

GRAVITATIONAL-WAVE ASTRONOMY^{*†}WILLIAM H. PRESS[‡] and KIP S. THORNE

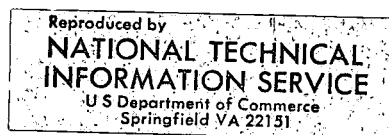
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1. INTRODUCTION

The "windows" of observational astronomy have become broader. They now include -- along with photons from many decades of the electromagnetic spectrum -- extraterrestrial "artifacts" of other sorts: cosmic rays, meteorites, particles from the solar wind, samples of the lunar surface, and neutrinos. With gravitational-wave astronomy, we are on the threshold -- or just beyond the threshold -- of adding another window; it is a particularly important window because it will allow us to observe phenomena which cannot be studied adequately by other means: gravitational collapse, the interiors of supernovae, black holes, short-period binaries, and perhaps new details of pulsar structure. There is the further possibility that gravitational-wave astronomy will reveal entirely new phenomena -- or familiar phenomena in unfamiliar guise -- in trying to explain the observations of Joseph Weber.

The future of gravitational-wave astronomy looks bright whether or not Weber (1969; 1970a,b,c; 1971a,b) is actually detecting gravitational radiation. If Weber's events are indeed produced by gravitational waves, then activity in the coming decade will focus on measurements of the polarization, the spectrum, and the wave-form of those waves -- and on theoretical attempts

to explain their source. If Weber's events are not gravitational waves, their explanation may be astronomically interesting in its own right, and they at least will have helped generate enough gravitational-wave technology to bring waves from well-understood sources within experimental reach by 1980.

We (the authors) find Weber's experimental evidence for gravitational waves fairly convincing. But we also recognize that there are as yet no plausible theoretical explanations of the waves' source and observed strength. Thus, we feel we must protect this review against being made irrelevant by a possible "disproof" of Weber's results. We have done this by relegating to the end of the article (§6) all ideas, issues, and discussion which hinge upon Weber's observations.

2. PROPERTIES OF GRAVITATIONAL WAVES

Physical reality of waves.-- Einstein's theory of gravity ("general relativity") predicts, unequivocally, that gravitational waves must exist; that they must be generated by any nonspherical, dynamically changing system; that they must produce radiation-reaction forces in their source; that those radiation-reaction forces must always extract energy from the source; that

the waves must carry off energy at the same rate as they extract it; and that the energy in the waves can be redeposited in matter (e.g., in gravitational-wave antennas). (For detailed mathematical derivations of these predictions see, e.g., Misner, Thorne, and Wheeler (1972), hereafter denoted "MTW".)

Regretably, there was an era (1925-1955) when many relativity theorists doubted whether general relativity actually made these predictions. But those doubts, one now realizes, had no foundation. They were generated by defective viewpoints and analyses. Not only does Einstein's theory of gravity predict the existence of gravitational waves; so does the theory of Brans and Dicke (1961) and its generalizations [cf. Morganstern (1967), Morganstern and Chiu (1967), O'Connell and Salmona (1967), Wagoner (1971)], and so does every other theory of gravity which today is experimentally viable. [For discussions of currently viable theories see Thorne, Will, and Ni (1971), Ni (1972a), and Nordtvedt and Will (1972).] Moreover, it appears likely -- though it is unproved as yet -- that the power and spectrum of the gravitational waves emitted by any nonspherical source are theory-independent, in order of magnitude. [See, e.g., Trautman (1965).] The strength of the

waves is probably fixed by the local validity of special relativity, by the nature of gravity in the Newtonian limit, and by theory-independent principles of physics (conservation of total energy, etc.). For perfectly spherical sources, some theories -- those with a scalar gravitational field -- allow monopole radiation, which is forbidden in (purely tensor!) general relativity. However, the strength of the monopole waves is comparable to the strength of the quadrupole waves that the same source would emit in general relativity -- if it were made somewhat nonspherical (Ni 1972b; Morganstern and Chiu 1967).

The detailed formulas and numbers given in this article will be based on the predictions of general relativity.

What is a gravitational wave? -- The answer can be given clearly and quantitatively without any appeal to the formalism of general relativity.

In Newtonian theory, the gravitational field is fully described by the gravitational potential ϕ . In the neighborhood of some fiducial point (e.g., the center of mass of a gravitational-wave receiving antenna), the potential can be expanded in a power series,

$$\phi(\underline{x}) = \phi_0 - \sum_j g_j x_j + \sum_{j,k} \frac{1}{2} R_{j0k0} x_j x_k + \dots \quad 1.$$

Here x_j are the components of the vector $\underline{x}$ from the fiducial point to the

measuring point; the numbers g_j are the components of the "local acceleration of gravity," and the numbers R_{j0k0} measure the inhomogeneity in the gravitational field at the fiducial point. In the language of Einstein, R_{j0k0} are components of the "Riemann curvature tensor." (Actually there are additional components, corresponding to indices other than zero in the second and fourth positions of R_{j0k0} ; but they will be ignored in this review article.) In the language of Newton, R_{j0k0} are second derivatives of the potential ϕ ,

$$R_{j0k0} = \partial^2 \phi / \partial x_j \partial x_k \quad 2.$$

The gravitational force which acts on a mass m at location $\underline{x}$ is given by

$\underline{F} = - \underline{\nabla} \phi$ and has the components

$$F_j = - m \partial \phi / \partial x_j = m g_j - \sum_k m R_{j0k0} x_k \quad 3.$$

Notice that the force $-\sum_k m R_{j0k0} x_k$ depends linearly on the mass position $\underline{x}$. It is a "relative force" (sometimes also called a "tidal force" or "stress") between the position $\underline{x}$ and the fiducial point. This relative force is responsible for the ocean tides (relative to their pull on the earth, the moon and sun pull harder on near oceans, weaker on far oceans, making two tidal bulges); it is also responsible for the general precession

of the equinoxes (moon and sun pull harder on that part of the earth's equatorial bulge nearest them than on that part farthest away; this causes a torque which precesses the earth's rotation axis).

Gravitational waves can be thought of as a "field of (relative) gravitational forces which propagate with the speed of light." They are a contribution to R_{j0k0} of which Newton was unaware, and which can be added straightforwardly to the Newtonian contribution (at least in nearly Newtonian regions of spacetime like the solar system):

$$R_{j0k0} = \partial^2 \Phi / \partial x_j \partial x_k + R_{j0k0}^{(GW)} \quad 4.$$

Einstein's theory dictates the form of $R_{j0k0}^{(GW)}$. For example, a (locally) plane gravitational wave propagating in the z direction has

$$R_{x0x0}^{(GW)} = - R_{y0y0}^{(GW)} = - \frac{1}{2} \ddot{h}_+ (t - z/c)$$

$$R_{x0y0}^{(GW)} = R_{y0x0}^{(GW)} = - \frac{1}{2} \ddot{h}_\times (t - z/c)$$

all other components vanish 5.

Here h_+ and $h_\times$ are arbitrary dimensionless functions, which represent the momentary amplitude¹ of the wave in the two orthogonal polarizations "+" and "×"; dots denote derivatives with respect to t ; and c is the speed of

In general relativity h_+ and $h_\times$ are the magnitude of the perturbations in the metric tensor $g_{\mu\nu} = \text{diag}(-1, 1, 1, 1) + h_{\mu\nu}$. (See, e.g., MTW where h_+ and $h_\times$ are denoted A_+ and $A_\times$.) This fact motivates the notation, but need not concern us here.

light. Notice that the relative forces, $F_j = - \sum_k m R_{j0k0} x_k$, are entirely perpendicular to the propagation (z) direction. In this sense, gravitational waves, like electromagnetic waves, are transverse. Figure 1 represents the relative forces of a gravitational wave by a line-of-force diagram. An object placed in this force field will experience time varying stresses due to the wave's relative gravitational forces, and those stresses will produce mechanical strains. This is the essence of the interaction of the wave with matter. We shall see below that the magnitude of the strain produced is typically of the order of the dimensionless wave amplitude h.

Energy carried by waves. -- Like electromagnetic waves, gravitational waves carry energy with the speed of light [(energy flux) = (energy density) $\times$ (speed of light)]. For a gravitational wave the energy flux is well defined when one averages over several wavelengths, but one cannot say unambiguously whether the energy is located in the "trough" of the wave or in its "crest" (Isaacson 1968). The energy flux, expressed in terms of the amplitude and an average " $\langle \rangle$ " over several wavelengths is (Isaacson 1968; MTW)

$$\mathcal{F} = \frac{c^3}{16\pi G} \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle = \frac{L_0}{16\pi} \left\langle \left(\frac{1}{c} \dot{h}_+ \right)^2 + \left(\frac{1}{c} \dot{h}_\times \right)^2 \right\rangle \quad 6,$$

where G is Newton's gravitation constant and L_0 is a natural unit for power in gravitation theory:

$$L_0 \equiv c^5/G = 3.63 \times 10^{59} \text{ erg/sec} = 2.03 \times 10^5 M_\odot c^2/\text{sec} \quad 7.$$

This energy flux has all the properties one would expect from experience with electromagnetic theory: It is conserved (amplitude dies as $1/r$, flux as $1/r^2$ when one recedes from source); it can be deposited in detectors; and it acts as a source for gravitation (e.g., it helps produce the cosmological curvature of the universe). For further details see Isaacson or MTW.

Propagation of waves. -- Once emitted, a gravitational wave propagates, virtually unimpeded, forever. It is harder to stop than a neutrino! The only significant modifications in the wave as it propagates are redshifts (doppler, gravitational, and cosmological; identical to those for an electromagnetic wave), and decrease in amplitude due to "inverse-square-law" spreading of wave fronts (also identical to electromagnetic case). Other modifications (dispersion; backscatter; tails; etc.) occur in principle but are negligible except near highly relativistic sources.

3. GENERATION OF GRAVITATIONAL WAVES

Fundamental regimes. -- In analyzing a source of gravitational waves, two issues are important: (i) Is the source slowly changing or rapidly changing? "Slow change" (or "slow motion") means that the reduced wavelength $\lambda = \lambda/2\pi$ of typical waves produced by the source is much larger than the size of the source, $\lambda \gg L$ -- i.e., that the source lies deep inside the near (induction) zone of its own fields. This is typically (but not always) true if the characteristic internal velocities of the source (relative to its center of mass) are much less than the speed of light, $\bar{v} \ll c$. "Fast motion" or "rapid change" means that the source lies partly in its own wave zone $\lambda \lesssim L$; this is necessarily true if $\bar{v} \sim c$. (ii) Are the gravitational fields inside the source weak or strong? "Weak" means size of source large compared to Schwarzschild radius, $L \gg 2GM/c^2 \approx 3 \text{ km } (M/M_\odot)$; "strong" means $L \sim 2GM/c^2$.

Slowly changing sources. -- If $\lambda \gg L$, a set of simple formulas describes the emission process. These formulas apply to strong-field sources as well as to weak-field sources [cf. § 10⁴ of Landau and Lifshitz (1962), or §§ 36.9 and 36.10 of MTW]. The simplicity of the radiation theory for $\lambda \gg L$ arises from the fact that, like electromagnetic radiation, gravita-

tional radiation admits the "poor-antenna" or "lowest-multipole" approximation. (In electromagnetism this is also called the "dipole approximation.")

A source much smaller than a wavelength is a very inefficient radiator, and (aside from fractional corrections of order $[L/\lambda]^2$) emits only radiation in the lowest allowed multipole. For gravitational radiation this is quadrupole radiation, and the radiation from slowly changing sources is completely

determined by the time evolution of its "reduced quadrupole-moment tensor" I_{jk} . For sources with weak fields (e.g., the solar system but not pulsars),

I_{jk} has the familiar form

$$I_{jk} = \left(\begin{array}{c} \text{trace-free part} \\ \text{of moment of inertia} \end{array} \right) = \int \rho x_j x_k d^3x - \frac{1}{3} \delta_{jk} \int \rho r^2 d^3x \quad 8.$$

For sources with strong fields I_{jk} cannot be calculated this way except in rough order of magnitude. Instead, it is operationally defined by an examination of the Newtonian potential ϕ outside the source (at $r > L$ and $r \gg GM/c^2$), but in the rear zone ($r \ll \lambda$). An accurate calculation requires general relativity [see, e.g., Ipser (1970) who treats the case of a rotating, deformed neutron star -- i.e., a pulsar].

In terms of I_{jk} , however calculated, the total power radiated in quadru-

pole waves by a slowly changing source is

$$L_{GW} = \frac{G}{c^5} \frac{1}{5} \sum_{j,k} \langle (\ddot{\mathbb{I}}_{jk})^2 \rangle \sim L_0 (2GM_{\text{eff}}/c^2 L)^2 (L/\lambda)^6 \quad 9.$$

$$\sim L_0 (2GM_{\text{eff}}/c^2 L)^2 (\bar{v}/c)^6 \sim (L_{\text{internal}}) \cdot (L_{\text{internal}}/L_0)$$

Here M_{eff} is the "effective mass" in the changing quadrupole moment, defined by (amplitude of changes in $\mathbb{I}_{jk}$) = $M_{\text{eff}} L^2$; $\bar{v} = cL/\lambda$ is the characteristic internal velocity, and L_{internal} is the "internal power flow" associated with the quadrupole motions

$$L_{\text{internal}} = (\frac{1}{2} M_{\text{eff}} \bar{v}^2) (L/\bar{v}) \quad 10.$$

The power is radiated in a typical quadrupole pattern (amplitude a quadratic function of angle; roughly isotropic). More particularly, the flux emitted in a given direction (unit vector n_j) is

$$\mathcal{F} = \frac{G}{c^5} \frac{1}{8\pi r^2} \sum_{j,k} \langle (\ddot{\mathbb{I}}_{jk}^{\text{TT}})^2 \rangle_{\text{ret}} \quad 11.$$

where "ret" means "evaluated at retarded time (t-r), and $\mathbb{I}_{jk}^{\text{TT}}$ is the "transverse traceless part of $\mathbb{I}_{jk}$ ":

$$\mathbb{I}_{jk}^{\text{TT}} \equiv \sum_{\ell,m} (\delta_{j\ell} - n_j n_\ell) \mathbb{I}_{\ell m} (\delta_{mk} - n_m n_k) \quad 12.$$

The field of relative forces, R_{j0k0} , produced by the waves is

$$R_{j0k0} = - \frac{G}{c^4} \frac{1}{r} \left[\frac{d^4 I_{jk}}{dt^4} \right]_{\text{ret}} \quad 13.$$

corresponding to a dimensionless amplitude with order-of-magnitude

$$h_{+ \text{ or } \times} \sim \left(\frac{GM_{\text{eff}}/c^2}{r} \right) \left(\frac{v}{c} \right)^2 \approx 10^{-16} \left(\frac{M_{\text{eff}}}{M_{\odot}} \right) \left(\frac{v}{c} \right)^2 \left(\frac{1 \text{ kpc}}{r} \right) \quad 14.$$

Rapidly changing, weak-field sources.-- When $L \gtrsim \lambda$, quadrupole radiation does not generally dominate over radiation of octupole and higher order, so the above formulas cannot be used. Instead, one must use the full formalism of general relativity, or else the "linearized theory" (linear approximation to general relativity).

Only a few rapidly changing, weak-field sources have yet been analyzed in the literature. One is the small-angle "Coulomb scattering" of a rapidly moving, light star by a heavy star (Peters 1970). During the encounter and slight deflection, the light star emits "gravitational bremsstrahlung" radiation. For stellar velocities near the speed of light, the radiation is strongly peaked in the direction of the star's motion [(half-angle) $\sim (1 - v^2)^{\frac{1}{2}}$]. A second example (Peters 1972) treats masses in close orbits, but the attractive force between them must be non-gravitational. (If it

were gravitational it would be "strong" and the weak-field limit would not apply). Here there is also a forward beaming of the radiation.

Rapidly changing, strong-field sources. -- (Examples: the fall of matter down a black hole; neutron stars in close orbits at relativistic velocities.) For these cases there is no standard technique of analysis. The slow-motion formalism is invalid -- though one hopes that, with an ad hoc "cutoff" of radiation at the Schwarzschild radius, it will give a rough indication of the energy, spectrum, and duration of the waves (see e.g., Ruffini and Wheeler 1971). Linearized theory is also invalid -- but is also often used, with cutoff, to get rough estimates. The only fully reliable calculations yet performed for rapidly changing, strong-field sources are calculations of small perturbations about stationary equilibrium configurations: small-amplitude pulsations of fully relativistic neutron stars (Thorne 1969); the gravitational collapse of an object, with small non-spherical perturbations, to form a black hole (de la Cruz, Chase, and Israel 1970; Price 1972a,b); the fall of a small object down a much larger black hole (Zerilli 1970; Davis, Ruffini, Press, and Price 1971); small objects in unbound, hyperbolic orbits near a black hole (Misner 1972). Such

calculations are often simplified -- for order-of-magnitude estimates -- by replacing the gravitational wave equations by a much simpler scalar wave equation (Christodoulou 1971, Price 1972a).

Equation (9) indicates that a rapidly changing, strong-field source will emit a far greater power in gravitational radiation than will a slowly changing or weak-field source of the same mass. The power, in order of magnitude, may be as large as the "natural" power L_0 (eq. 7), but it probably cannot become much greater than L_0 . Much effort should be put into the development of new techniques for analyzing rapidly changing, strong-field sources.

4. ASTROPHYSICAL SOURCES OF GRAVITATIONAL WAVES

This section describes the authors' theoretical estimate of the characteristics of the gravitational-wave flux at the earth. Our estimate (--- "guess" is probably a better word---) is based on a survey of the literature on theoretical analyses of astrophysical sources of gravitational waves. We advance our estimate with a full expectation that it is wrong in many, if not most respects. (One is by now accustomed to startling surprises in observational astronomy -- some more fantastic even than the wilder dreams

of theorists!') However, we feel that an estimate is needed to act as a "foil" against which to plan, design, and analyze experiments.

In our discussion of the expected radiation (this section) and of methods of detection (§§ 5-6), we shall divide the gravitational-wave spectrum into bands, ranging from the "extra-low frequency" (ELF) band of 10^{-7} to 10^{-4} Hz, up to the "very-high frequency" (VHF) band of 10^8 - 10^{11} Hz.

Table I lists the bands and their characteristics, while Table II summarizes the expected and hoped-for radiation in each band. The ideas and calculations underlying Table II are described in the text below -- beginning with sources which certainly exist, and working down to sources which could exist but seem unlikely.

A. Sources known to Exist

Nuclear bomb explosions and other terrestrial sources. With the possible exception of highly sophisticated nuclear explosions at very close range (Wood et al. 1971), and the barely-conceivable exception of certain laser-like devices (Nagibarov and Kopvillem 1967a,b, 1969; Braginskii and Rudenko 1970), all terrestrial sources of gravitational waves are far too weak for any detector which has yet been invented. (See Weber 1961; Ruffini and Wheeler 1971; MTW.)

Binary star systems. -- All known binary star systems have periods

longer than one hour, corresponding to $\lambda/L \sim (c^2 L/GM)^{\frac{1}{2}} \gtrsim 10^3$. Thus, they

change so slowly and have such weak internal fields that to high accuracy

one can analyze them using equations (8)-(14). Such an analysis (Peters and

Mathews 1963) predicts a power output of

$$L = \frac{32}{5} \left(\frac{G^5}{c^{10}} \frac{\mu^2 M^3}{a^5} \right) f(e) L_0 = \left(3.0 \times 10^{33} \frac{\text{erg}}{\text{sec}} \right) \left(\frac{\mu}{M_\odot} \right)^2 \left(\frac{M}{M_\odot} \right)^{4/3} \left(\frac{P}{1 \text{ hour}} \right)^{-10/3} f(e) \quad 15.$$

Here M and μ are the total and reduced masses of the system

$$M = m_1 + m_2, \quad \mu = m_1 m_2 / M; \quad 16.$$

a is the orbit's semi-major axis; P is the period; and $f(e)$ is the following

function of orbital eccentricity

$$f(e) = (1 + \frac{73}{24} e^2 + \frac{37}{96} e^4) / (1 - e^2)^{7/2} \quad 17.$$

The radiation is emitted at a "fundamental" frequency equal to twice the orbital

frequency, and at harmonics of the fundamental up to order ~ 3 for $e = 0.5$ and ~ 10

for $e = 0.7$. The radiation is strongest at periastron, and thus radiation

reaction tends to circularize the orbit. If gravitational radiation is the

dominant force changing the orbital period, and if the orbit is nearly

circular, then the orbital period will decrease at the rate

$$\frac{1}{P} \frac{dP}{dt} = - \frac{96}{5} \frac{G^3}{c^5} \frac{\mu M^2}{a^4} = \left(\frac{1}{2.8 \times 10^7 \text{ yr}} \right) \left(\frac{M}{M_{\odot}} \right)^{2/3} \left(\frac{\mu}{M_{\odot}} \right) \left(\frac{1 \text{ hr}}{P} \right)^{8/3} \quad 18.$$

However, the problem for short-period binaries is more complex: As the orbit shrinks by radiation reaction, one star may encroach on the other's Roche surface, leading to a mass transfer from one star to the other, which can markedly effect the evolution of the system (Faulkner 1971, Vila 1971). There may also be mass loss to infinity.

As received on earth, the energy flux and dimensionless amplitude of the waves from a binary system are

$$\mathcal{F} = \left(2.6 \times 10^{-9} \frac{\text{erg}}{\text{cm}^2 \text{ sec}} \right) \left(\frac{\mu}{M_{\odot}} \right)^2 \left(\frac{M}{M_{\odot}} \right)^{4/3} \left(\frac{P}{1 \text{ hour}} \right)^{-10/3} \left(\frac{r}{100 \text{ pc}} \right)^{-2} f(e) \quad 19.$$

$$h = \left[(h_{+ \text{ max}})^2 + (h_{\times \text{ max}})^2 \right]^{\frac{1}{2}}$$

$$= 1.4 \times 10^{-20} \left(\frac{\mu}{M_{\odot}} \right) \left(\frac{M}{M_{\odot}} \right)^{2/3} \left(\frac{P}{1 \text{ hour}} \right)^{-2/3} \left(\frac{r}{100 \text{ pc}} \right)^{-1} f(e) \quad 20.$$

Braginski (1965) and Ruffini and Wheeler (1971, p. 128) have compiled small, incomplete lists of spectroscopic binaries which emit strongly; but no one has attempted a thorough compilation. The most powerful emitters in the lists have orbital periods $P \sim 8$ hours and produce fluxes at Earth of

$\mathcal{F} \approx 10^{-12}$ to 10^{-10} erg/cm² sec, corresponding to amplitudes h of 10^{-22} to 10^{-21} . Mironovski (1965) has calculated the total flux bathing Earth from all binary stars with $P \gtrsim 1$ hour. Assuming that the Galaxy contains $\sim 2 \times 10^7$ WUMa-type binaries, he finds $\mathcal{F}_{\text{total}} \sim 10^{-7}$ ergs/cm² sec, with a spectrum peaked at a wave period of about 4 hours. Binary stars with periods shorter than 1 hour will be destroyed so quickly by fusion and/or radiation damping that (i) the failure of astronomers to find any such systems is not surprising, and (ii) one cannot with any confidence expect even a single binary star with $P < 1$ hour, close enough to produce $\mathcal{F} > 10^{-10}$ erg/cm² sec.

Pulsars. -- To a high degree of precision, one expects the neutron stars in pulsars to be symmetric about their rotation axes. This is unfortunate, because only deformations from axial symmetry can produce a time-changing quadrupole moment and thereby radiate gravitational waves. Ipser (1970) presents a detailed mathematical treatment of the radiation produced by a given deformation; but for our purposes order-of-magnitude estimates will suffice. [These estimates are due to Melosh (1969), Ostriker and Gunn (1969), Ferrari and Ruffini (1969), Shklovski (1969).] If one

idealizes the neutron star as a slightly deformed, homogeneous sphere with moment of inertia I , rotation period P , and ellipticity

$$\epsilon \equiv \frac{e^2}{2} = \frac{(\text{difference in two equatorial radii})}{(\text{mean equatorial radius})}$$

one obtains for the power radiated

$$L = \frac{32}{5} \frac{G}{c} I^2 \epsilon^2 \left(\frac{2\pi}{P}\right)^6 \sim \left(10^{38} \frac{\text{erg}}{\text{sec}}\right) \left(\frac{I}{4 \times 10^{44} \text{ g cm}^2}\right)^2 \left(\frac{P}{0.033 \text{ sec}}\right)^{-6} \left(\frac{\epsilon}{10^{-3}}\right)^2 \quad 21.$$

By far the most promising pulsar is NP0532 (the pulsar in the Crab nebula);

it has the shortest period (0.033 sec) and is the most likely to be deformed.

The crucial issue is the magnitude of the non-axial deformation ϵ . An upper

limit of $\epsilon < 10^{-3}$ comes from the demand that gravitational radiation reaction

brake the star's rotation more strongly than the observed braking. A lower

limit of $\epsilon \gtrsim 10^{-11}$ comes from the deformation due to poloidal magnetic pressure but note the error in equation (4) of Melosh and hence in his numerical results). (Melosh 1969; Theoretical analyses of the strength of a neutron-star crust,

and the theoretical interpretation of jitter and glitches in the period of

NP0532 as due to starquakes, suggest an ellipticity in the equatorial plane

of $\epsilon \sim 10^{-5}$ to 10^{-7} [cf. Ruderman (1969), Baym and Pines (1971), Pines and

Shaham (1972)]. The corresponding values of flux and amplitude at Earth are

$$\mathcal{F} \lesssim 3 \times 10^{-7} \frac{\text{erg}}{\text{cm}^2 \text{ sec}},$$

$$h \lesssim 0.7 \times 10^{-24} \text{ (slowdown rate)}$$

$$\mathcal{F} \sim 3 \times 10^{-11} \text{ to } 3 \times 10^{-15} \frac{\text{erg}}{\text{cm}^2 \text{ sec}}, \quad h \sim 10^{-26} \text{ to } 10^{-28} \text{ (crystal strength and starquakes)}$$

$$\mathcal{F} \gtrsim 3 \times 10^{-23} \frac{\text{erg}}{\text{cm}^2 \text{ sec}}, \quad h \gtrsim 0.7 \times 10^{-32} \text{ (magnetic pressure)} \quad 22.$$

Because the luminosity varies as P^{-6} , the gravitational waves from other known pulsars should be at least ~ 400 times weaker (in flux $\mathcal{F}$) than those from the

Crab. Correspondingly, a "newborn" neutron star will emit much more strongly:

At a time t after its birth, its gravitational wave luminosity is roughly estimated by

$$L \sim (10^{45} \text{ erg/sec}) \left(\frac{4 \times 10^{44} \text{ g cm}^2}{I} \right)^{1/2} \left(\frac{10^{-3}}{\epsilon} \right) \left(\frac{10^6 \text{ sec}}{t + 10^4 \text{ sec}} \right)^{3/2} \quad 23.$$

(cf. Ostriker and Gunn 1969). This estimate begins to fail for $t \gtrsim 10$ years as electromagnetic braking processes become important. Note that

$L \gtrsim 10^{45} \text{ erg/sec}$ holds for days after formation. For pulsars in our galaxy

(distance $\sim$ few kpc) this corresponds to $\mathcal{F} \sim 1 \text{ erg/cm}^2 \text{ sec}$, $h \sim 10^{-22}$. In

the Virgo cluster neutron stars should be born about once each month,

giving $\mathcal{F} \sim 10^{-6} \text{ erg/cm}^2 \text{ sec}$, $h \sim 10^{-25}$.

Supernovae and the birth of neutron stars. -- Some, if not all, supernovae

produce rotating neutron stars (pulsars). The gravitational binding energies of rapidly rotating neutron stars are typically in the range 0.01 to $0.3 M_{\odot} c^2$ (Hartle and Thorne 1968; Baym, Pethick, and Sutherland 1971). A sizable fraction of this binding energy is probably emitted as gravitational waves during and shortly after the collapse which triggers the supernova. Ruffini and Wheeler (1971, pp. 127-140) list a variety of processes which might contribute to the radiation: (i) initial asymmetric implosion of the stellar core if asymmetric; (ii) possible fragmentation of the core into several large "chunks", due to its rapid rotation and high degree of flattening; (iii) the orbital chase of chunk around chunk; (iv) the collision and coalescence of chunks as the angular momentum of the system is carried away by gravitational waves; (v) the birth of neutron stars out of core or chunks. In its first seconds a neutron star could be in a non-axisymmetrical Jacobi-ellipsoid type configuration with $\epsilon \sim 1/2$, period $P \sim 1$ msec, and gravitational luminosity $\sim 10^{51}$ erg/sec [Ruffini and Wheeler (1971) p. 146; for detailed treatment of radiation from Jacobi ellipsoids, see Chandrasekhar (1970a,b,c)]. Its pulsations might also generate significant radiation (Chau 1967, Thorne 1969). Whatever the processes which actually occur, the waves will probably come off in several

broad-band bursts with frequency $\nu \sim 10^3$ Hz to 10^4 Hz, with duration for each burst $\sim 10^{-3}$ sec to 1 sec, and with total duration for the entire process of a few seconds. (The reason for the short duration is the high effectiveness of radiation-reaction forces for a system so near its Schwarzschild radius.) If the endproduct of the stellar collapse is a black hole rather than a neutron star, the radiation emitted will be similar. Note that for a burst of frequency $\sim 10^3$ Hz, which carries off Mc^2 of energy in a time interval Δt , the flux and amplitude at Earth will be

$$\mathcal{F} \sim \left(0.5 \times 10^8 \frac{\text{ergs}}{\text{cm}^2 \text{ sec}} \right) \left(\frac{M}{0.03 M_{\odot}} \right) \left(\frac{0.1 \text{ sec}}{\Delta t} \right) \left(\frac{10^4 \text{ pc}}{r} \right)^2$$

$$\langle h \rangle_{\text{rms}} \sim \left(0.5 \times 10^{-18} \right) \left(\frac{M}{0.03 M_{\odot}} \right)^{1/2} \left(\frac{0.1 \text{ sec}}{\Delta t} \right)^{1/2} \left(\frac{10^4 \text{ pc}}{r} \right)$$
24.

Once a neutron star has been formed, its rotation can produce gravitational waves of gradually increasing period and decreasing amplitude (see previous section).

Explosions in quasars and nuclei of galaxies. -- For a (nonspherical!) explosion of energy E and characteristic duration τ , equation (9) predicts the gravitational-wave luminosity

$$L \sim (1/L_0) (E^2/\tau^2) \quad 25.$$

(As before $L_0 = c^5/G = 3.63 \times 10^{59}$ erg/sec.) Ozernoi (1965)-- using a more elaborate model than our rough order-of-magnitude formula -- conceives of quasar explosions with $E \sim 10^{59}$ ergs, $\tau \sim 10^8$ sec, and a resulting gravitational-wave luminosity $L \sim 10^{45}$ ergs/sec. For explosions in the nuclei of galaxies (e.g. M82) he takes $E = 10^{55}$ ergs, $\tau = 10^8$ sec and obtains $L \sim 10^{37}$ ergs/sec. Given that our present theoretical understanding of quasars and galactic nuclei is essentially nil, these estimates must be considered as only suggestive. On the other hand, the observational evidence for "explosions" on galactic scales seems uncontestable.

Atomic and molecular processes. -- The interactions of particles, atoms, and molecules generate gravitons by processes qualitatively the same as those which generate photons. Unfortunately, photon processes typically dominate by a ratio $\sim Gm^2/e^2 \sim 10^{-40}$; thus 10^{40} photons are produced for each graviton. (Of course, this is not true for the "classical" gravitons generated by the bulk motion of electrically neutral matter.) If they are of no practical interest, microscopic gravitational interactions are nonetheless fascinating

in principle: For analyses of thermal bremsstrahlung from a hot gas see Weinberg (1965), Mironovski (1965), Carmelli (1967), Barker, Gupta, and Kashkas (1969); for gravitational waves from lattice vibrations in solids see Halpern (1969); for gravitational waves from particle-antiparticle annihilation see Ivanenko and Sokolov (1947, 1952), and Ivanenko and Brodski (1953); for gravitational synchrotron radiation from charged particles spiralling in magnetic fields, see Pustovoit and Gertsenshtein (1962). It is possible that microscopic interactions might someday be useful in detecting supra-VHF (e.g. optical-frequency) gravitons, if any could be generated. A transition stimulated by a graviton (in rotational levels of a molecule, say) might be followed by an electromagnetic transition and by detection of the resultant photon. (See Nagibarov and Kopvillem 1967a,b, 1969; Braginskii and Rudenko 1970.)

B. Sources Which Probably Exist

Stellar collapse with little optical display. -- When one tries to build computer models of supernovae triggered by stellar collapse, one often achieves collapse without producing a supernova-type optical "display" [see, e.g., Arnett (1969); Wilson (1969)]. It is quite possible that

stellar collapse without brilliant optical display is more common than supernova explosions. Assuming that the distribution of stellar masses is the same throughout the Universe as in the solar neighborhood, and ignoring the effects of mass ejection in late stages of stellar evolution, one obtains (Zel'dovich and Novikov 1971, §13.13) an upper limit of 7 stellar collapses per galaxy per year. In the nuclei of galaxies, where conditions are quite different, the frequency of collapse might be higher than this. Each stellar collapse will produce bursts of gravitational waves similar to those from supernovae -- though in the case of a massive star ($M \gtrsim 20 M_{\odot}$) the energy output might be several solar rest masses rather than several tenths. Once a black hole has formed, it can swallow surrounding matter, emitting a chirp of gravitational radiation each time it does so (Zel'dovich and Novikov 1964; Davis, Ruffini, Press, and Price 1971). But black holes produced by normal stars (mass $< 100 M_{\odot}$) are so small (< 300 km) that, before they can swallow an object, they must

break it up into "bite-sized" pieces. As a result, the radiation from each swallow should be far less than from the original collapse.

Condensation of galaxies. -- Ruffini and Wheeler (1971, p. 141) have made a rough estimate of the gravitational waves generated when galaxies condensed out of the expanding primordial gas:

$$\lambda \sim 10^{23} \text{ cm}, \quad \mathcal{F} < 10^{-2} \text{ erg/cm}^2 \text{ sec}, \quad h < 1 \times 10^{-7} \quad 26.$$

The flux and amplitude might be considerably less than these limits. Note that over a human lifetime these gravitational "waves" will be essentially static, a constant gravitational stress-field.

Primordial gravitational radiation. -- In the earliest stages of the universe gravitational radiation may have been in thermal equilibrium with other forms of matter and energy. Thus one might expect a cosmological black-body spectrum of gravitons like the 3 °K photon background. Unfortunately, as Matzner (1968) has pointed out, the current temperature of the graviton background should be much less than that of the photon background:

$$T_{\text{grav}} \sim T_{\text{photon}} (2/N)^{1/3} \lesssim 1.6 \text{ } ^\circ\text{K} \quad 27.$$

Here N is the number of modes (including, e.g., particle-antiparticle pairs),

which were in equipartition at the time the gravitons decoupled, but which decayed to photons in the subsequent expansion. If all known particles were in equilibrium, then N is $\gtrsim 10^2$ to 10^4 ; Matzner's lower limit is $N \gtrsim 16$, derived from the number of quark states. A thermal graviton background of this type is certainly undetectable with current or foreseeable technology.

It is conceivable, however, that the Universe began sufficiently chaotic that there were large-amplitude modes of gravitational waves which never became thermalized. [Cf., e.g., Misner (1969); Zel'dovich and Novikov (1972).] It seems likely that any such waves will by now have suffered such great redshifts that they are undetectable and play no significant role in the Universe (cf. Ruffini and Wheeler, p. 143). But one is so ignorant of conditions in the initial big-bang that it is dangerous to claim any firm conclusions.

C. Sources Which Might Exist

Huge black holes in nuclei of galaxies. -- Lynden-Bell (1969) has suggested that violent activity in the nuclei of galaxies may produce (--- or may be produced by ---) huge black holes, which subsequently accrete

matter from their surroundings. In particular (Lynden-Bell 1969, Lynden-Bell and Rees 1971) our own galaxy might contain a black-hole nucleus of $\sim 10^4$ to $10^8 M_{\odot}$. As any object falls into such a black hole, it will emit a burst of gravitational radiation. For simple radial infall into a non-rotating black hole, the total energy radiated is

$$E^{GW} = 0.01 (m/M) mc^2 = (10^{44} \text{ ergs})(m/M_{\odot})^2 (M/10^8 M_{\odot})^{-1} \quad 28.$$

where m is the mass of the infalling object and M is the mass of the hole (Davis, Ruffini, Press, and Price 1971, Zerilli 1970). If the fall is non-radial or the hole is rotating (Bardeen 1970), the numerical constant 0.01 is probably somewhat larger, but the dependence on m and M is probably the same. The duration of the burst emitted during infall is $\Delta t \sim 10 \text{ GM}/c^3 \approx 10^4 \text{ sec}(M/10^8 M_{\odot})$; its frequency is probably not much higher than $1/\Delta t$; and its bandwidth is $\sim 1/\Delta t$ (Misner and Chrzanowski 1972, Bardeen et al. 1972, Ruffini et al. 1972).

These results make such a source seem fairly mundane. However Misner (1972) points out that the radiation will be quite different if somehow one can inject an object into a highly energetic trajectory (much more energetic than simple fall from infinity can provide). Then the object can emit strong,

beamed gravitational synchrotron radiation with frequency much higher than c^3/GM . (Cf. Press 1971.) Misner would like to explain Weber's observations by means of such radiation, but the model faces very serious difficulties: How can one achieve the large initial injection energy? How can one avoid difficulties with the Roche limit?

Black holes in globular clusters. -- Wyller (1970), Cameron et al. (1971), and Peebles (1972) have discussed the possibility that large black holes might be formed in globular clusters and might congregate in the centers of the clusters. Gravitational waves would result from the infall of other objects into the holes (see above), or from collisions or near encounters between the holes and between holes and stars.

Superdense clusters. -- More extreme models (motivated by Weber's observations) have been constructed by Kafka (1970) and by Bertotti and Cavalieri (1971). They imagine a very dense cluster of black holes and/or compact stars, in which near encounters occur frequently (several times per day), producing strong bursts of gravitational radiation. Of course, the model clusters are so designed that their output resembles what Weber sees. The difficulty with these models (G. Greenstein 1969) is that a

cluster dense enough for frequent collisions must evolve so rapidly that its active lifetime would be far shorter than 10^9 years. Conversely, collisions between black holes in a normal, non-relativistic cluster would be extremely rare.

When two black holes do collide -- whether in a superdense cluster or elsewhere -- they probably release a substantial fraction of their rest mass in a gravitational-wave burst of duration $\sim GM/c^3$, and of frequency $\sim$ bandwidth $\sim (\text{duration})^{-1}$, where M is the total mass of the holes. Hawking (1971) has derived an upper limit on the energy radiated: for two ^{nonrotating} black holes of equal mass m , $E_{\text{rad}} < (2 - \sqrt{2}) mc^2$.

Coherent conversion of electromagnetic waves into gravitational waves. --

Gertsenshtein (1962) and Vladimirov (1964) have pointed out that, when an electromagnetic wave propagates through a region with a static electric or magnetic field, the electromagnetic wave gets coherently (but slowly) converted into a gravitational wave. Unfortunately the effect is so weak that it is probably of no practical interest. However, if strongly charged black holes [$e \sim M$ in the notation of Christodoulou and Ruffini (1971)] can exist, despite their intense electrostatic pull on surrounding plasma, then as an

electromagnetic wave propagates outward from near the surface of the hole toward "infinity", its conversion into a gravitational wave will be near 100 per cent effective.

5. GRAVITATIONAL-WAVE RECEIVERS

We turn now from the speculative to the practical: How can gravitational waves be detected? Weber (1960, 1961), is responsible for the pioneering detection schemes, which involve vibrations of the Earth and vibrations of cylinders. More recently, since 1969, Weber's apparent success has generated vigorous activity by perhaps 15 other research groups to design new detection schemes and improve on Weber's old ones. In this section we will review the various schemes which have been proposed, we will describe their relationships to each other, and the current state-of-the-art in each, and we will speculate about the future prospects of each. As background for the discussion we will have to review a number of basic ideas, well known to the experts in the field, which do not seem to have appeared explicitly in the literature before.

A gravitational wave is in essence a propagating field of stresses.

When this field acts on a physical system ("antenna"), it produces displacements and motion; the stresses produces strains. Any device which monitors these strains we shall call a "displacement sensor". The sensor and the antenna together make up a gravitational-wave receiver.

Free-mass antennas. -- The simplest antenna for gravitational waves consists of two free masses separated by a distance ℓ_0 . Although such an antenna is not terribly practical, we shall discuss it in detail because it points the way toward more sophisticated and more practical antennas.

Locate the masses in a plane perpendicular to the direction of wave propagation; if the wave is that of equation (5), for example, the masses could be at $x = \pm \ell/2$, $y = z = 0$. Then the stresses of the wave will produce a relative motion of the masses; their separation will vary as,

$$\ell = \ell_0 + h_+(t) \ell_0 \quad 29.$$

[Eqs. (5) and (3), plus Newton's law $F = ma$]. Thus, the dimensionless wave amplitude $h_+(t)$ determines the system's strain directly:

$$\Delta\ell/\ell_0 = h_+(t) \quad 30.$$

If the masses were oriented differently, the antenna would respond to a linear combination of the two polarizations h_+ and $h_\times$ instead of purely to h_+ (see Fig. 1). If the separation were not normal to the propagation direction,

the displacement $\Delta\ell$ would be reduced by a factor $\sin^2\theta$. [See below; also Ruffini and Wheeler (1971), p. 113.] It is quite general that the dimensionless field strength $h \equiv \left[(h_+)^2 + (h_\times)^2 \right]^{1/2}$ sets the scale of the dimensionless strain $\Delta\ell/\ell$ which one must measure. In the special case of monochromatic gravitational waves (e.g. from binary stars or pulsars), one can use resonance effects and sophisticated antennas to make $\Delta\ell/\ell$ somewhat larger than h . However, for signals of wide bandwidth (e.g. for waves from any collision, collapse, or explosion; for Weber bursts; for waves of cosmological origin) $\Delta\ell/\ell$ is not much larger than h , no matter how sophisticated the antenna. We will discuss this point in detail below.

How far apart should one locate the free masses? The answer depends on how one proposes to measure their displacements; but it is generally optimal to space the masses as distant as the displacement sensor will allow, but no more than half of a wavelength of the gravitational wave. Consider, for example, two masses separated by astronomical distances (the Earth and Moon, or the Earth and a spacecraft), with displacement monitored by radar or laser techniques. If the wavelength of the gravitational wave is much larger than the separation, the analysis of equation (29) completely describe

the system, and the motions of the masses generate doppler shifts which are measurable in the ordinary way. As the size of the system approaches half a wavelength, the analysis becomes more complicated, because the gravitational wave changes appreciably during the time that photons are in transit between the masses, and cancels all except "half a wavelength's worth", or less, of their doppler shift. Thus, the magnitude of the observed displacement is typically maximal for half a wavelength separation and varies sinusoidally for larger distances. (See, e.g. Kaufmann 1970.)

For laboratory or earthbound experiments, the condition (apparatus size) $\ll$ (wavelength) is essentially automatic, since all important astrophysical sources lie in the MF band and below (wavelengths > 3 km). Henceforth we will assume tacitly that (apparatus size) $\ll$ (wavelength), unless stated otherwise.

Non-mechanical displacement sensors. -- How can one measure the separation of free masses? Over Earth-size distances and larger, the only useful techniques would appear to be radar ranging, laser ranging, and laser interferometry.

Spacecrafts are routinely tracked by radar with precision in velocity ("Doppler") of several millimeters per second and precision in distance ("range") of ~ 10 meters. Either method of tracking, range or Doppler, permits the detection of strains $h \gtrsim 10^{-11}$ in the VLF region and below ($\nu_{\text{GW}} \lesssim 10^{-2}$ Hz). However, such radiation can be ruled out on energetic grounds with fair confidence. For example, the tracking residuals reported by Anderson (1971) -- if due to gravitational waves as he suggests and we strongly doubt -- would correspond to an integrated energy flux of $\gtrsim 6 \times 10^{13}$ ergs/cm² per event (Gibbons 1971). If they were to originate in the galactic center, such waves would carry $3 \times 10^5 M_{\odot} c^2$ per event -- many orders greater than even Weber's events. (The waves could not be cosmological: their energy density would be inconsistent by many orders of magnitude with the observational limits on the Hubble constant, age, and deceleration parameter of the Universe.) Radar technology, therefore, is not a very good detection scheme -- not even with the most optimistic estimates of improvements during the coming decade.

Laser ranging via lunar reflector is now performed routinely with precision of ~ 30 cm. Such ranging can give information on waves with periods

of a few seconds, and $h \gtrsim 1 \times 10^{-9}$; but again the existence of such waves can be ruled out on energetic and cosmological grounds.

Laser interferometry is considerably more promising for experiments in near space (earth orbit) or for ground-based measurements (Moss, Miller, and Forward 1971). It is straightforward to measure displacements of one interference fringe, i.e. $\sim$ one wavelength of laser light, over moderately large distances. However this sensitivity compares poorly to other displacement sensors: for example Weber detects strains of $\sim 10^{-16}$ piezoelectrically, while 10^{16} laser wavelengths is 6×10^6 km! To be useful in gravitational-wave detection, laser interferometers must measure very small fractions of an interference fringe. The theoretical limit on interferometers of this sort is determined by photon fluctuation noise

$$\Delta l_{\min} \sim \lambda/\sqrt{N} \sim \left(1 \times 10^{-12} \frac{\text{cm}}{\text{Hz}^{1/2}}\right) \left(\frac{\lambda}{6000 \text{ \AA}}\right)^{1/2} \left(\frac{\text{laser power}}{1 \text{ mw}}\right)^{-1/2} (\text{Bandwidth})^{1/2} \quad 31.$$

where λ is the wavelength and N is the number of photons in a measurement.

This precision improves with an increase in the laser power, or with an increase in the averaging time (i.e. narrower bandwidth). The bandwidth factor suggests that laser techniques may find application to pulsar (highly monochromatic) waves in the LF band, or to VLF signals in general.

As of 1971, the limiting sensitivity (eq. 31) has been achieved experimentally in order of magnitude with laboratory-sized apparatus, fractional milliwatt lasers, and bandwidths of a few Hz [Moss, Miller, and Forward (1971) and references cited therein; see also Moss (1971)]. This corresponds to measured distances of $\sim 10^{-12}$ cm or 5×10^{-8} fringe. Such a sensitivity, if it could be achieved in earth orbit over a baseline of 10^3 km, could detect the radiation from known short-period binaries (e.g. i Boo with $h \sim 6 \times 10^{-21}$).

Almost-free antennas. -- We begin the transition to more complicated antennas with the question: How "free" must the masses be in a free-mass antenna?

Only for experiments in space can one imagine anything like ideal free masses. Otherwise, the masses must be held in place by a suspension which allows them to move in response to the wave (Fig. 2b). There may also be a mechanical connection between the masses, part of the suspension proper or part of the displacement measuring device. For example, one might place a piezoelectric rod between the masses and measure their displacement by monitoring the strain in the rod. One can analyze how the suspension and

mechanical coupling affect the antenna by studying the system's normal modes of oscillation. Some normal modes have no influence on the wave-induced displacements, so one can ignore them. [Example: the modes associated with vibrations in the x-z plane for the detector of Fig. 2b.] Compare ^{the} frequencies ν_n of the remaining modes with the characteristic frequency of ν_{GW} of the gravitational waves. If $\nu_n \ll \nu_{GW}$ for all ν_n , then the system will respond to the waves as if the masses were free. If some ν_n have $\nu_n \gg \nu_{GW}$, their modes can be treated as rigid, but the masses will be "free" in the remaining modes ($\nu_n \ll \nu_{GW}$). In practical work it is often sufficient to satisfy the inequalities by factors of 3 or 5. If there are modes for which neither inequality holds, $\nu_n \approx \nu_{GW}$, then the system is no longer "almost-free". Rather, one says that it is resonant. We will treat resonant systems below.

A promising example of an almost-free antenna is a dumbbell-shaped bar (Rasband et al. 1971) or hollow square (Douglass 1971) monitored in the frequency band between its fundamental ν_0 and its first harmonic ν_1 . (Note: for such antennas $\nu_0/\nu_1 \ll 1$.)

Mechanical dissipation in the suspension and coupling of an almost-free antenna produces thermal noise fluctuations in the distance between the

masses. If the conditions for an almost-free detector are met, so that the wave frequency ν_{GW} is not near any of the detector frequencies ν_n , then this noise fluctuation at temperature T is given roughly by

$$\Delta l_{\text{thermal}} \sim (4 \times 10^{-16} \text{ cm}) \left(\frac{10^3 \text{ Hz}}{\nu_{\text{GW}}} \right)^2 \left(\frac{T}{300 \text{ }^\circ\text{K}} \right)^{\frac{1}{2}} \left(\frac{10 \text{ sec}}{\tau_n} \right)^{\frac{1}{2}} \left(\frac{10^3 \text{ kg}}{M} \right)^{\frac{1}{2}} \left(\frac{\text{BW}}{10^3 \text{ Hz}} \right)^{\frac{1}{2}} \quad 31b.$$

Here τ_n is a typical dissipation time for those normal modes with frequencies $\ll \nu_{\text{GW}}$ (but driven at ν_{GW}), M is the mass of the detector, and BW is the bandwidth monitored.

Mechanical displacement sensors. -- Free-mass antennas require non-mechanical displacement sensors (e.g., lasers); but almost-free and resonant antennas permit a mechanical link between the test masses. This opens the way for other types of displacement sensors. Braginskii (1968, 1970) divides displacement sensors into two classes: transducers, which convert the mechanical energy of the detector's motion to some other form of energy; and modulators, which make use of the detector's mechanical

motion to control an external source of energy. The output of a modulator is not limited to the energy extracted from the gravitational wave. Examples: a piezoelectric crystal, a bar magnet and moving coil are transducers; a laser interferometer, and a resonant circuit with mechanically varied capacitor are modulators. Although Weber's experiment uses piezoelectric transducers, most experiments designed subsequently make use of modulators (Braginskii 1971; Hamilton 1970a,b).

The most useful measure of a displacement sensor's performance is the function $\Delta \ell_{\min}(\tau)$, the minimum detectable displacement in an averaging time τ (with signal/noise = 1). In many cases the sensor noise will be "white" and the function of averaging time will be the typical square-root random walk

$$\Delta \ell_{\min}(\tau_1)/\Delta \ell_{\min}(\tau_2) = (\tau_2/\tau_1)^{1/2} \quad 32.$$

In these cases the useful figure of merit is the constant

$$S \equiv \Delta \ell_{\min}(\tau) \tau^{1/2} \quad 33.$$

with units $\text{cm}/(\text{Hz})^{1/2}$, which we call the displacement sensitivity. (Notice that the inverse time resolution τ^{-1} is the bandwidth $\Delta \omega$ of the displacement sensor, not the frequency at which it operates which is usually much higher.

For example, Weber's piezoelectric transducers measure displacements of 10^{-15} cm at a frequency of 1660 Hz, with a bandwidth $\Delta\omega = \tau^{-1}$ of a few Hz.

Gibbons and Hawking (1971) have considered in some detail the theoretical limits on piezoelectric sensors, and similar considerations limit other transducer sensors. The key idea is that the electrical output of a transducer is subject to thermal ("Johnson"; "Nyquist") noise, which increases with decreasing averaging time (i.e. with increasing bandwidth). This noise power per unit bandwidth is a constant ($\sim kT$), while the signal power is proportional to the volume of piezoelectric crystal. As the crystal volume is increased, it comes to store more and more of the antenna's mechanical energy. A limit is reached when the crystal stores all the mechanical energy, and this translates into a rigorous limiting sensitivity for piezoelectric sensors:

$$\begin{aligned}
 S_{\min} &\sim d_{\text{piezo}} (kT B \tan \delta / M \omega^3)^{1/2} \\
 &\approx \left(1.5 \times 10^{-16} \frac{\text{cm}}{\text{Hz}^{1/2}} \right) \left(\frac{d_{\text{piezo}}}{10^{-5} \text{ cm/statv}} \right) \left(\frac{T}{300 \text{ }^\circ\text{K}} \right)^{1/2} \left(\frac{\tan \delta}{5 \times 10^{-3}} \right)^{1/2} \left(\frac{B}{10^{12} \text{ dyne/cm}^2} \right)^{1/2} \\
 &\quad \left(\frac{10^3 \text{ kg}}{M} \right)^{1/2} \left(\frac{10^4 \text{ rad/sec}}{\omega} \right)^{3/2}
 \end{aligned} \tag{34}$$

Here d_{piezo} (the piezoelectric strain constant), B (the elastic modulus)

and $\tan \delta$ (the dissipation factor) are properties of the material; and ω is the frequency of the wave. The experimenter is able to adjust only T (the temperature) and M (roughly, the total mass of the gravitational-wave antenna).

Modulator-type displacement sensors are also limited in principle in their sensitivities (Braginskii 1968, 1970). However, the limits of principle are many orders of magnitude below current technological limits, so we will not consider them here.

One cannot understand the technological limits on modulator-type sensors without first exploring their possible configurations. Modulator-type sensors require three elements: an oscillator, which supplies a highly monochromatic, oscillating electromagnetic signal; a resonator, which is coupled to the gravitational-wave antenna, and which modulates the oscillator output, and an electromagnetic detector, a nonlinear component which detects the modulated signal. The electromagnetic signal may be at any frequency -- optical, microwave, radio. In the optical regime the oscillator is a laser, and the resonator is an interferometer cavity with the separation between its mirrors

modulated by the gravitational wave [Moss et al. (1971), see above]. In the microwave regime one might use as the resonator a microwave cavity, perhaps superconducting. Flexing of the cavity (produced by antenna displacements) will change its resonant frequency and modulate its output. [Dick and Press (1970) have designed displacement sensors based on this principle.] For electromagnetic signals of radio frequency one can use an L-C circuit as the resonator. Antenna displacements produced by gravitational waves can be used either to vary the distance between the capacitor plates [Braginskii's (1971) sensor works this way], or to vary the inductor, say by moving it with respect to a ground plane [a sensor designed by Fairbank and Hamilton works this way -- see, e.g. Hamilton (1970a,b)]. In either case the output is a modulated electromagnetic signal. It is worth noting the essential unity of the above three resonators, and the possibility of constructing intermediate devices: as the wavelength λ of the resonator's oscillating (standing) electromagnetic wave increases relative to the size L of the resonator, one slides continuously from laser interferometer ($\lambda \ll L$) to microwave cavity ($\lambda \sim L$) to L-C circuit ($\lambda \gg L$).

A number of displacement-sensing configurations can be built with

oscillators, resonators, and detectors -- some with AM modulation, others with FM modulation, and others with more complicated schemes. The displacement sensitivity is limited by two factors: the oscillator noise at frequencies close to the oscillator frequency where the modulated sidebands will appear, and the noise in the demodulating detector. Thermal electromagnetic noise ($1/2 kT$) in the resonator is almost always much smaller, so 1971 sensors are only state-of-the-art limited. It appears that 1971 technology in the radio and microwave (superconducting cavity) region can achieve a factor of ~ 10 better displacement sensitivity than piezoelectric technology; and one expects that this number will increase with time as the materials limit on piezoelectric transducers is reached, and as oscillators and electromagnetic detectors with lower noise are developed.

Table III gives typical parameters for three displacement sensors which have actually been built. For modulator-type sensors we can expect large improvements over the currently measurable strains ($\sim 10^{-16}$) during the coming decade.

Acoustical systems: the uses and abuses of resonance. -- Thus far we have estimated the strength h of incident gravitational waves from various

astrophysical sources; we have seen that when a wave of strength h acts on a free-mass or almost-free gravitational wave antenna of size ℓ , a displacement $\Delta\ell \approx h\ell$ is produced; and we have surveyed displacement sensors and have found that given a resolution time τ , one can measure a displacement as small as $\Delta\ell_{\min} \approx S \tau^{-1/2}$, where S is the displacement sensitivity. How should one choose τ , the resolution time?

Ideally one would like to take τ as small as possible so as to examine the actual waveform of the gravitational wave as it passes. [A wide-band (small τ) gravitational wave receiver extracts more information from the wave than does a narrow-band (large τ) receiver.] But τ is limited by the condition of detectability $(\Delta\ell)_{\text{due to wave}} > \Delta\ell_{\min}$. Thus, to detect a wave of amplitude h one must measure for a time τ larger than

$$\tau_{\min} = \left(\frac{S}{h\ell}\right)^2 = (10^4 \text{ sec}) \left(\frac{S}{10^{-16} \text{ cm/Hz}^{1/2}}\right)^2 \left(\frac{1 \text{ m}}{\ell}\right)^2 \left(\frac{10^{-20}}{h}\right)^2 \quad 35.$$

For "burst" gravitational radiation (from collapse, explosion, collision, etc.) $\tau_{\min}$ may be longer than the duration of the burst, so that not enough averaging time is available to see the burst at all. Even for highly monochromatic waves (e.g. pulsars) $\tau_{\min}$ may be unfeasably long, say years. Can anything be done in these cases?

Yes: one can utilize a resonant mechanical system as the antenna. For burst radiation, a resonant system "remembers" that it has been hit by a burst (the way a bell "remembers" that it has been struck by a hammer), and allows averaging times τ much longer than the duration of the burst. For monochromatic waves, the resonance "remembers" the last Q_{res} cycles of the wave (Q_{res} is the antenna's resonance quality factor), and superimposes them so that the displacement is increased by a factor Q_{res} and τ_{min} is decreased by a factor Q_{res}^2 :

$$\tau_{\text{min}} = \left(\frac{S}{h \ell Q_{\text{res}}} \right)^2 = (10^4 \text{ sec}) \left(\frac{S}{10^{-16} \text{ cm Hz}^{1/2}} \right)^2 \left(\frac{1 \text{ m}}{\ell} \right)^2 \left(\frac{10^6}{Q_{\text{res}}} \right)^2 \left(\frac{10^{-26}}{h} \right)^2 \quad 36.$$

Notice (and we will prove below) that these two effects are disjoint. For burst radiation, resonance does not increase the detector response $\Delta \ell$; it only allows longer resolution times and hence less sensor noise.

The benefits of resonance are obtained at a tremendous cost -- the loss of all information about the wave except one single number, its fourier component (i.e. spectral energy density) at one single frequency, the frequency of mechanical resonance. Only wide-band detectors can give detailed information on the waveform or spectrum of a burst, or precise time-of-arrival

information which can determine the source direction. If resolution time τ is increased to take advantage of the resonant antenna, one decreases the bandwidth $\Delta\omega = \tau^{-1}$ accordingly. Resonance is a technique of the last resort, to be used to detect gravitational signals which could otherwise not have been detected at all.

The force field of the gravitational wave acts independently on each normal mode of a general resonant antenna. Describe the n 'th normal mode by its angular frequency ω_n , its damping time τ_n , and its "eigenfunction" $\tilde{u}_n(\tilde{x})$. Thus, vibrating freely in this mode the antenna exhibits the displacements

$$\Delta\tilde{x} = \tilde{u}_n(\tilde{x}) \sin(\omega_n t) \exp(-t/\tau_n) \quad 37a.$$

To make the eigenfunctions $\tilde{u}_n$ dimensionless with magnitude of order unity, impose the normalization

$$\int \rho |\tilde{u}_n|^2 d^3x = M \quad 37b.$$

where ρ is the density and M is the mass of the antenna. If $B_n(t)$ is the amplitude of the n 'th mode, defined by

$$\Delta\tilde{x} = \tilde{u}_n(\tilde{x}) B_n(t) \quad 38a.$$

then the action of the wave on the mode is described by the equation for a

forced damped harmonic oscillator (Fig. 2c; see MTW exercise 37.11):

$$\ddot{B}_n + \dot{B}_n/\tau_n + \omega_n^2 B_n = R_n(t) \quad 38b.$$

The forcing term is related to the components of the gravitational wave by

$$R_n(t) \equiv - \sum_{j,k} R_{joko}^{GW}(t) \int (\rho/M) \tilde{u}_n^j x^k d^3x \quad 39.$$

Note that for an antenna of fixed mass M and fixed characteristic size l , one can maximize the displacement $\Delta l = B_n u_n$ to be measured by making the measurement at a point where the eigenfunction $\tilde{u}_n$ is large. In principle one can obtain an arbitrary amount of amplification by designing the antenna so that $\tilde{u}_n$ is huge somewhere [but not where much mass is; cf. eq. (37b)]; for an example see Lavrent'ev (1969a,b). Unfortunately, this type of amplification is not usually practical -- it is easy to draw a long, massless lever (the perfect displacement amplifier), but not so easy to construct one; and other mechanical amplifiers suffer similar drawbacks. Note also that unless the normal mode-displacements $\tilde{u}_n$ "look something like" the force diagram of Figure 1, various parts of the integral will largely cancel, and the driving force $R_n(t)$ will be very small; in other words, the gravitational wave will couple only poorly to that mode. For example, the coupling to the longitudinal modes of a vibrating cylinder decreases as the inverse square of the

mode number n , for odd n ; for even n the coupling is zero, since these modes are precisely orthogonal to the force of the gravitational wave [Ruffini and Wheeler (1971), §7.3]. A similar power law holds for high modes of general mechanical systems; for example, it is unlikely that gravitational waves could excite high-mode free oscillations of the Earth without exciting the lower modes preferentially (this point is sometimes overlooked; cf. Tuman 1971).

Figure 3 shows the familiar Green's function solution to equation (38b). One imagines the wave's driving force $R_n(t)$ propagating rightward and the (damped sine-wave) Green's function held fixed. The momentary displacement $B_n(t)$ is the integrated product of R_n and G . In Figure 3a the wave has not yet reached the antenna, and there is no antenna response. Skip now to Figure 3c; this is after the wave has gone by. The waveform lies completely within the nearly sinusoidal part of the Green's function: the amplitude of the detector's ringing measures the product of wave and sine-wave, i.e. it measures one fourier component of the wave. Its magnitude, the quantity the displacement sensor must measure, is $\Delta l \approx B_n(t) \sim h l \exp(-t/\tau_n)$. (To obtain this integrate Fig. 3c twice by parts, thereby turning components

of R_n into h .) As the wave marches on through the Green's function, the ringing dies away with time constant τ_n -- this is the time during which one must ferret the signal from the noise in order to detect the wave at all. Go back to Figure 3b. This is during the time that the gravitational wave is driving the apparatus. The response depends in a complicated way on the incident waveform: if one could measure the response with good time resolution during this period, one could in principle reconstruct the entire incident wave. (More exactly: the wave high-pass filtered at ω_n , since the antenna is essentially rigid to frequencies much below ω_n .) Here again one faces the issue of wide- vs. narrow-band antennas. If the resolving time determined by system noise and sensor noise is shorter than the duration of the wave, then one can resolve the wave's structure; if it is longer than the duration of the wave, but less than τ_n , one can see only a single fourier component of the wave; if it is longer than τ_n , one cannot detect the wave at all. The free-mass and almost-free antennas are special cases of this discussion with $\omega_n \rightarrow 0$. Their conceptual advantages are their simple relation between detector response and incident waveform ($\Delta l/l$ measures h directly and instantaneously), and absence of the "high-pass filter" effect.

Their disadvantage is that they cannot "remember" the wave for a long time τ_n , as a resonance can.

Figure 3 is drawn for burst radiation. For a long monochromatic train, one would have a picture with two intersecting sine trains, and the response would be of order

$$\Delta l \approx B_n \sim h \ell Q_{\min} \quad 40.$$

where $Q_{\min}$ is the "number of peaks" in the product, therefore the minimum of wave "Q" and detector "Q".

In analyses of resonant antennas the concept of cross-section,

$$\sigma \equiv (\text{energy absorbed by detector})/(\text{energy flux in wave})$$

has sometimes been introduced. However, the cross section is irrelevant and useless (i) when one deals with free-mass and almost-free antennas, and (ii) when one uses or designs even a resonant antenna to measure more than the single Fourier component of the wave at the resonant frequency. Thus, a designer of gravitational-wave antennas should focus his attention on cross sections no more than does a designer of radio-wave antennas. Cross section is far too narrow a concept to be central in antenna design.

For detailed discussions of cross sections see, e.g. MTW or Ruffini and

Wheeler (1971).

Thermal noise in resonant antennas. -- We mentioned above the effects of thermal noise on an almost-free antenna. In a resonant antenna the thermal noise fluctuations are of crucial importance. To analyze the effect of thermal noise, one need only notice that the antenna's oscillating displacement $B_n(t)$ is linear in the driving force (eq. 38b or 40); and that therefore the displacement $B_n^{\text{thermal}}(t)$ produced by Brownian (thermal) forces adds linearly to the displacement $B_n^{\text{GW}}(t)$ produced by the gravitational wave. The thermal-noise displacement oscillates sinusoidally,

$$B_n^{\text{thermal}}(t) = \beta_n^{\text{thermal}}(t) e^{i\omega_n t}$$

with a slowly fluctuating, complex amplitude $\beta_n^{\text{thermal}}(t)$ that has typical magnitude corresponding to $\frac{1}{2} kT$ energy in the mode:

$$|\beta_n^{\text{thermal}}| \approx \left(\frac{kT}{M\omega_n^2} \right)^{1/2} \approx (2 \times 10^{-14} \text{ cm}) \left(\frac{T}{300 \text{ K}} \right)^{1/2} \left(\frac{10^3 \text{ kg}}{M} \right)^{1/2} \left(\frac{10^4 \text{ rad/sec}}{\omega_n} \right) 41.$$

β_n^{thermal} moves about in the complex plane (varying magnitude and phase) on a characteristic time scale τ_n , which is the same as the damping time for free oscillations far above thermal noise. In shorter times Δt the fluctuations

obey a stochastic square-root law

$$|\Delta \beta_n^{\text{thermal}}| \sim (\Delta t / \tau_n)^{1/2} |\beta_n^{\text{thermal}}| \quad 42.$$

(See Braginskii 1970 for more details.) Now an important point: if over a

time Δt one tries to measure a signal $B_n^{\text{GW}}(t)$, one need not have

$$B_n^{\text{GW}} \gtrsim |\beta_n^{\text{thermal}}|; \text{ rather one need only have } B_n^{\text{GW}} \gtrsim |\Delta \beta_n^{\text{thermal}}|. \text{ In}$$

other words, the thermal noise level is not the $\frac{1}{2} kT$ thermal-oscillation dis-

placement; is the fluctuation in thermal oscillation over the time of the

measurement. This explains why high Q_n (large τ_n) resonances are favorable

for burst radiation: not that the high Q_n increases the size of the signal

$\Delta l \approx B_n^{\text{GW}}$ (it does so for monochromatic waves, but not for bursts); nor

that it decreases the amplitude of thermal $\frac{1}{2} kT$ oscillations (it never does

so!); rather, the high Q_n lengthens the time scale over which the thermal

oscillations change amplitude, so that a smaller burst $B_n^{\text{GW}}(t)$ can be picked

out against the smooth thermal oscillations. This thermal noise advantage

is in addition to the advantage of resonance previously mentioned, the

permitted lengthening of the signal resolution time.

The fact that fluctuations, not absolute magnitudes, determine the

noise level also explains why feedback schemes to "cool one mode of a

detector instead of the whole detector" will not work. A feedback loop with a characteristic timescale τ_{fb} ($\approx$ resolution time of displacement sensor) will reduce the magnitude of the thermal oscillations by a factor $(\tau_{fb}/\tau_n)^{1/2}$. But it will leave completely unaffected the magnitude $|\Delta \beta_n^{\text{thermal}}|$ of fluctuations on timescales $\Delta t < \tau_{fb}$ and will therefore not improve the noise problems for gravitational-wave bursts shorter than τ_{fb} . For bursts longer than τ_{fb} the feedback will destroy the signals along with the noise -- essentially by increasing the antenna's effective inertial mass, while leaving unchanged the passive gravitational mass which feels the wave.

What is the optimal sensor resolution time τ to barely detect the smallest possible burst with a resonant detector? The battle against thermal fluctuations makes a short τ desirable; but sensor noise $\Delta \ell_{\min} \sim S \tau^{-1/2}$ favors large τ . The optimal point is in between:

$$\tau_{\text{optimal}} \sim \frac{S \tau_n^{1/2}}{|\beta_n^{\text{thermal}}|} \approx (0.15 \text{ sec}) \cdot \left(\frac{S}{10^{-15} \text{ cm/Hz}^{1/2}} \right) \left(\frac{\tau_n}{10 \text{ sec}} \right)^{1/2} \left(\frac{2 \times 10^{-14} \text{ cm}}{|\beta_n^{\text{thermal}}|} \right) \quad 43.$$

(Gibbons and Hawking 1971). For wideband experiments one seeks smaller resolution times $\tau < \tau_{\text{optimal}}$ (hence needs stronger waves), so sensor noise

increases while thermal mechanical noise becomes less troublesome. The interesting point is that in narrow-band experiments, one need not take τ any greater than τ_{optimal} .

Classes of resonant antennas. -- Here is a brief catalog of configurations which have been suggested for resonant antennas.

(a) Distributed resonant antennas. The restoring forces and inertial forces are distributed more or less uniformly throughout the antenna mass. The resonant period is determined (approximately) by the sound travel time across the mass. Examples: Weber's cylinders, rods, discs, the Earth.

(Douglass 1971, Douglass and Tyson 1971 call these "Class I" antennas.)

(b) Lumped resonant antennas. The main restoring force and main inertial force are contributed by different parts of the system. The resonant period of the fundamental mode can be made much longer than the typical sound travel time, but the periods of higher modes are usually of the order of that time. Examples: hollow squares, rings, tuning forks

(Douglass 1971; Douglass and Tyson 1971 call these "Class II"); also dumbbells

(Rasband et al. 1972); also two pendula, well separated but suspended from a common support [Braginskii and Rudenko (1970)]; this antenna looks promising

for detecting waves from pulsars; it has the advantage of a very large $Q \sim 10^9$.] A lumped, resonant antenna, monitored between its low fundamental frequency and its much higher "harmonic" frequencies, would function as a wideband almost-free antenna.

(c) Acoustical transmission lines. Here a smoothly distributed mass is used not as the primary antenna, but rather to carry a displacement to a convenient place for sensing. Examples: Braginskii's (1971) cylinder has "horns" which carry the full displacement of the cylinder ends to a capacitive sensor in the center. Vali and Filler (1972) have proposed using a long resonant rail to transmit rigidly a (gravitational-wave-induced) displacement over a distance of several kilometers. (The key idea is that a resonant rail acts as if it were "infinitely rigid" between nodes of its resonant frequencies.) This technique may find application in detecting monochromatic pulsar waves in the LF band.

(d) Rotational resonances -- heterodyne antennas. These have been devised by Braginskii [see Braginskii et al. (1969), Braginskii and Nazarenko (1971)]. For a circularly polarized gravitational wave, the force diagram of Figure 1 rotates with time. If a dumbbell rotates at half the frequency

of the gravitational wave in a plane perpendicular to the wave, it will always stay fixed with respect to the lines of force and be continuously accelerated. Two independent dumbbells, rotating in the same direction, but 90° out of phase, will experience opposite accelerations. The experimenter can search for the constant relative angular acceleration of the two rods (constant so long as the angle between them does not depart significantly from 90°). Better yet, the experimenter can adjust the rods' rotation rate so that it does not quite match the waves' frequency (all too easy to do!); the resulting frequency beating will give oscillations in the relative orientation of the rods. One need not worry about the other circular polarization marring the experiment. Since the other polarization does not rotate with the rods, its angular accelerations average out over one cycle; hence such a detector also works for linearly polarized or unpolarized waves. Heterodyne antennas, particularly in earth orbit, may be the most practical means of detecting waves from pulsars. They may also have application in threshold detection of bursts, with a very long resolution time available to detect the relative rotation after the burst has gone by (Braginskii and Nazarenko 1971).

A similar antenna has been proposed by Sakharov (1969). A nonrotating

dumbbell is driven in its vibrational mode in resonance with a gravitational wave. When maximally distended it experiences a torque in one direction, and a torque in the opposite direction acts when it is minimally contracted. Hence it experiences a net angular acceleration relative to local inertial frames (gyroscopes).

(e) Surface interactions with matter. A gravitational wave interacts with the free surface of an elastic body, producing elastic waves (Dyson 1968, Esposito 1971). In principle, the surface could be the surface of the earth or moon, and the waves could be detected seismically. In practice this method is not sensitive enough to be useful for astronomical sources. However there are possibilities for improvements, e.g. using resonances (elastic waves reflected between two surfaces) in the antarctic sheet ice or in lunar mascons (de Sabbata 1970). These techniques might have application for monochromatic LF waves.

Other gravitational-wave antennas. -- Fluid-in-pipe antennas, where the force field of the gravitational wave causes a fluid to flow around the inside of a closed pipe of appropriate configuration (e.g. figure-eight shaped), have been considered by Press (1970). These antennas are related

to free-mass antennas in a way that is similar to the relation between magnetic-loop and electric-dipole antennas in electromagnetism. In the gravitational case, however, the size of the loop is limited by the speed of sound in the fluid, and fluid-in-pipe detectors are typically only (v_{sound}/c) as efficient as other mechanical detectors. (See MTW for further details.)

This disadvantage might not be debilitating if the "pipe" is a superconducting wire and the "fluid" consists of conducting electrons. The wave would induce a weak alternating current with the same frequency as the wave. Papini (1970), DeWitt (1966), and others have considered the action of a gravitational wave on superconducting and normal metals, from somewhat different points of view. Papini's detector is primarily for HF and VHF waves.

Braginskii and Menskii (1971) have devised a gravitoelectric detector consisting of a toroidal wave guide with a monochromatic electromagnetic wave train propagating around it. Gravitational waves, passing through the plane of the wave guide, act on its EM wave train (much as they do on the rods in the mechanical heterodyne detector; see above), producing frequency

and phase shifts between different parts of the train. (See Box 37.6 of MTW.) This detector might be useful with highly monochromatic waves in the VHF band; unfortunately there are no known astrophysical sources of this character.

Other gravitoelectric antennas have been described by Lupanov (1967), Vadyanitskii and Dimanshtein (1968), Boccaletti and colleagues (1970, 1971); these also seem ill-suited to predicted waves of astronomical origin.

Table IV summarizes the various proposed types of gravitational-wave antennas.

Directionality of antennas; arrays. -- All gravitational-wave antennas have quadrupole patterns of directionality: the amplitude of the response to a given wave is a quadratic function of the antenna's orientation [Exercise 37.13 and Box 37.4 of MTW; p. 115 of Ruffini and Wheeler (1971); Weber (1970b, 1971)]. The particular form of the quadrupole pattern (coefficients in quadratic expression) depends on the shape of the antenna and the polarization of the waves. For example, the patterns of a disc (Weber 1971) and a sphere (Forward 1971) are somewhat less directional than those of a cylinder.

The step from "antennas" to "telescopes" requires either antennas as

big as a fraction of a wavelength (impractical), or arrays of individual antennas spaced over such a distance. Much detail can, in principle, be derived from such an array. Since the frequencies are low (compared to radio astronomy), it is not impractical to apply sophisticated numerical techniques on-line to the output of an array. For example the directionality of an array will not be "diffraction limited", rather it will only be "noise limited".

Natural antennas. -- Nature provides one with a number of "natural" antennas for detecting gravitational waves. One (earth-moon separation) was discussed in some detail above. Others (the Earth's vibrations and seismic activity; anomalies in the Earth's rotation; fluctuations in the relative velocities of stars) are discussed in Braginskii's (1965) review and in references cited therein. None of these natural antennas look promising. None give limits on gravitational-wave flux that are markedly tighter than one gets from cosmological considerations (observed expansion rate, deceleration, and age of Universe demand mass density $\rho \lesssim 10^{-28} \text{ g/cm}^3$, corresponding to flux of waves $\mathcal{F} \lesssim 10^3 \text{ erg/cm}^2 \text{ sec}$).

Winterberg (1968) and Bergmann (1971) have argued that one might search

for gravitational waves of LF, VLF, ELF and even lower frequency by their action in interstellar space to produce fluctuations in the intensity of starlight. Unfortunately, the predicted fluctuations are far smaller than estimated by Winterberg and Bergmann. For the errors in Winterberg's analysis see Zipoy and Bertotti (1968). Bergmann erred in assuming that the waves produced fluctuations directly [so (amplitude of fluctuations) $\propto$ (amplitude of waves)]. Rather, it is only the energy carried by the waves that can affect the starlight intensity;² and Bergmann's equation (3) should be corrected to read (cf. Penrose 1966)

$$\begin{aligned} \langle \alpha^2 \rangle^{1/2} &= (\text{amplitude of starlight intensity fluctuations}) \\ &\sim \frac{G}{c^4} \left(\begin{array}{c} \text{energy per unit area} \\ \text{in one coherence} \\ \text{length of waves} \end{array} \right) \times \left(\begin{array}{c} \text{distance} \\ \text{to} \\ \text{star} \end{array} \right) \times \left(\begin{array}{c} \text{number of coherence} \\ \text{lengths between} \\ \text{Earth and star} \end{array} \right)^{1/2} \quad 44. \\ &\sim \left(\frac{h}{\lambda} \right)^2 \ell \times L \times \left(\frac{L}{\ell} \right)^{1/2} = \left[\left(\frac{h}{\lambda} \right)^2 L_H^2 \right] \left(\frac{L}{L_H} \right)^2 \left(\frac{\ell}{L} \right)^{1/2} \end{aligned}$$

Here L is distance to star, ℓ is coherence length of gravitational waves, λ is wavelength of gravitational waves, and $(c^4/G)(h/\lambda)^2$ is energy density in waves. The last formula introduces the Hubble radius L_H . Cosmological observations demand $(h/\lambda)^2 L_H^2 \lesssim 1$ (i.e. $\rho \lesssim 10^{-28} \text{ g/cm}^3$). Thus, the last

The oscillating Riemann tensor produces a shear of the light rays; the square of the shear then focusses the rays. The net focussing is proportional to the energy density of the gravitational waves and is the same as if the waves had been electromagnetic or neutrino; see Penrose (1966).

formula shows explicitly that the amplitude of the fluctuations can never exceed ~ 1 and under all reasonable circumstances will be $\ll 1$. The effect is not at all promising. (See Zipoy 1966 for a more complete treatment, which is basically correct but overly difficult.)

6. THE WEBER EXPERIMENT

Since 1969, Joseph Weber (1969, 1970a,b,c, 1971a) has observed sudden, coincident excitations of two resonant gravitational-wave antennas spaced 1000 km apart, one in Maryland, the other near Chicago. If these excitations are caused by gravitational radiation, then the characteristics of each burst are about what one expects from a "strong" supernova or stellar collapse somewhere in our Galaxy; but the number of bursts observed is at least 1000 times greater than current astrophysical ideas predict! Weber's observations lead one to consider the possibility that gravitational-wave astronomy will yield not just new data on known astrophysical phenomena (binary stars, pulsars, supernovae) but might discover entirely new phenomena (colliding black holes, cosmological gravitational waves, ???). In fact, one is offered the tantalizing possibility that these new phenomena might dominate all other forms of energy generation and might force a major restructuring of our understanding of galactic and cosmological evolution.

The possible resolutions of the present theoretical and experimental crisis fall into five inclusive categories: (i) Weber's events are not caused by gravitational waves. (ii) The events are caused by gravitational

waves, but the flux is somehow much less than it appears. (iii) The deduced flux is correct, but the deduced total luminosity is wrong (i.e., the source is either nearer us than we believe, or the radiation is "beamed" or focussed in our direction). (iv) The deduced luminosity is correct, so in the present epoch (at least) our Galaxy (?) emits orders of magnitude more gravitational radiation than electromagnetic. (v) The waves are of cosmological origin. Here we briefly summarize the observations as reported in the literature and elaborate on the possibilities.

Weber's detectors and the events. -- The detectors are aluminum cylinders, typical size 66 cm diameter by 153 cm length. The end-to-end strain is monitored by piezoelectric crystals bonded around the girth of the cylinder (Table III). In our terminology, (see above) the cylinders are distributed resonant antennas with $\omega_0/2\pi = 1661$ Hz, $\tau_0 \sim 10$ sec. The antenna output is monitored with a resolution time $\tau \gtrsim 0.1$ sec and strains of $\sim 10^{-17}$ are detected, so the implied sensitivity S is roughly $\sim 5 \times 10^{-16}$ cm/Hz $^{\frac{1}{2}}$. The thermal noise displacement is $|\mathfrak{B}_0^{\text{thermal}}| \sim 10^{-14}$ cm, so the resolution times chosen are about optimal for this device (eq. 42).

The observed events occur ~ 1 per day. The coincidences disappear

when one introduces a time delay of 2 seconds into the output of one detector.

Since no structure within the time resolution τ has been reported, an experimental limit on Q_{wave} (the wave's ratio of frequency to bandwidth) is

$Q_{\text{wave}} \lesssim 200$. Recently, Weber (1971b) has observed coincident excitations on another antenna at 1580 Hz. This would indicate $Q_{\text{wave}} \lesssim 20$.

The coincident events exhibit typical displacements of $\Delta l = B_0^{\text{GW}} \sim 5 \times 10^{-15}$ cm. Coincidences occur most frequently when the axes of the cylinders are perpendicular to the direction of the galactic center. The observations are consistent with the hypothesis of a single point source ^{of randomly polarized waves} in that direction (or in the opposite direction -- waves propagate through the earth unimpeded). A source $\lesssim 10^\circ$ from these directions cannot (in late 1971) be excluded; but sources farther away can unless they are consistently polarized.

The case for gravitational waves. -- Weber has tested for the possibilities of seismic excitation of his detectors, and excitation by cosmic rays and by radio waves, all with negative results. Nevertheless, in excluding non-gravitational sources there is always the possibility that something has been overlooked. Therefore it is important to find direct evidence that the excitation is gravitational.

One such bit of evidence is offered by Weber's scalar-wave experiment (1971a). There a disc antenna (not a cylinder) was used to search for scalar gravitational radiation [excluded in Einstein's theory, but predicted by, e.g., the theory of Brans and Dicke (1961)]. However, a disc is not a "perfect" scalar antenna; it also responds to ordinary tensor gravitational waves, but with a somewhat different directionality than a cylinder. Weber's evidence for experiment found no \scalar radiation; perhaps more interesting, the response of the disc was consistent with a point source of \tensor gravitational waves in the center of the galaxy. Since it would not be easy for a non-gravitational mechanism to "mimic" the different directionalities of disc and cylinder, this is direct -- if weak -- evidence that the excitation mechanism is a tensor gravitational wave.

The deduced wave strength. -- If the excitation is caused by gravitational waves, equation (40) must hold in order of magnitude, so

$$h \sim 3 \times 10^{-17} / Q_{\text{wave}} \quad 45.$$

As remarked above, the 1971 experimental limit is $Q_{\text{wave}} \lesssim 20$. However,

Q_{wave} is probably not even this large -- if it were so large, then one

would conclude that either Weber was fortunate enough to guess the "universal"

wave band (1580-1661 Hz), or else Weber misses many bursts at other frequencies. It is of crucial importance that good experimental limits be obtained for Q_{wave} ; the waves must be examined with wideband antennas, or with narrow-band antennas at various frequencies.

The luminosity of the source. -- Using equations (45) and (6) we can calculate the mass M associated with the total energy Mc^2 of each Weber burst:

$$M \sim 0.5 M_{\odot} \left(\frac{r}{10 \text{ kpc}} \right)^2 \left(\frac{1}{Q_{\text{wave}}} \right) \left(\frac{\Delta\Omega}{4\pi} \right), \quad 46.$$

where r is the distance to the source, and $\Delta\Omega/4\pi$ is the solid-angle beaming factor, about unity for a typical quadrupole source of waves. If we take $Q_{\text{wave}} \sim 10$ and suppose the source is at the center of the Galaxy, and that Weber observes 10 per cent of all events, then the rate of mass loss to gravitational waves is $\sim 150 M_{\odot}/\text{yr}$. (For $Q_{\text{wave}} \sim 1$, it is $\sim 1500 M_{\odot}/\text{yr}$; for contrast, the total luminosity of the Galaxy in E.M. radiation is $\sim 10^{-2} M_{\odot}/\text{yr}$.) To reduce this value we can either bring the source much closer to us, or suppose that $\Delta\Omega/4\pi$ is small so that the radiation is "beamed" in our direction or into a narrow range of galactic latitude

(Misner 1972). Another idea is to look for a "focussing" mechanism which would decrease the effective distance to the source (Lawrence 1971). No theoretical model has yet been devised which exploits any of those possibilities in a plausible way.

A different line of reasoning tries to find limits on the mass loss which are consistent with other observations. The best limit is that of Field, Rees, and Sciama (1969), Sciama (1969), Sciama et al. (1969) who find that $70 M_{\odot}/\text{yr}$ is the maximum admissible loss for periods of $\sim 10^9$ yr. A greater loss would produce runaway stars in our galactic neighborhood, which are not observed.

The puzzle remains. -- Our assessment, in terms of the original five possibilities, is that the ultimate answer will ^{probably} lie in (i) (events not gravitational waves), (iii) (beaming or focussing of waves), or (iv) (Galaxy overly active today). Possibility (iii) is attractive, but will require theoretical models which do not exist today; possibility (iv) will require this and more -- either we live in an exceptionally active epoch, or our present cosmological understanding is wildly defective. (Note that the epoch must be peculiarly active in gravitational waves alone: there is

no evidence for coincident radio bursts [Partridge (1971), Charman et al. (1970)] or neutrino bursts [Bahcall and Davis (1971)].

It is characteristic of important scientific puzzles that before the solution is known all possibilities look equally implausible. Certainly the puzzle of Weber's observations passes this test.

7. CONCLUSIONS

What progress can one expect in the course of the next ten or fifteen years? With 1971 technology (strains $\sim 10^{-17}$ measurable on MF resonant antennas) one could observe gravitational waves from a supernova at a distance of a few kiloparsecs -- hardly an event that one should count on. To evaluate the possibilities for other known sources of waves one must project technological progress: perhaps an improvement of 10 or 100 in the sensitivity of displacement sensors with the routine use of cryogenic temperatures? Perhaps another factor of 10 or 100 with improved basic technology? These estimates could expand one's range from kiloparsecs to tens of megaparsecs, where one may hope to detect "monthly" events (individual supernovae or stellar collapses among thousands of galaxies).

For known monochromatic sources (pulsars, binaries) one must project the technological prospects for high-Q antennas (cf. eq. 36). Here one foresees that space experiments may become particularly important: only rotational resonances are not limited by materials properties (e.g., the dissipation in a vibrating aluminum cylinder); and a weightless vacuum environment is the only "perfect" answer to suspension and isolation problems. Space experiments may also allow the long baselines necessary to detect VLF or ELF waves with free-mass detectors and laser interferometry. With conceivable improvements in technology, one has hope in the next 10 or 15 years of detecting waves from short-period binaries as well as from the Crab pulsar.

If Weber's events are gravitational waves, one projects a more rapid development of gravitational-wave astronomy: the events can be detected with current methods; and further technological improvements, particularly with wide-band devices, will yield immediate returns in greater observational detail. The impetus of the experimental results on further theoretical developments will also be considerable.

As a tonic to optimism (or perhaps only as wishful thinking) one recalls

Jansky's (1933) paper: "Electromagnetic waves of unknown origin were detected during a series of experiments at high frequencies. Directional records have been taken of these waves for a period of over a year The time at which these waves are at a maximum changes gradually throughout the year in a manner that is accounted for by the rotation of the earth around the sun [This fact] leads to the conclusion that the direction of arrival of the waves is fixed in space, i.e., that the waves come from some source outside the solar system." Jansky correctly guessed that the source might be in the direction of the galactic center.

Radio astronomy was the first of the "unconventional" additions to 20th century observational astronomy, and took more than 15 years to reach fruition. By now the precedents have been set and the time scale for advance has been shortened. One hopes -- and expects -- that the development of gravitational-wave astronomy will be rapid.

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TABLE I. GRAVITATIONAL-WAVE FREQUENCY BANDS

Designation	Frequency (Period)	Wavelength	Typical sources	Length of acoustically resonant antenna	Useful antennas
extremely low frequency ELF	(10^7 sec) to (10^4 sec)	$\approx .1 \text{ pc}$ to $\approx 20 \text{ A.U.}$	Cosmological? Explosions in quasars and galactic nuclei Binaries		
very low frequency VLF	(10^4 sec) to (10 sec)	$\approx 20 \text{ A.U.}$ to $3 \times 10^6 \text{ km}$	Short-period binaries Huge black holes ($\sim 10^5$ to $10^8 M_\odot$)	$\sim 25000 \text{ km}$ to $\sim 25 \text{ km}$	Planetary resonances Free masses in deep space
low frequency LF	$1/10 \text{ Hz}$ to 100 Hz	$3 \times 10^6 \text{ km}$ to 3000 km	Pulsars	$\sim 25 \text{ km}$ to $\sim 25 \text{ m}$	Lumped resonant antennas Heterodyne antennas Free masses in near space
medium frequency MF	100 Hz to 100 kHz	3000 km to 3 km	Black holes ($1 - 10^3 M_\odot$) Collapse of stars Weber bursts Supernovae	$\sim 25 \text{ m}$ to $\sim 2.5 \text{ cm}$	Resonant antennas (Weber) Laboratory almost-free masses
high frequency HF	100 kHz to 100 MHz	3 km to 3 m	Man made?		Laboratory almost-free masses
very high frequency VHF	100 MHz to 100 GHz	3 m to 3 mm	Black-body Cosmological?		Gravitoelectric detectors

TABLE II. THE GRAVITATIONAL WAVES WHICH BATHE THE EARTH
(See text for references and discussion)

Region of Spectrum	Source of Waves	Characteristics of Waves
Wavelength $\gtrsim$ size of galaxies	Primordial	Unknown; but must not carry an average energy density larger than $\rho_{\max} \sim 10^{-28} \text{ g/cm}^3$ (more would produce too great a deceleration of the expansion of the Universe). Thus, $\mathcal{F} < 3 \times 10^3 \text{ erg/cm}^2 \text{ sec}$, $h < 2 \times 10^{-7} \times (\lambda/10^6 \text{ l. yr.})^2$.
	Galaxy condensation	$\lambda \sim 10^5 \text{ l. yr.}$, $\mathcal{F} < 10^{-2} \text{ erg/cm}^2 \text{ sec}$. $h < 10^{-7}$.
ELF ($P \sim 100 \text{ days}$) (to $\sim 3 \text{ hours}$)	Explosions in distant quasars and galaxy nuclei	Huge explosions (e.g., those which create strong radio sources) might produce broad-band bursts with $P \sim 100 \text{ days}$, $\mathcal{F} \sim 10^{-12} \text{ erg/cm}^2 \text{ sec}$, $h \sim 10^{-21}$. Parameters could be rather different depending on nature and nearness of explosions. The short-term outbursts of quasars (energy release $\sim 10^{53} \text{ ergs}$ in time $\sim$ one day may produce waves of $\mathcal{F} \sim 10^{-21} \text{ erg/cm}^2 \text{ sec}$ — far weaker than flux from binary stars.
	Binary stars in our galaxy	Too weak to be of interest for $P > 10 \text{ days}$. Each source emits highly monochromatic waves at a fundamental frequency $\nu_o^{\text{GW}} = 1/(\text{orbital period})$ and at its harmonics, $\nu_n = (n + 1) \nu_o^{\text{GW}}$.

TABLE II. (cont.)

		Brightest known source, i Boo, produces at Earth $\nu_o^{\text{GW}} = 7.5/\text{day}$, $\mathcal{F} = 1 \times 10^{-10} \text{ erg/sec}$, $h = 6 \times 10^{-21}$. Other source with similar ν_o^{GW} but $\mathcal{F} \sim 10^{-12}$ to $10^{-11} \text{ erg/cm}^2 \text{ sec}$ are listed by Branginsky (1965) and by Ruffini and Wheeler (1971). Total Flux at Earth in ELF band, due to binary stars, is $\mathcal{F} \sim 1 \times 10^{-7}$ $\text{erg/cm}^2 \text{ sec}$, with spectrum peaked at $\nu^{\text{GW}} \sim 6/\text{day}$.
VLFF $\left(\begin{array}{l} P \sim 10^4 \text{ sec} \\ \text{to } \sim 10 \text{ sec} \end{array} \right)$	Huge black holes ($M \sim$ 10^5 to $10^8 M_{\odot}$)	Such a black hole <u>might</u> exist in the nucleus of our Galaxy. If so, each time it swallows a star of $M \sim M_{\odot}$, it emits a broad-band burst of VLF waves (energy $\sim 10^{45} \text{ ergs}$, $\mathcal{F} \sim 10^{-3}$ $\text{erg/cm}^2 \text{ sec}$, $h \sim 10^{-19}$)
LF $\left(\begin{array}{l} \nu \sim 0.1 \text{ Hz} \\ \text{to } 100 \text{ Hz} \end{array} \right)$	Pulsars	Crab pulsar (NP0532) emits highly monochromatic waves at $\nu \approx 60 \text{ Hz}$ and $\mathcal{F} < 3 \times 10^{-7} \text{ erg/cm}^2 \text{ sec}$, $h < 0.7 \times 10^{-24}$. Our "best guess" (probable error: ~ 2 orders of mag- nitude in $\mathcal{F}$) is $\mathcal{F} \sim 3 \times 10^{-13} \text{ erg/cm}^2$ sec, $h \sim 10^{-27}$. Other known pulsars are weaker by a factor of 400 or more in $\mathcal{F}$.
MF $\left(\begin{array}{l} \nu \sim 100 \text{ Hz} \\ \text{to } 100 \text{ kHz} \end{array} \right)$	Supernovae and collapse of stars with little optical display	Occur in our Galaxy at least once every ~ 100 years; perhaps as often as once each ~ 1 year. Should pro- duce several broad-band bursts with $\nu \sim 1$ to 10 kHz, with duration

TABLE II. (cont.)

	$\sim 10^{-3}$ sec to 1 sec, and with $\mathcal{F} \sim 10^7$ to 10^{10} ergs/cm ² sec, $h \sim 2 \times 10^{-19}$ to 10^{-17} .
	In galaxies out to distance of Virgo cluster such events should occur at least once each month, with $\mathcal{F} \sim 10$ to 10^4 ergs/cm² sec, $h \sim 2 \times 10^{-22}$ to 10^{-20}. After collapse, if a neutron star is formed, its rotation should produce monochromatic waves (during the first few days of its life) with $\nu \sim 1$ kHz; $\mathcal{F} \sim 1$ erg/cm² sec, $h \sim 10^{-22}$ in our galaxy; $\mathcal{F} \sim 10^{-6}$ erg/cm² sec, $h \sim 10^{-25}$ in Virgo.
Superdense clusters	See text for discussion. Such sources seem unlikely from conventional 1971 viewpoints.
Huge black hole in center of Galaxy	Gravitational synchrotron radiation, from objects injected into hole with high energy, can come off in the MF region of spectrum; see text. Such a source seems unlikely from con- ventional 1971 viewpoints.

TABLE III. THREE DISPLACEMENT SENSORS IN OPERATION IN 1971

Type of sensor	Role in gravitational-wave receiver	Measured displacement and baseline	Frequency " ν_{GW} " of measured displacement	Bandwidth $\Delta\omega$ of measured displacement, = $1/(\text{resolution time})$	References	Future improvements
Piezoelectric crystal (transducer)	Bonded to surface of Weber's vibrating cylinder 	$\sim 5 \times 10^{-15}$ cm in ~ 1 m (strains of $\sim 5 \times 10^{-17}$)	1660 Hz	~ 10 radians/sec	Weber (1969; 1970a, b, c; 1971a, b)	Lower temperature
Capacitor in resonant L-C circuit at ~ 10 MHz (modulator)	Placed between "horns" of Braginskii's vibrating cylinder 	$\sim 5 \times 10^{-15}$ cm in ~ 1 m (strains of $\sim 5 \times 10^{-17}$)	$\sim 10^3$ Hz	~ 10 radians/sec	Braginskii (1971)	Better oscillators. Lower EM detector noise. Superconducting resonant circuit. Change of resonator to superconducting microwave cavity. Measurable strains may well improve in next decade, to $h < 10^{-20}$.
Laser interferometer (modulator)	Not yet used in gravitational-wave receiver	$\sim 10^{-12}$ cm in ~ 1 m (strains of $\sim 10^{-14}$)	5×10^3 Hz	~ 1 radian/sec	Moss et al. (1971)	Higher laser power. Same or better displacement over much much larger baseline. Measurable strains may well improve in next decade, to $h < 10^{-20}$.

TABLE IV. POSSIBLE TYPES OF GRAVITATIONAL-WAVES ANTENNAS
(See text for details and references)

General type	Description	Potentially Astrophysically useful?	Frequency Band; burst (B) or monochromatic (M)?	Feasible within a decade?
Free masses	Masses in earth orbit Spacecraft tracking or lunar ranging	yes no	VLF or ELF (M)(B)	?
Almost-free masses	Dumbbells Other resonant systems far above resonance	yes yes	LF or MF (B)	yes
Resonant systems	Distributed resonators Cylinders Discs Sphere (Planets) Lumped resonators Dumbbells Hollow squares, etc. Planetary surface interactions Acoustical transmission lines (resonant rails) Heterodyne detectors (rotational resonances)	yes no (?) yes yes	MF (B) (M) (VLF) LF or MF (B) (M) LF (M) LF (M) LF or MF (B) (M)	yes (yes) yes no yes yes
Gravitoelectric	Toroidal waveguide Direct action on superconductors Scintillation of starlight	no no no	VHF (M) ? (effect too small)	no no no
Gravitovisquantum	Stimulated emission of gravitons	no	VHF and above	?

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FIGURE CAPTIONS

Fig. 1. Lines-of-Force diagram for the relative forces produced by a gravitational wave (Press 1970). The fiducial point, relative to which one measured the forces, is at the origin of coordinates. The direction of the relative force at any point is the direction of the arrow there; the magnitude of the force is proportional to the density of force lines. The force lines are hyperbolae, and their density is proportional to distance from the fiducial point [cf. eq. (3) and (15)]. The diagram for polarization "+" corresponds to equation (5) with $\ddot{h}_x = 0$, $\ddot{h}_+ > 0$; that for polarization "x" corresponds to $\ddot{h}_+ = 0$, $\ddot{h}_x > 0$. When the wave changes phase by 180° , the directions of all arrows reverse.

Fig. 2. Three types of gravitational-wave detectors illustrated by idealized examples: (a) Free-mass detector (e.g. two masses in "free-fall" orbit above the Earth). The displacement sensor (e.g. laser interferometer) must leave the masses free. (b) Almost-free detector. The masses are coupled to their surroundings, and perhaps also to each other, by (i) a suspension system and/or (ii) the displacement sensor. However, the motions excited by the gravitational waves (here displacements of suspended masses in y

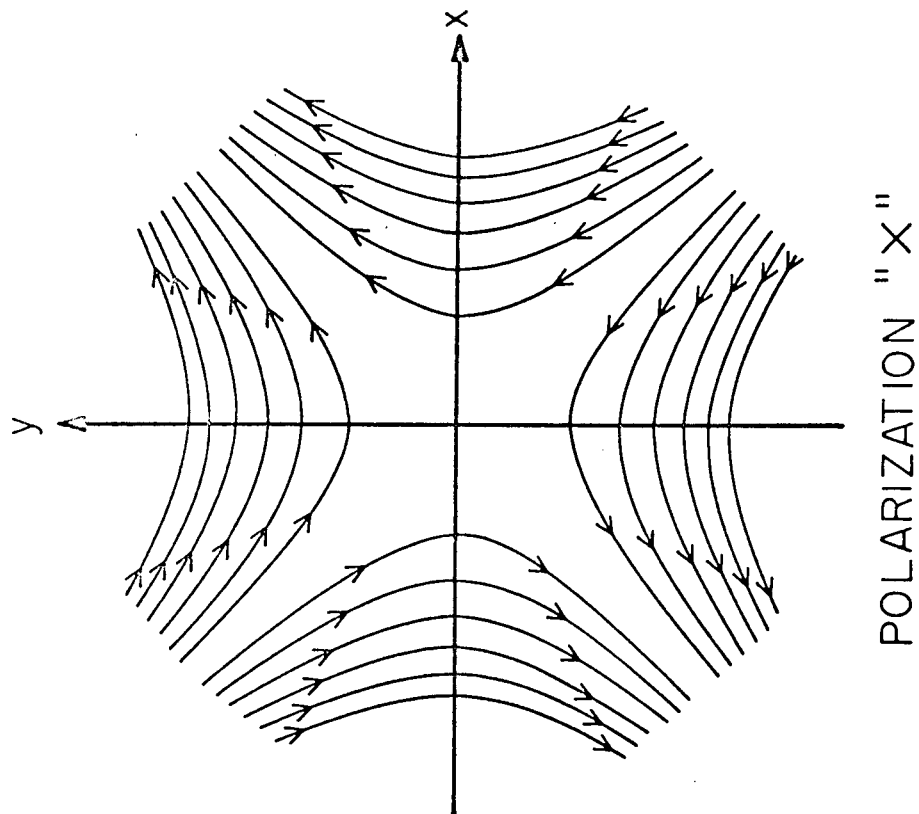
direction) are essentially free. (Free motion here requires that the wave frequency ν_{GW} be far larger than the "pendulum" frequency ν_0 in the y-direction, $\nu_{\text{GW}} \gg \nu_0$; and also large compared to characteristic frequencies ν_{MDS} of the coupled mass-displacement-sensor system, $\nu_{\text{GW}} \gg \nu_{\text{MDS}}$.) (c) Resonant detector. The masses are strongly coupled, and vibrate in a resonant mode at the frequency ν_{GW} of the gravitational wave.

Fig. 3. Graphical evaluation of the effect of a burst-type wave on a resonant antenna (see text for details). R_n is the wave's driving force, G is the antenna's Green's function

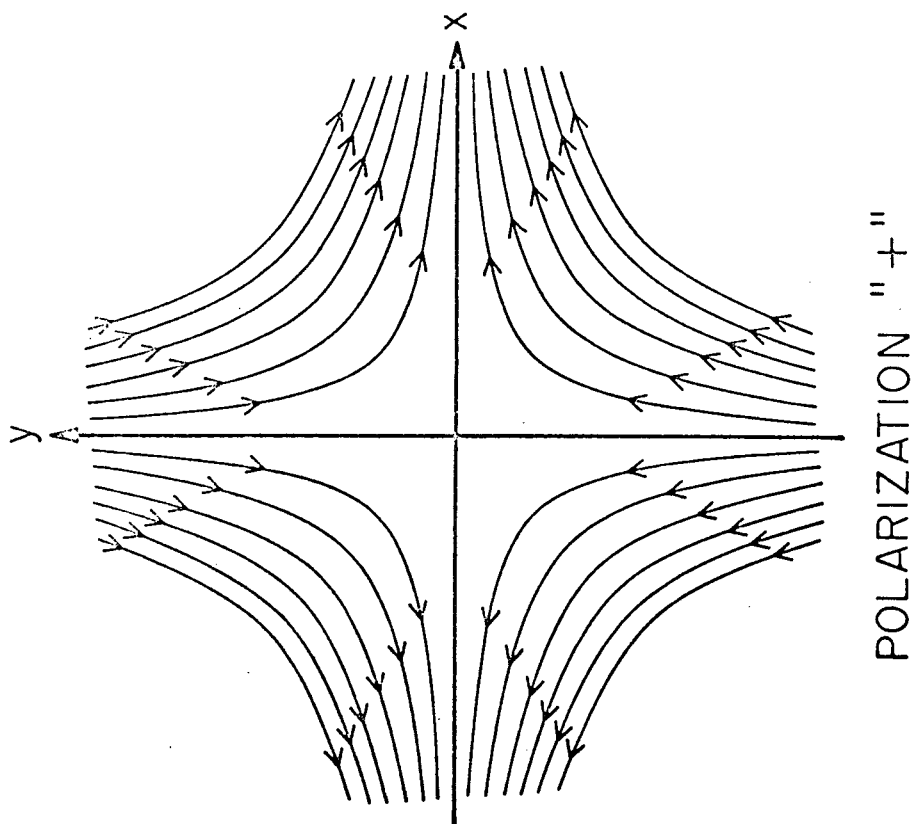
$$G(\xi) = \begin{cases} (1/\omega_n) \sin(\omega_n \xi) \exp(-\xi/\tau_n), & \xi > 0 \\ 0, & \xi < 0 \end{cases},$$

and the response of the antenna to the wave is $B_n(t) = \int_{-\infty}^{+\infty} R_n(\xi) G(\xi) d\xi$.

The three plots correspond to times t that are (a) before the burst reaches the antenna, (b) while the burst is exciting the antenna, and (c) after the burst has passed.



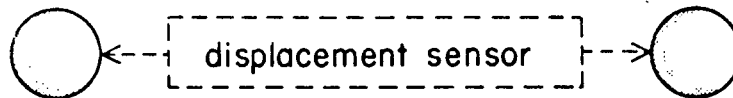
POLARIZATION "+"



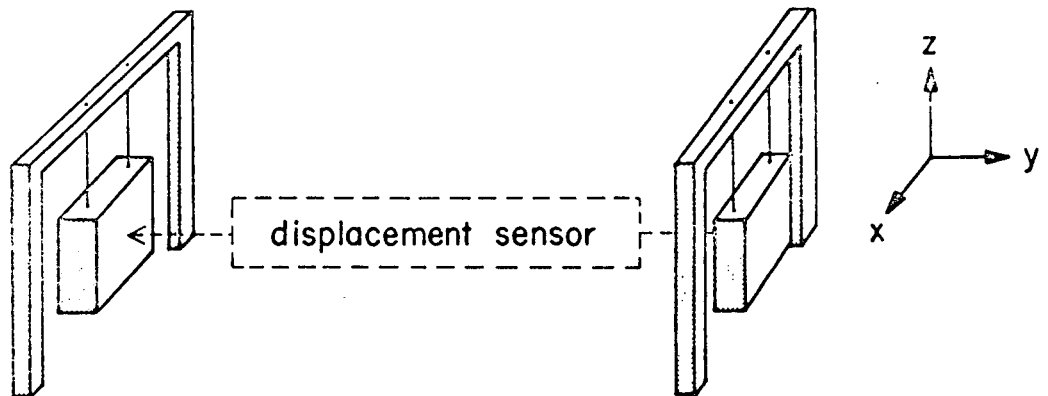
POLARIZATION "X"

Fig. 1

(a) FREE-MASS
ANTENNA



(b) ALMOST-
FREE
ANTENNA



(c) RESONANT
ANTENNA

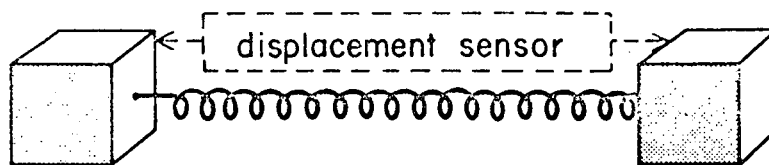


Fig. 2

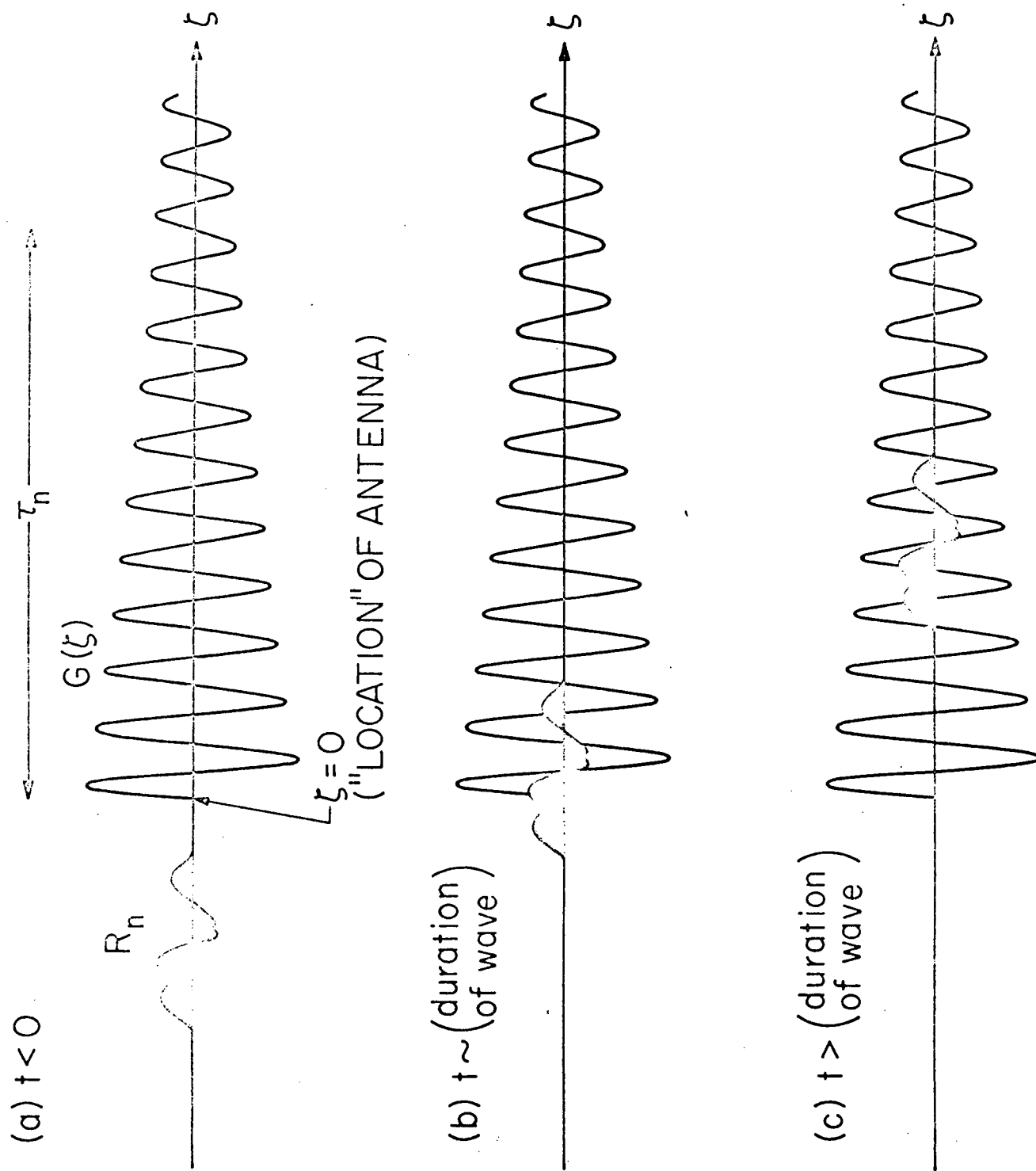


Fig. 3