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THEORETICAL INTERPRETATIONS OF SCATTER PROPAGATION

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The discovery of microwave propagation well beyond the radio horizon in the late forties created an immediate interest in its communication possibilities. The first decade (1950 - 1960) was characterized by a vigorous experimental and theoretical program aimed at mapping out those propagation features which were most relevant to the design of radio relay systems. Long term averages of mean signal level and their rough scaling with frequency and distance were of primary interest. Highly reliable signal levels were emphasized at the expense of unusually large, though infrequent signals. The fine structure of the signal was explored in a rough way, in order to set conservative bounds on the communication channel capacity. Space correlations of the signal were measured so as to establish appropriate separation distances for diversity transceivers. Theoretical explanations were tied either to a layered reflection model or to turbulence scattering schemes. The single scattering far field or cross section approximation was used almost exclusively to describe the latter, while signal variability had to be embroidered onto the stable layer theories by qualitative concepts of moving glint or reflection points. The two theories were first considered as competitive explanations of the same phenomenon, although gradually it was appreciated that a variable mixture of the two was probably responsible. Whatever the deficiencies of this program, one must admit that the combination of rough experiment and theory met the communication engineering needs of the first decade rather well.

The second decade beginning in 1960 is probably best described as a renaissance of the scientific interest which first stimulated the subject. The attractive possibility of using short term radio measurements as a means for inferring the instantaneous structure of the atmosphere through which the waves had passed was gradually appreciated. Important new experimental techniques also became available to improve such capabilities. Very stable oscillators were developed which made it possible to measure round trip signal phase on scatter paths (1). The time domain correlation or RAKE technique used by the Sylvania group(2) provides a unique means for examining particular range slices of the scattering volume. Multiple arrays of receiving antennas were used by the Stanford group (3) to study the azimuth and elevation fine structure of the propagation volume by the electronic synthesis of very narrow beams. Over and above these measurement techniques, data processing capabilities have improved importantly since the early days when much of the data was hand reduced. The capacity to do high speed digital processing of wide band signals is central to the availability of substantial amounts of precision data on signal fine structure. Taken together, these experimental techniques provide a means for studying the atmospheric structure in three space dimensions and time with exciting precision. Nevertheless, our ability to design specific measurement programs to exploit this capability depends in large measure on our intrinsic understanding of the phenomenon.

Unfortunately, our theoretical understanding of the propagation mechanism has not kept pace with experimental progress. No adequate description of the signal fine structure has been developed thus far. We must still depend on two qualitative, competitive theories (layers vs. scatter) to explain the general features of the transmission, as discussed in Dr. Cox's review (4). Either a reconciliation or fusion of the two theories is needed to suggest the next step in an evolving understanding of the propagation. It may even be that the two models are simply different ways of describing the same basic mechanism which is as yet undiscovered. It is vital to separate the geometrical considerations of the propagation path from the physics of the layers and/or turbulent irregularities. Gaussian correlation models, chosen for their integrability, are no longer an adequate starting point. Nor is it appropriate to try to force all available data into universal turbulence theories which may or may not be primarily responsible at various times. Finally, the familiar restrictions on stationarity, homogeneity, and isotropy should be relaxed in a consistent way.

There has been a great deal of advanced theoretical research on line-of-sight propagation, both in Russia and the United States, stimulated in large measure by optical propagation using lasers. However, the geometries and signal statistical considerations for line-of-sight and transhorizon propagation are sufficiently different to make the recent research largely irrelevant to scatter propagation.

The general features of a theory for single scattering by turbulent irregularities which meets most of the above needs were described in 1959 (5). The fundamental physical notion is that the received signal is the integrated result of scattering/reflection events throughout the common volume. This leads naturally to an integral (equation) relation between the measured signal and the refractive structure producing it.

$$E_s(R) = -k^2 \int_V d^3r\, G(R,r)\, E_o(r,T)\, \Delta\varepsilon(r,t) \qquad (1)$$

where $G(R,r)$ is the free space Green's function, V the common volume, $\lambda = \frac{2\pi}{k}$ the electromagnetic wavelength, R the receiver and T the transmitter locations. Formally, this is just the Born approximation for scattering by a dielectric perturbation $\Delta\varepsilon$. The problem is that $\Delta\varepsilon$ is the function that we wish to infer, not assume. Furthermore, it is usually a stochastic function of position and time, which is defined only by its statistical averages. The essential trick is to introduce a three dimensional Fourier transformation

$$\Delta \varepsilon(r,t) = \int d^3\kappa\, \xi(\kappa,t)\, e^{i\vec{\kappa}\cdot\vec{r}} \tag{2}$$

in which the turbulence wavenumber $\vec{\kappa}$ decomposition of the turbulent irregularities $\xi(\kappa,t)$ is related to the more familiar wavenumber spectrum $S(\kappa)$ by:

$$\langle \xi(\kappa,t)\, \xi(\kappa',t) \rangle = \frac{\delta(\vec{\kappa}-\vec{\kappa}')}{(2\pi)^3}\, S(\kappa) . \tag{3}$$

If we substitute (2) into (1) and interchange the orders of integration, we achieve the crucial separation of propagation geometry and turbulence physics.

$$E_s(R) = \underbrace{-k^2 \int d^3\kappa\, \xi(\kappa,t)}_{\text{Physics}} \int_V \underbrace{d^3r\, e^{i\vec{\kappa}\cdot\vec{r}}\, G(R,r)\, E_0(r,T)}_{\text{Geometry}} \tag{4}$$

Finally one can establish an explicit expression for the average scattered power by taking the magnitude of (4) and introducing Equation (3).

$$\langle |E_s|^2 \rangle = \frac{k^4}{(2\pi)^3} \int d^3\kappa\, S(\kappa) \left| \int_V d^3r\, e^{i\vec{\kappa}\cdot\vec{r}}\, G(R,r)\, E_0(r,T) \right|^2 \tag{5}$$

It is this expression, rather than the cross section approximation, which provides the starting points for a more precise theory.

All of the electromagnetic and path geometrical features of the propagation are contained in the bracketed quantity in (5), which appears as a weighting function or kernel of the turbulent spectrum S (κ). The frequency and distance dependence of the scattered field are implicit in this weighting function via their appearance in the incident wave E_0, the Green's function, and their convolution with $\exp i\vec{\kappa}\cdot\vec{r}$ over the scattering volume V. One can also exploit Equation (5) to describe more complicated experiments. For instance, by inserting appropriate Delta functions in the volume integration, it is possible to isolate those elements of the received signal corresponding to particular multipath components. An inhomogenous field in which the intensity of turbulent irregularities varies with altitude is relevant to the real atmosphere and can be included in the above expression as height dependent shaping factors (i.e. $\exp - z/h$) in the geometrical integrals. The influence of a transmitter antenna pattern is contained naturally in the incident wave E_0, and can be included for the receiver by multiplying the free space Green's function G(R, r) by the pattern function describing ray propagation from the various scattering points r to the receiver R. The usual approximation for narrow beam geometries, in which the integrations are replaced by V times an average value of the integrand, can thus be checked. By further complicating the geometry of Equation (5), one can describe beam swinging within or off the great circle plane. Unfortunately, this formalism has not yet been reduced to explicit expressions useful for comparison with experiment, and these calculations represent an ambitious program. However the important point to make is that it is essentially an exercise in integral calculus, one which can reliably provide the relationship between the scattered power and the still arbitrary spectrum $S(\kappa)$ for any propagation path. The hope, of course, is that such integral relations can be inverted to deduce the spectrum $S(\kappa)$ from the measured quantities.

It is of some interest to note that the bracketed weighting function in Equation (5) is a function of the three vector components of the wavenumber vector $\vec{K}$. This occurs because of the different ways in which the propagation geometry integrals emphasizes the x, y, and z projections of $\vec{K}$ via the scattering mechanism. Through its argument, this produces a directional emphasis of the spectrum $S(\kappa_x \kappa_y \kappa_z)$, which must be considered together with non-isotropy of the turbulent field itself. The possible confluence of flat layers with large horizontal anisotropic blobs having short vertical correlation has been suggested before (6), and these two observations may provide an avenue to a common understanding of the phenomenon.

The basic signal statistical distribution of the received field should also provide a mechanism for distinguishing between: (1) scatter, and (2) layers plus scatter or glint. The usual assumption that the amplitude is Rayleigh distributed is not confirmed in any detail by experiment - nor should it be expected. The central limit theorem for the contributions of a large number of scattering elements does tell one that the orthogonal signal components of the scattered field

$$E_s = x(t) + i y(t) \tag{6}$$

ought to be distributed as follows:

$$dP(x,y) = \frac{dx\,dy}{2\pi\sigma^2\sqrt{1-\rho^2}} \exp -\left[\frac{x^2+y^2-2\rho xy}{2\sigma^2(1-\rho^2)}\right] \tag{7}$$

where ρ is the cross correlation coefficient of x and y and σ is the variance of each. Converting to polar amplitude and phase coordinates and integrating over the phase

$$dP(R) = \frac{R\,dR}{\sigma^2\sqrt{1-\rho^2}} I_0\left[\frac{\rho R^2}{2\sigma^2(1-\rho^2)}\right] \exp -\left[\frac{R^2}{2\sigma^2(1-\rho^2)}\right] \tag{8}$$

If the correlation ρ were zero, as usually assumed, this would reduce to the Rayleigh distribution. However, one can generalize Equation (5) to write an explicit expression for

$$\sigma^2_{\rho} = \frac{k^4}{(2\pi)^3} \int d^3\kappa\, S(\kappa) \left[\int_V d^3r\, e^{i\vec{K}\cdot\vec{r}}\, \mathrm{Re}(G E_0) \right] \times \left[\int_V d^3r\, e^{i\vec{K}\cdot\vec{r}}\, \mathrm{Im}(G E_0) \right] \tag{9}$$

In a mature theory this should be calculated as a function of geometry and compared with experiment. However, we must also note that a constant vector A added to a scattered component x + iy also produces a non-Rayleigh distribution. Starting from (6) and setting ρ = o for contrast and temporary simplicity, we find

$$dP'(R) = \frac{R\,dR}{\sigma^2}\, I_0\left[\frac{RA}{\sigma^2}\right] \exp-\left[\frac{R^2+A^2}{2\sigma^2}\right] \tag{10}$$

which is what one ought to expect from a steady layer reflection plus a turbulent scatter component or a random glint from the layer. The hope is that a detailed comparison of short sample experimental data with the two predictions can distinguish between the mechanisms.

The relationship of signal amplitude and phase to one another at succeeding instants of time provides one of the most useful measures of signal fine structure. Included in such relationships are the time auto-correlation of amplitude and phase, plus their corresponding power spectra. The derivatives of these quantities lead to signal fading rates and doppler shifts, both of which are now available experimentally. The joint probability distribution of two time-displaced complex signals

$$E_s(t) = x_1 + i y_1$$

and

$$E_s(t+\tau) = x_2 + i y_2 \tag{11}$$

has been given before in its most general form (5). By converting to polar signal coordinates, it is possible to write an explicit expression for the joint distribution of amplitude R_1, R_2, and phase ϕ_1 and ϕ_2. From this one can compute all relevant experimental quantities by integration; viz $\langle R_1 R_2 \rangle$, $\langle |\phi_1 - \phi_2|^2 \rangle$, etc. The coefficients in this probability distributions are functions of propagation geometry and the time displacement τ. Explicit expressions for the coefficients which are time-displaced generalizations of equations like Equation (8) have been established (5) using the appropriate generalization of Eq. (3).

$$\langle \delta(\kappa,t)\,\delta(\kappa',t+\tau) \rangle = \frac{\delta(\kappa-\kappa')}{(2\pi)^3}\, S(\kappa)\, e^{i\vec{K}\cdot\vec{u}\tau}\, C(\kappa,\tau) \tag{12}$$

Here $\vec{u}$ is the drift velocity of a frozen irregular structure and $C(k,\tau)$ describes the time correlation of the turbulent internal rearrangement of the structure. Using this general decomposition, it is possible to write

$$\langle x(t)\,x(t+\tau)\rangle = \langle x_1 x_2\rangle = \frac{k^4}{(2\pi)^3}\int d^3k\, S(k)\, e^{i\vec{u}\cdot\vec{k}\tau}$$

$$\times\, C(k,\tau)\left|\int_V d^3r\, e^{i\vec{k}\cdot\vec{r}}\, \mathrm{Re}\left[G(R,r)\,E_0(r,T)\right]\right|^2 \qquad (13)$$

and so on for the other components of the moment matrix which defines the coefficients in the general probability distribution function (5). Notice that these expressions also provide explicit separation of the turbulent physics and the path geometry. In point of fact, the integrals which occur in the time-displaced signal component correlations are the same as those required for the ordinary variances of x and y plus the cross correlation $\langle xy\rangle$.

The novel physical ingredient is the time autocorrelation of the turbulent structure C (K,τ) describing the self motion. Most qualitative theories of fading on scatter paths have ignored this and dealt only with the horizontal wind vector $\vec{u}$. The intuitive justification is that the vertical transport velocity is negligible and that only off great circle scattering elements have significant horizontal components of $\vec{k}$ via the weighting integrals in (13). This may be, but it is worth verifying by explicit calculation. The self-motion has no such projection, so that its vertical component is effective in producing time variability in the great circle plane. There is some reason (5) to believe that C (K,τ) ought to be a function of $K^{2/3}\cdot\tau$, which would suggest a fading rate variation with carrier frequency of $f^{2/3}$, as compared to a linear variation for straight drift motion f. Experiment shows a sliding variability between the two extremes, suggesting a mixture. This kind of experiment ought to provide an exceedingly good means for studying the effect and relative strengths of the various propagation mechanisms. Furthermore, time variability measurements are probably easier to perform than beam swinging experiments because they do not require long times for antenna alignment and give one a chance to examine true snapshots of the atmospheric structure.

A final word about space and frequency correlations is in order. The basic probability distribution for signal components gathered at displaced antennas or at the same site on separated frequencies is given by the same expression referenced (5) above for time-correlated signals. The coefficients which occur in such distributions change by virtue of the modification of the geometrical integrals viz:

$$\langle \chi(R)\,\chi(R+\Delta R)\rangle = \langle \chi_1 \chi_2 \rangle = \frac{k^4}{(2\pi)^3}\int d^3\kappa\, S(\kappa)\, \times$$
$$\times \left\{ \int_V d^3r\, e^{i\vec{\kappa}\cdot\vec{r}}\, \mathrm{Re}\left[G(R,r)\, E_0(r,\tau) \right] \right\}$$
$$\times \left\{ \int_V d^3r\, e^{i\vec{\kappa}\cdot\vec{r}}\, \mathrm{Re}\left[G(R+\Delta R,r)\, E_0(r,\tau) \right] \right\} \quad (14)$$

while the separated frequency expressions involve different electromagnetic wavenumbers k in the Green's function and incident field. The calculation of these coefficients is part of the same unified analytical program suggested above.

Having pointed the way to a wider theory, it is appropriate to emphasize its deficiencies. Firstly, it refers only to the turbulent scatter component and not to a steady layer reflection. A comparable sophisticated theory is needed for the latter, plus a consistent means for predicting its time variability, space correlation, etc. We have also assumed single scattering in the common volume, and some have suggested that multiple scattering may be the dominant mode . While our expressions do allow for a non-isotropic turbulent spectrum $S(\vec{\kappa})$ depending individually on the three components of $\vec{\kappa}$, we have no clear means for dealing with a non-stationary process. However, the primary deficiency of this proposal is that it is a plan and not a finished thesis. Perhaps the availability of new experimental data and the renaissance of genuine scientific interest in the subject will stimulate such research.

REFERENCES

(1) William P. Birkemeier, "Identification of Atmospheric Structure by Coherent Microwave Scattering", April 1968 (this volume).

(2) B. B. Barrow, L. Abraham, S. Stein, and D. Bitzer, "Preliminary Report on Tropospheric-Scatter Propagation Tests Using a Rake Receiver", Proc. IEEE, Vol. 53, No. 6, pp. 649-651; June 1965.

(3) Donald C. Cox, "Phase and Amplitude Measurements of Microwaves Propagated Beyond the Horizon", Stanford Electronics Laboratories Technical Report No. 2275-1, December 1967; and J. W. Strohbehn and A. T. Waterman, Jr., "Transhorizon Propagation Measurements from a Simultaneous Frequency and Angle Scan Experiment", Radio Science, Vol. 7, pp. 729-741, (1966).

(4) Donald C. Cox, "A Review of Transhorizon Propagation as a Possible Tool for Probing of the Atmosphere", March 1968 (this volume).

(5) Albert D. Wheelon, "Radio Wave Scattering by Tropospheric Irregularities", J. Research NBS Part D, 63 D, p. 205, 1959.

(6) Harold Staras and Albert D. Wheelon, "Theoretical Research on Tropospheric Scatter Propagation in the United States 1954-1957", IRE Trans. PGAP AP-7, 80 (1959).