A Transient Solution for Domain Wall Motion *

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Abstract - *An analytic solution of transient wall motion in a bulk ferromagnetic material is obtained from the dynamic torque equations with a Gilbert damping term. The steady state solution reduces to previously obtained, well known solutions.*

Introduction

A one dimensional Bloch wall is assumed to exist in a bulk specimen of ferromagnetic material described by a uniaxial anisotropy constant K, an exchange constant A, and a saturation magnetization M. With the coordinate system shown in Fig. 1 the following torque equations may be derived from the vector equation of motion containing a viscous damping parameter α:

$$\frac{M\sin\theta}{\gamma}\frac{\partial\theta}{\partial t} + \frac{\alpha M}{\gamma}\sin^2\theta\,\frac{\partial\varphi}{\partial t} = H_y M\sin\theta\cos\varphi + 2A\sin^2\theta\,\nabla^2\varphi + 4A\sin\theta\cos\theta\,(\nabla\theta\cdot\nabla\varphi), \tag{1a}$$

$$\frac{M\sin\theta}{\gamma}\frac{\partial\varphi}{\partial t} - \frac{\alpha M}{\gamma}\frac{\partial\theta}{\partial t} = -H_y M\cos\theta\sin\varphi + H_z M\sin\theta + 2K\sin\theta\cos\theta + 2A\sin\theta\cos\theta(\nabla\varphi)^2 - 2A\,\nabla^2\theta. \tag{1b}$$

The fields H_y and H_z permit the introduction of stray and applied fields and γ is the gyromagnetic ratio. The z-axis is the easy direction. In the absence of an applied field in the hard or y-direction

$$H_y = -\frac{M}{\mu_o}\sin\theta\sin\varphi\ .$$

The following trial functions are introduced:

$$\varphi \neq f(y),\ \frac{\partial\theta}{\partial t} = C_1\sin\theta,\ \frac{\partial\theta}{\partial y} = C_2\sin\theta,\ \frac{\partial^2\theta}{\partial y^2} = C_2^{\,2}\sin\theta\cos\theta\ ,$$

in which φ, C_1, and C_2 may be functions of time but not of y. Substituting these functions into eqs. 1 and equating coefficients of $\sin\theta$ and $\sin\theta\cos\theta$ yields:

*This work was supported in part by the National Aeronautics and Space Administration under Research Grant NGR-44-006-001 and by the National Science Foundation under Grant GK3827.

$$C_1 + \alpha \frac{\partial\varphi}{\partial t} = -\frac{\gamma M}{\mu_o} \sin\varphi \cos\varphi \tag{2a}$$

$$\frac{\partial\varphi}{\partial t} - \alpha C_1 = \gamma H_z \tag{2b}$$

$$0 = \frac{M}{\mu_o} \sin^2\varphi + H_k - C_2^{\,2} C_A \tag{2c}$$

in which $H_k = 2K/M$, $C_A = 2A/M$, and H_z is the applied field.

Approximate Solution

An approximate solution is obtained if the additional assumption is made that φ, the angle associated with the component of M perpendicular to the wall plane, is small. From eqs. 2a and 2b with $\sin\varphi \cong \varphi$ and $\cos\varphi \cong 1$, a linear differential equation in φ results, and the solution for a step function applied field is

$$\varphi = \frac{\mu_o H_z}{\alpha M}\left(1 - e^{-\frac{\alpha}{1+\alpha^2}\frac{\gamma M}{\mu_o} t}\right)$$

and

$$C_1 = -\alpha \gamma H_z - \frac{\gamma H_z}{\alpha}\left(1 - e^{-\frac{\alpha}{1+\alpha^2}\frac{\gamma M}{\mu_o} t}\right).$$

From eq. 2c, C_2 is obtained as

$$C_2 = \left[\frac{\mu_o H_z^{\,2}}{\alpha^2 M}\left(1 - e^{-\frac{\alpha}{1+\alpha^2}\frac{\gamma M}{\mu_o} t}\right)^2 + H_k\right]^{\frac{1}{2}} / C_A^{\frac{1}{2}}.$$

Finally, the velocity is obtained from $v = dy/dt = -(\partial\theta/\partial t)/(\partial\theta/\partial y) = -C_1/C_2$ as

$$v = \frac{\gamma H_z}{\alpha}(A/K)^{\frac{1}{2}}\left(1 - e^{-\alpha\frac{\gamma M}{\mu_o} t}\right)\left[1 + \frac{h_z^{\,2}}{\alpha^2 h_k}\left(1 - e^{-\alpha\frac{\gamma M}{\mu_o} t}\right)^2\right]^{-\frac{1}{2}},$$

with $h_z = \mu_o H_z/M$, and $h_k = \mu_o H_k/M$,

and $\alpha^2 << 1$.

The steady-state velocity corresponds to the solution previously obtained by Feldtkeller [1]. The effective time constant for $\alpha = 0.01$ and $M = 1.0$ w/m^2 is 0.5 nanosecond. The transient behavior of C_2 corresponds to the wall contraction that takes place as the wall accelerates. C_2 may also be displayed as a function of velocity.

$$C_2 = [A/K - \mu_o v^2/2\gamma^2 K]^{\frac{1}{2}} .$$

For small v or small H_z, the wall contraction is negligible and the velocity increases to the final value as a simple exponential.

"Exact" Solution

Eqs. 2 may also be integrated directly to yield:

$$\sin^2\varphi = \frac{AB(1 - e^{-\beta t})^2}{(A - Be^{-\beta t})^2 + AB(1 - e^{-\beta t})^2}$$

and

$$\sin\varphi\cos\varphi = \frac{(AB)^{\frac{1}{2}}(1 - e^{-\beta t})(A - Be^{-\beta t})}{(A - Be^{-\beta t})^2 + AB(1 - e^{-\beta t})^2} ,$$

in which

$$A = 1 + [1 - (\frac{2h_z}{\alpha})^2]^{\frac{1}{2}} , \quad \beta = \frac{\alpha}{1+\alpha^2} [1 - (\frac{2h_z}{\alpha})^2]^{\frac{1}{2}} \frac{\gamma M}{\mu_o} ,$$

$$B = 1 - [1 - (\frac{2h_z}{\alpha})^2]^{\frac{1}{2}} , \quad h_z = \mu_o H_z/M .$$

The time dependence of the velocity constant, the structure constant, and the velocity are given by

$$C_1 = \frac{1}{1+\alpha^2} [- \alpha\gamma H_z - \frac{\gamma M}{\mu_o} \sin\varphi\cos\varphi] ,$$

$$C_2 = (\frac{M}{\mu_o}\sin^2\varphi + H_k)^{\frac{1}{2}} C_A^{\frac{1}{2}} ,$$

$$v = - C_1/C_2 .$$

For $2h_z/\alpha << 1$ but not $(h_z/\alpha)^2 << h_k$ with $h_k = \mu_o H_k/M$, the solution reduces to the previous approximate solution. The steady-state velocity is given by

$$v = \frac{\gamma H_z}{\alpha} (A/K)^{\frac{1}{2}} [1 + \frac{1}{2h_k} (1 - \sqrt{1 - (\frac{2h_z}{\alpha})^2})]^{-\frac{1}{2}}$$

which is the Walker solution [2] discussed in detail by Schlömann [3],[4]. The solution is limited by $2h_z/\alpha = 1$ corresponding to $\varphi = 45°$ and

$$v\Big|_{h_z = \alpha/2} = \gamma(\frac{2A}{\mu_o})^{\frac{1}{2}}(2 + 4h_k)^{-\frac{1}{2}} .$$

Note that the effective damping constant, β , in the transient solution is zero at this point although the overall time response remains finite. The peak velocity and the corresponding applied field are given by

$$v_{pk} = \gamma\left(\frac{2A}{\mu_o}\right)^{\frac{1}{2}}[(1 + h_k)^{\frac{1}{2}} - h_k^{\frac{1}{2}}],$$

$$h_z\Big|_{v = v_{pk}} = \alpha\, h_k^{\frac{1}{4}}(1 + h_k)^{\frac{1}{4}}[(1 + h_k)^{\frac{1}{2}} - h_k^{\frac{1}{2}}] .$$

The motion predicted by this "exact" solution for a planar wall is well-behaved for $2h_z/\alpha \leq 1$. Schlömann [4] points out that there may be reason to doubt that the solution is valid for fields larger than $h_z\Big|_{v = v_{pk}}$. Slonczewski [5] reasons that if the wall is described by a negative differential mobility then instabilities characterizerized by corrugated variations along the wall may develop and the assumption of a planar wall is violated. The "exact" transient solution predicts that velocity changes in the negative differential mobility region are characterized by long time constants possibly requiring that the instabilities develop slowly. Experimental evidence in permalloy films indicates that a corrugated structure is much more likely to appear with relatively long pulses and that the wall remains planar for relatively short pulses.

A family of curves of the steady-state solution is shown in Fig. 2 for various h_k and normalized drive field, $2h_z/\alpha$. A corresponding transient response for a particular h_k and various drives is shown in Fig. 3. Notice that overshoots in the velocity are predicted for small h_k and large drives.

Summary

Analytic solutions are found for transient domain wall velocity in bulk uniaxial ferromagnetic materials excited by a step function easy-axis field. The solutions reduce to previously obtained steady-state solutions. The response time(s) associated with wall contraction and velocity is of the order of nanoseconds for permalloy materials but may be much longer for ortho-ferrite materials.

References

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Figure Captions

Fig. 1. Coordinate system.

Fig. 2. Normalized velocity versus normalized applied field from the exact steady-state solution.

Fig. 3. Normalized velocity versus normalized time from the exact theory with normalized drive as a parameter.

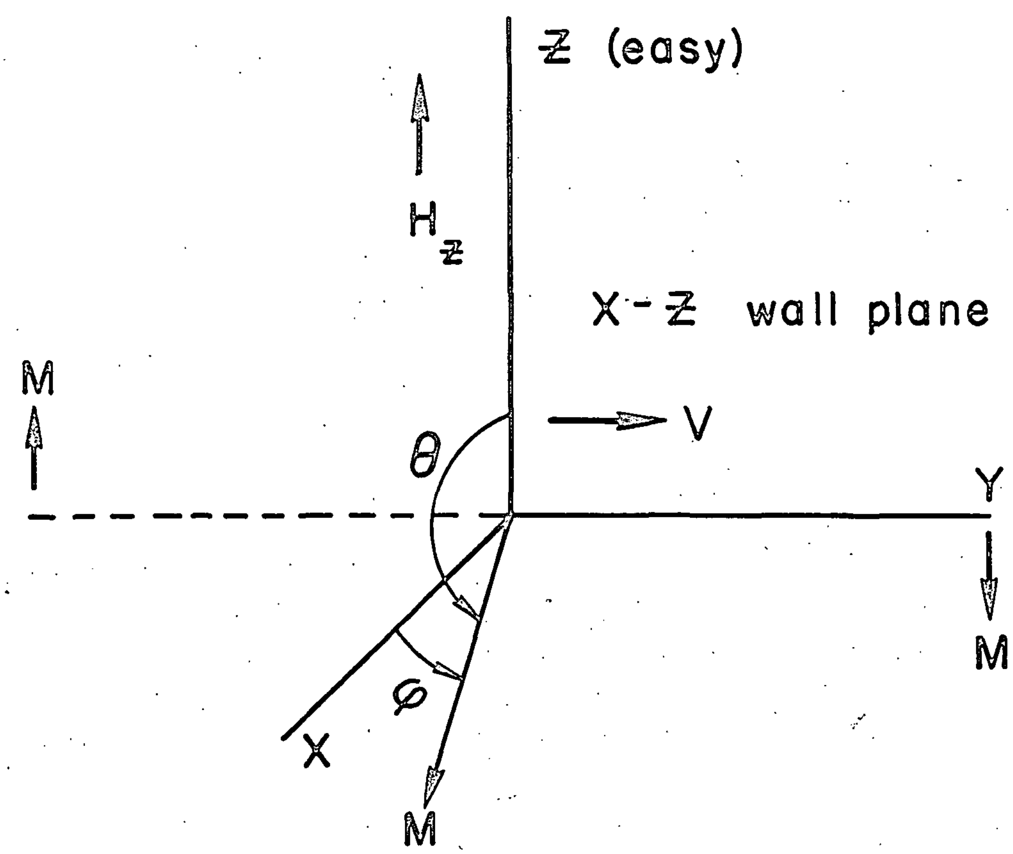

Fig. 1
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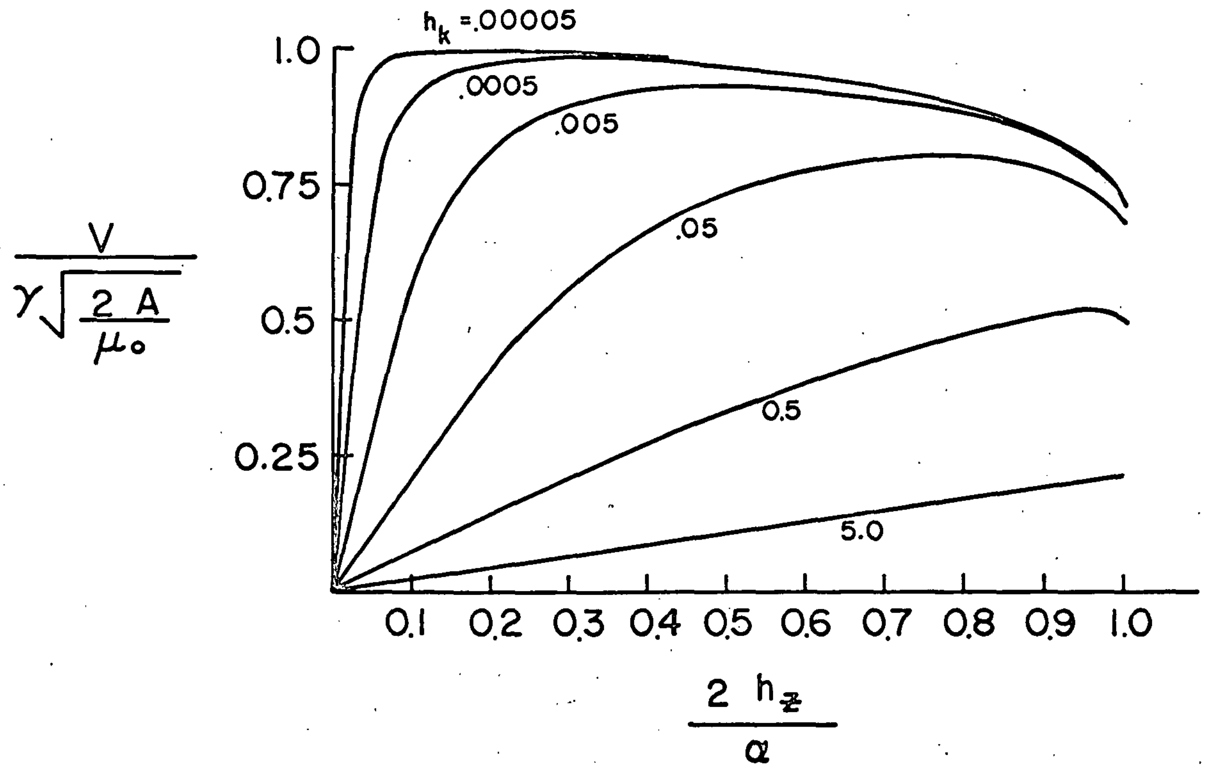

Fig 2
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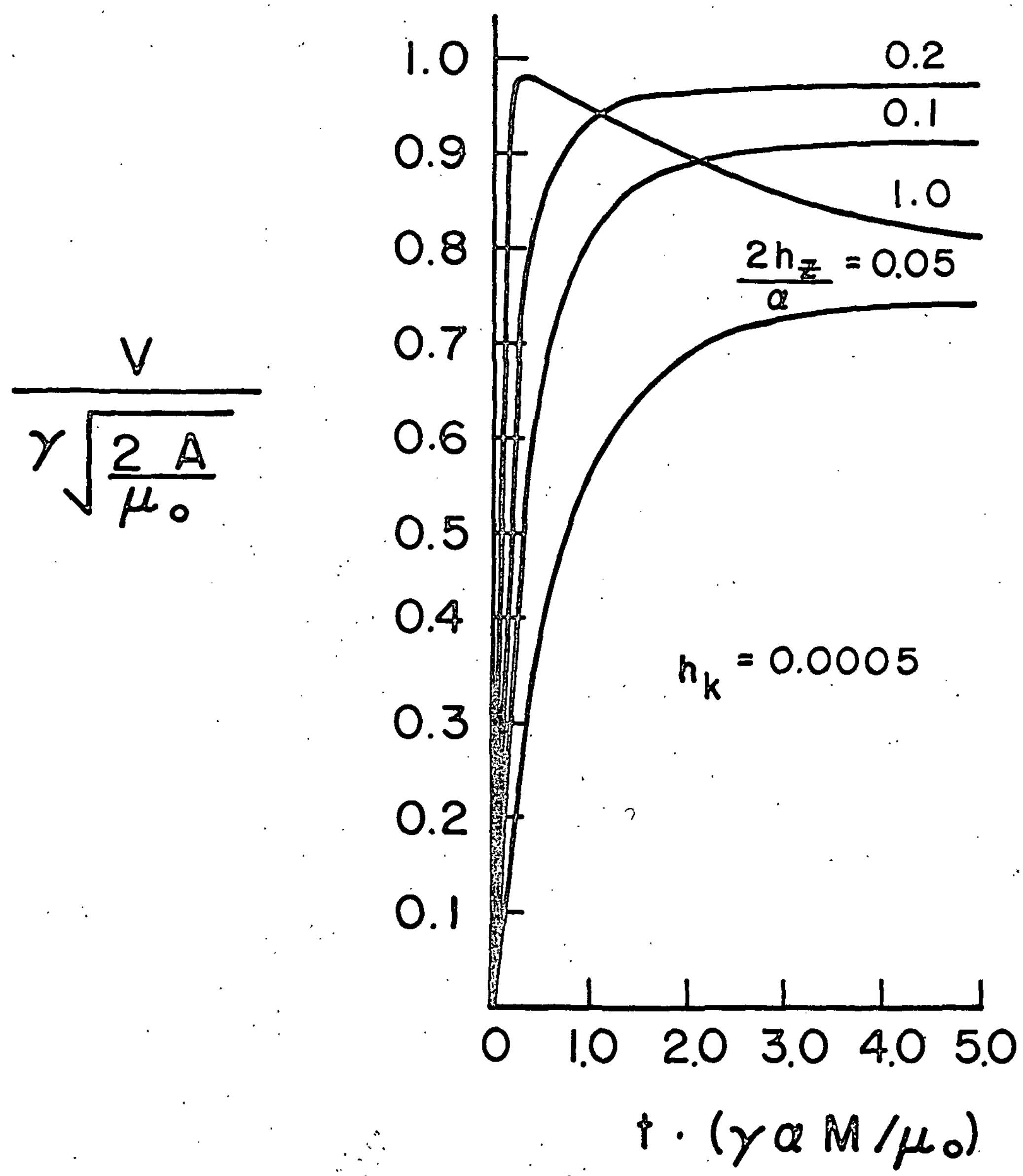
V / γ√(2A/μo)
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.2
0.1
1.0
2hz/α = 0.05
hk = 0.0005
0
1.0
2.0
3.0
4.0
5.0
t · (γαM/μo)

Fig 3.