N72-25372

GROUND-BASED MICROWAVE PROBING

Ed R. Westwater and Martin T. Decker

Environmental Science Services Administration
Research Laboratories
Boulder, Colorado

ABSTRACT

This report reviews the theoretical and experimental activities in remote sensing conducted by the Millimeter Wave Program, Wave Propagation Laboratory, of the ESSA Research Laboratories. The projects reported here are (1) the linear statistical inversion method, (2) determination of vertical temperature profiles from microwave emission measurements near 60 GHz, (3) determination of radio path length corrections from emission measurements at 20.6 GHz, and (4) determination of liquid water content of thunderstorm cells by emission measurements at 10.7 GHz.

1. INTRODUCTION

The possibilities of remote inference of temperature and water vapor profiles from microwave thermal emission measurements are well known (Meeks and Lilley, 1963; Barrett and Chung, 1962). The problems of inverting radiometer observations to obtain profiles has been solved for two frequently occurring situations:

(1) a priori statistical knowledge of both the profile and experimental noise level, and complete knowledge of absorption coefficients are available (Rodgers, 1966; Strand and Westwater, 1968a, b).

(2) simultaneous measurements of thermal emission and direct measurements of the profile can be obtained

over a suitable ensemble of data. This technique, which is a multidimensional regression analysis, does not require knowledge of the absorption coefficient. The observational requirements of this technique, however, are not always practical.

Sections 2 and 3 of this report describe the linear inversion technique (1) and its use in evaluating the potential of ground - based probing in inferring temperature structure. A passive technique which corrects for range errors due to the refractive index of water vapor is described in Section 4. The remote inference of the liquid water content of discrete cloud structures by passive radiometry is discussed in Section 5.

2. THE LINEAR STATISTICAL INVERSION METHOD

Attempts to infer atmospheric profiles from measurements of emitted, absorbed, or scattered radiation have been described in the literature. Many of these problems can be reduced to solving an integral equation of the first kind in the presence of error in the measured quantity. Some of the difficulties in solving this type of equation are:

a. Finite measurements. Only a finite (and usually small) number of measurements can be taken from which to infer an entire profile.

b. Instability. Direct attempts to solve the equation by standard methods of matrix inversion can yield unstable solutions because of ubiquitous measurement errors, i.e., small errors in the measured quantity can yield large, physically unrealistic estimates of the inferred quantity.

c. Nonuniqueness. In a mathematical sense, the solutions are not unique, since any function that can be integrated to yield the observed value to within the noise level of the sensor is mathematically (but not necessarily physically) a legitimate solution.

d. Ill-Conditioning. Because kernel functions encountered in practical applications are smooth functions of physical parameters, measurements are often dependent, in the sense that certain measurements can be obtained from linear combinations of the others to within the noise level of the observations.

e. As Twomey (1965) has shown, methods that attempt to approximate the profile by approximating its transform are based on the incorrect premise that "nearness" to the transform implies "nearness" to the profile.

Some investigators, e.g., King (1964) have advocated nonlinear techniques to achieve useable results; others (Twomey, 1965) have used linear methods incorporating smoothing to reduce the instability. More recently (Alishouse, et al., 1967) empirical orthogonal functions have been used to maximize the information obtainable from the small number of determinable parameters. The minimum-rms inversion method (Rodgers, 1966; Strand and Westwater, 1968a, b) is linear, achieves smoothing by filtering the signal from the noise in a statistically optimum manner, and estimates the profile from the measurements at hand and the a priori statistical knowledge of both the profile and the noise level of the sensor. The essentials of this method may be described as follows:

The Fredholm integral equation of the first kind is written

$$g_e(x) = g(x) + \epsilon(x) = \int_a^b K(x, y) f(y)\, dy + \epsilon(x), \tag{1}$$

where $g_e(x)$ is the measured value and is the sum of the amount contributed by the profile alone, $g(x)$, and the instrumental noise $\epsilon(x)$; $K(x, y)$ is the kernel (assumed to be known); and $f(y)$ is the unknown. From measurements of $g_e(x)$ at some set of values of x, say x_i, $i = 1, 2, \ldots, n$, it is wished to infer $f(y)$. Introduction of a suitable quadrature approximation to (1) yields the matrix equation

$$g_e = Af + \epsilon, \tag{2}$$

where

$$g_e = [\, g_e(x_1), \ g_e(x_2), \ \ldots \ g_e(X_n)]^T,$$

$$(A)_{ij} = K(x_i, \ y_j)\, w_j,$$

$$f = [\, f(y_1), f(y_2), \ \ldots, \ f(y_m)]^T,$$

$$\epsilon = [\, \epsilon(x_1), \ \epsilon(x_2), \ \ldots, \ \epsilon(x_n)]^T;$$

w_j = quadrature weight associated with the point y_j, and the superscript T means matrix transposition. The minimum-rms inversion method does not attempt to solve (2); rather it attempts to use the data vector, g_e, to estimate the solution such that the average mean-square error in the estimation is as small as possible. Let

$$\eta \equiv f - f_o, \tag{3}$$

where f_o is the mean of f, obtained by averaging over a representative ensemble of profiles. If one assumes that the estimate to the profile, $\hat{\eta}$, can be expressed as a linear combination of the data, g_e, then the estimate that minimizes the expected mean-square error of $\hat{\eta} - \eta$ is given by (Strand and Westwater, 1968b)

$$\hat{\eta} = S_f A^T H^{-1} (g_e - Af_o), \tag{4}$$

where

$$H = S_\epsilon + AS_f A^T,$$

$$S_f = \text{known covariance matrix of the profile}$$

$$\equiv E \left\{ (f - f_o)(f - f_o)^T \right\}$$

(E denotes expected value operator),

and

$$S_\epsilon = \text{known covariance matrix of experimental observations.}$$

In addition, the covariance matrix of the solution, $S_{\hat{\eta} - \eta}$, is given by

$$S_{\hat{\eta} - \eta} = X^{-1} \tag{5}$$

where

$$X = S_f^{-1} + A^T S_\epsilon^{-1} A.$$

The sum of the diagonal elements of $S_{\hat{\eta} - \eta}$ is m times the expected mean-square error per quadrature point. The essential properties and requirements of the solution may be summarized as follows:

a. The method requires knowledge of S_f and S_ϵ. We can estimate S_f from past observations of f (usually direct observations); S_ϵ can be determined experimentally by calibration of the sensor.

b. Data with correlated errors can be treated by this method.

c. The instability problem is completely eliminated: as $S_\epsilon \to \infty$, $\hat{\eta} \to 0$, i.e., the best estimate of the profile is its mean.

d. The average error of the solution can be determined immediately from properties of the equation of transfer, the measurement noise level, and the a priori information.

e. The introduction of basis vectors to represent the solution is neither necessary nor desirable. A completely adequate representation of the solution is its values at each of the quadrature abscissas.

f. The solution yields the best fit to the profile in the sense of minimum mean-square linear unbiased estimation when averaged over the joint probability distribution of f and ϵ, where the components of f are assumed uncorrelated with those of ϵ.

g. A criterion for optimum observation ordinates (such as frequency or angle) may be given. From a large set of possible measurement locations, the optimum subset is the set that yields the minimum overall expected mean-square error ($\mathrm{Tr}\, S_{\hat{\eta} - \eta} = \mathrm{Tr} X^{-1}$). The optimum subset is a function of the kernel, the noise level of observations, the a priori information and the number of elements in the subset (Westwater and Strand, 1968).

h. The method estimates the solution at each point along the profile. Thus, questions of "height resolution" can be answered only by reference to the correlation properties of the medium. For example, a profile "spike" of arbitrarily small thickness and lying outside the interval in which the kernel contributes to the integral could be inferred strictly on the basis of conditional probability if the medium had a correlation coefficient of unity between the two regions.

i. The linear estimate is the optimum estimate, in the expected mean-square sense, if the profile and measurement error are jointly gaussian distributed and if the integral equation is exactly a Fredholm equation (C. D. Rodgers, 1966).

3. GROUND-BASED TEMPERATURE PROBING BY PASSIVE MICROWAVE TECHNIQUES

Under clear-sky conditions, ground-based measurements of microwave thermal emission near 60 GHz can provide useful information about the temperature structure of the lower troposphere. Such information, obtainable on a time scale much finer than that provided by conventional radiosonde launches, could be useful in studying development of low altitude inversion layers, and in air pollution studies. In addition, ground-based passive probing for water vapor profiles requires knowledge of tropospheric temperature structure which could be provided by this technique. In this section, theoretical results of Westwater and Strand (1968) are presented to indicate the accuracies that might be expected from such a technique under clear-sky conditions.

Data vectors (i.e., thermal emission measurements) from which profiles can be extracted can be generated by either frequency or angular variation or both. A complete discussion of the comparative experimental accuracies achievable at present and in the future is beyond the scope of this paper. However, with statistical inversion methods, various choices of random experimental error, as embodied in the covariance matrix, S_ϵ, can be made and the average effect of these measurement errors on the inferred profile can be determined. As an example of this type of calculation, Figures 1 and 2 show accuracies obtainable by ground-based probing using vertical emission measurements at five frequencies in the oxygen complex. In these figures, "constrained" refers to the use of the surface temperature to modify the estimate of the profile. With measurement accuracies of 1^{o}K, only two, or at most three, of the measurements are independent (cannot be predicted to within the noise level by linear combinations of the remaining measurements), so that similar results could be obtained by a suitable choice of two or three channels. Figure 3 shows the accuracy expected from a fixed frequency variable-angle scheme at 53.8 GHz. The initial elevation angles ranged from 1^{o} to 90^{o} in equal increments of the cosecant of the elevation angle, spherical temperature stratification was assumed, and ray tracing based on Snell's law for a spherical atmosphere was used. The results indicate that, for the noise levels assumed here, the temperature profile can be determined to a standard deviation of 2^{o} up to about 5 km. Above the 5 km, the results deteriorate rapidly, and the uncertainty approaches the _a priori_ uncertainty.

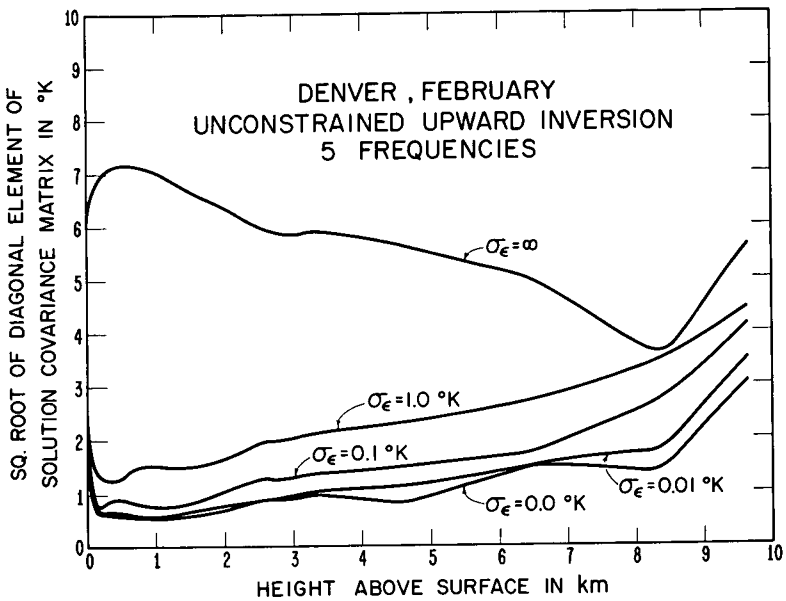

Figure 1. Accuracies obtainable by ground-based probing in the oxygen complex (unconstrained).

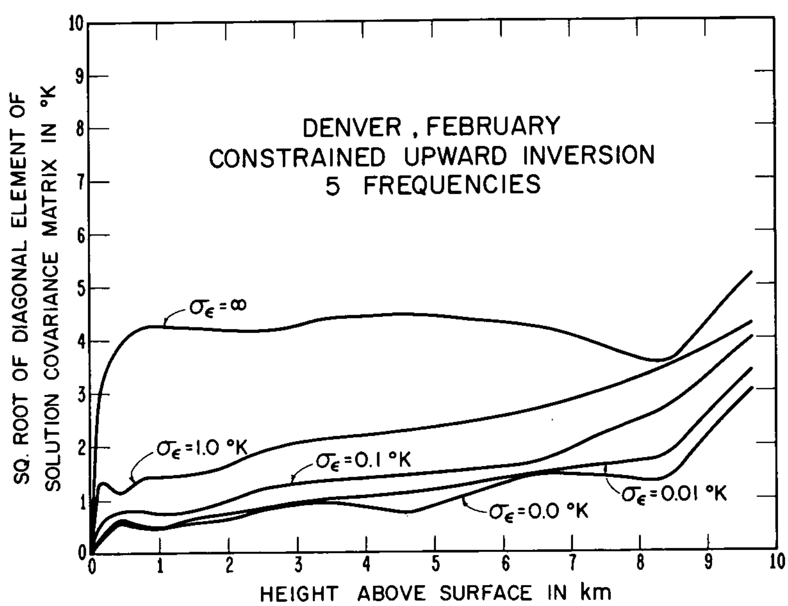

Figure 2. Accuracies obtainable by ground-based probing in the oxygen complex (constrained).

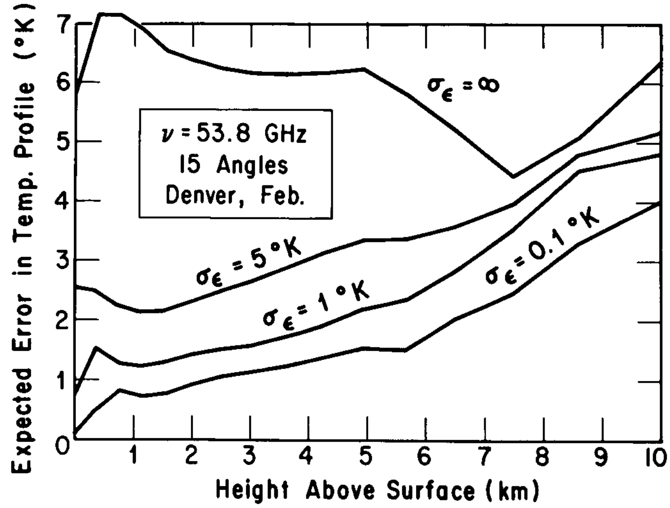

Figure 3. Information content of angular emission measurements in determining temperature profiles.

4. DETERMINATION OF ATMOSPHERIC RANGE ERROR CORRECTION BY PASSIVE MICROWAVE RADIOMETRY

The accuracy of baseline-type tracking systems which measure range to missiles is limited by atmospheric refraction. The MARCOR (microwave atmospheric range correction) technique (Menius, et al., 1964) corrects for range errors due to the wet component of atmospheric refractivity by using thermal emission measurements near the 1.35 cm water vapor line. These measurements are made by a radiometer pointing in the approximate direction of the tracked object, and related to the line integral of the wet refractivity. Theoretical results indicate that this technique can improve surface estimates of range correction by a factor of 5 to 10 (Westwater, 1967).

The Millimeter Wave Program Area of the Wave Propagation Laboratory and Tropospheric Physics Program Area of the Tropospheric Telecommunications Laboratory, ESSA Research Laboratories, have initiated a joint experimental effort to establish the feasibility of the MARCOR technique. Phase measurement techniques were used to measure apparent path length at frequencies near 10 GHz over a 64 km path from Upolu Point (elevation 30 m) on the island of Hawaii to Mt. Haleakala (elevation 3025 m) on the island of Maui. Variations of several meters in this measured path length, resulting from the changing atmospheric refractivity, were observed. Simultaneous measurements of atmospheric thermal emission were made at 20.6 GHz with a conical horn reflector having an aperture diameter of 1.2 m and directed along a path near that mentioned above. Meteorological data were taken at the surface, through the atmosphere with radiosondes, and with an aircraft instrumented for refractivity and temperature soundings. Supplementary atmospheric emission and absorption measurements were made at 15, 31, and 53.8 GHz. The data are being analyzed to determine to what extent the radiometric data can predict the changes in apparent path length.

5. LIQUID WATER CONTENT OF THUNDERSTORM CELLS

Measurements of the thermal noise emission from Colorado thunderstorms by means of a 10.7 GHz radiometer and a 18 m diameter steerable parabolic antenna have been used to prepare contours of integrated liquid water content along radio rays passing through the storm (Decker and Dutton, 1968).

The experimental data consist of horizontal radiometric scans across the thunderstorm that make it possible to measure the difference in antenna temperature when the antenna is pointed at the clear atmosphere and the storm. In addition to the radiometric data, radar, radiosonde, and surface meteorological data were used to estimate the absorption that must have been occurring along a line of sight toward the clear sky and also when looking toward the storm. Subtracting the absorption due to the clear sky from that occurring when looking at the storm yielded an estimate of the absorption due to the storm itself. Since the storm absorption is proportional to the liquid water content with a known constant of proportionality dependent upon the temperature, the water content of the storm along the line of sight can be estimated.

Examples of contours of liquid water content, as seen from the radiometer site, plotted on an azimuth angle-elevation angle grid are shown in Figure 4. The contour values indicate the number of kilograms of liquid in a column of one square meter cross section along the entire path through the storm. Alternatively, one may estimate the average liquid water density in grams per cubic meter over the path length by dividing the contour values by an estimated length (in kilometers) through the storm.

The technique shows promise as a tool in the study of liquid water in clouds, although further work is needed to delineate its limitations. In particular, the effects of scattered energy should be evaluated, as well as the effects of ice particles with their different emitting and scattering properties.

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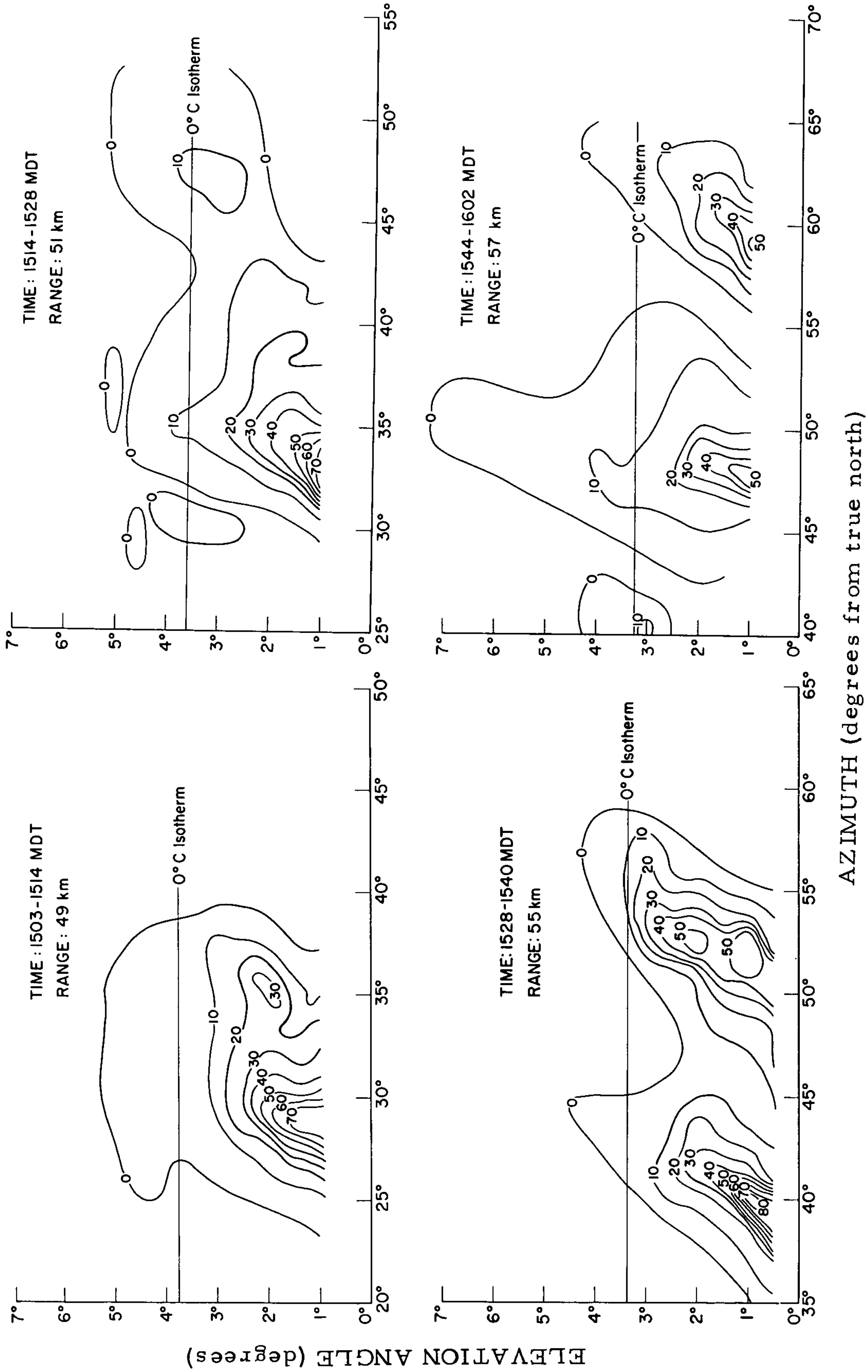

Figure 4. Liquid water content (kg/m^2 column) of a July 18, 1967 thunderstorm near Boulder, Colorado.

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