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REMOTE PROBING OF THE OPTICAL STRENGTH
OF ATMOSPHERIC TURBULENCE AND OF WIND VELOCITY

by

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ABSTRACT

A procedure for determining the optical strength of turbulence of the atmosphere and the wind velocity at various altitudes by measuring the spatial and temporal covariance of scintillation is developed. Emphasis is placed on the development of the formal relationships that have to be inverted to obtain the desired results. For determination of optical strength of turbulence, it is a linear integral equation that is developed. However, for determination of remote wind velocity, a nonlinear integral equation is obtained. A computer approach for solving each of the equations is suggested. The configuration and performance requirements of the measurement apparatus are discussed.

1. INTRODUCTION

It is the objective of this paper to explore analytic techniques for utilizing measurements of turbulence-induced effects on optical propagation as a basis for remote probing of the atmosphere. Our attention is centered on the problem of vertical rather than horizontal probing, although the analytic techniques are sufficiently general that they might be applied with certain changes to either case.

In the interest of establishing the basis for an economical and a truly remote probing technique, we have eschewed the possible use of a balloon terminal either as an optical target or as one end of an optical propagation link. We have restricted our attention to the use of a stellar source, and have placed heavy reliance on the potential of a sophisticated computer-signal-processing oriented ground station. With this approach, we have not only been able to establish the basis for remote measurement of the optical strength of atmospheric turbulence-- a quantity of great interest to those of us concerned with optical propagation in the atmosphere, but have also been able to lay the conceptual and analytic groundwork for remote measurement of wind velocities aloft -- a matter of very real interest in a variety of fields of endeavor. Our work on the remote measurement

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of wind velocity is, in a sense, an attempt to formalize and extend the earlier work of Barnhart, Keller, and Mitchell (1959), and to lift the requirement for their assumption that all stellar scintillation is generated by turbulence in a fairly narrow altitude range with a single characteristic wind velocity.

We have approached our problem in two steps: first, by studying the relationship between the spatial covariance of scintillation, as measured by the conventional log-amplitude covariance, and the distribution along the propagation path of the optical strength of turbulence, as measured by the refractive-index structure constant. From this examination, we have found that we can obtain a linear integral equation relating the two functions. This considers the log-amplitude covariance as the known function -- it is a quantity we plan to measure -- and treats the refractive-index structure constant as an unknown function of position along the path of propagation. By inverting this integral equation, we are able to solve directly for the optical strength of turbulence along the propagation path. When the propagation path is that for star light, we have, in effect, a vertical probe of the atmosphere.

In the second part of our approach to the remote probing problem, to obtain information on the wind velocities in the atmosphere, we have generalized our interest in the log-amplitude covariance function to include its temporal as well as spatial dependence. The temporal dependence provides us with a "handle" on the wind velocities. (It is an interesting feature of a computer-oriented signal processing set-up that obtaining the temporal as well as the spatial dependence of the log-amplitude covariance, as compared with obtaining only the spatial dependence, requires only a change in the computer program and no additional hardware or data-taking time.) In this case again, an integral equation is obtained relating the spatial and temporal dependence of the log-amplitude covariance function to the wind velocity function. In this treatment, the wind velocity function is a vector function with functional dependence on position along the propagation path. (The vector function is the two-dimensional vector projection of the actual wind velocity vector on a plane perpendicular to the propagation direction. A correction for the projection of the actual wind feature can be introduced after the vector function has been obtained, by assuming that the true winds are horizontal, or nearly so.) It is an awkward aspect of this part of the work that the integral equation from which we expect to obtain the wind velocity function is very nonlinear. As a consequence, the task of inverting the equation, even approximately, on a computer is expected to be formidable. However, we see no reason in principle that the integral equation can not be adequately approximated by a set of nonlinear simultaneous equations and these solved by the use of relaxation techniques. With a modern, high speed computer, this should be quite economical.

In the next section, we shall define the quantities of interest with sufficient detail to provide a basis for planning their measurements. We will follow this with a pair of sections that develop the formal relationships of interest. Finally, we shall devote several sections to problems related to consideration of the actual implementation of the necessary measurement processes.

2. DEFINITIONS

Basically, the measurements we intend to make are of the irradiance produced by star light from some star at a variety of points with various separations. The measurements are to be continuous in time so that we have data on the temporal as

well as the spatial aspects of the star light irradiance. Although we will get into more detail on the measurement equipment configuration in a subsequent section, for the present it is sufficient to consider that we are dealing with a pair of photoelectric detection telescopes, each of whose signals is recorded on magnetic tape for subsequent computer data processing. Each telescope has a very restricted field-of-view, which is guided by the telescope mount to keep the star of interest in the field-of-view. The two telescopes are placed on the same mount in such a manner that the separation of their collection apertures can be adjusted. The collection apertures are intended to be of zero dimension, although for practical reasons a one-millimeter aperture is contemplated. The optical train in each telescope contains an interference filter which permits only radiation in a narrow spectral band to be observed. Ideally, the band should be of zero width, but for practical reasons we plan to use a 10% bandwidth. It can be shown (Fried (1967)) that the effect of using a non-zero spectral bandwidth is virtually negligible as far as scintillation effects are concerned, although it may be desirable at some later date to incorporate any slight changes in theory that go with the non-monochromatic signal. For this paper we shall use a monochromatic theory without reservation. We denote the center of the wavelength band by λ , with an associated wavenumber, $k = 2\pi/\lambda$.

We shall use the notation $\vec{\rho}$ to denote the vector separation between the pair of miniature telescope collection apertures. We will let z denote a distance from the telescope along the path of propagation. Working with a stellar source at a zenith angle θ , the height of a point at z is approximately $h = z \cos \theta$.

We denote the photocurrent out of each of the telescopes by $i_1(t)$ and $i_2(t)$, t being the time at which the photocurrent is observed. The logarithmic quantities $\ell_1(t)$ and $\ell_2(t)$, called the log-amplitudes, are computed from the relationships

$$\ell_1(t) = 1/2 \ln [i_1(t) / \langle i_1 \rangle] , \quad (1a)$$

$$\ell_2(t) = 1/2 \ln [i_2(t) / \langle i_2 \rangle] , \quad (1b)$$

where $\langle i_1 \rangle$ and $\langle i_2 \rangle$ are the average values of $i_1(t)$ and $i_2(t)$, respectively. (The angle brackets $\langle \rangle$ are used here and hereafter to denote an ensemble average, although in practice we shall feel free to invoke the ergodic hypothesis and consider the brackets to denote a time average.) From $\ell_1(t)$ and $\ell_2(t)$ we calculate the spatial-temporal log-amplitude covariance, $C_\ell(\vec{\rho}, \tau)$, according to the equation

$$C_\ell(\vec{\rho}, \tau) = \langle [\ell_1(t) - \langle \ell_1 \rangle] [\ell_2(t + \tau) - \langle \ell_2 \rangle] \rangle . \quad (2)$$

For convenience in dealing with the spatial dependence of log-amplitude covariance for zero time delay, we shall use the notation $C_\ell(\rho)$ rather than $C_\ell(\vec{\rho}, 0)$. Because of the isotropy of the statistics of turbulence, we are able to suppress the dependence on the vector aspect of $\vec{\rho}$ and merely show a dependence on the scalar value ρ . (For a non-zero value of τ the vector wind velocity becomes significant and it is the interaction of this vector with $\vec{\rho}$ that required the dependence on the vector nature of $\vec{\rho}$.)

The optical strength of turbulence is measured by the refractive-index structure constant. The refractive-index structure constant is defined in relation to the

Kolmogorov theory of turbulence in the inertial subrange. According to this theory, the difference of two turbulently varying quantities has a mean square value which varies in proportion to the two-thirds power of their separation. The refractive-index structure constant is the constant of proportionality when the turbulently varying quantity is the refractive index. We denote the refractive-index structure constant at the position z along the propagation path by $C_N^2(z)$. At present there is very little data on the value of C_N^2 as a function of altitude and effectively no data on how it varies with time of day and year, and with geographic location, or even how and if it varies statistically under apparently similar conditions. It is the first objective of the suggested measurements to permit compilation of data on C_N^2 at various altitudes.

We shall use the notation $\vec{V}(z)$ to denote the projection of the actual wind velocity vector at a distance z along the propagation path upon a plane perpendicular to the propagation direction. We shall speak of this as the wind velocity, although in all cases we shall eventually want to know what the vector in a horizontal plane is whose projection is $\vec{V}(z)$, for that vector is the true wind velocity. It is the second objective of the stellar scintillation measurements that we recommend, to provide data from which $\vec{V}(z)$, and ultimately the true wind velocity, can be determined.

3. DETERMINATION OF THE REFRACTIVE-INDEX STRUCTURE CONSTANT

In this section, we will establish the basic mathematical relationship between the refractive-index structure constant and the log-amplitude covariance for zero time delay, i.e., for $\tau = 0$. This will eventually permit determination of C_N^2 from stellar scintillation data. We start with the basic relationship obtainable from Tatarski (1961), that

$$C_\ell(\rho) = 0.652 k^2 \int_0^L dz C_N^2(z) \int_0^\infty d\sigma \sigma^{-8/3} J_0(\sigma\rho) \left(1 - \cos \frac{\sigma^2 z}{k}\right) \quad (3)$$

which applies for propagation of an infinite plane wave* of wavenumber $k = 2\pi/\lambda$ traveling a distance L . (The z -integration is from collector to source.) For a stellar source we replace L by infinity, but keep carefully in mind the fact that C_N^2 falls to zero about as rapidly as the square of the atmospheric density, so there is no problem of convergence of the integral.

It is a surprising fact that the double integral in Eq. (3) has never been evaluated by doing the σ -integration first. (For some reason an assumption about the z -dependence of C_N^2 has always been made and the z -integration performed first.) This time we wish to perform the σ -integration first. This will yield some function of k , z , and ρ , which will appear as the kernel in the z -integration side of an integral equation connecting $C_\ell(\rho)$ and $C_N^2(z)$.

* It should be carefully noted that most ground-to-ground propagation paths involve a source quite different from an infinite plane wave source and that the results developed here from Eq. (3) will be grossly inadequate for interpretation of such measurements. Often the theory for a spherical wave source will be applicable.

To perform the σ -integration, we use the two integral formulas that

$$\int_0^{\infty} u^{\alpha} J_0(\beta u^{1/2}) du = \left(\frac{4}{\beta^2}\right)^{\alpha+1} \frac{\Gamma(\alpha+1)}{\Gamma(-\alpha)}, \quad (4a)$$

and that

$$\int_0^{\infty} u^{\alpha-1} \cos u J_0(\beta u^{1/2}) du = \Gamma(\alpha) \operatorname{Re} \left[\exp\left(-i\frac{\pi\alpha}{2}\right) {}_1F_1\left(\alpha; 1; i\frac{\beta^2}{4}\right) \right]. \quad (4b)$$

Eq. (4b), which apparently does not exist in any of the tables of definite integrals, can be obtained by a slight modification of the application of Ramanujan's formula presented in Appendix A of the paper by Fried and Cloud (1966). In fact, it can be recognized as the conjugate result to that developed in the reference. Eq. (4a) comes directly from that reference.

By making the transformation of $u = \sigma^2 z/k$ in Eq. (3), and replacing L by ∞ to restrict our attention to propagation of star light, we get

$$C_{\mathcal{L}}(\rho) = 0.326 k^{7/6} \int_0^{\infty} dz z^{5/6} C_N^2(z) \int_0^{\infty} du u^{-11/6} J_0\left(\sqrt{\frac{k\rho^2}{z}} u\right) \times (1 - \cos u). \quad (5)$$

With the aid of Eq.'s (4a) and (4b), we perform the u -integration, thus obtaining

$$C_{\mathcal{L}}(\rho) = 0.326 k^{7/6} \int_0^{\infty} dz z^{5/6} C_N^2(z) \left\{ \left(\frac{k\rho^2}{4z}\right) \frac{\Gamma(-5/6)}{\Gamma(11/6)} - \Gamma(-5/6) \operatorname{Re} \left[\exp\left(i\frac{5\pi}{12}\right) {}_1F_1\left(-5/6; 1; i\frac{k\rho^2}{4z}\right) \right] \right\}. \quad (6)$$

We recognize that everything in the curly brackets in Eq. (6) may simply be considered as some function of $k\rho^2/4z$. Thus we can rewrite Eq. (6) in the desired integral equation form, namely

$$C_{\mathcal{L}}(\rho) = k^{7/6} \int_0^{\infty} dz z^{5/6} C_N^2(z) \mathfrak{F}\left(\frac{k\rho^2}{4z}\right), \quad (7)$$

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where the function $\mathfrak{F}(x)$ is defined in terms of the power series

$$\mathfrak{F}(x) = \frac{-x^{5/6}}{\Gamma(11/6)} + \sum_{n=0}^{\infty} (a_n + b_n x) x^{2n}/(2n)! \quad (8)$$

The power series coefficients are defined by the recurrence relationships

$$a_n = -a_{n-1} \left[\frac{(2n - 17/6)(2n - 11/6)}{(2n - 1)(2n)} \right], \quad (9a)$$

$$b_n = -b_{n-1} \left[\frac{(2n - 11/6)(2n - 5/6)(2n - 1)}{(2n)(2n + 1)^2} \right], \quad (9b)$$

and the initial values

$$a_0 = (-0.326) \cos(5\pi/12) \Gamma(-5/6) \quad (10a)$$

$$b_0 = (11/6) (-0.326) \sin(5\pi/12) \Gamma(-5/6). \quad (10b)$$

The first few values of a_n and b_n are listed in Table I.

TABLE I
Coefficients for the Power Series Determination of
 $\mathfrak{F}(x)$ According to Eq. (8)

The notation a(b) denotes $a \times 10^b$

n	a_n	b_n
0	5.403 (-1)	3.696 (0)
1	3.752 (-2)	-3.993 (-2)
2	-7.903 (-3)	8.219 (-3)
3	3.476 (-3)	-3.009 (-3)
4	-1.978 (-3)	1.437 (-4)
5	1.286 (-3)	-7.999 (-4)
6	-9.080 (-4)	4.926 (-4)
7	6.778 (-4)	-3.257 (-4)
8	-5.268 (-4)	2.270 (-4)
9	4.221 (-4)	-1.648 (-4)
10	-3.464 (-4)	1.236 (-4)

From this point on, we may consider $\mathfrak{F}(x)$ to be a known and tabulated function, which makes Eq. (7) a very straightforward linear integral equation connecting the measured function, $C_\ell(\rho)$ to the function $C_N^2(z)$, which is to be determined. As an integral equation, we have somewhat of a problem in obtaining $C_N^2(z)$. However, by replacing the integration in Eq. (7) with a summation over a set of finite ranges of z , and by replacing the functions $C_\ell(\rho)$ and $C_N^2(z)$ with a finite set of values, known or to be determined, we convert Eq. (7) to a set of linear simultaneous equations. This is an easy matter for a computer to handle and we can expect data on $C_N^2(z)$ whose quality is limited only by how well we have chosen the set of ρ 's for measurement of $C_\ell(\rho)$, by how high our signal-to-noise ratio is for determination of $C_\ell(\rho)$, and by how effectively we designate the ranges of z in replacing the integration by a summation. The choice of ranges will depend on our a priori estimate of how $C_N^2(z)$ varies with altitude. This should be nearly proportional to the variation of the square of the density of the atmosphere -- with allowance for a possible anomaly in the vicinity of the tropopause, as has recently been suggested by Hufnagel (1966). It will also depend on how $\mathfrak{F}(x)$ varies and on our choice of values of ρ . Optimally, the choice of the set of ρ 's will be governed by the values of z for which $C_N^2(z)$ is desired. For initial work, however, we expect that almost any choice of values for ρ for the measurement of $C_\ell(\rho)$ and any reasonable separation of the range of z to replace the integral with a sum will yield a reasonably representative set of results. The really important matter will be the signal-to-noise ratio achieved in measuring $C_\ell(\rho)$. Of all of the adjustable features of the data-taking and data reduction procedure, this will have the most pronounced effect on the quality of the set of C_N^2 that we obtain. The signal-to-noise ratio can be changed by varying the length of the data-taking. We shall have to be careful in the design of the actual experiment that we have enough equipment to take all the data we need in a short enough total period, and yet allow enough time for each measurement.

4. DETERMINATION OF WIND VELOCITY

For this section, let us assume that the procedure suggested in the previous section has been successfully implemented -- that measurements of the log-amplitude covariance, $C_\ell(\rho)$ have been obtained and that the appropriate linear simultaneous equations have been solved so that we now have accurate results for the refractive-index structure constant, $C_N^2(z)$. We now wish to consider what we could do with measurements of the temporal dependence of the refractive-index structure constant, i.e., with $C_\ell(\rho, \tau)$ to compute the projected wind velocity, $\vec{V}(z)$. The key equation to carry this out is the time-dependent version of Eq. (3), which has the form

$$C_\ell(\vec{\rho}, \tau) = 0.652 k^2 \int_0^\infty dz C_N^2(z) \int_0^\infty d\sigma \sigma^{-8/3} J_0(\sigma |\vec{\rho} - \vec{V}(z) \tau|) \times (1 - \cos \frac{\sigma^2 z}{k}). \quad (11)$$

This equation does not appear in any of the published literature. Although the most straightforward way to obtain it is to derive it in detail from the stochastic propagation equations for log-amplitude at points 1 and 2 and times t and $t + \tau$,

utilizing the hypothesis of frozen turbulence (i.e., that the wind transports the turbulent structure across the propagation path more rapidly than the structure breaks up and reforms), for the purpose of this presentation we shall argue that Eq. (11) follows as an obvious extension of Eq. (3) when we consider the contribution to the log-amplitude of the turbulence at any particular value of z . With the hypothesis of frozen turbulence, we note the equivalence of the contribution to the scintillation pattern at time (t) and position (\vec{R}) , to the contribution at time $(t + \tau)$ and position $(\vec{R} + \vec{V}\tau)$. With this in mind, the Eq. (11) follows directly from Eq. (3).

We can now use the same mathematical manipulations as in Section 3. This time they yield the result, in analogy to Eq. (7), that

$$C_{\ell}(\vec{\rho}, \tau) = k^{7/6} \int_0^{\infty} dz z^{5/6} C_N^2(z) \mathfrak{F}\left(\frac{k|\vec{\rho} - \vec{V}(z)\tau|}{4z}\right) . \quad (12)$$

Recognizing that now our problem is, given $C_{\ell}(\vec{\rho}, \tau)$ and $C_N^2(z)$ -- solve for $\vec{V}(z)$, we see that we are dealing with a rather formidable nonlinear integral equation. If we had to solve this equation, in the formal sense, it is doubtful that any further progress could be made on the problem. However, the use of numerical techniques reduces the problem to one of approximation, and that we may expect to be able to accomplish. By replacing the integration with a summation over a set of ranges of z , we obtain a set of nonlinear simultaneous equations to solve. Although there is no closed form solution possible for the set of general nonlinear simultaneous equations, as there is for the linear simultaneous equations, (for which matrix inversion produces a solution-generating operator), by the use of a relaxation procedure or some other iterative method, it should be possible to solve for $|\vec{\rho} - \vec{V}(z)\tau|$ as a function of z . How quickly the solution procedure will converge will depend on the quality of the starting estimate for $\vec{V}(z)$.

The calculations would start with data on $C_N^2(z)$ obtained from measurements of $C_{\ell}(\vec{\rho})$ and would probably use the data for $C_{\ell}(\vec{\rho}, \tau)$ and $C_{\ell}(\vec{\rho}', \tau)$ where $\vec{\rho}$ and $\vec{\rho}'$ have the same magnitude but are perpendicular. Values of τ would range from $\tau = 0$ to a value of τ sufficiently large as to make $C_{\ell}(\vec{\rho}, \tau)$ vanish. This will be sufficient to permit calculation of $\vec{V}(z)$. Since data for other values of $\vec{\rho}$ and $\vec{\rho}'$ will be available, these measurements will also be processed to provide a consistency check and to improve the accuracy of the solutions for $\vec{V}(z)$.

5. MEASUREMENT OF SIGNAL-TO-NOISE RATIO

At this point, it is appropriate to examine the actual measurement and determine what kind of signal-to-noise ratio we can expect. Stellar scintillation measurements have previously been done with optics of one or more inches diameter. For order of magnitude evaluation of the intensity covariance, such a large aperture might have been acceptable, and it certainly helped the signal-to-noise ratio; but for the precision application which we have in mind for the data, we must plan to use much smaller apertures. Our apertures must be small enough that they do not average over the irradiance pattern. Examining the data in reference 1, it appears that a one-millimeter diameter would be suitable. As we have indicated

earlier, to avoid the complication in the theory of spectral spread, we plan to use a fairly narrow spectral bandwidth. Based on the results in reference 2, we have elected a 10% bandwidth at $5250 \text{ \AA} \pm 250 \text{ \AA}$. To be certain that we do not miss any of the rapid scintillation fluctuations, we plan to use a 1 kHz electronic bandwidth. These three quantities, the aperture diameter, the spectral bandwidth, and the electronic bandwidth, virtually determine the signal-to-noise ratio we will have on the photocurrent.

We plan to use an S-20 photocathode image dissector type photomultiplier, with a quantum efficiency in this spectral range of $\eta = 4 \times 10^{-2}$ amps/watt. The use of the image dissector is simply to provide a very small photocathode area and in that way reduce the dark current to a negligible amount. By working with a very limited field-of-view, we can keep the background photocurrent down to a negligible amount even for daytime operation.

Since we are using a photomultiplier, there will be no significant amount of noise generated after the photocathode, so we can identify the shot noise in the average stellar current as the only noise. If the average stellar signal photocurrent is I , then the signal-to-noise ratio in dB will be

$$S/N = 10 \log_{10} (I/2 e \Delta f) \quad (13)$$

where $e = 1.6 \times 10^{-19}$ coulombs is the electron charge and $\Delta f = 10^3$ Hz is the electronics bandwidth. We consider as typical stellar sources the stars Vega or Arcturus, both of nearly zero-magnitude. A zero-magnitude star produces an irradiance of about

$$W = 10^{-11} \text{ watts/m}^2 - \text{\AA}$$

in the spectral range of interest. With combined atmospheric and optics transmission of $T = .5$, a collector diameter $D = 10^{-3}$ m, and a spectral bandwidth $\Delta\lambda = 500 \text{ \AA}$, we see that the stellar signal average photocurrent will be

$$I = \eta \left(\frac{\pi}{4} D^2 \right) \Delta\lambda T W, \quad (14)$$

from which it follows that the photocurrent is $I \approx 7.9 \times 10^{-17}$ amps. This yields a signal-to-noise ratio of -6.1 dB.

At first one is inclined to say that this proves that the measurement can't be made -- there isn't enough signal-to-noise ratio. However, we must remember that we are trying to make an accurate determination of the log-amplitude covariance, not of the instantaneously fluctuating signal! To see the difference, consider the fact that every one of the two thousand samples per second that goes with $\Delta f = 10^3$ Hz contributed coherently to the final value of the log-amplitude covariance itself, but the noise in each sample, being independent of the noise in each other sample, contribute in quadrature to the error in the measured covariance.

After 300 seconds of operation, we may expect that the noise voltage will be up above the instantaneous voltage by $\sqrt{300 \times 2000} \approx 800$, while the covariance voltage will accumulate about $(800)^2$ for a net improvement in signal-to-noise ratio of about 58.1 dB. Thus, after five minutes of operation we should have a value of the log-amplitude covariance which has a signal-to-noise ratio of about 52 dB. This should be entirely adequate.

6. MEASUREMENT PROCEDURE

In the preceding sections, we have spoken of the measurement of the log-amplitude covariance, $C_L(\vec{\rho}, \tau)$ in terms of the use of a pair of one-millimeter aperture photoelectric telescopes on a tracking mount and pointed at a star. The implication was that the signals from the two telescopes were directly correlated by some suitable electronic device. In practice, this would be much too slow and inefficient a procedure. By the time a complete set of data was obtained, so much time would have elapsed that it is doubtful that conditions would be the same for the first and last part of the set. It is questionable whether or not there would be any self-consistency within the data set. Instead of such a one-at-a-time type of measurement, we visualize the entire set of data being obtained simultaneously in about five minutes of running time.

There would be no real-time conversion of the scintillation signals to log-amplitude covariances. Instead, all of the raw data would be stored on magnetic tape and later entered into and reduced in a digital computer. This would, amongst other things, permit a limited number of telescopes to provide data for calculation of log-amplitude covariance for a much larger number of combinations of values of $\vec{\rho}$ and τ . The output from any one telescope can be paired in the computer with several other telescopes to provide data for several values of $\vec{\rho}$. For example, a string of thirteen telescopes spaced 1, 1, 1, 1, 1, 1, 7, 7, 7, 7, 7, 7 units between each would permit the computer to select pairs covering any separation from one-unit to 48-units apart -- 48 values of $\vec{\rho}$ covered with thirteen telescopes. Another string of thirteen crosswise to the first string (using a common element so there is a total of only 25-telescopes) would provide all possible data of interest. The computer, by shifting the data from any telescope, can simulate a time delay τ and in that way calculate all the time dependent features of the log-amplitude covariance without increasing the equipment or measurement time requirements above the requirements for determination of just $C_L(\rho)$.

The tape recorder capacity to accommodate all of this data is not very extensive. By using three twenty-channel multiplexers, we can record all of the data on three 20 kHz FM tape recorder channels, with a blank space recorded between each signal sample, just to insure the impossibility of data crosstalk between telescopes.

Although we have spoken of the equipment as a collection of 25 one-millimeter diameter telescopes on a common mount, with the implication that each unit involved separate optics and separate boresighting, in practice it will probably prove to be much more economical to use a single large telescope, perhaps twelve inches in diameter, with a single narrow band filter and focal plane stop to define the limited field-of-view. The light passing through the focal plane stop would be allowed to diverge and when it had reached some suitably large diameter, would be sampled by positioning the 25 photomultipliers (or by positioning 25 one-millimeter diameter diagonal mirrors or fiber optics tubes to "tap-off" the light for the photodetectors.) The balance of the light could be used to provide a signal for tracking the star. The total configuration would be relatively simple and trouble-free. There would be, for instance, no significant pointing problems for any of the individual sensors.

7. COMMENTS

All of this analysis is based on two assumptions which, while we believe they are satisfied, should be pointed out here. First, all of our analysis assumes that the Kolmogorov spectrum for turbulence in the inertial subrange is the appropriate spectrum. There is quite reliable evidence to support this for the high spatial frequencies of interest in optical propagation, for regions near the ground. For much lower spatial frequencies, this has also been demonstrated for higher altitudes. While it seems quite plausible to extend this information to higher altitudes with the higher spatial frequencies associated with optical propagation, it must be recognized that the applicability of the Kolmogorov spectrum is not demonstrated by any measurements.

The second assumption is that the theory for optical propagation in a randomly inhomogeneous medium, as we have used it, is valid. Recently there has been considerable controversy about this. However, it is almost universally agreed that for small enough scintillation effects, the results of theory should be valid. As Hulett (1967) has pointed out, measured values of scintillation for stars near the zenith are quite small. For this reason, we believe there is no reason to question the accuracy of propagation theory as we have applied it in this paper.

We believe that a firm foundation exists for exploiting our understanding of optical propagation and the flexibility of modern data handling electronics to develop an economical method of remote probing of the atmosphere. The expense of development of this method will be fully justified by the cost reduction in obtaining winds aloft data. We further believe that the reduced cost and the ease with which this data is obtained will result in expanded coverage of winds aloft measurements and will thereby promote a better understanding of meteorological interactions.

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CRITIQUE OF PAPER BY D. L. FRIED

R. E. Hufnagel

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Dr. Fried is to be congratulated for his skill and perseverance in solving this difficult inversion problem. A few months before this meeting, I learned of a similar work by Arthur Peskoff (1968) who has independently solved the same problem. It will be interesting to compare these two methods. My comments below apply to both methods.

I believe that it will be very difficult in practice to get accurate and detailed inversions from real-life experimental data. To explain why this might be so, we note, first, that all scintillation experimental data tend to look somewhat alike (thus implying that it can't contain much information of interest). To illustrate this, we need only note that the autocorrelation function for scintillation as computed by Chernov (1960) using a Gaussian shaped turbulence spectrum is very similar to the scintillation autocorrelation function obtained by Tatarsky (1961) who used a Kolmogoroff description of turbulence. A second difficulty arises from the often observed nonstationarity of turbulence statistics.* This means that one cannot reliably average observations over long time periods to gain effective signal to noise enhancement. On the other hand, since the theory in its proposed form requires the use of small diameter telescopes, and a narrow spectral filter, the noise problem will be severe, and accurate inversion will be difficult. Still another difficulty is that present inversion theories assume isotropic Kolmogoroff turbulence statistics. It is very likely that above the atmospheric boundary layer that these statistics are often not valid -- especially in thin turbulent strata, which do appear to exist (Hufnagel, 1966).

On a constructive note, however, there are some ways in which we can make this remote sensing method more accurate and reliable. The first and foremost principle to employ is to make use of all available *a priori* information. For example, it is relatively easy to learn the wind velocity profile through standard radiosonde techniques. Rather than ask this proposed remote inversion method to determine the wind velocity profile in addition to the turbulence (as has been proposed), one should instead use an independently measured wind profile as *a priori* information to

* If the turbulence is indeed nonstationary, it would be well to inquire again about just what descriptors you are trying to measure anyway.

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aid the inversion method. Secondly, we know something already of the general distribution of turbulence with altitude and should use this information. Also, since the ground layer turbulence can be easily measured *in situ*, one could use this information to subtract its known contribution from the total observed scintillation statistics.

It is certainly apparent that an error analysis for this inversion method is required. Also, I would strongly recommend that a limited program of *in situ* measurements be made to learn more about the true nature of the turbulence before we attempt to rely on indirect remote sensing methods, such as proposed here.

On a different subject, we should note that in the discussions today we have talked only about using intensity scintillation measurements as an indicator of remote effects. Similar methods have been developed for other types of optical disturbances, such as, image blurring (Hufnagel, 1967). These alternate methods can well complement the scintillation methods described here.

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SESSION 3

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