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SOLAR MODULATION OF GALACTIC COSMIC RAYS, III: IMPLICATIONS OF THE COMPTON-GETTING COEFFICIENT

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SOLAR MODULATION OF GALACTIC COSMIC RAYS, III: IMPLICATIONS OF THE COMPTON-GETTING COEFFICIENT

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ABSTRACT

It is useful to express spectra of modulated galactic cosmic rays in terms of the Compton-Getting coefficient C. This parameter can reveal the energy range over which the force-field approximation is valid, and the range where convection effects dominate over those of diffusion. A value of C near zero over an extended low energy range, which according to recent observations is the case for protons, implies that the radial gradient at low energies can not be large. This small gradient may imply, in turn, that the diffusion coefficient increases beyond 1 AU less rapidly than proportional to heliocentric radial distance and/or there is essentially no scattering for a sizeable distance from the Sun to earth. The behavior of C with rigidity (or energy) is discussed in terms of the omni-directional distribution function f_o. Contours of constant f_o in the heliocentric distance vs rigidity plane are useful for illustrating the mean rigidity loss experienced by cosmic rays in the interplanetary medium.

Introduction

To understand completely the solar modulation of galactic cosmic rays, we must measure, among other quantities, the differential energy spectrum, the radial gradient, and the particle diffusion coefficient. These quantities, however, are interrelated. The shape of the energy spectrum, which is the most readily available of the measurements, can reveal the likely behavior of other quantitites. As we shall discuss, it is useful to express the spectrum in terms of the Compton-Getting coefficient C, a coefficient that was originally derived to transform the differential streaming of cosmic rays between frames of reference moving relative to each other at constant velocity (Compton and Getting, 1935; Gleeson and Axford, 1968a; Forman, 1970). The magnitude of the differential streaming due to convection by the solar wind, and also the rate at which the differential number density changes due to energy loss in the expanding wind, can be expressed in terms of C. The behavior of C with energy can reveal the relative importance of diffusion, convection, and energy loss processes in determining the modulated spectrum, as well as indicate the likely magnitude of the radial gradient and some features of the radial dependence of the diffusion coefficient.

Forman (1970) noted that in the energy range $\sim$ 30 - 200 MeV the Compton-Getting coefficient for the cosmic ray proton spectrum at solar minimum is essentially zero, which corresponds at these non-relativistic energies to a differential intensity spectrum with a spectral index near unity. An extended energy range where C is essentially zero is apparently a fundamental feature of the modulated spectrum at low energies. Recently, Rygg (1970) and Rygg and Earl (1971) have reported balloon observations that, taken together with satellite observations, show that the Compton-Getting coefficient is near zero for protons in the energy range ~ 30-300 MeV from 1965-1969, i.e. from solar minimum to solar maximum. It is difficult to determine from the observations in 1966-1969 exactly how C varies in the range 150-300 MeV. The observations can be interpreted to imply C near zero only below ~ 150 MeV. Also, the balloon observations during solar minimum are in disagreement with satellite observations in the range ~ 150-300 MeV, which predict a slightly larger value of C (Gleeson and Axford, 1968a). The Compton-Getting coefficient for the helium spectrum obtained by Rygg (1970) and Rygg and Earl (1971) exceeds zero at solar minimum, corresponding to a spectrum flatter than that of protons, but it too approaches zero near solar maximum.

In this paper we discuss the implications of the behavior of the Compton-Getting coefficient with energy, and, in particular, we consider conditions in the interplanetary medium that can lead to a near zero Compton-Getting coefficient at energies below ~ 200 MeV/nucleon. We show that an energy range where C passes through zero, from positive values at high energies to negative at low, is a consequence of the fact that particles observed at low energies were decelerated in the interplanetary medium from higher energies (Goldstein et al., 1970; Gleeson and Urch, 1971). The observation that C is near zero over an extended energy range implies that the radial gradient of the intensity must be relatively small, and we consider various radial dependences for the diffusion coefficient that can lead to the required small gradients. We show that the explanations for zero C proposed by Rygg and Earl (1971) and by O'Gallagher (1972), although able to account for many features of the problem, contain some inaccuracies. Rygg and Earl consider that zero C results from balancing the effects of convection against those of energy loss, but this explanation, which uses the approximate equations developed by Fisk and Axford (1969) and by Gleeson (1971), is unlikely to be applicable in the energy range they consider (~ 30-300 MeV/nuclon). The radial gradients that we find are necessary for C to be near zero over an extended energy range are inconsistent with the large, energy independent gradients predicted by O'Gallagher (1972).

Modulation equations and the distribution function

The behavior of the cosmic ray differential number density U(r,T) in the interplanetary medium can be described in terms of a spherically-symmetric Fokker-Planck equation (Parker, 1965; Gleeson and Axford, 1967):

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 VU) - \frac{1}{3r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial}{\partial T}(\alpha TU) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \kappa \frac{\partial U}{\partial r}\right) \qquad (1)$$

Here, r is heliocentric radial distance and T is particle kinetic energy. The solar wind speed is given by V(r), the particle diffusion coefficient by $\kappa(r,T)$, and $\alpha(T) = (T+2T_o)/(T+T_o)$, with T_o the rest energy of a particle. The terms in (1) describe from left to right the convection, energy loss in the expanding solar wind, and diffusion of the particles. The differential streaming S(r,T) (radial current density) is given by Gleeson and Axford (1967):

$$S = CVU - \kappa \frac{\partial U}{\partial r} \qquad (2)$$

where $C = 1 - \frac{1}{3U} \frac{\partial}{\partial T} (\alpha TU)$

is the Compton-Getting coefficient. The energy loss term in (1) can also be expressed in terms of C, or (1) becomes

$$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \kappa \frac{\partial U}{\partial r}) - V \frac{\partial U}{\partial r} = \frac{C}{r^2} \frac{\partial}{\partial r} (r^2 V) U \qquad (3)$$

We will discuss the behavior of the Compton-Getting coefficient with energy (or rigidity), and the implications, not in terms of the differential number density or intensity (intensity $j_o = vU/4\pi$, where v is particle speed), but rather in terms of the omni-directional distribution function f_o. The function f_o represents the number of particles per unit volume of phase space ($d^3\bar{r}\ d^3\bar{p}$ where $\bar{p}$ is particle momentum) averaged over particle direction, and is related to j_o by $f_o = j_o/p^2$. For convenience in relating the discussion given here with other treatments of cosmic ray problems, we express f_o as a function of particle rigidity $P = pc/Ze$, where c is the speed of light and Ze, the particle charge. It is useful to discuss the behavior of C in terms of f_o since these two are simply related (Forman, 1970; Rygg and Earl, 1971).

$$C = - \frac{P}{3f_o} \frac{\partial f_o}{\partial P} \qquad (4)$$

e.g. C = 0 corresponds to $\partial f / \partial P = 0$. The Fokker-Planck equation (1) or (3) can also be expressed in terms of f_o, or

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\kappa\frac{\partial f_o}{\partial r}\right) - V\frac{\partial f_o}{\partial r} = -\frac{P}{3}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2V)\frac{\partial f_o}{\partial P} \qquad (5)$$

We will see that the physics involved in the various approximations that can be made to (1,2,3,and 5) is clearly illustrated when these approximate equations are expressed in terms of f_o. In addition, we will find that the contours of constant density in phase space (the contours in the P-r plane along which f_o is a constant) are useful for illustrating the energy (or rigidity) range over which C can be expected to be zero, as well as for discussing the mean rigidity loss experienced by particles in the interplanetary medium.

High Energies

Gleeson and Axford (1968b) argued that the streaming S can be neglected in (2) at relatively high energies, yielding a simple first-order equation for the number density known as the force-field equation. In terms of f_o this approximate equation becomes:

$$v\frac{\partial f_o}{\partial r} + \frac{VPv}{3\kappa}\frac{\partial f_o}{\partial P} \simeq 0 \qquad (6)$$

This equation is in the form of a one-dimensional Liouville equation with a "force" $VPv/3\kappa$ - hence, the name force-field equation. This "force" is of course not a real force, but merely a convenient representation for the accumulated effects of convection, diffusion, and energy loss. The concept of modulation by a force-field is only valid when we consider the behavior of the entire distribution function, not the behavior of individual particles. However, as we will discuss below, the contours of constant f_o predicted by (6) can reveal, to a certain approximation, the mean rigidity loss experienced by particles.

The solution to (6) is

$$f_o = F(P'(P,r,Z)) \qquad (7)$$

where F(P) is the unmodulated distribution function, the distribution function determined at some radius r = R where the modulation is negligibly small. The function P'(P,r,Z) is the result of integrating the equation

$$dP = \frac{VP}{3\kappa}dr \qquad (8)$$

Subject to the condition P' = P at r = R. The integral is straightforward when κ is a separable function of P and r, or $\kappa = \beta\kappa_1(P)\kappa_2(r)$, where $\beta = v/c$; however, the force-field approximation holds and (7) is valid irrespective of whether κ has this form. When κ is separable, it can also be shown that

$$C(r,P) = \frac{\kappa(r,P)}{\kappa(r,P')}C(R,P') \qquad (9)$$

where C(r,P) denotes the Compton-Getting coefficient at rigidity P, determined from the distribution function at radius r.

Fisk and Axford (1969) showed that the condition that must be satisfied for the force-field equation to hold is:

$$\left|\frac{\tilde{C}}{2}\,(\tilde{C}-1)\,\frac{\widetilde{Vr}}{\tilde{\kappa}}\right| << 1 \qquad (10)$$

where the tilde denotes characteristic value. The parameter $\widetilde{Vr}/\tilde{\kappa}$ is a measure of the modulation, small values indicating small modulation, large values indicating that the particles are strongly influenced by the interplanetary medium. The parameter $\widetilde{Vr}/\tilde{\kappa}$ is expected to be small at high energies since κ increases with increasing energy. According to (10), however, the force-field equation can remain valid even for moderate values of $\tilde{\tilde{V}}r/\tilde{K}$ provided that $|\tilde{C}(\tilde{C}-1)/2| << 1$. The Compton-Getting coefficient is therefore a useful quantity to measure since it can reveal the energy range over which the force-field equation can be expected to hold. Note, however, that we should not expect that the force-field solution will remain an accurate approximation if C passes through zero from positive to negative values. From (9), it can be seen that the force-field solution can yield only positive values of C (provided C(R, P') is positive, as it is for those forms of the unmodulated spectrum normally considered). However, the force-field solution may remain valid down to small positive values of C, i.e. $|C(C-1)/2| << 1$, $C > 0$.

It is instructive, as we do below, to plot contours of constant f_o in the P-r plane. When the force-field equation holds these contours are determined by P'(P,r,Z). They have a positive slope

$$\frac{dP}{dr} = - \frac{\partial f_o/\partial r}{\partial f_o/\partial P} = \frac{VP}{3K} \tag{11}$$

and intersect the boundary r = R at P = P'.

Low Energies

Near 1 AU the force-field equation is expected to hold for cosmic ray nuclei with energies down to ~ 150 MeV/nucleon during solar minimum (Fisk, 1971) and only down to somewhat larger energies during higher levels of solar activity. To cover the low energy range, where presumably $\tilde{\tilde{V}}r/\tilde{K}$ is large, Fisk and Axford (1969) showed that two approximate equations are possible:

$$VU \simeq K \frac{\partial U}{\partial r} \tag{12}$$

and

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 VU) - \frac{1}{3r^2}\frac{\partial}{\partial r}(r^2 V)\ \frac{\partial}{\partial T}(\alpha T U) \simeq 0 \qquad (13)$$

Equation (12), which is in the form of a simple convection-diffusion equation (Parker, 1963), has the additional requirement that the energy spectrum can not be too steep. The essential difference between these two approximate equations lies in the source of the low energy particles. When the parameter $\widetilde{Vr}/\widetilde{K} >> 1$, the modulation is large and it is diffuclt for low energy interstellar particles to penetrate to the vicinity of earth. However, if there is a sufficient number of low energy interstellar particles so that despite the substantial modulation, these are the main source of low energy particles seen at earth, (12) is the appropriate approximate equation to use. On the other hand, when $\widetilde{Vr}/\widetilde{K} >> 1$, higher energy particles are effectively cooled to lower energies in the interplanetary medium. If these higher energy particles are the main source of low energy particles, (13) is the appropriate approximate equation. In practice, (13) is probably correct since the number of low energy interstellar particles required for (12) to hold, in reasonable models for the interplanetary medium, appears to result in an energy density for interstellar cosmic rays inconsistent with that required to account for the observed half-thickness of the gaseous disk of the galaxy (Goldstein et al., 1970a, b; Gleeson and Urch, 1971; Kellman, 1972).

Equation (13) and its implications have been discussed extensively by Gleeson (1971). The effects of diffusion are negligibly small ((13) is obtained from (1) by simply neglecting the diffusion term), and the particles behave in this approximation as if they were essentially "frozen-into the solar wind." The particles are simply convected outward, with the resulting energy loss in the expanding solar wind, and hence we call (13) the "convection approximation." In terms of f_o, with V constant, (13) becomes

$$V \frac{\partial f_o}{\partial r} - \frac{2VP}{3r}\frac{\partial f_o}{\partial P} \simeq 0 \qquad (14)$$

which is again in the form of a one-dimensional Liouville equation, where now the "force" is -2vP/3r. This "force", unlike the "force" in (6), has a direct physical interpretation in that it describes the action of the expanding solar wind in cooling the particles. Note that

$$\text{force} = \frac{dT}{dr} = -\frac{2vp}{3r} \qquad (15)$$

where here we have used the actual momentum p as opposed to the rigidity. Since the particles are "frozen-into" the wind d/dr = (1/V)d/dt, or (15) becomes

$$\frac{dT}{dt} = -\frac{2V\alpha T}{3r} \qquad (16)$$

on noting that vp = αT. As can be seen from the second term in (1) or (13), taking V constant, this is just the rate of change in kinetic energy due to the expansion of the solar wind (Parker, 1965).

The solution to (14) can be expressed

$$f_o = F'(r(P/P_1)^{3/2}) \qquad (17)$$

where F'(r) is the distribution function at some rigidity P_1. This solution holds, of course, only for those values of r and P (and P_1) that lie in the regime where (14) is valid, i.e. where $\widetilde{Vr}/\widetilde{\kappa} >> 1$. Note that (14) is independent of V and κ, although it could depend on gradients in V. The solution (17), however, depends on the behavior of V and κ at values of r and P outside the regime where (14) holds, since F'(r) depends on this behavior. Nevertheless, the distribution function (or the number density) in the convection approximation should be relatively insensitive to any short-term fluctuations in V and κ, particularly when contrasted with the sensitivity to V and κ predicted for U by (12). It may thus be possible observationally, by comparing fluctuations in V and κ with fluctuations in the intensity, to see whether (13) and not (12) is in fact the appropriate approximate equation to use at low energies. However, as is discussed below, the convection approximation may only be valid at very low energies (< 30 Mev), and hence such an observation may be difficult to perform due to the dominant presence of solar cosmic rays (Kinsey, 1970).

Note from (14) that in the convection regime

$$C \simeq - \frac{r}{2f_o}\frac{\partial f_o}{\partial r} = \frac{-r}{2j_o}\frac{\partial j_o}{\partial r} \qquad (18)$$

The Compton-Getting coefficient can thus be used to determine the gradient at low energies (Fisk and Axford, 1969, 1970; Gleeson, 1971). Note, as was pointed out by O'Gallagher (1972), that the gradient in the convection approximation is not a measure of local interplanetary conditions (i.e. V and κ), as it is in, for example, the force-field approximation. Rather, as is illustrated below, the gradient at low energies is determined by interplanetary conditions throughout the inner solar system. Note also that the gradients in the convection approximation are expected to be small, since C may be near zero

(Rygg, 1970; Rygg and Earl, 1971). However, the intensity of low energy particles near earth is substantially reduced over that present in the interstellar medium. Consequently, there must be a region beyond the orbit of earth, near the "boundary" of the modulating region, where the gradient is large. This region is effectively a "boundary layer" of width $\sim \tilde{\kappa}/\tilde{V}$, where the particle behavior is described by (12). Most of the particles here still come in directly from the interstellar medium, rather than cooling down from higher energies. This large gradient might well serve as a useful marker of the end of the modulating region for deep space probes.

Rygg and Earl (1971) discuss the observation that $C \simeq 0$ for protons in the energy range $\sim$ 30-300 MeV in terms of (13), which, as can be seen from (18), is consistent with small C provided that the radial gradient is essentially zero. The difficulty with using (13), however, is that it is valid only when $\tilde{V}\tilde{r}/\tilde{\kappa} >> 1$. This is the appropriate condition to use, not the condition $(\frac{\tilde{\kappa}}{\tilde{V}\tilde{r}})(\frac{r}{U}\frac{\partial U}{\partial r}) << 1$ also considered by Fisk and Axford (1969), which for small gradients is less stringent. We noted above that (13) is derived from (1) or (3) simply by neglecting diffusion terms. When expanded, (3) becomes

$$\kappa \frac{\partial^2 U}{\partial r^2} + [\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\kappa) - V]\frac{\partial U}{\partial r} = \frac{C}{r^2}\frac{\partial}{\partial r}(r^2 V)U \qquad (19)$$

Hence, provided that κ and the radial gradient of U are not strong functions of r, the condition for (13) to hold is $Vr/\kappa >> 1$, i.e. it is appropriate to use the actual values of V, r, and κ for their characteristic values. In practice, we anticipate that Vr/κ is large near earth only for very low energies. Using the diffusion coefficient obtained by Jokipii and Coleman (1968) from power spectra of magnetic field fluctuations observed near solar minimum, we find that Vr/κ for protons attains a value only ~ 5 when $T \sim 5$ MeV. Even allowing for the possibility that κ decreases with increasing solar activity Vr/κ is unlikely ever to be sufficiently large, or equivalently (13) is unlikely to hold, in the energy range considered by Rygg and Earl (1971) (30-300 MeV for protons).

It follows from (17) and (18) that in the convection approximation

$$C(r,P) = -\frac{1}{2}\left(\frac{P}{P_1}\right)^{3/2}\left(\frac{r}{F'}\frac{dF'}{dr}\right|_{r\left(\frac{P}{P_1}\right)^{3/2}} \tag{20}$$

where the radial gradient of F' is to be evaluated at $r(P/P_1)^{3/2}$.

For $P < P_1$, C is related to the radial gradient at smaller radial distances. If the gradient is positive, C is negative, and it will decrease in magnitude with decreasing P provided that gradient, as r becomes smaller, does not increase too rapidly. In practice, we anticipate that the radial gradient actually becomes quite small at small values of r. There is no particularly good reason to assume that there is extensive scattering ($\kappa \to 0$) at small r. Consequently Vr/κ is small at small r and a gradient can not be maintained.

In the convection approximation, the contours of constant f_o in the P-r planes follow the curves $rP^{3/2}$ = constant, with a negative slope

$$\frac{dP}{dr} = -\frac{2P}{3r} \tag{21}$$

Since the particles are effectively "frozen-into" the solar wind in this approximation, these contours actually describe the mean trajectories of particles in phase space. The contours will always occur so that the particles are being convected radially outward and cooled down in energy.

Transition Region

The contours of constant f_0 are presumably continuous from the regime where the force-field approximation holds ($Vr/K \leqslant 1$), through some transition regime, into the regime of the convection approximation ($Vr/K >> 1$). In the force-field approximation the contours have a positive slope, connecting to the boundary ($r = R$) at some rigidity P'. In the convection approximation the slope is negative and the contours run back towards the boundary and down in rigidity, consistent with the requirement that the particles are convected outward and cooled. It follows, therefore, that in the transition regime there must be a point where the contours are vertical, i.e. where $dP/dr \to \infty$ or from (11) where $\partial f/\partial P = 0$. Thus, in the transition regime between the force-field and convection approximation there must be a point where the Compton-Getting coefficient is zero. It is our contention that the observations of Rygg (1970) and Rygg and Earl (1971) apply to the transition regime, and we discuss below conditions that can lead to C essentially zero over an extended rigidity (or energy) range. It is a regime not in which the effects of convection balance those of energy loss as claimed by Rygg and Earl (1971), but rather where the terms on the left side of (3) or (19) cancel. The effects of diffusion are just as important as those of convection or energy loss.

An Illustrative Numerical Solution

As an illustration of the above considerations consider a model for the interplanetary medium in which V is a constant and $\kappa = 5\text{x}10^{17}\beta P\exp(r-1)/\text{cm}^2/\text{sec}$, where P is in units of MV, and r in units of AU. This diffusion coefficient, with its βP and exp(r) dependences, is typical of those used in discussing modulation problems (e.g. Fisk, 1971). Its magnitude, however, is smaller than many previous choices for κ, but is chosen so that the maximum of the modulated intensity spectrum for protons at 1 AU occurs at ~ 400 MeV, in agreement with spectra observed near solar maximum (e.g. Lezniak and Webber, 1971). With these forms for V and κ, and assuming that the unmodulated intensity spectrum is a power law in total energy with a spectral index of -2.65, we have solved (1) using the numerical technique developed by Fisk (1971). On the left side of Figure 1, the unmodulated spectrum and the modulated spectrum at 1 AU are plotted vs. kinetic energy, assuming that the particles are protons. Shown also are the corresponding force-field solution and Compton-Getting coefficient at 1 AU. Note that the force-field solution is scarely distinguishable from the numerical solution down to energies where C is small, but positive. Note also that C is positive at high energies, passes through zero (corresponding to an intensity spectrum roughly proportional to T), attains a minimum value, and then approaches zero from the negative side at low energies. The energy of the minimum value (~ 35 MeV) roughly marks the entry into the regime where the convection approximation holds, since, as we saw above, a negative C decreasing in magnitude with decreasing energy is characteristic of the convection approximation. This passage of C through a minimum value, in principle, could be detected observationally. If so, it would confirm the existence of a regime where the convection approximation is valid, and thereby would place an upper limit on acceptable values of κ at low energies ($\kappa << Vr$). However, there are conditions, discussed below, that result in a negligible variation of C away from zero at low energies, and would, therefore, make this observation practically impossible.

On the right side of Figure 1 are plotted the contours of constant f_o in the P-r plane, obtained using these forms of V, κ, and the unmodulated spectrum. Shown also are the contours predicted by the force-field and by the convection approximation, and the contours along which Vr/κ is 1 and 30. As can be seen the force-field approximation is valid even for moderate values of Vr/κ and the convection approximation, where $Vr/\kappa \geqslant 30$. Note that the force-field contour becomes vertical for small κ (see (9)), corresponding to C = 0, but only after it is no longer an adequate approximation. Note also that the convection contours do not extend indefinitely out in radial distance. As we discussed above, there is a regime in which (12) is the appropriate approximation equation lying between where the convection approximation holds and where the modulation ceases.

Mean Rigidity Loss

The contours of constant f_o are also useful for illustrating the mean loss in rigidity experienced by particles in the interplanetary medium. Note first that the solution $f_o(r,P)$ can be determined, in general, from a Green's function $G(Z, R, \mathcal{P}, r, P)$ or

$$f_o(r,P) = \int_P^\infty F(\wp)G(Z,\ R,\wp\ ,\ r,\ P)\wp^2 d\wp \qquad (22)$$

where F(P) is the unmodulated distribution function (at r = R). The solution can also be expressed

$$f_o\ (r,P) = F(P''(r,\ P,\ Z)) \qquad (23)$$

when it is possible to define contours of constant f_o that intersect the boundary (r = R), or

$$F(P''(r,\ P,\ Z)) = \int_P^\infty F(\wp)G(Z,\ R,\wp\ ,\ r,\ P)\ \wp^2 d\wp \qquad (24)$$

The contour P"(r, P, Z) = constant, along which f_o is constant, is identical to the force-field contour (P'(r, P, Z) = constant) in the regime where that approximation is valid, and to the contour $rP^{3/2}$ = constant in the convection approximation. The contours intersect the boundary r = R at P = P". It is shown in the Appendix to this paper that to a good approximation the contours of constant f_o are relatively insensitive to whether F(P) or PF(P) is chosen for the unmodulated distribution function, at least for those forms of F(P) normally used. Thus

$$P''(r,\ P,\ Z)\ F(P''(r,\ P,\ Z)) \simeq \int_P^\infty \wp\ F(\wp)G(Z,\ R,\wp\ ,r,\ P)\ \wp^2 d\wp \qquad (25)$$

where P"(r, P, Z) is the same as in (23) and (24). Dividing (25) by (24) we find then that

$$P''(r,P,Z) \simeq \frac{\int_P^\infty \wp\ F(\wp)G(Z,R,\wp\ ,r,P)\wp^2 d\wp}{\int_P^\infty F(\wp)G(Z,R,\wp,r,P)\wp^2 d\wp} \qquad (26)$$

The Green's function, however, is simply the probability that a particle of charge Ze, entering the interplanetary medium at rigidity , will reach r with rigidity P. Consequently P"(r,P,z) is approximately the mean rigidity in the interstellar medium of particles that arrive at r with rigidity P. Particles arriving at values of r and P connected by the same contour of constant f_o come from the same mean rigidity in the interstellar medium. For a given r, we can determine the mean rigidity loss experienced by particles at rigidity P by comparing P with P"(r,P,Z), i.e. by comparing P with the intersection at the boundary r = R of the contour through r and P (provided such a contour exists).

It is important to realize that contours of constant f_o do not describe, in general, the mean trajectory of particles that enter the interplanetary medium at rigidity P". In general, the contours are sensitive to the behavior of particles that enter at rigidities other than P", i.e. they are sensitive to F(P). The contours of constant f_o in the force-field approximation are an exception to this, to a certain extent. The contours in this regime are determined by an equation (equation (6)) that contains only first-order derivatives of f_o, and thus are independent of F(P). However, the condition necessary for the force-field equation to hold,(10),depends on C which in turn will depend on F(P). The contours of constant f_o in the convection approximation are mean particle trajectories since here the particles behave as if they were "frozen-into" the solar wind. However, particles can arrive on a given contour in this regime from different values of P".

Note, in Figure 1, that at lower rigidities the contours of constant f_0 intersect $r = 1$ AU both above and below the rigidity where $C = 0$. Consequently at these lower rigidities we observe particles from the same mean rigidity in the interstellar medium at two different rigidities. This behavior follows from the requirement that particles arrive at low rigidities principally by being convected outward with the solar wind. We thus sample particles with the same mean interstellar rigidity as they penetrate into the region $r \leq 1$ AU at higher rigidities and then again at lower rigidities as they are convected outward having lost considerable rigidity between the Sun and earth. Note that particles arriving at progressively lower rigidities below where $C = 0$ come from progressively higher mean rigidities in the interstellar medium. For particles to arrive at low rigidites at 1 AU they must penetrate into small radial distances, the lower the rigidity at 1 AU, the smaller the radial distance attained by the particles, on the average. The penetration of particles into small radial distances, however, depends on their mean interstellar rigidity, the particles penetrating further with increasing mean interstellar rigidity.

It might be possible to show that low rigidity particles arrive at 1 AU principally from the region $r < 1$ AU by comparing fluctuations in the low rigidity intensity with fluctuations in V and the appropriate values of κ (although the intensity fluctuations are not expected to be large). For this argument to hold, we require that irregularities in the interplanetary magnetic field are, for the most part, convected outward with the solar wind, and not locally generated or damped. Then, since the intensity is sensitive to conditions at $r < 1$ AU, it may show a tendency to vary with fluctuations in V and κ that are displaced in time by up to a few days following the intensity observations. If the intensity were sensitive to conditions at $r > 1$ AU, it should respond to changes in V and κ that occur at some earlier time. Note that the streaming (anisotropy) is a poor measure of the tendency of low rigidity particles to arrive at 1 AU from the sunward side. The Compton-Getting coefficient at low rigidities is zero or negative and the radial gradient is in general positive. Thus, from (2) the streaming is negative, i.e. more particles will be observed propagating towards the Sun than away from it. A zero or negative Compton-Getting coefficient obscures the dominant importance of convection at low rigidities.

Compton-Getting coefficient near zero for low energy protons

The Compton-Getting coefficient plotted in Figure 1 is not in good agreement with that found by Rygg (1970) and Rygg and Earl (1972) in that it is not essentially zero over an extended energy range. As noted above, C is zero when the terms on the left side of (3) or (19) cancel each other. We can preserve the approximate balance between these terms by requiring that $\partial U/\partial r$ and $\partial^2 U/\partial r^2$ are relatively small. It is better to relie simply on a small gradient to preserve the balance, rather than on, for example, some peculiar energy dependence for κ. The latter should vary with solar cycle contrary to the observation that C is always roughly zero at low energies (Rygg, 1970; Rygg and Earl, 1971). One possible way to achieve the required small gradient is with a diffusion coefficient whose radial dependence is such that the ratio Vr/κ increases with increasing r, i.e. κ increases less rapidly than proportional to r (out to the boundary $r = R$). With this dependence, the behavior of even moderate energy particles (e.g. $\sim$ 30 -300 MeV) at values of r near the boundary $r = R$ can be dominated by convection effects (large Vr/κ), while these particles near 1 AU can still diffuse relatively easily (smaller Vr/κ). These particles, which were cooled down from higher energies, must pass through the region $r \leqslant R$ convected outward at the solar wind speed, i.e. they can leave the interplanetary medium only at a fixed rate. However, since the particles can diffuse at smaller r, they will tend to come into equilibrium, i.e. $\partial U/\partial r$ and $\partial^2 U/\partial r^2$ tends toward small values. Note that in this case the gradient is determined by conditions throughout the interplanetary medium, not simply the local values of V and κ.

In Figure 2 we have plotted the same quantities as in Figure 1, using $\kappa = 7.5 \times 10^{17} \beta P/cm^2/sec$ for $r \leq 2.5$ AU and infinite thereafter; P is in units of MV. Here κ is independent of r out to the boundary $R = 2.5$ AU, and thus Vr/κ increases with increasing r. The magnitude of κ is chosen again so that the maximum of the modulated intensity spectrum for protons at 1 AU occurs at roughly 400 MeV. As can be seen by comparing Figure 2 with 1, the radial gradient at 1 AU for this second case is considerably smaller, than in the first, and the variation in C away from zero at low energies is quite small. Note that C is near zero at all low energies, and gets progressively smaller in magnitude with decreasing energy. Hence, the spectrum of galactic cosmic rays can be extrapolated down to energies where it can not be observed directly because of the dominant presence of solar cosmic rays simply by assuming that the slope is unity.

The contours of constant f_0 plotted on the right side of Figure 2 remain nearly vertical over a wider rigidity range as r decreases. Thus C will be near zero at low energies not only at 1 AU but for all $r < 1$ AU. The magnitude of the minimum value attained by C increases for $r > 1$ AU, but close to 1 AU will still be approximately zero. Thus a Compton-Getting coefficient that is essentially zero over an extended energy range is characteristic of the behavior of low energy cosmic rays throughout the inner solar system, in agreement with the observation that C is near zero at low energies during all levels of solar activity (Rygg, 1970; Rygg and Earl, 1971).

We can reduce the radial gradient still further, and as a result reduce the variation of C away from zero at low energies, by requiring that there is little cosmic ray scattering for a sizeable distance from the Sun to earth. This behavior is illustrated in Figure 3 where we have plotted the relevant quantities using a diffusion coefficient identical to that used in Figure 2 except now there is essentially no scattering ($\kappa \to \infty$) from 0 - 0.7 AU. As noted above, particles tend to arrive at energies below where $C = 0$ principally by losing energy as they are convected outward with the solar wind. The rate of energy loss, which depends inversely on r (see (16)), is fairly uniform in the region $r < 1$ AU, since there is only scattering for $0.7 \text{ AU} < r < 1$ AU. The radial gradient is thus smaller at low energies and C scarely distinguishable from zero. The radial gradient plotted in Figure 3 is in good agreement with the gradient observations of Webber (private communication) obtained recently from Pioneers 8 and 9, as well as with the theoretical predictions of Gleeson and Urch (1971).

Alpha particles

Finally, we note that the behavior of the alpha particle spectrum, which yields a $C > 0$ at low energies for solar minimum, but $C \simeq 0$ for solar maximum (Rygg, 1970; Rygg and Earl, 1971), is consistent with the above considerations. In the models presented here, the Compton-Getting Coefficient only attains a value near zero at relatively large values of Vr/κ. At a given energy per nucleon, the rigidity of an alpha particle is twice that of a proton, or κ is in general larger. Thus, for alpha particles at solar minimum Vr/κ may not be sufficiently large for C to be zero at the energy range observed by Rygg (1970) and Rygg and Earl (1971) (100-260 MeV/nucleon), even though it is sufficiently large for protons. Near solar maximum, Vr/κ at low energies apparently also attains the required large values for alpha particles.

Summary

We have shown in this paper that the behavior of the Compton-Getting Coefficient C with energy (or rigidity) is a useful indicator of how galactic cosmic rays are modulated in the interplanetary medium.

1) The behavior of the parameter can reveal the energy range over which the force-field approximation is valid (at energies down to where $|C(C-1)/2| << 1$, $C > 0$ and where the convection approximation holds (at energies below where C is a minimum).

2) At 1 AU, the observation of a negative (but small) value of C at low energies implies that, on the average, low energy particles have

penetrated into the region between the Sun and earth prior to observation.

3) An extended low energy range where C is near zero implies that the radial gradient at low energies is small. This small gradient may imply, in turn, that the diffusion coefficient increases beyond 1 AU less rapidly than proportional to heliocentric radial distance and/or there is essentially no scattering for a sizeable distance from the Sun to earth.

We have shown that it is useful to discuss the behavior of the Compton-Getting coefficient, and the various approximations that can be made to the modulation equations, in terms of the omni-directional distribution function f_o. Contours of constant f_o in the heliocentric distance vs. rigidity plane are useful for illustrating the rigidity (or energy) range over which C can be expected to be zero, as well as for discussing the mean rigidity loss experienced by particles in the interplanetary medium.

Appendix

We wish to show that contours of constant density are relatively insensitive to whether F(P) or PF(P) is used for the unmodulated distribution function, at least for those forms of F(P) normally assumed. Let $P''(r,P)$ define the contour of constant f_o corresponding to F(P), and $\bar{P}''(r,P)$, the contour corresponding to PF(P). On using (5), $P''(r,P)$ and $\bar{P}''(r,P)$ satisfy, respectively

$$\kappa \frac{d^2F}{dP^2} \left(\frac{\partial P''}{\partial r}\right)^2 + \left(\kappa \frac{\partial^2 P''}{\partial r^2} + \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\kappa) - V\right)\frac{\partial P''}{\partial r} + \frac{P}{3}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial P''}{\partial P}\right)\frac{dF}{dP} = 0 \quad \text{(A1)}$$

and

$$\kappa \left(\frac{d^2F}{dP^2} + \frac{2}{P}\frac{dF}{dt}\right)\left(\frac{\partial \bar{P}''}{\partial r}\right)^2 + \left(\kappa\frac{\partial^2 \bar{P}''}{\partial r^2} + \left(\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\kappa) - V\right)\frac{\partial \bar{P}''}{\partial r} + \frac{P}{3}\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 V)\frac{\partial \bar{P}''}{\partial P}\right)\left(\frac{dF}{dP} + \frac{F}{P}\right) = 0 \quad \text{(A2)}$$

It is a straight-forward matter to show that these equations will be approximately the same, i.e. $P''(r,P) \simeq \bar{P}''(r,P)$, provided that

$$\left| \left[\frac{2}{P}\left(\frac{dF}{dP}\right)^2 - \left(\frac{F}{P}\right)\left(\frac{d^2F}{dP^2}\right) \right] \Big/ \left[\frac{2}{P}\left(\frac{dF}{dP}\right)^2 + \left(\frac{dF}{dP}\right)\left(\frac{d^2F}{dP^2}\right) \right] \right| << 1 \quad \text{(A3)}$$

With $F \propto P^\gamma$, (A3) is satisfied provided that $|1/\gamma| << 1$. We noted above that $f_o = j_o/p^2$ (p is momentum; $P = pc/ze$), and that, typically, the unmodulated intensity spectrum is taken to be a power law in total energy with spectral index 2.65. Thus γ runs from $\gamma \simeq -4.65$ at large rigidities (corresponding to $T >> T_o$) to $\gamma \simeq -2$ at small rigidities. However, the contours that connect the boundary ($r = R$) with regions in the inner solar system ($r \leqslant 1$ AU) intersect the boundary at relatively large rigidities ($P \simeq 1$ BV, as can be seen in Figures 1 and 2). Thus, $3 \leqslant |\gamma| < 4.65$ and (A3) is roughly satisfied.

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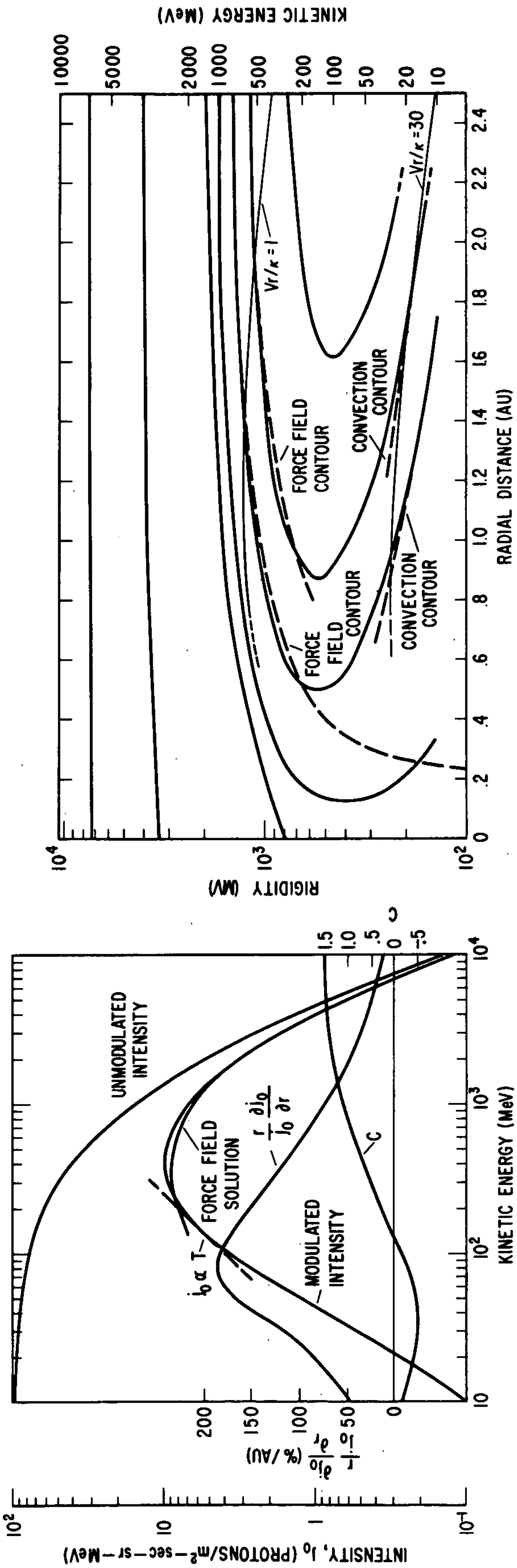
Figure 1. The unmodulated energy spectrum and modulated energy spectrum at 1 AU (left side), and contours of constant f_o in the P-r plane (right side). These quantities are determined from a numerical solution to (1) using $\kappa = 5\text{x}10^{17}\beta P\exp(r-1)\text{cm}^2/\text{sec}$ (P is in units of MV; r, in units of AU). Shown also on the left side is the behavior with energy at 1 AU of the corresponding force-field solution, Compton-Getting coefficient, and radial gradient. Shown also on the right side are the contours predicted by the force-field and by the convection approximation, and the contours along which Vr/κ equals 1 and 30.

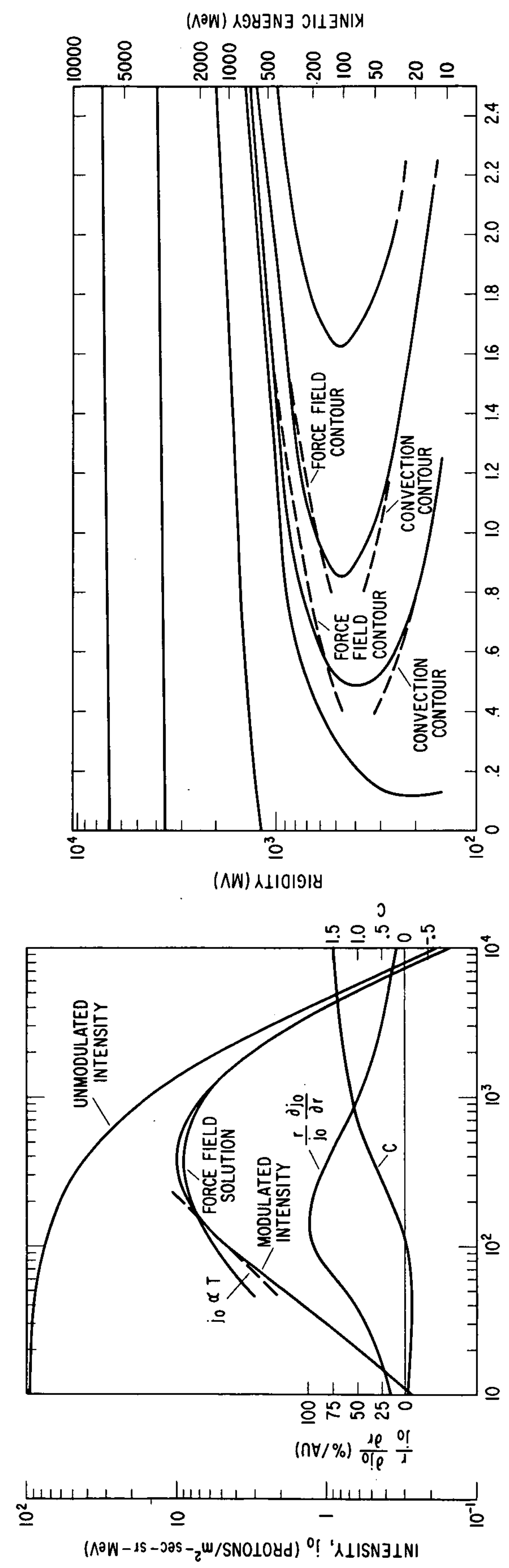
Figure 2. The unmodulated energy spectrum and modulated energy spectrum at 1 AU (left side), and contours of constant f_o in the P-r plane (right side). These quantities are determined from a numerical solution to (1) using $\kappa = 7.5\text{x}10^{17}\beta P\text{cm}^2/\text{sec}$ (P is in units of MV; r, in units of AU). Shown also on the left side is the behavior with energy at 1 AU of the corresponding force-field solution, Compton-Getting coefficient, and radial gradient. Shown also on the right side are the contours predicted by the force-field and by the convection approximation.

Figure 3. The unmodulated energy spectrum and modulated energy spectrum at 1 AU, and the behavior with energy at 1 AU of the corresponding Compton-Getting coefficient and radial gradient. These quantities are determined from a numerical solution to (1), using a value for κ identical to the one used for Figure 2, except that here there is essentially no scattering ($\kappa \rightarrow \infty$) from 0 - 0.7 AU.



INTENSITY, j_0 (PROTONS/m²–sec–sr–MeV)
$\frac{r}{j_0}\frac{\partial j_0}{\partial r}$ (%/AU)
UNMODULATED INTENSITY
FORCE FIELD SOLUTION
MODULATED INTENSITY
$j_0 \propto T$
C
KINETIC ENERGY (MeV)
RIGIDITY (MV)
RADIAL DISTANCE (AU)
KINETIC ENERGY (MeV)
FORCE FIELD CONTOUR
CONVECTION CONTOUR
FORCE FIELD CONTOUR
CONVECTION CONTOUR
$Vr/\kappa = 1$
$Vr/\kappa = 30$

Figure 1



INTENSITY, j_0 (PROTONS/m^2-sec-sr-MeV)
UNMODULATED INTENSITY
FORCE FIELD SOLUTION
MODULATED INTENSITY
$j_0 \propto T$
$\frac{r}{j_0}\frac{\partial j_0}{\partial r}$ (%/AU)
C
KINETIC ENERGY (MeV)
RIGIDITY (MV)
FORCE FIELD CONTOUR
CONVECTION CONTOUR
RADIAL DISTANCE (AU)

Figure 2

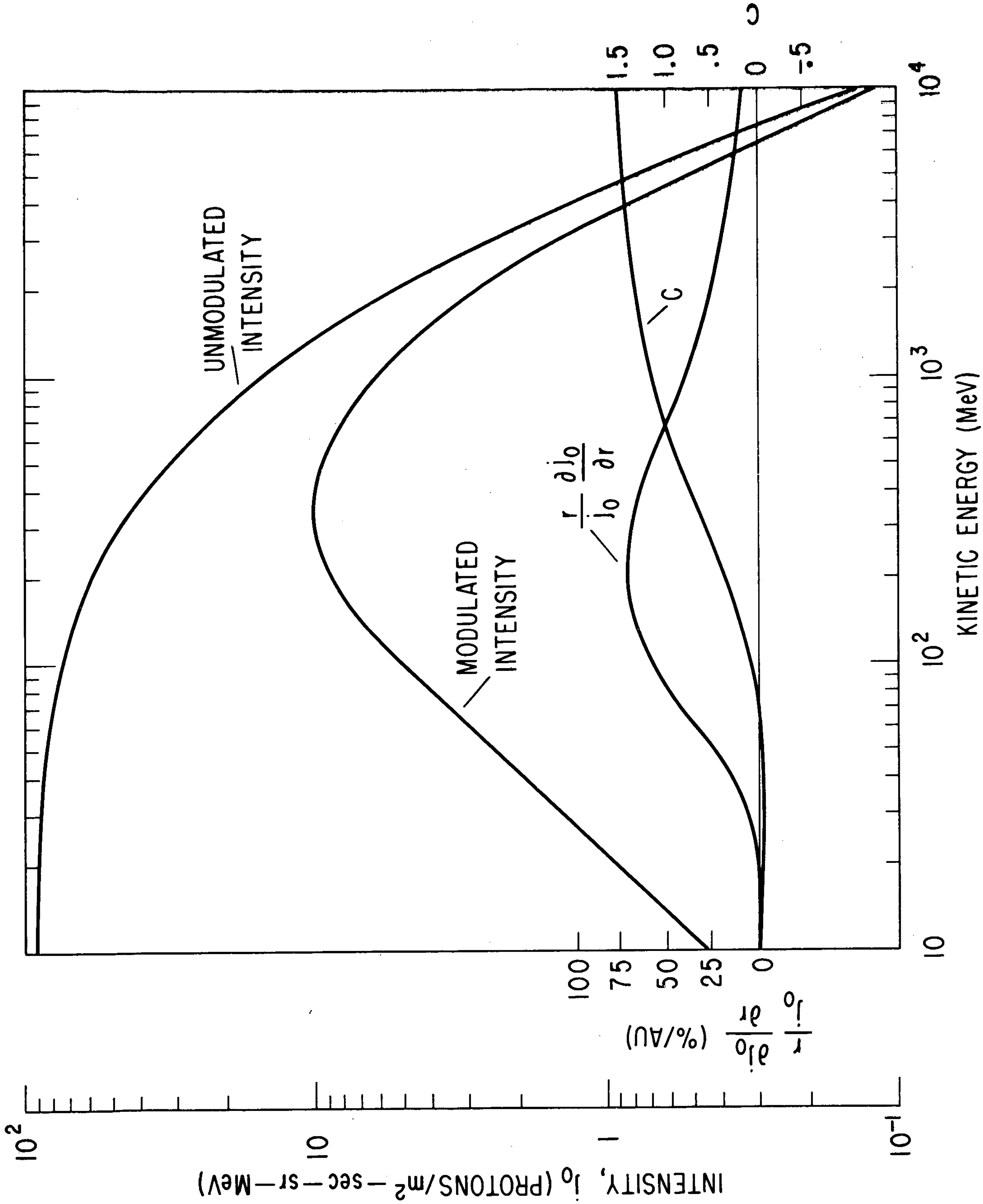

UNMODULATED INTENSITY
MODULATED INTENSITY
C
r/j0 ∂j0/∂r
KINETIC ENERGY (MeV)
10
10^2
10^3
10^4
INTENSITY, j0 (PROTONS/m^2 — sec — sr — MeV)
10^2
10
1
10^-1
r/j0 ∂j0/∂r (%/AU)
100
75
50
25
0
C
1.5
1.0
.5
0
-.5

Figure 3