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SUPPLEMENT TO THE ICRPG TURBULENT BOUNDARY LAYER NOZZLE ANALYSIS COMPUTER PROGRAM

by Satoaki Omori, Klaus W. Gross, and Alfred Krebsbach
Astronautics Laboratory

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* Dr. Omori is a research associate with the National Research Council, National Academy of Science, Washington, D.C.

16. ABSTRACT
This document is a supplement to the Turbulent Boundary Layer (TBL) Nozzle Analysis Computer Program, developed by Pratt & Whitney Aircraft and recommended by the Joint Army, Navy, NASA, Air Force (JANNAF) Performance Standardization Working Group as a standard reference. It describes the program calculation sequence and presents a detailed documentation of each subroutine. Important equations are derived explicitly, and future improvements to the program are discussed.

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

LIST OF SYMBOLS

A	Area
a	Speed of sound
C_f	Skin friction coefficient
C_{fa}	Adiabatic skin friction coefficient
C_H	Stanton number
C_p	Specific heat at constant pressure
C_v	Specific heat at constant volume
D	Drag
$\dot{E}$	Enthalpy flux
F_{ax}, F_x	Force in axial direction
F_{norm}, F_y	Force in direction normal to axis
g	Acceleration of gravity
H	Total enthalpy
h	Static enthalpy within boundary layer
h_o	Total enthalpy within boundary layer
h_g	Heat transfer coefficient
I	Integral
J	Conversion factor between thermal and work units
M	Mach number at boundary layer outer edge
$\dot{M}$	Momentum flux

LIST OF SYMBOLS (Continued)

m	Exponent in viscosity-temperature relation
$\dot{m}$	Mass flow rate
$\mathfrak{M}$	Mean molecular weight
n	Exponent in velocity and enthalpy profiles
$\tilde{n}$	Interaction exponent in Stanton number equation
P	Pressure
Pr	Prandtl number
Q_w	Integrated rate of heat transfer to the wall
q_w	Specific heat transfer rate
R	Gas constant ($\mathcal{R}/\mathfrak{M}$)
R_L	Reynolds number based on contour length
R_T	Recovery factor
R_x	Reynolds number based on axial length
$R_{\delta *}$	Reynolds number based on displacement thickness
R_θ	Reynolds number based on momentum thickness
R_ϕ	Reynolds number based on energy thickness
$\mathcal{R}$	Universal gas constant
r	Radius of the nozzle wall at x
S	Entropy, length along the wall or $(y/\delta)^{1/n}$
T	Temperature

LIST OF SYMBOLS (Continued)

t	Temperature inside boundary layer
U	Velocity at boundary layer outer edge
$\bar{u}$	Velocity within boundary layer
$\tilde{u}$	Internal energy
w	$(y/\Delta)^{1/n}$
x	Axial distance (contour length in appendix)
y	Distance from wall
z	Axial length in appendix
α	Angle of wall to axial direction
β	Angle of wall to axial direction
γ	Specific heat ratio
Δ	Temperature thickness
ΔF	Thrust loss
δ	Velocity thickness
$\delta *$	Displacement thickness
δ_p'	Distance from n^{th} streamline to potential wall
δ_r'	Distance from n^{th} streamline to real wall
ζ	$(\Delta/\delta)^{1/n}$
θ	Momentum thickness
μ	Viscosity

LIST OF SYMBOLS (Concluded)

ν	Kinetic viscosity
ρ	Density
τ_w	Shear stress at the wall
ϕ	Energy thickness

Subscripts

o	Stagnation condition
aw	Adiabatic wall
c	Chamber stagnation
e	Boundary layer outer edge
f	Fluid bulk
p	Potential flow
r	Real flow
w	Wall
∞	Boundary layer outer edge
$-$	Conditions inside boundary layer

TECHNICAL MEMORANDUM X- 64663

SUPPLEMENT TO THE ICRPG TURBULENT BOUNDARY LAYER NOZZLE ANALYSIS COMPUTER PROGRAM

INTRODUCTION

This document serves as a supplement to Reference 1 and contains an additional detailed description of the Turbulent Boundary Layer (TBL) Nozzle Analysis Computer Program, developed by Pratt & Whitney Aircraft and recommended by the Joint Army, Navy, NASA, Air Force (JANNAF) Performance Standarization Working Group as a standard reference.

Among other losses occurring in a rocket thrust chamber, the viscous effects in the boundary layer contribute significantly to the performance degradation. It is the objective of this analytical model to determine the necessary boundary layer parameters which permit the calculation of the thrust deficiency. In order to treat the boundary layer behavior, the edge conditions, the wall temperature distribution, and the nozzle geometry must be known. Results can only be obtained for a boundary layer development along a solid-wall surface area without any film or transpiration coolant flow introduction. Heat transfer calculations which are also performed in this analysis are not very precise and have only a secondary effect on the performance degradation.

This supplementary document starts with a description of the basic calculation sequence. Then the necessary computer program input parameters and calculation options are characterized, followed by a summary of the analytical results printed by the program. The subsequent section headed by a flow chart showing the coordination of all subroutines in the calculation process contains a detailed description of each subroutine. After a brief narrative, outlining the purpose of the subroutine accompanied by important physical equations, the subroutine from a programming aspect is presented. All common blocks used in the subroutine, the subroutine calling, the subject subroutine, as well as the subroutines called by this subroutine, computer library routines, and program built-in library routines, and the calling sequence are specified. The underlined parameters in the calling sequence represent the terms solved for in the subroutine called. Then the step-by-step calculation process in accordance with the computer program listing is reported. The symbols used in the expressions in the subroutine are listed in the front of this report. In the appendix a detailed derivation of the important equations used in the analysis [1] is presented. This document in connection with Reference 1 should provide the user of the subject computer program all necessary information to apply the program successfully or to incorporate modifications for specific problems.

ASSUMPTIONS

The analytical model and associated equations are based upon the following assumptions:

1. The flow is steady, two-dimensional, or axisymmetric.

2. The boundary layer is confined to a distance from the wall which is small compared with either the distance from an axis of symmetry or the height of a two-dimensional channel.

3. The only forces acting on the gas are those caused by pressure gradients and skin friction on the wall.

4. The only changes in total enthalpy are those caused by heat flux through the wall.

5. The flow immediately outside the boundary layer is isentropic and one-dimensional parallel to the wall.[1]

6. Pressure is constant through the boundary layer perpendicular to the wall.

7. The gas follows the perfect gas law and either has constant specific heat (calorically perfect) or is thermally perfect [C_p is a function of temperature only and $H = \int_o^T C_p(t)\,dt$].

8. The gas has a constant Prandtl number, a viscosity which varies as a power of the temperature, and an adiabatic recovery factor equal to Prandtl number to the one-third power.[1]

[1]These assumptions are to be improved.

9. The skin friction coefficient is the same as for constant-pressure, constant-wall temperature flow on a flat plate at the same free-stream conditions, wall temperature, and momentum thickness.[2]

10. The Stanton number is the same as for constant-pressure, constant-wall temperature flow on a flat plate at the same free-stream conditions, wall temperature, and energy and momentum thicknesses.[2]

11. The Stanton number for unequal momentum and energy thicknesses is that for equal thickness multiplied by $(\phi/\theta)^{\tilde{n}}$ where $\tilde{n}$ is a small interaction exponent.[2]

12. Heat transfer has no effect on skin friction. The term C_f is the same as for adiabatic flow.[2]

13. The Stanton number for equal momentum and energy thicknesses is related to the skin friction coefficient by von Karman's form of Reynold's analogy.

14. Any chemical reactions in the boundary layer affect only the driving enthalpy potential for heat flux which can be reflected in the C_p versus T curve.

15. The boundary layer shape parameters θ/δ, Δ/δ, and δ^*/θ are those for 1/n-power profiles of velocity and of the difference between stagnation and wall enthalpy. The exponent n is automatically set equal to seven unless otherwise specified in the input.

CALCULATION PROCESS

The objective of the analysis is the generation of the necessary parameters to determine the thrust decrement $\Delta F_{B.L.}$ resulting from the existence of a turbulent boundary layer along the nozzle wall. The thrust deficiency relationship

[2]These assumptions are to be improved.

$$\Delta F_{B.L.} = \left(2\pi r \rho u^2 \theta \cos \alpha\right)_{exit} \left(1 - \frac{\delta *}{\theta} \frac{P}{\rho u^2}\right)_{exit}$$

which is presently not included in the computer program basically represents the summation of the shear stress along the nozzle wall. However, this summation can be correlated to nozzle exit flow parameters as the derivation in the appendix indicates. In the previous equation only the momentum thickness θ and displacement thickness $\delta *$ are representative expressions resulting from a boundary layer analysis, while the remaining parameters identify potential flow conditions at the boundary layer edge.

The heat transfer rate to the nozzle wall at any local station, expressed by the following relationship

$$q_w = C_H \rho_e U_e (H_{aw} - H_w) \quad ,$$

is another important result of this program. Its integrated quantity along the wall indicates the energy which is either lost or recovered, depending on the cooling concept applied.

Subsequently, the important equations are listed which must be solved within the computer program.

<u>Momentum equation</u>

$$\frac{d\theta}{dx} = \frac{\bar{C}_f}{2}\sqrt{1+\left(\frac{dr}{dx}\right)^2} - \theta\left[\frac{1 + \delta */\theta}{U_e}\frac{dU_e}{dx} + \frac{1}{\rho_e U_e}\frac{d(\rho_e U_e)}{dx} + \frac{1}{r}\frac{dr}{dx}\right] .$$

<u>Energy equation</u>

$$\frac{d\phi}{dx} = C_H \left(\frac{H_{aw} - H_w}{H_o - H_w}\right)\sqrt{1 + \left(\frac{dr}{dx}\right)^2}$$

$$- \phi\left[\frac{1}{\rho_e U_e}\frac{d(\rho_e U_e)}{dx} + \frac{1}{r}\frac{dr}{dx} - \frac{1}{H_o - H_w}\frac{dH_w}{dx}\right] ,$$

where

<u>Displacement thickness:</u>

$$\delta * = \int_o^{\delta_r} \left(1 - \frac{\overline{\rho u}}{\rho_e U_e}\right) dy \quad ,$$

Momentum thickness:

$$\theta = \int_{o}^{\delta' r} \frac{\bar{\rho}\bar{u}}{\rho_e U_e} \left(1 - \frac{\bar{u}}{U_e}\right) dy \quad ,$$

Energy thickness:

$$\phi = \int_{o}^{\delta' r} \frac{\bar{\rho}\bar{u}}{\rho_e U_e} \left(1 - \frac{\bar{h}_o - H_w}{H_o - H_w}\right) dy \quad .$$

The power law profiles assuming a 1/7 exponent are used for the velocity and enthalpy profiles.

$$\frac{\bar{u}}{U_e} = \left(\frac{y}{\delta}\right)^{1/7} \qquad \text{for } y \leq \delta \quad ;$$

$$\frac{\bar{u}}{U_e} = 1 \qquad \text{for } y > \delta \quad ;$$

$$\frac{\bar{h}_o - H_w}{H_o - H_w} = \left(\frac{y}{\Delta}\right)^{1/7} \qquad \text{for } y \leq \Delta \quad ;$$

$$\frac{\bar{h}_o - H_w}{H_o - H_w} = 1 \qquad \text{for } y > \Delta \quad .$$

With a given nozzle contour $r = r(x)$, the free-stream velocity U_e and density ρ_e are calculated at each axial distance x using the input Mach number $M_e = M_e(x)$. The skin friction coefficient $\bar{C}_f$ and Stanton number C_H are also obtained using empirical relationships.

Computation of the following parameters

1. Momentum thickness $\theta = \theta(x)$,
2. Energy thickness $\phi = \phi(x)$,
3. Displacement thickness $\delta^* = \delta^*(x)$,
4. Temperature thickness $\Delta = \Delta(x)$,
5. Velocity thickness $\delta = \delta(x)$,
6. Temperature or density distribution at the boundary layer edge
7. Adiabatic wall enthalpy $H_{aw} = H_{aw}(x)$

is performed by using the ideal gas equation

$$P = \rho \bar{R} T$$

and enthalpy relations

$$H = \int_o^T C_p(t)\, dt,$$

$$H = C_p T, \qquad \text{for calorically perfect gas,}$$

or

$$H_{aw} = H + (Pr)^{1/3} \frac{U_e^2}{2\, gJ} \quad ,$$

$$\bar{h}_o = h + \frac{u^2}{2\, gJ} \quad .$$

$$\therefore \bar{h} = H_w + (H_o - H_w) \left(\frac{y}{\Delta}\right)^{1/7} - \frac{\bar{u}^2}{2gJ} \quad .$$

Sequence of Calculation

All equations used in the calculation process are presented in Reference 1. In the appendix of this document a detailed derivation of important equations is given, and the equation number is related to the same one used in Reference 1. Subsequently, the calculation sequence used in the computer program is outlined and schematically shown in Figure 1.

1. Input values such as thrust chamber operating conditions and stagnation properties (P_o, T_o, γ_o, μ_o, and P_r), the nozzle contour $r = r(x)$, the free-stream Mach number distribution $M_e = M_e(x)$, and the specific heat versus temperature relationship $C_p = C_p(t)$ are necessary requirements. To start the computer program calculation, initial assumptions for the momentum and energy thickness must be made, and appropriate option indicators must be set.

2. The following intermediate values M_e, $\frac{dM_e}{dx}$, T_e, $\frac{dT_e}{dx}$, P_e, C_{pe}, H_e, $\frac{dH_e}{dx}$, U_e, $\frac{dU_e}{dx}$, ρ_e, $\frac{d\rho_e}{dx}$, μ_e, H_{aw}, H_w, and $\frac{dH_w}{dx}$ are then internally determined by the program using the associated equations outlined in the following section.

3. The important parameter $\zeta = \left(\frac{\Delta}{\delta}\right)^{1/7}$, varied internally during iteration loops, is utilized to calculate the boundary layer thicknesses such as Δ, δ, δ^*, and δ^*/θ at a specific location of the nozzle wall.

4. With empirical relationships the friction coefficient $\bar{C}_f$ and the Stanton number C_H are determined by iteration. These parameters must be known to calculate the heat transfer rate q_w.

5. Now the Runge-Kutta-Gill method is applied to predict an energy and momentum thickness (ϕ and θ) as a new approximation for a new downstream location $x = x + \Delta x$.

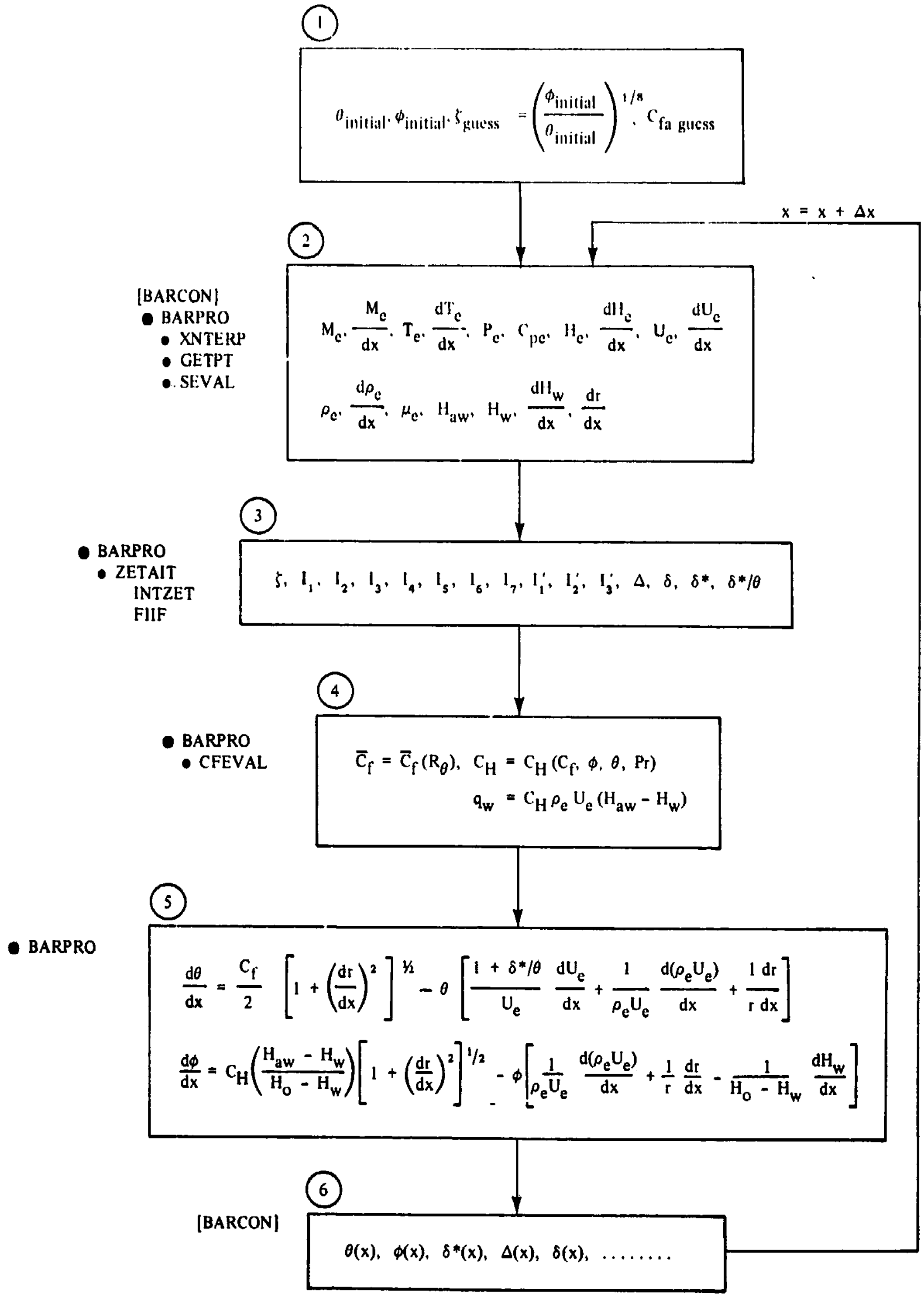


Figure 1. Calculation sequence used in the computer program.

6. The previously outlined computation procedure is repeated using the initial approximations from paragraph 5.

In Figure 1 the previously outlined steps are shown in a diagram format. On the left-hand side of each step, the calling subroutine (in brackets), the executing subroutine (marked by a solid bubble), and the assisting subroutines during the calculation process are outlined.

Boundary Layer Edge Conditions

From the chamber stagnation data, the Mach number distribution along the nozzle wall and specific heat versus temperature relationship additional static flow and thermodynamic properties (γ_e, T_e, P_e, U_e, ρ_e) must be generated to permit the calculation of various integrals as shown in Block 3 of Figure 1. The local specific heat is determined in subroutine GETPT. At any local station x the Mach number M_e, its derivative $\frac{dM_e}{dx}$, and the first derivative of T_e, dT_e/dx are obtained from a quintic spline interpolation routine. In addition, using the definition of enthalpy,

$$H_o = H_e + \frac{U_e^2}{2gJ},$$

the free-stream velocity is calculated

$$U_e = \sqrt{2Jg\,(H_o - H_e)},$$

where

$$H_e = H_i + \int_{T_i}^{T_e} C_p(t)\, dt \,.$$

Differentiating the previous equation with respect to x, one obtains

$$\frac{dH_e}{dx} = C_{pe}\frac{dT_e}{dx}.$$

Differentiating the original enthalpy equation results in

$$U_e \frac{dU_e}{dx} = \frac{dH_e}{dx} gJ \quad .$$

Substituting the right-hand term by the previous expression, the following equation used in Block 2 of Figure 1 is obtained.

$$U_e \frac{dU_e}{dx} = - C_{pe} \frac{dT_e}{dx} gJ \ .$$

The derivative of the free-stream density $\rho_e = \rho_e(x)$ is determined by means of the ideal gas equation

$$\rho_e = \frac{P_e}{R \quad T_e} \ .$$

If the density change is considered to be very small for a small distance Δx along the boundary layer edge, the derivative can be approximated by

$$\frac{d\rho_e}{dx} = \frac{\rho_e(x) - \rho_e(x - \Delta x)}{\Delta x} \quad .$$

In order to solve the momentum equations in step 5 of the preceding section (Sequence of Calculation), the value of δ^*/θ must be given. Equations (89) and (119) of Reference 1 are used to determine these quantities.

Future Improvements

1. The TBL program uses only the free-stream Mach number M_e as a function of axial nozzle length to describe the boundary layer edge condition. All the other important thermodynamic and fluid dynamic properties are calculated from the Mach number profile. Furthermore, the molecular weight, represented by the specific gas constant, is assumed to remain constant throughout the calculation process.

It is recommended that the free-stream parameters such as static temperature T_e, static pressure P_e, local density ρ_e, static enthalpy H_e, local specific heat ratio γ_e, and the local molecular weight should be input to the program which more accurately represents the boundary layer edge condition.

2. The equation representing the thrust decrement caused by the boundary layer should be implemented into the program.

3. The relation between the skin friction coefficient and the Reynolds number used in the TBL program is based upon low-speed flow data without a pressure gradient. Furthermore, operation of the computer program revealed that during numerical iteration the skin friction coefficient showed different values when the Reynolds number was greater than 10^4. In order to avoid this problem, it is recommended that the Blasius equation [2,3] be used because it satisfies experimental low-speed flow data quite well. The Blasius equation reads

$$\frac{C_f}{2} = \frac{0.0128}{Re_\theta^{0.25}} \quad \text{with} \quad Re_\theta = \frac{\rho\ U_e \theta}{\mu} \quad ,$$

where the density ρ and the dynamic viscosity μ are based upon either the free-stream temperature or Eckert's reference temperature [4,5]. Cebeci [6,7] provides another analytical approach to determine the skin friction coefficient, which is compared to some test data.

For supersonic turbulent flow with a pressure gradient, the following modified Blasius equation is recommended

$$C_f = \frac{0.018}{Re_\theta^{0.25}} \quad ,$$

where

$$Re_\theta = \frac{\rho_\infty \quad U_\infty \quad \theta}{\mu_\infty} \quad .$$

This equation describes experimental results obtained by Brott [8] adequately.

4. The presently used constant power law associated with the velocity and enthalpy does not adequately represent the flow condition especially at high Mach numbers and for a cooled wall [8-10]. Therefore, it is recommended that a variable exponent $1/n$ as a function of the Mach number be incorporated into the calculation process.

5. Presently a constant Prandtl number is input to the program. Since the specific heat and the molecular weight change during the expansion process in a rocket nozzle, it is recommended that the Prandtl number be internally calculated according to the equation

$$Pr = \frac{\mu C_p}{\lambda} = \frac{C_p}{C_p + \frac{5}{4} R} ,$$

where the specific gas constant

$$R = \frac{\mathcal{R}}{\mathfrak{M}} ,$$

where $\mathcal{R}$ represents the universal gas constant and $\mathfrak{M}$ the mean molecular weight.

TBL COMPUTER PROGRAM DOCUMENTATION

Tables 1 and 2 describe the TBL computer program input variables and the results printed out by the program. Figure 2 shows the TBL computer program subroutine linkage. A description of the individual subroutines in the computer program is given in the next section.

TABLE 1. TBL PROGRAM INPUT

Symbol		Description	Units
1. Physical Properties at Stagnation Condition			
P_o	P0	Stagnation pressure	lbf/ft^2
T_o	T0	Stagnation temperature	°R
γ_o	GAM0	Specific heat ratio at stagnation condition	
μ_o	ZMU0	Viscosity at stagnation condition	lbm/(sec-ft)
Pr	PRANDT	Prandl number	—
R	RBAR	Gas constant	(ft-lbf)/(lbm-°R)
2. Contour Geometry $r = r(x)$			
r	YITAB	Radius table	ft
x	XITAB	Axial distance table	ft
3. Mach Number in Free Stream $M_e = M_e(x)$			
M_e	ZMTAB	Mach number table	—
4. Table of Specific Heat $C_p = C_p(T)$			
C_p	CPTAB	Specific heat table at constant pressure	Btu/(lbm-°R)
T	TCTAB	Temperature table corresponding to the above CPTAB	°R
5. Constant Values			
J	FJ	Conversion factor between thermal and work units	(ft-lbf)/Btu
g	G	Proportionality constant in equation $F = (M/g)a$ (acceleration of gravity)	(ft/sec^2) (lbm/lbf)

TABLE 1. TBL PROGRAM INPUT (Continued)

Symbols		Description	Units
5. Constant Values (Concluded)			
n	MZETA	Velocity power law exponent $\bar{u}/U_e = (y/\delta)^{1/n}$	—
m	ZMVIS	Exponent in viscosity-temperature relation $\mu/\mu_o = (T/T_o)^m$	—
$\tilde{n}$	ZNSTAN	Interaction exponent in Stanton number relation	—
6. Initial conditions			
$\theta_{initial}$	THETAI	Initial momentum thickness	ft
$\phi_{initial}$	PHII	Initial energy thickness	ft
7. Wall Temperature (for Option ITWTAB = 1)			
T_w	TWTAB	Wall temperature table	°R
8. Maximum Length of Step Size			
	DXMAX	Maximum length of step size	ft
9. Tolerances for Iteration Loops			
	TOLCFA	Tolerance in C_f - $C_f R_\theta$ iteration loop	—
	TOLZET	Tolerance in ζ iteration loop	—
	TOLZMF	Tolerance used in gas property evaluation loops	—

TABLE 1. TBL PROGRAM INPUT (Concluded)

Option Indicators		
1. Wall Temperature Options		
ITWTAB	= -1	T_w = adiabatic wall temperature (°R)
	= 0	T_w = constant (°R)
	= 1	$T_w = T_w(x)$ (°R)
2. Plane or Axisymmetric Flow Option		
EPSZ	= 0.	Two-dimensional
	= 1.	Axisymmetric
3. Print Option		
IPRINT	= 0	Prints only at input values of x
	= 1	Prints at all calculated subintervals
4. Number of Points in r, x, and M Tables		
IXTAB		$4 \leq \text{IXTAB} \leq 500$
5. Specific Heat Options		
ICTAB	= 0	C_p = constant
	= $(3 \leq \text{ICTAB} \leq 20)$ Number of points in C_p versus T table	
6. Multiplier to Dimensionalize r and x Tables		
The computer program operates with dimensional quantities of r and x. If normalized values r and x are input, the multiplier SCALE must be input in feet to internally calculate dimensionalized values of r and x.		

At every printout point the program prints six groups of quantities called for by subroutine BARPRO.

TABLE 2. TBL PROGRAM OUTPUT

Symbol	Description	Units
1. Contour Properties		
X	Axial distance	ft
XLARC	Actual nozzle wall length referenced to the first x value	ft
YR	Radius or height of contour at X	ft
YRP	Slope of contour	—
2. Flow Properties		
ZME	Mach number	-
TE	Static temperature	°R
TW	Wall temperature	°R
TAW	Adiabatic wall temperature	°R
ZMEP	Mach number gradient	1/ft
UE	Free-stream velocity	ft/sec
PE	Static pressure	lbf/ft^2
3. Boundary Layer		
DELTA	Velocity thickness δ	ft
BDELTA	Temperature thickness Δ	ft
DELSTR	Displacement thickness δ^*	ft
THETA	Momentum thickness θ	ft
PHI	Energy thickness ϕ	ft
DELSOT	Shape factor δ^*/θ	-
4. Heat Transfer		
HG	Heat transfer coefficient h_g	$Btu/(ft^2\text{-sec-}°R)$
QW	Local rate of heat transfer q_w to the wall	$Btu/(ft^2\text{-sec})$
SUMQDA	Integrated heat transfer Q_w rate to point X	Btu/sec (axisymmetric) Btu/(sec-ft) (planar)

TABLE 2. TBL PROGRAM OUTPUT (Concluded)

Symbol	Description	Units
4. Heat Transfer (Concluded)		
FORCE	Drag force in axial F_x direction	lbf
FLAT	Force normal to X direction F_y for two-dimensional planar flow	lbf
5. Internal Integrals		
ZETA, ZI1, ZI2, ZI3, ZI2P, ZI3P, ZI4, ZI5, ZI6, ZI7, ZI1P		
6. Coefficients		
CF	Skin friction coefficient C_f	—
CH	Stanton number C_H	—
RTHE	Reynolds number based on momentum thickness R_θ	—
RXLN	Reynolds number based on wall length R_x	—
RPHI	Reynolds number based on energy thickness R_ϕ	—
RDLS	Reynolds number based on displacement thickness $R_{\delta *}$	—

If the sonic point start procedure is used, the initial values of momentum thickness, THETAI, and energy thickness, PHII, are printed.

A new contour table is printed, corrected by displacement thickness as follows:

XCCP — Array of corrected normalized axial distance points

YCCP — Array of corrected normal contour points normalized by the potential throat radius.

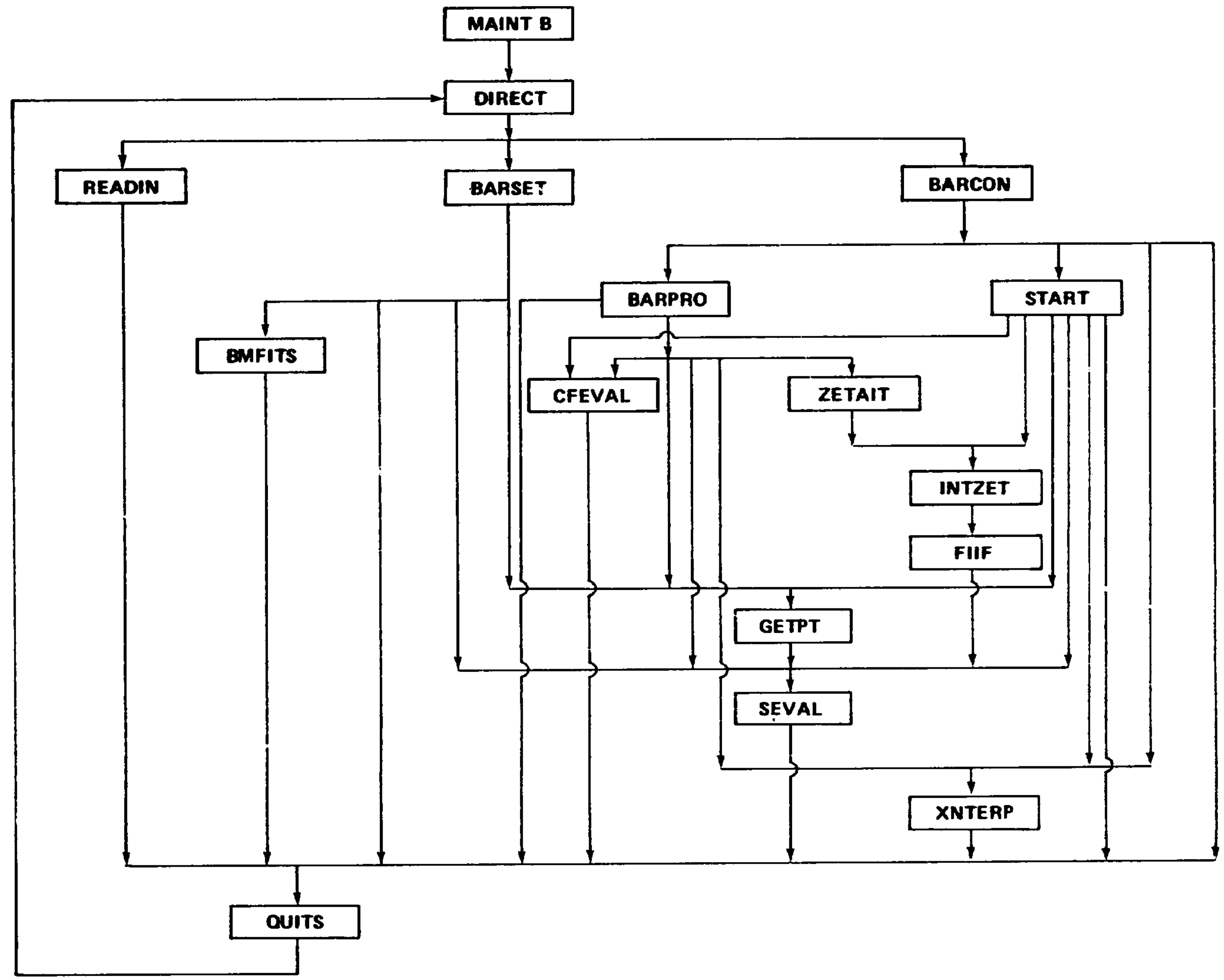


Figure 2. TBL computer program subroutine linkage.

Description of TBL Program Subroutines

SUBROUTINE BARCON

Subroutine BARCON accepts initial conditions and executes a Runge-Kutta solution for the boundary layer along the contour. The initial conditions are set

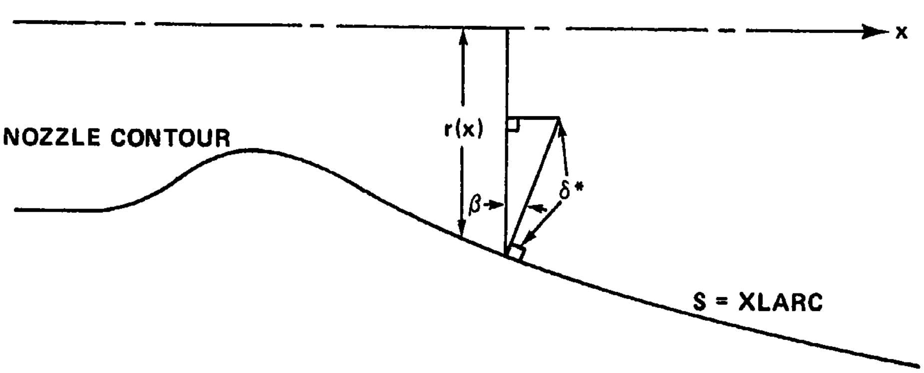
$$\phi = \phi_o, \theta = \theta_o, \text{ at } x = x_o .$$

A step-size Δx, used to increment the axial coordinate x, is determined from the input quantity Δx_{max} and the local entries in the x-table. Having two first-order ordinary differential equations

$$\left[\frac{d\theta}{dx} = f(x, \theta, \phi) \text{ and } \frac{d\phi}{dx} = g(x, \theta, \phi)\right],$$

and using subroutine BARPRO to evaluate the derivatives, a four-term, Runge-Kutta numerical solution is used.

Derivation of corrected axial distance and contour radius is shown in the diagram below.



The corrected contour radius is

$$r'(x) = r(x) - \delta^* \cos \beta ,$$

where

$$\cos\beta = \frac{dx}{ds},$$

$$ds^2 = dr^2 + dx^2,$$

and

$$\frac{ds}{dx} = \sqrt{1 + \left(\frac{dr}{dx}\right)^2}$$

Thus, one obtains

$$r'(x) = r(x) - \frac{\delta^*}{\sqrt{1 + \left(\frac{dr}{dx}\right)^2}} \leftarrow \text{YCCP}.$$

The corrected axial distance is

$$x' = x + \delta^* \sin\beta,$$

where

$$\sin\beta = \frac{dr}{ds} = \frac{dr/dx}{ds/dx} = \frac{dr/dx}{\sqrt{1 + \left(\frac{dr}{dx}\right)^2}}.$$

Thus, one obtains

$$x' = x + \frac{\delta^* \frac{dr}{dx}}{\sqrt{1 + \left(\frac{dr}{dx}\right)^2}} \leftarrow \text{XCCP}.$$

COMMON BLOCKS

COMMON blocks INPUT, INTER, LOOKUP, OUTPUT, and TABLES are used.

TBL SUBROUTINES

Subroutine DIRECT calls BARCON.

BARCON calls subroutines BARPRO, QUITS, START, AND XNTERP.

FORTRAN SYSTEM ROUTINES

FORTRAN library routines ALOG and SQRT are used.

Built-in FORTRAN library routine ABS is used.

CALLING SEQUENCE

The subroutine calling sequence is:

CALL BARCON

SOLUTION METHOD

Compute the cubic root of the Prandtl number.

1. PRE1O3 = $(\text{PRANDT})^{1/3}$

Compute term in denominator of the Stanton number.

2. CHPAR1 = 1.0 - PRANDT + Ln ((6.0)/((5.0)(PRANDT) + 1.0))

3. MZETAM = MZETA - 1

Save the value of exponent in velocity profile.

4. ZMZETA = MZETA

5. ZMZETP = ZMZETA + 1.0

6. ZMZETM = ZMETA - 1.0

7. RMZETA = (1.0)/(ZMZETP)

8. OOMZET = (1.0)/(ZMZETA)

Set the initial value of axial distance.

9. X = XITAB(1)

Set the initial value of the difference of axial distances.

10. DX = 0.0

Set the initial integrated heat transfer rate.

12. SUMQDA = 0.0

Set the initial drag force.

13. FORCE = 0.0

Set the initial normal force.

14. FLAT = 0.0

Set the initial local heat transfer rate.

15. QW = 0.0

Set the initial heat transfer coefficient.

16. HG = 0.0

Set the following parameters.

17. IXPOS = 1

18. IMX = 0

19. ITX = 0

20. IPX = 0

21. IYX = 0

22. ITWX = 0

23. DXRHO = 0.0

24. IBEG = 2

Initial assumption of adiabatic skin friction coefficient.

25. CFAGT = 0.002

Set the initial value of the sonic point start indicator.

26. ISTART = 0

Check whether the sonic point start procedure is to be used.

27. If THETAI $\leq$ 0.0, go to 30

If THETAI > 0.0, go to 28

Calculate the shape factor ξ based upon initial assumptions.

28. ZETA = $((\text{PHII})/(\text{THETAI}))^{\text{RMZETA}}$

29. Go to 32

Use the sonic point start procedure.

30. CALL START

Set the sonic point start indicator.

31. ISTART = 1

32. CFAGP = CFAGT

Set the energy thickness equal to the input value.

33. PHI = PHII

Set the momentum thickness to the input value.

34. THETA = THETAI

Set the start value of axial distance to the first value in the axial distance table.

35. XIBASE = XITAB(1)

Set the last value of axial distance to the last value in the axial distance table.

36. XIEND = XITAB(IXTAB)

Check whether the Mach number table contains at least two values.

37. If IXTAB ≤ 1, go to 39

If IXTAB > 1, go to 38

38. DXRHO = (XITAB(2) - XIBASE)/(10.0)

Call subroutine BARPRO to obtain C_h, $\overline{C}_f$, $d\theta/dx$, $d\phi/dx$, q_w, h_g, FORCE, SUMQDA, XLARC, and so on.

39. CALL BARPRO(1)

Call subroutine BARPRO to print the output of BARPRO.

40. CALL BARPRO(5)

Call subroutine XNTERP to obtain the radius YR and its first derivative YRP.

41. CALL XNTERP (X, YR, YRP, IYX, XITAB, YITAB, IXTAB, CYX, IMX)

Save initial displacement thickness and radius of the contour.

42. DEL = DELSTR

43. YMIN = YR

44. ONOC = SQRT (1.0 + (YRP)(YRP))

Compute the first corrected axial distance point and contour point.

45. XCCP(1) = X + (DELSTR)(YRP)/(ONOC)

46. YCCP(1) = YR - (DELSTR)/(ONOC)

Check whether the mach number table contains at least two values.

47. If IXTAB ≤ 1, return

    If IXTAB > 1, go to 48

48. Do 101, I = IBEG, IXTAB

Compute next value of axial distance.

49. XNEW = XITAB (I)

50. XMAG = (|XNEW| + |X|)/(2.0)

51. DXINT = XNEW - X

52. NX = (DXINT)/(DXMAX) + 0.99

53. If NX > 0, go to 55

    If NX ≤ 0, go to 54

54. NX = 1

Obtain real value of NX.

55. ZNX = NX

Compute "weighted" difference of axial distance values.

56. DX = (DXINT)/(ZNX)

57. DXO2 = (DX)/(2.0)

58. DXRHO = (DX)/(10.0)

59. Do 94, INX = 1,NX

Save the values of energy thickness ϕ and momentum thickness θ.

60. PHIOLD = PHI

61. THEOLD = THETA

Save the value of the axial distance.

62. XOLD = X

63. DPHIRK(1) = (DX)(PHIP)

64. DTHERK(1) = (DX)(THETAP)

Compute new value of axial distance.

65. X = XOLD + DXO2

66. Do 80, IRK = 2,4

Check for last time through the Do-loop.

67. If IRK $\neq$ 4, go to 74

If IRK = 4, go to 68

Compute new value of axial distance.

68. X = XOLD + DX

69. If $|(X - XNEW)/(XMAG)| > 1.0E-6$, go to 71

If $|(X - XNEW)/(XMAG)| \leq 1.0E-6$, go to 70

70. X = XNEW

Compute new value of energy thickness and momentum thickness.

71. PHI = PHIOLD + DPHIRK (IRK - 1)

72. THETA = THEOLD + DTHERK (IRK - 1)

73. Go to 76

Compute energy thickness and momentum thickness.

74. PHI = PHIOLD + (DPHIRK(IRK -))(0.50)

75. THETA = THEOLD + (DTHERK(IRK -1))(0.50)

Check whether the energy thickness is negative or zero.

76. If PHI $\leq$ 0.0, go to 85

If PHI > 0.0, go to 77

Check whether the momentum thickness is negative or zero.

77. If THETA $\leq$ 0.0, go to 85

If THETA > 0.0, go to 78

Call subroutine BARPRO to obtain $d\phi/dx$ and $d\theta/dx$.

78. CALL BARPRO (IRK)

79. DPHIRK(IRK) = (DX)(PHIP)

80. DTHERK(IRK) = (DX)(THETAP)

Compute energy thickness and momentum thickness according to the Runge-Kutta-Gill method.

81. PHI = PHIOLD + (DPHIRK(1) + (2.0)(DPHIRK(2))
+ (2.0)(DPHIRK(3)) + DPHIRK(4))/(6.0)

82. THETA = THEOLD + (DTHERK(1) + (2.0)(DTHERK(2))

+ (2.0)(DPHIRK(3)) + DPHERK(4))/(6.0)

Check whether the energy thickness is negative or zero.

83. If PHI $\leq$ 0.0, go to 85

If PHI > 0.0, go to 84

Check whether the momentum thickness is negative or zero.

84. If THETA > 0.0, go to 88

If THETA $\leq$ 0.0, go to 85

An error has been made in the calculations; write out an error message.

85. WRITE X, ZME, THETA, PHI

Call subroutine BARPRO to obtain q_w, h_g, FORCE, SUMQDA, and XLARC.

86. CALL BARPRO (5)

87. CALL QUITS

Call subroutine BARPRO to obtain new C_H, $\overline{C}_f$, $d\theta/dx$, and $d\phi/dx$.

88. CALL BARPRO (1)

Select the minimum contour radius and its corresponding displacement thickness.

89. IF YR > YMIN, go to 92

IF YR $\leq$ YMIN, go to 90

90. DEL = DELSTR

91. YMIN = YR

Check for printout of all calculated subintervals.

92. If IPRINT $\leq$ 0, go to 94

If IPRINT > 0, go to 93

Print data at the calculated subinterval.

93. CALL BARPRO (5)

End of INX Do-loop.

94. Continue

Call subroutine XNTERP to obtain YR and YRP corresponding to X.

95. CALL XNTERP (X, YR, YRP, IYX, XITAB, YITAB, IXTAB,
CYX, IMX)

96. ONOC = SQRT(1.0 + (YRP)(YRP))

Compute corrected axial distance points and contour points.

97. XCCP(I) = X + (DELSTR)(YRP)/(ONOC)

98. YCCP(I) = YR - (DELSTR)/(ONOC)

Check for printout at input intervals.

99. If IPRINT > 0, go to 101.

If IPRINT $\leq$ 0, go to 100.

Print out data at input intervals.

100. CALL BARPRO (5)

End of Mach number table Do-loop.

101. Continue

Compute the potential throat radius.

102. RPOT = YMIN - DEL

Print out the potential throat radius.

103. WRITE RPOT

Normalize the table of corrected contour points, using the potential throat radius.

Nomalized axial distance:

104. XCCP(1) = (XCCP(1))/(RPOT)

Nomalized radius:

105. YCCP(1) = (YCCP(1))/(RPOT)

106. Do 108, I = IBEG, IXTAB

107. XCCP(I) = (XCCP(I))/(RPOT)

108. YCCP(I) = (YCCP(I))/(RPOT)

109. WRITE heading for normalized contour point table

Check whether the sonic point start procedure has been used.

110. IF ISTART ≤ 0, go to 113.

IF ISTART > 0, go to 111

Print output from sonic point start procedure.

111. WRITE XCCP(1), YCCP(1), (I,XCCP(I), YCCP(I), I = IBEG, IXTAB)

112. Return

Print regular output.

113. WRITE (I, XCCP(I), YCCP(I), I = 1, IXTAB)

114. Return

Table 3 gives subroutine BARCON nomenclature.

TABLE 3. SUBROUTINE BARCON NOMENCLATURE

Symbol	Description	Units	Reference
CFAGP	Guess value of skin friction coefficient		/INTER/, 32
CFAGT	Guess value of skin friction coefficient		/INTER/, 25, 32
CHPAR1	Term in the denominator of the Stanton number equation		/INTER/, 2
CYX	Array of parabola coefficients for the nozzle radius table		/LOOKUP/, 41, 95
DEL	Initial value of displacement thickness or that corresponding to the minimum nozzle radius	ft	42, 90, 102
DELSTR	Boundary layer displacement thickness	ft	/OUTPUT/, 42, 45, 46, 90, 97, 98
DPHIRK	Array of the first derivative of the potential energy thickness times the axial step size		DIM, 63, 71, 74, 79, 81
DTHERK	Array of the first derivative of the potential momentum thickness times the axial step size		DIM, 64, 72, 75, 80, 82
DX	Weighted difference of table values of the axial distance	ft	/INTER/, 10, 56-58, 68, 79, 80
DXINT	Difference between axial distance and table values of axial distance	ft	51, 52, 56
DXMAX	Maximum length of step size	ft	/INPUT/, 52
DXO2	One-half DX	ft	57, 65

TABLE 3. SUBROUTINE BARCON NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
DXRHO	One-tenth the difference between axial distances	ft	/INTER/ 23, 38, 58
FLAT	Force normal to x-direction for two-dimensional planar flow	lbf	/OUTPUT/, 14
FORCE	Drag force in axial or x-direction	lbf	/OUTPUT/, 13
HG	Heat transfer coefficient	Btu/(ft^2-sec-°R)	/OUTPUT/, 16
I	Do-loop counter and printout counter		48, 49, 106-108, 111,113
IBEG	Sonic line subscript counter		/INTER/, 24, 48, 106, 111
IMX	Mach number table entry indicator and saved subscript counter		/LOOKUP/, 18, 41, 95
INX	Do-loop counter		59
IPRINT	Printout indicator		/INPUT/, 92, 99
IPX	Pressure table entry indicator and saved subscript counter		/LOOKUP/, 20
IRK	Do-loop counter		66, 67, 71, 72, 74, 75, 78-80
ISTART	Indicator for sonic point start procedure		26, 31, 110
ITWX	Wall temperature table entry indicator and saved subscript counter		/LOOKUP/, 22
ITX	Free-stream temperature table entry indicator and saved subscript counter		/LOOKUP/, 19
IXPOS	Array position indicator		/LOOKUP/, 17

TABLE 3. SUBROUTINE BARCON NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
IXTAB	Number of points in the x, y, and Mach tables ($4 \leq IXTAB \leq 500$)		/INPUT/, 36, 37, 41, 47, 48, 95, 106, 111, 113
IYX	Nozzle radius table entry indicator and saved subscript counter		/LOOKUP/, 21, 41, 95
MZETA	Exponent of velocity distribution		/INPUT/, 3, 4
MZETAM	MZETA minus one		/INTER/, 3
NX	Weighting factor for axial distance increment		52-55, 59
ONOC	Square root of one plus the slope of the contour squared		44-46, 96-98
OOMZET	One divided by ZMZETA		/INTER/, 8
PHI	Boundary layer energy thickness	ft	/OUTPUT/, 33, 60, 71, 74, 76, 81, 83, 85
PHII	Initial value of energy thickness	ft	/INPUT/, 28, 33
PHIOLD	Saved value of energy thickness	ft	60, 71, 74, 81
PHIP	Slope of energy thickness		/INTER/, 63, 79
PRANDT	Prandtl number		/INPUT/, 1, 2
PRE1O3	Recovery factor - cubic root of the Prandtl number		/INTER/, 1
QW	Local heat transfer rate to the wall	Btu/(ft^2-sec)	/OUTPUT/, 15
RMZETA	One divided by ZMZETP		/INTER/, 7, 28

TABLE 3. SUBROUTINE BARCON NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
RPOT	Potential throat radius corrected for displacement thickness	ft	102-105, 107, 108
SUMQDA	Integrated heat transfer rate to point X	Btu/sec(axial)	/OUTPUT/, 12
THEOLD	Saved value of momentum thickness	ft	61, 72, 75, 82
THETA	Boundary layer momentum thickness	ft	/OUTPUT/, 34, 61, 72, 75, 77, 82, 84, 85
THETAI	Input value of momentum thickness; if = - 1, sonic point start procedure will be used	ft	/INPUT/, 27, 28, 34
THETAP	Slope of momentum thickness	ft	/INTER/, 64, 80
X	Axial distance in monotonically increasing order	ft	/OUTPUT/, 9, 41, 45, 50, 51, 62, 65, 68-70, 85, 95, 97
XCCP	Array of normalized corrected axial distance points		DIM, 45, 97, 104, 107, 111, 113
XIBASE	First value in axial distance table	ft	/INTER/, 35, 38
XIEND	Last value in axial distance table	ft	/INTER/, 36
XITAB	Table of IXTAB values (axial distance, x) in monotonically increasing order	ft	/TABLES/, 9, 35, 36, 38, 41, 49, 63, 64, 95
XLARC	Arc length of the contour up to point x	ft	/OUTPUT/, 11
XMAG	One-half the sum of XNEW and X	ft	50, 69

TABLE 3. SUBROUTINE BARCON NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
XNEW	Successive values of axial distance from the XITAB table	ft	49-51, 69, 70
XOLD	Saved value of axial distance	ft	62, 65, 68
YCCP	Array of corrected contour points normalized by the potential throat radius		DIM, 46, 98, 105, 108, 111, 113
YITAB	Nozzle contour radius table related to IXTAB array	ft	/TABLES/, 41, 95
YMIN	Saved value of initial or minimum radius or height of contour	ft	43, 89, 91, 102
YR	Nozzle radius or contour height	ft	/OUTPUT/, 41, 43, 46, 89, 91, 95, 98
YRP	Slope of contour		41, 44, 45, 95-97
ZETA	Shape factor		/OUTPUT/, 28
ZME	Mach number		/OUTPUT/, 85
ZMZETA	Real value of MZETA		/INTER/, 4-6, 8
ZMZETM	ZMZETA minus one		/INTER/, 6
ZMZETP	ZMZETA plus one		/INTER/, 5, 7
ZNX	Real value of NX		55, 56

SUBROUTINE BARPRO

Subroutine BARPRO calculates the boundary layer parameters from initial or previously determined energy thickness ϕ, momentum thickness θ, and $\frac{d\theta}{dx}$ and $\frac{d\phi}{dx}$ along the contour at a point x.

The interpolation routine XNTERP is used to evaluate inviscid flow and contour properties and derivatives at point x, such as M_e, $\frac{dM_e}{dx}$, T_e, $\frac{dT_e}{dx}$, r, and $\frac{dr}{dx}$. Then subroutine ZETAIT is called and the shape factor ζ and boundary layer thicknesses Δ, δ, and δ^* are computed. An iteration procedure is used to calculate the skin friction coefficient $\bar{C}_f$ and the Stanton number C_H. This procedure is as follows:

1. An initial guess C_{fag} is made.

2. The term $\bar{C}_f \bar{R}_{\bar{\theta}} = \left(\frac{T_{aw}}{T_e}\right)^{1-m} C_{fag} R_\theta$ is calculated.

3. Subroutine CFEVAL is used to evaluate $\bar{C}_f$ as a function of $\bar{C}_f \bar{R}_{\bar{\theta}}$.

4. The terms $\left(\frac{T_s}{T_{aw}}\right)$ and $C_{fa} = \frac{\bar{C}_f}{\left(\frac{T_{aw}}{T_e}\right)\left(\frac{T_s}{T_{aw}}\right)^m}$ are calculated.

5. A relative error comparison is made. If

$$\left|\frac{C_{fa} - C_{fag}}{C_{fag}}\right| \leq \text{TOLCFA},$$

convergence is satisfactory. Otherwise a form of Wegstein's method is used to calculate a new guess C_{fa}; and steps 2 through 5 are repeated up to a maximum of 50 iterations.

The derivatives $\frac{d\phi}{dx}$ and $\frac{d\theta}{dx}$ are now evaluated, from the differential equations, at the point x. For the point x at the end of a Runge-Kutta step (IND = 1), total heat transfer and drag quantities are also computed.

This subroutine also provides the data for the printout if called for by the input parameter IPRINT.

The values obtained in BARPRO are as follows:

M_e, $\frac{dM_e}{dx}$, T_e, $\frac{dT_e}{dx}$, P_e, C_{pe}, H_e, $\frac{dH_e}{dx}$, γ_e,

U_e, $\frac{dU_e}{dx}$, ρ_e, $\frac{d\rho_e}{dx}$, $\rho_e U_e$, $\frac{d(\rho_e U_e)}{dx}$, μ_e, H_{aw}, T_{aw},

I_1, I_2, I_3, I_4, I_5, I_6, I_7, I'_1, I'_2, I'_3,

ζ, Δ, δ, $\delta*$, $\overline{C_f}$, C_H, $\frac{d\theta}{dx}$, $\frac{d\phi}{dx}$, $\frac{dr}{dx}$,

h_g, q_w, Q_w, R_θ, R_x, R_L, R_ϕ, $R_{\delta*}$, F_x, F_y.

COMMON BLOCKS

COMMON blocks COFIIF, CSEVAL, INPUT, INTER, LOOKUP, OUTPUT, SAVED, and TABLES are used.

TBL SUBROUTINES

Subroutine BARCON calls BARPRO.

BARPRO calls subroutines CFEVAL, CETPT, QUITS, SEVAL, XNTERP, and ZETAIT.

FORTRAN SYSTEM ROUTINES

FORTRAN library function SQRT is used.

Built-in FORTRAN function ABS is used.

CALLING SEQUENCE

The subroutine calling sequence is:

CALL BARPRO (IND)

where

IND = program loop control indicator.

SOLUTION METHOD

Determine which program option is desired.

1. If IND = 1, go to 2

   If IND = 2, go to 2

   If IND = 3, go to 58

   If IND = 4, go to 2

   If IND = 5, go to 150

Call subroutine XNTERP to obtain the free-stream Mach number M_e and its gradient dM_e/dx for a given x.

2. CALL XNTERP (X, ZME, ZMEP, IMX, XITAB, ZMTAB, IXTAB, CMX, IXPOS)

Save IMX.

3. IXPOS = IMX

Call subroutine XNTERP to obtain the free-stream temperature T_e and its gradient dT_e/dx.

4. CALL XNTERP (X, TE, TEP, ITX, XITAB, TITAB, IXTAB, CTX, IXPOS)

Call subroutine GETPT to obtain the free-stream pressure P_e by using M_e and T_e.

5. CALL GETPT (ZME, <u>PE</u>, <u>TE</u>)

Call subroutine SEVAL to obtain the specific heat C_{pe} and enthalpy H_e by using T_e.

6. CALL SEVAL (1, TE, <u>CPE</u>, <u>HE</u>)

Determine the enthalpy gradient $\left(\frac{dH_e}{dx} = C_{pe} \frac{dT_e}{dx} \times gJ\right)$.

7. HEP = (FJG) (CPE) (TEP)

Compute the specific heat ratio $\left(\gamma_e = \frac{C_{pe}}{C_{pe} - R/J}\right)$.

8. GAME = (CPE)/(CPE - ROJ)

Obtain the dynamic enthalpy $(U_e^2/2)$.

9. UE2O2 = H0 - HE

The free-stream velocity U_e is obtained from equation (9).

10. UE = SQRT ((2.)(UE2O2))

Determine the velocity gradient $\left(\frac{dU_e}{dx} = -\frac{1}{U_e}\frac{dH_e}{dx}\right)$.

11. UEP = -(HEP)/(UE)

Obtain the free-stream density ρ_e $\left(\rho_e = \frac{P_e}{RT_e}\right)$.

12. RHOE = (PE)/(TE)/(RBAR)

13. If DXRHO $\neq$ 0.0, go to 16

If DXRHO = 0.0, go to 14

Save the initial density gradient.

14. RHOEP = 0.0

15. Go to 32

16. If X > XIBASE, go to 20

If X $\leq$ XIBASE, go to 17

Save the density ρ_e.

17. Z1 = RHOE

Put one as:

18. Z3 = 1.0

19. Go to 24

Call subroutine XNTERP to determine M_e = (ERASE1) and dM_e/dx = (ERASE2) for (X-DXRHO).

20. CALL XNTERP (X-DXRHO, ERASE1, ERASE2, IMX, XITAB, ZMTAB, IXTAB, CMX, IXPOS)

Call subroutine GETPT to obtain P_e = (Z4) and T_e = (Z5) corresponding to M_e = (ERASE1).

21. CALL GETPT (ERASE1, Z4, Z5)

Compute the density ρ_e corresponding to M_e. $\left(\rho_e = \frac{P_e}{RT_e}\right)$

22. Z1 = (Z4)/(Z5)/(RBAR)

Set point one as:

23. Z3 = 0.5

24. If X < XIEND, go to 28

If X ≥ XIEND, go to 25

Set

25. Z2 = RHOE

26. Z3 = 1.0

27. Go to 31

Call subroutine XNTERP to obtain M_e = (ERASE1) and dM_e/dx = (ERASE2) corresponding to X + DXRHO.

28. CALL XNTERP (X + DXRHO, ERASE1, ERASE2, IMX, XITAB, ZMTAB, IXTAB, CMX, IXPOS)

Call subroutine GETPT to obtain P_e = (Z4) and T_e = (Z5) using the above M_e = (ERASE1).

29. CALL GETPT (ERASE1, Z4, Z5)

Compute the density ρ_e = (Z2). ($\rho_e = P_e/\bar{R}T_e$)

30. Z2 = (Z4)/(Z5)/(RBAR)

Approximate the density gradient.

31. RHOEP = ((Z2 - Z1)/(DXRHO))(Z3)

Obtain the mass flow density $\rho_e U_e$.

32. RHOUE = (RHOE)(UE)

The first derivative of mass flow density: $\dfrac{d(\rho_e U_e)}{dx}$

33. RHOUEP = (RHOE) (UEP) + (UE) (RHOEP)

Evaluate the viscosity $\mu_e = \mu_o \left(\dfrac{T_e}{T_o}\right)^m$.

34. ZMU = (ZMUO) $((TE)/(T0))^{ZMVIS}$

Compute the adiabatic wall enthalpy $H_{aw} = H_e + Pr^{1/3} \dfrac{U_e^2}{2}$.

35. HAW = (HE) + (PRE1O3)(UE2O2)

Call subroutine SEVAL to determine T_{aw} and C_{pw} = (ERASE3) using the known H_{aw}.

36. CALL SEVAL (2, TAW, ERASE3, HAW)

ITWTAB = - 1: adiabatic wall temperature

= 0: constant wall temperature

= 1: input wall temperature (variable)

37. If ITWTAB < 0, go to 38

If ITWTAB = 0, go to 42

If ITWTAB > 0, go to 46

Consider the case of adiabatic wall temperature.

Set

38. TW = TAW

39. HW = HAW

The enthalpy gradient along the wall:

40. HWP = HEP + (PRE1O3)(UE)(UEP)

41. Go to 49

Consider the case of constant wall temperature.

42. TW = TWTAB(1)

The wall enthalpy is calculated at 86 of BARSET:

43. HW = TWTAB(2)

The gradient of wall enthalpy is zero for constant wall temperature option.

44. HWP = 0.0

45. Go to 49

Call subroutine XNTERP to find T_w and obtain dT_w/dx in the case of variable wall temperature option.

46. CALL XNTERP(X, TW, TWP, ITWX, XITAB, TWTAB, IXTAB, CTWX, IXPOS)

Call subroutine SEVAL to obtain C_{pw} and H_w using T_w.

47. CALL SEVAL(1, TW, CPW, HW)

The enthalpy gradient along the wall:

48. HWP = (FJG)(CPW)(TWP)

49. If TW ≤ TAW, go to 54

If TW > TAW, go to 50

Write T_w and T_{aw}.

50. WRITE TW, TAW

Write the axial distance x, Mach number M_e, momentum thickness θ, and energy thickness ϕ at the point where T_w exceeds T_{aw}.

51. WRITE X, ZME, THETA, PHI

52. WRITE error indication message

Stop the calculation by calling QUITS.

53. CALL QUITS

Save the wall enthalpy H_w.

54. A = HW

Stagnation enthalpy minus wall enthalpy:

55. B = H0 - HW

Save the minus sign of dynamic enthalpy $(-U_e^2/2)$.

56. C = - UE2O2

Save the free-stream temperature T_e.

57. TFINT = TE

Call subroutine ZETAIT to calculate the shape parameter $[\zeta = (\Delta/\delta)^{1/n}]$ and boundary layer thickness Δ, δ, and $\delta*$ at point x, for given values of θ and ϕ.

58. CALL ZETAIT

Save the value of $\rho_e U_e/\mu_e$.

59. CREY = (RHOUE)/(ZMU)

Obtain the Reynolds number based on momentum thickness θ.

60. RTHE = (CREY)(THETA)

The Reynolds number based on energy thickness ϕ:

61. RPHI · (CREY)(PHI)

Adiabatic ⁄ emperature T_{aw} divided by free-s.ream temper ⁄ T_e:

62. E1 = (TAW)/(TE)

Save (1 - m) power of the above value $\left(\frac{T_{aw}}{T_e}\right)^{1-m}$.

63. ERASE2 = $(ERASE1)^{(1.0 - ZMVIS)}$

Set

64. ERASE3 = (17.2)(T0 - TAW)/(TAW)

65. ERASE4 = (305.0)(TE - T0)/(TAW)

66. ICFCH = 0

Save the guess value of skin friction coefficient C_{fagt} = (0.002).

67. CFA = CFAGT

Save the Reynolds number based on momentum thickness.

68. RSUB = RTHE

The following calculation, down to step 95, determines the friction coefficient by iterations.

69. Do 95, I = 1, 50

Save

70. CFAG = CFA

Check the value of CFAGTP.

71. If CFAGTP(ICFCH + 1) = 0, go to 110

If CFAGTP(ICFCH + 1) ≠ 0, go to 72

Calculate the following expression from the relationship

$$(\bar{C}_f \bar{R}_\theta)_{guess} = \frac{\rho_e \mu_e}{\rho_{aw} \mu_{aw}} (C_{fa})_{guess} \; R_\theta .$$

72. CFRT = (ERASE2)(CFAG)(RSUB)

The turbulent skin friction coefficient $\bar{C}_f$ is obtained from subroutine CFEVAL, in which empirical relations between $\bar{C}_f$ and $(\bar{C}_f \bar{R}_{\bar{\theta}})$ are tabulated.

73. CFBAR = CFEVAL(CFRT)

Sublayer temperature T_s divided by adiabatic wall temperature T_{aw}:

74. TSOTAW = 1.0 + (ERASE3)(SQRT((CFBAR)/(2.)))
+ (ERASE4)((CFBAR)/(2.))

Check the sign of T_s/T_{aw}.

75. If TSOTAW > 0, go to 83

If TSOTAW ≤ 0, go to 76

Write error message.

76. WRITE error message

Write X, M_e, θ, and ϕ in the case of BARPRO FAILURE.

77. WRITE X, ZME, THETA, PHI

78. WRITE cause of error message

Set

79. CFAGTP ∞ (ICFCH + 1) = 0.0

80. CFAG = 0.0

81. CFA = 0.0

82. Go to 101

Save the adiabatic skin friction coefficient

83. CFA = (CFBAR)/(ERASE1)/(TSOTAW)^{ZMVIS}

Check the tolerance in the present $\bar{C}_f$ - $\bar{C}_f \bar{R}_{\bar{\theta}}$ iteration loop.

84. If |(CFA-CFAG)/(CFAG)| < TOLCFA, go to 100

If |(CFA-CFAG)/(CFAG)| ≥ TOLCFA, go to 85

Check the number of iteration.

85. If I ≥ 2, go to 89

If I < 2, go to 86

Set

86. Z4 = CFA

87. Z2 = CFAG

88. Go to 95

Save the following values.

89. Z3 = Z4

90. Z1 = Z2

91. Z4 = CFA

92. Z2 = CFAG

93. Z5 = (Z4 - Z3)/(Z2 - Z1)

94. CFA = (Z4 - (Z5)(Z2))/(1. - Z5)

Go to 69 to continue the iteration.

95. Continue

Write that the skin friction coefficient $\overline{C}_f$ could not be obtained.

96. WRITE error message

97. WRITE X, ZME, THETA, PHI

98. WRITE Z1, Z2, CFA, Z3, Z4

99. WRITE cause of error message

Save C_{fa}.

100. CFAG = CFA

101. If ICFCH > 0, go to 111

If ICFCH ≤ 0, go to 102

Save

102. CFAGT = CFA

103. CF = CFAG

104. CFA = CFAGP

105. RSUB = RPHI

Set ICFCH equal one.

106. ICFCH = 1

Check whether the wall temperature is adiabatic or not.

107. If ITWTAB $\geq$ 0, go to 69

If ITWTAB < 0, go to 108

Set

108. CH = 0.0

109. Go to 113

Check the value of ICFCH.

110. If ICFCH $\leq$ 0, go to 106

If ICFCH > 0, go to 113

Set

111. CFAGP = CFAG

Calculate the Stanton number.

112. CH = ((PHI)/(THETA))$^{\mathrm{ZNSTAN}}$((CFAG)/(2.))/(1. - (5.)
(SQRT((CFAG)/(2.0))(CHPAR1))

Save $\left[\dfrac{1}{\rho_e U_e} \dfrac{d(\rho_e \overline{U}_e)}{dx}\right]$.

113. ERASE1 = (RHOUEP)/(RHOUE)

Put $\left(\dfrac{1 + \delta^*/\theta}{U_e} \dfrac{dU_e}{dx}\right)$ as

114. ERASE2 = (UEP)(1.0 + DELSOT)/(UE)

Call subroutine XNTERP to obtain the radius r and its gradient dr/dx at the axial distance x.

115. CALL XNTERP(X, YR, YRP, IYX, XITAB, YITAB, IXTAB,
CYX, IXPOS)

Calculate the value $\sqrt{1 + \left(\frac{dr}{dx}\right)^2}$.

116. DARC = SQRT(1.0 + (YRP)²)

Put

117. CDFORC = ((RHOUE)/(G))(UE)/(DARC)(CF)/(2.)

Check the geometry indicator EPSZ (= 0: Two-dimensional planar flow, = 1: Axisymmetric flow).

118. If EPSZ ≤ 0.0, go to 120

If EPSZ > 0.0, go to 119

Save the value $\left[\frac{1}{\rho_e U_e} \frac{d(\rho_e U_e)}{dx} + \frac{1}{r} \frac{dr}{dx}\right]$.

119. ERASE1 = ERASE1 + (EPSZ)/(YR)(YRP)

The gradient of momentum thickness $\left(\frac{d\phi}{dx}\right)$:

120. THETAP = (CF)/(2.0)(DARC)-
(THETA)(ERASE2 + ERASE1)

Set

121. ERASE2 = H0 - HW

The gradient of energy thickness $\left(\frac{d\phi}{dx}\right)$:

122. PHIP = (CH)(DARC)/(ERASE2)(HAW - HW)-
(PHI)(ERASE1 - (HWP)/(ERASE2))

Check the indicator IND.

123. If IND ≠ 1, return

If IND = 1, go to 124

Check whether adiabatic wall temperature option is used.

124. If ITWTAB < 0, go to 127

If ITWTAB ≥ 0, go to 125

Local rate of heat transfer to wall (q_w):

125. QW = (RHOUE)/(FJ)(CH)/(G)(HAW - HW)

Heat transfer coefficient (h_g):

126. HG = (QW)/(TAW - TW)

Set

127. QDAO = QDA

128. DFORCO = DFORCE

129. DFLATO = DFLAT

Check whether axisymmetric flow (EPSZ = 1.) or two-dimensional planar flow (EPSZ = 0.) is used.

130. If EPSZ ≤ 0.0, go to 136

If EPSZ > 0.0, go to 131

Set πr

131. ERASE1 = (PIE)(YR)

132. QDA = (ERASE1)(QW)

133. DFORCE = (ERASE1)(CDFORC)

134. DFLAT = 0.0

135. Go to 139

Set

136. QDA = (QW)/(2.)

137. DFORCE = (CDFORC)/(2.)

138. DFLAT = (DFORCE)(YRP)

139. Y0ARC = Y2ARC

140. Y2ARC = DARC

Check the value of DX.

141. If DX $\leq$ 0.0, return

If DX > 0.0, go to 142

Call subroutine XNTERP to obtain the radius r = (ERASE1) and its gradient $\frac{dr}{dx}$ = (ERASE2) corresponding to x = X - DX/2.

142. CALL XNTERP(X - (DX)/(2.0), ERASE1, ERASE2, IYX, XITAB, YITAB, IXTAB, CYX, IXPOS)

143. Y1ARC = SQRT(1.0 + (ERASE2)^2)

Increment of contour length:

144. DXLARC = (DX)(Y0ARC + (4.)(Y1ARC) + Y2ARC)/(6.)

Contour length:

145. XLARC = XLARC + DXLARC

Integrated rate of heat transfer to the wall:

146. SUMQDA = SUMQDA + (DXLARC)(QDA + QDAO)

Drag in axial direction:

147. FORCE = FORCE + (DXLARC)(DFORCE + DFORCO)

Drag normal to the axial direction:

148. FLAT = FLAT + (DXLARC)(DFLAT + DFLATO)

Return to the main routine.

149. Return

Store the results obtained in subroutine BARPRO for printout. Save the Reynolds number based on the contour length.

150. RXLN = (CREY)(XLARC)

Reynolds number based on the displacement thickness:

151. RDLS = (CREY)(DELSTR)

Check the value of the shape factor ζ.

152. If ZETA $\geq$ 1.0, go to 160

If ZETA < 1.0, go to 153

Set

153. I = 1

Save the following integrals.

154. Z1 = ZI4

155. Z2 = ZI5

156. Z3 = ZI6

157. Z4 = ZI7

158. Z5 = ZI1P

159. Go to 166

Set

160. I = 6

Save the following inegrals.

161. Z1 = ZI1

162. Z2 = ZI2

163. Z3 = ZI3

164. Z4 = ZI2P

165. Z5 = ZI3P

Print out

166. WRITE heading for output values

167. WRITE X, ZME, DELTA, HG, ZETA, CF

168. WRITE XLARC, TE, BDELTA, QW, ZINTPR (I), Z1, CH

169. WRITE YR, TW, DELSTR, SUMQDA, ZINTPR (I + 1), Z2, RTHE

170. WRITE YRP, TAW, THETA, FORCE, ZINTRP (I + 2), Z3, RXLN

171. WRITE ZMEP, PHI, FLAT, ZINTPR (I + 3), Z4, RPHI

172. WRITE UE, DELSOT, ZINTPR (I + 4), Z5, RDLS

173. WRITE PE

174. Return

Table 4 gives subroutine BARPRO nomenclature.

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE

Symbol	Description	Units	Reference
A	Saved value of wall enthalpy H_w	ft^2/sec^2	/SAVED/, 54
B	Stagnation enthalpy minus wall enthalpy $H_o - H_w$	ft^2/sec^2	/SAVED/, 55
BDELTA	Temperature thickness Δ	ft	/OUTPUT/, 168
C	Minus sign of dynamic enthalpy $-\frac{U_e^2}{2}$	ft^2/sec^2	/SAVED/, 56
CDFORC	Local drag force per unit area	lbf/ft^2	117, 133, 137
CF	Skin friction coefficient	—	/OUTPUT/, 103, 117, 120, 167
CFA	Adiabatic skin friction coefficient C_{fa}	—	67, 70, 81, 83, 84, 86, 94, 100, 102, 104
CFAG	Guess value of adiabatic skin friction coefficier C_{fag}	—	70, 72, 80, 84, 87, 100, 103, 111, 112
CFAGP	Skin friction coefficient obtained by using the Reynolds number R_ϕ	—	/INTER/, 104, 111
CFAGT	Initial value of skin friction coefficient	—	/INTER/, EQUIV, 67, 102
CFAGTP	Array equivalenced to CFAGT and CFAGP		DIM, EQUIV, 71, 79
CFBAR	Skin friction coefficient obtained from empirical relationship	—	73, 74, 83

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
CFRT	Turbulent skin friction coefficient multiplied by experimental Reynolds Number $\bar{C}_f \bar{R}_{\bar{\theta}}$	—	72, 73
CH	Stanton number	—	/OUTPUT/, 108, 112, 122, 125, 168
CHPAR1	$1 - Pr + \ln\left(\frac{6}{5Pr + 1}\right)$	—	/INTER/, 112
CMX	Array of parabola coefficients for Mach number table	—	/LOOKUP/ 2, 20, 28
CPE	Specific heat in free stream C_{pe}	Btu/(1bm - °R)	6, 7, 8
CPW	Specific heat at the wall temperature C_{pw}	Btu/(1bm - °R)	47, 48
CREY	Mass flow density divided by viscosity in free stream $\rho_e U_e / \mu_e$	1/ft	59, 60, 61, 150, 151
CTWX	Array of parabola coefficients for the wall temperature table	—	/LOOKUP/, 46
CTX	Array of parabola coefficients for the free stream temperature table	—	/LOOKUP/, 4
CYX	Array of parabola coefficients for the nozzle radius table	—	/LOOKUP/, 115, 142
DARC	$\sqrt{1 + (dr/dx)^2}$	—	116, 117, 120, 122, 140
DELSTR	Displacement thickness $\delta *$	ft	/OUTPUT/, 151, 169
DELSOT	Displacement thickness divided by momentum thickness $\delta * / \theta$	—	/OUTPUT/, 114, 172

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
DELTA	Velocity thickness δ	ft	/OUTPUT/, 167
DFLAT	One-half the local drag force normal to x-axis	lbf/ft^2	128, 130, 138, 148
DFLATO	Saved value of DFLAT	lbf/ft^2	129, 148
DFORCE	One-half the local drag force (for axisymmetric flow) or (for two-dimensional planar flow)	lbf/ft lbf/ft^2	133, 147 128, 137, 138, 147
DFORCO	Saved value of DFORCE	lbf/ft^2	128, 147
DX	Weighted difference of table values of axial distance	ft	/INTER/, 141, 142, 144
DXLARC	Increment of the length along contour	ft	144, 145, 146, 147, 148
DXRHO	One-tenth the difference between axial distance	ft	/INTER/, 13, 28, 31
EPSZ	Geometry indicator EPSZ = 0: two-dimensional planar flow EPSZ = 1: axisymmetric flow	—	/INPUT/, 118, 119, 130
ERASE1	1. Free-stream Mach number M_e 2. Saved value of T_{aw}/T_e 3. Saved value of $\frac{1}{\rho_e U_e}\frac{d(\rho_e U_e)}{dx}$ or $\frac{1}{\rho_e U_e}\frac{d(\rho_e U_e)}{dx}+\frac{1}{r}\frac{dr}{dx}$ 4. πr 5. Contour radius r	— — ft ft	20, 21, 28, 29 62, 63, 72, 83 113, 119, 120, 122 131, 132, 133 142

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
ERASE2	1. Gradient of free-stream Mach number dM_e/dx	1/ft	20, 28
	2. Saved value of $\left(\frac{T_{aw}}{T_e}\right)^{1-m}$	—	63
	3. Saved value of $\frac{1 + \delta^*/\theta}{U_e} \frac{dU_e}{dx}$	1/ft	114, 120
	4. Enthalpy difference $H_o - H_w$	ft^2/sec^2	121, 122
	5. dr/dx	—	142, 143
ERASE3	1. Specific heat at adiabatic wall temperature C_{paw}	—	36
	2. Saved value of 17.2 $(T_o - T_{aw})/T_{aw}$	—	64, 74
ERASE4	Saved value of 305 $(T_e - T_o)/T_{aw}$	—	65, 74
FJ	Conversion factor between thermal and work units	(ft-lbf)/Btu	/INPUT/, 125
FJG	Conversion factor between thermal and work units multiplied by acceleration of gravity used as a proportionality constant gJ	$lbm/(Btu\text{-}sec^2)$	/CSEVAL/, 7, 48

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
FLAT	Force normal to x-direction for two-dimensional planar flow	lbf	/OUTPUT/, 148, 171
FORCE	Drag force in axial or x-direction	lbf	/OUTPUT/, 147, 170
G	Acceleration of gravity used as a proportionality constant	lbm/lbf ft/sec^2	/INPUT/, 117, 125
GAME	Specific heat ratio in free stream	—	8
H0	Stagnation enthalpy H_o	ft^2/sec^2	/CSEVAL/, 9, 55, 121
HAW	Adiabatic wall enthalpy H_{aw}	ft^2/sec^2	35, 36, 39, 122, 125
HE	Enthalpy of free stream in work units H_e	ft^2/sec^2	/INTER/, 6, 9, 35
HEP	Enthalpy gradient dH_e/dx	ft/sec^2	7, 11, 40
HG	Heat transfer coefficient h_g	Btu/(ft^2-sec° R)	/OUTPUT/, 126, 167
HW	Enthalpy at the wall H_w	ft^2/sec^2	/INTER/, 39, 43, 47, 54, 55, 121, 122, 125
HWP	Enthalpy gradient dH_w/dx	ft/sec^2	40, 44, 48
I	Subscript counter	—	153, 160, 168-172
ICFCH	Option indicator	—	66, 71, 79, 101, 106, 110
IMX	Mach number table entry indicator and saved subscript counter	—	/LOOKUP/, 2, 20, 28
IND	Program loop control indicator	—	CALL, 1, 123

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
ITWTAB	= - 1: T_w = adiabatic wall temperature = 0: T_w = constant = 1: IXTAB values of T_w will be input	— —	/INPUT/ 37, 107, 124
ITWX	Wall temperature table entry indicator and saved subscript counter	—	/LOOKUP/, 46
ITX	Free-stream temperature table entry indicator and saved subscript counter	—	/LOOKUP/, 4
IXPOS	Array position indicator	—	/LOOKUP/, 2, 4, 20, 28, 46, 115, 142
IXTAB	Number of points in the x, y, and Mach number tables (4 ≤ IXTAB ≤ 500)	—	/INPUT/, 2, 4, 20, 28, 46, 115, 142
IYX	Nozzle radius table entry indicator and saved subscript counter	—	/LOOKUP/, 115, 142
PE	Static pressure of the free stream	lbf/ft^2	/OUTPUT/, 5, 12, 173
PHI	Energy thickness ϕ	ft	/OUTPUT/, 51, 61, 77, 97, 112, 122, 171
PHIP	Gradient of energy thickness $d\phi/dx$	—	/INTER/, 122
PIE	Circumferential constant π	—	/INPUT/, 131
PRE1O3	Cubic root of Prandtl number	—	/INTER/, 35, 40
QDA	One-half the local heat transfer to the wall (axisymmetric flow) (two-dimensional planar flow)	 Btu/(ft-sec) $Btu/(ft^2-sec)$	 127, 132, 146 127, 136, 146
QDAO	Saved value of QDA	Btu/(ft-sec)	127, 146

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
QW	Local heat transfer rate to wall	Btu/(ft^2-sec)	/OUTPUT/, 125, 126, 132, 136, 168
RBAR	Gas constant in work units	(ft-lbf)/(lbm-°R)	/INPUT/, 12, 30
RDLS	Reynolds number based on displacement thickness $R_{\delta*} = \rho_e U_e \delta*/\mu_e$	—	151, 172
RHOE	Free-stream density ρ_e	lbm/ft^3	/INTER/, 12, 32, 33
RHOEP	Density gradient $d\rho_e/dx$	lbm/ft^4	14, 31, 33
RHOUE	Mass flow density $\rho_e U_e$	lbm/(ft^2-sec)	/INTER/, 32, 59, 113, 117
RHOUEP	Gradient of mass flow density $d(\rho_e U_e)/dx$	lbm/(ft^3-sec)	33, 113
ROJ	Gas constant in thermal units	Btu/(lbm-°R)	/CSEVAL/, 8
RPHI	Reynolds number based on energy thickness $R_\phi = \rho_e U_e \phi/\mu_e$	—	61, 105, 171
RSUB	Saved value of Reynolds number R_θ Saved value of Reynolds number R_ϕ	—	68, 72, 105
RTHE	Reynolds number based on momentum thickness $R_\theta = \rho_e U_e \theta/\mu_e$	—	60, 68, 169
RXLN	Reynolds number based on arc length $R_L = \rho_e U_e L/\mu_e$	—	150, 170
SUMQDA	Integrated heat transfer rate to the wall up to station x (axisymmetric flow) (two-dimensional planar flow)	 Btu/sec Btu/(sec-ft)	/OUTPUT/, 149, 169

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
T0	Stagnation temperature T_o	°R	/INPUT/, 34
TAW	Adiabatic wall temperature T_{aw}	°R	36, 38, 49, 50, 62, 126, 170
TE	Free-stream temperature T_e	°R	/OUTPUT/, 4, 5, 6, 12, 34, 57, 62, 168
TEP	Temperature gradient in free-stream dT_e/dx	°R/ft	4, 7
TFINT	Saved value of free-stream temperature	°R	/COFIIF/, 57
THETA	Momentum thickness θ	ft	/OUTPUT/, 51, 60, 77, 97, 112, 120, 170
THETAP	Gradient of momentum thickness $d\theta/dx$	—	/INTER/, 120
TITAB	Free-stream temperature table related to IXTAB array. This table is determined by subroutine GETPT via BARSET.	°R	/TABLES/, 4
TOLCFA	Tolerance in $\bar{C}_f - \bar{C}_f \bar{R}_{\bar{\theta}}$ iteration	—	/INPUT/, 84
TSOTAW	Sublayer temperature divided by adiabatic wall temperature T_s/T_{aw}	—	74, 75, 83
TW	Wall temperature	°R	/OUTPUT/, 38, 42, 46, 47, 49, 50, 126, 169
TWP	Temperature gradient on the wall	°R/ft	46, 48
TWTAB	Wall temperature table related to IXTAB array	°R	/TABLES/, 42, 43, 46

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
UE	Free-stream velocity	ft/sec	/OUTPUT/, 10, 11, 32, 33, 40, 114, 117, 172
UE2O2	Square of free-stream velocity divided by two (dynamic enthalpy) $U_e^2/2$	ft^2/sec^2	9, 10, 56
UEP	Velocity gradient along free stream	1/sec	11, 33, 40, 114
X	Axial distance or distance in free-stream direction	ft	/OUTPUT/, 2, 4, 16, 20, 24, 28, 46, 51, 77, 97, 115, 142, 167
XIBASE	First value in axial distance table	ft	/INTER/, 16
XIEND	Last value in axial distance table	ft	/INTER/, 24
XITAB	Table of IXTAB values (axial distance x) in monotonically increasing order	ft	/TABLES/, 2, 20, 28, 46, 115, 142
XLARC	Arc length of contour corresponding to x	ft	/OUTPUT/, 145, 150, 168
Y0ARC	Saved value of Y2ARC	—	139, 144
Y1ARC	$\sqrt{1 + \left(\frac{dr}{dx}\right)^2}$ corresponding to $\left(x - \frac{\Delta x}{2}\right)$	—	143, 144
Y2ARC	Saved value of DARC	—	139, 140, 144
YITAB	Nozzle radius or contour height table related to IXTAB array	ft	/TABLES/, 115, 142
YR	Radius or height of contour r	ft	/OUTPUT/, 115, 119, 131, 169

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
YRP	dr/dx	—	115, 116, 119, 138, 170
Z1	1. Saved value of free-stream density ρ_e	lbm/ft^3	17, 22, 31
	2. Saved value of C_{fag}	—	90, 98
	3. Saved value of I_1 = (ZI1) or I_4 = (ZI4)	—	/OUTPUT/, 154, 161, 168
Z2	1. Saved value of ρ_e	lbm/ft^3	25, 30, 31
	2. Saved value of C_{fag} (CFAG)	—	87, 92, 94, 98
	3. Saved value of integral I_2 = (ZI2) or I_5 = (ZI5)	—	/OUTPUT/, 155, 162, 169
Z3	1. Saved value of one or point five	—	18, 23, 26, 31
	2. Saved value of C_{fa}	—	89, 98
	3. Saved value of integral I_3 = (ZI3) or I_6 = (ZI6)	—	/OUTPUT/, 156, 163, 170
Z4	1. Free-stream pressure P_e	lbf/ft^2	21, 22, 29, 30
	2. Saved value of C_{fa} (CFA)	—	86, 91, 94, 98
	3. Saved value of integral I_2' = (ZI2P)	—	/OUTPUT/, 157, 164, 172
Z5	1. Free-stream temperature T_e	°R	21, 22, 29, 30
	2. Saved value of (Z4 - Z3)/(Z2 - Z1)	—	93, 94
	3. Saved value of integral I_1' = (ZI1P) or I_3' = (ZI3P)	—	/OUTPUT/, 158, 165, 172
ZETA	Shape factor: $\zeta = (\Delta/\delta)^n$	—	/OUTPUT/, 152, 167

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
ZI1	Integral $I_1 = \int_0^1 \frac{\rho}{\rho_e} S^n(1 - S)dS$	—	/OUTPUT/, 161
ZI2	Integral $I_2 = \int_0^1 \frac{\rho}{\rho_e} S^n dS$	—	/OUTPUT/, 162
ZI3	Integral $I_3 = \int_0^\zeta \frac{\rho}{\rho_e} S^{n-1} dS$	—	/OUTPUT/, 163
ZI4	Integral $I_4 = \int_0^\zeta \frac{\rho}{\rho_e} S^n(1 - S)dS$	—	/OUTPUT/, 154
ZI5	Integral $I_5 = \int_\zeta^1 \frac{\rho}{\rho_e} S^n(1 - S)dS$	—	/OUTPUT/, 155
ZI6	Integral $I_6 = \int_0^\zeta \frac{\rho}{\rho_e} S^n dS$	—	/OUTPUT/, 156
ZI7	Integral $I_7 = \int_\zeta^1 \frac{\rho}{\rho_e} S^n dS$	—	/OUTPUT/, 157
ZI1P	Integral $I_1' = \int_0^1 \frac{\rho}{\rho_e} W^n(1 - W)dW$	—	/OUTPUT/, 158

TABLE 4. SUBROUTINE BARPRO NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
ZI2P	Integral $I_2' = \int_0^{1/\zeta} \frac{\rho}{\rho_e} W^n (1 - W) dW$	—	/OUTPUT/, 164
ZI3P	Integral $I_3' = \int_{1/\zeta}^{1} \frac{\rho}{\rho_e} W^{n-1} (1 - W) dW$	—	/OUTPUT/, 165
ZINTPR	Array of Hollerith headings for printout of integrals	—	DATA, DIM, 168, 169, 170, 171, 172
ZME	Mach number of free-stream M_e	—	/OUTPUT/, 2, 4, 51, 77, 97, 167
ZMEP	dM_e/dx	1/ft	2, 171
ZMTAB	Mach number table related to IXTAB array	—	/TABLES/, 2, 20, 28
ZMU	Viscosity of free stream μ_e	lbm/(sec-ft)	34, 59
ZMU0	Stagnation viscosity μ_o	lbm/(sec-ft)	/INPUT/, 34
ZMVIS	Exponent in viscosity - temperature law	—	/INPUT/, 34, 63, 83
ZNSTAN	Interaction exponent in Stanton number relation $\tilde{n}$	—	/INPUT/, 112

SUBROUTINE BARSET

Subroutine BARSET computes program constants and sets up specific heat C_p and enthalpy H versus temperature T tables and tables of pressure P and temperature T as a function of axial distance x for the inviscid flow. The entropy S for T and P can be determined.

Evaluation of enthalpy is as follows:

$$H(T) = H_i + \int_{T_i}^{T} C_p(t)\,dt \quad ,$$

$$H(T) = \int_{0}^{T_i} C_p(t)\,dt + \int_{T_i}^{T} C_p(t)\,dt \quad ,$$

$$H(T) = \int_{0}^{T_1} C_p(t)\,dt + \int_{T_1}^{T_2} C_p(t)\,dt + \int_{T_2}^{T_3} C_p(t)\,dt + \ldots + \int_{T_{i-1}}^{T_i} C_p(t)\,dt$$

$$+ \int_{T_i}^{T} C_p(t)\,dt \quad .$$

The first integral is evaluated assuming that the C_p - T curve is linear from T = 0 to $T = T_1$.

The value of C_p at $T = T_1$ is C_{p1}. The value of dC_p/dT at $T = T_1$ is BCP_1. Thus for

$$0 < T < T_1 \quad ,$$

$$C_p = C_{p1} + BCP_1(T - T_1) \quad ,$$

$$\int_{0}^{T_1} C_p(t)\,dt = \int_{0}^{T_1} [C_{p1} + BCP_1(T - T_1)]\,dT \quad .$$

Therefore, one obtains

$$\int_{o}^{T_1} C_p(t)\,dt = C_{p1}T_1 - \frac{BCP_1}{2} T_1^2 \quad .$$

The remaining integrals can be evaluated gradually from the expression for C_p.

$$\int_{T_i}^{T_{i+1}} C_p(t)\,dt = \int_{T_i}^{T_{i+1}} [C_{pi} + BCP_i(t - T_i) + CCP_i(t - T_i)^2$$

$$+ DCP_i(t - T_i)^3]\,dt \quad ,$$

where

$$CCP_i = \frac{1}{2!}\left(\frac{d^2C_p}{dt^2}\right)_{t = T_i} \quad ,$$

$$DCP_i = \frac{1}{3!}\left(\frac{d^3C_p}{dt^3}\right)_{t = T_i} \quad .$$

Therefore,

$$\int_{T_i}^{T_{i+1}} C_p(t)\,dt = C_{pi}(T_{i+1} - T_i) + BCP_i \frac{(T_{i+1} - T_i)^2}{2}$$

$$+ CCP_i \frac{(T_{i+1} - T_i)^3}{3} + DCP_i \frac{(T_{i+1} - T_i)^4}{4} \quad ,$$

and

$$\int_{T_i}^{T} C_p(t)\,dt = C_{pi}(T - T_i) + BCP_i \frac{(T - T_i)^2}{2} + CCP_i \frac{(T - T_i)^3}{3}$$

$$+ DCP_i \frac{(T - T_i)^4}{4} \quad .$$

By assuming that

$$H_1 = \int_{o}^{T_1} C_p\,(t)\,dt$$

$$H_2 = H_1 + \int_{T_1}^{T_2} C_p(t)\,dt$$

$$\vdots$$

$$H_i = H_{i-1} + \int_{T_{i-1}}^{T_i} C_p(t)\,dt \quad ,$$

then

$$H(T) = H_i + C_{pi}(T - T_i) + BCP_i \frac{(T - T_i)^2}{2} + CCP_i \frac{(T - T_i)^3}{3} + DCP_i \frac{(T - T_i)^4}{4} \quad .$$

The values of H_i are calculated and stored in BARSET under the name HTAB (I).

Evaluation of entropy is as follows.

$$S = S_o + \int_{T_o}^{T} \frac{Cp(t)}{t}\,dt - R\ln\left(\frac{P}{P_o}\right) = S_o + \int_{T_o}^{T_1} \frac{Cp(t)}{t}\,dt + \int_{T_1}^{T} \frac{Cp(t)}{t}\,dt$$

$$- R\ln\left(\frac{P}{P_o}\right) \quad .$$

Arbitrarily $S = 0$ can be set at $T = T_1$ and $P = P_o$ where T_1 is the first point of the $C_p - T$ table and P_o is the stagnation pressure.

$$0 = S_o + \int_{T_o}^{T_1} \frac{Cp(t)}{t}\,dt + \int_{T_1}^{T=T_1} \frac{Cp(t)}{t}\,dt - R\ln\left(\frac{P_o}{P_o}\right) \quad .$$

$$0 = S_o + \int_{T_o}^{T_1} \frac{C_p(t)}{t}\, dt$$

Thus the entropy will be evaluated from the above equation.

$$S = \int_{T_1}^{T} \frac{C_p(t)}{t}\, dt - R \ln \left(\frac{P}{P_o}\right) \quad .$$

By definition

$$P'R' \equiv R \ln \left(\frac{P}{P_o}\right) \quad ,$$

then

$$S = \int_{T_1}^{T} \frac{C_p(t)}{t}\, dt - P'R' \quad .$$

$$S = \int_{T_1}^{T_2} \frac{C_p(t)}{t}\, dt + \int_{T_2}^{T_3} \frac{C_p(t)}{t}\, dt + \ldots$$

$$+ \int_{T_{i-1}}^{T_i} \frac{C_p(t)}{t}\, dt + \int_{T_i}^{T} \frac{C_p(t)}{t}\, dt - P'R' \quad .$$

$$S = \sum_{j=1}^{i-1} \int_{T_j}^{T_{j+1}} \frac{C_p(t)}{t}\, dt + \int_{T_i}^{T} \frac{C_p(t)}{t}\, dt - P'R' \quad .$$

From the expression for C_p,

$$S = \sum_{j=1}^{i-1} \int_{T_j}^{T_{j+1}} \frac{C_{pj} + BCP_j\,(t - T_j) + CCP_j(t - T_j)^2 + DCP_j(t - T_j)^3}{t}\,dt$$

$$+ \int_{T_i}^{T} \frac{C_{pi} + BCP_i(t - T_i) + CCP_i(t - T_i)^2 + DCP_i(t - T_i)^3}{t}\,dt - P'R' \quad .$$

$$S = \sum_{j=1}^{i-1} \int_{T_j}^{T_{j+1}} \left[\frac{C_{pj}}{t} + \frac{BCP_j(t - T_j)}{t} + \frac{CCP_j(t - T_j)^2}{t} + \frac{DCP_j(t - T_j)^3}{t}\right] dt$$

$$+ \int_{T_i}^{T} \left[\frac{C_{pi}}{t} + \frac{BCP_i(t - T_i)}{t} + \frac{CCP_i(t - T_i)^2}{t} + \frac{DCP_i(t - T_i)^3}{t}\right] dt - P'R' \quad .$$

$$S = \sum_{j=1}^{i-1} \left[C_{pj} \ln t + BCP_j(t - T_j \ln t) + CCP_j\left(\frac{t^2}{2} - 2tT_j + T_j^2 \ln t\right)\right.$$

$$\left. + DCP_j\left(\frac{t^3}{3} - \frac{3t^2T_j}{2} + 3tT_j^2 - T_j^3 \ln t\right)\right]_{T_j}^{T_{j+1}}$$

$$+ \left[C_{pi} \ln t + BCP_i(t - T_i \ln t) + CCP_i\left(\frac{t^2}{2} - 2tT_i + T_i^2 \ln t\right)\right.$$

$$\left. + DCP_i\left(\frac{t^3}{3} + \frac{3t^2T_i}{2} + 3tT_i^2 - T_i^3 \ln t\right)\right]_{T_i}^{T} - P'R' \quad .$$

$$S = \sum_{j=1}^{i-1} \Big[C_{pi} \ln T_{j+1} + BCP_j (T_{j+1} - T_j \ln T_{j+1})$$

$$+ CCP_j \left(\frac{T_{j+1}^2}{2} - 2T_{j+1} T_j + T_j^2 \ln T_{j+1} \right)$$

$$+ DCP_j \left(\frac{T_{j+1}^3}{3} - \frac{3T_{j+1}^2 T_j}{2} + 3T_{j+1} T_j^2 - T_j^3 \ln T_{j+1} \right)$$

$$- C_{pj} T_j - BCP_j (T_j - T_j \ln T_j) - CCP_j \left(\frac{T_j^2}{2} - 2T_j T_j + T_j^2 \ln T_j \right)$$

$$- DCP_j \left(\frac{T_j^3}{3} - \frac{3T_j^2 T_j}{2} + 3T_j T_j^2 - T_j^3 \ln T_j \right) \Big]$$

$$+ C_{pi} \ln T + BCP_i (T - T_i \ln T) + CCP_i \left(\frac{T_2}{2} - 2TT_i + T_i^2 \ln T \right)$$

$$+ DCP_i \left(\frac{T^3}{3} - \frac{3T^2 T_i}{2} + 3TT_i^2 - T_i^3 \ln T \right)$$

$$- C_{pi} \ln T_i - BCP_i (T_i - T_i \ln T_i) - CCP_i \left(\frac{T_i^2}{2} - 2T_i T_i + T_i^2 \ln T_i \right)$$

$$- DCP_i \left(\frac{T_i^3}{3} - 3 \frac{T_i^2 T_i}{2} + 3T_i T_i^2 - T_i^3 \ln T_i \right) - P'R' \quad .$$

$$S = \sum_{j=1}^{i-1} \left[\ln \frac{T_{j+1}}{T_j} (C_{pj} - BCP_j T_j + CCP_j T_j^2 - DCP_j T_j^3) \right.$$

$$+ (T_{j+1} - T_j)(BCP_j - 2CCP_j T_j + 3DCP_j T_j^2)$$

$$\left. + (T_{j+1}^2 - T_j^2)\left(\frac{CCP_j}{2} - \frac{3}{2} DCP_j T_j\right) + \left(\frac{T_{j+1}^3}{3} - \frac{T_j^3}{3}\right) DCP_j \right]$$

$$+ \ln T (C_{pi} - BCP_i T_i + CCP_i T_i^2 - DCP_i T_i^3)$$

$$+ T (BCP_i - 2CCP_i T_i + 3DCP_i T_i^2)$$

$$+ T^2\left(\frac{CCP_i}{2} - \frac{3DCP_i}{2} T_i\right) + \frac{T^3}{3} DCP_i$$

$$- \ln T_i (C_{pi} - BCP_i T_i + CCP_i T_i^2 - DCP_i T_i^3)$$

$$- T_i (BCP_i - 2CCP_i T_i + 3DCP_i T_i^2)$$

$$- T_i^2\left(\frac{CCP_i}{2} - \frac{3DCP_i}{2} T_i\right) - T_i^3 \frac{DCP_i}{3} - P'R' \quad .$$

By letting

$$BARB1_i = C_{pi} - BCP_i T_i + CCP_i T_i^2 - DCP_i T_i^3 \quad ,$$

$$BARB2_i = BCP_i - 2CCP_i T_i + 3DCP_i T_i^2 \quad ,$$

$$BARB3_i = \frac{CCP_i}{2} - \frac{3}{2} DCP_i T_i \quad ,$$

then

$$S = \sum_{j=1}^{i-1} \left[\ln \frac{T_{j+1}}{T_j} \, BARB1_j + (T_{j+1} - T_j) \, BARB2_j \right.$$

$$\left. + (T^2_{j+1} - T^2_j) \, BARB3_j + (T^3_{j+1} - T^3_j) \, \frac{DCP_j}{3} \right]$$

$$+ \ln T \, BARB1_i + T \, BARB2_i + T^2 \, BARB3_i + T^3 \, \frac{DCP_i}{3}$$

$$- \ln T_i \, BARB1_i - T_i \, BARB2_i - T_i^2 \, BARB3_i - T_i^3 \, \frac{DCP_i}{3} - P'R' \quad .$$

The terms G1 and G2 are defined as follows:

$$G1 = \sum_{j=1}^{i-1} \left[\ln \frac{T_{j+1}}{T_j} \, BARB1_j + (T_{j+1} - T_j) \, BARB2_j \right.$$

$$\left. + (T^2_{j+1} - T^2_j) \, BARB3_j + (T^3_{j+1} - T^3_j) \, \frac{DCP_j}{3} \right] .$$

$$G2 = \ln T_i \, BARB1_i + T_i \, BARB2_i + T_i^2 \, BARB3_i + T_i^3 \, \frac{DCP_i}{3} \quad .$$

Then

$$S = G1 + \ln T \, BARB1_i + T \, BARB2_i + T^2 \, BARB3_i + \frac{T^3 DCP_i}{3} - G2 - P'R' \quad .$$

By letting

$$G_i = G1 - G2 \quad ,$$

one finally obtains

$$S = G_i + BARB1_i \ln T + BARB2_i T + BARB3_i T^2 + DCP_i \frac{T^3}{3} - P' R' \quad .$$

The terms G1, G2, G_i, $BARB1_i$, $BARB2_i$, $BARB3_i$ are evaluated in BARSET.

COMMON BLOCKS

COMMON blocks CSEVAL, INPUT, and TABLES are used.

TBL SUBROUTINES

Subroutine DIRECT calls BARSET.

BARSET calls subroutines BMFITS, SEVAL, QUITS, and GETPT.

FORTRAN SYSTEM ROUTINES

FORTRAN library routine ALOG is used.

CALLING SEQUENCE

The subroutine calling sequence is:

CALL BARSET

SOLUTION METHOD

Set the circular constant π.

1. PIE = 3.14159265

Multiply sea-level acceleration with the conversion factor relating thermal and work units.

2. FJG = (FJ)(G)

Divide the specific gas constant by the conversion factor relating thermal and work units.

3. ROJ = (RBAR)/(FJ)

Set TMAX equal to the free-stream stagnation temperature.

4. TMAX = T0

Set indicator I1 for nominal entry.

5. I1 = 1

Test whether adiabatic wall temperature (ITWTAB = -1), constant wall temperature (ITWTAB = 0), or tabular wall temperature (ITWTAB = 1) is to be used.

6. If ITWTAB > 0, go to 7

   If ITWTAB = 0, go to 8

   If ITWTAB < 0, go to 12

Set I1 equal to the number of points in x, y, and M tables (IXTAB).

7. I1 = IXTAB

Check and save the maximum value of the wall temperature tabulated (TWTAB).

8. Do 11 I = 1, I1

9. If TWTAB (I) < TMAX, go to 11

   If TWTAB (I) ≧ TMAX, go to 10

Save the maximum value of TWTAB.

10. TMAX = TWTAB (I)

11. Continue.

Test whether constant specific heat calculation (ICTAB = 0) or specific heat table is required (ICTAB > 0).

12. If ICTAB = 0, go to 47

    If ICTAB ≠ 0, go to 13

Check whether the temperature input value (TMAX) exceeds the table upper limit.

13. If TMAX < TCTAB(ICTAB), go to 16

If TMAX ≥ TCTAB(ICTAB), go to 14

WRITE TCTAB(ICTAB) and TMAX, when the table upper limit TCTAB(ICTAB) is exceeded.

14. WRITE TMAX, TCTAB(ICTAB)

Stop the calculation by calling QUITS.

15. Go to 49

Save the number of points in C_p versus T table (ICTAB).

16. NOCTAB = ICTAB

The following calculation, down to step 43, is only used for specific heat polynomial, enthalpy, and entropy equations. Call subroutine BMTAB to determine polynomial coefficients BCP, CCP, and DCP.

17. CALL BMFITS (TCTAB, CPTAB, ICTAB, <u>BCP</u>, <u>CCP</u>, <u>DCP</u>)

Calculate temperature and various powers thereof.

18. TIE1 = TCTAB(1)

19. TIE2 = $(\text{TCTAB}(1))^2$

20. TIE3 = $(\text{TCTAB}(1))^3$

Calculate the enthalpy at T_1.

21. HTAB(1) = (CPTAB(1))(TIE1) - (BCP(1))(TIE2)/(2.0)

A term in entropy equation

22. BARB1(1) = CPTAB(1) - (BCP(1))(TIE1) + (CCP(1))(TIE2)-
(DCP(1))(TIE3)

First derivative of the above equation with negative sign

23. BARB2(1) = BCP(1) - (2.0)(CCP(1))(TIE1)
+ (3.0)(DCP(1))(TIE2)

Second derivative of the equation 22 times (-0.25).

24. BARB3(1) = ((CCP(1)) - (3.0)(DCP(1))(TIE1)/(2.0)

First term of the entropy equation

25. GTAB(1) = -(BARB1(1))(Ln(TIE1)) - (BARB2(1))(TIE1)-
(BARB3(1))(TIE2) - (DCP(1))(TIE3)/(3.0)

Set entropy summation term to zero.

26. G1 = 0.0

Store the enthalpy and coefficients of entropy equation at each temperature corresponding to C_p - T table.

27. Do 43, I = 2, ICTAB

Save T1 = TIE1, $T1^2$ = TIE2, $T1^3$ = TIE3.

28. TME1 = TIE1

29. TME2 = TIE2

30. TME3 = TIE3

Set new values from C_p - T table.

31. TIE1 = TCTAB (I)

32. TIE2 = (TIE1)(TIE1)

33. TIE3 = (TIE1)(TIE2)

Determine the temperature difference $(T_i - T_{i-1})$.

34. DELT = TIE1-TME1

Enthalpy at temperature T_i from C_p - T table.

35. $HTAB(I) = HTAB(I - 1) + (CPTAB(I - 1))(DELT)$
$+ (BCP(I - 1))(DELT)^2/(2.0)$
$+ (CCP(I - 1))(DELT)^3/(3.0)$
$+ (DCP(I - 1))(DELT)^4/(4.0)$

Check whether the value of I equals or exceeds ICTAB.

36. If $I \geq ICTAB$, go to 43

If $I < ICTAB$, go to 37

Determine the coefficients used in the entropy equation:

$$S = GTAB(I) + BARB1(I)\ Ln\ T + BARB2(I)\ T + BARB3(I)\ T^2 + DCP(I)\ T^3/3 - R\ Ln(P/Po).$$

37. $BARB1(I) = CPTAB(I) - (BCP(I))(TIE1) + (CCP(I))(TIE2) - (DCP(I))(TIE3)$

First derivative of the previous equation

38. $BARB2(I) = BCP(I) - (CCP(I))(TIE1)/(0.5) + (3.0)(DCP(I))(TIE2)$

Second derivative of 37 times (-0.25)

39. $BARB3(I) = (CCP(I) - (3.0)(DCP(I))(TIE1))/(2.0)$

Sum up terms in entropy equation.

40. $G1 = G1 + (BARB1(I - 1))(Ln((TIE1)/(TME1)))$
$+ (BARB2(I - 1))(DELT)$
$+ (BARB3(I - 1))(TIE2 - TME2)$
$+ (DCP(I - 1))(TIE3 - TME3)/(3.0)$

41. $G2 = (BARB1(I))(Ln(TIE1)) + (BARB2(I))(TIE1)$
$+ (BARB3(I))(TIE2) + (DCP(I))(TIE3)/(3.0)$

42. $GTAB(I) = G1 - G2$

43. Continue

Determine the number of input intervals minus one.

44. $IS = ICTAB - 1$

Stagnation enthalpy H_o and specific heat C_{po} are obtained from subroutine SEVAL for a given stagnation temperature.

45. CALL SEVAL(1, T0, <u>CP0</u>, <u>H0</u>)

Calculation of specific heat ratio at stagnation point

$(\gamma_o = C_{po}/(C_{po} - \overline{R}/J)$

46. GAM0 = (CP0)/(CP0 - ROJ)

When this specific heat ratio γ_o is less than 1, check for inconsistant units and stop the calculation. Otherwise continue.

47. If GAM0 > 1.0, go to 50

    If GAM0 ≤ 1.0, go to 48

48. WRITE GAM0

Subroutine QUITS stops the computation.

49. CALL QUITS

Set $(\gamma_o - 1)/2$ as

50. GM1O2 = (GAM0 - 1.)/(2.)

51. GOGM1 = (GAM0)/(GAM0 - 1.0)

Save the stagnation pressure.

52. P0MAX = P0

Generate the table of free-stream pressure P_e and temperature T_e. P_e and T_e are obtained for a given M_e by using subroutine GETPT.

53. Do 54, I = 1, IXTAB

54. CALL GETPT(ZMTAB(I), PITAB(I), TITAB(I))

Calculation of the free-stream enthalpy in the case of C_p = constant. This calculation continues to step 85.

55. If ICTAB > 0, go to 85

If ICTAB ≤ 0, go to 56

Set

56. NOCTAB = 6

Save NOCTAB minus one.

57. NOCTM1 = NOCTAB - 1

Save NOCTM1.

58. IS = NOCTM1

Compute the specific heat for stagnation condition $C_{po} = \dfrac{\gamma_o \bar{R}}{J(\gamma_o - 1)}$.

59. CP0 = (GOGM1)(RBAR)/(FJ)

Calculate the specific heat in work units $C'_p = \dfrac{\gamma_o g R}{\gamma_o - 1}$.

60. CJG = (CP0)(FJG)

Compute stagnation enthalpy in work units.

61. H0 = (CJG)(T0)

If the free-stream temperature exceeds the stagnation temperature, set the former as TMAX.

62. Do 66, I = 1, IXTAB

Set the free-stream temperature equal to the table value T_e.

63. TE = TITAB(I)

64. If TE $\leq$ TMAX, go to 66

If TE > TMAX, go to 65

Save T_e.

65. TMAX = TE

66. Continue

Set the maximum temperature in the table to TMAX plus a hundred.

67. TCTAB(NOCTAB) = TMAX + 100.0

Set the first temperature table value to 10^{-10}.

68. TCTAB(1) = 1.0E-10

Obtain NOCTM1 as a real number.

69. Z1 = NOCTM1

Definition of DELT = $\frac{\text{TMAX + 100 - (1.0E - 10)}}{5}$

70. DELT = (TCTAB(NOCTAB) - TCTAB(1))/(Z1)

Compute first term in entropy equation.

71. ERASE1 = -(CP0)(Ln(TCTAB(1))

Compute the coefficients for the specific heat polynomial and entropy equation.

72. Do 84, I = 1, NOCTAB

Save ERASE1.

73. GTAB(I) = ERASE1

Set the specific heat C_{po} in C_p table to a constant value.

74. CPTAB(I) = CP0

Set coefficients in the C_p = polynomial equal to zero.

75. BCP(I) = 0.0

76. CCP(I) = 0.0

77. DCP(I) = 0.0

Set coefficients in the entropy polynomial.

78. BARB1(I) = CP0

79. BARB2(I) = 0.0

80. BARB3(I) = 0.0

Compute table values of enthalpy corresponding to TCTAB.

81. HTAB(I) = (CP0)(TCTAB(I) - TCTAB(1))

82. If I ≥ NOCTM1, go to 84

If I < NCOTM1, go to 83

Generate the temperature table TCTAB.

83. TCTAB(I + 1) = TCTAB(I) + DELT

84. Continue

Check whether variable wall temperature option (ITWTAB ≠ 0) is used.

85. If ITWTAB ≠ 0, go to 87

If ITWTAB = 0, go to 86

Obtain the specific heat and enthalpy for constant wall temperature by calling the subroutine SEVAL.

86. CALL SEVAL(1, TWTAB(1), <u>ERASE1</u>, <u>TWTAB(2)</u>)

Obtain the stagnation entropy from stagnation temperature and pressure by calling the subroutine SEVAL.

87. CALL SEVAL (0, T0, P0, S0)

88. Return

Table 5 gives subroutine BARSET nomenclature.

TABLE 5. SUBROUTINE BARSET NOMENCLATURE

Symbol	Description	Units	Reference
BARB1	Coefficient in polynominal equation	Btu/(lbm-°R)	/CSEVAL/, 22, 25, 37, 40, 41, 78
BARB2	First derivative of BARB1 (negative sign)	Btu/(lbm-°R^2)	/CSEVAL/, 23, 25, 38, 40, 41, 79
BARB3	Second derivative of BARB1 times -0.25	Btu/(lbm-°R^3)	/CSEVAL/, 24, 25, 39-41, 80
BCP	Coefficient in the C_p-T relationship determined by BMFITS	Btu/(lbm-°R^2)	/CSEVAL/, 17, 21-23, 35, 37, 38, 75
CCP	Coefficient in the C_p-T relationship determined by BMFITS	Btu/(lbm-°R^3)	/CSEVAL/, 17, 21-24, 35, 37-39, 76
CJG	Specific heat at stagnation condition in work units	ft^2/(sec^2-°R)	/CSEVAL/, 60, 61
CP0	Specific heat at constant pressure at stagnation condition	Btu/(lbm-°R)	/CSEVAL/, 45, 46, 59, 60, 71, 74, 81
CPTAB	Array of C_p values corresponding to the values in the temperature table	Btu/(lbm-°R)	/CSEVAL/, 17, 21, 22, 35, 37, 74
DCP	Coefficient in the C_p-T relationship determined by BMFITS	Btu/(lbm-°R^4)	/CSEVAL/, 17, 22-25, 35, 37-41, 77

TABLE 5. SUBROUTINE BARSET NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
DELT	Temperature difference between two values in C_p-T table	°R	34, 35, 40, 70, 83
ERASE1	Minus value of stagnation specific heat multiplied by the natural logarithm of TCTAB (1)	Btu/(lbm-°R)	71, 73, 86
FJ	Conversion factor between thermal and work units	(ft-lbf)/Btu	/INPUT/, 2, 3, 59
FJG	FJ multiplied by G	$\frac{(ft_2\text{-lbm})}{(Btu\text{-}sec^2)}$	/CSEVAL/, 2, 60
G	Acceleration of gravity used as a proportionality constant	$\frac{(lbm\text{-}ft)}{(lbf\text{-}sec^2)}$	/INPUT/, 2
G1	Summation of entropy terms	Btu/(lbm-°R)	26, 40, 42
G2	Intermediate term for entropy equation	Btu/(lbm-°R)	41, 42
GAM0	Specific heat ratio at stagnation condition	—	/INPUT/, 46-48, 50, 51
GM1O2	One-half the specific heat ratio for stagnation condition minus one	—	/CSEVAL/, 50
GOGM1	Specific heat ratio at stagnation condition divided by the specific heat ratio minus one	—	/CSEVAL/, 51, 59
GTAB	Array of terms used by SEVAL	Btu/(lbm-°R)	/CSEVAL/, 25, 42, 73
H0	Stagnation enthalpy	ft^2/sec^2	/CSEVAL/, 45, 61
HTAB	Array of enthalpy values	Btu/lbm	/CSEVAL/, 21, 35, 81
I	Do loop counter	—	8-10, 27, 31, 35-42, 53, 54, 62, 63, 72-83

TABLE 5. SUBROUTINE BARSET NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
I1	Saved value of one or IXTAB	—	5, 7, 8
ICTAB	Indicator; = 0, constant specific heat calculation; = (3 ≤ ICTAB ≤ 20), number of C_p values in input table	—	/INPUT/, 12-14, 16, 17, 27, 36, 44, 55
IS	Number of C_p values in input table minus one	—	/CSEVAL/, 44, 58
ITWTAB	Indicator; = -1, T_w = adiabatic wall temperature; = 1, IXTAB input values of T_w are used	—	/INPUT/, 6, 85
IXTAB	Number of points in the x, y, and Mach number tables	—	/INPUT/, 7, 53, 62
NOCTAB	Saved value of ICTAB	—	/CSEVAL/, 16, 56, 57, 67, 70, 72
NOCTM1	NOCTAB minus one	—	57, 58, 69, 82
P0	Stagnation pressure	lbf/ft^2	/INPUT/, 52, 87
P0MAX	Saved value of stagnation pressure	lbf/ft^2	/CSEVAL/, 52
PIE	Circular constant π	—	/INPUT/, 1
PITAB	Pressure table for nozzle contour points, x, y, corresponding to IXTAB values; obtained from GETPT	lbf/ft^2	/TABLES/, 54
RBAR	Specific gas constant: universal gas constant divided by molecular weight	$\frac{(ft\text{-}lbf)}{(lbm\text{-}^\circ R)}$	/INPUT/, 3, 59

TABLE 5. SUBROUTINE BARSET NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
ROJ	Specific gas constant in thermal units, RBAR divided by FJ	Btu/(lbm-°R)	/CSEVAL/, 3, 46
S0	Stagnation entropy	Btu/(lbm-°R)	/CSEVAL/, 87
T0	Stagnation temperature	°R	/INPUT/, 4, 45, 61, 87
TCTAB	Temperature values corresponding to ICTAB values in increasing order defining C_p-T tables	°R	/CSEVAL/, 13, 14, 17-20, 31, 67, 68, 70, 71, 81, 83
TE	Saved value of TITAB	°R	63-65
TIE1	Saved value of TCTAB (I)	°R	18, 21-25, 28, 31-34, 37-41
TIE2	Squared value of TIE1	$°R^2$	19, 21-23, 29, 33, 37, 38, 40, 41
TIE3	Cubed value of TIE1	$°R^3$	20, 22, 30, 33, 37, 40, 41
TITAB	Temperature table corresponding to IXTAB values for contour points x, y; obtained from GETPT	°R	/TABLES/, 54, 63
TMAX	Saved value of maximum temperature	°R	4, 9, 10, 13, 14, 64, 65, 67
TME1	Saved value of TIE1	°R	28, 34, 40
TME2	Saved value of TIE2	$°R^2$	29, 40
TME3	Saved value of TIE3	$°R^3$	30, 40
TWTAB	Wall temperature table corresponding to IXTAB values for wall locations x, y.	°R	/TABLES/, 9, 10, 86
Z1	Real value of NOCTM1		69, 70

SUBROUTINE BMFITS

Subroutine BMFITS computes the coefficients from a cubic spline fit routine using tablular data.

COMMON BLOCKS

No COMMON blocks are used.

TBL SUBROUTINES

Subroutine BARSET calls BMFITS.

BMFITS does not call any TBL subroutines.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines or built-in FORTRAN functions are used.

CALLING SEQUENCE

The subroutine calling sequence is

CALL BMFITS (X, Y, N, <u>BL</u>, <u>CL</u>, <u>DL</u>)

where

X = array of independent table values,

Y = array of dependent table values,

N = number of values in the X and Y arrays,

BL = array of coefficients for first-power term,

CL = array of coefficients for second-power term,

DL = array of coefficients for third-power term.

SOLUTION METHOD

Initialize array subscript counter to one.

1. I = 1

Compute difference between adjacent values in X array.

2. FL(I) = X(I + 1) - X(I)

Increment subscript counter.

3. I = I + 1

Check whether subscript counter has covered the complete X array.

4. If I < N, go to 2

   If I ≥ N, go to 5

Set subscript counter to two.

5. I = 2

Compute intermediate terms.

6. B(I) = -(FL(I - 1))/(FL(I))

7. A(I) = -(2.0)(FL(I) + FL(I - 1))/(FL(I))

8. C(I) = ((6.0)/(FL(I))((Y(I + 1) - Y(I))/(FL(I)) - (Y(I)
   -Y(I - 1))/(FL(I - 1)))

Increment subscript counter.

9. I = I + 1

Check whether subscript counter has covered the complete array.

10. If I < N, go to 6

    If I ≥ N, go to 11

Initialize intermediate terms.

11. G(2) = 1.0

12. F(2) = 0.0

Set subscript counter to three.

13. I = 3

Compute intermediate terms.

14. G(I) = A(I - 1) + (B(I - 1))/(G(I .. 1))

15. F(I) = -((B(I - 1))(F(I - 1))/(G(I - 1)) + C(I - 1))

Increment the subscript counter.

16. I = I + 1

Check whether the subscript counter has covered the complete array.

17. If I ≤ N, go to 14

    If I > N, go to 18

Compute intermediate array terms starting with the last value.

18. YPP(N) = (F(N))/(G(N) - 1.0)

19. YPP(N - 1) = YPP(N)

20. I = N - 2

21. YPP(I) = (YPP(I + 1) + F(I + 1))/(G(I + 1))

Decrement subscript counter.

22. I = I - 1

Check whether subscript counter has reached zero.

23. If I > 0, go to 21

If I ≤ 0, go to 24

Set subscript counter to one.

24. I = 1

Compute array of polynomial coefficients.

25. BL(I) = (Y(I + 1) - Y(I))/(FL(I)) - ((FL(I))(YPP(I + 1) + (2.0)(YPP(I))))/(6.0)

26. CL(I) = (YPP(I))/(2.0)

27. DL(I) = (YPP(I + 1) - YPP(I))/((6.0)(FL(I)))

Increment subscript counter.

28. I = I + 1

29. If I < N, go to 25

If I ≥ N, go to 30

30. Return

Table 6 gives subroutine BMFITS nomenclature.

TABLE 6. SUBROUTINE BMFITS NOMENCLATURE

Symbol	Description	Units	Reference
A	Array of intermediate terms	—	DIM, 7, 14
B	Array of intermediate terms	—	DIM, 6, 14, 15
BL	Output array of polynomial coefficients	Btu/(lbm-°R^2)	CALL, DIM, 25
C	Array of intermediate terms	—	DIM, 8, 15
CL	Output array of polynomial coefficients	Btu/(lbm-°R^3)	CALL, DIM, 26

TABLE 6. SUBROUTINE BMFITS NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
DI.	Output array of polynomial coefficients	Btu/(lbm-°R^4)	CALL, DIM, 27
F	Array of intermediate terms	—	DIM, 12, 15, 18, 21
FL	Array of intermediate terms	°R	DIM, 2, 6-8, 25, 27
G	Array of intermediate terms	—	DIM, 11, 14, 15, 18, 21
I	Array subscript counter	—	1-10, 13-17, 20-29
N	Input dimension of table arrays	—	CALL, 4, 10, 17-20, 29
X	Input array of independent table values	°R	CALL, DIM, 2
Y	Input array of dependent table values	Btu/(lbm-°R)	CALL, DIM, 8, 25
YPP	Array of intermediate terms	—	DIM, 18, 19, 21, 25-27

SUBROUTINE DIRECT

Subroutine DIRECT controls the overall flow of the program. It calls subroutines which successively read in data, compute program constants, fit curves, and then obtain the boundary layer solution.

COMMON BLOCKS

No COMMON blocks are used.

TBL SUBROUTINES

Subroutines MAINTB and QUITS call DIRECT.

DIRECT calls BARCON, BARSET, and READIN.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines or built-in FORTRAN functions are used.

CALLING SEQUFNCE

The subroutine calling sequence is: CALL DIRECT.

Call subroutine to read input data.

1. CALL READIN

Call subroutine to compute constants and initialize data.

2. CALL BARSET

Obtain the boundary layer solution .

3. CALL BARCON

Read in the next case.

4. Go to 1

SUBROUTINE CFEVAL

Subroutine CFEVAL evaluates the skin friction coefficient $\overline{C}_f$ as a function of $\overline{C}_f\ \overline{R}_{\bar{\theta}}$ based on Coles' relationship. For values of $\overline{C}_f\ \overline{R}_{\bar{\theta}}$ such that $0 < \overline{C}_f\ \overline{R}_{\bar{\theta}} < 2.51$, $\overline{C}_f$ is evaluated from

$$C_f = \frac{0.009896}{(\overline{C}_f\ \overline{R}_{\bar{\theta}})^{0.562}} \quad .$$

For values of $\overline{C}_f\ \overline{R}_{\bar{\theta}}$, such that $2.51 \leq \overline{C}_f\ \overline{R}_{\bar{\theta}} < 1982759.2$, $\overline{C}_f$ is evaluated from a cubic spline fit of log $(\overline{C}_f)$ versus log $(\overline{C}_f\ \overline{R}_{\bar{\theta}})$ from input data. All other values of $\overline{C}_f\ \overline{R}_{\bar{\theta}}$ are considered out of range.

COMMON BLOCKS

No COMMON blocks are used.

TBL SUBROUTINES

Subroutines BARPRO and START call CFEVAL.

CFEVAL calls subroutine QUITS.

FORTRAN SYSTEM ROUTINES

FORTRAN library routines ALOG and EXP are used.

No built-in FORTRAN functions are used

CALLING SEQUENCE

CFEVAL is a function routine and is called as:

CFBAR = CFEVAL (CFRT),

where CFRT is the skin friction coefficient multiplied by the Reynolds number based on the momentum thickness, and CFBAR is the skin friction coefficient.

SOLUTION METHOD

Save $\overline{C}_f\ \overline{R}_{\bar{\theta}}$ = (CFRT).

1. Z = CFRT

Check the sign of $\bar{C}_f \bar{R}_{\bar{\theta}}$.

2. If $Z \leq 0$, go to 6

   If $Z > 0$, go to 3

Check whether the integer IZ of $Z = (\bar{C}_f \bar{R}_{\bar{\theta}})$ is greater, equal, or smaller than the integer IX(J) of the X table values.

3. If IZ > IX(J), go to 18

   If IZ = IX(J), go to 13

   If IZ < IX(J), go to 4

Set

4. J = J - 1

Check

5. If J > 0, go to 3

   If J = 0, go to 9

   If J < 0, go to 6

Set J equals one and print a message.

6. J = 1

7. WRITE Z, ZERO, X(8)

Stop calculation and go to next case.

8. CALL QUITS

Check the sign of Z = CFRT.

9. If Z ≤ 0.0, go to 6

   If Z > 0.0, go to 10

Set

10. J = 1

Compute $\overline{C}_f$ = (Y) from the relation $\overline{C}_f = \dfrac{0.009896}{(\overline{C}_f\ \overline{R}_{\bar{\theta}})^{0.562}}$.

11. Y = $(0.009896)/(Z)^{0.562}$

12. Go to 16

Save the natural logarithm of Z = CFRT.

13. ZL = Ln(Z)

Compute $\ln(\overline{C}_f)$ = YL by using the polynomial equation obtained from Coles' experimental data, in the case $2.51 \leq \overline{C}_f\ \overline{R}_{\bar{\theta}} < 1982759.2$.

14. YL = D(J) + (ZL)(C(J) + (ZL)(B(J) + (ZL)(A(J))))

Obtain $\overline{C}_f$ = Y from $\ln(\overline{C}_f)$.

15. Y = Exp (YL)

Define CFEVAL representing C_f.

16. CFEVAL = Y

17. Return

Compare the integers of IZ and IX(J + 1).

18. If IZ ≤ IX(J + 1), go to 13

If IZ > IX(J + 1), go to 19

Set

19. J = J + 1

Check whether J exceeds eight.

20. If $J \geq 8$, go to 6

If $J < 8$, go to 18

Table 7 gives subroutine CFEVAL nomenclature.

TABLE 7. SUBROUTINE CFEVAL NOMENCLATURE

Symbol	Description	Units	Reference
A	Coefficient in $\ln(\overline{C}_f)$ versus	—	DATA, DIM, 14
	$\ln(\overline{C}_f \overline{R}_{\overline{\theta}})$ polynomial		
B	Coefficient in $\ln(\overline{C}_f)$ versus	—	DATA, DIM, 14
	$\ln(\overline{C}_f \overline{R}_{\overline{\theta}})$ polynomial		
C	Coefficient in $\ln(\overline{C}_f)$ versus	—	DATA, DIM, 14
	$\ln(\overline{C}_f \overline{R}_{\overline{\theta}})$ polynomial		
CFEVAL	Saved value of Y, skin friction coefficient $\overline{C}_f$	—	16
CFRT	$\overline{C}_f \overline{R}_{\overline{\theta}}$	—	CALL, 1
D	Coefficient in $\ln(\overline{C}_f)$ versus $\ln(\overline{C}_f \overline{R}_{\overline{\theta}})$ polynomial	—	DATA, DIM, 14
IX	Integer value of X	—	DIM, EQUIV, 3, 18
IZ	Integer value of Z	—	EQUIV, 3, 18
J	Indicator to pick up data	—	DATA, 3-6, 10, 14, 18-20
X	Input value of $\overline{C}_f \overline{R}_{\overline{\theta}}$ (data)	—	DATA, DIM, EQUIV, 7
Y	$\overline{C}_f$, Skin friction coefficient	—	11, 15, 16
YL	$\ln(\overline{C}_f)$	—	14, 15
Z	Saved value of CFRT = $\overline{C}_f \overline{R}_{\overline{\theta}}$	—	EQUIV, 1, 2, 7, 9, 11, 13
ZERO	0.0	—	DATA, 7
ZL	$\ln(\overline{C}_f \overline{R}_{\overline{\theta}})$	—	13, 14

SUBROUTINE FIIF

Subroutine FIIF defines the function (of S) to be evaluated by the numerical integration routine INTZET. Three second-degree coefficients (A_F, B_F, C_F) and an exponent value n are determined by subroutine ZETAIT. Enthalpy value $\bar{h}$ is computed from

$$\bar{h} = H_w + \frac{H_o - H_w}{\zeta} S + \left(- \frac{U_e^2}{2} \right) S^2$$

$$= A_F + B_F\ S + C_F\ S^2 \quad .$$

(AFINT) (BFINT) (CFINT)

Entering subroutine SEVAL with $\bar{h}$, the temperature $\bar{t}$ is obtained. The appropriate form of the function is then evaluated for

$$\text{IFINT} = 1,\ f(s) = \left(\frac{T_e}{\bar{t}} \right) S^h\ (1 - S)\ \ldots\ \text{FIIF (S)}$$

and

$$\text{IFINT} = 2,\ f(s) = \left(\frac{T_e}{\bar{t}} \right) S^h\ \ldots\ \text{FIIF (S)} \quad ,$$

where $T_e/\bar{t}$ has been used for $\bar{\rho}/\rho_e$.

COMMON BLOCKS

COMMON block COFIIF is used.

TBL SUBROUTINES

Subroutine INTZET calls FIIF.

FIIF calls SEVAL.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines or built-in FORTRAN functions are used.

CALLING SEQUENCE

FIIF is a function routine and is called as

Y = FIIF(S),

where S is an independent variable $0 \le S \le 1$ and defined as

$$S = s = \frac{\bar{u}}{\bar{U}} = \left(\frac{y}{\delta}\right)^{1/n}$$

or

$$S = W = \frac{S}{\zeta} = \frac{\bar{h}_o - H_w}{H_o - H_w} = \left(\frac{y}{\Delta}\right)^{1/n} .$$

FIIF (s) represents

$$\frac{T_e}{\bar{t}} S^n \ (1 - S) \text{ or } \frac{T_e}{\bar{t}} S^n .$$

SOLUTION METHOD

Initialize power summation term to 1.0.

1. STOM = 1.0

Check the sign of power index n used in the velocity and enthalpy distribution equations. MMINT = (n) is defined in subroutine ZETAIT.

2. If MMINT $\le$ 0, go to 5

   If MMINT > 0, go to 3

Save "S^n" as STOM.

3. Do 4, I = 1, MMINT

4. STOM = (STOM)(S)

Calculate the enthalpy.

$$\bar{h} = H_w + \frac{H_o - H_w}{\zeta} S + \left(-\frac{U_e^2}{2}\right) S^2 \quad .$$

5. FDEN = AFINT + (BFINT)(S) + (CFINT)(S)^2

Check the option indicator.

6. If IFINT $\geq$ 2, go to 9

   If IFINT < 2, go to 7

Calculate $S^n(1 - S)$.

7. FNUM = (STOM)(1.0 - S)

8. Go to 10

Save S^n.

9. FNUM = STOM

Call subroutine SEVAL to obtain the temperature $\bar{t}$ = (T) and specific heat C_p = (ϕ) by using the previously determined enthalpy $\bar{h}$ = (FDEN).

10. CALL SEVAL (2, T, O, FDEN)

Calculate the function FIIF.

11. FIIF = (FNUM)(TFINT)/(T)

Where TFINT is the free-stream temperature T_e determined in BARPRO and defined in ZETAIT.

12. Return

Table 8 gives subroutine FIIF nomenclature.

TABLE 8. SUBROUTINE FHF NOMENCLATURE

Symbol	Description	Units	Reference
AFINT	Coefficient in enthalpy equation representing H_w (wall enthalpy).	ft^2/sec^2	/COFHF/, 5
BFINT	Coefficient in enthalpy equation representing $(H_o - H_w)/\zeta$	ft^2/sec^2	/COFHF/, 5
CFINT	Coefficient in enthalpy equation representing $(-U_e^2/2)$.	ft^2/sec^2	/COFHF/, 5
FDEN	Enthalpy $\bar{h} = H_w + [(H_o - H_w)/\zeta]S - (\bar{U}_e^2/2)S^2$	ft^2/sec^2	5, 10
FNUM	S^n or $S^n(1 - S)$	—	7, 9, 11
IFINT	= 1; Calculation of $\frac{T_e}{t} S^n(1 - S)$ = 2; Calculation of $\frac{T_e}{t} S^n$	—	/COFHF/, 6
MMINT	Velocity power law exponent n	—	/COFHF/, 2, 3
O	Specific heat C_p	Btu/(lbm-°R)	10
S	$S = \frac{\bar{u}}{U_e} = \left(\frac{y}{\delta}\right)^{1/n}$ or $S = \frac{\bar{h}_o - H_w}{H_o - H_w} = \left(\frac{y}{\Delta}\right)^{1/n}$	—	CALL, 4, 5, 7

TABLE 8. SUBROUTINE FIIF NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
STOM	s^n	—	1, 4, 7, 9
T	Temperature $\bar{t}$	°R	10, 11
TFINT	Free-stream temperature T_e	°R	/COFIIF/, 11

SUBROUTINE GETPT

Subroutine GETPT computes pressure P_e and temperature T_e from C_p, H, and T relations for a given Mach number M_e.

For constant specific heat $\gamma_e = \gamma_o$

$$T_e = \frac{T_o}{1 + \frac{\gamma_e - 1}{2} M_e} \quad , \quad P_e = \frac{P_o}{\left(1 + \frac{\gamma_e - 1}{2} M_e^2\right)^{\frac{\gamma_e}{\gamma_e - 1}}}$$

For variable C_p the following iteration is used.

1. An approximation is made for temperature T_{eg} using

$$T_{eg} = \frac{T_o}{1 + \frac{\gamma_o - 1}{2} M_e^2}$$

2. In subroutine SEVAL, C_{pe} and H_e are computed at temperature T_{eg}.

$$C_p = A + BT + CT^2 + DT^3 \quad ;$$

$$H(T) = H_i + \int_{T_i}^{T} C_p(t)\,dt \quad .$$

3. The specific heat ratio γ_e is then evaluated from

$$\gamma_e = \frac{C_{pe}}{C_{pe} - \frac{R}{J}} \quad .$$

4. Temperature T_e is then calculated.

$$T_e = \frac{2(H_o - H_e)}{\gamma_e RM_e} ,$$

since

$$a^2 = \gamma_e RT_e ,$$

$$\gamma_e RM_e^2 = \gamma_e R \frac{U_e^2}{a^2} = \frac{U_e^2}{T_e} ,$$

$$T_e = \frac{U_e^2}{\gamma_e RM_e^2} ,$$

$$H_o = H_e + \frac{U_e^2}{2} ,$$

$$U_e^2 = 2(H_o - H_e) = T_e \gamma_e RM_e^2 ,$$

$$\therefore T_e = \frac{2(H_o - H_e)}{\gamma_e RM_e^2} .$$

5. A relative error comparison is then made. If

$$\left| \frac{T_e - T_{eg}}{T_{eg}} \right| \leq \text{TOLZME} ,$$

convergence is obtained, and subroutine SEVAL is used to evaluate pressure P_e at temperature T_e. If the convergence criterion is not satisfied, a form of Wegstein's method is used to approximate T_{eg} again, and steps 2 through 5 are repeated. A maximum of 50 iterations are possible.

COMMON BLOCKS

COMMON blocks CSEVAL and INPUT are used.

TBL SUBROUTINES

Subroutines BARPRO, BARSET, and START call GETPT. GETPT calls subroutine SEVAL.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines are used.

Built-in FORTRAN function ABS is used.

CALLING SEQUENCE

The subroutine calling sequence is:

CALL GETPT (ZME, $\underline{\text{PI}}$, $\underline{\text{TI}}$)

where

ZME = free-stream Mach number,

PI = free-stream pressure,

TI = free-stream temperature.

SOLUTION METHOD

Determine the square of Mach number.

1. ZME2 = $(\text{ZME})^2$

Calculate $\frac{2}{gRM_e^2}$.

2. PROD1 = (2.)/(RBAR)/(ZME2)/(G)

Compute $\left(1 + \frac{\gamma_e - 1}{2} M_e^2\right)$.

3. DENM2 = 1.0 + (GM1O2)(ZME2)

Obtain free-stream temperature $T_e = \dfrac{T_o}{1 + \dfrac{\gamma_o - 1}{2} M_e^2}$.

4. TE = (T0)/(DENM2)

Check the value of ICTAB [= 0: constant specific heat calculation, = ($3 \leq$ ICTAB ≤ 20): number of points in variable C_p versus T table].

5. If ICTAB > 0, go to 10

   If ICTAB ≤ 0, go to 6

Calculate

$$P_e = \frac{P_o}{\left(1 + \frac{\gamma_e - 1}{2} M_e^2\right)^{\gamma_e/\gamma_e - 1}} \quad \text{for } \gamma_e = \text{constant.}$$

6. PE = $(P0)/(DENM2)^{GOGM1}$

Set

7. PI = PE

8. TI = TE

Return to the main routine or continue the calculation unless C_{pe} is constant.

9. Return

Set the iteration counter to zero initially.

10. ITER = 0

Define the tolerance.

11. TOL = (TOLZME)/(ZME)

Save the previous approximation of free-stream temperature T_{eg}.

12. TEO = TEG

Save $T_c = \frac{2(H_o - H_e)}{\gamma_e R M_e^2}$ as calculated in step 17.

13. TCO = TC

Approximate T_{eg}.

14. TEG = TE

Call subroutine SEVAL to obtain C_{pe} and H_e for the known T_e.

15. CALL SEVAL(1, TE, <u>CPE</u>, <u>HE</u>)

Calculate specific heat ratio γ_e.

16. GAME = (CPE)/(CPE - ROJ)

Calculate new free-stream temperature $T_e = \frac{2(H_0 - H_e)}{\gamma_e R M_e^2}$.

17. TC = (H0 - HE)(PROD1)/(GAME)

Test whether the temperature falls within the tolerance.

18. If |(TC - TE)/(TE)| ≤ TOL, go to 33

If |(TC - TE)/(TE)| > TOL, go to 19

Check on the number of iterations.

19. If ITER > 0, go to 22

If ITER ≤ 0, go to 20

Determine

20. TE = ((2.0)(TE) + TC)/(3.0)

21. Go to 30

Check on the number of iterations, and print out the error message if convergence was not obtained within 50 iterations.

22. If ITER ≤ 50, go to 25

If ITER > 50, go to 23

Print out error message.

23. WRITE ZME, TC, TCO, TE, TEO

24. Go to 33

Use a form of Westein's method to approximate a new value for T_e.

25. ZK = (TC - TCO)/(TE - TEO)

26. TE = (TC - (ZK)(TE))/(1.0 - ZK)

Check whether convergence has been achieved.

27. If |(TE - TEG)/(TE)| < TOL, go to 32

If |(TE - TEG)/(TE)| ≥ TOL, go to 28

Check on the number of iterations.

28. If ITER < 10, go to 30

If ITER ≥ 10, go to 29

Check whether the calculated number falls within the tolerance.

29. If $|(TE - TEO)/(TE)| < TOL$, go to 32

If $|(TE - TEO)/(TE)| \geq TOL$, go to 30

Add one to the iteration counter.

30. ITER = ITER + 1

Repeat the above method until a convergence is obtained.

31. Go to 12

Determine the new free-stream temperature.

32. TE = (TE + TEG)/(2.0)

Call subroutine SEVAL to obtain the free-stream pressure P_e, using temperature T_e and stagnation entropy S_o.

33. CALL SEVAL (-1, TE, PE, S0)

34. Go to 7

Table 9 gives subroutine GETPT nomenclature.

TABLE 9. SUBROUTINE GETPT NOMENCLATURE

Symbol	Description	Units	Reference
CPE	Specific heat at constant pressure in free-stream C_{pe}	Btu/(lbm-°R)	15, 16
DENM2	$1 + \frac{\gamma_e - 1}{2} M_e^2$	—	3, 4, 6
G	Acceleration of gravity used as a proportionality constant	(lbm/lbf)/(ft/sec^2)	/INPUT/, 2

TABLE 9. SUBROUTINE GETPT NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
GAME	Specific heat ratio in free stream	—	16, 17
GM1O2	$\frac{\gamma_e - 1}{2}$	—	/CSEVAL/, 3
GOGM1	$\frac{\gamma_e}{\gamma_e - 1}$	—	/CSEVAL/, 6
H0	Stagnation enthalpy H_o	ft^2/sec^2	/CSEVAL/, 17
HE	Free-stream enthalpy H_e	ft^2/sec^2	15, 17
ICTAB	Indicator; = 0, constant specific heat calculation; = $(3 \leq ICTAB \leq 20)$, dimension of variable C_p versus T table	—	/INPUT/, 5
ITER	Iteration counter	—	10, 19, 22, 28, 30
P0	Stagnation pressure P_o	lbf/ft^2	/INPUT/, 6
PE	Free-stream pressure P_e	lbf/ft^2	6, 7, 33
PI	Saved value of PE	lbf/ft^2	CALL,7
PROD1	$\frac{2}{gRM_e^2}$	$(°R\text{-}sec^2)/ft^2$	2, 17
RBAR	Specific gas constant: universal gas constant divided by molecular weight	(ft-lb)/(lbm-°R)	/INPUT/, 2

3

TABLE 9. SUBROUTINE GETPT NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
ROJ	Specific gas constant in thermal units, RBAR divided by FJ	Btu/(lbm-°R)	/CSEVAL/, 16
SC	Stagnation entropy S_o	Btu/(lbm-°R)	/CSEVAL/, 33
T0	Stagnation temperature T_o	°R	/INPUT/, 4
TC	Saved value of free-stream temperature	°R	13, 17, 18, 20, 23, 25, 26
TCO	Saved value of TC	°R	13, 23, 25
TE	Free-stream temperature T_e	°R	4, 8, 14, 15, 18, 20, 23, 25, 26, 27, 29, 32, 33
TEG	Approximation of free-stream temperature in iteration loop	°R	12, 14, 27, 32
TEO	Saved value of TEG	°R	12, 23, 25, 29
TI	Saved value of TE	°R	CALL, 8
TOL	Tolerance value divided by free-stream Mach number	—	11, 18, 27, 29
TOLZME	Tolerance	—	/INPUT/, 11
ZK	Intermediate term in free-stream temperature equation	—	25, 26
ZME	Free-stream Mach number M_e	—	CALL, 1, 11, 23
ZME2	M_e^2	—	1, 2, 3

SUBROUTINE INTZET

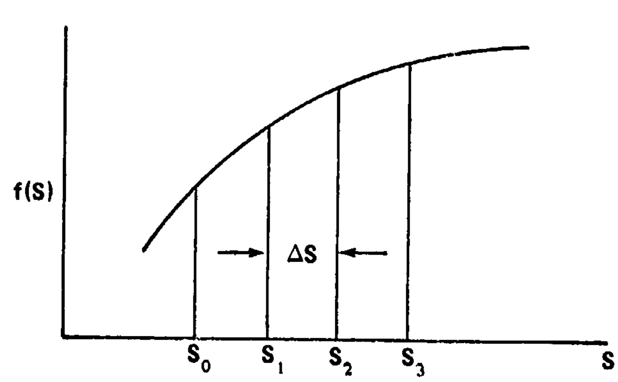
Subroutine INTZET uses a numerical technique in connection with a quintic spline fit to evaluate an integral within the limits $X_1 = XC(1)$ and $X_2 = XC(21)$. The integration interval is divided into 20 coarse subintervals. Since the integration limits are restricted to certain values $(0,\ 1,\ \zeta,\ 1/\zeta)$, a test on the interval distance $(X_2 - X_1)$ is performed. If $(X_2 - X_1) \leq 0.1$, the function is evaluated at 21 subinterval endpoints, and the integral is approximated by the quintic technique with Simpson's rule for the outer subintervals. For $(X_2 - X_1) > 0.1$ the maximum function value within the interval is determined, and an approximation of the integral is performed over the coarse subintervals up to a point where the function value reaches 80 percent of the maximum value. Beyond this point the remaining coarse subintervals are further subdivided individually into three fine subintervals. Solving the function at the endpoints of the fine subintervals, the integral approximation is finally completed using these finer subinterval step sizes.

For a given function $f(S)$, evaluated at equally spaced (ΔS) points S_o, S_1, S_2, S_3, the quintic approximation of the integral is prepresented by

$$\int_1^2 f(S)\,dS = \frac{\Delta S}{24}\left[-f(S_o) + 13f(S_1) + 13f(S_2) - f(S_3)\right] \quad .$$

The one-sided Simpson rule is expressed by the following equation and diagram.

$$\int_o^1 f(S)\,dS = \frac{\Delta S}{12}\left[5f(S_o) + 8f(S_1) - f(S_2)\right] \quad .$$



COMMON BLOCKS

No COMMON blocks are used.

TBL SUBROUTINES

Subroutines START and ZETAIT call INTZET.

INTZET calls FIIF.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines are used.

Built-in FORTRAN function FLOAT is used.

CALLING SEQUENCE

The subroutine calling sequence is:

CALL INTZET (X1, X2, ZINT)

where

X1 = lower limit of integration

X2 = upper limit of integration

ZINT = integral value.

SOLUTION METHOD

Set the length of interval for integral.

1. DX21 = X2 - X1

Set the initial value of integral as zero.

2. SUMINT = 0.0

Check whether the interval length is zero.

3. If DX21 = 0.0, go to 22

If DX21 ≠ 0.0, go to 4

Divide the interval over which the integration is performed into 20 coarse subintervals.

4. DXC = (DX21)/(20.0)

Set maximum value of integral and its subscript counter to large negative values.

5. IMAX = -9999

6. FMAX = -1.0E30

Determine the independent variable, the integrand of the function, and the maximum value of the integrand.

7. Do 14, I = 1, 21

Independent variable:

8. XC(I) = X1 + (FLOAT(I - 1))(DXC)

Call subroutine FIIF to determine the integrand.

9. YC(I) = FIIF(XC(I))

Check for the maximum value of the integrand.

10. If YC(I) ≤ FMAX, go to 14

    If YC(I) > FMAX, go to 11

Save subscript counter at the maximum value of the integrand.

11. IMAX = I

Save independent variable corresponding to maximum integrand.

12. XMAX = XC(I)

Save maximum value of integrand.

13. FMAX = YC(I)

14. Continue

Check for the interval over which the coarse or fine integration must be performed.

15. If DX21 > 0.10, go to 24

    If DX21 ≤ 0.10, go to 16

Determine 24 times the quantity of the integral according to the one-sided Simpson's Rule:

16. SUMINT = (10.0)(YC(1)) + (16.0)(YC(2)) - (2.0)(YC(3))

Calculate the quintic approximation of Simpson's rule.

17. Do 19, I = 2, 19

18. PARINT = (13.0)(YC(I) + YC(I + 1)) - YC(I - 1) - YC(I + 2)

19. SUMINT = SUMINT + PARINT

20. SUMINT = SUMINT + (10.0)(YC(21)) + (16.0)(YC(20))-
    (2.0)(YC(19))

Obtain the integral.

21. SUMINT = (SUMINT)/(24.0)(DXC)

Save the integral

22. ZINT = SUMINT

Go back to main routine.

23. Return

Determine the function value 20 percent off the maximum value.

24. FBRK = (FMAX)(0.20)

Set the initial value of the integral equal to zero.

25. SUAINT = 0.0

26. SUBINT = 0.0

Check the value of the subscript counter for the maximum value of the integrand.

27. If IMAX ≤ 2, go to 35

    If IMAX > 2, go to 28

Determination of IBRK corresponding to FBRK is made from step 28 to step 33.

28. Do 32, I = 2, IMAX

Check whether the integrand has reached the switch-over value.

29. If YC(I) ≤ FBRK, go to 32

    If YC(I) > FBRK, go to 30

Save the subscript counter for the switch-over value.

30. IBRK = I - 1.

31. Go to 34

32. Continue

Set the swich-over subscript counter to one less than the maximum.

33. IBRK = IMAX - 1

Check the value of IBRK.

34. If IBRK > 1, go to 38

    If IBRK ≤ 1, go to 35

Set the switch-over subscript counter and switch-over indicator.

35. IBRK = 1

36. IBRKM1 = 0

37. Go to 45

Save IBRK minus one.

38. IBRKM1 = IBRK - 1

Calculate 24 times the quantity of the integral according to the one-sided Simpson's rule.

39. SUAINT = (10.0)(YC(1)) + (16.0)(YC(2)) - (2.0)(YC(3))

Check the value of IBRK.

40. If IBRK $\leq$ 2, go to 44

If IBRK > 2, go to 41

Continue integration.

41. Do 43 I = 2, IBRKM1

Compute 24 times the quantity of the integral for each interval using Simpson's rule.

42. PARINT = (13.0)(YC(I) + YC(I + 1)) - YC(I - 1) - YC(I + 2)

Sum the individual interval integrals.

43. SUAINT = SUAINT + PARINT

Obtain the integral up to where the integrand is 20 percent of the maximum value.

44. SUAINT = (SUAINT)(DXC)/(24.0)

Obtain one-third DXC to consider finer subintervals.

45. DXM = (DXC)/(3.0)

Check the value of IBRKM1.

46. If IBRKM1 > 0, go to 50

If IBRKM1 ≤ 0, go to 47

Set temporary integers.

47. K = 2

48. JS = 2

49. Go to 52

Set temporary integers.

50. K = 3

51. JS = 1

Determine the independent variables corresponding to fine subintervals.

52. Do 54, I = 2, 4

53. XM = XC(IBRK) + (FLOAT(I - K))(DXM)

Call subroutine FIIF to obtain integrand.

54. YM(I) = FIIF(XM)

Check the value of IBRKM1.

55. If IBRKM1 > 0, go to 57

If IBRKM1 ≤ 0, go to 56

Calculate 24 times the quantity of the integral corresponding to the fine subinterval.

56. SUBINT = (10.0)(YM(2)) - (2.0)(YM(4)) + (16.0)(YM(3))

Continue the integration dividing the coarse subintervals into three fine subintervals.

57. Do 67, I = IBRK, 19

58. Do 65, J = JS, 3

59. YM(1) = YM(2)

60. YM(2) = YM(3)

61. YM(3) = YM(4)

62. XM = XM + DXM

Call subroutine FIIF to obtain integrand.

63. YM(4) = FIIF(XM)

Compute 24 times the quantity of the integral for fine subintervals.

64. PARINT = (13.0)(YM(2) + YM(3)) - YM(1) - YM(4)

Sum the subinterval integrals.

65. SUBINT = SUBINT + PARINT

Set

66. JS = 1

Independent variable:

67. XM = XC(I + 1) + DXM

So far, 24 times the quantity of the integral up to XC(20) has been obtained. Calculate the remaining integral between XC(20) and XC(21).

68. Do 75, J = 1, 2

69. YM(1) = YM(2)

70. YM(2) = YM(3)

71. YM(3) = YM(4)

72. XM = XM + DXM

Call subroutine FHF to obtain integrand.

73. YM(4) = FHF(XM)

74. PARINT = (13.0)(YM(2) + YM(3)) - YM(1) - YM(4)

Calculate 24 times the quantity of the integral up to the independent variable of XC(20) + (2.0)(DXM).

75. SUBINT = SUBINT + PARINT

Determine 24 times the quantity of the integral up to XC(21).

76. SUBINT = SUBINT + (10.0)(YM(4)) + (16.0)(YM(3)
- (2.0)(YM(2))

Calculate the final integral beyond the point where the value of the integrand is 20 percent of the maximum value.

77. SUBINT = (SUBINT)(DXM)/(24.0)

Obtain the total integral.

78. SUMINT = SUAINT + SUBINT

79. Go to 22

Table 10 gives subroutine INTZET nomenclature.

TABLE 10. SUBROUTINE INTZET NOMENCLATURE

Symbol	Description	Units	Reference
DX21	Upper limit of integration minus lower limit	—	1, 3, 4, 15
DXC	Coarse subinterval	—	4, 8, 21, 44, 45
DXM	Fine subinterval	—	45, 53, 62, 67, 72, 77
FBRK	20% value of maximum integrand	—	24, 29
FMAX	Saved value of maximum integrand	—	6, 10, 13, 24
I	Do-loop counter	—	7-13, 17, 18, 28-30, 41, 42, 52-54, 57
IBRK	Subscript counter for switch-over value of the integrand	—	30, 33-35, 38, 40, 53, 57
IBRKM1	IBRK minus one	—	36, 38, 41, 46, 55
IMAX	Subscript counter value corresponding to maximum integrand	—	5, 11, 27, 28, 33
J	Do-loop counter	—	58, 68
JS	Starting do-loop value	—	48, 51, 58, 66
K	Switch-over point counter	—	47, 50, 53
PARINT	24 times the quantity of integral	—	18, 19, 42, 43, 64, 65, 74, 75
SUAINT	Integral value up to 20% of maximum integrand	—	25, 39, 43, 44, 78
SUBINT	Integral value beyond 20% of maximum integrand	—	26, 56, 65, 75-78
SUMINT	Integral	—	2, 16, 19-22, 78
X1	Lower limit of integration	—	CALL, 1, 8
X2	Upper limit of integration	—	CALL, 1
XC	Independent variable		DIM, 8, 9, 12, 53, 67

TABLE 10. SUBROUTINE INTZET NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
XM	Independent variable corresponding to fine subintervals	—	53, 54, 62, 63, 67, 72, 73
XMAX	Saved value of the independent variable corresponding to maximum value of integrand	—	12
YC	Integrand	—	DIM, 9, 10, 13, 16, 18, 20, 29, 39, 42
YM	Integrand corresponding to fine subintervals	—	DIM, 54, 56, 59-61, 63, 64, 69-74, 76
ZINT	Saved value of integral	—	CALL, 22

SUBROUTINE MAINTB

Subroutine MAINTB provides the main control for the turbulent boundary layer program.

COMMON BLOCKS

COMMON block INPUT is used.

TBL SUBROUTINES

MAINTB calls subroutine DIRECT.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines or built-in FORTRAN functions are used.

SOLUTION METHOD

Initialize step-size indicator.

1. IDXMAX = 0

Call the overall flow control program.

2. CALL DIRECT

Table 11 gives the nomenclature for subroutine MAINTB.

TABLE 11. SUBROUTINE MAINTB NOMENCLATURE

Symbol	Description	Units	Reference
IDXMAX	Indicator for previous value of step size or to compute the step size = 0, compute step size; = 1, use previous step size	—	/INPUT/, 1

SUBROUTINE QUITS

Subroutine QUITS is entered when an error has been detected in the input or if an unreasonable number has been calculated during execution. It prints out appropriate error statements and the contents of the COMMON blocks. If consecutive cases are considered, the next case is called for.

COMMON BLOCKS

COMMON blocks COFIIF, CSEVAL, INPUT, INTER, LOOKUP, OUTPUT, SAVED, and TABLES are used.

TBL SUBROUTINES

Subroutines BARCON, BARPRO, BARSET, CFEVAL, READIN, SEVAL, STARTS, and XNTERP call QUITS.

QUITS calls subroutine DIRECT.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines or built-in FORTRAN functions are used.

CALLING SEQUENCE

The subroutine calling sequence is:

CALL QUITS

SOLUTION METHOD

Print out error statement.

1. WRITE error message

Print out COMMON block /COFIIF/.

2. WRITE IFINT, AFINT, BFINT, CFINT, MMINT, TFINT

Print out COMMON block /INPUT/.

3. WRITE IDXMAX, ICTAB, IPRINT, ITWTAB, IXTAB, MZETA, DXMAX, EPSZ, FJ, G, GAM0, P0, PHII, PIE, PRANDT, RBAR, SCALE, T0, THETAI, TOLCFA, TOLZET, TOLZME, ZMU0, ZMVIS, ZNSTAN

Print out COMMON block /OUTPUT/.

4. WRITE BDELTA, CF, CH, DELTA, DELSOT, DELSTR, FLAT, FORCE, HG, PE, PHI, QW, SUMQDA, TE, THETA, TW, UE, X, XLARC, YR, Z1, Z2, Z3, Z4, Z5, ZETA, ZME

Print out COMMON block /CSEVAL/.

5. WRITE NOCTAB, IS, ROJ, FJG, CJG, GM1O2, GOGM1, P0MAX, CP0, H0, S0, TCTAB, CPTAB, BCP, CCP, DCP, GTAB, HTAB, BARB1, BARB2, BARB3

Print out COMMON block /INTER/.

6. WRITE CFAGT, CFAGP, CHPAR1, DX, DXRHO, HE, HW, IBEG, MZETAM, OOMZET, PHIP, PRE1O3, RHOE, RHOUE, RMZETA, THETAP, XIBASE, XIEND, ZETATM, ZMZETA, ZMZETM, ZMZETP

Print out COMMON block /SAVED/.

7. WRITE A, B, C, ZI1, ZI1P, ZI2, ZI2P, ZI3, ZI3P, ZI4, ZI5, ZI6, ZI7

Print out COMMON block /LOOKUP/.

8. WRITE IMX, IPX, ITX, ITWX, IXPOS, IYX, CMX, CPX, CTX, CTWX, CYX

Check whether Mach number tables have been input.

9. If IXTAB $\leq$ 0, go to 34

   If IXTAB > 0, go to 10

Check whether dimension of Mach tables supersedes the maximum value.

10. If IXTAB < 500, go to 13

If IXTAB ≥ 500, go to 11

Set number of table values to be printed.

11. I3 = 495

12. Go to 14

Set number of table values to be printed to input table dimension.

13. I3 = IXTAB

Increase the number of table values to be printed.

14. I3 = (10)((I3)/(10) + 1)

Print out heading for tables printout.

15. WRITE message for tables output

Print out all the values in the axial distance table.

16. Do 18, I = 1, I3, 10

Compute printout counter.

17. K = I + 9

18. WRITE I, (XITAB(J), J = I, K)

Print out all the values in the nozzle radius table.

19. Do 21, I = 1, I3, 10

Compute printout counter.

20. K = I + 9

21. WRITE I, (YITAB(J), J = I, K)

Print out all the values in the Mach number table.

22. Do 24, I = 1, I3, 10

Compute printout counter.

23. K = I + 9

24. WRITE I, (ZMTAB(J), J = I, K)

Print out all the values in the wall temperature table.

25. Do 27, I = 1, I3, 10

Compute printout counter.

26. K = I + 9

27. WRITE I, (TWTAB(J), J = I, K)

Print out all the values of the calculated pressure table.

28. Do 30, I = 1, I3, 10

29. K = I + 9

30. WRITE I, (PITAB(J), J = I, K)

Print out all the values of the calculated temperature table.

31. Do 33, I = 1, I3, 10

Compute printout counter.

32. K = I + 9

33. WRITE I, (TITAB(J), J = I, K)

Execute the next case.

34. CALL DIRECT

Subroutine QUITS nomenclature is given in Table 12.

TABLE 12. SUBROUTINE QUITS NOMENCLATURE

Symbol	Description	Units	Reference
A	Wall enthalpy H_w	ft^2/sec^2	/SAVED/, 7
AFINT	Wall enthalpy for integral evaluation	ft^2/sec^2	/COFIIF/, 2
B	Difference between stagnation enthalpy and wall enthalpy	ft^2/sec^2	/SAVED/, 7
BARB1	C_p polynomial equation	Btu/(lbm-°R)	/CSEVAL/, 5
BARB2	Negative value of the first derivative of BARB1	$Btu/(lbm\text{-}°R^2)$	/CSEVAL/, 5
BARB3	Minus one-fourth the value of the second derivative of BARB1	$Btu/(lbm\text{-}°R^3)$	/CSEVAL/, 5
BCP	Coefficients in the C_p - T relationship	Btu/(lbm-°R)	/CSEVAL/, 5
BDELTA	Boundary layer temperature thickness	ft	/OUTPUT/, 4
BFINT	Difference between stagnation and wall enthalpy	ft^2/sec^2	/COFIIF/, 2
C	Difference between static and stagnation enthalpy	ft^2/sec^2	/SAVED/, 7
CCP	Coefficient in the C_p - T relationship	$Btu/(lbm\text{-}°R^3)$	/CSEVAL/, 5
CF	Skin friction coefficient	—	/OUTPUT/, 4
CFAGP	Skin friction coefficient obtained by using the Reynolds number R_ϕ	—	/INTER/, 6
CFAGT	Initial value of skin friction coefficient	—	/INTER/, 6
CFINT	Difference between static and stagnation enthalpy	ft^2/sec^2	/COFIIF/, 2

TABLE 12. SUBROUTINE QUITS NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
CH	Stanton number	—	/OUTPUT/, 4
CHPAR1	Term in the denominator of the Stanton number equation	—	/INTER/, 6
CJG	Specific heat at stagnation condition in work units	$ft^2/(sec^2-°R)$	/CSEVAL/, 5
CMX	Array of parabola coefficients for the Mach number table (ZMTAB)	—	/LOOKUP/, 8
CP0	Specific heat coefficient at constant pressure for stagnation condition	Btu/(lbm-°R)	/CSEVAL/, 5
CPTAB	Array of C_p values corresponding to the values in the temperature table	Btu/(lbm-°R)	/CSEVAL/, 5
CPX	Array of parabola coefficients for the contour point pressure table (PITAB)	—	/LOOKUP/, 8
CTWX	Array of parabola coefficients for the wall temperature table (TWTAB)	—	/LOOKUP/, 8
CTX	Array of parabola coefficients for the contour point temperature table (TITAB)	—	/LOOKUP/, 8
CYX	Array of parabola coefficients for the nozzle radius table (YITAB)	—	/LOOKUP/, 8

TABLE 12. SUBROUTINE QUITS NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
DCP	Coefficient in the C_p - T relationship	Btu/(lbm-°R)	/CSEVAL/, 5
DELSOT	Boundary layer shape factor	—	/OUTPUT/, 4
DELSTR	Boundary layer displacement thickness	ft	/OUTPUT/, 4
DELTA	Boundary layer velocity thickness	ft	/OUTPUT/, 4
DX	Weighted difference of table values of the axial distance	ft	/INTER/, 6
DXMAX	Maximum length of step size	ft	/INPUT/, 3
DXRHO	One-tenth the difference between axial distance points	ft	/INTER/, 6
EPSZ	Flow geometry indicator	—	/INPUT/, 3
FJ	Conversion factor between thermal and work units	(ft-lbf)/Btu	/INPUT/, 3
FJG	FJ multiplied by G	$(ft^2\text{-lbm})/(Btu\text{-}sec^2)$	/CSEVAL/, 5
FLAT	Force normal to x-direction for two-dimensional planar flow	lbf	/OUTPUT/, 4
FORCE	Drag force in axial or x-direction	lbf	/OUTPUT/, 4
G	Acceleration of gravity used as a proportionality constant	$(lbm\text{-}ft)/(lbf\text{-}sec^2)$	/INPUT/, 3
GAM0	Specific heat ratio at stagnation condition	—	/INPUT/, 3
GM1O2	One-half the specific heat ratio at stagnation condition minus one	—	/CSEVAL/, 5

TABLE 12. SUBROUTINE QUITS NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
GOGM1	Specific heat ratio at stagnation condition divided by the specific heat ratio minus one	—	/CSEVAL/, 5
GTAB	Array of terms used by SEVAL	Btu/(lbm-°R)	/CSEVAL/, 5
H0	Stagnation enthalpy	ft^2/sec^2	/CSEVAL/, 5
HE	Enthalpy at static temperature	ft^2/sec^2	/INTER/, 6
HG	Heat transfer coefficient	Btu/(ft^2-sec-°R)	/OUTPUT/, 4
HTAB	Array of enthalpy values	Btu/lbm	/CSEVAL/, 5
HW	Wall enthalpy at wall temperature	ft^2/sec^2	/INTER/, 6
I	Do-loop counter	—	16-33
I3	Number of table values to be printed out	—	11, 13, 14, 16, 19, 22, 25, 28, 31
IBEG	Subscript counter at which Mach number table exceeds one	—	/INTER/, 6
ICTAB	Specific heat indicator or table dimension	—	/INPUT/, 3
IDXMAX	Step-size indicator	—	/INPUT/, 3
IFINT	Integral evaluation indicator	—	/COFHF/, 2
IMX	Position or start indicator for Mach number table	—	/LOOKUP/, 8
IPRINT	Printout indicator	—	/INPUT/, 3
IPX	Position or start indicator for contour point pressure table	—	/LOOKUP/, 8
IS	One less than the dimension of the C_p - T	—	/CSEVAL/, 5

TABLE 12. SUBROUTINE QUITS NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
ITWTAB	Wall temperature indicator	—	/INPUT/, 3
ITWX	Position or start indicator for wall temperature table	—	/LOOKUP/, 8
ITX	Position or start indicator for contour point temperature table	—	/LOOKUP/, 8
IXPOS	Array position indicator	—	/LOOKUP/, 8
IXTAB	Dimension of x, y, Mach tables	—	/INPUT/, 3
IYX	Position or start indicator for nozzle radius table	—	/LOOKUP/, 8
J	Subscript printout counter	—	18, 21, 24, 27, 30, 33
K	Printout counter	—	17, 18, 20, 21, 23, 24, 26, 27, 29, 30, 32, 33
MMINT	Exponent for integral evaluation	—	/COFIIF/, 2
MZETA	Exponent of velocity profile	—	/INPUT/, 3
MZETAM	MZETA minus one	—	/INTER/, 6
NOCTAB	Saved value of ICTAB	—	/CSEVAL/, 5
OOMZET	One divided by ZMZETA	—	/INTER/, 6
P0	Stagnation pressure	lbf/ft^2	/INPUT/, 3
P0MAX	Saved value of stagnation pressure	lbf/ft^2	/CSEVAL/, 5
PE	Static pressure	lbf/ft^2	/OUTPUT/, 4
PHI	Boundary layer energy thickness	ft	/OUTPUT/, 4
PHII	Input or initial value of energy thickness	ft	/INPUT/, 3

TABLE 12. SUBROUTINE QUITS NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
PHIP	Slope of energy thickness	—	/INTER/, 6
PIE	Circular constant π	—	/INPUT/, 3
PITAB	Array of pressures at contour points x, y	lbf/ft^2	/TABLES/, 30
PRANDT	Prandtl number	—	/INPUT/, 3
PRE1O3	Recovery factor	—	/INTER/, 6
QW	Local heat transfer rate to the wall	$Btu/(ft^2\text{-}sec)$	/OUTPUT/, 4
RBAR	Specific gas constant	(ft-lbf)/(lbm-°R)	/INPUT/, 3
RHOE	Density at Boundary layer edge	lbm/ft^3	/INTER/, 6
RHOUE	Flow density ($\rho \cdot v$) at boundary layer edge	$lbm/(ft^2\text{-}sec)$	/INTER/, 6
RMZETA	One divided by ZMZETP	—	/INTER/, 6
ROJ	Specific gas constant	Btu/(lbm-°R)	/CSEVAL/, 5
S0	Stagnation entropy	Btu/(lbm-°R)	/CSEVAL/, 5
SCALE	Contour scale factor	or ft	/INPUT/, 3
SUMQDA	Integrated heat transfer rate	Btu/sec	/OUTPUT/, 4
T0	Stagnation temperature	°R	/INPUT/, 3
TCTAB	Temperature table for C_p values	°R	/CSEVAL/, 5
TE	Static temperature	°R	/OUTPUT/, 4
TFINT	Static temperature for integral evaluation	°R	/COFIIF/, 2
THETA	Boundary layer momentum thickness	ft	/OUTPUT/, 4
THETA	Boundary layer momentum thickness	ft	/OUTPUT/, 4

TABLE 12. SUBROUTINE QUITS NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
THETAI	Input or initial value of momentum thickness	ft	/INPUT/, 3
THETAP	Slope of momentum thickness	—	/INTER/, 6
TITAB	Array of temperatures at contour points, x, y	°R	/TABLES/, 33
TOLCFA	Tolerance in $C_f - C_f R_\theta$ iteration	—	/INPUT/, 3
TOLZET	Tolerance in the zeta interation	—	/INPUT/, 3
TOLZME	Tolerance in the gas property evaluation loop	—	/INPUT/, 3
TW	Wall temperature	°R	/OUTPUT/, 4
TWTAB	Array of wall temperatures at contour points x, y	°R	/TABLES/, 27
UE	Velocity of fluid	ft/sec	/OUTPUT/, 4
X	Axial distance	ft	/OUTPUT/, 4
XIBASE	First value of axial distance table	ft	/INTER/, 6
XIEND	Last value of axial distance table	ft	/INTER/, 6
XITAB	Array of axial distance values	ft	/TABLES/, 18
XLARC	Arc length of the contour	ft	/OUTPUT/, 4
YITAB	Array of nozzle radius or contour height	ft	/TABLES/, 21
YITAB	Array of nozzle radius or contour height	ft	/TABLES/, 21

TABLE 12. SUBROUTINE QUITS NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
YR	Nozzle radius or contour height	ft	/OUTPUT/, 4
Z1	Previous "guess" value of zeta	—	/OUTPUT/, 4
Z2	Saved "guess" value of zeta	—	/OUTPUT/, 4
Z3	Previous computed value of zeta	—	/OUTPUT/, 4
Z4	Saved value of computed zeta	—	/OUTPUT/, 4
Z5	Zeta iteration convergence term	—	/OUTPUT/, 4
ZETA	Shape factor $\zeta = (\Delta/\delta)^n$	—	/OUTPUT/, 4
ZETATM	Shape factor raised to MZETA power	—	/INTER/, 6
ZI1	$I_1 = \int_0^1 \frac{\rho}{\rho_e} S^n (1 - S) dS$	—	/SAVED, 7
ZI1P	$I_1' = \int_0^1 \frac{\rho}{\rho_e} w^n (1 - w) dw$	—	/SAVED/, 7
ZI2	$I_2 = \int_0^1 \frac{\rho}{\rho_e} S^n dS$	—	/SAVED/, 7
ZI2P	$I_2' = \int_1^{1/\zeta} \frac{\rho}{\rho_e} w^n (1 - w) dw$	—	/SAVED/, 7
ZI3	$I_3 = \int_0^{\zeta} \frac{\rho}{\rho_e} S^{n-1} dS$	—	/SAVED/, 7
ZI3P	$I_3' = \int_{1/\zeta}^1 \frac{\rho}{\rho_e} w^{n-1} (1 - w) dw$	—	/SAVED/, 7
ZI4	$I_4 = \int_0^{\zeta} \frac{\rho}{\rho_e} S^n (1 - S) dS$	—	/SAVED/, 7
ZI5	$I_5 = \int_{\zeta}^1 \frac{\rho}{\rho_e} S^n (1 - S) dS$	—	/SAVED/, 7

TABLE 12. SUBROUTINE QUITS NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
ZI6	$I_6 = \int_0^{\zeta} \frac{\rho}{\rho_c} S^n dS$	—	/SAVED/, 7
ZI7	$I_7 = \int_{\zeta}^{1} \frac{\rho}{\rho_c} S^n dS$	—	/SAVED/, 7
ZME	Mach number	—	/OUTPUT/, 4
ZMTAB	Array of Mach numbers at the contour points, x, y	—	/TABLES/, 24
ZMU0	Viscosity at stagnation temperature	lb/(ft-sec)	/INPUT/, 3
ZMVIS	Exponent in viscosity-temperature law	—	/INPUT/, 3
ZMZETA	Real value of MZETA	—	/INTER/, 6
ZMZETM	ZMZETA minus one	—	/INTER/, 6
ZMZETP	ZMZETA plus one	—	/INTER/, 6
ZNSTAN	Interaction exponent in Stanton number relationship	—	/INPUT/, 3

SUBROUTINE READIN

Subroutine READIN first calls for single item variables such as integers, constants, and gas constants. It then reads in tabular values such as contour points, Mach number, and wall temperature. It prints out the input and performs diagnostics to detect obvious inconsistencies in input quantities.

Before any input is read in, the following constants are set to their nominal values and need be input only if different values are desired.

1.	Contour scale factor	SCALE = 1.0
2.	Velocity power law exponent	MZETA = 7
3.	Interaction exponent in Stanton number relation	ZNSTAN = 0.10
4.	Conversion factor between thermal and work units	FJ = 778.20 (ft-lb)/Btu
5.	Sea level acceleration of gravity	G = 32.1740 (ft-lbm)/(lbf-sec^2)
6.	Tolerance in skin friction coefficient iteration loop	TOLCFA = 1.0×10^{-4}
7.	Tolerance in gas property evaluation loops	TOLZME = 1.0×10^{-7}
8.	Tolerance in zeta iteration loop	TOLZET = 0.00030

The first input card for each case is a title card containing Hollerith information in columns 1 through 78. Input data are read in by the NAMELIST name NAM1. The variables that are included in the NAM1 NAMELIST are as follows: CPTAB, DXMAX, EPSZ, FJ, G, GAM0, ICTAB, IPRINT, ITWTAB, IXTAB, MZETA, P0, PHII, PRANDT, RBAR, SCALE, T0, TCTAB, THETAI, TOLCFA, TOLZET, TOLZME, TWTAB, XITAB, ZMTAB, ZMU0, ZMVIS, ZNSTAN

COMMON BLOCKS

COMMON blocks CSEVAL, INPUT, and TABLES are used.

TBL SUBROUTINES

Subroutine DIRECT calls READIN.

READIN calls subroutine QUITS.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines are used.
Built-in FORTRAN function IABS is used.

CALLING SEQUENCE

The subroutine calling sequence is:

CALL READIN

SOLUTION METHOD

Set constants to nominal values.

1. SCALE = 1.0

2. MZETA = 7

3. ZNSTAN = 0.10

4. FJ = 778.20

5. G = 32.1740

6. TOLCFA = 1.0×10^{-4}

7. TOLZME = 1.0×10^{-7}

8. TOLZET = 0.00030

Save value of maximum step size from previous case.

9. DXMAXO = DXMAX

Set maximum step size to check whether saved value from previous case is to be used.

10. DXMAX = -28982.0

Read in title for this case.

11. READ TITLE

Read in name list data for this case.

12. READ ($NAM1)

Check whether a new value for the maximum step size has been input.

13. If DXMAX ≠ -28982.0, go to 17

If DXMAX = -28982.0, go to 14

Check whether step size from previous case or computed value must be used.

14. If IDXMAX = 0, go to 21

If IDXMAX ≠ 0, go to 15

Set maximum step size to saved value from previous case.

15. DXMAX = DXMAXO

16. Go to 22

Step size has been input; check whether actual step size must be computed.

17. If DXMAX ≤ 0.0, go to 20

If DXMAX > 0.0, go to 18

Set step size indicator to use the same step size.

18. IDXMAX = 1

19. Go to 22

Set step size indicator to use a new step size.

20. IDXMAX = 0

Compute value of maximum step size to be used.

21. DXMAX = ((XITAB(IXTAB) - XITAB(1))/(100.))(SCALE)

Write out the title heading for this case.

22. WRITE TITLE

Initialize error indicator to zero.

23. IERROR = 0

Print out the velocity profile exponent.

24. WRITE MZETA

Check value of velocity profile exponent.

25. If MZETA $\geq$ 0, go to 28

If MZETA < 0, go to 26

If the velocity profile exponent is in error, print out message.

26. WRITE error message about input value

Set input error indicator to one.

27. IERROR = 1

Write out printout indicator.

28. WRITE IPRINT

Check value of printout indicator.

29. If IPRINT = 1 or IPRINT = 0, go to 32

If IPRINT ≠ 1 and IPRINT ≠ 0, go to 30

The printout indicator is in error; print out message.

30. WRITE error message about input value

Set input error indicator to one.

31. IERROR = 1

Print out dimension of x, y, and M tables.

32. WRITE IXTAB

Check whether value of dimension indicator is within limits.

33. If IXTAB ≥ 4 and IXTAB ≤ 500, go to 36

If IXTAB < 4 or IXTAB > 500, go to 34

Dimension indicator is below minimum limit or above maximum limit.

34. WRITE error message about input value

Set input error indicator to one.

35. IERROR = 1

Print out specific heat indicator.

36. WRITE ICTAB

Check specific heat option to be used.

37. If ICTAB = 0, go to 41

If ICTAB ≠ 0, go to 38

Tabular specific heat option is called for, check whether dimension is within limits.

38. If ICTAB ≥ 3 and ICTAB ≤ 20, go to 41

If ICTAB < 3 or ICTAB > 20, go to 39

Dimension indicator is out of limits.

39. WRITE error message about input value

Set input error indicator to one.

40. IERROR = 1

Write out wall temperature option indicator.

41. WRITE ITWTAB

Check on value of wall temperature option indicator.

42. If |ITWTAB| = 1 or ITWTAB = 0, go to 45

If |ITWTAB| ≠ 1 and ITWTAB ≠ 0, go to 43

Wall temperature indicator is in error.

43. WRITE error message about input value

Set input error indicator to one.

44. IERROR = 1

Print out the value of the stagnation temperature.

45. WRITE T0

Check value of stagnation temperature for error.

46. If T0 > 0.0, go to 49

If T0 ≤ 0.0, go to 47

Stagnation temperature value is in error.

47. WRITE error message about input value

Set input error indicator to one.

48. IERROR = 1

Print out the value of the stagnation pressure.

49. WRITE P0

Check value of stagnation pressure for error.

50. If P0 > 0.0, go to 53

    If P0 ≤ 0.0, go to 51

Stagnation pressure value is in error.

51. WRITE error message about input value

Set input error indicator to one.

52. IERROR = 1

Print out stagnation value of the specific heat ratio.

53. WRITE GAM0

Check on specific heat ratio option indicator.

54. If ICTAB ≠ 0, go to 58

    If ICTAB = 0, go to 55

Constant specific heat is to be used, check value of stagnation specific heat ratio.

55. If GAM0 > 1.0, go to 58

    If GAM0 ≤ 1.0, go to 56

Stagnation specific heat ratio is in error.

56. WRITE error message about input value

Set input error indicator to one.

57. IERROR = 1

Print out the value of the Prandtl number.

58. WRITE PRANDT

Check on value of the Prandtl number.

59. If PRANDT > 0.0, go to 62

If PRANDT ≤ 0.0, go to 60

The Prandtl number is in error.

60. WRITE error message about input value

Set input error indicator to one.

61. IERROR = 1

Print out the stagnation viscosity.

62. WRITE ZMU0

Check on value of the stagnation viscosity.

63. If ZMU0 > 0.0, go to 66

If ZMU0 ≤ 0.0, go to 64

The stagnation viscosity is in error.

64. WRITE error message about input value

Set input error indicator to one.

65. IERROR = 1

Print out the temperature-viscosity relation exponent.

66. WRITE ZMVIS

Print out the Stanton number interaction exponent.

67. WRITE ZNSTAN

Print out the maximum length of the step size.

68. WRITE DXMAX

Check whether the sonic point start procedure is to be used.

69. If THETAI < 0.0, go to 72

If THETAI ≥ 0.0, go to 70

Print out the initial value of the momentum thickness.

70. WRITE THETAI

Print out the initial value of the energy thickness.

71. WRITE PHII

Print out the flow geometry indicator.

72. WRITE EPSZ

Check value of the flow geometry indicator.

73. If EPSZ = 0.0 or EPSZ = 1.0, go to 76

If EPSZ ≠ 0.0 and EPZS ≠ 1.0, go to 74

The flow geometry indicator is in error.

74. WRITE error message about input value

Set input error indicator to one.

75. IERROR = 1

Print out the value of the specific gas constant.

76. WRITE RBAR

Check value of specific gas constant.

77. If RBAR > 0.0, go to 80

If RBAR ≤ 0.0, go to 78

The specific gas constant is in error.

78. WRITE error message about input value

Set input error indicator to one.

79. IERROR = 1

Print out the conversion factor between thermal and work units.

80. WRITE FJ

Check value of the conversion factor.

81. If FJ > 0.0, go to 84

If FJ ≤ 0.0, go to 82

The conversion factor is in error.

82. WRITE error message about input value

Set input error indicator to one.

83. IERROR = 1

Print out the value of the acceleration of gravity at sea level.

84. WRITE G

Check value of the acceleration of gravity.

85. If $G > 0.0$, go to 88

If $G \leq 0.0$, go to 86

The value of the acceleration of gravity is in error.

86. WRITE error message about input value

Set input error indicator to one.

87. IERROR = 1

Print out the nozzle contour scale factor.

88. WRITE SCALE

Check whether the skin friction coefficient tolerance has been input.

89. If TOLCFA $= 1.0 \times 10^{-4}$, go to 91

If TOLCFA $\neq 1.0 \times 10^{-4}$, go to 90

Print out the input value of the skin friction coefficient tolerance.

90. WRITE TOLCFA

Check whether the zeta iteration tolerance has been input.

91. If TOLZET = 0.00030, go to 93

If TOLZET $\neq$ 0.00030, go to 92

Print out the input value of the zeta iteration tolerance.

92. WRITE TOLZET

Check whether the gas property evaluation tolerance has been input.

93. IF TOLZME $= 1.0 \times 10^{-7}$, go to 95

IF TOLZME $\neq 1.0 \times 10^{-7}$, go to 94

Print out the input value of the gas property evaluation tolerance.

94. WRITE TOLZME

Check whether the specific heat table has been input.

95. If ICTAB ≤ 0, go to 112

If ICTAB > 0, go to 96

Print out a heading for the specific heat/temperature tables.

96. WRITE heading for specific heat/temperature tables

Print out the specific heat/temperature tables.

97. WRITE (I, CPTAB(I), TCTAB(I), I = 1,ICTAB)

Set limit for do-loop.

98. II = ICTAB - 1

99. Do 103, I = 1, II

Check whether temperature table values are monatonically increasing.

100. If TCTAB (I + 1) > TCTAB (I), go to 103

If TCTAB (I + 1) ≤ TCTAB (I), go to 101

Temperature table values are not in the proper order.

101. WRITE error message about temperature table

Set input error indicator to one.

102. IERROR = 1

103. Continue

Check the first value in the temperature table.

104. If TCTAB (1) > 0.0, go to 107

If TCTAB (1) ≤ 0.0, go to 105

The first temperature table value is in error.

105. WRITE error message about temperature table

Set input error indicator to one.

106. IERROR = 1

107. Do 111, I = 1, ICTAB

Check whether specific heat table values are greater than zero.

108. If CPTAB (I) > 0.0, go to 111

If CPTAB (I) ≤ 0.0, go to 109

Specific heat table values are in error.

109. WRITE error message about specific heat table

Set input error indicator to one.

110. IERROR = 1

111. Continue

112. Do 116, I = 1, IXTAB

Check Mach number table values.

113. If ZMTAB (I) > 0.0, go to 116

If ZMTAB (I) ≤ 0.0, go to 114

The Mach number table is in error.

114. WRITE error message about Mach number table

Set input error indicator to one.

115. IERROR = 1

116. Continue

Set limit for do-loop.

117. II = IXTAB - 1

118. Do 122, I = 1, II

Check whether axial distance table values are monatonically increasing.

119. If XITAB (I + 1) ≥ XITAB (I), go to 122

If XITAB (I + 1) < XITAB (I), go to 120

The axial distance table values are not in proper order.

120. WRITE error message about the axial distance table

Set input error indicator to one.

121. IERROR = 1

122. Continue

Check wall temperature option indicator.

123. If ITWTAB < 0, go to 133

If ITWTAB = 0, go to 130

If ITWTAB > 0, go to 124

Tabular wall temperatures are to be used.

124. Do 128, I = 1, IXTAB

Check whether wall temperature values are greater than zero.

125. If TWTAB (I) > 0.0, go to 128

If TWTAB (I) ≤ 0.0, go to 126

The wall temperature table is in error.

126. WRITE error message about wall temperature table

Set input error indicator to one.

127. IERROR = 1

128. Continue

129. Go to 133

Constant wall temperature option is to be used; check for proper values.

130. If TWTAB(1) > 0.0, go to 133

If TWTAB(1) ≤ 0.0, go to 131

Input value of constant wall temperature is in error.

131. WRITE error message about wall temperature

Set input error indicator to one.

132. IERROR = 1

Check whether contour scale factor has been input.

133. If SCALE = 1.0, go to 137

If SCALE ≠ 1.0, go to 134

Scale factor has been input; calculate real values for x and y.

134. Do 136, I = 1, IXTAB

135. XITAB(I) = (XITAB(I))(SCALE)

136. YITAB (I) = (YITAB(I))(SCALE)

Check wall temperature option indicator.

137. If ITWTAB < 0, go to 139

If ITWTAB = 0, go to 138

If ITWTAB > 0, go to 142

Print out value of constant wall temperature to be used.

138. WRITE TWTAB(1)

Print out heading for x, y, and M tables.

139. WRITE heading for x, y, and M tables

Print out input values of x, y, and M tables.

140. WRITE (I, XITAB(I), YITAB(I), ZMTAB(I), I = 1, IXTAB)

141. Go to 144

Print out heading for x, y, M, and T_w tables.

142. WRITE heading for x, y, M, and T_w tables

Print out input values of x, y, M, and T_w tables.

143. WRITE (I, XITAB(I), YITAB(I), ZMTAB(I),
TWTAB(I), I = 1, IXTAB)

Check dimension indicator for x, y, and M tables.

144. If IXTAB ≤ 1, go to 150

If IXTAB > 1, go to 145

Check whether axial distance values are in monatonically increasing order.

145. Do 149, I = 2, IXTAB

146. If XITAB (I) > XITAB (I - 1), go to 149

If XITAB (I) $\leq$ XITAB (I - 1), go to 147

The axial distance table is in error.

147. WRITE error message about axial distance table

Set input error indicator to one.

148. IERROR = 1

149. Continue

Check wall temperature option indicator.

150. If ITWTAB < 0, go to 154

If ITWTAB $\geq$ 0, go to 151

Check for sonic point start procedure option.

151. If THETAI $\geq$ 0.0, go to 154

If THETAI < 0.0, go to 152

The sonic point start option is incompatible with constant or tabular wall temperature options.

152. WRITE error message about input options

Set input error indicator to one.

153. IERROR = 1

Check for any errors in the input values.

154. If IERROR $\leq$ 0, return

If IERROR > 0, go to 155

There is an error in the input; print out COMMON blocks, and go to the next case.

155. CALL QUITS

Subroutine READIN nomenclature is given in Table 13.

TABLE 13. SUBROUTINE READIN NOMENCLATURE

Symbol	Description	Units	Reference
CPTAB	Array of specific heat values	Btu/(lbm-°R)	/CSEVAL/, /NAM1/, 97, 108
DXMAX	Maximum length of step size	ft	/INPUT/, /NAM1/, 9, 10, 13, 15, 17, 21, 68
DXMAXO	Maximum step size from last case	ft	9, 15
EPSZ	Flow geometry indicator	—	/INPUT/, /NAMI/, 72, 73
FJ	Conversion factor between thermal and work units	(ft-lbf)/Btu	/INPUT/, /NAM1/, 4, 80, 81
G	Acceleration of gravity at sea level	(ft-lbm)/(lbf-sec^2)	/INPUT/, /NAM1/, 5, 84, 85
GAM0	Specific heat ratio at stagnation condition	—	/INPUT/, /NAM1/, 53, 55
I	Do-loop and print-out counter	—	99, 100, 107, 108, 112, 113, 118, 119, 124, 125, 134-136, 140, 143, 145, 146
ICTAB	Specific heat indicator and/or table dimension	—	/INPUT/, /NAM1/, 36-38, 54, 95, 97, 98, 107

TABLE 13. SUBROUTINE READIN NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
IDXMAX	Step size indicator	—	/INPUT/, 14, 18, 20
IERROR	Input error indicator	—	23, 27, 31, 35, 40, 44, 48, 52, 57, 61, 65, 75, 79, 83, 87, 102, 106, 110, 115, 121, 127, 132, 148, 153, 154
II	Do-loop limit	—	98, 99, 117, 118
IPRINT	Print-out indicator	—	/INPUT/, /NAM1/, 28, 29
ITWTAB	Wall temperature option indicator	—	/INPUT/, /NAM1/, 41, 42, 123, 137, 150
IXTAB	Dimension indicator for x, y, and M tables	—	/INPUT/, /NAM1/, 32, 33, 112, 117, 124, 134, 140, 143-145
MZETA	Velocity profile exponent	—	/INPUT/, /NAM1/, 2, 24, 25
P0	Stagnation pressure	lbf/ft^2	/INPUT/, /NAM1/, 49, 50
PHII	Initial value of energy thickness	ft	/INPUT/, /NAM1/, 71
PRANDT	Prandtl number	—	/INPUT/, /NAM1/, 58, 59

TABLE 13. SUBROUTINE READIN NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
RBAR	Specific gas constant	(ft-lbf)/(lbm-°K)	/INPUT/, /NAM1/, 76, 77
SCALE	Contour scale factor	—	/INPUT/, /NAM1/, 1, 21, 88, 133, 135, 136
T0	Stagnation temperature	°R	/INPUT/, /NAM1/, 45, 46
TCTAB	Array of temperature values corresponding to the specific heat table values	°R	/CSEVAL/, /NAM1/, 97, 100, 104
THETAI	Initial value of momentum thickness	ft	/INPUT/, /NAM1/, 69, 70, 151
TITLE	Array of Hollerith characters input as the title heading for the case being run	—	DIM, 11, 22
TOLCFA	Skin friction iteration tolerance	—	/INPUT/, /NAM1/, 6, 89, 90
TOLZET	Zeta iteration tolerance	—	/INPUT/, /NAM1/, 8, 91, 92
TOLZME	Gas property evaluation tolerance	—	/INPUT/, /NAM1/, 7, 93, 94
TWTAB	Array of wall temperature values	°R	/TABLES/, /NAM1/, 125, 130, 138, 143

TABLE 13. SUBROUTINE READIN NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
XITAB	Array of axial distance values	ft	/TABLES/, /NAM1/, 119, 135, 140, 143, 146
YITAB	Array of nozzle radius values	ft	/TABLES/, /NAM1/, 136, 140, 143
ZMTAB	Array of Mach number values	—	/TABLES/, /NAM1/, 113, 140, 143
ZMU0	Stagnation viscosity	lb/(ft-sec)	/INPUT/, /NAM1/, 62, 63
ZMVIS	Viscosity-temperature law exponent	—	/INPUT/, /NAM1/, 66
ZNSTAN	Stanton number interaction exponent	—	/INPUT/, /NAM1/, 67

SUBROUTINE SEVAL

Subroutine SEVAL provides the evaluation of several thermodynamic properties under the basic assumptions that the enthalpy is a function of temperature (thermally perfect) only and the C_p is expressed as an analytic function between intervals of a C_p - T table, the function being a piecewise cubic.

The basic relations connecting the thermodynamic quantities are:

$$C_p = C_{pi} + BCP_i (T - T_i) + CCP_i (T - T_i)^2 + DCP_i (T - T_i)^3 \quad , \quad (1)$$

$$H (T) = H_i + \int_{T_i}^{T} C_p (t) dt \quad , \quad (2)$$

$$S(T, P) = S(T_o, P_o) + \int_{T_o}^{T} \frac{C_p (t)}{t} dt - R' \ln \frac{P}{P_o} \quad . \quad (3)$$

These functions are solved for several modes of input to SEVAL by an option indicator IND1.

IND1 = 0 — S is evaluated directly from equation (3) for given values of P and T .

$S = S (T, P)$.

- SEVAL (0, T, P, <u>S</u>)

IND1 = 1 — H is evaluated directly from equation (2) for a given T, and C_p is evaluated directly from equation (1).

$H = H(T), \; C_p = C_p(T)$.

- SEVAL (1, T, <u>CP</u>, <u>H</u>)

IND1 = 2 — T is evaluated from equation (2) for a given H in an inverse manner using a modified pseudo-position procedure. Then C_p is evaluated directly from equation (1).

$T = T(H),\ C_p = C_p(T)$.

- SEVAL (2, <u>T</u>, <u>CP</u>, H)

IND1 = 3 T is evaluated from equation (3), for given S and P, in an inverse manner using a modified pseudo-position procedure.

$T = T\ (S, P)$.

- SEVAL (3, <u>T</u>, P, S)

IND1 = -1 P is evaluated from equation (3) written in the form

$$\ln\left(\frac{P}{P_o}\right) = \left[\int_{T_o}^{T} \frac{C_p(t)}{t}\,dt + S(T_o, P_o) - S(T, P)\right] / R' \ .$$

$P = P\ (T, S)$.

- SEVAL (-1, T, <u>P</u>, S)

COMMON BLOCKS

COMMON blocks CSEVAL and INPUT are used.

TBL SUBROUTINES

Subroutines BARPRO, BARSET, FIIF, GETPT, and START call SEVAL.

SEVAL calls subroutine QUITS.

FORTRAN SYSTEM ROUTINES

FORTRAN library routines ALOG and EXP are used.

Built-in FORTRAN function ABS is used.

CALLING SEQUENCE

The subroutine calling sequence is

CALL SEVAL (IND1, AA, BB, CC)

where

IND1 = option indicator (0, 1, 2, 3 or -1),

AA = temperature T,

BB = pressure P or specific heat C_p,

CC = entropy S or enthalpy H.

Calling sequence depends on the option indicator represented by the first term in the parentheses.

SEVAL (0, T, P, <u>S</u>)
SEVAL (1, T, <u>CP</u>, <u>H</u>)
SEVAL (2, <u>T</u>, <u>CP</u>, H)
SEVAL (3, <u>T</u>, P, S)
SEVAL (-1, T, <u>P</u>, S)

SOLUTION METHOD

Define the function of entropy at temperature T and pressure P.

1. GAPF(T,G,A1,B1,C1,D1) = G + (A1)(Ln(T)) + (B1)(T) +(C1)(T)^2 + (D1)(T)^3/(3.0) - PR

Set three input values as

2. T = AA

3. A = BB

4. B = CC

Check the option indicator IND1.

5. If IND1 < 2, go to 11

If IND1 = 2, go to 6

If IND1 > 2, go to 16

Save the input enthalpy in thermal units

(IND1 = 2; H → T̲, C̲P̲) .

6. B = (B)/(FJG)

Check whether variable or constant specific heat is considered.

7. If ICTAB > 0, go to 55

If ICTAB ≤ 0, go to 8

Save temperature and specific heat in the case of constant specific heat $(C_p = C_{po})$.

8. T = (B)/(CP0)

9. A = CP0

10. Go to 106

Check the option indicator IND1.

11. If IND1 < 1, go to 33

If IND1 ≥ 1, go to 12

Check whether variable or constant specific heat (IND1 = 1; T $\underline{C_p}$, $\underline{H}$) is used.

12. If ICTAB > 0, go to 33

If ICTAB ≤ 0, go to 13

Save specific heat.

13. A = CP0

Calculate enthalpy in work units: $(C_{po}gJT_{c})$.

14. B = (CJG)(T)

15. Go to 106

Calculate the last term in the entropy equation (thermal units).

$$\frac{R}{J} \ln\left(\frac{P}{P_o}\right) \quad (\text{IND1} = 3;\ P,\ S \rightarrow \underline{T}) \quad .$$

16. PR = (ROJ)(Ln((A)/(P0MAX)))

Calculate entropy to search the temperature table.

17. STAB = GAPF(TCTAB(IS), GTAB(IS), BARB1(IS),
BARB2(IS), BARB3(IS), DCP(IS))

Check the values between input entropy and calculated entropy above.

18. If B ≥ STAB, go to 22

If B < STAB, go to 19

Decrement subscript counter to use next lower table values.

19. IS = IS - 1

Check whether subscript counter is still greater than zero.

20. If IS ≤ 0, go to 42

If IS > 0, go to 21

21. Go to 17

Calculate the entropy corresponding to TCTAB(IS + 1).

22. STAB = GAPF(TCTAB(IS + 1), GTAB(IS), BARB1(IS),
BARB2(IS), BARB3(IS), DCP(IS))

Check whether the calculated entropy above exceeds the input entropy value.

23. If B < STAB, go to 27

    If B ≥ STAB, go to 24

Increment the subscript counter to use the next higher table values.

24. IS = IS + 1

Check whether the subscript counter has exceeded the table dimension.

25. If IS ≥ 20, go to 42

    If IS < 20, go to 26

Repeat the above search until B < STAB is obtained.

26. Go to 22

Check the value of IS.

27. If IS ≥ NOCTAB or IS ≤ 0, go to 42

    If IS < NOCTAB and IS > 0, go to 28

Save values of temperature and entropy.

28. TTP = TCTAB(IS)

29. FP = GAPF(TCTAB(IS), GTAB(IS), BARB1(IS), BARB2(IS), BARB3(IS), DCP(IS))

30. TTPP = TCTAB(IS + 1)

31. FPP = STAB

32. Go to 68

Temperature T is checked in the following steps down to 44. (IND1 = 0 or 1).

33. If T $\geq$ TCTAB(IS), go to 37

If T < TCTAB(IS), go to 34

34. IS = IS - 1

35. If IS $\leq$ 0, go to 42

If IS > 0, go to 36

36. Go to 33

37. If T < TCTAB(IS + 1), go to 41

If T $\geq$ TCTAB(IS + 1), go to 38

38. IS = IS + 1

39. If IS $\geq$ 20, go to 42

If IS < 20, go to 40

40. Go to 37

41. If IS > 0, go to 44

If IS $\leq$ 0, go to 42

Print errors and corresponding values.

42. WRITE IS, IND1, T, A, B

Stop calculation by calling subroutine QUITS.

43. CALL QUITS

Check whether IS exceeds NOCTAB.

44. If IS $\geq$ NOCTAB, go to 42

If IS < NOCTAB, go to 45

Check the indicator for option IND1.

45. If IND1 < 0, go to 53

If IND1 = 0, go to 50

If IND1 > 0, go to 46

Set T minus tabulated value TCTAB(IS)

(IND1 = 1; T → $\underline{\text{CP}}$, $\underline{\text{H}}$) .

46. DELT = T - TCTAB(IS)

Calculate enthalpy in thermal units according to the equation in subroutine BARSET.

47. B = HTAB(IS) + (CPTAB(IS))(DELT)
+ (0.5)(BCP(IS))(DELT)^2 + (CCP(IS))(DELT)^3/(3.0)
+ (0.25)(DCP(IS))(DELT)^4

Save enthalpy in work units.

48. B = (B)(FJG)

49. Go to 105

Calculate entropy in this case (IND1 = 0; T, P → $\underline{\text{S}}$) .

50. PR = (ROJ)(Ln((A)/ P0MAX))

Entr opy:

51. B = GAPF(T,GTAB(IS), BARB1(IS), BARB2(IS), BARB3(IS), DCP(IS))

52. Go to 1υ6

Since

$$S_e = G_i + BARB1_i Ln\ T + BARB2_i T + BARB3_i T^2 + DCP_i \frac{T^3}{3} - \frac{R}{J} Ln \frac{P}{P_o} ,$$

$$P = P_o \exp\left[\frac{G_i + BARB1_i Ln\ T + BARB2_i T + BARB3_i T^2 + DCP_i \frac{T^3}{3} - S}{R/J}\right] .$$

The pressure P can be obtained, using entropy S and temperature T in the case of (IND1 = -1; T, S, → P), where B is the input entropy.

53. A = (P0MAX)(Exp((GTAB(IS) + (BARB1(IS))(Ln(T)) + (BARB2(IS))(T) + (BARB3(IS))(T)^2 + (DCP(IS))(T)^3/(3.0) - B)/ROJ))

54. Go to 106

Variable specific heat is considered from 55, when IND1 = 2. Temperature T and specific heat C_p are obtained using given enthalpy H = (B).

(IND1 = 2; H → T, CP)

Check whether the input enthalpy exceeds HTAB(IS) to search for temperature T_e.

55. If B ≥ HTAB(IS), go to 59

If B < HTAB(IS), go to 56

56. IS = IS - 1

Check the value of IS.

57. If IS ≤ 0, go to 42

If IS > 0, go to 58

58. Go to 55

59. If B < HTAB(IS + 1), go to 63

If B ≥ HTAB(IS + 1), go to 60

60. IS = IS + 1

61. If IS ≥ 20, go to 42

If IS < 20, go to 62

62. Go to 59

Check the value of IS.

63. If IS ≥ NOCTAB or IS ≤ 0, go to 42

If IS < NOCTAB and IS > 0, go to 64

Determine the table temperature and enthalpy corresponding to IS and IS + 1.

64. TTP = TCTAB(IS)

65. FP = HTAB(IS)

66. TTPP = TCTAB(IS + 1)

67. FPP = HTAB(IS + 1)

Determine the following quantity.

68. TT0 = ((TTP)(FPP - B) - (TTPP)(FP - B))/(FPP - FP)

Check the option indicator IND1.

69. If IND1 > 2, go to 73

If IND1 ≤ 2, go to 70

Define DELT (IND1 = 2; H $\rightarrow$ $\underline{T}$, $\underline{CP}$).

70. DELT = TT0 - TCTAB(IS)

Compute enthalpy.

71. $F0 = HTAB(IS) + (CPTAB(IS))(DELT) + (0.5)(BCP(IS))$
$\times (DELT)^2 + (CCP(IS))(DELT)^3/(3.0)$
$+ (0.25)(DCP(IS))(DELT)^4$

72. Go to 74

Compute entropy for IND1 = 3 (P, S $\rightarrow$ $\underline{T}$).

73. F0 = GAPF(TT0,GTAB(IS),BARB1(IS),BARB2(IS),BARB3(IS),
DCP(IS))

Determine the temperature.

74. TT1P = ((TT0)(FPP - B) - (TTPP)(F0 - B))/(FPP - F0)

75. TT1PP = ((TT0)(FP - B) - (TTP)(F0 - B))/(FP - F0)

Set switch indicator to minus one.

76. N = -1

Save temperature.

77. TAU = TT0

Save entropy.

78. SF = F0

Check whether the entropy or enthalpy calculated is nearly equal to the input one, using a tolerance of 10^{-7}.

79. If $|(SF - B)/(B)| \leq 10^{-7}$, go to 84

If $|(SF - B)/(B)| > 10^{-7}$, go to 80

Check whether the entropy or enthalpy computed is equal to or less than the input value.

80. If SF $\leq$ B, go to 86

If SF > B, go to 81

Set

81. TTPP = TAU

82. FPP = SF

83. Go to 88

Save the temperature in case convergence is achieved in step 79.

84. T = TAU

85. Go to 103

Store temperature and entropy or enthalpy in case convergence is not obtained and the computed entropy or enthalpy is less than the input values.

86. TTP = TAU

87. FP = SF

Check the switch indicator.

88. If N < 0, go to 89

If N = 0, go to 92

If N > 0, go to 101

Set the switch indicator to zero.

89. N = 0

Store the temperature.

4

90. TAU = TT1P

91. Go to 94

Set N equal to one.

92. N = 1

Store temperature TAU.

93. TAU = TT1PP

Check temperature TAU.

94. If TAU $\leq$ TTP or TAU $\geq$ TTPP, go to 88

If TAU > TTP and TAU < TTPP, go to 95

Check the option indicator IND1.

95. If IND1 $\leq$ 2, go to 98

If IND1 > 2, go to 96

Calculate entropy using temperature TAU, (IND1 = 3; P, S $\rightarrow$ $\underline{T}$).

96. SF = GAPF(TAU, GTAB(IS), BARB1(IS), BARB2(IS), BARB3(IS), DCP(IS))

Repeat the iteration.

97. Go to 79

Define DELT to obtain enthalpy at temperature TAU (IND1 = 2; H $\rightarrow$ $\underline{T}$, $\underline{CP}$).

98. DELT = TAU - TCTAB(IS)

Enthalpy at temperature TAU.

99. SF = HTAB(IS) + (CPTAB(IS))(DELT) + (0.5)(BCP(IS))(DELT)^2 + (CCP(IS))(DELT)^3/(3.0) + (0.25)(DCP(IS))(DELT)^4

100. Go to 79

Check whether convergence is obtained.

101. If $(FPP - FP)/(FP + FPP) > 0.0010$, go to 68

If $(FPP - FP)/(FP + FPP) \leq 0.0010$, go to 102

Calculate the temperature.

102. $T = ((TTP)(FPP - B) - (TTPP)(FP - B))/(FPP - FP)$

Check the option indicator.

103. If $IND1 > 2$, go to 106

If $IND1 \leq 2$, go to 104

Compute DELT, ($IND1 = 2$, $H \rightarrow \underline{T}, \underline{CP}$).

104. $DELT = T - TCTAB(IS)$

Compute specific heat at temperature T.

105. $A = CPTAB(IS) + (BCP(IS))(DELT) + (CCP(IS))(DELT)^2$
$+ (DCP(IS))(DELT)^3$

Save the temperature.

106. $AA = T$

Save the specific heat or pressure.

107. $BB = A$

Check the option indicator.

108. If $IND1 = 2$, return

If $IND1 \neq 2$, go to 109

Save enthalpy or entropy depending on the option indicator IND1.

109. CC = B

110. Return

Subroutine SEVAL nomenclature is given in Table 14.

TABLE 14. SUBROUTINE SEVAL NOMENCLATURE

Symbol	Description	Units	Reference
A	Saved value of pressure (IND1 = -1, 0,3) or specific heat (IND1 = 1, 2)	lbf/ft^2 Btu/(lbm-°R)	3, 16, 42, 50, 53, 107 3, 9, 13, 105, 107
A1	Argument of entropy function	Btu/(lbm-°R)	1
AA	Saved value of temperature	°R	CALL, 2, 106
B	Saved value of entropy (IND1 = -1, 0, 3) Saved value of enthalpy (IND1 = 1, 2)	Btu/(lbm-°R) (ft^2/sec^2) or Btu/(lbm-°R)	4, 18, 23, 42, 53, 79, 80, 109 4, 6, 8, 14, 42, 47, 48, 55, 59, 68, 74, 75, 79, 80, 109
B1	Argument of entropy function	Btu/(lbm-°R^2)	1
BARB1	Polynomial equation for $C_p = C_{pi} - BCP_iT_i + CCP_iT_i^2 - DCP_iT_i^3$	Btu/(lbm-°R)	/CSEVAL/, 17, 22, 29, 51, 53, 73, 96
BARB2	First derivative of BARB1 with negative sign	Btu/(lbm-°R^2)	/CSEVAL/, 17, 22, 29, 51, 53, 73, 96
BARB3	Second derivative of BARB2 times - 1/4	Btu/(lbm-°R^3)	/CSEVAL/, 17, 22, 29, 51, 53, 73, 96

TABLE 14. SUBROUTINE SEVAL NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
BB	Specific heat in the case of IND1 = 1 or 2 Pressure in the case of IND1 = -1, 0 or 3	Btu/(lbm-°R) lbf/ft^2	CALL, 3, 107
BCP	Coefficient in the C_p - T relationship determined by subroutine BMFITS	$(Btu/lbm)-°R^2$	/CSEVAL/, 47, 71, 99, 105
C1	Argument of entropy function	$Btu/(lbm-°R^3)$	1
CC	Enthalpy in the case of IND1 = 1 or 2 Entropy in the case of IND1 = -1, 0 or 3	ft^2/sec^2 Btu/(lbm-°R)	CALL, 4 CALL, 109
CCP	Coefficient in the C_p - T relationship determined by subroutine BMFITS	$Btu/(lbm-°R^3)$	/CSEVAL/, 47, 71, 99, 105
CJG	Specific heat at stagnation condition in work units	$ft^2/(sec^2-°R)$	/CSEVAL/, 14
CP0	Specific heat at constant pressure at stagnation condition	Btu/(lbm-°R)	/CSEVAL/, 8, 9, 13
CPTAB	Array of C_p values corresponding to the values in the temperature table	Btu/(lbm-°R)	/CSEVAL/, 71, 99, 105
D1	Argument of entropy function	$Btu/(lbm-°R^4)$	1
DCP	Coefficient in the C_p - T relationship determined by subroutine BMFITS	$Btu/(lbm-°R^4)$	/CSEVAL/, 17, 22, 29, 47, 51, 53, 71, 73, 96, 99, 105

TABLE 14. SUBROUTINE SEVAL NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
DELT	Temperature difference	°R	46, 47, 70, 71, 98, 99, 104, 105
F0	Saved value of enthalpy Saved value of entropy	Btu/lbm Btu/(lbm-°R)	71, 74, 75, 78 73, 74, 75, 78
FJG	Conversion factor between thermal and work units multiplied by acceleration of gravity used as a proportionality constant	$(ft^2\text{-lbm})/(Btu\text{-sec}^2)$	/CSEVAL/, 6, 48
FP	Saved value of table enthalpy	ft^2/sec^2	65, 68, 75, 87, 101, 102
FPP	Saved value of table enthalpy	ft^2/sec^2	67, 68, 74, 82, 101, 102
G	Argument of entropy function	Btu/(lbm-°R)	1
GAPF	Entropy function	Btu/(lbm-°R)	1, 17, 22, 29, 51, 73, 96
GTAB	Array of terms used in entropy function	Btu/(lbm-°R)	/CSEVAL/, 17, 22, 29, 51, 53, 73, 96
HTAB	Array of enthalpy values	Btu/lbm	/CSEVAL/, 47, 55, 59, 65, 67, 71, 99
IND1	Option indicator	—	CALL, 5, 11, 42, 45, 69, 95, 103, 108
ICTAB	Indicator = 0, constant specific heat calculation = $(3 \leq ICTAB \leq 20)$, dimension values in C_p versus T tables	—	/INPUT/, 7, 12

TABLE 14. SUBROUTINE SEVAL NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
IS	Tabulated position in flow direction for temperature table	—	/CSEVAL/, 17, 19, 20, 22, 24, 25, 27-30, 33-35, 37-39, 41, 42, 44, 46, 47, 51, 53, 55-57, 59-61, 63-67, 70, 71, 73, 96, 98, 99, 104, 105
N	Switch indicator	—	76, 88, 89, 92
NOCTAB	Saved value of ICTAB	—	/CSEVAL/, 27, 44, 63
P0MAX	Saved value of stagnation pressure	lbf/ft^2	/CSEVAL/, 16, 50, 53
PR	$R' \ln \left(\frac{P}{P_o}\right)$	Btu/(lbm-°R)	1, 16, 50
ROJ	Specific gas constant in thermal units, RBAR divided by FJ	Btu/(lbm-°R)	/CSEVAL/, 16, 50, 53
SF	Saved value of enthalpy or Save value of entropy	Btu/lbm Btu/(lbm-°R)	78-80, 82, 87, 99 78-80, 96
STAB	Computed value of entropy	Btu/(lbm-°R)	17, 18, 22, 23
T	Temperature	°R	1, 2, 8, 14, 33, 37, 42, 46, 51, 53, 84, 102, 104, 106
TAU	Saved value of temperature	°R	77, 81, 84, 86, 90, 93, 94, 96, 98
TCTAB	ICTAB values of temperature in increasing order defining C_p - T table	°R	/CSEVAL/, 17, 22, 28-30, 33, 37, 46, 64, 66, 70, 98, 104

TABLE 14. SUBROUTINE SEVAL NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
TT0	Assumed temperature for iteration on the temperature	°R	68, 70, 73-75, 77
TT1P	Assumed temperature for iteration on the temperature	°R	74, 90
TT1PP	Assumed temperature for iteration on the temperature	°R	75, 93
TTP	Saved value of TCTAB (IS)	°R	64, 68, 75, 86, 94, 102
TTPP	Saved value of TCTAB (IS + 1)	°R	66, 68, 74, 81, 94, 102

SUBROUTINE START

Subroutine START locates the sonic point ($M = 1$) from the input Mach number versus x table and then computes initial values for momentum and energy thicknesses.

The table of Mach numbers is first scanned to locate an exact input value of $M = 1.$ If none is found, the sonic point location is determined from the following iteration procedure.

1. Locate the i^{th} interval, in the table, which contains $M = 1$, and assume $x_g = x_i$.

2. Using subroutine XNTERP, evaluate M_g at x_g.

3. Make a tolerance comparison.

   If $|M_g - 1.0| \leq$ TOLZME, convergence is satisfactory, and appropriate flow quantities are avaluated at x_g. If outside the tolerance, a secant method is used to make another assumption for x_g and steps 2 to 3 are repeated up to a maximum of 50 iterations.

Using the flow quantities just computed, subroutine INTZET is used to evaluate integrals I_1 and I_2 for $\zeta = 1$. The shape factor δ^*/θ is then evaluated from equation (125) [1]. An iteration for the skin friction coefficient C_f is performed (similar to that in subroutine BARPRO), and a value of momentum thickness θ is computed. These values are assumed to be the initial values for θ and ϕ at x_g.

COMMON BLOCKS

COMMON blocks COFIIF, CSEVAL, INPUT, INTER, LOOKUP, OUTPUT, SAVED, and TABLES are used.

TBL SUBROUTINES

Subroutine BARCON calls START.

START calls CFEVAL, GETPT, INTZET, QUITS, SEVAL, and XNTERP.

FORTRAN SYSTEM ROUTINES

FORTRAN library routine SQRT is used.

Built-in FORTRAN function ABS is used.

CALLING SEQUENCE

The subroutine calling sequence is:

CALL START

SOLUTION METHOD

Set table position indicators for a first time entry indication.

1. IMX = -1

2. IYS = -1

3. ITWX = -1

Initialize subscript counter to zero.

4. I = 0

Increment subscript counter.

5. I = I + 1

Check whether Mach number table includes M = 1.0 .

6. If ZMTAB(I) < 1.0 , go to 7

   If ZMTAB(I) = 1.0 , go to 10

   If ZMTAB(I) > 1.0 , go to 13

Check whether subscript counter is equal to the table dimension.

7. If $I < IXTAB$, go to 5

If $I \geq IXTAB$, go to 8

The Mach number table does not contain $M = 1.0$.

8. WRITE error message about Mach number table

Print out all the COMMON blocks and go to next case.

9. CALL QUITS

Obtain axial distance at which $M = 1.0$.

10. X = XITAB(I)

Set subscript counter for the Mach number that just exceeds one.

11. IBEG = I + 1

12. Go to 29

Check whether the first Mach number table value is greater than one.

13. If $I \leq 1$, go to 8

If $I > 1$, go to 14

Save value of axial distance at which Mach number becomes greater than one.

14. XG = XITAB(I)

Obtain the lowest number table value greater than one.

15. ZME = ZMTAB(I)

Approximate the axial distance at which $M = 1.0$.

16. X = XITAB(I - 1) + (XITAB(I) - XITAB(I - 1))/
(ZMTAB(I) - ZMTAB(I - 1))(1.0 - ZMTAB(I - 1))

Initialize Mach number iteration counter to zero.

17. J = 0

Set subscript counter for which Mach number exceeds one.

18. IBEG = I

Increment Mach number iteration counter.

19. J = J + 1

Save previous value of axial distance at which M > 1.0 .

20. XO = XG

Save previous value of Mach number obtained from table lookup.

21. ZMO = ZME

Save present value of axial distance for which M > 1.0 .

22. XG = X

Determine value of Mach number for approximated value of axial distance.

23. CALL XNTERP(X, ZME, ZMEP, IMX, XITAB, ZMTAB, IXTAB, CMX, IMX)

Check whether the Mach number falls within the desired tolerance band.

24. If |ZME - 1.0| ≤ TOLCFA , go to 29

If |ZME - 1.0| > TOLCFA , go to 25

Compute a new value of the Mach number.

25. ZMX = (XG - XO)/(ZME - ZMO)

Compute a new value of the axial distance.

26. X = XO + (1.0 - ZMO)(ZMX)

Check whether there have been more than 50 iterations to determine the Mach number equal to one.

27. If J ≤ 50, go to 19

If J > 50, go to 28

There have been more than 50 iterations on the Mach number.

28. WRITE error message about Mach number iteration failure

Determine the value of Mach number at final value of axial distance.

29. CALL XNTERP (X, ZME, ZMEP, IMX, XITAB, ZMTAB, IXTAB, CMX, IMX)

Determine the nozzle radius at this value of axial distance.

30. CALL XNTERP (X, YR, YRP, IYS, XITAB, YITAB, IXTAB, CYX, IMX)

Compute the pressure and temperature for the computed Mach number.

31. CALL GETPT (ZME, PSE, TE)

Compute the enthalpy and specific heat at this temperature.

32. CALL SEVAL (1, TE, CPE, HE)

Compute the ratio of specific heats.

33. GAME = (CPE)/(CPE - (RBAR)/(FJ))

Compute the difference between stagnation and static enthalpy.

34. HB = HO - HE

Compute the fluid velocity.

35. UE = SQRT((2.0)(HB))

Compute the fluid density.

36. RHSE = (PSE)/(TE)/(RBAR)

Compute viscosity at the static temperature.

37. $\text{ZMU} = (\text{ZMU0})((\text{TE})/(\text{T0}))^{\text{ZMVIS}}$

Compute the adiabatic wall enthalpy.

38. $\text{HAW} = \text{HE} + ((\text{PRANDT})^{1/3})(\text{HB})$

Compute the adiabatic wall temperature and specific heat.

39. CALL SEVAL(2, $\underline{\text{TAW}}$, $\underline{\text{CPAW}}$, HAW)

Check for temperature option to be used.

40. If ITWTAB < 0 , go to 41

If ITWTAB = 0 , go to 44

If ITWTAB > 0 , go to 47

Adiabatic wall temperature option is used; set wall enthalpy and wall temperature to adiabatic values.

41. HW = HAW

42. TW = TAW

43. Go to 49

Constant wall temperature option is used; set wall enthalpy to previously computed value.

44. HW = TWTAB(2)

Set wall temperature to input value.

45. TW = TWTAB(1)

46. Go to 49

Tabular wall option is used; determine the value of wall temperature.

47. CALL XNTERP(X, TW, TWP, ITWX, XITAB, TWTAB, IXTAB,
CTWX, IMX)

Compute enthalpy and specific heat at the wall temperature.

48. CALL SEVAL(1, TW, CPW, HW)

Obtain flow properties to evaluate integrals I_1 and I_2 .

49. AFINT = HW

50. BFINT = H0 - HW

51. CFINT = - HB

52. TFINT = TE

53. MMINT = MZETA

Set indicator to evaluate first integral I_1 .

54. IFINT = 1

Evaluate first integral I_1 .

55. CALL INTZET(0.0, 1.0, ZI1)

Set indicator to evaluate second integral I_2 .

56. IFINT = 2

Evaluate second integral I_2 .

57. CALL INTZET(0.0, 1.0, ZI2)

Compute the boundary layer shape factor.

58. DELSOT = ((1.0)/(ZMZETA) - ZI2)/(ZII)

Check for type of flow to be considered.

59. If EPSZ = 0.0, go to 62

If EPSZ ≠ 0.0, go to 60

Axisymmetric flow is considered; set intermediate term to ratio of nozzle radius slope to the nozzle radius.

60. ERASE5 = (YRP)/(YR)

61. Go to 63

Two-dimensional flow is considered; set intermediate term to one.

62. ERASE5 = 1.0

Compute intermediate term.

63. ERASE4 = (1.0 + DELSOT)/(1.0 + (GAME - 1.0)/(2.0))(ZMEP)
+ ERASE 5

Check on value of intermediate term.

64. If ERASE4 ≠ 0.0, go to 66

If ERASE4 = 0.0, go to 65

The intermediate term is in error.

65. WRITE error message and possible remedy

Compute ratio of difference between stagnation and adiabatic wall temperature to the adiabatic wall temperature.

66. ERASE1 = (17.20)(T0 - TAW)/(TAW)

Compute ratio of difference between static and stagnation temperature to adiabatic wall temperature.

67. ERASE2 = (305.0)(TE - T0)/(TAW)

Compute term for calculating the momentum thickness.

68. CTHET = (0.50)(SQRT(1.0 + YRP^2))/(ERASE4)

Compute skin friction coefficient intermediate term.

69. CRT2 = $((TAW)/(TE))^{(1.0-ZMVIS)}$(RHSE)(UE)/((ZMU)(CTHET)

Compute ratio of adiabatic wall temperature to the static temperature.

70. ERASE3 = (TAW)/(TE)

Set initial assumption for the skin friction coefficient.

71. CFA = 0.0010

Initialize skin friction coefficient iteration counter to zero.

72. JE = 0

Increment skin friction coefficient iteration counter.

73. JE = JE + 1

Save previous skin friction coefficient quantity

74. CFG = CFA

Compute momentum thickness based on computed skin friction coefficient.

75. THETA = (CFG)(CTHET)

Compute term for obtaining skin friction coefficient based on Coles' relationship.

76. CR = $(CRT2)(THETA)^2$

Compute skin friction coefficient based on Coles' relationship.

77. CFB = CFEVAL(CR)

Compute intermediate term.

78. TTAW = 1.0 + (ERASE1)(SQRT((CFB)/(2.0))
+ (ERASE2)(CFB)/(2.0)

Check value of intermediate term.

79. If TTAW > 0.0, go to 86

If TTAW ≤ 0.0, go to 80

Compute new value of the skin friction coefficient.

80. CFA = (3.0)(CFG)

Save computed value of skin friction coefficient.

81. Z4 = CFA

Save approximation value of skin friction coefficient.

82. Z2 = CFG

Check whether there have been over 50 interations to determine the skin friction coefficient.

83. If JE ≤ 50, go to 73

If JE > 50, go to 84

There have been more than 50 iterations on the skin friction coefficient.

84. WRITE error message about skin friction coefficient iteration failure.

85. Go to 96

Compute new value of the skin friction coefficient.

86. $CFA = (CFB)/((ERASE3)(TTAW)^{ZMVIS})$

Check whether the skin friction coefficient is within the tolerance.

87. If $|(CFA - CFG)/(CFA + CFG)| \leq TOLCFA$, go to 96

If $|(CFA - CFG)/(CFA + CFG)| > TOLCFA$, go to 88

Check whether the iteration counter is less than two.

88. If $JE < 2$, go to 81

If $JE \geq 2$, go to 89

Save previous computed value of skin friction coefficient.

89. Z3 = Z4

Save previous assumed value of skin friction coefficient.

90. Z1 = Z2

Save new computed value of skin friction coefficient.

91. Z4 = CFA

Save new approximation of skin friction coefficient.

92. Z2 = CFG

Compute skin friction coefficient convergence term.

93. $ZS5 = (Z4 - Z3)/(Z2 - Z1)$

Compute new value of the skin friction coefficient.

94. $CFA = (Z4 - (ZS5)(Z2))/(1.0 - ZS5)$

Loop back to continue iteration on skin friction coefficient.

95. Go to 83

Compute initial value of momentum thickness.

96. THETAI = (CFA)(CTHET)

Set initial value of momentum thickness equal to energy thickness.

97. PHII = THETAI

Save final computed value of skin friction coefficient.

98. CFAGT = CFA

Set shape factor to one.

99. ZETA = 1.0

Write out the initial values of momentum and energy thicknesses at the throat.

100. WRITE X, YR, THETAI, PHII

101. Return

Subroutine START nomenclature is given in Table 15.

TABLE 15. SUBROUTINE START NOMENCLATURE

Symbol	Description	Units	Reference
AFINT	Wall enthalpy for integral evaluation	ft^2/sec^2	/COFIIF/, 49
BFINT	Difference between stagnation enthalpy and wall enthalpy	ft^2/sec^2	/COFIIF/, 50
CFA	Computed value of skin friction coefficient	—	71, 74, 80, 81, 86, 87, 91, 94, 96, 98
CFAGT	Saved value of computed skin friction coefficient	—	/INTER/, 98
CFB	Value of skin friction coefficient from CFEVAL	—	77, 78, 86

TABLE 15. SUBROUTINE START NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
CFG	Approximation of skin friction coefficient	—	74, 75, 80, 82, 87, 92
CFINT	Difference between static enthalpy and stagnation enthalpy	ft^2/sec^2	/COFHF/, 51
CMX	Array of parabola coefficients for Mach number table (ZMTAB)	—	/LOOKUP/, 23, 29
CPAW	Specific heat at adiabatic wall conditions	Btu/(lbm-°R)	39
CPE	Specific heat at static temperature	Btu/(lbm-°R)	32, 33
CPW	Specific heat at wall temperature	Btu/(lbm-°R)	48
CR	Term for computing skin friction coefficient in CFEVAL	lbm/(ft-sec)	76, 77
CRT2	Term for computing CR.	$lbm/(ft^3-sec)$	69, 76
CTHET	Term for computing momentum thickness	ft	68, 75, 96
CTWX	Array of parabola coefficients for the wall temperature table (TWTAB)	—	/LOOKUP/, 47
CYX	Array of parabola coefficients for the nozzle radius table (YITAB)	—	/LOOKUP/, 30
DELSOT	Boundary layer shape factor $\delta*/\theta$	—	/OUTPUT/, 58, 63
EPSZ	Flow geometry indicator	—	/INPUT/, 59

TABLE 15. SUBROUTINE START NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
ERASE1	Ratio of stagnation and adiabatic wall temperature	—	66, 78
ERASE2	Ratio of static, stagnation, and adiabatic wall temperatures	—	67, 78
ERASE3	Ratio of adiabatic wall temperature to the static temperature	—	70, 86
ERASE4	Intermediate term for computing momentum thickness	ft^{-1}	63, 64, 68
ERASE5	Intermediate term for computing momentum thickness	ft^{-1}	60, 62, 63
FJ	Thermal-to-work unit conversion factor	(ft-lbf)/Btu	/INPUT/, 33
GAME	Ratio of specific heats	—	33, 63
H0	Stagnation enthalpy	ft^2/sec^2	/CSEVAL/, 34, 50
HAW	Adiabatic wall enthalpy	ft^2/sec^2	38, 39, 41
HB	Difference between stagnation and static enthalpy	ft^2/sec^2	34, 35, 38, 51
HE	Static enthalpy	ft^2/sec^2	/INTER/, 32, 34, 38
HW	Wall enthalpy	ft^2/sec^2	/INTER/, 41, 44, 48-50
I	Array subscript counter	—	4-7, 10, 11, 13-16, 18
IBEG	Subscript counter at which the Mach number table exceeds one	—	/INTER/, 11, 18

TABLE 15. SUBROUTINE START NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
IFINT	Integral evaluation indicator	—	/COFIIF/, 54, 56
IMX	Position or start indicator for Mach number table (ZMTAB)	—	/LOOKUP/, 1, 23, 29, 30, 47
ITWTAB	Wall temperature indicator	—	/INPUT/, 40
ITWX	Position or start indicator for wall temperature table (TWTAB)	—	/LOOKUP/, 3, 47
IXTAB	Number of points in x, y, and M tables	—	/INPUT/, 7, 23, 29, 30, 47
IYS	Position or start indicator for nozzle radius table (YITAB)	—	2, 30
J	Mach number iteration counter	—	17, 19, 27
JE	Skin friction coefficient iteration counter	—	72, 73, 83, 88
MMINT	Exponent for integral evaluation	—	/COFIIF/, 53
MZETA	Exponent of velocity profile	—	/INPUT/, 53
PHII	Initial value of energy thickness	ft	/INPUT/, 97, 100
PRANDT	Prandtl number	—	/INPUT/, 38
PSE	Static pressure at M = 1.0	lbf/ft^2	31, 36
RBAR	Specific gas constant	(ft-lbf)/(lbm-°R)	/INPUT/, 33, 36
RHSE	Fluid density at M = 1.0	lbm/ft^2	36, 69
T0	Stagnation temperature	°R	/INPUT/, 37, 66, 67
TAW	Adiabatic wall temperature	°R	39, 42, 66, 67, 69, 70

TABLE 15. SUBROUTINE START NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
TE	Static temperature at M = 1.0	°R	/OUTPUT/, 31, 32, 36, 37, 52, 67, 69, 70
TFINT	Static temperature for integral evaluation	°R	/COFIIF/, 52
THETA	Boundary layer momentum thickness	ft	/OUTPUT/, 75, 76
THETAI	Initial value of momentum thickness	ft	/INPUT/, 96, 97, 100
TOLCFA	Tolerance in skin friction coefficient iteration	—	/INPUT/, 24, 87
TTAW	Intermediate term in skin friction coefficient iteration	—	78, 79, 86
TW	Wall temperature	°R	/OUTPUT/, 42, 45, 47, 48
TWP	Derivative of wall temperature	°R/ft	47
TWTAB	Array of wall temperatures at contour points	°R	/TABLES/, 47
UE	Velocity of fluid at M = 1.0	ft/sec	/OUTPUT/, 35, 69
X	Axial distance at which M = 1.0	ft	/OUTPUT/, 10, 16, 22, 23, 26, 29, 30, 47, 100
XG	Approximation of axial distance at which M = 1.0	ft	14, 20, 22, 25
XITAB	Array of axial distance values	ft	/TABLES/, 10, 14, 16, 23, 29, 30, 47
XO	Previous assumption of axial distance at which M = 1.0	ft	20, 25, 26
YITAB	Array of nozzle radius values	ft	/TABLES/, 30

TABLE 15. SUBROUTINE START NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
YR	Nozzle radius or contour height	ft	/OUTPUT/, 30, 60, 100
YRP	Derivative of nozzle radius	—	30, 60, 68
Z1	Saved value of skin friction coefficient assumption	—	/OUTPUT/, 90, 93
Z2	Saved value of skin friction coefficient assumption	—	/OUTPUT/, 82, 90, 92-94
Z3	Saved value of computed skin friction coefficient	—	/OUTPUT/, 89, 93
Z4	Saved value of computed skin friction coefficient	—	/OUTPUT/, 81, 89, 91, 93, 94
ZETA	Shape factor $\zeta = (\Delta/\delta)^{1/n}$	—	/OUTPUT/, 99
ZI1	Value of integral I_1	—	/SAVED/, 55, 58
ZI2	Value of integral I_2	—	/SAVED/, 57, 58
ZME	Mach number	—	/OUTPUT/, 15, 21, 23-25, 29, 31
ZMEP	Derivative of Mach number	ft^{-1}	23, 29, 63
ZMO	Saved value of Mach number for iteration on M = 1.0	—	21, 25, 26
ZMTAB	Array of Mach numbers associated with contour points	—	/TABLES/, 6, 15, 16, 23, 29
ZMU	Fluid viscosity at M = 1.0	lbm/(ft-sec)	37, 69
ZMU0	Viscosity at stagnation temperature	lbm/(ft-sec)	/INPUT/, 37
ZMVIS	Exponent in viscosity temperature law	—	/INPUT/, 69, 86

TABLE 15. SUBROUTINE START NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
ZMX	Computed value of Mach number in iteration on M = 1.0	—	25, 26
ZMZETA	Real value of velocity profile exponent	—	/INTER/, 58
ZS5	Ratio of difference between computed values of skin friction coefficient and approximations	—	93, 94

SUBROUTINE XNTERP

Subroutine XNTERP determines the dependent variable y and its first derivative y' for a dependent variable x based upon a functional relationship $y = f(x)$ represented in tabular form. A local quintic spline interpolation is used in the solution process as described below.

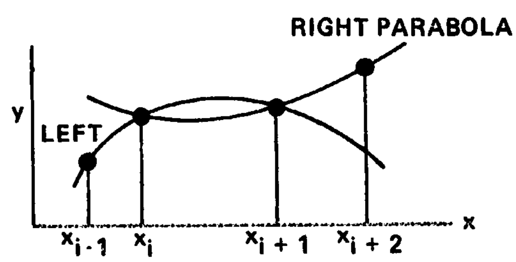
The independent variable table is searched until the interval containing x is located. The associated interval table values x_i and x_{i+1} and two additional ones x_{i-1} and x_{i+2}, one on either side of the specified interval, are then selected for the quintic spline fit. Using either the first three $\left(x_{i-1}, x_i, x_{i+1}\right)$ or the last three $\left(x_i, x_{i+1}, x_{i+2}\right)$ table values from the ones previously specified, two parabolas and their corresponding coefficients are determined satisfying the following relationship.

$$\overline{Y}_L = C_1 + C_2 (x - x_{i-1}) + C_3 (x - x_{i-1})^2 \quad ,$$

$$\overline{Y}_R = C_4 + C_5 (x - x_i) + C_6 (x - x_i)^2 \quad ,$$

$$\overline{Y}'_L = C_2 + 2C_3 (x - x_{i-1}) \quad ,$$

$$\overline{Y}'_R = C_5 + 2C_6 (x - x_i) \quad .$$



A cubic weighting function α and its derivative α'

$$\alpha(x) = 3U^2 - 2U^3 \quad ,$$

$$\alpha'(x) = (6U - 6U^2) \cdot U'(x) \quad ,$$

where

$$U(x) = \frac{x - x_i}{x_{i+1} - x_i} \quad ,$$

$$U'(x) = \frac{1}{x_{i+1} - x_i} \quad ,$$

provide a scheme to solve for the dependent variable y and its derivative y' according to the relationship below.

$$y(x) = [1 - \alpha(x)][C_1 + C_2 (x - x_{i-1}) + C_3 (x - x_{i-1})^2]$$

$$+ \alpha(x) [C_4 + C_5 (x - x_i) + C_6 (x - x_i)^2]$$

$$= [1 - \alpha(x)] \bar{y}_L + \alpha(x) \bar{y}_R \quad ;$$

$$y'(x) = \alpha'(x)(\bar{y}_R - \bar{y}_L) + [1 - \alpha(x)] \bar{y}'_L + \alpha(x) \bar{y}'_R \quad .$$

COMMON BLOCKS

No COMMON blocks are used.

TBL SUBROUTINES

Subroutines BARCON, BARPRO, and START call XNTERP.

XNTERP calls subroutine QUITS.

FORTRAN SYSTEM ROUTINES

No FORTRAN library routines or built-in FORTRAN functions are used.

CALLING SEQUENCE

The subroutine calling sequence is

CALL XNTERP (X, Y, YP, IXIN, XAR, YAR, IAR, CAR, IPOS),

where

X = input value of the independent variable

Y = output value of the corresponding dependent variable

YP = derivative of the dependent variable

IXIN = table position or start indicator

XAR = input array of independent variables

YAR = input array of dependent variables

IAR = dimension of the independent and dependent arrays

CAR = input or output array of parabola coefficients

IPOS = array position indicator

SOLUTION METHOD

Set interval subscript counter limit.

1. IXO = IXIN

Set working maximum length of input arrays.

2. IXMAX = IAR - 1

Set internal subscript counter to array position indicator.

3. IX = IPOS

Set array of parabola coefficients to those used previously.

4. Do 5, I = 1, 6

5. C(I) = CAR(I)

Check whether this is a first time entry.

6. If IXO > 0, go to 10

   If IXO ≤ 0, go to 7

This is a first time entry; set the indicator to one.

7. IF1RST = 1

Set subscript counter limit to IAR + 1.

8. IXO = IXMAX + 2

Set array subscript counter to one.

9. IX = 1

Check whether subscript counter is zero or negative.

10. If IX ≤ 0, go to 7

    If IX > 0, go to 11

Check whether the input independent value is greater than or equal to table value.

11. If X ≥ XAR(IX), go to 16

    If X < XAR(IX), go to 12

Decrement the array subscript counter.

12. IX = IX - 1

Check whether the subscript counter is still greater than zero.

13. If IX > 0, go to 11
    If IX ≤ 0, go to 14

The input value is out of range of the table; print out values.

14. WRITE X, XAR(1), XAR(IXMAX + 1), YAR(1),
YAR(IXMAX + 1)

Print out the COMMON blocks, and go to the next case.

15. CALL QUITS

Check whether the input value is less than or equal to the next table value.

16. If $X \le XAR(IX + 1)$, go to 19

If $X > XAR(IX + 1)$, go to 17

Increment the array subscript counter.

17. IX = IX + 1

Check whether the subscript counter has exceeded the maximum value.

18. If $IX \le IXMAX$, go to 16

If $IX > IXMAX$, go to 14

The points bracketing x have been found; obtain the four surrounding points for x and y.

19. Do 22, I = 1, 4

Set subscript counter to obtain x_{i-1}, x_i, x_{i+1}, and x_{i+2}.

20. I1 = IX - 2 + I

Obtain the four surrounding points from the x table and the corresponding points from the y table.

21. XI(I) = XAR(I1)

22. YI(I) = YAR(I1)

Compute difference between x and x_i.

23. DX2 = X - XI(2)

Compute difference between x_{i+1} and x_i.

24. DX32 = XI(3) - XI(2)

Check subscript counter against subscript limit.

25. If IX < IXO, go to 39

If IX = IXO, go to 26

If IX > IXO, go to 62

The subscript limit has been reached; set subscript limit indicator.

26. IXOGO = 0

Check whether subscript counter is greater than one.

27. If IX > 1, go to 30

If IX ≤ 1, go to 28

Subscript counter is less than or equal to one; set indicator.

28. IGO = -1

29. Go to 74

Check whether subscript counter is less than the maximum.

30. If IX < IXMAX, go to 37

If IX ≥ IXMAX, go to 31

Check for first time entry into the dependent array.

31. If IFIRST = 0, go to 35

If IFIRST ≠ 0, go to 32

This is a first time entry; reset entry indicator, and set loop indicator.

32. IFIRST = 0

33. IGO = 1

34. Go to 54

This is not a first time entry; set the loop indicator.

35. IGO = 1

36. Go to 70

Subscript counter is less than the maximum; set loop indicator.

37. IGO = 0

38. Go to 70

Subscript counter is below the limit; set subscript limit indicator.

39. IXOGO = -1

Check whether the subscript counter is one less than the limit.

40. If IX < IXO -1, go to 45

If IX ≥ IXO - 1, go to 41

Set left parabola coefficients to the right parabola coefficients.

41. C(4) = C(1)

42. C(5) = C(2)

43. C(6) = C(3)

44. Go to 52

Compute coefficients for the left parabola.

45. C(4) = YI(2)

Compute difference between x_{i+2} and x_i and y_{i+1} and y_i.

46. DX42 = XI(4) - XI(2)

47. DY32 = YI(3) - YI(2)

48. DYOX32 = (DY32)/(DX32)

49. C(6) = (DYOX32 - (YI(4) - YI(2))/(DX42))/(XI(3) - XI(4))

50. C(5) = DYOX32 - (C(6))(DX32)

Check subscript limit indicator.

51. If IXOGO > 0, go to 70

If IXOGO ≤ 0, go to 52

Check whether subscript counter is greater than one.

52. If IX ≤ 1, go to 28

If IX > 1, go to 53

Subscript is less than the limit and greater than one; set indicator.

53. IGO = 0

Compute coefficients for the left parabola.

54. C(1) = YI(1)

Compute difference between x_i and x_{i-1}.

55. DX21 = XI(2) - XI(1)

Compute difference between x_{i+1} and x_{i-1}.

56. DX31 = XI(3) - XI(1)

Compute difference between y_i and y_{i-1}.

57. DY21 = YI(2) - YI(1)

58. DYOX21 = (DY21)/(DX21)

59. C(3) = (DYOX21 - (YI(3) - YI(1))/(DX31))/(XI(2) - XI(3))

60. C(2) = DYO X21 - (C(3))(DX21)

Check subscript limit indicator.

61. If IXOGO $\leq$ 0, go to 70

If IXOGO > 0, go to 67

The subscript counter is above the limit; set subscript limit indicator.

62. IXOGO = 1

Check whether the subscript counter is more than one above the limit.

63. If IX > IXO + 1, go to 54

If IX $\leq$ IXO + 1, go to 64

Set coefficients of left parabola to those of right parabola.

64. C(1) = C(4)

65. C(2) = C(5)

66. C(3) = C(6)

Check whether subscript counter is less than the maximum.

67. If IX $\geq$ IXMAX, go to 35

If IX < IXMAX, go to 68

Subscript counter is less than the maximum set loop indicator.

68. IGO = 0

69. Go to 45

Compute difference between x and x_{i-1}.

70. DX1 = X - XI(1)

Compute dependent variable value from the left parabola.

71. YB1 = ((C(3))(DX1) + C(2))(DX1) + C(1)

Compute derivative of dependent variable.

72. YPB1 = (C(3))(DX1)/(0.50) + C(2)

Check loop control indicator.

73. If IGO > 0, go to 89

If IGO ≤ 0, go to 74

Compute dependent variable value from the right parabola.

74. YB2 = ((C(6))(DX2) + C(5))(DX2) + C(4)

Compute derivative of dependent variable.

75. YPB2 = (C(6))(DX2)/(0.50) + C(5)

Check loop control indicator.

76. If IGO < 0, go to 92

If IGO ≥ 0, go to 77

Compute U (x), U^2, and U^3 for the cubic weighting function.

77. U1 = (DX2)/(DX32)

78. U2 = (U1)(U1)

79. U3 = (U1)(U2)

Compute the cubic weighting function α .

80. A1 = (3.0)(U2) - (2.0)(U3)

Compute the derivative of the cubic weighting function.

81. A1P = (6.0)(U1 - U2)/(DX32)

Compute the interpolated value of the dependent variable.

82. Y = (1.0 - A1)(YB1) + (A1)(YB2)

Compute the derivative of the dependent variable.

83. YP = (1.0 - A1)(YPB1) - (A1P)(YB1 - YB2) + (A1)(YPB2)

Save value of the array subscript counter for the next entry into the dependent table.

84. IXIN = IX

Check whether the subscript counter limit has been reached.

85. If IXOGO = 0, return

If IXOGO ≠ 0, go to 86

Save the values of the parabola coefficients for the next entry into the same tables.

86. Do 87, I = 1, 6

87. CAR(I) = C(I)

Return to the calling subroutine.

88. Return

Set the value of the dependent variable to that computed from the left parabola.

89. Y = YB1

Set the derivative to that computed from the left parabola.

90. YP = YPB1

Save array position indicator and parabola coefficients.

91. Go to 84

Set the value of the dependent variable to that computed from the right parabola.

92. Y = YB2

Set the derivative to that computed from the right parabola.

93. YP = YPB2

Save array position indicator and parabola coefficients.

94. Go to 84

Table 16 gives the subroutine XNTERP nomenclature.

TABLE 16. SUBROUTINE XNTERP NOMENCLATURE

Symbol	Description	Units	Reference
A1	Cubic weighting function α (x)		80, 82, 83
A1P	Derivative of the cubic weighting function α' (x)		81, 83
C	Internal array of coefficients for right and left parabolas		DIM, 5, 41-43, 45, 49, 50, 54, 59, 60, 64-66, 71, 72, 74, 75, 87
CAR	Input or output array of parabola coefficients for input tables		CALL, DIM, 5, 87
DX1	Difference between value of independent variable x and the second table point less than x $(x - x_{i-1})$		70, 71
DX2	Difference between value of independent variable x and the first table point less than or equal to x $(x - x_i)$		23, 72, 74, 75, 77

TABLE 16. SUBROUTINE XNTERP NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
DX21	Difference between first t ble point less than x and the second table point less than x $(x_i - x_{i-1})$		55, 58, 60
DX31	Difference between first table point greater than x and second table point less than x $(x_{i+1} - x_{i-1})$		56, 59
DX32	Difference between first table point greater than x and first table point less than x $(x_{i+1} - x_i)$		24, 48, 50, 77, 81
DX42	Difference between second table point greater than x and first table point less than x $(x_{i+2} - x_i)$		46, 49
DY21	Difference between dependent table point corresponding to x_i and the table point corresponding to $x_{i-1} (y_i - y_{i-1})$		57, 58
DY32	Difference between dependent table point corresponding to x_{i+1} and the table point corresponding to x_i $(y_{i+1} - y_i)$		47, 48
DYOX21	Ratio of DY21 to DX21 $[(y_i - y_{i-1})/(x_i - x_{i-1})]$		58-60
DYOX32	Ratio of DY32 to DX32, $[(y_{i+1} - y_i)/(x_{i+1} - x_i)]$		48-50
I	Do-loop counter		4, 5, 19-22, 86, 87
I1	Subscript counter		20-22
IAR	Dimension of the independent and dependent tables		CALL, 2
IFIRST	First time entry indicator		7, 31, 32
IGO	Internal loop control indicator		28, 33, 35, 37, 53, 68, 73, 76

TABLE 16. SUBROUTINE XNTERP NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
IPOS	Input array position indicator		CALL, 3
IX	Array subscript counter		3, 9-13, 16-18, 20, 25, 27, 30, 40, 52, 63, 67, 84
IXIN	First time entry indicator or saved value of subscript counter from previous entry		CALL, 1, 84
IXMAX	Working maximum length of the input tables IAR - 1		2, 8, 14, 18, 30, 67
IXO	Internal limit for the subscript counter		1, 6, 8, 25, 40, 63
IXOGO	Subscript limit indicator		26, 39, 51, 61, 62, 85
U1	Ratio of DX2 to DX32 $[(x - x_i)/(x_{i+1} - x_i)]$		77-79, 81
U2	U1 squared		78-81
U3	U1 cubed		79, 80
X	Input value of the independent variable		CALL, 11, 14, 16, 23, 70
XAR	Input array of independent variables		CALL, DIM, 11, 14, 16, 21
XI	Array of table points surrounding the input value of the independent variable		DIM, 21, 23, 24, 46, 49, 55, 56, 59, 70
Y	Output value of the dependent variable corresponding to the independent variable		CALL, 82, 89, 92
YAR	Input array of dependent variables		CALL, DIM, 14, 22
YB1	Value of dependent variable on the left parabola		71, 82, 83, 89
YB2	Value of dependent variable on the right parabola		74, 82, 83, 92
YI	Array of dependent table points corresponding to the XI values		DIM, 22, 45, 47, 49, 54, 57, 59

TABLE 16. SUBROUTINE XNTERP NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
YP	Output value of the derivative of the dependent variable		CALL, 83, 90, 93
YPB1	Derivative of dependent variable from the left parabola		72, 83, 90
YPB2	Derivative of dependent variable from the right parabola		75, 83, 93

SUBROUTINE ZETAIT

Subroutine ZETAIT calculates the shape parameter $\zeta = (\delta / \Delta)^{1/n}$ and boundary layer thicknesses Δ, δ, and $\delta*$ at an axial point x for given values of θ and ϕ.

For $\zeta \geq 1$

$$\frac{\delta *}{\theta} = \left(\frac{\zeta^n - I_2 - I_3}{I_1} \right), \tag{4}$$

$$\delta = \frac{\theta}{nI_1} \quad , \tag{5}$$

where

$$\zeta = \left(\frac{\phi}{\theta} \frac{I_2}{I_2' + I_3'/\zeta} \right)^{\frac{1}{n+1}} \quad . \tag{6}$$

For $\zeta < 1$

$$\frac{\delta *}{\theta} = \left(\frac{1/n - I_6 - I_7}{I_4 + I_5} \right) \quad , \tag{7}$$

$$\delta = \frac{\theta}{n(I_4 + I_5)} \quad , \tag{8}$$

where

$$\zeta = \left(\frac{\phi}{\theta} \frac{I_4 + I_5}{I_1'} \right)^{\frac{1}{n+1}} \quad . \tag{9}$$

An iteration procedure is used to evaluate ζ.

1. An initial guess ζ_g is made.

2. For the value of ζ_g ($\geq$ 1 or < 1) appropriate functions of $\bar{\rho}/\rho_e$ have been defined in subroutine FIIF, and the proper integrals are evaluated using subroutine INTZET.

3. The term ζ is calculated by the appropriate formula above.

4. A relative error comparison is made. If

$$\left| \frac{\zeta - \zeta_g}{\zeta_g} \right| \leq \text{TOLZET} \quad ,$$

convergence is considered to be satisfactory. If not converged, a form of Wegstein's method (see step 42) is used to calculate a new guess for ζ_g, and steps 2 to 4 are repeated up to a maximum of 50 iterations.

The boundary layer thicknesses δ^*, δ, and Δ are then calculated according to equations (4), (5), (7), (8), and $\Delta = \zeta^n \delta$.

COMMON BLOCKS

COMMON blocks COFIIF, INPUT, INTER, OUTPUT, and SAVED are used.

TBL SUBROUTINES

Subroutine BARPRO calls ZETAIT.

ZETAIT calls INTZET.

FORTRAN SYSTEM ROUTINE

No FORTRAN library routines are used.

Built-in FORTRAN function ABS is used.

CALLING SEQUENCE

The subroutine calling sequence is

CALL ZETAIT

SOLUTION METHOD

Save the value of ϕ/θ

1. ERASE1 = (PHI)/(THETA)

Set indicator to evaluate integrals.

2. IFINT = 1

An iteration procedure is executed to evaluate $\zeta = (\Delta/\delta)^{1/n}$ in the following Do-loop, using an initial guess ζ_g.

3. Do 43 I = 1, 50

Save n = (MZETA) used in the exponent of the velocity equation

4. MMINT = MZETA

Save the wall enthalpy H_w = (A) defined in subroutine BARPRO.

5. AFINT = A

Approximation of ζ, initially using $\zeta_g = (\phi/\theta)^{1/8}_{initial}$ calculated in subroutine BARCON.

6. ZETAG = ZETA

Check whether the temperature thickness Δ exceeds velocity thickness δ.

7. If ZETA $\geq$ 1.0, go to 19

If ZETA < 1.0, go to 8

Save $H_o - H_w$ = (B) which is determined in BARPRO.

8. BFINT = B

Calculate the dynamic enthalpy multiplied by square of ζ with minus sign.

9. CFINT = (C((ZETA)^2

10. CALL INTZET (0.0, 1.0, ZI1P)

Divide B = $H_o - H_w$ by ζ.

11. BFINT = (B)/(ZETA)

Save dynamic enthalpy with minus sign $-\frac{U_e}{2}$.

12. CFINT = C

Call subroutine INTZET to obtain $I_4 = \int_0^{\zeta} \frac{\rho}{\rho_e} (1 - S)S^n dS$.

13. CALL INTZET(0.0, ZETA, ZI4)

Save stagnation enthalpy H_o, where $A = H_w$ and $B = H_o - H_w$.

14. AFINT = A + B

Set

15. BFINT = 0.0

16. CALL INTZET(ZETA, 1.0, ZI5)

Calculate ζ according to $\zeta = \left(\frac{\phi}{\theta} \frac{I_4 + I_5}{I_1'}\right)^{1/n+1}$

17. ZETA = ((ERASE1)(ZI4 + ZI5)/(ZI1P))^{RMZETA}

18. Go to 31

Save $(H_o - H_w)/\zeta$.

19. BFINT = (B)/(ZETA)

Save $\left(-\frac{U_e^2}{2}\right)$.

20. CFINT = C

Call subroutine INTZET to obtain $I_1 = \int_0^1 \frac{\rho}{\rho_e} S^n (1 - S)dS$.

21. CALL INTZET(0.0, 1.0, ZI1).

Save $H_o - H_w = (B)$.

22. BFINT = B

Compute $-\frac{U_e^2}{2}\zeta^2$.

23. CFINT = (C)(ZETA)²

Set $(1/\zeta)$.

24. ERASE2 = (1.0)/(ZETA)

Call subroutine INTZET to obtain $I_2' = \int_0^{1/\zeta} \frac{\rho}{\rho_e} W^n (1 - W)dW$.

25. CALL INTZET(0.0. ERASE2, ZI2P).

Save $(n - 1)$.

26. MMINT = MZETAM

Calculate $(H_w - U_e^2/2)$.

27. AFINT = A + C

Set

28. CFINT = 0.0

Call subroutine INTZET to obtain $I_3' = \int_{1/\zeta}^{1} \frac{\rho}{\rho_e} W^{n-1}(1 - W)dW$.

29. CALL INTZET(ERASE2, 1.0, ZI3P)

Calculate ζ by an appropriate formula $\zeta = \frac{\phi}{\theta}\left(\frac{I_2}{I_2' + \frac{I_3'}{\zeta}}\right)^{\frac{1}{n+1}}$.

30. ZETA = ((ERASE1)(ZI1)/((ZI2P + ZI3P)/(ZETA)))$^{\text{RMZETA}}$

Determine the relative error $(\zeta - \zeta_g)/\zeta_g$.

31. DZETA = (ZETA - ZETAG)/(ZETAG)

Check whether convergence is obtained.

32. If |DZETA| < TOLZET, go to 46

If |DZETA| ≥ TOLZET, go to 33

Check the Do-loop counter.

33. If I ≥ 2, go to 37

If I < 2, go to 34

Save ζ in the case of I = 1.

34. Z4 = ZETA

Save ζ_g.

35. Z2 = ZETAG

36. Go to 43

Save Z4 and Z2 when I ≥ 2.

37. Z3 = Z4

38. Z1 = Z2

Set

39. Z4 = ZETA

40. Z2 = ZETAG

41. Z5 = (Z4 - Z3)/(Z2 - Z1)

Approximate a new ζ_g using a form of Wegstein's method.

42. ZETA = (Z4 - (Z5)(Z2))/(1.0 - Z5)

43. Continue

Print out the error indication of shape parameter iteration failure.

44. WRITE X, ZME, THETA, PHI

45. WRITE Z1, Z2, ZETA, Z3, Z4

Proceed with calculation when convergence is obtained.

Set IFINT equal to two.

46. IFINT = 2

Save n in the exponent of velocity profile.

47. MMINT = MZETA

Save wall enthalpy H_w.

48. AFINT = A

Save $(H_o - H_w)/\zeta$.

49. BFINT = (B)/(ZETA)

Save $(-U_e^2/2)$.

50. CFINT = C

Calculate ζ^n.

51. ZETATM = $(\text{ZETA})^{\text{ZMZETA}}$

Check whether ζ exceeds one.

52. If ZETA $\geq$ 1.0, go to 60

    If ZETA < 1.0, go to 53

Call subroutine INTZET to obtain $I_6 = \int_0^{\zeta} \frac{\rho}{\rho_e} S^n dS$ in the case $\zeta < 1.0$.

53. CALL INTZET(0.0, ZETA, <u>ZI6</u>).

Save stagnation enthalpy H_o.

54. AFINT = A + B

Set

55. BFINT = 0.0

Call subroutine INTZET to obtain $I_7 = \int_{\zeta}^{1} \frac{\rho}{\rho_e} S^n dS$.

56. CALL INTZET(ZETA, 1.0, <u>ZI7</u>).

Calculate $(I_4 + I_5)$.

57. ERASE2 = ZI4 + ZI5

Compute $\delta^*/\theta = \dfrac{\frac{1}{n} - I_6 - I_7}{I_4 + I_5}$ in the case of $\zeta \leq 1$, i.e., $\delta \geq \Delta$.

58. DELSOT = (OOMZET - ZI6 - ZI7)/(ERASE2)

Obtain the velocity thickness $\delta = \dfrac{\theta}{n\ (I_4 + I_5)}$.

59. Go to 67

Call subroutine INTZET to obtain $I_2 = \int_0^1 \frac{\rho}{\rho_e} S^n dS$ in the case of $\zeta > 1$, i, e., $\zeta < \Delta$.

60. CALL INTZET(0.0, 1.0, ZI2).

Save $(n - 1)$.

61. MMINT = MZETAM

Save $(H_w - U_e^2/2)$.

62. AFINT = A + C

Set

63. CFINT = 0.0

Call subroutine INTZET to obtain $I_3 = \int_1^\zeta \frac{\rho}{\rho_e} S^{n-1} dS$.

64. CALL INTZET(1.0, ZETA, ZI3).

Obtain the velocity thickness $\delta = \frac{\theta}{nI_1}$.

65. DELTA = (THETA)/(ZMZETA)/(ZI1)

Compute $\frac{\delta*}{\theta} = \frac{\zeta^n/n - I_2 - I_3}{I_1}$.

66. DELSOT = ((ZETATM)/(ZMZETA) - ZI3 - ZI2)/(ZI1)

Obtain the temperature thickness $\Delta = \zeta^n \delta$.

67. BDELTA = (ZETATM)(DELTA)

Obtain the displacement thickness $\delta*$.

68. DELSTR = (THETA)(DELSOT)

69. Return

Subroutine ZETAIT nomenclature is given in Table 17.

TABLE 17. SUBROUTINE ZETAIT NOMENCLATURE

Symbol	Description	Units	Reference
A	Wall enthalpy H_w defined in subroutine BARPRO	ft^2/sec^2	/SAVED/, 5, 14, 27, 48, 54, 62
AFINT	Saved value of A	ft^2/sec^2	/COFIIF/, 5, 14, 27, 48, 54, 62
B	Stagnation enthalpy minus wall enthalpy calculated in BARPRO	ft^2/sec^2	/SAVED/, 8, 11, 14, 19, 22, 49, 54
BDELTA	Temperature thickness Δ	ft	/OUTPUT/, 67
BFINT	Saved value of B or B/ZETA	ft^2/sec^2	/COFIIF/, 8, 11, 15, 19, 22, 49, 55
C	Dynamic enthalpy with minus sign $(-U_e^2/2)$ calculated in BARPRO	ft^2/sec^2	/SAVED/, 9, 12, 20, 23, 27, 50, 62
CFINT	Saved value of zero $(-U_e^2/2)$ or $(-\zeta^2 U_e^2/2)$	ft^2/sec^2	/COFIIF/, 9, 12, 20, 23, 28, 50, 63
DELSOT	Displacement thickness divided by momentum thickness δ^*/θ	—	/OUTPUT/, 58, 66, 68
DELSTR	Displacement thickness δ^*	ft	/OUTPUT/, 68
DELTA	Velocity thickness δ	ft	/OUTPUT/, 65, 67
DZETA	Relative error $(\zeta - \zeta_g)/\zeta_g$	—	31, 32
ERASE1	Energy thickness divided by momentum thickness ϕ/θ	—	1, 17, 30
ERASE2	Temporary storage variable	—	24, 25, 29, 57, 58
I	Do-loop counter	—	3, 33

TABLE 17. SUBROUTINE ZETAIT NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
IFINT	Switch indicator for calculating integrands in subroutine FIIF	—	/COFIIF/, 2, 46
MMINT	Exponent for integral evaluation	—	/COFIIF/, 4, 26, 47, 61
MZETA	n in the exponent of the velocity profile	—	/INPUT/, 4, 47
MZETAM	n - 1	—	/INTER/, 26, 61
OOMZET	1/n	—	/INTER/, 58
PHI	Energy thickness ϕ	ft	/OUTPUT/, 1, 44
RMZETA	1/(n + 1)	—	/INTER/, 17, 30
THETA	Momentum thickness θ	ft	/OUTPUT/, 1, 44, 65, 68
TOLZET	Tolerance for ζ iteration	—	/INPUT/, 32
X	Axial distance	ft	/OUTPUT/, 44
Z1	Previous value of ζ_g	—	/OUTPUT/, 38, 41, 45
Z2	Saved value of ζ_g	—	/OUTPUT/, 35, 38, 40, 41, 42, 45
Z3	Previous value of ζ	—	/OUTPUT/, 37, 41, 45
Z4	Saved value of ζ	—	/OUTPUT/, 34, 37, 39, 41, 42, 45
Z5	(Z4 - Z3)/(Z2 - Z1)	—	/OUTPUT/, 41, 42
ZETA	Shape parameter $\zeta = (\delta/\Delta)^{1/n}$	—	/OUTPUT/, 6, 7, 9, 11, 13, 16, 17, 19, 23, 24, 30, 31, 34, 39, 42, 45, 49, 51-53, 56, 64
ZETAG	Approximation for ζ	—	6, 31, 35, 40
ZETATM	ζ^n	—	/INTER/, 51, 66, 67
ZI1	$I_1 = \int_0^1 \frac{\rho}{\rho_e} S^n(1 - S)dS$	—	/SAVED/, 21, 30, 65, 66

TABLE 17. SUBROUTINE ZETAIT NOMENCLATURE (Continued)

Symbol	Description	Units	Reference
ZI2	$I_2 = \int_0^1 \frac{\rho}{\rho_e} S^n dS$	—	/SAVED/, 60, 66
ZI3	$I_3 = \int_0^\zeta \frac{\rho}{\rho_e} S^{n-1} dS$	—	/SAVED/, 64, 66
ZI4	$I_4 = \int_0^\zeta \frac{\rho}{\rho_e} S^n(1 - S) dS$	—	/SAVED/, 13, 17, 57
ZI5	$I_5 = \int_\zeta^1 \frac{\rho}{\rho_e} S^n(1 - S) dS$	—	/SAVED/, 16, 17, 57
ZI6	$I_6 = \int_0^\zeta \frac{\rho}{\rho_e} S^n dS$	—	/SAVED/, 53, 58
ZI7	$I_7 = \int_\zeta^1 \frac{\rho}{\rho_e} S^n dS$	—	/SAVED/, 56, 58
ZI1P	$I_1' = \int_0^1 \frac{\rho}{\rho_e} W^n(1 - W) dW$	—	/SAVED/, 10, 17
ZI2P	$I_2' = \int_0^{1/\zeta} \frac{\rho}{\rho_e} W^n(1 - W) dW$	—	/SAVED/, 25, 30

TABLE 17. SUBROUTINE ZETAIT NOMENCLATURE (Concluded)

Symbol	Description	Units	Reference
ZI3P	$I_3' = \int_{1/\zeta}^{1} \frac{\rho}{\rho_c} W^{n-1}(1 - W)\,dW$	—	/SAVED/, 29, 30
ZME	Mach number in free stream	—	/OUTPUT/, 44
ZMZETA	Real value of n	—	/INTER/, 51

APPENDIX. DERIVATION OF EQUATIONS [1]

This appendix is a complement to Reference 1 and covers the derivation of important equations in detail from section 2.2 to 2.9. The equation numbers are identical to the ones in Reference 1.

Derivation of Equation (7)(Displacement Thickness)

The potential nozzle flow can be considered identical to the flow in a real nozzle from the nozzle axis to the n^{th} streamline close to the nozzle wall. In addition the flow field would extend a distance δ'_p from the n^{th} streamline to a fictitious potential wall if the flow would possess the core flow properties. The real flow, however, extends a distance δ'_r from the n^{th} streamline because of the existence of a boundary layer. The mass flow rate $\dot{m}$ through the potential flow-field extension δ'_p must be equal to the flow-field extension δ'_r considering real-flow behavior.

$$\dot{m}_p = \dot{m}_r \quad .$$

This postulation provides the key relating the real-flow condition to a potential-flow equivalent and makes it possible to determine the displacement thickness δ^* which identifies the dislocation of the potential wall to account for the real-flow behavior. Introducing equations (1) and (4) into the previous equation results in

$$2\pi r \;\rho_\infty U_\infty \delta'_p = 2\pi r \int_0^{\delta'_r} \rho u \, dy \quad ,$$

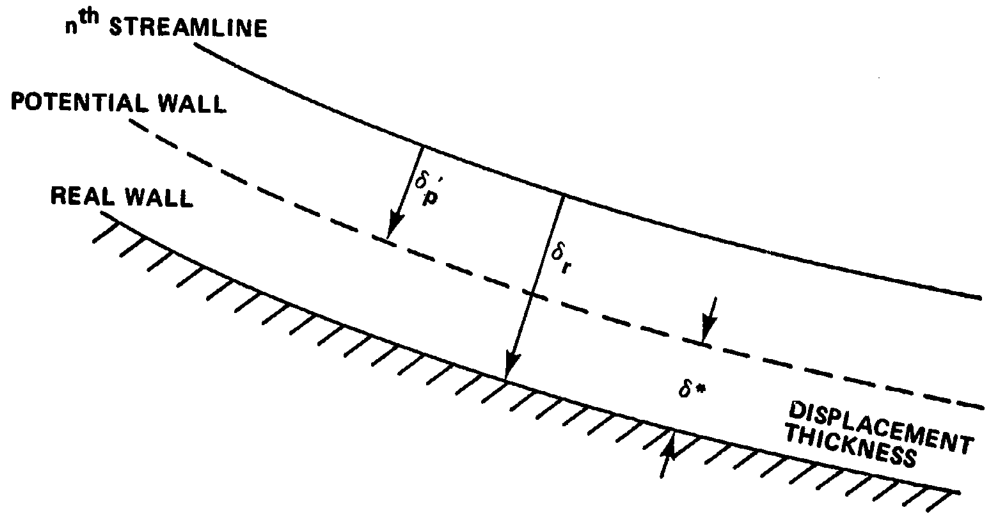
$$\delta'_p = \frac{1}{\rho_\infty U_\infty} \int_0^{\delta'_r} \rho u \, dy \quad ,$$

$$\delta'_r - \delta'_p = -\int_0^{\delta'_r} \frac{\rho u}{\rho_\infty U_\infty}\, dy + \delta'_r$$

$$= \int_0^{\delta'_r} dy - \int_0^{\delta'_r} \frac{\rho u}{\rho_\infty U_\infty}\, dy \quad ,$$

so that one obtains the displacement thickness $\delta*$

$$\delta^* = \delta'_r - \delta'_p = \int_0^{\delta'_r} \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy \quad .$$



Derivation of Equation (10) (Momentum Thickness)

Using equations (2) and (5), the deficiency of momentum flux in the real flow as compared to the potential flow can be represented by

$$\dot{M}_p - \dot{M}_r = 2\pi r \; \rho_\infty U_\infty^2 \; \delta'_p - 2\pi r \int_o^{\delta'_r} \rho u^2 \, dy$$

$$= 2\pi r \; \rho_\infty U_\infty^2 \left(\delta'_p - \int_o^{\delta'_r} \frac{\rho u^2}{\rho_\infty U_\infty^2} \, dy \right) \quad .$$

Introducing equation (7) yields

$$\delta'_p = \delta'_r - \int_o^{\delta'_r} \left(1 - \frac{\rho u}{\rho_\infty U_\infty} \right) dy \quad ,$$

$$\dot{M}_p - \dot{M}_r = 2\pi r \; \rho_\infty U_\infty^2 \left[\delta'_r - \int_o^{\delta'_r} \left(1 - \frac{\rho u}{\rho_\infty U_\infty} \right) dy - \int_o^{\delta'_r} \frac{\rho u^2}{\rho_\infty U_\infty^2} \, dy \right]$$

$$= 2\pi r \; \rho_\infty U_\infty^2 \int_o^{\delta'_r} \frac{\rho u}{\rho_\infty U_\infty} \left(1 - \frac{u}{U_\infty} \right) dy \quad ,$$

where the integral represents the momentum thickness θ.

Derivation of Equation (13) (Energy Thickness)

Applying equations (3) and (6) yields the deficiency of enthalpy flux of the real flow as compared to the potential flow.

$$\dot{E}_p - \dot{E}_r = 2\pi r \; \rho_\infty U_\infty \; (H_o - H_w) \; \delta'_p - 2\pi r \int_o^{\delta'_r} \rho u (h_o - H_w) \; dy$$

$$= 2\pi r \; \rho_\infty U_\infty \; (H_o - H_w) \left[\delta'_p - \int_o^{\delta'_r} \frac{\rho u}{\rho_\infty U_\infty} \left(\frac{h_o - H_w}{H_o - H_w} \right) dy \right] \; .$$

Using equation (7) results in

$$\delta'_p = \delta'_r - \int_o^{\delta'_r} \left(1 - \frac{\rho u}{\rho_\infty U_\infty} \right) dy \quad ,$$

$$\dot{E}_p - \dot{E}_r = 2\pi r \; \rho_\infty U_\infty \; (H_o - H_w) \left[\delta'_r - \int_o^{\delta'_r} \left(1 - \frac{\rho u}{\rho_\infty U_\infty} \right) dy - \int_o^{\delta'_r} \frac{\rho u}{\rho_\infty U_\infty} \left(\frac{h_o - H_w}{H_o - H_w} \right) dy \right]$$

$$= 2\pi r \; \rho_\infty U_\infty \; (H_o - H_w) \int_o^{\delta'_r} \frac{\rho u}{\rho_\infty U_\infty} \left(1 - \frac{h_o - H_w}{H_o - H_w} \right) dy \quad .$$

The integral represents the energy thickness ϕ.

Definition of Boundary Layer Thicknesses

<u>Displacement Thickness</u>

$$\delta^* = \delta'_r - \delta'_p = \int_o^{\delta'_r} \left(1 - \frac{\rho u}{\rho_\infty U_\infty} \right) dy \quad .$$

Momentum Thickness

$$\theta = \int_0^{\delta'_r} \frac{\rho u}{\rho_\infty U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy \quad .$$

Energy Thickness

$$\phi = \int_0^{\delta'_r} \frac{\rho u}{\rho_\infty U_\infty}\left(1 - \frac{u}{U_\infty}\right)dy \quad .$$

Derivation of Equation (16) (Skin Friction Coefficient)

The friction coefficient is defined by the force in direction along the wall per unit area $\frac{F}{A} = \tau_w$ made dimensionless by the dynamic pressure

$$C_f = \frac{2\tau_w}{\rho_\infty U_\infty^2} \quad .$$

Equation (17) (Stanton Number)

This dimensionless parameter can be interpreted as the ratio of the heat flow density emerging from a wall for each degree of temperature difference between the wall and the fluid and the heat flow density convected by the flowing medium for each degree of temperature difference.

$$St = \frac{Nu}{RePr} \quad ,$$

where

$$\mathrm{Nu} = \frac{h_g \ell}{\lambda} \quad ,$$

$$\mathrm{Re} = \frac{U\ell}{\nu} \quad ,$$

$$\mathrm{Pr} = \frac{\mu C_p}{\lambda} \quad ,$$

h_g = specific heat transfer coefficient (contains all influences caused by properties and flow conditions),

λ = heat conduction coefficient (is a material constant),

ℓ = characteristic length

Therefore, the Stanton number can be expressed by

$$\mathrm{St} = \frac{h_g}{\lambda U_\infty} \frac{\lambda}{C_p \rho_\infty} \quad ,$$

$$= \frac{h_g}{C_p \rho_\infty U_\infty} \quad ,$$

C_p = specific heat at constant pressure,

ρ_∞ = density.

5

The total heat flow between a surface and a fluid is

$$Q = h_g A (T_F - T_w) \quad ,$$

$$\frac{Q}{A} = q_w = h_g (T_F - T_w) \quad ,$$

$$h_g = \frac{q_w}{T_F - T_w} \quad .$$

Substituting this expression in the Stanton number yields

$$St = \frac{q_w}{C_p \rho_\infty U_\infty (T_F - T_w)} \quad ,$$

or since

$$h = C_p T \text{ for } C_p = \text{constant},$$

$$St = \frac{q_w}{\rho_\infty U_\infty (H_{aw} - H_w)} \quad .$$

Since T_F is the temperature which a specific location on the surface assumes when the convective heat transfer is zero, it can be considered as the recovery temperature. The corresponding enthalpy expression is the adiabatic wall enthalpy.

Equation (18) (Adiabatic Wall Enthalpy)

When a gas flows past an insulated or adiabatic surface, the temperature at the surface will rise above the temperature of the free-stream gas but will not quite reach the total temperature. The temperature at an adiabatic surface T_{aw} is called the adiabatic wall temperature.

In practice it has been found convenient to relate T_{aw} and T_∞ by the recovery factor R_T, which is a measure of the fraction of the free-stream, dynamic-temperature rise recovered at the wall. The term R_T is defined as

$$R_T = \frac{T_{aw} - T_\infty}{T_o - T_\infty} = \frac{T_{aw} - T_\infty}{\dfrac{U_\infty^2}{2 c_p J g}} = \frac{2 J}{(\gamma - 1) M_\infty^2} \left(\frac{T_{aw}}{T_\infty} - 1\right) .$$

The derivation of this equation is shown below.

$$\frac{T_{aw} - T_\infty}{\dfrac{U_\infty^2}{2 c_p g J}} = \frac{(T_{aw} - T_\infty) 2 c_p g J}{U_\infty^2 \dfrac{a^2}{a^2}} ,$$

where speed of sound is $a = \sqrt{\gamma g R T_\infty}$ Mach number $M_{a_\infty} = \dfrac{U_\infty}{a}$.

Therefore,

$$\frac{T_{aw} - T_\infty}{\dfrac{U_\infty^2}{2 c_p g J}} = \frac{(T_{aw} - T_\infty) 2 c_p g J}{M_{a_\infty}^2 \gamma g R T_\infty} . . .$$

Introducing the thermodynamic relationship

$$R = C_p - C_v$$

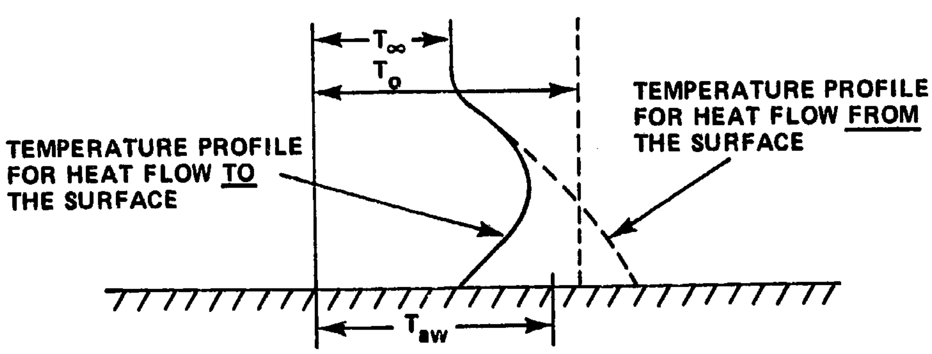
results in

$$\frac{T_{aw} - T_\infty}{\dfrac{U_\infty^2}{2 c_p g J}} = \left(\frac{T_{aw} - T_\infty}{T_\infty}\right) \frac{2 c_p J}{\gamma (C_p - C_v) M_{a_\infty}^2}$$

$$= \left(\frac{T_{aw} - T_\infty}{T_\infty}\right) \frac{2 J}{\gamma \left(1 - \dfrac{C_v}{C_p}\right) M_{a_\infty}^2}$$

$$= \left(\frac{T_{aw} - T_\infty}{T_\infty}\right) \frac{2 J}{\gamma \left(1 - \dfrac{1}{\gamma}\right) M_{a_\infty}^2}$$

$$= \left(\frac{T_{aw} - T_\infty}{T_\infty}\right) \frac{2 J}{(\gamma - 1) M_{a_\infty}^2} \quad .$$



For turbulent boundary layers the recovery factor R_T can be approximated by the relation

$$R_T \simeq \sqrt[3]{Pr} \ .$$

The ratio of the adiabatic wall enthalpy and the stagnation enthalpy is defined as

$$\frac{H_{aw}}{H_o} = \frac{h + R_T \frac{U_\infty^2}{2}}{h + \frac{U_\infty^2}{2}} \ ,$$

where R_T = recovery factor [the fraction of dynamic enthalpy (or temperature) which is recovered],

$$\frac{U_\infty^2}{2} = C_p (T_o - T_\infty) \equiv \text{dynamic enthalpy},$$

$$h \equiv \text{free-stream static enthalpy.}$$

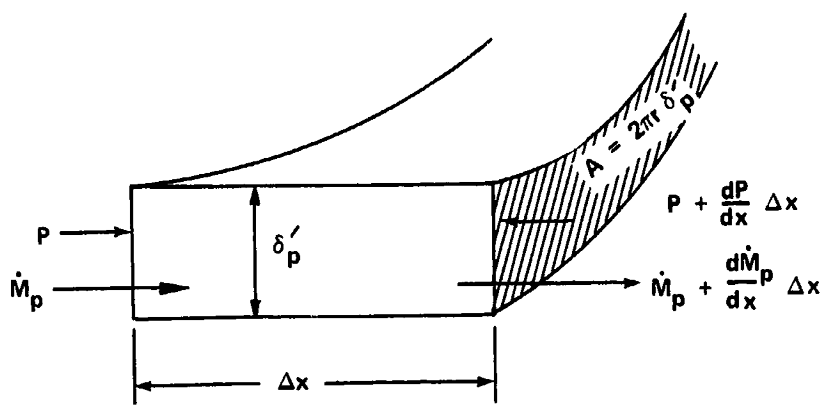
Rearranging the previous equation yields

$$\frac{H_{aw}}{H_o} = \frac{1 + R_T \frac{U_\infty^2}{2h}}{1 + \frac{U_\infty^2}{2h}} \ ,$$

and introducing the recovery factor $R_T = Pr^{1/3}$ for the turbulent boundary layer results in

$$\frac{H_{aw}}{H_o} = \frac{1 + Pr^{1/3} \frac{U_\infty^2}{2h}}{1 + \frac{U_\infty^2}{2h}} \quad .$$

Derivation of Equation (21)[3]



Since the gradient of momentum flux for the potential stream tube of area $A = 2\pi r\delta'_p$ should be balanced by the pressure gradient acting over the same area A , we obtain the following equation.

$$\left(\dot{M}_p + \frac{d\dot{M}_p}{dx} \Delta x \right) - \dot{M}_p = \left[P - \left(P + \frac{dP}{dx} \Delta x \right) \right] 2\pi r \, \delta'_p \quad .$$

Thus

$$\frac{d\dot{M}_p}{dx} = - 2\pi r \, \delta'_p \frac{dP}{dx} \quad .$$

[3]From now on the subscript ∞ is omitted for convenience.

Derivation of Equation (23)

For the real flow the viscosity effect appears. Therefore the first equation in the previous section is modified as

$$\left(\dot{M}_r + \frac{d\dot{M}_r}{dx}\,\Delta x\right) - \dot{M}_r = \left[P - \left(P + \frac{dP}{dx}\,\Delta x\right)\right] 2\pi r\, \delta'_r - 2\pi r\, \Delta x\, \tau_w \;.$$

Thus

$$\frac{d\dot{M}_r}{dx} = -2\pi r\, \delta'_r\, \frac{dP}{dx} - 2\pi r\, \tau_w \;.$$

Derivation of Equation (24)

Beginning with equations (22) and (23)

$$\frac{d}{dx}\left(\dot{M}_r + 2\pi r\, \rho U^2\, \theta\right) = -\,2\pi r\,(\delta'_r - \delta^*)\,\frac{dP}{dx}$$

$$\frac{d}{dx}\left(\dot{M}_r\right) = -\,2\pi r \tau_w - 2\pi r \delta'_r\,\frac{dP}{dx}$$

and subtracting these equations from each other yield

$$\frac{d}{dx}\left(2\pi r\, \rho U^2\, \theta\right) = 2\pi r \tau_w + 2\pi r \delta^*\,\frac{dP}{dx} \;.$$

After factoring out 2π, one obtains

$$\frac{d}{dx}\left(r \rho U^2 \theta\right) = r\tau_w + r\delta^*\,\frac{dP}{dx} \;.$$

Application of the Euler equation

$$\frac{dP}{dx} = -\rho U \frac{dU}{dx}$$

yields

$$\frac{d}{dx}\left(r\rho U^2\theta\right) = r\tau_w - r\delta^*\rho U \frac{dU}{dx} \quad .$$

Derivation of Equation (25)

The left-hand side of equation (24) is derived by parts, and τ_w is substituted by equation (16).

$$r\rho U^2 \frac{d\theta}{dx} + r\rho\theta \frac{dU^2}{dx} + rU^2\theta \frac{d\rho}{dx} + \rho U^2\theta \frac{dr}{dx} = \frac{r\rho U^2 C_f}{2} - r\delta^*\rho U \frac{dU}{dx} \quad ,$$

$$\frac{d\theta}{dx} = \frac{\frac{r\rho U^2 C_f}{2} - r\delta^*\rho U \frac{dU}{dx} - r\rho\theta \frac{dU^2}{dx} - rU^2\theta \frac{d\rho}{dx} - \rho U^2\,\theta \frac{dr}{dx}}{r\rho U^2} \quad ,$$

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \frac{\delta^*}{U}\frac{dU}{dx} - \frac{\theta}{U^2}\frac{dU^2}{dx} - \frac{\theta}{\rho}\frac{d\rho}{dx} - \frac{\theta}{r}\frac{dr}{dx} \quad ,$$

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \theta\left(\frac{\frac{\delta^*}{\theta}}{U}\frac{dU}{dx} + \frac{1}{U^2}\frac{dU^2}{dU}\frac{dU}{dx} + \frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{r}\frac{dr}{dx}\right) \quad ,$$

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \theta\left(\frac{\frac{\delta^*}{\theta}}{U}\frac{dU}{dx} + \frac{2U}{U^2}\frac{dU}{dx} + \frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{r}\frac{dr}{dx}\right) \quad ,$$

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \theta\left(\frac{\frac{\delta^*}{\theta}}{U}\frac{dU}{dx} + \frac{2}{U}\frac{dU}{dx} + \frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{r}\frac{dr}{dx}\right) \quad ,$$

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \theta \left(\frac{\frac{\delta*}{\theta}}{U} \frac{dU}{dx} + \frac{1}{U} \frac{dU}{dx} + \frac{1}{U} \frac{dU}{dx} + \frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{r} \frac{dr}{dx} \right) ,$$

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \theta \left(\frac{\frac{\delta*}{\theta} + 1}{U} \frac{dU}{dx} + \frac{1}{U} \frac{dU}{dx} + \frac{1}{\rho} \frac{d\rho}{dx} + \frac{1}{r} \frac{dr}{dx} \right) .$$

Consider only the two terms

$$\frac{1}{U} \frac{dU}{dx} + \frac{1}{\rho} \frac{d\rho}{dx} .$$

According to the mathematical definition

$$\frac{d \ln x}{dx} = \frac{1}{x} ,$$

one can write

$$\frac{1}{U} \frac{dU}{dx} + \frac{1}{\rho} \frac{d\rho}{dx} = \frac{d \ln U}{dU} \frac{dU}{dx} + \frac{d \ln \rho}{d\rho} \frac{d\rho}{dx}$$

$$= \frac{d \ln U}{dx} + \frac{d \ln \rho}{dx}$$

$$= \frac{d}{dx} (\ln U + \ln \rho)$$

$$= \frac{d}{dx} [\ln (U\rho)]$$

$$= \frac{d [\ln (U\rho)]}{dU\rho} \frac{dU\rho}{dx}$$

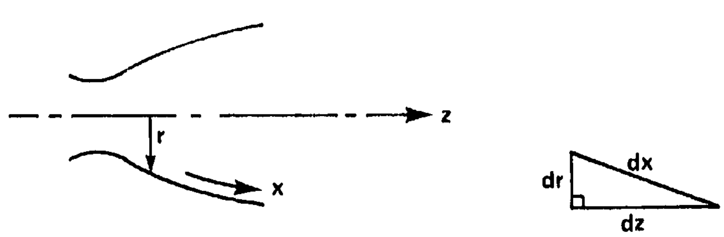
$$= \frac{1}{\rho U} \frac{d\rho U}{dx} .$$

Substituting this into the previous equation results in

$$\frac{d\theta}{dx} = \frac{C_f}{2} - \theta\left(\frac{\frac{\delta^*}{\theta} + 1}{U}\frac{dU}{dx} + \frac{1}{\rho U}\frac{d\rho U}{dx} + \frac{1}{r}\frac{dr}{dx}\right) .$$

Derivation of Equations (26) and (27)

Geometry



$$(dx)^2 = (dr)^2 + (dz)^2$$

$$dx = [(dz)^2 + (dr)^2]^{1/2}$$

$$\frac{dx}{dz} = \left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2} \rightarrow dx = dz\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2} .$$

Substituting this into equation (25) yields

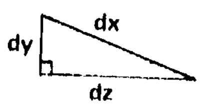
$$\frac{d\theta}{dz\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2}} = \frac{C_f}{2} - \theta\left\{\frac{1 + \frac{\delta^*}{\theta}}{U} \cdot \frac{dU}{dz\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2}} + \frac{1}{\rho U}\frac{d\rho U}{dz\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2}} + \frac{1}{r}\frac{dr}{dz\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2}}\right\}$$

$$\frac{d\theta}{dz} = \frac{C_f}{2}\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2} - \theta\left(\frac{1 + \frac{\delta^*}{\theta}}{U}\frac{dU}{dz} + \frac{1}{\rho U}\frac{d\rho U}{dz} + \frac{1}{r}\frac{dr}{dz}\right) . \quad (26)$$

This equation is applicable for axisymmetric flow.

For two-dimensional flow

$$r \to \infty \quad \text{and} \quad \frac{1}{r} \to 0 \quad ,$$



$$dx = \left[(dy)^2 + (dz)^2\right]^{1/2} \quad ,$$

$$\frac{dx}{dz} = \left[\left(\frac{dy}{dz}\right)^2 + 1\right]^{1/2} \quad .$$

Substituting this into equation (26) yields

$$\frac{d\theta}{dz}\,\frac{1}{\left[1 + \left(\frac{dy}{dz}\right)^2\right]^{1/2}} = \frac{C_f}{2} - \theta \left\{ \frac{1 + \frac{\delta^*}{\theta}}{U\left[1 + \left(\frac{dy}{dz}\right)^2\right]^{1/2}}\,\frac{dU}{dz} + \frac{1}{\rho U\left[1 + \left(\frac{dy}{dz}\right)^2\right]^{1/2}}\,\frac{d\rho U}{dz} \right\} ,$$

$$\frac{d\theta}{dz} = \frac{C_f}{2}\left[1 + \left(\frac{dy}{dz}\right)^2\right]^{1/2} - \theta\left(\frac{1 + \frac{\delta^*}{\theta}}{U}\,\frac{dU}{dz} + \frac{1}{\rho U}\,\frac{d\rho U}{dz}\right) \quad .$$

Derivation of Equation (30)

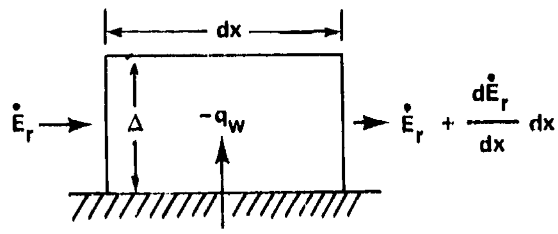
The change in energy along a streamline for real flow is equal to the heat transferred between the fluid and the wall

$$d\dot{E}_r = 2\pi r q_w \, dx$$

$$\frac{d\dot{E}_r}{dx} = 2\pi r q_w \quad .$$

(The sign of the right-hand term depends on the direction of q_w.)

Heat transfer from the wall to the fluid is indicated by a negative sign. Another way of the derivation is given below.



$$\dot{E}_r - \left(\dot{E}_r + \frac{d\dot{E}_r}{dx}\,dx \right) = -2\pi r q_w \, dx$$

$$\therefore \quad \frac{d\dot{E}_r}{dx} = 2\pi r q_w \qquad (30)$$

Derivation of Equations (32) through (34)

Beginning with equation (31) and differentiating by parts yield

$$\frac{d}{dx}\left[r\rho U \, (H_o - H_w)\,\phi\right] = r q_w \quad ,$$

$$r\rho U \, (H_o - H_w) \frac{d\phi}{dx} + r\rho U\phi \frac{d(H_o - H_w)}{dx} + r\rho(H_o - H_w)\phi \frac{dU}{dx}$$

$$+ \, rU(H_o - H_w)\phi \frac{dp}{dx} + \rho U(H_o - H_w)\phi \frac{dr}{dx} = r q_w \quad .$$

Substituting q_w by equation (17)

$$q_w = C_H \rho U (H_{aw} - H_w)$$

and solving for $\frac{d\phi}{dx}$ yield

$$\frac{d\phi}{dx} = \frac{r C_H \rho U (H_{aw} - H_w) - r\rho U\phi \frac{d(H_o - H_w)}{dx} - r\rho(H_o - H_w)\phi \frac{dU}{dx} - rU(H_o - H_w)\phi \frac{d\rho}{dx} - \rho U(H_o - H_w)\phi \frac{dr}{dx}}{r\rho U (H_o - H_w)}$$

$$\frac{d\phi}{dx} = C_H \frac{H_{aw} - H_w}{H_o - H_w} - \phi \left[\frac{1}{H_o - H_w} \frac{d(H_o - H_w)}{dx} + \frac{1}{U}\frac{dU}{dx} + \frac{1}{\rho}\frac{d\rho}{dx} + \frac{1}{r}\frac{dr}{dx} \right]$$

$$\frac{d(H_o - H_w)}{dx} = \frac{dH_o}{dx} - \frac{dH_w}{dx} \quad .$$

Since H_o is assumed to be constant,

$$\frac{dH_o}{dx} = 0 \quad ;$$

$$\frac{1}{U}\frac{dU}{dx} + \frac{1}{\rho}\frac{d\rho}{dx} = \frac{1}{\rho U}\frac{d\rho U}{dx} \quad .$$ [See derivation of equation (25).]

Substituting these terms into the previous equation yields

$$\frac{d\phi}{dx} = C_H \frac{H_{aw} - H_w}{H_o - H_w} - \phi \left(- \frac{1}{H_o - H_w} \frac{dH_w}{dx} + \frac{1}{\rho U}\frac{d\rho U}{dx} + \frac{1}{r}\frac{dr}{dx} \right) \quad . \quad (32)$$

For axisymmetric flow

$$dx = (dr^2 + dz^2)^{1/2} \quad ,$$

$$\frac{dx}{dz} = \left[\left(\frac{dr}{dz}\right)^2 + 1 \right]^{1/2} \quad ,$$

$$dx = \left[\left(\frac{dr}{dz}\right)^2 + 1 \right]^{1/2} dz \quad .$$

This equation substituted into the previous equation yields

$$\frac{d\phi}{dz} = C_H \left(\frac{H_{aw} - H_w}{H_o - H_w} \right) \left[1 + \left(\frac{dr}{dz}\right)^2 \right]^{1/2} - \phi \left(- \frac{1}{H_o - H_w} \frac{dH_w}{dx} + \frac{1}{\rho U}\frac{d\rho U}{dz} + \frac{1}{r}\frac{dr}{dz} \right) \quad . \quad (33)$$

For two-dimensional planar flow $r \rightarrow \infty$,

$$\frac{1}{r} = 0 \quad ,$$

$$dx = (dy^2 + dx^2)^{1/2} \quad ,$$

$$\frac{dx}{dz} = \left[\left(\frac{dy^2}{dz} \right) + 1 \right]^{1/2} \quad ,$$

$$dx = \left[\left(\frac{dy}{dz} \right) + 1 \right]^{1/2} dz \quad .$$

This equation substituted into equation (32) yields

$$\frac{d\phi}{dz} = C_H \left(\frac{H_{aw} - H_w}{H_o - H_w} \right) \left[1 + \left(\frac{dy}{dz} \right)^2 \right]^{1/2} - \phi \left(- \frac{1}{H_o - H_w} \frac{dH_w}{dz} + \frac{1}{\rho U} \frac{d\rho U}{dz} \right) . \qquad (34)$$

Derivation of Equations (36) through (39)

The skin friction coefficient in a nozzle is assumed to be the same as for a flat plate at the same free-stream conditions ρ , U , μ , T , M , the same wall temperature T_w , and the same momentum thickness θ . Unfortunately, even this drastic assumption does not permit a completely reliable evaluation of C_f since only the adiabatic skin friction coefficient C_{fa} , obtained when $T_w = T_{wa}$, is known accurately. The relationship between C_f and C_{fa} for severely cooled turbulent boundary layers, when gas properties vary greatly between the free stream and the wall, is uncertain.

The most accurate method for computing the adiabatic skin friction coefficient C_{fa} is judged to be that of Coles [11]. He employs a transformation method which gives C_{fa} as a function of Mach number M and the Reynolds number R_θ based upon the momentum thickness as defined by

$$R_\theta = \frac{\rho U\theta}{\mu} = \frac{U\theta}{\nu} \qquad (\mu = \rho\nu) \quad .$$

For large temperature differences the properties of gases cannot be assumed constant. They depend very much on the temperature. Using the Prandtl number, which contains basically only the property parameters of a fluid,

$$Pr = \frac{\nu}{a'} \quad , \qquad a' = \frac{\lambda}{C_p \rho} \quad ,$$

$$Pr = \frac{\nu C_p \rho}{\lambda} = \frac{\mu C_p}{\lambda} \quad .$$

Since the dynamic viscosity μ and the conductivity λ vary considerably faster with temperature, one can rearrange

$$\frac{\mu}{\lambda} = \frac{Pr}{C_p} \quad .$$

This equation shows that for a constant Prandtl number Pr and constant specific heat C_p, the heat conductivity λ varies proportionally to the viscosity μ. Therefore the law of temperature dependence has to be prescribed for one property only

$$\mu \sim T^{\omega} \quad .$$

For ideal gases Reference 12 gives an equation for the dynamic viscosity

$$\mu = K\sqrt{\mathfrak{M}T}$$

K is a constant
$\mathfrak{M}$ = mclecular weight

or

$$\mu \sim T^m \quad .$$

Assuming a reference condition (0) the above equations yield

$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^m \quad . \tag{36}$$

The exponent m is dependent on the temperature and varies between 1.0 and 0.5.

It is interesting to note that for $m = 1$ the dependence of the friction factor on the Mach number and the ratio of T_{aw} (wall temperature) and T_{∞} (stream temperature) vanishes. It may therefore be expected that property values introduced at a proper reference temperature situated somewhere between the temperature extremes, encountered within the boundary layer, will cause the variation of the friction factor with M and T_{aw}/T_{∞} to vanish. It has been shown that such a reference temperature exists. Using the latter relationship

$$\frac{\mu}{\mu_o} = \left(\frac{T}{T_o}\right)^m ,$$

the equation for R_{θ} can be rewritten

$$R_{\theta} = \frac{\rho U \theta}{\mu_o} \left(\frac{T_o}{T}\right)^m .$$

In this equation the term $\left(\frac{T_o}{T}\right)$ and (ρU) can be replaced by relationships as derived below.

Using the equation derived in step 7 yields

$$\frac{T_{aw} - T_{\infty}}{T_{o\infty} - T_{\infty}} = \frac{(T_{aw} - T_{\infty})}{T_{\infty}} \frac{2J}{(\gamma - 1)\ M^2} ,$$

$$1 = \left(\frac{T_{o\infty}}{T_{\infty}} - 1\right) \frac{2J}{(\gamma - 1)\ M^2} ,$$

$$\frac{T_{o\infty}}{T_{\infty}} = 1 + \frac{\gamma - 1}{2J} M^2 .$$

The expression ρU is derived below. Starting with the speed of sound relationship

$$a^2 = \gamma R T$$

and the ideal gas equation

$$\frac{p}{\rho} = RT$$

results in

$$a^2 = \frac{\gamma p}{\rho} \quad .$$

Applying the Mach number definition

$$M = \frac{U}{a} \quad \text{or} \quad M^2 = \frac{U^2}{a^2} \quad ,$$

$$M^2 = \frac{U^2\rho}{\gamma p}$$

(multiply and divide by ρ)

$$M^2 = \frac{U^2\rho^2}{\gamma\rho p} \quad ,$$

$$\rho^2 U^2 = \gamma\rho p M^2 \quad ,$$

$$\rho U = M \ (\gamma\rho p)^{1/2} \quad .$$

Modification of the term in parentheses results in

$$\gamma\rho p = \gamma\rho p \ \frac{\rho_o p_o}{\rho_o p_o}$$

$$= \gamma \ \frac{\rho p}{\rho_o p_o} \ \rho_o p_o \quad ,$$

$$\frac{p_o}{\rho_o} = RT_o \quad ,$$

$$\rho_o = \frac{p_o}{RT_o} \quad ,$$

$$\gamma \rho p = \gamma \frac{\rho p}{\rho_o p_o} \frac{p_o p_o}{RT_o}$$

$$= \frac{\gamma^2}{\gamma} \frac{p_o^{\,2}}{RT_o} \frac{\rho p}{\rho_o p_o} \quad .$$

Application of the adiabatic relation between density and pressure

$$\frac{\rho}{\rho_o} = \left(\frac{p}{p_o}\right)^{1/\gamma}$$

yields

$$\gamma \rho p = \frac{\gamma^2 p_o^{\,2}}{\gamma RT_o} \frac{p}{p_o} \left(\frac{p}{p_o}\right)^{1/\gamma}$$

$$= \frac{\gamma^2 p_o^{\,2}}{\gamma RT_o} \frac{p}{p_o}^{\frac{\gamma+1}{\gamma}} \quad .$$

Using the following thermodynamic relationship

$$C_p - C_v = R \quad ,$$

$$\frac{C_p - C_v}{C_p} = \frac{R}{C_p} \quad ,$$

$$1 - \frac{1}{\gamma} = \frac{R}{C_p} \quad ,$$

$$\gamma R = C_p \, (\gamma - 1) \quad ,$$

together with the adiabatic relation of temperature and pressure

$$\frac{T_o}{T} = \left(\frac{p_o}{p}\right)^{\frac{\gamma-1}{\gamma}} \quad ,$$

and the previously derived equation

$$\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

yields

$$\left(\frac{p_o}{p}\right)^{\frac{\gamma-1}{\gamma}} = 1 + \frac{\gamma - 1}{2} M^2 \quad ,$$

$$\frac{p_o}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \quad .$$

Substituting these terms yields

$$\gamma \rho p = \frac{\gamma^2 p_o^{\ 2}}{\gamma R T_o} \left(\frac{p}{p_o}\right)^{\frac{\gamma+1}{\gamma}}$$

$$= \frac{\gamma^2 p_o^{\ 2}}{C_p (\gamma - 1) T_o} \left[\left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}}\right]^{\frac{\gamma+1}{\gamma}} .$$

Introducing this expression in the original equation (page 244) yields

$$\rho U = M \left[\frac{\gamma^2 p_o^{\ 2}}{C_p (\gamma - 1) T_o} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{-\frac{\gamma+1}{\gamma-1}}\right]^{1/2}$$

$$= \frac{M \gamma p_o}{\left[C_p (\gamma - 1) T_o\right]^{1/2} \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma + 1}{2(\gamma - 1)}}} \quad .$$

Coles [11] shows that the available data on the variation of C_{fa} (adiabatic skin friction coefficient) with R_θ and M_a can be correlated within a few percent by a single curve

$$C_f = \frac{\mu_s \rho}{\mu_{aw} \rho_{aw}} C_{fa} \quad . \tag{37}$$

Multiplying by R_θ yields

$$C_f R_\theta = \frac{\rho \mu}{\rho_{aw} \mu_{aw}} C_{fa} R_\theta \quad , \tag{38}$$

where ρ_{aw} and μ_{aw} are the density and viscosity evaluated at T_{aw} and μ_s is the viscosity at a mean sublayer temperature T_s given by

$$\frac{T_s}{T_{aw}} = 1 + 17.2 \left(\frac{T_o}{T_{aw}} - 1\right)\left(\frac{C_f}{2}\right)^{1/2} - 305 \left(\frac{T_o}{T_{aw}} - \frac{T}{T_{aw}}\right)\frac{C_f}{2} \quad . \tag{39}$$

It is evident that C_f and R_θ are merely values of C_{fa} and R_θ for low-speed flow. Coles' relation between C_f and $C_f R_\theta$ is shown in the Figure A-1.

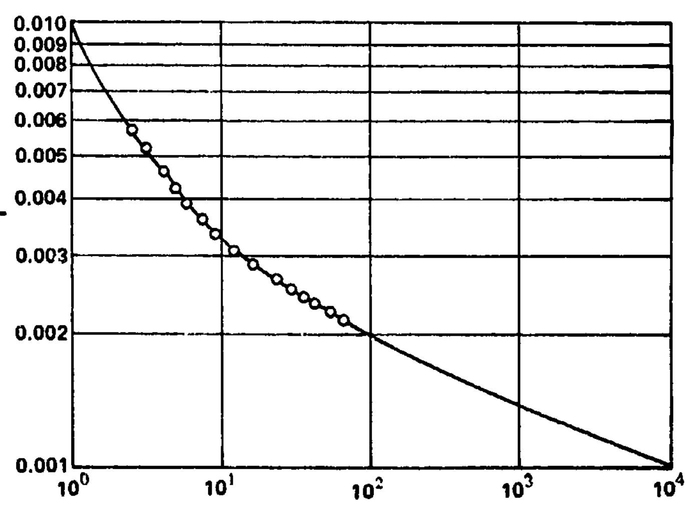


Figure A-1. Coles' relation between C_f and $C_f R_\theta$.

For values of $C_f R_\theta$ above 64.8, the relation employed is the asympotic one given by Coles [11].

$$\left(\frac{2}{C_f}\right)^{1/2} = 2.44 \ln \left\{ \frac{C_f R_\theta}{C_f \left[3.781 - \frac{25.104}{\left(\frac{2}{C_f}\right)^{1/2}} \right]} \right\} + 7.68 \quad .$$

To permit the choice of a small initial boundary layer thickness the range below $C_f R_\theta = 2.51$ is extended by

$$C_f = \frac{0.009896}{(C_f R_\theta)^{0.562}} \quad .$$

Writing density and viscosity in terms of temperature, using the ideal gas equation

$$\frac{P}{\rho} = RT$$

$$\frac{P_{aw}}{\rho_{aw}} = RT_{aw}$$

and considering that at the same location $P = P_{aw}$, one obtains

$$\frac{\rho}{\rho_{aw}} = \frac{T_{aw}}{T} \quad .$$

Applying the previously derived relationship of viscosity and temperature yields

$$\frac{\mu}{\mu_{aw}} \frac{\rho}{\rho_{aw}} = \left(\frac{T}{T_{aw}}\right)^m \frac{T_{aw}}{T} = \left(\frac{T_{aw}}{T}\right)^{1-m} \quad .$$

Introducing the proper terms in equations (37) and (38) results in

$$C_{fa} = \frac{C_f}{\left(\frac{T_{aw}}{T}\right)\left(\frac{T_s}{T_{aw}}\right)^m} , \qquad (37a)$$

$$C_f R_\theta = \left(\frac{T_{aw}}{T}\right)^{1-m} C_{fa} R_\theta \quad . \qquad (38a)$$

Thus given all the gas properties outside the boundary layer and given θ, then R_θ can be calculated from equation (35). A value of C_{fa} is then assumed and equation (38) is used to find $C_f R_\theta$. However, $C_f R_\theta$ is related to C_f by the data in Figure A-1 and equations (37a) and (38a). Therefore, C_f can be found from $C_f R_\theta$; but C_{fa} is related to C_f by equation (37). Therefore, the calculated C_{fa} is compared to the assumed C_{fa} and an iteration is performed until convergence is obtained. The skin friction coefficient is then set equal to C_{fa}, which means that there is no effect of heat transfer on the skin friction coefficient.

Derivation of Equation (40)

It is assumed that the Stanton number C_H in a nozzle is the same as on the flat plate at the same free-stream conditions ρ, U, μ, T, M; the same wall temperature T_w; and the same energy and momentum thickness ϕ and θ. The most accurate available relation for computing the flat plate Stanton number is von Karman's form of Reynolds' analogy which relates Stanton number to skin friction coefficient, with a secondary correction for non unity Prandtl number as follows [13].

$$C_H = \frac{\frac{C_f}{2}}{1 - 5\left(\frac{C_f}{2}\right)^{1/2} \left[1 - Pr + \ln\left(\frac{6}{5\ Pr + 1}\right)\right]} \quad .$$

This equation is valid when the momentum thickness and energy thickness are equal $\theta = \phi$. Under the same circumstances $R_\theta = R_\phi$, where R_ϕ is the Reynolds number based upon the energy thickness defined by

$$R_\phi = \frac{\rho U \phi}{\mu} \quad .$$

If R_ϕ is employed instead of R_θ in the procedure for computing the skin friction coefficient, a number is obtained which is designated $C_f(R_\phi)$. If $\phi = \theta$, then $C_f(R_\phi) = C_f$ and the Stanton number in this special case is

$$C_H = \frac{\dfrac{C_f(R_\phi)}{2}}{1 - 5\left[\dfrac{C_f(R_\phi)}{2}\right]^{1/2}\left[1 - Pr + \ln\left(\dfrac{6}{5\ Pr + 1}\right)\right]} \quad .$$

Since C_H must depend more on ϕ than on θ, this equation is considered as the first approximation to C_H for unequal θ and ϕ as well. (The term C_H must approach infinity as ϕ approaches zero, regardless of the value of θ.) When test data were compared with the previous equation, it was found that the Stanton number and skin friction coefficient are insensitive to T_{aw}/T_o for cooling, which was assumed when $C_f \cong C_{fa}$.

The effect of nonunity ϕ/θ cannot be large for $\phi < \theta$. However, test data show same scatter, and a larger effect could be present for nozzles, since here $\phi > \theta$. To include such an effect, the final equation for C_H is multiplied by $(\phi/\theta)^{\tilde{n}}$ allowing for a correction.

$$\boxed{C_H = \frac{\dfrac{C_f(R_\phi)}{2}\left(\dfrac{\phi}{\theta}\right)^{\tilde{n}}}{1 - 5\left[\dfrac{C_f(R_\phi)}{2}\right]^{1/2}\left[1 - Pr + \ln\left(\dfrac{6}{5\ Pr + 1}\right)\right]}} \quad . \qquad (40)$$

While $C_f(R_\phi)$ varies approximately with the $-\frac{1}{4}$ power of ϕ (Blasius equation), the last equation for C_H contains the following approximate dependence on ϕ and θ.

$$C_H \simeq \frac{1}{\theta^{\frac{1}{4}-\tilde{n}}\ \theta^{\tilde{n}}} \quad .$$

Since C_H must depend more on ϕ than on θ and, in particular, must approach infinity as $\phi \to 0$, it is evident that the upper limit for $\tilde{n}$ is $\tilde{n} = \frac{1}{4}$. An estimate of the actual value of $\tilde{n}$ was made with a relationship of

$$C_H \simeq \frac{1}{\theta^{\frac{1}{4}} \left(\frac{\phi}{\theta}\right)^{\frac{1}{7}}}$$

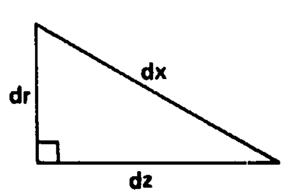
which is the same as the previous equation for $\tilde{n} = 3/28 \simeq 0.1$. Thus $\tilde{n}$ lies between zero and 0.25 with some basis to choose 0.1.

Derivation of Equations (43) and (44)

$$Q_w = q_w A = 2\pi r \times q_w \quad ,$$

$$dQ_w = 2\pi r\ q_w\ dx \quad .$$

For axisymmetric flow



$$dx = \left(dr^2 + dz^2\right)^{\frac{1}{2}} \quad ,$$

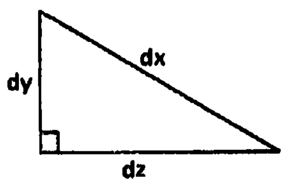
$$\frac{dx}{dz} = \left[\left(\frac{dr}{dz}\right)^2 + 1\right]^{\frac{1}{2}}$$

$$dx = \left[\left(\frac{dr}{dz}\right)^2 + 1\right]^{\frac{1}{2}} dz \quad ,$$

$$dQ_w = 2\pi r\ q_w \left[\left(\frac{dr}{dz}\right)^2 + 1\right]^{\frac{1}{2}} dz \quad .$$

$$\boxed{Q_w = 2\pi \int_0^z r\, q_w \left[\left(\frac{dr}{dz}\right)^2 + 1\right]^{1/2} dz} \quad . \tag{43}$$

For two-dimensional planar flow



$$dx = (dy^2 + dz^2)^{1/2} \quad ,$$

$$\frac{dx}{dz} = \left[\left(\frac{dy}{dz}\right)^2 + 1\right]^{1/2} \quad ,$$

$$dx = \left[\left(\frac{dy}{dz}\right)^2 + 1\right]^{1/2} dz \quad ,$$

$$dQ_w = 2\pi r\, q_w \left[\left(\frac{dy}{dz}\right)^2 + 1\right]^{1/2} dz \quad ,$$

$$Q_w = 2\pi \int_0^z r\, q_w \left[\left(\frac{dy}{dz}\right)^2 + 1\right]^{1/2} dz \quad , \qquad \begin{pmatrix} r \to \infty \\ A = 2\pi r \to \infty \end{pmatrix} \quad .$$

Therefore dividing by $2\pi r$ yields the heat transfer per unit width

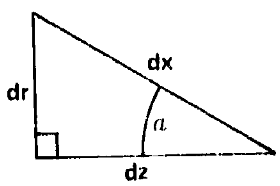
$$\boxed{Q_w = \int_0^z q_w \left[\left(\frac{dy}{dz}\right)^2 + 1\right]^{1/2} dz} \quad . \tag{44}$$

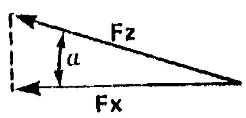
Derivation of Equations (45) through (47)

The total drag along a surface is represented by

$$D = C_f A \frac{\rho}{2} U^2 \quad .$$

For axisymmetric flow the drag in z-direction can be determined if the angle between the wall and the z-direction is known.





$$\tan\alpha = \frac{dr}{dz} \quad ,$$

$$1 + \tan^2\alpha = \frac{1}{\cos^2\alpha} \quad ,$$

$$\therefore \cos\alpha = \frac{1}{\sqrt{1 + \left(\frac{dr}{dz}\right)^2}} \quad .$$

$$A = 2\pi r \; x \quad ,$$

$$D = 2\, r\pi C_f \frac{\rho}{2} U^2 \; x \quad ,$$

The force in z-direction is

$$F_z = F_x \cos\alpha \quad ,$$

$$dD = 2\pi r \; C_f \frac{\rho}{2} U^2 \cos\alpha \; dx \quad ,$$

$$dx = \left[\left(\frac{dr}{dz}\right)^2 + 1\right]^{1/2} dz \quad ,$$

$$dD = 2\pi r \; C_f \frac{\rho}{2} U^2 \left[\left(\frac{dr}{dz}\right)^2 + 1\right]^{1/2} \cos\alpha \; dz \quad .$$

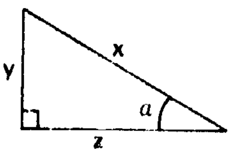
$$\boxed{D = \int_0^z 2\pi r \; C_f \frac{\rho}{2} U^2 \left[\left(\frac{dr}{dz}\right)^2 + 1\right]^{1/2} \cos\alpha \; dz} \; . \qquad (45)$$

For two-dimensional planar flow

$$r \to \infty$$

$$2\pi r \to \infty \quad .$$

Dividing the equation by $2\pi r$ the drag per unit width is determined. Furthermore



$$dx = \left[\left(\frac{dy}{dz} \right)^2 + 1 \right]^{1/2} dz \quad .$$

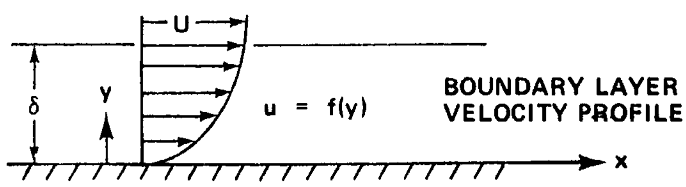
The drag in axial direction is

$$\boxed{F_{ax} = \int_0^z C_f \frac{\rho}{2} U^2 \left[\left(\frac{dy}{dz} \right)^2 + 1 \right]^{1/2} \cos \alpha \; dz} \quad . \tag{46}$$

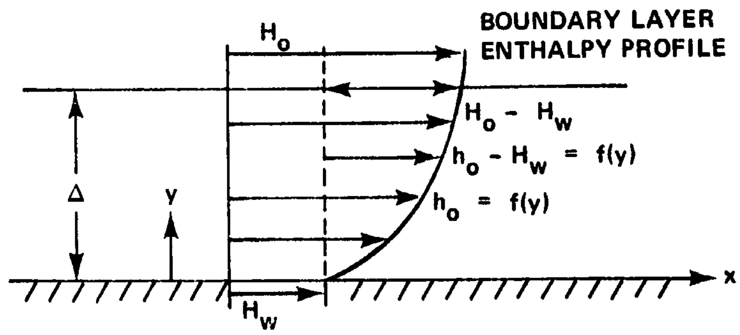
The drag normal to the axial direction is

$$\boxed{F_{norm} = \int_0^z C_f \frac{\rho}{2} U^2 \left[\left(\frac{dy}{dz} \right)^2 + 1 \right]^{1/2} \sin \alpha \; dz} \quad . \tag{47}$$

Velocity and Enthalpy Profiles Related to Equations (48) and (50)

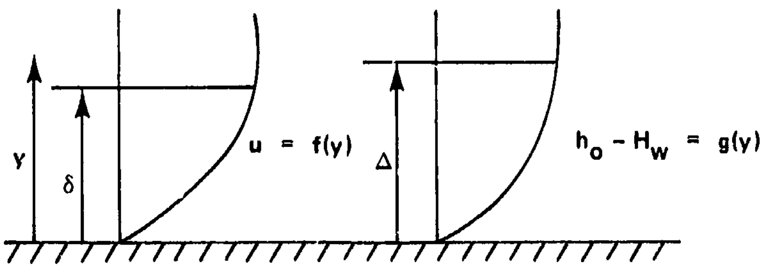


$$\frac{\bar{u}}{U} = \left(\frac{y}{\delta} \right)^{1/n} \qquad \text{for} \quad y \leq \delta \quad . \tag{48}$$



$$\frac{h_o - H_w}{H_o - H_w} = \left(\frac{y}{\Delta} \right)^{1/n} \quad \text{for } y \leq \Delta \quad . \tag{50}$$

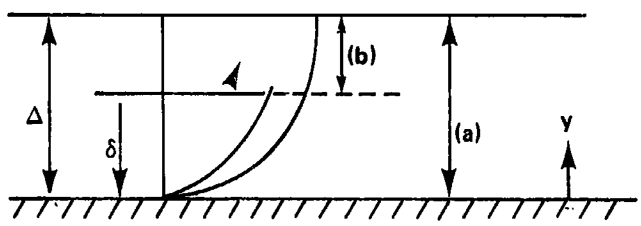
Geometrical Representation of Possible Velocity and Temperature Thickness Combinations for the Boundary Layer and the Derivation of Appropriate Equations



I. $\delta \leq \Delta$

(a) $y \leq \Delta$

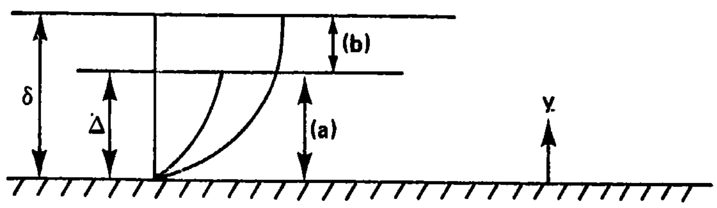
(b) $\delta < y \leq \Delta$



II. $\delta \geq \Delta$

(a) $y \leq \Delta$

(b) $\Delta < y \leq \delta$



Case I

$\delta \leq \Delta$: The velocity boundary layer is smaller than or equal to the temperature (enthalpy) boundary layer.

(a) $y \leq \Delta$ In this case y can be within or outside of the velocity boundary layer.

For the whole range the velocity boundary layer is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/n} = S \leq 1.$$

If $y \geq \delta \rightarrow S = 1$ Outside velocity boundary layer.

If $y < \delta \rightarrow S < 1$ Inside velocity boundary layer.

For the whole range the enthalpy (or temperature) boundary layer is

$$\frac{h_o - H_w}{H_o - H_w} = \left(\frac{y}{\Delta}\right)^{1/n} = W \leq 1 \quad .$$

If $y < \delta \rightarrow W < 1$ Inside velocity boundary layer.

If $\delta < y \leq \Delta \rightarrow w \leq 1$ Outside velocity boundary layer.

When the velocity and enthalpy (temperature) boundary layers are compared,

$$\left(\frac{\Delta}{\delta}\right)^{1/n} = \frac{\left(\frac{y}{\delta}\right)^{1/n}}{\left(\frac{y}{\Delta}\right)^{1/n}} = \frac{S}{W} = \zeta \quad ;$$

and the following definitions result.

$$S = \left(\frac{y}{\delta}\right)^{1/n} \quad ,$$

$$W = \left(\frac{y}{\Delta}\right)^{1/n} \quad ,$$

$$\zeta = \left(\frac{\Delta}{\delta}\right)^{1/n} \quad .$$

Therefore the velocity boundary layer can be expressed by

$$u = U\,S \quad \text{for} \quad S < 1 \quad ; \tag{59}$$

whereas the enthalpy (temperature) boundary layer yields

$$\frac{h_o - H_w}{H_o - H_w} = W = \frac{S}{\zeta} \quad , \tag{60}$$

$$h_o = H_w + \frac{S}{\zeta}\,(H_o - H_w) \tag{61}$$

with h_o the stagnation enthalpy within the velocity boundary layer. The stagnation enthalpy is composed of a static and dynamic portion.

$$h_o = h + \frac{u^2}{2} \quad . \tag{62}$$

Substituting this in the last equation and making use of $u = U\,S$ equation (65) is obtained.

$$h = H_w + \frac{S}{\zeta}\,(H_o - H_w) - \frac{U^2\,S^2}{2} \quad . \tag{65}$$

The density variation can now be determined from the universal gas equation

$$\frac{P}{\rho} = R\bar{t} \qquad \text{(Pressure is constant across the boundary layer.)}$$

$$\frac{\dfrac{P}{\rho} = RT}{\dfrac{\rho}{\rho_{-}} = \dfrac{\bar{t}}{T}} \quad , \tag{66}$$

where $\bar{t}$, temperature in the boundary layer, can be calculated from h by means of

$$h = C_p\bar{t} \quad .$$

However the results are only valid for

$$S \leq 1 \quad ,$$

$$W \leq \frac{1}{\zeta} \quad .$$

(b) $\delta < y \leq \Delta$ In this case y is greater than the velocity boundary layer but smaller or equal to the temperature boundary layer.

For this range the velocity boundary layer is

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/n} = S = 1 \quad . \tag{69}$$

The second integral of equation (75) is zero since it considers a portion outside the boundary layer, whereas the first integral is modified according to

$$S = \left(\frac{y}{\delta}\right)^{1/n} \quad ,$$

$$S\delta^{1/n} = y^{1/n} \quad ,$$

$$S^n \delta = y \quad ,$$

$$\frac{dy}{ds} = nS^{n-1}\delta \quad ,$$

$$dy = nS^{n-1}\delta \; dS \quad ,$$

and finally reduces to

$$\theta = n \int_0^1 \frac{\bar{\rho}}{\rho} S^n \; (1 - S) \; \delta \; ds \quad . \tag{76}$$

In order to obtain the equation for the displacement thickness δ^*, the relationship used in the derivation of equation (76) is used. Furthermore the first integral on the right-hand side of equation (80) is evaluated.

$$n \int_0^{\zeta} S^{n-1}\, ds = n \left(\frac{S^n}{n}\right) \Bigg|_0^{\zeta}$$

$$= n \frac{\zeta^n}{n} - 0$$

$$= \zeta^n \quad ,$$

and the result appears in equation (81). In the third integral on the right-hand side of equation (80) the relationship

$$\frac{u}{U} = 1$$

is applied since y is greater than the velocity boundary layer.

To obtain the energy thickness in equation (85), use is made of the relationship as de,eloped for equation (76). Furthermore the following relation is applied.

$$S = W\zeta \quad ,$$

$$\frac{ds}{dw} = \zeta \quad ,$$

$$ds = \zeta \; dw \quad .$$

Case II

$\delta \geq \Delta$	The velocity boundary layer is equal to or greater than the temperature boundary layer.
(a) $y \leq \Delta$	In this case y is smaller than or at the most equal to the temperature boundary layer.
(b) $\Delta < y \leq \delta$	Here y is greater than the temperature boundary layer but smaller than or equal to the velocity boundary layer.

Equation (101) is equal to one because y is outside the temperature boundary layer.

For the momentum equation (106), the relationship shown in connection with equation (76) is utilized.

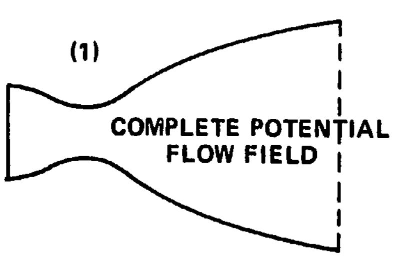
Also equation (111) representing the displacement thickness δ^* uses the relationship applied in equation (76). The first integral on the right-hand side is solved and results in

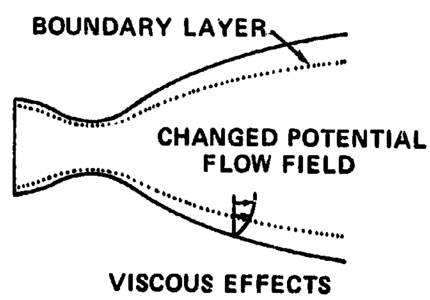
$$\int_0^1 nS^{n-1}\, ds = S^n \Big|_0^1 = 1 \quad .$$

For the momentum equation leading finally to equation (116), the second integral is equal to zero since it represents a quantity which is outside the temperature boundary layer. The first integral on the right-hand side is reduced by substitution of the relationships developed in equations (76) and (85) resulting finally in equation (116).

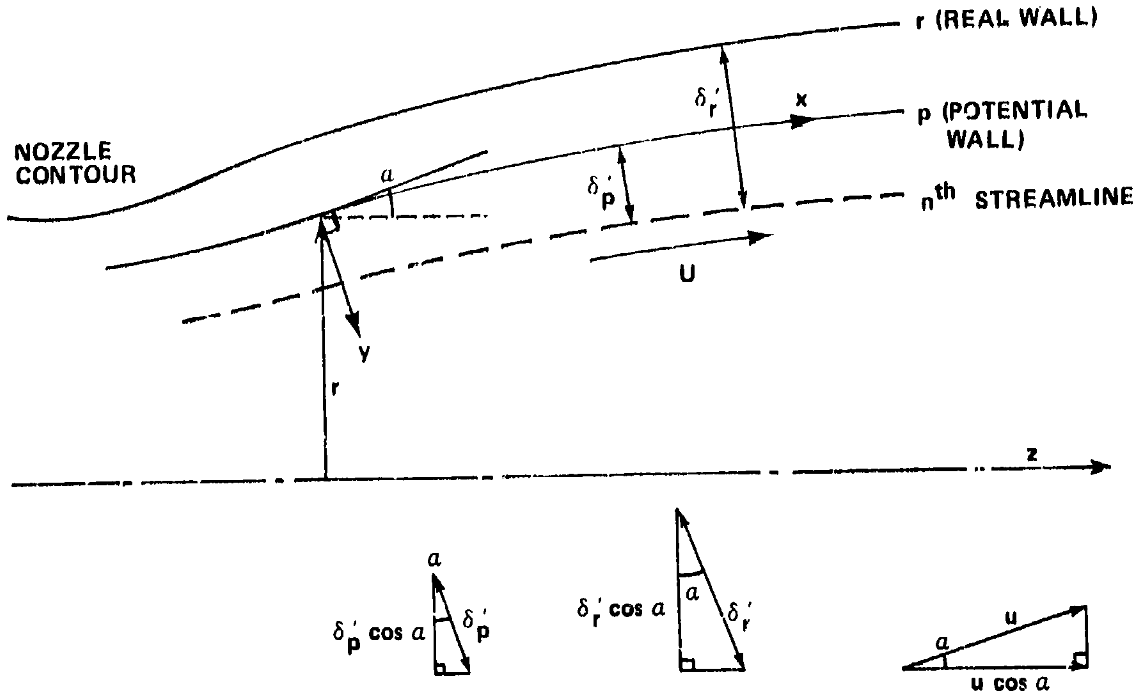
Thrust Degradation

In a real flow the boundary layer and its viscous effects cause an I_{SP} degradation. Furthermore, the potential flow field is narrowed because of the presence of the boundary layer.





Subsequently the derivation of the equation representing the degradation of thrust caused by the viscous flow effects in the boundary layer is presented in the following diagram, where p represents a contour of the potential (inviscid) flow and r represents a contour of an exactly equivalent viscous flow such that the pressure P along the normal to the wall y is the same on r as on p and the mass flow through r - n is the same as through p - n. The velocity distribution is confined between n - r.



The mass flow rate of the potential flow through the area between p and n is

$$\dot{m}_p = 2\pi r \delta'_p \, U\rho \quad .$$

The mass flow rate of the viscous flow through an area between r and n is

$$\dot{m}_r = 2\pi r \delta'_r \, U\rho \quad .$$

However U and ρ are varying with y such that

$$\delta'_r \, U\rho = \int_0^{\delta'_r} \bar{\rho}\bar{u} \, dy$$

and

$$\dot{m}_r = 2\pi r \int_0^{\delta'_r} \bar{\rho}\bar{u} \, dy \quad .$$

The momentum flux in z-direction is

$$\dot{M} = \dot{m}U$$

$$\dot{M}_Z = \dot{m}U_Z$$

such that for the potential and viscous flow one obtains

$$(\dot{M}_p)_z = (2\pi r\ \delta'_p\ U\rho_\infty)\ (U \cos \alpha)$$

$$(\dot{M}_r)_z = 2\pi r \int_0^{\delta'_r} \bar{\rho}\bar{u}^2 \cos \alpha \ dy \quad .$$

Because of the definition, α is assumed to be constant between n and r, because $\frac{\delta'_r}{r} << 1$; therefore,

$$(\dot{M}_r)_z = 2\pi r \cos \alpha \int_0^{\delta'_r} \bar{\rho}\bar{u}^2\, dy \quad .$$

Since the same mass flows through n - p and r - n, one obtains

$$\dot{m}_p = \dot{m}_r \quad ,$$

$$2\pi r \delta'_p\ U\rho_\infty = 2\pi r \int_0^{\delta'_r} \bar{\rho}\bar{u}\ dy \quad ,$$

$$\delta'_p = \int_0^{\delta'_r} \frac{\bar{\rho}u}{\rho_\infty U}\, dy \quad .$$

Subtracting δ'_r on either side

$$\delta'_r - \delta'_p = \delta'_r - \int_0^{\delta'_r} \frac{\bar{\rho}u}{\rho U}\, dy$$

or

$$\delta_r' - \delta_p' = \int_0^{\delta_r'} dy - \int_0^{\delta_r'} \frac{\bar{\rho}\bar{u}}{\rho_\alpha U} dy$$

$$= \int_0^{\delta_r'} \left(1 - \frac{\bar{\rho}\bar{u}}{\rho\ U}\right) dy \quad .$$

As defined earlier

$$\delta^* = \delta_r' - \delta_p'$$

$$\delta^* = \int_0^{\delta_r'} \left(1 - \frac{\bar{\rho}\bar{u}}{\rho_\alpha U}\right) dy \quad .$$

Now, both momentum equations will be subtracted from each other

$$(\dot{M}_p)_z - (\dot{M}_r)_z = 2\pi r\delta_p' \ U\rho \ U \cos\alpha - 2\pi r \cos\alpha \int_0^{\delta_r'} \bar{\rho}\bar{u}^2 \ dy$$

$$= 2\pi r\rho \ U^2 \cos\alpha \left(\delta_p' - \int_0^{\delta_r'} \frac{\bar{\rho}\bar{u}^2}{\rho\ U^2} dy\right)$$

$$= 2\pi r\rho \ U^2 \cos\alpha \left(\delta_r' - \delta^* - \int_0^{\delta_r'} \frac{\bar{\rho}\bar{u}^2}{\rho U^2} dy\right)$$

$$= 2\pi r\rho \ U^2 \cos\alpha \left[\int_0^{\delta_r'} dy - \int_0^{\delta_r'} \left(1 - \frac{\bar{\rho}\bar{u}}{\rho\ U}\right) dy - \int_0^{\delta_r'} \frac{\bar{\rho}\bar{u}^2}{\rho\ U^2} dy\right]$$

$$= 2\pi r\rho \ U^2 \cos\alpha \int_0^{\delta_r'} \left[1 - \left(1 - \frac{\bar{\rho}\bar{u}}{\rho\ U}\right) - \frac{\bar{\rho}\bar{u}^2}{\rho\ U^2}\right] dy$$

$$= 2\pi r\rho \ U^2 \cos \alpha \int_0^{\delta' r} \left(\frac{\bar{\rho}\bar{u}}{\rho \ U} - \frac{\bar{\rho}\bar{u}^2}{\rho \ U^2} \right) dy$$

$$= 2\pi r\rho \ U^2 \cos \alpha \int_0^{\delta' r} \frac{\bar{\rho}\bar{u}}{\rho \ U} \left(1 - \frac{\bar{u}}{U} \right) dy \quad .$$

The integral expression however represents just the momentum thickness as derived in equation (12).

$$\theta = \int_0^{\delta' r} \frac{\bar{\rho}\bar{u}}{\rho \ U} \left(1 - \frac{\bar{u}}{U} \right) dy \quad .$$

Therefore

$$(\dot{M}_p)_z - (\dot{M}_r)_z = 2\pi r\rho_x U^2 \cos \alpha \ \theta \quad .$$

Since the momentum flux is equal to the forces

$$\dot{M} = F$$

and the forces F can be expressed by pressure P times area A

$$F = PA \quad ,$$

one obtains

$$\dot{M} = PA \quad .$$

Differentiation yields for the z-direction

$$\frac{d\dot{M}}{dz} = A \frac{dP}{dz} + P \frac{dA}{dz} \quad .$$

Assuming that the variation of the cross-sectional area at a station z to be very small and negligible results in

$$\frac{d\dot{M}}{dz} = A \frac{dP}{dz} \quad .$$

For the potential flow

$$A = 2\pi r \delta_p' \quad ,$$

$$A_z = 2\pi r \delta_{pz}' = 2\pi r \delta_p' \cos \alpha \quad .$$

For the viscous flow

$$A = 2\pi r \delta_r' \quad ,$$

$$A_z = 2\pi r \delta_{rz}' = 2\pi r \delta_r' \cos \alpha \quad .$$

Furthermore the shear forces must be considered for the viscous flow with

$$\tau_w A' = \dot{m} u = \dot{M} \quad .$$

In this case the surface area along x must be considered

$$A' = 2\pi r x \quad .$$

Differentiating the momentum equation yields

$$\frac{d\dot{M}}{dx} = \tau_w \ 2\pi r \ \frac{dx}{dx} \quad ,$$

$$\frac{d\dot{M}}{dx} = \tau_w \ 2\pi r \quad .$$

Using the previously derived relationships results in the following momentum equation in z-direction

$$\frac{d \ (\dot{M}_p)_z}{dz} = - \ 2\pi r \delta_p' \cos \alpha \ \frac{dP}{dz} \quad .$$

(A negative sign was chosen since the pressure decreases with increasing z.)

$$\frac{d \ (\dot{M}_r)_z}{dz} = - \ 2\pi r \delta_r' \cos \alpha \ \frac{dP}{dz} - 2\pi r \ \tau_w \quad .$$

Using the previously derived relationships yields

$$(\dot{M}_p)_z - (\dot{M}_r)_z = 2\pi r\rho_\infty U^2 \cos\alpha\ \theta$$

$$\delta_r' - \delta_p' = \delta^*$$

for the potential case results in

$$\frac{d\ (\dot{M}_p)_z}{dz} = \frac{d}{dz}\left[(\dot{M}_r)_z + 2\pi r\rho_\infty U^2\ \theta\cos\alpha\right] = -\ 2\pi r\ (\delta_r' - \delta^*)\cos\alpha\ \frac{dP}{dz}\ .$$

Subtracting the equation representing the viscous case

$$\frac{d\ (\dot{M}_r)_z}{dz} = -2\pi r\delta_r'\ \cos\alpha\ \frac{dP}{dz} - 2\ r\pi\ \tau_w$$

from this one yields

$$\frac{d}{dz}\ (2\pi r\rho_\infty U^2\ \theta\cos\alpha) = 2\pi r\delta^*\cos\alpha\ \frac{dP}{dz} + 2\pi r\ \tau_w\ .$$

Multiplying the equation by $\frac{dz}{2\pi}$ results in

$$d\ (r\rho\ U^2\ \theta\cos\alpha) = r\delta^*\cos\alpha\ dP + r\ \tau_w\ dz\ .$$

Solving for the τ_w term and integrating result in

$$\int r\ \tau_w\ dz = \int d\ (r\rho\ U^2\ \theta\cos\alpha) - \int r\ \delta^*\cos\alpha\ dP\ .$$

According to a previous derivation, the force acting on the exit plane is for

1. Potential flow

$$(F_p)_{ze} = \dot{m}\ U + P_c A_c + \int_{A_c}^{A_{ep}} P\ dA_p$$

2. Viscous flow with the inclusion of shear forces

$$(F_r)_{ze} = \dot{m}\, U + P_c A_c + \int_{A_c}^{A_{er}} P\, dA_r - \int_{A_c}^{A_{er}} \tau_w\, dA'_\tau \quad .$$

(The subscript c refers to chamber.)

The degradation in force caused by the viscosity and the shear forces is equal to the difference between the potential and the viscous flow

$$(\Delta F)_{ze} = (F_p)_{ze} - (F_r)_{ze} = \int_{A_c}^{A_{ep}} P\, dA_p - \int_{A_c}^{A_{er}} P\, dA_r$$

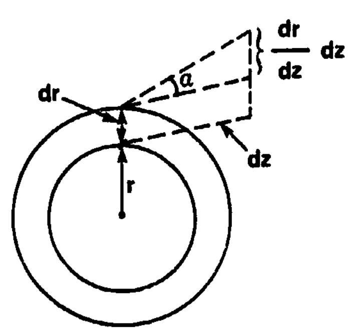
$$+ \int_{A_c}^{A_{er}} \tau_w\, dA'_\tau \quad .$$

The change in area normal to the z-direction as a function of z is

$$dA = 2\pi r\, dr \quad ,$$

$$\frac{dA}{dz} = 2\pi r\, \frac{dr}{dz} \quad ,$$

$$dA = 2\pi r\, \frac{dr}{dz}\, dz \quad .$$



For potential flow

$$dA_p = 2\pi r\, \frac{dr}{dz}\, dz \quad .$$

For viscous flow

$$dA_r = 2\pi(r + \delta^* \cos\alpha) \frac{d(r + \delta^* \cos\alpha)}{dz} dz \quad .$$

For shear forces

$$dA'_\tau = 2\pi(r + \delta^* \cos\alpha) dz \quad .$$

Introducing these expressions in the previous equation yields

$$(\Delta F)_{z_e} = \int_o^{z_e} P\, 2\pi r \frac{dr}{dz} dz - \int_o^{z_e} P\, 2\pi(r + \delta^* \cos\alpha) \frac{d(r + \delta^* \cos\alpha)}{dz} dz$$

$$+ \int_o^{z_e} \tau_w\, 2\pi(r + \delta^* \cos\alpha) dz$$

$$= \int_o^{z_e} P\, 2\pi r \frac{dr}{dz} dz - \left[\int_o^{z_e} P\, 2\pi r \frac{dr}{dz} dz + \int_o^{z_e} P\, 2\pi\delta^* \cos\alpha \frac{dr}{dz} dz \right.$$

$$+ \int_o^{z_e} P\, 2\pi r \frac{d(\delta^* \cos\alpha)}{dz} dz$$

$$\left. + \int_o^{z_e} P\, 2\pi\delta^* \cos\alpha \frac{d(\delta^* \cos\alpha)}{dz} dz \right]$$

$$+ \int_o^{z_e} \tau_w\, 2\pi(r + \delta^* \cos\alpha) dz \quad .$$

The first two integrals cancel each other.

$$(\Delta F)_{z_e} = -\int_o^{z_e} 2\pi P \left[\delta^* \cos\alpha \frac{dr}{dz} + r \frac{d(\delta^* \cos\alpha)}{dz} \right.$$

$$\left. + \delta^* \cos\alpha \frac{d(\delta^* \cos\alpha)}{dz} \right] dz$$

$$+ \int_o^{z_e} \tau_w\, 2\pi(r + \delta^* \cos\alpha) dz \quad .$$

In the last equation the following terms can be expressed differently.

$$\delta^* \cos\alpha \frac{dr}{dz} + r \frac{d(\delta^* \cos\alpha)}{dz} = \frac{d}{dz}(r\delta^* \cos\alpha) \quad ,$$

$$\delta^* \cos\alpha \frac{d(\delta^* \cos\alpha)}{dz} = \frac{1}{2}\left[\frac{d}{dz}(\delta^{*2} \cos^2\alpha)\right] \quad ,$$

such that

$$(\Delta F)_{z_e} = -\int_0^{z_e} 2\pi P \left[\frac{d}{dz}(r\delta^* \cos\alpha) + \frac{1}{2}\frac{d}{dz}(\delta^{*2} \cos^2\alpha)\right] dz$$

$$+ \int_0^{z_e} 2\pi \tau_w (r + \delta^* \cos\alpha) dz \quad .$$

Since terms of $\frac{\delta^* \cos\alpha}{r} << 1$, they can be neglected especially when higher order terms are used.

$$(\Delta F)_{z_e} = -\int_0^{z_e} 2\pi p \, d(r\delta^* \cos\alpha) + \int_0^{z_e} 2\tau_w \pi r \, dz \quad .$$

The previously derived expressions on page 266

$$\int r \tau_w dz = \int d(r\rho_\infty U^2\theta \cos\alpha) - \int r\delta^* \cos\alpha \, dP$$

is substituted in the equation above, such that

$$(\Delta F)_{z_e} = -\int_0^{z_e} 2\pi p \, d(r\delta^* \cos\alpha) + \int_0^{z_e} 2\pi d(r\rho_\infty U^2\theta \cos\alpha)$$

$$-\int_0^{z_e} 2\pi r\delta^* \cos\alpha \, dP \quad .$$

Combining the following integrals yields

$$\int Pd\ (r\delta^* \cos\alpha) + \int r\delta^* \cos\alpha\, dP = \int d\ (Pr\delta^* \cos\alpha) \quad .$$

The equation for $(\Delta F)_{z_e}$ can be written

$$(\Delta F)_{z_e} = -2\pi \int_0^{z_e} d\ (Pr\delta^* \cos\alpha) + 2\pi \int_0^{z_e} d\ (r\rho_\infty U^2\theta \cos\alpha)$$

$$= -2\pi Pr\delta^* \cos\alpha \Big|_0^{z_e} + 2\pi r\rho\ U^2\theta \cos\alpha \Big|_0^{z_e} \quad .$$

Since the displacement thickness δ^* and the momentum thickness θ are zero at $z = 0$, the thrust degradation is obtained as

$$(\Delta F)_{z_e} = (-2\pi p\ r\delta^* \cos\alpha + 2\pi r\rho_\infty U^2\theta \cos\alpha)_{exit} \quad ,$$

or

$$\boxed{(\Delta F)_{z_e} = \left[2\pi r\rho\ U^2\theta \cos\alpha \left(1 - \frac{\delta^* P}{\theta\rho\ U^2}\right)\right]_{exit}} \quad .$$

The result is only a function of exit plane conditions.

Derivation of Equation (122)

First the term $\frac{1 + \delta^*/\theta}{U} \frac{dU}{dz}$ of equation (121) will be derived. The energy equation states that

$$dh = C_p\ dt$$

$$c_p - c_v = R \quad .$$

Multiplying both sides by γ yields

$$\gamma(c_p - c_v) = \gamma R \quad ,$$

$$\gamma = \frac{c_p}{c_v} \quad ,$$

$$c_p(\gamma - 1) = \gamma R \rightarrow c_p = \frac{\gamma R}{\gamma - 1} \quad .$$

Substituting this expression in the energy equation yields

$$dh = \frac{\gamma R}{\gamma - 1}\, dT \quad .$$

The steady-flow energy equation states

$$Q - W = \tilde{u}_1 - \tilde{u}_2 + \frac{P_1}{\rho_1} - \frac{P_2}{\rho_2} + \frac{U_1^2 - U_2^2}{2} + Z_1 - Z_2$$

with

$W = 0$	no work,
$Q = 0$	no heat addition,
$Z = 0$	no gravity effect.

The equation reduces to

$$0 = \tilde{u}_1 - \tilde{u}_2 + \frac{P_1}{\rho_1} - \frac{P_2}{\rho_2} + \frac{U_1^2 - U_2^2}{2} \quad .$$

By definition

$$h = \tilde{u} + \frac{P}{\rho} \quad ,$$

$$0 = h_1 - h_2 + \frac{U_1^2 - U_2^2}{2} \quad ,$$

$$0 = dh + UdU \quad .$$

Substituting the previously derived term yields

$$0 = \frac{\gamma R}{\gamma - 1} dT + UdU \quad .$$

Dividing the equation by $a^2 = \gamma RT$ yields

$$0 = \frac{1}{\gamma - 1} \frac{dT}{T} + \frac{U^2}{a^2} \frac{dU}{U} \quad ,$$

$$0 = \frac{1}{\gamma - 1} \frac{dT}{T} + M^2 \frac{dU}{U} \qquad \text{with } M = \frac{U}{a} \quad ,$$

$$\frac{dT}{T} = (1 - \gamma) M^2 \frac{dU}{U} \quad .$$

Now a relationship for $\frac{dU}{U}$ will be derived. Starting with the Mach number definition

$$M = \frac{U}{a} \quad \text{or} \quad M^2 = \frac{U^2}{a^2}$$

and representing the speed of sound a by

$$a = \sqrt{\gamma RT}$$

result in

$$M^2 = \frac{U^2}{\gamma RT} \quad .$$

Differentiating this equation in logarithmic form yields

$$2 \ln M = 2 \ln U - \ln \gamma - \ln R - \ln T \quad ,$$

$$2 \frac{d(\ln M)}{dz} = 2 \frac{d(\ln U)}{dz} - \frac{d(\ln \gamma)}{dz} - \frac{d(\ln R)}{dz} - \frac{d(\ln T)}{dz} \quad ,$$

$$2\,\frac{d(\ln\ M)}{dM}\;\frac{dM}{dz} = 2\,\frac{d(\ln U)}{dU}\;\frac{dU}{dz} - \frac{d(\ln \gamma)}{d\gamma}\;\frac{d\gamma}{dz}\quad \frac{d(\ln R)}{dR}\;\frac{dR}{dz}$$

$$- \frac{d(\ln T)}{dT}\;\frac{dT}{dz} \;,$$

$$\frac{2}{M}\;\frac{dM}{dz} = \frac{2}{U}\;\frac{dU}{dz} - \frac{1}{\gamma}\;\frac{d\gamma}{dz} - \frac{1}{R}\;\frac{dR}{dz} - \frac{1}{T}\;\frac{dT}{dz} \quad .$$

Assuming that $R =$ constant and $\gamma =$ constant and multiplying the equation by dz result in

$$\frac{dM}{M} = \frac{dU}{U} - \frac{1}{2}\;\frac{dT}{T} \quad ;$$

and rearranging yields

$$\frac{dU}{U} = \frac{dM}{M} + \frac{1}{2}\;\frac{dT}{T} \quad .$$

Substituting the previously derived expression for dT/T yields

$$\frac{dU}{U} = \frac{dM}{M} + \frac{1}{2}\,(1\ -\ \gamma)\,M^2\;\frac{dU}{U} \quad ,$$

$$\frac{dU}{U}\left[1 - \frac{1}{2}\,(1 - \gamma)\ M^2\right] = \frac{dM}{M} \quad ,$$

$$\frac{dU}{U} = \frac{1}{\left(1 - \dfrac{1 - \gamma}{2}\ M^2\right)M}\,dM \quad .$$

The first term in equation (121) finally results in

$$\boxed{\frac{(1\ +\ \delta^*/\theta)}{U}\;\frac{dU}{dz} = \frac{1\ +\ \delta^*/\theta}{\left(1\ -\ \dfrac{1\ -\ \gamma}{2}\ M^2\right)M}\;\frac{dM}{dz}} \quad .$$

The second term of equation (121) can be expressed by

$$\frac{1}{\rho U}\;\frac{d(\rho U)}{dz} = \frac{1}{U}\;\frac{dU}{dz} + \frac{1}{\rho}\;\frac{d\rho}{dz} \quad .$$

The first right-hand term has already been expressed by the Mach number expression. The second right-hand term is derived from the ideal gas equation as follows.

$$\rho = \frac{P}{RT} \quad .$$

Differentiating by parts in logarithmic form yields

$$d(\ln \rho) = d(\ln P) - d(\ln R) - d(\ln T) \quad ,$$

$$\frac{d(\ln \rho)}{dz} = \frac{d(\ln P)}{dz} - \frac{d(\ln R)}{dz} - \frac{d(\ln T)}{dz} \quad ,$$

$$\frac{d(\ln \rho)}{d\rho}\frac{d\rho}{dz} = \frac{d(\ln P)}{dP}\frac{dP}{dz} - \frac{d(\ln R)}{dR}\frac{dR}{dz} - \frac{d(\ln T)}{dT}\frac{dT}{dz} \quad ,$$

$$\frac{1}{\rho}\frac{d\rho}{dz} = \frac{1}{P}\frac{dP}{dz} - \frac{1}{R}\frac{dR}{dz} - \frac{1}{T}\frac{dT}{dz} \quad ,$$

with R = constant and multiplication by dz results in

$$\frac{d\rho}{\rho} \quad \frac{dP}{P} - \frac{dT}{T} \quad .$$

Another expression for $d\rho/\rho$ can be derived from the thermodynamic adiabatic relationship

$$\frac{P}{\rho^{\gamma}} = \text{const} \quad .$$

Differentiating by parts in logarithmic form yields

$$d(\ln P) - \gamma\, d(\ln \rho) \quad d(\ln \text{const}) \quad .$$

Assuming that the specific heat ratio γ = constant and dividing by dz result in

$$\frac{d(\ln P)}{dz} - \gamma \frac{d(\ln \rho)}{dz} = \frac{d(\ln \text{const})}{dz} \quad ,$$

$$\frac{d(\ln P)}{dP}\frac{dP}{dz} - \gamma \frac{d(\ln \rho)}{d\rho}\frac{d\rho}{dz} = 0 \quad ,$$

$$\frac{1}{P}\frac{dP}{dz} - \gamma\frac{1}{\rho}\frac{d\rho}{dz} = 0 \quad ,$$

$$\frac{dP}{P} - \gamma\frac{d\rho}{\rho} = 0 \quad .$$

Final rearrangement results in

$$\frac{dP}{P} = \gamma\frac{d\rho}{\rho} \quad .$$

Substitution yields

$$\frac{d\rho}{\rho} = \gamma\frac{d\rho}{\rho} - \frac{dT}{T} \quad ,$$

or

$$\frac{d\rho}{\rho}(1 - \gamma) = -\frac{dT}{T} \quad .$$

The previously derived temperature-Mach number relationship yields

$$\frac{d\rho}{\rho} = -\frac{1}{1 - \gamma}(1 - \gamma)\, M^2\frac{dU}{U} \quad .$$

Thus

$$\frac{d\rho}{\rho} = -M^2\frac{dU}{U} \quad .$$

Also derived was the velocity-Mach number relationship:

$$\frac{d\rho}{\rho} = -M^2\frac{1}{\left[1 - \left(\frac{1 - \gamma}{2}\right)M^2\right]M}\, dM \quad .$$

Therefore the second term in equation (121) yields

$$\frac{1}{\rho U}\,\frac{d(\rho U)}{dz} = \frac{1}{U}\,\frac{dU}{dz} + \frac{1}{\rho}\,\frac{d\rho}{dz} \quad ,$$

$$= \frac{1}{\left(1 - \frac{1-\gamma}{2}\,M^2\right) M}\,\frac{dM}{dz} - \frac{M^2}{\left(1 - \frac{1-\gamma}{2}\,M^2\right) M}\,\frac{dM}{dz} \; ,$$

$$\boxed{\frac{1}{\rho U}\,\frac{d(\rho U)}{dz} = \frac{1 - M^2}{\left(1 - \frac{1-\gamma}{2}\,M^2\right) M}\,\frac{dM}{dz}} \quad .$$

In replacing the appropriate terms of equation (121) by the just derived expression yields

$$\frac{d\theta}{dz} = \frac{C_f}{2}\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2} - \theta\left[\frac{1 + \delta^*/\theta}{\left(1 - \frac{1-\gamma}{2}\,M^2\right) M}\,\frac{dM}{dz} + \frac{1 - M^2}{\left(1 - \frac{1-\gamma}{2}\,M^2\right) M}\,\frac{dM}{dz} + \frac{1}{r}\,\frac{dr}{dz}\right] \; ,$$

$$\boxed{\frac{d\theta}{dz} = \frac{C_f}{2}\left[1 + \left(\frac{dr}{dz}\right)^2\right]^{1/2} - \theta\left[\frac{2 - M^2 + \delta^*/\theta}{\left(1 - \frac{1-\gamma}{2}\,M^2\right) M}\,\frac{dM}{dz} + \frac{1}{r}\,\frac{dr}{dz}\right]} \quad . \tag{122}$$

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APPROVAL

SUPPLEMENT TO THE ICRPG TURBULENT BOUNDARY LAYER NOZZLE ANALYSIS COMPUTER PROGRAM

By Satoaki Omori, Klaus W. Gross,
and Alfred Krebsbach

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.

Hans G. Paul

HANS G. PAUL
Chief, Propulsion
and Thermodynamics Division

K. L. Heimburg

K. L. HEIMBURG
Director, Astronautics
Laboratory

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