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RESONANCE-INERT STABILIZATION FOR SPACE STATIONS

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16. ABSTRACT

A new approach of stabilizing control systems is presented that structures controllers like passive mechanical systems. Conventional concepts, which equate feedback with loops, are abandoned here and the controller is visualized as a structural part with a passive behavior similar to springs, dashpots, and masses. If such a controller is connected by a proper feedback arrangement, then a passive mechanical plant cannot upset stability regardless of masses, resonances, and three-dimensional coupling.

The design of attitude control systems for flexible space vehicles and space stations (e.g, Skylab) usually requires a tailoring of the controller response according to the resonances of the flexible body. However, the rather low resonant frequencies often interfere with the control purpose of providing stiffness to a desired attitude. Extensive simulations with a good dynamic model of the flexible body are then needed to ensure overall stability. But dynamic complexity often defies extensive modeling efforts, and vehicle structures must be dynamically tested in full size to verify the model and to determine empirically the structural damping. The efforts are characterized by a general uncertainty on resonances and a continuous updating process whose impact is felt all through the control system design, which is boxed in between late updating and launch dates.

The concept of resonance-inert stabilization is explained by structuring a controller of a simple feedback loop. Reactive functions, connections, and matrices are defined and used in proving the stabilization concept. The realization of a possible Skylab control system is discussed and compared with the present design. This example demonstrates the applicability to three-dimensional problems with lagging controllers.

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DEFINITION OF SYMBOLS

Symbol	Definition
A	General transfer matrix
a_0, a_1, a_2	Plant coefficients
$a_{ik}(s)$	Elements of transfer matrix
B	General transfer matrix
b_0, b_1	Controller coefficients
C	Control gain matrix
c_0, c_1, c_2	Unified Skylab control gains
c_{01}, c_{11}, c_{21}; c_{02}, c_{12}, c_{22}; c_{03}, c_{13}, c_{23}.	Skylab control gains for body coordinates 1, 2, and 3, respectively.
D_1, D_2	Denominators
G	Control moment gyro matrix
I	3 by 3 unit matrix
II	Inertial cross coupling coefficient
I_0	Unified moment of inertia
I_1, I_2, I_3	Diagonal elements of decomposed inertial matrix
J	Inertial matrix

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
J_1, J_2, J_3	Diadic components of decomposed inertial matrix
k	General matrix factor
K	Resonance matrix of flexible Skylab
m	Minimum exponent
m_1, m_2, . . . m_n	Generalized masses
M	Moment vector
M_1, M_2, M_3	Moment vector components
N_1, N_2	Numerators
N_π	Number of 180-deg encirclements of the origin
P	Number of poles and flexible body matrix
$p_1 - p_6$	Decomposition parameters
Q	Angular sensor matrix
q_1, q_2	Quadratic form of flexible body matrix and controller, respectively
q_{21}, q_{22}, q_{23}	Diagonal elements of controller
R	Radius of large quarter circle
$r_1 - r_9$	Remainders of decomposition
s	Laplace operator

DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
S	General passive mechanical system
y	General vector
Y	Mode matrix
$Y_{11}, Y_{12}, \ldots Y_{1n}$; $Y_{21}, Y_{22}, \ldots Y_{2n}$; $Y_{31}, Y_{32}, \ldots Y_{3n}$	Modes of slopes at resonance 1, 2, ..., n about axes 1, 2, and 3, respectively
z	General vector
Z	Number of zeros
α_{ik}	Matrix elements
γ	Small angle vector of vehicle attitude
ϵ	Positive real part of bound open at the positive imaginary axis
$\zeta, \zeta_{10}, \zeta_1, \zeta_2, \ldots \zeta_n$	Damping of control moment gyros, rigid body, and vehicle resonances, respectively
ρ	Radius of small quarter circle
τ	Time constant of controller
ϕ	Attitude command
ω	Circular frequency
ω_0	Control moment gyro resonance

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
ω_{10}, ω_{20}	Rigid body resonances excluding damping effect
Ω	Modal resonance matrix
$\times$	Reduction symbol
$\backslash$	Subduction symbol

RESONANCE-INERT STABILIZATION FOR SPACE STATIONS

INTRODUCTION

Space vehicles and space stations are rather flexible bodies whose resonances can destabilize attitude control systems, even though the control plant (i.e., the flexible body) is passive. The controller consists of motion sensors (e.g., platforms or rate gyros), signal processors, amplifiers, actuators, and control force devices (e.g., thrusters, gimbaled engines, torque motors, or control moment gyros). Regenerative undamping of the structure by the controller is possible and must be avoided through a good control design.

The same situation exists for the Skylab where the present attitude control system design depends also on a flexible body which must be properly described. The dynamic complexity of the body is overcome through an extensive analysis with progressive updating and a full-scale dynamic test of the short stack, which is used to verify and correct the modeling of the dynamically most-difficult Skylab section encompassing the Apollo Telescope Mount, Multiple Docking Adapter, and the Airlock Module as shown in Figure 1. The corrected model is then analytically combined with other parts, such as the S-IVB tank and the solar panels whose dynamic behavior is given by math models. All possible Skylab configurations (e.g., undocked and docked) are thus obtained for designing the attitude control system. Obviously, present design methods do not take advantage of the flexible body's passivity.

The method proposed herein departs from the conventional feedback loop concept and uses a controller with a response equivalent to a passive mechanical system consisting of springs, dashpots, and masses. The passivity of the controller yields a passive attitude control system which provides stiffness to the heading or the orientation of the vehicle and consequently satisfies the main control purpose, but also makes the system inert to resonances of the flexible body. The method was successfully used in 1966 in stabilizing a hydraulic support for dynamic tests with the Saturn-V space vehicle [1] at near free-flight conditions.

The problem posed here is similar to the task of realizing certain filter characteristics by a passive electrical network, as it has been abundantly

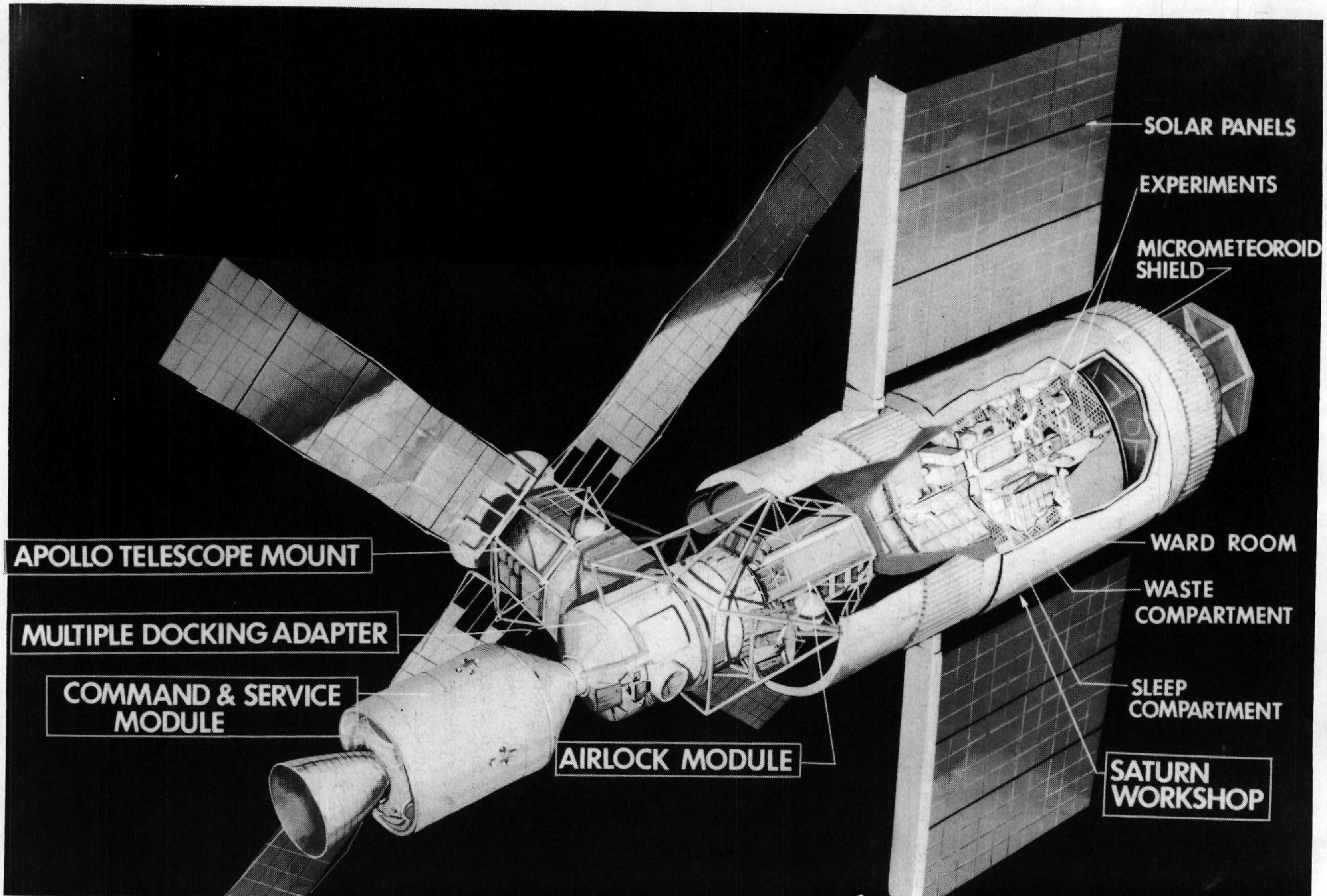

Figure 1. Skylab cluster.

treated in the literature [2-14] for the past 40 years. The dynamic description of mechanical and electrical systems is analogous [15], such that the electrical network theory is applicable to both cases, at least for one-dimensional problems. Also, the analogy permits a discussion in mechanical terms, which are preferred here because control problems are thus directly interpreted. The simple feedback loop shown in Figure 2 serves as a brief explanation of the concept. The plant of Figure 2 is a passive mechanical system with a torsion spring, a_0, damping, a_1, mass moment of inertia, a_2, and an additional spring-mass system, S, where S can also be described by a quotient. The controller consists of displacement and velocity sensors, their respective signal amplifications, b_0 and b_1, and the control force which lags with a time constant, τ. If the system is constant (linear and time invariant) and initially at zero rest, then the response of the feedback of Figure 2 yields an equation in Laplace transform:

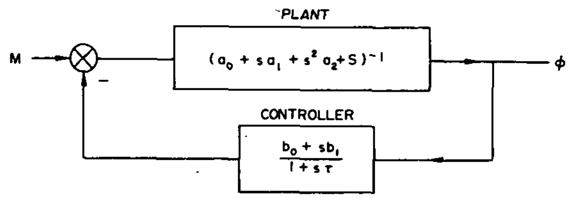

Figure 2. Feedback loop with lead-lag controller.

$$M = \phi\left(\frac{b_0 + sb_1}{1 + s\tau} + a_0 + sa_1 + s^2a_2 + S\right) \quad . \tag{1}$$

The factor on the right of equation (1) usually delivers the characteristic polynomial, whose roots determine the stability. Here, however, a reactive stabilization (discussed in the next three sections) will be applied which describes the structure of systems with two elementary connective operations, the addition and the reduction

$$a \times b = \frac{a \cdot b}{a + b} \quad . \tag{2}$$

For example, addition represents parallel arrangements of springs, where the total stiffness is the sum of the individual stiffnesses; reduction is a series connection of springs, where the total stiffness results from the reduction of the individual stiffnesses. These connections can also be used for dashpots and masses, where the latter is always connected to an inertial reference. The same connective operations are also found in electrical series and parallel circuits.

In equation (1), M is a moment and ϕ is an angular displacement; the objective is now to shape the response of the right side such that it behaves like a damped spring-mass system. Since a_0, a_1, a_2, and S represent a passive spring-mass arrangement, the system can be made stable, if the controller $(b_0 + sb_1)/(1 + s\tau)$ also behaves like a damped spring system. The main question now is how to structure the controller. With the help of a long division, we decompose first the quotient

$$\frac{b_0 + sb_1}{1 + s\tau} = b_0 + \frac{s\,(b_1 - b_0\tau)}{1 + s\tau} \tag{3}$$

into two additive parts and then with the help of the reduction into the structure

$$\frac{b_0 + sb_1}{1 + s\tau} = b_0 + \left[s\left(b_1 - b_0\,\tau\right) \times \frac{b_1 - b_0\tau}{\tau}\right] \quad . \tag{4}$$

Structure (4) is now completely decomposed such that mechanical elements can be recognized: spring b_0, dashpot $b_1 - b_0\tau$, and again a spring $(b_1 - b_0\tau)/\tau$. The plus indicates that b_0 is in parallel to the component contained in the brackets, where a reduction symbol indicates that $b_1 - b_0\tau$ and $(b_1 - b_0\tau)/\tau$ are connected in series. However, structure (4) cannot consist of springs and dashpots unless the resolved parameters are all positive:

$$b_0 > 0,\; b_1 - b_0\tau > 0,\; \tau > 0 \quad . \tag{5}$$

If inequalities (5) are satisfied, then structure (4) behaves like a damped spring system, which is not only stable and passive but also remains stable when connected to another spring-mass system (e.g., S). A substitution in equation (1) yields

$$M = \phi\left\{b_0 + \left[s\left(b_1 - b_0\,\tau\right) \times \frac{b_1 - b_0\tau}{\tau}\right] + a_0 + sa_1 + s^2 a_2 + S\right\} \quad , \tag{6}$$

which with conditions (5) represents a damped spring-mass system as demonstrated in Figure 3.

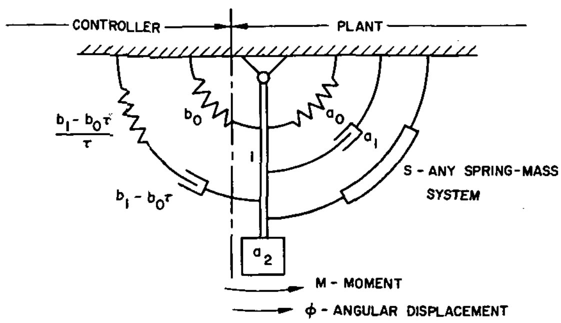

Figure 3. Mechanical equivalent of equation (6).

The system in Figure 2 is certainly stable, if conditions (5) are satisfied, which is intuitively clear from the mechanical equivalence in Figure 3. The mathematical proof is based on half-plane properties of the frequency response as defined and shown in the sections called reactive functions and reactive operations.

If the system in Figure 2 is now treated the conventional way (e.g., after Hurwitz), then a characteristic polynomial is extracted from equation (1) where ϕ is the output. This means that S, which can also be a quotient, must be considered. Stating simply that S is passive will not suffice; all details of S must be known. If

$$S = 0 \tag{7}$$

is assumed, then a third-order polynomial

$$p(s) = a_0 + b_0 + s\left(a_1 + a_0\tau + b_1\right) + s^2\left(a_2 + a_1\tau\right) + s^3 a_2 \tau \tag{8}$$

is found, which results in the Hurwitz conditions

$$\begin{vmatrix} a_2 + a_1\tau & a_0 + b_0 \\ a_2\tau & a_1 + a_0\tau + b_1 \end{vmatrix} > 0, \quad a_1 + a_0\tau + b_1 > 0 \tag{9}$$

or the inequality

$$b_1 > \frac{b_0 a_2 \tau - a_1 \left(a_2 + a_1 \tau + a_0 \tau^2\right)}{a_2 + a_1 \tau} \quad , \tag{10}$$

where all parameters are positive.

The Hurwitz case gives a lower bound for the rate gain, b_1, as the concept of mechanical equivalence which, according to inequalities (5), results in the simple condition

$$b_1 > b_0 \tau \quad . \tag{11}$$

A comparison of inequalities (10) and (11) shows that mechanical equivalence is more conservative than the conventional approach. But mechanical equivalence accepts any passive plant with numerous and unknown resonances, while the conventional approach requires an exact specification of the plant (e.g., S = 0) as in this example. If the plant has no damping ($a_1 = 0$) then Hurwitz condition (10) and inequality (11) become equally restrictive.

Mechanical equivalence is based on the following reactive stabilization principle.

Definition 1: A system, connected by addition, is reactively stable when the system is stable in the decomposed and the assembled state.

Reactive stabilization is similar to the absolute stability of electrical networks [16] which are stable regardless of terminating impedances. Here, the elements or building blocks are reactive functions as defined in the next section, structural connections are reactive operations as given in the second section, and n-dimensional presentations are treated with reactive matrices in the third section.

Stability analysis on subsystems (here, the controller) is thus possible and plant changes caused by different passive configurations do not affect stability; this apparently is an advantage in docking and deployment situations of space stations.

REACTIVE FUNCTIONS

Reactive stabilization is applicable to constant systems and represents systems or subsystems by mappings of the right half-plane's contour with Nyquist-type plots [17] relative to the origin. Reactive connections [i.e., additions and reductions (equation (2)] are important because they propagate half-plane properties of mappings of the contour domain depicted in Figure 4.

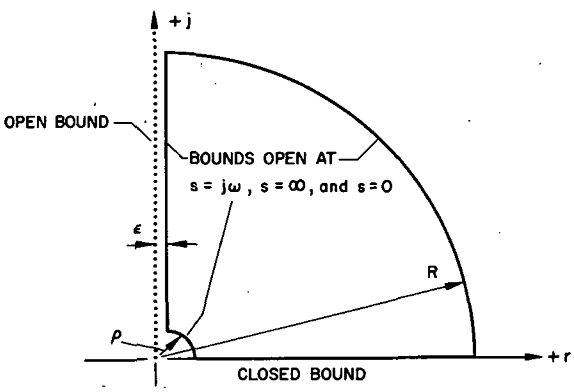

Figure 4. Contour of the first quadrant with $\rho > 0$, $\lim \rho = 0$, $\epsilon > 0$, $\lim \epsilon = 0$, $\infty > R > 0$, and $\lim R = \infty$.

The contour about the first quadrant can also be used to enclose the fourth quadrant by taking the complex conjugate. Any poles and zeros, enclosed in the right half-plane and the contour of the first quadrant, are thus related to the mapping's encirclements of the origin in multiples of 180 deg (N_π). From the residue theorem [18, 19] follows

$$N_\pi = Z - P, \tag{12}$$

where Z and P are the zeros and poles, respectively, within the contour of Figure 4 and the contour's complex conjugate. Figure 4 describes one domain for the contour mapping; however, three mapping ranges were selected according to Figures 5 through 7 and the definitions of three different reactive functions: positive imaginary, positive real, and negative imaginary.

The domain and the ranges of Figures 4 through 7 are defined as follows.

Definition 2: The domain of contour mappings by reactive functions is the first quadrant with a closed bound on the positive real axis and open bounds at the origin, the point of infinity, and the positive imaginary axis (Fig. 4).

Definition 3: The range of contour mappings of positive imaginary (denoted + j) functions is the upper half-plane with a closed bound on the positive real axis and open bounds at the origin, the point of infinity, and the negative real axis (Fig. 5).

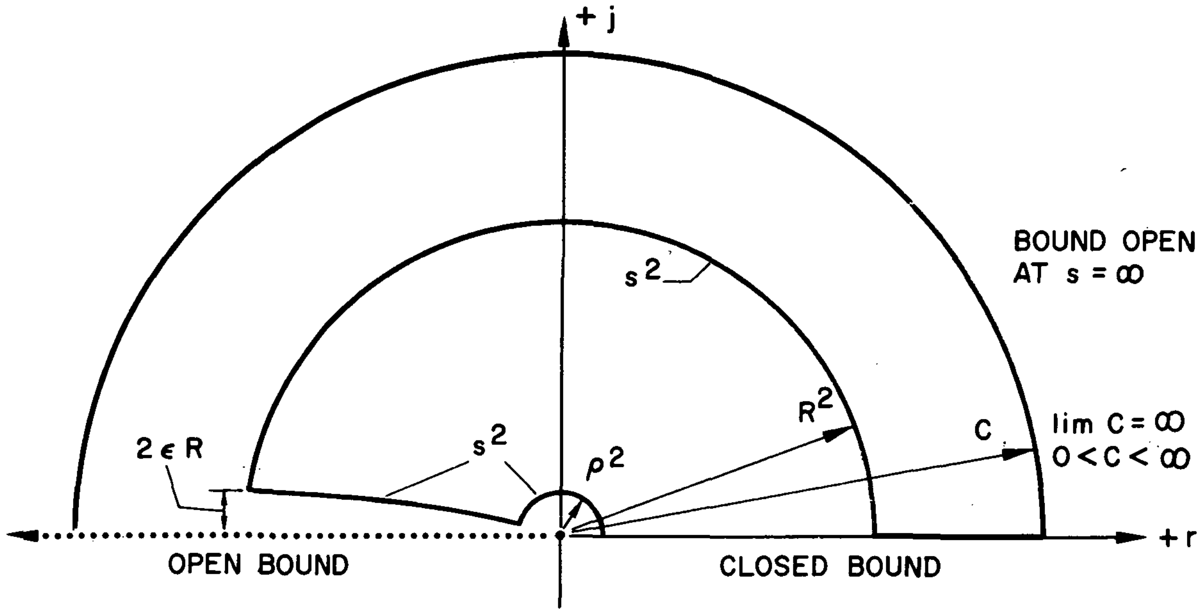

Figure 5. Range of contour mappings for positive imaginary functions with example s^2.

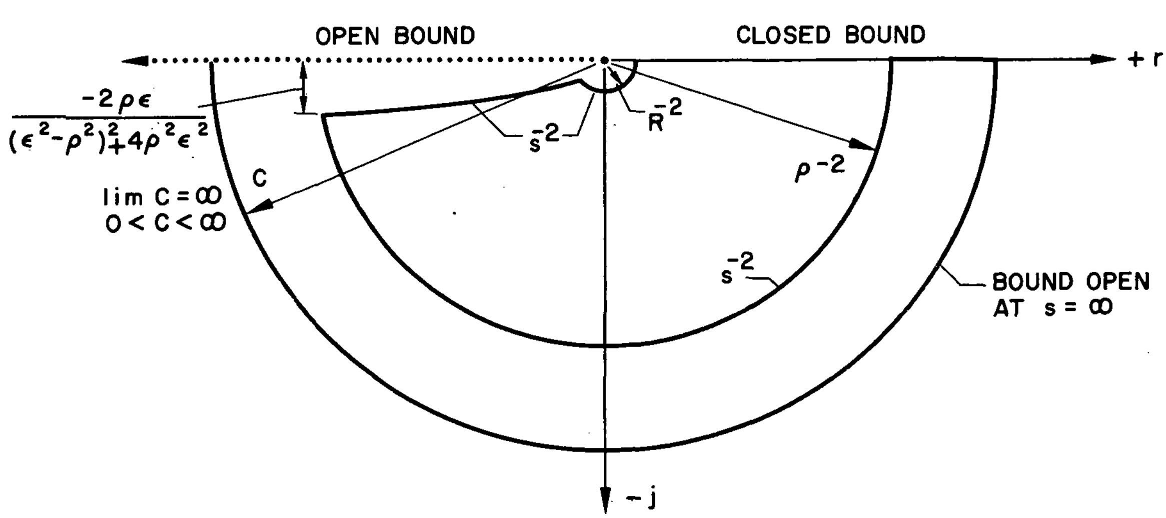

Figure 6. Range of contour mappings for negative imaginary functions with example s^{-2}.

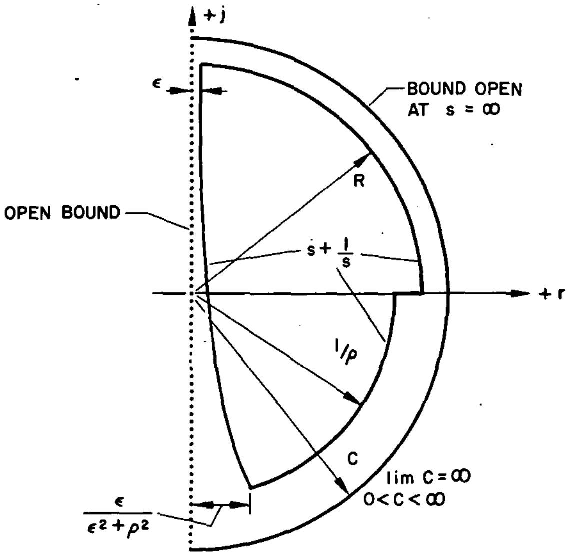

Figure 7. Range of contour mappings of positive real functions with example $s + 1/s$.

Definition 4: The range of contour mappings of positive real (denoted + r) functions is the right half-plane with open bounds at the origin, the point of infinity, and the whole imaginary axis (Fig. 7).

Definition 5: The range of contour mappings of negative imaginary (denoted - j) functions is the lower half-plane with a closed bound on the positive real axis and open bounds at the origin, the point of infinity, and the negative real axis (Fig. 6). The point of infinity considered here is one pole on the Riemann sphere [18, 19].

Definition 6: Functions are reactive iff[1] they have no poles within the contour of definition 2 and the contour's complex conjugate, iff they map the positive real axis on itself, and iff they map the contour of definition 2 within one of the ranges of definitions 3 through 5.

Theorem 1: All reactive functions imply no zeros within the contour of definition 2 and the contour's complex conjugate.

Theorem 1 is proven in denying the implication and assuming that at least one zero ($Z = 1$) exists. But the contour mappings of definitions 3 through 5 do not encircle the origin ($N_\pi \doteq 0$), reactive functions of definition 6 do not contain any poles within the contour of definition 2 ($P = 0$), and the hypothesis of reactive functions yields with equation (12) a $Z = 0$. Therefore, the denial of the implication ($Z = 1$) leads to a contradiction and theorem 1 is proven.

Theorem 2: All (+ j)-functions imply for $s = 0$ and $s = \infty$ a limit cs^n ($c > 0$), where $n \in \{0, 1, 2\}$.

Theorem 3: All (+ r)-functions imply for $s = 0$ and $s = \infty$ a limit cs^n ($c > 0$), where $n \in \{-1, 0, 1\}$.

1. If and only if.

Theorem 4: All $(-j)$-functions imply for $s = 0$ and $s = \infty$ a limit cs^n $(c > 0)$, where $n \in \{-2, -1, 0\}$.

Theorems 2 through 4 are proven by denying the implications which means in the order of the theorems that $n \notin \{0, 1, 2\}$; $n \notin \{-1, 0, 1\}$, and $n \notin \{-2, -1, 0\}$. A substitution of the boundaries $\rho > 0$, $\lim \rho = 0$, $\epsilon > 0$, $\lim \epsilon = 0$, $\infty > R > 0$, and $\lim R = \infty$ into s^n leads to contradictions with reactive functions as given by definitions 2 through 5. The open bounds of Figure 4. are represented for the origin by a small quarter circle with $s = \rho e^{j\delta}$ $(\rho > 0$; $\lim \rho = 0$, $0 \leq \delta < \pi/2)$, for the point of infinity by a large quarter circle with $s = Re^{j\delta}$ $(\infty > R > 0$, $\lim R = \infty$, $0 \leq \delta < \pi/2)$, and for the positive imaginary axis by a line with $s = \epsilon + j\omega$ $(\epsilon > 0$, $\lim \epsilon = 0$, $\rho \leq \omega \leq R)$. A substitution of these boundaries into s^n proves theorems 7 through 9.

Theorem 5: All $(+j)$-functions $q(s)$, excluding constants, imply a mapping of the line $s = \epsilon + j\omega$ $(\epsilon > 0$, $\lim \epsilon = 0$, $\rho \leq \omega \leq R)$ such that $\mathrm{Im}\,[q(\epsilon + j\omega)] > 0$.

Theorem 6: All $(+r)$-functions $q(s)$ imply a mapping of the line $s = \epsilon + j\omega (\epsilon > 0$; $\lim \epsilon = 0$, $\rho \leq \omega \leq R)$ such that $\mathrm{Re}\,[q(\epsilon + j\omega)] > 0$.

Theorem 7: All (-j)-functions q(s), excluding constants, imply a mapping of the line $s = \epsilon + j\omega$ $(\epsilon > 0$, $\lim \epsilon = 0$, $\rho \leq \omega \leq R)$ such that $\mathrm{Im}\,[q(\epsilon + j\omega)] < 0$.

Theorems 5 through 7 are proved by denying again the implications which produce in the order of the theorems $\mathrm{Im}\,[q(\epsilon + j\omega] \leq 0$, $\mathrm{Re}\,[q(\epsilon + j\omega)] \leq 0$, and $\mathrm{Im}\,[q(\epsilon + j\omega)] \geq 0$. Such mappings would lie outside the range specified in definitions 3 through 5, thus contradicting the hypothesis of reactive functions, and theorems 5 through 7 are proven.

Theorem 8: A function q(s) is a reactive iff q(s) is stable in the sense of definition 6, iff all coefficients are positive real, and iff at least one of the theorem-pairs 2 and 5, 3 and 6, or 4 and 7 holds.

The proof begins with the "only if" part which equates reactive functions with definitions 2 through 6. The latter produces all theorems (2 through 7) used in theorem 8. Definition 6 implies (asymptotic or semi-) stability and positive real coefficients. Definitions 2 through 5 imply that the limit cases ($s = 0$, $s = \infty$) and the mapping of the open bound on the positive imaginary axis must be in the same half-plane, which results in the pairing of theorems 2 and 5, 3 and 6, and 4 and 7. Definitions 3 through 5 give three choices, implying at least one of the three pairs must be valid.

The proof continues with the "if" part. Stability and positive coefficients imply definition 6. The pairing of theorems 2 through 7 implies that the limit cases of theorems 2 through 4 must lie in the same half-plane as the mapping of the open bound on the positive imaginary axis (theorems 5 through 7). Therefore, the contour of definition 2 is mapped at least within one of the ranges of definitions 3 through 5. Thus, definitions 3 through 6 are met which implies that the function $q(s)$ is a reactive function. Both proof parts show that theorem 8 gives necessary and sufficient conditions for reactive functions.

The stability of reactive functions is found directly from the denominator's polynomial which is also related to the following mapping property.

Theorem 9: Reactive (+ j)-, (+ r)-, and (- j)-functions imply respectively that the minimum of all imaginary parts, the minimum of all real parts, and the maximum of all imaginary parts lie only on the contour of definition 2.

The proof of theorem 9 follows from the maximum principle of harmonic functions [19] which relates the analytic property of these functions to the maximum at the boundary of a simply connected s-domain. For example, in the case of a (+ j)-function $q(s) = u(s) + j\,v(s)$ [$v(s) \geq 0$], we use the auxiliary form

$$w(s) = e^{jq(s)} = e^{-v(s)} e^{ju(s)} \tag{13}$$

and find from the differential

$$w'(s) = jq'(s)e^{jq(s)} \tag{14}$$

that function $w(s)$ is analytic if the function $q(s)$ is analytic and satisfies the Cauchy-Riemann differential equations [18, 19]. Since reactive functions $q(s)$

are analytic, $w(s)$ must be analytic. If we deny now the implication for $(+j)$-functions and state that the imaginary part's minimum $v(a) = m$ $(m > 0)$ lies at a point "a" inside the contour of definition 2, then the analyticity yields with Gauss' mean-value theorem the inequality

$$e^{-m} \le \frac{1}{2\pi} \left| \int_0^{2\pi} w\left(a + re^{j\phi}\right) d\phi \right| = \frac{1}{2\pi} \int_0^{2\pi} e^{-v(a + rej\phi)} d\phi \quad . \tag{15}$$

Since $-m$ was assumed to be a maximum, there must be at least one angular section $\Delta\phi = \phi_2 - \phi_1$ where

$$e^{-m} > e^{-v(a + rej\phi)} = e^{-m-\epsilon}, \quad \epsilon > 0, \text{ and } \phi_1 < \phi < \phi_2 \tag{16}$$

holds. Consequently, inequality (15) results in

$$e^{-m} \le e^{-m}\left[1 - \frac{\Delta\phi}{2\pi}\left(1 - e^{-\epsilon}\right)\right] < e^{-m} \quad , \tag{17}$$

which is obviously a contradiction. Therefore, the maximum cannot be at a point "a" interior to the contour of definition 2, and theorem 9 is proven for $(+j)$-functions. The proofs for $(+r)$- and $(-j)$-functions are similar, if the respective functions $w(s) = e^{-q(s)} = e^{-u(s)}\, e^{-jv(s)}$ and $w(s) = e^{-jq(s)} = e^{v(s)}\, e^{-ju(s)}$ are used.

Theorem 10: A function $q(s)$ is reactive iff real values $s = r$ are mapped as real values $q(r)$ and iff one of the ranges of definitions 3 through 5 is occupied by the mappings of all the contours within the contour of definition 2.

The proof begins with the "only if" part that a function is reactive which, according to definition 6, implies that real values $s = r$ are mapped as real values $q(r)$. Reactive functions also map the contour of definition 2 within the ranges specified by one of definitions 3 through 5. Definition 6 and theorem 1 show that the contour of definition 2 contains no poles and zeros; therefore, mappings of any contours interior to definition 2 will not encircle the origin and with theorem 9 will stay within one of the ranges of definitions 3 through 5. Therefore, all implications of the "only if" part are met.

The second half of the proof continues with the "if" part of mapping the positive real axis onto itself; this satisfies the real part statement of definition 6. The "if" part of the ranges satisfies also definitions 3 through 5 for all contours within definition 2, which implies that the function does not have any poles and zeros within the contour of definition 2. All conditions of definition 6 are met and the function is reactive. Both halves of the proof are thus completed and theorem 10 gives necessary and sufficient conditions.

Well known in network theory [2-14] are positive real functions which are the (+ r)-functions of definition 4 when definition 6 is also satisfied.

Definition 7: A function $q(s)$ is positive real iff real values $s = r$ result in $\mathrm{Im}\ [q(r)] = 0$ and iff $s \in [s: \mathrm{Re}(s) > 0]$ produces $\mathrm{Re}\ [q(s)] > 0$.

Definition 7 and theorem 9 result in the properties of definitions 2, 4, and 6. Since definition 7 and theorem 9 are not easily applied, the contour approach of definitions 2 through 6 is preferred here. Note also that (+ r)-functions result in (+ j)- or (- j)-functions when multiplied by s or when divided by s, respectively.

Definitions 3 through 5 and Figures 5 through 7 represent three different types of reactive functions, depending on the half-plane property of a transfer function or a quotient. For example, a constant element is a (+ j)-, (+ r)-, and (- j)-function; an element s is a (+ j)- and a (+ r)-function; an element s^{-1} is a (+ r)- and a (- j)-function; an element s^{+2} is a (+ j)-function; an element $s + s^{-1}$ is a (+ r)-function; and an element s^{-2} is a (- j)-function. These half-plane properties are preserved when the elements are properly connected and thus permit the construction of stable systems because the resulting reactive functions have no poles and no zeros within the contour of definition 2 according to definition 6 and theorem 1.

REACTIVE OPERATIONS

Reactive stabilization is a building-block approach, where the elements belong to one reactive function set, specified by definitions 3 through 5, and where the structure is given by reactive operations.

Definition 8: Operations producing reactive functions from functions of the same reactive type are denoted as reactive operations.

If an operation of definition 8 connects two (+ j)-functions, a (+j)-function will result. The example applies also to (+ r)- and (- j)-functions; but mixing between the three function sets is not allowed here and will not be discussed, even through mixing is advantageous in certain cases [1].

Theorem 11: The addition is a reactive operation.

Theorem 12: The reduction, defined in equation (2), is a reactive operation.

In proving theorem 11 we observe that the mappings of the contour from definition 2 are contours which are described by a complex vector. Therefore, the addition of reactive functions is a vector addition where the resultant is given by the addition of the vector's components. From the half-plane properties reflected in theorems 2 through 7 and definitions 3 through 5, we see that the vectors are either positive real or complex. When complex, the vectors have at least one (either positive or negative) definite component. The addition of positive real parts results always in positive real parts and the addition of positive (negative) imaginary parts will always produce positive (negative) imaginary parts. Therefore, the addition preserves the half-plane property of reactive functions as long as the function types are not mixed. Also, the addition preserves stability because only stable elements are added. Thus, all conditions for reactive functions (definitions 2 through 6) are met, definition 8 is satisfied, and theorem 11 is proven.

For the proof of theorem 12 we consider first the reductive vector presentation

$$z = a \times jb = a \frac{b^2}{a^2 + b^2} + jb \frac{a^2}{a^2 + b^2} = u + jv \qquad , \qquad (18)$$

where a, b, u, and v are real numbers. Equation (18) shows that the reductive and the additive components are of the same polarity, which means that the positive or negative definitiveness of one component, as given by a reactive function set, will not be changed by the choice of the vector presentation. For reductive connections it is advantageous to use the reductive presentation

$$z = z_1 \times z_2 = a_1 \times jb_1 \times a_2 \times jb_2 = (a_1 \times a_2) \times j(b_1 \times b_2) = a \times jb \quad , \qquad (19)$$

because the resultant components

$$a = a_1 \times a_2 \qquad \text{and} \qquad b = b_1 \times b_2 \qquad (20)$$

are simply determined. Equation (2) shows that the reduction does not change the polarity when real numbers with the same sign are reduced; e.g., if b_1 and b_2 are both positive (negative) definite, their reduction will also be positive (negative) definite. Thus, the half-plane characteristic of reactive functions is also preserved by this operation. Further, we find that the reduction of two reactive quotients

$$\frac{N_1(s)}{D_1(s)} \times \frac{N_2(s)}{D_2(s)} = \frac{N_1(s) \cdot N_2(s)}{D_1(s)N_2(s) + D_2(s)N_1(s)} \qquad (21)$$

produces a stable common numerator N_1N_2 with only stable zeros $(Z = 0)$, since the individual numerators N_1 and N_2 have only stable roots (theorem 1). The stability of the zeros $(Z = 0)$ and the half-plane property $(N_\pi = 0)$ causes only stable poles in equation (21) because equation (12) results in $P = Z = 0$. All requirements of reactive functions (definitions 2 through 6) are thus met, definition 8 is satisfied, and theorem 12 is proven.

Analogous to the subtraction we introduce now the negative reduction or the subduction

$$a \setminus b = \frac{ab}{b - a} \quad , \qquad (22)$$

which together with the subtraction is not a reactive operation. Between the addition and the reduction exists a duality which is listed in Table 1. The numbers a, b, and c are complex and are unequal to zero or infinity, where infinity represents the point of infinity on the Riemann sphere [18, 19]. Addition and reduction form two Abelian groups which are coupled through the unification rule. Both groups are not distributive among each other, but are distributive with multiplication and form two rings [20] which are again coupled through the unification.

TABLE 1. DUALITY OF ADDITION AND REDUCTION

Associativity:	$a + (b + c) = (a + c) + c$	$a \times (b \times c) = (a \times b) \times c$
Distributivity:	$a(b + c) = ab + ac$	$a(b \times c) = ab \times ac$
Commutativity:	$a + b = b + a$	$a \times b = b \times a$
Repetition:	$a + a + a = 3a$	$a \times a \times a = a/3$
Perpetuation:	$a + a + a + \ldots = \infty$	$a \times a \times a \times \ldots = 0$
Identity:	$a + 0 = a$	$a \times \infty = a$
Cancellation:	$a - a = 0$	$a \setminus a = \infty$
Unification:	$(a + a) \times (a + a) = a$	$(a \times a) + (a \times a) = a$
Inversion:	$(a + b)^{-1} = a^{-1} \times b^{-1}$	$(a \times b)^{-1} = a^{-1} + b^{-1}$

Multiplication cannot be identified as a reactive operation, since it can rotate a complex vector outside the range of a specified half-plane. In special cases, multiplication produces reactive functions, but generally this is not true. However, the inversion always preserves reactive functions.

Theorem 13: Inverse reactive functions are reactive functions.

Theorem 14: Inversions are reactive (+ r)-operations.

Inversions produce vectors pointing in the direction of the complex conjugate and thus do not change the real part, but they reverse the polarity of the imaginary part. The half-plane properties are preserved for (+ r)-functions, but (+ j)-functions exchange the half-planes with (- j)-functions and conversely. Poles and zeros are only rotated, but are stable for reactive functions. Thus, definitions 3 through 6 are satisfied and theorem 13 is proven. For (+ r)-functions, definition 8 also holds and theorem 14 is proven.

The synthetic approach to stability was first published in 1966 [1] with a different notation (e.g., reduction was then called inverse addition). In 1967 [21], Lewis suggested a complementary algebra for describing electrical switching circuits, where he denotes a "complementary addition" or "sulp" for the operation which is called reduction here. The algebraic features of systems are obviously present and can be a valuable tool if properly defined as in the case of the addition and reduction which propagate stability.

REACTIVE MATRICES

In multivariable feedback cases [22] inputs and outputs are related by a transfer matrix A(s) whose elements $a_{ik}(s)$ are transfer functions or quotients. Also, an attitude control system of a space station belongs to this category, because of the three-dimensional feedback from the structure as shown for the Skylab in the next section. This section brings the theory of reactive stability for n-dimensional cases based on quadratic forms y^*Ay (asterisk means complex conjugate transpose).

Definition 9: A transfer matrix A(s) is stable, when all elements $a_{ik}(s)$ are (asymptotic or semi-) stable transfer functions.

Definition 10: Matrix A(s) is complex definite, when the quadratic form $q(s) = y^* Ay$ has for all vectors $y \neq 0$ or ∞ at least one complex component which is either positive or negative definite.

Definition 11: Matrix A(s) is reactive, when A is complex definite and nonsingular, and when the quadratic form $q(s) = y^* Ay$ is a reactive function for all vectors $y \neq 0$ or ∞.

Theorem 15: Reactive matrices are stable.

The proof uses definition 11, where the quadratic form $y^* Ay$ is a reactive function for all vectors $y \neq 0$ or ∞. The quadratic form combines linearly all matrix elements $a_{ik}(s)$ and therefore draws all its poles from these elements, except for possible pole-zero cancellations. General cancellations involve a factor k common to all matrix elements $a_{ik} = k\ \alpha_{ik}$, such that k is still a part of each element; special cancellations for certain y-vectors cannot hide any poles because other y-vectors will show them. Therefore, the quadratic form $y^* Ay$ for all vectors $y \neq 0$ or ∞ represents all poles of the elements a_{ik} which must be stable by definition 6, since $y^* Ay$ is a reactive function. With definition 9, stable elements lead to a stable matrix, and theorem 15 is proven.

Theorem 16: Nonsingular complex definite matrices have complex definite inverses.

The proof uses the transformation

$$z = Ay \tag{23}$$

and the quadratic form

$$q^*(s) = y^* A^* y = z^* A^{-1} z \tag{24}$$

By hypothesis, matrix A was complex definite. Since the complex conjugate of the quadratic form $q^* = (y^* Ay)^* = y^* A^* y$ only changes the sign of the imaginary part, matrix A^* must be also complex definite. Since all vectors $y \neq 0$ or ∞ are involved (definition 10) and matrix A is nonsingular by hypothesis, transformation (23) produces all possible vectors $z \neq 0$ or ∞. Equation (24) is therefore valid for all possible vectors y and z, and the quadratic form $q^* = z^* A^{-1} z$ must be complex definite for all vectors $z \neq 0$ or ∞, thus proving theorem 16.

Theorem 17: Inverse reactive matrices are reactive.

The proof begins with the form $y^* A^* y$ of equation (24) which is a reactive function for a mapping of the complex conjugate contour of definition 2. This means that $y^* A^* y$ has the half-plane and the stability properties of reactive functions. Theorem 10 is therefore applicable even though complex conjugate contours are used here. All contours within the complex conjugate contour of definition 2 must result in one and the same half-plane property for all vectors $y \neq 0$ or ∞. Since transformation (23) is not singular and equation (24) holds, the quadratic form $z^* A^{-1} z$ must maintain the same half-plane properties as $y^* A^* y$ for any vector $z \neq 0$ or ∞. Additionally, all real values $s = r$ are mapped as real values $q^*(r)$, thus satisfying all conditions of theorem 10 and the form $z^* A^{-1} z$ is a reactive function for any vectors $z \neq 0$ or ∞. Therefore, matrix A^{-1} is by definition 10 complex definite, by assumption nonsingular, and consequently reactive by definition 11. This proves theorem 17. Note that by theorem 15, matrix A^{-1} is also stable, which is important in proving overall stability for n-dimensional cases, especially when connections are used as emphasized by the next statements.

Definition 12: The reduction of matrices A and B is defined as $A \times B = (A^{-1} + B^{-1})^{-1}$.

Definition 13: Operations producing reactive matrices from matrices of the same reactive type are denoted as reactive matrix operations.

Theorem 18: Addition is a reactive matrix operation.

Theorem 19: Reduction, according to definition 12, is a reactive matrix operation.

The proof of theorem 18 uses the quadratic form

$$y^* (A + B) y = y^* Ay + y^* By \quad , \tag{25}$$

where A and B are reactive matrices. The right of equation (25) is a sum of reactive functions which again is a reactive function. This is valid for all vectors $y \neq 0$ or ∞. Equation (25) equates all properties of reactive matrices with the quadratic form of matrix A + B which therefore must be reactive, and theorem 18 is proven.

The proof of theorem 19 employs the quadratic form

$$z^* (A^{-1} + B^{-1}) z = z^* A^{-1} z + z^* B^{-1} z \quad , \tag{26}$$

where A and B are reactive matrices. By theorem 17, A^{-1} and B^{-1} are also reactive matrices and by theorem 18, $A^{-1} + B^{-1}$ is also a reactive matrix. Definition 12 leads to the quadratic form

$$y^* (A \times B) y = y^* (A^{-1} + B^{-1})^{-1} y \quad , \tag{27}$$

where $A^{-1} + B^{-1}$ is a reactive matrix whose inverse is by theorem 17 reactive again. Therefore, the reduction of matrices of the same reactive kind preserve the reactive property, definition 13 is satisfied, and theorem 19 is proven. Note that the reduction represents a difficult matrix operations, but permits a rather easy stability assessment if matrices A and B are reactive.

ATTITUDE CONTROL LOOP

The following sections demonstrate that reactively stabilized attitude control systems are feasible for space stations. The Skylab's control moment gyro (CMG) loop [23-27] has been chosen to show a possible solution with rate gyros and angular accelerometers as attitude sensors. The sensors are mounted near the CMGs on the racks of the Apollo Telescope Mount (ATM) as depicted in Figure 8. The discussion is limited to the CMG loop, even though the principle can be extended to include the experiment pointing control system (EPCS) and the thruster attitude control system (TACS). The CMG control loop is shown in Figure 9 by a vector signal flow diagram.

The commanded attitude is given by the vector ϕ, C is a control gain matrix, G is the CMG response matrix, M is the moment vector applied to the Skylab structure, P is the Skylab's structural response matrix, γ is the attitude vector measured by three rate gyros and three angular accelerometers, and Q is the sensor response matrix feeding back to the control gain matrix G. Because the Skylab rotates slowly, coriolis and centrifugal forces are neglected and the dynamic behavior is expressed by the linearized vector form

$$\gamma = PM \tag{28}$$

The plant response is given in Laplace transform by the transfer matrix P which is the sum

$$P = \frac{J^{-1}}{s^2} + K \tag{29}$$

of the moment of inertia matrix

$$J = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} - \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} [J_1\ J_2\ J_3] \tag{30}$$

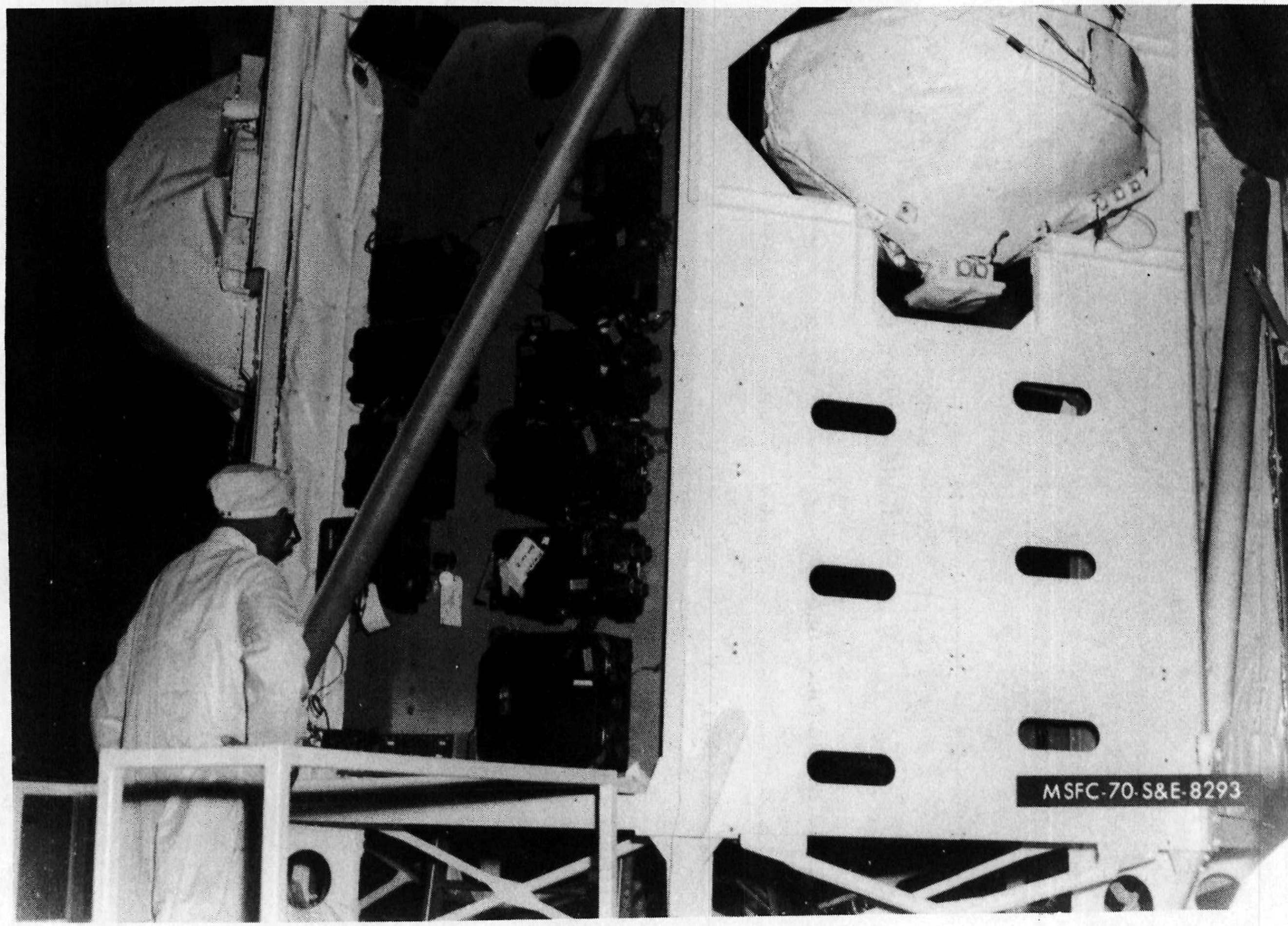

Figure 8. ATM rack with CMG.

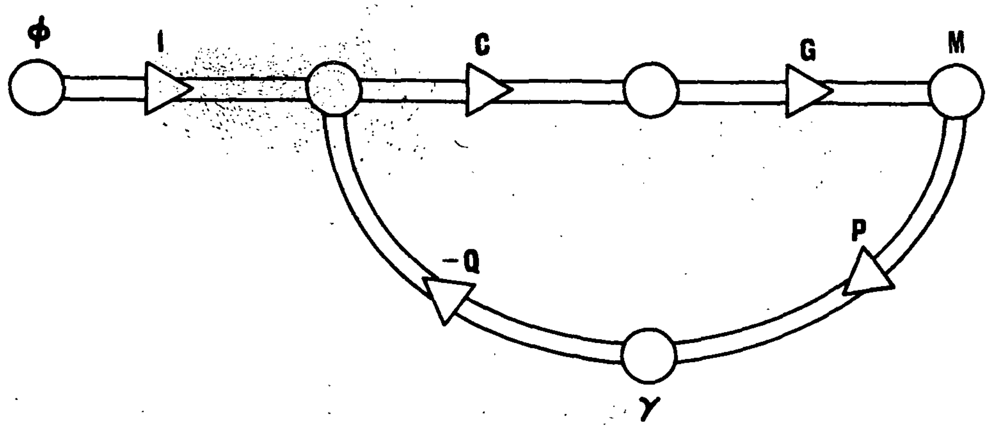

Figure 9. Vector signal flow diagram of a CMG attitude control system.

and the resonance matrix

$$K = Y'\Omega^{-1}Y \quad . \tag{31}$$

Matrix K is the product of the transpose of the mode matrix

$$Y' = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & \dots & Y_{1n} \\ Y_{21} & Y_{22} & Y_{23} & \dots & Y_{2n} \\ Y_{31} & Y_{32} & Y_{33} & \dots & Y_{3n} \end{bmatrix} \tag{32}$$

(' means transposed) and the modal resonance matrix

$$
\Omega = \begin{bmatrix}
m_1(s^2 + s2\zeta_1\omega_1 + \omega_1^2) & 0 & 0 & \cdots & 0 \\
0 & m_2(s^2 + s2\zeta_2\omega_2 + \omega_2^2) & 0 & \cdots & 0 \\
0 & 0 & m_3(s^2 + s2\zeta_3\omega_3 + \omega_3^2) & \cdots & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & m_n(s^2 + s2\zeta_n\omega_n + \omega_n^2) &
\end{bmatrix} . \tag{33}
$$

The inertial matrix, equation (30), is always the sum of a diagonal and a diadic matrix. The resultant elements are moments and produce moments of inertia. The mode matrix elements, equation (32), are modal slopes at the area of the CMGs and the nearby located attitude sensors (rate gyros and angular accelerometers). The first index of the elements represents body-fixed coordinates 1, 2, or 3, and the second index is related to the modal number. The modal resonance matrix has only diagonal elements which represent resonators with generalized masses m_i, damping factors ζ_i, and resonant frequencies ω_i.

QUADRATIC FORMS

The passivity of the Skylab structure follows from the quadratic form

$$
M^* \gamma = M^* PM = \frac{1}{s^2} M^* J^{-1} M + M^* Y' \Omega^{-1} Y M = q_1 \quad , \tag{34}
$$

where M^* is the conjugate transpose of the vector M. Since the inertial matrix J is always positive definite, we find with $M = J\mathbf{z}$ that

$$
M^* J^{-1} M = z^* Jz \tag{35}
$$

and therefore the form $M^* J^{-1} M$ is positive definite. Since $(YM)^* = M^* Y'$, the form $(YM)^* \Omega^{-1} (YM)$ results in a positive sum of second-order systems. Therefore, matrix P is complex definite (definition 10) and has a quadratic form which is a reactive (-j)-function (Fig. 6, definitions 2, 5, and 6). Consequently, matrix P is reactive as specified in definition 11. Since the plant P is a reactive (- j)-matrix, the controller must also be a reactive (- j)-matrix. The system in Figure 8 now yields the vector equation

$$\phi = (QP + C^{-1}G^{-1})M \tag{36}$$

which is changed to

$$Q^{-1}\phi = (P + Q^{-1}C^{-1}G^{-1})M \quad . \tag{37}$$

A new quadratic form

$$q(s) = M^* (P + Q^{-1}C^{-1}G^{-1})M = q_1 + q_2 \tag{38}$$

results where q_1 is already given by equation (34) and q_2 is defined by

$$q_2 = M^* Q^{-1} C^{-1} G^{-1} M \quad . \tag{39}$$

Equation (39) is the quadratic form of the controller whose reactive stability property must be further investigated to ensure that q_2 becomes a reactive (- j)-function. This principle permits a separation of the controller from the plant for a stability analysis similar to the example of the introduction (Figs. 2 and 3).

CONTROLLER RESPONSE

The response of the CMGs[2] is given by a diagonal matrix

$$G = \frac{I}{1 + s2\zeta/\omega_0 + s^2/\omega_0^2} \quad , \tag{40}$$

2. Data contributed by H. L. Shelton, S&E-ASTR-SD

where I is a 3 by 3 identity matrix. The angular sensor response is assumed to be negligible as expressed by the matrix

$$Q = I \quad . \tag{41}$$

The control gain matrix is

$$C = \begin{bmatrix} c_{01} + sc_{11} + s^2c_{21} & 0 & 0 \\ 0 & c_{02} + sc_{12} + s^2c_{22} & 0 \\ 0 & 0 & c_{03} + sc_{13} + s^2c_{23} \end{bmatrix} \tag{42}$$

where c_{0i} is the integrated rate gyro gain, c_{1i} is the rate gyro gain, and c_{2i} is the angular accelerometer gain; therefore, the total block $(GCQ)^{-1}$ has the gain

$$Q^{-1}C^{-1}G^{-1} = \begin{bmatrix} q_{21} & 0 & 0 \\ 0 & q_{22} & 0 \\ 0 & 0 & q_{23} \end{bmatrix} \qquad \begin{aligned} q_{21} &= \frac{1 + s2\zeta / \omega_0 + s^2 / \omega_0^2}{c_{01} + sc_{11} + s^2c_{21}} \\ q_{22} &= \frac{1 + s2\zeta / \omega_0 + s^2 / \omega_0^2}{c_{02} + sc_{12} + s^2c_{22}} \\ q_{23} &= \frac{1 + s2\zeta / \omega_0 + s^2 / \omega_0^2}{c_{03} + sc_{13} + s^2c_{23}} \end{aligned} \quad . \tag{43}$$

DECOMPOSITION OF CONTROLLER RESPONSE

The controller is represented by the quadratic form of equation (39) which with equation (43) produces the sum

$$q_2 = |M_1|^2 q_{21} + |M_2|^2 q_{22} + |M_3|^2 q_{23} \tag{44}$$

where all quotients q_{21} have only positive factors M_i^2. Therefore, q_2 will be a (- j)-function if all q_{2i} are reactive (- j)-functions like q_1, equation (34). The shaping is found by a decomposition into reactive (- j)-elements which are reactively connected (definition 8). First, we separate from q_{2i} a constant

$$p_1 = \frac{1}{c_{2i}\omega_0^2} \tag{45}$$

and obtain a remainder

$$r_1 = q_{2i} - p_1 = \frac{1 - c_{0i}p_1 + s\,(2\zeta/\omega_0 - c_{1i}p_1)}{c_{0i} + sc_{1i} + s^2c_{2i}} = r_2 \times r_3 \tag{46}$$

which is split by a reduction into two components

$$r_2 = \frac{1 - c_{0i}p_1 + s\,(2\zeta/\omega_0 - c_{1i}p_1)}{s^2c_{2i}} = r_4 + r_5 \tag{47}$$

and

$$r_3 = \frac{1 - c_{0i}p_1 + s\,(2\zeta/\omega_0 - c_{1i}p_1)}{c_{0i} + sc_{1i}} = r_6 + r_7 \quad . \tag{48}$$

From r_2 and r_3, we obtain the new components

$$r_4 = \frac{1 - c_{0i}p_1}{s^2c_{2i}} = \frac{p_2}{s^2} \quad , \tag{49}$$

$$r_5 = \frac{2\zeta/\omega_0 - c_{1i}p_1}{sc_{2i}} = \frac{p_3}{s} \quad , \tag{50}$$

$$r_6 = \frac{2\zeta/\omega_0 - c_{1i}p_1}{c_{1i}} = p_4 \quad , \tag{51}$$

and

$$r_7 = r_3 - r_6 = \frac{1 - c_{0i}p_1 - c_{0i}r_6}{c_{0i} + sc_{1i}} = r_8 \times r_9 \quad . \tag{52}$$

The remainder r_7 is decomposed further by a reduction which produces the elements

$$r_8 = \frac{1 - c_{0i}p_1 - c_{0i}r_6}{c_{0i}} = p_5 \tag{53}$$

and

$$r_9 = \frac{1 - c_{0i}p_1 - c_{0i}r_6}{sc_{1i}} = \frac{p_6}{s} \quad . \tag{54}$$

This completes the decomposition which yields the following structure

$$q_{2i} = p_1 + \left\{ \left(\frac{p_2}{s^2} + \frac{p_3}{s} \right) \times \left[p_4 + \left(p_5 \times \frac{p_6}{s} \right) \right] \right\} \tag{55}$$

where the elements have factors s^n with $n \in \{-2, -1, 0\}$ as given by equations (45), (49), (50), (51), (53), and (54). If all parameters p_i are positive, then q_{2i} are $(-j)$-functions and q_2 is a $(-j)$-function, because reactive operations connect $(-j)$-function elements.

REACTIVE STABILITY CONDITIONS

Decomposition (55) results in the same conditions as required to make the imaginary part of $q_{2i}(j\omega)$ negative. Equation (43) and theorem 7 yield for $\omega > 0$ the condition

$$\mathrm{Im}\,[q_{2i}(j\omega)] = \omega \; \frac{\frac{2\zeta}{\omega_0}(c_{0i} - \omega^2 c_{2i}) - c_{1i}\left(1 - \frac{\omega^2}{\omega_0^2}\right)}{(c_{0i} - \omega^2 c_{2i})^2 + \omega^2 c_{1i}^2} < 0 \quad . \tag{56}$$

Additionally, if all coefficients (c_{0i}, c_{1i}, c_{2i}, ζ, and ω_0) are positive real, then the limits for $s = 0$ and $s = \infty$ will be positive constants, and the poles and zeros will also be stable. Theorems 4, 7, and 8 are satisfied and all q_{2i} are reactive (-j)-functions. Inequality (56) results in the negative definite polynomial

$$\frac{2\zeta}{\omega_0} c_{0i} - c_{1i} + \omega^2 \left(\frac{c_{1i}}{\omega_0^2} - \frac{2\zeta}{\omega_0} c_{2i} \right) < 0 \tag{57}$$

which yields the inequalities

$$c_{2i} \omega_0^2 > \frac{c_{1i} \omega_0}{2\zeta} > c_{0i} > 0, \qquad i \in \{1, 2, 3\} \quad . \tag{58}$$

The same conditions are derived from decomposition (55), which is therefore a necessary and sufficient decomposition. Often, decompositions yield only sufficient (conservative) conditions which are unnecessarily restrictive and therefore must be avoided; e.g., by a proper structural pattern. Since the decomposition is rather difficult, an evaluation by complex component parts is recommended.

Inequalities (58) produce for Skylab's CMG control system $\omega_0 = 34$ and $\zeta = 0.5$ the following reactive stability condition:

$$c_{2i}\ 1160 > c_{1i}\ 34 > c_{0i} > 0 \quad . \tag{59}$$

The coefficients c_{0i} and c_{1i} are determined from the rigid-body resonances which are found by an approximation for small s values. Equations (28), (29), and (36) yield the rigid-body response

$$C\ \phi = (C + s^2 J)\ \gamma \tag{60}$$

with the control matrix

$$C = \begin{bmatrix} c_{01} + sc_{11} & 0 & 0 \\ 0 & c_{02} + sc_{12} & \\ 0 & 0 & c_{03} + sc_{13} \end{bmatrix} \tag{61}$$

and the inertial matrix of equation (30). The Skylab's inertial matrix [25] in kgm^2 is

$$J \cdot 10^{-4} = \begin{bmatrix} 101.6 & -2.155 & -38.4 \\ -2.155 & 615 & -3.45 \\ -38.4 & -3.45 & 605 \end{bmatrix} =$$

$$\begin{bmatrix} 125.6 & 0 & 0 \\ 0 & 615.2 & 0 \\ 0 & 0 & 666.5 \end{bmatrix} - \begin{bmatrix} 4.9 \\ 0.44 \\ 7.8 \end{bmatrix} [4.9 \ 0.44 \ 7.8] \quad . \tag{62}$$

Equation (60) has the form

$$C\ \phi = \left\{ \begin{bmatrix} c_{01} + sc_{11} + s^2 I_1 & 0 & 0 \\ 0 & c_{02} + sc_{12} + s^2 I_2 & 0 \\ 0 & 0 & c_{03} + sc_{13} + s^2 I_3 \end{bmatrix} - s^2 \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix} \right\} \gamma \tag{63}$$

where the diagonal elements are assumed to be proportionally designed:

$$c_{0i} + sc_{1i} + s^2 I_i = \frac{I_i}{I_0} (c_0 + sc_1 + s^2 I_0) \quad , \ i \in \{1,\ 2,\ 3\} \tag{64}$$

such that equation (63) takes the simple form

$$C\ \phi = \left\{ \frac{c_0 + sc_1 + s^2 I_0}{I_0} \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_3 \end{bmatrix} - s^2 \begin{bmatrix} J_1 \\ J_2 \\ J_3 \end{bmatrix} \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix} \right\} \gamma \quad . \tag{65}$$

The inverse of equation (65) is now

$$\gamma = \left\{ I + \begin{bmatrix} J_1/I_1 \\ J_2/I_2 \\ J_3/I_3 \end{bmatrix} \frac{[J_1\ J_2\ J_3]\ s^2 I_0}{c_0 + sc_1 + s^2 I_0 (1 - \mathrm{II})} \right\} \frac{c_0 + sc_1}{c_0 + sc_1 + s^2 I_0} \quad \phi \tag{66}$$

with the abbreviation

$$\mathrm{II} = \frac{J_1^2}{I_1} + \frac{J_2^2}{I_2} + \frac{J_3^2}{J_3} \quad . \tag{67}$$

Equation (66) relates the commanded orientation ϕ with the space station's angular position γ and shows two undamped resonances

$$\omega_{10} = \sqrt{\frac{c_0}{I_0}}$$

and

$$\omega_{20} = \sqrt{\frac{c_0}{I_0\ (1 - \mathrm{II})}} \tag{68}$$

which are rather close. For example, Skylab has a $\mathrm{II} = 0.285$ and the resonances differ only by approximately 14 percent.

The following approximate parameters are assumed for Skylab: $c_{01} = 16\ 000$ Nm, $c_{02} = 79\ 000$ Nm, $c_{03} = 85\ 000$ Nm, and $c_{1i} = 10\ c_{0i}$ Nms. Now inequalities (59) yield the reactive stability condition

$$c_{21}\ 1.16 > 5400 > 16 > 0$$

$$c_{22}\ 1.16 > 27\ 000 > 79 > 0 \tag{69}$$

$$c_{23}\ 1.16 > 29\ 000 > 85 > 0 \quad .$$

The Skylab case satisfies all conditions except for the c_{2i}'s which are zero, since no angular accelerometers are used. Skylab has been conventionally stabilized by attenuating sufficiently the controller's response at structural resonances. The lowest rigid-body resonance is approximately

$$f_{10} = 0.018 \text{ Hz}, \quad \zeta_{10} = 0.6 \quad . \tag{70}$$

The present control system of Skylab uses the conventional approach with rather low control gains when compared to launch vehicles. However, these gains give the Skylab sufficient attitude stiffness and damping against disturbances; e.g., crew motion. The control gains are changed depending on the docking of the Apollo command and service module. Since reactively stabilized systems are insensitive to docking configuration, the new concept appears to be ideally suited for such applications; however, scheduling problems make any new design changes prohibitive, especially when the conventional approach has already produced a workable attitude control system.

CONCLUSION

This report demonstrates that system structures can make controllers inert to plant resonances if the principle of reactive stabilization is applied. Overall stability can thus be predicted by analyzing only the controller without knowing the resonances of the passive plant. This is especially advantageous for space stations (Fig. 10), where configurations vary widely with deployment and docking and where conventional methods may be impractical because of the analysis and bookkeeping problems associated with flexible body resonances.

Today's sophisticated computer programs have only partially succeeded in math-modeling dynamic behavior of complex systems, and full-scale dynamic tests are still used for model verification. Also, structural damping is difficult to predict and must usually be determined by tests. Small damping factors cause resonance amplification after which controllers must be designed if conventional methods are followed. However, resonance amplifications caused by small or even zero damping have no effect if a system is reactively stable.

The proposed method was studied for the Skylab case, where the lag of the CMGs was compensated with rate gyros and angular accelerometers such that reactive stability was obtainable. However, the case has been simplified; e.g., by neglecting sensor lag, CMG cross-coupling, and sampled signal processing. This negligence may be permissible, but generally must be evaluated by a more detailed analysis than presented here.

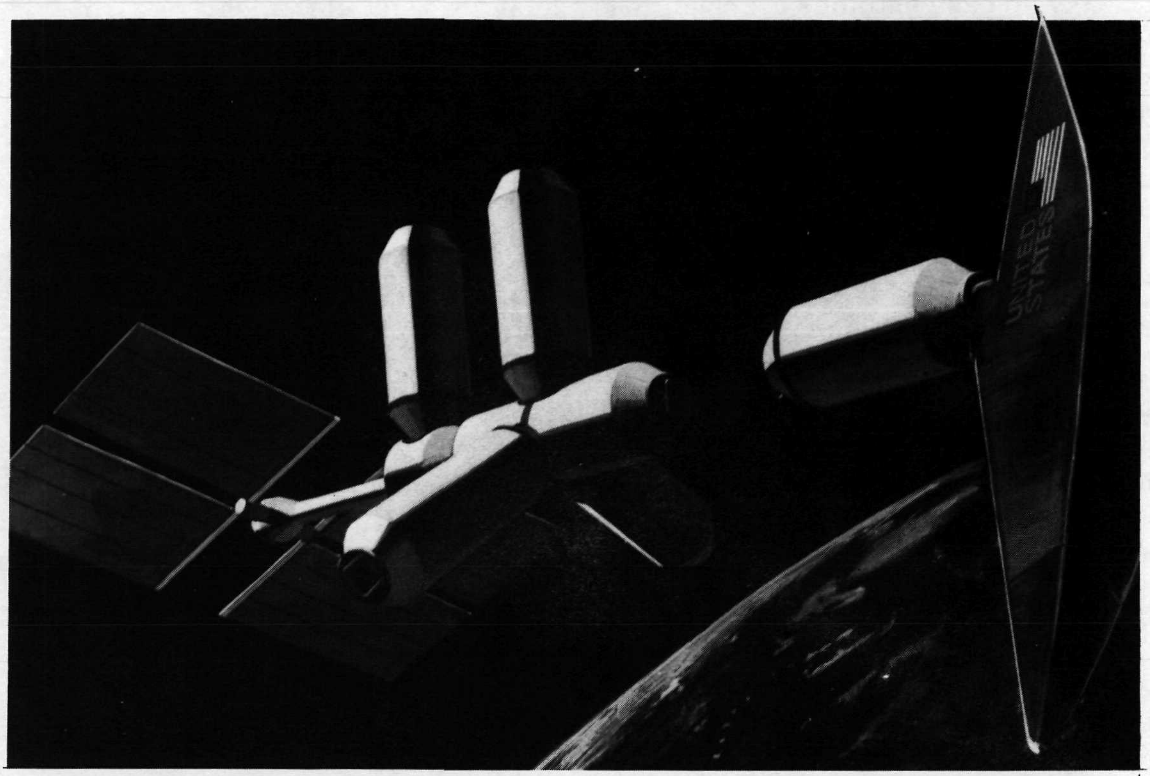

Figure 10. Modular space station configuration.

The algebraic structure of reactive stabilization leads to a building block approach and stability can be predicted by rather simple subsystem analyses. The method is ideally suited for syntheses of control systems, which is important for multivariable cases with high degrees of freedom. The given definitions and theorems are related to the frequency response and result in simple criteria similar to Nyquist plots. The method promises to simplify design, to untangle different control areas, and to reduce overall cost.

George C. Marshall Space Flight Center
National Aeronautics and Space Administration
Marshall Space Flight Center, Alabama 35812, October 6, 1971
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REFERENCES

1. von Pragenau, G. L.: A Hydraulic Support for Free-Flight Simulation With the Saturn-V Apollo Vehicle. AIAA/JACC Guidance and Control Conference, American Institute of Aeronautics and Astronautics, New York, 1966, p. 426.

2. Foster, R. M.: A Reactance Theorem. The Bell System Tech. J., vol. 3, 1924, p. 259.

3. Cauer, W.: Die Verwirklichung von Wechselstromwiderständen Vorgeschriebener Frequenzabhängigkeit. Arch. Elektrotech., vol. 17, 1927, p. 355.

4. Brune, O.: Synthesis of a Finite Two-Terminal Network Whose Driving Point Impedance is a Predescribed Function of Frequency. J. Math. Phy., vol. 10, August 1931, p. 191.

5. Brune, O.: Sc. D. Dissertation. M.I.T., Cambridge, Mass., 1930.

6. Cauer, W.: Theorie der linearen Wechselstromschaltungen. Akad. Verlag, 1941.

7. Su, K. L.: Active Network Synthesis. McGraw-Hill, New York, 1953.

8. Guillemin, E. A.: Synthesis of Passive Networks. John Wiley, New York, 1957.

9. Cauer, W.: Synthesis of Linear Communication Networks. vols. I and II, McGraw-Hill, New York, 1958.

10. Bello, P.: Extension of Brune's Energy Function Approach to Study LLF Networks. IRE Trans. CT-7, September 1960, p. 270.

11. Carlin, H. J., and Giordano, A. B.: Network Theory, an Introduction to Reciprocal and Nonreciprocal Circuits. Prentice Hall, Englewood Cliffs, 1964.

12. Karni, S.: Network Theory, Analysis and Synthesis. Allyn and Bacon, Rockleich, N. J., 1966.

REFERENCES (Continued)

13. Newcomb, R. W.: Linear Multiport Synthesis. McGraw-Hill, New York, 1966.

14. Belevitch, V.: Classical Network Theory. Holden Day, San Francisco, 1968.

15. Mac Farlane, A. G. J.: Dynamical System Models. George G. Harrap and Co. LTD, London, 1970.

16. Wolf, H.: Lineare Systeme und Netzwerke. Springer Verlag, Berlin, New York, 1971.

17. Nyquist, H.: Regeneration Theory. Bell System Techn. J., American Telephone and Telegraph Co., vol. 11, 1932, p. 126.

18. Duschek, A.: Vorlesungen über höhere Mathematik, Bd. III. Springer-Verlag, Wien, 1953.

19. Kryszig, E.: Advanced Engineering Mathematics. John Wiley, New York, 1962.

20. Birkhoff, G., and Mac Lane, S.: A Survey of Modern Algebra. The Macmillan Co., New York, 1965.

21. Lews, F. M.: Complementary Algebra. The Electronics Engineer, August 1967, p. 38.

22. von Pragenau, G. L.: Stability of Constant-Gain Systems With Vector Feedback. National Aeronautics and Space Administration, Washington, D. C., Technical Note NASA TN D-5714, March 1970.

23. Kennel, H. F.: A Control Law for Double-Gimbaled Control Moment Gyros Used for Space Vehicle Attitude Control. George C. Marshall Space Flight Center, Huntsville, Ala., NASA TM X-64536, July 20, 1970.

24. Fikes, E. H.: Control Moment Gyro for Skylab. ION National Space Meeting, The Institute of Navigation, Washington, D. C., February 1971, p. 277. (Also George C. Marshall Space Flight Center, Huntsville, Ala., NASA TM X-64583, March 19, 1971.)

25. McCullough, A. C., and Rupp, C. C.: Skylab Nested Attitude Control System Concept. ION National Space Meeting, The Institute of Navigation, Washington, D.C., February 1971, p. 305.

26. Rupp, S. C.: Skylab Strapdown Attitude Navigation System. ION National Space Meeting, The Institute of Navigation, Washington, D.C., February 1971, p. 315.

27. Chubb, W. B., and Seltzer, S. M.: Skylab Attitude and Pointing Control System. National Aeronautics and Space Administration, Washington, D. C., Technical Note NASA TN D-6068, February 1971.

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