (The graphics terminal mentioned above permits the operator to discard any spurious points.) These coordinates are used in the mathematical subroutine. The mathematical program minimizes the errors in an RMS sense to the surface of interest. The computer then prints out the OPD map, RMS and peak-valley errors. The fringe scan coordinates are also written on magnetic tape for use with later calculations.

The technique has great potential, not only for specification and test of a single piece, but also for complete system test. The orthogonal polynomials permit rapid computer determination of alignment errors as well as the usual optical figure errors.

In the next section, we shall discuss some of the important aspects of testing a 3-meter primary mirror, as well as the two possible types of secondary mirror (concave and convex) that have been investigated.

6.2 TESTING

Although interference test techniques are reasonably well established, one should not overlook the fact that a physical measurement is being performed and that there is an experimental error associated with that measurement. Furthermore, with large mirrors, there is some question as to what constitutes sufficient sampling density. With an aperture of 120 inches and, as an example, with 20 fringes uniformly spaced across that aperture, means that there is one fringe for every six inches of surface. Although it is a straightforward procedure to compute an RMS surface error from this low density, the question remains whether this error is really a dependable measure of the performance of the mirror. Experience has shown that the answer to this question is dependent upon the nature of the errors on the surface and on the repeatability of the test. Errors of reasonable magnitude that appear at a frequency higher than the sampling rate will have an effect on performance depending upon their distribution. Included in the test repeatability are errors associated with mounting and remounting the piece in the test chamber and scanning and analysis of the same interferogram. Perkin-Elmer has for some time made it a practice to associate an error band with interferometric test results and would strongly recommend that this practice be included in the RMS error specification.

If the optical path difference error, δ_i , associated with each OPD point of the average error map is known, then one can calculate a quantity entitled RMS Worst Case for the average map by using the following relation which describes the RMS value which would be obtained if the OPD error always contributed so as to increase the absolute value of the OPD at each point:

$$\text{RMS (Worst Case)} = \sqrt{\frac{\sum_{i=1}^N (\overline{\text{OPD}}_i + \delta_i)^2}{N}}$$

Knowing this quantity and the RMS of the average map, one can obtain an idea of the extent to which uncertainty in individual OPD values may affect the determination of the overall RMS, which is often the quantity of prime interest.

A large number of parameters are potentially responsible for limitations on the achievable accuracy, and their relative effect should be known. These parameters may be divided into two groups including those which affect the accuracy of the interpretation of this recorded information:

Parameters Involving Recording
of Interferograms

Mount
Vibration
Vacuum Testing
Thermal Stabilization
Time
Optics and Cleanliness
of Interferometer
Film
Source Stability

Parameters Affecting Interpretation
of Interferograms

Interpolation Procedure
Fiducializing Procedure
Scanning Mode
Number of Scans
Size of Negative
Contrast of Negative
(Photographic Quality)

Orientation of Fringes
Number of Fringes
Number of Interferograms
Fringe Finesse

Background Artifacts or
Fringe Patterns
Uniformity of Background
Illumination

6.2.1 Primary Mirror Testing

An area of great importance is the method used to mount the primary mirror. It may be possible to develop a mount that simulates a uniform support condition, but eventually the mirror will be assembled into flight hardware. If the flight hardware cannot (in some orientation) provide support consistent with that used during fabrication, then it will be impossible to perform a meaningful system test. Consideration should be given in the early design stages to providing a flight-qualified mount that can also serve to support the mirror uniformly for test purposes. Use of this same mount will make possible the subsequent system tests.

The primary mirror initially will be made spherical by grinding with loose abrasive. Measurements require use of a large spherometer in order to ensure the radius is within the required tolerance. It is important at this stage of fabrication to be certain that the surface is relatively free of astigmatism. This will require a polishing cycle and testing from the center of curvature with a knife edge or interferometer.

6.2.2 Difficulty in Testing Mirrors of This Type

One of the difficulties associated with the fabrication of large light weight parabolic mirrors is the asphericity. The departure of these mirrors from the best fitting spherical surfaces can be expressed as $CY/(f/No.)^3$ where C = constant, Y = diameter of the mirror and $f/No.$ is the relative aperture. This function in terms of number of fringes over various aperture sizes is shown in Figure 6-3. The 48-inch mirror and 88 inch mirror shown have recently been fabricated by Perkin-Elmer. Slash marks on the curves indicate the relative apertures of these mirrors and some of the recommended candidate designs in the 120 inch size.

Two comments can be made relative to the above functional relationship. The maximum departure depends both on diameter and relative aperture and therefore this is the pacing consideration with regard to testing. More correction is needed in the auxiliary optics for null testing or more fringes need to be measured if testing is to be done at the center of curvature directly such as with a scatterplate interferometer. Manufacturability on the other hand is affected by the rate of change of surface and this would be the first derivative of the above expression with respect to Y . This shows that for a given $f/no.$, the rate of change of surface is independent of the diameter. The point is that the degree of difficulty in fabricating an $f/2$ of smaller size such as 48 inches is comparable to that in fabricating a larger size of the same $f/no.$ The difference is in the greater amount of material that must be removed assuming that material stability, mounting difficulties, etc. are equal.

6.2.3 Test Plans

Because of the large asphericity, the test plan becomes dependent on the state of completion of the surface. The conventional fabrication approach will be to generate the blank to the edge radius and then deepen the center by alternately grinding and polishing. This progress can be followed most efficiently by knife edge testing or wire testing at the center of curvature and aided by test plates over smaller areas where the most rapid slope changes exist. Initial aspherizing to $\lambda/2$ can be assisted with an Offner type null corrector.

Since final figuring won't commence until the errors are of the order of $\lambda/2$ or smaller distributed over areas of approximately 12 inches in

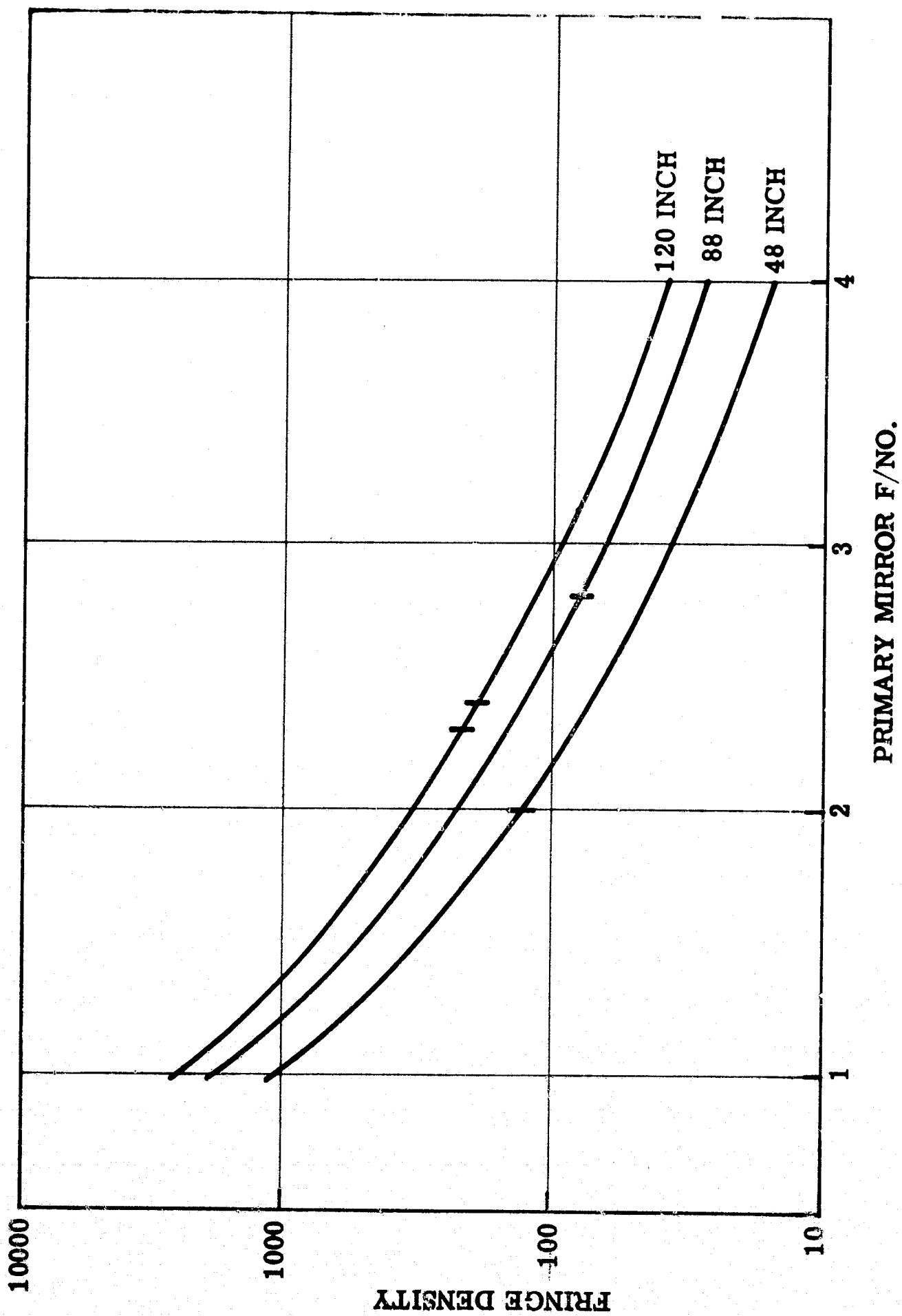


Figure 6-3. Departure from Best Fit Sphere of Parabolic Surfaces

diameter, a simple support would be used initially. It will be necessary to allow the optician to follow his progress in roughing out the aspheric with a short turn around time. Ideally, it would be preferable that the glass remain in its mount during fabrication and test at this stage.

Final figuring requires interferometric testing with quick turn around time of data since the actual polishing time will be shorter for each correction. If a null lens is used for the final test, then it would be possible to use either a SWIM^{*} or scatterplate interferometer.

The SWIM, described by Herriott and by Polster and Heintze, consist of a concave reference surface and field lens used to image the surface being tested onto the reference sphere. The fringe pattern formed at this surface is then re-imaged onto a photographic plate.

The scatterplate interferometer allows direct testing of the surface. No auxiliary high quality reference surfaces are required. The fringe pattern is formed at the surface under test and must be re-imaged by a low distortion photographic system onto a plate.

Alternate forms of testing without the use of a scatterplate are described below.

The use of null testers offers an effective method provided that an accurate null is achieved. The basis for the null test technique is given in Appendix F.

Since the asphericity of the surface under test is so great, the contribution to figure errors due to the null lens becomes an important consideration. In the first place, it is conceivable that perfect correction may not be achieved by the designer. This condition would not alter the fundamental approach toward determining the surface errors, but would most certainly impact on the amount of data reduction required. If the designer is allowed the extra degree of freedom provided by an aspheric surface, then complete correction could be achieved. The lens can be made very precisely since it is relatively small (3 inch class) and can be measured direct using a scanning

^{*}Spherical wavefront multiple beam interferometer.

microscope to examine the fringe pattern between its surface and that of a spherical test plate. Current software for data reduction could be used (in expanded form) thereby assuming quick turn around.

Without the use of an aspheric, it is possible that a perfect null cannot be achieved. This could impact on the turn around time for data reduction if the errors made are necessary to use an in-house Grant measuring engine.

6.2.4 Sensitivities in Use of the Null Lens

If a perfect null can be achieved from a design point of view, then it will be necessary to consider the following sources of error in the real lens.

- (a) surface quality
- (b) radii and thickness variations
- (c) glass inhomogeneity
- (d) physical location of lens assembly relative to center of curvature of primary

The location of the lens in turn depends on an accurate determination of the vertex radius of the blank. This determination becomes complicated by the fact that the center may be removed. There is a possibility that error contributions due to glass inhomogeneity may be overcome by a reflecting null lens arrangement.

Regardless of what the solution is in terms of design and fabrication of the lens, it remains to test and verify its performance. The lens, in fact, has aberration equivalent to the mirror being tested and must, therefore, be calibrated to a comparable degree of precision.

Alternate tests without a null lens would provide a direct interferogram from the surface. Direct scatterplate interferometry is very attractive because no intermediate optics are required and the only errors introduced are those due to the fringe position measurements. The problem would be the large number of fringes caused by the surface departure from a sphere. It is conceivable that large photographs would have to be made for precise measurements of the fringe locations. In order to obtain sufficiently dense information over the whole surface, a large number of diameters would have to be scanned and new programming would be required. The most serious effect of these requirements would be to increase the turnaround time.

Another approach that could be considered would be to make a hologram which is representative of the final aspheric surface. This could be achieved by fabricating a lens with the same amount of over-corrected spherical aberration and making a hologram of the emerging wavefront. The interference pattern would result from the combination of a wavefront from the system and that of the hologram. See Reference 11.

Another test method would employ a sub aperture (1.5 to 2m) autocollimating flat. This technique has been described (for one dimension) by Saunders¹² and can readily be extended to two dimensions. Software development for this test may be less extensive than for the two previous methods discussed.

A full size autocollimating flat should also be given serious consideration for a number of reasons. In the first place, all of the previous methods with one exception yield the type of information that would be obtained at the focus in autocollimation. Furthermore, they are useful only when the blank is a few fringes from the correct surface so that other means of testing are required to bring the surface to this point. The flat, on the other hand, could be used almost constantly after the very crude stages of initial parabolization.

Secondly, the availability of a large flat would also provide a means by which subsequent system testing could be done. This would certainly be useful not only for initial system tests, but if the system is to be retrieved via a shuttle, then subsequent system tests can be performed for direct comparison. Additional discussion on the fabrication and test of this flat is given in Section 6.2.7.

The available means for testing the primary are summarized in Figure 6-4.

6.2.5 Test Facility

In order to achieve useful results with any of the above techniques a suitable controlled environment is required. A vertical test facility is best suited for this purpose and should be capable of being evacuated. There currently exists a facility of this type at Perkin-Elmer.

The existing facility consists of a vertical vacuum tank approximately 45 ft. long and composed of four sections. Basically, it would be required to modify the uppermost 10 foot long section. The modification will be suitable for accommodating the knife edge, null lens, and interferometer.

INITIAL ASPHERIZING

- KNIFE EDGE TEST FOR ZONAL CORRECTION
- INTERFEROMETER CHECK WITH NULL LENS
- INTERFEROMETER CHECK WITH AUTOCOLLIMATING FLAT

FINAL FIGURING

- INTERFEROMETRY FROM CENTER OF CURVATURE
- SYNTHETIC HOLOGRAM
- SUB-APERTURE AUTOCOLLIMATING FLAT
- INTERFEROMETRY WITH NULL LENS
- FULL SIZE AUTOCOLLIMATING FLAT

Figure 6-4. Test Plans for Primary

6.2.6 Secondary Test

Two fundamental approaches are available for testing the secondaries. For the convex hyperboloid used in the Cassegrain, a Hindle test would be the most straightforward, but requires a large fast spherical reference mirror. For the Gregorian secondary which is a concave ellipse, a smaller convex or concave reference sphere can be placed with its center of curvature at the short conjugate of the mirror. Testing could then be achieved at the other conjugate.

The most significant advantage to the concave secondary mirror (as is used in the Gregorian system) is that the secondary mirror can be completely finished in parallel with the primary mirror with no major auxiliary test optics required. In the case of the convex secondary mirror, either a large Hindle sphere (to be used in the Hindle test) must first be fabricated, which may require nearly as much time as the primary, or, as has often been done, the secondary can be finished using the primary mirror itself as the test optic. In the latter case, program time will obviously be extended since the finishing of the secondary mirror must follow in series the completion of the primary mirror. For this reason, Perkin-Elmer strongly recommends that the Gregorian Aplanat system be given serious consideration in the selection of the final optical system for the Large Space Telescope.

6.2.7 System Test

One of the most difficult problems associated with building a large telescope is the provision of means for adequately testing its performance on the ground. Such testing is subject to errors caused by atmospheric turbulence, vibration, thermal gradients, gravitational forces, test equipment, measurement and interpretation of test results.

Figure 6-5 indicates a suggested method by which the testing may be done which should minimize the errors associated with such a procedure. An especially built facility provides space for the telescope, T, to be tested with an interferometer, I, by autocollimation against a flat mirror, M, supported from a carriage, C. The flat mirror must first be fabricated and tested to the required precision. This is accomplished in the same space by use of a polishing machine, a test sphere, S, and an interferometer used successively at I_2 and I_3 . The carriage

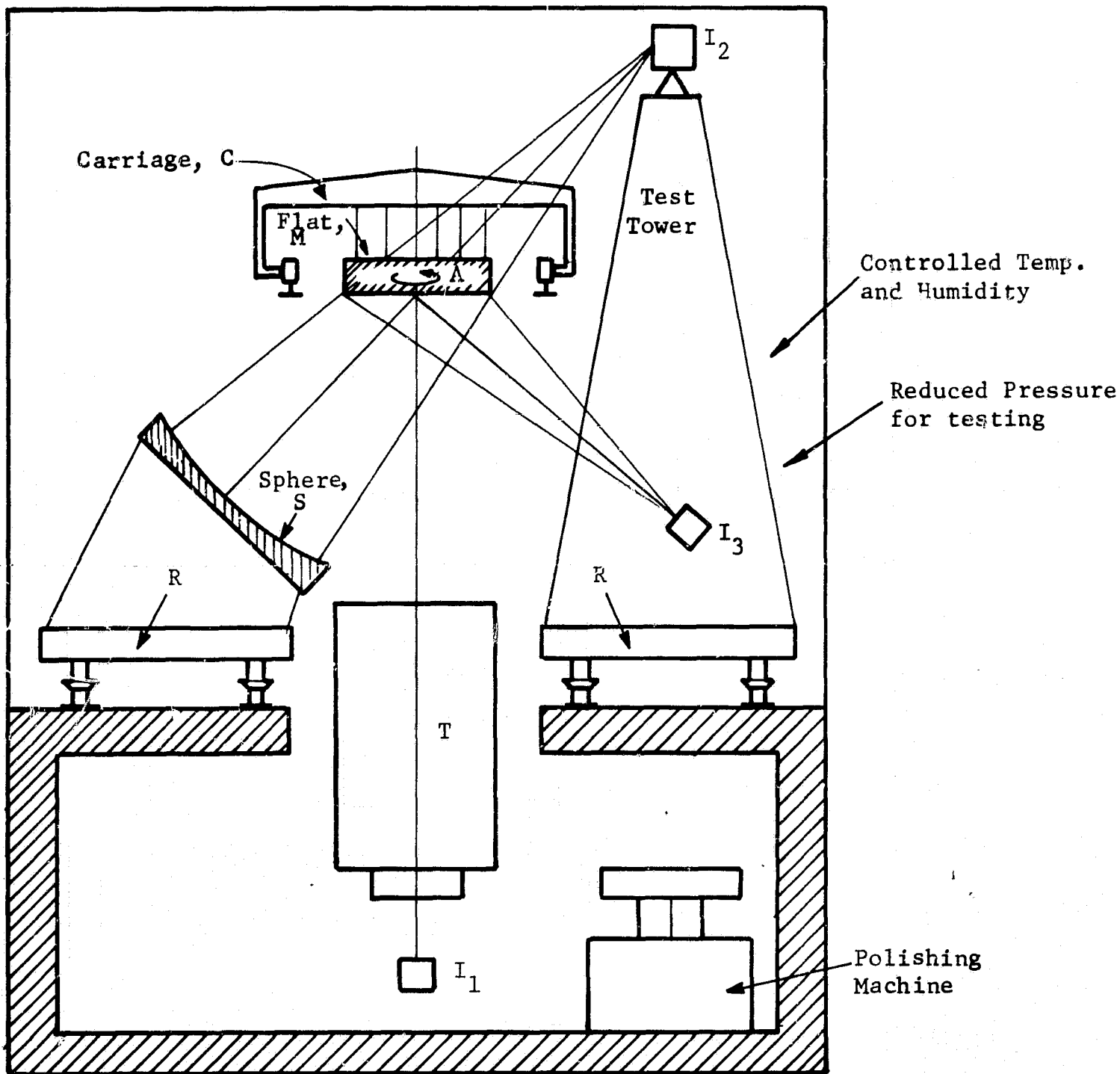


Figure 6-5. Large Telescope Test Setup

C, is on rails for removing the mirror, M, from the path between S and I_2 for accomplishing independent tests of the sphere, S, which would first be brought to a desired state of correction on the polishing machine. For testing the mirror, M, either it must be ~~rotatable~~ on the carriage as indicated by the arrow, A, or the sphere and test tower must be mounted on a rotary test table, R. Although provision for the latter is awkward it may be preferable to the process of rotating the flat because of unknown effects such as a rotation may conceivably induce. Though similar objections can be raised against the movement of the sphere the effects, if any, of such a motion may be assessed periodically when the flat is removed and assumptions as to its stability upon replacement of the flat are much more justifiable than an unassessed impairment of the flat due to its rotation.

The flat mirror need not be perfectly flat in that a small amount of residual power merely changes the focus of the telescope slightly. This circumstance can be used to alleviate the inherent difficulties associated with making perfectly flat mirrors. In addition the multiplicity of supports necessary for this vertical support provide additional means of "tuning" M to an optimum figure by use of the active optics techniques developed at Perkin-Elmer. This tuning of the flat may be accomplished on a mirror made up of segments or otherwise as may become most desirable.

The spherical mirror should preferably be sufficiently large to cover the full diameter of the flat with one interferogram. Though this would require a mirror about 150" in diameter the assurance this would provide in the assessment of the figure of the flat and hence of the system would probably warrant its use over that of a smaller aperture. The process of figuring the sphere to sufficient precision is also not required in that small residuals up to a certain level may be more economically handled by the data reduction process in the computer programming than by placing overstringent requirements on the manufacture of the test sphere.

Depending on the requirements for testing the space telescope itself, the figuring and stability of the surface on the flat mirror may be quite severe indeed. Though this may have a slight residual concavity or convexity which may be accommodated by a slight change in focus, as mentioned above, any departures

from conic form would degrade the imagery formed by a perfect test article. Though such known degradation could be accommodated in the data reduction process without undue impairment of performance in space, the degraded laboratory imagery may mitigate against accomplishment of tests such as of associated guidance equipment and the like, which may require imagery as near perfection as possible for proper assessment.

It should be noted that the test facility envisioned places the telescope and test mirror in a vertical position to more readily eliminate uncertainties due to unsymmetrical gravitational effects making zero-g simulation more tractable than would be had with equipment arranged otherwise.

Not indicated in the sketch are cranes, handling equipment, vacuum tank for coating the mirrors which would be major pieces of equipment needed in any such facility.

6.3 THE SCATTERED LIGHT PROBLEM

Another aspect of the problem of designing and manufacturing the primary mirror for a Large Space Telescope is that of determining the probable amount of light scatter, setting tolerances for scatter over the wavelength region, and then investigating the mechanisms which cause the scattering of light with the objective of controlling and holding to an acceptable level this unwanted phenomenon. In Appendix G we examine the problem of obtaining a photometric measure of the brightness of a 29th magnitude star with a specified accuracy and obtain upper limits on the amount of scatter (scatter coefficient) which can be permitted for this type of measurement. In the next paragraph some of the scatter mechanisms are examined and some tentative means for the control of scatter are set forth. Very little experimental data is available concerning the scatter of large mirrors in the vacuum ultraviolet part of the spectrum, but additional information should be forthcoming as more operational experience is obtained in the OAO series. As this information is obtained, the limits that have been set in this study should be re-examined in conjunction with the new data to determine whether or not light scatter in the Large Space Telescope is a disabling problem. The results of this study suggest that is probably is not, at least in the visual wavelength region, except for observations close to very bright stars.

6.3.1 Surface Specification

For specification of the surface it can be said, to a first approximation, that the scatter function variation with angle should show no sharp peaks and that the total fraction scattered should be less than a certain amount-dependent on the star field brightness. The prime importance is to minimize scatter within small angles.

The scatter can be connected with the spatial frequency of errors on the surfaces, but this is an indirect way of expressing the scatter. The scattering associated with these errors is important over an angular spread up to 12° for the primary and $1/2^\circ$ for the secondary. Furthermore this scatter is probably associated with sub millimeter errors. Ordinary test methods associated with determining the overall surface errors of the primary will not suffice for these errors. It may be necessary to apply high finesse multiple beam interferometric techniques for examining selected regions of the primary and/or secondary.

It should be noted that smoothness of finish is of great importance and fine scratches should be avoided (giving low spatial frequencies).

It can also be noted it is better to finish the primary first and to do any necessary hand corrections on the secondary because of the relative dominance of the scatter of the primary.

6.3.2 Characterization of Mirror Surface

In general, the more accurately a mirror is figured the more difficult it will be to hold the scattering to some low level. To establish quantitative predictions of what the scatter will be prior to going through some polishing cycle is not possible at this moment in time. Nor is it possible to obtain scattering data from interferometric measurements, since the sampling frequencies are not in the correct range. The spatial frequencies for all points on the mirror would need to be known.

The best that could be done would be to establish a dig-scratch tolerance and polish the mirror by techniques which in the past have yielded low scatter surfaces. Some tradeoff must be established between figure and scatter which would be dependent on usage.

A point to be borne in mind is that the telescope may be required to yield results in the UV down to $1000\text{\AA}$. Here the scatter would probably become of dominant importance as opposed to the more usual $6328\text{\AA}$ test wavelengths. Also the coatings and coating life would have to be considered.

Another point is that whatever figure the scatter is reduced to there will be some light acceptance angle and star brightness combination where faint stars will not be observable as shown in Figure G-1 of Appendix G.

A program to establish empirical relationships between polishing techniques and resultant scatter should be set up to perhaps minimize the product (root mean square variation) x (scatter into angles of interest). This would basically involve the manufacture of mirrors, which must be curved, of various sizes, using various techniques, and then measuring the scatter coefficient accurately. Hence the best techniques or techniques could be empirically established.

6.4 CONCLUSIONS AND RECOMMENDATIONS

Based on the foregoing discussion, Perkin-Elmer makes the following recommendations with regard to the manufacture of the optics for a 3-meter telescope:

- (1) The speed of the primary mirror can be set as fast as $f/2$ without incurring exceptional time and cost requirements.
- (2) The tolerancing of the primary mirror can be much more sophisticated than has been the practice in the past. The application of the new techniques requires that the entire optical design task be integrated and handled as a single system design function, however. The overall optical design task should include the instrumentation relay optics as well as the primary light-collecting optics.
- (3) A Gregorian Aplanatic optical system should be adopted, primarily because of the reduction in the time and money required to fabricate the optics.

- (4) Serious consideration should be given to the design and fabrication of a composite test flat which could be used in testing the primary mirror as well as in final testing of the completed telescope.
- (5) Experiments on near-field scattering in the vacuum U.V. should be performed and correlated with analysis.

SECTION 7

TRADEOFF DISCUSSION

7.1 OPTICAL DESIGN RANGE FOR ACCEPTABLE FIGURE OF MERIT VALUES

We have now discussed the principle effects that will determine the optical performance, manufacture, and operation of the Large Space Telescope. We have seen that choosing an optical design with a fast primary mirror will permit a minimal central obscuration that will maximize imaging performance, that manufacture of a 3-meter mirror as fast as $f/2$ is feasible, and that the operational constraints imposed by presently-contemplated structural designs are commensurate with the alignment tolerances inherent in a fast-primary system. Two other aspects of the optical design problem need to be considered before the design can be considered final; namely, the optimum $f/\text{no.}$ for the instrument section, and the desired field curvature. Unfortunately, the latter two considerations are outside the scope of this study, and it is recommended that additional work be authorized to continue the optimization of the entire optical system. However, we shall apply what knowledge is available at this time to the tradeoff analysis.

A study of the scientific instrumentation has recently been completed under NASA Contract and published as GSFC Report No. X-670-70-480 by the Kollsman Instrument Corporation.¹³ This report indicates that the focal ratio at the Cassegrain focus should be $f/12$. The reasons for this selection are not clear, but it is assumed that they stem from space considerations inside the instrument section housing. In recognition of this, one of the options in the optical design tradeoff will be a final focal ratio constrained to $f/12$.

The question of field curvature is much more difficult to deal with since it is only the field curvature at the detector plane which matters in the final analysis, and many compensating curvatures can be introduced by the optical designer in the design of the instrumentation optics. However, the following general statements may be made regarding field curvature in the two systems we have been considering:

- a.) The curvature of field at the telescope focal plane is increased for faster primary mirrors and for greater magnification in the secondary mirror
- b.) The direction of field curvature in the radial direction will be reversed when going from a concave to a convex secondary mirror, and the tangential curvature may reverse depending upon the relative focal length of the primary mirror.
- c.) The most desirable direction and amount of field curvature at the telescope focal plane should be determined by the size of the field at the detector focal plane, the amount of residual field curvature which is tolerable, and the design of the relay optics used in the scientific instruments.

Generally speaking, the direction of field curvature in the concave secondary mirror system is easier to correct with relay optics than is the field curvature inherent in the convex mirror system. In passing, it is suggested that the optical design for the instrumentation package should be tested in the light of the figure of merit criteria developed in this study. Additional work in this area is highly recommended as additional performance can very probably be achieved by some changes in the optical design of the relay optics.

We shall now consider examples which illustrate the optical design limits. Figure 7-1 lists the design which are considered to define the optical design limits. The four situations given are

- a.) Maximum performance
- b.) Constrained primary mirror speed
- c.) Constrained final focal ratio, and
- d.) Slow primary mirror system similar to that discussed in other studies.

The performance figure of merit varies from 0.88 down to 0.7, with the slow primary mirror system having the lowest performance. Selection of a performance level to recommend is difficult since none of the centrally-obscured systems achieve performance levels above the "knee" of the performance vs. central obscuration curve shown in Figure 3-4. This point of diminishing returns

<u>HIGHEST FIGURE OF MERIT</u>		<u>CONSTRAINED PRIMARY MIRROR FOCAL RATIO ($f \sim 2.3$)</u>	
GREGORIAN	CASSEGRAIN	GREGORIAN	
$N_p = 1.72$	$N_p = 2.3$	$N_p = 2.3$	$N_p = 2.3$
$N = 16$	$N = 20$	$N = 20$	$N = 20$
INTRAVERTEX = 6M	INTRAVERTEX = 6M	INTRAVERTEX = 6M	INTRAVERTEX = 8.7M
FIGURE OF MERIT = 0.88	FIGURE OF MERIT = 0.84	FIGURE OF MERIT = 0.84	FIGURE OF MERIT = 0.85
<u>CONSTRAINED FINAL FOCAL RATIO ($N \leq 12$)</u>		<u>SLOW PRIMARY SYSTEM</u>	
CASSEGRAIN	GREGORIAN	CASSEGRAIN	
$N_p = 2.56$	$N_p = 2.22$	$N_p = 4.0$	$N_p = 4.0$
$N = 12$	$N = 12$	$N = 12$	$N = 12$
INTRAVERTEX = 6M	INTRAVERTEX = 8.7M	INTRAVERTEX = 5.7M	INTRAVERTEX = 5.7M
FIGURE OF MERIT = 0.81	FIGURE OF MERIT = 0.81	FIGURE OF MERIT = 0.7	FIGURE OF MERIT = 0.7

Figure 7-1. Examples Illustrating Optical Design Limits

may be thought of as occurring where the slope of this curve is between $1/3$ and $1/2$ of the maximum slope. This occurs for obscuration ratios between 0.1 and 0.2, and this cannot be achieved within the stated ground rules. Thus, significant gains can be achieved for any decrease in primary mirror focal ratio. In the vicinity of focal ratios of $f/2$, the achievable Strehl ratio (barring optical aberrations) is 0.86, which leaves a margin over the commonly accepted value of 0.80 ("well corrected optical system"). The total allowable wavefront error which could then be used for the tolerancing of the optical design and manufacturing errors would be 0.06λ . Though stringent, this is consistent with the current state of the art. Any increase in the focal ratio of the primary mirror would impose more stringent tolerances on the optical design and manufacturing quality, and the achievement of an 80 percent of the theoretical performance would become problematical at best.

7.2 TELESCOPE TYPE

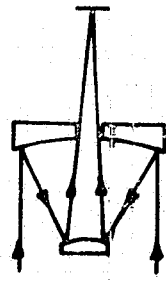
Throughout the study, we have considered the possibility of two telescope forms; the Cassegrain and the Gregorian (or their optimized counterparts, the Ritchey-Chretien and the Gregorian Aplanat). In view of the discovery that the maximum performance achievable is identical for the two types of system, the tradeoff between them rests more on secondary considerations such as manufacture and weight. Figure 7-2 lists the comparative advantages and disadvantages of the two types of system. Many of the points are self-explanatory while others require elaboration.

The Cassegrain has the disadvantage that the convex secondary mirror cannot form a real image without the assistance of auxiliary optics. This creates problems in manufacture as has already been pointed out, and this fact also makes Cassegrains difficult to align during assembly. Aligning a high performance telescope by making evaluations of the imagery (as opposed to interferometric wave front analysis) produced is generally a risky business because of the subjectivity which can come into play. It is much more straightforward to use an alignment theodolite in aligning the system, making use of various fiduciary marks on the major optical elements and the real images of these marks which the mating optics produce. Another possible advantage to having real images inside the telescope system is the accessibility to the prime focus. It is often desirable to introduce masks or stops (such as a Lyot stop to absorb the diffraction from the edge of the primary mirror) at an

ADVANTAGES

DISADVANTAGES

CASSEGRAIN



LIGHTER, SHORTER SYSTEM

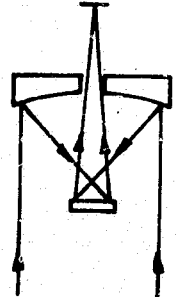
REQUIRES EXTENSIVE AUXILIARY

MORE EXPERIENCE WITH TYPE

OPTICS FOR COMPONENT
MANUFACTURE

DIFFICULT TO ALIGN

GREGORIAN



GREATER EASE IN MANUFACTURE

WEIGHT

PETZVAL SUM EASIER TO

CORRECT TO OBTAIN FLAT FIELD

PROBABLY LESS SUSCEPTABLE TO

THERMAL DEFLECTION

PRIME FOCUS

ACCESSABLE

EASIER TO BAFFLE

Figure 7-2. Optical Design Comparison

intermediate focal plane, and this becomes a possibility in the Gregorian form that does not exist in Cassegrainian systems.

Under advantages for the Gregorian system is listed "probably less susceptible to thermal deflection". The reason for this is illustrated in Figure 7-3. Because the Gregorian system has an internal shared focal point, the bowing of the telescope tube due to unsymmetrical temperatures does not move the focal points as far apart as would be the case when the shared focal points are external to the tube. In order to evaluate this effect, actual telescope tube shapes, mirror mounting arrangements and the geometry of the spacers between the primary and secondary mirror supports need to be known.

The Gregorian is easier to baffle since the design permits a tubular baffle surrounding the secondary mirror, surmounted by a conical baffle similar to that used in the Cassegrain. Since this type of baffle has more area than the conical baffle used with the convex Cassegrain secondary mirror, there is more opportunity to absorb stray light.

Most of the above items would seem to favor the Gregorian type, but there is one disadvantage to the Gregorian which could prove to be overriding, and that is weight. The weight would tend to be greater because the telescope length* needed to house the telescope is longer in the case of the Gregorian form. However, the recommended Gregorian forms still fit within the space envelope given in the ground rules, and in any case, the telescope tube should be as long as possible to provide the maximum baffle and shielding possible. If a Cassegrain is chosen, the tube construction beyond the secondary mirror could be lightened since no structural loads need be taken beyond this point (except for its own weight).

Before proceeding to the final recommendations, the sensitivity of performance to changes in some of the design parameters will be examined.

7.3 DISCUSSION OF SENSITIVITIES

In this section we shall discuss the relative sensitivity of the wave front-related parameters to the primary design-related parameter, central obscuration (which is determined in turn by the primary mirror focal ratio).

* The ratio of the length of the Gregorian, L_G , to that of the Cassegrain, L_C , is given approximately by $\frac{L_G}{L_C} = \frac{1+\epsilon_s}{1-\epsilon_s}$, where ϵ_s is the unbaffled obscuration ratio.

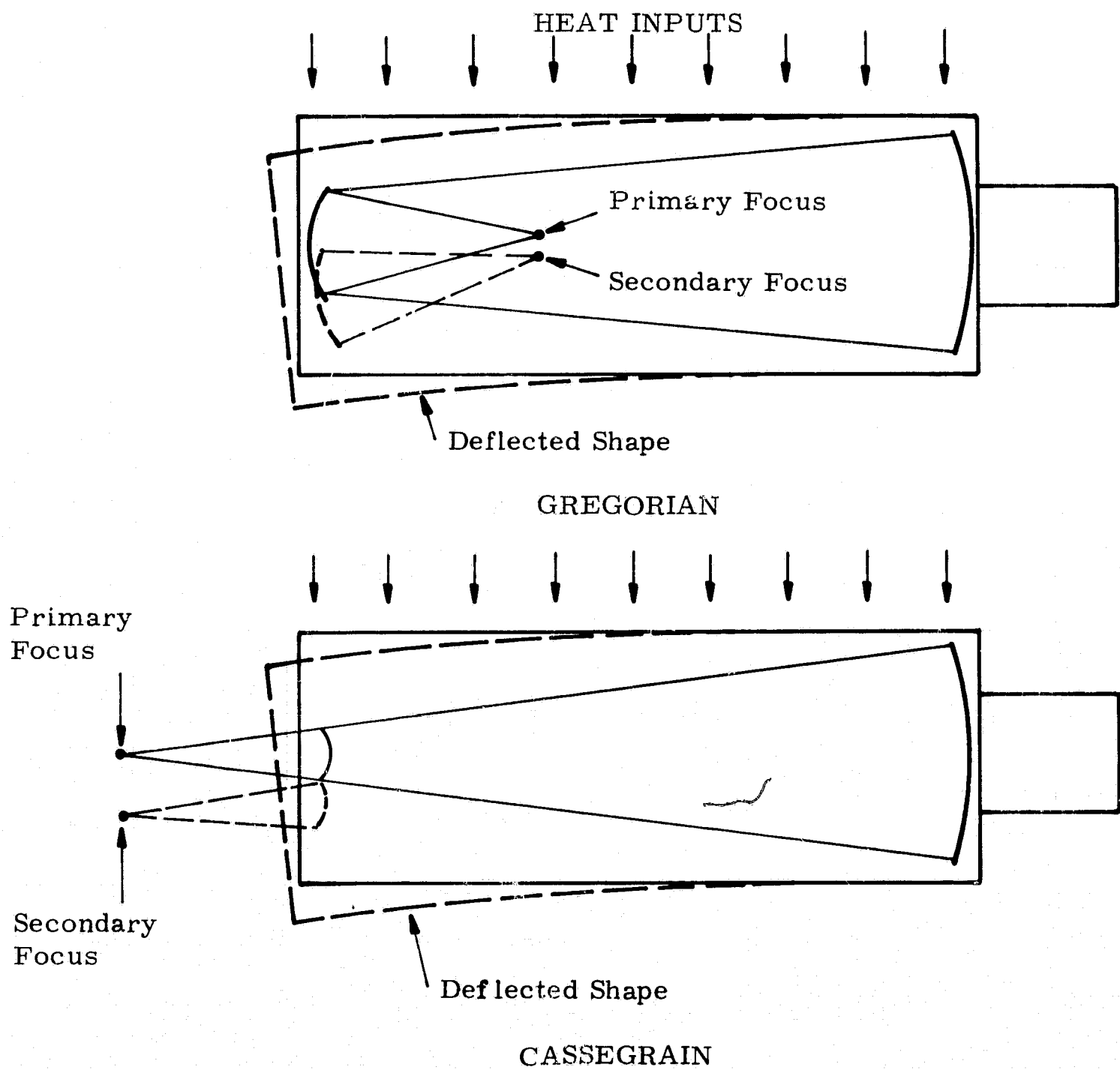


Figure 7-3. Comparison of Effects of Thermally Induced Deflections on Optical Alignment

The modified Strehl ratio which we have been using in the parameter variation studies was given in the equation of Figure 3-15 which is repeated here:

$$S = (1 - \epsilon^2 - \frac{2\pi^2}{\lambda^2} (\Delta\Phi)^2)^2$$

By expanding this equation, evaluating the terms, and eliminating those which are negligible, we can arrive at an equation which is useful in assessing the sensitivity or rate at which required tolerances would vary as a given design parameter is varied. By neglecting terms in ϵ^4 , $\Delta\Phi^4$, and $\epsilon^2 \Delta\Phi^2$, and regrouping

$$S = \left[1 - \epsilon^2 - \frac{2\pi^2}{\lambda^2} (\Delta\Phi)^2 \right]^2$$

Now, to obtain a given level of performance, the quantity subtracted from the 1 must be held constant. Therefore, the relationship

$$C = \epsilon^2 + \frac{2\pi^2}{\lambda^2} (\Delta\Phi)^2$$

can be deduced, where C is related to the desired performance of modified Strehl value. Note that the above equation is of the same form as the equation of an ellipse or circle. In Figure 7-4, we have plotted the relationship for several Strehl values.

The points corresponding to a central obscuration set by an f/2 primary mirror and by a $\lambda/25$ tolerance on the wavefront error (commensurate with the current state of the art) are indicated on the graph. Note that the intersection for these two points occurs just about on the contour for a Strehl value of 0.80, indicating that this design choice is essentially the only one possible which can meet a requirement of 0.8 Strehl. This suggests that off-axis systems or very simple Newtonian systems such as discussed in Offner's paper in Appendix A are the next step in the development of larger space telescopes. No exploration of off-axis systems was included in this study, because the ground rules would be violated in one way or another. However, the necessary design and manufacturing capability is in existence today at Perkin-Elmer, and as better focal plane figure error sensors are developed, many of the problems which currently exist with off-axis optical systems can be eliminated.

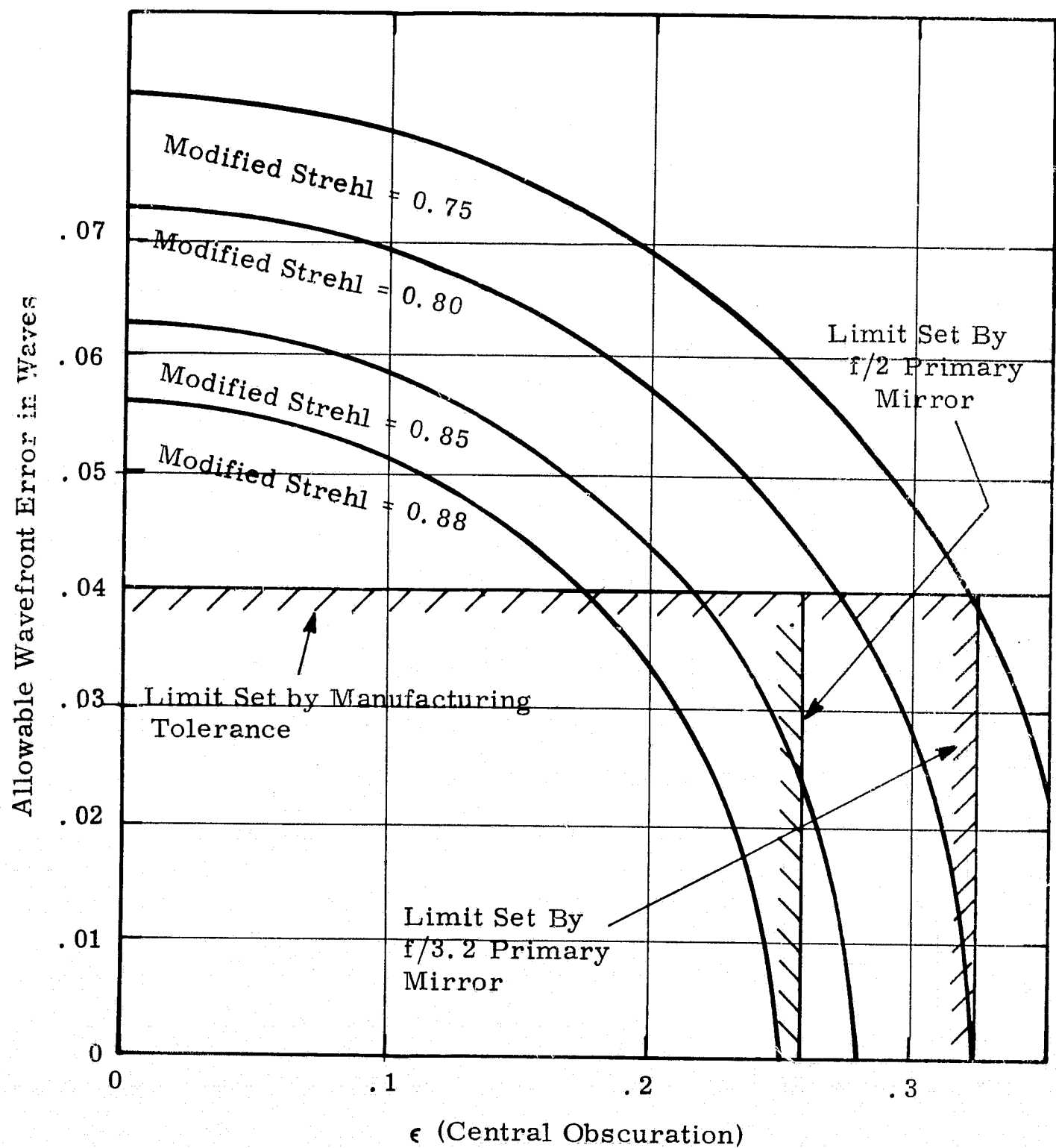


Figure 7-4. Tradeoff Between Central Obscuration and Allowable Wavefront Aberrations

7.4 RECOMMENDATIONS

The tradeoff analysis shows that there are strong reasons for preferring a telescope system with a fast primary mirror (in the vicinity of $f/2$) since this will give the highest performance for the astronomy research function of the Large Space Telescope. The sensitivity analysis shows that the primary mirror recommendation is compatible with today's state of the art in large optics fabrication and that it is possible to achieve the performance level which most authorities consider to be a worthwhile achievement ($\text{Strehl} = 0.8$). We have not dealt with the problems of telescope misalignment in the tradeoff and sensitivity analysis because the next section will allude to some developments currently underway that promise to alleviate the problem and eliminate it from consideration as one of the design-limiting factors.

The choice between the Cassegrain and Gregorian forms, as we have seen, rests more on subsidiary factors such as schedule time, cost, and system weight, than it does on overall system performance. Since this study was not to deal with the overall systems problem, a recommendation which has telescope weight as one of the decisive factors cannot be made at this time. However, because of other factors which favor the Gregorian form, Perkin-Elmer recommends serious consideration of a Gregorian Aplanat form (a design corrected for spherical and comatic aberrations, yielding the same performance as the Ritchey-Chrétien).

SECTION 8

FIGURE SENSING TECHNIQUES

The scientists and astronomers responsible for the design of the LST have the design goal of maximizing the scientific information capability of the telescope under the constraint of minimizing cost. The capability of measuring the performance of the telescope optical system will be a requirement if this design goal is to be approached. A number of interferometers using lasers and operating at the primary center of curvature have been designed, fabricated, and successfully tested.^{14,15,16} These units are primarily designed to monitor the figure of the primary mirror. However, these interferometers may be modified to permit one to obtain information about the primary-secondary alignment. Thus, the center of curvature type interferometers satisfy all the requirements of measuring both alignment errors and intrinsic errors in the primary mirror. Since these devices have been reduced to practice and have been adequately discussed in the literature, we will not consider them further.

Another class of instruments operate in the telescope focal plane and obtain the required information by analyzing the point spread function of an unresolved (or nearly resolved) stellar source. Two possible devices that operate in the focal plane are the Koesters prism¹⁷ and the **In-flight Alignment (IFA) System of Stratoscope II**.¹⁸ The latter device transformed the point spread function profile into an electrical signal by scanning a slit in front of a star image formed on the face of a phototube.

The ultimate output of a Focal Plane Figure Sensor and its related electronics will be an Optical Path Difference (OPD) map of the telescope exit pupil. That is, for each point in the exit pupil there will exist a number equal to the number of waves (or portion thereof) that the wavefront emitted from the telescope differs from some reference (but not necessarily optimum) wavefront.

These data are in the form of an array and may be operated upon by an existing software package¹⁹ utilizing a digital computer. The technique decomposes the wavefront and matches it to a 22 term polynomial.

By judicious choice of the polynomial terms, it is possible to obtain by inspection the data required for the following diverse functions.

1. Align secondary to primary
2. Obtain diagnostics on alignment and intrinsic mirror aberrations
3. Provide data on the aberrated wavefront that is crucial for post exposure image processing
4. Monitor mirror figure for various mounting loads or thermal environments
5. Provide control signals for Active Optics servo electronics (optional)

These functions are shown conceptually in Figure 8-1.

Note that the focal plane figure sensor truly optimizes the complete optical system since it operates directly on the wavefront containing the scientific information. This makes it possible to balance wavefront errors resulting from misalignments, mirror distortions, and optical design in such a manner as to maximize the overall system performance. As discussed in Section 3, the criterion by which we judge the optical system performance may be different for different functions. This change of criterion may be easily implemented by modifying the weighting functions in the computer software. Thus the focal plane figure sensor is both versatile and truly optimizes the overall optical performance.

Another important advantage of the focal plane figure sensor is that it eliminates the barrier against using fast (small f/D ratios) primaries. Classical technique of aligning the primary-secondary mirrors have limited accuracies which in turn limit the speed of the primary mirror. However, the perfection of the focal plane figure sensor allows one to align the secondary mirror relative to the primary mirror independent of the primary mirror f-number. The limiting factors are now the capability to accurately position the secondary and maintain it in position over the required exposure time. The sophistication of microinch positioners²⁰ and orbiting telescope mechanical designs²¹ will permit one to consider faster primaries for the LST.

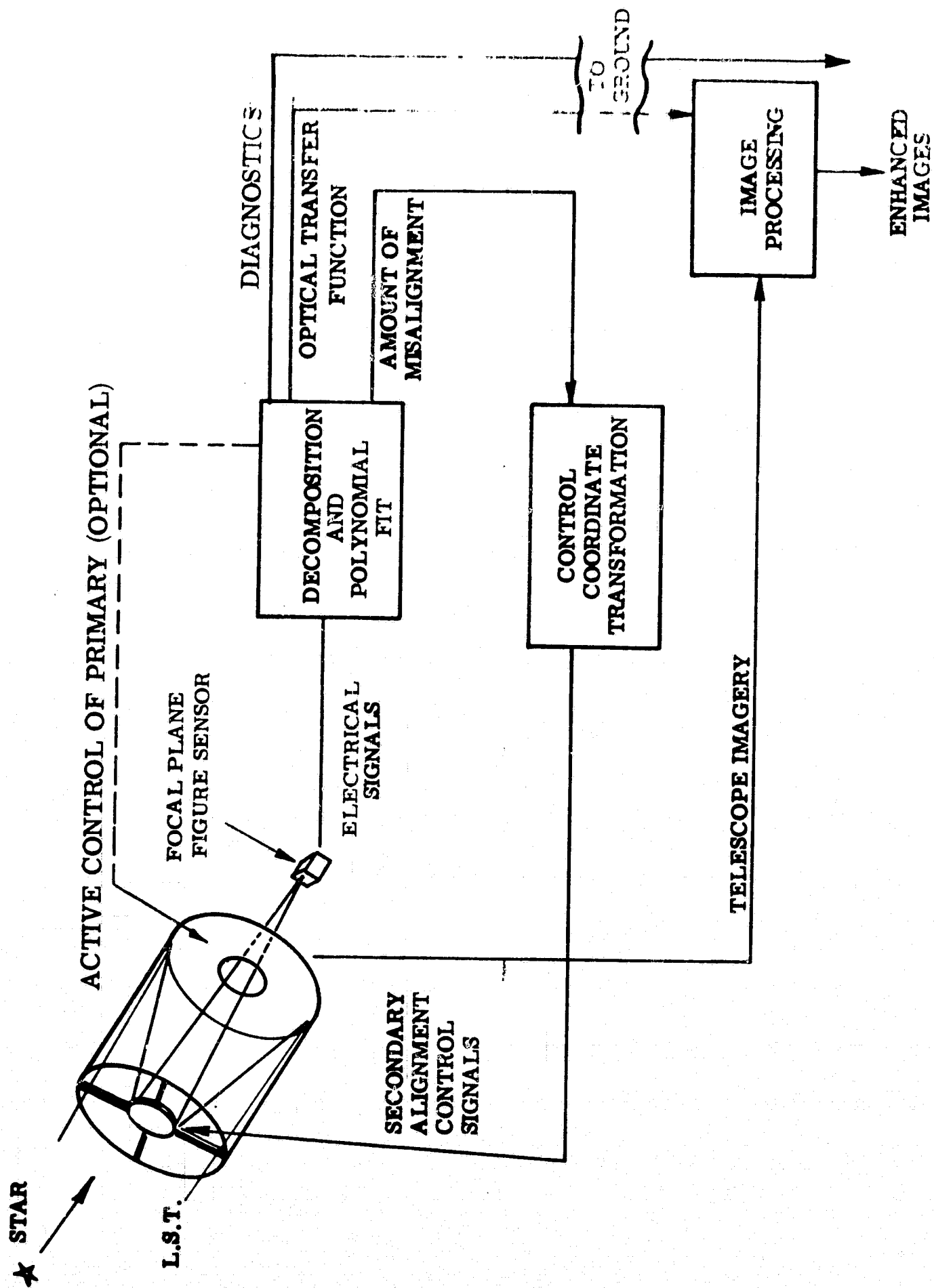


Figure 8-1. Figure Sensing Concept

SECTION 9

CONCLUSIONS AND RECOMMENDATIONS

The maximum diameter of the collecting aperture of the Large Space Telescope is a fixed quantity, determined by the diameter of the launch vehicle. An important task of the telescope designer is to maximize the scientific utility of this constrained telescope collecting aperture. In this study we have ascertained a means whereby the effect of various changes in the most important parameters of a three-meter aperture space astronomy telescope can be evaluated to determine design trends and to optimize the design.

Three figures of merit were derived based on three different areas of astronomy interest: 1) the class of instruments in which the resolving power is set by a slit or small aperture in the focal plane; 2) those investigations involving the extreme in distance penetrations or detection of faint objects; and 3) the imaging of extended objects such as planets or gas clouds having scenic detail which should be reproduced with the utmost in fidelity. The three figures of merit derived from each of the three classes of useage are termed, respectively, the Normalized Relative Energy, the Modified Strehl Ratio, and the Relative Edge Response. (The modification of the Strehl Ratio was to include the effect of central obscuration.) Each of these figures of merit shows a similar trend with increases in central obscuration: the value of each figure of merit falls off with increasing central obscuration, slowly at first, but at a maximum rate when half the aperture is obscured. Based upon a reasonable design goal of achieving 80 percent of the theoretically maximum performance, the central obscuration diameter should not exceed 32.5 percent of the diameter of the primary mirror, and should be less than this to allow for other performance degrading effects such as manufacturing tolerances, alignment errors, etc.

The effects of the basic Seidel aberrations (coma, astigmatism, and spherical aberration) in centrally obscured systems were also investigated.

The approach taken was to calculate the amount of each of the aberrations that is tolerable to reach a given performance level (or a given Strehl value) as the amount of central obscuration was varied. The results of the procedure show that the constraint on central obscuration should be even tighter, as the tolerable amount of aberrations is reduced by 20 to 30 percent when the central obscuration is about 0.25 of the aperture diameter, and the tolerance curves have a noticeable decrease at 0.2 central obscuration (see Figures 3-10 through 3-13).

The conclusion reached in this part of the study is that the central obscuration should be less than 0.25 if reasonable possible, but certainly less than 0.325.

The next phase of the study determined the relationship between primary mirror focal ratio and central obscuration, including the baffles needed to prevent direct rays from striking any part of the focal plane of the telescope. It was found that for any given primary and secondary mirror separation there is an optimum primary mirror focal ratio and secondary mirror magnification that will yield the minimum central obscuration. The parametric investigation included telescopes with convex secondary mirrors (as in the Cassegrain) as well as those with concave secondary mirrors (as in the Gregorian). Interestingly enough, the Gregorian systems had smaller central obscurations for a given mirror separation than did the Cassegrainian systems, although the optimum primary mirror focal ratio was always lower for the Gregorian systems. This observation led to an investigation in which the mirror separation was varied to minimize the central obscuration, and the resulting primary mirror focal ratios were determined. A very significant result was obtained, namely, that in both the Cassegrain and Gregorian forms, the same minimum central obscuration is obtained for each value of primary mirror focal ratio, subject to maximum overall length constraints (see Figure 4-15). Furthermore, the trend of central obscuration with primary mirror focal ratio is nearly linear, with lower focal ratios yielding smaller central obscurations. In order to achieve the previously mentioned central obscuration values of 0.25 and 0.325, the primary mirror focal ratios would have to be approximately 1.7 and 3.2, respectively, making no allowance for manufacturing tolerances, uncompensated aberrations in the optical design, or

misalignment between the primary and secondary mirrors. The modified Strehl values for the $f/1.7$ system is 0.88 and for the $f/3.2$ system it is just 0.8, when fully baffled.

The conclusion drawn from this investigation is that performance will be maximized if the primary mirror is made as fast as budgetary and technology constraints allow, but should certainly be faster than $f/3.2$.

The operational problem of optical alignment was investigated, using the modified Strehl ratio as a measure of the effect of misalignment. The importance of maintaining the coincidence of the geometric focal points of the primary and secondary mirrors was developed, and the resulting unimportance of tilt alignment was shown. A new development currently being exploited at Perkin-Elmer has the capability of sensing misalignments through interferometric measurements at the focal plane of the wave front forming the images of stars. This development holds great promise for alleviating the tightened alignment tolerances which faster primary mirror focal ratios require.

Because the selection of a recommended primary mirror focal ratio is heavily weighted by cost and risk factors, discussions of the current state of the art in large optics fabrication was included, showing the trend in mirror manufacturing time as focal ratio is varied. Based on extrapolations of recent shop experience, the minimum focal ratio appears to be in the vicinity of $f/1.5$ and mirrors as fast as $f/2.0$ are certainly within the state of the art for a nominal three-meter diameter.

The final selection of the primary mirror focal ratio should be based on many of the factors that were investigated in this study, as well as others such as the requirements of the scientific instrumentation behind the primary mirror, the total allowable system weight, the longitudinal length available for the telescope in the launch vehicle, and other factors that were outside the scope of this study. Most of the factors just mentioned favor shorter, lighter telescopes, and hence faster mirrors, while a diversity of requirements is presented by the scientific instrumentation and pointing subsystems.

With the ground rules of the study in mind, but with due consideration being given to the scientific useage of a Large Space Telescope and state of the art manufacturing tolerances, the tradeoff study shows that a desireable primary mirror focal ratio would be about $f/2$ in order to obtain a Strehl ratio of 0.8. With the same manufacturing tolerances, the Strehl ratio reduces to 0.75 at $f/3.2$. If weight allowances permit, considerations should be given to a concave secondary mirror (or Gregorian) system because of the economies that could be realized in the fabrication of an image forming secondary mirror. Such a telescope would be about 40 percent longer than a Cassegrain with the same primary mirror focal ratio. The telescope with the $f/3.2$ mirror would be about 50 percent longer than the optimum $f/2$ telescope.

Since budgetary pressures are extremely important in the selection of the optical design for a Large Space Telescope, it is recommended that NASA authorize additional economic analysis to determine the trends in optical performance as a function of total program cost, and to determine which of the design options discussed in this study is least expensive irrespective of performance, and the performance level that corresponds to this configuration.

APPENDIX A

OPTICAL TELESCOPE TECHNOLOGY:

SOME OPTICAL SYSTEMS FOR A SPACEBORNE TELESCOPE

APPENDIX A

OPTICAL TELESCOPE TECHNOLOGY: SOME OPTICAL SYSTEMS FOR A SPACEBORNE TELESCOPE*

Abe Offner
The Perkin-Elmer Corporation

In choosing a form for a space telescope, we can make use of the knowledge and experience gained from the long history of astronomical telescopes. Since, however, each successful design is a compromise in which the limitations imposed by its environment and mode of operation are taken into account, it is useful to reexamine the candidate optical systems for attaining the goals of the National Aeronautics and Space Administration. Following Munch (ref. 1), we assume that a diffraction-limited guidance field of 3 arc minutes will satisfy all the guidance requirements and will also be sufficient for most high-resolution astronomical programs with a 3-meter aperture telescope.

Before comparing optical systems, it is necessary to define the term "diffraction-limited" more precisely. In absence of aberration, a system with an unobscured, unapodized, circular aperture forms an image of a star that consists of a central disc surrounded by the well-known ring pattern in which 84 percent of the energy is within the first dark ring. Small amounts of aberration reduce the proportion of the energy within the first dark ring of the diffraction pattern without affecting its diameter. Because central obscuration has a similar influence on the diffraction pattern, it may be treated as an aberration.

Historically, a system has been called "diffraction-limited" if the wavefront produced by it, when forming the image of a star, departs by no more than one-fourth the wavelength, λ , of the image-forming light from a reference sphere that approximates it most closely. In the case of spherical aberration, this results in a decrease of 20 percent in the normalized intensity at the diffraction focus. For other aberrations, the $\lambda/4$ criterion results in values of the normalized intensity (or Strehl ratio), which may differ appreciably

*Reprint of an article presented at Marshall Space Flight Center Workshop, Huntsville, Alabama, April 29 to May 1, 1969.

from 0.8. For this reason, a criterion based upon the value of the Strehl ratio has been proposed by Maréchal (ref. 2). For diffraction patterns formed by unobscured apertures in which the radius of the first dark ring is constant, the relative energy within the first dark ring is closely approximated by 84 percent of the Strehl ratio so that an equivalent to Maréchal's criteria for unobscured apertures is obtained by substituting for the Strehl ratio the normalized relative energy within the first dark ring (i.e., $1/0.84$ times the relative energy).

The advantage of this criterion is that the effects of obscuration as well as aberration, figure, and the like can be taken into account. A diffraction-limited system can now be defined as one in which the normalized relative energy within the first dark ring of the diffraction pattern of the image of a star formed by the system is greater than some number, say 0.8. This value, which we may call NRE, corresponds quite closely to the so-called Rayleigh criterion of $\lambda/4$ in the case of spherical aberration and coma. Since the reduction in the relative energy within the first dark ring is proportional to the mean square of the wave aberration, a reasonable tolerance for the residual amounts of these aberrations in a design is $\lambda/8$, which uses up one-fourth of the system tolerance.

A system with obvious advantages from the point of view of manufacture, testing, and alignment consists of a single spherical mirror with a detector at its focus. If we choose a sufficiently large focal length for the mirror, it forms diffraction-limited images on a spherical surface. The f-number, N , and diameter, D , corresponding to $\lambda/8$ spherical aberration (or a loss of 5 percent of the relative energy inside the first dark ring) is given by the expression

$$N = \frac{D^{1/3}}{(256\lambda)^{1/3}} \quad (1)$$

At $\lambda = 5 \times 10^{-7}$ meter, $N = 20 D^{1/3}$ so that for $D = 3$ meters, $N = 29$. (Allowing a loss of 20 percent of the relative energy in the central disc would reduce N to 23.) An $f/29$ system has a resolving power of about 60 cycles per millimeter in the visual region of the spectrum and is thus suited to the capabilities of likely detectors; however, the distance from the mirror to the focal plane of the system is 87 meters. Although such a length is prohibitive for an earth-based telescope, this is not necessarily true in space. The

addition of a small Newtonian diagonal to this system would cause negligible obscuration (less than 10 percent for a 3-minute field). The diagonal could be tilted for fine guidance. The advantages of the availability of a wider-than-minimal field for unusual experiments, minimal alignment problems, low obscuration ratio, and ease of baffling make this a very strong candidate for a diffraction-limited, 3-meter space telescope. The costs and probability of success of all other candidate systems should be compared with those of this simple system.

A shorter optical system can be achieved by substituting a paraboloidal mirror for the spherical one. In this case, the only aberration of importance for the small required field is coma. The semifield angle, θ , at which the coma of a paraboloid results in a wavefront error of $\lambda/8$, is given by the expression

$$\theta = 8N^2 (\lambda/D) \quad (2)$$

To achieve $2\theta = 3$ minutes at $\lambda = 5 \times 10^{-7}$ meter and $D = 3$ meters requires $N = 18.4$. If the requirements are relaxed to allow at the edge of the guidance field an NRE of 0.8 due to the inherent coma of the primary mirror, then $N = 13$. The single mirror system in which the mirror is paraboloidal retains many of the advantages of the single spherical mirror. The field can be made shorter, but this results in the requirement of a detector with greater resolving power. While the diffraction-limited field of the spherical mirror can be extended appreciably above 3 minutes, this cannot be done in the case of the paraboloid mirror without adding optical elements.

In a two-mirror optical system, the advantages of a short physical length and a long equivalent focal length can be simultaneously achieved in the well-known Cassegrain and Gregorian arrangements. Such systems can be sufficiently well corrected so that over a 3-minute field the loss in the NRE due to aberrations is negligible. This imagery is achieved if two mirrors of proper figure are maintained at the proper separation and are aligned so that their axes are coincident. Tolerances for departures from the nominal situation can be obtained in terms of the amount by which they reduce the proportion of the energy in the central disc. These tolerances are functions primarily of the f-number of the primary mirror. We have computed them (see Appendix) for a Cassegrain system in which the final image is at the primary mirror so that

the separation, d , between the two mirrors is also the back focus. No plausible systems of this type have tolerances that differ significantly from those calculated.

Since a hyperboloidal mirror forms an aberration-free image of a (virtual) point object at its geometrical near focus, a misalignment of the axes of the primary and secondary mirrors has no effect on the axial imagery at the Cassegrain focus, provided that the focus of the primary mirror is at the (geometric) focus of the secondary hyperboloid (fig. 1). In the case of off-axis aberrations, a difference in the compensation between primary and secondary contributions is introduced by the angle between the two image surfaces at the virtual image. For an $f/1$ primary mirror and a 2-degree angle between the primary and secondary mirror axes, the change in coma at the edge of a 3-minute total field is $\lambda/16$ at $\lambda = 5 \times 10^{-7}$ meter and $D = 3$ meters. Thus, the tolerances on decentration, tilt, and separation of the two mirrors can be reduced to tolerances on the departure from coincidence of their foci if the angle between the axes of the two mirrors is reasonably small (fig. 2). (For systems, such as the Ritchie-Chrétien, in which the two foci do not coincide, these tolerances can be expressed in terms of the departure from nominal separations of the two foci.)

A longitudinal separation, δx , of the foci of the primary and secondary mirrors results in spherical aberration at a displaced image position. The longitudinal tolerance, δx_t , corresponding to a maximum departure, W_t , from the closest sphere is derived in the Appendix. In terms of the secondary magnification, m , and Cassegrain f -number, N , it is given by the expression

$$\delta x_t = \frac{512 N^4}{m^2(m^2-1)} W_t \quad (3)$$

For values of m for which $m^2 \gg 1$,

$$\delta x_t = 512 N_p^4 W_t \quad (4)$$

where N_p is the f -number of the primary mirror.

A lateral displacement, y_t , of the focus of the primary mirror from that of the secondary mirror results in coma on an image plane that is tilted by m times the angle between the axes of the two mirrors. The lateral tolerance,

y_t corresponding to a maximum departure, W_t , from the closest reference sphere is given by the expression

$$y_t = \frac{64 N^3}{m(m^2-1)} W_t \quad (5)$$

When $m^2 \gg 1$, this can be approximated by the expression

$$y_t = (64 N_p^3) W_t \quad (6)$$

For an $f/2$ primary mirror and $W_t = \lambda/8$ at $\lambda = 5 \times 10^{-7}$ meter, $\phi_{x_t} = 0.512$ millimeter for $m^2 \gg 1$. The same system has a lateral tolerance $y_t = 0.032$ millimeter. The value of the lateral tolerance is proportional to the cube of the primary mirror f -number. The lateral tolerance on a system with an $f/1$ primary mirror is thus 0.004 millimeter. Increasing the value of W_t to a value that reduces the NRE to 0.8 as a result of this misalignment alone merely doubles this tolerance. Since the reduction in length achieved by a two-mirror system is approximately N_p/N , large reductions in length are accompanied by very tight tolerances in the permissible lateral separations of the foci of the two mirrors.

A further restriction in the design of a two-mirror system is the need to keep the obscuration ratio low. Examination of figure 3 shows that for an obscuring aperture whose diameter is r in units of the system diameter, the NRE is closely approximated by the expression for the relative intensity at the center of the diffraction pattern

$$\text{NRE} = 1 - r^2 (2 - r^2) \approx 1 - 1.8r^2 \quad (7)$$

It can be seen that, in terms of NRE, the effect of an obscuration ratio of 1 to 6 is the same as that of one-eighth wave of third-order spherical aberration. Although it is not difficult to keep the obscuration by the secondary mirror below this figure, the requirements for baffling in the presence of the earth, sun, and moon may either increase the obscuration ratio or restrict the use of the system.

Appendix

The following are the computations of positioning tolerances for a Cassegrain system whose back focus is at the primary mirror.

Longitudinal Tolerance

The hyperbola, S, of figure A-1 is the intersection of the hyperboloidal secondary mirror with a plane containing the axes of the hyperbola. Its foci are at F and F', and its center is at C. In terms of rectangular coordinates with origin, O, at the pole of the hyperbola and the x-axis along the optical axis, the equation of the hyperbola is

$$\frac{(x+a)^2}{a^2} - \frac{y^2}{ar} = 1 \quad (A1)$$

where a is the distance CO and r is the radius of curvature of the hyperbola at its pole. Solving equation (A1) for x, we have

$$x = \frac{y^2}{2r} - \frac{y^4}{8ar^2} + 0 \left(\frac{y^6}{a^2 r^3} \right) \quad (A2)$$

When the primary focus is at F, the secondary mirror forms an image at F' at the magnification m. If the primary focus is displaced a distance, δx , to $\bar{F}$, the secondary image is formed at $\bar{F}$, which is at the distance $m^2 \delta x$ from F. A hyperboloidal mirror with pole at O and foci at $\bar{F}$ and $\bar{F}'$ would form an aberration-free image at $\bar{F}'$. Using barred variables to refer to the correct hyperbola, we can describe it by an expression similar to equation (A3)

$$\bar{x} = \frac{\bar{y}^2}{2r} - \frac{\bar{y}^4}{8a\bar{r}^2} \quad (A3)$$

where the r is not barred because it must have the same value for the two hyperbolae. Since $\bar{a}$ is the distance $\bar{C}O$, we have

$$\bar{a} = a + (m^2 - 1) \delta x / 2 \quad (A4)$$

For $\bar{y} = y$, we have from equations (A2), (A3), and (A4)

$$x - \bar{x} = \frac{y^4}{8r^2} \left(\frac{1}{a} - \frac{1}{\bar{a}} \right) = y^4 \frac{(m^2 - 1) \delta x}{16a^2 r^2} \quad (A5)$$

where we have made the approximation $a\bar{a} = a^2$. For the Cassegrain image at the primary mirror, the separation, d, between the two mirrors is equal to the back focal length $\bar{F}O$. From the properties of the hyperbola,

$$d = \sqrt{a^2 + ar} + a = m \left[\sqrt{a^2 + ar} - a \right] \quad (A6)$$

Hence

$$d^2 / m = ar \quad (A7)$$

For the value of y at the edge of the mirror, the f-number, N , of the Cassegrain system is given by

$$N = d/2y \quad (A8)$$

Making the substitutions from equations (A7) and (A8) in equation (A5), we obtain the relation

$$x - \bar{x} = \frac{m^2(m^2-1)}{256 N^4} \delta x \quad (A9)$$

This is the departure of the edge of the actual hyperboloid from the surface that would give aberration-free imagery. Since the image defect is third-order spherical aberration, the departure of the aberrated wave from the nearest sphere is given by the expression

$$W = \frac{2(x-\bar{x})}{4} = \frac{m^2(m^2-1)}{512 N^4} \delta x \quad (A10)$$

The tolerance on longitudinal separation of the foci of the two mirrors, δx_t , can then be related to the tolerance on the resultant wave aberration, W_t , by the expression

$$\delta x_t = \frac{512 N^4}{m^2(m^2-1)} W_t \quad (A11)$$

For $m^2 \gg 1$, this reduces to the expression

$$\delta x_t = 512 N_p^4 W_t \quad (A12)$$

where N_p is the f-number of the primary mirror.

Lateral Tolerance

For moderate angles between the axes of the two mirrors, the image deterioration caused by either lateral shift or tilt is proportional to the resultant lateral displacement, y_t , of the primary focus from the secondary focus (fig. 2). The only significant aberration introduced by a small displacement is the coma of the hyperboloid for an object point at this height. For Cassegrain f-number, N , the maximum departure, W , of the comatic wavefront from the best-fitting reference sphere is given by the expression

$$W = \frac{m(m^2-1)y}{64 N^3} \quad (A13)$$

The tolerance on the lateral separation of the foci of the two mirrors, y_t , is related to the tolerance on the resultant wave aberration, W_t , by the expression

$$y_t = \frac{64 N^3 W_t}{m(m^2 - 1)} \quad (A14)$$

For $m^2 \gg 1$, this can be approximated by the expression

$$y_t = (64 N_p^3) W_t \quad (A15)$$

If we set equal tolerances of W_t due to longitudinal and lateral separations of the foci of the two mirrors,

$$y_t = \delta x_t / 8N_p \quad (A16)$$

References

1. Münch, G.: Space Research, No. 172, National Academy of Sciences - National Research Council, Washington, D.C., 1961.
2. Maréchal, A.: Rev. d' Optique, vol. 26, no. 257, 1947.

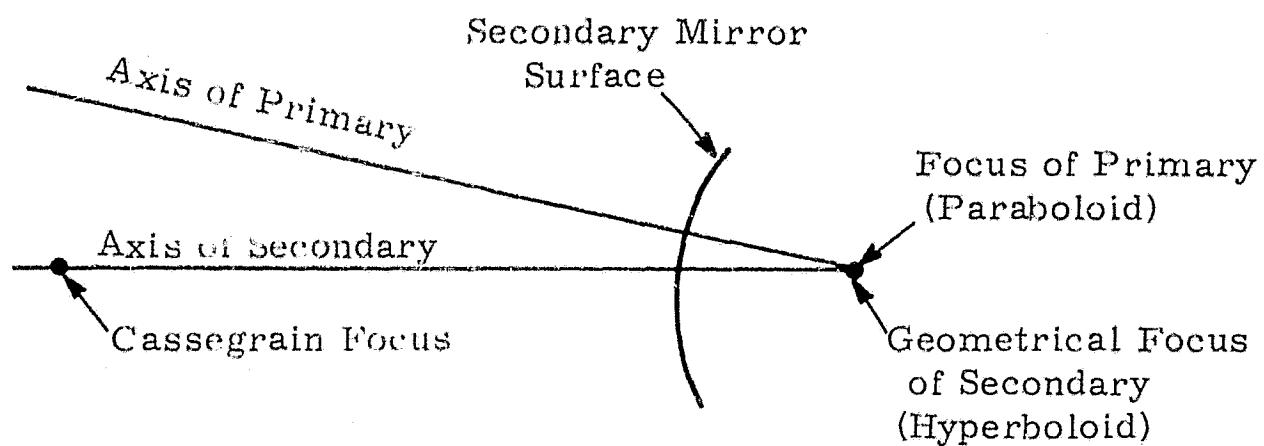


Figure 1. Permissible Misalignment of Mirror Axes in Cassegrain Telescope

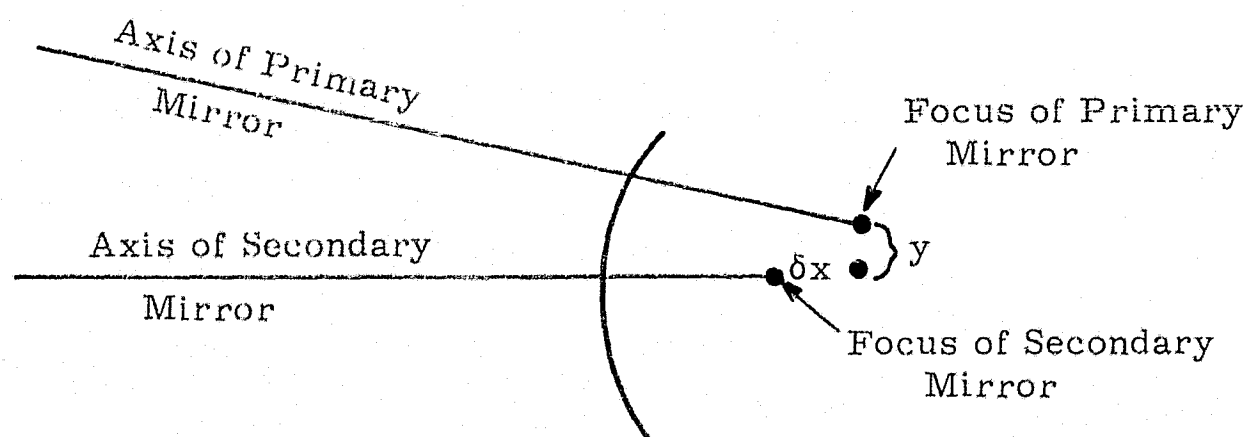


Figure 2. Parameters for Mirror-Positioning Tolerances

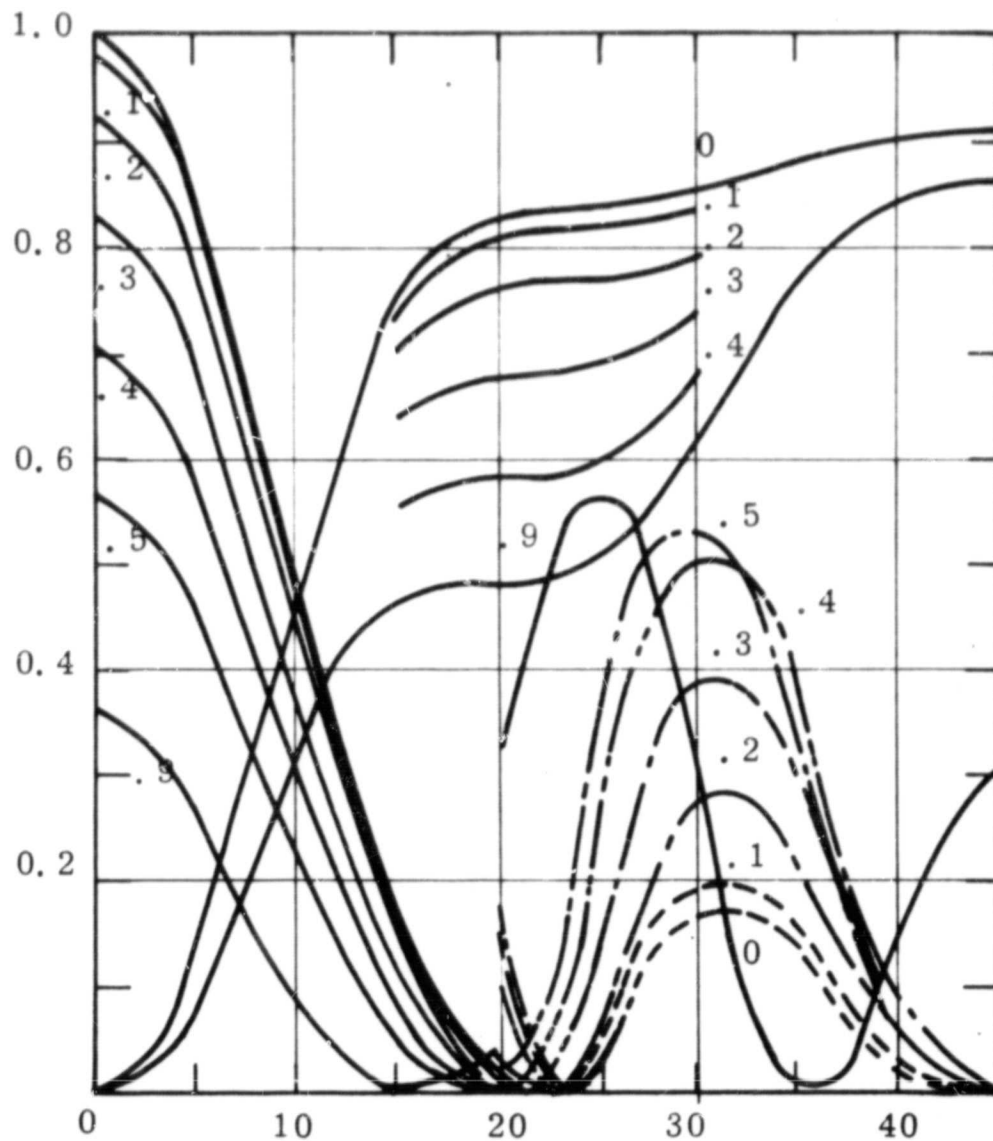


Figure 3. Energy Distribution in Diffraction Pattern Corresponding to Apertures with Central Obscuration

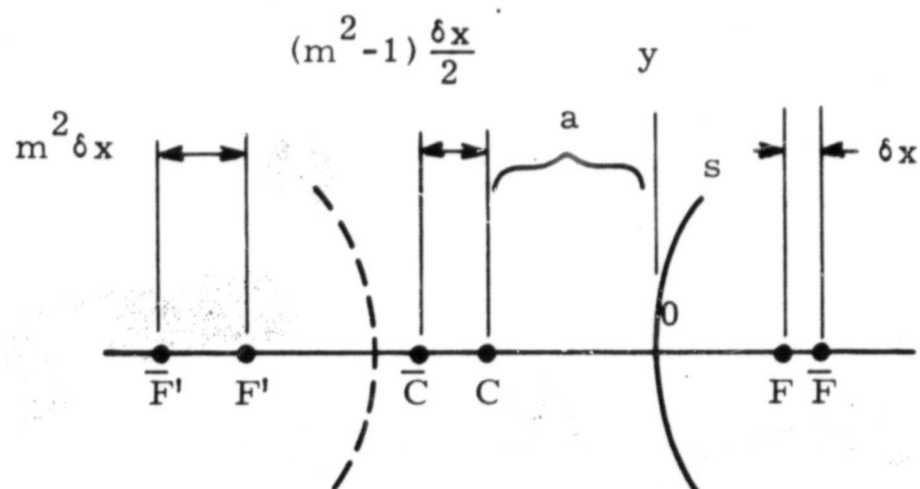


Figure A1. Geometry for Computing the Effect of Longitudinal Separation of Foci

APPENDIX B

CALCULATIONS OF NRE FOR OBSCURED APERTURES

APPENDIX B

CALCULATION OF NRE FOR OBSCURED APERTURES

This Technical Appendix will discuss the rate at which the Normalized Relative Energy (NRE) is reduced as the diameter of a circular central obscuration is increased. To expedite the calculations, compatible notation and existing results from Born and Wolf⁷ will be used wherever possible.

The NRE is defined as .

$$NRE = L_{\odot}(w_1)/L_o(w_1) \quad (1)$$

where

- w_1 = diameter of first dark ring for unobscured aperture
- $L_o(w_1)$ = fraction of energy contained within first dark ring of the point spread of an unobscured aperture
- $L_{\odot}(w_1)$ = fraction of energy contained within radius of w_1 of the point spread of the obscured aperture

Referring to Born and Wolf, Section 8.5.2, equation 18, we note:

$$L_o(w_1) = 1 - J_o^2(kaw_1) \quad (2)$$

and

$$w_1 = 0.610\lambda/a$$

$$2a = \text{diameter of aperture}$$

$$k = 2\pi/\lambda$$

The calculation now follows from the following definition (equation 17, Section 8.5.2, Born and Wolf)

$$L_{\odot}(w_1) = \frac{1}{E} \int_0^{2\pi} \int_0^{w_1} I_{\odot}(w) w dw d\psi \quad (3a)$$

$$= \frac{2\pi}{E} \int_0^{w_1} I_{\odot}(w) w dw \quad (3b)$$

where $I(w)$ is derived from equation 26, Section 8.6.2, Born and Wolf

$$\begin{aligned}
 I_{\odot}(w) &= \frac{ED}{\lambda^2} \left[\frac{2J_1(kaw)}{kaw} - \epsilon^2 \frac{2J_1(ka\epsilon w)}{ka\epsilon w} \right]^2 \\
 &= \frac{ED}{\lambda^2} \left[\frac{2J_1(kaw)}{kaw} \right]^2 + \epsilon^4 \left[\frac{2J_1(ka\epsilon w)}{ka\epsilon w} \right]^2 \\
 &\quad - 2\epsilon^2 \left[\frac{2J_1(kaw)}{kaw} \cdot \frac{2J_1(ka\epsilon w)}{ka\epsilon w} \right]
 \end{aligned} \tag{4}$$

and

$2\epsilon a$ = the diameter of the central obscuration

$$E = \int_0^{2\pi} \int_0^\infty I_{\odot}(\epsilon=0) w dw d\psi = \text{total energy through unobscured aperture}$$

D = area of unobscured pupil

Substitution of (4) into (3) yields the following:

$$\begin{aligned}
 L(w_1) &= \frac{2\pi}{E} \cdot \frac{ED}{\lambda^2} \int_0^{w_1} \left\{ \left[\frac{2J_1(kaw)}{kaw} \right]^2 + \epsilon^4 \left[\frac{2J_1(ka\epsilon w)}{ka\epsilon w} \right]^2 \right. \\
 &\quad \left. - 2\epsilon^2 \left[\frac{2J_1(kaw)}{kaw} \cdot \frac{2J_1(ka\epsilon w)}{ka\epsilon w} \right] \right\} w dw
 \end{aligned} \tag{5}$$

The first two integrals are solved by again referring to Born and Wolf, Section 8.5.2 (18) as follows:

$$\begin{aligned}
 \frac{2\pi D}{\lambda^2} \int_0^{w_1} \left[\frac{2J_1(kaw)}{kaw} \right]^2 w dw &= 1 - J_0^2(kaw_1) \\
 &= L_o(w_1)
 \end{aligned} \tag{6}$$

$$\frac{2\pi D \epsilon^4}{\lambda^2} \int_0^{w_1} \left[\frac{2J_1(ka\epsilon w)}{ka\epsilon w} \right]^2 w dw = \epsilon^4 \left\{ 1 - J_0^2(ka\epsilon w_1) - J_1^2(ka\epsilon w_1) \right\} \tag{7}$$

The third integral of (5) required some approximations

$$\frac{4\pi D\epsilon^2}{\lambda^2} \int_0^{w_1} \frac{2J_1(kaw)}{kaw} \cdot \frac{2J_1(ka\epsilon w)}{ka\epsilon w} w dw \quad (8a)$$

$$\approx \frac{4\pi D\epsilon^2}{\lambda^2} \int_0^{w_1} \frac{2J_1(kaw)}{kaw} \cdot \frac{2w}{\gamma} \cdot \left[\frac{\gamma}{2} - \frac{\gamma^3}{16} + \frac{\gamma^5}{384} + \dots \right] dw \quad (8a)$$

where

$$\gamma = ka\epsilon w$$

since*

$$J_1(\gamma) = \frac{\gamma}{2} \sum_{k=0}^{\infty} \frac{(-1/4 \gamma^2)^k}{k_0! \Gamma(k+2)}$$

For $\epsilon < 0.4$ the first two terms of the expansion give sufficiently accurate results which infers

$$\approx \frac{8\pi D\epsilon^2}{\lambda^2 ka} \left[\frac{1}{ka} \int_0^{kaw_1} J_1(x) dx - \frac{\epsilon^2 k^2 a^2 w_1^3}{8} \int_0^1 J_1(kaw_1 y) y^2 dy \right] \quad (8c)$$

where

$$x = kaw$$

$$dx = kadw$$

$$y = \frac{w}{w_1}$$

$$dy = \frac{dw}{w_1}$$

These functions are easily integrated as follows:

$$\int_0^b J_1(x) dx = 1 - J_0(b) \quad (9)$$

$$\int_0^1 y^2 J_1(by) dy = \frac{1}{b} J_2(b) \quad (10)$$

* Abramowitz, M. Handbook of Mathematical Functions,
NBS - AMS #55, p360.

Which allow us to rewrite (8c), after simple algebra,

$$= 2\epsilon^2 \left[1 - J_0^2(kaw_1) \right] - (0.61 \pi \epsilon^2)^2 J_2^2(kaw_1) \quad (11)$$

We have thus solved (5) inclosed form by noting (5), (6), (7), and (11).

$$L(w_1) = \frac{2\pi D}{\lambda^2} \left\{ 1 - J_0^2(kaw_1) + \epsilon^4 \left\{ 1 - J_0^2(ka\epsilon w_1) - J_1^2(ka\epsilon w_1) \right\} \right. \\ \left. - 2\epsilon^2 \left[1 - J_0^2(kaw_1) \right] - (0.61 \pi \epsilon^2)^2 J_2^2(kaw_1) \right\} \quad (12)$$

which infers via equations 1 and 6 that for sufficiently small ϵ we have

$$NRE = \left[(1-\epsilon^2)^2 - J_0^2(kaw_1) + \epsilon^2 \left\{ 2 J_0^2(kaw_1) \right\} \right. \\ \left. - \epsilon^4 \left\{ J_0^2(ka\epsilon w_1) + J_1^2(ka\epsilon w_1) \right\} + \dots \right] / \left\{ 1 - J_0^2(kaw_1) \right\} \quad (13)$$

Examination of (13) for small ϵ reveals that the NRE follows the following law as one introduces a small circular central obscuration.

$$NRE \approx \frac{(1-\epsilon^2)^2 - J_0^2(kaw_1) + 2\epsilon^2 J_0^2(kaw_1)}{1 - J_0^2(kaw_1)} \cong (1 - \epsilon^2)^2 \quad (14)$$

since

$$J_0(kaw_1) = J_0\left(\frac{2\pi}{\lambda} a \frac{0.61\lambda}{a}\right) = J_0(3.83) = -0.40$$

For larger central obscurations, refer to the plot of NRE verses ϵ in Figure B-1 which was derived by computer operation on (1) and (4a). The figure also demonstrates the relation between the Strehl ratio S and ϵ which is computed from the following equation.

$$S = (1-\epsilon^2)^2 \quad (15)$$

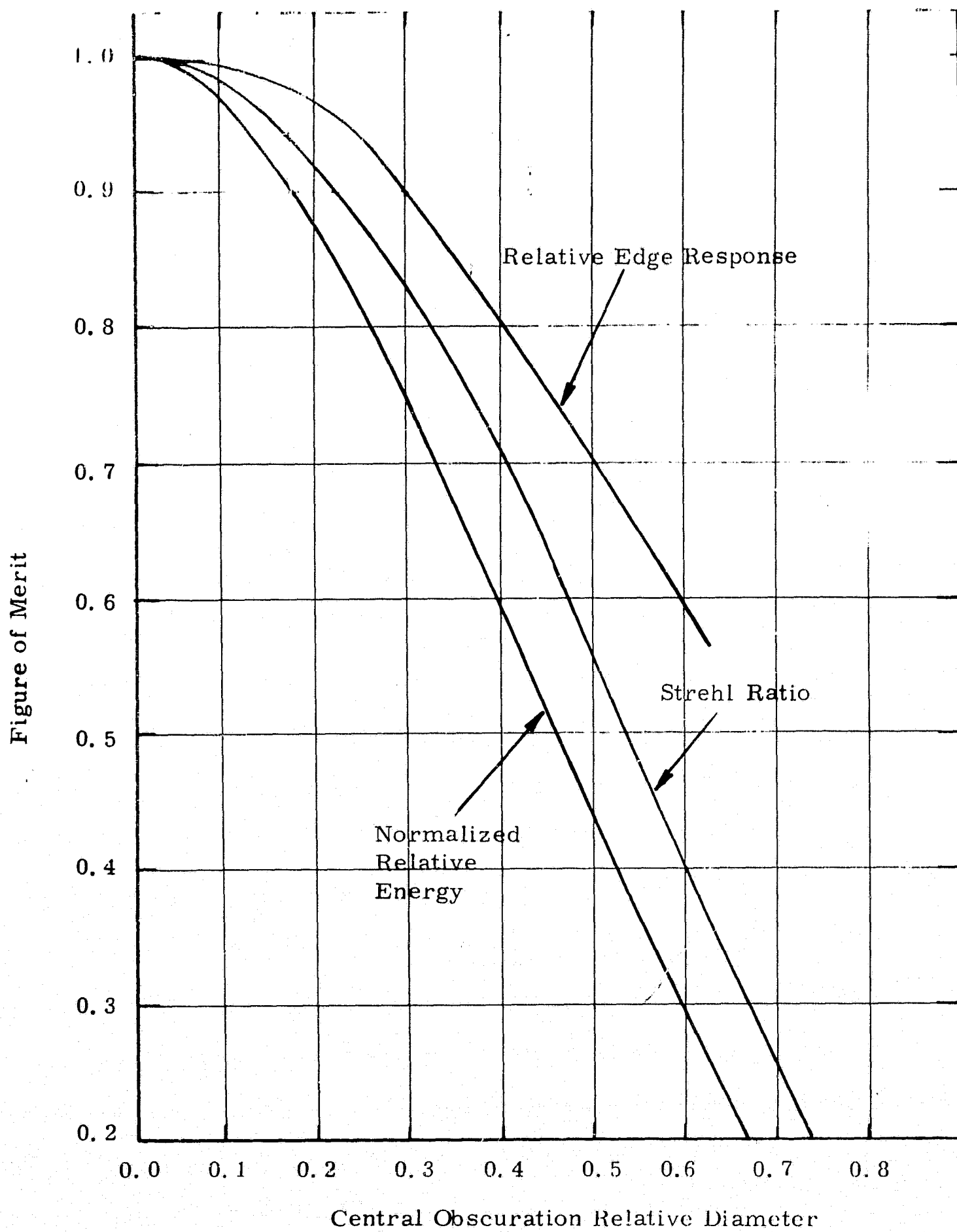


Figure B-1. NRE vs. ϵ

APPENDIX C

DIGITAL IMAGE PROCESSING

APPENDIX C

DIGITAL IMAGE PROCESSING

I INTRODUCTION

Perkin-Elmer has developed a sophisticated capability in the area of digital image processing. Computer programs have been developed for image evaluation, restoration, noise filtering, and image enhancement. A Perkin-Elmer Line Scan Image Generator has been designed, built, and used as an input-output device between the photographic transparency and the digital computer. This work is described and depicted in the following paragraphs.

II IMAGE EVALUATION

The Modulation Transfer Function (MTF) of an optical system supplies a useful evaluation in terms of the system's spatial frequency response. Edge gradient techniques permit the determination of the MTF's of optical systems from laboratory test exposures or normal operational exposures. This is accomplished by examining an edge in the photographic image which corresponds to an exposure step in the object plane. An exposure versus distance step in the object plane would result in a step edge on the negative for a perfect photographic system. In actuality, the step edge is degraded by the atmosphere, lens system, image motion, and other degradations. The resulting edge on the negative is not a step edge, but is representation of the step response of the photographic system. Edge gradient techniques are methods for obtaining the MTF's of photographic systems from degraded images of object exposure steps.

Perkin-Elmer has developed an Edge Gradient Analysis Computer Program which directs the IBM 360/67 high speed computer to operate on input edge scan and sensitometric data, and compute the resultant MTF for the photographic systems which produce the edge image. The edge scan and the sensitometric data are collected on the Mark II Microdensitometer and stored on magnetic tape. This instrument with its associated electronics is shown in figure 1.

Certain known anomalies such as microdensitometer degradation, noise introduced by film granularity and microdensitometer electronics, and the nonlinearity of the image recording on film must be removed to insure a valid MTF determination. This is achieved in the computer program by an accurate

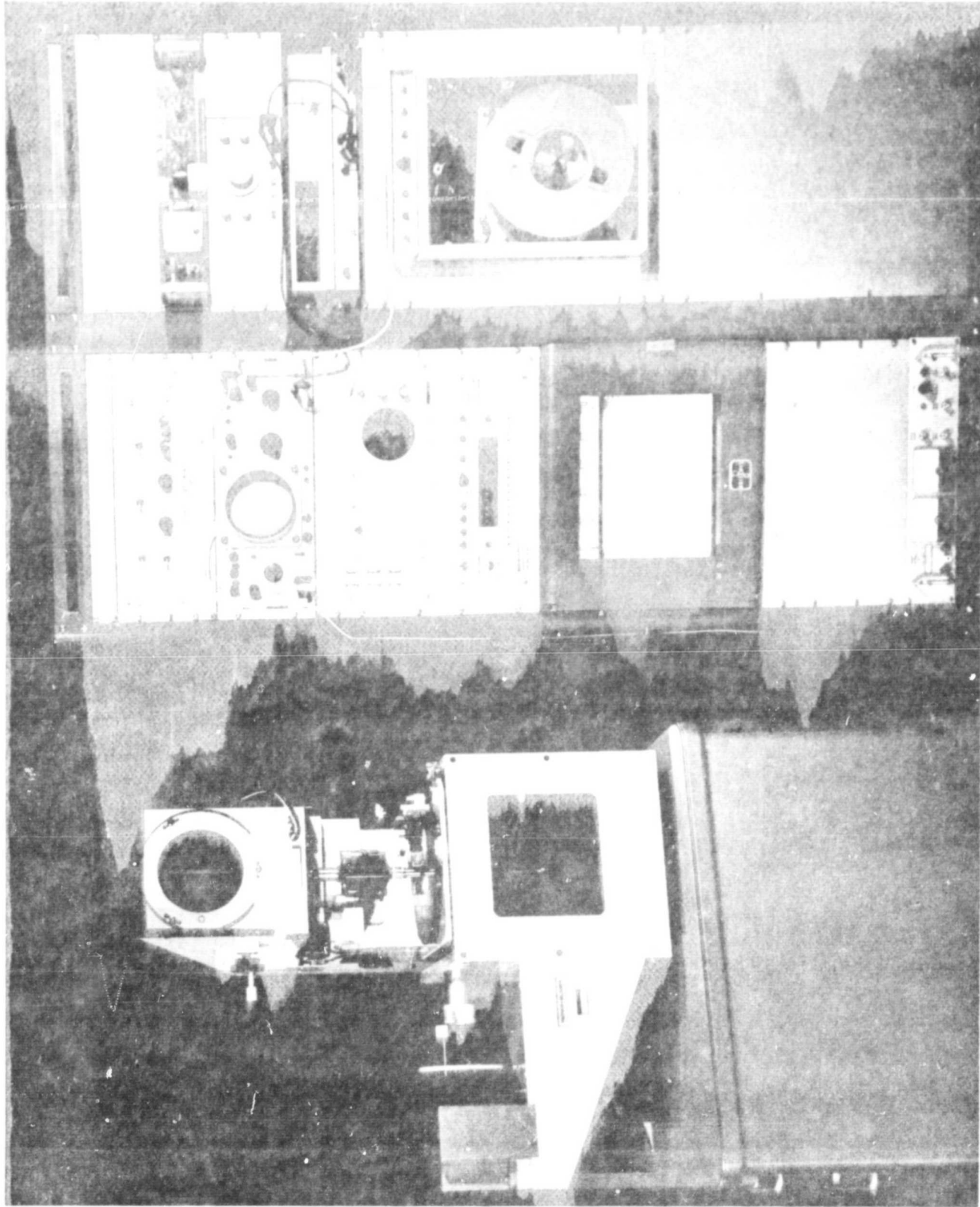


Figure 1. Mark II Microdensitometer

correction for the microdensitometer degradation on the transmittance side of the sensitometric conversion, a precise sensitometric conversion of transmittance to exposure values and an accurate procedure for properly smoothing noisy edge data.

The evaluation of a photographic system by edge gradient techniques is useful as a diagnostic tool in determining the type and degree of degradation present when the photograph was taken. Once this information is extracted from a photograph, restoration techniques, as described below, can be used to compensate for the degradations.

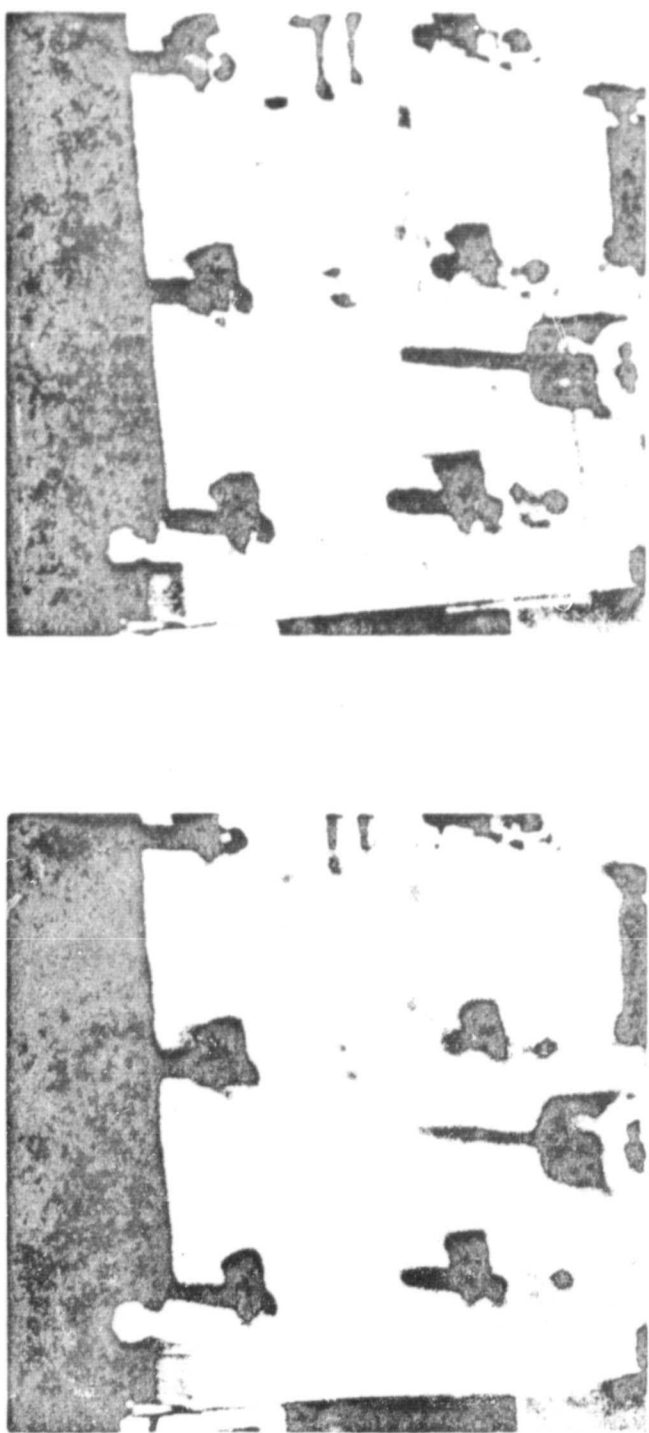
III IMAGE RESTORATION

Computer software has been developed for linearly filtering images that have been linearly degraded. Typical degradations include blur due to limitations in the optics, and smear caused by linear image motion. Linear filtering is performed in the spatial domain by convolving a "processing spot" with the digital image data. This spatial domain approach is used instead of Fourier domain filtering because it requires relatively small computer memory and thus can be applied to real-time wide-bandwidth situations. Pictorial results of this process are shown in figure 2 where an original photograph is deliberately blurred digitally and then restored (deblurred). The deblurring results in a significant increase in high spatial frequency content.

IV NOISE FILTERING

The objective of the noise filtering program is to develop software to reduce the effects of additive image noise. For the case of Gaussian noise, an algorithm was developed which determines a statistical estimate of the true scene value, given the statistical properties of the scene and noise. Noise filtering experiments with normal aerial imagery have indicated that a simple zero-memory technique produces a measurable increase in the signal-to-noise ratio. The use of memory in noise filtering, presently being investigated, should improve the performance of the procedure.

The results of a noise filtering experiment are shown in figure 3. The original scene is shown in figure 3a, and the scene corrupted by the addition of noise (with standard deviation $\sigma = 228$ on a transmittance scale of 0-4095) is shown in figure 3b. The standard deviation of the scene itself is



↑
45 FT.
↓
SCALE

AFTER

BEFORE

SAMPLING DENSITY

2 SAMPLES/FT.

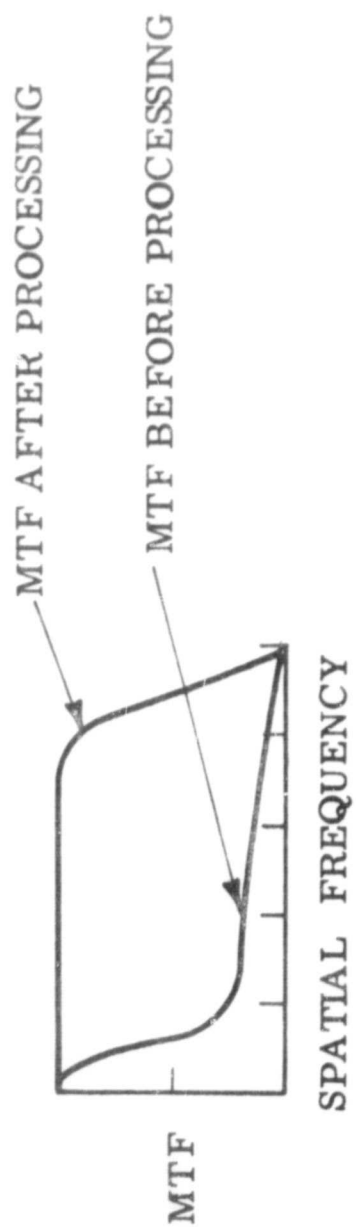


Figure 2. Digital Image of Rooftop, Blurred and Deblurred Digitally

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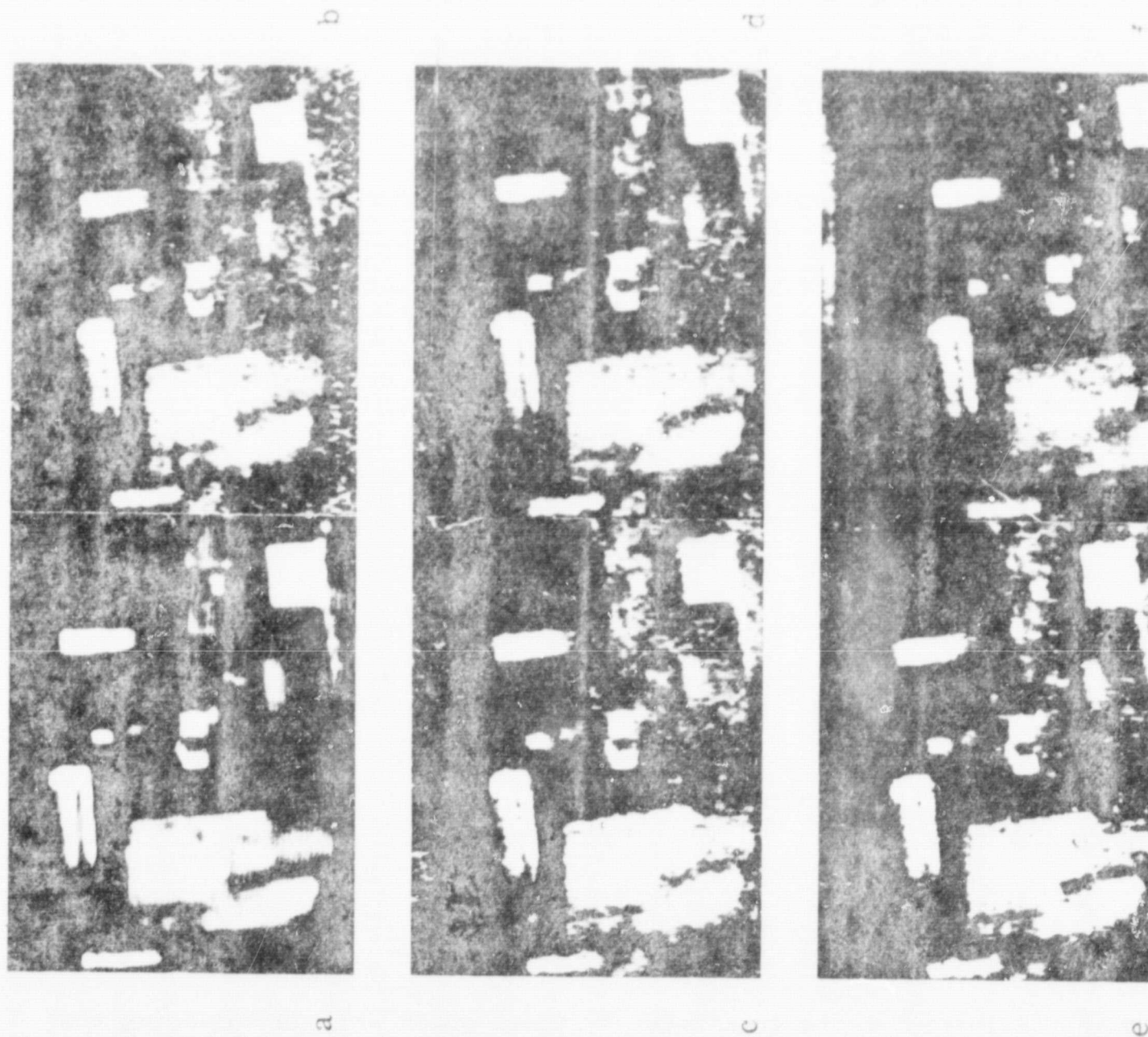


Figure 3. Noise Filtering Experiment, Noise $\sigma = 228$

is approximately 1000. Figures 3c-3f show the scene filtered by nonlinear filtering algorithms assuming values of σ of 116, 171, 228 and 370, respectively. Therefore, figure 3e was correctly processed, while figures 3c and 3d were underprocessed and figure 3f overprocessed. From this last figure, it is seen that the effect of overestimating σ is the production of a "clipped" scene, with two predominant levels of transmittance. There is a noticeable reduction in noise content, but this is accompanied by a loss of scene detail.

V IMAGE ENHANCEMENT

Image enhancement is an attempt to process an image for the purpose of making the information content of that image more readily extractable by an observer. One basic enhancement operation which has been successfully performed is edge enhancement by derivative techniques. In this technique, one of several derivative operators is used to obtain the derivative of a digital image; this derivative is then added to the original image to yield an enhanced image. Digital approximations to two derivative operators were founded to yield encouraging results. The gradient, a bipolar derivative with a preferred direction and zero mean, is defined in this context as the finite difference operator,

$$\nabla u_{i,j} = 2u_{i,j} - u_{i-1,j} - u_{i,j+1}$$

where $u_{i,j}$ is the image sample located at coordinates (i,j) . The second derivative operator successfully used is the Laplacian. As defined below, the Laplacian is a bipolar two-dimensional second derivative, having zero mean and no preferred direction

$$\nabla^2 u_{i,j} = 4u_{i,j} - u_{i-1,j} - u_{i+1,j} - u_{i,j-1} - u_{i,j+1}$$

Both of these derivative techniques have performed quite well in enhancing the appearance of imagery. Figure 4 presents an example of Image Enhancement by derivative techniques. The original image is shown in 4a, and the Laplacian in 4b. The remaining four images are enhanced by the addition of various amounts of Laplacian to the original image.

VI THE LSIG

The Perkin-Elmer Line Scan Image Generator (LSIG) is a drum scanner used to convert photographic data to digital values for use in computer proces-

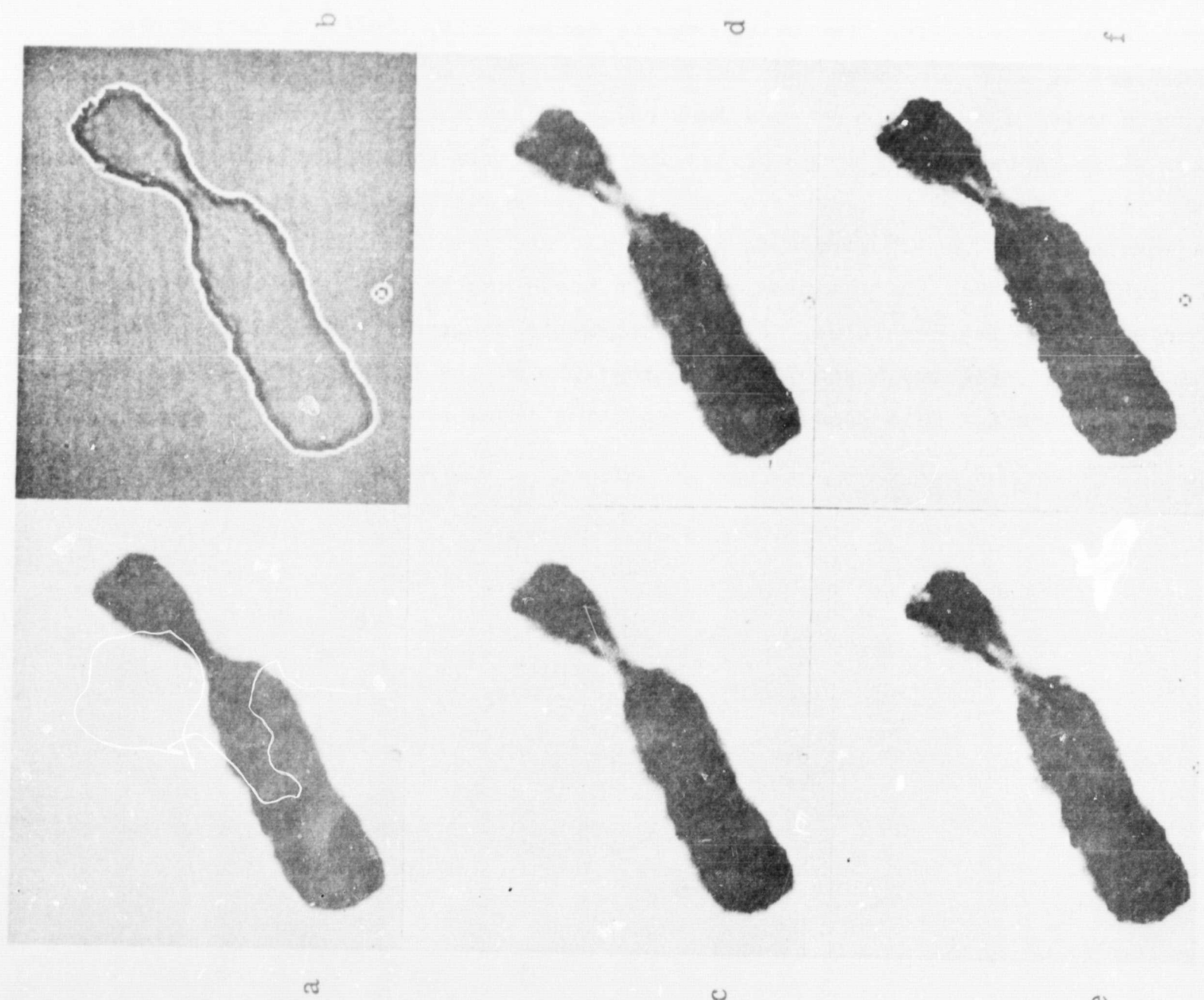


Figure 4. Digital Image Enhancement,
Electron Microscope Image
of Chromosome

- a. Original Image
- b. Image Laplacian
- c. Original Image + 0.125 x Laplacian
- d. Original Image + 0.25 x Laplacian
- e. Original Image + 0.5 x Laplacian
- f. Original Image + 1.0 x Laplacian

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sing, and similarly in the reverse process of producing a photographic image from digital data. It is unique in its ability to sense (read) and record (write) photographic images with a desired point spread function. The LSIG consists of three primary components; a scanning head, a writing head, and a small digital computer for data handling. The scanning head, which is used to scan or digitize a given scene, consists of a glass drum on which a photographic transparency is mounted. The drum rotates about its axis and translates along its axis, carrying the transparency in a helical fashion past a light source and detector assembly. The write head is mechanically identical to the read head and is used for image reconstruction. An exposed piece of film is mounted on the write drum, and is scanned in a helical fashion past an illuminating system which exposes the film by projecting a precisely shaped spot of light on the film. The LSIG is capable of scanning or reconstructing images of 10^6 elements per $10'' \times 10''$ format in 22 minutes; the sampling intervals in both directions on both the scanning and write heads can be varied to obtain a wide range of sampling densities. A basic schematic diagram of the LSIG is shown in figure 5. Figures 6 and 7 show photographs of the LSIG and the Varian 6201 computer used for data handling and real time image processing.

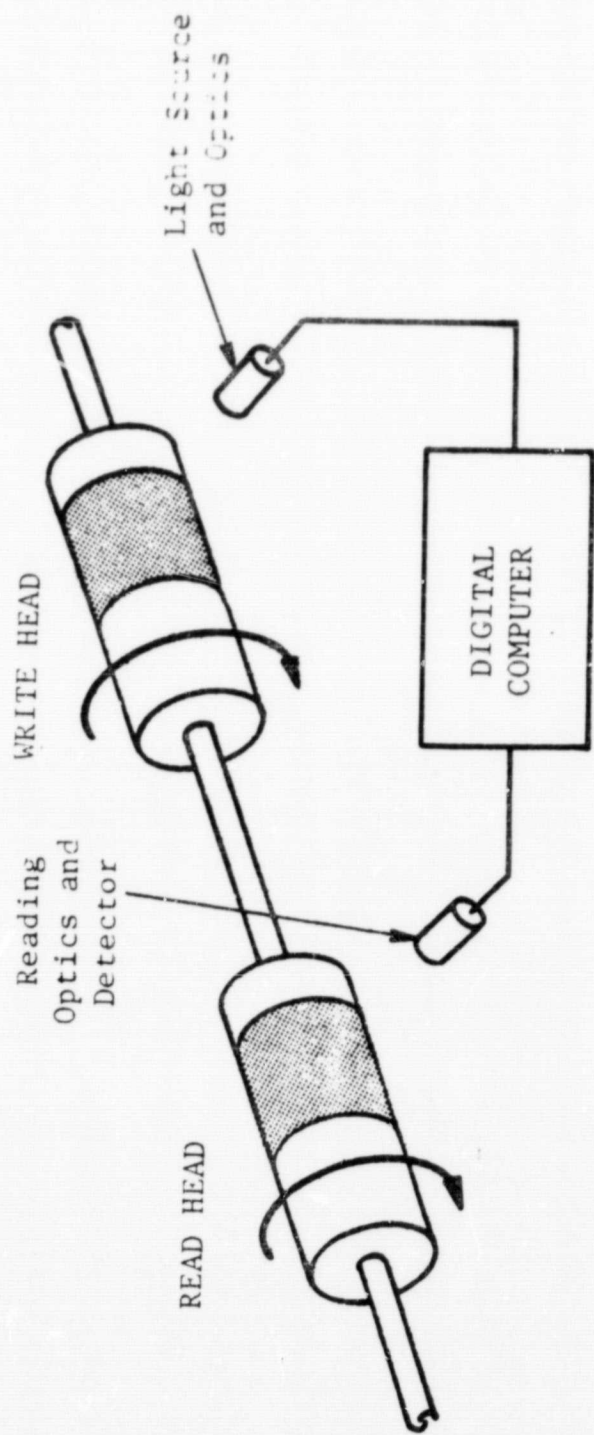


Figure 5. Schematic Diagram, LSIG

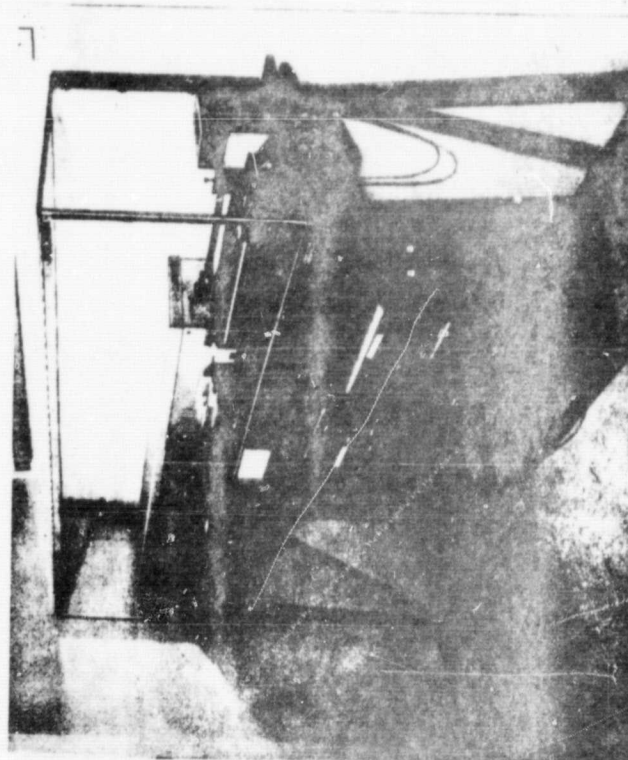


Figure 6. Line Scan Image Generator

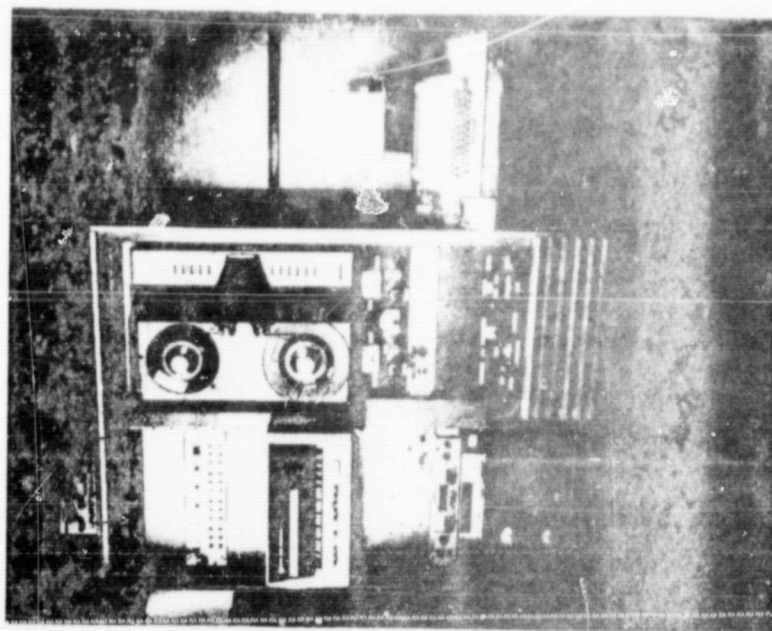


Figure 7. Varian 620i Computer

APPENDIX D

LARGE APERTURE INTERFEROMETER

FOR TESTING OPTICAL FLATS

APPENDIX D

LARGE APERTURE INTERFEROMETER FOR TESTING OPTICAL FLATS*

In order to facilitate the production of large optical flats, Perkin-Elmer designed and fabricated a 76 cm diameter multiple beam Fizeau interferometer. This interferometer has been operating now for the past three years. The major optical elements shown in Figure 1 consist of an f/6 aspheric collimating objective and a transmission flat on which the reference surface is polished. The reference surface has a highly reflective coat so that sharp interference bands can be obtained. The optical axis of the interferometer is parallel to the gravity vector. With a vertical axis, the test article can be supported in a way which minimizes distortion. Any distortion that does occur will be symmetrical if the article is supported on a fluid, air bag or multi-point mount. (Figure 2.) The support of the transmission flat is partially achieved by an inflatable silicon rubber 'O' ring around the periphery.

During the initial design of the interferometer, it was intended to make the reference surface of the transmission flat appropriately concave to compensate for the sag due to gravity. Subsequently, it was determined that evacuation of the region between this (transmission) flat and the objective would provide an active means by which this bending could be overcome. This approach yielded a number of benefits. In the first case, the reference surface no longer had to be fabricated to some precise concavity, but only had to be smooth relative to a mild spherical surface. Second, variation of the pressure in the evacuated region provided a very sensitive means by which the interferometer could be focused. This allows determination of flatness of test pieces independent of power. Another mechanical feature is that the folding mirrors are adjustable in such a way that slight tilts will not change the focal distance during alignment.

The entire interferometer is supported on air isolators. In order to lift the test article so that it is isolated, a transport mechanism on three ball screws is used. This also allows a variable interference cavity where required.

* paper presented by W. Augustyn, Jr. at OSA Meeting, Hollywood, Florida, October 2, 1970

As mentioned previously, the curvature, or power of the reference surface is a function of the pressure between the evacuated region and ambient atmosphere. It is also affected to a small degree by thermal changes with time in the test area. The thermal changes are of the order of 0.1 to $0.2^{\circ}\text{F}/\text{hour}$ and the associated power change is less than 0.01λ RMS.* The power variation as a function of pressure was obtained from interferometric data using a 22-inch test flat as part of the interference cavity. This data is shown in Figure 3. Since this flat had a small amount of curvature, it was necessary to subtract this systematic error from the raw data. The corrected plot is shown in Figure 2.

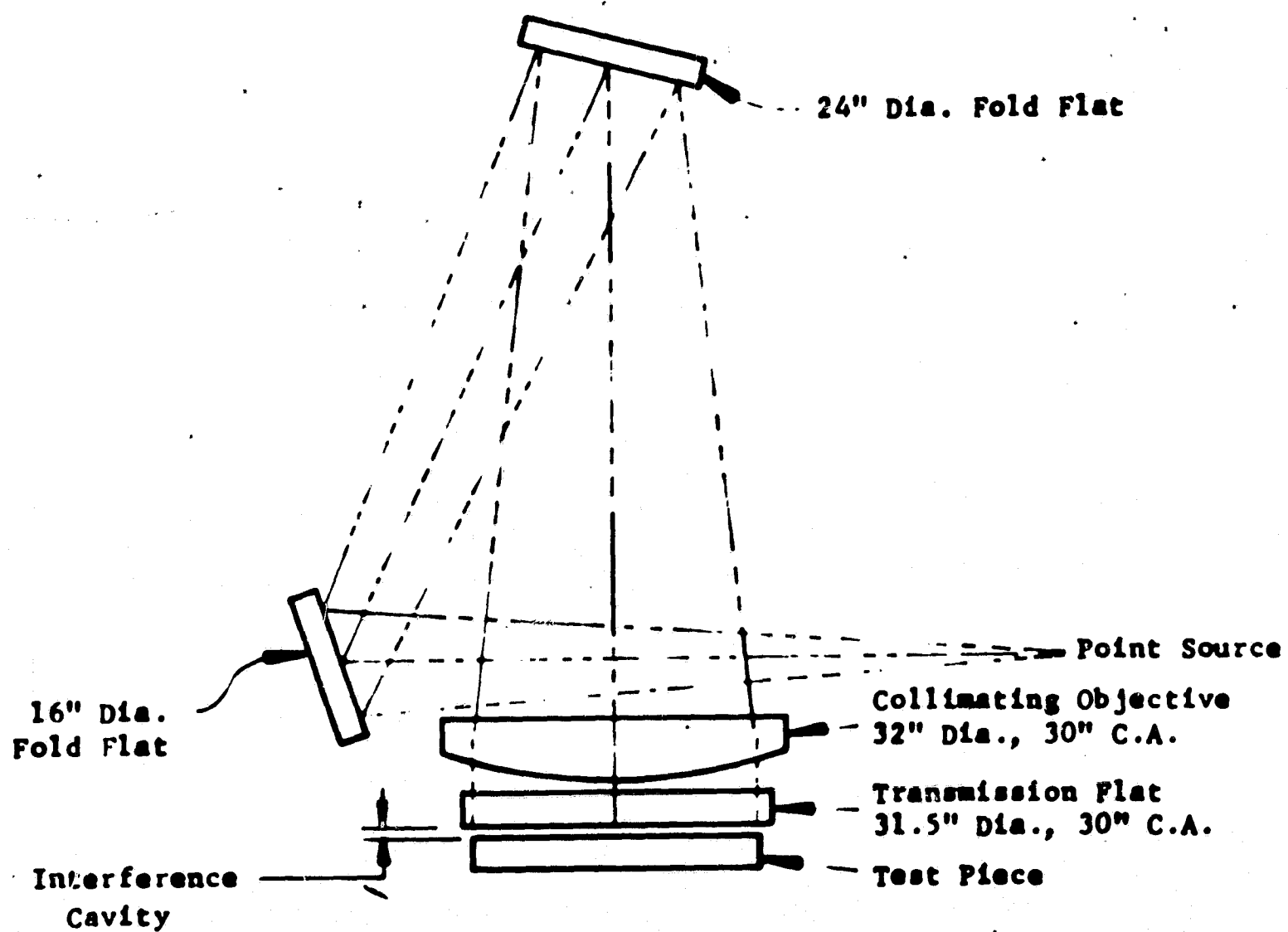
Smoothness and astigmatism were evaluated along two orthogonal diameters using a 12-inch glancing incidence or "skip" interferometer. (Figure 4.) This interferometer consists of a 12-inch aspheric collimating objective and transmission flat. The transmission flat has a highly reflecting coat so that multiple beam interference can be obtained between this surface and the end flat. By choosing the proper angle of incidence, this technique allows measurement of the full diameter of the 30-inch interferometer. The "skip" interferometer may be calibrated directly by measuring the end flat surface.

A series of "skip" interferograms of the Fizeau reference surface was obtained with each taken at a slightly different pressure. Power was analytically subtracted from each one and the resultant data averaged. The "skip" interferometer errors were next subtracted and the resultant smoothness along the two diameters determined. (Figure 5.) Since astigmatism would be manifest as a difference in power in two orthogonal directions, examination of the power calculated earlier as a function of pressure would yield this information. As a result, it can be concluded that the amount of astigmatism is less than $\lambda/60$ peak to valley. This value is consistent with independent experimental data obtained using the 22-inch test flat. The errors associated with all of the measurements which were made for evaluating the reference surface are tabulated in Figure 6. The values shown are the residual random errors and the root sum square of those errors.

*All wavelength values are a $\lambda = 6328\text{nm}$.

To further corroborate these results, a 24-inch autocollimating flat was measured using the Common test which has been described in the April 1970, issue of Applied Optics by H. D. Polster. The RMS smoothness evaluated in this test was 0.0001. With an RMS value of this magnitude, the assumption was made that an interferogram obtained in the Fizeau interferometer using this flat would be representative of the errors of the Fizeau reference surface. To the extent that one can neglect the errors of this 24-inch piece, good correlation with the original smoothness data should be obtained. (Figure 7.) Examination of this slide shows that the correlation between the two sets of data is well within the 0.95 confidence level.

(Figure 8.) This slide provides an example of the fine quality optical flats that have been fabricated by Perkin-Elmer and tested in this interferometer. Figure 9 shows a photograph of the actual interferometer test stand.



30" Fizeau Interferometer

Figure 1.

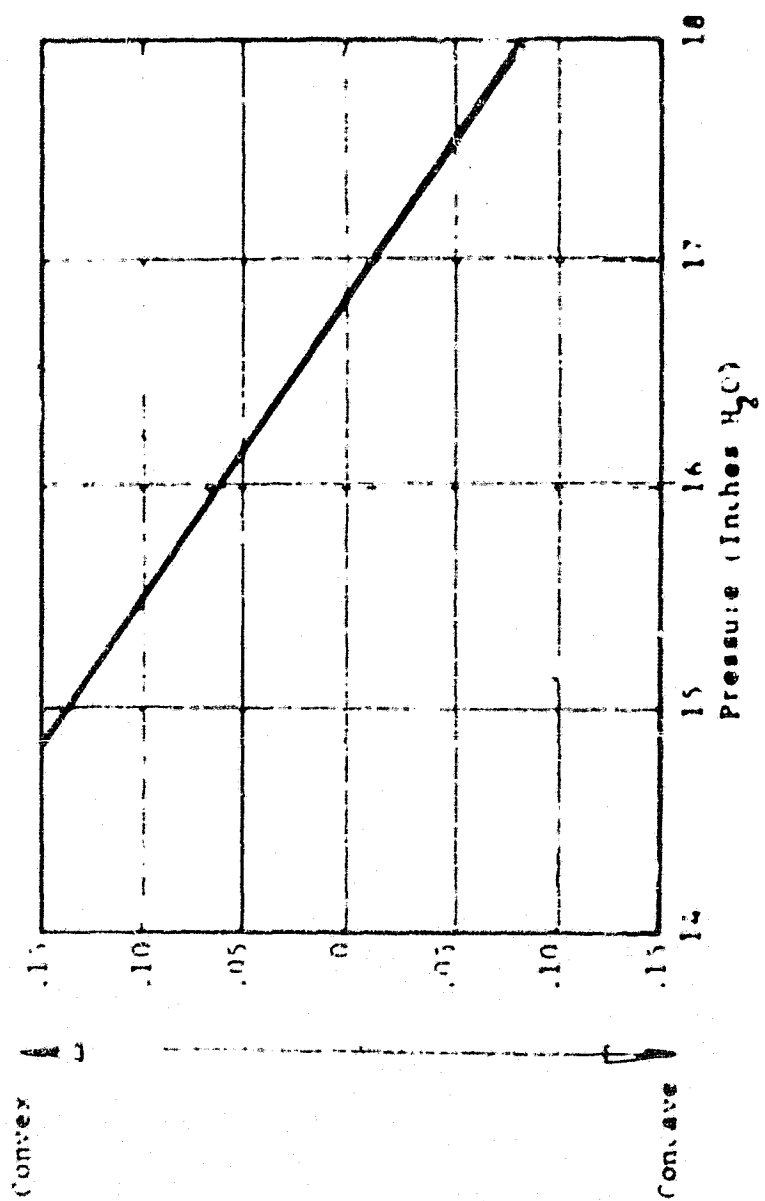
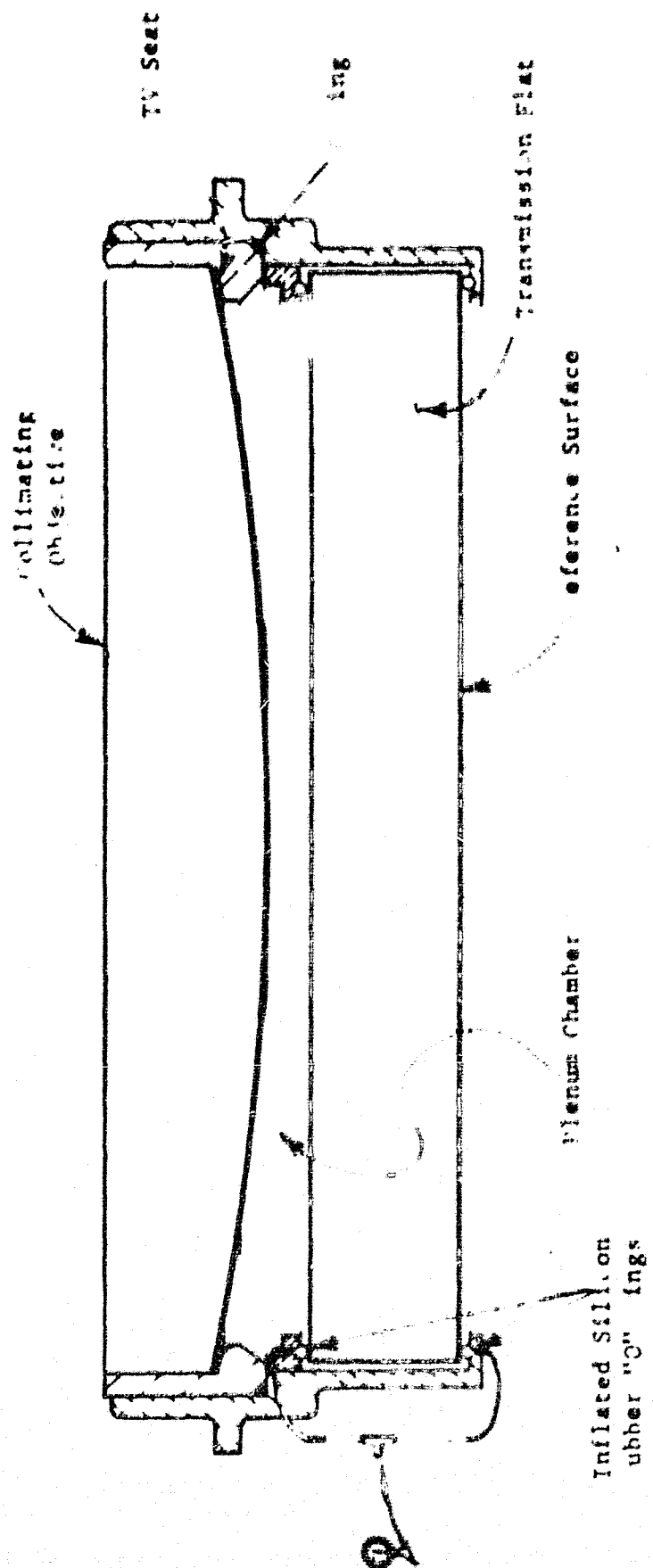
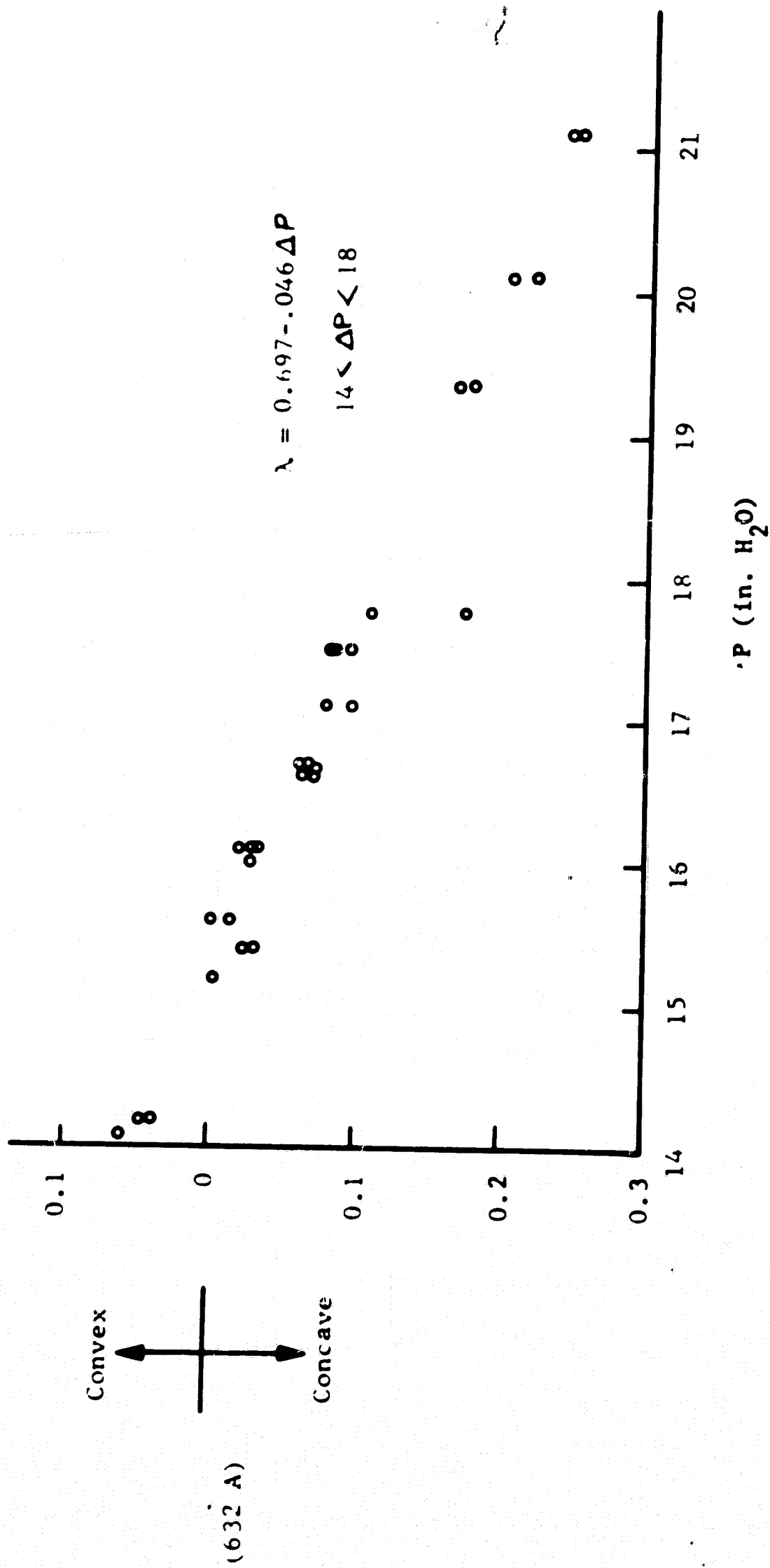


Figure 2.



Power vs Differential Pressure

Figure 3.

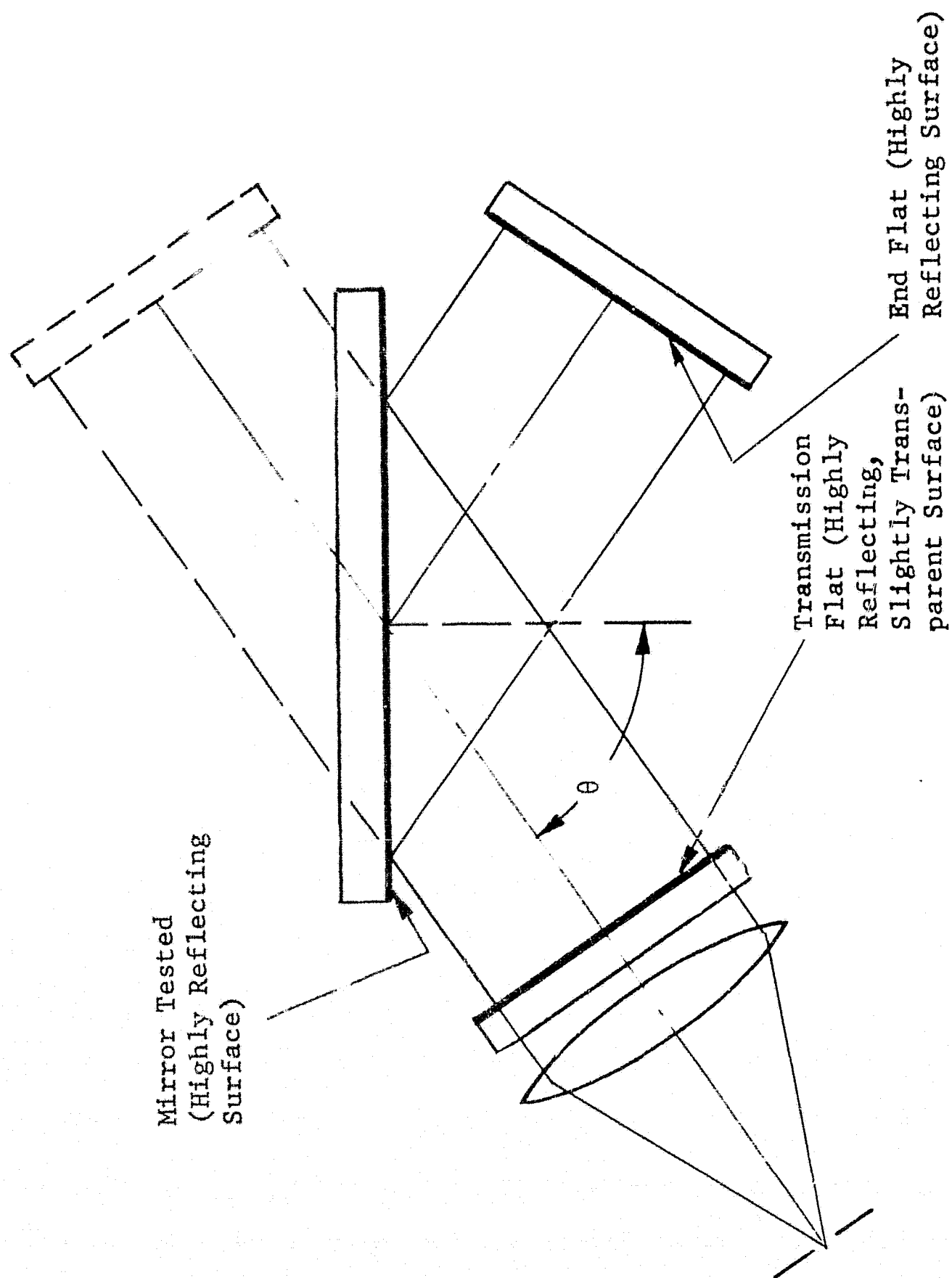
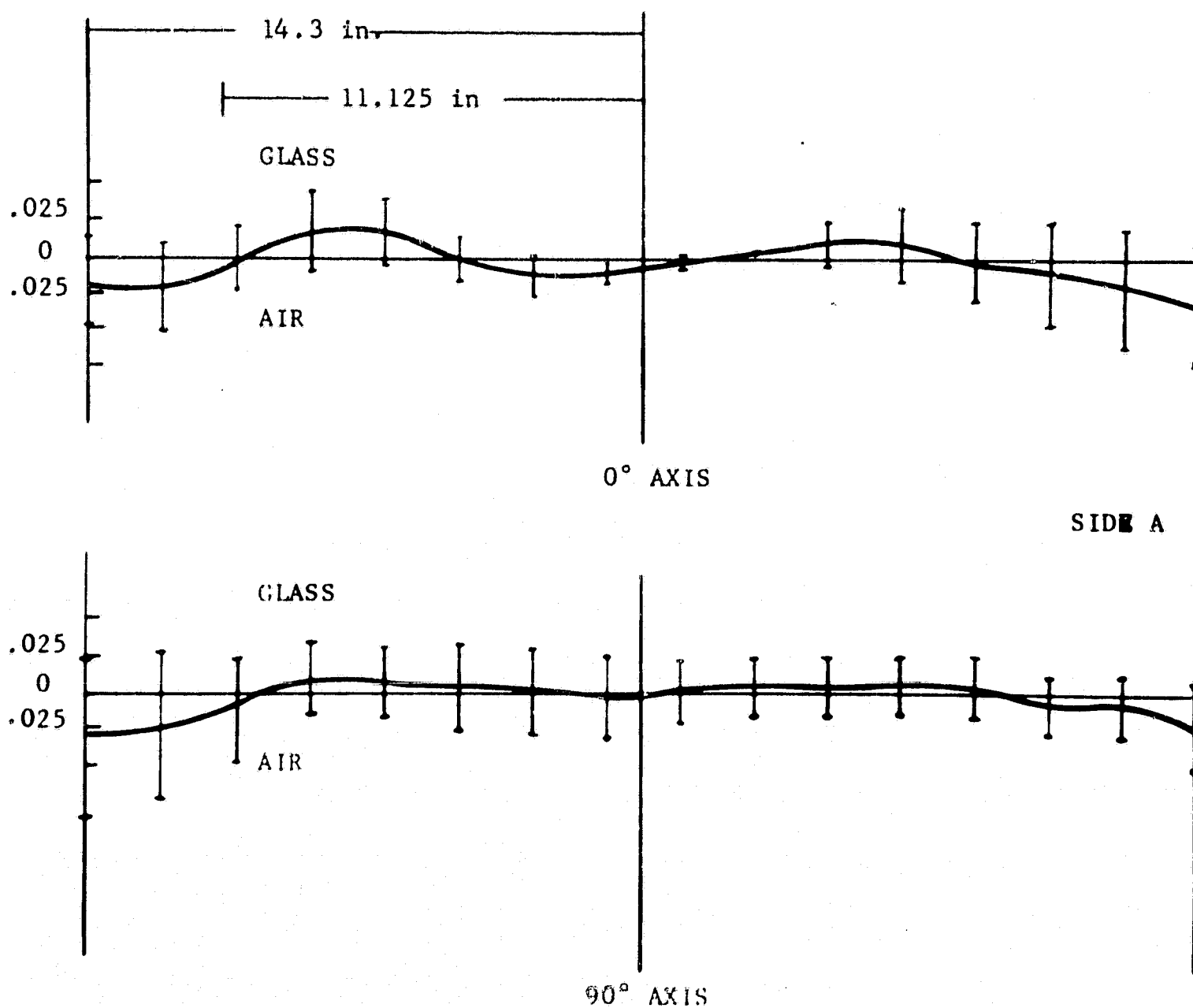


Figure 4.

INDICATES THAT THE CURVE IS CONTAINED WITHIN THE LIMIT
SHOWN TO A PROBABILITY OF .95



Smoothness of Fizeau Flat reference surface

Figure 5.

C. 4
Power at 16.85 inch (H_2O)

Thermal error (RMS)

33 measurements----- $1\sigma = \pm .0084\lambda$

Gervit 22.25 inch flat S/N 4

17 measurements----- $1\sigma = \pm .0077\lambda$

Cervit and Fizeau ; $\Delta P = 17$ inch (H_2O)

10 measurements----- $1\sigma = \pm .0063\lambda$

Total RMS error----- $= \pm .0130\lambda (\lambda/77)$

Astigmatism $\leq \lambda/62$ peak to peak 0° to 90° as shown in Figure 5.

Smoothness:

23 inch diameter, 0° ; peak to peak $.03\lambda$

28.6 inch diameter, 0° ; peak to peak $.05\lambda$

23 inch diameter, 90° ; peak to peak $.025\lambda$

28.6 inch diameter, 90° ; peak to peak $.04\lambda$

Figure 6.

I

INDICATES THAT THE CURVE IS CONTAINED WITHIN THE LIMIT
SHOWN TO A PROBABILITY OF .95

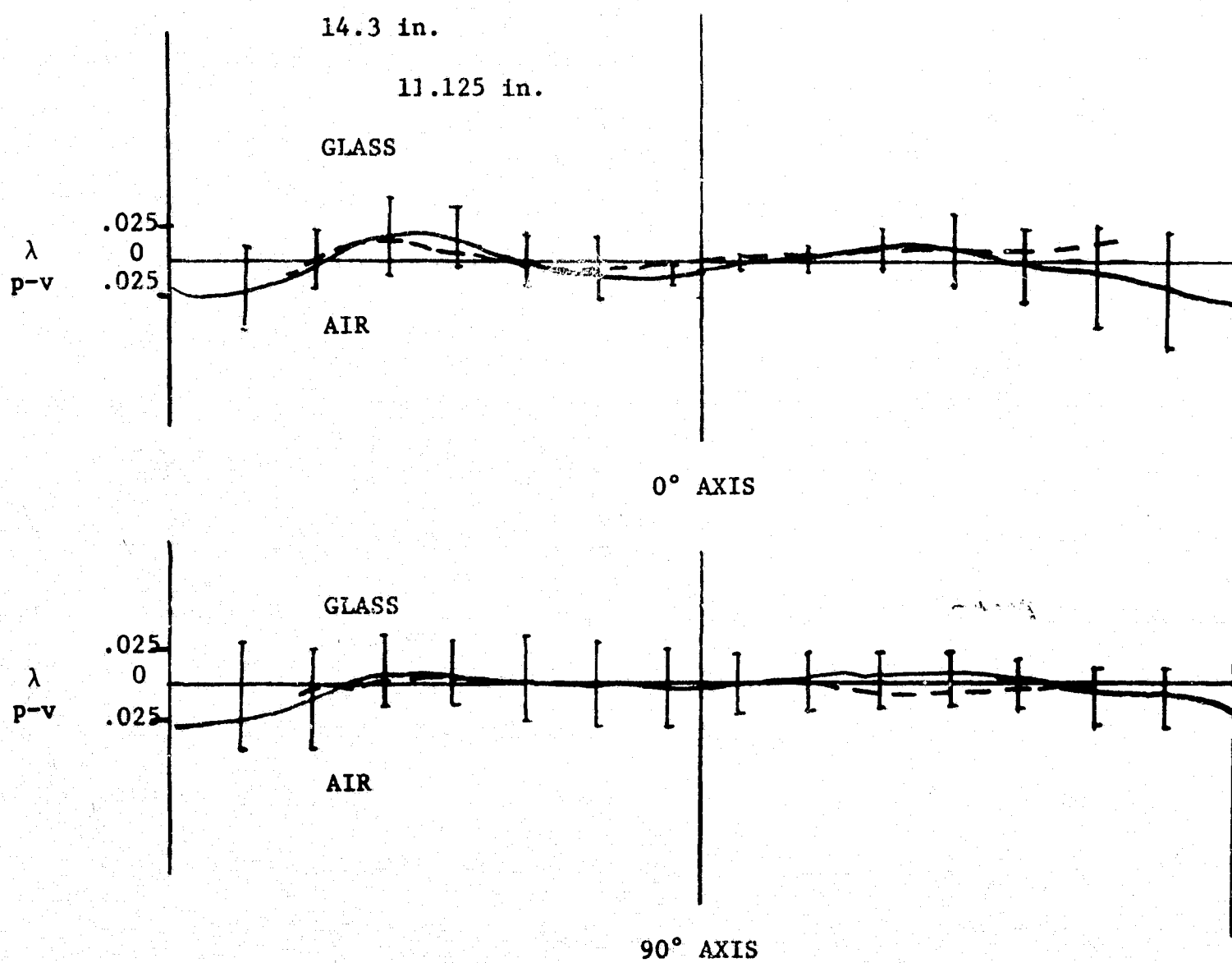
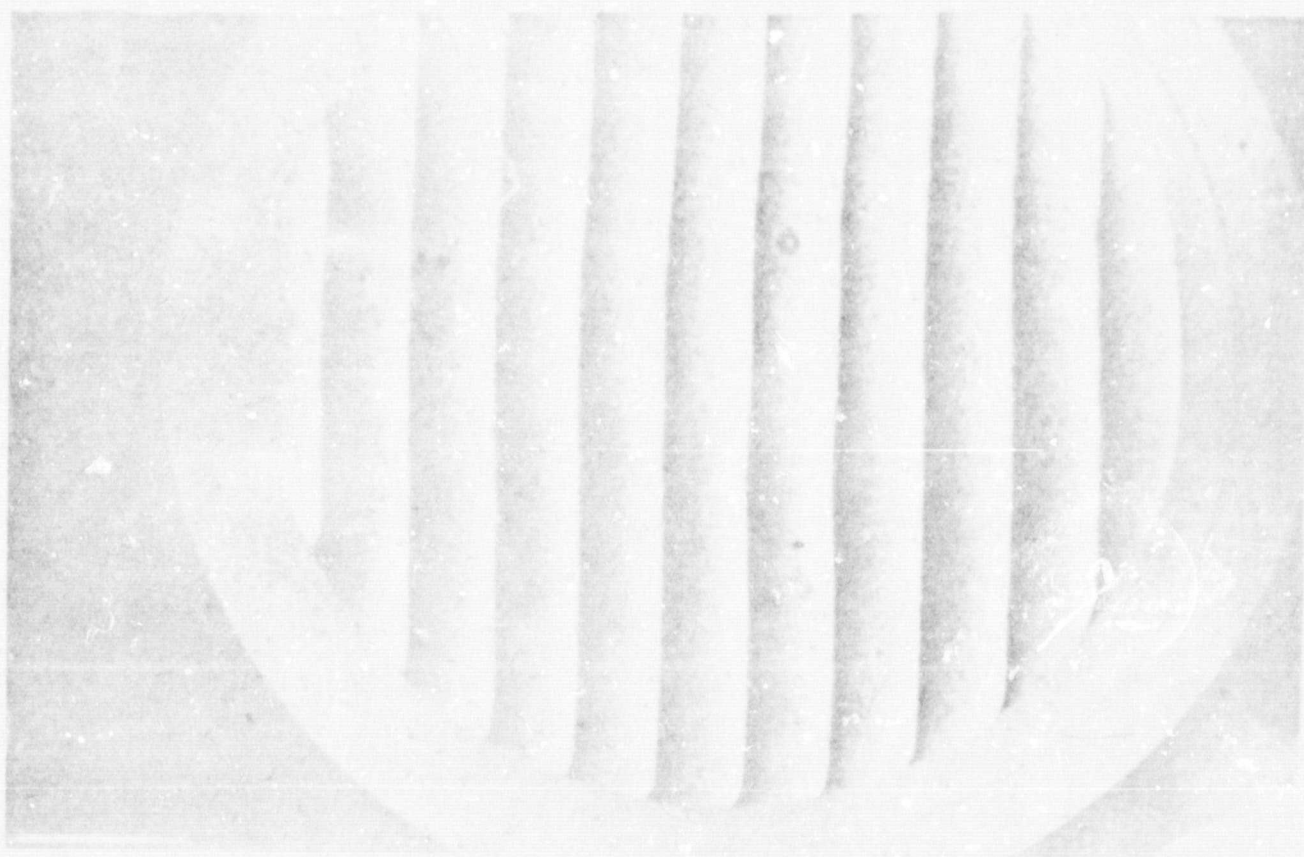
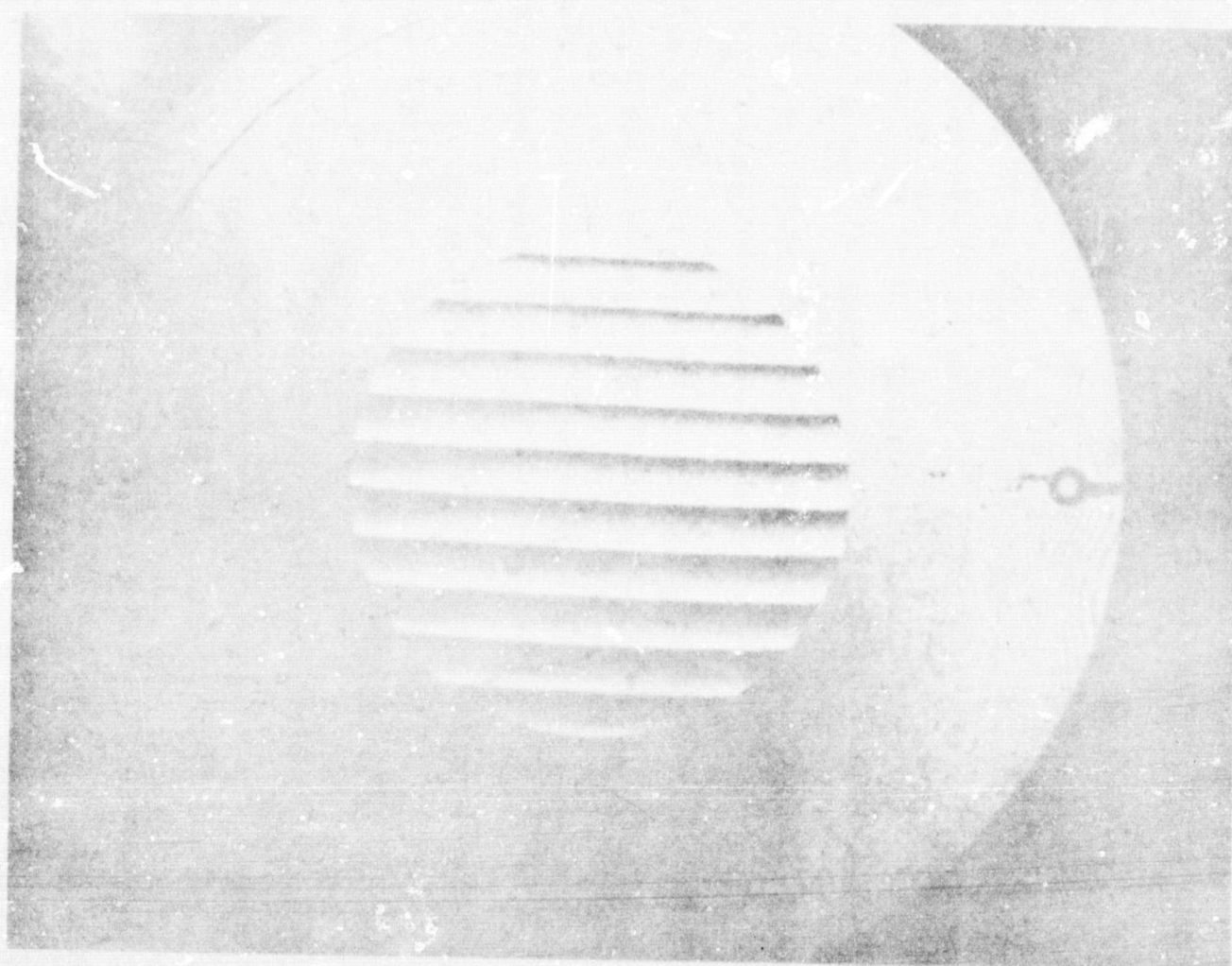


Figure 7.



24-Inch Diameter
 $\lambda/60$ RMS



15-Inch Diameter
 $\lambda/100$ RMS

Figure 8.

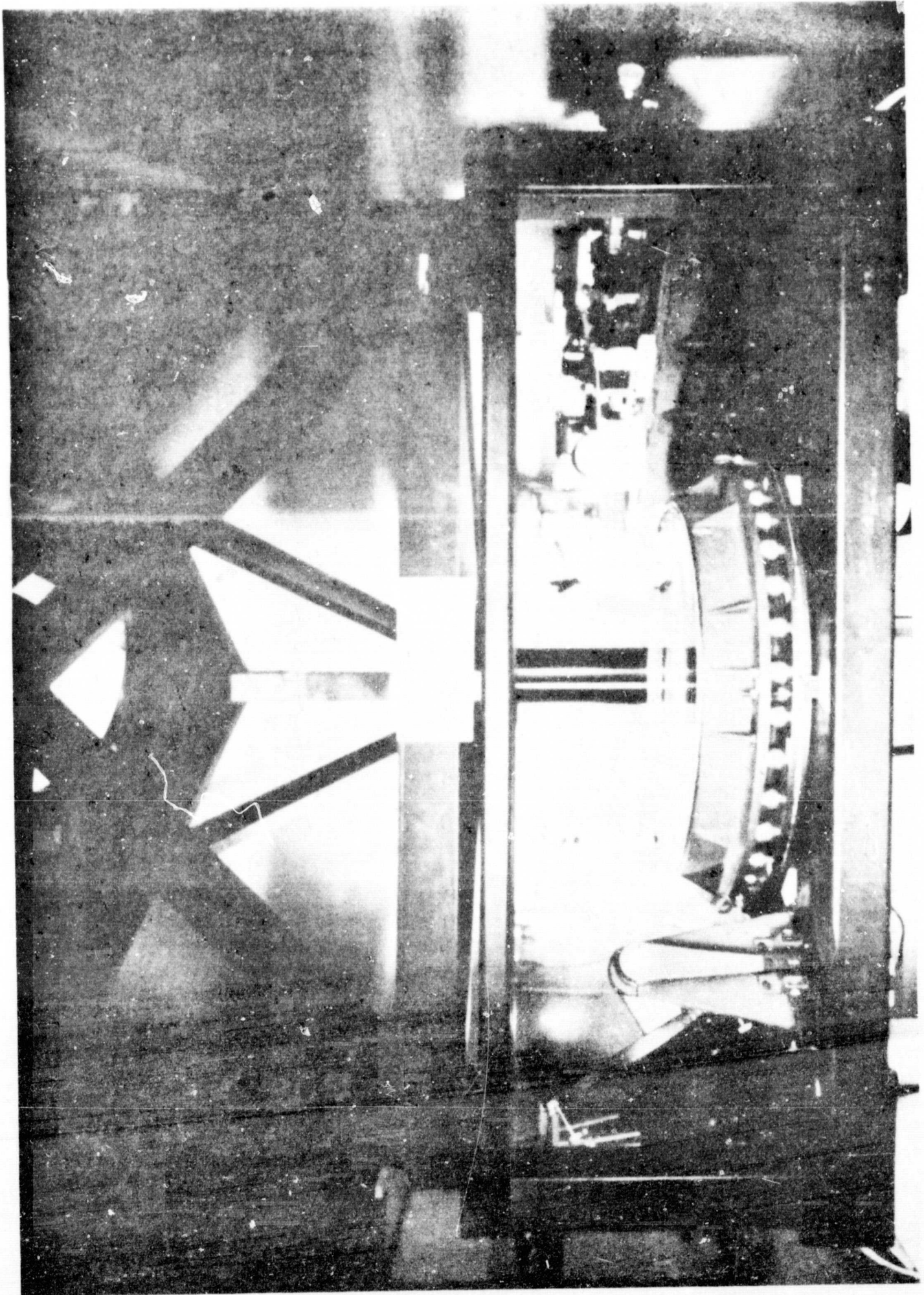


Figure 9.

APPENDIX E

DERIVATION OF ABERRATION FUNCTIONS

APPENDIX E

DERIVATION OF ABERRATION FUNCTIONS

ANALYSIS

DEFINITION AND APPLICATION OF ORTHONORMALITY

If there exists a set of polynomials $P_n(x,y)$ defined over a region, $A(x,y)$, such that the condition

$$\frac{1}{A(x,y)} \int_{A(x,y)} P_m^*(x,y) P_n(x,y) dA(x,y) = \delta_{mn} \quad (1)$$

where δ_{nm} is the Kronecker symbol and the asterisk denotes the complex conjugate, is fulfilled, the polynomials are said to form an orthonormal set within that region.

Given such a set which is also complete and given a function $W(x,y)$ defined over the region of orthonormality, it is permissible to write $W(x,y)$ as an expansion in the polynomials, or

$$W(x,y) = \sum_{n=1}^{\infty} C_n P_n(x,y) \quad (2)$$

The orthogonality condition may then be used to determine the value of the coefficient. Multiplying both sides of equation by $P_m^*(x,y)/A(x,y)$ and integrating over the area yields

$$\frac{1}{A(x,y)} \int_{A(x,y)} W(x,y) P_m^*(x,y) dA(x,y) = \frac{1}{A(x,y)} \int_{A(x,y)} \sum_{n=0}^{\infty} C_n P_n(x,y) P_m^*(x,y) dA(x,y) \quad (3)$$

Interchanging the summation and integration,

$$\begin{aligned} \frac{1}{A(x,y)} \int_{A(x,y)} W(x,y) P_m^*(x,y) dA(x,y) &= \sum_{n=1}^{\infty} C_n \frac{1}{A(x,y)} \int_{A(x,y)} P_n(x,y) P_m^*(x,y) dA(x,y) \\ &= \sum_{n=1}^{\infty} C_n \delta_{nm} \\ &= C_m \end{aligned} \quad (4)$$

ZERNIKE POLYNOMIALS

One such complete set of orthonormal polynomials when the region of convergence is a unit circle are the circle polynomials of Zernike. The treatment here is based entirely on the treatment of Born and Wolf⁴, where the Zernike polynomials, their derivation and properties, are discussed in detail. Using Equation 10 of the reference (repeated here):

$$U_n^m = R_n^m(\rho) \cos n \theta \quad (5)$$

$$U_n^{-m} = R_n^m(\rho) \sin m \theta$$

where the sub- and superscripts have the restrictions of Reference 4,

U_n^m and U_n^{-m} are the circle polynomials of Zernike and

$R_n^m(\rho)$ are the radial polynomials of Zernike

and the tabulated expressions for the radial polynomials (Table XXI of Reference 4), the first 22 circle polynomials were expressed in terms of the aperture coordinates x and y . These expressions appear in Table I. In this treatment, since no rotational symmetry was assumed (contrary to the application in Reference 1), the sine terms were retained. The square root factors in Table I are renormalization factors introduced to make (1) hold exactly. In this, the usage differs somewhat from that of Reference 4. In Born and Wolf's notation the factor is

$$\begin{aligned} \sqrt{n+1}, & \quad m = 0 \\ \sqrt{2(n+1)}, & \quad m \neq 0 \end{aligned} \quad (6)$$

this normalizes the "modified" polynomials so that they have a mean square value of one rather than a peak absolute value of one.

In the evaluation of the modified Zernike circle polynomials, these definitional relations were assumed:

$$\begin{aligned} x &= \rho \cos \theta \\ y &= \rho \sin \theta \end{aligned} \quad (7)$$

Hence,

$$\rho^2 = x^2 + y^2$$

TABLE I

ZERNIKE POLYNOMIALS FOR WHICH COEFFICIENTS ARE EVALUATED

Radial Polynomial *	Circle Polynomial	Corresponding
$R_n^m(\rho)$	$R_n^m(\rho) \cos m\theta$ or $R_n^m(\rho) \sin m\theta$	Aberration Designation**
R_0^0 1	$R_0^0 = 1$	#1 constant error.
R_1^1 ρ	$R_1^1 \cos \theta = 2x$	#2 x-tilt
	$R_1^1 \sin \theta = 2y$	#3 y-tilt
R_2^0 $2\rho^2 - 1$	$R_2^0 = \sqrt{3} (2\rho^2 - 1)$	#4 focus error
R_2^2 ρ^2	$R_2^2 \sin 2\theta = \sqrt{6} 2xy$	#5 45° astigmatism
	$R_2^2 \cos 2\theta = \sqrt{6} (x^2 - y^2)$	#6 0° astigmatism
R_3^1 $\rho(3\rho^2 - 2)$	$R_3^1 \sin \theta = \sqrt{8} y(3\rho^2 - 2)$	#7 y-coma
	$R_3^1 \cos \theta = \sqrt{8} x(3\rho^2 - 2)$	#8 x-coma
R_3^3 ρ^3	$R_3^3 \sin 3\theta = \sqrt{8} y(3x^2 - y^2)$	#9 y-clover
	$R_3^3 \cos 3\theta = \sqrt{8} x(x^2 - 3y^2)$	#10 x-clover
R_4^0 $6\rho^2(\rho^2 - 1) + 1$	$R_4^0 = \sqrt{5} [6\rho^2(\rho^2 - 1) + 1]$	#11 3rd-order spherical aberrati
R_4^2 $\rho^2(4\rho^2 - 3)$	$R_4^2 \cos 2\theta = \sqrt{10} (x^2 - y^2)(4\rho^2 - 3)$	#12 sphero-astigmatism
	$R_4^2 \cos 2\theta = \sqrt{10} 2xy(4\rho^2 - 3)$	#13 sphero-astigmatism

TABLE I (Continued)

ZERNIKE POLYNOMIALS FOR WHICH COEFFICIENTS ARE EVALUATED

Radial Polynomial*	Circle Polynomial	Corresponding
$R_n^m(\rho)$	$R_n^m(\rho) \cos m\theta$ or $R_n^m(\rho) \sin m\theta$	Aberration Designation**
$R_4^4 \quad \rho^4$	$\sqrt{10} R_4^4 \cos 4\theta = \sqrt{10} [x^4 - 6x^2y^2 + y^4]$	#14 Ashtray Aberration
	$\sqrt{10} R_4^4 \sin 4\theta = \sqrt{10} 4xy(x^2 - y^2)$	#15 Ashtray Aberration
$R_5^1 \quad \rho(10\rho^4 - 12\rho^2 + 3)$	$\sqrt{12} R_5^1 \cos \theta = \sqrt{12}(10\rho^4 - 12\rho^2 + 3)$	#16
	$\sqrt{12} R_5^1 \sin \theta = \sqrt{12}y(10\rho^4 - 12\rho^2 + 3)$	#17
$R_5^3 \quad \rho^3(5\rho^2 - 4)$	$\sqrt{12} R_5^3 \cos 3\theta = \sqrt{12}x(x^2 - 3y^2)(5\rho^2 - 4)$	#18
	$\sqrt{12} R_5^3 \sin 3\theta = \sqrt{12}y(3x^2 - y^2)(5\rho^2 - 4)$	#19
$R_5^5 \quad \rho^5$	$\sqrt{12} R_5^5 \cos 5\theta = \sqrt{12}x(x^4 - 10x^2y^2 + 5y^4)$	#20
	$\sqrt{12} R_5^5 \sin 5\theta = \sqrt{12}y(5x^4 - 10x^2y^2 + y^4)$	#21
$R_6^0 \quad 20\rho^6 30\rho^4 + 12\rho^2 - 1$	$\sqrt{7} R_6^0 = \sqrt{7} [20\rho^6 - 30\rho^4 + 12\rho^2 - 1]$	#22 5th-order spherical aberration

* From Table XXI, Reference 4

**Designations supplied by Dr. R.E. Hufnagel

APPENDIX F

A NULL CORRECTOR FOR PARABOLOIDAL MIRRORS

A Null Corrector for Paraboloidal Mirrors

Abe Offner

A simple optical system consisting of two small lenses can be designed to take the place of a large flat in testing paraboloidal mirrors. Other aspheric concave mirrors can also be tested in this manner. The procedure for computing such a null corrector and the accuracy required in its manufacture and use are discussed.

In making a large mirror, the limit of the accuracy which a good optician can achieve is set by the magnitude of the mirror errors which he can see or measure. When a large enough flat is available, it is customary to test paraboloidal mirrors during the progress of the work by the well-known Foucault knife-edge test. The light source and knife edge are placed at the focus of the mirror and the flat is used to reflect the collimated light coming from the mirror back to the mirror. This method is very sensitive and gives easily interpretable results if it is assumed that the errors of the flat are small compared to the maximum error allowable in the mirror.

In the project known as Stratoscope II, sponsored by ONR, NSF, and NASA, the Perkin-Elmer Corporation is building a 91-cm telescope system for Princeton University. The telescope is to be carried by an unmanned balloon to a height at which the effect of the earth's atmosphere on its performance is essentially removed. A tolerance analysis indicated that if the system were to achieve its desired performance, the root mean square departure of the primary mirror surface from a paraboloid had to be no more than one fiftieth of a wave. To measure this magnitude of mirror error we felt it desirable to have auxiliary optics which are more easily made and calibrated than a 91-cm flat.

A spherical mirror can be tested easily with no auxiliary optics by putting the light source and knife edge at the center of curvature. All rays from the center of curvature proceed along normals to the spherical surface and are reflected back along the same paths so that even cutoff indicates a perfect sphere. Our object was

to obtain a similar setup for paraboloidal mirrors with the help of small auxiliary optical elements which are easily made and measured.

It has been suggested that the spherical aberration of a paraboloidal mirror used at unit magnification can be compensated by a convex lens placed between the paraboloidal mirror and its "center of curvature,"^{1,2} or by a spherical mirror placed beyond the "center of curvature."² In such tests it is desirable to place the compensating mirror or lens near the "center of curvature" of the paraboloid in order to keep its size down. F. E. Ross² showed, however, that one can decrease the residual uncompensated aberration over the aperture by increasing the distance between the compensating mirror or lens and the "center of curvature." This comes about because the third order spherical aberration introduced by the paraboloidal mirror can be balanced only by a combination of third and higher order aberration if the balancing is done at any position other than at the paraboloid, where the corrector would have to be as large as the paraboloidal mirror itself.

To get around this difficulty it was decided to correct the paraboloid by a lens which was placed at the paraboloid optically but not physically. A system for doing this is shown in Fig. 1. Here the field lens forms an image of the imaging lens at the paraboloid. This results in exact compensation provided that the spherical aberration introduced by the imaging lens is pure third order and of the proper amount to balance the "spherical aberration" of the normals to the paraboloid.

In practice it is not necessary that the spherical aberration of the imaging lens be pure third order, a requirement which is not always easy or convenient to attain. What is required is that the spherical aberration of the correcting system match the "aberration" of the normals to the paraboloidal mirror.

Figure 2 shows the "aberration" of the normals to a paraboloid. C is the center of curvature of the oscu-

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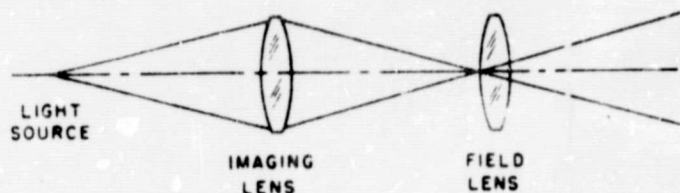


Fig. 1. Optical system for obtaining spherical aberration following a desired law.

lating sphere at P. The desired aberration of the null corrector is given by the relationship,

$$CD = (-R/2) \tan^2 U.$$

In the optical system of Fig. 1, the aberration will not be proportional to the square of the tangent of the slope angle, as desired, for any arbitrary shape of the imaging lens. If, now, the power of the field lens is changed, the third order spherical aberration of the system is unchanged but its "offense against the sine condition" is altered. This means that the relationship between the spherical aberration and $\tan U$ is changed. The power of the field lens can be varied until the spherical aberration of the system follows the desired law. The system can now be scaled to match the spherical aberration of the desired paraboloid.

The operation of this null corrector depends on the fact that a real image of the light source is formed at the center of curvature of the paraboloid. This affords a position for the field lens at an image plane at which spherical aberration is present. The shapes of the elements of the null corrector and the magnification at which it is used can be varied between wide limits.

The system actually used with the 91-cm $f/4$ paraboloidal primary mirror of Stratoscope II is shown in Fig. 3. The test system was designed to be used with mercury green light. Both elements are plano-convex. The larger one has a clear aperture of 45 mm.

It is interesting to compare the residual uncompensated aberrations of previous null correctors with those which can be achieved by the present arrangement. The H. E. Dall¹ corrector restricts the shape of the com-

pensating lens to plano-convex. The resulting compensation is adequate only for relatively small aperture mirrors if a high degree of precision is required.

F. E. Ross² used the shape of the compensating lens as one of his parameters and was able to reduce the residual aberrations considerably. As a compensator for the 508-cm Mt. Palomar paraboloid, he used a lens of 25-cm aperture which, according to his computations, would have resulted in a paraboloid with a circle of confusion of 3.2 arc sec diameter. An aspheric corrector plate was added to this null corrector to make it adequate.

Scaling the null corrector of Fig. 3, to make it work with a 508-cm aperture paraboloid, results in a clear aperture of 25 cm for the larger lens, the same as the aperture of the null corrector lens used by F. E. Ross. The residual aberration of a paraboloid made with this null corrector would result in a geometrical circle of confusion of less than 0.01 arc sec diameter. Allowing a factor of 3 for the fact that the Mt. Palomar mirror is

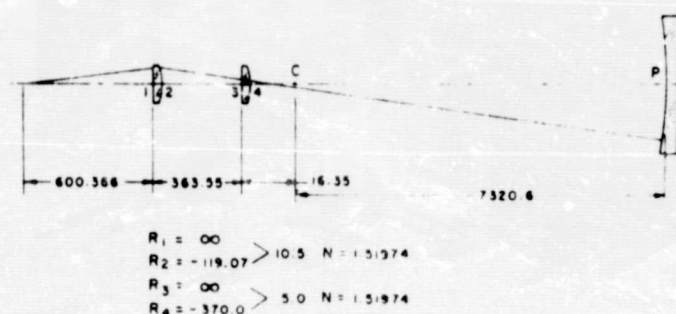


Fig. 3. Optical system used for null testing a 91-cm $f/4$ parabolic mirror at center of curvature.

$f/3.33$ while the Stratoscope II mirror is $f/4$, one can see that the use of a field lens has resulted in a decrease of the uncompensated residual aberration by a factor of 100. If required, further improvement could be obtained by altering the shape of the imaging lens.

An investigation was made of the effects of small departures of the parameters of this system from nominal on the imagery obtained with a perfect paraboloid.

From its center of curvature, the paraboloid subtended an $f/8$ cone. The light source and knife edge were at an image plane which was magnified 1.57 times with respect to the center of curvature of the paraboloid so that the convergence at the knife edge was $f/12.6$. The radius of the first dark ring of the diffraction pattern in mercury e-light was therefore 8.4μ .

A change of index of refraction of the elements of the corrector system of 0.0002 caused an enlargement of the geometric circle of confusion to 0.3μ radius.

A change of radius of the large lens of 0.050 mm, which corresponds to a change of 1μ in its saggita over its clear aperture, caused a geometric circle of confusion of 0.4μ radius.

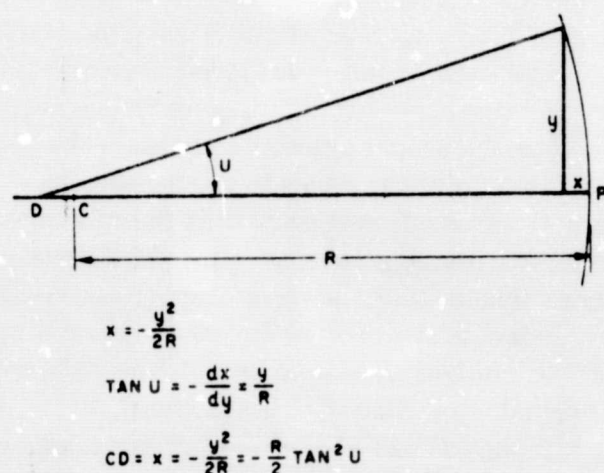


Fig. 2. "Aberration" of normals to a parabola.



Fig. 4. Focogram of 91-cm mirror made by autocollimation off 100-cm flat.

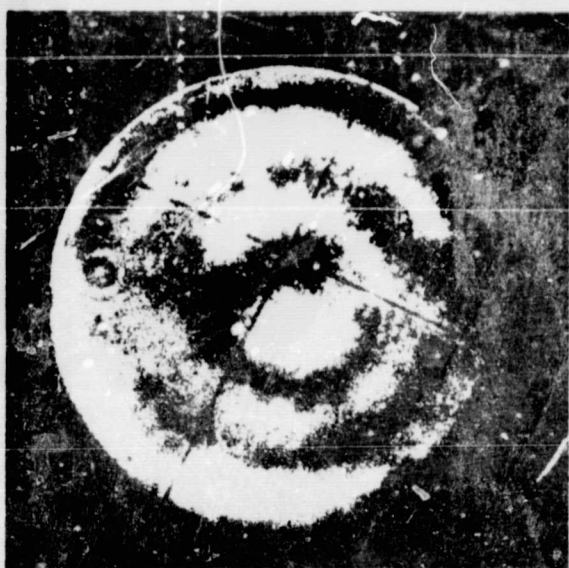


Fig. 5. Focogram of 91-cm mirror made with null corrector shown in Fig. 3.

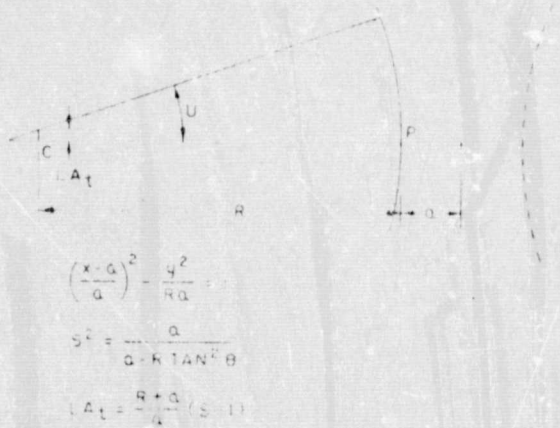


Fig. 6. "Aberration" of normals to a hyperbola.

A change in the separation of the two elements of 0.1 mm caused an increase of the geometrical circle of confusion of 0.6 μ radius.

Similar changes would be caused by changes in the thickness of either element by 0.15 mm.

An uncertainty in the location of the point C, the center of curvature of the osculating sphere to the paraboloid at its pole, would result in a parallel surface to a paraboloid in place of the paraboloid. A simple calculation showed that, for an error of 5 mm in the location of C, the resulting surface at perfect cutoff would depart from a paraboloid with the same radius of curvature at its pole by $1/59$ of a wave.

The null corrector can therefore be built to the required degree of accuracy without unusually tight tolerances. It must be made of highly homogeneous glass, and the figure of the large element must be better than that aimed for in the paraboloid. To meet the first of these requirements, the system was designed with Schott BK-7 glass which is obtainable with very good quality. Measurements with a Herriot type interferometer indicated a figure of better than $1/20$ of a wave in the elements as made.

Figures 4 and 5 show focograms of the primary mirror, one of which was made with the null corrector and the other of which was made by autocollimation with a 100-cm flat.

This method is of course not confined to the testing of paraboloidal mirrors. It is particularly useful in the manufacture and testing of hyperboloidal concave mirrors which are used as the primaries in Ritchey-Chrétien systems, since such mirrors cannot be tested by autocollimation with flats. One reason astronomers have avoided specifying these systems in the past in spite of their superior performance was the difficulty of manufacturing them. With the new null corrector, Ritchey-Chrétien primaries can be made and tested almost as easily as paraboloidal mirrors. Figure 6 shows the required spherical aberration of a system to be used for null testing a concave hyperboloidal mirror at its "center of curvature."

A null corrector was designed for the 102-cm primary of a Ritchey-Chrétien telescope being built for Mt. Stromlo Observatory in Australia by the Perkin-Elmer Corporation. The primary is an $f/3.5$ hyperboloid with an eccentricity of 1.026. The larger lens of the null corrector system has a clear aperture of 52 mm.

References

1. H. E. Dall, "A null test for paraboloids," *Astronomical Telescope Making* (Book Three) (Scientific American, New York, 1953), pp. 149-153.
2. F. E. Ross, *Astrophys. J.* **98**, 341-346 (1943).

APPENDIX G

SCATTERED LIGHT PROBLEM FEASIBILITY
OF PHOTOMETRY ON 29TH
MAGNITUDE STARS

APPENDIX G

SCATTERED LIGHT PROBLEM FEASIBILITY OF PHOTOMETRY ON 29TH MAGNITUDE STARS

INTRODUCTION

The ability of the LST to detect a 29th magnitude star will be limited by the scattering characteristics of the primary mirror for the case where a relatively bright star is in the LST's 30-arc-minute field of view. Since the tracking function requires at least one 10th magnitude star within that field of view, this case will be used to make a first order estimate of the primary mirror scattering coefficient required for a S/N ratio of 10, for a photometric measurement of a 29th magnitude star. In addition, an average sky brightness of 23rd visual magnitude per arc-second squared will also be considered as a source of scattered light.

COMPUTATION OF SIGNAL FROM 29TH MAGNITUDE STAR

We know that the entrance aperture irradiance due to a 29th magnitude star is found from the defining equation for visual magnitudes, as shown below.

$$m_v = -2.5 \log_{10} \left[\frac{I(m_v)}{I(0)} \right] \quad (1)$$

where

$$I(0) = 3.1 \times 10^{-13} \text{ w/cm}^2$$

thus

$$H_{29} = \frac{I(0)}{10^{(29/2.5)}} = 7.7 \times 10^{-25} \text{ w/cm}^2 \quad (2)$$

Now we can find the total signal through the aperture of the LST as (ignoring transmission losses, obscuration ratio, etc.),

$$P_{29} = \frac{\pi}{4} D_p^2 H_{29} \text{ (watts)} \quad (3)$$

where

$$\begin{aligned} D_p &= \text{diameter of primary mirror} \\ &= 3 \times 10^2 \text{ cm} \end{aligned}$$

COMPUTATION OF SCATTERED LIGHT FROM AVERAGE SKY BACKGROUND

We know that the average sky background is given as

$$B_{\text{sky}} = 23 \text{rd m}_V / \text{sec}^2 \quad (4)$$

Thus

$$H_{\text{sky}} = \frac{(3.1 \times 10^{-13})}{10^{(23/2.5)}} = 1.94 \times 10^{-22} \text{ w/cm}^2 \quad (5)$$

Now let

$$1 \text{ sec}^2 = 2.35 \times 10^{-11} \text{ steradian} \quad (6)$$

Thus (4) becomes

$$B_{\text{sky}} = \frac{(1.94 \times 10^{-22})}{(2.35 \times 10^{-11})} = 8.3 \times 10^{-12} \text{ w/cm}^2\text{-st} \quad (7)$$

If we assume that 6° of the sky is seen by the primary mirror due to the baffling of the telescope tube, the solid angle subtended by the sky from the primary is

$$\Omega_{\text{sky}} \approx \left[\frac{6}{57.3} \right]^2 = 0.011 \text{ steradian} \quad (8)$$

Thus the irradiance on the primary, H_p , is given by

$$H_p = B_{\text{sky}} \cdot \Omega_{\text{sky}} [\text{w/cm}^2] \quad (9)$$

Now we can find the brightness, or radiance, of the primary, assuming that some fraction ρ of the incident light is scattered uniformly into a hemisphere (assumption of Lambertian scatter).

$$B_p = \frac{\rho H_p}{\pi} [\text{w/cm}^2\text{-ster}] \quad (10)$$

where

ρ = primary mirror integrated scatter coefficient

We may now find the flux density of the image plane due to the scatter from the primary as

$$H_{ip} = \frac{\pi B_p}{4N^2} \quad (11)$$

where

$$N = \text{system f/number}$$

Thus the total scattered light captured by the photometer is given by

$$P_{sc} = H_{ip} \cdot A_d \quad (12)$$

where

$$A_d = \text{detector area}$$

Now expressing the detector area as a function of its angular subtense as

$$A_d = (F\theta_d)^2 \quad (13)$$

where

$$F = \text{system focal length}$$

$$\theta_d = \text{angular subtense}$$

but

$$A_d = (F\theta_d)^2 = (N\theta_d)^2 D_p^2 \quad (14)$$

Now combine the various equations above to find

$$P_{sc} = \frac{1}{4} \rho \Omega_{sky} \cdot B_{sky} \cdot \theta_d^2 \cdot D_p^2 \quad (15)$$

SKY BACKGROUND LIMITED S/N RATIO

If we define a signal-to-noise ratio as

$$S/N = \frac{P_{29}}{P_{sc}} = \frac{\pi}{4} \cdot \frac{H_{29}}{\rho \Omega_{sky} B_{sky} \theta_d^2} \quad (16)$$

We may now solve equation (16) for the primary mirror scattering coefficient required to achieve some minimum S/N ratio, as follows

$$\rho_{req'd} = \pi \frac{H_{29}}{(S/N)_{min} (\Omega_{sky} B_{sky} \theta_d^2)} \quad (17)$$

If we let $(S/N)_{\min} = 10.0$, $\theta_d = 0.4$ arc-second, then we have

$$\rho_{\text{req'd}} = 7.80 \quad (18)$$

Thus the primary mirror can scatter virtually all of the incident energy, in a Lambertian fashion, and still permit a S/N greater than 10, for the case of a uniform sky background of 23rd m_v/sec^2 .

SCATTERED LIGHT DUE TO A RELATIVELY BRIGHT STAR WITHIN THE FIELD OF VIEW

For a 10th magnitude star within $1/2$ degree of the much dimmer 23rd magnitude star, we will assume that all of the scattered energy will be narrow angle forward scatter, having an equivalent scattering angle of ϕ radians.

Now find the irradiance on the primary due to a 10th magnitude star.

$$H_{10} = \frac{(3.1 \times 10^{-13})}{10^4} = 3.1 \times 10^{-17} \text{ w/cm}^2 \quad (19)$$

Thus the brightness of the primary, due to forward scatter, is

$$B_p = \frac{\rho_F \cdot H_{10}}{\phi^2} \text{ w/cm}^2\text{-ster} \quad (20)$$

where

ρ_F = forward scattering coefficient

ϕ = forward scattering angle

Once again, the irradiance on the photometer detector is given by

$$H_{ip} = \frac{\pi B_p}{4N^2} = \frac{\pi \rho_F H_{10}}{4N^2 \phi^2} \quad (21)$$

Now we derive the total scattered light on the photometer detector

as

$$P_{sc} = \frac{\pi \rho_F H_{10} \theta_d^2 D_p^2}{4\phi^2} \quad (22)$$

BRIGHT STAR LIMITED S/N RATIO

Now examine the signal-to-noise ratio defined by

$$S/N = \frac{P_{29}}{P_{sc}} = \frac{H_{29} \phi^2}{\rho_F H_{10} \theta_d^2} \quad (23)$$

We may not solve equation (23) for the required forward scattering primary mirror scattering coefficient required to achieve some minimum S/N ratio, as follows;

$$\rho_{F \text{ req'd}} = \frac{H_{29} \phi^2}{(S/N)_{\min} H_{10} \theta_d^2} \quad (24)$$

Figure G-1 illustrates the relationship between the required scattering coefficient and values of the forward scattering angle, for various bright star magnitudes.

As shown, for stellar magnitudes of +7.5 or greater, the required forward scattering coefficient is not difficult to achieve for visible wavelengths. However, for relatively bright stars, the requirements on the primary mirror scatter (both the scattering coefficient, and the equivalent forward scattering angle) become quite stringent, and may well limit the ability of the LST to perform photometry on 29th magnitude stars.

OTHER SOURCES OF SCATTER

A major source of scatter will arise from the baffling systems, support structures used, etc. It is impossible to proceed any further on this point because information is unavailable and experimental work may be necessary.

One thing that could be done is that baffles be put on the inside of the shielding tube attached to the secondary in such a way that their faces would be approximately parallel to the light incident on them. This is a three dimensional 'razor blade stack' and would give better absorption than simple perpendicular baffles, in the sense that any light which entered this baffling system would be re-emitted with high attenuation whereas light that strikes ordinary baffles would be mainly redirected rather than absorbed due to the large angles of incidence involved.

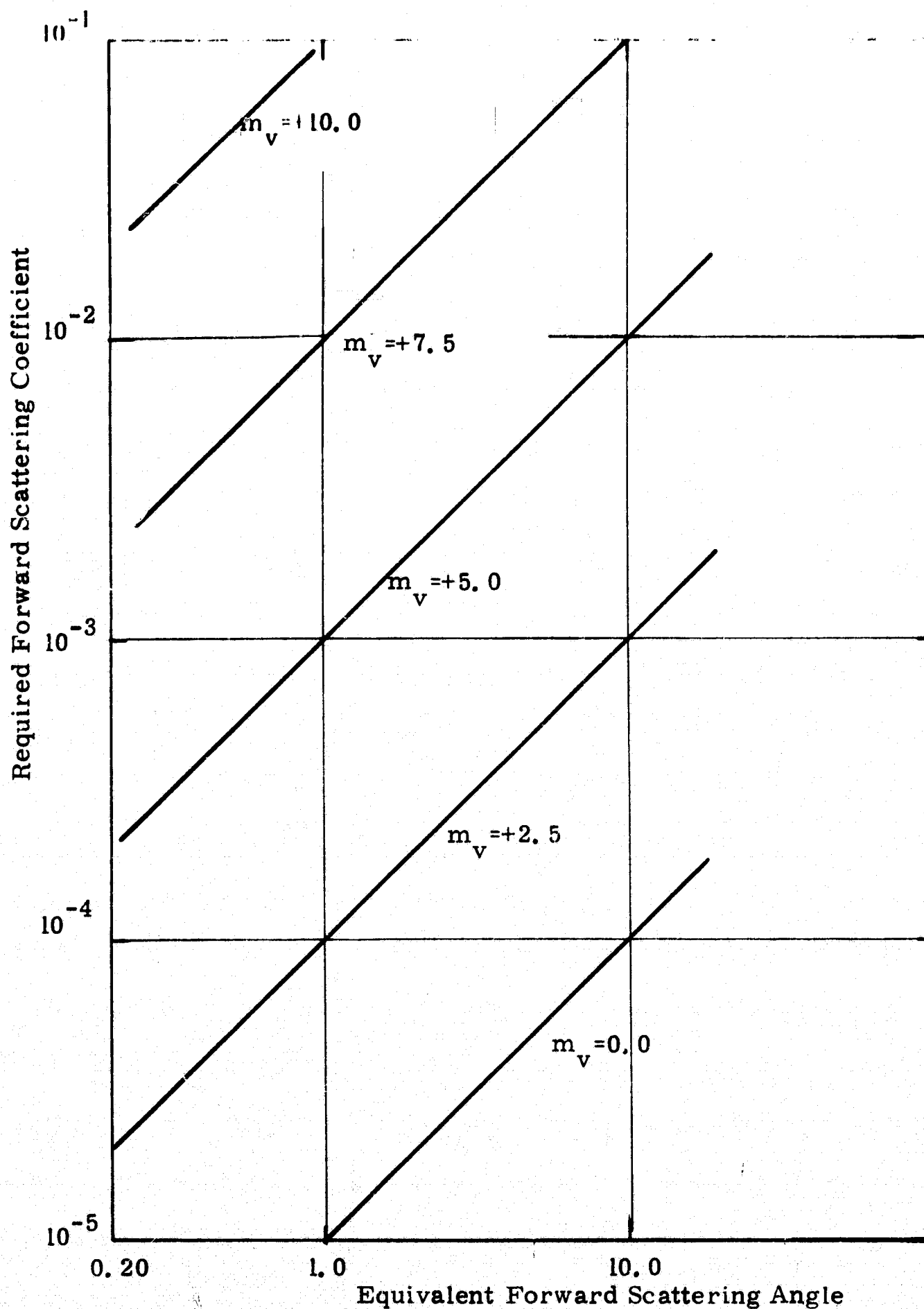
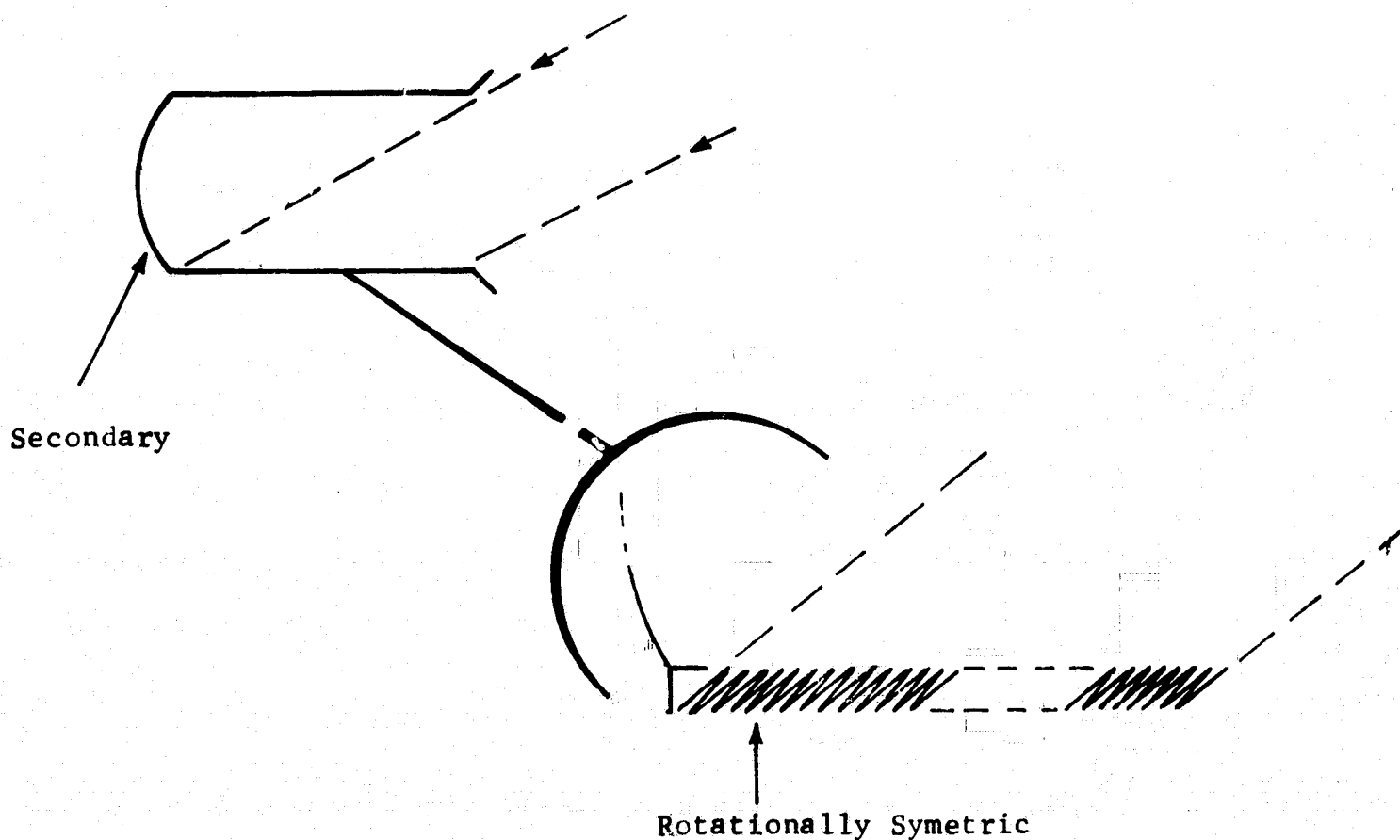


Figure G-1. Limiting Scattering Coefficient

The basic idea is to use an array of baffles aligned approximately parallel to the beam.

For example on the secondary tube baffle



a large amount of light will fall along the dotted lines - both from stars outside the field of view of the telescope and from scatter from the primary.

To ensure a minimum of this light being reflected along some path back into the system a 'razor blade' stack could be used. Very little of this light would escape and the quantity that does will radiate into a large angle.

The optimum depth, spacing and 'sharpness' of the baffles would need to be determined from the required absorption. The angles of the baffles would be determined by the location within the system by consideration of the most probable angles of unwanted light.

DIFFRACTION

Some light from nearby bright stars will be diffracted onto the detector. Light will be diffracted by simple diffraction by all edges that can be seen looking out from within the detector aperture. Other edges may diffract light onto the detector by second order effects such as diffraction from two edges or diffraction and scattering. Such multiple processes will be ignored.

The light diffracted can usually be associated with the edge of apertures. Thus the amount of diffracted light will be reduced if the number of edges seen from the detector aperture is minimized and if the edges are as smooth as possible. It probably does not matter if the edges are sharp sheet metal or round rods as long as they can be considered black and specular reflections are ignored. The exposed edges should be smooth. The degree of smoothness needs some further study but an optically smooth surface is presumably not required because it can serve as a specular reflector. Any edge that is smooth with respect to the width of a Fresnel fringe as seen from within the detector aperture is probably smooth enough. Needless to say no small objects such as bolts should stick into the aperture. The edges should appear as gentle curves not straight sections because straight sections will diffract light very strongly in the perpendicular direction while a curved surface diffracts light more nearly isotropically.

In principle baffles could eliminate all diffraction except diffraction from the outer edge of the primary and secondary and the secondary supporting structure or spider. Light diffracted by these three edges will be considered. First we write the diffracted light for the obscured primary without a spider. The point spread function gives the diffracted light. The point spread function is

$$I_1(\alpha) = \left(\frac{2}{1-\epsilon^2} \right)^2 \left(\frac{J_1(x)}{x} - \epsilon^2 \frac{J_1(\epsilon x)}{\epsilon x} \right)^2 \quad (25)$$

where

$$k = \frac{2\pi}{\lambda} \quad (26)$$

$$x = kR\alpha$$

and $J_1(x)$ is the first order Bessel function and R is the radius of the primary. The intensity is normalized to unity for $\alpha \neq 0$. The average diffraction at large angles can be evaluated with the aid of the approximation

$$\frac{J_1(x)}{x} = \frac{1}{x} \left(\frac{2}{\pi x} \right)^{\frac{1}{2}} \cos \left(x - \frac{3}{4} \pi \right) \quad (27)$$

The fringes represented by the cosine term and the two terms in (25) are averaged out at large angles because of the spread in wavelengths. The average diffraction of large angles becomes

$$I_1(\alpha) = \frac{4(1+\epsilon)}{\pi(1-\epsilon^2)^2 (kR\alpha)^3} \quad (28)$$

Next we compute the diffraction due to one spider that supports a secondary. The diffraction for the spider is largest in the direction perpendicular to the spider. For convenience we will ignore the obscuration and let $\epsilon = 0$. The spider is a narrow straight bar of length $2R$ and width $2d$. The angle α , is perpendicular to the spider. The diffraction from the spider in this direction may be written

$$I_2(\alpha_1) = \left(\frac{4d}{\pi R} \right)^2 \text{sinc}^2(kd\alpha_1) \quad (29)$$

This may be expanded for large angles in analogy to (4) to yield

$$I_2(\alpha_1) = \frac{8}{(\pi k R \alpha_1)^2} \quad (30)$$

as the large angle diffraction. Equations (25) and (29) are normalized properly for $\epsilon = 0$ so that we can add them directly to compute the total diffracted light at large angles. The sum of (28) and (30) shows that the spider is the dominant source of diffraction in the α direction perpendicular to the spider.

If this is a problem curved spiders might be used to cause the spider diffraction to be more nearly isotropic and fall off with the cube of the angle instead of only the square.

The ratio of primary and secondary circular apertures' diffracted light from a magnitude m star a large angle α away from a magnitude 30 star is

$$R_1(\alpha) = I_1(\alpha) (2.51)^{30-m} \quad (31)$$

at the center of the magnitude 30 star image. The corresponding diffracted light to star light ratio for one spider is

$$R_2(\alpha_1) = I_2(\alpha_1) (2.51)^{30-m} \quad (32)$$

in the direction perpendicular to the spider.

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