

OPTICAL PROPAGATION IN A NEAR-EARTH ENVIRONMENT

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ABSTRACT

This paper describes optical measurements of the magnitude of scintillation as a function of the range and refractive index structure coefficient, C_n . The results are compared with the Tatarski and deWolf geometrical optics saturation equations. In addition, a variation in the Kolmogorov turbulence model is suggested to account for certain observed optical and meteorological parameters.

1. INTRODUCTION

During the past year, experiments were initiated to test the well-known log-amplitude variance equation of Tatarski, describing the effect of scintillation. Tests made first with a pulsed ruby laser as a source were later extended to include measurements with a helium-neon laser. During a majority of the tests, the refractive index structure coefficient, C_n , was measured by means of a high-speed, temperature-differential system. These experiments clearly show a saturation of scintillation magnitude with increasing range and C_n . In addition, some of the characteristics of propagation under temperature inversion conditions were investigated. A change in the Kolmogorov turbulence model is suggested to account for certain observed optical and meteorological parameters.

2. THEORY AND BACKGROUND

Central to the development of the log-amplitude variance equation is the employment by Tatarski of the Rytov approximation. Higher order terms in the Born series formulation of the wave equation are dropped. Thus Tatarski (1960) obtains in this approximation an expression giving the log-intensity variance of a plane wave as

$$\sigma^2 = 1.23 C_n^2 k^{7/6} L^{11/6} \quad (1)$$

where C_n is the index structure coefficient, k is the wave number of the light, and L is the path length. The solution of the wave equation for spherical wave propagation using the same approximations, given by Fried (1967), is identical to Eq. (1) except for the constant which is evaluated as 0.496.

This formulation went essentially untested until experiments were performed by Gracheva and Gurvich (1965) in which temporal fluctuations of intensity were monitored over ranges from 125 to 1750 meters using a mercury arc source. Simultaneous with the optical data, Richardson's number was measured. Through this parameter, C_n was inferred by a method reported by Tsvang (1962). These experiments showed that the variance did not increase indefinitely with L and C_n , but reached a maximum. Tatarski (1967) developed formulation to show that saturation could be achieved theoretically. One such evaluation gives the log-intensity variance as

$$\sigma^2 = 4\{1 - (1 + 6\sigma_1^2)^{-1/6}\} \quad (2)$$

where

$$\sigma_1^2 = \frac{\pi^2 \cdot 0.033 \Gamma(7/6)}{24} C_n^2 L^3 k_m^{7/3} \quad (3)$$

Here Γ indicates a gamma function, C_n is the index structure coefficient, L is the path length, and k_m is the Obukov scale. Tatarski finds the product of k_m and the inner scale, l_0 , equal to 5.48.

More recently, deWolf (1968) has predicted a saturation of scintillation magnitude by evaluating higher order terms in the Born series formulation of the scalar integral wave equation. He gives the variance as

$$\sigma^2 = \ln\{2 - \exp(-\sigma_1^2)\} \quad (4)$$

where σ_1^2 is the variance for the plane wave solution given earlier in Eq. (1).

3. EXPERIMENTAL METHOD

The optical measurement method employed here utilizes a photo-optical technique to record directly a 61-cm cross section of a received optical beam. The receiving system used is essentially a telescope with the image plane located at a position conjugate to the entrance aperture of the system rather than to the far field. Thus, a photograph is made of the distribution of energy across a 61-cm aperture. After appropriate calibration, the film is scanned with a densitometer and computer-corrected to give unidirectional scans of intensity versus distance (in object space). The scanning aperture of the densitometer corresponds to an aperture a few millimeters in object space so that the highest spatial frequencies can be resolved.

In order to measure C_n , instrumentation was utilized which incorporates two high-speed thermometers. These are arranged in a Wheatstone bridge circuit to give a difference signal proportional to the temperature gradient between the two probes. The RMS value of this signal can be used to compute a mean value of C_T , the thermal structure coefficient, and hence C_n .

4. MEASUREMENT RESULTS

A pulsed ruby laser was used to make measurements over ranges of 200 to 1500 meters at a beam height of 2 meters above ground. Figure 1 shows a typical cross-section photograph taken at a range of 1000 meters. The magnification of the image can be judged from the diameter of the observable field (61 cm). One-dimensional scans from such photographs, converted to energy as a function of distance, were used to compute a variance using the formulation

$$\sigma^2 = \ln\left\{1 + \frac{\sigma_1^2}{(\bar{I})^2}\right\} \quad (5)$$

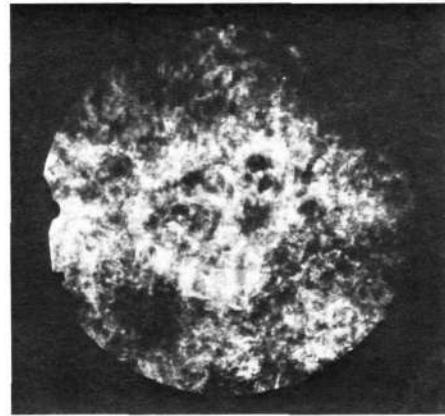


Figure 1. Cross Section of Pulsed Ruby Laser at 1000 Meters

where σ_1 is the normal variance and \bar{I} is the mean intensity.

Tests were made in which the log-intensity variance was measured for various path length and C_n values. The results of these tests are shown in Figure 2. Here the measured standard deviations, derived using Eq. (5), are plotted against the square root of Eq. (1), corrected to the Fried coefficient. The Tatarski and deWolf formulations (Eqs. (2) and (4)) are also shown. One of the indications

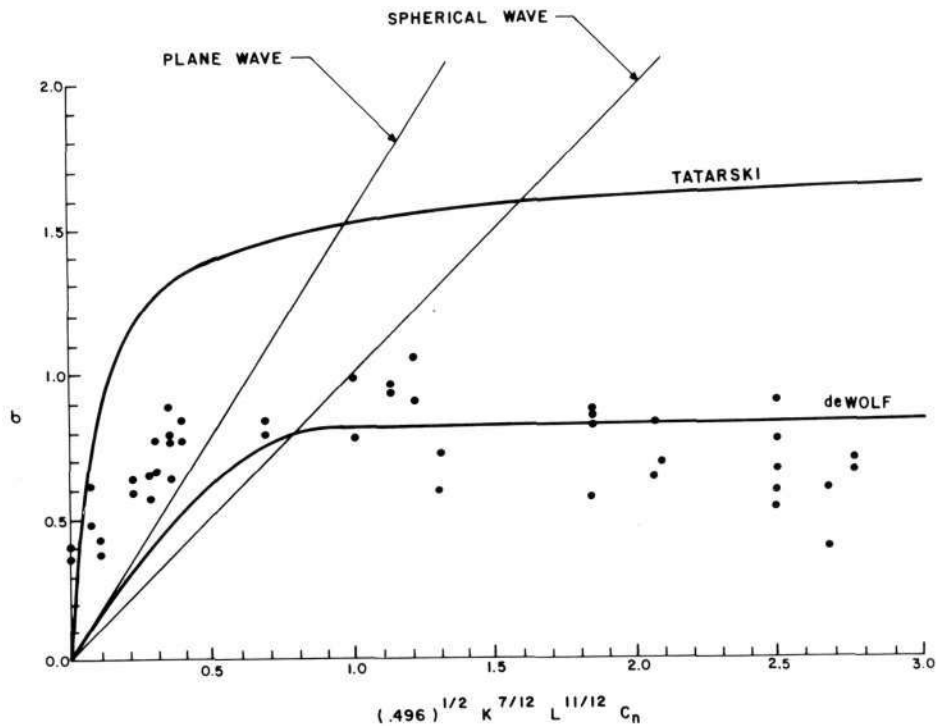


Figure 2. Measured Standard Deviations as a Function of Range, L, and Refractive Index Structure Coefficient, C_n

of these data is that beyond a range of about 500 meters (depending on the magnitude of C_n) the strength of the optical turbulence (C_n) cannot be quantified by an optical measurement using, for instance, Eq. (1).

5. METEOROLOGICAL MODEL

The cross sectional energy distribution in an optical beam (such as indicated in Figure 1) may be thought of as the realization of the spectral interaction of an optical filter function and a three-dimensional spectrum of turbulence. Tatarski (1960: see Fig. 11, p. 140) has formulated an optical filter function for an infinite plane wave which acts as a high bandpass filter on the turbulence spectrum (which is a low bandpass filter) given by Kolmogorov (1941). Carlson and Ishimaru (1968) have indicated that the optical filter function for beam-wave propagation is different than for infinite wave propagation; i.e., for high wave numbers the optical filter function does not damp to unity, but rather peaks at some wave number and then diminishes to zero. The peak of the filter function is set by the beam diameter of the diffraction-limited transmitter and moves to lower wave numbers for larger diameters. Thus, depending on the beam width, various turbulence domains are sharply weighted as to their contribution to the beam modulation. These results suggest a method of probing the index medium to ascertain the strength of fluctuations in various turbulence domains (integrated over the optical path within the range of saturation).

The importance of these spectral interactions is indicated by the character of data gathered using a helium-neon laser. Figure 3 shows a beam cross-section photograph made of a CW laser (2-millisecond exposure) at a range of 650 meters. This picture was taken during a summer afternoon under high temperature lapse conditions. The image is composed of small intensity cells with sharp edges.

Figure 4 shows a photograph made with the identical optical configuration after nightfall during a temperature inversion. There are pronounced variations in the intensity regions, but the scale for the variations is larger and the edges of the intensity cells are not as sharp. In addition, a low-contrast fringe pattern can be observed in the image. This effect is caused by an interference filter element in the optical receiver which is acting as a Fabry-Perot

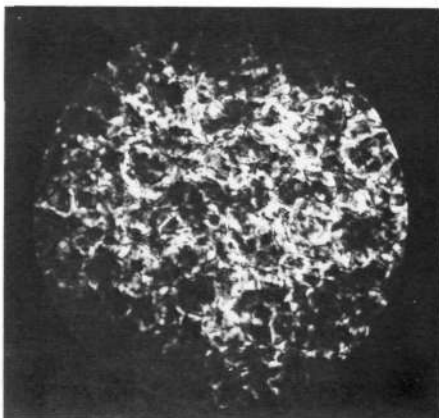


Figure 3. Cross Section of Helium-Neon Laser at 650 Meters, Lapse Conditions

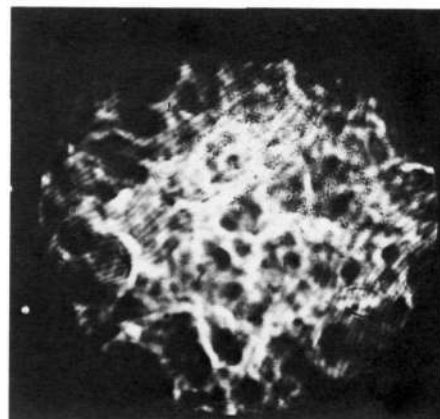


Figure 4. Cross Section of Helium-Neon Laser at 650 Meters, Inversion Conditions

etalon. The fringes indicate a large transverse coherence width made observable by a large temporal coherence length. Secondly, there are diffraction patterns of high spatial frequency in the upper right-hand corner of the image due apparently to dust particles or a smudge on one of the lenses of the receiving system.

It may be conjectured that a change in the statistics of the turbulence is responsible for the variation in the optical parameters. Measurements taken with the C_n apparatus under inversion conditions indicate an outer scale of turbulence (here meant to be the largest scale size for which the "two-thirds law" applies) on the order of or less than 30 cm. It can be expected that as the outer scale of turbulence decreases less energy is injected into the medium, and viscous forces tend to dissipate the energy at a larger inner scale. Measurements of temperature spectra by Tsvang (1960) lend support to this view. During inversion conditions, both a decrease in the outer scale and an enlargement of the inner scale are indicated.

Such a change in the Kolmogorov model may explain the observed optical characteristics. The amplitude and phase filter functions of Tatarski (1960: Eqs. (7.87) and (7.88), p. 151) indicate that the high wave number turbulence contributes most to amplitude fluctuations and that the low wave number turbulence contributes most to phase fluctuations. An expansion of the inner scale will tend to increase the intensity correlation interval. Spatial correlations of beam photographs have shown this shift. A reduction in size of the outer scale will tend to increase the dimension over which the phase front is unperturbed. As this dimension approaches the size of the diffracting object (dust particle or lens anomaly referenced to object space), diffraction images will begin to appear.

6. SUMMARY

Log-intensity variances have been measured for optical paths from 200 to 1500 meters over a range of C_n values. For high C_n conditions, the variance is seen to saturate at about 500 meters. Beyond this range, the optical variance cannot be used to compute a strength parameter for the turbulence.

In addition, the significance of the optical filter function and the Kolmogorov turbulence model is discussed. A modification of the Kolmogorov spectrum is suggested to account for the character of optical data gathered during temperature inversion conditions. Because of the close relationship between the optical propagation effects and the underlying meteorology, the importance is stressed of performing careful experiments in which both meteorological and optical parameters are examined.

7. ACKNOWLEDGMENT

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