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CALCULATION OF LINEARIZED SUPERSONIC FLOW OVER SLENDER CONES OF ARBITRARY CROSS SECTION

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16. Abstract

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SUMMARY

Supersonic linearized conical-flow theory is used to determine the flow over slender pointed cones having horizontal and vertical planes of symmetry. The geometry of the cone cross sections and surface velocities are expanded in Fourier series. The symmetry condition permits the uncoupling of lifting and nonlifting solutions. The present method reduces to Ward's theory for flow over a cone of elliptic cross section. Results are also presented for other shapes. Results by this method diverge for cross-sectional shapes where the maximum thickness is large compared with the minimum thickness. However, even for these slender-body shapes, lower order solutions are good approximations to the complete solution.

INTRODUCTION

The solution to supersonic flow over a cone of arbitrary cross section has been treated extensively in the literature. References 1, 2, and 3, for example, treat this problem with nearly exact inviscid formulations, which admit the existence of shocks and vortical singularities induced by large crossflows. The present solution to linearized supersonic flow over a cone of arbitrary cross section can be superimposed to obtain the flow over a body which changes shape longitudinally. An example of this approach is the classical solution for the body of revolution obtained by the superposition of circular-cone solutions. (See ref. 4.) The superposition approach could possibly be applied to the solution of the flow over wings, fuselages, and wing-body combinations.

A general theory for the solution of slender bodies as well as cones was given by Ward in reference 5. Ward's theory indicates that for slender bodies, the velocity potential satisfies Laplace's equation in the cross-plane coordinates; and thus, the methods of classical hydrodynamics can be used to obtain solutions. Ward's theory has been used in detail only to obtain the flow over an elliptic body (refs. 6 and 7), since the transformation from ellipse to circle by the Joukowski transformation is well known.

The purpose of this paper is to present a method for determining the flow over a cone of arbitrary cross section based on the assumptions of supersonic linearized theory.

The method of solution will not depend upon incompressible cross flow (slender-body theory); therefore, results by complete linearized theory can be obtained. The method uses the conical-flow solutions to the wave equation first classified in reference 8. These solutions individually have the property of vanishing at the Mach cone. The cone geometry and velocities on the surface are expanded in a Fourier series to satisfy the surface boundary condition.

The validity of this method is demonstrated by comparing the present results with those from Ward's theory for the elliptic cone. Results from the present method are shown to diverge when the maximum thickness is large compared with the minimum thickness. Results are also presented for two cones of arbitrary cross section with slender-body approximations. Finally, the present method is applied to the elliptic cone without slender-body approximations, and results are compared with an extension of Ward's theory presented in reference 9.

The method of this paper is presently restricted to cones with horizontal as well as vertical planes of symmetry. With this restriction, the lifting and nonlifting solutions can be treated separately and superimposed to obtain a complete solution.

SYMBOLS

A_k kth coefficient of body-geometry expansion

a,b cone cross-section parameters (see fig. 1)

$B(x,r,\theta)$ function describing body surface

C_n nth coefficient in a series of superimposed potential solutions

C_p pressure coefficient

G body-geometry function, $G = \sum\limits_{k=0,1,2}^{k=\infty} A_k \cos(k\theta)$

M free-stream Mach number

m number of terms required to approximate body geometry

N total number of superimposed solutions

n individual solution to be superimposed

P_k kth coefficient of boundary equation

$\bar{q}$ velocity vector in flow field

$R_{k,n}$ kth coefficient of expansion, $\cos n\theta \, G^n$

$S_{k,n}$ kth coefficient of expansion, $\sin n\theta \, G^n$

U   free-stream velocity

X,Y,Z   body-axis system

x, r, θ cylindrical coordinate system (see fig. 1)

α angle of attack, deg

$\beta = \sqrt{m^2 - 1}$

ϕ perturbation velocity potential

Subscripts:

k,m,n   integer indices

METHOD OF SOLUTION

Governing Equations

A uniform supersonic stream of Mach number M flowing over a cone at an angle of attack α with respect to the X-axis (fig. 1) is considered. Under the restrictions of inviscid flow and small velocity perturbations, the flow field can be described in terms of the perturbation velocity potential ϕ which obeys the equation

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} - \beta^2 \frac{\partial^2 \phi}{\partial x^2} = 0$$

The general solution for the flow over a pointed body is obtained from reference 10, equation (18), as

$$\phi(x,r,\theta) = -\frac{1}{2\pi}\sum_{n=0,1,2}^{n=\infty} \cos n\theta \int_0^{x-\beta r} \frac{F_n(\xi)\cosh\left(n\cosh^{-1}\frac{x-\xi}{\beta r}\right)d\xi}{\sqrt{(x-\xi)^2 - \beta^2 r^2}}$$

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where $F_n(\xi)$ are functions to be determined by the surface boundary condition. If $\xi = x - \beta r \cosh z$, then $d\xi = 1 - \beta r \sinh z\, dz$ and since properties along a ray are constant for conical flow,

$$F_n(\xi) = F_n \xi = F_n(x - \beta r \cosh z)$$

Then the general solution for a cone becomes

$$\phi(x,r,\theta) = \frac{1}{2\pi} \sum_{n=0,1,2}^{n=\infty} F_n \cos n\theta \int_{\cosh^{-1}\frac{x}{\beta r}}^{0} (x - \beta r \cosh z)\cosh nz\, dz \qquad (1)$$

The integrated forms of equation (1) were first given individually in reference 8 as solutions to conical flow having the property that the perturbation potential vanishes at the Mach cone. These perturbation velocity components are

$$\frac{\partial \phi}{\partial x} = \frac{1}{2\pi} \sum_{n=0,1,2}^{n=\infty} F_n \cos n\theta \int_{\cosh^{-1}\frac{x}{\beta r}}^{0} \cosh nz\, dz \qquad (2a)$$

$$\frac{\partial \phi}{\partial r} = -\frac{\beta}{2\pi} \sum_{n=0,1,2}^{n=\infty} F_n \cos n\theta \int_{\cosh^{-1}\frac{x}{\beta r}}^{0} \cosh z \cosh nz\, dz \qquad (2b)$$

and

$$\frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{1}{2\pi r} \sum_{n=0,1,2}^{n=\infty} F_n n \sin n\theta \int_{\cosh^{-1}\frac{x}{\beta r}}^{0} (x - \beta r \cosh z)\cosh nz\, dz \qquad (2c)$$

Integrating equations (2) yields

$$\frac{\partial \phi}{\partial x} = -C_0 z - C_1 \cos \theta \sinh z - \sum_{n=2,3,4}^{n=\infty} C_n \cos n\theta \frac{\sinh nz}{n} \qquad (3a)$$

$$\frac{\partial \phi}{\partial r} = C_0 \beta \sinh z + \frac{\beta}{2} C_1 \cos \theta \left[(\cosh z \sinh z) + z \right] + \beta \sum_{n=2,3,4}^{n=\infty} C_n \cos n\theta \left[\frac{\sinh(n+1)z}{2(n+1)} + \frac{\sinh(n-1)z}{2(n-1)} \right] \qquad (3b)$$

and

$$\frac{1}{r}\frac{\partial \phi}{\partial \theta} = \frac{\beta}{2} C_1 \sin \theta \left[(\cosh z \sinh z) - z \right] - \beta \sum_{n=2,3,4}^{n=\infty} n C_n \sin n\theta \left[\frac{\sinh(n+1)z}{2(n+1)} + \frac{\sinh(n-1)z}{2(n-1)} - \frac{\cosh z \sinh nz}{n} \right] \qquad (3c)$$

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where

$$C_n = \frac{F_n}{2\pi}$$

and

$$z = \cosh^{-1} \frac{x}{\beta r}$$

With the appropriate boundary condition, the term $n = 0$ is the required linearized theory solution for the nonlifting circular cone and the term $n = 1$ is the solution for the lifting circular cone (ref. 4, pp. 215 and 223). The infinite series represents a general solution to a cone with an arbitrary cross section. It is expected that as the cross-sectional shape approaches a circle, a lesser number of solutions will be required.

If the cone is slender, that is,

$$\frac{x}{\beta r} \gg 1$$

then

$$z = \cosh^{-1} \frac{x}{\beta r} \approx \ln \frac{2x}{\beta r}$$

$$\sinh z = \sqrt{\frac{x^2}{\beta^2 r^2} - 1} \approx \frac{x}{\beta r}$$

and

$$\cosh z = \frac{x}{\beta r}$$

With the aid of hypergeometric identities, equations (3) become

$$\frac{\partial \phi}{\partial x} = -C_0 \ln \frac{2x}{\beta r} - \sum_{n=1,2,3}^{n=\infty} \frac{n+1}{n} C_n \cos n\theta \left(\frac{x}{r}\right)^n \tag{4a}$$

$$\frac{\partial \phi}{\partial r} = C_0\left(\frac{x}{r}\right) + C_1 \cos \theta \left(\frac{x}{r}\right)^2 + \sum_{n=2,3,4}^{n=\infty} C_n \cos n\theta \left(\frac{x}{r}\right)^{n+1} \tag{4b}$$

and

$$\frac{1}{r} \frac{\partial \phi}{\partial \theta} = C_1 \sin \theta \left(\frac{x}{r}\right)^2 + \sum_{n=2,3,4}^{n=\infty} C_n \sin n\theta \left(\frac{x}{r}\right)^{n+1} \tag{4c}$$

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In agreement with the slender-body theory of Ward (ref. 5), the velocity components $\frac{\partial \phi}{\partial r}$ and $\frac{1}{r}\frac{\partial \phi}{\partial \theta}$ satisfy Laplace's equation:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial \theta} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} = 0$$

Surface Boundary Condition

Since any body shape can be described as $B(x,r,\theta) = 0$, the surface boundary condition for steady flow is given by $\bar{q} \cdot \nabla B = 0$ where $\bar{q}$ is the velocity vector. Combining these expressions gives

$$\left(U \cos \alpha + \frac{\partial \phi}{\partial x}\right)\frac{\partial B}{\partial x} + \left(U \sin \alpha \cos \theta + \frac{\partial \phi}{\partial r}\right)\frac{\partial B}{\partial r}$$

$$+ \left(-U \sin \alpha \sin \theta + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\right)\frac{1}{r}\frac{\partial B}{\partial \theta} = 0$$

where all quantities are evaluated at the body surface. For flow over a cone

$$B = r - \frac{x}{G} = 0$$

and the boundary condition becomes with rearranging

$$U \cos \alpha + \frac{\partial \phi}{\partial x} = \left(U \sin \alpha \cos \theta + \frac{\partial \phi}{\partial r}\right)G$$

$$+ \left(-U \sin \alpha \sin \theta + \frac{1}{r}\frac{\partial \phi}{\partial \theta}\right)\frac{\partial G}{\partial \theta}$$

With the assumptions that α is small and $U \cos \alpha \gg \frac{\partial \phi}{\partial x}$ and with the perturbation velocities normalized by U,

$$1 - \alpha\left(\cos \theta G - \sin \theta \frac{\partial G}{\partial \theta}\right) = G\frac{\partial \phi}{\partial r} + \frac{\partial G}{\partial \theta}\frac{1}{r}\frac{\partial \phi}{\partial \theta} \tag{5}$$

If G is expressed by

$$G = \frac{x}{r} = \sum_{k=0,1,2}^{k=\infty} A_k \cos k\theta$$

the velocity components (eqs. (4)) can be expanded in Fourier series and the boundary condition can be put in the form

$$\sum_{k=0,1,2}^{k=\infty} P_k \cos k\theta = 0$$

Since $\cos 0\theta$, $\cos \theta$, $\cos 2\theta$, . . ., $\cos k\theta$ are linearly independent, setting $P_0 = 0$, $P_1 = 0$, . . ., $P_k = 0$ will give k linear equations for the constants C_0, C_1, C_2, . . ., C_k.

If

$$G = \sum_{k=0,2,4}^{k=0} A_k \cos k\theta \tag{6}$$

then the cone cross section is horizontally as well as vertically symmetric. The n-even velocity contributions contain only even cosine terms, and the n-odd velocity contributions contain only odd cosine terms. Rewriting the boundary condition (eq. (5)) gives

$$\underbrace{1}_{\text{Constant}} - \underbrace{\alpha\left(\cos\theta\, G - \sin\theta\, \frac{\partial G}{\partial\theta}\right)}_{\text{Odd cosine series}} = \underbrace{\left(G\,\frac{\partial\phi}{\partial r}\right)_{\text{even}} + \left(\frac{\partial G}{\partial\theta}\,\frac{1}{r}\,\frac{\partial\phi}{\partial\theta}\right)_{\text{even}}}_{\text{Even cosine series}} + \underbrace{\left(G\,\frac{\partial\phi}{\partial r}\right)_{\text{odd}} + \left(\frac{\partial G}{\partial\theta}\,\frac{1}{r}\,\frac{\partial\phi}{\partial\theta}\right)_{\text{odd}}}_{\text{Odd cosine series}}$$

Therefore, with the assumption of equation (6), the n-even solutions are nonlifting solutions and the n-odd solutions are lifting solutions, and the two problems can be solved separately and independently. The nonlifting solutions obtained from equations (4) are

$$\frac{\partial\phi}{\partial x} = -C_0 \ln\frac{2x}{\beta r} - \sum_{n=2,4,6}^{n=\infty} \frac{n+1}{n}\, C_n \cos n\theta\, G^n \tag{7a}$$

$$\frac{\partial\phi}{\partial r} = \sum_{n=0,2,4}^{n=\infty} C_n \cos n\theta\, G^{n+1} \tag{7b}$$

and

$$\frac{1}{r}\,\frac{\partial\phi}{\partial\theta} = \sum_{n=0,2,4}^{n=\infty} C_n \sin n\theta\, G^{n+1} \tag{7c}$$

where the boundary condition from equation (5) is

$$1 = G\,\frac{\partial\phi}{\partial r} + \frac{\partial G}{\partial\theta}\,\frac{1}{r}\,\frac{\partial\phi}{\partial\theta} \tag{7d}$$

The lifting solutions obtained from equations (4) are

$$\frac{\partial\phi}{\partial x} = -\sum_{n=1,3,5}^{n=\infty} \frac{n+1}{n}\cos n\theta\, G^n \tag{8a}$$

$$\frac{\partial \phi}{\partial r} = \sum_{n=1,3,5}^{n=\infty} C_n \cos n\theta\, G^{n+1} \tag{8b}$$

and

$$\frac{1}{r} \frac{\partial \phi}{\partial \theta} = \sum_{n=1,3,5}^{n=\infty} C_n \sin n\theta\, G^{n+1} \tag{8c}$$

where the boundary condition from equation (5) is

$$-\alpha \left(\cos\theta\, G - \sin\theta\, \frac{\partial G}{\partial \theta} \right) = G \frac{\partial \phi}{\partial r} + \frac{\partial G}{\partial \theta} \frac{1}{r} \frac{\partial \phi}{\partial \theta}$$

The ability to split the boundary condition into two parts also holds for equations (3) and has previously been demonstrated only for the circular cone in reference 11, page 241. With slender-body assumptions, the nonlifting solution contains a Mach number variation only in the first term of the $\partial \phi / \partial x$ expression, which does not enter into the solution of the boundary condition. The lifting solution is entirely independent of Mach number.

Expansion Procedure

There remains the necessity of stating the procedure by which the infinite series are truncated and a finite solution obtained. Results of the expansion must approach those obtained by Ward's theory as n approaches infinity. Since the expansion procedures for the lifting and nonlifting solutions are similar, only the procedure for the non-lifting solution will be given.

(1) Expand G in the form $\displaystyle\sum_{k=0,2,4}^{k=m} A_k \cos k\theta$

(2) Choose $n = 0$, $n = 2$, $n = 4$, . . ., $n = N$ number of terms and expand the velocity equations (eqs. (7a) to (7c)) in the form

$$\frac{\partial \phi}{\partial x} = -C_0 \ln \frac{2G}{\beta} - \sum_{n=2,4,6}^{n=\infty} \frac{n+1}{n} C_n \cos n\theta\, G^n = -\sum_{n=0,2,4}^{n=N} \frac{n+1}{n} C_n \sum_{k=0,2,4}^{k=N} R_{k,n} \cos k\theta$$

$$\frac{1}{G} \frac{\partial \phi}{\partial r} = \sum_{n=0,2,4}^{n=\infty} C_n \cos n\theta\, G^n = \sum_{n=0,2,4}^{n=N} C_n \sum_{k=0,2,4}^{k=N} R_{k,n} \cos k\theta$$

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and

$$\frac{1}{Gr}\frac{\partial\phi}{\partial\theta} = \sum_{n=0,2,4}^{n=\infty} C_n \sin n\theta \, G^n = \sum_{n=0,2,4}^{n=N} C_n \sum_{k=0,2,4}^{k=N} S_{k,n} \sin k\theta$$

Although the function $\cos n\theta \, G^n$ is completely defined by a series to $n(m + 1)$, it is truncated at N.

(3) Solve for the boundary condition in the form

$$1 = G^2\left(\frac{1}{G}\frac{\partial\phi}{\partial r}\right) + \frac{1}{2}\frac{\partial}{\partial\theta}(G^2)\left(\frac{1}{Gr}\frac{\partial\phi}{\partial\theta}\right)$$

(4) Solve for pressure coefficients by

$$C_p = -2\frac{\partial\phi}{\partial x} - G^2\left[\left(\frac{1}{G}\frac{\partial\phi}{\partial r}\right)^2 + \left(\frac{1}{Gr}\frac{\partial\phi}{\partial\theta}\right)^2\right]$$

This expansion procedure is best illustrated by presenting, in detail, the solutions for $N = 0$ and $N = 2$. These solutions are presented in the section entitled "Nonlifting Solutions."

RESULTS AND DISCUSSION

The expansion procedures described previously must converge to well-known solutions based on the same assumptions. Therefore, a comparison of results from this theoretical method for the elliptic cone with those from well-known solutions is an important test.

Nonlifting Solutions

Elliptic cone.- As an illustration of the expansion procedure, the lowest order solution ($N = 0$) and the next to lowest order solution ($N = 2$) will be presented in detail for the elliptic cone. The geometry function G for the elliptic cone is

$$G = \frac{x}{r} = \sqrt{A_0 + A_2 \cos 2\theta}$$

where

$$A_0 = \frac{a^2 + b^2}{2a^2b^2}, \qquad A_2 = \frac{b^2 - a^2}{2a^2b^2}$$

and a and b are the semimajor and semiminor axis, respectively. For $N = 0$ the velocity components obtained from equations (7a) to (7c) are

$$\frac{\partial\phi}{\partial x} = -C_0 \ln\frac{2G}{\beta} = -C_0 \ln\frac{a + b}{\beta ab} + \dots$$

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$$\frac{1}{G}\frac{\partial\phi}{\partial r} = C_0 + \ldots$$

and

$$\frac{1}{Gr}\frac{\partial\phi}{\partial\theta} = 0 + \ldots$$

The boundary condition is

$$1 = G^2\left(\frac{1}{G}\frac{\partial\phi}{\partial r}\right) + \frac{1}{2}\frac{\partial}{\partial\theta}\left(G^2\right)\left(\frac{1}{Gr}\frac{\partial\phi}{\partial\theta}\right) \qquad (9)$$

By substituting in velocity components and geometry, this equation becomes

$$1 = \left(A_0 + A_2\cos 2\theta\right)C_0$$

or

$$1 - A_0 C_0 - C_0 A_2 \cos 2\theta = 0$$

and the boundary condition is now in the form

$$\sum_{k=0,1,2}^{k=N} P_k \cos k\theta = 0$$

Therefore,

$$P_0 = 1 - A_0 C_0 = 0$$

or

$$C_0 = \frac{1}{A_0} = \frac{2a^2 b^2}{a^2 + b^2}$$

and the velocity components are

$$\frac{\partial\phi}{\partial x} = -\frac{2a^2 b^2}{a^2 + b^2}\ln\frac{a+b}{\beta ab}$$

$$\frac{1}{G}\frac{\partial\phi}{\partial r} = \frac{2a^2 b^2}{a^2 + b^2}$$

and

$$\frac{1}{Gr}\frac{\partial\phi}{\partial\theta} = 0$$

The pressure coefficient then becomes

$$C_p = -2\frac{\partial\phi}{\partial x} - G^2\left[\left(\frac{1}{G}\frac{\partial\phi}{\partial r}\right)^2 + \left(\frac{1}{Gr}\frac{\partial\phi}{\partial\theta}\right)^2\right] \qquad (10)$$

or

$$C_p = \frac{4a^2b^2}{a^2 + b^2}\left(\ln \frac{a + b}{\beta ab} - 1\right) + \frac{2a^2b^2}{a^2 + b^2}\left(1 + \frac{a^2 - b^2}{a^2 + b^2}\cos 2\theta\right) \tag{11}$$

For   a = b   (slender circular cone),

$$C_p = 2a^2\left(\ln \frac{2}{\beta a} - \frac{1}{2}\right) \tag{12}$$

Equation (12) agrees with that presented in reference 11, page 234. Ward's slender-body theory as given by Van Dyke in reference 9 is

$$C_p = ab\left[2 \ln \frac{4}{\beta(a + b)} - 2 + \frac{ab}{a^2\sin^2\eta + b^2\cos^2\eta}\right] \tag{13}$$

where

$$\tan \eta = \frac{a}{b}\tan \theta$$

This expression can be shown to have the following Fourier series expansion in θ (see ref. 9):

$$C_p = 2ab\left[\ln \frac{4}{\beta(a + b)} - 1\right] + \frac{2a^2b^2}{a^2 + b^2}\sum_{k=0,2,4}^{k=\infty}\left(\frac{a^2 - b^2}{a^2 + b^2}\right)\cos k\theta$$

For   k = 2, the expression becomes

$$C_p = 2ab\left[\ln \frac{4}{\beta(a + b)} - 1\right] + \frac{2a^2b^2}{a^2 + b^2}\left(1 + \frac{a^2 - b^2}{a^2 + b^2}\cos 2\theta\right) \tag{14}$$

The expression for the present method (eq. (11)), where   N = 0, is repeated here for convenience:

$$C_p = \frac{4a^2b^2}{a^2 + b^2}\left(\ln \frac{a + b}{\beta ab} - 1\right) + \frac{2a^2b^2}{a^2 + b^2}\left(1 + \frac{a^2 - b^2}{a^2 + b^2}\cos 2\theta\right)$$

Comparison of the present method (eq. (11)) with Van Dyke's solution (eqs. (13) and (14)) is shown in figure 2 for $M = \sqrt{2}$,   x = 1,   a = tan 30^0 = 0.57735, and   b = 0.57735, 0.5, 0.3, and 0.1. Although the geometry shown in this figure is far beyond the range of linear theory, and certainly slender-body theory, the purpose of this figure is simply to compare theoretical results.

In the present method for   N = 2, the velocity components (given previously as eqs. (7)) are

$$\frac{\partial \phi}{\partial x} = -C_0 \ln \frac{2G}{\beta} - \frac{3}{2} C_2 \cos 2\theta \, G^2$$

$$= -C_0 \left(\ln \frac{a+b}{\beta ab} - \frac{a-b}{a+b} \cos 2\theta \right) - \frac{3}{2} C_2 \left(\frac{A_2}{2} + A_0 \cos 2\theta \right) \tag{15a}$$

$$\frac{1}{G} \frac{\partial \phi}{\partial r} = C_0 + C_2 \cos 2\theta \, G^2$$

$$= C_0 + C_2 \left(\frac{A_2}{2} + A_0 \cos 2\theta \right) \tag{15b}$$

and

$$\frac{1}{Gr} \frac{\partial \phi}{\partial \theta} = 0 + C_2 \sin 2\theta \, G^2$$

$$= 0 + C_2 A_0 \sin 2\theta \tag{15c}$$

Substituting equations (15) into the boundary condition (eq. (7d)) yields

$$1 = \left(A_0 + A_2 \cos 2\theta \right) \left(C_0 + \frac{C_2 A_2}{2} + C_2 A_0 \cos 2\theta \right)$$

$$- A_2 \sin 2\theta \left(C_2 A_0 \sin 2\theta \right) \tag{16}$$

By expanding equation (16)

$$\left[1 - A_0 \left(C_0 + \frac{C_2 A_2}{2} \right) \right] - \left(C_2 A_0{}^2 + A_2 C_0 + \frac{C_2 A_2{}^2}{2} \right) \cos 2\theta$$

$$- \left(C_2 \frac{A_0 A_2}{2} + C_2 A_0 A_2 \right) \cos 4\theta = 0$$

is obtained, and the boundary condition is now in the form:

$$\sum_{k=0,1,2}^{k=N} P_k \cos k\theta = 0$$

Therefore,

$$P_0 = 0 = 1 - A_0 \left(C_0 + \frac{C_2 A_2}{2} \right)$$

$$P_2 = 0 = C_2 A_0{}^2 + A_2 C_0 + C_2 \frac{A_2{}^2}{2}$$

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or

$$C_0 = \frac{1}{A_0}\left[1 + \frac{1}{2}\left(\frac{A_2}{A_0}\right)^2\right] \quad \text{and} \quad C_2 = -\frac{A_2}{A_0^3}$$

A comparison of the present method for $N = 2$ with Van Dyke's solution is presented in figure 3.

The expansion procedure just illustrated has been generalized and programed so that higher order solutions can be obtained. Figures 4 and 5 show results for $N = 8$ and $N = 32$, respectively. Although the calculation procedure does not converge for small eccentricity, lower order solutions are good approximations to the complete solution.

Arbitrary cone.- The present method has been used to calculate the pressure distribution over cones with the geometry shown in figure 6. These results are shown in figures 7 and 8 for $N = 8$ and $N = 32$, respectively. The results for the most winglike cone are diverging.

Lifting Solutions

The lifting solution has been shown to be independent and separable from the non-lifting solution. Solutions are obtained by superimposing n-odd solutions in the identical manner as was done with the nonlifting solution. Again the agreement between the results from the analytic solution to the lifting elliptic cone and those from the present method will be the important test of the expansion procedure. It should be noted that the lifting solutions are independent of Mach number.

Results for the elliptic cones previously examined are presented in figures 9 and 10 for $N = 7$ and $N = 31$ at $\alpha = 10^O$, respectively. These results also show signs of divergence in the thin-wing limit.

Figure 11 shows results for the cone of arbitrary cross section at $\alpha = 10^O$ for $N = 32$. Large changes in pressure are shown in the region where the straight-line geometry changes to circular geometry. This trend is also indicated by the experimental data of reference 12.

A listing of the computer program to calculate the pressure distribution around lifting cones of arbitrary cross section with slender-body theory is presented and discussed in the appendix. Computational time for obtaining pressures with this program has been estimated at 1 minute per case for $N = 32$ on the Control Data series 6600 computer system at the Langley Research Center.

Results Without Slender-Body Assumptions

The present method of Fourier series expansion can be applied to the solution of the cone with arbitrary cross section without slender-body assumptions. The velocity components, in this case, satisfy equations (3). Results from this solution are presented in figure 12 for several elliptic cones. The solid line represents results obtained using the "not-so-slender" solution of reference 9. The solution of reference 9 does not reduce to the well-known circular-cone solution of reference 4, page 214. However, the present method does reduce to identically the correct circular-cone solution.

Theoretical Limitation

For all the results shown, the present method diverges in the thin-wing limit; however, the lower order solutions gave good approximations to the actual solution.

Ward, in reference 5, indicates that slender-body theory should not be applied to cross-sectional shapes where the local radius of curvature is small compared with the maximum thickness. This restriction could explain the divergence of the present method as the geometry approaches a wing. It is important to recognize that the analytic solution to the elliptic cone given in reference 6 was achieved only after the transformation from ellipse to circle by the Joukowski transformation. The results of the present paper would indicate that in order to achieve converged results for arbitrary winglike cross sections, an initial transformation would be necessary.

CONCLUDING REMARKS

A Fourier series expansion procedure has been developed to solve for the flow around slender cones at supersonic speeds. The results for an elliptic cone reduce to those obtained by Ward's theory. Both lifting and nonlifting solutions to the arbitrarily shaped cone can be obtained, except in the thin-wing limit where the method diverges. The present method has been programed and applied to cone solutions without slender-body assumptions with good results.

Langley Research Center,
National Aeronautics and Space Administration,
Hampton, Va., June 12, 1972.

APPENDIX

COMPUTER PROGRAM FOR CALCULATING THE PRESSURE DISTRIBUTION
AROUND SLENDER CONES

The calculation procedure described in the main body of the paper for obtaining pressure distributions around slender cones has been programed for high-speed digital computation. The purpose of this appendix is to provide a description of the necessary input and available output as well as a FORTRAN IV (ref. 13) listing on the source program. An example input case and the resulting output listing are included.

Description of Program

The program reads in the necessary number of lifting and nonlifting solutions to be superimposed. The coefficients of the required geometry function are computed by standard Fourier expansion series techniques (numerical integration). The matrices given by the boundary condition are formed and inverted using Gaussian elimination. The values for C_n that result are used to compute the velocity components and pressure distribution. The program listing that follows has the geometry of an elliptic cone built in. However, this geometry can readily be replaced by any arbitrary description of geometry.

Program Listing

The FORTRAN IV listing of the source program used on the Control Data series 6600 computer system at the Langley Research Center is as follows:

```
      COMMON THETA(181),Q(181),II,PI,Q0,QQ(640),A(17,18),B(17),K
      DIMENSION G(181),RRO(32),          RO(32),R(320,32),
     1P(32), RR(32,32),                  TT(32,32),F(181),H(181),
     2FF(181),HH(181),C(32),CX(32),CP(32),CT(32),CP(181)
     3,AR(32,32),AS(32,32),AT(32,32)
      REAL M,LAM
      NAMELIST/NUM1/MM,N,NN
      NAMELIST/NUM2/M,ALPHA
      READ(5,NUM1)
      WRITE(6,NUM1)
      DO 10 I=1,N
      RO(I)=0.
      LL=MM*N
      DO 10 J=1,LL
   10 R(J,I)=0.
      PI=3.141592653589793
      AA=.57735
      BB=.3
      THETA(1)=0.
      DO 1 I=2,181
```

```
  1 THETA(I)=THETA(I-1)+1.
    DO 2 I=1,IR1
    THETR=THETA(I)*PI/180.
  2 G(I)=SQRT((AA**2+BB**2+(BB**2-AA**2)*COS(2.*THETR))/(2.*AA**2*BB**
   12))
100 FORMAT(2X3E16.8)
    DO 3 K=1,2
    DO 4 I=1,IR1
  4 Q(I)=G(I)**K
    II=MM*K
    CALL INTER
    RO(K)=QQ
    DO 5 J=2,II,2
  5 R(J,K)=QQ(J)
  3 CONTINUE
    DO 7 I=1,IR1
  7 Q(I)=ALOG(2.*G(I))
    II=N
    CALL INTER
    PO=QQ
    DO 8 J=2,II,2
```

```
  8 P(J)=QQ(J)
    DO 20 L=1,N
    PR0(L)=0.
    DO 24 K=1,N
 24 PR(K,L)=0.
    DO 21 I=1,IR1
    THETR=THETA(I)*PI/180.
 21 Q(I)=COS(L*THETR)*G(I)**L
    II=N
    CALL INTER
    PR0(L)=QQ
    DO 22 J=2,II,2
 22 PR(J,L)=QQ(J)
 20 CONTINUE
    DO 40 L=1,N
    DO 44 K=1,N
 44 TT(K,L)=0.
    DO 41 I=1,IR1
    THETR=THETA(I)*PI/180.
 41 Q(I)=SIN(L*THETR)*G(I)**L
    II=N
    CALL INTER
    DO 42 J=2,II,2
 42 TT(J,L)=QQ(J)
 40 CONTINUE
    DO 54 I=1,IR1
    THETR=THETA(I)*PI/180.
    H(I)=0.
    F(I)=RO(2)
    KK=2*MM
    DO 50 K=1,KK
    H(I)=H(I)-K*R(K,2)*SIN(K*THETR)
 50 F(I)=F(I)+R(K,2)*COS(K*THETR)
 54 CONTINUE
    DO 51 I=1,IR1
```

```
      THETR=THETA(I)*PI/180.
      FF(I)=P0(1)
      DO 51 K=1,N
  51  FF(I)=FF(I)+P(K,1)*COS(K*THETR)
      DO 52 I=1,181
  52  Q(I)=F(I)
      II=N
      CALL INTER
      A(1,1)=QC
      DO 53 K=2,II,2
      L=K/2+1
  53  A(L,1)=QQ(K)
      DO 63 J=2,N,2
      JJ=J/2+1
      DO 61 I=1,181
      THETR=THETA(I)*PI/180.
      FF(I)=RRO(J)
      HH(I)=0.
      DO 61 K=1,N
      FF(I)=FF(I)+RR(K,J)*COS(K*THETR)
  61  HH(I)=HH(I)+TT(K,J)*SIN(K*THETR)
      DO 62 I=1,181
  62  Q(I)=FF(I)*F(I)+.5*H(I)*HH(I)
      II=N
      CALL INTER
      A(1,JJ)=OO
      DO 63 K=2,II,2
      L=K/2+1
  63  A(L,JJ)=QQ(K)
      I=N/2+2
      A(1,I)=1.
      DO 65 J=2,N,2
      K=J/2+1
  65  A(K,I)=0.
      I=2*MM+2
      IF(I.GT.N)GO TO 67
      DO 64 J=1,N,2
      K=J/2+1
  66  A(K,1)=0.
      DO 66 L=2,N,2
      LL=L/2+1
      I=2*MM+MM*L+L+2
      IF(I.GT.N)GO TO 67
      DO 68 J=1,N,2
      K=J/2+1
  68  A(K,LL)=0.
  67  CONTINUE
      K=N/2+1
      K=N/2+1
      CALL MATRIX
      C0=B(1)
      DO 70 L=2,N,2
      J=L/2+1
  70  C(L)=B(J)
      DO 124 L=1,NN
      DO 124 K=1,NN
 124  AR(K,1)=0.
      DO 121 I=1,181
      THETR=THETA(I)*PI/180.
 121  Q(I)=COS(L*THETR)*G(I)**L
      II=NN
      CALL AINTER
```

```
         DO 122 J=1,NN,2
122 AR(J,L)=QQ(J)
120 CONTINUE
         DO 130 L=1,NN
         DO 134 K=1,NN
134 AS(K,L)=0.
         DO 131 I=1,181
         THETR=THETA(I)*PI/180.
131 Q(I)=COS(L*THETR)*G(I)**1
         II=NN
         CALL AINTER
         DO 132 J=1,NN,2
132 AS(J,L)=QQ(J)
130 CONTINUE
         DO 140 L=1,NN
         DO 144 K=1,NN
144 AT(K,L)=0.
         DO 141 I=1,181
         THETR=THETA(I)*PI/180.
141 Q(I)=SIN(L*THETR)*G(I)**1
         II=NN
         CALL AINTER
         DO 142 J=1,NN,2
142 AT(J,L)=QQ(J)
140 CONTINUE
         DO 163 J=1,NN,2
         JJ=(J+1)/2
         DO 161 I=1,181
         THETR=THETA(I)*PI/180.
         FF(I)=0.
         HH(I)=0.
         DO 161 K=1,NN
         FF(I)=FF(I)+AS(K,J)*COS(K*THETR)
161 HH(I)=HH(I)+AT(K,J)*SIN(K*THETR)
         DO 162 I=1,181
162 Q(I)=FF(I)*F(I)+.5*H(I)*HH(I)
         II=NN
         CALL AINTER
         DO 163 K=1,NN,2
         L=(K+1)/2
163 A(L,JJ)=QQ(K)
500 READ(5,NUM2)
         IF(EOF,5) 200,9
  9 WRITE(6,NUM2)
         LAM=SQRT(M**2-1.)
         PP=P0+ALOG(1./LAM)
         XCONF=-1.*C0*PP
         YCONF=C0
         DO 80 J=2,N,2
         XCONF=-1.*FLOAT(J+1)/FLOAT(J)*PRQ(J)*C(J)+XCONF
80 YCONF=C(J)*PRU(J)+YCONF
         TCONF=0.
         DO 82 L=2,N,2
         CX(L)=-1.*C0*P(L)
         CR(L)=0.
         CT(L)=0.
         DO 82 K=2,N,2
         CX(L)=-1.*FLOAT(K+1)/FLOAT(K)*PP(L,K)*C(K)+CX(L)
         CR(L)=C(K)*PR(L,K)+CR(L)
82 CT(L)=C(K)*TT(L,K)+CT(L)
         DO 90 LLL=1,181
         THETR=THETA(LLL)*PI/180.
```

```
      U=XCONE
      V=YCONE*G(LLL)
      W=0.
      DO 91 L=2,N,2
      U=U+CX(L)*COS(L*THETR)
      V=V+CR(L)*COS(L*THETR)*G(LLL)
   91 W=W+CT(L)*SIN(L*THETR)*G(LLL)
      CP(LLL)=-2.*(U+(V**2+W**2)/2.)
   90 CONTINUE
      ALPHR=ALPHA*PI/180.
      DO 101 I=1,181
      THETR=THETA(I)*PI/180.
  101 G(I)=G(I)*COS(THETR)-.5*SIN(THETR)*H(I)/G(I)
      II=NN
      CALL AINTER
      L=(NN+1)/2+1
      DO 102 J=1,NN,2
      K=(J+1)/2
  102 A(K,1)=-1.*ALPHR*QQ(J)
      K=(NN+1)/2
      CALL MATRIX
      DO 170 L=1,NN,2
      J=(L+1)/2
  170 C(L)=P(J)
      DO 182 L=1,NN,2
      CX(L)=-2.*AR(L,1)*C(1)
      CR(L)=C(1)*AS(L,1)
      CT(L)=C(1)*AT(L,1)
      DO 182 K=3,NN,2
      CX(L)=-1.*(K+1)/K*AR(L,K)*C(K)+CX(L)
      CR(L)=C(K)*AS(L,K)+CR(L)
  182 CT(L)=C(K)*AT(L,K)+CT(L)
      WRITE(6,202)
  202 FORMAT(7X5HTHETA,10X6HCP,A=0,9X11HCP,A-CP,A=0,7X7HCPTOTAL/)
      DO 190 LLL=1,181
      THETR=THETA(LLL)*PI/180.
      AU=0.
      AV=0.
      AW=0.
      DO 191 L=1,NN,2
      AU=AU+CX(L)*COS(L*THETR)
      AV=AV+CR(L)*COS(L*THETR)*G(LLL)
  191 AW=AW+CT(L)*SIN(L*THETR)*G(LLL)
      CPA=ALPHR**2-2.*AU-(ALPHR*COS(THETR)+AV)**2-(ALPHR*SIN(THETR)
     1-AW)**2
      CPT=CPA+CP(LLL)
  190 WRITE(6,201)THETA(LLL),CP(LLL),CPA,CPT
  201 FORMAT(2X4F16.8)
      GO TO 500
  200 STOP
      END
```

```
SUBROUTINE INTER
COMMON THETA(181),Q(181),II,PI,QU,QQ(640),A(17,18),B(17),K
DIMENSION F(181)
AREB=0.
DO 1 I=2,181
1 AREB=AREB+(Q(I)+Q(I-1))/2.
QQ=AREB/180.
DO 2 J=2,II,2
AREB=0.
F(1)=Q(1)
DO 3 I=2,181
THETR=THETA(I)*PI/180.
F(I)=Q(I)*COS(J*THETR)
3 AREB=AREB+(F(I)+F(I-1))/2.
2 QQ(J)=2.*AREB/180.
RETURN
END
```

```
SUBROUTINE ONTER
COMMON THETA(181),Q(181),II,PI,QU,QQ(640),A(17,18),B(17),K
DIMENSION F(181)
DO 2 J=2,II,2
AREB=0.
F(1)=Q(1)
DO 3 I=2,181
THETR=THETA(I)*PI/180.
F(I)=Q(I)*SIN(J*THETR)
3 AREB=AREB+(F(I)+F(I-1))/2.
2 QQ(J)=2.*AREB/180.
RETURN
END
```

```
SUBROUTINE MATRIX
COMMON THETA(181),Q(181),II,PI,QU,QQ(640),A(17,18),B(17),K
L=K+1
DO 4 II=1,K
DO 1 I=II,K
IF(A(I,II).EQ.0.)GO TO 1
C=A(I,II)
DO 2 I=1,L
2 A(I,I)=A(I,I)/C
1 CONTINUE
IF(II.EQ.K)GO TO 4
JJ=II+1
DO 1 I=JJ,K
IF(A(I,II).EQ.0.)GO TO 10
DO 3 I=1,L
3 A(I,I)=A(II,I)-A(I,J)
10 CONTINUE
4 CONTINUE
DO 5 I=1,K
5 B(I)=0.
B(K)=A(K,K+1)/A(K,K)
KK=K-1
DO 6 J=1,KK
I=K-J
B(I)=A(I,K+1)
LL=I+1
DO 6 L=LL,K
6 B(I)=B(I)-B(L)*A(I,L)
RETURN
END
```

```
SUBROUTINE AINTER
COMMON THETA(181),Q(181),II,PI,QO,QQ(640),A(17,18),B(17),K
DIMENSION F(181)
QO=0.
DO 2 J=1,II,2
AREB=0.
F(1)=Q(1)
DO 3 I=2,181
THETR=THETA(I)*PI/180.
F(I)=Q(I)*COS(J*THETR)
3 AREB=AREB+(F(I)+F(I-1))/2.
2 QQ(J)=2.*AREB/180.
RETURN
END
```

```
SUBROUTINE AONTER
COMMON THETA(181),Q(181),II,PI,QO,QQ(640),A(25,26),B(25),K
DIMENSION F(181)
DO 2 J=1,II,2
AREB=0.
F(1)=Q(1)
DO 3 I=2,181
THETR=THETA(I)*PI/180.
F(I)=Q(I)*SIN(J*THETR)
3 AREB=AREB+(F(I)+F(I-1))/2.
2 QQ(J)=2.*AREB/180.
RETURN
END
```

21

Input

A single case consists of the determination of the pressure distribution around a given cone at a given Mach number and angle of attack. It is necessary to input the number of terms in a Fourier expansion series to accurately represent the cross-sectional shape and the number of lifting and nonlifting solutions to be superimposed. For the loading routine used in the program, any column except the first may be used on the input cards. A decimal format is used for the input quantities. A description of the required inputs in the correct order and the FORTRAN variables used by the source program are as follows:

FORTRAN variable	Description
$NUM1	Arbitrary name required by the loading routine to define the first input data block
MM	Number of terms necessary to accurately define cross-section geometry
N	Number of nonlifting solutions to be superimposed
NN	Number of lifting solutions to be superimposed
$	Denotes end of case (column 2)
$NUM2	Arbitrary name required by the loading routine to define the second input block
M	Free-stream Mach number
ALPHA	Angle of attack, deg
$	Denotes end of case (column 2)

The system loading subroutine in the program (name list) is quite flexible in that the order of the input cards is unimportant and successive cases can be run by repeating the identification and $NUM cards followed by only the changed parameters and a $ card. An example of a set of input cards is given by the following listing of the inputs necessary to compute the pressure distribution around an elliptic cone (a = 0.57735 and b = 0.3) at M = 1.414 and $\alpha = 10^{\circ}$.

```
$NUM1
M,4      =    2 .
IJ       =    3 .
IJN      =    31 .
$END)
$NUM2
N        =    0.14156+ 1 .
ALPHA    =    0.1 + 2 .
$END)
```

Output

An output listing for the example input is as follows:

THETA	CP,A=0	CP,A-CP,A=0	CPTOTAL
0.	5.10787096E-01	-2.74783668E-01	2.36000342E-01
1.00000000E+00	5.10032387E-01	-2.74110494E-01	2.35921893E-01
2.00000000E+00	5.07740580E-01	-2.72090694E-01	2.35649846E-01
3.00000000E+00	5.03845408E-01	-2.68729705E-01	2.35115704E-01
4.00000000E+00	4.98286770E-01	-2.64055587E-01	2.34231183E-01
5.00000000E+00	4.91068529E-01	-2.58140302E-01	2.32928227E-01
6.00000000E+00	4.82309618E-01	-2.51117846E-01	2.31191772E-01
7.00000000E+00	4.72270014E-01	-2.43191597E-01	2.29078417E-01
8.00000000E+00	4.61340232E-01	-2.34626219E-01	2.26714013E-01
9.00000000E+00	4.49993740E-01	-2.25723695E-01	2.24270046E-01
1.00000000E+01	4.38712943E-01	-2.16787628E-01	2.21925314E-01
1.10000000E+01	4.27907672E-01	-2.08083648E-01	2.19824024E-01
1.20000000E+01	4.17848042E-01	-1.99805274E-01	2.18042768E-01
1.30000000E+01	4.08629992E-01	-1.92053683E-01	2.16576309E-01
1.40000000E+01	4.00182960E-01	-1.84836471E-01	2.15346488E-01
1.50000000E+01	3.92317372E-01	-1.78085752E-01	2.14231620E-01
1.60000000E+01	3.84798509E-01	-1.71690974E-01	2.13107535E-01
1.70000000E+01	3.74259884E-01	-1.65538083E-01	2.11887901E-01
1.80000000E+01	3.70096773E-01	-1.59545069E-01	2.10551704E-01
1.90000000E+01	3.62834760E-01	-1.53585090E-01	2.09149669E-01
2.00000000E+01	3.55779686E-01	-1.47091772E-01	2.07787914E-01
2.10000000E+01	3.49140187E-01	-1.42546162E-01	2.06594026E-01
2.20000000E+01	3.43125946E-01	-1.37449754E-01	2.05676191E-01
2.30000000E+01	3.37879739E-01	-1.32791653E-01	2.05088096E-01
2.40000000E+01	3.33429788E-01	-1.28619340E-01	2.04810448E-01
2.50000000E+01	3.29676410E-01	-1.24921302E-01	2.04755108E-01
2.60000000E+01	3.26416590E-01	-1.21625296E-01	2.04790294E-01
2.70000000E+01	3.23398763E-01	-1.18519273E-01	2.04779490E-01
2.80000000E+01	3.20391223E-01	-1.15769360E-01	2.04621863E-01
2.90000000E+01	3.17243765E-01	-1.12261939E-01	2.04281826E-01
3.00000000E+01	3.13924440E-01	-1.10125865E-01	2.03798580E-01
3.10000000E+01	3.10520958E-01	-1.07248302E-01	2.03272655E-01
3.20000000E+01	3.07206915E-01	-1.04373341E-01	2.02833574E-01
3.30000000E+01	3.04183490E-01	-1.01584913E-01	2.02598577E-01
3.40000000E+01	3.01614371E-01	-9.89792194E-02	2.02635152E-01
3.50000000E+01	2.99573433E-01	-9.66344619E-02	2.02938972E-01
3.60000000E+01	2.98020557E-01	-9.45464794E-02	2.03434077E-01
3.70000000E+01	2.96812324E-01	-9.28169832E-02	2.03995341E-01
3.80000000E+01	2.95743783E-01	-9.12574917E-02	2.04486291E-01
3.90000000E+01	2.94608211E-01	-8.98075533E-02	2.04800658E-01
4.00000000E+01	2.93256605E-01	-8.83418447E-02	2.04894760E-01
4.10000000E+01	2.91638092E-01	-8.68393133E-02	2.04800674E-01
4.20000000E+01	2.89615403E-01	-8.51993590E-02	2.04616044E-01
4.30000000E+01	2.87933656E-01	-8.34601536E-02	2.04473500E-01

4.40000000E+01	2.86181187E-01	-8.16819421E-02	2.04499245E-01
4.50000000E+01	2.84725846E-01	-7.99524677E-02	2.04773374E-01
4.60000000E+01	2.83663754E-01	-7.83592945E-02	2.05304459E-01
4.70000000E+01	2.82989472E-01	-7.69637355E-02	2.06026137E-01
4.80000000E+01	2.82599965E-01	-7.57428024E-02	2.06817062E-01
4.90000000E+01	2.82322206E-01	-7.47841959E-02	2.07538010E-01
5.00000000E+01	2.81970341E-01	-7.38955575E-02	2.08074784E-01
5.10000000E+01	2.81398732E-01	-7.30251064E-02	2.08373635E-01
5.20000000E+01	2.80545660E-01	-7.20375762E-02	2.08458084E-01
5.30000000E+01	2.79449029E-01	-7.10279288E-02	2.08422000E-01
5.40000000E+01	2.78236696E-01	-6.98360714E-02	2.08400624E-01
5.50000000E+01	2.77077174E-01	-6.85484784E-02	2.08528695E-01
5.60000000E+01	2.76134841E-01	-6.72364258E-02	2.08898415E-01
5.70000000E+01	2.75514769E-01	-6.59344174E-02	2.09530352E-01
5.80000000E+01	2.75231556E-01	-6.48651319E-02	2.10366424E-01
5.90000000E+01	2.75205440E-01	-6.39181587E-02	2.11287281E-01
6.00000000E+01	2.75287483E-01	-6.31385903E-02	2.12148883E-01
6.10000000E+01	2.75305742E-01	-6.24786949E-02	2.12827047E-01
6.20000000E+01	2.75118034E-01	-6.18618464E-02	2.13256188E-01
6.30000000E+01	2.74654095E-01	-6.12148244E-02	2.13450171E-01
6.40000000E+01	2.73940290E-01	-6.04416478E-02	2.13498642E-01
6.50000000E+01	2.73081475E-01	-5.95422047E-02	2.13539771E-01
6.60000000E+01	2.72237957E-01	-5.85202804E-02	2.13717677E-01
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6.80000000E+01	2.71176610E-01	-5.63436767E-02	2.14832934E-01
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1.80000000E+02	5.10787096E-01	3.35707150E-01	8.46494246E-01

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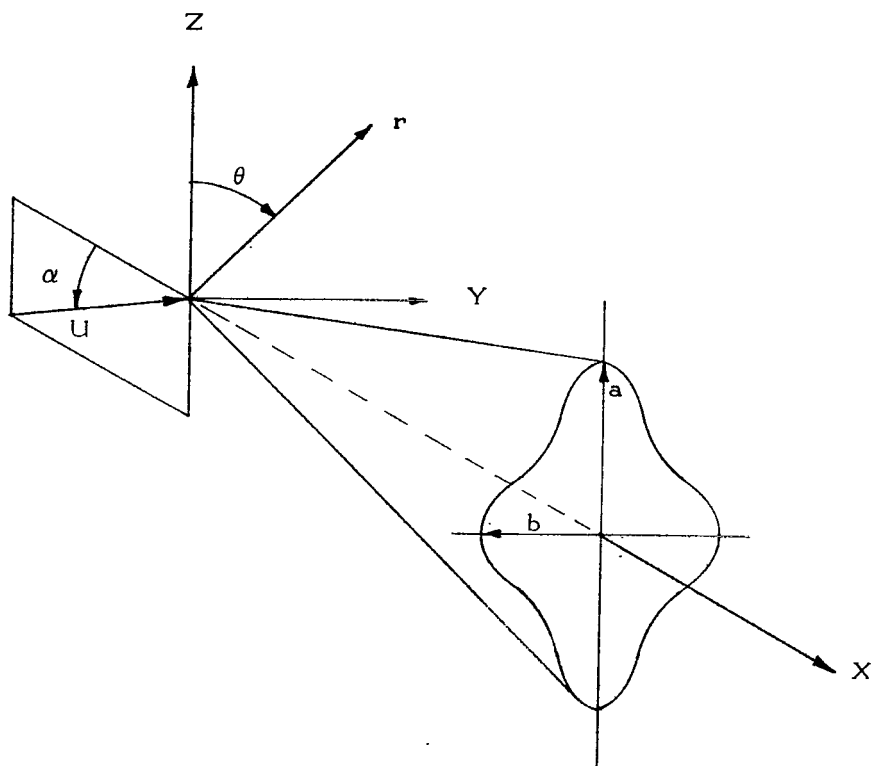13. McCracken, Daniel D.: A Guide to FORTRAN Programming. John Wiley & Sons, Inc., c.1961.
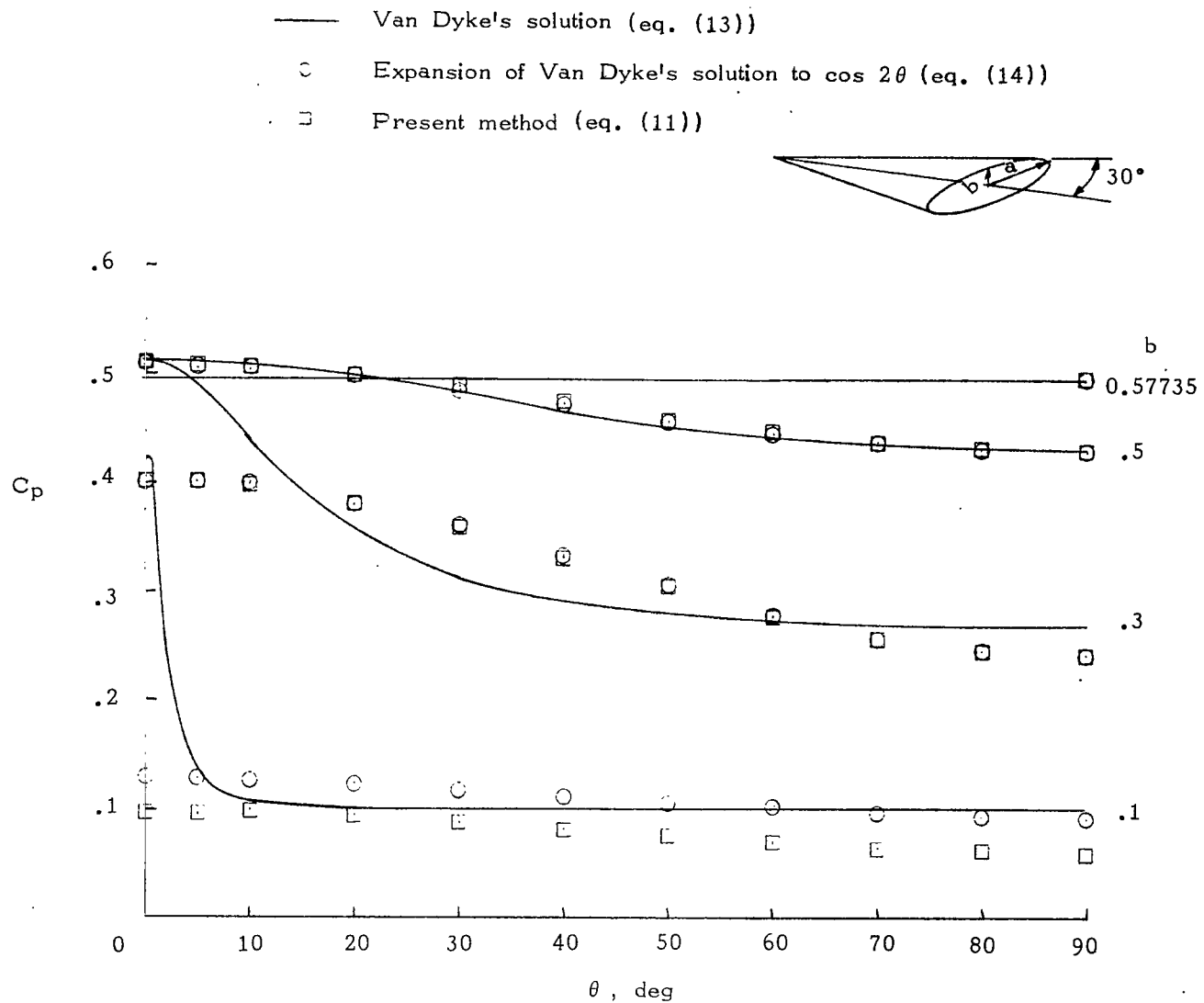
Figure 1.- Cylindrical coordinate system.

Figure 2.- Comparison of results for an elliptic cone from the present method with those from Van Dyke's solution (ref. 9). $N = 0$; $M = \sqrt{2}$; $\alpha = 0^0$; $a = 0.57735$.
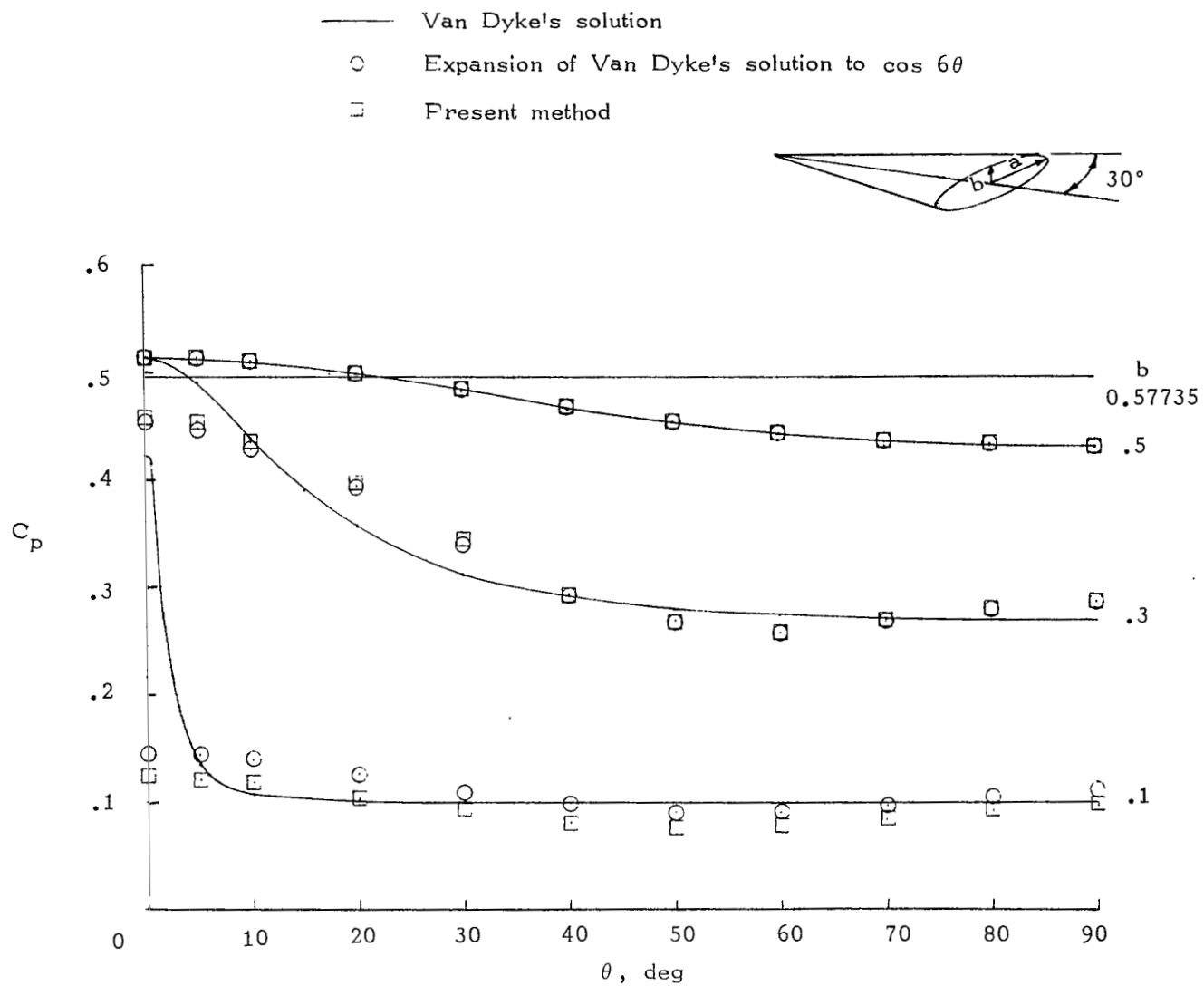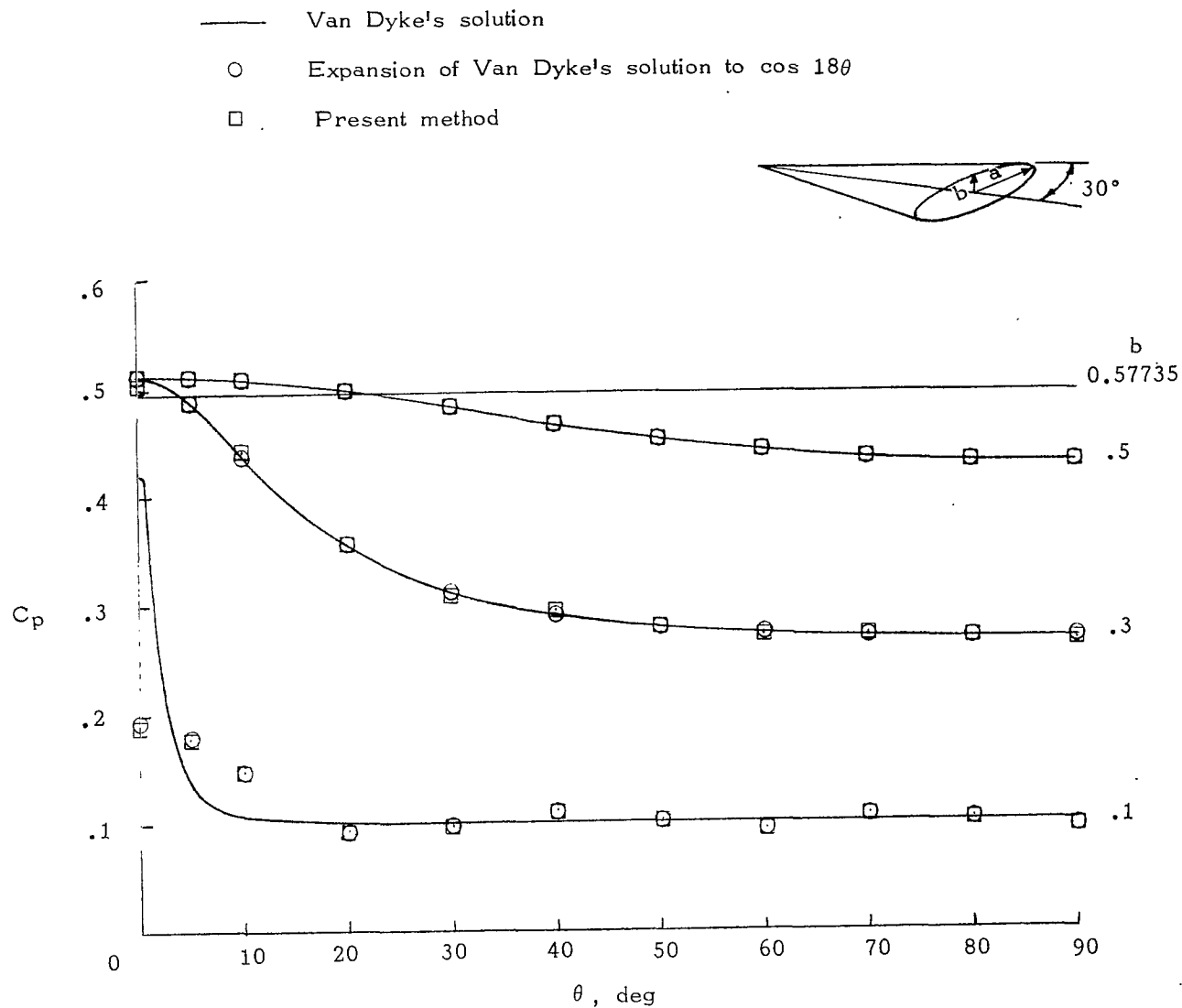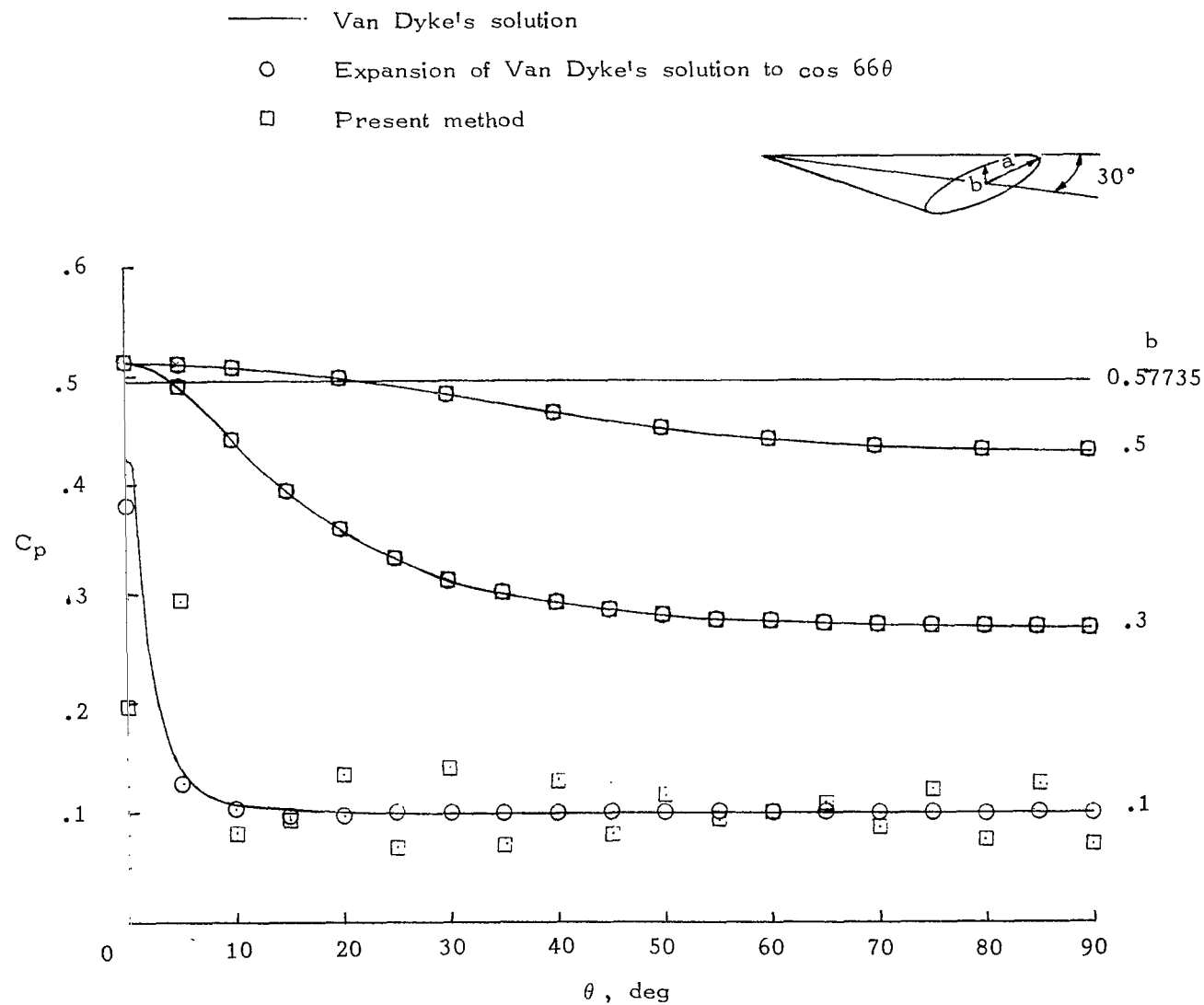
Figure 3.- Comparison of results for an elliptic cone from the present method with those from Van Dyke's solution (ref. 9). $N = 2$; $M = \sqrt{2}$; $\alpha = 0^O$; $a = 0.57735$.
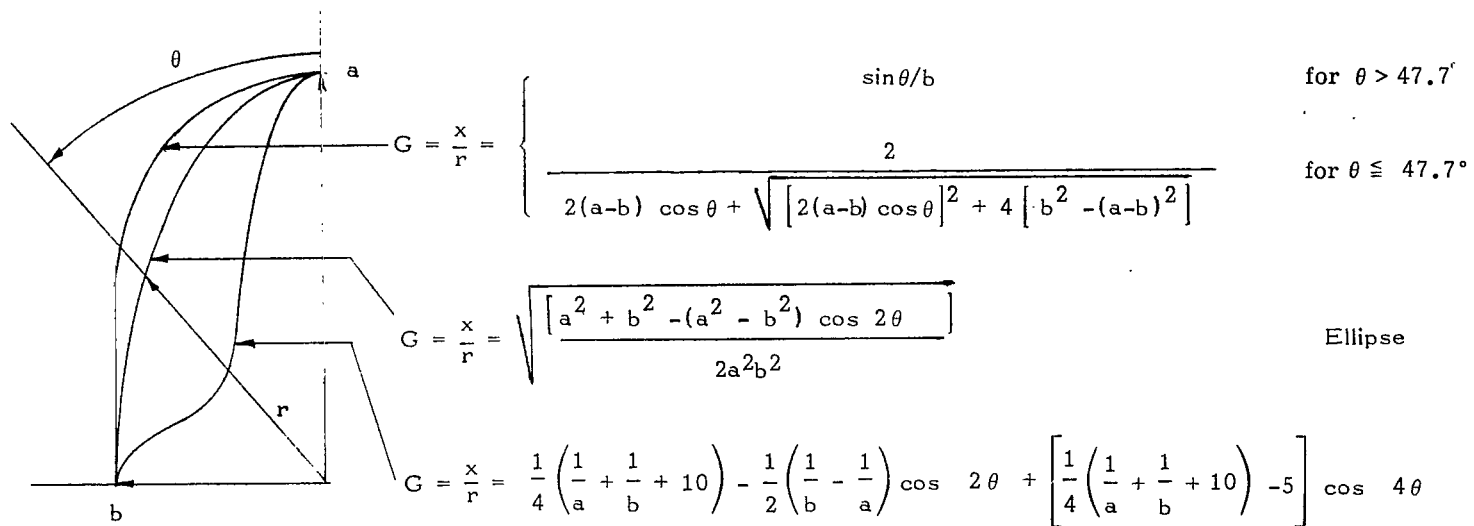
Figure 4.- Comparison of results for an elliptic cone from the present method with those from Van Dyke's solution (ref. 9). $N = 8$; $M = \sqrt{2}$; $\alpha = 0^O$; $a = 0.57735$.

Figure 5.- Comparison of results for an elliptic cone from the present method with those from Van Dyke's solution (ref. 9). N = 32; M = $\sqrt{2}$; $\alpha = 0^O$; a = 0.57735.

$$G = \frac{x}{r} = \begin{cases} \sin\theta/b & \text{for } \theta > 47.7° \\[2em] \dfrac{2}{2(a-b)\cos\theta + \sqrt{\left[2(a-b)\cos\theta\right]^2 + 4\left[b^2 - (a-b)^2\right]}} & \text{for } \theta \leqq 47.7° \end{cases}$$

$$G = \frac{x}{r} = \sqrt{\frac{\left[a^2 + b^2 - (a^2 - b^2)\cos 2\theta\right]}{2a^2b^2}} \qquad \text{Ellipse}$$

$$G = \frac{x}{r} = \frac{1}{4}\left(\frac{1}{a} + \frac{1}{b} + 10\right) - \frac{1}{2}\left(\frac{1}{b} - \frac{1}{a}\right)\cos 2\theta + \left[\frac{1}{4}\left(\frac{1}{a} + \frac{1}{b} + 10\right) - 5\right]\cos 4\theta$$

Figure 6.- Cone geometry for flow calculations. a = 0.57735; b = 0.3.
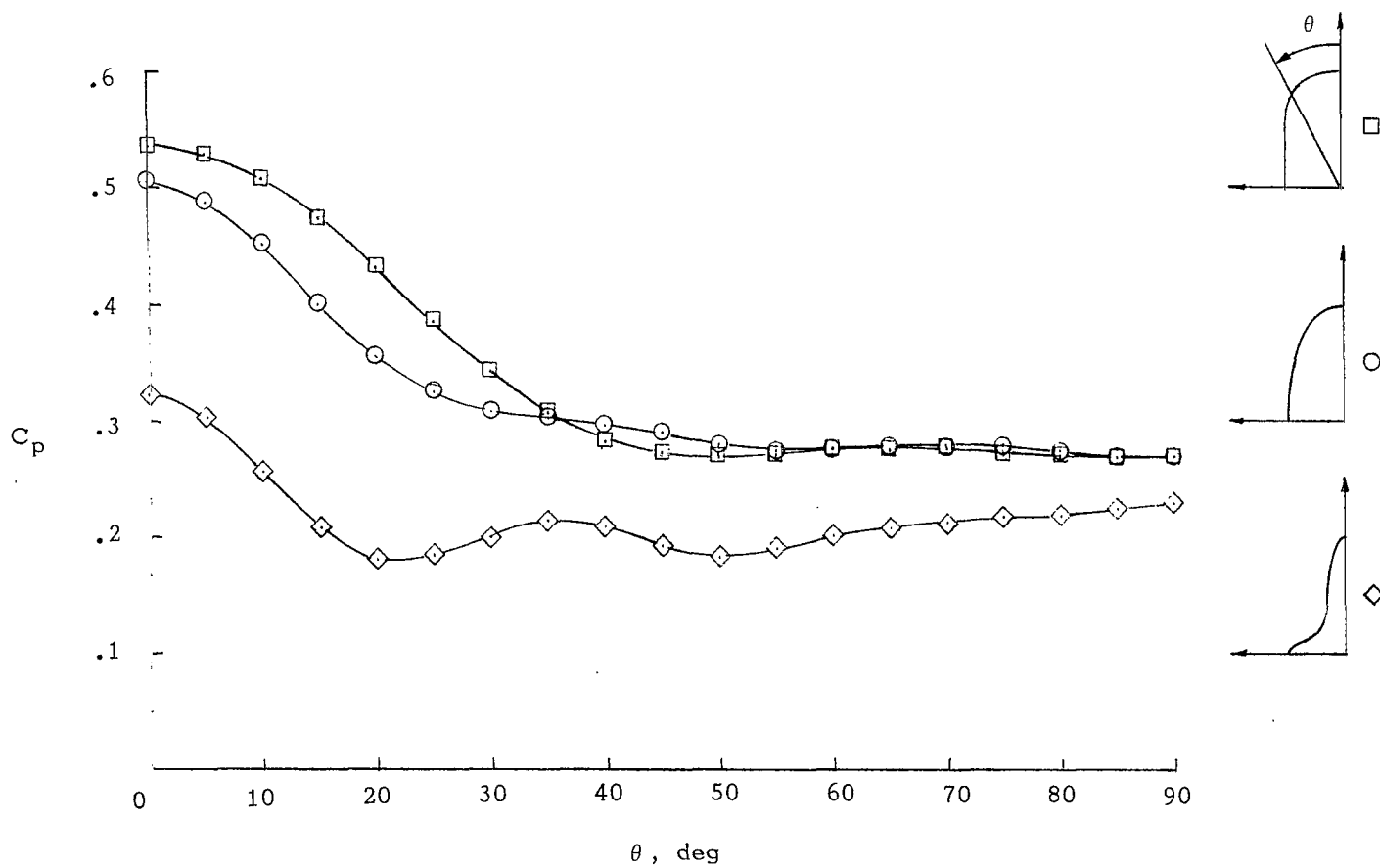
Figure 7.- Results for cones of arbitrary cross section from figure 6. N = 8; M = $\sqrt{2}$; $\alpha = 0^{0}$.
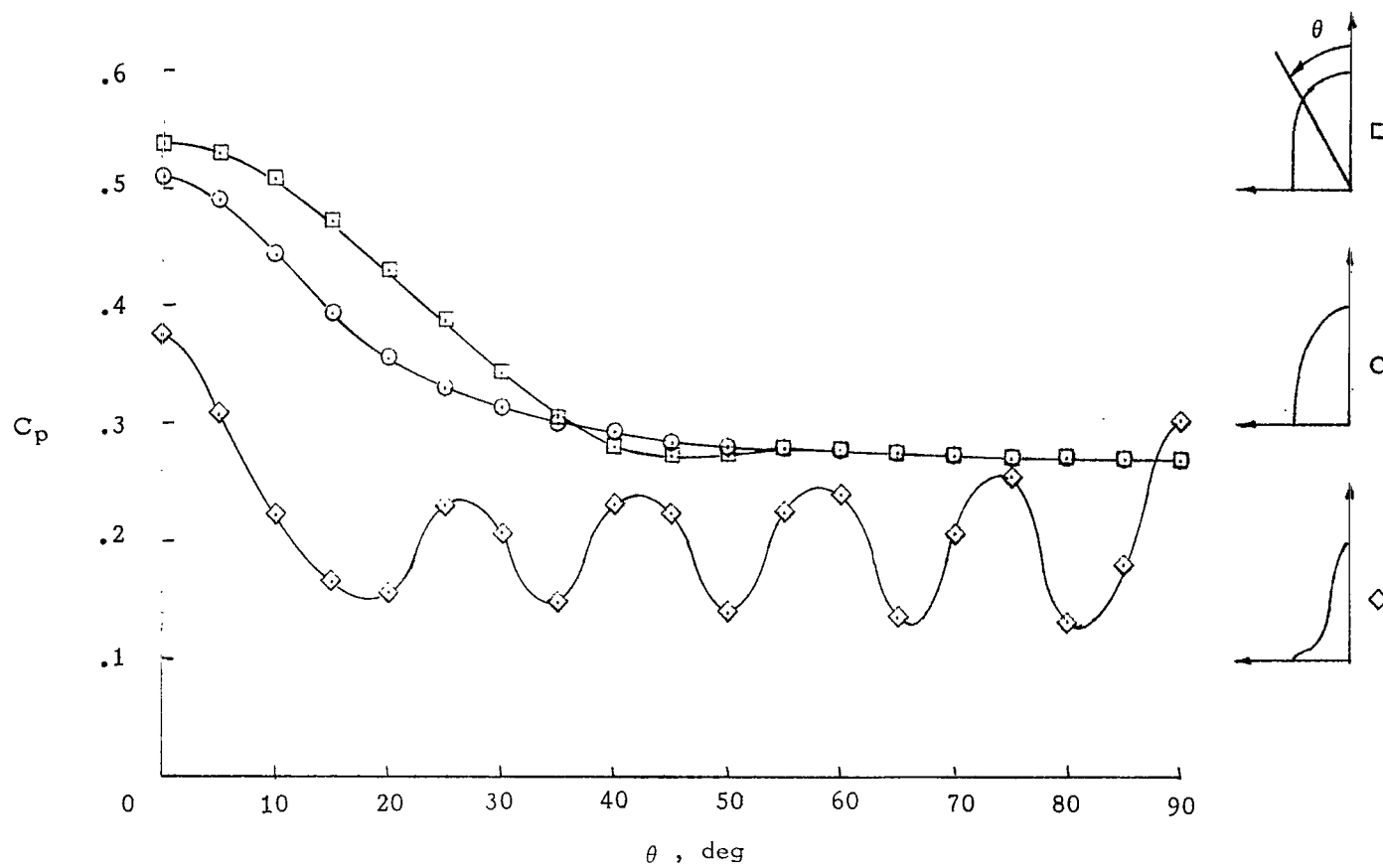
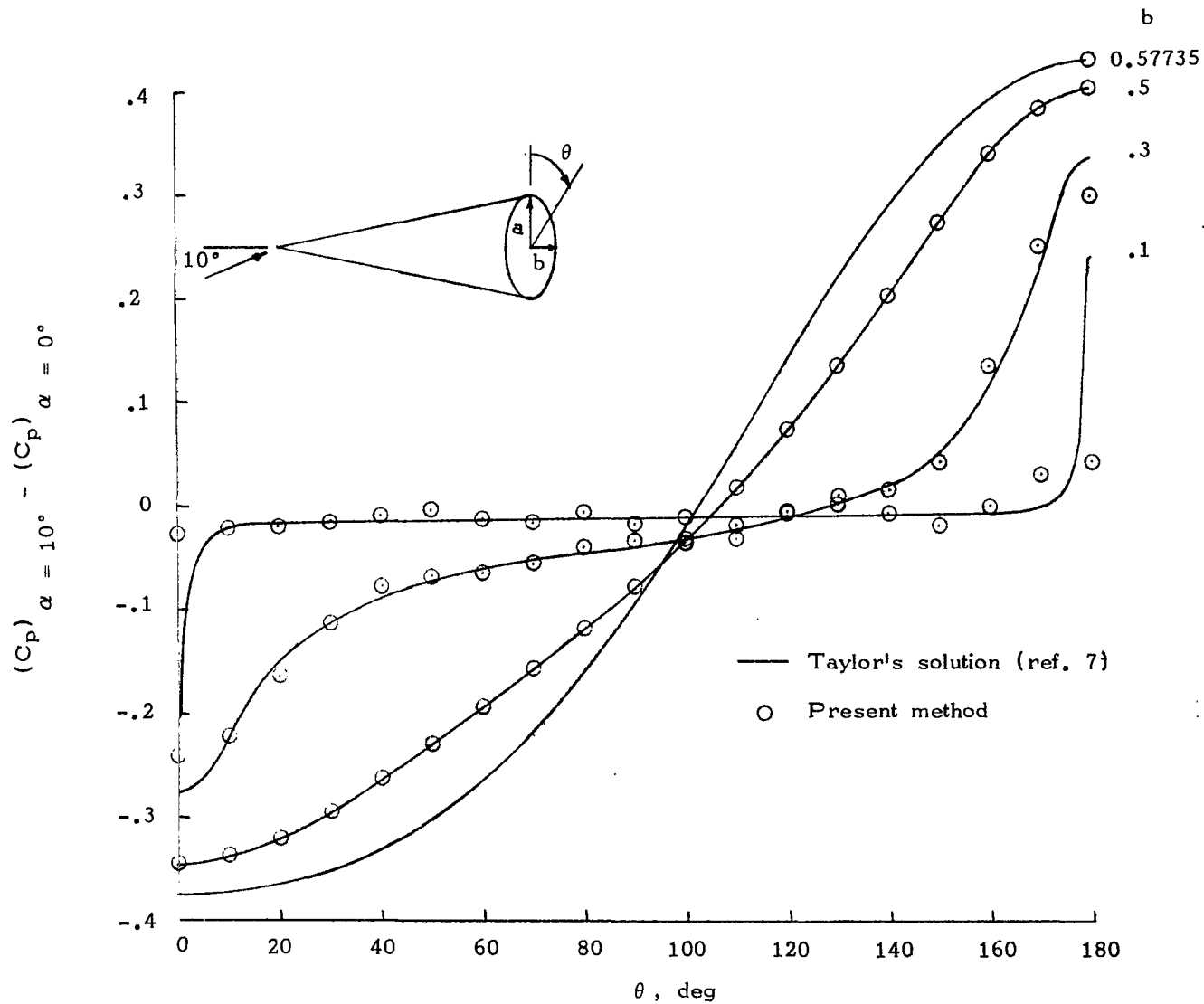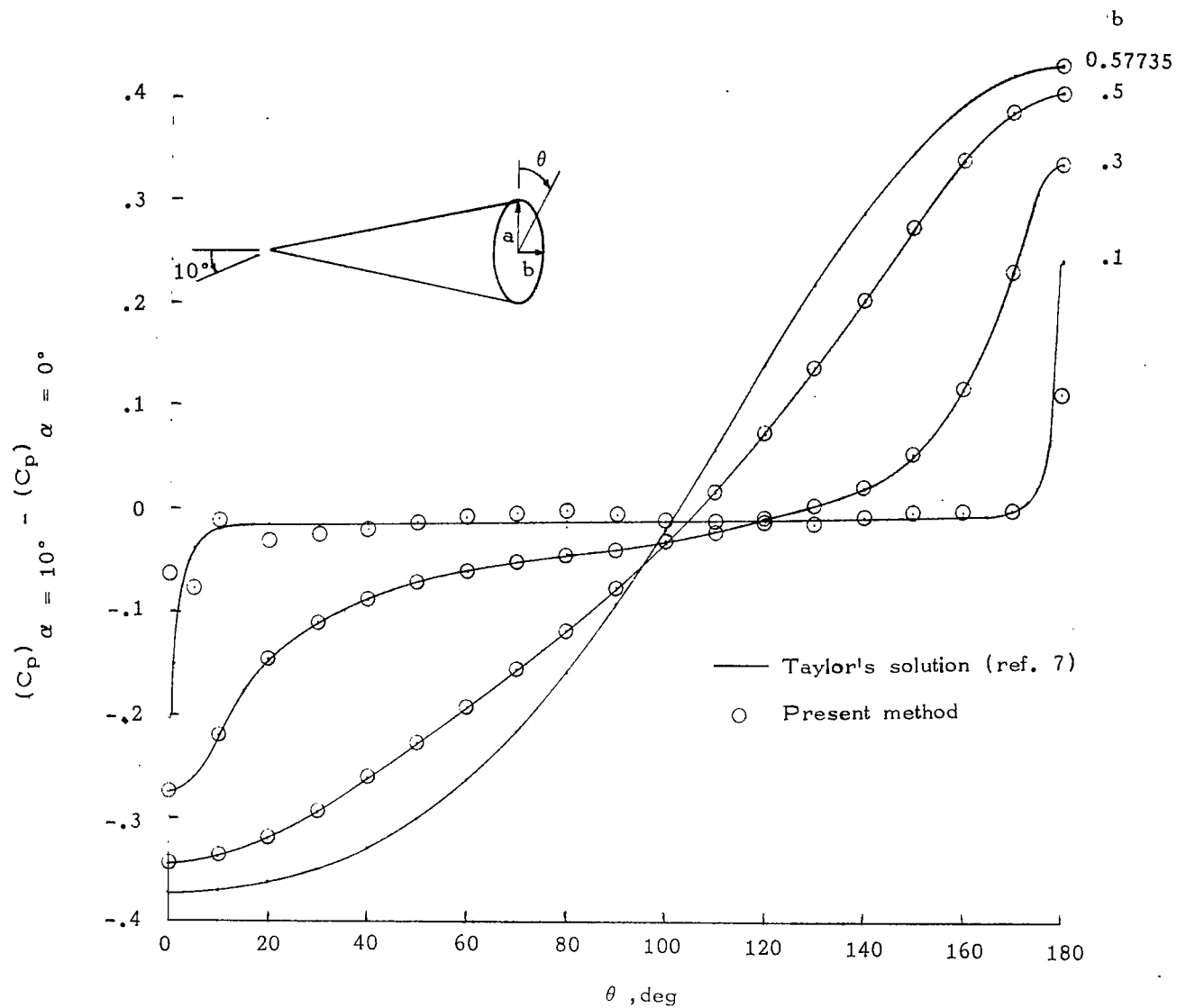Figure 8.- Results for cones of arbitrary cross section from figure 6. N = 32; M = $\sqrt{2}$; $\alpha = 0^O$.

Figure 9.- Comparison of results for lifting elliptic cones. N = 7; $\alpha = 10^{\circ}$; a = 0.57735.

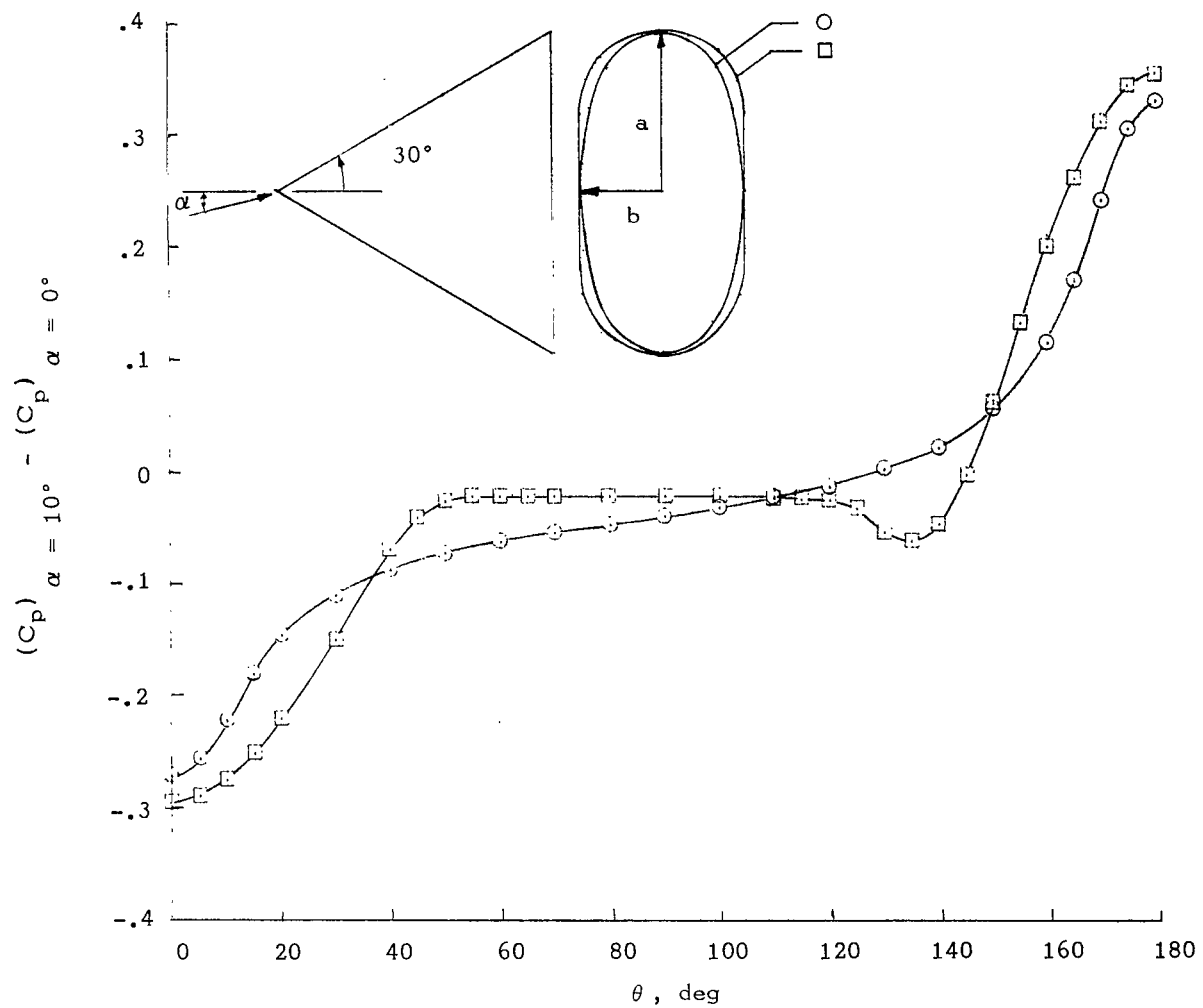Figure 10.- Comparison of results for lifting elliptic cones. N = 31; α = 10°; a = 0.57735.

Figure 11.- Comparison of results for lifting cones. N = 32; $\alpha = 10^{O}$; a = 0.57735; b = 0.3.

——— Van Dyke's "not-so-slender" solution (ref. 9)

▢   Present method

C_p

.6 —

.5 —

.4 —

.3 —

.2 —

.1 —

0

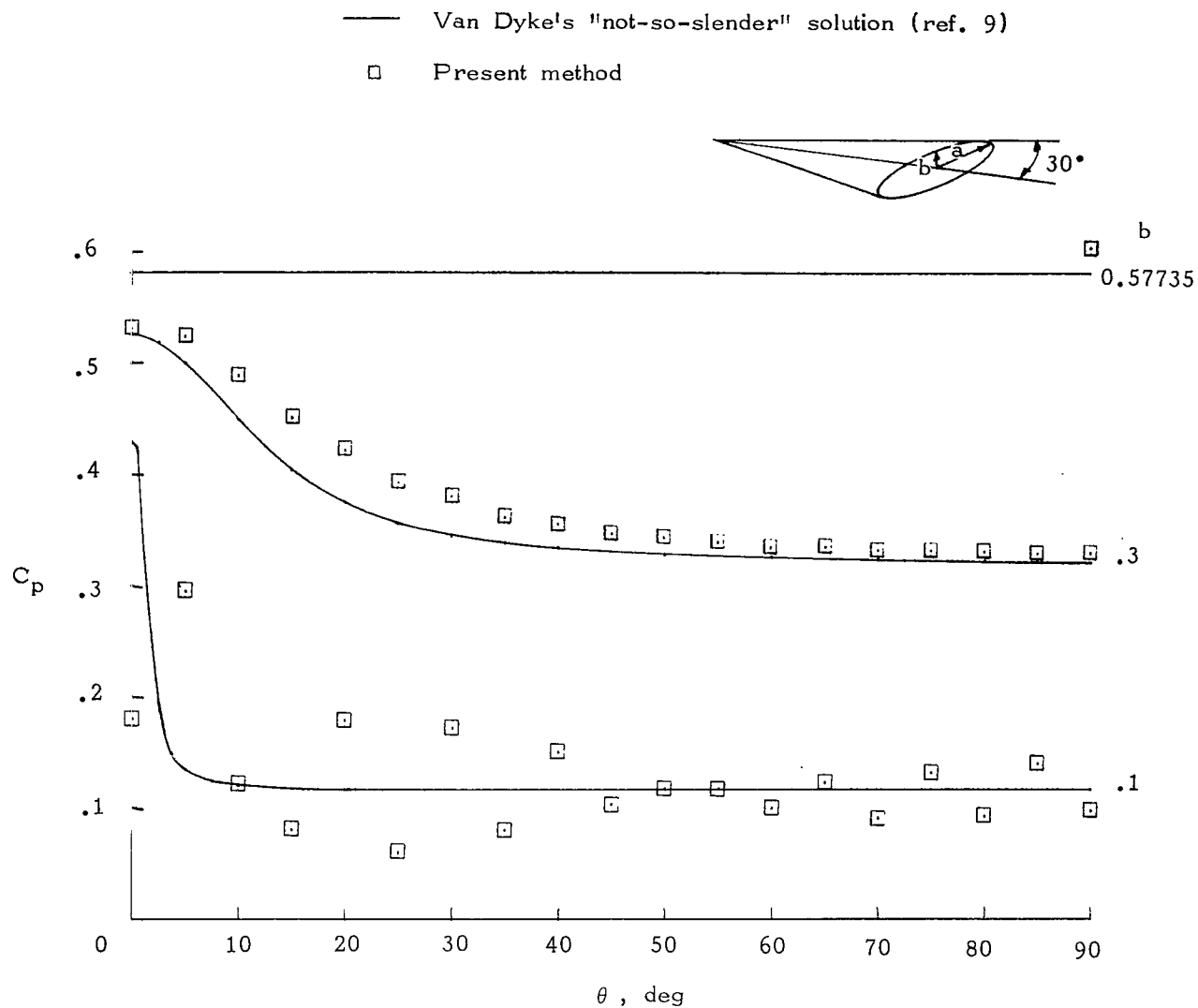| 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 |

b

0.57735

.3

.1

θ , deg

Figure 12.- Comparison of results for elliptic cones without slender body assumptions.
$N = 32$; $M = \sqrt{2}$; $\alpha = 0^{0}$; $a = 0.57735$.

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