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ORBIT SELECTION CONSIDERATIONS FOR EARTH OBSERVATORY SATELLITES

J. L. COOLEY

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ABSTRACT

The purpose of this paper is to display all the possible circular sun-synchronous orbits for earth observatory satellites having orbital altitudes between 740 and 1,115 kilometers (approximately 400 to 600 nautical miles) and having either a 16, 17, or 18 day repeat cycle for the ground trace. The requirement of having a circular, sun-synchronous orbit in the given altitude region with the given repeat cycle time defines all but the node of the orbits; the right ascension of the ascending node is determined by a desired sun elevation angle criteria.

It is found that there are 9 orbits with a 16 day repeat cycle, 17 orbits with a 17 day repeat cycle, and 7 orbits with an 18 day repeat cycle meeting the requirements. For each of these admissible orbits (one of which is the nominal ERTS orbit), various characteristics such as ideal ground trace patterns, swathing patterns, and daily drift are displayed. To aid in selection of the node, or local time of the orbit, the solar elevation angle along the orbit and the change in the solar elevation angle with season are given. This presentation of general orbital characteristics has application to finding approximate orbital elements and selecting orbits for many types of earth sensing satellite missions.

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INTRODUCTION

The problem of orbit selection for earth observatory satellites can be separated into two parts. First, after specifying a desired range of orbital altitudes and repeat cycle periods, a preliminary study would reveal the entire set of possible orbits and their general characteristics. The paper by King (reference 1) represents such a generalized investigation. Then, after specifying a desired orbit and deciding upon a desired node, the approximate orbital elements may be obtained. Secondly, the effect of all known dynamic perturbations (such as drag, gravitational perturbations, maneuvers, etc.) on the mission criteria must be analyzed so that the precise orbital parameters can be obtained. The paper by Fuchs and Strafella (reference 2) represents such a detailed investigation for the Earth Resources Technology Satellite (ERTS) project.

This paper addresses the first part of the orbit selection problem in detail for the particular specifications of the Earth Observatory Satellites (EOS). The two important criteria for the preliminary EOS orbit selection study are the range of orbital altitudes and the repeat cycle periods. The goal is to determine all circular, sun-synchronous orbits with

1. height above the Earth over the equator between 740 and 1,115 kilometers (approximately 400 to 600 nautical miles), and

2. repeat cycles of either 16, 17, or 18 days.

When all possible orbits have been found, then various characteristics of each, such as ground-trace patterns, daily drift, swathing patterns, etc., will be displayed. Finally some sun-illumination angle criteria, an important consideration in selection of the node, will be given.

Many of the calculations were done originally on a desk calculator. Many of the results were then checked and refined with a computer program for "Satellite Trajectory Analysis for Remote Sensing" (STARS). This computer program considers the trade offs between sensor resolution, sensor field of view, cycle times, planetary coverage, and the satellite orbit (see reference 3). Important general papers are given by references 4 and 5.

ADMISSIBLE CIRCULAR SUN-SYNCHRONOUS ORBITS

Several requirements will be made on the orbits to be generated. The first requirement is that only circular, sun-synchronous orbits be considered for approximate earth-sensing satellite orbits. The orbits for earth-resources missions should be nearly circular in order to keep a uniform distance between the earth's surface and the spacecraft instruments which are generally optical. Uniformity in distance from lens to object standardizes optical characteristics and aids interpretation of visual data. A constant angle of illumination at a given spacecraft latitude can also assist visual interpretations. To achieve this condition requires an orbital plane that rotates about the earth at the same angular rate as the mean angular velocity of the earth about the sun, known as a sun-synchronous orbit. The requirement that the orbit be circular specifies that the eccentricity of the orbit be nearly zero. The requirement that the orbit be sun-synchronous specifies the angle of inclination of the orbital plane with the equator for any given semi-major axis (computed for each admissible orbit in a later section).

With eccentricity specified and inclination (for any given semi-major axis) thus determined, and the argument of perigee and mean or true anomaly supplying no information about orbit shape, there are only two remaining free parameters to consider. These are 1) the semi-major axis of the orbit and 2) the right ascension of the ascending node. The node can be varied to meet local time or sun-illumination angle requirements; this will be considered in a later section. In controlling the subsatellite ground tracks and determining the set of admissible circular sun-synchronous orbits, the semi-major axis, or equivalently in this case the orbital altitude, is the key parameter. The orbital altitude specifies orbital period, which in turn governs the frequency of earth-surface coverage and the subsatellite track and pattern of coverage.

Two requirements will now be placed on the orbits which will serve to define the semi-major axis. First, an orbital altitude region of interest will be set between 740 kilometers (approximately 400 nm) and 1,115 kilometers (approximately 600 nm) in order to limit the semi-major axis. The altitude of the operational orbit to be employed by the initial Earth Resources Technology Satellite (ERTS) (reference 2) is nearly in the middle of this region. Second, it will be required that an admissible orbit have repeatability of its ground-trace over selected areas of the earth after 16, 17, or 18 days. Repeating the traces furnishes a predictable pattern of coverage and permits direct comparison of similar data taken at regular intervals. The operational orbit for ERTS has repeatability every 18 days. The orbital altitude requirement will limit the semi-major axis, and the repeatability requirement will provide specific semi-major axis values.

The problem is now set up. The goal of the next sections is to find all circular sun-synchronous orbits with

1. height above the Earth over the equator between 740 and 1,115 kilometers (approximately 400 to 600 nm) and

2. repeat cycles of either 16, 17, or 18 days.

Orbital Altitude Criteria

Consider circular, sun-synchronous orbits. Requiring the orbital altitude above the equator to be between 740 and 1,115 kilometers places bounds on the semi-major axis (a) of the orbits, such that

$$7118\,\text{km.} < a < 7494\,\text{km.}$$

This in turn places bounds on the period and mean motion (η) of the orbits. The maximum for the mean motion (η_{max}) and minimum for the mean motion (η_{min}) are:

$$\eta_{max} = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{398603.2}{(7118)^3}} = 5204.3 \text{ degrees/day}$$

$$\eta_{min} = \sqrt{\frac{\mu}{a^3}} = \sqrt{\frac{398603.2}{(7494)^3}} = 4817.6 \text{ degrees/day.}$$

This is equivalent to bounds on the number of revolutions per day, as follows

$$13.38 < \text{number of revolutions per day} < 14.46.$$

Thus the desired range of orbital altitudes translates into bounds on the number of revolutions of the earth per day.

Next consider the requirement of having repeat cycles of 16 days, 17 days, or 18 days. This requires an integer number of revolutions in the repeat cycle time (16, 17, or 18 days). Take a fraction with the numerator as the number of revolutions in the repeat cycle time and the denominator as the whole number of days for the repeat cycle. This fraction, representing the number of revolutions per day, must be within the bounds given above. Also this fraction must not be reducible; if the fraction is reducible then the repeat cycle is actually smaller. The possible number of revolutions for each repeat cycle (16, 17, or 18 days), and thus the admissible orbits, may be found from the bounds on the number of revolutions per day above by examining for irreducible fractions.

16 Day Repeat Cycle Criteria

It was found that the number of revolutions per day to satisfy the orbital altitude criteria must be between 13.38 and 14.46. Consider fractions with a denominator of 16:

$$214/16 < 13.38 \text{ revolutions per day} < 215/16$$

$$231/16 < 14.46 \text{ revolutions per day} < 232/16.$$

Then 214 revolutions in the 16 days is not admissible since that would mean less than 13.38 revolutions per day, or a higher satellite orbital altitude than allowable. Similarly 232 revolutions in the 16 days would mean a lower satellite orbital altitude than allowable. Thus,

$$215 \leq \text{number of revolutions in 16 days} \leq 231.$$

Now each fraction with a numerator between 215 and 231 and a denominator of 16 may be examined to see whether or not it is reducible. If reducible, then the repeat cycle is actually less than 16 days and the corresponding orbit not admissible.

Number of Revolutions in 16 Days	Fraction	Repeat Cycle
215	13-7/16	16 days
216	13-8/16 = 27/2	2 days
217	13-9/16	16 days
218	13-10/16 = 109/8	8 days
219	13-11/16	16 days
220	13-12/16 = 55/4	4 days
221	13-13/16	16 days
222	13-14/16 = 111/8	8 days
223	13-15/16	16 days
224	14 = 14/1	1 day
225	14-1/16	16 days
226	14-2/16 = 113/8	8 days
227	14-3/16	16 days
228	14-4/16 = 57/4	4 days
229	14-5/16	16 days
230	14-6/16 = 115/8	8 days
231	14-7/16	16 days

Thus there are 9 circular sun-synchronous orbits with orbital altitudes between 740 and 1,115 kilometers which will produce a 16 day repeat cycle.

17 Day Repeat Cycle Criteria

Consider fractions with a denominator of 17:

$$227/17 < 13.38 \text{ revolutions per day} < 228/17$$

$$245/17 < 14.46 \text{ revolutions per day} < 246/17.$$

Thus admissible orbits must have the characteristic that

$$228 \leq \text{number of revolutions in 17 days} \leq 245.$$

Examining each possible fraction with numerator between 228 and 245 and a denominator of 17 for reducibility gives:

Number of Revolutions in 17 Days	Fraction	Repeat Cycle
228	13-7/17	17 days
229	13-8/17	17 days
230	13-9/17	17 days
231	13-10/17	17 days
232	13-11/17	17 days
233	13-12/17	17 days
234	13-13/17	17 days
235	13-14/17	17 days
236	13-15/17	17 days
237	13-16/17	17 days
238	14 = 14/1	1 day
239	14-1/17	17 days
240	14-2/17	17 days
241	14-3/17	17 days
242	14-4/17	17 days
243	14-5/17	17 days
244	14-6/17	17 days
245	14-7/17	17 days

Thus there are 17 circular sun-synchronous orbits with orbital altitudes between 740 and 1,115 kilometers which will produce a 17 day repeat cycle.

18 Day Repeat Cycle Criteria

Consider fractions with a denominator of 18:

$$240/18 < 13.38 \text{ revolutions per day} < 241/18$$

$$260/18 < 14.46 \text{ revolutions per day} < 261/18.$$

Thus admissible orbits must have the characteristic that

$$241 < \text{number of revolutions in 18 days} < 260.$$

Examining each possible fraction with numerator between 241 and 260 and a denominator of 18 for reducibility gives:

Number of Revolutions in 18 Days	Fraction	Repeat Cycle
241	13-7/18	18 days
242	13-8/18 = 121/9	9 days
243	13-9/18 = 27/2	2 days
244	13-10/18 = 122/9	9 days
245	13-11/18	18 days
246	13-12/18 = 41/3	3 days
247	13-13/18	18 days
248	13-14/18 = 124/9	9 days
249	13-15/18 = 83/6	6 days
250	13-16/18 = 125/9	9 days
251	13-17/18	18 days
252	14 = 14/1	1 day
253	14-1/18	18 days
254	14-2/18 = 127/9	9 days
255	14-3/18 = 85/6	6 days
256	14-4/18 = 128/9	9 days
257	14-5/18	18 days
258	14-6/18 = 43/3	3 days
259	14-7/18	18 days
260	14-8/18 = 130/9	9 days

Thus there are 7 circular sun-synchronous orbits with orbital altitudes between 740 and 1,115 kilometers which will produce an 18 day repeat cycle.

Summary of 33 Admissible Orbits

The previous sections have found all circular sun-synchronous orbits with

1. height above the Earth over the equator between 740 and 1,115 kilometers (approximately 400 to 600 nm) and

2. repeat cycles of either 16, 17, or 18 days.

There are 33 orbits satisfying these requirements, 9 having a 16 day repeat cycle, 17 having a 17 day repeat cycle, and 7 having an 18 day repeat cycle.

The fraction associated with each orbit gives the number of revolutions per day. From this the orbital altitude, semi-major axis of the orbit, and the revolution period may be computed. The approximate inclination required for a sun-synchronous orbit, the inclination which produces a nodal regression rate equal to the mean angular rate of the earth-sun line, is computed from equation (18) in reference 1:

$$i = \cos^{-1}\left[\frac{2\dot{\Omega}}{3R^2J_2}\mu^{-1/2}a^{7/2}\right] \quad \text{or} \quad i = \cos^{-1}\left[1.50948\, a^{7/2}\right].$$

where a is the semi-major axis of the orbit. Table 1 gives the fraction, repeat cycle time, orbital height above the equator, revolution period, and orbital inclination for the 33 admissible orbits. The orbits are arranged in order of increasing orbital height. This also serves to arrange the orbits in terms of decreasing fraction magnitude, increasing revolution period, and increasing inclination.

It is seen that the admissible orbits are well distributed within the region, with the greatest gap in the Table occuring near the integer 14 number of orbits per day. There is a maximum gap of 38.6 kilometers in orbital height, 0.81 minutes in revolution period, and 0.17 degrees in orbital inclination. These 33 orbits may be enclosed inside a box on Figure 2 in the paper by King, reference 1, based on the cycle period of 16-18 days and orbital period between 99.75 and 107.56 minutes.

CHARACTERISTICS OF ADMISSIBLE ORBITS

Various characteristics of the 33 admissible orbits (9 for the 16 day repeat cycle, 17 for the 17 day repeat cycle, and 7 for the 18 repeat cycle) can now be

Table 1

33 Admissible Circular Sun-Synchronous Orbits
Arranged for Increasing Orbital Height

Fraction	Repeat Cycle in days	Orbital Height km (nm)	Revolution Period min.	Inclination degrees
14-7/16	16	745.1 (402)	99.75	98.38
14-7/17	17	753.6 (407)	99.93	98.41
14-7/18	18	761.2 (411)	100.08	98.44
14-6/17	17	773.1 (417)	100.34	98.50
14-5/16	16	786.6 (425)	100.62	98.55
14-5/17	17	792.7 (428)	100.75	98.58
14-5/18	18	798.2 (431)	100.86	98.60
14-4/17	17	812.5 (439)	101.16	98.66
14-3/16	16	828.6 (447)	101.51	98.73
14-3/17	17	832.4 (449)	101.58	98.75
14-2/17	17	852.4 (460)	102.01	98.83
14-1/16	16	871.3 (470)	102.41	98.91
14-1/17	17	872.6 (471)	102.43	98.92
14-1/18	18	873.7 (472)	102.46	98.92
13-17/18	18	912.3 (493)	103.27	99.09
13-16/17	17	913.3 (493)	103.30	99.10
13-15/16	16	914.6 (494)	103.33	99.10
13-15/17	17	933.9 (504)	103.74	99.19
13-14/17	17	954.7 (515)	104.18	99.28
13-13/16	16	958.6 (518)	104.26	99.30
13-13/17	17	975.5 (527)	104.62	99.37
13-13/18	18	990.7 (535)	104.95	99.44
13-12/17	17	996.6 (538)	105.07	99.47
13-11/16	16	1003.2 (542)	105.21	99.50
13-11/17	17	1017.8 (550)	105.53	99.57
13-11/18	18	1030.8 (557)	105.80	99.63
13-10/17	17	1039.1 (561)	105.98	99.66
13-9/16	16	1048.5 (566)	106.18	99.71
13-9/17	17	1060.6 (573)	106.44	99.76
13-8/17	17	1082.3 (584)	106.91	99.86
13-7/16	16	1094.5 (591)	107.17	99.92
13-7/17	17	1104.1 (596)	107.38	99.97
13-7/18	18	1112.6 (601)	107.56	100.01

investigated. Various features of the ground-trace patterns and swathing patterns produced will distinguish each orbit. The sensor characteristics and experiments will often require or favor certain ground-trace patterns.

Ground-Trace Patterns

One characteristic is the westward longitudinal displacement per revolution (ΔN). This is the number of degrees in longitude measured along the equator between 2 successive satellite revolutions (i.e., between 2 successive northbound equatorial crossings). Equivalently this can be converted to kilometers (1 degree = 60 nautical miles on the equator, or approximately 111 kilometers). This orbit to orbit westward motion also represents the nodal distance measured on the equator between 2 successive nodes on a given day. To get daily global coverage would require that an onboard sensor have such a width that this region be covered on each revolution. The nodal distance ΔN is computed from (reference 1):

$$\Delta N \text{ (degrees)} = \text{orbital period (hours)}/24 \times 360$$

or

$$\Delta N \text{ (degrees)} = 15 \times \text{orbital period in hours.}$$

The above nodal distance measures the distance between 2 successive revolutions. However it is the revolution on the following day, following close to the revolution 24 hours earlier, that defines the particular swathing pattern. If the orbital period is such that the satellite falls slightly short of completing an integer (such as 14) number of revolutions in 24 hours, then the subsatellite swaths will be displaced somewhat westward on each succeeding day. Conversely if the orbital period is such that the satellite completes an integer number of revolutions in less than 24 hours, then the satellite swaths will be displaced somewhat eastward on each succeeding day. This longitudinal displacement of the swaths eastward or westward each day is called the daily drift. The computation of the daily drift depends on the time excess of the integer number of orbit periods relative to 24 hours. Let "S" be the integer number of revolutions per day times the orbital period. Then the daily drift ($\delta\lambda$) is computed from (reference 1):

$$\delta\lambda \text{ (degrees)} = \text{S (hours)} -24/24 \times 360$$

or

$$\delta\lambda \text{ (degrees)} = 15 \times (S - 24).$$

It is the daily drift which determines swathing patterns. To get daily coverage to the nearest revolution on the previous day would require that an onboard sensor have such a width that the daily drift region be covered. This is needed in order to produce overlapping pictures in a swath on consecutive days.

Two different time intervals have been used in the above — namely 1) the orbital period between 2 consecutive revolutions and 2) 24 hours. The third important time increment is the repeat cycle time (16, 17, or 18 days). Consider a swath as the region between any 2 consecutive revolutions. On the following day, due to the eastward or westward daily drift, a revolution will cut through this swath. One revolution will cut through this swath per day until, at the end of the repeat cycle time, the first revolution is repeated. By following the daily drift through a swath the swathing pattern may be produced. The distance between 2 adjacent swaths, whether produced on consecutive days or not, is called the minimum gap, or the trace spacing. A sensor of this width would be needed to produce detailed pictures of a swath in the repeat cycle time. Other parameters of interest are related to the time; the number of days between the adjacent crossings, the number of days it takes to cover, or go through, a swath (revisit time) and the number of times the pattern crosses through a swath in the repeat cycle time.

These characteristics, for the 33 admissible orbits, are given in Tables 2, 3, and 4 for the 16 day, 17 day, and 18 day repeat cycle orbits respectively. These characteristics distinguish each of the 33 orbits.

Swathing Patterns

The swathing patterns produced by each of the 33 admissible orbits can be displayed on charts. In the repeat cycle time (16, 17, or 18 days) a certain pattern of equatorial crossings is set up to cover any region. This pattern is shown on Figures 1-3, where 0 represents the start, 1 represents the crossing point for the first day in the given swath, 2 the second day, etc. In this way the pattern may be viewed and the minimum time interval between adjacent swaths found. A distinction is made between westward daily drift (to the left) and eastward daily drift (to the right). The scale is also converted from degrees, as used in the previous tables, to kilometers and nautical miles (1 degree equals 60 nm on the equator) in order to compare with the field of view needed by the various sensors for certain coverage. For each pattern the associated N (the fraction, or number of orbits per day), ht (satellite height above the equator), and i (satellite inclination with respect to the equator) are given.

Table 2

Ground Trace Characteristics of the 16 Day Repeat Orbits

Fraction	Ht (km)	Orbit-to-Orbit Nodal Distance (deg)	Daily Drift (deg, east or west)	Minimum Gap (deg)	Minimum Time Between Adjacent Crossings (days)	Revisit Time or Time to go Through Swath (days)	Times Through Swath in 16 days
14-7/16	745	24.94	10.91 E	1.56	7	2-3	7
14-5/16	787	25.15	7.86 E	1.57	3	3-4	5
14-3/16	829	25.37	4.76 E	1.59	5	5-6	3
14-1/16	871	25.60	1.60 E	1.60	1	16	1
13-15/16	915	25.83	1.61 W	1.61	1	16	1
13-13/16	959	26.06	4.89 W	1.63	5	5-6	3
13-11/16	1003	26.30	8.22 W	1.64	3	3-4	5
13-9/16	1049	26.54	11.61 W	1.66	7	2-3	7
13-7/16	1095	26.79	11.72 E	1.67	7	2-3	7

Table 3

Ground Trace Characteristics of the 17 Day Repeat Orbits

Fraction	Ht (km)	Orbit-to-Orbit Nodal Distance (deg)	Daily Drift (deg, east or west)	Minimum Gap (deg)	Minimum Time Between Adjacent Swaths (days)	Revisit Time or Time to go Through Swath (days)	Times Through Swath in 17 days
14-7/17	754	24.98	10.28 E	1.47	5	2-3	7
14-6/17	773	25.08	8.85 E	1.48	3	2-3	6
14-5/17	793	25.19	7.41 E	1.48	7	3-4	5
14-4/17	813	25.29	5.95 E	1.49	4	4-5	4
14-3/17	832	25.39	4.48 E	1.49	6	5-6	3
14-2/17	852	25.50	3.00 E	1.50	8	8-9	2
14-1/17	873	25.61	1.51 E	1.51	1	17	1
13-16/17	913	25.82	1.52 W	1.52	1	17	1
13-15/17	934	25.93	3.05 W	1.53	8	8-9	2
13-14/17	955	26.04	4.60 W	1.53	6	5-6	3
13-13/17	976	26.15	6.15 W	1.54	4	4-5	4
13-12/17	997	26.27	7.73 W	1.55	7	3-4	5
13-11/17	1018	26.38	9.31 W	1.55	3	2-3	6
13-10/17	1039	26.49	10.91 W	1.56	5	2-3	7
13-9/17	1061	26.61	12.52 W	1.57	2	2-3	8
13-8/17	1082	26.73	12.58 E	1.57	2	2-3	8
13-7/17	1104	26.84	11.05 E	1.58	5	2-3	7

Table 4

Ground Trace Characteristics of the 18 Day Repeat Orbits

Fraction	Ht (km)	Orbit-to-Orbit Nodal Distance (deg)	Daily Drift (deg, east or west)	Minimum Gap (deg)	Minimum Time Between Adjacent Swaths (days)	Revisit Time or Time to go Through Swath (days)	Times Through Swath in 18 days
14-7/18	761	25.02	9.73 E	1.39	5	2-3	7
14-5/18	798	25.21	7.01 E	1.40	7	3-4	5
14-1/18	874	25.61	1.42 E	1.42	1	18	1
13-17/18	912	25.82	1.43 W	1.43	1	18	1
13-13/18	991	26.24	7.29 W	1.46	7	3-4	5
13-11/18	1031	26.45	10.28 W	1.47	5	2-3	7
13-7/18	1113	26.89	10.46 E	1.49	5	2-3	7

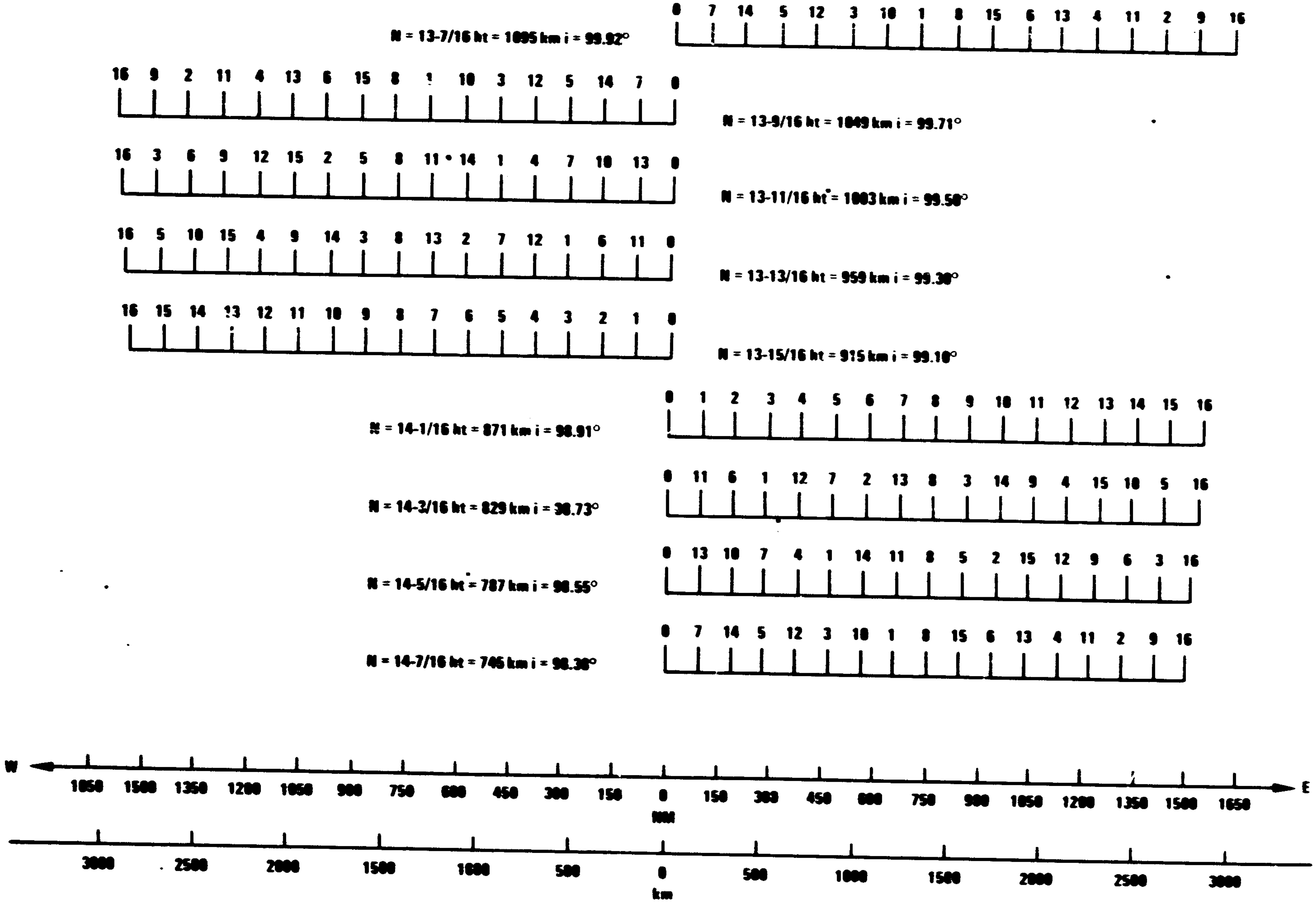

Figure 1. Swathing Patterns for 16 Day Repeat Cycle (9 Orbits)

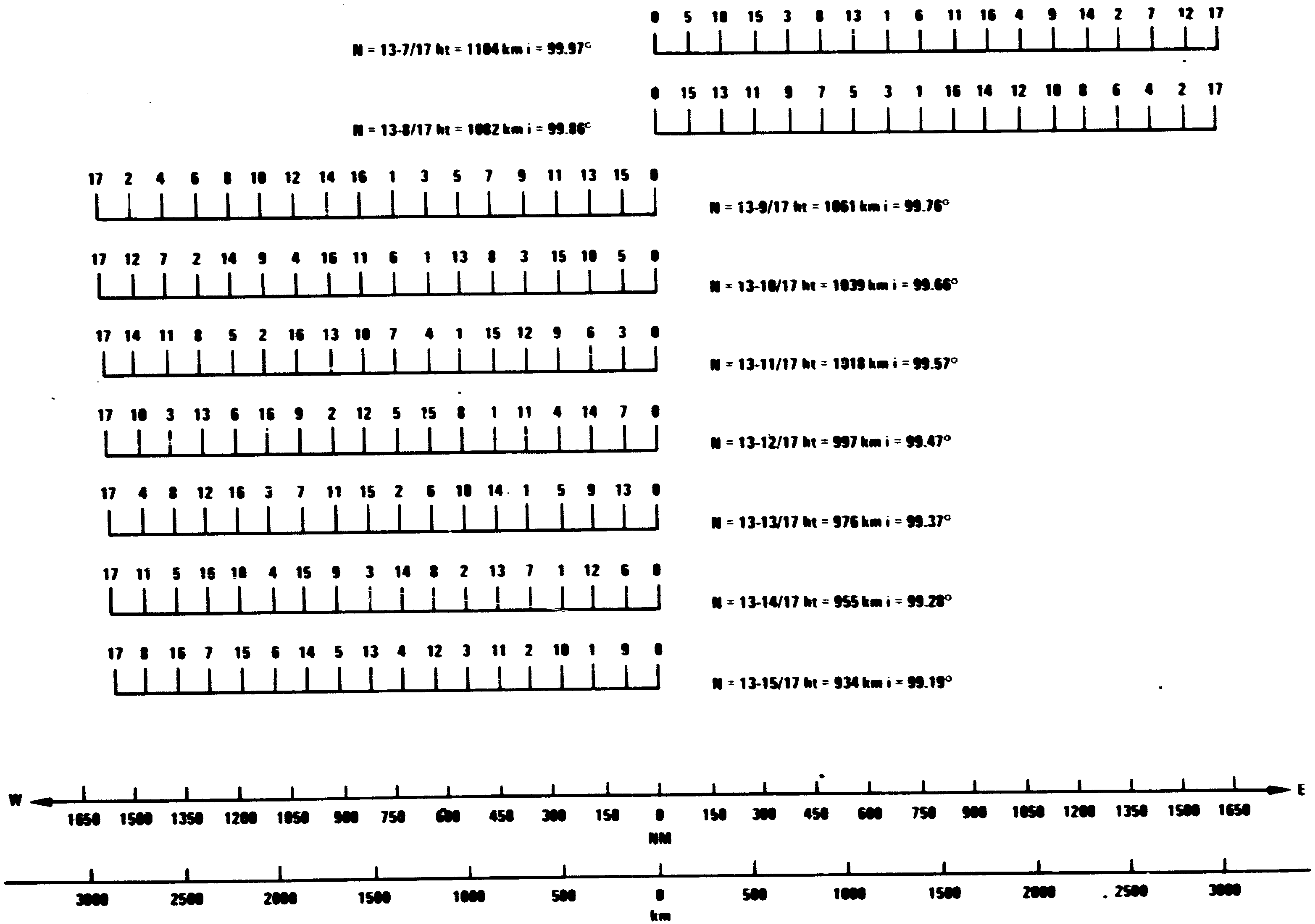

Figure 2A. Swathing Patterns for 17 Day Repeat Cycle (9 of the 17 Orbits)

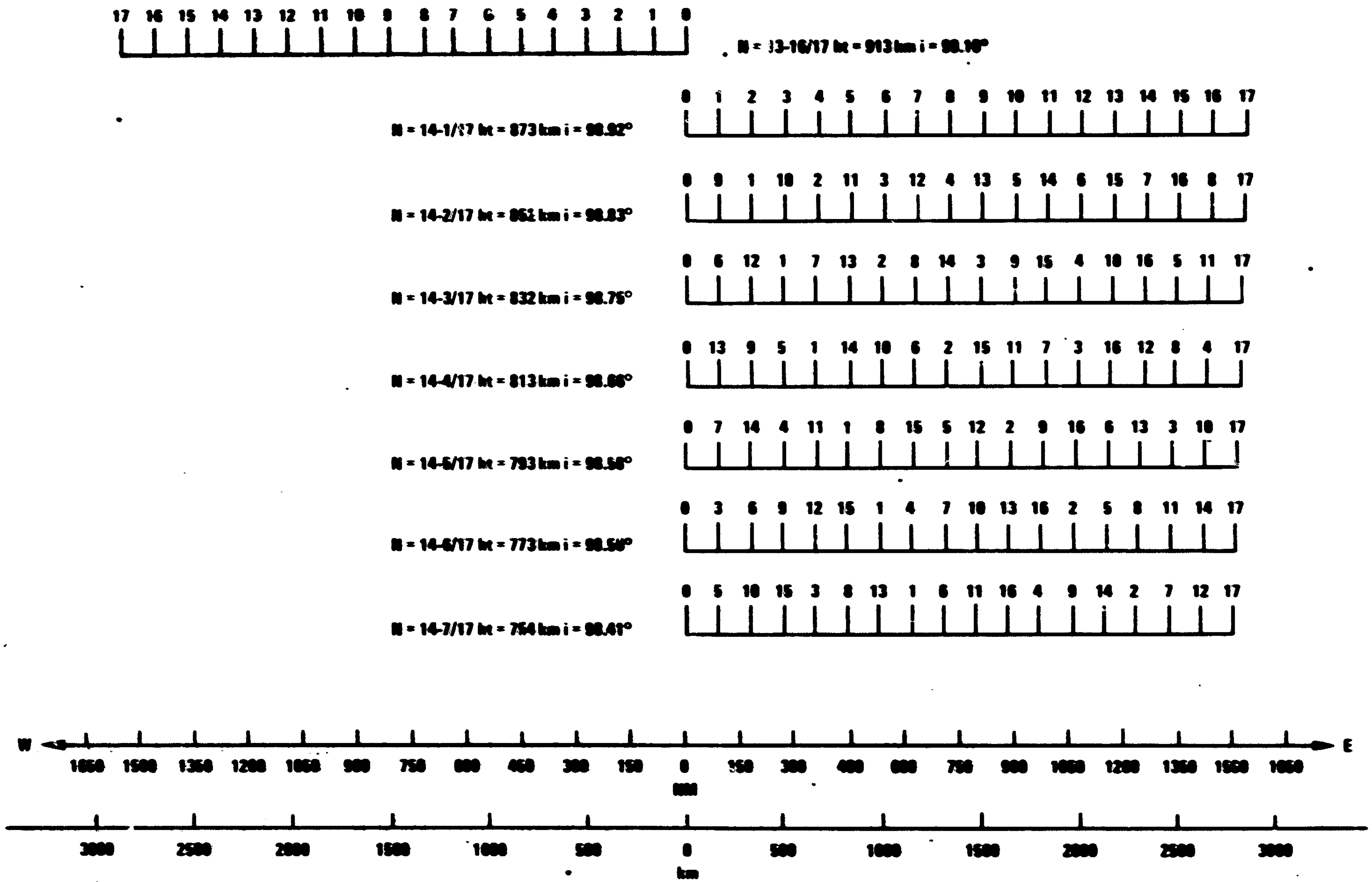

Figure 2B. Swathing Patterns for 17 Day Repeat Cycle (8 of the 17 Orbits)

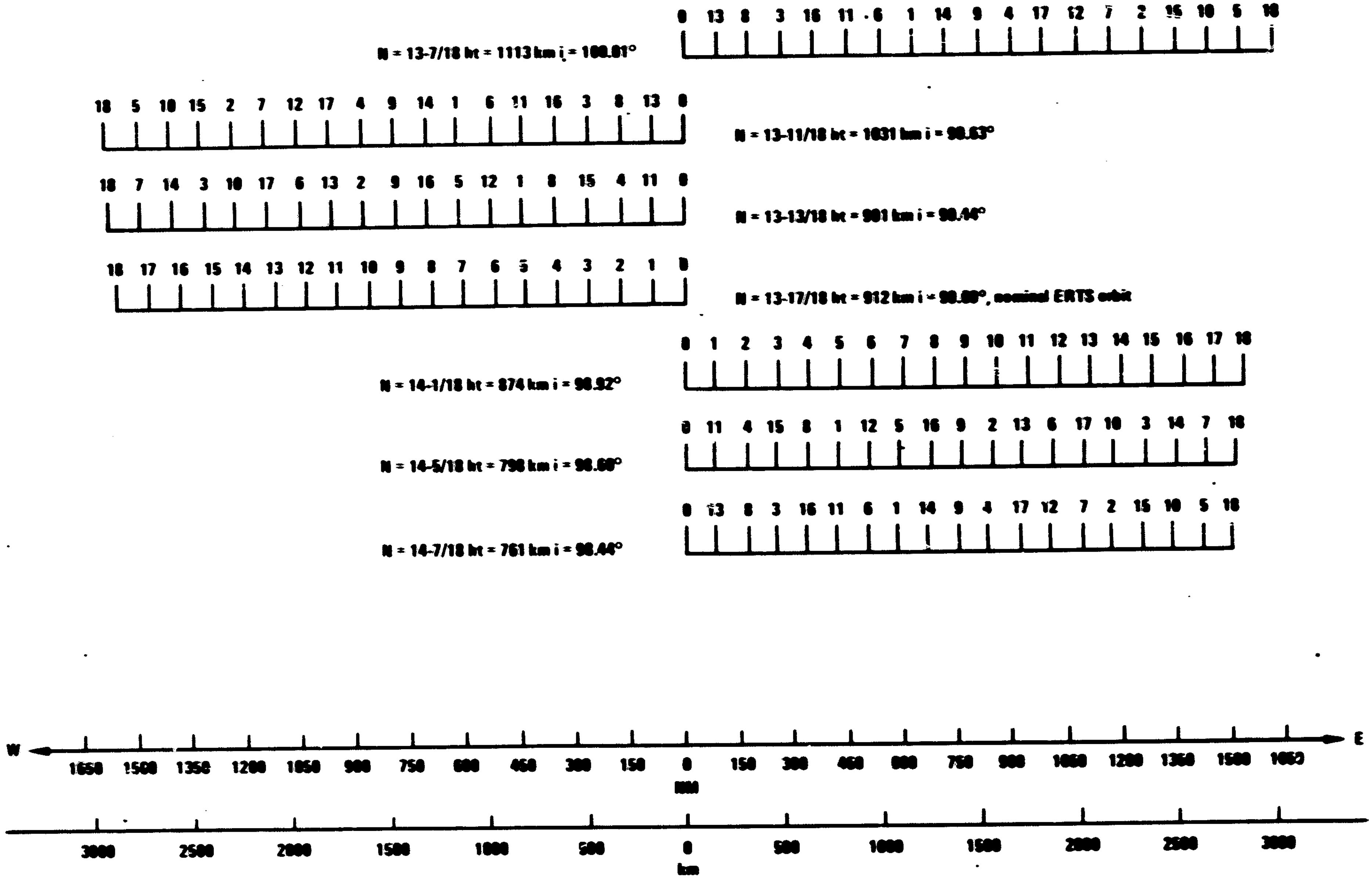

Figure 3. Swathing Patterns for 18 Day Repeat Cycle (7 Orbits)

Summary of Characteristics of Admissible Orbits

Orbital selection depends in part on sensor characteristics. Thus the information is next put in tabular form with an emphasis on the sensor width values. The following tables repeat the fraction N (or number of orbits per day), ht (satellite height above the equator), and i (satellite inclination required for a sun-synchronous orbit) for each admissible orbit. The nodal distance ΔN between 2 nodes on a given day, or the orbit-to-orbit westward motion, is given. A sensor of this ground width would be needed to cover each swath, or give global coverage, each day. The daily drift, $\delta\lambda$, gives the eastward or westward motion in a swath on two successive days. A sensor of this ground width would be needed to produce overlapping pictures in a swath on consecutive days. Finally the minimum gap is presented — this is the distance between 2 adjacent swaths. A sensor of this ground width would be needed to cover a swath in the repeat cycle time. Tables 5-7 present this information, which illustrate the 3 different sensor widths needed for 3 different types of ground coverage. Again, angular drifts are converted to kilometers and nautical miles.

SELECTION OF THE NODE

The previous section has outlined the characteristics of 33 admissible orbits, based on the consideration of the semi-major axis of the orbit as a free parameter. After the choice of any orbit from these 33 selections, there still remains the problem of specifying the remaining free parameter, the right ascension of the ascending node.

The node can be varied to meet local time or sun-illumination angle requirements. The node essentially gives the position of the orbit with respect to the sun. Since a sun-synchronous orbit has been specified, every equatorial ascending (or descending) crossing will occur at the same local time. The only variation then in the sun elevation angle (measured from the local tangent plane) for the ground-trace occurs due to the sun motion due to the seasons. The solar elevation angle along the orbit, both for ascending and descending nodes in daylight, the solar elevation angle for orbits with different nodes, and the change in solar elevation angle with season must be investigated.

Ascending Node and Descending Node

It must be specified whether the ascending node or the descending node should be in daylight. Which possibility gives the better sun elevation angles for the northern hemisphere? To find an answer, the ground-trace, with associated subsatellite solar elevation angles, was generated for a 9 AM orbit.

Table 5

Summary of Characteristics of 16 Day Repeat Orbits

N	Ht		INC	Nodal Distance ΔN		Daily Drift δλ		Minimum Gap	
(orbits/day)	km	(nm)	(degrees)	km	(nm)	km	(nm)	km	(nm)
13-7/16	1095	(591)	99.92	2977	(1607)	1302	(703)	186	(100)
13-9/16	1049	(566)	99.71	2950	(1593)	1290	(697)	184	(100)
13-11/16	1003	(542)	99.50	2923	(1578)	913	(493)	183	(99)
13-13/16	959	(518)	99.30	2896	(1564)	543	(293)	181	(98)
13-15/16	915	(494)	99.10	2870	(1550)	179	(97)	179	(97)
14-1/16	871	(470)	98.91	2845	(1536)	178	(96)	178	(96)
14-3/16	829	(447)	98.73	2820	(1522)	529	(285)	176	(95)
14-5/16	787	(425)	98.55	2795	(1509)	873	(472)	175	(94)
14-7/16	745	(402)	98.38	2771	(1496)	1212	(655)	173	(94)

Table 6

Summary of Characteristics of 17 Day Repeat Orbits

N	Ht		INC	Nodal Distance ΔN		Daily Drift δλ		Minimum Gap	
(orbits/day)	km	(nm)	(degrees)	km	(nm)	km	(nm)	km	(nm)
13-7/17	1104	(596)	99.97	2983	(1611)	1228	(663)	175	(95)
13-8/17	1082	(584)	99.86	2970	(1604)	1397	(755)	175	(94)
13-9/17	1061	(573)	99.76	2957	(1597)	1391	(751)	174	(94)
13-10/17	1039	(561)	99.66	2944	(1590)	1212	(655)	173	(94)
13-11/17	1018	(550)	99.57	2931	(1583)	1035	(559)	172	(93)
13-12/17	997	(538)	99.47	2919	(1576)	858	(464)	172	(93)
13-13/17	976	(527)	99.37	2906	(1569)	684	(369)	171	(92)
13-14/17	955	(515)	99.28	2894	(1563)	511	(276)	170	(92)
13-15/17	934	(504)	99.19	2882	(1556)	339	(183)	170	(92)
13-16/17	913	(493)	99.10	2869	(1549)	169	(91)	169	(91)
14-1/17	873	(471)	98.92	2845	(1536)	167	(90)	167	(90)
14-2/17	852	(460)	98.83	2834	(1530)	333	(180)	167	(90)

Table 6 (Continued)

N	Ht		INC	Nodal Distance ΔN		Daily Drift $\delta\lambda$		Minimum Gap	
(orbits/day)	km	(nm)	(degrees)	km	(nm)	km	(nm)	km	(nm)
14-3/17	832	(449)	98.75	2822	(1524)	498	(269)	166	(90)
14-4/17	813	(439)	98.66	2810	(1517)	661	(357)	165	(89)
14-5/17	793	(428)	98.58	2799	(1511)	823	(444)	165	(89)
14-6/17	773	(417)	98.50	2787	(1505)	984	(531)	164	(89)
14-7/17	754	(407)	98.41	2776	(1499)	1143	(617)	163	(88)

Table 7

Summary of Characteristics of 18 Day Repeat Orbits

N	Ht		INC	Nodal Distance ΔN		Daily Drift $\delta\lambda$		Minimum Gap	
(orbits/day)	km	(nm)	(degrees)	km	(nm)	km	(nm)	km	(nm)
13-7/18	1113	(601)	100.01	2988	(1613)	1162	(627)	166	(90)
13-11/18	1031	(557)	99.63	2939	(1587)	1143	(617)	163	(88)
13-13/18	991	(535)	99.44	2915	(1574)	810	(437)	162	(87)
13-17/18	912	(493)	99.09	2869	(1549)	159	(86)	159	(86)
14-1/18	874	(472)	98.92	2846	(1537)	158	(85)	158	(85)
14-5/18	798	(431)	98.60	2802	(1513)	778	(420)	156	(84)
14-7/18	761	(411)	98.44	2780	(1501)	1081	(584)	154	(83)

In Figure 4 the descending node is in daylight with the local time being 9 AM at the equator crossings. Because of the inclination, the daylight portion of the orbit proceeds from north to south in a westerly direction. On the first complete orbit shown, when the latitude is 70° N, the subsatellite longitude is 40° W. At the final point shown (40 minutes later), when the latitude is 70° S, the subsatellite longitude is 100° W. Thus, for a pass the subsatellite point of the orbit goes through about 60 degrees in longitude. At the same time, the sun in 40 minutes passes through 10 degrees in longitude (nearly 15 degrees/hour). This difference in sublongitude movement between the orbit and the sun accounts for different solar elevation angles along the orbit in the 2 hemispheres.

The orbit shown in Figure 4 is a 9 AM orbit — at each equator crossing the local time is 9 AM. Thus at each equator crossing the sun is east of the orbit by 45 degrees (3 hours). For example, for the equator crossing at 70° W, the sun is at 25° W (a 45 degree difference). Now back up the orbit by 20 minutes; the longitude of the orbit is then 40° W and of the sun is 20° W (only a 20 degree difference). Going forward (south) 20 minutes, the longitude of the orbit is 100° W and of the sun 30° W (a 70 degree difference). Thus it follows that the sun elevation angle at a northern latitude under the orbit will be greater than at a southern latitude under the orbit. For example it is seen that points on the orbits near 70° N latitude have about a 19 degree sun elevation angle (on a March 22 date), while points on the orbits near 70° S latitude have about a 6 degree solar elevation angle. The difference then in solar elevation angle at the same latitude north or south is upwards of 13 degrees. Or, the same sun elevation angle occurs north or south at a difference in latitude of about 15 degrees (i.e., a 38 degree sun elevation angle occurs about 20° S latitude and 35° N latitude).

Figure 5 shows sun elevation angles for the same 9 AM orbit with ascending node (south to north equator crossing) in daylight. In this case the better solar elevation angles occur in the southern hemisphere, since the orbit goes from south to north in a westwardly direction. Thus, of the 2 possibilities the descending node in daylight favors the northern hemisphere for morning 9 AM equator crossings. It is noted here that the nominal ERTS orbit has a morning descending node in daylight.

Now consider the different equator crossing times. Increasing the time from 9 AM toward noon decreases the eastward longitude separation of the orbit from the sun. This lessens the differences in sun elevation angle between the hemispheres. At noon the hemispheres are treated equally (except for seasonal variations). In a PM orbit, the sun is westward of the orbit and the situation reverses; the descending node in daylight then favors the southern hemisphere, and the ascending node in daylight the northern hemisphere. The 3 PM orbit is similar to the 9 AM orbit (at the vernal or autumnal equinox) with the solar elevation angles for the hemispheres reversed.

In conclusion,

1. for a morning AM orbit, the descending node in daylight gives the better solar elevation angles in the northern hemisphere under the ground-trace.

2. for an afternoon PM orbit, the ascending node in daylight gives the more favorable solar elevation angles along the ground-trace in the northern hemisphere.

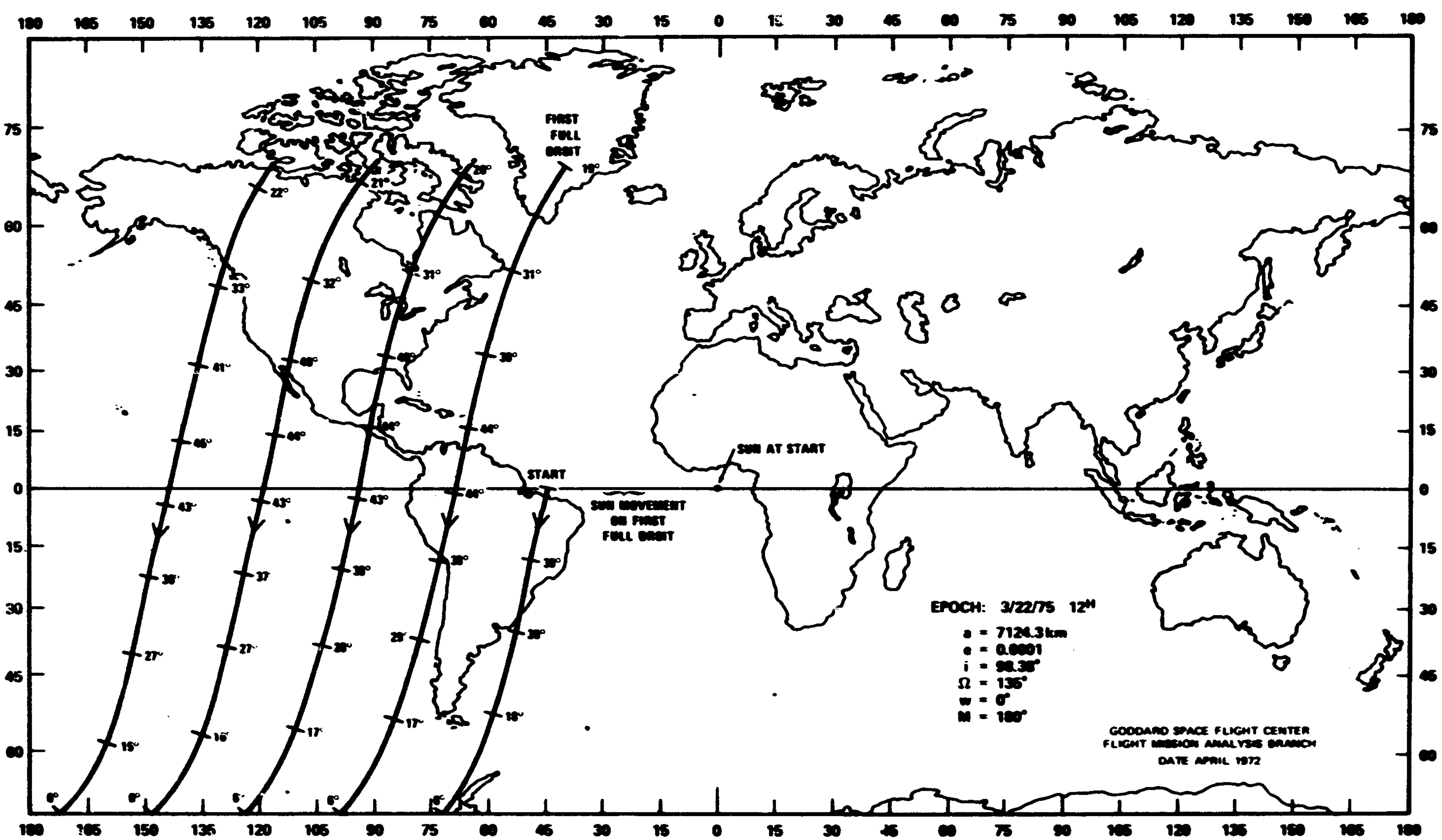

Figure 4. Solar Elevation Angles Along 9 AM Descending Node Orbits

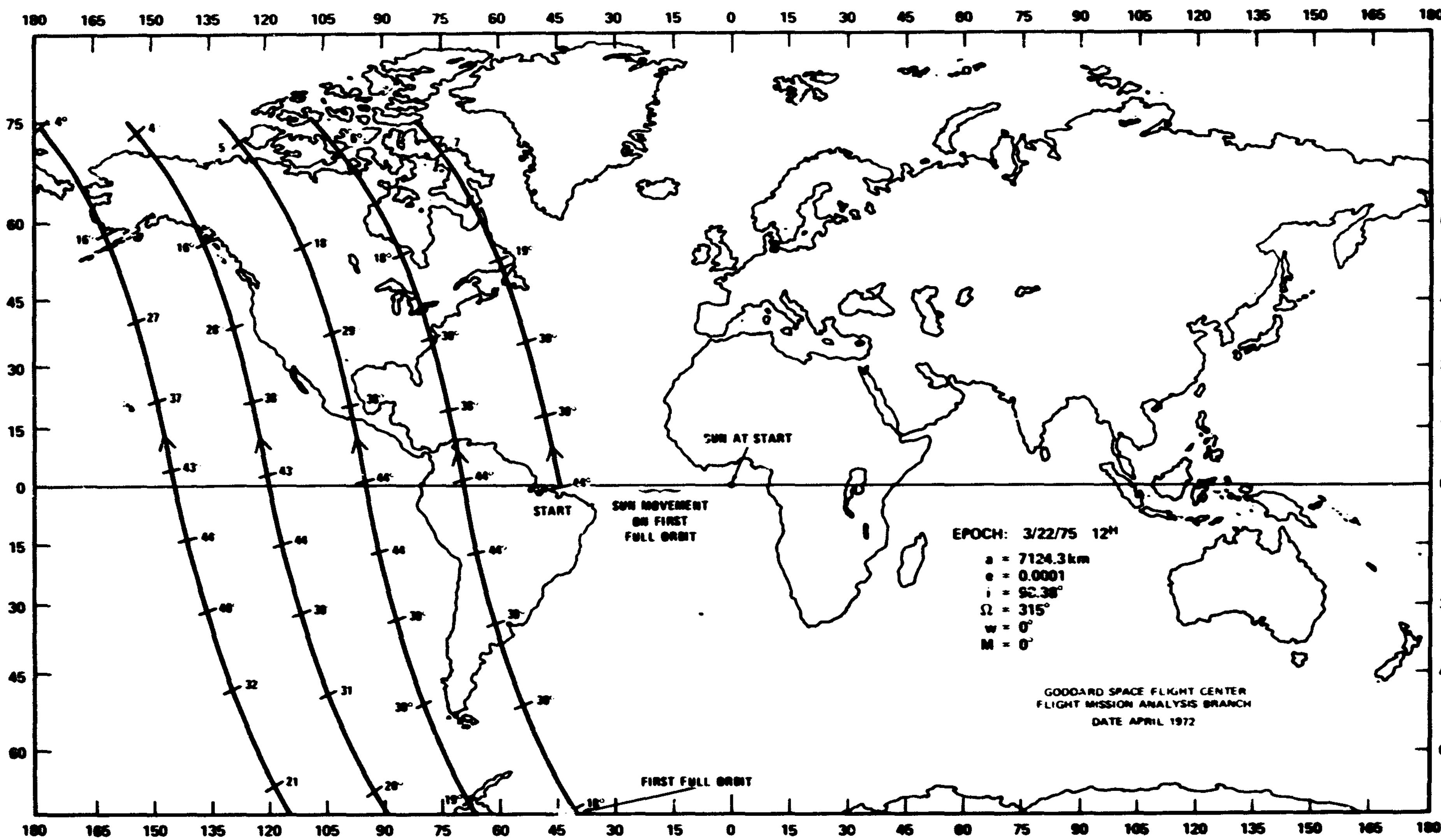

Figure 5. Solar Elevation Angles Along 9 AM Ascending Node Orbits

Computation of Initial Node

Often certain sun lighting conditions are desired so that the sensors will have a given sun elevation angle at a given latitude at a given time. Such conditions will define the value of the ascending node.

Given a desired subsatellite latitude L, orbit inclination i, and sun-local vertical-spacecraft angle η (which is 90 degrees minus the sun elevation angle), the initial longitude of the ascending node, Ωo, required at the vernal equinox can be found using spherical trigonometry:

$$\Omega o = \sin^{-1} [-\tan L \cot i] - \cos^{-1} [\cos\eta / \cos L].$$

For example, the initial longitude of the ascending node can be found so that the sun appears 30 degrees above the horizon ($\eta = 60°$) at 50 degrees north latitude (L = 50 degrees) at the vernal equinox for the 13-13/17 sun-synchronous trajectory (i = 99.37 degrees). Then,

$$\Omega o = \sin^{-1} (0.19665) - \cos^{-1} (0.77786),$$

$$\Omega o = \begin{Bmatrix} 11.341 \\ 168.659 \end{Bmatrix} - \begin{Bmatrix} 38.935 \\ 321.065 \end{Bmatrix},$$

or

$$\Omega o = \begin{Bmatrix} -152.406° \\ -\ 27.594° \\ +\ 50.276° \\ +129.724° \end{Bmatrix}.$$

Translating this into local time gives,

$$\Omega o = \begin{cases} \text{1:50 PM for a descending node} \\ \text{10:10 AM for an ascending node} \\ \text{3:21 PM for an ascending node} \\ \text{8:39 AM for a descending node} \end{cases}$$

This formula may be used for any combination of L, η, and i.

For each of the 33 admissible orbits, there are 4 combinations of node and local time of equator crossing which will satisfy any requirement for specific sun lighting conditions — a) ascending node in daylight, morning orbit, b) descending node morning orbit, c) ascending node afternoon orbit, d) descending node afternoon orbit. The sun's position is east of the orbit's equator crossing in the morning and west in the afternoon. The ascending node in daylight means a south to north equator crossing with the sublatitude points in the northern hemisphere being west from those in the southern hemisphere. The descending node in daylight means a north to south equator crossing with the sublatitude points in the southern hemisphere being west from those in the northern hemisphere. The relationship of the sun to the subsatellite points determines the solar elevation angles and thus the node required to meet certain conditions.

Variation of Sun Elevation Angle With Season

For the 13-13/17 sun-synchronous orbit, with the node adjustod to provide a 30 degree sun elevation angle over the 50 degree north sub-latitude point at the vernal equinox, the sun elevation angle is found at each equinox and solstice to give an idea of the variation of the sun elevation angle with season.

The 13-13/17 orbit has semi-major axis 7354 kilometers, eccentricity 0.0001 argument of perigee 0.0, and inclination 99.37 degrees. For morning orbits, there are 8 combinaticns of longitude of ascending node and mean anomaly which give the elements at each equinox and solstice. For the ascending node in daylight, a 10:10 AM equator crossing achieves a 30 degree sun elevation angle at 50 degrees north at the vernal equinox, while for the descending node in daylight an 8:39 AM equator crossing achieves the same requirement. The 8 combinations of node and mean anomaly are then as follows:

Date(1)	Longitude of Ascending Node	Mean Anomaly
March 21 12^H	332.4°	0.0 (ascending node)
June 21 12^H	62.4°	0.0 (ascending node)
Sept. 21 12^H	152.4°	0.0 (ascending node)
Dec. 21 12^H	242.4°	0.0 (ascending node)
March 21 12^H	129.7°	180.0 (descending node)
June 21 12^H	219.7°	180.0 (descending node)
Sept. 21 12^H	309.7°	180.0 (descending node)
Dec. 21 12^H	39.7°	180.0 (descending node)

(1) These are approximate dates for the equinox and solstices, which vary yearly. In 1972 the dates are (references 6 and 7)

Spring equinox	March 20 12^H 22^M
Summer solstice	June 21 7^H 6^M
Autumn equinox	Sept. 22 22^H 33^M
Winter solstice	Dec. 21 18^H 13^M

Figure 6 gives the elevation angle of the sun at the sub-latitude point as a function of the latitude for the above 8 cases.

The sun elevation angle requirement determines the crossing point of the ascending and descending node orbits on March 21; at latitude 50 degrees north the sun elevation angle is restricted to be 30 degrees. Thereafter the ascending and descending orbits cross the 50 degree northern latitude point at the same time and thus have the same sun elevation angle. However, this angle varies with the season between about 10 degrees and 50 degrees.

The morning ascending node orbit (10:10 AM) has the greater eastward position and thus the higher sun elevation angles for all latitudes below 50 degrees north. It is especially noticeable when the sun is in the southern latitudes, December 21, that the sun elevation angle in the southern hemisphere for the ascending node morning orbit substantially increases. The descending node morning orbit (8:39 AM) has the higher sun elevation angles above 50 degrees north latitude, and also shows an increase in the sun elevation angle for the northern hemisphere when the sun is northward, June 21. For ascending nodes, the sun being in the southern hemisphere means that the sun is closer in longitude to the trajectory. A sun position of 23 degrees south latitude means that the sun is about 10 degrees closer in longitude to the trajectory than when the sun is at 23 degrees north latitude — this accounts for the nonsymmetry in the ascending node cases between the June 21 and December 21 dates. The same situation exists for the AM descending and PM ascending orbits and for the AM ascending and PM descending orbits.

Figure 7 gives the sun elevation angles for a noon orbit. Here, the sun elevation angles at a given latitude on a given date are the same for the ascending node and descending node orbits. The sun elevation angle is a maximum at about 23-1/2 degrees north on the summer solstice, and at about 23-1/2 degrees south on the winter solstice.

CONCLUSIONS

There are 33 circular, sun-synchronous orbits having orbital altitudes between 740 and 1,115 kilometers (approximately 400 to 600 nautical miles) and having either a 16, 17, or 18 day repeat cycle for the ground-trace. Of the 33 orbits, 9 orbits have a 16 day repeat cycle, 17 orbits have a 17 day repeat cycle, and 7 orbits have an 18 day repeat cycle. These orbits are well distributed within the altitude region of interest and have revolution periods between 99.7 and 107.6 minutes and orbital inclinations between 98.3 degrees and 100.1 degrees.

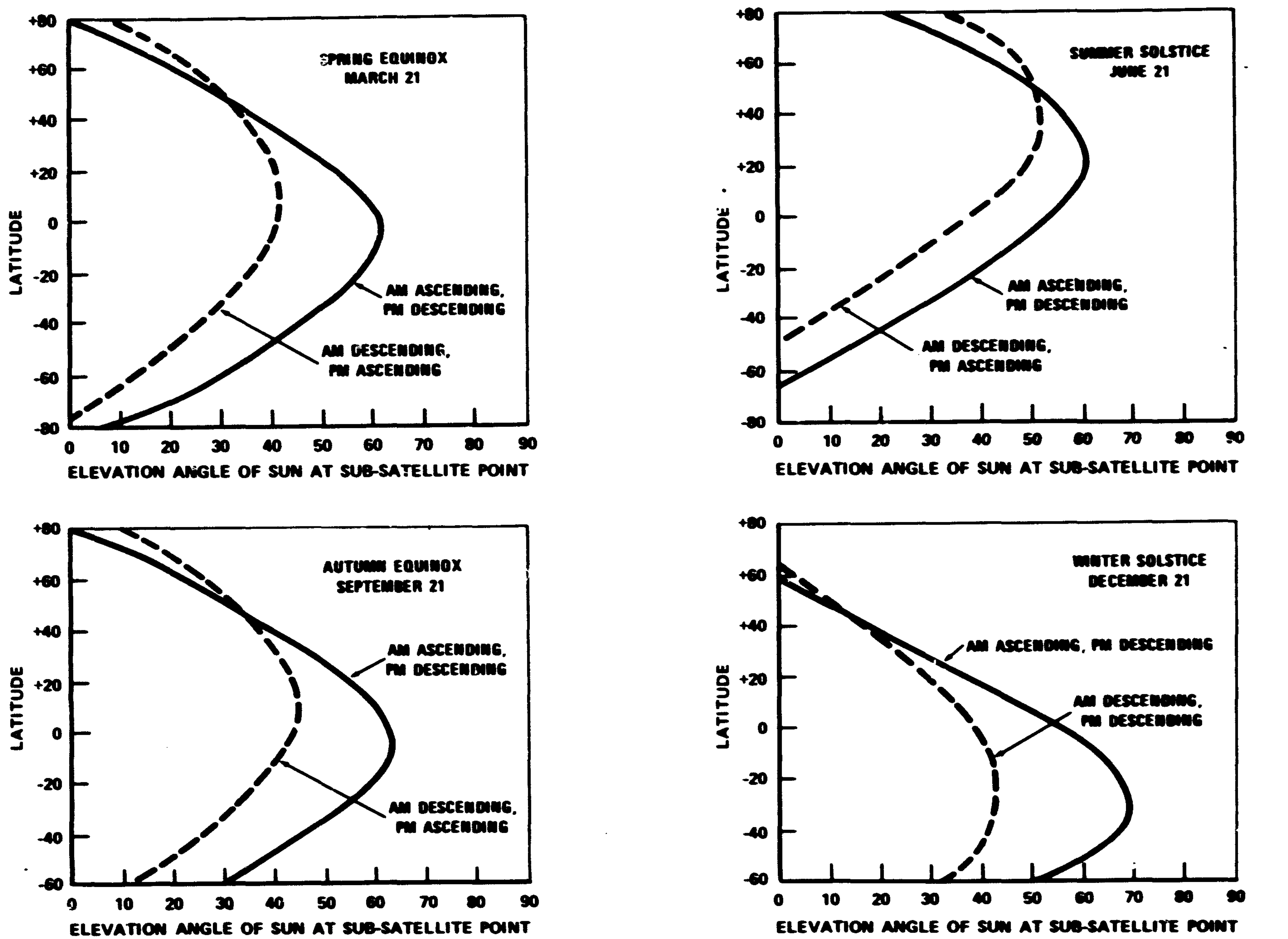

Figure 6. Sun Elevation Angles, Variation With Season for AM and PM Orbits

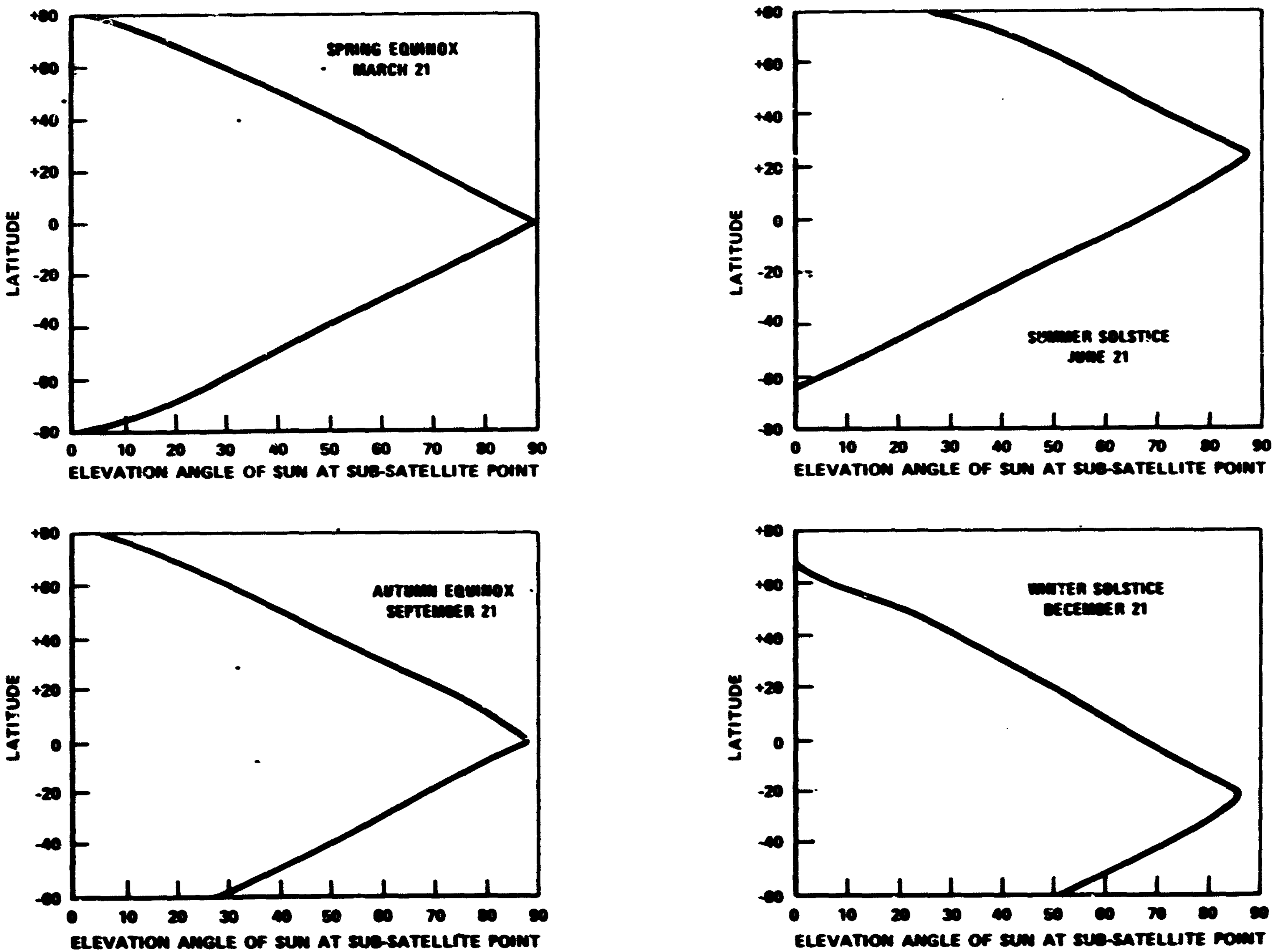

Figure 7. Sun Elevation Angles, Variation With Season for Noon Orbits

Various features of the ground-trace patterns and swathing patterns distinguish each of the 33 orbits. The orbits vary between minimum drift patterns (such as the ERTS nominal orbit) where the traces are adjacent and a swath is covered only once in the repeat cycle time, to patterns which go through a swath 8 times in the repeat cycle time. The nodal distance, daily drift, and minimum gap characteristics of an orbit can be compared with sensor characteristics for orbit selection.

Once an orbit is selected, the only remaining parameter to determine is the node. Selection of a node, or equivalently a local time of equator crossing or the solar position with respect to the orbit, depends upon the illumination and subsatellite solar elevation angles that optimize the use of the sensors. Variation of sun elevation angle with season is given.

Information is presented to determine approximate orbital elements (semimajor axis, eccentricity, inclination, node, argument of perigee, and mean anomaly) for any of 33 earth observatory satellite missions having altitudes between 740 and 1,115 kilometers and having either a 16, 17, or 18 day repeat cycle. The next step will be an analysis of dynamic perturbations in order to refine these approximate orbital elements and obtain precise orbital parameters for any orbital mission selected.

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