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DETERMINATION OF THE TEMPERATURE PROFILE IN AN ATMOSPHERE FROM ITS OUTGOING RADIATION

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ABSTRACT

A highly convergent "relaxation" method for the inversion of the full radiative transfer equation has been developed. The results of the iterative solution indicate that convergence can be achieved over a wide range of initial "guesses," enabling the temperature profile of a relatively unknown atmosphere to be unambiguously determined. The method is illustrated by examples for the outgoing radiance in the earth's atmosphere for the region of the 4.3μ CO_2 band, but can be similarly applied in other frequency ranges.

1. INTRODUCTION

The total upward radiance $I(\nu,\overline{P})$, arriving vertically in a small solid angle ($\omega \approx 0$) at a pressure level $\overline{P}$ of an atmosphere in a given wave-number interval ν, is obtained by formal integration of the fundamental radiative transfer equation (RTE), subject to a black-body boundary condition, and is conveniently reduced to read

$$I(\nu,\overline{P}) = \varepsilon\, B\left[\nu,\, T(P_0)\right]\tau(\nu,P_0) + \int_{P_0}^{\overline{P}} B\left[\nu,T(P)\right] \frac{\partial\tau(\nu,P)}{\partial P}\, dP, \qquad (1)$$

where B is the Planck function, τ is the transmission function dependent on ν,P, and the temperature distribution T(P) between $\overline{P}$ and P, P_0 is the pressure at the lower boundary, and ε is the emissivity at P_0.

Following a proposal made by Kaplan (1959) that the vertical distribution of atmospheric temperature, T(P), can be inferred from measurements of outgoing radiance, $\tilde{I}(\nu_i, \overline{P})$, i = 1, 2, ... n, a number of sophisticated but inconclusive attempts have been made to solve Eq. (1) for T(P). The history of these attempts is extensively discussed and referenced in a series of articles by Wark and Fleming (1966). A review of these attempts further discloses the need for discovering and employing to advantage other physical and mathematical properties of the RTE that relate the measured radiance to the desired atmospheric parameter, and to a general method of solution.

Past attempts to solve this problem, by linearizing the Planck function and reducing Eq. (1) to an $n \times n$ system of linear equations, failed because the resulting system is always "ill-conditioned." In fact, our extensive studies of the above method applied to the ν_3 CO_2 band show that, without introducing any damping functions, the iterative solution converges only if the initial guess is made accurate to at least 10^{-8} in T(P). However, this author's past experience (Chahine, 1965) with the same family of Eq. (1) strongly suggests that the RTE in its integral form is "well-behaved," and that its reduction to a linear system of equations is improper.

The purpose of this work is to explore briefly an important property of the radiance emitted vertically from different layers of an atmosphere, and to apply it to the numerical solution of the complete RTE for the determination of the temperature profile.

2. GENERAL FORMULATION

The physical properties of outgoing radiation from various layers of an atmosphere are such that at any pressure level P within a wide range of pressures, $P_2<P<P_1$, the total upward radiance can be related (e.g., from the mean value theorem) to the Planck function at P by

$$\frac{I(\nu,\bar{P})}{I^1(\nu,\bar{P})} \approx \frac{B[\nu,T(P)]}{B[\nu,T^1(P)]}, \qquad (2)$$

where T(P) and $T^1(P)$ are two different temperature profiles in the same atmosphere, resulting in radiance values I and I^1, respectively. This general feature is directly related to the fact that over a wide range of temperature profiles, the dependence of $\tau(\nu,P)$ on changes in T(P) is negligible compared to that of the Planck function. Thus, if we select a set of i frequencies that satisfy Eq. (2) at an appropriate set of i pressure levels, we can readily use the above approximation to solve Eq. (1). In practice, it is more convenient to proceed first by factorizing the Planck function in terms of a function of temperature only and then use the latter as a universal variable of iteration.

To factorize the temperature in the Planck function we write

$$B(\nu,T) = a\nu^3/(e^{b\nu/T} - 1) \qquad (3)$$

$$= X(T)\beta(\nu)\delta(\nu,T), \qquad (4)$$

with $\delta(\nu,T) = O(1.0)$. Since the temperature dependence of Eq. (3) appears as $\sim e^{-b\nu/T}$, we replace the argument $-b\nu/T$, within a small ν-T range, by its least-squares fit as

$$\frac{\partial}{\partial C_j}\int_{\nu'}^{\nu''}\int_{T'}^{T''}\left[\frac{b\nu}{T} - C_1 - C_2\nu - \frac{C_3}{T}\right]^2 dT\, d\nu = 0, \quad (j = 1,\ 2,\ 3.) \qquad (5)$$

The limits of integration reflect roughly the frequency range in use and the expected temperature variations in the atmosphere. Solution of the above 3×3 system of equations gives

$$C_1 = -b \frac{\ln(T''/T')}{T'' - T'} \frac{\nu'' + \nu'}{2} , \qquad (6\text{-a})$$

$$C_2 = b \frac{\ln(T''/T')}{T'' - T'} \qquad (6\text{-b})$$

and

$$C_3 = b \frac{\nu'' + \nu'}{2} . \qquad (6\text{-c})$$

The scaled universal variable of iteration is then

$$X(T) = \exp\left(-C_1 - \frac{C_3}{T}\right) \qquad (7)$$

The factorized Planck function becomes

$$\frac{B(\nu,T)}{a\nu^3} \equiv X(T) \left[\frac{\exp\left(-\frac{b\nu}{T} + C_1 + \frac{C_3}{T}\right)}{1 - e^{-b\nu/T}} \right], \qquad (8)$$

in which the variation of X(T) with temperature can be several orders of magnitude larger than that of the second factor, as in the 4.3μ region of our example.

By substituting Eq. (8) into Eq. (1) and grouping, the RTE takes its final form as

$$I(\nu,\bar{P}) = X[T(P_0)]\; A(\nu,P_0) + \int_{P_0}^{\bar{P}} X\; [T(P)]\; K(\nu,P)\; dp, \qquad (9)$$

in which the relatively weak dependence of A and K on changes in T(P) is not explicitly shown. Thus, the problem reduces now to solving Eq. (9) for X_i when $\bar{P}$ is given together with a selected set of radiance values $\tilde{I}_i\ (\nu,\bar{P})$ measured at i frequencies.

3. METHOD OF SOLUTION

We propose to solve Eq. (9) by iteration: We make a first guess $T^0(P)$ for X_i^0, and using this as input to Eq. (9), evaluate $I_i^0\ (\nu,\bar{P})$ for the given atmosphere. The mechanism for generating the iterative solution is obtained from Eqs. (8) and (2) as prescribed in the following relaxation equation:

$$X_i^n = X_i^{n-1} \frac{\tilde{I}_i}{I_i^{n-1}} , \qquad (10)$$

where n is the order of iteration. From the above equation we determine X_i^1, then $T^1(P_i)$, substitute it into Eq. (9), and evaluate I_i^1. Using Eqs. (10) and (7) again, we evaluate X_i^2, then $T^2(P_i)$, and repeat the same procedure for the following iterations. It is important to point out here that when $X_i^n(T)$ is reduced to temperature according to Eq. (7), the resulting values are temperatures at specific altitudes and are not average values taken over an atmospheric layer.

The rate of convergence is determined from the residuals

$$R_i^n = \frac{\left|\tilde{I}_i - I_i^n\right|}{\tilde{I}_i} \tag{11}$$

or their average value

$$\left\langle R^n \right\rangle_{av} = \frac{1}{i} \sum R_i^n \, . \tag{12}$$

The solution converges when $R_i^n \to 0$ for all ν_i.

4. APPLICATIONS AND DISCUSSION

In the remaining part of this work we will demonstrate the way the above scheme can be used to determine the temperature distribution in the earth's atmosphere from the spectral distribution of vertically outgoing radiance in the region of the 4.3μ CO_2 band.

4.1 Procedure

To set up the problem, we assume a temperature profile $\tilde{T}(P)$, determine the transmission function τ, and use an accurate quadrature formula to evaluate the radiance $\tilde{I}_i(\nu,P)$ for $\varepsilon = 1$, $\overline{P}/P_0 = 0.009$, and for the 39 frequencies from 2180 (5) 2370 cm^{-1} (because our transmission subroutine had been designed to give $\tau(\nu,P)$ at intervals of ν equal to 5 cm^{-1} in the above range). The aim now is to take the radiance at a set of selected frequencies as simulated experimental data and to investigate how accurately we can retrieve the temperature profile $\tilde{T}(P)$.

The results presented here were obtained according to the following sequence:

1. Select a set of 10 sounding frequencies (ν_i = 2180, 2235, 2245, 2260, 2290, 2295, 2305, 2315, 2370, 2360), corresponding to 10 pressure levels (P_i/P_0 = 1.0, 0.85, 0.60, 0.40, 0.20, 0.10, 0.04, 0.025, 0.0135, 0.009). The selection of this set is not unique and the choice of its size is optional. The most <u>efficient</u> sounding frequencies, for numerical computation purposes, are those that satisfy Eq. (2) in the region where their contribution to the integrand of Eq. (9) is substantial.

2. Make an initial guess $T^0(P)$ and solve Eq. (9); check the residuals R_i^n and $\langle R^n \rangle_{av}$, and satisfy convergence.

3. Compare the results with the exact answer, $\tilde{T}(P)$.

Other pertinent details of the above sequence will be presented along with our examination of the following results.

4.2 Discussion of Numerical Results

Figure 1 shows the results of the fifth iteration using the U.S. Standard Atmosphere as initial guess. The temperature profile computed at 10 pressure levels (not layers) has an average accuracy of 1^oK. In the calculations, the transmission function τ was left unchanged from that of the Standard Atmosphere temperature profile. When τ was recomputed after every iteration, to account for changes in the temperature profile, the results improved slightly. When other sets of ν_i were used as sounding frequencies, the corresponding results compared favorably with those shown in Fig. 1, indicating the general prevalence of the feature given in Eq. (2). The numerical evaluation of the integral in Eq. (9) was performed by using a modified Simpson's rule and a second-order interpolation formula for the intermediate values of temperature.

The rate of convergence at each of the selected 10 sounding frequencies, as well as that at each of the remaining 29ν, was checked after every iteration. The average rates of convergence $\langle R^n \rangle_{av}$, listed in Table I, were taken over all 39ν. Examination of this list clearly reveals the two most important characteristic features of the iterative solution: stability and convergence.

A similar examination of the average rates of convergence of the temperature profile $T^n(P)$ indicates that an average accuracy of 1^oK is achieved when $\langle R^n \rangle_{av} < 3\%$, and that an average accuracy of $2\text{-}3^oK$ can be achieved when $5\% < \langle R^n \rangle_{av} < 7\%$. The above results offer an initial indication of the effect on the accuracy of the final profile of errors in the measurements of radiance values.

Figure 2 shows the results of the eighth iteration using the same sounding frequencies as in Fig. 1, but using as initial guess any one of three isothermal temperature profiles, 280, 240, and 200^oK. The transmission function τ was recomputed during the first five iterations only and was kept unchanged afterward. After the fifth iteration, the results for all three (very different) initial guesses converged to the same profile, and became indistinguishable on the scale of Fig. 2. The fact that the same set of sounding frequencies gave equally good results for all the above cases demonstrates the strong persistence of the basic feature given in Eq. (2).

The rates of convergence for these three initial guesses were remarkably similar and the residuals decreased very sharply. The results listed in Table I show a spectacular example of convergence for the case of $T^0(P) = 280^oK$; there, the average residual decreased from an initial value of 920.10% to 21.74% in just one iteration.

It was further noticed that convergence occurred first at the lower levels of the atmosphere, then spread upward. Thus, higher order iterations tended to improve convergence at the upper levels at the expense of accumulating truncation errors at the lower levels. We note in conclusion that the numerical results shown here do not represent the best in the art of computational refinements; we believe this method is capable of far greater accuracy.

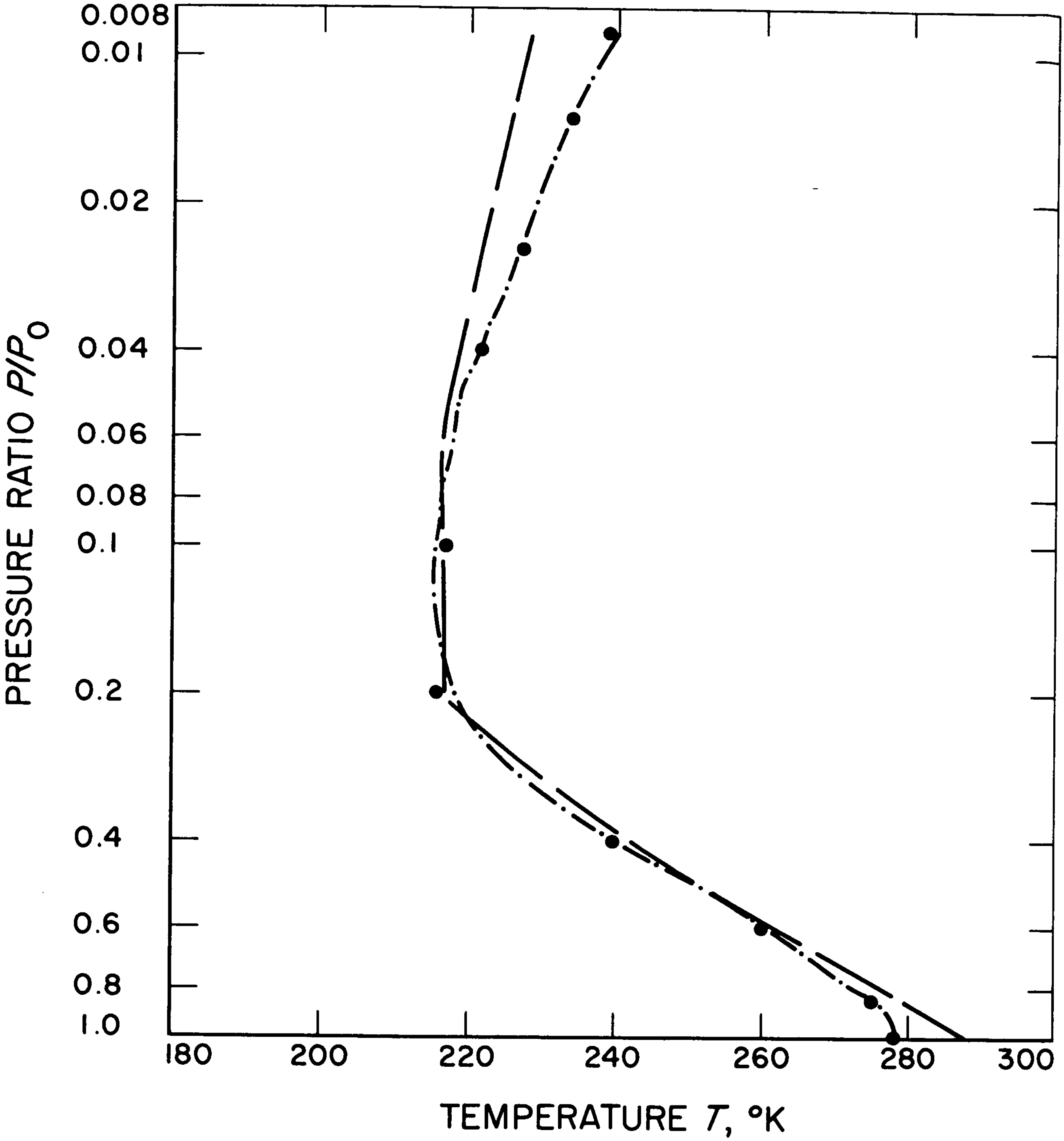

Fig. 1. Temperature profile, from 10 sounding frequencies, at the fifth iteration using the U.S. Standard Atmosphere as initial guess. • • • , iterative solution. —•—•, exact profile. — —, initial guess.

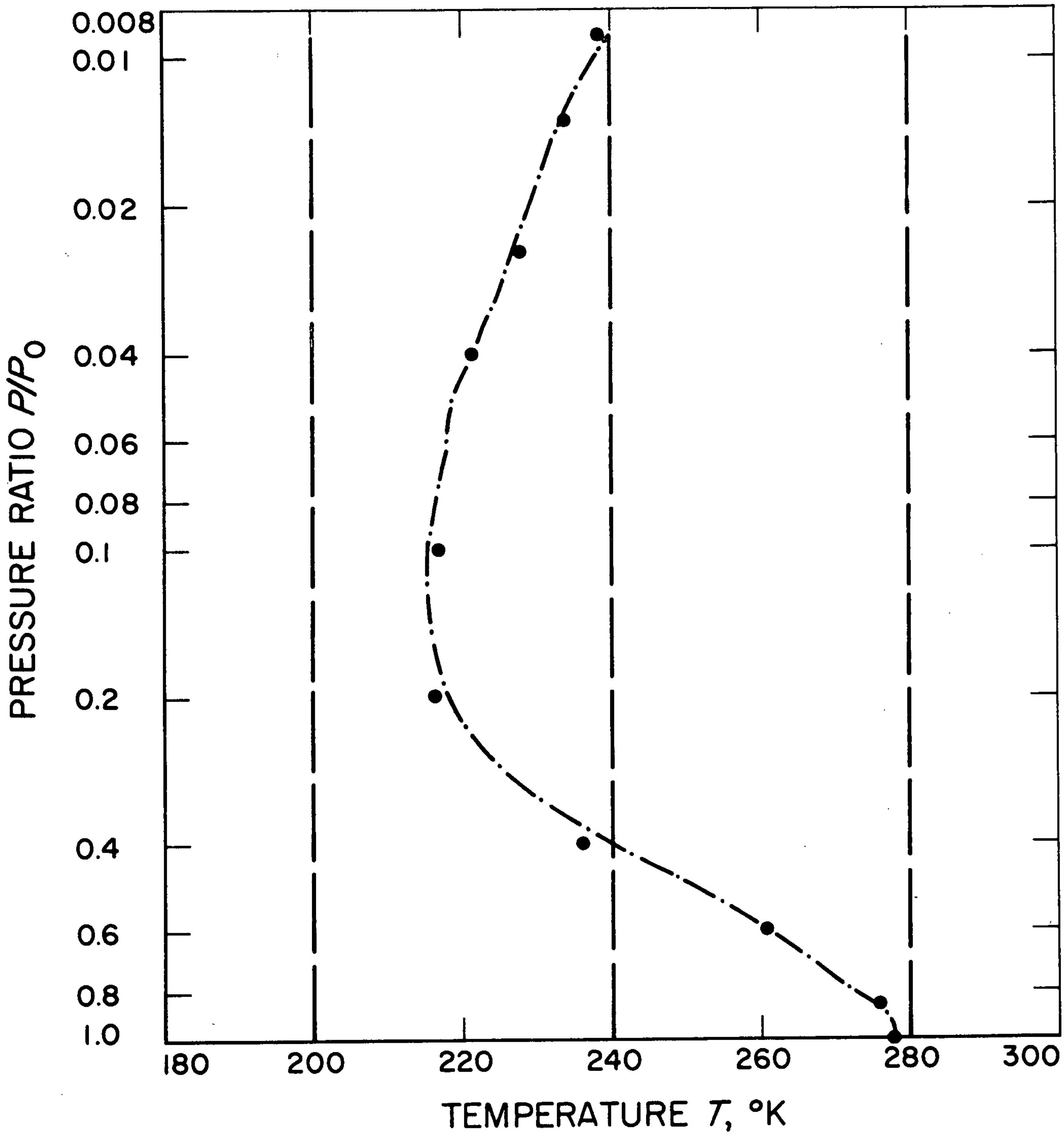

Fig. 2. Temperature profile, from 10 sounding frequencies, at the eighth iteration using as initial guess any one of three isothermal profiles: 280, 240, or 200°K. ••• , iterative solution. —•—•, exact profile. — — , initial guess.

TABLE I. Average Residuals R^n_{av}, %

Initial guess $T^0(P)$	Order of iteration, n									
	0	1	2	3	4	5	6	7	8	9
U.S. Standard Atmosphere	26.30	6.97	2.25	1.52	1.02	0.73	0.54	0.40	*	*
280^oK	920.10	21.74	10.11	5.49	2.88	1.35	0.81	0.54	0.42	*
240^oK	80.23	21.86	10.33	5.49	2.85	1.33	0.79	0.54	0.43	*
200^oK	90.89	22.42	10.76	5.53	2.84	1.31	0.79	0.55	0.46	0.41

*The relative error caused by the numerical quadrature precludes making the average residuals less than 0.39%.

5. CONCLUSION

The great advantage of the approach discussed here lies in the fact that it has exploited a dominant physical property of radiance occurring over a wide range of frequencies, and related this property to an efficient relaxation method of solution, and that it can be easily adapted for use in different frequency ranges, such as the 15 μ CO_2 band and the microwave regions.

The resulting inversion scheme provides a reliable tool for practical sounding of temperature with the accuracy required for use in numerical weather forecasting in the earth's atmosphere. Perhaps more important, this method is also suitable for unambiguous determination of temperature profiles of the comparatively unknown atmospheres of other planets. The time required for the reduction of a set of measured radiance values to the temperature profile shown here is very small, and the calculations are simple and can even be carried out on a desk calculator. Thus, a readily feasible system for the reduction of the measured radiance values can be arranged to produce temperature profiles in "real time" as data are accumulated.

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REFERENCES

Kaplan, Lewis D., 1959: Interference of atmospheric structure from remote radiation measurements. J. Opt. Soc. Am., 49, 1004-1007.

Wark, D. Q. and H. E. Fleming, 1966: Indirect measurements of atmospheric temperature profiles from satellites: I. Introduction. Monthly Weather Review, 94, 351-362.

Chahine, Moustafa T., 1965: "Numerical Solution of the Complete Krook-Boltzmann Equation for Strong Shock Waves," in Methods in Computational Physics, Vol. 4, E. Alder, S. Fernbach, and M. Rotenberg, Eds. (Academic Press, New York).