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SOME ASPECTS OF REMOTE ATMOSPHERIC SENSING  
BY LASER RADAR

Richard M. Schotland  
New York University\*

ABSTRACT

An analysis has been made of some aspects of atmospheric sensing by laser radar spectroscopy. A discussion of the laser radar equation is presented which leads to methods for obtaining atmospheric information by spectroscopic means. Calculations are presented which investigate the potential of Doppler, Raman and differential absorption techniques. It is concluded that atmospheric measurements based upon Doppler and Raman approaches are restricted to qualitative studies at the present time primarily because of the limitations imposed by existing laser sources and detectors. It is probable that these limitations will be removed in the near future and it is recommended that exploratory experiments be continued. The differential absorption method for obtaining the spatial distribution of atmospheric gases is well developed for such gases as water vapor and oxygen where the basic laser frequency lies close to the center of the particular absorption line. Further investigation is recommended in the field of laser frequency shifting. Additional information is also required on the shape of the absorption line, particularly in the region where the line reflects both pressure and Doppler broadening.

1. INTRODUCTION

The purpose of this paper is to consider the potential of the laser radar as a ground based remote sensor of the earth's atmosphere. Perhaps the simplest approach to this subject may be had by a consideration of the transfer equation which relates the power transmitted by the laser to that which is captured by coaxial receiving optics.

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$$P_r = \left( \frac{P_T}{\pi Z^2 \theta^2} \right) \cdot \left( \sigma_\pi \cdot n(Z) \right) \cdot \left( \frac{\Delta Z}{2} \cdot \pi Z^2 \theta^2 \right) \cdot \tau \cdot \frac{A_r}{Z^2} \quad (1)$$

Radiance at Z	Scatter cross section per unit length	Scattering volume	Transmis- sion of at- mosphere	Capture solid angle
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where

- $P_r$  = received power
- $P_T$  = transmitted power
- $A_r$  = receiver aperture
- $Z$  = slant range
- $\sigma_\pi$  = molecular backscatter cross section
- $n(Z)$  = molecular number density
- $\Delta Z$  = pulse length
- $\tau$  = two way atmospheric transmission
- $\theta$  = receiver beam width

When this equation is applied to the atmosphere it takes the normalized form:

$$Z^2 \frac{\bar{H}_{r\lambda}}{P_{T\lambda}} = \Delta Z \left[ \frac{2\pi^2 \gamma^2}{\lambda^4} n(Z) + \frac{\lambda^2}{8\pi^2} \int_0^\infty i(r, \lambda) N(r) dr \right] \tau \quad (2)$$

$$\tau = \exp -2 \left[ \int_0^Z \beta_G dz + \int_0^Z \beta_A dz + \int_0^Z \rho(Z) K_\lambda dz \right] \quad (3)$$

In this equation the following definitions hold:

- $\bar{H}_{r\lambda}$  = average irradiance at the receiver at wavelength  $\lambda$
- $P_{T\lambda}$  = pulse power of the transmitter at wavelength  $\lambda$
- $Z$  = slant range to the pulse volume
- $\lambda$  = laser wavelength
- $i(r, \lambda)$  = Mie scattering function
- $n(Z)$  = molecular number density at range  $Z$

$N(r, Z)$  = spectral distribution of aerosol particles at range  $Z$

$\beta_G$  = molecular scatter cross section/unit volume

$\beta_A$  = aerosol scatter cross section/unit volume

$K_\lambda$  = molecular absorption cross section (area/unit mass)

$\gamma$  =  $2\pi$  molecular polarizability

$\rho(Z)$  = atmospheric density

### 1a. Atmospheric Constituents

The atmosphere is composed primarily of nitrogen (78.5%) and oxygen (20.95%). These gases are well mixed and maintain the same percentages on a volume basis throughout the troposphere and stratosphere. The major variable gases of meteorological interest are water vapor, ozone, and carbon dioxide. These gases can vary rapidly both in time and in space.

The surface air contains large numbers of small particles whose sizes range from  $10^{-2}$  microns to 2 microns. Total number densities may be as large as  $10^5$  per cubic centimeter. The bulk of these particles have radii less than 0.1 micron and a typical count for particles greater than 0.5 micron would be 1 per cubic centimeter. The particle density normally decreases with height in an exponential manner as shown in Figure 1. The refractive index of these particles is complex with a real part in the range 1.5 to 1.7. Little is known of the spectral variability of either the real or imaginary portions of this quantity.

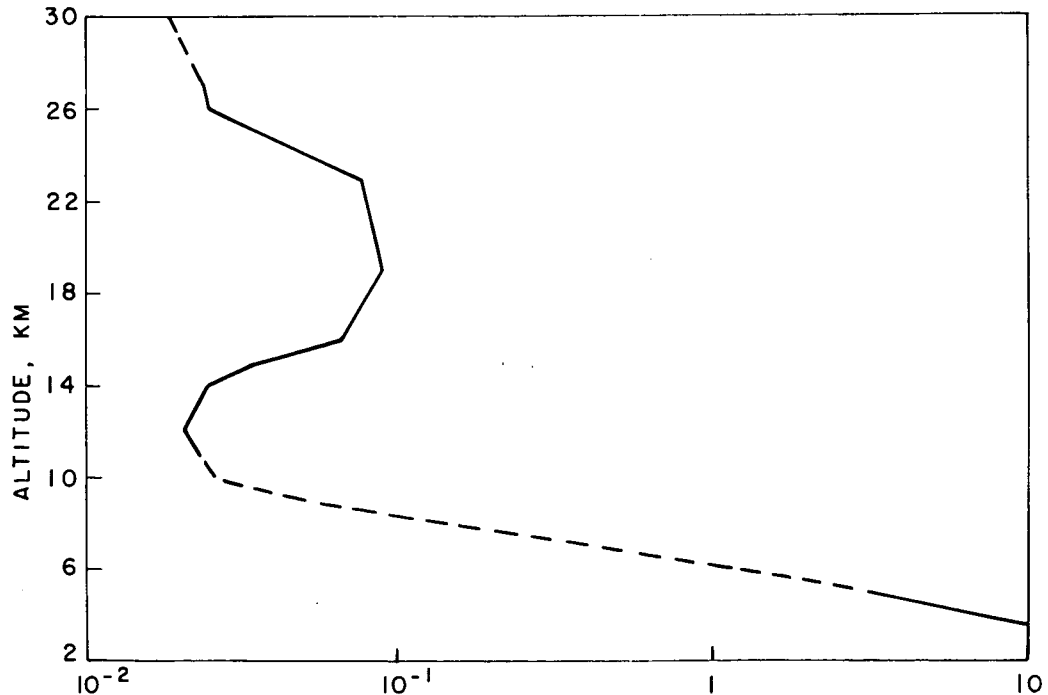


Figure 1. Variation of particle concentration with height.

## 1b. Atmospheric Spectral Properties

Elterman has computed the sea level particle scattering coefficient as a function of wavelength. These values have been plotted by Breece, *et al.* (1966) and are presented here as Figure 2. It should be noted that the scattering coefficient changes very slowly in the region below 1 micron. At wavelengths near 10 microns the aerosol particles will be in the Rayleigh range and the wavelength dependence will be  $\lambda^{-4}$ .

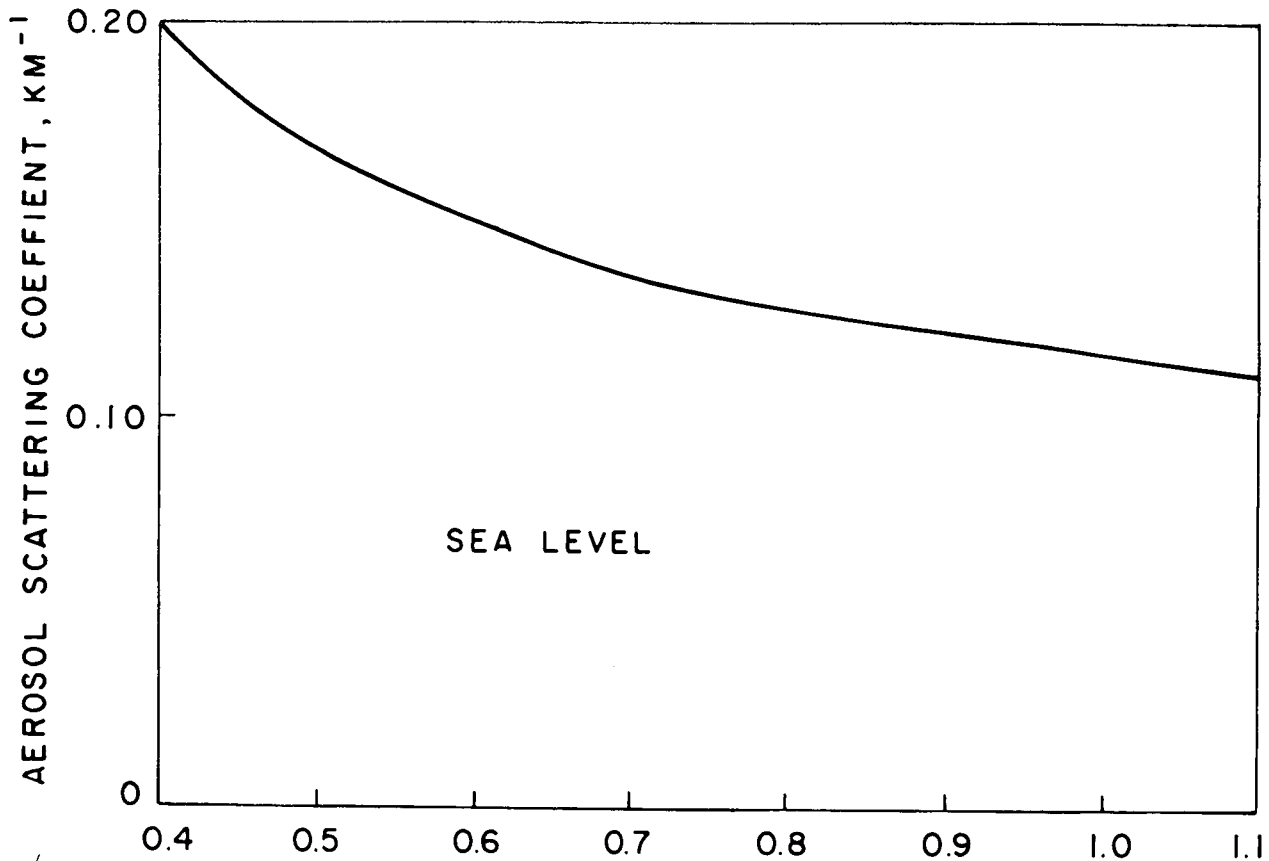


Figure 2. Variation of aerosol particles scattering coefficient with wavelength.

The magnitude of the particle backscatter at  $\lambda = .7\mu$  is an order of magnitude greater than that due to molecular scatter at ground level. However, due to the rapid decrease of particle density with height the two terms become essentially equal at 3 kilometers and remain so throughout the troposphere as shown in Figure 3 (Breece, *et al.* 1966).

The strong absorption bands of water vapor, carbon dioxide and ozone in the infrared and of ozone in the ultraviolet regions of the spectrum are well known to the meteorologist as they play important roles in the thermodynamics of the atmosphere. It is normally assumed that the atmosphere is essentially transparent from 0.3 to 0.9 microns. However, reference to high resolution telluric spectra shows that narrow, intense absorption lines due to water vapor, oxygen, ozone are dispersed throughout this region of the spectrum. A sample of such a spectrum taken at N. Y. U. by Bradley is shown in Figure 4. It should be noted that the atmospheric absorption for water vapor (one way) at  $6943.80\text{\AA}$  is nearly 60% and the line width is essentially  $0.05\text{\AA}$  at half width. Reference will be made to this isolated line in a latter portion of this paper.

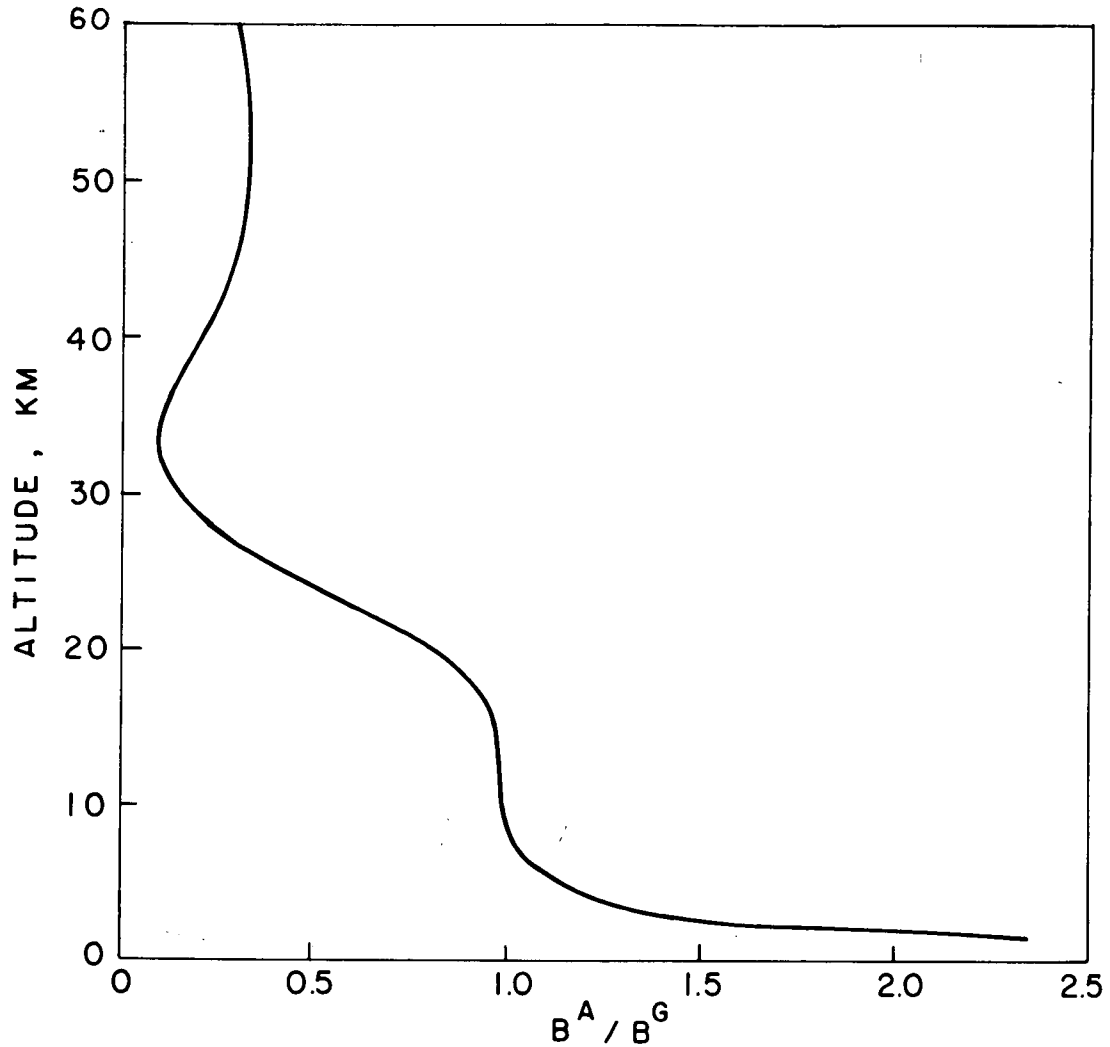


Figure 3. Ratio of particle to Rayleigh scattering coefficients as a function of height.

#### 1c. Energy Background Level

In the discussion which follows it will be necessary to compute signal to noise ratios of various proposed systems in order to establish their feasibility. Generally, these systems, if they are to be considered useful from a meteorological standpoint, must be operable during day as well as night hours. Consequently, the noise contribution due to the presence of background radiation must be known. The background radiation may be computed from the relationship

$$P_B(\lambda) = A_R J(\lambda) \Delta \omega$$

where

- $P_B(\lambda)$  = background radiation (watts)
- $A_R$  = receiving aperture ( $\text{cm}^2$ )
- $J(\lambda)$  = radiance ( $\text{watts}/\text{cm}^2/\text{sr}/\text{micron}$ )
- $\Delta \omega$  = solid viewing angle

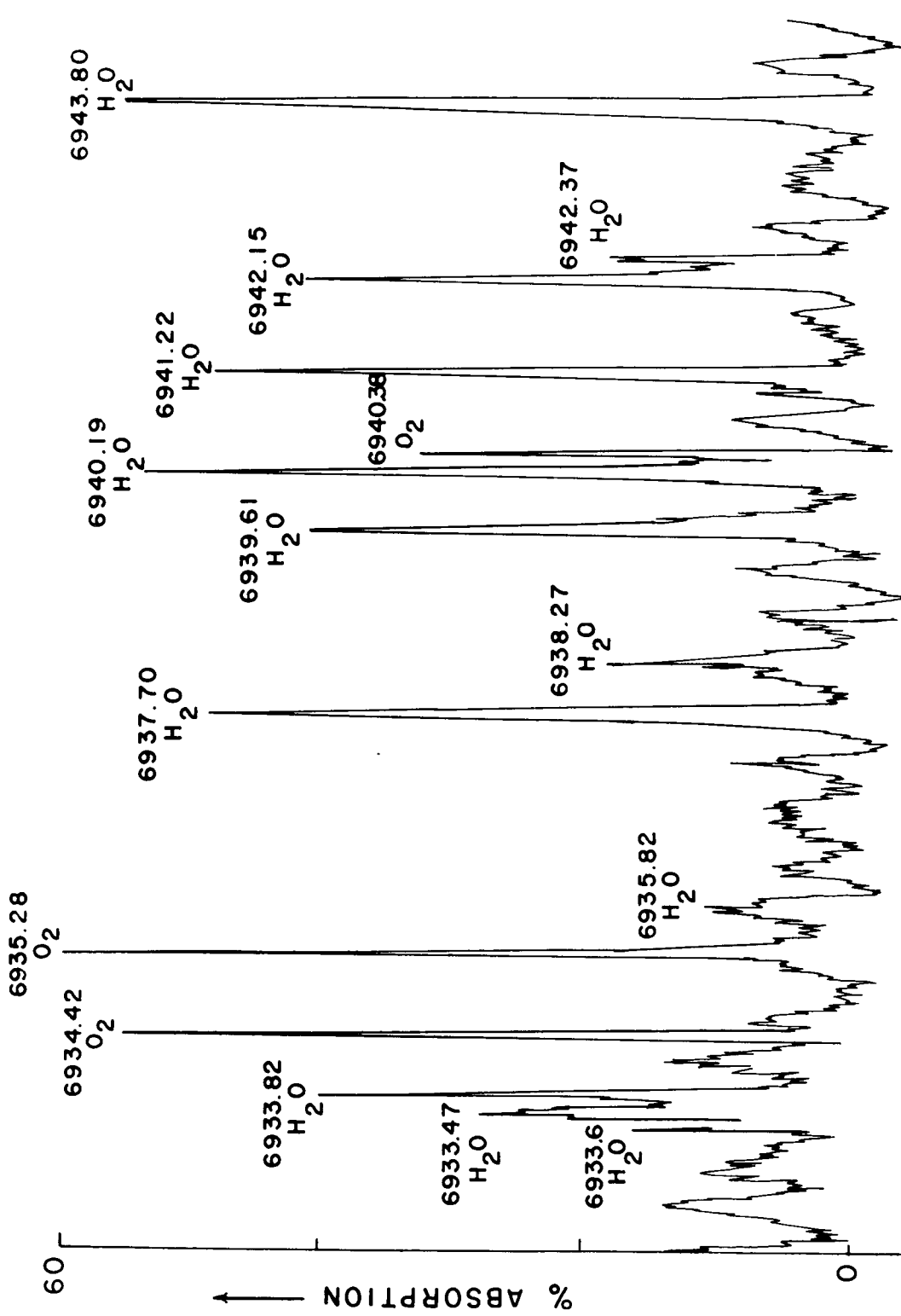


Figure 4. Telluric absorption spectrum (6933-6945Å) April 10, 1967.

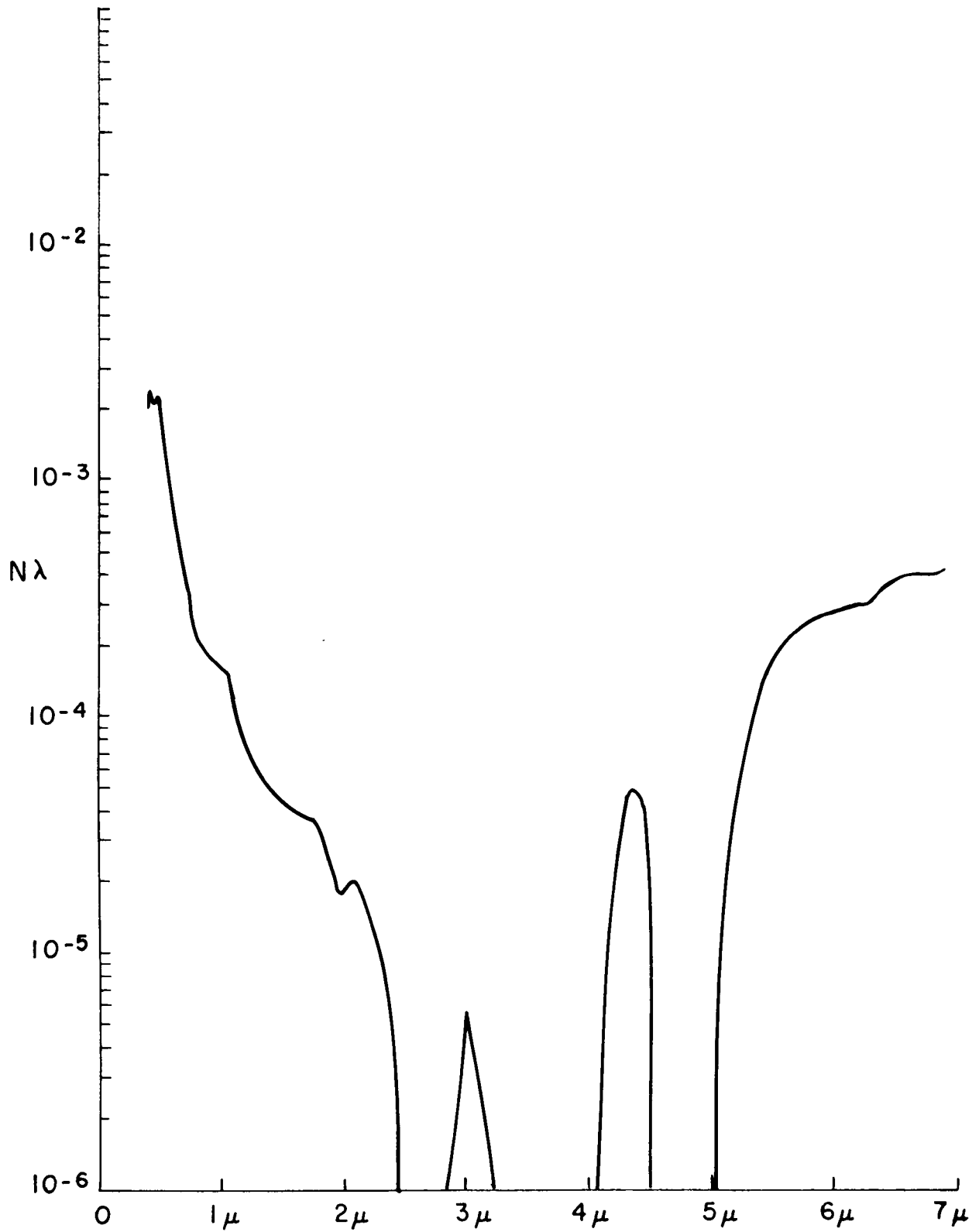


Figure 5. Spectral radiance of background radiation for a blue sky as a function of wavelength.

A graph of  $J(\lambda)$  versus  $\lambda$  of blue sky observed by Vanderhei and Taylor (1960) is given in Figure 5. A value of  $10^{-3} \text{w/cm}^2/\text{sr}/\text{micron}$  is characteristic of the ruby laser region.

## 2. CAPABILITIES OF LASER SYSTEMS

One of the goals of a remote sensing device would be to provide information equivalent to that obtained by the standard radiosonde under all weather conditions. It is clear that the laser radar will not penetrate thick cloud decks. This limitation is imposed by the available energy in laser pulses. For example, assume a cloud deck of 300 meters thickness and of  $3 \text{ gram/meter}^3$  liquid water content. In order to penetrate this cloud and provide a path for the return scattered energy it would be necessary to evaporate the water in the path of the beam. If the beam were 0.1 meter x 0.1 meter in cross section this would require a pulse containing approximately 21,000 joules of energy. Present day systems provide energy levels (Q switch) of a maximum of 10 joules. It does not seem possible in the foreseeable future to provide energy of the requisite amount through combinations of increased pulse energy and repetition rates.

What can the laser radar do aside from deriving qualitative indications of  $n(Z)$  and  $N(Z)$ ? We would like to operate the laser radar in a manner which would permit the separation of the quantities  $n(Z)$ ,  $\rho(Z)$  and  $T(Z)$ . The following approaches to such measurements will be considered in the remainder of the paper

$n(Z)$  and  $N(Z)$  via Raman scattering

$\rho(Z)$  via differential absorption of scattered energy

$T(Z)$  via Doppler power spectrum of the scattered energy.

$v(Z)$  via Doppler power spectrum of the scattered energy.

### 2a. Doppler Power Spectrum

The transfer equation stated in the introduction of this paper relates the energy returned to the laser radar to that which is transmitted. The scattering process is an interaction mechanism between the traveling laser energy and the molecules and particles which comprise the atmosphere. These constituents are in random motion and their velocities are governed by the Maxwell-Boltzmann relationship

$$dN(v) = \frac{1}{N} \frac{dv}{\sqrt{2\pi}} \sigma_v \exp \left[ - \frac{(v - \bar{v})^2}{2\sigma_v^2} \right] \quad (4)$$

where

$\bar{v}$  = mean wind

$v$  = random particle velocity

$\sigma_v$  = standard deviation of the thermal wind due to Brownian motion

$N$  = number of particles/unit volume

$dN(v)$  = number of particles in the range  $dv$  at the velocity  $v$



The scattered radiation although initially nearly monochromatic will be Doppler broadened by virtue of the velocities of the scattering components. The Doppler shift may be expressed for these particles as

$$f_d = \frac{\vec{v}_i}{\lambda} \cdot (\hat{r} + \hat{t})$$

where

$\vec{v}_i$  = particle velocity

$\hat{r}$  = unit vector to the receiver

$\hat{t}$  = unit vector to the transmitter

for a coaxial system

$$f_d = \frac{2v_i}{\lambda}$$

This relationship may be used with the Maxwell-Boltzmann expression to transform the normalized transfer equation into a power spectrum for the backscattered radiation. Use is made at this point of the relationship

$$\sigma_v^2 = \frac{kT}{m} \quad (5)$$

where

$k$  = Boltzmann's constant

$T$  = temperature

$m$  = scattering mass

$$I = \frac{R^2}{P_T} \frac{\partial H_R}{\partial f_d} = \frac{\gamma^3 \pi^2}{\lambda^2} \frac{\Delta Z_n(Z)}{[2\pi R^* T]^{1/2}} \exp \left[ -\frac{\lambda_o^2 f_d^2}{8R^* T} \right] \quad (6)$$

This relationship (Schotland, et al. 1962) is plotted in Figures 6 and 7. It is assumed that the plot has been centered about  $\bar{f}_d = \frac{\bar{v}}{\lambda}$ .

It should be said at this point that the spectrum may not transform simply as shown. Dicke (1953) has suggested a collision narrowed Lorentz form. This expression does not deviate appreciably at halfwidth from the Boltzmann form for molecular scatter, but does so markedly for particles larger than  $0.1\mu$  (Schotland, et al. 1967). The Boltzmann form also has been used by Zirkel (1966) and Breece, et al. (1966). It is important that this point be clarified.

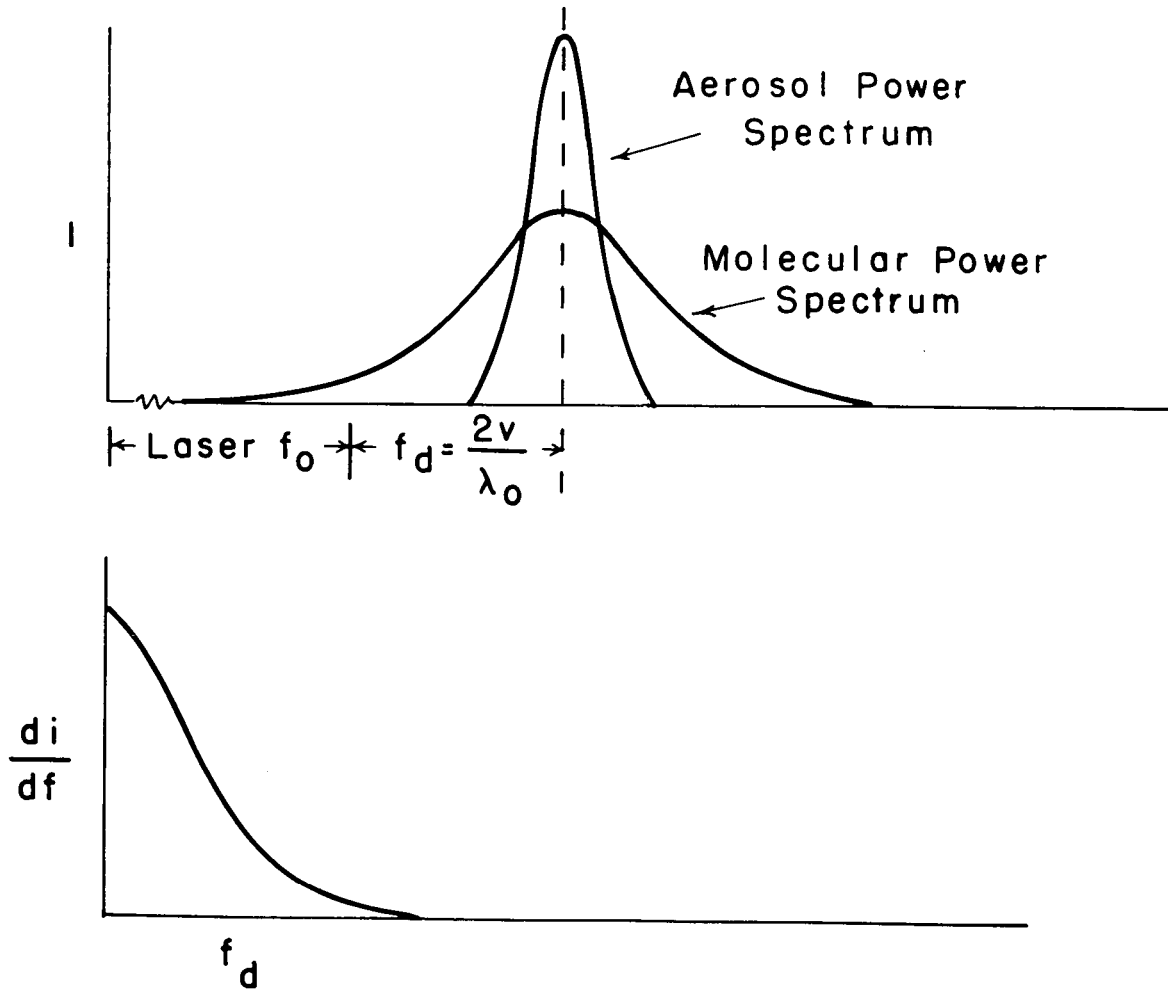


Figure 6. Representation of molecular and aerosol power spectra.

The normalized power spectrum for particles in the Rayleigh range is

$$R^2 \frac{\partial H_R}{\partial f_d} = \frac{9\pi^2 K^2}{\lambda^3} \Delta Z V^2 N(Z) \frac{1}{\sqrt{2\pi}\sigma_v} \exp\left[-\frac{\lambda^2 f_d^2}{8\sigma_v^2}\right] \quad (7)$$

where  $V$  = volume of scattering particle.

$$K = \frac{m - 1}{m + 2}, \quad m = \text{refractive index.}$$

This form of the equation is useful at the  $\text{CO}_2$  laser wavelength ( $10.6\mu$ ).

In order to observe the power spectrum it is necessary to limit the receiving aperture to the coherence area of the sources. This area,  $A_c$  is of the order of  $\lambda^2/\Omega$  where  $\Omega$  is the solid angle of the scattering volume viewed from the receiver. This restriction implies that the beamwidths of the transmitted signal must be extremely narrow if practical values of  $A_c$  are to be obtained. Fried (see Breece, et al. 1966)

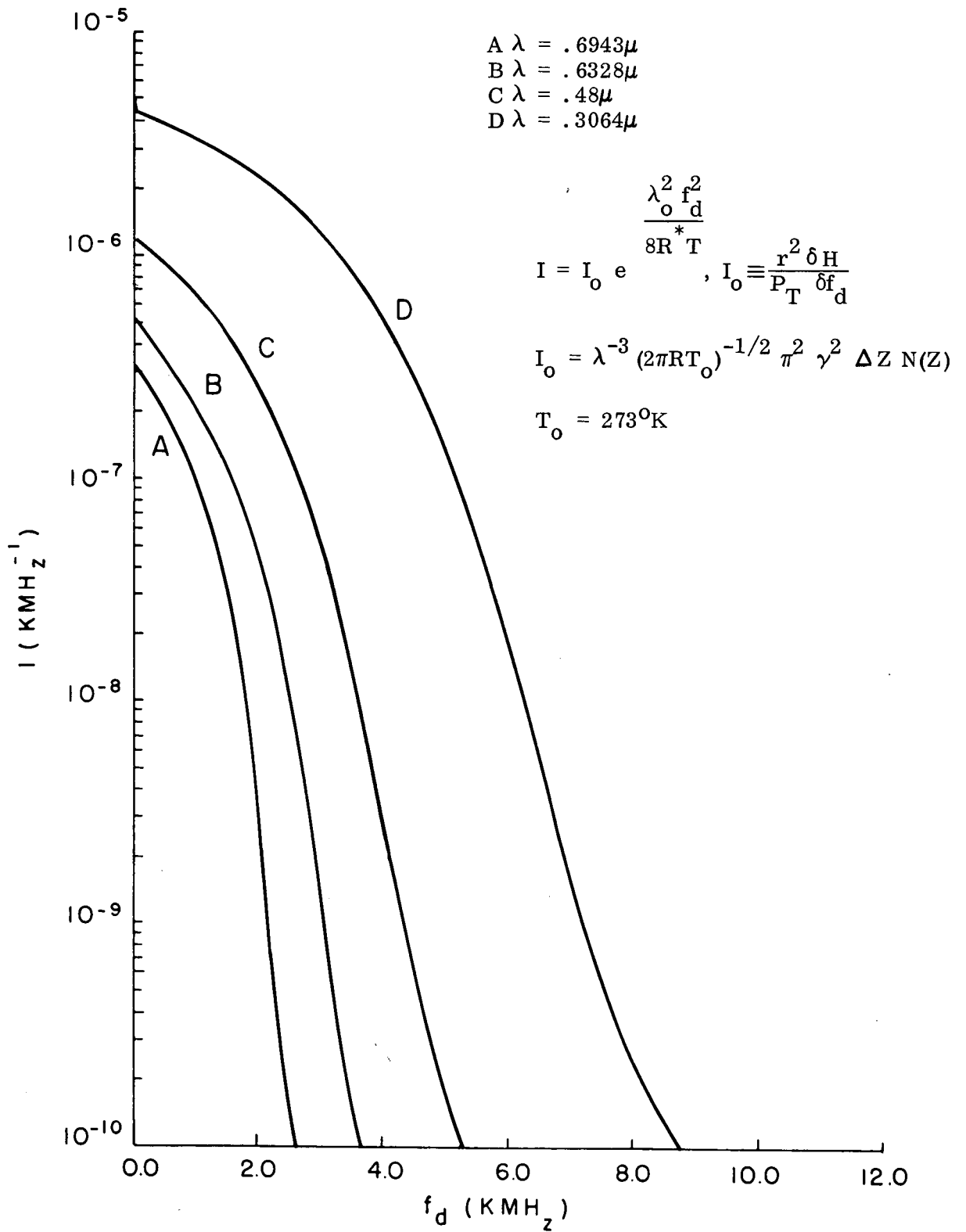


Figure 7. Normalized molecular Doppler spectrum.

has demonstrated that the transmitting optics should be diffraction limited and equal in area to that of the receiver for optimum results.

Another problem area in the application of Doppler techniques to the atmosphere results from the perturbation of the wavefront of the radiation by the atmosphere. The diameter of the receiving telescope should not exceed a value  $r_o$  given by Fried (1965) as

$$r_o = 1.2 \times 10^{-8} \lambda^{6/5} R^{-3/5} C_N^{-6/5} \quad (8)$$

where

$\lambda$  = wavelength

$R$  = range (two way)

$C_N^2$  = Tatarski's structure constant

The strong dependency on  $\lambda$  should be noted. As an example  $r_o$  for day operation is of the order of 7 cm for  $\lambda = .7\mu$  while it may be as large as  $10^2$  cm at  $10\mu$ .

In order to discuss the potential of the laser radar for Doppler application, the line purity of typical lasers must be considered. At the present time two classes of lasers can be considered useful. They are the pulsed crystal and the  $CO_2$  gas laser.

According to Hercher (private communication) the line width of pulsed crystal lasers can be of the order of  $30 \times 10^6$  Hz, when operated in the Q-switch mode. This line width is of the order of the pulse spectrum so that no improvement appears possible except through the use of longer pulses. The energy available is approximately 0.1 joule in projected laser systems.

From a discussion with equipment manufacturers it appears that the line width of a  $CO_2$  gas laser is of the order of  $0.008\text{\AA}$  ( $2.4 \times 10^6$  Hz) at a power level of 100 watts.

Modulation schemes have been proposed whereby the Doppler shift occurs at the modulation frequency. This technique has the twin advantages of strong spectral purity and a coherence area dictated by the modulation frequency. However, present modulation schemes are limited to  $10^9$  Hz ( $\lambda = 30$  cm). Therefore the Doppler shifts are generally small:  $f_d/v = 6$  Hz/m/sec.

### 3. APPLICATIONS OF THE DOPPLER SYSTEM

It has been suggested that the Doppler technique be used to measure the following parameters.

1. Temperature from the width of the molecular spectrum
2. Wind velocity via the mean Doppler shift using heterodyne techniques
3. Velocity gradient using square-law detection of the backscattered radiation from two laser pulses separated in range

Problems 1 and 2 were analyzed by Schotland, et al. (1962) and although times have improved and a better understanding of the problem is available, the conclusion is still pessimistic.

Consider the temperature problem. If

$$I = \frac{Z^2}{P_T} \frac{\partial H}{\partial f_d}$$

$$I(f_0) = 3.4 \times 10^{-16} \text{ for } \lambda = 0.7\mu$$

Then at the half power point which is sufficiently removed from the aerosol contribution,

$$I\left(\frac{1}{2}\right) \sim 1.7 \times 10^{-16}$$

$$\Delta P_R = 1.7 \times 10^{-16} \times \frac{P_T}{Z^2} A_c \times \Delta f$$

Phase limitations limit  $A_c$  to approximately  $50 \text{ cm}^2$ . Assume  $\Delta f = 10^7 \text{ Hz}$  and a range of  $1 \text{ km}$ .

$$\Delta P_R = 1.7 \times 10^{-16} \times 10^7 \times 10^{-10} \times 50 \times 10^7$$

(I<sub>0</sub>)                      (Δf)              (Z<sup>2</sup>)              (A)              (P)

$$\Delta P_R = 8.5 \times 10^{-11} \text{ watts}$$

Substitution of this power into the S/N relation assuming  $P_B = 0$  yields  $S/N = 3$  which is not very satisfactory. It should be noted that with coherent detection the S/N ratio is basically equivalent to that obtained with  $P_B = 0$ .

An estimate of the desired signal/noise value for thermal measurement can be obtained by logarithmically differentiating the expression for I.

$$\frac{dI}{I} = \frac{dT}{T} \left[ \frac{\lambda_o^2 f_d^2}{8R^* T} - \frac{1}{2} \right]$$

Thus for  $\frac{\lambda_o^2 f_d^2}{8R^* T} = 1$ , a  $2^0$  temperature precision at the level of  $300^0\text{K}$  requires a precision in the measurement of I of  $1/300$ . This level of accuracy could be obtained at a maximum range of only  $10 \text{ m}$ .

Bistatic systems using c. w. lasers such as the Argon or the  $\text{CO}_2$  units can be used with synchronous detection techniques for the temperature measurements. The difficulty with this approach is that with present capabilities the integration times necessary to yield reasonable S/N ratio are of the order of minutes. Another method which can be used to obtain the change in Doppler width involves the direct use of an interferometer. The advantage of this approach is that the conditions on phase coherence

are relaxed - permitting much larger receiving apertures. The interferometer provides information only on the spectral width and therefore this method cannot be used for velocity studies.

Doppler techniques to measure the mean wind and the radial wind gradient are basically dependent upon the ability of the system to "see" these shifts in the presence of the spectrum variability of the laser source. The Doppler shift sensitivity may be expressed as  $f_d/v = 2/\lambda$ .

$\lambda$	$.7\mu$	$10.6\mu$
$f_d/v$	$3 \times 10^6 \text{ m}^{-1}$	$2 \times 10^5 \text{ m}^{-1}$

If a velocity measurement accurate to 1 m/sec is desired, the expected shift will be of the order of 0.1 of the spectral noise of state of the art laser systems. This uncertainty does not lead to a useful measurement. At the present time a laser is not available which will deliver 0.1 joule with a bandwidth of  $30 \times 10^6$  Hz. It appears that such a system could be developed with existing technology.

Recently Fiocco and DeWold (private communication) performed an experiment in which they used a pressure scanned Fabry-Perot interferometer to observe the Doppler spectrum of an aerosol. Although this system cannot be used as it stands for atmospheric probing, the work does demonstrate the reality of the Doppler spectra.

Similar conclusions were also reached by Cummins, et al. (1964) who used optical mixing techniques to study the Doppler power spectrum in liquids.

It is recommended that an experimental program be initiated to study the thermal Doppler spectrum. In particular the shape of the spectrum should be considered in relationship to the Lorentz versus Gaussian forms.

### 3b. Raman Scattering Technique

In general, the radiation which is backscattered is returned at a wavelength similar, except for Doppler shift, to the existing wavelength. At times the scattering molecule will undergo a change of state during the scattering process and the scattered energy will be returned at a different wavelength. This process is known as Raman scattering. The average Raman cross section is of the order of  $4 \times 10^{-3}$  that of the Rayleigh cross section for atmospheric gases. The changes of molecular state are related to the vibration and rotation spectra of the molecules and the magnitude and wavelength shift will be specific for a given molecule. Since the Raman cross section (non-stimulated) is small, the technique will probably be useful only for the major atmospheric components such as  $N_2$  and  $O_2$ .

An example of the signal to noise ratio in the measurement of the vertical distribution of  $O_2$  is calculated below. It is assumed that a ruby laser operating at  $6943\text{\AA}$  has been frequency doubled with an efficiency of 25 percent. The  $\nu = 0$  to  $\nu = 1$  vibration state corresponding to a line shift of  $1550 \text{ cm}^{-1}$  will be used in the computation.

System parameters

$$A_R = 4 \times 10^3 \text{ cm}^2$$

$$\Delta\Omega = 10^{-5} \text{ sr}$$

$$\Delta\lambda = 4 \times 10^{-4} \mu$$

$$J_\lambda = 2 \times 10^{-3} \text{ watts/sr/}\mu\text{/cm}^2$$

$$P_T = 0.5 \times 10^8 \text{ watts}$$

$$\Delta f = 10^6 \text{ hz}$$

$$C = \text{photomultiplier responsivity} = .15 \text{ amp/w.}$$

The signal to noise ratio for shot noise limited system such as a photomultiplier is given by the following expression:

$$\frac{S}{N} = \frac{P_R C \tau}{[2e \Delta f C \tau (P_R + P_B)]^{1/2}} \quad (9)$$

where  $e$  = electronic charge

Substitution of the above parameters into the S/N relationship results in the following values: for ranges  $Z = 3$  and  $6 \text{ Km.}$ ,  $S/N \approx 80$  and  $18$ , respectively.

It appears that the measurement is feasible. Some work has been done by Cooney at R.C.A. (private communication) using a straight ruby laser system and filter suitable for the  $N_2$  molecule. His results have not been published but it appears that he has observed the Raman profile.

There is a difficulty with this model. While Raman scattering is specific for a particular molecule, the attenuation function is not. Thus the returned signal will be modulated by the total atmospheric cross section. A possible solution to this problem would be to monitor the unshifted backscatter (or an additional line) and use this quantity as a correction to the Raman measurement.

Consider the relationship

$$I_1 = \frac{R^2_{H_R}}{P_T} = \frac{L}{2} [\sigma_{RAY} + \sigma_P] \tau^2 \quad \text{for the unshifted line} \quad (10)$$

$$I_2 = \frac{R^2_{H_{RAM}}}{P_T} = \frac{L}{2} [\sigma_{RAM}] \bar{\tau}^2 \quad \bar{\tau}^2 = \tau \text{ for the Raman line} \quad (11)$$

$$\frac{I_1}{I_2} = k_1 + k_2 \frac{\sigma_P}{\sigma_{RAY}} \quad \text{i. e. the ratio of particle to Rayleigh cross section is available} \quad (12)$$

Cooney (private communication) points out that temperature information is incorporated in the ratio of the intensities of the Stokes and anti-Stokes lines. It is not clear at the present time if the intensity of the anti-Stokes line is of sufficient magnitude to insure a useful measurement.

### 3c. Differential Absorption of Scattered Energy

The expression for atmospheric transmission which appears in the radar transfer equation includes a term  $\exp -2 \int_0^Z \rho K_\lambda dz$ . This function represents the molecular

absorption due to atmospheric gases. The atmospheric spectrum presents an abundance of narrow lines ( $0.1\text{\AA}$ ) of moderate strength due to  $\text{H}_2\text{O}$ ,  $\text{O}_2$ ,  $\text{O}_3$ , and  $\text{CO}_2$ . Figure 4 represents a portion of the telluric spectrum from 6934 to 6944 $\text{\AA}$  obtained by Bradley. The predominant lines in this region are due to  $\text{H}_2\text{O}$  and  $\text{O}_2$ . A technique has been developed at N. Y. U. over the past 10 years to measure the vertical distribution of absorbing gases by measuring the differential absorption of the backscattered radiation from a pulse emitted by an optical radar. The technique was initially applied to  $\text{O}_3$  in 1958 using a pulsed Xenon source. However, the results were marginal due to the poor spectral brightness of the source. The development of the laser has provided an energy source which is ideal for this type of measurement. Basically the laser is tuned to an absorption line center and then to an adjacent window region.

An example of simulated radar data is presented in Figure 8 which represents the transfer equation evaluated for 6943.80 $\text{\AA}$  water vapor line. Line parameters are due to Benedict.

The inversion of the optical return to obtain the vertical profile of water vapor proceeds as follows:

$$R(Z) = \frac{P_R(\lambda_L)}{P_R(\lambda_W)} = \frac{F_T(\lambda_L)}{F_T(\lambda_W)} \exp \left[ -2 \int_0^Z \rho_v(Z) K_\lambda(\lambda_i) dz \right] \quad (13)$$

$$\frac{\partial}{\partial Z} \ln R(Z) = -2 \rho_v(Z) K_w(\lambda_i) dz \quad (14)$$

$$\therefore \rho_v(Z) = \frac{1}{2K_w(\lambda_i)} \frac{\partial}{\partial Z} \ln R(Z) \quad (15)$$

The accuracy of the determination of the water vapor profile by the differential absorption technique is limited primarily to the uncertainties in the measurement of the gradient of the power return and by the incomplete knowledge of the water vapor absorption coefficient. The power measurement, at least from the first four kilometers of the atmosphere, does not represent a problem area.

The variance of the deduced water vapor density may be expressed in terms of the variances of the absorption coefficient and returned power gradients as follows:

$$\sigma_w^2 = \frac{\rho^2(Z)}{K^2} \sigma_K^2 \left[ \frac{1}{P_L^2} \sigma^2 (\partial P_L / \partial Z) + \frac{1}{P_W^2} \sigma^2 (\partial P_W / \partial Z) \right] \quad (16)$$



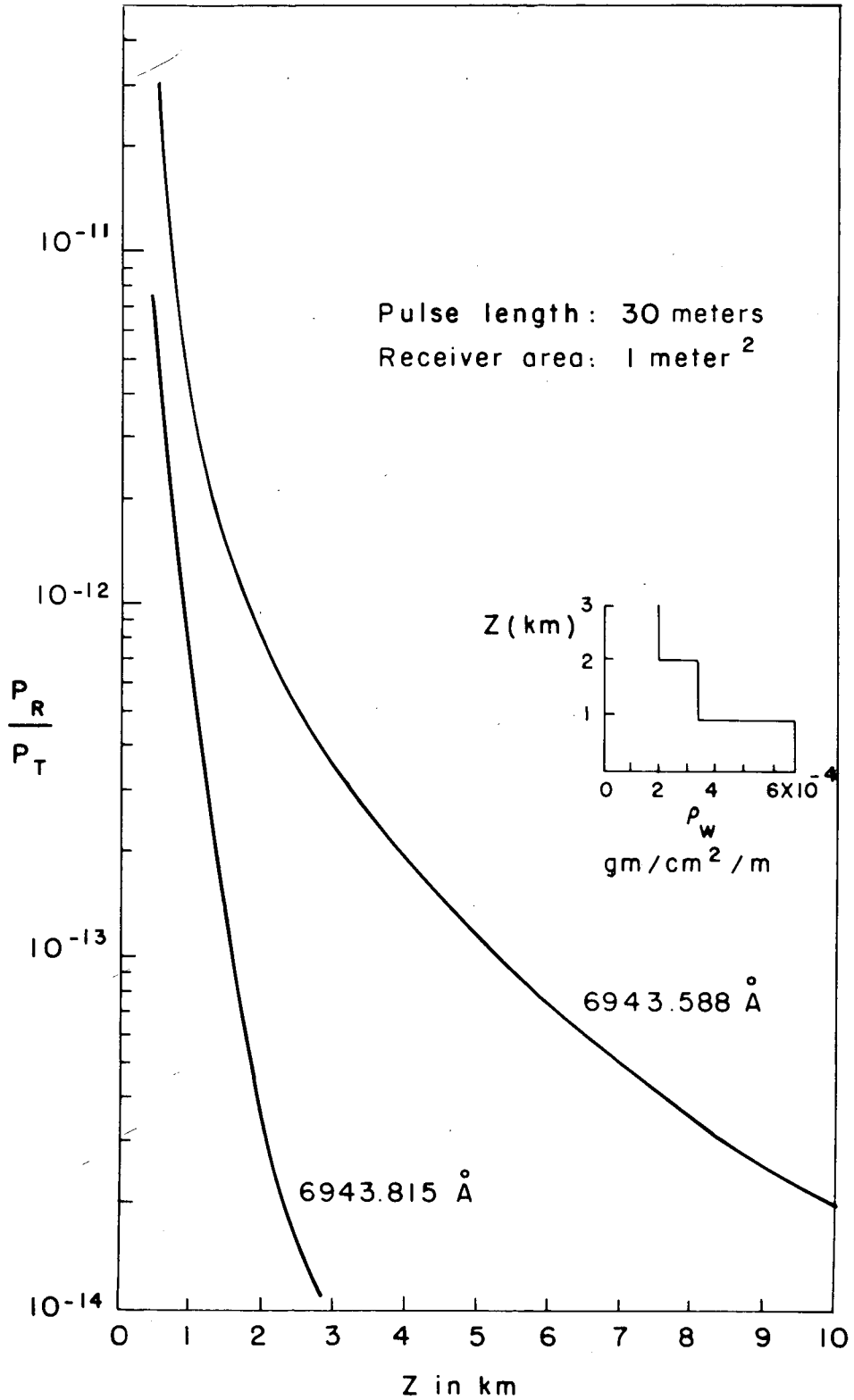


Figure 8. Transfer function for .7μ

where

- $\sigma_w^2$  = water vapor variance
- $\sigma_K^2$  = the absorption coefficient variance
- $\sigma^2(\partial P/\partial Z)$  = the power gradient variance

The variance of the absorption coefficient is due to two factors. The first factor is that an uncertainty exists in the measurements of the line parameters and in the effects of pressure and temperature broadening upon line shape. The values for the line width and strength according to Benedict are thought to be correct to  $\pm 33\%$  while the line strength has an uncertainty of  $\pm 25\%$ . The second factor that contributes to the variance in the absorption coefficient is the uncertainty in the frequency of the laser. From the work that has been done at N. Y. U., it appears that a laser frequency setting uncertainty of  $\pm 0.01\text{\AA}$  is possible. If the laser is tuned to line center and the half width of the water vapor is taken to be  $0.04\text{\AA}$ , then the fractional uncertainty in  $K\lambda$  is of the order of 5 percent. However, if the laser should be tuned to the inflection point of the curve, the fractional uncertainty can be as large as 29%.

The variance of the water vapor density due to the power gradient term arises primarily from the shot noise produced by the photomultiplier-detecting system. The variance of radiant noise power may be written

$$\sigma_N^2 = \frac{2e}{R} (P_r + P_B) \Delta f \tag{17}$$

where

- e = electron charge
- C = photomultiplier responsivity
- $\Delta f$  = measurement system bandwidth
- $P_B$  = background power

The variance of  $\frac{\partial P}{\partial Z}$  can be expressed as

$$\sigma^2(\partial \rho / \partial P) \cong \frac{2\sigma_N^2}{\Delta Z^2} \tag{18}$$

Substitution of the expression for variance of  $(\partial P/\partial Z)$  into the equation for the variance of  $\rho_w$  yields

$$\sigma_w^2 = \frac{\rho^2(Z)}{\Delta K^2} \sigma_K^2 + \frac{e \Delta f}{R \Delta K^2 \Delta Z^2} \left[ \frac{P_L + P_B}{P_L^2} + \frac{P_w + P_B}{P_w^2} \right] \tag{19}$$

The value of  $\sigma_w$  has been computed using the model atmosphere of Figure 8 and the following constants:

$$P_B = 5 \times 10^{-8} \text{ watts}$$

$$P_t = 2 \times 10^7 \text{ watts (over } 25 \times 10^{-9} \text{ sec)}$$

$$A = 10^3 \text{ cm}^2$$

$$\Delta f = 2\Delta Z/c = 4$$

$$\Delta Z = 120 \text{ meters}$$

The results are plotted in Figure 9 separately for the contributions due to  $\sigma_{\partial P/\partial Z}$  and to  $\sigma_K$ . The uncertainty in  $\sigma_K$  was assumed to arise solely from the frequency stability of the laser. It is apparent that in the first two kilometers the uncertainty in  $\sigma_w$  arises from  $\sigma_K$ . However, the contribution from  $\sigma_{\partial P/\partial Z}$  increases rapidly above two kilometers and is the dominating factor at three kilometers. The uncertainty in  $\rho_w$  due to  $\sigma_{\partial P/\partial Z}$  can be reduced by increasing both the output energy of the laser and the aperture of the receiver.

The major problem area in developing a field laser radar system of this type centers about the frequency stability of the laser. It is necessary to center the laser on line center with an accuracy of  $\pm .01\text{\AA}$ . Initial experiments were undertaken using the thermal tuning characteristic of the ruby laser to locate the line center. The thermal sensitivity of a ruby laser is  $0.065\text{\AA}/^\circ\text{C}$  which implies a rod thermal stability of  $0.1^\circ\text{C}$  if the wavelength tolerance of  $.01\text{\AA}$  is to be maintained. This proved difficult to achieve at high laser repetition rates and consequently resonant cavity frequency stabilizers are now used. These devices also permit rapid frequency shifting of the laser. A sample of  $R(Z)$  data for water vapor obtained using a thermally tuned laser is given in Figure 10.

It is possible in principle to generate both the window and line center wavelength by using an appropriate Raman shifter as suggested by Dobbins and La Grone, (1967). However, no calculations are available in which the power output and line width of the appropriate Raman shifted energy are available. This work should be continued since Raman shifting will provide access to a variety of atmospheric lines.

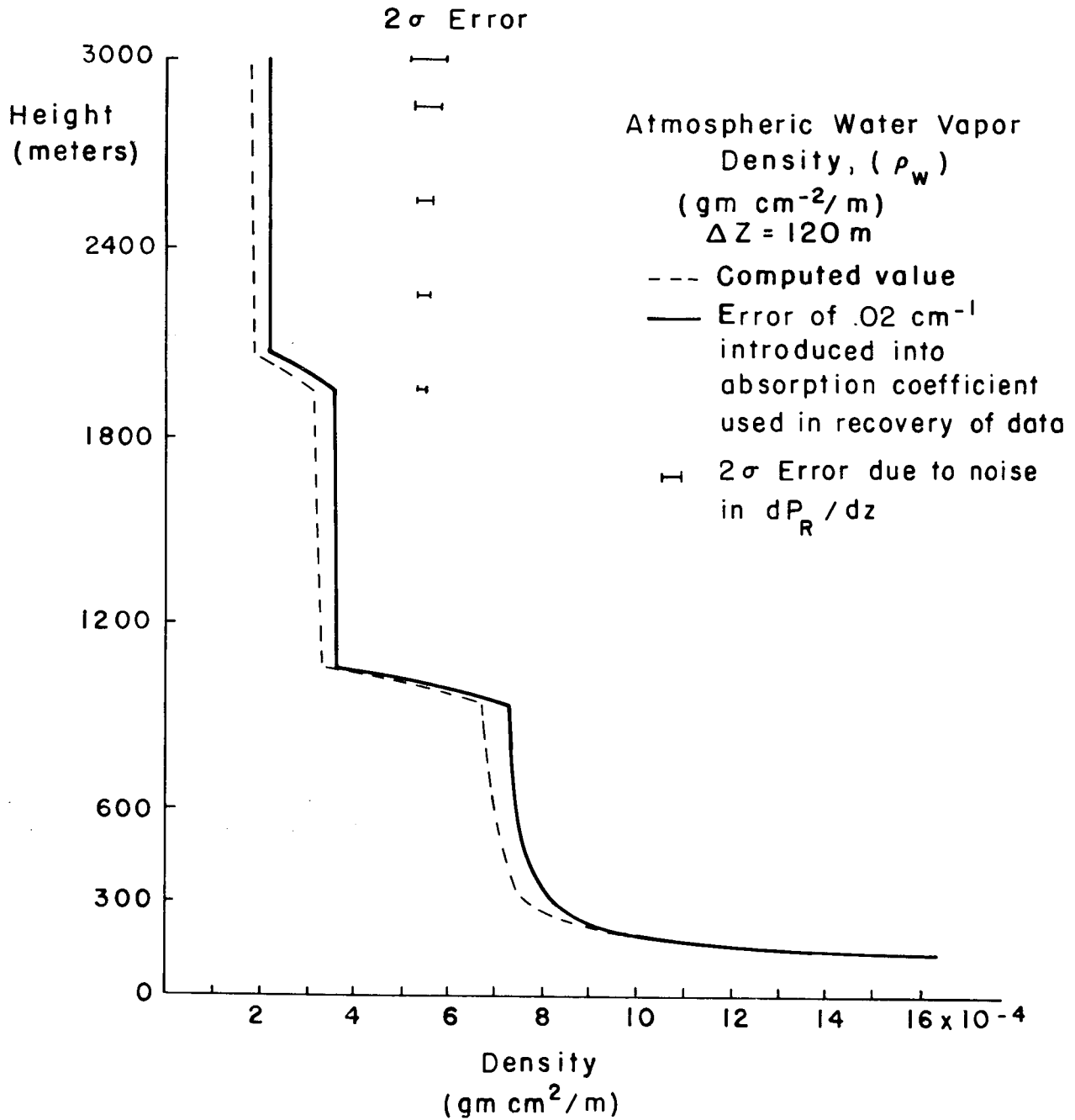


Figure 9. Uncertainty in atmospheric water vapor density measurement.

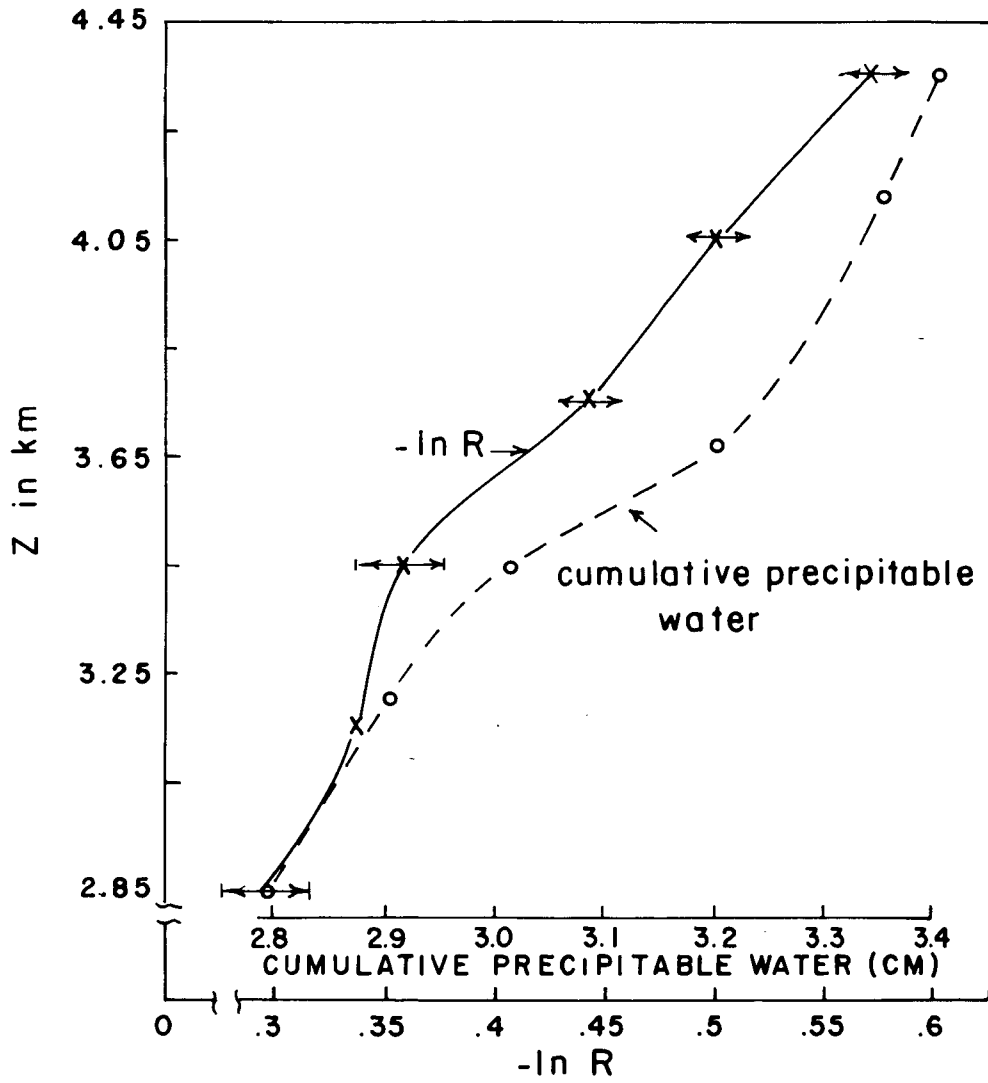


Figure 10. Comparison of observed cumulative precipitable water as a function of height with laser return.

REFERENCES

- Breece, R. C., D. L. Fried, J. B. Seidman, 1966: Remote measurement of differential atmosphere. Institute of Navigations Proc. Conf. Clear Air Turbulence, Society of Automotive Engineers, Inc., Washington, D. C., 135-162.
- Cummins, H. Z., N. Knabe, Y. Yeh, 1964: Observation of diffusion broadening of Rayleigh scattered light. Columbia University, Physics Review Letters, 12, 150-153.
- Dicke, R. H., 1953: Physical Review, 89, 472.
- Dobbins, D. L., A. H. LaGrone, 1967: Number density determination of O<sub>2</sub>, H<sub>2</sub>O and CO<sub>2</sub> by high intensity laser beam. Conference Record, South Western Institute of Electrical and Electronics Engineers, Institute of Electrical and Electronics Engineers, Inc., New York, 5-3-1-5-3-10.
- Fried, D. L., 1965: Conference on atmospheric limitations to optical propagation. Central Radio Propagation Laboratory and National Center for Atmospheric Research, Boulder, Colorado, 192-264.
- Schotland, R. M., J. Bradley, A. M. Nathan, 1967: Optical sounding III. Technical Report 67-2, ECOM-02207-F, Fort Monmouth, New Jersey.
- Schotland, R. M., A. M. Nathan, E. A. Chermack, E. E. Uthe, D. Chang, 1962: Optical sounding. Technical Report No. 3, U.S. Army Electronic Research Development Laboratory, Fort Monmouth, New Jersey, 58 pp.
- Vandehei, P. T., B. J. Taylor, 1960: Spectral ground and sky backgrounds. Background Measurements IRMP-56, Geophysical Research Directorate Research Notes, No. 46, Geophysical Research Directorate, Bedford, Massachusetts.
- Zirkel, R. E., 1966: The feasibility of optical radar to detect clear air turbulence. Institute of Navigation, Proc. Conf. Clear Air Turbulence, Society of Automotive Engineers, Inc., Washington, D. C., pp. 55-68.