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SPHERICAL BALLOON RESPONSE TO THREE-DIMENSIONAL TIME-DEPENDENT FLOWS

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16. Abstract

The dynamic response of a balloon to atmoshperic flow is a function of the aerodynamic drag and lift forces, virtual mass effects, the Archimedean buoyancy forces, and dynamic buoyancy forces. The equations of motion for a spherical balloon which include these effects are derived by examining the various forces that can act on a spherical balloon in relation to the atmospheric equations of motion. It is assumed that the atmosphere affects the balloon but the balloon does not affect the atmosphere. An analysis shows that Coriolis effects in the balloon equations of motion can be neglected for most rising or falling balloons.

The concept of the Lagrangian displacement of a balloon is introduced. It is shown that the general balloon response problem is extremely complicated because the wind-forcing functions in the balloon equations of motion are functions of the wind velocity vector and its Eulerian first derivatives evaluated at the location of the balloon. The balloon location is the dependent variable of the problem.

The linear perturbation equations for a spherical balloon are derived by perturbing the components of velocity of the balloon about a terminal velocity state which is in equilibrium with a space-time invariant mean horizontal flow. The atmospheric flow is also perturbed such that the resulting equations can be used to analyze the responses of spherical balloons to three-dimensional time-dependent flows. The wind field is represented in terms of a four-fold Fourier integral that involves three orthogonal wave numbers and a frequency, while the balloon components of velocity are represented as Fourier integrals involving a frequency which, in turn, is a function of the wind field wave numbers and frequency and the unperturbed flow components of velocity. The general nonstationary spectra of the balloon components of velocity are obtained in terms of integrals of the nonhomogeneous and nonstationary spectra of the wind field. The flow associated with a statistically stationary and homogeneous atmospheric velocity field is treated as a special case. The introduction of Taylor's frozen eddy hypothesis reduces the response integrals to algebraic equations which are relatively simple forms for the balloon transfer functions.

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SUMMARY

To fully understand what a balloon wind observation means, it is necessary to know the response properties of the balloon relative to the imposed wind field. In this report, the basic equations of motion for a spherical balloon are derived with the assumption that the atmosphere affects the balloon, but the balloon does not affect the atmosphere. The response of the balloon to the wind field is affected by aerodynamic drag and lift forces, virtual mass effects, the Archimedean buoyancy forces, and dynamic buoyancy forces. An analysis shows that the Coriolis terms in the equations of motion can be neglected for most rising or falling balloons.

The equations of motion for a spherical balloon are linearized about a terminal velocity state in which the horizontal velocity is in equilibrium with the mean horizontal velocity of the atmosphere, and the rise or fall rate is that which is obtained when the Archimedean buoyancy force balances the aerodynamic drag force. The mean atmospheric flow is assumed to be a space-time invariant horizontal flow. The atmospheric flow and the balloon velocity vector are perturbed by superimposing small perturbations such that the square and higher-order terms in perturbation quantities can be neglected with respect to first-order terms. The environmental perturbation velocity field is a three-dimensional time-dependent vector field, which is represented as a four-fold Fourier integral involving three mutually orthogonal wave numbers and a frequency. The components of balloon velocity are, in turn, represented in terms of Fourier integrals which involve a frequency that is a function of the wind field wave numbers, frequency, and the velocity vector of the unperturbed wind field. The combination of these Fourier representations with the equations of motion results in expressions which relate the Fourier amplitudes of the balloon to the Fourier amplitudes of the wind field via a three-fold integral. This integral convolves the wind field Fourier amplitudes with the ballon system function over the domains of the wind field horizontal wave numbers and frequency.

The nonstationary spectra of the balloon velocity vector are calculated in terms of the nonhomogeneous and nonstationary spectra of the wind field. The results are simplified by assuming that the wind field is statistically stationary and homogeneous. The stationary balloon spectra are thus obtained as three-fold integrals of the wind field spectra modified by the balloon transfer function. The introduction of Taylor's hypothesis reduces these integrals to algebraic expressions which show that the spectra of the corresponding components of velocity of the balloon and wind field are proportional and that the function of proportionality is a function of the balloon parameters and the frequency of the wind perturbations relative to the balloon.

INTRODUCTION

It is widely recognized that meteorological balloons do not respond perfectly to the wind fields they traverse. A number of analyses have been published during the last 10 years which attempt to define the nature of balloon responses to the wind. Among these works are those by Reed [1], Lewis and Engler [2], Zartarian and Thompson [3], Eckstrom [4], Scoggins [5], and Fichtl [6]. However, in all cases the analyses have an incomplete set of equations of motion as their starting point. In addition, the analyses are restricted to rather simplified atmospheric flows in which only vertical variations of the wind are permitted. This report presents what are believed to be complete equations of motion for a spherical balloon and also presents a linear theory of spherical balloon response to three-dimensional, time-dependent atmospheric flows.

EQUATION OF MOTION

The equations of motion for a spherical balloon of mass m (skin and inflation gas) relative to a Cartesian frame of reference attached to the earth with the x_3-axis directed toward the zenith and the x_1 and x_2 axes in a plane tangent to the earth (e.g., directed toward the east and north) are given in tensor form by

$$m\left(\frac{dv_k}{dt} + 2\Omega_i v_j e_{ijk}\right) = -gm\, \delta_{k3} + D_k + L_k$$

$$+ m_a \left[\frac{d}{dt} (u_k - v_k) + \Omega_i (u_j - v_j) e_{ijk} \right]$$

$$- V \frac{\partial p}{\partial x_k} \quad , \qquad (1)$$

where t denotes time and all subscripts and superscripts take on values 1, 2, 3. We shall use the Einstein summation convention; i.e., repeated sub- or superscripts imply summation from 1 to 3. The quantities v_k and u_k denote the k components of velocity of the center of gravity (cg) of the balloon and the atmosphere at the location of the cg in the absence of the balloon.

The first term on the right-hand side of this equation denotes the gravitational body force on the balloon, where g is the magnitude of the gravity vector and

$$\delta_{k\,3} = \begin{Bmatrix} 1, & k = 3 \\ 0, & k \neq 3 \end{Bmatrix} .$$

The second and third terms denote the k components of the aerodynamic drag and lift forces. The drag and lift forces act parallel and normal to the relative wind vector $u_k - v_k$. The standard assumed forms of these forces which have been used in analyses of balloons are

$$D_k = \frac{1}{2} \rho A C_D \left[(u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2 \right]^{1/2} (u_k - v_k) \quad , \qquad (1a)$$

$$L_k = \frac{1}{2} \rho A C_{Lk} \left[(u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2 \right] \qquad (1b)$$

where ρ is the density of air, C_D is the aerodynamic drag coefficient, and C_{Lk} is the k component of the aerodynamic lift coefficient vector. All available evidence appears to show that these representations are valid for the Reynolds number region of typical balloons; i.e., $10^4 \lesssim Re < 10^6$, where $Re = \overline{v}_3 D/\nu$, $\overline{v}_3$ being the mean rise or fall rate, D the balloon diameter, and ν the coefficient of kinematic viscosity.

The fourth term is the force which results from the apparent or virtual mass effect, where m_a is the apparent mass and Ω_i is the i component of the angular velocity vector associated with the rotation of the earth. The quantity e_{ijk} is the alternating tensor which has the following properties:

$$e_{ijk} = \begin{Bmatrix} 1\ , & i \neq j \neq k, \text{ even permutation} \\ -1\ , & i \neq j \neq k, \text{ odd permutation} \\ 0\ , & \text{otherwise.} \end{Bmatrix}$$

The fifth term is the force on the balloon resulting from atmospheric pressure gradients, $\partial p/\partial x_k$, where V is the volume of the balloon. It is reasonable to hypothesize that the pressure gradient force on the balloon should be proportional to the atmospheric pressure gradient at the balloon cg and the volume of the balloon. However, it should be remembered that the pressure gradient is that of the atmophere in the absence of the balloon. As in the specification of the atmospheric velocity vector at the balloon cg, it is assumed that the atmosphere affects the balloon but the balloon does not affect the atmosphere. Thus, the form of the pressure gradient force in equation (1) is a hypothesis. However, as we shall see, it leads to the accepted form of the Archimedean buoyancy force for balloons.

The net vector sum of the forces on the right side of equation (1) must be balanced by the inertial reaction, which is the left side of this equation. The two Coriolis terms in equation (1) which contain Ω_i result from the fact that the frame of reference, attached to a point on the surface of the earth, is not an inertial frame. These terms are a natural consequence of the transformation of a Lagrangian time derivative of a vector relative to an absolute or inertial frame of reference to an accelerating frame of reference.

ELIMINATION OF THE PRESSURE GRADIENT FORCE

The inviscid equations of motion of the atmosphere will be used to obtain an expression for the pressure gradient $\partial p/\partial x_k$. These equations are given by

$$\frac{Du_k}{Dt} = -\frac{1}{\rho}\frac{\partial p}{\partial x_k} - 2\,\Omega_i\, u_j\, e_{ijk} - g\,\delta_{3k}\ . \qquad (2)$$

Elimination of the pressure gradient between equations (1) and (2) yields

$$(m + m_a) \frac{dv_k}{dt} + (2m + m_a)\, \Omega_i v_j e_{ijk} = - g (m - m_o)\, \delta_{k3}$$
$$+ D_k + L_k + m_a \left(\frac{du_k}{dt} + \Omega_i u_j e_{ijk} \right) \tag{3}$$
$$+ m_o \left(\frac{Du_k}{Dt} + 2\, \Omega_i u_j e_{ijk} \right) ,$$

where $m_o (= \rho V)$ is the mass of the atmosphere displaced by the balloon. The first term on the right side of this equation is the well-known Archimedean buoyancy force which results from combination of the gravitational body forces in equation (1) and (2). The last term, the dynamic buoyancy force, results from inertial accelerations of the atmosphere.

At this point, a brief discussion of the various derivatives in equation (3) is appropriate. The derivative in the last term on the right side of equation (3) is the material derivative following the atmospheric particle at the balloon cg. It can be expressed as

$$\frac{Du_k}{Dt} = \frac{\partial u_k}{\partial t} + u_j \frac{\partial u_k}{\partial x_j} \quad . \tag{4}$$

To calculate Du_k/Dt, we perform the Eulerian operations on the right side of equation (4) and then evaluate at the location of the balloon cg. The differentiation indicated on the left side of equation (3) and in the apparent mass term on the right side denotes a change following the balloon. To evaluate the derivative of u_k in the apparent mass term, we note that

$$\frac{du_k}{dt} = \frac{\partial u_k}{\partial t} + v_j \frac{\partial u_k}{\partial x_j} \quad . \tag{5}$$

Thus, to determine the change of u_k following the balloon, we execute the Eulerian operations on the right side of equation (5) and then evaluate at the location of the balloon. The difference between the operators d/dt and D/Dt becomes obvious by comparing the right sides of equations (4) and (5).

Utilization of equations (4) and (5) permits equation (3) to be expressed as

$$(m + m_a) \frac{dv_k}{dt} + (2m + m_a) \Omega_i v_j e_{ijk} = -g(m - m_o) \delta_{k3}$$

$$+ D_k + L_k + (m_a + m_o) \frac{\partial u_k}{\partial t} + (m_a v_j + m_o u_j) \frac{\partial u_k}{\partial x_j}$$

$$+ (m_a + 2 m_o) \Omega_i u_j e_{ijk} \quad . \tag{6}$$

GENERALIZED RESPONSE PROBLEM

The motivation behind the formulation of the equations of motion for a balloon is the calculation of the responses of balloons to atmospheric fields of velocity. Accordingly, this permits one to determine how well a balloon can measure the wind and what that measurement means. We denote the Lagrangian displacements of the balloon cg relative to the frame of reference attached to the earth with $X_k(t)$. The Lagrangian displacements are functions of time and by definition are related to the balloon components of velocity through the expression

$$v_k(t) = \frac{dX_k}{dt} \quad . \tag{7}$$

Substitution of equation (7) into (6) yields

$$(m + m_a) \frac{d^2X_k}{dt} + (2m + m_a) \Omega_i \frac{dX_j}{dt} e_{ijk} = -g(m - m_o) \delta_{k3}$$

$$+ D_k\left(\frac{dX_m}{dt}, u_m\right) + L_k\left(\frac{dX_m}{dt}, u_m\right) + (m_a + m_o) \frac{\partial u_k}{\partial t}$$

$$+ m_o u_j \frac{\partial u_k}{\partial x_j} + m_a \frac{\partial u_k}{\partial x_j} \frac{dX_j}{dt} + (m_a + 2m_o) \Omega_i u_j e_{ijk} \quad . \tag{8}$$

This tensor equation represents three equations for the balloon coordinates.

To solve these equations, we must specify the initial conditions and the wind field. Substitution of the known wind field into equation (8) and evaluation at $x_m = X_m(t)$ yields the equations for the Lagrangian displacements. The solution of these equations yields the functions $X_m(t)$ and hence $(v_m = dX_m/dt)$. The balloon velocities $v_m(t)$ can be compared with the specified winds along the trajectory $u_m [X_k (t), t]$ to determine how the balloon responds to the prescribed wind field.

The inverse problem is to specify the functions $X_m(t)$ for a single balloon track and determine the wind field $u_m[X_k (t), t]$ along the balloon track. Equation (8) is a partial differential equation in u_k, and because it is a partial differential equation, the stated inverse problem cannot be solved. To solve this equation for u_k, one must do the impossible; namely, release an infinite number of balloons so that X_k can be determined as a function of time and space. One could then solve equation (8) for $u_m(x_k, t)$ and then evaluate for the particular balloon in question to obtain $u_m[X_k(t), t]$.

SOME SIMPLE BALLOON PROBLEMS – DEFINITION OF A BALLOON TIME CONSTANT

Let us consider the simple situation of a vertically rising or falling balloon in a quiescent nonrotating atmosphere. Furthermore, the aerodynamic lift is assumed to be zero. Under these assumptions, equation (6) reduces to

$$(m + m_a) \frac{dv_3}{dt} = -\frac{1}{2} \rho A C_D |v_3| v_3 + g (m_o - m) \quad . \tag{9}$$

Let us assume

$$v_3 = \overline{v}_3 + v_3' \quad , \tag{10}$$

where $\overline{v}_3$ is a mean rise or fall rate in equilibrium with the mean flow drag force and buoyancy force, such that

$$\frac{1}{2} \rho A C_D |\overline{v}_3| \overline{v}_3 = g (m_o - m) \quad . \tag{11}$$

The quantity v_3' is a superimposed perturbation which is sufficiently small in magnitude such that second-order terms in v_3' can be neglected with respect to first-order terms. Substitution of equation (10) and (11) into equation (9) yields

$$(m + m_a) \frac{dv_3'}{dt} = - \frac{1}{2} \rho A C_D |\overline{v}_3| v_3' \quad . \tag{12}$$

Thus, we are considering small balloon velocity perturbations superimposed on a mean terminal vertical rise or fall rate. Integration of equation (12) yields

$$v_3' (t) = v_3' (0) e^{-t/T} \quad , \tag{13}$$

where

$$T = \frac{2(m + m_a)}{\rho A C_D |\overline{v}_3|} \quad , \tag{14a}$$

or in view of equation (11)

$$T = \frac{m + m_a}{|m - m_o|} \frac{|\overline{v}_3|}{g} \quad . \tag{14b}$$

The quantity T is the balloon time constant and is that time required for an initial balloon vertical velocity perturbation from terminal velocity conditions to be reduced by a factor of e^{-1}. The balloon time constant is in the order of a few seconds for most meteorological balloons, and it commonly occurs in various theories of the responses of balloons to atmospheric motions [6, 7].

For a neutrally buoyant balloon, the time constant is defined in terms of the solution of equation (9) for $m = m_o$. We take v_3 to be initially positive

$[v_3(0) > 0]$ and assume that v_3 is positive definite, so that upon being displaced the balloon will immediately decelerate as it passes through the atmosphere. This means we can drop the absolute value sign in equation (9) and obtain the solution

$$\frac{v_3(t)}{v_3(0)} = \frac{1}{1 + t/T} \quad . \tag{15}$$

When $t/T = 1$, we find from equation (15) that $v_3(t)/v_3(0) = 1/2$. Thus, the quantity T is the time required for the initial velocity of a neutrally buoyant balloon to be reduced by a factor of one-half when the only effect present is that of aerodynamic drag.

THE CORIOLIS TERMS

Let us consider the Coriolis terms in equation (6). To neglect the Coriolis terms on the left side of equation (6) we could require

$$\frac{\left|\frac{dv_1}{dt}\right|}{|\Omega_2 v_3 - \Omega_3 v_2|} >> \frac{2m + m_a}{m + m_a} \tag{16}$$

$$\frac{\left|\frac{dv_2}{dt}\right|}{|\Omega_3 v_1|} >> \frac{2m + m_a}{m + m_a} \tag{17}$$

$$\frac{g|m - m_o|}{(m + m_a)\,|\Omega_2 v_1|} >> \frac{2m + m_a}{m + m_a} \quad . \tag{18}$$

In the analysis that follows, we shall set $(2m + m_a)/(m + m_a)$ equal to its maximum value of 2. If these inequalities are satisfied for this value of $(2m + m_a)/(m + m_a)$, they will be satisfied for all values of this parameter. In the case of an ascending or descending balloon, the vertical velocity should be near terminal conditions, so that we can cast equation (18) into the form

$$T\ \Omega_2 << \frac{2|v_3|}{|v_1|} \quad . \qquad (19)$$

Now, $|v_3|/|v_1|$ is in the order of unity, T has value approximately equal to a few seconds, and $\Omega_2 \sim 10^{-4}\ s^{-1}$. Thus, condition (19) is satisfied for most rising or falling balloons.

To estimate the left side of equation (17), we use the frequency Ω_B of the wind perturbation relative to a rising or falling balloon to estimate the operator d/dt in equation (17) and thus require

$$\frac{\Omega_B}{|\Omega_3|} >> \frac{2|v_1|}{|v_2|} \quad . \qquad (20)$$

Now, $|v_1|/v_2| \sim 1$ so that equation (20) requires $\Omega_B/|\Omega_3| >> 1$. The quantity Ω_B is on the order of $2\pi \times 10^{-2}$ for a balloon with rise or fall rate equal to approximately 5 $m-s^{-1}$ ascending or descending through a wind perturbation with a wavelength approximately equal to 500 m and $|\Omega_3| \sim 10^{-4}\ s^{-1}$, so that $\Omega_B/\Omega_3 \sim 2\pi \times 10^{+2}$. Thus condition (20) or (17) appears to be satisfied for typical perturbations on the wind profile.

Let us now consider condition (16). For a rising or falling balloon, v_3 tends to have one sign with magnitude approximately equal to the terminal velocity. It is highly unlikely that $\Omega_2 v_3 = \Omega_3 v_2$ for all t, so that the possibility of the denominator vanishing in condition (16) will only occur at a few brief instants at most. If the denominator vanished, the x_1 component of the Coriolis effect would then be zero, and we would not have to be concerned with condition (16). Since

$$\Omega_2 |v_3| + |\Omega_3 v_2| \geq |\Omega_2 v_3 - \Omega_3 v_2| \quad ,$$

we replace condition (16) with

$$\frac{\Omega_B |v_1|}{\Omega_2 |v_3| + |\Omega_3 v_2|} >> 2, \qquad (21)$$

where we have used $\Omega_B |v_1|$ as an estimate of $|dv_1/dt|$. Because $\Omega_2 \simeq |\Omega_3|$ in mid-latitudes and $|v_1| \simeq |v_2|$, we have

$$\frac{\Omega_B}{|\Omega_3|} >> 2 \left(1 + \frac{|v_3|}{|v_2|}\right) \quad . \tag{22}$$

It is clear from the previous comments that this expression is satisfied for the rising or falling balloon.

Let us now consider the Coriolis terms on the right side of equation (6). To neglect these terms we could require that

$$\frac{\left|\frac{dv_1}{dt}\right|}{|\Omega_2 u_3 - \Omega_3 u_2|} >> 2 \quad , \tag{23}$$

$$\frac{\left|\frac{dv_2}{dt}\right|}{|\Omega_3 u_1|} >> 2, \tag{24}$$

and

$$\frac{g\,|m - m_o|}{2(m_a + m)\ \Omega_2\ |u_1|} >> 2 \quad . \tag{25}$$

Since $|u_1| \sim |v_1|$, conditions (24) and (25) are satisfied because the replacement of $|u_1|$ with $|v_1|$ yields conditions (17) and (18), which have already been shown to be satisfied by the balloon motions. We replace condition (23) with

$$\frac{\Omega_B\ |v_1|}{\Omega_2\ |u_3| + |\Omega_3\ u_2|} >> 2 \ , \tag{26}$$

as in the case of condition (16), we have estimated $|dv_1/dt|$ with $\Omega_B |v_1|$. Now, $|u_2|$ approximates $|v_2|$, and $|v_3| \gtrsim |u_3|$ for most rising or falling balloons. Thus,

$$\frac{\Omega_B \, |v_1|}{\Omega_2 \, |v_3| + |\Omega_3 \, v_2|} \lesssim \frac{\Omega_B \, |v_1|}{\Omega_2 \, |u_3| + |\Omega_3 \, u_2|} \quad , \tag{27}$$

so that condition (26) is satisfied because condition (21) was found to be valid. Thus, it appears that the horizontal Coriolis terms in equation (6) can be safely neglected against the horizontal balloon acceleration terms referenced to the relative frame of reference and the vertical Coriolis terms can be neglected against the Archimedean buoyancy term.

LINEAR PERTURBATION EQUATIONS

In the standard linearized balloon response problem, one considers the response of the balloon to infinitesimal wind perturbations superimposed on an unperturbed wind state which is spatially and temporally invariant. The assumed infinitesimal nature of the wind perturbations permits one to assume that the balloon responses are infinitesimal and thus allows one to pose the balloon response problem in a linear context. The motivation for this procedure is the simplicity of the resulting equations which facilitate a relatively straightforward calculation of the response properties of the balloon. The analysis herein neglects the Coriolis terms in equation (6).

We assume that the wind field can be represented in terms of a spatially and temporally invariant mean component and a superimposed wind perturbation which can depend on both space and time:

$$u_i \, (x_k, \, t) = \overline{u}_i + u_i' \, (x_k, \, t) \, . \tag{28}$$

Henceforth, overbar and prime indicate a mean or unperturbed state and a superimposed fluctuation. The assumed mean wind field executes horizontal flow, so that

$$\overline{u}_3 = 0 \; . \tag{29}$$

The balloon velocity fluctuations are represented in a similar way; i.e.,

$$v_i(t) = \overline{v}_i + v_i' \, (t) \qquad . \tag{30}$$

Unlike the assumed mean wind field, $\overline{v}_3 \neq 0$. It is assumed that the mean horizontal motion of the balloon is in equilibrium with the environmental wind, so that

$$\overline{v}_i = \overline{u}_i, \quad i = 1, 2 \quad . \tag{31}$$

The aerodynamic lift coefficient vector is a time-dependent vector. The mean aerodynamic lift coefficients are taken to be equal to zero,

$$\overline{C}_{Lk} = 0 \quad . \tag{32}$$

Thus, the life coefficient vector is assumed to be a perturbation vector. Fichtl, DeMandel, and Krivo [7] have shown for a sufficiently fast rising or falling balloon that if the horizontal lift coefficients and the wind and balloon velocity perturbations are of first-order smallness, then the vertical lift coefficient is a second-order quantity. In other words, to within first order, the lift force acts in the horizontal plane for a sufficiently fast mean rise or fall rate. Because the lift coefficients can vary in time, it is reasonable to allow for the drag coefficient to depend on time. Thus, we shall assume that

$$C_D = \overline{C}_D + C_D' \quad . \tag{33}$$

It is assumed that the mean or unperturbed quantities satisfy the equations of motion, equation (6), so that in view of the above assumptions, the horizontal equations of motion of the balloon are identically satisfied. The vertical equation of motion for the balloon reduces to

$$-\frac{1}{2} \rho A C_D \, |\overline{v}_3| \, \overline{v}_3 - g\,(m - m_o) = 0 \quad . \tag{34a}$$

Substitution of equations (28), (30), and (33) into the balloon equations of motion [equation (6)], utilization of conditions (29), (31), (32), and (34a), and the neglect of second- and higher-order terms yields

$$\frac{dv_i'}{dt} + \gamma(i)\,\frac{v_i'}{T} = \mu \left(\frac{\partial u_i'}{\partial t} + \overline{u}_k \frac{\partial u_i'}{\partial x_k} \right) + \alpha\, \overline{v}_3 \frac{\partial u_i'}{\partial x_3}$$

$$+ \gamma\,(i)\,\frac{u_i'}{T} + [2 - \gamma(i)]\,\frac{|\overline{v}_3|}{T\overline{C}_D}\, C_{Li}' - \delta_{i3}\,\frac{\overline{v}_3}{T\overline{C}_D}\, C_D' \ , \tag{34b}$$

where

$$\gamma(i) = \begin{cases} 1, & i = 1, 2 \\ 2, & i = 3 \end{cases} , \tag{35}$$

$$T = \frac{2\,(m + m_a)}{\rho A \overline{C}_D \, |\overline{v}_3|} = \frac{(m + m_a)\,|\overline{v}_3|}{|m_o - m|\, g} , \tag{36}$$

$$\mu = \frac{m_o + m_a}{m + m_a} , \tag{37}$$

and

$$\alpha = \frac{m_a}{m + m_a} . \tag{38}$$

The Einstein summation convention does not apply to terms containing $\gamma(i)$. In deriving equation (34b), we have assumed that all velocity perturbations and drag and horizontal lift coefficient fluctuations are first-order quantities, while the vertical lift coefficient is a second-order quantity.

For a sphere, the apparent mass is related to the displaced air mass through the relationship

$$m_a = \frac{1}{2} m_o , \tag{39}$$

so that the parameters μ and α can be expressed in terms of one parameter, ϵ; thus,

$$\mu = \frac{3}{1 + 2\epsilon} , \tag{40}$$

$$\alpha = \frac{1}{1 + 2\epsilon} , \tag{41}$$

where

$$\epsilon = \frac{m}{m_o} . \tag{42}$$

Equations (34a) and (34b) are the basic equations of the linear response problem. To solve equation (34b), one must specify the environmental wind perturbations. In the special case in which u_i' is a function of x_3, only the term involving μ in equation (34b) vanishes identically. This means that, in the context of the linear response problem, the dynamic buoyancy effect vanishes when the wind perturbations have only vertical variations.

FOURIER INTEGRAL REPRESENTATION OF WIND FIELD AND AERODYNAMIC COEFFICIENTS

We hypothesize that the wind perturbations can be represented with a Fourier integral as follows:

$$u_i'(x_j, t) = \int \int \int_{-\infty}^{\infty} \int \hat{u}_i(\kappa_m, \omega)\; e^{i(\kappa_k x_k - \omega t)}\; d\kappa\, d\omega \quad , \tag{43}$$

where $(\hat{\ })$ denotes Fourier amplitude and $d\kappa = d\kappa_1\, d\kappa_2\, d\kappa_3$. The quantity i in the exponent is the $\sqrt{-1}$ and whenever i occurs as a coefficient it is understood to be this quantity; otherwise it is an index. The quantity κ_k denotes the k component of the radian wave number vector and ω is the radian frequency.

All quantities which involve u_i' in equation (34b) are interpreted to mean that we perform the indicated operation on u_i' and then evaluate at $x_k = X_k(t)$. To do this, we must perturb the Lagrangian coordinates of the balloon. Integration of equation (7) yields

$$X_k(t) = \int_0^t v_k(t)\; dt \quad , \tag{44}$$

where, without loss of generality, we have taken the initial position of the balloon to be at the origin of the coordinate system $[X_k(0) = 0]$. Substitution of equation (30) into equation (44) yields

$$X_k(t) = \bar{v}_k t + X_k'(t) \quad , \tag{45}$$

where

$$X_k'(t) = \int_0^t v_k'(t)\; dt \quad . \tag{46}$$

The quantity $X_k'(t)$ should be a perturbation quantity of first or higher order of smallness because the sign of $v_k'(t)$ will fluctuate in time, thus tending to cause the positive and negative contributions to the integral to cancel. We will assume that $X_k'(t)$ is a quantity of first-order smallness.

Substitution of equation (43) into the right side of equation (34b) and evaluation at the location of the balloon given by equation (45) yield

$$\frac{dv_i'}{dt} + \gamma(i)\,\frac{v_i'}{T} = [2 - \gamma(i)]\,\frac{|\overline{v}_3|}{T\overline{C}_D}\,C_{Li}' - \delta_{i3}\,\frac{\overline{v}_3}{T\overline{C}_D}\,C_D'$$

$$+ \iint\!\!\iint_{-\infty}^{\infty} \left\{ i\,[\mu(\overline{u}_k\,\kappa_k - \omega) + \alpha\,\overline{v}_3\,\kappa_3] + \frac{\gamma(i)}{T} \right\} \qquad (47)$$

$$\times\,\hat{u}_i(\kappa_j,\ \omega)\,e^{i(\kappa_k\overline{v}_k - \omega)t}\,d\kappa\,d\omega \quad ,$$

where we have neglected second- and higher-order terms in fluctuation quantities. Let us set

$$\Omega = \kappa_k\,\overline{v}_k - \omega \quad , \qquad (48)$$

so that equation (47) can be expressed as

$$\frac{dv_i'}{dt} + \gamma(i)\,\frac{v_i'}{T} = [2 - \gamma(i)]\,\frac{|\overline{v}_3|}{T\overline{C}_D}\,C_{Li}' - \delta_{i3}\,\frac{v_3}{T\overline{C}_D}\,C_D'$$

$$+ \int_{-\infty}^{\infty} B_i(\Omega)\,e^{i\Omega t}\,d\Omega \quad , \qquad (49)$$

where

$$B_i(\Omega) = \frac{1}{\overline{v}_3}\iiint_{-\infty}^{\infty} \left\{ i\,[(\mu - \alpha)\,\Omega_1 + \alpha\,\Omega] + \frac{\gamma(i)}{T} \right\}$$

$$\times\,\hat{u}_i\left(\kappa_1,\ \kappa_2,\ \frac{\Omega - \Omega_1}{\overline{v}_3},\ \omega\right) d\kappa_1\,d\kappa_2\,d\omega \quad , \qquad (50)$$

and

$$\Omega_1 = \kappa_k \overline{u}_k - \omega \quad .$$

The quantities C'_{Li} and C'_D are functions of time which can also be represented as Fourier integrals:

$$C'_{Li} = \int_{-\infty}^{\infty} \hat{C}_{Li} \, e^{i\Omega t} \, d\Omega \tag{51}$$

and

$$C'_D = \int_{-\infty}^{\infty} \hat{C}_D \, e^{i\Omega t} \, d\Omega \quad . \tag{52}$$

Substitution of equations (51) and (52) into equation (49) yields

$$\frac{dv'_k}{dt} + \gamma(k) \frac{v'_k}{T} = \int_{-\infty}^{\infty} G_k(\Omega) \, e^{i\Omega t} \, d\Omega , \tag{53}$$

where

$$G_k(\Omega) = B_k(\Omega) + [2 - \gamma(k)] \frac{|\overline{v}_3|}{T\overline{C}_D} \hat{C}_{Lk} - \delta_{k3} \frac{\overline{v}_3}{T\overline{C}_D} \hat{C}_D \quad . \tag{54}$$

FORCED RESPONSE OF BALLOONS

To obtain the forced response of a balloon, we assume that

$$v'_k(t) = \int_{-\infty}^{\infty} \hat{v}_k(\Omega) \, e^{i\Omega t} \, d\Omega \quad . \tag{55}$$

Substitution of equation (55) into equation (53) yields

$$\hat{v}_k(\Omega) = \frac{T}{\gamma(k) + i\Omega t} G_k(\Omega) \quad . \tag{56}$$

Thus, the forced response of a balloon could be calculated by substituting equation (56) back into equation (55) to yield

$$v_k'(t) = \int_{-\infty}^{\infty} \frac{T}{\gamma(k) + i\Omega t} G_k(\Omega)\, e^{i\Omega t}\, d\Omega \quad . \tag{57}$$

The power spectra of the balloon velocities are useful statistics for describing the properties of the balloon motions. To obtain the generalized power spectrum of $v_k'(t)$, we multiply equation (56) by its complex conjugate evaluated at Ω', so that

$$\left\langle \hat{u}_k(\Omega)\, \hat{v}_k^*(\Omega') \right\rangle = \frac{T^2 \left\langle G_k(\Omega)\, G_k^*(\Omega') \right\rangle}{\gamma^2(k) + i\gamma(k)\, T\,(\Omega - \Omega') + \Omega\, \Omega'\, T^2} , \tag{58}$$

where <( )> denotes the ensemble average operator. Substitution of equation (54) into equation (58) yields

$$\begin{aligned} \left\langle \hat{v}_k(\Omega)\, \hat{v}_k^*(\Omega') \right\rangle &= \frac{T^2 \left\langle B_k(\Omega)\, B_k^*(\Omega') \right\rangle}{\gamma^2(k) + i\,\gamma(k)\, T\,(\Omega - \Omega') + \Omega\, \Omega'\, T^2} \\ &+ [2 - \gamma(i)] \left(\frac{\bar{v}_3}{T\bar{C}_D} \right)^2 \frac{\left\langle \hat{C}_{Li}(\Omega)\, \hat{C}_{Li}^*(\Omega') \right\rangle}{\gamma^2(k) + i\,\gamma(k) T\,(\Omega - \Omega') + \Omega\, \Omega'\, T^2} \\ &+ \delta_{i3} \left(\frac{\bar{v}_3}{T\bar{C}_D} \right)^2 \frac{\left\langle \hat{C}_D(\Omega)\, \hat{C}_D^*(\Omega') \right\rangle}{\gamma^2(k) + \gamma(k)\, T\,(\Omega - \Omega') + \Omega\, \Omega'\, T^2} \quad , \end{aligned} \tag{59}$$

where we have assumed that the Fourier amplitudes $\hat{C}_{Lk}(\Omega)$ and $\hat{C}_D(\Omega)$ are uncorrelated with $B(\Omega)$. The second and third terms on the right side of equation (59) are the contributions to the balloon response spectra from the fluctuations in the aerodynamic lift and drag coefficients. These terms have been analyzed in detail for the Jimsphere balloon [5] by Fichtl, DeMandel, and Krivo [7]. The first term which we shall denote as $S(\Omega, \Omega')$ is the contribution to the balloon response sepçtra from the wind perturbations along the balloon track. The remainder of this report will be concerned with an analysis of this term.

Substitutuion of equation (50) into $S(\Omega, \Omega')$ yields the balloon response spectrum in terms of the spectrum of u_i':

$$S_k(\Omega, \Omega') = \int \cdots \int H_k \, \Phi \left(\kappa_1, \kappa_2, \frac{\Omega - \Omega_1}{\overline{V}_3}, \omega, \kappa_1', \kappa_2', \frac{\Omega - \Omega'}{\overline{V}_3}, \omega\right)$$

$$\times \, d\kappa_1 \, d\kappa_2 \, d\omega \, d\kappa_1' \, d\kappa_2' \, d\omega', \qquad (60)$$

where

$$H_k = \frac{\gamma^2(k) + [(\mu - \alpha)\,\Omega_1 + \alpha\Omega]\,[(\mu - \alpha)\,\Omega_1' + \alpha\Omega']\,T^2 + i\,\gamma(k)\,[(\mu - \alpha)\,(\Omega_1 - \Omega_1') + \alpha(\Omega - \Omega')]\,T^2}{\gamma^2(k) + i\gamma(k)\,T\,(\Omega - \Omega') + \Omega\,\Omega'\,T^2}, \qquad (61)$$

$$\Phi_k = \frac{1}{\overline{V}_3^2} \left\langle \hat{u}_k\left(\kappa_1, \kappa_2, \frac{\Omega - \Omega_1}{\overline{V}_3}, \omega\right) \hat{u}_k^*\left(\kappa_1', \kappa_2', \frac{\Omega' - \Omega_1'}{\overline{V}_3}, \omega'\right)\right\rangle . \qquad (62)$$

The spectrum $S_k(\Omega, \Omega')$ is the generalized power spectrum as defined by Bendat and Piersol [8]. Another useful representation of the spectrum is

$$\Psi(\Omega, t) = \int_{-\infty}^{\infty} S(\Omega, \Omega') \, e^{i(\Omega - \Omega')t} \, d\Omega'. \qquad (63)$$

This spectrum is the nonstationary spectrum as defined by Kinsman [9] in his analysis of ocean waves.

STATIONARY BALLOON RESPONSE

If the balloon response is statistically stationary, then Ψ_k must be time invariant, which means $S(\Omega, \Omega')$ must be of the form

$$S(\Omega, \Omega') = \Psi_k(\Omega)\,\delta(\Omega - \Omega'). \tag{64}$$

This implies

$$\Phi_k\left(\kappa_1, \kappa_2, \frac{\Omega-\Omega_1}{\overline{v}_3}, \omega, \kappa_1', \kappa_2', \frac{\Omega'-\Omega_1'}{\overline{v}_3}, \omega'\right) = \Phi_k\left(\kappa_1, \kappa_2, \frac{\Omega-\Omega_1}{\overline{v}_3}, \omega\right)$$

$$\times\,\delta(\kappa_1 - \kappa_1')\,\delta(\kappa_2 - \kappa_2')\,\delta(\Omega - \Omega')\,\delta(\omega)\quad . \tag{65}$$

Substitution of equations (64) and (65) into equation (60) yields

$$\Psi_k(\Omega) = \int\int\int_{-\infty}^{\infty} \frac{\gamma^2(k) + [(\mu - \alpha)\ \Omega_1 + \alpha\Omega]^2\, T^2}{\gamma^2(k) + (\Omega T)^2}\, \Phi_k\left(\kappa_1, \kappa_2, \frac{\Omega - \Omega_1}{\overline{v}_3}, \omega\right) d\kappa_1\, d\kappa_2\, d\omega. \tag{66}$$

This result is the stationary spectrum for the k component of balloon velocity resulting from a statistically stationary and homogeneous wind field.

WIND FIELD WITH ONLY VERTICAL VARIATION

The result given by equation (66) shows that the spectrum of a component of balloon velocity is related to a three-fold integral of the four-dimensional spectrum of the corresponding component of the wind. In the special case in which the wind only has vertical variation, we must have

$$\Phi_k\left(\kappa_1, \kappa_2, \frac{\Omega - \Omega_1}{\overline{v}_3}, \omega\right) = \Phi_k(\Omega)\,\delta(\kappa_1)\,\delta(\kappa_2)\,\delta(\omega)\quad , \tag{67}$$

so that equation (66) reduces to

$$\Psi_k(\Omega) = \frac{\gamma^2(k) + (\alpha\Omega T)^2}{\gamma^2(k) + (\Omega T)^2}\,\Phi_k(\Omega)\quad . \tag{68}$$

The quantity $\Phi_k(\Omega)$ is the one-dimensional wave-number spectrum of the wind along the vertical transformed to a frequency spectrum via the transformation

$$\Omega = \overline{v}_3\,\kappa_3\quad . \tag{69}$$

A detailed account of equation (68) can be found in Reference 6.

RESPONSE OF BALLOONS TO ATMOSPHERIC FLOWS IN WHICH TAYLOR'S HYPOTHESIS IS VALID

In many instances, atmospheric turbulence and other types of stochastic flows seem to obey Taylor's frozen eddy hypothesis [10], which states that local changes are produced by advective effects or

$$\omega = \overline{u}_k \kappa_k \quad . \tag{70}$$

This means that the spectrum of the wind must be of the form

$$\Phi_k\left(\kappa_1, \kappa_2 \frac{\Omega - \Omega_1}{\overline{v}_3}, \omega\right) = \Phi_k\left(\kappa_1, \kappa_2, \frac{\Omega - \Omega_1}{\overline{v}_3}\right)\delta(\omega - \overline{u}_j \kappa_j), \tag{71}$$

so that integration of equation (66) with this representation yields

$$\Psi_k(\Omega) = \frac{\gamma^2(k) + (\alpha\Omega T)^2}{\gamma^2(k) + (\Omega T)^2} \Phi_k(\Omega) \quad . \tag{72}$$

The quantity $\Phi(\Omega)$ is a one-dimensional spectrum given by

$$\Phi_k(\Omega) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \Phi_k\left(\kappa_1, \kappa_2, \frac{\Omega}{\overline{v}_3}\right) d\kappa_1\, d\kappa_2 \quad . \tag{73}$$

The result given by equation (72) is identical to the form of the result given by equation (66). The assumptions, however, which lead to the two results are different. The result [equation (68)] is a special case of equation (72) and was obtained by assuming the wind field can vary only in the vertical. This assumption is rather restrictive so that the result given by equation (68) is of only a limited interest. On the other hand, the result given by equation (72) is intriguing in that, except for the basic assumptions of the perturbation scheme, we have only assumed that Taylor's hypothesis is valid at each level (eddies are transported by the mean wind). Taylor's hypothesis is not a very restrictive assumption and is commonly invoked in studies of atmospheric turbulence. The hypothesis appears to be valid for a wide variety of situations [10]. The result [equation (72)] says that, if the mean wind is height-invariant, then it is possible to compute the one-dimensional spectrum of the k component of the turbulence velocity vector directly from the spectrum of the corresponding components of balloon velocity. The result offers many possibilities with regard to turbulence measurements with balloons. If the mean wind is not reasonably constant with height or if the Fourier components propagate relative to the mean flow, thus voiding the application of Taylor's hypothesis, the simple result given by equation (72) fails.

Evaluation of equation (43) at the location of the balloon and utilization of equation (70) yields

$$u_i' (\overline{v}_3 t) = \int_{-\infty}^{\infty} \left[\iint_{-\infty}^{\infty} \hat{u}_i (\kappa_k) \, d\kappa_1 \, d\kappa_2 \right] e^{i\overline{v}_3 \kappa_3 t} \, d\kappa_3 \quad , \tag{74}$$

where we have put

$$\hat{u}_i (\kappa_k, \omega) = \hat{u}_i(\kappa_k) \, \delta (\omega - \overline{u}_j \kappa_j) \quad . \tag{75}$$

Now, $\overline{v}_3 t = x_3$, so that equation (74) states that the horizontal components of turbulence (u_1' and u_2') along a balloon track in a space-time invariant horizontal mean flow are lateral velocity fluctuations and the vertical velocity perturbations also along the track are longitudinal fluctuations if Taylor's hypothesis is valid. This means that Φ_1 and Φ_2 in the context of equation (72) are lateral spectra and Φ_3 is a logitudinal spectrum.

CONCLUSIONS

The interpretation of balloon data is not a straightforward procedure. This is especially true if the balloon tranverses all or a large portion of a flow field Fourier component. If the waves move sufficiently slow relative to the balloon rise rate and if the wave lengths of the Fourier components are sufficiently long, then the balloon essentially measures a mean flow velocity. However, if the waves are sufficiently short and have phase velocities on the order of the balloon rise rate, then the interpretation of a balloon wind measurement becomes difficult. In this report, we have attempted to define a complete set of equations for spherical balloon response studies and have calculated the linear response properties of a spherical balloon to a three-dimensional time-varying wind field. We found that, in the case of stationary and homogeneous flows, the power spectra of the balloon components of velocity are related to a three-fold integral of the three-dimensional frequency spectrum of the corresponding components of the wind modified by a balloon system function [equation (66)]. We found that, in the special case in which Taylor's hypothesis is valid at each level, the above-stated integral can be evaluated to yield the result that the power spectra of the balloon components of velocity are directly proportional to the one-dimensional spectra of the corresponding components of air velocity along the balloon track relative to the mean flow. The function of proportionality in this case is the system transfer function of the balloon which is a known function. This result is identical in form to that obtained by previous investigators (noted in the Introduction) for the case in which only vertical variations of the wind field are permitted. Our analysis showed that the horizontal and vertical balloon velocity spectra are lateral and longitudinal. These results offer intriguing possibilities for turbulence research with balloons because Taylor's hypothesis is not an unduly restrictive assumption. This analysis shows that it is possible, in principle, to obtain longitudinal and lateral spectra of turbulence, provided the underlying assumptions of the analysis are reasonably satisfied.

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