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Technical Memorandum 33-542

The Least-Squares Process of MEDIA for Computing DRVID Calibration Polynomials

R. K. Leavitt

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PREFACE

The work described in this report was performed by the Data Systems Division of the Jet Propulsion Laboratory in support of the Tracking System Analytic Calibration activities.

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ABSTRACT

This document describes and evaluates a process for computing a least-squares polynomial approximation of data points in which the optimum degree of the polynomial is automatically determined. An iterative smoothing technique is used to replace every point with the value taken on by a moving least-squares polynomial computed from a subset of points centered at the point to be replaced. The optimum degree of the resulting polynomial approximation is determined by analyzing the finite differences of each successive set of smoothed points. To evaluate the process, this document uses both artifically constructed data and actual Mariner Mars 1971 (MM'71) tracking data.

This process has been incorporated into the Transmission Media Calibration computer program (MEDIA), which calibrates radiometric data to overcome the effects on the tracking signal of charged-particle media. MEDIA was used in support of MM'71.

I. INTRODUCTION

The radio signal going to and from a spacecraft at planetary distances travels through several media containing charged particles. These particles affect the propagation of the radio signal and, in turn, the radiometric data recorded at the DSN tracking station. The charged-particle content of the media through which the radio signal passes is determined at the tracking stations by differencing range and integrated doppler (DRVID) observables (Ref. 1). To correct the recorded radiometric data for these effects, the digital computer program MEDIA (Refs. 2 and 3) computes calibration polynomials from the DRVID data for each tracking pass of the stations. To compute these polynomials, MEDIA employs a least-squares curve fitting process that automatically determines the appropriate degree required to optimize the fit.

This curve fitting process uses an iterative technique in which the data points are smoothed with a moving least-squares polynomial. In each successive iteration, every datum is replaced by the value taken on by this polynomial, which is computed from a subset of k points centered at the point to be replaced. Its degree and the number of points k of the subsets are determined from the total number of data points in the pass.

After each smoothing iteration, the first, second, etc., finite differences of the resulting smoothed points and each of their variances are computed until the minimum variance is determined. When the minimum variances from two successive iterations occur for the same (say, the m^{th}) finite differences, then a statistical F-test is used to compare them.

The smoothing process is ended when the F-test is satisfied. A least-squares polynomial of degree m is then fitted to the smoothed points. A statistical runs test is performed on the original data relative to this fit to detect those cases in which the DRVID data do not resemble a polynomial. When the test fails, the degree is increased, a new fit is computed, and the runs test is again performed with the new fit. This process is repeated until either the test has been satisfied or the degree of fit exceeds its maximum allowable value.

The following two conditions must be satisfied by the data in order to use this process:

1. The data must occur at equal time intervals.
2. The data should usually resemble a polynomial.

The first condition is imposed to simplify and to speed up the processing logic. Without it, the computer run time required to process a pass of data would be prohibitive.

The second condition is obvious, since the objective of the process is to fit a polynomial to the points. Violations of this condition should be detected and overcome by the runs test that is applied. Such occurrences, however, should be infrequent; otherwise, the processing time would become excessive.

This curve fitting technique is not applied unless there are at least 16 data points. If there are fewer then 16 points but at least three, a least-squares polynomial is fit directly to the points. Its degree is set at the largest integer contained in the square root of the number of points.

II. METHOD

A. Least-Squares Smoothing

A moving least-squares-fitting iterative technique is used to smooth the raw data. Each successive iteration smooths the data until the smoothed points resemble closely the resulting polynomial that is to be fit to the data. Each smoothing iteration replaces every data point with the value taken on by the moving polynomial, which is computed from a subset of k points centered at the point to be replaced.

The number of points k in this subset is computed from the total number of points in the pass. In general, the more points there are in a pass, the greater the sample rate, and the more likely that low-frequency noise will appear in the data. Increasing k results in more smoothing of the data, which is necessary to filter the low-frequency noise. Hence, k is increased as the number of points in the pass increases. It is set equal to the largest integer contained in the square root of the total number of points in the pass and then forced to be odd by adding one if necessary. For computing ease, k must always be odd.

To preserve local trend in the data, the degree of the moving polynomial is increased also as the total number of points increases. It is set equal to the largest integer contained in the square root of k. This compensates somewhat for the extra smoothing that results from increasing k.

The independent variables t_i, $i = 1, 2, \cdots, k$, in each subset are expressed in seconds past January, 0, 1950. They are scaled so that they range in value from -1 to +1 within each subset to assure a solution to the least-squares equation and to simplify the computations. These scaled quantities x_i are defined as follows:

$$x_i = \frac{t_i - \dfrac{t_1 + t_k}{2}}{\dfrac{t_k - t_1}{2}}, \quad i = 1, \cdots, k$$

The x take on the same values in all subsets once k is determined. This follows from the condition that the data points occur at equal time intervals. Thus, for $k = 3$, $x = \{-1, 0, 1\}$; for $k = 5$, $x = \{-1, -1/2, 0, 1/2, 1\}$; for $k = 7$, $x = \{-1, -2/3, -1/3, 0, 1/3, 2/3, 1\}$; etc.

The least-squares equation can be expressed as follows (Ref. 4):

$$\begin{pmatrix} k & \sum x & \cdots \sum x^n \\ \sum x & \sum x^2 & \cdots \sum x^{n+1} \\ \cdots & \cdots & \cdots \\ & & \\ \sum x^n & \sum x^{n+1} & \cdots \sum x^{2n} \end{pmatrix} \begin{pmatrix} C_0 \\ C_1 \\ \cdot \\ \cdot \\ \cdot \\ C_n \end{pmatrix} = \begin{pmatrix} \sum y \\ \sum xy \\ \cdot \\ \cdot \\ \cdot \\ \sum x^n y \end{pmatrix}$$

where

1. $C_0, C_1, \cdots, C_n$ are the unknown coefficients of the least-squares polynomial.
2. n is the degree of the least-squares polynomial.
3. All sums are from 1 to k, e.g.,

$$\sum x^{n+1} = \sum_{i=1}^{k} x_i^{n+1}$$

4. The set of values y represent the dependent variables in each of the subsets, i.e.,

$$\{y\} = y_1, y_2, \cdots, y_k$$

By substituting into the above equation

$$\begin{pmatrix} \sum y \\ \sum xy \\ \cdot \\ \cdot \\ \cdot \\ \sum x^r y \end{pmatrix} = \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_k \\ \cdots & \cdots & \cdots & \cdots \\ x_1^n & x_2^n & \cdots & x_k^n \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_k \end{pmatrix}$$

and transposing terms, we can express the coefficients as follows:

$$\begin{pmatrix} C_0 \\ C_1 \\ \cdot \\ \cdot \\ \cdot \\ C_k \end{pmatrix} = \left[\begin{pmatrix} k & \sum x & \cdots \sum x^n \\ \sum x & \sum x^2 & \cdots \sum x^{n+1} \\ \cdots & \cdots & \cdots \\ \sum x^n & \sum x^{n+1} & \cdots \sum x^{2n} \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_k \\ \cdots & \cdots & \cdots & \cdots \\ x_1^n & x_2^n & \cdots & x_k^n \end{pmatrix} \right] \begin{pmatrix} y_1 \\ y_2 \\ \cdot \\ \cdot \\ \cdot \\ y_k \end{pmatrix}$$

The matrix inside the brackets can be computed before entering the main smoothing loop. All of the values included are known and depend only on the total number of points in the pass.

Since each of the subsets is centered at the point corresponding to $x = 0$, only the constant term coefficient C_0 need be calculated. At $x = 0$, all of the other terms of the polynomial vanish; i.e., $y(0) = C_0$. Thus, in the smoothing process, the coefficient C_0 replaces the y-value at the center of each subset. It is computed by simply taking the dot product of the first row of the matrix in the brackets times the vector of y-values contained in each subset.

The end points of the pass are replaced by the values taken on by least-squares lines that are fit to the first k-points and the last k-points. The additional smoothing that results is necessary to prevent the smoothed function from curling at its ends.

B. Finite Differences of the Smoothed Data

The first, second, etc., finite differences of the smoothed points are analyzed after each smoothing iteration to determine (1) when the smoothing process is completed and (2) the degree of the resulting least-squares calibration polynomial. The key parameters are the variances of these finite differences and, in particular, the minimum variance.

The j^{th} finite differences $\delta_j y_i$ are defined as follows:

$$\delta_j y_i = \delta_{j-1} y_{i+1} - \delta_{j-1} y_i, \qquad \text{for } i = 1, 2, \cdots, d_j$$

where $\delta_o y_i = y_i$ and d_j is the number of j^{th} finite differences and is always j less than the number p of points in the tracking pass; i.e.,

$$d_j = p - j$$

The nature of the finite differences in the case of polynomials is that each successive set (j = 1, 2, etc.) of finite differences only has meaning up to the degree of the polynomial. For example, in the case of a cubic, only the first, second, and third finite differences have any meaning. In an absolute (or ideal) sense, the fourth and higher differences would all vanish. Practically, however, almost all numbers are truncated. Therefore, the fourth differences of a cubic are random truncation errors, and each succeeding set of differences tends to double in value and to alternate in sign.

The variances s_j^2 of the j^{th} finite differences can be estimated with the usual equation:

$$s_j^2 = \frac{\sum_{i=1}^{d_j} \left[\delta_j y_i - \frac{\sum_{k=1}^{d_j} \delta_j y_k}{d_j} \right]^2}{d_j - 1}$$

This parameter is a good measure of the finite differences. It decreases in value as j increases as long as the finite differences are meaningful, and then it increases. Therefore, the minimum variance of this set is the dividing point between meaningful and not meaningful finite differences. It is also a convenient measure of the amount of smoothing that has been accomplished.

The analysis of the finite difference is summarized as follows. The finite differences and their corresponding variances are computed for $j = 1, 2, 3$, etc., until the minimum variance is determined. This is done for each smoothing iteration until the minimum variances for two successive iterations occur for the same, say the m^{th}, finite differences. The F-test is then used to compare these two minimum variances. If there is a significant difference between them, the smoothing process is continued; if there is no significant difference, it is terminated. The value of m is used as the degree of the least-squares polynomial that is fit to the set of smoothed points. This polynomial becomes the charged-particle calibration equation for the pass if it satisfies the statistical runs test described below.

C. Statistical Runs Test

A statistical runs test is performed on the data relative to the fit to ensure that the resulting polynomial does indeed represent the raw data. This test is necessary in the event that the raw data do not resemble a polynomial. Although such occurrences could be tolerated occasionally, too many could increase the computer processing time to an unacceptable level.

A run is defined as a set of consecutive points, all lying on the same side of the polynomial (either above or below). It must contain at least one point. The total number of runs is one plus the number of times two consecutive points lie on opposite sides of the polynomial and can range from one, if all points lie on one side, to the total number of data points in the pass. In the latter case the points would have to alternate from one side to the other.

The cumulative sampling distribution z of the number u of runs can be approximated from the normal tables using the following equation (Ref. 5):

$$z = \frac{u - \overline{z} + 1/2}{s_z}$$

where

$$u = \text{number of runs}$$

$$\overline{z} = q + 1$$

$$s_z^2 = \frac{q\,(q-1)}{p-1}$$

and

$$q = \frac{2p_1\,p_2}{p_1 + p_2}$$

In these equations,

p = total number of points

p_1 = number of points on one side of the polynomial

and

p_2 = number of points on the other side of the polynomial

Note that $p_1 + p_2 = p$.

The runs test is satisfied if z is greater than the lower 5% level of the cumulative normal distribution, i.e., if $z > -1.654$. This is a one-sided test, since we are only interested in detecting runs that are too long. Long runs (i.e., too few runs) indicate that the fit does not adequately follow the flow of the data points.

If the runs test is not satisfied, then the degree of fit is increased by one, a new polynomial is fit to the smoothed points, and the runs test is performed again. This process is repeated until the test has been satisfied. The resulting polynomial then becomes the calibration equation.

III. APPLICATION

The least-squares process described above was applied to various sets of both simulated and real data to evaluate and to demonstrate its curve-fitting ability. The simulated sets provided a controlled situation in which the parameters were known. The real sets were used to demonstrate how the process would function in a real-life situation. A comparison with Faraday rotation calibration polynomials was also performed.

The following paragraphs describe various statistical tests that are performed on the least-squares fits that result from this process. These tests include the chi-square over degree of freedom, linear regression, and analysis of covariance. The primary parameter used in these tests is the variance of the points about the polynomial fit to them. For simulated data, this variance is compared to the known variance that was artificially constructed in the data. For real data, it is compared with the variance of the actual noise in the radiometric data. In comparing the resulting polynomials with Faraday rotation calibration polynomials, the mean and variance of the differences of the derivatives of the corresponding polynomials are analyzed.

A. Simulated Data

Two equations were used to synthesize DRVID data: a cubic and a tenth-degree polynomial. These equations are defined as follows:

$$\Delta\rho(t) = 1 + t + t^2 + t^3$$

$$\Delta\rho(t) = 0.15 + 0.15t + 0.15t^2 + \cdots + 0.15t^{10}$$

The cubic equation is used to produce three cases of 100 points each by superimposing on it 0-, 5-, and 10-ns Gaussian noise. In the first case, where no noise is present, a step function results. Because of format constraints imposed on the DRVID values, such functions are anticipated when the spacecraft is very close to earth. The noise level will be so small that it will be below the imposed precision level of the data and, hence, undetectable. The 5- and 10-ns noise levels are considered representative of the noise that will be present as the spacecraft gradually moves away from earth.

Figures 1a, 1b, and 1c, respectively, are graphs showing the data points for the three cases and the least-squares polynomials that were fit to them. The derivatives of these polynomials are also shown in each of the corresponding figures. The fit to the step function is presented to demonstrate that such functions are adequately processed. However, step functions could create problems for the statistical runs test because it assumes random noise. For this reason the runs test has been made a program option for the user.

For the 5- and 10-ns noise cases, the variance s^2 of the data points about the least-squares polynomial is an efficient parameter for testing the fit (Ref. 4). The statistic used is s^2/σ^2, where σ is either 5 or 10 ns. The distribution of s^2/σ^2 is the chi-squared over degree of freedom (χ^2/df) density function (Ref. 5). The 10% level of confidence is used for this test so that the requirements for rejecting the fit are not too stringent.

The acceptance criteria for 100 points are

$$0.777 < s^2/\sigma^2 < 1.24$$

The following tabulation summarizes the results of this test:

σ	s	s^2/σ^2
5	4.53	0.821
10	10.41	1.083

Both are within the acceptance criteria; therefore, both fits are acceptable according to the test.

The tenth-degree polynomial was chosen to produce a curve with a relatively steep slope. Figure 2a illustrates this equation and its derivative. A 10-ns noise was superimposed on this curve to obscure its slope. It is believed that DRVID data may behave in a similar manner in extreme cases.

Samples of 125, 250, and 500 evenly spaced points over the interval $-1 < t < 1$ were taken, converted to DRVID data, and used as input. Figures 2b, 2c, and 2d illustrate each set of points, their resulting least-squares polynomials, and their derivatives. The variances about these polynomials were subjected to the χ^2/df test. The results are summarized below.

Points	s	s^2/σ^2	Criteria
125	11.23	1.26	0.801 - 1.22
250	10.24	1.05	0.858 - 1.15
500	10.38	1.08	0.898 - 1.11

Although s^2/σ^2 is within the acceptance criteria for the samples of 250 and 500, it is not for the 125-point sample. These results seem to indicate that the sample rate should be increased as the noise increases to detect local or irregular trends in the data.

In conclusion, the results from the simulated data demonstrate that the process is performing in an acceptable manner, except possibly for extreme and unlikely cases. In such cases, the user will have to override the process by specifying the degree of fit. Input options built into the process make such action possible.

B. Real Data

MM'71 is the primary source of real data used to evaluate the curve-fitting ability of the process in an actual data-processing environment. The early part of the flight, when MM'71 was relatively close to earth, was selected for the evaluation to minimize the number and effect of the variables involved and to make the analysis easier. Appendix A contains the graphs of the least-squares fits for the passes used in this evaluation. They are included to illustrate the operation of the process on typical data.

The least-squares polynomials in these graphs appear to represent reasonably the data points in each pass, and their derivatives seem believable, except possibly at some of the ends of the passes. The data points in these cases exhibit slight trends at the ends of the passes that probably would disappear in the smoothing process if the passes were extended. This is particularly apparent in the relatively short pass of day 195 (Fig. A-4), where the effects of the end points are magnified and result in what would appear to be an unrealistic derivative. This "end points phenomenon" should be kept in mind during the numerical evaluation of the fits that follows.

The variances of the deviations of the points from the fitted polynomials are used to evaluate the fits as in the case of simulated data. They are a measure of the dispersion of the data caused by noise in the received signal. The fit is considered to be good if the resulting variance is an accurate measure of this dispersion. Table 1 lists the statistics for the tracking passes early in the flight that are used in this evaluation. In the last column on the right, the variances have been converted to dBm units (Ref. 6), which are comparable to units of power, i.e.,

$$\mu_2 = 10 \log_{10} \frac{s^{-2}}{0.001}$$

where μ_2 is the inverse of the variance in dBm units and s^2 is the variance.

The dispersion due to noise is related to the power received at the tracking stations; the less power received, the greater the dispersion (assuming everything else equal). The following tabulation lists the expected power received at Station 12 at 5-day intervals:

Day	Power, dBm
186	-163.7
191	-165.3
196	-166.8
201	-168.9
206	-170.9
211	-173.0

The relationship between these values and the variances of the deviations from the least-squares polynomials is what will be evaluated.

Linear regression lines and correlation coefficients (Ref. 5) were computed for both the inverse of the variances (dBm units) and the power received vs. days. Their slopes b, correlation r, and 95 percentiles ρ_{95} of the r distribution are listed below:

	Points	b	r	ρ_{95}
Power	6	-0.3737	-0.9977	0.729
Variance	14	-0.4163	-0.9931	0.457

Since $|r| > \rho_{95}$ in both cases, it is concluded that both sets are correlated with days.

The slopes of the two regression lines were compared using the analysis of covariance technique (Ref. 7). If there is no significant difference between these slopes, then the variances are related to the dispersion of the tracking data received at the stations. The F-distribution is used to test the slopes. The required equations are summarized below.

	Sum of squares	Degrees of freedom
Denominator	$\sum_{i=1}^{k} \sum_{j=1}^{n_i} (y_{ij} - a_i - b_i x_{ij})^2$	$\sum_{i=1}^{k} (n_i - 2)$
Numerator	$\sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 (b_i - \bar{b})^2$	$k - 1$

where

$k = 2$ is the number of data sets

$n_i = 6, 14$ is the number of points in the i^{th} set

(x_{ij}, y_{ij}) are the points

$\bar{x}_i$ is the mean of the x-values in the i^{th} set

a_i is the y-intercept of the i^{th} set

b_i is the slope of the i^{th} set

and

$$\bar{b} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2 b_i}{\sum_{i=1}^{k} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2}$$

The F-ratio that results from these equations is

$$F = \frac{1.993}{0.655} = 3.93$$

From the F-distribution tables at the 95% confidence level,

$$F(16, 1) = 4.50$$

Since $3.93 < 4.50$, it can be concluded that the two slopes are the same. Thus, the dispersion of the deviations about the fit is an accurate measure of the noise in the data, and the fits adequately represent DRVID data.

C. Derivatives of the Least-Squares Polynomials

The derivatives of the least-squares polynomials must accurately represent the rate of change of the charged-particle media. It is the rate of change that is important in the calibration of the doppler tracking data. The accuracy of these derivatives is evaluated here using Faraday rotation data (Ref. 8), which measure the charged-particle content of the ionosphere. They are obtained by passing a signal from a stationary satellite through the ionosphere to a tracking station and mapping the results to the line-of-sight of the spacecraft. They should be comparable to DRVID data when the spacecraft is close to earth and the ionosphere is practically the only source of charged particles. The Faraday calibration polynomials used for this comparison were computed by the digital computer program ION (Ref. 8).

Appendix B contains the graphs that illustrate the derivatives that result from the MM'71 DRVID and Faraday rotation data and the differences between them for the days included in this evaluation. The letter F identifies the polynomials that represent the Faraday derivatives. The ends of the Faraday derivatives have been deleted from the graphs and from the evaluation because of their erratic and unrealistic movements. The oscillations that are so pronounced in the graphs of the differences are caused almost entirely by the Faraday derivatives. The relatively large differences at the ends of the curves probably result from the end points phenomenon of the DRVID derivatives, as noted earlier. This effect is magnified again in the relatively short pass of day 195 (Fig. B-4). Table 2 lists the statistical parameters from these graphs that are used in the numerical analysis that follows. The last column on the right presents the inverses of the variances in dBm units. A linear regression line and correlation coefficient were computed for these variances (dBm units) vs. days to determine whether there is any relation between them and the dispersion of DRVID data. The results are summarized below:

$$N = 14$$
$$b = -0.141$$
$$r = -0.261$$
$$\rho_{95} = 0.457$$

Since $|r| < \rho_{95}$, the variances are not correlated with days; i.e., the slope b of the regression line is not significantly different from zero. Therefore, the differences of the derivatives are not related to the dispersion of the DRVID data or to the signal noise, both of which (as noted above) are correlated with days and have the same slope. This result indicates that the least-squares process has successfully filtered the noise from the data.

From the values in Table 2, the total mean and standard deviation of the differences of derivatives are -0.0031 and 0.0881 mm/s, respectively. Although the mean is relatively small, the standard deviation does indicate a rather large spread considering the size of the derivatives. However, computing error percentages at this time does not seem worthwhile since it is not known whether DRVID or Faraday contributes the greater error. A more profitable approach might be to apply these calibrations to a simplified orbit determination process and evaluate its convergence.

In conclusion, the derivatives of the resulting DRVID calibration polynomials are acceptable provided the tracking passes are not overly short in duration. The successful filtering of random noise from the data has minimized the dangers inherent in computing derivatives of raw data such as these. This accomplishment is one of the most important results of the process. Another significant attribute of the process is the fact that it has not introduced into the fit such distortions as "ringing" or other similar deformations, which too often appear in least-squares polynomial approximations. Such distortions are often amplified in the derivatives of these polynomials to the extent that the subsequent application of the derivatives is impractical. Although the differences between the derivatives of DRVID and Faraday polynomials have what seems to be a relatively large dispersion, they do average out as indicated by the small total mean. This anomaly is not yet fully understood; however, there are several likely causes that might contribute to it:

1. The end points phenomenon.
2. The mapping of the Faraday rotation calibrations to the spacecraft line-of-sight.
3. The lack of pronounced trends in the DRVID data. (The data used in this evaluation were all sampled during the "flat" portion of the ionosphere's diurnal charged-particle content curve.)

D. Statistical Runs Test

The need for a statistical runs test is demonstrated in Fig. 3a. As shown in this figure, the least-squares process determined that a fourth-degree polynomial would fit the points. Their configuration, however, is such that this degree is not adequate. Probably, an exponential function would fit these points better than a polynomial.

The runs test, upon detecting such situations, causes the degree to be increased until the test is satisfied. In this example, a tenth-degree polynomial was required to fit the points adequately enough to satisfy the test. The resulting fit is shown in Fig. 3b.

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Table 1. Dispersion about the least-squares fits

Day	N	s	$1/s^2$	dBm
188	417	0.5183	3.7225	35.71
189	245	0.5639	3.1448	34.98
191	370	0.6387	2.4514	33.89
195	146	0.8261	1.4653	31.66
198	415	0.8681	1.3270	31.23
199	423	0.9024	1.2280	30.89
200	209	0.9500	1.1080	30.45
201	363	0.9992	1.0016	30.01
204	423	1.1967	0.6983	28.44
205	424	1.1850	0.7121	28.53
207	401	1.3839	0.5221	27.18
208	384	1.4145	0.4998	26.99
209	423	1.5839	0.3986	26.01
210	227	1.4439	0.4796	26.81

Table 2. Statistics of the differences between the derivatives of DRVID and Faraday rotation calibration polynomials

Day	N	$\bar{x}$	s	$1/s^2$	dBm
188	438	0.026	0.046	472.59	56.74
189	250	0.023	0.097	106.28	50.27
191	377	0.022	0.105	90.70	59.58
195	150	-0.176	0.150	44.44	46.48
198	432	0.022	0.066	229.57	53.61
199	432	0.005	0.043	540.83	57.33
200	215	-0.053	0.132	57.39	47.59
201	368	0.042	0.077	168.66	52.27
204	233	-0.105	0.159	39.56	45.97
205	123	0.132	0.077	168.66	52.27
207	436	-0.061	0.090	123.46	50.91
208	436	0.071	0.077	168.66	52.27
209	436	-0.036	0.061	268.74	54.29
210	298	-0.032	0.084	141.72	51.51

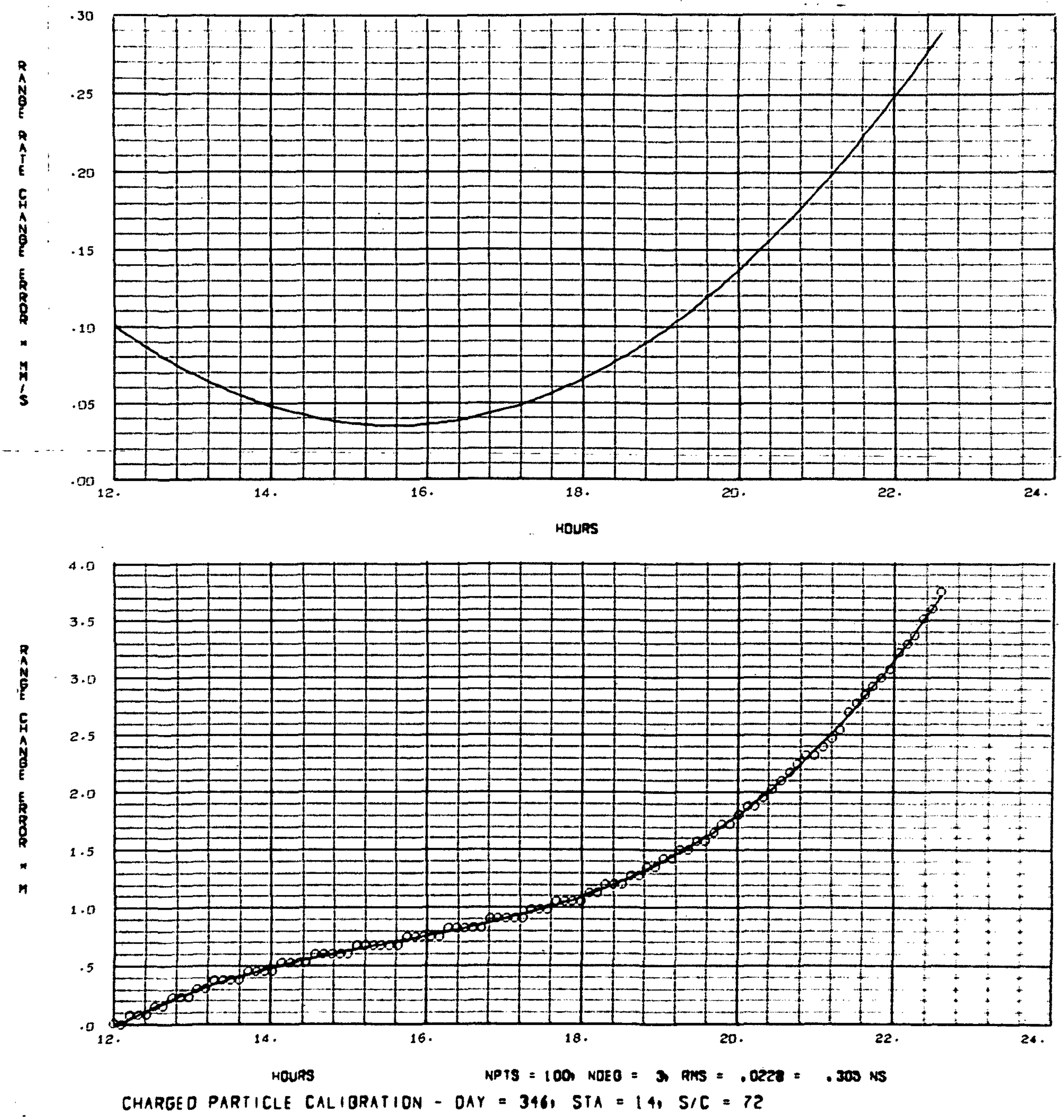


Fig. 1a. Cubic with step function

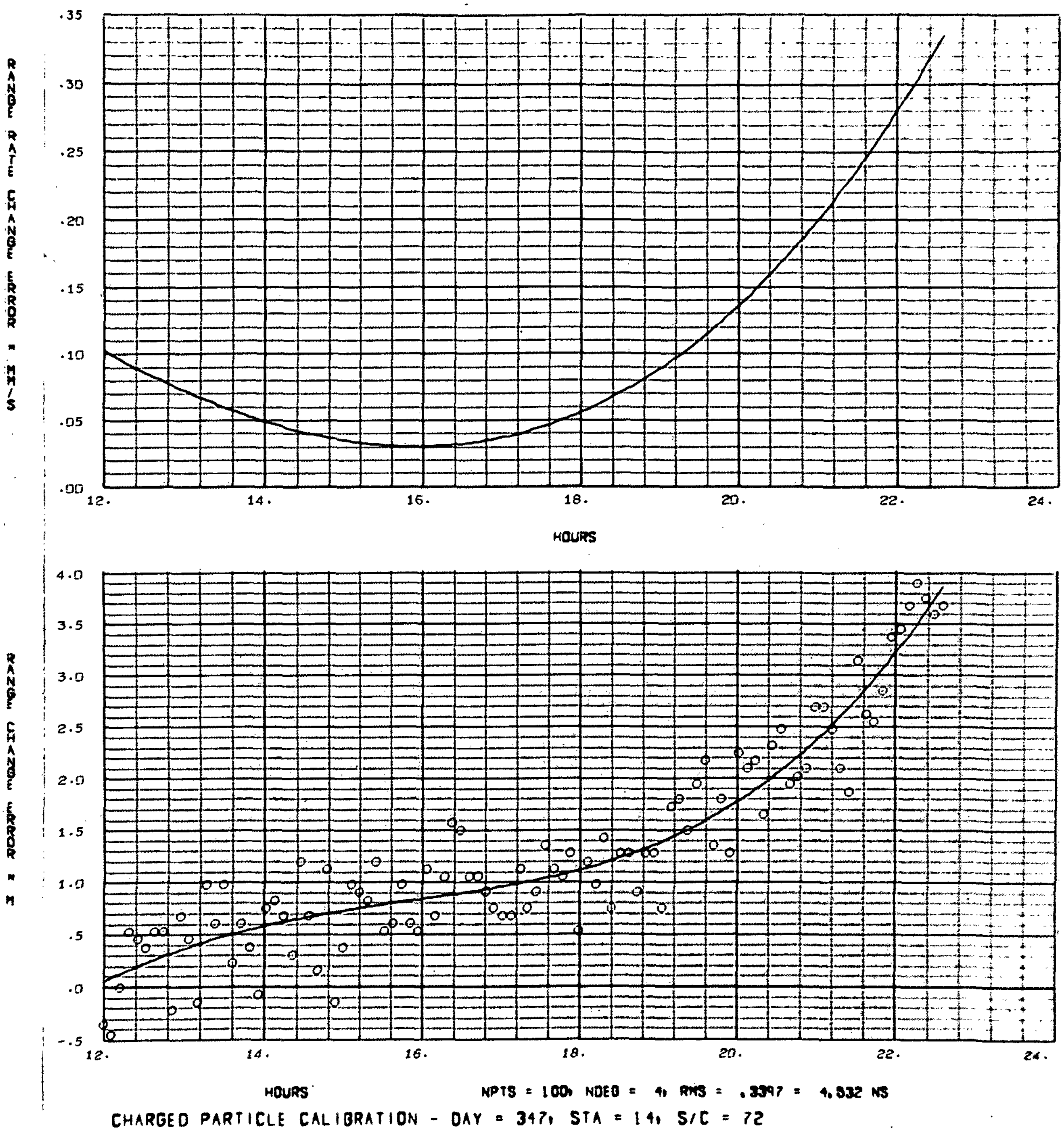


Fig. 1b. Cubic with 5-ns noise

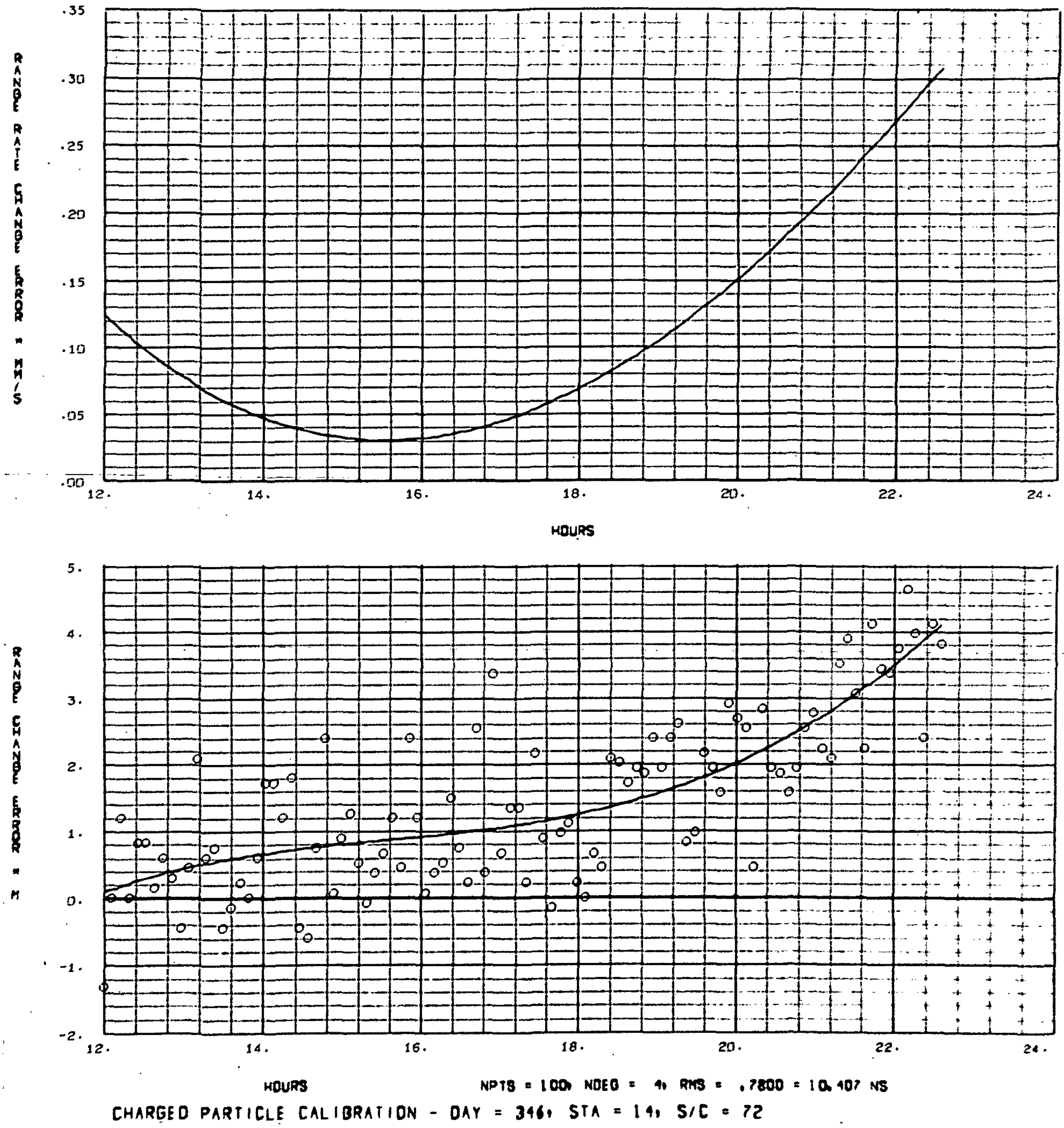


Fig. 1c. Cubic with 10-ns noise

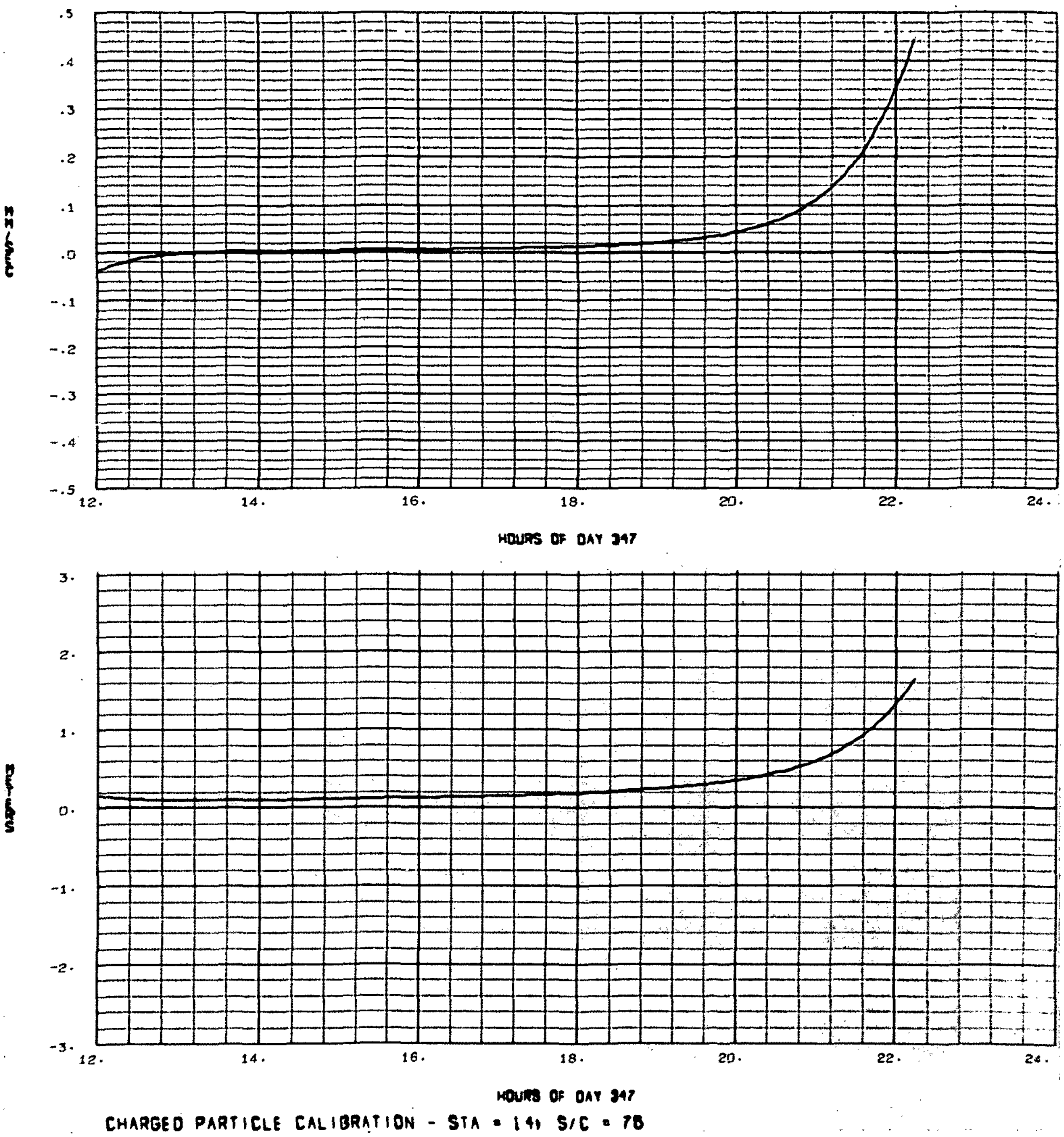


Fig. 2a. Tenth-degree polynomial

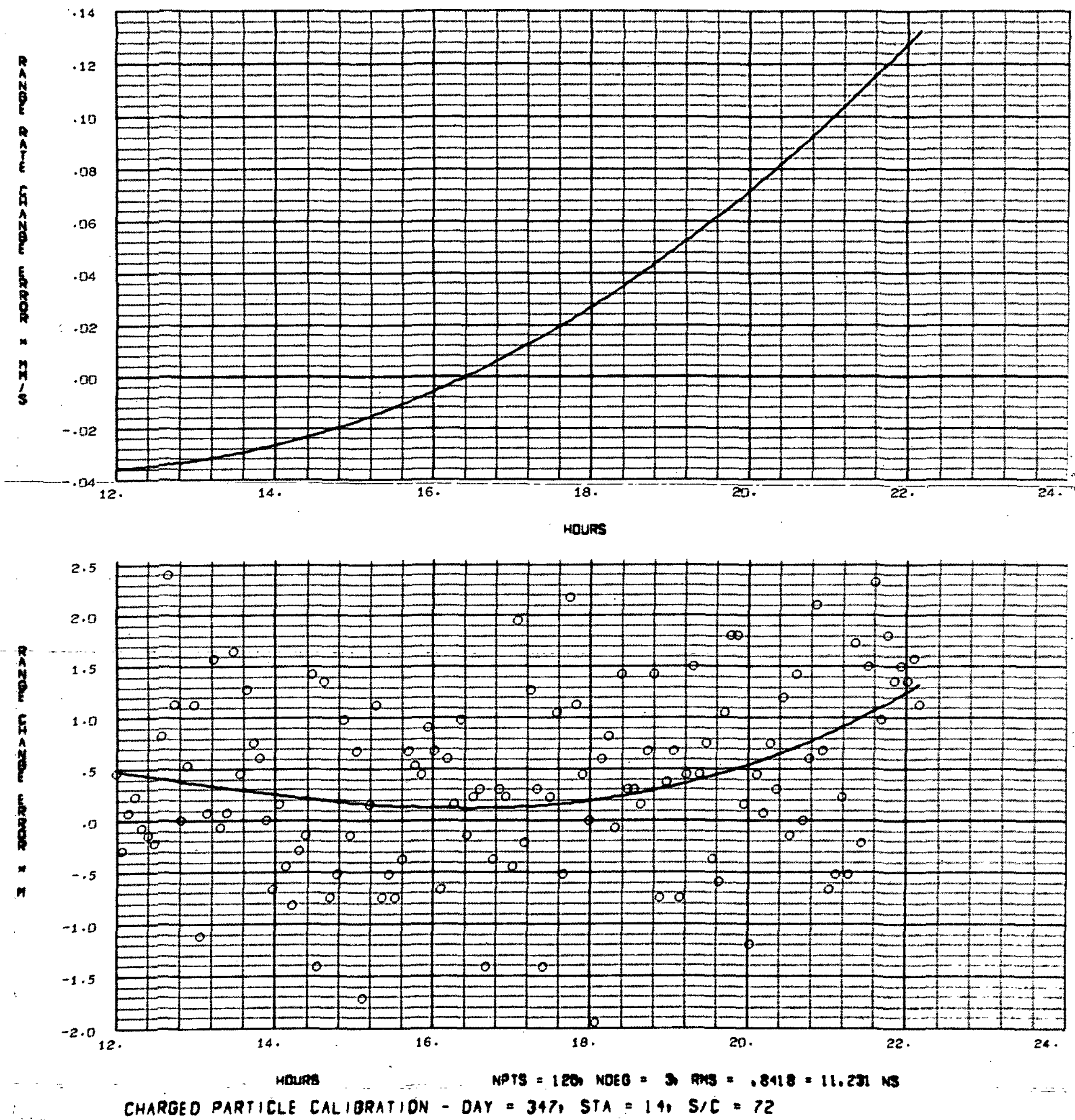


Fig. 2b. 125 points sampled from tenth-degree polynomial with 10-ns noise

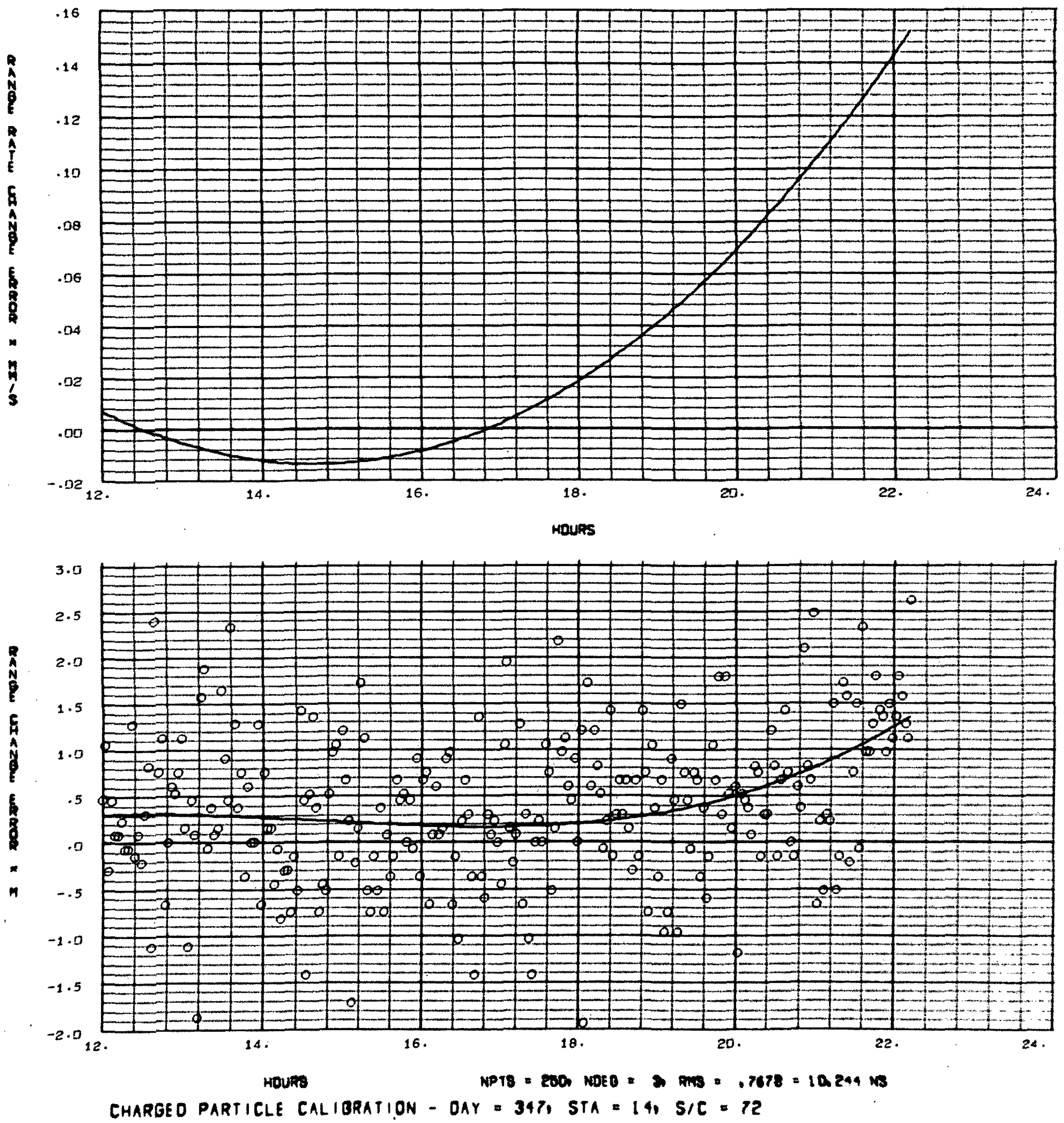


Fig. 2c. 250 points sampled from tenth-degree polynomial with 10-ns noise

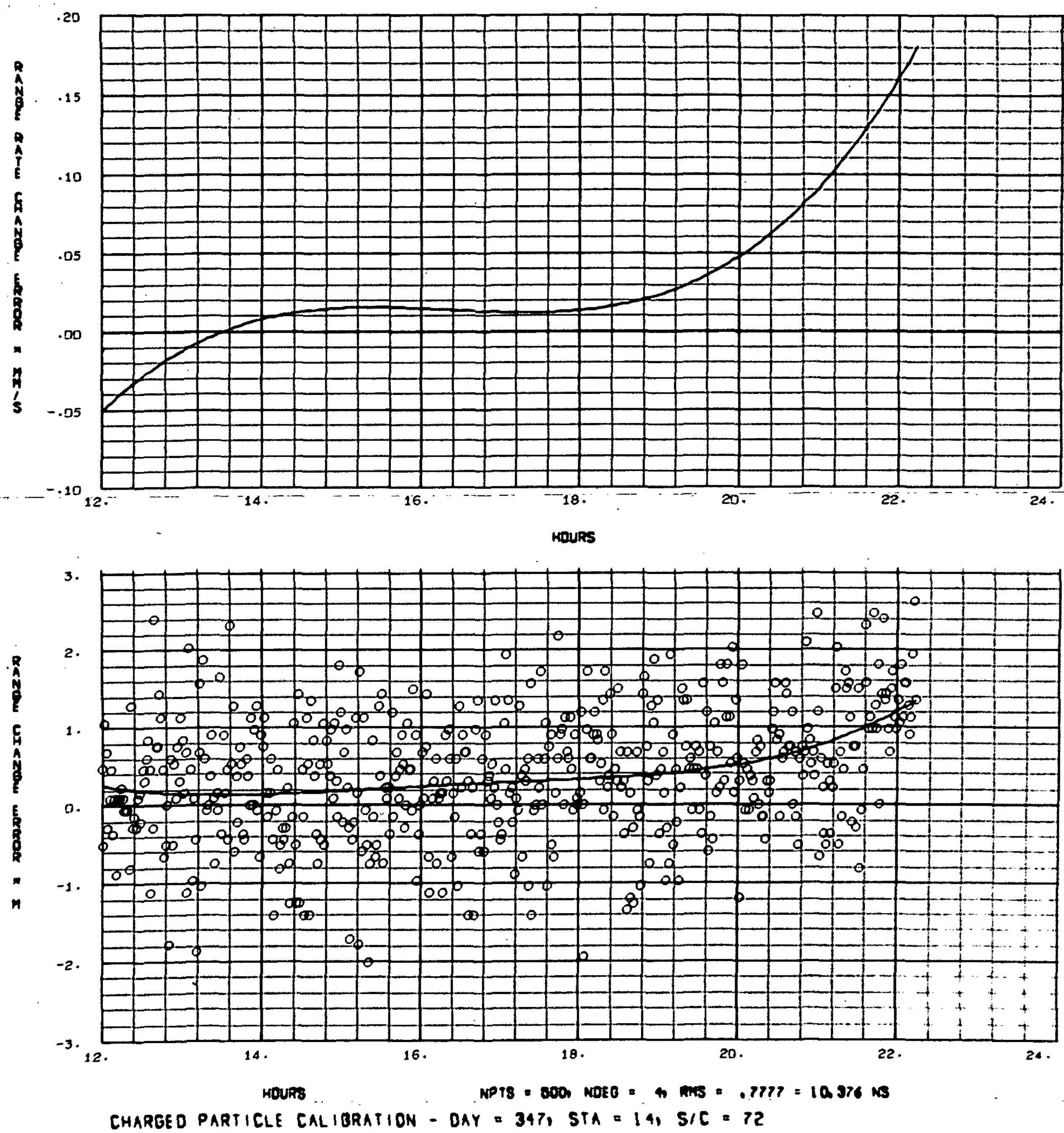


Fig. 2d. 500 points sampled from tenth-degree polynomial with 10-ns noise

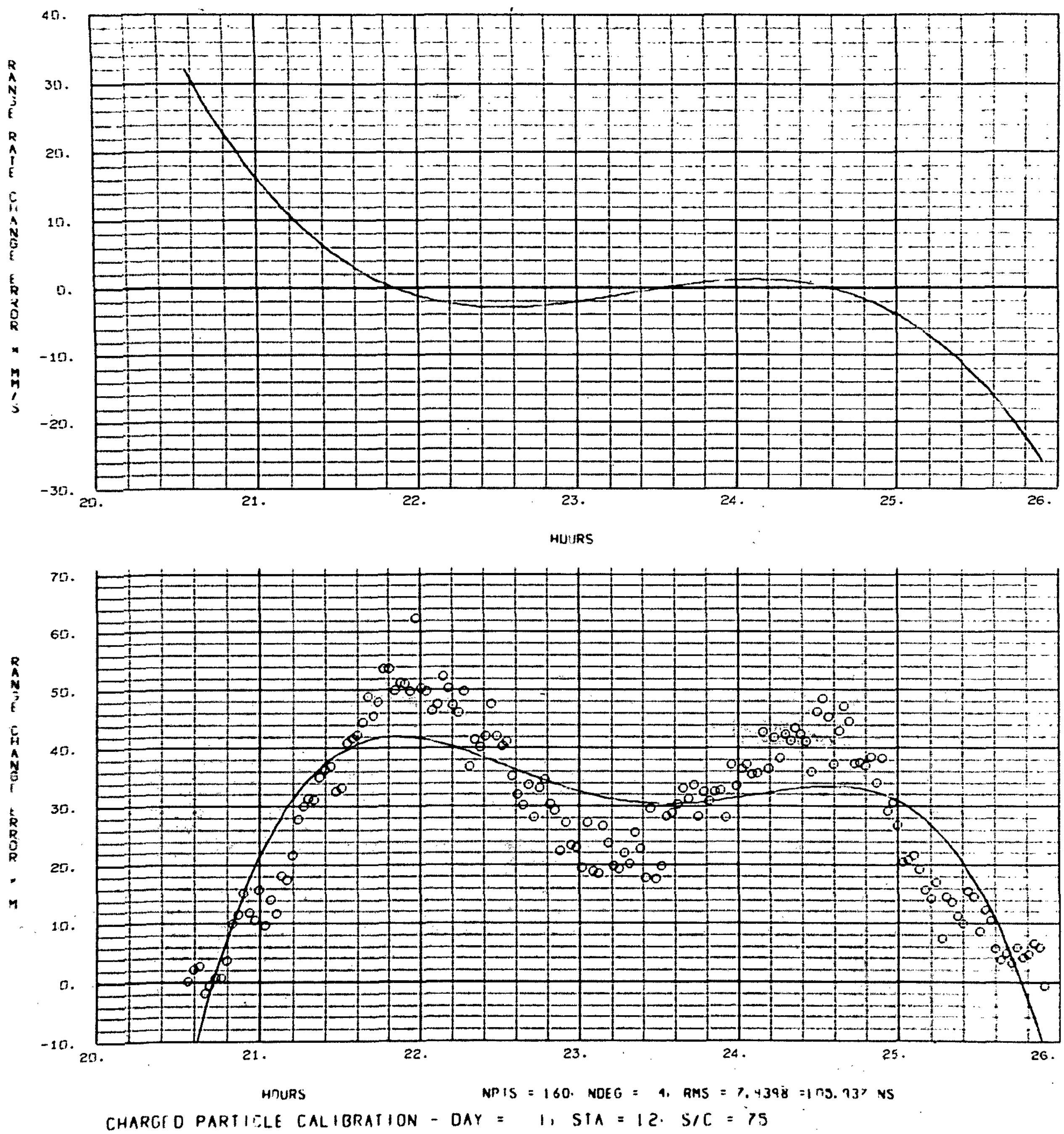


Fig. 3a. Least-squares fit without runs test

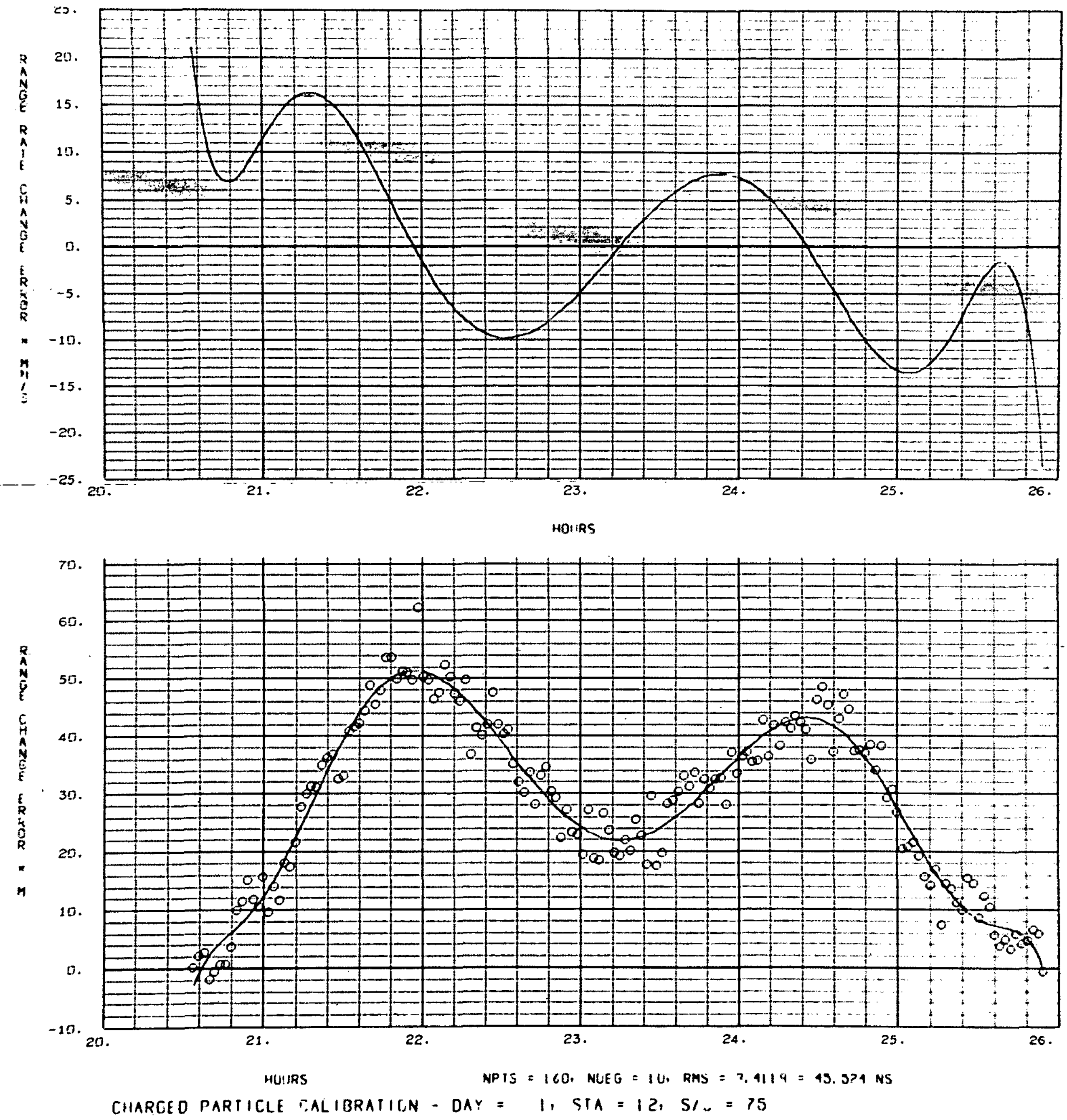


Fig. 3b. Least-squares fit with runs test applied

APPENDIX A

PLOTS ILLUSTRATING MM'71 CHARGED-PARTICLE CALIBRATION POLY-NOMIALS AND THEIR DERIVATIVES

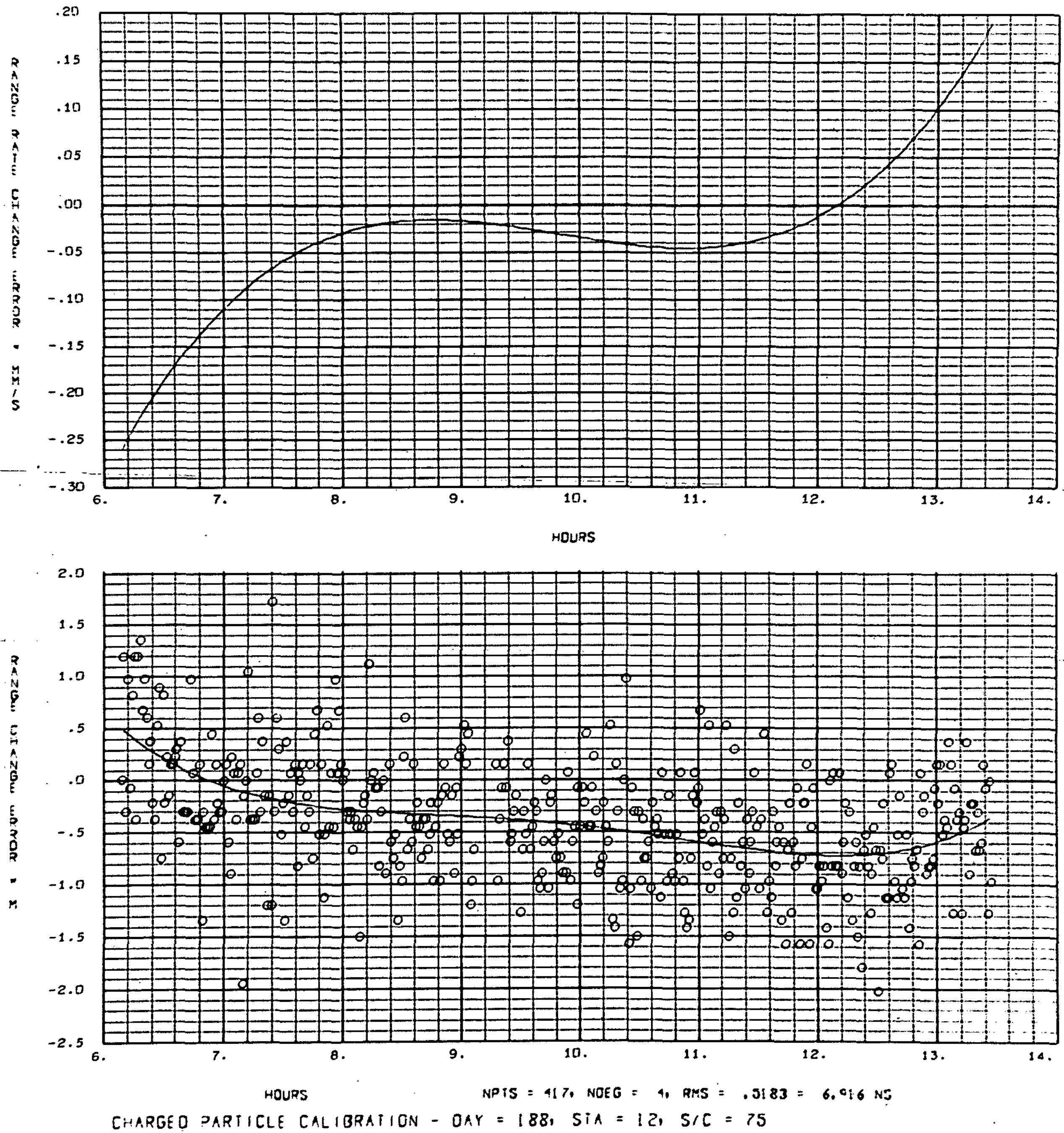

RANGE RATE CHANGE ERROR - MM/S
.20
.15
.10
.05
.00
-.05
-.10
-.15
-.20
-.25
-.30
6.
7.
8.
9.
10.
11.
12.
13.
14.
HOURS
RANGE CHANGE ERROR - M
2.0
1.5
1.0
.5
.0
-.5
-1.0
-1.5
-2.0
-2.5
6.
7.
8.
9.
10.
11.
12.
13.
14.
HOURS
NPTS = 417, NDEG = 4, RMS = .5183 = 6.916 NS
CHARGED PARTICLE CALIBRATION - DAY = 188, STA = 12, S/C = 75

Fig. A-1.

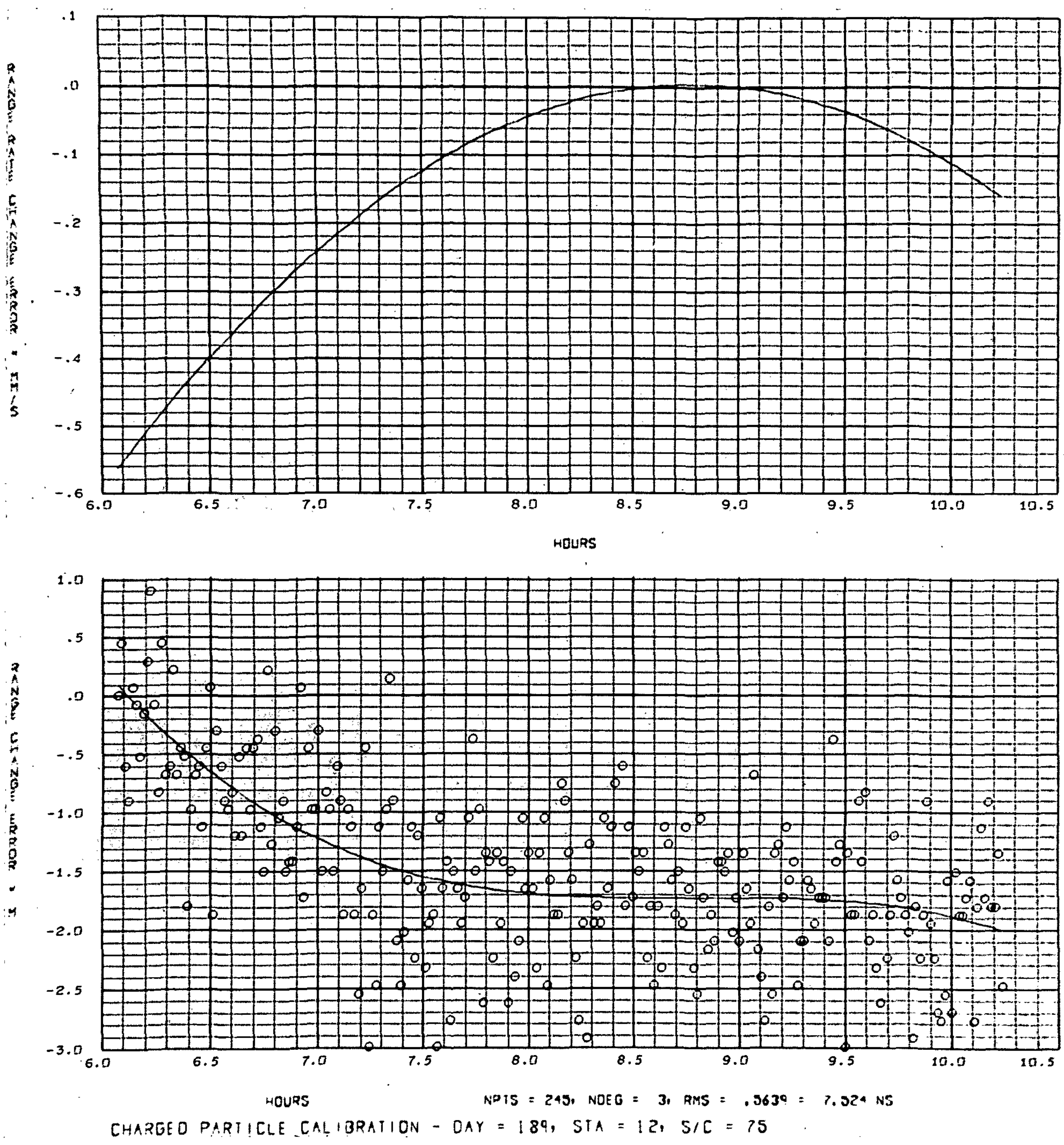
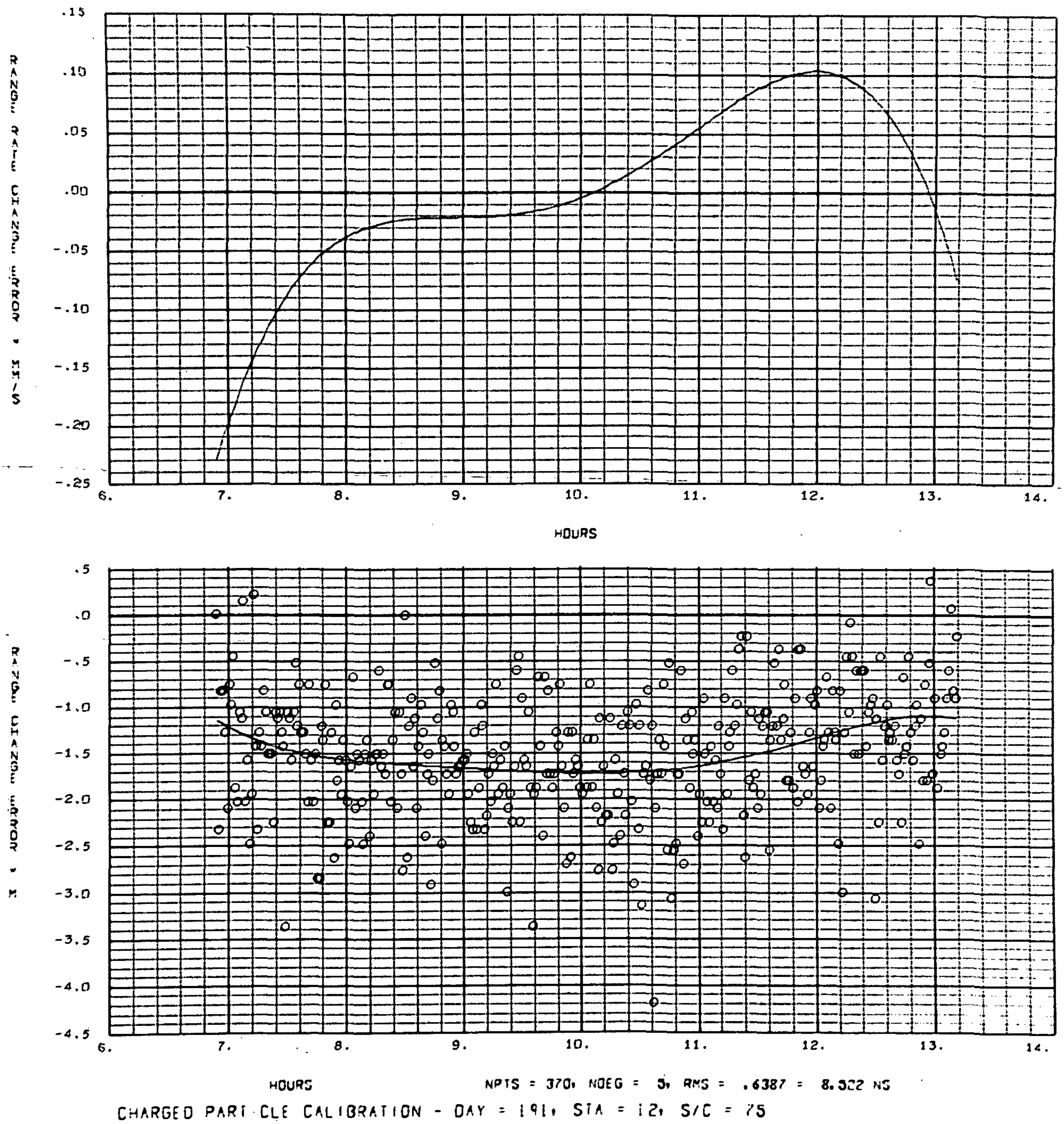

.1
.0
-.1
-.2
-.3
-.4
-.5
-.6
6.0
6.5
7.0
7.5
8.0
8.5
9.0
9.5
10.0
10.5
HOURS
1.0
.5
.0
-.5
-1.0
-1.5
-2.0
-2.5
-3.0
HOURS
NPTS = 245, NDEG = 3, RMS = .5639 = 7.524 NS
CHARGED PARTICLE CALIBRATION - DAY = 189, STA = 12, S/C = 75

Fig. A-2.



RANGE RATE CHANGE ERROR, MM/S
.15
.10
.05
.00
-.05
-.10
-.15
-.20
-.25
6.
7.
8.
9.
10.
11.
12.
13.
14.
HOURS
RANGE CHANGE ERROR, M
.5
.0
-.5
-1.0
-1.5
-2.0
-2.5
-3.0
-3.5
-4.0
-4.5
HOURS
NPTS = 370, NDEG = 5, RMS = .6387 = 8.522 NS
CHARGED PARTICLE CALIBRATION - DAY = 191, STA = 12, S/C = 75

Fig. A-3.

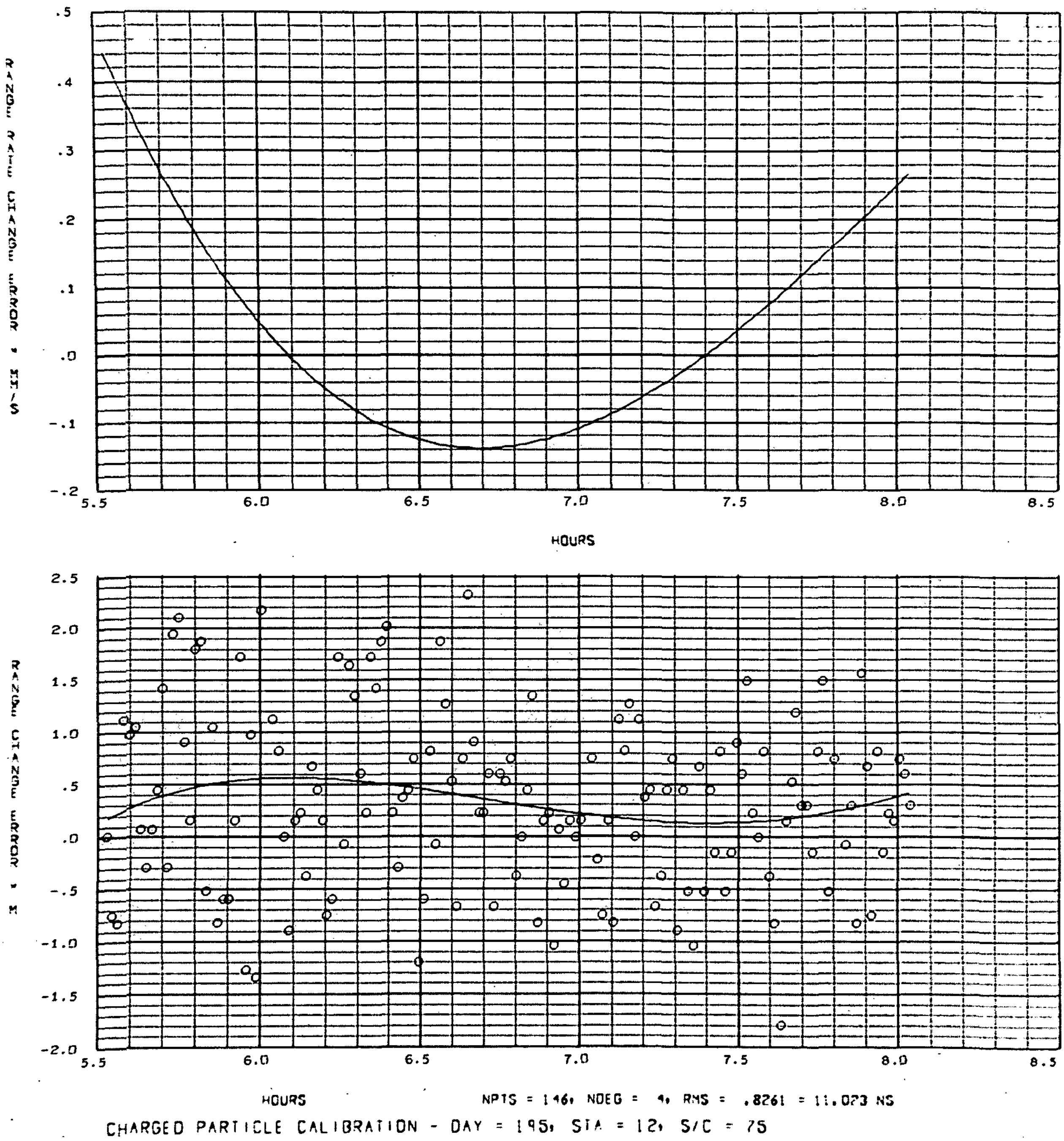

HOURS
HOURS
NPTS = 146, NDEG = 4, RMS = .8261 = 11.023 NS
CHARGED PARTICLE CALIBRATION - DAY = 195, STA = 12, S/C = 75

Fig. A-4.

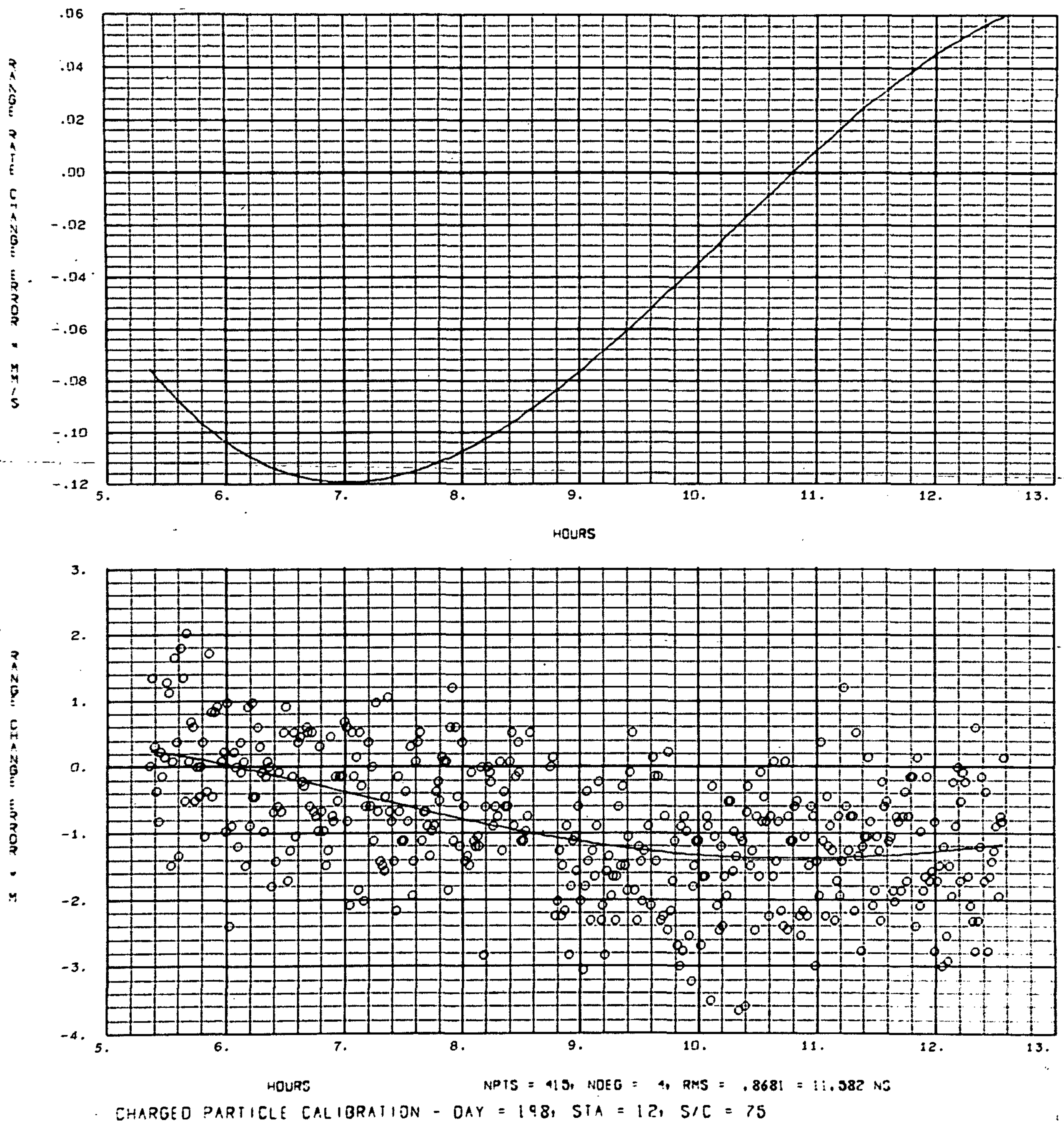
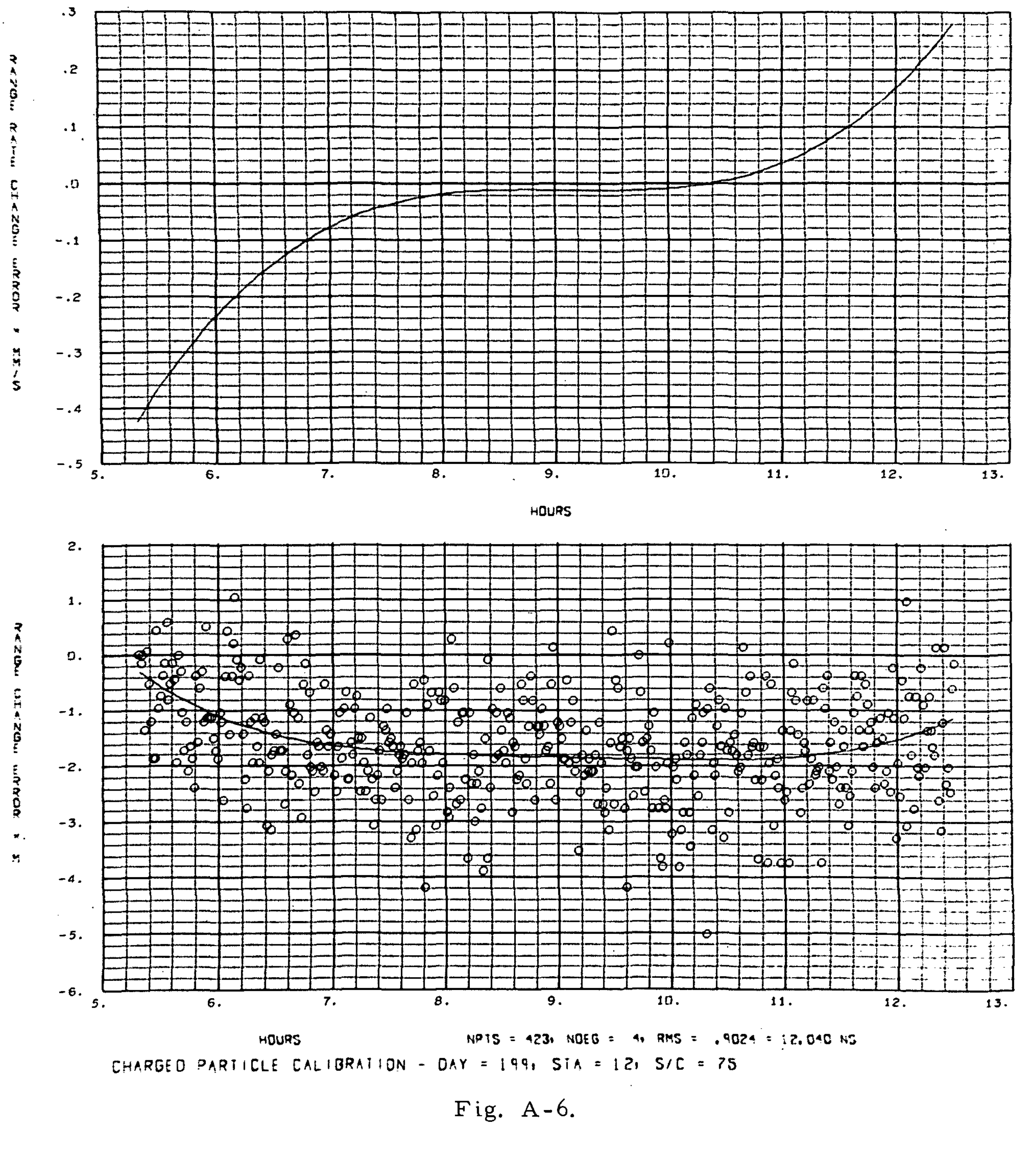
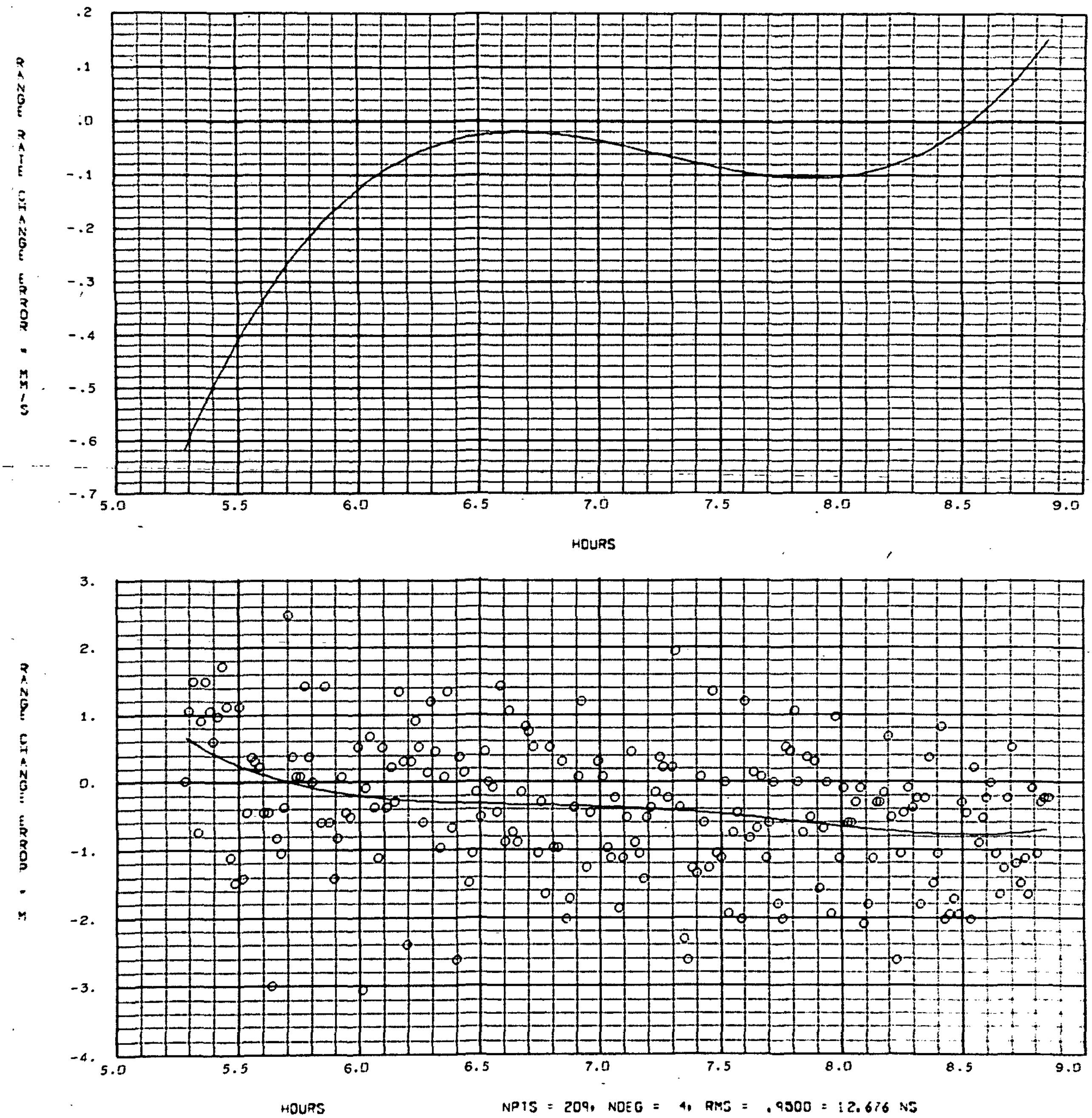

RANGE RATE CHANGE ERROR * MM/S
.06
.04
.02
.00
-.02
-.04
-.06
-.08
-.10
-.12
5.
6.
7.
8.
9.
10.
11.
12.
13.
HOURS
RANGE CHANGE ERROR * M
3.
2.
1.
0.
-1.
-2.
-3.
-4.
HOURS
NPTS = 415, NDEG = 4, RMS = .8681 = 11.582 NS
CHARGED PARTICLE CALIBRATION - DAY = 198, STA = 12, S/C = 75

Fig. A-5.



RANGE RATE CHANGE ERROR * MM/S
.3
.2
.1
.0
-.1
-.2
-.3
-.4
-.5
5.
6.
7.
8.
9.
10.
11.
12.
13.
HOURS
RANGE CHANGE ERROR * M
2.
1.
0.
-1.
-2.
-3.
-4.
-5.
-6.
HOURS
NPTS = 423, NDEG = 4, RMS = .9024 = 12.040 NS
CHARGED PARTICLE CALIBRATION - DAY = 199, STA = 12, S/C = 75

Fig. A-6.



CHARGED PARTICLE CALIBRATION - DAY = 200, STA = 12, S/C = 75

Fig. A-7.

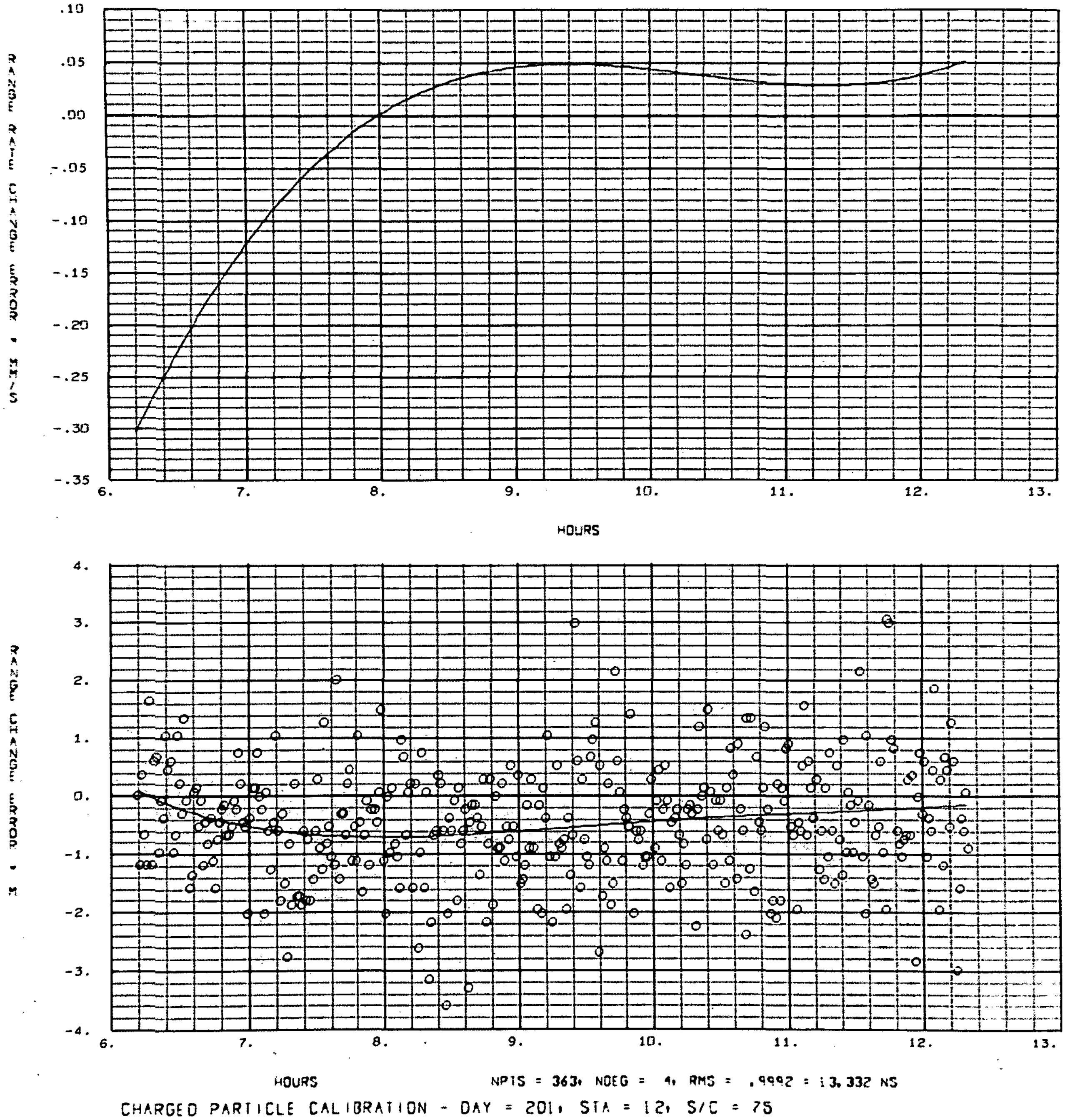
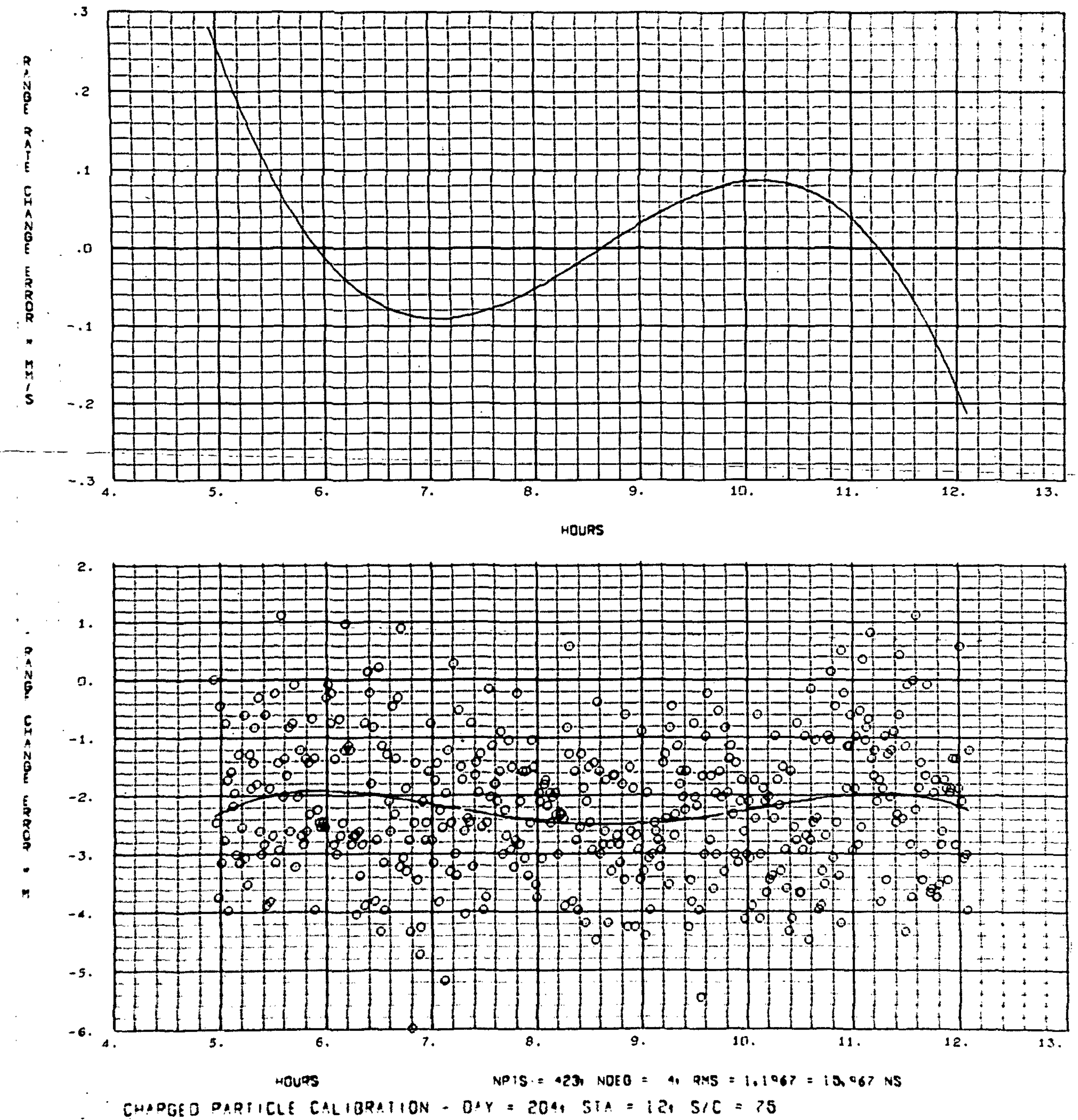

.10
.05
.00
-.05
-.10
-.15
-.20
-.25
-.30
-.35
6.
7.
8.
9.
10.
11.
12.
13.
HOURS
4.
3.
2.
1.
0.
-1.
-2.
-3.
-4.
HOURS
NPTS = 363, NDEG = 4, RMS = .9992 = 13.332 NS
CHARGED PARTICLE CALIBRATION - DAY = 201, STA = 12, S/C = 75

Fig. A-8.



RANGE RATE CHANGE ERROR * MM/S
.3
.2
.1
.0
-.1
-.2
-.3
4.
5.
6.
7.
8.
9.
10.
11.
12.
13.
HOURS
RANGE CHANGE ERROR * M
2.
1.
0.
-1.
-2.
-3.
-4.
-5.
-6.
HOURS
NPTS = 423, NDEG = 4, RMS = 1.1967 = 15.967 NS
CHARGED PARTICLE CALIBRATION - DAY = 204, STA = 12, S/C = 75

Fig. A-9.

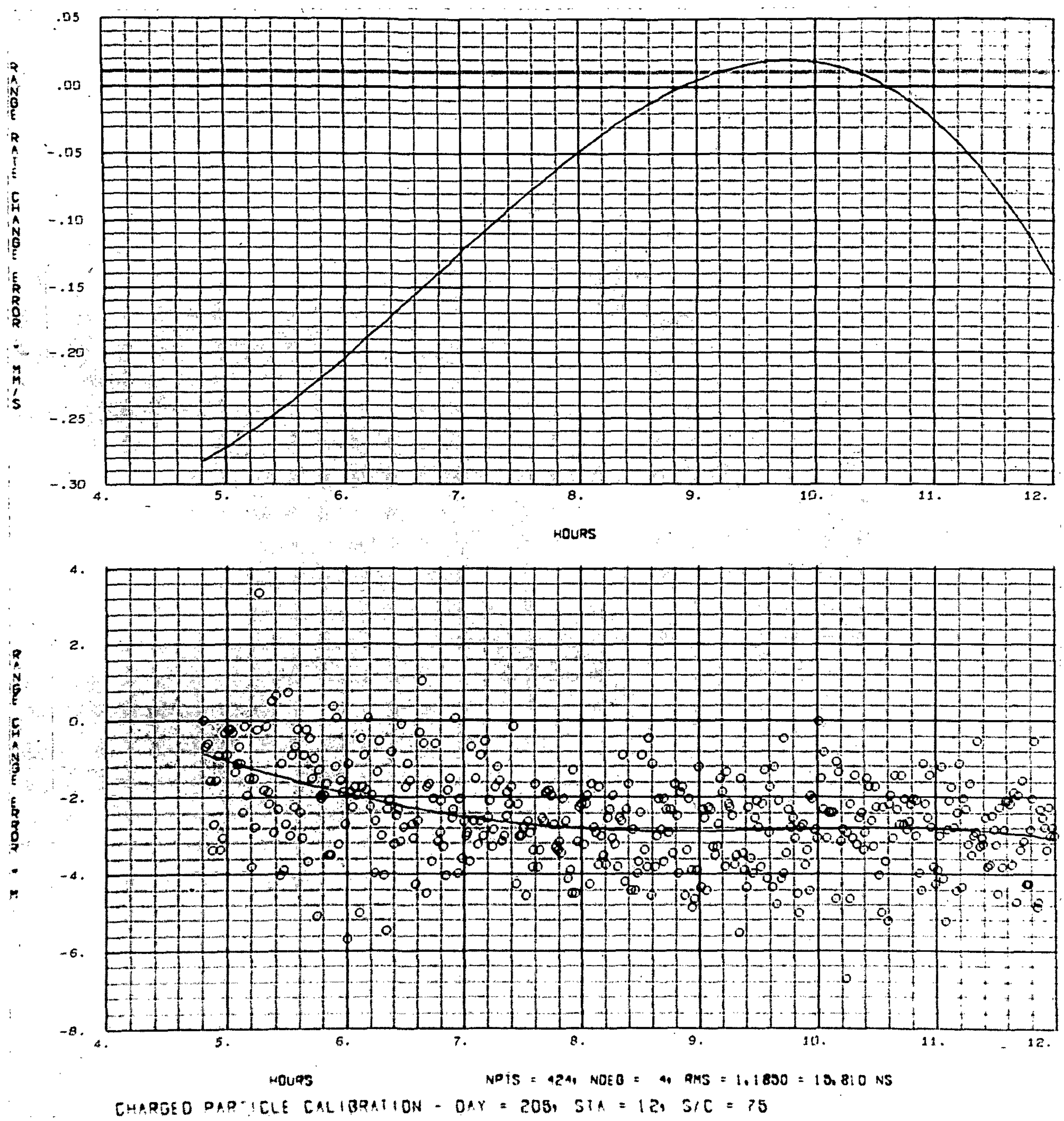

RANGE RATE CHANGE ERROR, MM/S
.05
.00
-.05
-.10
-.15
-.20
-.25
-.30
4. 5. 6. 7. 8. 9. 10. 11. 12.
HOURS
RANGE CHANGE ERROR, M
4.
2.
0.
-2.
-4.
-6.
-8.
4. 5. 6. 7. 8. 9. 10. 11. 12.
HOURS
NPTS = 424, NDEG = 4, RMS = 1.1850 = 15.810 NS
CHARGED PARTICLE CALIBRATION - DAY = 205, STA = 12, S/C = 75

Fig. A-10.

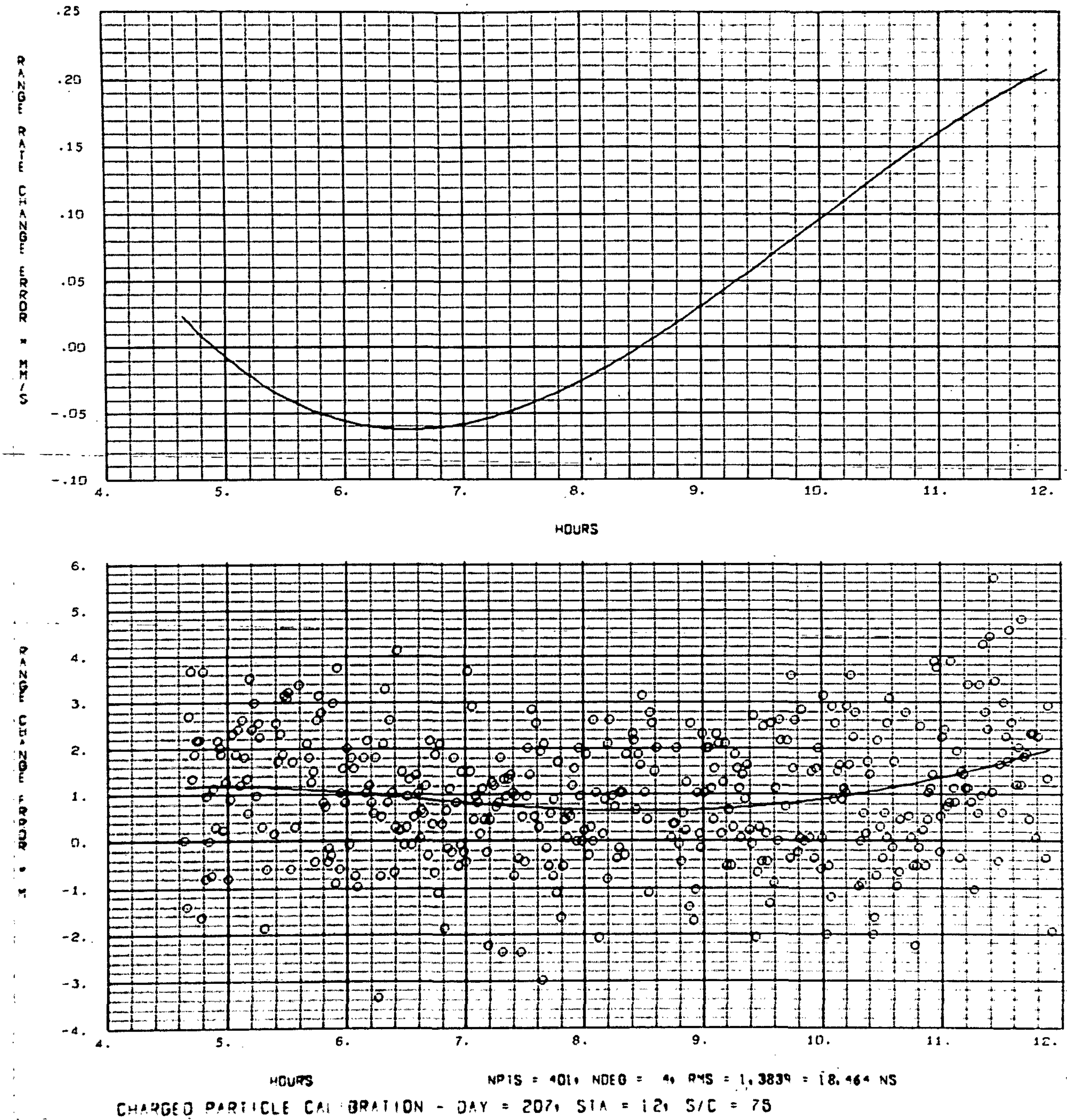

RANGE RATE CHANGE ERROR * MM/S
HOURS
RANGE CHANGE ERROR * M
HOURS
NPTS = 401, NDEG = 4, RMS = 1.3839 = 18.464 NS
CHARGED PARTICLE CALIBRATION - DAY = 207, STA = 12, S/C = 75

Fig. A-11.

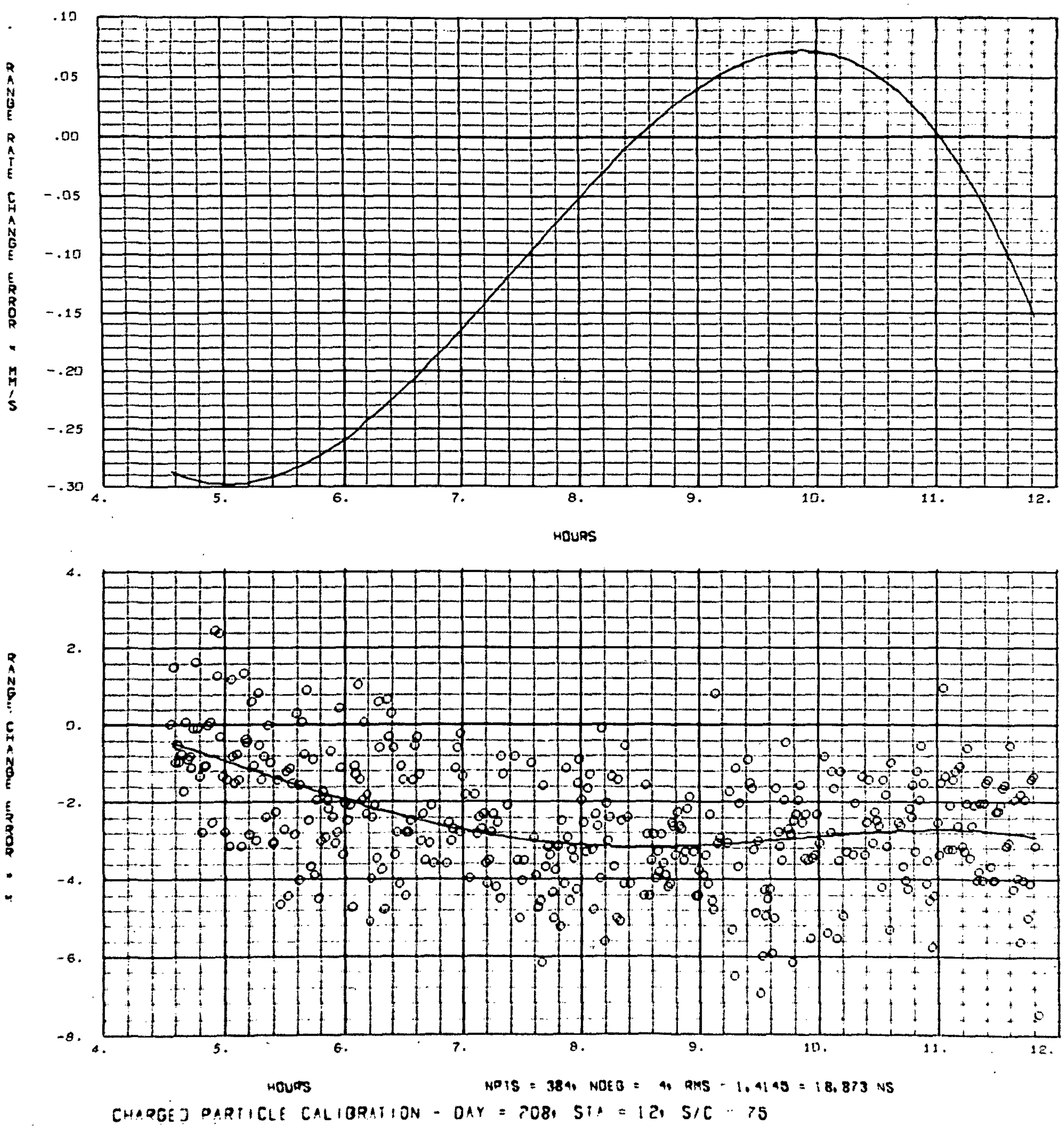

RANGE RATE CHANGE ERROR, MM/S
.10
.05
.00
-.05
-.10
-.15
-.20
-.25
-.30
4.
5.
6.
7.
8.
9.
10.
11.
12.
HOURS
RANGE CHANGE ERROR, M
4.
2.
0.
-2.
-4.
-6.
-8.
HOURS
NPTS = 384, NDEG = 4, RMS = 1.4145 = 18.873 NS
CHARGED PARTICLE CALIBRATION - DAY = 208, STA = 12, S/C = 75

Fig. A-12.

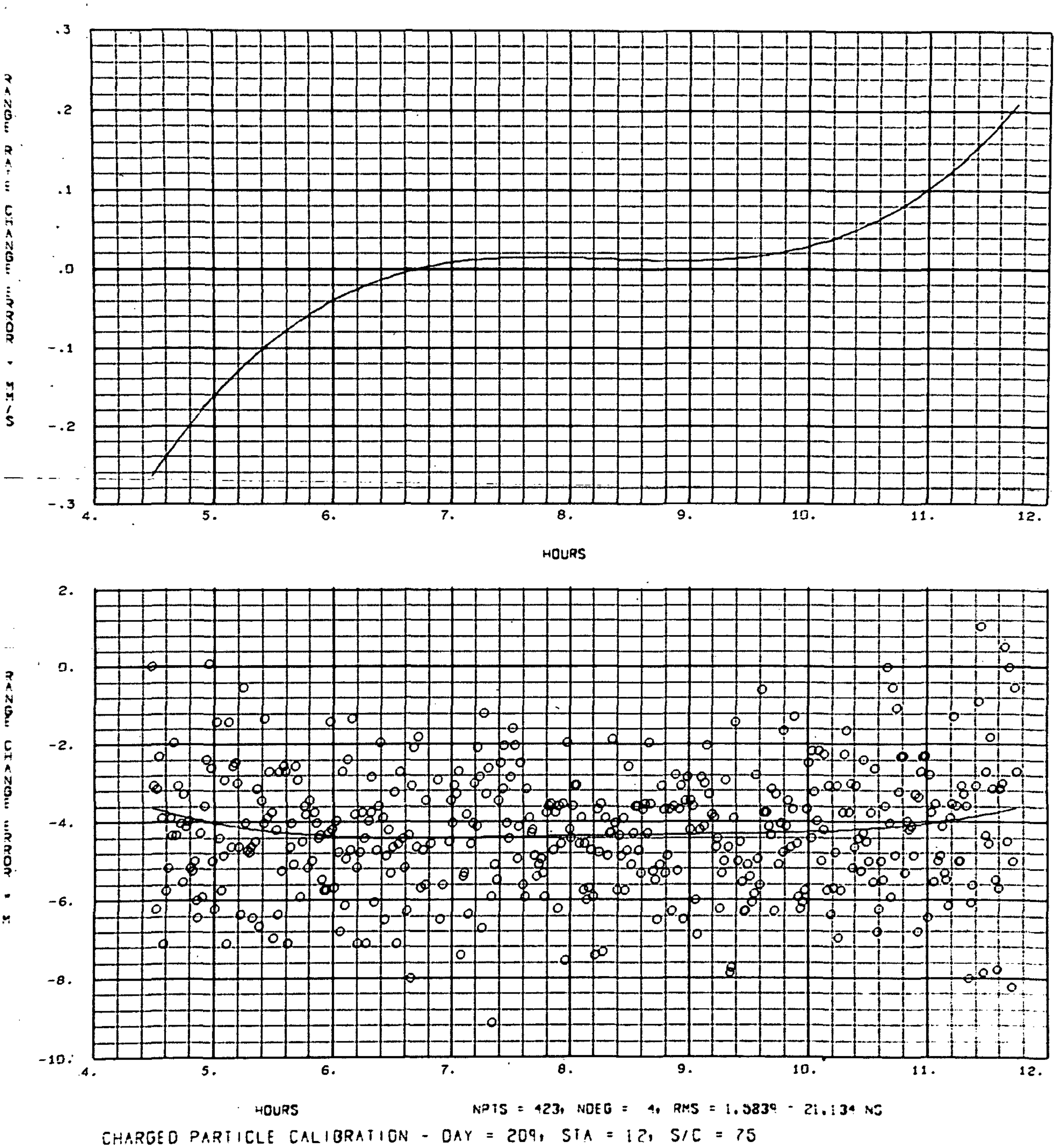

RANGE RATE CHANGE ERROR, MM/S
HOURS
RANGE CHANGE ERROR, M
HOURS
NPTS = 423, NDEG = 4, RMS = 1.5839 - 21.134 NS
CHARGED PARTICLE CALIBRATION - DAY = 209, STA = 12, S/C = 75

Fig. A-13.

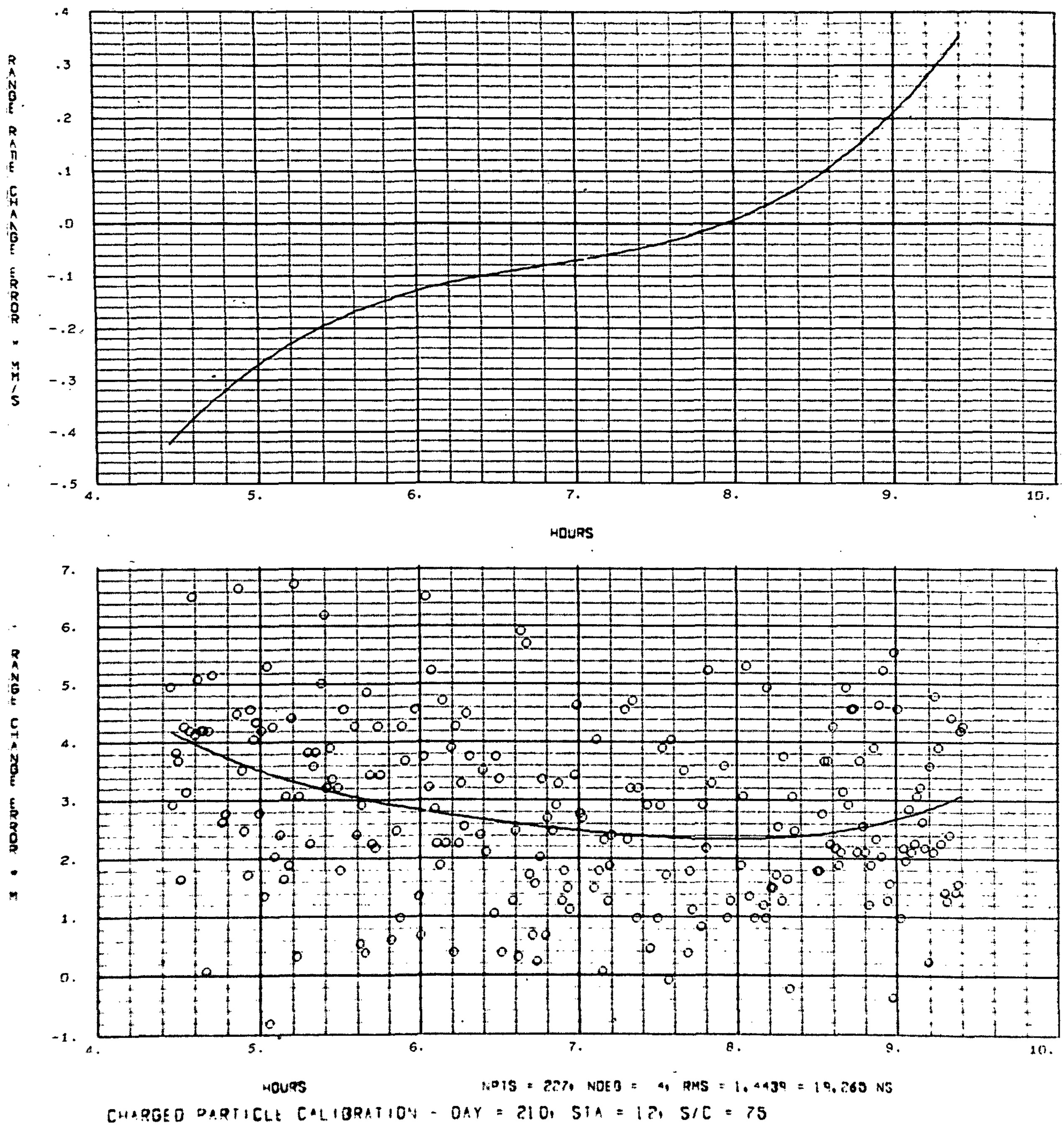

RANGE RATE CHANGE ERROR * MM/S
.4
.3
.2
.1
.0
-.1
-.2
-.3
-.4
-.5
4.
5.
6.
7.
8.
9.
10.
HOURS
RANGE CHANGE ERROR * M
7.
6.
5.
4.
3.
2.
1.
0.
-1.
HOURS
NPTS = 227, NDEG = 4, RMS = 1.4439 = 19.260 NS
CHARGED PARTICLE CALIBRATION - DAY = 210, STA = 12, S/C = 75

Fig. A-14.

APPENDIX B

COMPARING DERIVATIVES OF FARADAY ROTATION AND DRVID CALIBRATION POLYNOMIALS

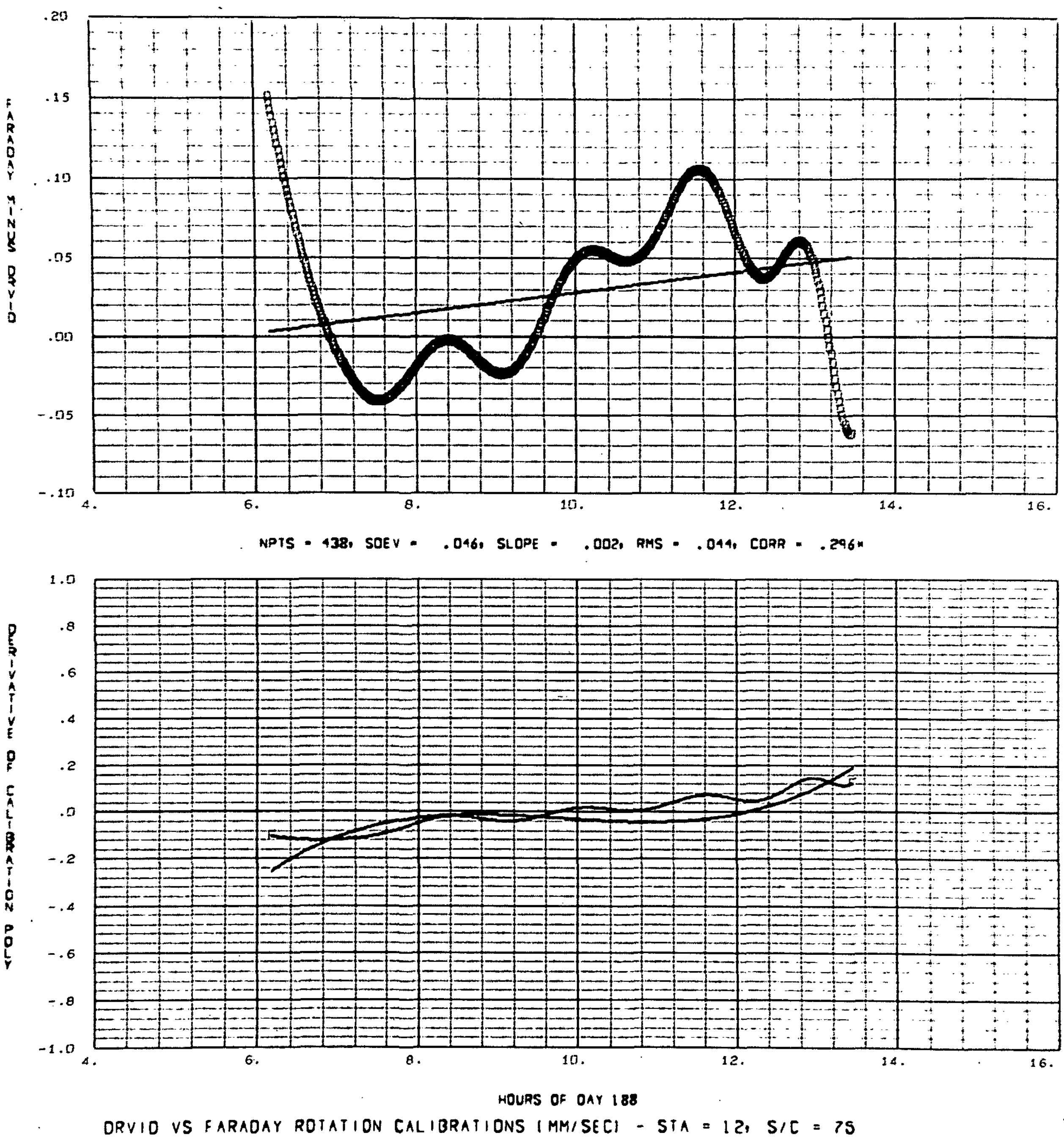
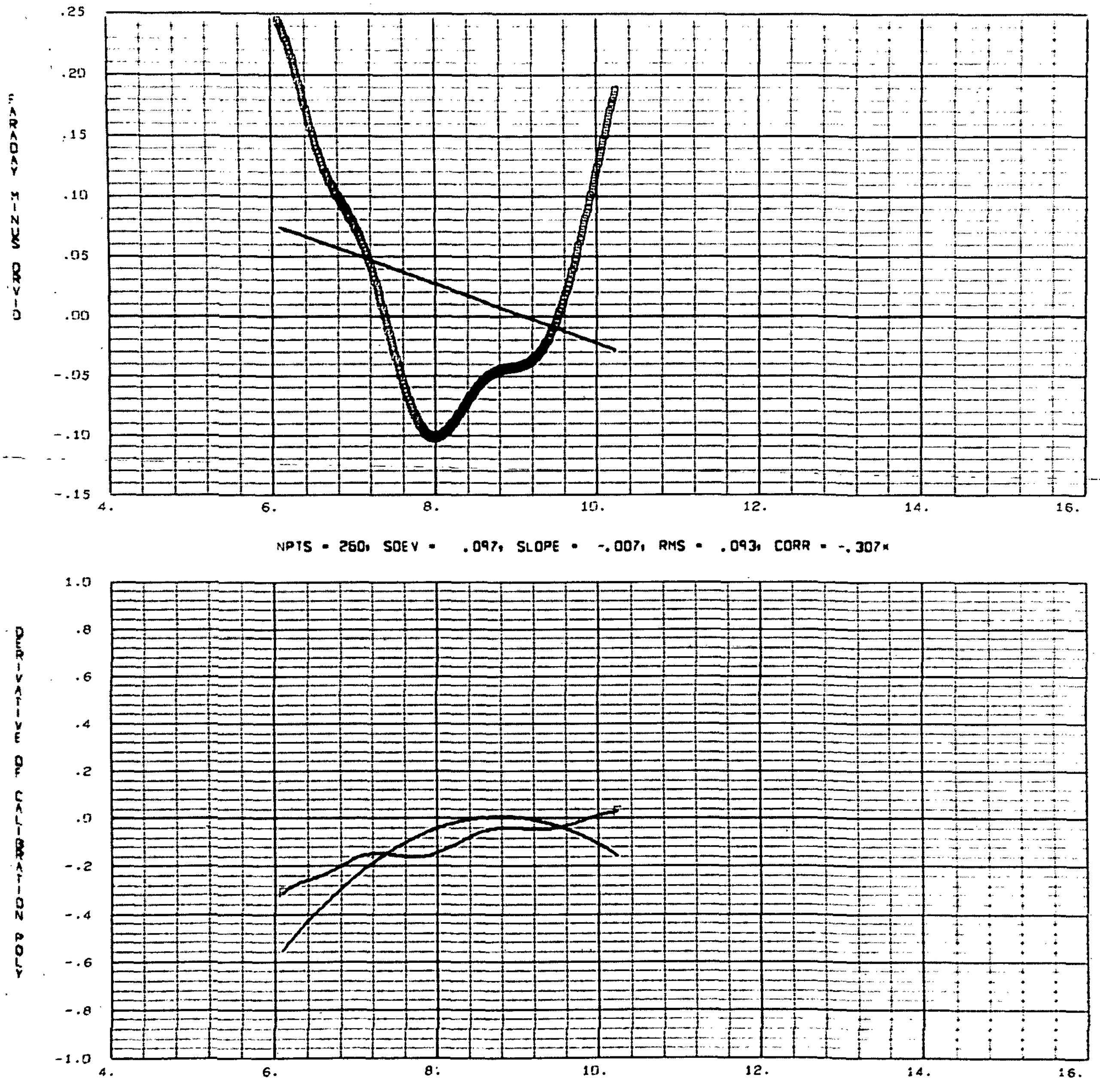
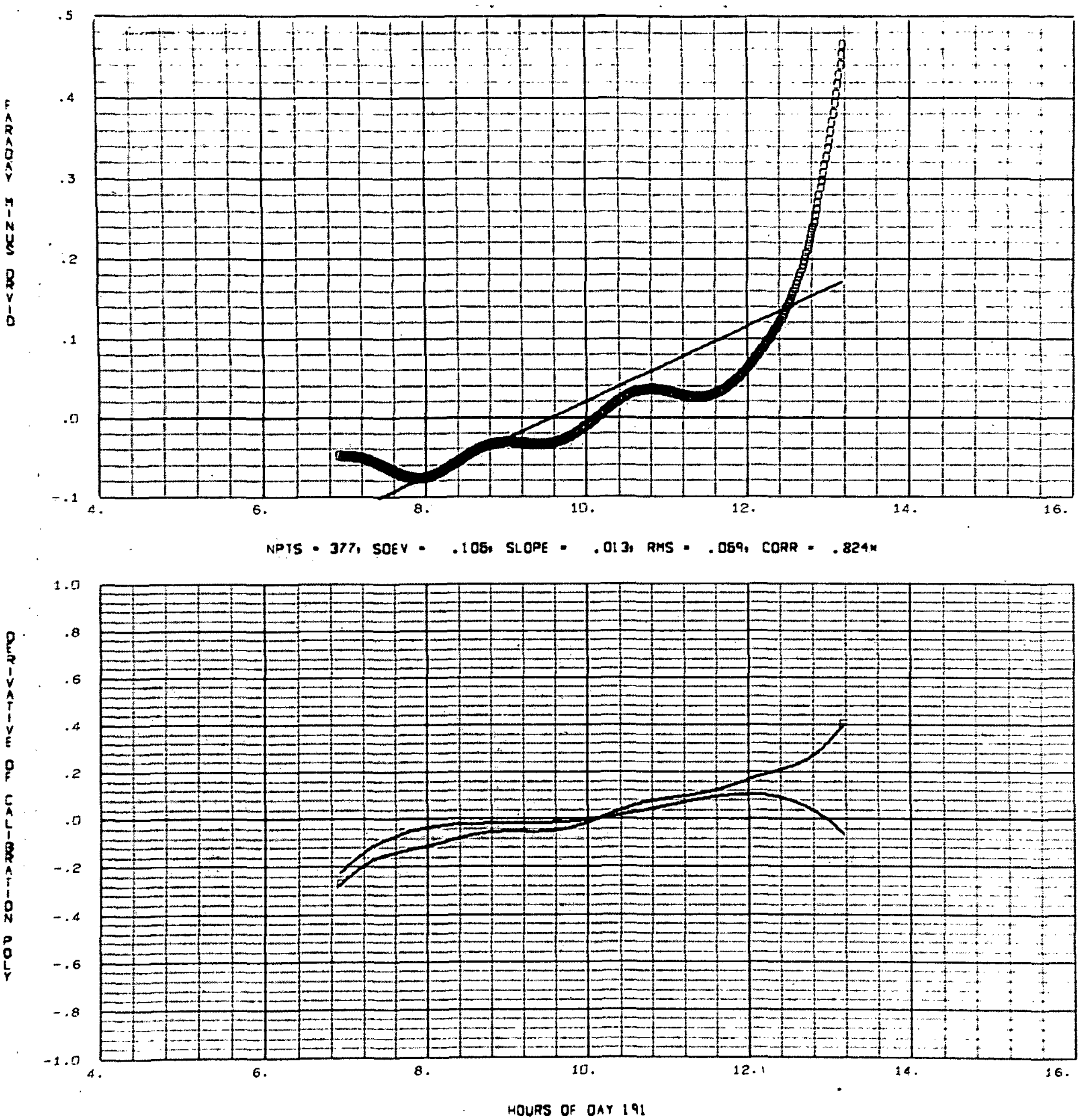

FARADAY MINUS DRVID
.20
.15
.10
.05
.00
-.05
-.10
4.
6.
8.
10.
12.
14.
16.
NPTS = 438, SDEV = .046, SLOPE = .002, RMS = .044, CORR = .296*
DERIVATIVE OF CALIBRATION POLY
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
HOURS OF DAY 188
DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-1.



DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-2.



DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-3.

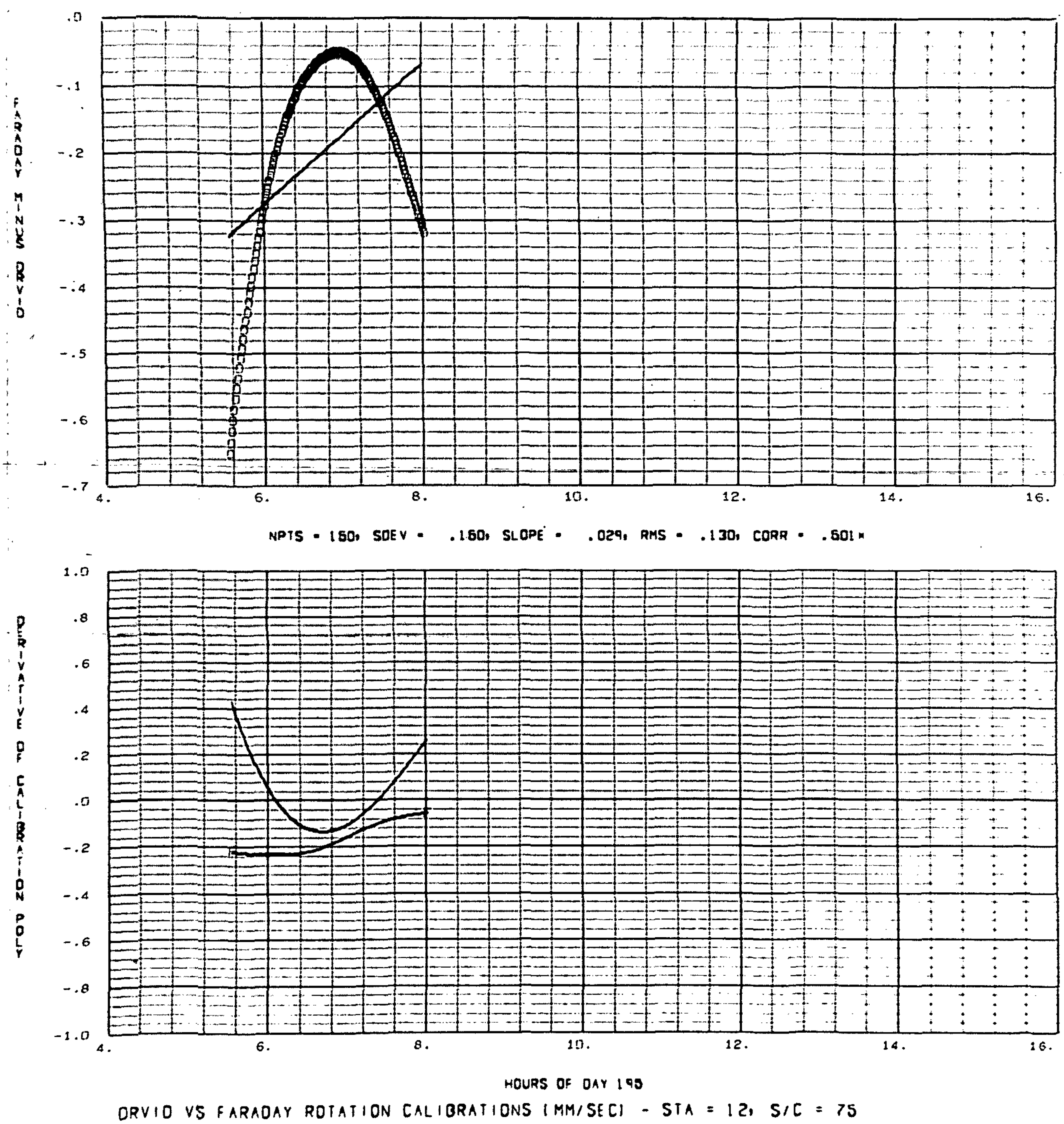

.0
-.1
-.2
-.3
-.4
-.5
-.6
-.7
FARADAY MINUS DRVID
4.
6.
8.
10.
12.
14.
16.
NPTS = 160, SDEV = .160, SLOPE = .029, RMS = .130, CORR = .601*
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
DERIVATIVE OF CALIBRATION POLY
HOURS OF DAY 195
DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-4.

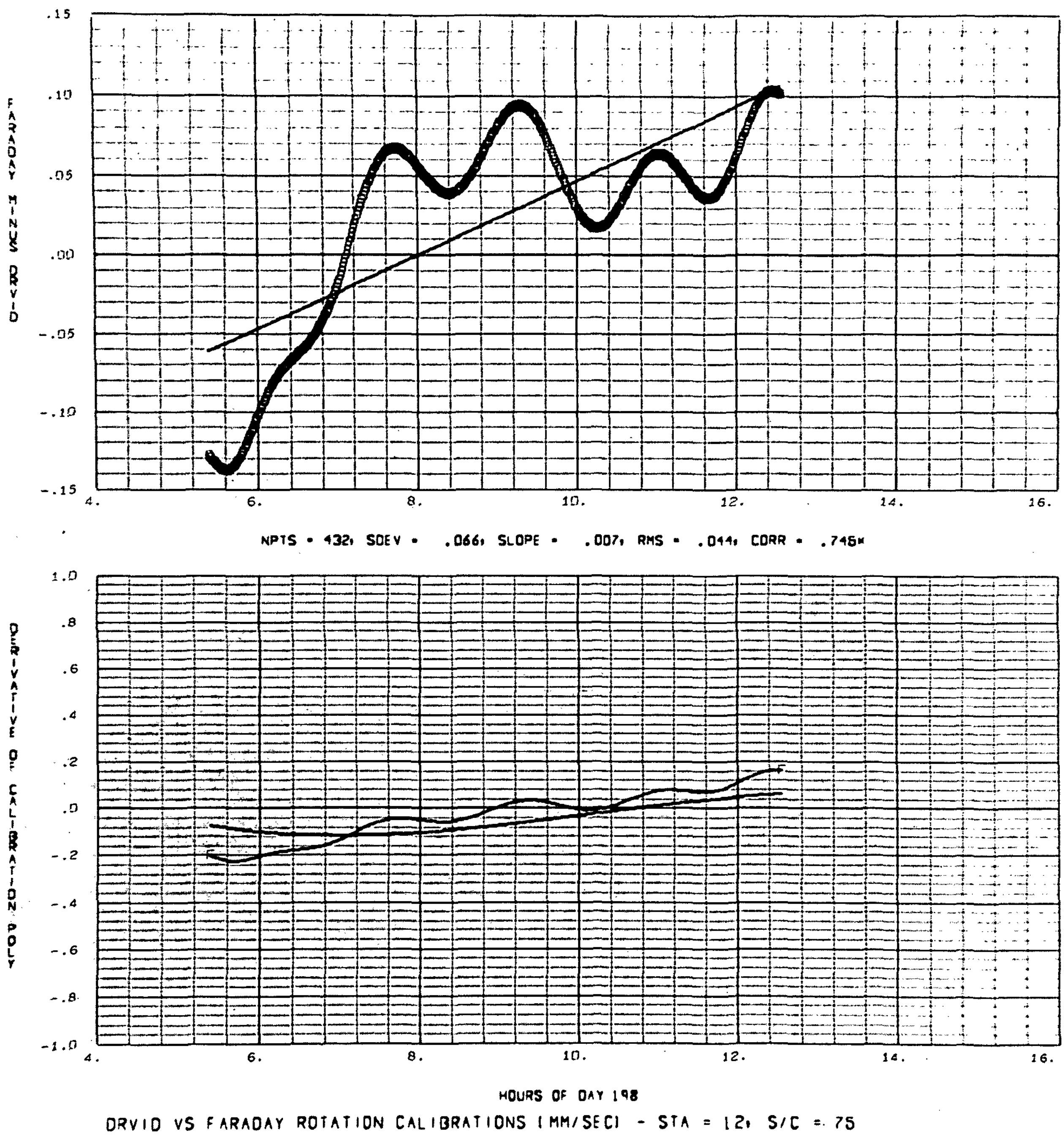

FARADAY MINUS DRVID
.15
.10
.05
.00
-.05
-.10
-.15
4.
6.
8.
10.
12.
14.
16.
NPTS = 432, SDEV = .066, SLOPE = .007, RMS = .044, CORR = .746
DERIVATIVE OF CALIBRATION POLY
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
HOURS OF DAY 198
DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-5.

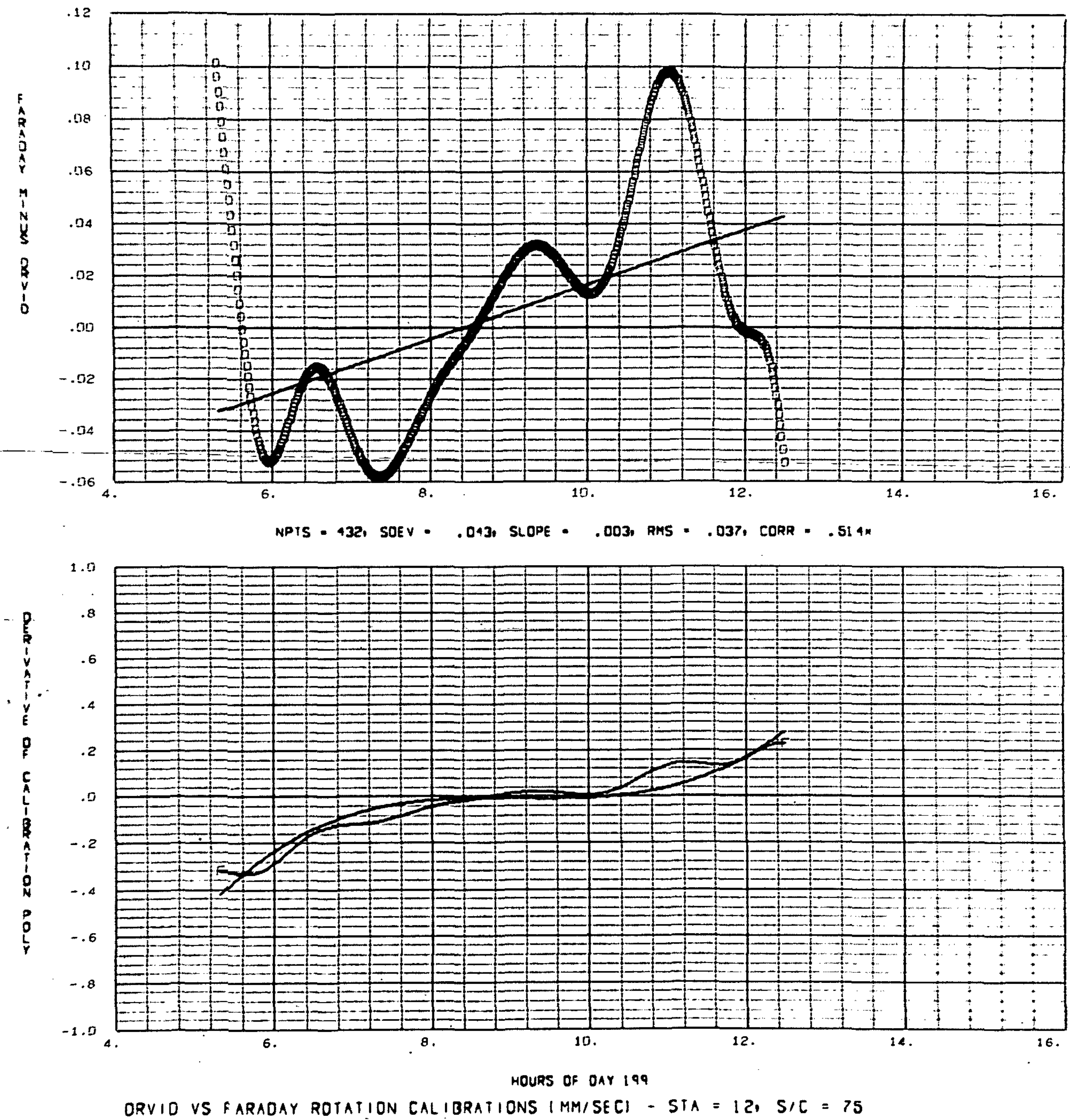
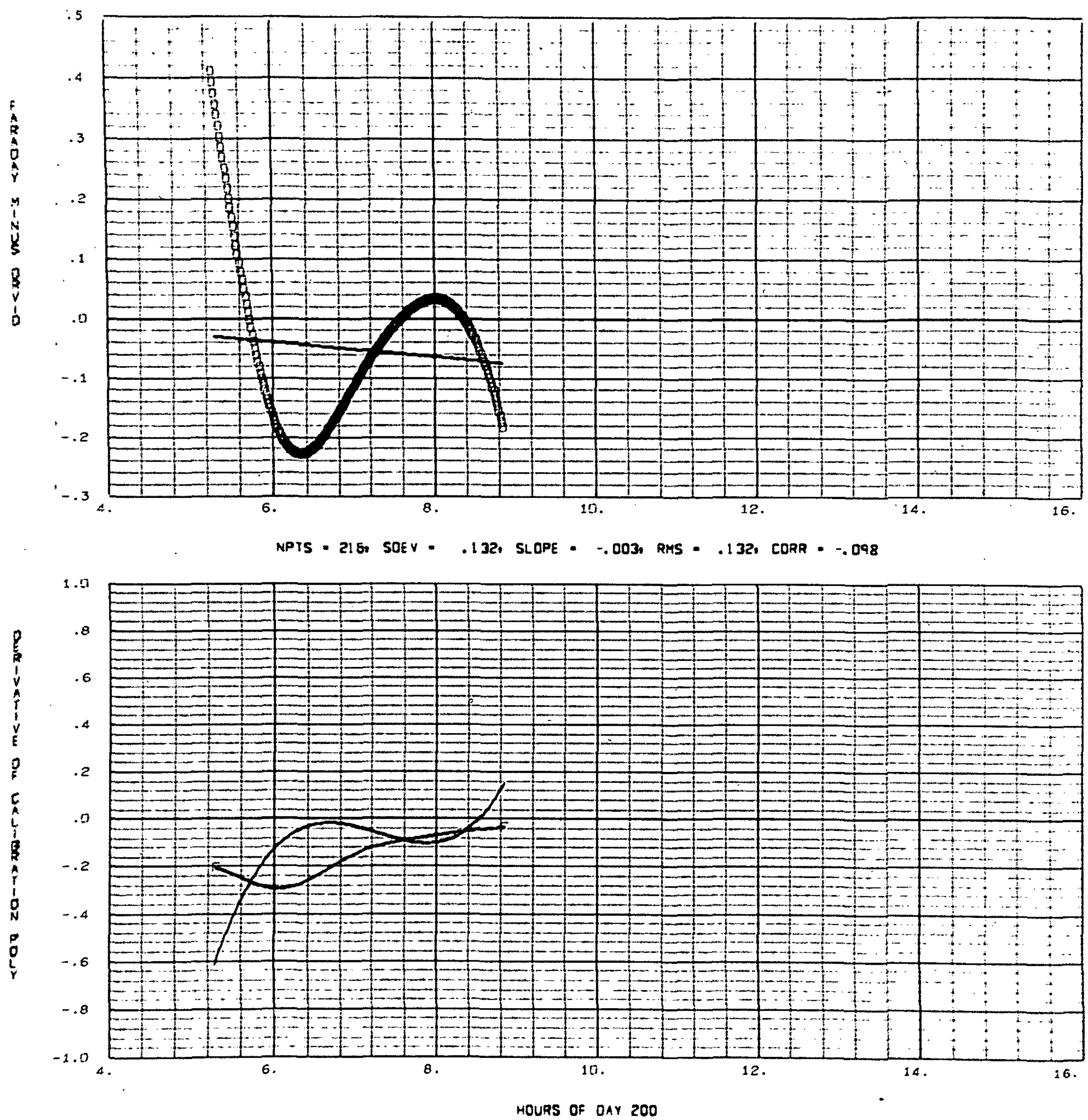
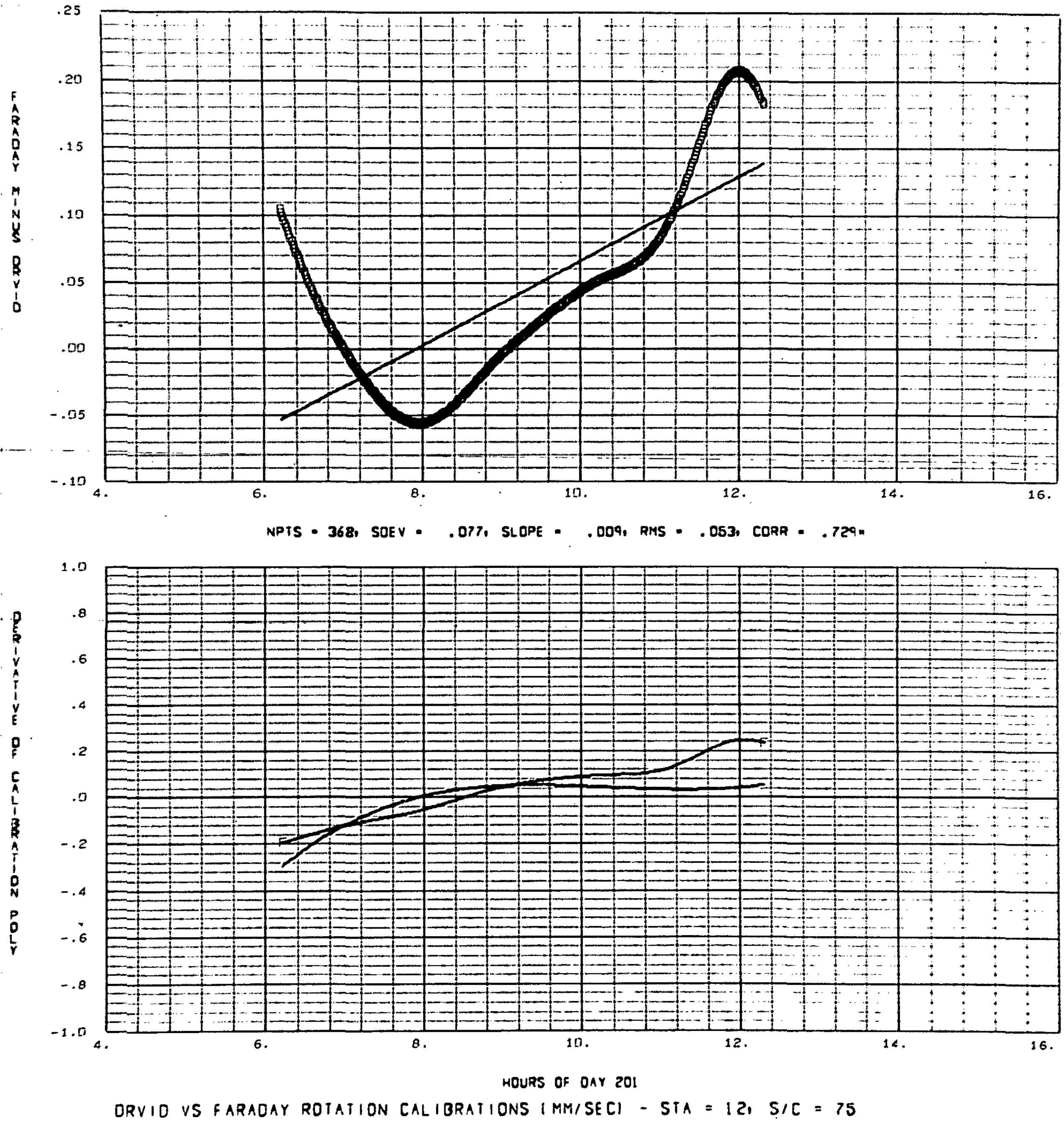

FARADAY MINUS ORVID
.12
.10
.08
.06
.04
.02
.00
-.02
-.04
-.06
4.
6.
8.
10.
12.
14.
16.
NPTS = 432, SDEV = .043, SLOPE = .003, RMS = .037, CORR = .514*
DERIVATIVE OF CALIBRATION POLY
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
HOURS OF DAY 199
ORVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-6.



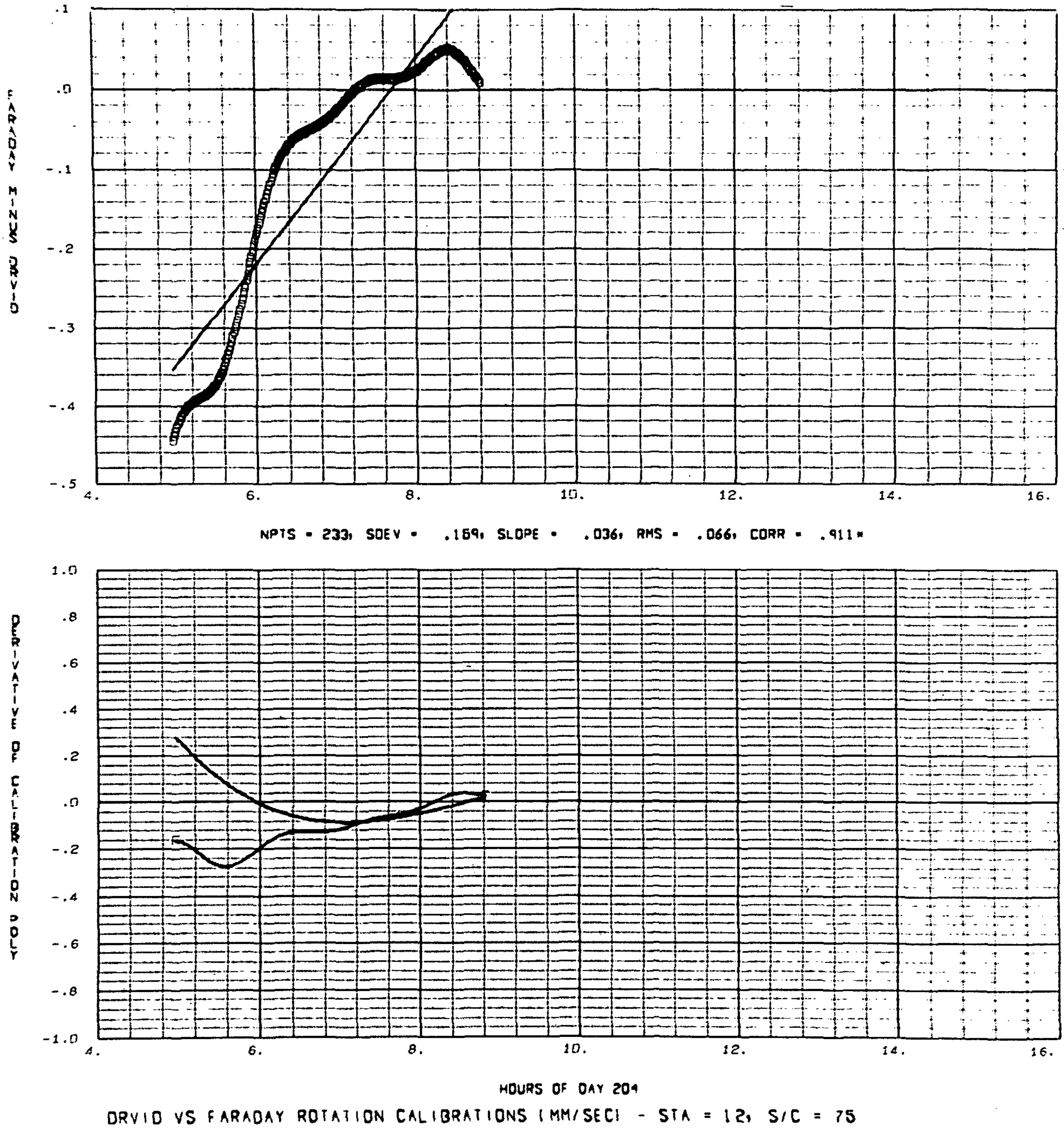
ORVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-7.



FARADAY MINUS DRVID
.25
.20
.15
.10
.05
.00
-.05
-.10
4.
6.
8.
10.
12.
14.
16.
NPTS = 368, SDEV = .077, SLOPE = .009, RMS = .053, CORR = .729*
DERIVATIVE OF CALIBRATION POLY
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
HOURS OF DAY 201
DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-8.



FARADAY MINUS DRVID
NPTS = 233, SDEV = .169, SLOPE = .036, RMS = .066, CORR = .911*
DERIVATIVE OF CALIBRATION POLY
HOURS OF DAY 204
DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-9.

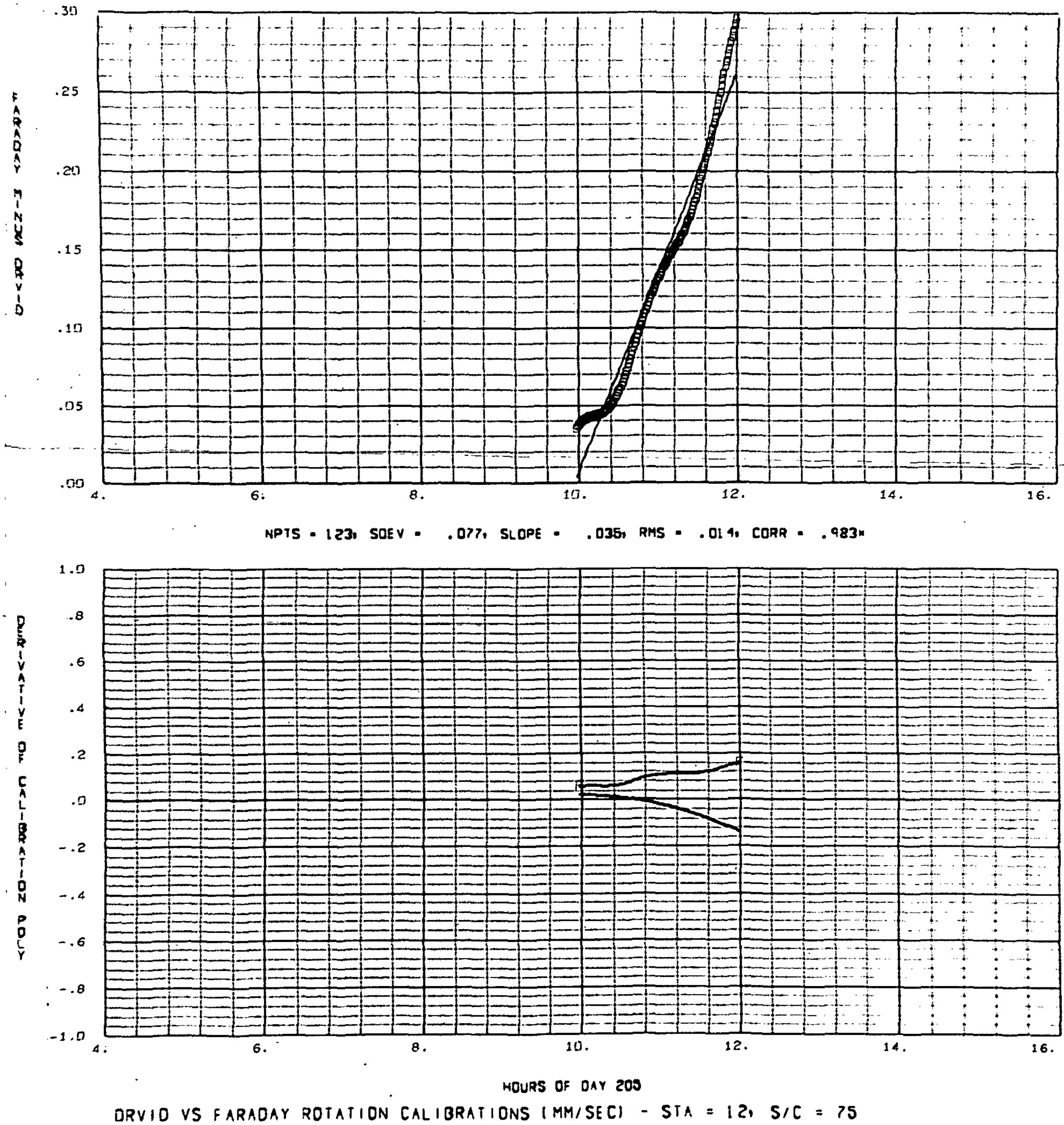
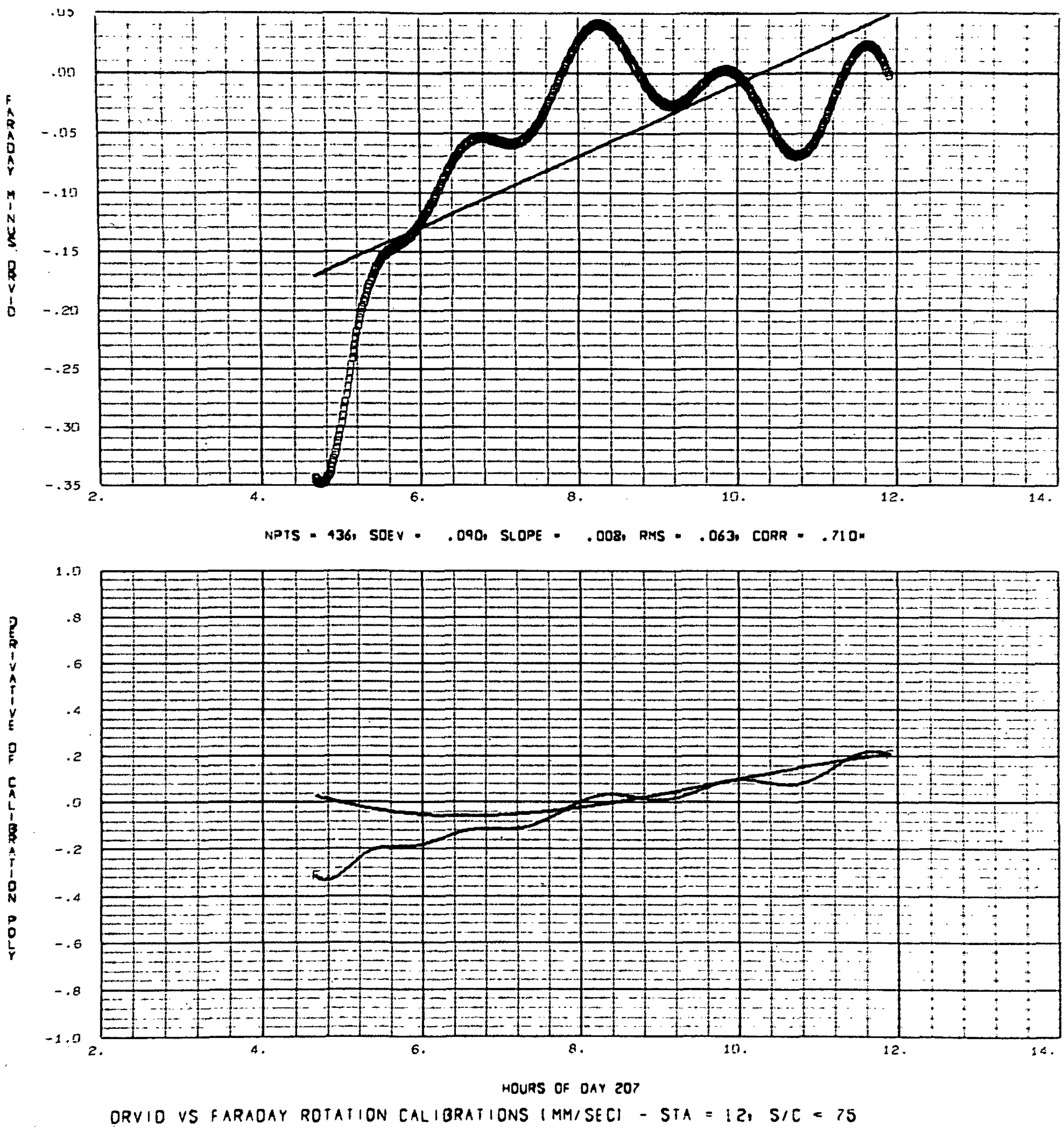
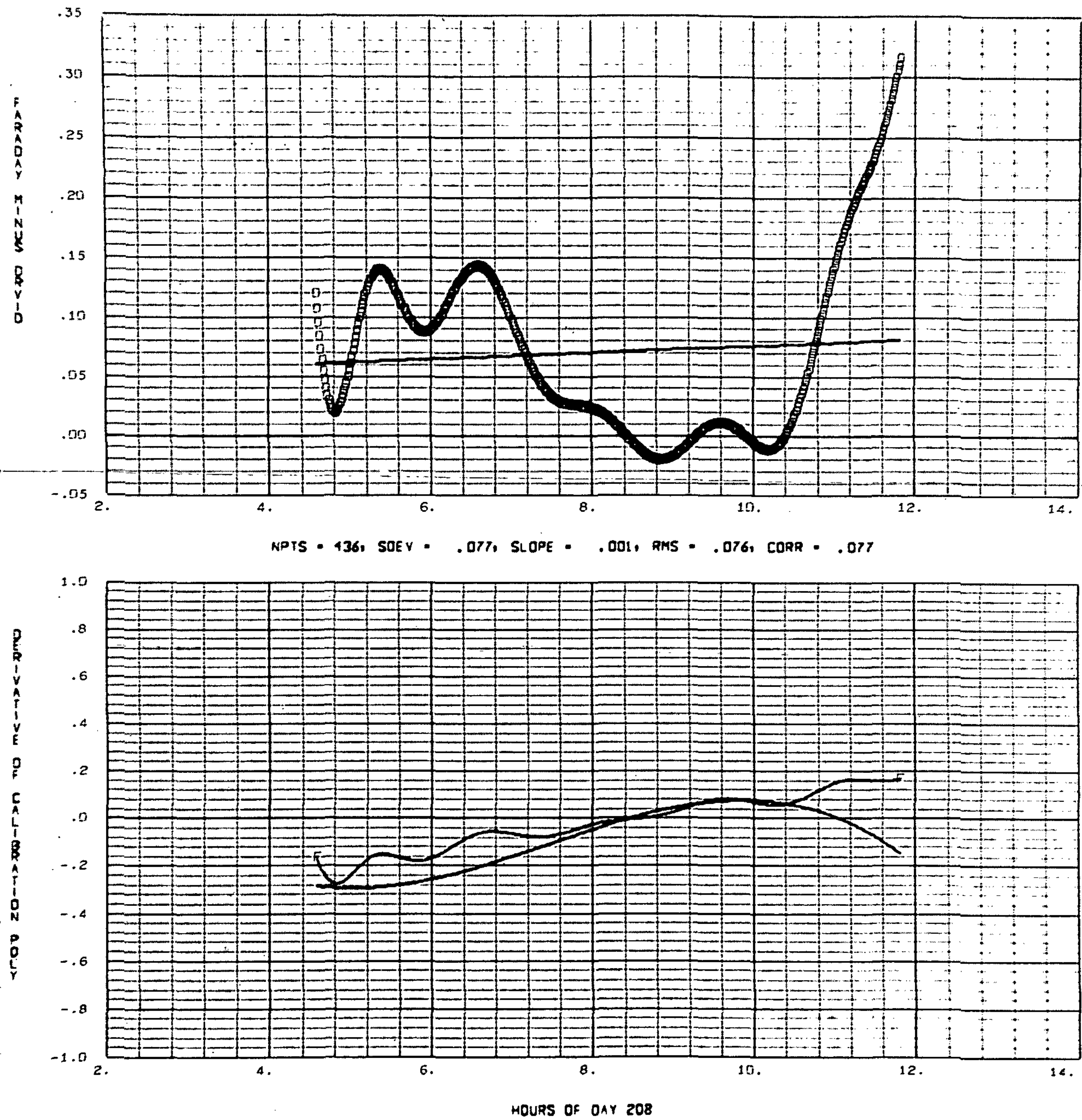

FARADAY MINUS DRVID
.30
.25
.20
.15
.10
.05
.00
4.
6.
8.
10.
12.
14.
16.
NPTS = 123, SDEV = .077, SLOPE = .035, RMS = .014, CORR = .983*
DERIVATIVE OF CALIBRATION POLY
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
4.
6.
8.
10.
12.
14.
16.
HOURS OF DAY 205
DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-10.



FARADAY MINUS DRVID
.05
.00
-.05
-.10
-.15
-.20
-.25
-.30
-.35
2.
4.
6.
8.
10.
12.
14.
NPTS = 436, SDEV = .090, SLOPE = .008, RMS = .063, CORR = .710*
DERIVATIVE OF CALIBRATION POLY
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
HOURS OF DAY 207
DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-11.



DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-12.

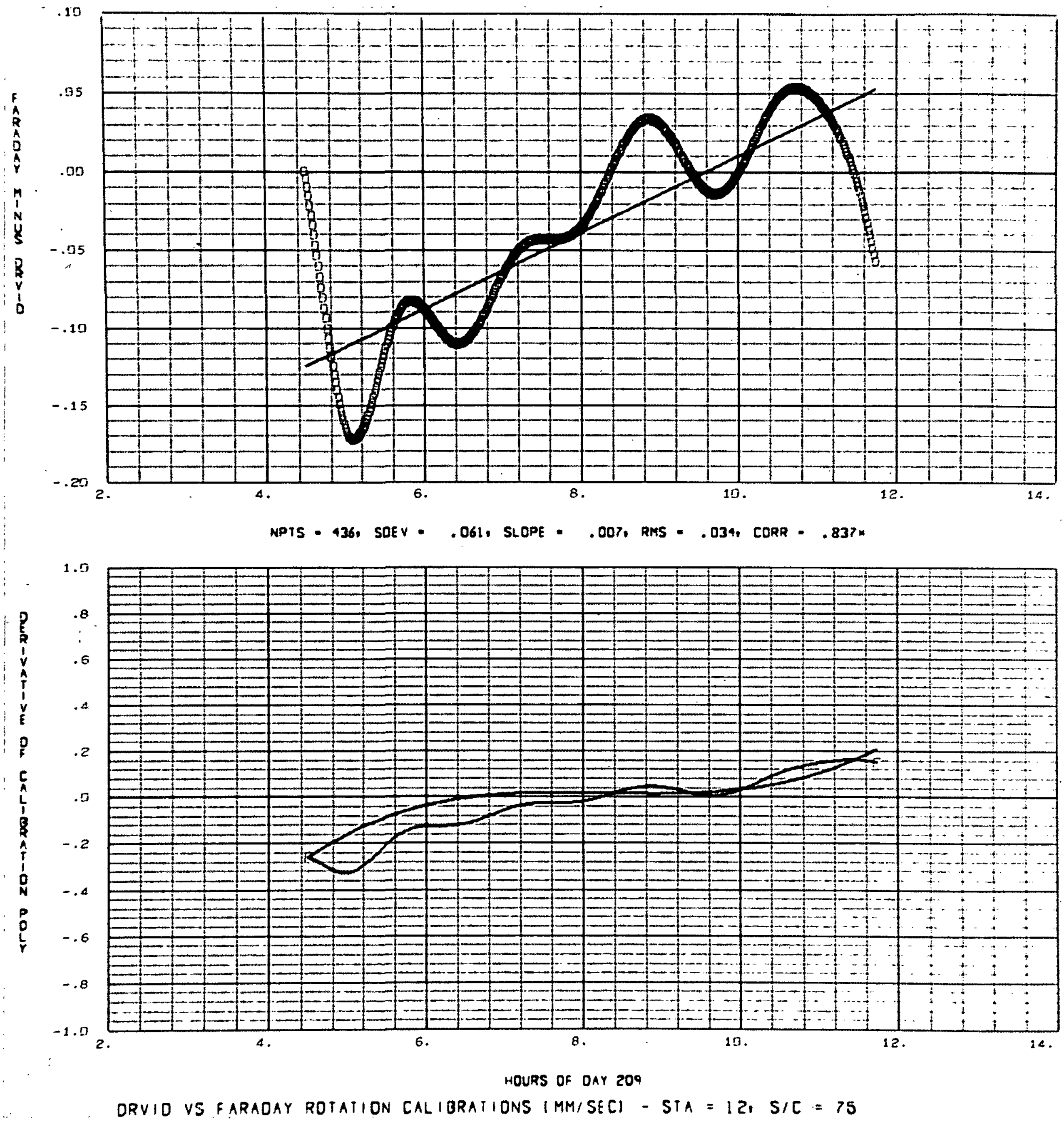

.10
.05
.00
-.05
-.10
-.15
-.20
2.
4.
6.
8.
10.
12.
14.
FARADAY MINUS DRVID
NPTS = 436, SDEV = .061, SLOPE = .007, RMS = .034, CORR = .837*
1.0
.8
.6
.4
.2
.0
-.2
-.4
-.6
-.8
-1.0
DERIVATIVE OF CALIBRATION POLY
HOURS OF DAY 209
DRVID VS FARADAY ROTATION CALIBRATIONS (MM/SEC) - STA = 12, S/C = 75

Fig. B-13.

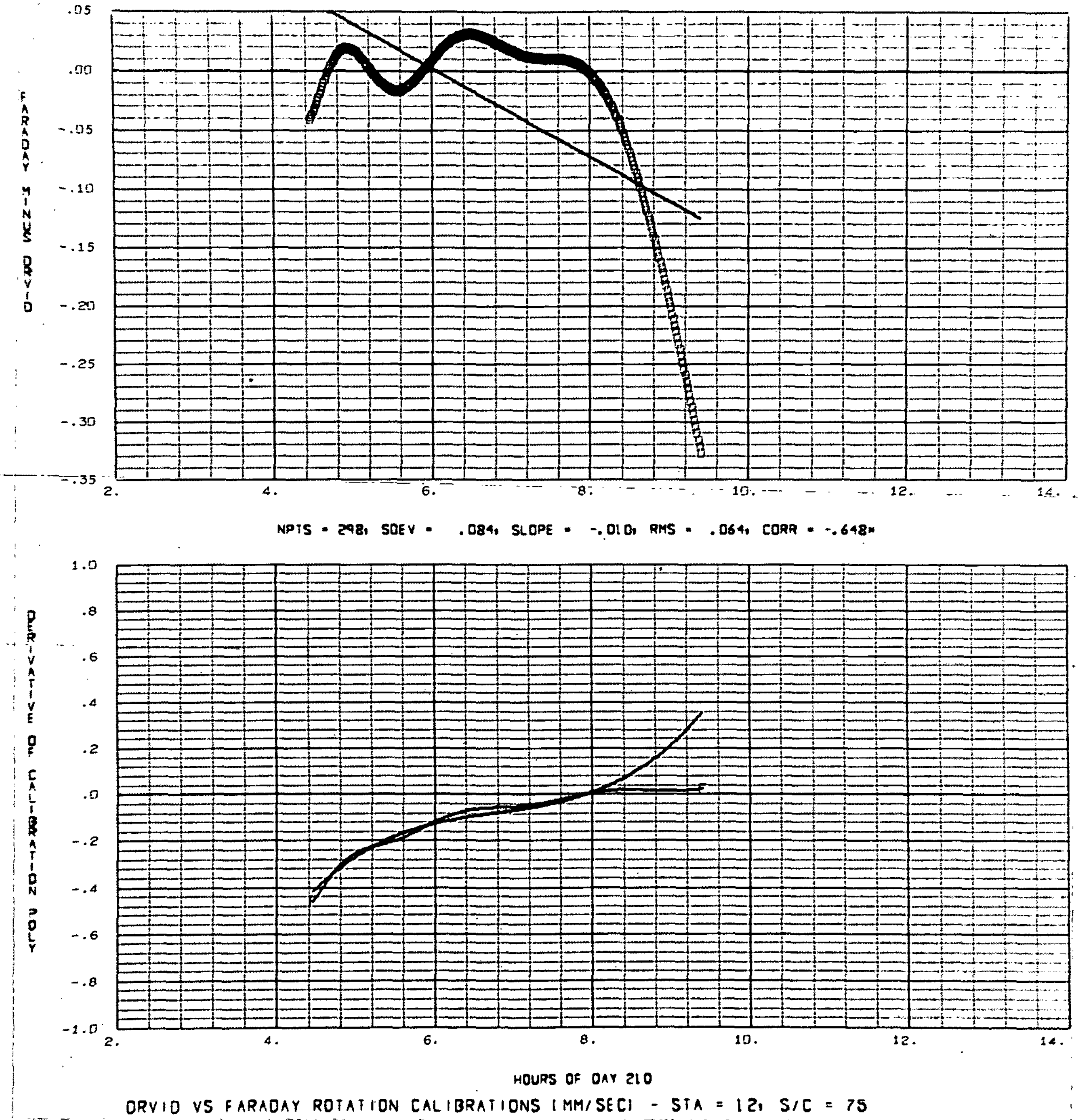


Fig. B-14.

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