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EXTRAPOLATION OF SONIC BOOM PRESSURE SIGNATURES BY THE WAVEFORM PARAMETER METHOD

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16. Abstract

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SYMBOLS

a	sound speed (includes sound speed perturbation due to wave passage)
a_o	ambient sound speed
A	ray tube area as cut by the wavefront
A_h	ray tube area as cut by a horizontal plane
c_n	speed that a wave propagates normal to itself, given by $a_o + \vec{V}_o \cdot \vec{N}$
c_o	defined as $c_n/\cos\theta$
C_1	atmospheric quantity defined by equation (34)
C_2	atmospheric quantity defined by equation (35)
$F(\xi)$	an F function
ΔF_i	discontinuity of $F(\xi)$ corresponding to Δp_i
m_i	slope of waveform segment i
m_i^o	initial value of m_i
M	Mach number
$\vec{N}$	wavefront unit normal
p	pressure perturbation due to wave passage
p_o	ambient pressure
Δp_i	pressure rise across shock at the juncture of waveform segments i and $i-1$
Δp_i^o	initial value of Δp_i
r	for a conical wave, the distance from the flight path of the projectile that is generating the wave; for a spherical wave, the distance from the point where the wave originated
r_o	initial value of r
S	for a plane wave, the distance of propagation
t	time

Δt propagation time required for the wave to traverse the distance between points on the ray path

T time associated with a waveform point; also, a time parameter defined by equation (39)

T_{ϵ_1} defined by equation (25)

T_{ϵ_2} defined by equation (26)

u fluid particle speed due to wave passage (positive in the direction of wave propagation)

$\vec{V}_o$ wind velocity

z distance above ground

z_1 initial altitude, where the pressure signature is known

z_2 altitude where the pressure signature is desired

ϵ_1 defined by sketch (c) and equation (20)

ϵ_2 defined by sketch (c) and equation (20) (if in eq. (20) ϵ_1 is replaced by ϵ_2 and ΔF_i is replaced by ΔF_{i+1})

γ ratio of specific heats

λ_i time duration of waveform segment i

λ_i^o initial value of λ_i

ρ_o ambient density

τ age variable

θ angle between $\vec{N}$ and horizontal plane

ξ phase, a variable used to identify or "tag" points on a waveform

Subscript Notation for Derivatives

$(\ )_z = \partial(\ )/\partial z$

$(\ )_\xi = \partial(\ )/\partial \xi$

$(\ )_{\xi z} = \partial^2(\ )/\partial z\, \partial \xi$

EXTRAPOLATION OF SONIC BOOM PRESSURE SIGNATURES

BY THE WAVEFORM PARAMETER METHOD

Charles L. Thomas

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SUMMARY

The waveform parameter method of sonic boom extrapolation is derived and shown to be equivalent to the F-function method. A computer program based upon the waveform parameter method is presented and discussed, with a sample case demonstrating program input and output.

INTRODUCTION

Sonic boom pressure waves created by high-flying supersonic aircraft propagate over extremely large distances through an atmosphere with temperature, pressure, and wind gradients before reaching the ground. The atmospheric gradients have a strong effect on both the wave amplitude and the nonlinear waveform distortion. Two sonic boom extrapolation methods that account for these atmospheric effects are the F-function method (ref. 1) and the waveform parameter method (ref. 2). These methods are based on the same fundamental concepts; both rely on a result from geometric acoustics for the wave amplitude, and both utilize isentropic wave theory to account for nonlinear waveform distortion. They differ, however, in the manner in which they account for nonlinear effects. The F-function method uses an age variable, calculated from the atmospheric properties, to distort the pressure signature. Shock waves are then located from the distorted signature by means of an area-balancing criterion. No shock-finding criterion is required in the waveform parameter method. Instead, a set of waveform parameters is defined that completely describes the waveform, and equations are obtained for the time rates of change of these parameters. These equations, which can be integrated over small time increments, are derived from the same basic equations used in the F-function method. No additional assumptions are required, and therefore the two methods are mathematically equivalent. The waveform parameter method appears to provide a more suitable approach for automatic computation because the necessity of using the area-balancing technique for locating shocks is eliminated. Also, in the special cases of plane, conical, and spherical pressure waves propagating in a uniform medium, closed-form expressions for the waveform parameters as functions of propagation distance can be obtained.

ANALYSIS AND DISCUSSION OF RESULTS

Both the F-function method and the waveform parameter method depend, for waveform amplitude, on a result derived from geometric acoustics—conservation of the Blokhintsev energy invariant along ray tubes (refs. 3 and 4). This conservation principle may be stated as

$$p\sqrt{\frac{c_n{}^2 A}{\rho_o a_o{}^3}} = F(\xi) \tag{1}$$

The quantities c_n, A, ρ_o, and a_o are functions only of z (altitude) and therefore vary along the ray tube. The acoustic pressure p is a function of both z and phase ξ. If the sonic boom pressure wave were truly an acoustic wave, the above expression would be sufficient for calculation of the sonic boom waveform at any point along the ray tube. However, because of the extremely large propagation distances involved, and because near the aircraft the wave is not extremely weak, the actual nonlinearity of pressure wave propagation causes a distortion of the waveform that must be taken into account. Both the F-function method and the waveform parameter method account for this nonlinear distortion by assuming that the speed of propagation of any point on the waveform is given by isentropic wave theory. The propagation speed of a point on a pressure wave is equal to the value of $u + a$ for the point. If the wave is assumed isentropic and of small amplitude, the propagation speed is given by

$$u + a = a_o\left(1 + \frac{\gamma + 1}{2\gamma}\,\frac{p}{p_o}\right) \tag{2}$$

or, in terms of the function $F(\xi)$

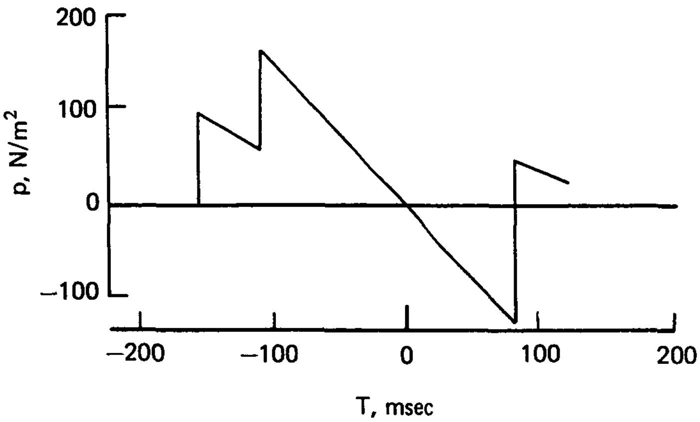
$$u + a = a_o + \frac{\gamma + 1}{2}\sqrt{\frac{a_o}{\rho_o A c_n{}^2}}\,F(\xi) \tag{3}$$

Let the pressure signature at some point along the ray tube be described by the pressure p versus time T plot that would be produced by a microphone located at that point. Also, let $T = 0$ correspond to a zero disturbance point on the waveform as shown by the example signature of sketch (a). Then the amount of nonlinear waveform distortion that occurs in a propagation time increment dt is given by

$$dT = \frac{a_o - (u + a)}{c_n}\,dt \tag{4}$$

If the altitude z of the wave is measured from the $T = 0$ point on the waveform, then

$$\frac{dz}{dt} = -a_o \sin\theta \tag{5}$$



Sketch (a)

and, using equation (3), the total nonlinear distortion that occurs during propagation from the initial altitude z_1 to the altitude of interest z_2 is given by

$$\Delta T(\xi) = \frac{\gamma + 1}{2} F(\xi) \int_{z_1}^{z_2} \frac{dz}{\sqrt{\rho_o a_o c_n{}^4 A \sin^2 \theta}} \tag{6}$$

The phase ξ is as yet undefined quantitatively. If ξ for a point on the waveform is taken to be the value of T corresponding to the same point on the initial signature—that is, the signature at z_1—then the required expression for $T(\xi)$ is

$$T = \xi - \tau F(\xi) \tag{7}$$

where the "age variable" τ is calculated from

$$\tau = -\frac{\gamma + 1}{2} \int_{z_1}^{z_2} \frac{dz}{\sqrt{\rho_o a_o c_n{}^4 A \sin^2 \theta}} \tag{8}$$

It is pointed out by Hayes (ref. 1) that in a horizontally stratified, steady atmosphere $c_n = c_o \cos\theta$ where c_o is constant along ray paths. It is possible to define the ray tube area as the area cut by a horizontal plane, rather than that cut by the wavefront. The relationship between the two areas can easily be shown to be

$$A_h = \frac{c_n}{a_o} \frac{A}{\sin\theta} \tag{9}$$

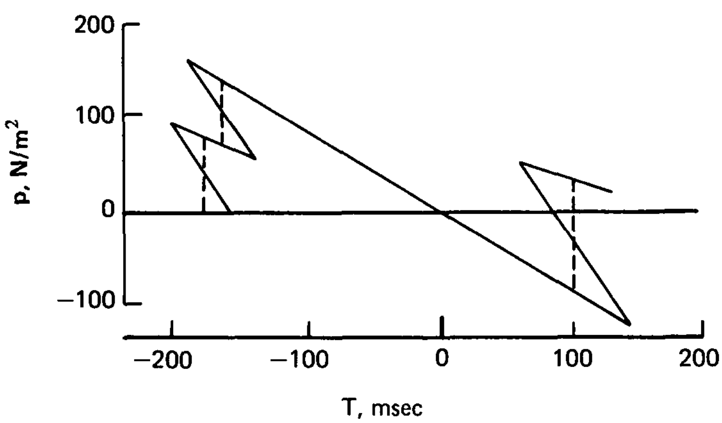
The expression for the age variable in terms of A_h is then

$$\tau \approx -\frac{\gamma + 1}{2c_o{}^{3/2}} \int_{z_1}^{z_2} \frac{dz}{\left(\rho_o a_o{}^2 A_h \sin^3 \theta \cos^3 \theta\right)^{1/2}} \tag{10}$$

In cases where sonic cutoff occurs, or nearly occurs, the use of A_h instead of A presents some difficulty due to $\sin\theta$ going to zero.

The above expression for τ is identical to that obtained in reference 1 (eq. (46)), except for the exponent of c_o, which in reference 1 is 1 instead of 3/2. This difference in exponent is due to a factor of $c_o^{-1/2}$ in the definition of $F(\xi)$ used in reference 1, where $F(\xi)$ is denoted $V_E(\xi)$. Since c_o is constant along ray tubes, including c_o in the definition of $F(\xi)$ is permissible.

The above equations are the fundamental equations of the F-function method. The F function $F(\xi)$ is first determined using a near-field signature and equation (1). The age variable corresponding to altitude z_2 is then calculated from equation (8) or equation (10). Then, using equations (1) and (7), one can plot p versus T at altitude z_2. The result will normally look as shown in sketch (b). It is seen that the signature (indicated by the solid line) is multivalued in p. The multivalued regions are eliminated by locating shock waves (indicated by dashed lines) according to the area-balancing rule. The area-balancing rule is a consequence of the fact that the propagation speed of a weak shock is equal to the average of the propagation speeds $(u + a)$ just ahead of and just behind the shock.



Sketch (b)

In the waveform parameter approach, the waveform is approximated by an arbitrary number of linear segments. For each segment the waveform parameters m_i, Δp_i, and λ_i are defined as follows: m_i is the slope $\partial p/\partial T$ of segment i, which may be positive or negative; Δp_i is the pressure rise across the shock at the juncture of segments i and $i-1$. Often there will be no shock at the juncture, in which case Δp_i is zero. Finally, λ_i is the time duration ΔT of segment i. A completely general waveform can be described using these waveform parameters.

Consider first the waveform parameter m_i, which is defined by

$$m_i = \frac{p_{\xi_i}}{T_{\xi_i}} \tag{11}$$

Then

$$\frac{dm_i}{dt} = -\frac{p_{\xi_i} T_{\xi z_i}}{T_{\xi_i}^2} \frac{dz}{dt} + \frac{p_{\xi z_i}}{T_{\xi_i}} \frac{dz}{dt} \tag{12}$$

The first term on the right-hand side of equation (12) is zero for a linear, nonplane wave, whereas the second term is zero for a nonlinear, plane wave propagating in a uniform, stationary medium. This means that the rate of change of the waveform parameter m_i can be obtained by superposition of the rate of change assuming the wave propagates as a linear, nonplane wave and the rate of change assuming the wave propagates as a nonlinear, plane wave. This concept of superposition of waveform parameter rates of change was first introduced, without proof, in reference 2. Equation (12) can be written as

$$\frac{dm_i}{dt} = -\frac{m_i^2 T_{\xi z_i}}{p_{\xi_i}} \frac{dz}{dt} + \frac{m_i}{p_{\xi_i}} \frac{\partial p_{\xi_i}}{\partial t} \tag{13}$$

Then, from

$$p_{\xi_i} = \sqrt{\frac{\rho_o a_o^3}{c_n^2 A}}\, F_i'(\xi) \tag{14}$$

$$T_{\xi z_i} = -F_i'(\xi)\tau_z \tag{15}$$

$$\tau_z = -\frac{\gamma + 1}{2} \sqrt{\frac{1}{\rho_o a_o c_n^4 A \sin^2 \theta}} \tag{16}$$

and equation (5), the time rate of change of the waveform parameter m_i is given by

$$\frac{dm_i}{dt} = \frac{\gamma + 1}{2\gamma} \frac{a_o}{p_o c_n} m_i^2 + \frac{1}{2}\left(\frac{3}{a_o} \frac{da_o}{dt} + \frac{1}{\rho_o} \frac{d\rho_o}{dt} - \frac{2}{c_n} \frac{dc_n}{dt} - \frac{1}{A} \frac{dA}{dt}\right) m_i \tag{17}$$

where the fractional time rates of change of the ambient properties a_o, ρ_o, and c_n and of the ray tube area A are understood to be the rates of change as seen by an observer moving down the ray tube with the wave.

The waveform parameter Δp_i can be expressed in terms of the jump in the F function across the shock as

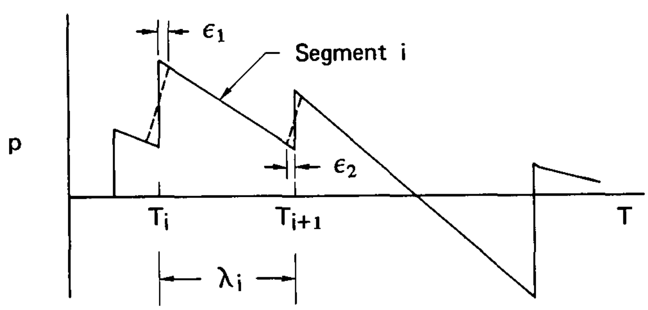
$$\Delta p_i = \sqrt{\frac{\rho_o a_o^3}{c_n^2 A}}\ \Delta F_i \tag{18}$$

Then

$$\frac{d\Delta p_i}{dt} = \frac{\Delta p_i}{\Delta F_i}\frac{d\Delta F_i}{dt} + \Delta p_i\sqrt{\frac{c_n^2 A}{\rho_o a_o^3}}\ \frac{d}{dt}\sqrt{\frac{\rho_o a_o^3}{c_n^2 A}} \tag{19}$$

The first term on the right-hand side of this equation is zero for a linear, nonplane wave, whereas the second term is zero for a nonlinear, plane wave propagating in a uniform, stationary medium. Therefore, the concept of superposition of waveform parameter rates of change is also valid for the parameter Δp_i The fractional time rate of change of ΔF_i can be determined as follows. Sketch (c) shows a typical signature at some point along a ray tube. The dashed lines are "equalizing lines" representing the shock waves at some time increment dt later. Using equation (3), the relationship between ϵ_1 and dt is

$$2\epsilon_1 c_n = \frac{\gamma + 1}{2}\sqrt{\frac{a_o}{\rho_o c_n^2 A}}\ \Delta F_i\, dt \tag{20}$$



Sketch (c)

Since from the area-balancing rule

$$\frac{d\Delta F_i}{dt} = \frac{\epsilon_1}{dt}\left(m_i + m_{i-1}\right)\sqrt{\frac{c_n^2 A}{\rho_o a_o^3}} \tag{21}$$

we have

$$\frac{1}{\Delta F_i}\frac{d\Delta F_i}{dt} = \frac{\gamma+1}{4\gamma}\frac{a_o}{p_o c_n}\left(m_i + m_{i-1}\right) \tag{22}$$

and therefore the time rate of change of the waveform parameter Δp_i is

$$\frac{d\Delta p_i}{dt} = \frac{\gamma+1}{4\gamma}\frac{a_o}{p_o c_n}\left(m_i + m_{i-1}\right)\Delta p_i + \frac{1}{2}\left(\frac{3}{a_o}\frac{da_o}{dt} + \frac{1}{\rho_o}\frac{d\rho_o}{dt} - \frac{2}{c_n}\frac{dc_n}{dt} - \frac{1}{A}\frac{dA}{dt}\right)\Delta p_i \tag{23}$$

The waveform parameter λ_i is defined as

$$\lambda_i = T_{i+1} - T_i \tag{24}$$

where T_i and T_{i+1} are the times corresponding to the shocks at each end of the waveform segment, as shown in sketch (c). Let

$$T_{\epsilon_1} = T_i + \epsilon_1 \tag{25}$$

$$T_{\epsilon_2} = T_{i+1} - \epsilon_2 \tag{26}$$

Since in time dt both ϵ_1 and ϵ_2 go to zero (by definition of ϵ_1 and ϵ_2), then

$$\frac{d\lambda_i}{dt} = \frac{d}{dt}\left(T_{\epsilon_2} - T_{\epsilon_1}\right) - \frac{\epsilon_1 + \epsilon_2}{dt} \tag{27}$$

Let F_{ϵ_1} and F_{ϵ_2} be the values of the F function $F(\xi)$ at T_{ϵ_1} and T_{ϵ_2}, respectively. Then, from equation (7)

$$\frac{d}{dt}\left(T_{\epsilon_2} - T_{\epsilon_1}\right) = -\left(F_{\epsilon_2} - F_{\epsilon_1}\right)\frac{d\tau}{dt} \tag{28}$$

$$= -m_i\lambda_i\sqrt{\frac{c_n{}^2 A}{\rho_o a_o{}^3}}\,\frac{d\tau}{dt} \tag{29}$$

Thus, from equations (5), (16), (18), and (20) the time rate of change of the waveform parameter λ_i is

$$\frac{d\lambda_i}{dt} = -\frac{\gamma+1}{4\gamma}\frac{a_o}{p_o c_n}\left(\Delta p_i + \Delta p_{i+1}\right) - \frac{\gamma+1}{2\gamma}\frac{a_o}{p_o c_n} m_i \lambda_i \tag{30}$$

We now have three ordinary, first-order, coupled differential equations for the waveform parameters m_i, Δp_i, and λ_i, which completely describe the deformation of the waveform. These equations can be written more concisely as

$$\frac{dm_i}{dt} = C_1 m_i^2 + C_2 m_i \tag{31}$$

$$\frac{d\Delta p_i}{dt} = \frac{1}{2} C_1 \Delta p_i \left(m_i + m_{i-1}\right) + C_2 \Delta p_i \tag{32}$$

$$\frac{d\lambda_i}{dt} = -\frac{1}{2} C_1 \left(\Delta p_i + \Delta p_{i+1}\right) - C_1 m_i \lambda_i \tag{33}$$

where

$$C_1 = \frac{\gamma+1}{2\gamma}\frac{a_o}{p_o c_n} \tag{34}$$

$$C_2 = \frac{1}{2}\left(\frac{3}{a_o}\frac{da_o}{dt} + \frac{1}{\rho_o}\frac{d\rho_o}{dt} - \frac{2}{c_n}\frac{dc_n}{dt} - \frac{1}{A}\frac{dA}{dt}\right) \tag{35}$$

The above equations differ from corresponding equations of reference 2 because the waveform parameters used here refer to a pressure versus time signature, while those of reference 2 refer to a pressure versus distance signature (the distance being measured normal to the wavefront).

In the general case of a wave with arbitrary wavefront shape propagating in a nonuniform atmosphere with winds the quantities C_1 and C_2 vary along the ray path. However, if these quantities are assumed to be constant over small time increments, the above waveform deformation equations can be integrated to obtain the following solutions.

$$m_i = \frac{m_i^o e^{C_2 \Delta t}}{1 - C_1 m_i^o T} \tag{36}$$

$$\Delta p_i = \frac{\Delta p_i^o e^{C_2 \Delta t}}{\left[\left(1 - C_1 m_i^o T\right)\left(1 - C_1 m_{i-1}^o T\right)\right]^{1/2}} \tag{37}$$

$$\lambda_i = \left(1 - C_1 m_i^o T\right)\left[\lambda_i^o - \frac{\Delta p_i^o}{m_i^o - m_{i-1}^o}\left(\sqrt{\frac{1 - C_1 m_{i-1}^o T}{1 - C_1 m_i^o T}} - 1\right)\right.$$

$$\left. - \frac{\Delta p_{i+1}^o}{m_i^o - m_{i+1}^o}\left(\sqrt{\frac{1 - C_1 m_{i+1}^o T}{1 - C_1 m_i^o T}} - 1\right)\right] \tag{38}$$

where

$$T = \frac{e^{C_2 \Delta t} - 1}{C_2} \tag{39}$$

The above solutions for m_i, Δp_i, and λ_i are used to determine the values of the waveform parameters at some point on the ray path, given m_i^o, Δp_i^o, and λ_i^o, which are the values of the parameters at the preceding point. The time increment Δt is the propagation time required for the wave to traverse the distance between points on the ray path. To use the above solutions for the waveform parameters one must first calculate the values of C_1 and C_2 at many points along the ray path of interest. The ray path can be calculated by the method outlined in reference 2. When the ray path is known, the fractional rates of change of the ambient properties a_o, ρ_o, and c_n can be determined at many points along the ray path. The fractional rate of change of ray tube area is also needed at each point along the ray path. Ray tube area may be calculated by theoretical methods, such as that of reference 1, or can be determined directly by independently calculating each of the four rays that bound the ray tube. Once the values of C_1 and C_2 are determined, the above solutions for the waveform parameters can be used to calculate the waveform at any point of interest along the ray path if the waveform near the aircraft is known.

The above expression for λ_i (eq. (38)) cannot be used when $m_i^o = m_{i-1}^o$ or when $m_i^o = m_{i+1}^o$. In the case when $m_i^o = m_{i-1}^o$, or, in general, when

$$\left|\frac{C_1\left(m_i^o - m_{i-1}^o\right)T}{1 - C_1 m_i^o T}\right| < 0.001 \tag{40}$$

we can use the approximation

$$\frac{\Delta p_i^o}{m_i^o - m_{i-1}^o}\left(\sqrt{\frac{1 - C_1 m_{i-1}^o T}{1 - C_1 m_i^o T}} - 1\right) \approx \frac{\Delta p_i^o}{2}\,\frac{C_1 T}{1 - C_1 m_i^o T} \tag{41}$$

A similar expression can be obtained when $m_i^o = m_{i+1}^o$.

As the wave propagates down the ray path, shocks often will coalesce and compression regions will steepen to form new shocks. When this occurs one or more of the λ_i will go to zero. When one of the λ_i does go to zero somewhere between two points on the ray path, the expression for λ_i given by equation (38) can be used to determine the value of Δt at which it goes to zero. The waveform parameters should then be incremented using this value of Δt to determine the waveform at the point where the waveform segment goes to zero. The waveform parameters must then be redefined so that the new waveform (which has one less segment and possibly one more shock) is represented correctly.

Extrapolations of plane, conical, and spherical pressure waves propagating in a uniform medium can be done easily by hand if the waveform is of a very simple shape. Solutions of the waveform deformation equations for plane, conical, and spherical waves are presented in appendix A.

A sonic boom extrapolation program, based on the waveform parameter method described above, is presented and discussed in appendix B. The program includes the effects of aircraft accelerations, aircraft flight path angle, and atmospheric temperature, pressure, and wind gradients. The program has been checked using the well-established Hayes program (ref. 1) and has been found to give nearly identical results for all types of flight conditions.

CONCLUDING REMARKS

The waveform parameter method for extrapolation of weak pressure waves has been shown to be valid, within the same limitations as the F-function theory. The waveform parameter method appears to provide a more suitable approach for automatic computation because the necessity of using the area-balancing technique for locating shocks is eliminated. The waveform parameter method also makes possible closed-form solutions for plane, conical, and spherical waves propagating in uniform media.

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APPENDIX A

SOLUTIONS OF THE WAVEFORM DEFORMATION EQUATIONS FOR PLANE, CONICAL, AND SPHERICAL WAVES

Plane, conical, and spherical pressure waves propagating in a uniform, stationary gas can be treated very simply by the waveform parameter method. The waveform deformation equations (31), (32), and (33) can be solved in closed form in these special cases. The solutions are found to be the following.

$$\frac{m_i}{m_i^o} = \frac{1}{Q_1\left(1 - \frac{\gamma + 1}{2\gamma}\frac{m_i^o}{a_o p_o} Q_2\right)}$$

$$\Delta p_i = \Delta p_i^o \sqrt{\frac{m_i}{m_i^o}}\sqrt{\frac{m_{i-1}}{m_{i-1}^o}}$$

$$\lambda_i = \frac{1}{Q_1}\frac{m_i^o}{m_i}\left[\lambda_i^o - \frac{\Delta p_i^o}{m_i^o - m_{i-1}^o}\left(\sqrt{\frac{m_i}{m_i^o}}\sqrt{\frac{m_{i-1}^o}{m_{i-1}}} - 1\right) - \frac{\Delta p_{i+1}^o}{m_i^o - m_{i+1}^o}\left(\sqrt{\frac{m_i}{m_i^o}}\sqrt{\frac{m_{i+1}^o}{m_{i+1}}} - 1\right)\right]$$

where the quantities Q_1 and Q_2 are given in the following table.

	Q_1	Q_2
Plane wave	1	S
Conical wave	$\sqrt{\frac{r}{r_o}}$	$\frac{2Mr_o}{\left(M^2 - 1\right)^{1/2}}\left(\sqrt{\frac{r}{r_o}} - 1\right)$
Spherical wave	$\frac{r}{r_o}$	$r_o \ln\frac{r}{r_o}$

For a plane wave, S is the distance of propagation. For a conical wave, r is the distance from the flight trajectory of the projectile that is generating the wave, and M is the Mach number of the projectile. For a spherical wave, r is the distance from the point where the wave originated. In both cases, r_o is the value of r corresponding to the initial pressure signature.

If the value of λ_i computed from the above equation is negative, the equation for λ_i must be used to determine the value of r or S at which λ_i becomes zero. The initial signature should then be redefined to be the signature at this value of r or S.

It should be remembered that the above solutions for the waveform parameters assume a weak, isentropic wave. If $p/p_o < 0.1$ over the entire signature, this assumption is justifiable. If the wave is conical or spherical and the initial signature is near the point where the wave originated, the waveform amplitude will attenuate rapidly due to a large fractional rate of change of ray tube area. In these cases, the maximum value of p/p_o can be somewhat larger than 0.1 and the above expressions for the waveform parameters will still be applicable.

APPENDIX B

A COMPUTER PROGRAM FOR SONIC BOOM EXTRAPOLATIONS

The sonic boom extrapolation program presented here is based on the waveform parameter method described in this report. This computer program has essentially the same capability as the extrapolation program of reference 1. Furthermore, both programs have the same limitations, namely, those of geometric acoustics and isentropic wave theory. The advantage of the waveform parameter approach is that it provides a more appropriate base for automatic computation since no shock-finding technique is needed. As a result, the program presented here is considerably shorter and simpler than the program of reference 1. Other advantages of the program presented here are the following. No F-function calculation is required, thus simplifying extrapolations of experimental signatures. No problems arise in extrapolating near field signatures corresponding to multivalued F functions. The atmospheric pressure variation with altitude is computed within the program, and therefore does not have to be input. Aircraft accelerations are more conveniently specified. Sonic boom "footprints" are more easily determined since both aircraft location and ground-ray intersections are specified in terms of geographical longitude and latitude. The sonic cutoff angle is computed, thus eliminating timely trial-and-error determinations of this angle. Signatures at more than one altitude can be output on each run. Also, signatures can be output in terms of Δp (psf), $\Delta p/p$, x (ft), or t (msec). Finally, the effect of varying aircraft length can be easily determined, without requiring recalculation of the aircraft F function for each aircraft length. Because of these advantages the extrapolation program presented here is considerably more convenient to use than the program presented in reference 1.

In this extrapolation program ray path calculations are performed by the method outlined in reference 2. Ray tube areas are determined directly by independently calculating each of the four rays that bound the ray tube.

Input to the program consists primarily of aircraft flight conditions, atmospheric properties, and near-field signature data. Flight conditions include Mach number, altitude, flight path angle, and aircraft accelerations. The three components of aircraft acceleration are expressed in terms of $\dot{M}$, $\dot{\gamma}$, and $\dot{\psi}$, which are the time rates of change of Mach number, flight path angle, and heading, respectively. Atmospheric properties are input in the form of temperature and wind velocity profiles. The atmospheric pressure variation with altitude is computed in the program from the input temperature profile, using the perfect gas law and the hydrostatic equation. Winds in both the northerly and easterly directions are input, allowing wind shear. However, vertical winds and atmospheric turbulence are not accounted for in this program. The near-field signature data consist of the signature itself and its corresponding location relative to the aircraft. The signature is input in the form of p/p_O (more commonly written as $\Delta p/p$) versus x, where x is a spatial coordinate measured parallel to the aircraft velocity vector. The signature that is input should be consistent with the flight conditions specified. The flight conditions that affect the near-field signature are Mach number and lift coefficient. Therefore, the lift coefficient should first be estimated from the aircraft weight, flight altitude, Mach number, and aircraft accelerations. The near-field signature corresponding to the Mach number and required lift coefficient can then be determined either experimentally by a wind tunnel test or by theoretical means. Normally, if near-field signatures are determined in the wind tunnel, signatures are obtained at several lift coefficients, or angles of attack, and in several azimuthal planes of the model. A near-field signature corresponding to a specific lift coefficient and location relative to the model can then be estimated by interpolation.

Program output consists of a listing of the input data and of output signatures at the desired altitudes. The location (latitude and longitude) where the output signature would be detected is also output so that sonic boom "footprints" can be determined. In the event that sonic cutoff occurs, the sonic cutoff altitude is output. Also, the minimum ϕ (azimuthal angle) for which sonic cutoff occurs is output. In the event that a ray tube area goes to zero, due to large aircraft accelerations, the program will output the altitude at which the ray tube area goes to zero and the location of the ground-ray intersection. The computation will then be terminated. Because the basic theory is one of geometric acoustics, the theory breaks down in regions where ray tube areas go to zero. All that can be said regarding the wave amplitude is that a "superboom" is likely to occur if the ray tube area goes to zero near the ground. Although the program cannot be used to predict the amplitude of a superboom, it can be used to predict where superbooms are likely to occur.

Program Input Variables

TITLE	title card. 80 columns available.
MACH	flight Mach number
FLTALT	flight altitude above ground in feet.
MDOT	time rate of change of MACH (/sec)
PSIDOT	time rate of change of HEAD (deg/sec)
GAMDOT	time rate of change of FPA (deg/sec)
FPA	aircraft flight path angle (deg)
LONGP	aircraft longitude (deg). West longitudes are taken to be positive. East longitudes are negative.
LATP	aircraft latitude (deg). North latitudes are positive. South latitudes are negative.
HEAD	aircraft heading in degrees from true north.
POG	atmospheric pressure on the ground in psf. Input POG = 1 if output signatures in terms of $\Delta p/p$ are desired. Input POG = 0 for uniform atmosphere extrapolations, or for any case where constant atmospheric pressure is desired.
NTEMP	number of atmospheric temperature values input. NTEMP $\geqslant$ 2
NWINDE	number of east wind values input. NWINDE $\geqslant$ 2
NWINDN	number of north wind values input. NWINDN $\geqslant$ 2
ZT(I)	altitudes (in thousands of feet) at which atmospheric temperatures are given. Program logic requires ZT(NTEMP) > FLTALT.
TO(I)	atmospheric temperatures in degrees Fahrenheit. The program assumes a linear variation of temperature between points.
ZWE(I)	altitudes (in thousands of feet) at which east wind values are given. Program logic requires ZWE(NWINDE) > FLTALT.
VOE(I)	wind speeds in east direction (blowing east) in ft/sec. The program assumes a linear variation of wind speed between points.
ZWN(I)	altitudes (in thousands of feet) at which north wind values are given. Program logic requires ZWN(NWINDN) > FLTALT.
VON(I)	wind speeds in north direction (blowing north) in ft/sec. The program assumes a linear variation of wind speed between points.

NALT number of altitudes at which output signatures are desired.

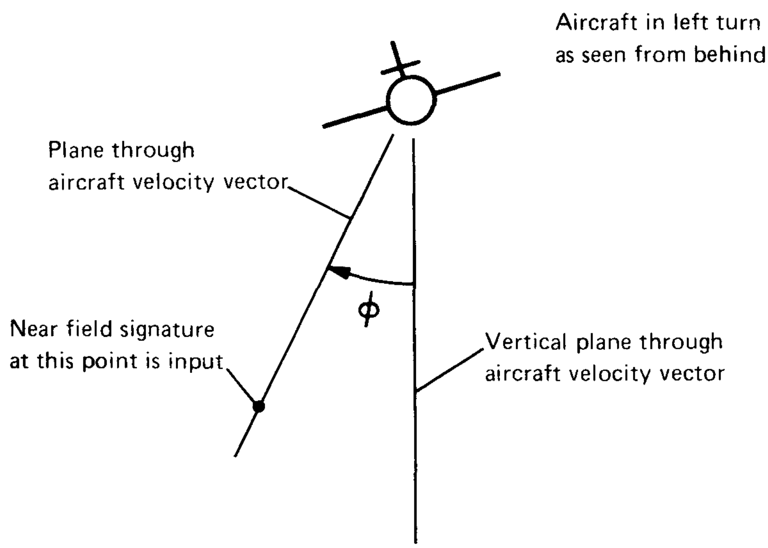
ALT(I) the altitudes (feet) at which signatures are desired, in descending order.

OCODE output code. OCODE = 1 for output signatures of pressure versus time (msec). OCODE = 2 for output signatures of pressure versus distance (feet), as measured along a line parallel to the horizontal component of the aircraft velocity (relative to the ground).

REFL sonic boom reflection factor. Signatures on the ground are multiplied by REFL.

ROVERL radial distance (nondimensionalized by the aircraft reference length) from the flight path corresponding to the input pressure signature.

PHI the lateral angle in degrees corresponding to the input pressure signature and aircraft bank angle. PHI is measured from the vertical plane passing through the aircraft velocity vector. Positive values of PHI correspond to rays that start out to the left of vertical, as seen from behind the aircraft.



Sketch (d)

NX number of values input for X(I) and DPP(I)

X(I) The near field pressure signature is input as a $\Delta p/p$ versus x signature, where x is a spatial coordinate measured parallel to the aircraft velocity vector. X(I) is a table of x values for the near field signature. X(I) can be input in any unit of length.

DPP(I) the value of $\Delta p/p$ corresponding to X(I). The program assumes a linear variation of $\Delta p/p$ between points.

AL	aircraft reference length in feet.
ML	model reference length in the same unit of length as used for X(I).

PROGRAM INPUT – SAMPLE CASE

```
SPACE SHUTTLE BOOSTER REENTRY    STANDARD ATMOSPHERE    EASTERLY WINDS
1.20    50400.
-.0197 -.359    1.013   -12.75
119.88 27.60    356.5
2116.2
     7     8    2
0.      36.2    65.8    105.5   155.5   172.0   202.0
59.0    -69.7   -69.7   -48.1   27.5    27.5    -4.8
0.      30.     40.     45.     55.     65.     83.     110.
5.      68.     84.     79.     36.     19.     16.     34.
0.      202.
0.      0.
     2
30000. 0.
     1
1.9
5.30    47.
    21
6.8     7.1     7.4     7.6     7.8     8.7     9.1     9.4     9.6     10.0
11.7    13.3    14.7    16.2    17.7    19.2    20.6    22.1    22.4    22.7
22.9
.000    .002    .015    .017    .018    .016    .021    .029    .034    .033
.022    .012    .004    -.003   -.008   -.014   -.022   -.029   -.022   -.004
.000
256.0   10.0
```

PROGRAM OUTPUT – SAMPLE CASE

SPACE SHUTTLE BOOSTER REENTRY   STANDARD ATMOSPHERE   EASTERLY WINDS

FLIGHT ALTITUDE = 50400. FEET

MACH NUMBER = 1.200

HEADING = 356.5 DEGREES

FLIGHT PATH ANGLE = -12.75 DEGREES

M-DOT = -.0197 /SEC

PSI-DOT = -0.359 DEGREES/SEC

GAMMA-DOT = 1.013 DEGREES/SEC

AIRCRAFT LONGITUDE = 119.880 DEGREES

AIRCRAFT LATITUDE = 27.600 DEGREES

AIRCRAFT LENGTH = 256.0 FEET

ATMOSPHERIC CONDITIONS

POG = 2116.20 PSF

ZT FEET	TO DEG F	ZWE FEET	VOE FT/SEC	ZWN FEET	VON FT/SEC
0.0	59.0	0.0	5.0	0.0	0.0
36200.0	-69.7	30000.0	68.0	202000.0	0.0
65799.9	-69.7	40000.0	84.0		
105500.0	-48.1	45000.0	79.0		
155500.0	27.5	55000.0	36.0		
172000.0	27.5	65000.0	19.0		
202000.0	-4.8	83000.0	16.0		
		110000.0	34.0		

INITIAL WAVEFORM R/L = 5.30 PHI = 47.00 DEGREES

X/L	DP/P
0.000	0.00000
0.030	0.00200
0.060	0.01500
0.080	0.01700
0.100	0.01800
0.190	0.01600
0.230	0.02100
0.260	0.02900
0.280	0.03400
0.320	0.03300
0.490	0.02200
0.650	0.01200
0.790	0.00400
0.940	-0.00300
1.090	-0.00800
1.240	-0.01400
1.380	-0.02200
1.530	-0.02900
1.560	-0.02200
1.590	-0.00400
1.610	0.00000

WAVEFORM AT ALTITUDE = 30000.0 FEET

LONGITUDE = 119.922 DEGREES
LATITUDE = 27.671 DEGREES

T, MSEC	P, PSF
0.0	0.000
0.0	1.477
12.5	1.412
136.5	0.770
238.4	0.257
333.8	-0.193
411.7	-0.513
498.3	-0.899
551.1	-1.164
551.1	0.000

GROUND-RAY INTERSECTION

LONGITUDE = 120.051 DEGREES
LATITUDE = 27.860 DEGREES
DISTANCE FROM GROUND TRACK = 9.31 MILES

WAVEFORM AT THE GROUND REFLECTION FACTOR = 1.90

T, MSEC	P, PSF
0.0	0.000
0.0	1.359
133.2	0.789
258.6	0.263
374.6	-0.197
467.1	-0.525
571.4	-0.919
608.2	-1.074
608.2	0.001

```
 1  FORMAT(20A4)
 2  FORMAT(10F7.0)
 3  FORMAT(3I5)
 4  FORMAT(////5X,24HSONIC CUTOFF ALTITUDE = ,F7.0,6H  FEET,
   1//5X,17HPHI FOR CUTOFF = ,F5.0,9H  DEGREES)
 5  FORMAT(5X,20A4,///)
 6  FORMAT(5X,18HFLIGHT ALTITUDE = ,F7.0,6H  FEET,//5X,14HMACH NUMBER
   1= ,F6.3,//5X,10HHEADING = ,F6.1,9H  DEGREES,//5X,20HFLIGHT PATH AN
   2GLE = ,F6.2,9H  DEGREES,//5X,8HM-DOT = ,F6.4,6H  /SEC,//5X,10HPSI-
   3DOT = ,F6.3,13H  DEGREES/SEC,//5X,12HGAMMA-DOT = ,F6.3,13H  DEGREE
   4S/SEC,//5X,21HAIRCRAFT LONGITUDE = ,F8.3,9H  DEGREES,//5X,20HAIRCR
   5AFT LATITUDE = ,F7.3,9H  DEGREES,//5X,18HAIRCRAFT LENGTH = ,F5.1,
   66H  FEET,////5X,22HATMOSPHERIC CONDITIONS)
 7  FORMAT(/5X,6HPOG = ,F7.2,5H  PSF)
 8  FORMAT(//9X,2HZT,11X,2HTO,11X,3HZWE,11X,3HVOE,11X,3HZWN,11X,3HVON/
   18X,4HFEET,9X,5HDEG F,9X,4HFEET,9X,6HFT/SEC,9X,4HFEET,9X,6HFT/SEC/)
 9  FORMAT(F14.1,F11.1,F15.1,F12.1,F16.1,F12.1)
10  FORMAT(25X,F15.1,F12.1,F16.1,F12.1)
11  FORMAT(25X,F15.1,F12.1)
12  FORMAT(52X,F16.1,F12.1)
13  FORMAT(F14.1,F11.1,27X,F16.1,F12.1)
14  FORMAT(F14.1,F11.1)
15  FORMAT(F14.1,F11.1,F15.1,F12.1)
16  FORMAT(//////5X,16HINITIAL WAVEFORM,13X,6HR/L = ,F5.2,13X,6HPHI =
   1,F7.2,8H DEGREES,///16X,3HX/L,17X,4HDP/P,//(F20.3,F21.5))
19  FORMAT(1H1)
20  FORMAT(///5X, 23HWAVEFORM AT ALTITUDE = ,F8.1,6H  FEET,//9X,12HLON
   1GITUDE = ,F8.3,9H  DEGREES,/9X,11HLATITUDE = ,F7.3,9H  DEGREES,//)
21  FORMAT(13X,7HT, MSEC,15X,6HP, PSF,//(F20.1,F20.3))
22  FORMAT(13X,7HX, FEET,14X,6HP, PSF,//(F20.2,F20.3))
23  FORMAT(13X,7HT, MSEC,15X,4HDP/P,//(F20.1,F20.6))
24  FORMAT(13X,7HX, FEET,14X,4HDP/P,//(F20.2,F20.6))
27  FORMAT(8X,34HRAY TUBE AREA GOES TO ZERO AT Z = ,F8.1,6H  FEET)
28  FORMAT(/5X,28HCONSTANT PRESSURE ATMOSPHERE)
29  FORMAT(///5X, 23HGROUND-RAY INTERSECTION,//9X,12HLONGITUDE = ,F8.3
   1,9H  DEGREES,/9X,11HLATITUDE = ,F7.3,9H  DEGREES,/9X,29HDISTANCE F
   2ROM GROUND TRACK = ,F6.2,7H  MILES,////5X,22HWAVEFORM AT THE GROUN
   3D,10X,20HREFLECTION FACTOR = ,F4.2,//)
    INTEGER   OCODE
    REAL   MACH,N(3,4),K1,M(100),LAMDA(100),ML,MAG,MDOT,LATP,LONGP
    DIMENSION  TITLE(20),R(3,4),ZT(100),TO(100),ZWE(100),VOE(100),
   1 ZWN(100),VON(100),ALT(50),X(100),DPP(100),F1(100),F2(100),
   2 P(100),DP(100),XX(100),PP(100),V0(3),A01(4),V01(3,4)
    DOUBLE PRECISION   MMACH,FFPA,HHEAD,MU
    WRITE(6,19)
40  READ(5,1)      TITLE
    READ(5,2)      MACH, FLTALT
    READ(5,2)      MDOT, PSIDOT, GAMDOT, FPA
    READ(5,2)      LONGP, LATP, HEAD
    READ(5,2)      POG
    READ(5,3)      NTEMP, NWINDE, NWINDN
    READ(5,2)      (ZT(I), I=1,NTEMP)
    READ(5,2)      (TO(I), I=1,NTEMP)
    READ(5,2)      (ZWE(I), I=1,NWINDE)
    READ(5,2)      (VOE(I), I=1.NWINDE)
    READ(5,2)      (ZWN(I), I=1,NWINDN)
    READ(5,2)      (VON(I), I=1,NWINDN)
    READ(5,3)      NALT
    READ(5,2)      (ALT(I), I=1,NALT)
```

```
      READ(5,3)      OCODE
      READ(5,2)      REFL
      READ(5,2)      ROVERL, PHI
      READ(5,3)      NX
      READ(5,2)      (X(I), I=1,NX)
      READ(5,2)      (DPP(I), I=1,NX)
      READ(5,2)      AL, ML
      DELTAT=.1
      PHISAV=PHI
      RO=ROVERL*AL
      G=32.26-0.17*COS(LATP/57.296)**2
      IF(POG .GT. 0.)   GO TO 45
      G=0.
      POG=1.
   45 ICUT=0
      DPHI=-10.
      IF(PHI .LT. 0.)   DPHI=10.
      XOFF = X(1)
      DO 50 I=1,NX
   50 X(I)=(X(I)-XOFF)/ML
      DO 60   I=1,NTEMP
   60 ZT(I)=1000.0*ZT(I)
      DO 70   I=1,NWINDE
   70 ZWE(I)=1000.0*ZWE(I)
      DO 80   I=1,NWINDN
   80 ZWN(I)=1000.0*ZWN(I)
      WRITE(6,5)   TITLE
      WRITE(6,6)  FLTALT,MACH,HEAD,FPA,MDOT,PSIDOT,GAMDOT,LONGP,LATP,AL
      IF(G .GT. 0.)   GO TO 85
      WRITE(6,28)
      GO TO 90
   85 WRITE(6,7)   POG
   90 WRITE(6,8)
      I=1
  100 IF(I .GT. NTEMP)   GO TO 110
      IF(I .GT. NWINDE)   GO TO 140
      IF(I .GT. NWINDN)   GO TO 160
      WRITE(6,9)  ZT(I),TO(I),ZWE(I),VOE(I),ZWN(I),VON(I)
      GO TO 170
  110 IF(I .GT. NWINDE)   GO TO 130
      IF(I .GT. NWINDN)   GO TO 120
      WRITE(6,10)  ZWE(I),VOE(I),ZWN(I),VON(I)
      GO TO 170
  120 WRITE(6,11)   ZWE(I),VOE(I)
      GO TO 170
  130 IF(I .GT. NWINDN)  GO TO 180
      WRITE(6,12)   ZWN(I),VON(I)
      GO TO 170
  140 IF(I .GT. NWINDN)   GO TO 150
      WRITE(6,13)   ZT(I),TO(I),ZWN(I),VON(I)
      GO TO 170
  150 WRITE(6,14)   ZT(I),TO(I)
      GO TO 170
  160 WRITE(6,15)  ZT(I),TO(I),ZWE(I),VOE(I)
  170 I=I+1
      GO TO 100
  180 IF(I .LT. 10)   WRITE(6,19)
      WRITE(6,16)   ROVERL,PHI,(X(I),DPP(I),I=1,NX)
      WRITE(6,19)
```

```
200   IF(ICUT .EQ. 0)   GO TO 230
      IF(PHISAV .LT. 0.)   GO TO 210
      IF(PHI .GT. 0.)   GO TO 230
      DPHI=1.
      GO TO 220
210   IF(PHI .LT. 0.)   GO TO 230
      DPHI=-1.
220   PHI=0.
230   MMACH=MACH
      FFPA=FPA/57.296
      HHEAD=HEAD/57.296
      PHI=PHI/57.296
      R(1,1)=0.
      R(2,1)=0.
      R(3,1)=FLTALT
      IZERO=0
      IFORK=1
      II=1
      IRAY=2
      H=FLTALT
240   DO 250  J=2,NTEMP
      IF(ZT(J) .LT. H)   GO TO 250
      A0=49.1*SQRT(TO(J-1)+(H-ZT(J-1))/(ZT(J)-Z T(J-1))*(TO(J)-TO(J-1))
     1+459.67)
      GO TO 260
250   CONTINUE
260   DO 270  J=2,NWINDE
      IF(ZWE(J) .LT. H)   GO TO 270
      VO(1)=VOE(J-1)+(H-ZWE(J-1))/(ZWE(J)-ZWE(J-1))*(VOE(J)-VOE(J-1))
      GO TO 280
270   CONTINUE
280   DO 290  J=2,NWINDN
      IF(ZWN(J) .LT. H)   GO TO 290
      VO(2)=VON(J-1)+(H-ZWN(J-1))/(ZWN(J)-ZWN(J-1))*(VON(J)-VON(J-1))
      GO TO 300
290   CONTINUE
300   IF(IRAY .GT. 1 .OR. ICUT .EQ. 1)   GO TO 340
      PO=POG
      DO 330  J=2,NTEMP
      Z2=ZT(J)
      IF(H .LE. Z2)   Z2=H
      IF(TO(J-1) .EQ. TO(J))   GO TO 310
      TOZ=(TO(J)-TO(J-1))/(ZT(J)-ZT(J-1))
      PO=PO*(1.+TOZ/(TO(J-1)+459.67)*(Z2-ZT(J-1)))**(-1./53.353/TOZ)
      GO TO 320
310   PO=PO*EXP((ZT(J-1)-Z2)/53.353/(TO(J-1)+459.67))
320   IF(Z2 .EQ. H)   GO TO 340
330   CONTINUE
340   GO TO (350,460,580),    IFORK
350   VOXH=VO(1)
      VOYH=VO(2)
      VX=MACH*A0*COS(FPA/57.296)*SIN(HEAD/57.296)
      VY=MACH*A0*COS(FPA/57.296)*COS(HEAD/57.296)
      IRAY=0
360   IRAY=IRAY+1
      GO TO (410,390,380,370,430),  IRAY
370   R(1,4)=R(1,3)
      R(2,4)=R(2,3)
      R(3,4)=R(3,3)
      GO TO 400
```

```
380  DT=0.17453*RO/AO/SQRT(MACH**2-1.0)
     MMACH=MMACH+MDOT*DT
     FFPA=FFPA+GAMDOT/57.296*DT
     HHEAD=HHEAD+PSIDOT/57.296*DT
     PHI=PHI-0.017453
     R(1,3)=R(1,1)+(VX+VO(1))*DT
     R(2,3)=R(2,1)+(VY+VO(2))*DT
     R(3,3)=R(3,1)+MACH*AO*SIN(FPA/57.296)*DT
     GO TO 410
390  R(1,2)=R(1,1)
     R(2,2)=R(2,1)
     R(3,2)=R(3,1)
400  PHI=PHI+0.017453
     GO TO 420
410  MU=DARSIN(1.0/MMACH)
420  N(1,IRAY)=DCOS(FFPA)*DSIN(HHEAD)*DSIN(MU)-DCOS(HHEAD)*DCOS(MU)
    1*SIN(PHI)+DSIN(FFPA)*DSIN(HHEAD)*DCOS(MU)*COS(PHI)
     N(2,IRAY)=DCOS(FFPA)*DCOS(HHEAD)*DSIN(MU)+DSIN(HHEAD)*DCOS(MU)
    1*SIN(PHI)+DSIN(FFPA)*DCOS(HHEAD)*DCOS(MU)*COS(PHI)
     N(3,IRAY)=DSIN(FFPA)*DSIN(MU)-DCOS(FFPA)*DCOS(MU)*COS(PHI)
     IF(IRAY .EQ. 4)   PHI=(PHI-.017453)*57.296
     GO TO 360
430  DO 440  IRAY=1,4
     IF(IRAY .EQ. 1)   DELT=RO/AO/DCOS(MU)
     IF(IRAY .EQ. 3)   DELT=DELT-DT
     R(1,IRAY)=R(1,IRAY)+(AO*N(1,IRAY)+VO(1))*DELT
     R(2,IRAY)=R(2,IRAY)+(AO*N(2,IRAY)+VO(2))*DELT
440  R(3,IRAY)=R(3,IRAY)+AO*N(3,IRAY)*DELT
     A1=((R(2,4)-R(2,1))*(R(3,3)-R(3,2))-(R(3,4)-R(3,1))*(R(2,3)
    1-R(2,2)))*N(1,1)+((R(3,4)-R(3,1))*(R(1,3)-R(1,2))-(R(1,4)-R(1,1))*
    2(R(3,3)-R(3,2)))*N(2,1)+((R(1,4)-R(1,1))*(R(2,3)-R(2,2))
    3-(R(2,4)-R(2,1))*(R(1,3)-R(1,2)))*N(3,1)
     IFORK=2
     IRAY=0
450  IRAY=IRAY+1
     H=R(3,IRAY)
     GO TO 240
460  AO1(IRAY)=AO
     VO1(1,IRAY)=VO(1)
     VO1(2,IRAY)=VO(2)
     IF(IRAY .LT. 4)   GO TO 450
     IF(ICUT .EQ. 1)   GO TO 530
     PO1=PO
     CN1=AO1(1)+VO1(1,1)*N(1,1)+VO1(2,1)*N(2,1)
     DO 470  I=1,NX
     P(I)=DPP(I)*PO
470  X(I)=X(I)/MACH*AL/CN1
     K=1
     J=2
     NN=NX-1
     M(1)=0.
     DO 520  I=1,NX
     IF(K .EQ. 2)   GO TO 510
     IF(I .EQ. NX)   GO TO 500
     IF(X(I) .EQ. X(I+1))   GO TO 480
     DP(J)=0.
     LAMDA(J)=X(I+1)-X(I)
     M(J)=(P(I+1)-P(I))/LAMDA(J)
     J=J+1
     GO TO 520
```

```
480   K=2
      DP(J)=P(I+1)-P(I)
      IF(I+2 .GT. NX)   GO TO 490
      LAMDA(J)=X(I+2)-X(I+1)
      M(J)=(P(I+2)-P(I+1))/LAMDA(J)
      NN=NN-1
      J=J+1
      GO TO 520
490   M(J)=0.
      NN=NN-1
      GO TO 520
500   DP(J)=0.
      M(J)=0.
510   K=1
520   CONTINUE
530   KRITE=0
      IF(DELTAT .EQ. .2)   GO TO 535
      DELTAT=.0002*SQRT(R(1,1)**2+R(2,1)**2+(FLTALT-R(3,1))**2)
      IF(DELTAT .GT. .2)   DELTAT=.2
535   IF(R(3,1)+A01(1)*N(3,1)*DELTAT-ALT(II))  540,540,550
540   DELT=(R(3,1)-ALT(II))/A01(1)/(-N(3,1))
      KRITE=1
      GO TO 560
550   DELT=DELTAT
560   IRAY=0
      A0S=A01(1)
570   IRAY=IRAY+1
      IF(IRAY .EQ. 5)   GO TO 610
      R(1,IRAY)=R(1,IRAY)+(A01(IRAY)*N(1,IRAY)+V01(1,IRAY))*DELT
      R(2,IRAY)=R(2,IRAY)+( A01(IRAY)*N(2,IRAY)+V01(2,IRAY))*DELT
      R(3,IRAY)=R(3,IRAY)+A01(IRAY)*N(3,IRAY)*DELT
      H=R(3,IRAY)
      IFORK=3
      GO TO 240
580   DZ=-A01(IRAY)*N(3,IRAY)*DELT
      A0Z=(A01(IRAY)-A0)/DZ
      V0XZ=(V01(1,IRAY)-V0(1))/DZ
      V0YZ=(V01(2,IRAY)-V0(2))/DZ
      A01(IRAY)=A0
      V01(1,IRAY)=V0(1)
      V01(2,IRAY)=V0(2)
      CC=-(N(1,IRAY)*V0XZ+N(2,IRAY)*V0YZ+A0Z)
      DN1=-N(1,IRAY)*N(3,IRAY)*CC*DELT
      DN2=-N(2,IRAY)*N(3,IRAY)*CC*DELT
      DN3=(N(1,IRAY)**2+N(2,IRAY)**2)*CC*DELT
      MAG=SQRT((N(1,IRAY)+DN1)**2+(N(2,IRAY)+DN2)**2+(N(3,IRAY)+DN3)**2)
      N(1,IRAY)=(N(1,IRAY)+DN1)/MAG
      N(2,IRAY)=(N(2,IRAY)+DN2)/MAG
      N(3,IRAY)=(N(3,IRAY)+DN3)/MAG
      IF(N(3,IRAY) .LT. -.01745)   GO TO 570
      IF(ICUT .EQ. 0)   GO TO 600
      IF(ABS(DPHI) .GT. 2.)   GO TO 590
      WRITE(6,4)   HCUT,PHI
      GO TO 1090
590   PHI=PHI+DPHI
      GO TO 200
600   HCUT=R(3,1)
      ICUT=1
      IPHI=PHI
      PHI=IPHI
      GO TO 590
```

```
610  IF(ICUT .EQ. 1 .OR. IZERO .EQ. 1)   GO TO 921
     CN =A01(1)+V01(1,1)*N(1,1)+V01(2,1)*N(2,1)
     A =((R(2,4)-R(2,1))*(R(3,3)-R(3,2))-(R(3,4)-R(3,1))*(R(2,3)
    1-R(2,2)))*N(1,1)+((R(3,4)-R(3,1))*(R(1,3)-R(1,2))-(R(1,4)-R(1,1))*
    2(R(3,3)-R(3,2)))*N(2,1)+((R(1,4)-R(1,1))*(R(2,3)-R(2,2))
    3-(R(2,4)-R(2,1))*(R(1,3)-R(1,2)))*N(3,1)
     IF(A .GT. 0.)   GO TO 620
     HZERO=R(3,1)
     IZERO=1
     GO TO 921
620  C1=1.714*(A01(1)+A0S)/(P01+P0)/(CN+CN1)
     C2=((A01(1)-A0S)/(A01(1)+A0S)+2.8*DZ*G/(A01(1)+A0S)**2-2.*(CN-CN1)
    1/(CN+CN1)-(A-A1)/(A+A1))/DELT
     P01=P0
     CN1=CN
     A1=A
630  DELTS=DELT
     TP=(EXP(C2*DELT)-1.0)/C2
     N1=NN+1
     N2=NN+2
     K=N2
640  J1=1
650  DO 660  J=1,N2
     IF(M(J) .EQ. M(K) .AND. J1 .EQ. 2 .AND. J .NE. K)  M(J)=.9999*M(J)
     F1(J)=1.0-C1*M(J)*TP
     IF(F1(J) .LE. 0. .AND. J .NE. K)   GO TO 670
660  CONTINUE
     GO TO 680
670  K=J
     J1=2
     TP=1.0/C1/M(J)
     GO TO 650
680  TK=C1*TP
     IF(J1 .EQ. 1)   GO TO 690
     IF(DP(K) .EQ. 0. .AND. DP(K+1) .EQ. 0.)   GO TO 690
     IF(DP(K) .EQ. 0.)   GO TO 760
     IF(DP(K+1) .EQ. 0.)   GO TO 780
     GO TO 800
690  KF=1
     J=1
695  J=J+1
     IF(J .GT. N1)   GO TO 840
     IF(J .EQ. K)   GO TO 695
700  IF(ABS(TK*(M(J)-M(J-1))/F1(J)) .GT. .001)    GO TO 710
     IF(ABS(TK*(M(J)-M(J+1))/F1(J)) .GT. .001)    GO TO 705
     F2(J)=LAMDA(J)-(DP(J)+DP(J+1))*TK/F1(J)/2.
     GO TO (730,820),  KF
705  F2(J)=LAMDA(J)-DP(J)*TK/F1(J)/2.-DP(J+1)/(M(J)-M(J+1))*(SQRT(F1(J+
    11)/F1(J))-1.)
     GO TO (730,820),  KF
710  IF(ABS(TK*(M(J)-M(J+1))/F1(J)) .GT. .001)    GO TO 720
     F2(J)=LAMDA(J)-DP(J)/(M(J)-M(J-1))*(SQRT(F1(J-1)/F1(J))-1.)-DP(J+1
    1)*TK/F1(J)/2.
     GO TO (730,820),  KF
720  F2(J)=LAMDA(J)-DP(J)/(M(J)-M(J-1))*(SQRT(F1(J-1)/F1(J))-1.)-DP(J+1
    1)/(M(J)-M(J+1))*(SQRT(F1(J+1)/F1(J))-1.)
     GO TO (730,820),  KF
730  IF(F2(J))   740, 735, 695
```

```
735   IF(J1 .EQ. 2)   LAMDA(J)=1.0001*LAMDA(J)
      IF(J1 .EQ. 2)   GO TO 700
      K=J
      GO TO 695
740   K=J
750   IF(DP(K) .GT. 0.)   GO TO 770
      IF(ABS(TK*(M(K)-M(K+1))/F1(K)) .GT. .001)    GO TO 760
      TP=2.*LAMDA(K)/C1/(DP(K+1)+2.*M(K)*LAMDA(K))
      GO TO 640
760   TP=(1.-(1.+LAMDA(K)/DP(K+1)*(M(K)-M(K+1)))**2)/(C1*(M(K+1)-M(K)*
     1(1.+LAMDA(K)/DP(K+1)*(M(K)-M(K+1)))**2))
      GO TO 640
770   IF(DP(K+1) .GT. 0.)   GO TO 790
      IF(ABS(TK*(M(K)-M(K-1))/F1(K)) .GT. .001)    GO TO 780
      TP=2.*LAMDA(K)/C1/(DP(K)+2.*M(K)*LAMDA(K))
      GO TO 640
780   TP=(1.-(1.+LAMDA(K)/DP(K)*(M(K)-M(K-1)))**2)/(C1*(M(K-1)-M(K)*(1.
     1+LAMDA(K)/DP(K)*(M(K)-M(K-1)))**2))
      GO TO 640
790   IF(ABS(TK*(M(K)-M(K-1))/F1(K)) .GT. .001)    GO TO 800
      IF(ABS(TK*(M(K)-M(K+1))/F1(K)) .GT. .001)    GO TO 800
      TP=2.*LAMDA(K)/C1/(DP(K)+DP(K+1)+2.*M(K)*LAMDA(K))
      GO TO 640
800   TP1=0.
      TP2=TP
      KF=2
      IC=0
810   IC=IC+1
      IF(IC .GT. 20)   GO TO 640
      TP=(TP1+TP2)/2.
      TK=C1*TP
      F1(J)=1.0-C1*M(J)*TP
      F1(J-1)=1.0-C1*M(J-1)*TP
      F1(J+1)=1.0-C1*M(J+1)*TP
      GO TO 700
820   IF(F2(K) .LE. 0.)   GO TO 830
      TP1=TP
      GO TO 810
830   TP2=TP
      GO TO 810
840   DELT=ALOG(TP*C2+1.0)/C2
      E2=EXP(C2*DELT)
      IF(K .EQ. N2)   GO TO 850
      DPSUM=DP(K)+DP(K+1)
      IF(K .EQ. 2)   GO TO 870
850   K11=K-1
      DO 860  J=2,K11
      M(J)=M(J)/F1(J)*E2
      DP(J)=DP(J)*E2/SQRT(F1(J)*F1(J-1))
860   LAMDA(J)=F1(J)*F2(J)
      IF(K .EQ. N2)   GO TO 920
870   K22=K+1
      IF(DPSUM .EQ. 0.)   GO TO 880
      DP(K)=E2*(DP(K)/SQRT(F1(K)*F1(K-1))+DP(K+1)/SQRT(F1(K)*F1(K+1)))
      GO TO 890
880   DP(K)=M(K)*LAMDA(K)*E2
890   M(K)=M(K+1)/F1(K+1)*E2
      IF(K .EQ. N1)   GO TO 910
      DO 900  J=K22,N1
      M(J)=M(J+1)/F1(J+1)*E2
      DP(J)=DP(J+1)*E2/SQRT(F1(J+1)*F1(J))
```

```
900   LAMDA(J-1)=F1(J)*F2(J)
910   NN=NN-1
      DELT=DELTS-DELT
      GO TO 630
920   DP(N2)=DP(N2)*E2/SQRT(F1(N2)*F1(N1))
921   IF(KRITE .EQ. 0)   GO TO 530
      IF(ICUT .EQ. 0)   GO TO 922
      II=II+1
      IF(II .LE. NALT)   GO TO 530
      DPHI=1.
      IF(PHISAV .LT. 0.)   DPHI=-1.
      GO TO 590
922   IF(IZERO .EQ. 0)   GO TO 926
      II=II+1
      IF(II .LE. NALT)   GO TO 530
      IF(ALT(NALT) .NE. 0.)   GO TO 924
      II=II-1
      GO TO 990
924   NALT=NALT+1
      ALT(NALT)=0.
      GO TO 530
926   XX(1)=0.
      PP(1)=0.
      J=2
      DO 940  L=2,N1
      IF(DP(L) .GT. 0.)   GO TO 930
      XX(J)=XX(J-1)+LAMDA(L)
      PP(J)=PP(J-1)+M(L)*LAMDA(L)
      J=J+1
      GO TO 940
930   XX(J)=XX(J-1)
      PP(J)=PP(J-1)+DP(L)
      XX(J+1)=XX(J)+LAMDA(L)
      PP(J+1)=PP(J)+M(L)*LAMDA(L)
      J=J+2
940   CONTINUE
      IF(DP(N2) .EQ. 0.)   GO TO 950
      XX(J)=XX(J-1)
      PP(J)=PP(J-1)+DP(N2)
      J=J+1
950   J=J-1
      IF(ALT(II) .NE. 0.)   GO TO 970
      DO 960  L=1,J
960   PP(L)=PP(L)*REFL
970   IF(POG .NE. 1.)   GO TO 990
      DO 980  L=1,J
980   PP(L)=PP(L)/PO
990   RLAT=R(2,1)/(20890070.+ALT(II))*57.296+LATP
      RLONG=-R(1,1)/(20890070.+ALT(II))*57.296/COS(RLAT/57.296)+LONGP
      IF(ALT(II) .NE. 0.)   GO TO 1000
      DGT=(R(2,1)*SIN(HEAD/57.296)-R(1,1)*COS(HEAD/57.296))/5280.
      WRITE(6,29)   RLONG,RLAT,DGT,REFL
      IF(IZERO)   1010, 1010, 1080
1000  WRITE(6,20)   ALT(II),RLONG,RLAT
1010  IF(OCODE .EQ. 2)   GO TO 1040
      DO 1020  L=1,J
1020  XX(L)=XX(L)*1000.
      IF(POG .EQ. 1.)   GO TO 1030
      WRITE(6,21)   (XX(L),PP(L),L=1,J)
      GO TO 1070
```

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1030  WRITE(6,23)    (XX(L),PP(L),L=1,J)
      GO TO 1070
1040  C=((VX+VOXH)*N(1,1)+(VY+VOYH)*N(2,1))/SQRT((VX+VOXH)**2+
     1(VY+VOYH)**2)
      DO 1050 L=1,J
1050  XX(L)=XX(L)*CN/C
      IF(POG .EQ. 1.)   GO TO 1060
      WRITE(6,22)  (XX(L),PP(L),L=1,J)
      GO TO 1070
1060  WRITE(6,24)  (XX(L),PP(L),L=1,J)
1070  II=II+1
      KRITE=0
      IF(II-NALT)    530, 530, 1090
1080  WRITE(6,27)    HZERO
1090  WRITE(6,19)
      GO TO 40
      END
```

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