CONSERVATION LAWS AND PREFERRED FRAMES IN RELATIVISTIC GRAVITY

I. PREFERRED-FRAME THEORIES AND AN EXTENDED PPN FORMALISM[*]

CLIFFORD M. WILL
California Institute of Technology, Pasadena, California

and

KENNETH NORDTVEDT, JR.[†]
Montana State University, Bozeman, Montana

and

II. EXPERIMENTAL EVIDENCE TO RULE OUT
PREFERRED-FRAME THEORIES OF GRAVITY[*]

KENNETH NORDTVEDT, JR.
Montana State University, Bozeman, Montana

and

CLIFFORD M. WILL
California Institute of Technology, Pasadena, California

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CLIFFORD M. WILL
California Institute of Technology, Pasadena, California

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I. INTRODUCTION AND SUMMARY

Because of the recent interest in high-precision experiments to test gravitational theories, a <u>theoretical framework</u> encompassing a very broad and populous class of metric gravitational theories has been developed to systematize the comparison between theory and experiment. This framework has taken the form of a <u>Parametrized Post-Newtonian (PPN) Formalism</u> [Eddington (1922), Robertson (1962), Schiff (1967), Nordtvedt (1968b), and Will (1971a)], which treats the post-Newtonian limits of metric theories of gravity in terms of a series of metric parameters whose values vary from theory to theory. Hand-in-hand with the PPN metric go equations of motion describing the response of matter to the metric (the usual "divergence of the stress-energy tensor vanishes" for stressed matter, or geodesic motion for test bodies). The PPN formalism has been used to study the "classical tests" of general relativity, to search for new tests [Nordtvedt (1968a,c; 1970a; 1971a,b), Will (1971b,d), Thorne, Will, and Ni (1971)], and to analyse a wide class of contemporary metric theories of gravity [Nordtvedt (1970b), Ni (1972)].

In this paper we introduce a revised, and in certain respects, extended version of the PPN formalism. We view the original PPN formalism as having an empirical emphasis or flavor; each parameter was simply a coefficient of a different term in the post-Newtonian metric. In the extended PPN we have found it desirable to regroup the metric parameters. The new parameters (algebraic combinations of old parameters) are now individually related to different physical or conceptual aspects of metric theories. In terms of these new parameters, classification of metric theories of gravity becomes

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simpler and more concise. Every metric theory of gravity[1] can be categorized

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[1]Exception: Whitehead's (1922) theory is too complex to fit into the PPN

formalism. However it has been shown to disagree violently with gravi-

meter measurements of the tides of the solid Earth (Will 1971d), and hence

is no longer a viable theory.

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by four attributes:

i) Curvature of Space-Geometry

Metric theories of gravity predict that the spatial metrical properties

of objects are describable by a non-Euclidean, or "curved" geometry, but

differ in their predictions for the strength of this curvature. The amount

of space-curvature which a standard mass produces in a given theory is mea-

sured by a parameter γ.

ii) Nonlinearity of Gravity

The superposition law for gravity in most theories is nonlinear (most

theories predict that gravity itself also produces gravity), and the para-

meter β measures the extent of nonlinearity of post-Newtonian gravity. Note

that γ and β are the usual Eddington-Robertson-Schiff parameters used to

describe the "classical tests" of general relativity.

iii) Preferred Universal Rest-Frame

Some theories of gravity single out or produce a "preferred" frame,

related to the mean rest-frame of the Universe. These theories have recently

been shown to predict observable Solar-System effects related quantitatively

to the Solar System's velocity through this frame [§ II, § III; see also

Will (1971d), and Paper II in this series]. Three parameters α_1, α_2, and

α_3 suffice to measure the strength and specific manner in which motion in the

preferred frame affects the post-Newtonian metric. All α's are zero in theories which have no such "preferred-frame" effects (such as general relativity).

iv) Conservation Laws for Energy-Momentum

Some metric gravitational theories do not possess a complete set of conserved energy-momentum quantities for isolated gravitating systems. A set of parameters ζ_i measures the effects resulting from a breakdown of conservation laws. For point mass gravitational systems, two parameters ζ_1 and ζ_2 suffice in the post-Newtonian metric; for perfect-fluid systems, four are needed, ζ_1, ζ_2, ζ_3, and ζ_4. More complicated models for matter may require additional ζ's to specify the complete post-Newtonian metric. Theories which do possess conservation laws (such as general relativity) predict that all ζ's should be zero [see Will (1971c) for a discussion of conservation laws using the original PPN formalism].

Table 1 presents a summary of this new version of the PPN formalism. Each of the new parameters has an equivalent expression in the old formalisms of Nordtvedt (1968b) and Will (1971a), and these expressions are given in this table.

Table 2 lists the values of these parameters for all viable metric theories which have been analysed to date (Ni 1972) and for one member of a new class of metric theories which are presented in this paper for the first time ($\S$ III).

In Table 3 the parameter dependences of a variety of experimental tests are presented using the new PPN formalism. Some of the effects listed in Table 3 are new and are discussed in Paper II of this series.

This then, is the new PPN formalism in summary, and the remainder of

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this paper is devoted to details. In § II we present and discuss the metric for the extended PPN formalism. This metric contains additional metric terms which are functions of the velocity of the observer's chosen coordinate system relative to some "preferred Universal rest-frame", and we show that these additional terms are necessary in order to handle "preferred-frame" theories of gravity from any frame of reference, more easily and consistently. Section III contains detailed discussions of two classes of "preferred-frame" theories: (a) theories which introduce the preferred frame as a basic postulate, called "stratified" theories (see Ni 1972), and (b) theories in which the preferred frame is established by cosmological fields (vector fields or second tensor fields) which are determined by the Universe as a whole. Concluding remarks are presented in § IV. An Appendix gives detailed calculation of the PPN metric of a vector-metric theory as an example of (b) above.

II. THE EXTENDED PPN FORMALISM

a) Key Features of the Old Formalism

We first outline PPN formalism as described in Nordtvedt (1968b) and Will (1971a). The PPN framework has the following key features:

i) Coordinate System: the formalism uses a coordinate system in which, at large distances from the matter, the metric becomes nearly Minkowskiian.

ii) Matter Variables: gravity is produced by matter and energy, and the various matter variables which enter into the metric are: masses of particles (m_i), rest-mass densities of fluid elements (ρ), specific internal energies (Π), and pressures (p), as measured in local inertial frames comoving with the matter. The metric also contains positions and velocities $(\underset{\sim}{v})$ of

elements of matter as measured <u>relative to the coordinate system.</u>

iii) <u>PPN Parameters:</u>[2] γ, β, α_1, α_2, α_3, ζ_1, ζ_2, ζ_3, ζ_4 (see Table 1).

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[2]Each of the authors of this paper originally used his own set of PPN parameters. By using the parameters given in Table 1, we have united our separate notations, in addition to using parameters with more conceptual significance. Therefore, here we use the "new" parameters to review the old formalism.

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iv) <u>Metric of the Old PPN Formalism:</u>[3]

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[3]In this series of papers, Greek indices will take the values 1, 2, and 3; Roman indices will take the values 0, 1, 2, 3; and summation over repeated indices will be employed. Commas will denote partial differentiation, and semicolons will denote covariant differentiation. We will use units for which the velocity of light is unity and the Newtonian gravitational constant in the outer regions of the solar system, as measured today, is unity.

A.   Perfect Fluid Metric.

$$g_{00} = 1 - 2U + 2\beta U^2 - (2\gamma + 2 + \alpha_3 + \zeta_1)\,\Phi_1$$

$$- 2\left[(3\gamma - 2\beta + 1 + \zeta_2)\,\Phi_2 + (1 + \zeta_3)\,\Phi_3 + 3(\gamma + \zeta_4)\Phi_4\right] + \zeta_1\,\alpha \,, \qquad (1)$$

$$g_{0\alpha} = \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1)\,V_\alpha + \frac{1}{2}(1 + \alpha_2 - \zeta_1)\,W_\alpha \,,$$

$$g_{\alpha\beta} = -(1 + 2\gamma U)\,\delta_{\alpha\beta} \,.$$

Here U is the Newtonian gravitational potential given by

$$U(\underset{\sim}{x}, t) = \int \frac{\rho(\underset{\sim}{x}', t)\, d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \qquad ; \tag{2}$$

the other potentials appearing in equations (1) are

$$\Phi_1 = \int \frac{\rho'\, v'^2\, d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \quad , \qquad \Phi_2 = \int \frac{\rho'\, U'\, d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \quad ,$$

$$\Phi_3 = \int \frac{\rho'\, \Pi'\, d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \quad , \qquad \Phi_4 = \int \frac{p'\, d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \quad , \tag{3}$$

$$\alpha = \int \frac{\rho'[\underset{\sim}{v}' \cdot (\underset{\sim}{x} - \underset{\sim}{x}')]^2\, d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|^3} \quad ,$$

$$V_\alpha = \int \frac{\rho'\, v'_\alpha\, d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \quad , \qquad W_\alpha = \int \frac{\rho'\, \underset{\sim}{v}' \cdot (\underset{\sim}{x} - \underset{\sim}{x}')(x - x')_\alpha\, d\underset{\sim}{x}'}{|\underset{\sim}{x} - \underset{\sim}{x}'|^3} \quad .$$

The amount of rest-mass contained in a proper colume $d\mathscr{V}$ of fluid is given by [see Will (1971a) for a discussion]

$$\rho d\mathscr{V} = \rho\left(1 + \frac{1}{2}\, v^2 + 3\gamma U\right)\, d\underset{\sim}{x} \quad . \tag{4}$$

B. Point-Mass Metric.

$$g_{00} = 1 - 2\, \sum_k \frac{m_k}{r_k} + 2\beta\left(\sum_k \frac{m_k}{r_k}\right)^2$$

$$- (2\gamma + 1 + \alpha_3 + \zeta_1)\, \sum_k \frac{m_k\, v_k^2}{r_k} - 2\, (1 - 2\beta + \zeta_2)\, \sum_k \frac{m_k}{r_k}\, \sum_{j \neq k} \frac{m_j}{r_{jk}}$$

$$+ \zeta_1\, \sum_k \frac{m_k}{r_k^3}\, (\underset{\sim}{v}_k \cdot \underset{\sim}{r}_k)^2 \quad , \tag{5}$$

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$$g_{0\alpha} = \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1) \sum_k \frac{m_k}{r_k} v_k^{\alpha} + \frac{1}{2}(1 + \alpha_2 - \zeta_1) \sum_k \frac{m_k}{r_k^3} (\underset{\sim}{v}_k \cdot \underset{\sim}{r}_k) r_k^{\alpha} ,$$

$$g_{\alpha\beta} = - \left(1 + 2\gamma \sum_k \frac{m_k}{r_k}\right) \delta_{\alpha\beta} . \qquad\qquad (5 \text{ con't.})$$

Here $\underset{\sim}{r}_k = \underset{\sim}{x} - \underset{\sim}{x}_k$, $\underset{\sim}{r}_{ik} = \underset{\sim}{x}_i - \underset{\sim}{x}_k$ and $r_k = |\underset{\sim}{r}_k|$, and

$$m_k = \int_{k \text{ th body}} \rho d\mathcal{V} . \qquad\qquad (6)$$

v) <u>Choice of Gauge</u>: an infinitesimal coordinate transformation

$$(x^i)^\dagger = x^i + \delta x^i , \qquad\qquad (7)$$

changes the metric to

$$(g_{ij})^\dagger = g_{ij} - \delta x_{i;j} - \delta x_{j;i} . \qquad\qquad (8)$$

We have chosen δx^1, δx^2, δx^3 in order to make the spatial part of the metric $(g_{\alpha\beta})$ diagonal and isotropic. We have chosen δx^0 of the form

$$\delta x^0 = \lambda \chi_{,0} , \qquad\qquad (9)$$

where λ is an adjustable parameter, and where

$$\chi(\underset{\sim}{x}, t) = - \int \rho(\underset{\sim}{x}', t) |\underset{\sim}{x} - \underset{\sim}{x}'| d\underset{\sim}{x}' \qquad [\text{fluid}] ,$$

$$= - \sum_k m_k |\underset{\sim}{x} - \underset{\sim}{x}_k| \qquad [\text{point mass}] , \qquad (10)$$

in order that the metric component g_{00} contain no term of the form

$$\beta(x,t) = \iint \frac{\rho' \; \rho'' \; (\underset{\sim}{x} - \underset{\sim}{x}') \cdot (\underset{\sim}{x}' - \underset{\sim}{x}'')}{|\underset{\sim}{x} - \underset{\sim}{x}'| \; |\underset{\sim}{x}' - \underset{\sim}{x}''|^3} \; d\underset{\sim}{x}' \; d\underset{\sim}{x}'' \qquad [\text{fluid}] \qquad ,$$

(11)

$$= - \sum_k \frac{m_k \; \underset{\sim}{r}_k \cdot d\underset{\sim}{v}_k/dt}{r_k} \qquad [\text{point mass}] \quad .$$

b) Lorentz Transformations and Preferred Frames

Recent theoretical evidence has led us to conclude that the PPN metric in the form of equations (1) and (5) is valid for any metric theory of gravity only in a coordinate system which is at rest relative to a "preferred Universal rest-frame". In fact, we will show (§ II.c) that there is a more general PPN metric which is valid in any chosen coordinate frame which moves at velocity $\underset{\sim}{w}$ relative to the "Universal rest-frame", and that the old PPN metric is a special case of this general metric corresponding to $\underset{\sim}{w} = [$speed of chosen frame relative to Universe$] = 0$. Three pieces of evidence support this conclusion:

i) A Lorentz transformation of the old PPN metric from one coordinate frame to another coordinate frame whose outer region moves with velocity $\underset{\sim}{w}$ ($|\underset{\sim}{w}|$ assumed small) relative to the first does not always yield the PPN metric in its initial form but introduces new metric functions which depend on $\underset{\sim}{w}$. Only when the PPN parameters satisfy the conditions

$$\alpha_1 = 0, \quad \alpha_2 = 0, \quad \alpha_3 = 0 \quad ,$$

(12)

do no such $\underset{\sim}{w}$-terms arise (Will 1971c, Nordtvedt 1969). (More precisely, when $\alpha_1 = \alpha_2 = \alpha_3 = 0$, any $\underset{\sim}{w}$-terms which do arise have no physical meaning; they can always be removed by an infinitesimal gauge transformation. See § II.c for details.)

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ii) A calculation by one of us (Will 1971d) showed that the Newtonian gravitational constant G as measured by a Cavendish experiment on Earth should depend on the Earth's velocity relative to some "preferred frame", for any theory whose PPN parameters violated equations (12).

iii) A compilation and study by Ni (1972) of contemporary metric theories of gravity gives several specific examples of theories which single out such a "preferred Universal rest-frame". These theories all predict that at least one of the α-parameters should be non-zero. We will discuss these theories in more detail in § III.a.

There is additional motivation for using a more general PPN metric which is valid in any coordinate frame. Many computations of solar-system effects are simplified by working in the rest frame of the system being considered. Also, comparison of experimental results with the predictions of theory becomes more straightforward when the experimental data and the theoretical predictions are analysed in the same coordinate system. There-fore, for use in any frame moving at speed $\underset{\sim}{w}$ relative to the Universal rest-frame, we will adopt a more general PPN metric which contains additional metric terms depending on $\underset{\sim}{w}$. Just what these additional terms are is spelled out in the next sub-section.

c) The General PPN Metric

We take as our starting point the PPN metric valid in the mean rest-frame of the Universe [eqs. (1) or (5)]. To obtain the form of the metric valid in a frame moving with velocity $\underset{\sim}{w}$ relative to this frame, we apply a Lorentz transformation to the metric, assuming throughout that $\underset{\sim}{w}$ is small, so that

our post-Newtonian expansion is still valid.[4] The transformation from the

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[4]The post-Newtonian metric is an "expansion" in terms of small dimention-less quantities which characterize the gravitating system: Newtonian gravi-tational potential $U \sim m_k/r_k$, velocity v^2, specific internal energy Π, pressure relative to density p/ρ, all of which are $< 10^{-6}$ in the solar system. In order to simplify the "bookkeeping" of the terms in the metric, one assigns them "orders of smallness" (Chandrasekhar 1965). In our notation $U \sim m_k/r_k$ is $O(2)$, v is $O(1)$, v^2 is $O(2)$ and so on. We will also assume that $\underset{\sim}{w}$ is $O(1)$.

Universal rest-frame coordinates $(\underset{\sim}{x}, t)$ to the moving coordinates $(\underset{\sim}{\xi}, \tau)$ has the form (to post-Newtonian order):

$$\tau = t(1 + \frac{1}{2} w^2 + \frac{3}{8} w^4) - \underset{\sim}{x} \cdot \underset{\sim}{w}(1 + \frac{1}{2} w^2) + O(5) \quad ,$$

$$\underset{\sim}{\xi} = \underset{\sim}{x} - (1 + \frac{1}{2} w^2) \underset{\sim}{w} t + \frac{1}{2}(\underset{\sim}{x} \cdot \underset{\sim}{w}) \underset{\sim}{w} + O(4) \quad .$$

(13)

The matter velocity in the two frames, $\underset{\sim}{V} = d\underset{\sim}{x}/dt$ and $\underset{\sim}{v} = d\underset{\sim}{\xi}/d\tau$ are related by

$$\underset{\sim}{v} = \underset{\sim}{v} - \underset{\sim}{w} + O(3) \quad .$$

(14)

In order to express the metric functions in terms of $(\underset{\sim}{\xi}, \tau)$ and $\underset{\sim}{v}$ we make use of the expression given by Chandrasekhar and Contopoulous (1967)

$$\frac{1}{|\underset{\sim}{\xi} - \underset{\sim}{\xi}'|} = \frac{1}{|\underset{\sim}{x} - \underset{\sim}{x}'|} \left[1 + \frac{1}{2}(\underset{\sim}{n}' \cdot \underset{\sim}{w})^2 - (\underset{\sim}{n}' \cdot \underset{\sim}{w})(\underset{\sim}{n}' \cdot \underset{\sim}{v}) + O(4) \right] \quad ,$$

(15)

where

$$\underset{\sim}{n}' = (\underset{\sim}{x} - \underset{\sim}{x}')/|\underset{\sim}{x} - \underset{\sim}{x}'| \quad .$$

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We use the standard transformation law ($x^0 = t$, $\xi^0 = \tau$),

$$g_{ij}(\xi,\tau) = \frac{\partial x^k}{\partial \xi_i} \frac{\partial x^\ell}{\partial \xi_j} g_{k\ell}(\underset{\sim}{x},t) \quad , \tag{16}$$

along with equations (1), (4), (13), (14), and (15) to obtain a metric which has the same functional form (in terms of $\underset{\sim}{\xi}$, τ, $\underset{\sim}{v}$, etc.) as the old metric (in terms of $\underset{\sim}{x}$, t, $\underset{\sim}{v}$, etc.) except for additional terms which depend on $\underset{\sim}{w}$ [we quote only the perfect-fluid case; an equivalent result is obtained using the point-mass metric eq. (5)]:

$$\delta g_{00} = (\alpha_1 - \alpha_2 - \alpha_3) \, w^2 \, U - (2\alpha_3 - \alpha_1 + \alpha_2 + \zeta_1 - 1) \, w^\alpha V_\alpha$$

$$+ (\alpha_2 + \zeta_1 - 1) \, w^\alpha W_\alpha + \alpha_2 \, w^\alpha w^\beta \, U_{\alpha\beta} \quad ,$$

$$\delta g_{0\alpha} = \frac{1}{2}(\alpha_1 - \alpha_2 + \zeta_1 - 1) \, w^\alpha \, U + \frac{1}{2}(\alpha_2 - \zeta_1 + 1) \, w^\beta \, U_{\alpha\beta} \quad , \tag{17}$$

$$\delta g_{\alpha\beta} = 0 \quad ,$$

where the potential $U_{\alpha\beta}$ is given by [cf. eq. (10)]

$$U_{\alpha\beta} = \chi_{,\alpha\beta} + \delta_{\alpha\beta} \, U \quad ,$$

$$= \int \frac{\rho(\underset{\sim}{\xi}',\tau)(\xi - \xi')_\alpha \, (\xi - \zeta')_\beta}{|\underset{\sim}{\xi} - \underset{\sim}{\xi}'|^3} \, d\underset{\sim}{\xi}' \quad , \quad [\text{fluid}] \tag{18}$$

$$= \sum_k \frac{m_k \, r_k^\alpha \, r_k^\beta}{r_k^3} \quad . \quad [\text{point mass}]$$

The introduction of the new variable $\underset{\sim}{w}$ in the PPN metric also introduces an additional gauge freedom. By making a coordinate transformation of the form

$$\tau^\dagger = \tau - \frac{1}{2}(\alpha_2 + \zeta_1 - 1) \, w^\alpha \chi_{,\alpha} \quad ,$$

$$\underset{\sim}{\xi}^\dagger = \underset{\sim}{\xi} \quad ,$$

(19)

and using equation (8), we can transform the metric into a gauge in which no term of the form $w^\alpha W_\alpha$ appears. Note that this gauge transformation is completely independent of the gauge change [eq. (9)] which led to the form of the old PPN metric in equation (1). The final result for the new PPN metric for perfect fluids and for point masses, is given in Table 4.

Table 4 shows clearly the usefulness of the new parameters α_1, α_2, and α_3. These parameters label the terms in the metric which depend on $\underset{\sim}{w}$ and indicate, for a given theory, how large and what kind of solar-system effects will be produced by motion relative to the Universal rest-frame. Paper II in this series (Nordtvedt and Will 1972) will catalogue such solar-system effects and present evidence to put experimental limits on the α's, thereby ruling out many theories of gravity which were thought at one time to be viable.

We close this section with a recipe for computing the PPN parameter values for any metric theory of gravity:

i) Pick a coordinate system whose metric becomes Minkowskiian far from the local distribution of matter. It doesn't matter whether the coordinate system is moving or at rest relative to the Universe, although for many theories (cf. § III, and Appendix) calculations are simplified using the "rest" coordinate frame of the Universe.

ii) Use the standard techniques of Chandrasekhar (1965) or Einstein, Infeld, and Hoffman (1938) to compute the post-Newtonian metric of the matter in this coordinate system.

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iii) Compare this metric term by term with the equations in Table 4 and read off the values of the PPN parameters. Even if you used the "rest" coordinate frame, in which $\underset{\sim}{w} = 0$, you will still be able to obtain values for all the PPN parameters.

iv) To obtain the metric valid in any chosen frame, use the equations in Table 4, along with the PPN parameter values just computed, the velocity $\underset{\sim}{w}$ of the chosen frame relative to the Universe, and the velocities $\underset{\sim}{v}$ of matter relative to the chosen frame (the other variables: mass, density, etc., are all determined in frames comoving with each element of matter and are unaffected by this change in coordinate frame).

III. THEORIES OF GRAVITY WITH A PREFERRED
UNIVERSAL REST-FRAME

There is a variety of gravitational theories which single out some "preferred frame" in the Universe, and which have been shown to predict observable effects due to motion through this frame. These theories fall into two classes: (a) Theories which postulate the existence of a preferred frame directly and incorporate it into their calculational rules. These theories endow the Universe with "prior geometry", and use this prior-geometric structure as a foundation for the theory.[5] In § III.a we discuss

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[5]For a detailed discussion of "prior-geometry" see Misner, Thorne, and Wheeler (1972).

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a particularly simple subclass of these theories, known as "stratified theories with time-orthogonal conformally flat space slices" (Ni 1972).

(b) Theories which assume no "prior geometry", but which possess, in addition to the metric, vector or tensor cosmological fields whose source is the matter in the Universe. In these theories, the particular structure of the vector and tensor fields determines (in a Machian sense) the "preferred frame". These theories are examined in § III.b and in an Appendix.

a) Stratified Theories With Time-Orthogonal
Conformally Flat Space Slices

This is a particulary simple subclass of theories of gravity which postulate a "prior geometry". These theories are devised using the following prescription:[6] The Universe's large-scale distribution of matter determines

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[6]This discussion is adapted from Ni (1972).

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a preferred reference frame whose space slices ("strata") are conformally flat, although the full spacetime is not. In this preferred frame, the metric has the form

$$ds^2 = e^{2f(\varphi)} dt^2 - e^{2g(\varphi)} (dx^2 + dy^2 + dz^2) \quad , \tag{20}$$

where φ is a scalar field. In geometric, coordinate-free language, such theories have (i) a background, flat metric $\boldsymbol{\eta}$; (ii) a Universal time coordinate t (a scalar field) which is convariantly constant and has timelike gradients with respect to $\boldsymbol{\eta}$; (iii) a scalar gravitational field φ; and (iv) a metric $\mathbf{g}$ constructed from $\boldsymbol{\eta}$, t, and φ by

$$\mathbf{g} = e^{2g(\varphi)} \boldsymbol{\eta} + [e^{2f(\varphi)} - e^{2g(\varphi)}] \mathbf{d}t \otimes \mathbf{d}t \quad . \tag{21}$$

These theories differ from one another by their choice of the function $f(\varphi)$

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and $g(\varphi)$ and by their field equations for φ.

Ni (1972) has analysed theories of this type, whose authors include: Papapetrou (1954a,b,c), Yilmaz (1958, 1962), Page and Tupper (1968), and Rosen (1971a,b). Two additional theories were devised by Ni himself: a Lagrangian Stratified Theory and a General Stratified Theory (Ni 1972). A special case of Ni's "General Stratified Theory" is Coleman's (1971) theory. Einstein (1912) (not general relativity!) and Whitrow and Morduch (1960, 1965) devised stratified theories of gravity, but their theories disagree violently with light deflection and time-delay experiments, and so will not be considered further.

We will not discuss any of these theories individually; the reader is referred to Ni (1972) for details. We will only present Ni's results, in the form of PPN parameter values for each theory (Table 2). For completeness, Table 2 also gives the PPN parameter values for general relativity and for scalar-tensor theories [generalizations of Brans-Dicke-Jordan theory; see Ni (1972) for details and references]. We conclude this subsection with a theorem concerning the PPN parameter α_1 for stratified theories:

In every stratified theory of gravity with time-orthogonal, conformally flat space slices, the PPN parameter α_1 has the value

$$\alpha_1 = - 4(1 + \gamma) \quad . \tag{22}$$

In order to agree with light deflection and radar time delay experiments [$\gamma \sim 1$; see Thorne, Will, and Ni (1971) for a review], these theories must therefore have

$$\alpha_1 \sim - 8 \quad . \tag{23}$$

Note that all the stratified theories in Table 2 obey equation (24). In Paper II of this series, we will make use of this result to show that <u>all</u> <u>the stratified theories in Table 2 disagree violently with experiment.</u>

The proof of this theorem goes as follows: Pick any stratified theory with time-orthogonal conformally flat space slices. Using a coordinate system at rest with respect to the Universe, compute the post-Newtonian metric due to an arbitrary configuration of matter. To put this metric into the "standard" PPN gauge, it may be necessary to apply an infinitesimal gauge transformation [eq. (9)]

$$t^{\dagger} = t + \lambda \, \chi_{,0} \, , \qquad \underset{\sim}{x}^{\dagger} = \underset{\sim}{x} \quad , \tag{24}$$

for some value of λ. Since $g_{0\alpha}$ was initially identically zero [by assumption, cf. eq. (20)], in the new gauge it becomes

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\begin{aligned}
(g_{0\alpha})^{\dagger} &= - \lambda \, \chi_{,0\alpha} \\
&= - \lambda \, V_{\alpha} + \lambda \, W_{\alpha} \quad .
\end{aligned}
\tag{25}
$$

By comparing this with the PPN metric in Table 4 we obtain

$$\cdot \frac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1) = - \lambda \quad , \tag{26}$$

$$\frac{1}{2}(1 + \alpha_2 - \zeta_1) = \lambda \quad . \tag{27}$$

Adding equations (26) and (27) we obtain, finally,

$$\alpha_1 = - (4\gamma + 4) \quad , \tag{28}$$

independent of gauge, λ. Q.E.D.

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b) Lagrangian-Based Metric Theories with Additional

Vector and Tensor Fields

Here we take an approach which is different then that of § III.a, and
analyse preferred-frame theories of gravity based on a Lagrangian formula-
tion which initially assumes no preferred frames (no "prior geometry").

Since gravitation appears to be a long-range, purely attractive force,
one might expect the Universe's global matter distribution to affect local
gravitational physics, and to play a dominant role in establishing any
"preferred" reference frame. In fact, from this "Machian" point of view,
the mystery seems to be: How can a metric theory of gravity avoid at all
having a preferred frame related to the Universe rest-frame? General
relativity and the Brans-Dicke theory obviously avoid preferred frame
effects (cf. their parameter values $\alpha_1 = \alpha_2 = \alpha_3 = 0$ in Table 2). Before
we create theories which do have preferred frames, we must first shed some
light on how these exceptional theories avoid them.

Consider a local gravitational system, such as the solar system, which
is embedded in the Universe. We separate the computation of the metric into
two parts: a Universal or cosmological solution, and a "local" solution.
From this viewpoint, the Universe affects the local gravitational physics of
the system by establishing the boundary conditions (at a boundary "far" from
the matter) for the various fields generated by the local system. The local
system "feels" its relationship to the Universe via the asymptotic field
values of the fields present; metric g_{ij}, scalar field φ, vector field K_i,
second-rank tensor field C_{ij}, and so on. Several conclusions follow:

1) A theory which contains solely a metric field yields local gravi-
tational physics which is identical in all asymptotic Lorentz frames, and

which does not change with Universe evolution. In addition, the Newtonian gravitational constant G is unaffected by the proximity of matter. All this follows from the invariance properties of η_{ij} (the asymptotic form of g_{ij}), the only field coupling the local system asymptotically to the Universe, and from general covariance, which allows us to find a coordinate system in which the metric field takes this Minkowskii form at the boundary between the Universe and the local system.

2) A theory which contains a metric field and a scalar field φ yields physics which is identical in all asymptotic Lorentz frames, but which may vary with Universe evolution; G may be affected by proximity of matter. These conclusions follow from invariance of both η_{ij} and φ under Lorentz transformations, but now φ may vary with Universe evolution and may depend on the proximity of matter.

3) A theory which contains a vector field K_i and/or an additional second-rank tensor field C_{ij} yields local physics which does depend on motion relative to a preferred Universe rest-frame, and which may vary with evolution of the Universe; G may be affected by proximity of matter. This follows because the asymptotic values of K_i and C_{ij} are <u>not</u> invariant under Lorentz transformations (an exception would be $C_{ij[\text{asymptotic}]}$ proportional to η_{ij}).

In summary, it is the Lorentz invariance of the asymptotic fields φ and η_{ij} which makes it impossible to have preferred-frame effects in theories containing solely those fields. Thus we must appeal to vector- or tensor-metric theories for preferred-frame effects.

It is commonly believed that cosmological vector fields and additional second-rank tensor fields are absent from physics. The Hughes-Drever

experiment (measurement of the isotropy of inertial mass) and a variety of laboratory "ether-drift" experiments have been used by Dicke [(1964); see also Peebles and Dicke (1962); Peebles (1962)] to rule out these fields. Closer examination of these arguments shows that <u>the experimental evidence</u> <u>rules out only those vector and "second tensor" cosmological fields which</u> <u>couple directly to matter</u>. Since these experiments were performed under conditions where the effects of gravity were negligible, <u>they do not rule</u> <u>out vector and second-tensor fields which couple only to gravity</u>.

We now proceed to devise such theories. We assume these theories are derivable from a coordinate-invariant Lagrangian, with no <u>a priori</u> assumption of a special coordinate frame or of any prescribed form for the tensors in the theory. All physical quantities (metric, vector fields, tensor fields, matter variables) must be calculated in terms of other physical quantities. The Lagrangian for such theories has the general form

$$L = L_m(q, p, g_{ij})$$

$$+ L_g(g_{ij}, g_{ij,k}, K_i, K_{i,j}, C_{ij}, C_{ij,k}, \cdots) \quad ,$$

(29)

where q and p symbolically represent the dynamical variables of matter, g_{ij} is the metric and K_i, C_{ij}, $\cdots$ are possible cosmological vector and tensor fields. The matter Lagrangian is related to the stress-energy tensor for matter and non-gravitational fields by the functional derivative

$$\frac{1}{\sqrt{-g}} \frac{\delta}{\delta g^{ij}} (\sqrt{-g} \, L_m) \equiv T_{ij} \quad .$$

(30)

The additional fields in the Lagrangian do not couple directly to matter; this leads to the equation of motion for matter

$$T^{ij}{}_{;j} = 0 \quad , \tag{31}$$

which yields geodesic motion for test bodies. This form for the Lagrangian thus guarantees that the theory satisfy the postulates of a "metric" theory [for a discussion of these postulates see Thorne and Will (1971)].

For simplicity, we restrict ourselves to theories which have linear field equations for all the supplementary fields; nevertheless, a wide variety of theories is possible. For vector-metric theories, the general Lagrangian has the form

$$
\begin{aligned}
L_{VM} = L_m &+ k_1 R + k_2 K_i K^i R + k_3 K_i K_j R^{ij} \\
&+ k_4 K_{i;j} K^{i;j} + k_5 K_{i;j} K^{j;i} + k_6 K_i{}^{;i} K_j{}^{;j} \quad ,
\end{aligned} \tag{32}
$$

where R_{ij} and R are the Ricci tensor and scalar formed from the metric, k_1, ..., k_6 are dimensionless coupling constants. Tensor-metric theories have even more structure (we assume C_{ij} is symmetric):

$$
\begin{aligned}
L_{TM} = L_m &+ k_1' R + k_2' C_{ij} C^{ij} R + k_3' (C_{ij} g^{ij})^2 R \\
&+ k_4' C_{ij} g^{ij} C_{k\ell} R^{k\ell} + k_5' C_{ik} C^k{}_j R^{ij} \\
&+ k_6' C_{ij;k} C^{ij;k} + k_7' C_{ij;k} C^{ik;j} + k_8' C_{ik;}{}^k C^{ij}{}_{;j} \\
&+ k_9' C_{ij;}{}^j g^{k\ell} C_{k\ell}{}^{;i} + k_{10}' C_{ij;k} C_{\ell m;}{}^k g^{ij} g^{\ell m} \quad .
\end{aligned} \tag{33}
$$

We point out here that the above equation (33) alone does not reveal the full richness of possible Lagrangian terms for tensor-metric theories. If C_{ij} is a non-singular field, one can define Christoffel symbols in terms of C_{ij}. Also, third-rank tensors $S^i{}_{jk}$ can be produced by taking the difference

of the Christoffel symbols

$$S^i_{\ jk} = \Gamma^i_{\ jk}(\mathbf{g}) - \Gamma^i_{\ jk}(\mathbf{C}) \quad .$$

Covariant derivatives can be defined with respect to each tensor, g_{ij} or C_{ij}, and tensor densities can be formed using either

$$\sqrt{-g}$$

or

$$\sqrt{|\det C|} \quad .$$

All these quantities can be used to construct Lagrangian terms.

A future paper in this series will give a complete study of these theories. Such a study will discuss consistency of the fields equations, the post-Newtonian limit and the PPN parameter values, cosmological solutions, and positive-definiteness of free-field energies.

To show that theories of this type can produce apparent agreement with the "three classical tests" of general relativity, yet also possess preferred-frame effects, we now quote the PPN parameter values for a simple illustrative case of a vector-metric theory. Detailed calculations have been relegated to an Appendix. This theory has as its action integral

$$A = \int d^4x \sqrt{-g} \, [G_0 \, L_m + R + K_{i;j} \, K_{k;\ell} \, g^{ik} \, g^{j\ell}] \tag{34}$$

where G_0 is the _unrenormalized_ gravitational constant. This theory corresponds to the case [eq. (32)] $k_1 = k_4 = G_0^{-1}$, $k_2 = k_3 = k_5 = k_6 = 0$.

The calculations in the Appendix yield a PPN metric with parameter values (see also Table 2)

$$\gamma = \beta = 1 \quad ,$$

$$\alpha_1 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0 \tag{35}$$

$$\alpha_2 = \kappa^2 / (1 + \tfrac{1}{2} \kappa^2) \quad .$$

Here κ^2 is the squared magnitude of the vector field, evaluated far from the matter, i.e.,

$$\kappa^2 = [g^{ij} K_i K_j]_{\text{far from matter}} \quad . \tag{36}$$

The gravitational constant is renormalized in this theory, and so changes in time as the Universe evolves:

$$G(t) = \frac{G_0}{1 + \tfrac{1}{2}[K(t)]^2} \quad . \tag{37}$$

IV. CONCLUSIONS

In this paper, we have presented a new version of the Parametrized Post-Newtonian Formalism, and have discussed preferred-frame theories of gravity. In Paper II of this series we will use this new formalism to compute observable solar-system effects predicted by preferred-frame theories and will present experimental evidence to rule many of these theories out.

APPENDIX

A VECTOR-METRIC THEORY OF GRAVITY

Here we discuss in detail the vector-metric theory described in § III.b. This theory has as its action integral [cf. eq. (34)]

$$\alpha = \int d^4x \sqrt{-g} \left[G_0 L_m + R + K_{i;j} K_{k;\ell} g^{ik} g^{j\ell} \right] , \tag{A1}$$

where L_m is the matter Lagrangian, R is the scalar curvature formed from the metric g_{ij}, K_i is the cosmological vector field and G_0 is the (unrenormalized) gravitational constant. By varying the action with respect to g_{ij} and K_i in the usual way, we obtain the field equations

$$K_{i;j}{}^{;j} = 0 , \tag{A2}$$

$$R_{ij} - \frac{1}{2} R g_{ij} = 8 \pi G_0 T_{ij} + \Theta_{ij} , \tag{A3}$$

where R_{ij} is the Ricci tensor, T_{ij} is the stress-energy tensor for matter and non-gravitational fields, and Θ_{ij} is the vector-field stress-energy tensor given by

$$\begin{aligned} \Theta_{ij} = &K_{i;m} K_j{}^{;m} + K_{m;i} K^m{}_{;j} - \frac{1}{2} K^m{}_{;n} K_m{}^{;n} g_{ij} \\ &+ \frac{1}{2} \left(K^m S_{ij} - K_i S^m{}_j - K_j S^m{}_i \right)_{;m} , \end{aligned} \tag{A4}$$

where

$$S_{ij} = K_{i;j} + K_{j;i} . \tag{A5}$$

It is straightforward, though tedious, to verify using equations (A2), (A4), and (A5) that

$$\Theta_{ij}{}^{;j} = 0 , \tag{A6}$$

and thus that

$$T_{ij}{}^{;j} = 0 \quad . \tag{A7}$$

We now use these equations to compute the metric produced by a configuration of perfect fluid, in the post-Newtonian approximation, using the standard techniques of Chandrasekhar (1965). [Equivalent results can be obtained for a system of point masses using the standard EIH (Einstein, Infeld, and Hoffman 1938) method.] The metric is expanded about the flat-space Minkowskii metric according to

$$g_{ij}(\underset{\sim}{x}, t) = \eta_{ij} + h_{ij}(\underset{\sim}{x}, t) \tag{A8}$$

where, far from the matter, h_{ij} tends asymptotically toward zero. We also expand the vector field K_i about its asymptotic value. However, in order to simplify the calculation, we will work in a coordinate system in which the asymptotic vector field has only a time component; i.e. asymptotically

$$K_\alpha = 0 \quad . \tag{A9}$$

Then, in this coordinate system, we have

$$|K|^2 \equiv \eta^{ij} K_i K_j = K_0{}^2 \quad , \tag{A10}$$

far from the configuration of fluid. The frame in which $K_0 = K$ and $K_\alpha = 0$ is presumably the rest-frame of the Universe's smoothed-out distribution of matter. The expansion of the vector field can thus be written

$$K_0 = K + \varphi(\underset{\sim}{x}, t) \quad , \tag{A11}$$

$$K_\alpha = k_\alpha(\underset{\sim}{x}, t) \quad , \tag{A12}$$

where φ is $0(2)$ and k_α is $0(3)$ (cf. footnote 4), and both go to zero far

from the fluid.

The perfect-fluid stress-energy tensor is given by (cf. Will 1971a)

$$T_{ij} = (\rho + \rho\Pi + p) \, u_i \, u_j - p \, g_{ij} \quad , \tag{A13}$$

where u_i is the four-velocity of the fluid. The field equation (A3) can be written in the form

$$R_{ij} = 8 \, \pi \, G_0(T_{ij} - \frac{1}{2} \, T \, g_{ij}) + (\Theta_{ij} - \frac{1}{2} \, \Theta \, g_{ij}) \quad , \tag{A14}$$

where T and Θ are the traces of T_{ij} and Θ_{ij} respectively.

We wish to obtain h_{00} to $O(4)$, $h_{0\alpha}$ to $O(3)$, and $h_{\alpha\beta}$ to $O(2)$. The calculation proceeds along the following lines:

i) Calculate h_{00} to $O(2)$. This corresponds to the theory's Newtonian limit. To the required order, we have

$$R_{00} = \frac{1}{2} \, \nabla^2 \, h_{00} \, , \quad T_{00} = T = \rho \quad ,$$

$$\Theta_{00} = - \frac{1}{2} \, \kappa^2 \, \nabla^2 \, h_{00} \, , \quad \Theta = - \kappa^2 \, R_{00} \quad . \tag{A15}$$

Equation (A14) then becomes

$$\nabla^2 \, h_{00} = 8 \, \pi \, \rho \left[G_0 \big/ (1 + \frac{1}{2} \, \kappa^2) \right] \quad , \tag{A16}$$

whose solution is

$$h_{00} = - 2 \, U \left[G_0 \big/ (1 + \frac{1}{2} \, \kappa^2) \right] \quad . \tag{A17}$$

Because we have chosen units so that the gravitational constant as measured far from the solar system, today is unity, we have

$$\frac{G_0}{1 + \frac{1}{2} \, \kappa^2} = G \equiv 1 \quad [\text{today}] \quad . \tag{A18}$$

Note that if K varies slowly as the Universe evolves, then the gravitational constant G may vary from its present value with time at a rate

$$\frac{1}{G}\left(\frac{dG}{dt}\right) = - \frac{K}{1 + \frac{1}{2} K^2}\left(\frac{dK}{dt}\right) \quad .$$ (A19)

ii) Calculate $h_{\alpha\beta}$ to $O(2)$. By making a particular choice of gauge:

$$h^i_{\ i,\alpha\beta} - h^\gamma_{\ \beta,\alpha\gamma} - h^\gamma_{\ \alpha,\beta\gamma} = 0 \quad ,$$ (A20)

we give the field equation for $R_{\alpha\beta}$ (eq. A14) the simple form (to the required order)

$$\nabla^2 h_{\alpha\beta} = 8 \pi \rho \delta_{\alpha\beta} \quad ,$$ (A21)

whose solution is

$$h_{\alpha\beta} = - 2 U \delta_{\alpha\beta} \quad .$$ (A22)

iii) Calculate φ and k_α to $O(2)$ and $O(3)$. The field equation (A2) for $i = 0$ becomes, to the necessary order

$$\nabla^2 (\varphi + K U) = 0 \quad .$$ (A23)

Since φ must vanish far from the fluid, we have

$$\varphi = - K U \quad .$$ (A24)

If we choose the gauge in which

$$\frac{1}{2} h^\beta_{\ \beta,0} - h^\beta_{\ 0,\beta} = U_{,0}\left[K^2/(1 + \frac{1}{2} K^2)\right] \quad ,$$ (A25)

Then equation (A2) for $i = \alpha$ becomes

$$\nabla^2\left[k_\alpha - \frac{1}{2} K h_{0\alpha} + \frac{1}{4} K\left(1 + \frac{K^2}{1 + \frac{1}{2} K^2}\right) \chi_{,0\alpha}\right] = 0 \quad ,$$ (A26)

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where χ is given by equation (10). Equation (A26) has the solution

$$k_\alpha = \frac{1}{2} K h_{0\alpha} - \frac{1}{4} K \left(1 + \frac{K^2}{1 + \frac{1}{2} K^2} \right) \chi_{,0\alpha} \quad . \tag{A27}$$

iv) Calculate $h_{0\alpha}$ to $O(3)$. With the choice of gauge (eq. A25), the field equation (A14) simplifies to

$$\nabla^2 h_{0\alpha} = - 16 \pi \rho v_\alpha - \frac{1}{2} \left(1 + \frac{K^2}{1 + \frac{1}{2} K^2} \right) \chi_{,0\alpha} \quad , \tag{A28}$$

or

$$h_{0\alpha} = 4 V_\alpha - \frac{1}{2} \left(1 + \frac{K^2}{1 + \frac{1}{2} K^2} \right) \chi_{,0\alpha} \quad . \tag{A29}$$

Using the fact that

$$\chi_{,0\alpha} = V_\alpha - W_\alpha \quad , \tag{A30}$$

we get

$$h_{0\alpha} = \frac{1}{2} \left(7 - \frac{K^2}{1 + \frac{1}{2} K^2} \right) V_\alpha + \frac{1}{2} \left(1 + \frac{K^2}{1 + \frac{1}{2} K^2} \right) W_\alpha \quad . \tag{A31}$$

v) Calculate h_{00} to $O(4)$. To the appropriate order, equation (A14) along with equations (A13), (A17), (A24), (A25), and (A27) gives

$$\nabla^2 h_{00} = 8 \pi \rho + 2 \nabla^2 (U^2) + 16 \pi \rho v^2 + 8 \pi \rho \Pi$$
$$+ 24 \pi p + 16 \pi \rho U \quad , \tag{A32}$$

whose solution is

$$h_{00} = - 2 U + 2 U^2 - 4 \Phi_1 - 4 \Phi_2 - 2 \Phi_3 - 6 \Phi_4 \quad . \tag{A33}$$

The final form of the metric is

$$g_{00} = 1 - 2\ U + 2\ U^2 - 4\ \Phi_1 - 4\ \Phi_2 - 2\ \Phi_3 - 6\ \Phi_4$$

$$g_{0\alpha} = \frac{1}{2}\left(7 - \frac{K^2}{1 + \frac{1}{2}\ K^2}\right) V_\alpha + \frac{1}{2}\left(1 + \frac{K^2}{1 + \frac{1}{2}\ K^2}\right) W_\alpha \qquad (A34)$$

$$g_{\alpha\beta} = - (1 + 2\ U)\ \delta_{\alpha\beta} \quad .$$

By comparing equation (A34) with the metric in Table 4, we can read off the values of the PPN parameters, as listed in Table 2.

Thus this theory does predict that local gravitational physics should depend on motion relative to the preferred Universal rest-frame (since $\alpha_2 \neq 0$), and should vary with evolution of the Universe [since K may vary with time; cf. eq. (A19)]. Notice that, in a Robertson-Walker universe with expansion parameter $a(t)$, K_i evolves according to ("dot" denotes d/dt)

$$\ddot{K}_0 + 3\ \dot{K}_0(\dot{a}/a) - 3(\dot{a}/a)^2\ K_0 = 0 \qquad (A35)$$

$$K_\alpha = 0$$

However, this theory predicts that the local gravitational constant G should be <u>unaffected</u> by nearby matter. In the presence of matter (and in the Universal rest-frame), this locally measured G is given by [see Nordtvedt (1970b), Will (1971d)]

$$G_{local} = 1 - (4\ \beta - \gamma - 3 - \zeta_2)\ U_{external} \quad . \qquad (A36)$$

Since $\beta = \gamma = 1$, and $\zeta_2 = 0$ for this theory, we find G_{local} is unaffected by nearby matter. This can be seen in another way, as follows: Consider a gravitating system in the presence of an external mass, whose Newtonian gravitational potential is U_{ext}. When the local system and the external mass

are analysed together in an asymptotically flat coordinate system, the metric and the vector field are given by, to first order [cf. eqs. (A24), (A34)]

$$g_{00} = 1 - 2\,U - 2\,U_{ext} \quad,$$

$$K_0 = K(1 - U - U_{ext}) \quad. \tag{A37}$$

The gravitational constant G is determined by the value of K_0 far from the system; i.e. [cf. eq. (A17)]

$$G = \frac{G_0}{1 + \frac{1}{2}\,K^2} \quad. \tag{A38}$$

We now look at the local gravitational system itself using a coordinate system which becomes asymptotically flat (to some desired precision) far from the local system but between the local system and the external mass. The transformation from the first coordinate system to this one has the form

$$t_{new} = t_{old}(1 - U_{ext}) \quad, \tag{A39}$$

which gives

$$(g_{00})_{new} = 1 - 2\,U \quad,$$

$$(K_0)_{new} = K(1 - U) \quad. \tag{A40}$$

Thus the transformation which removes the external field from the metric also removes the external field from the vector field. Therefore the gravitational constant as measured in the local system is unaffected by the external mass.

TABLE 1

THE NEW PPN PARAMETERS AND THEIR INTERPRETATION[†]

Parameter	What it Measured, Relative to General Relativity	Value in General Relativity	Equivalent Parameter Expressions in the Old Formalism*	
			a) Nordtvedt	b) Will
γ	How much space-curvature is produced by unit rest mass $(g_{\alpha\beta})$?	1	γ	γ
β	How much nonlinearity is there in the superposition law for gravity (g_{00})?	1	β	β
α_1	To what extent and in what manner does the theory single out a preferred Universal rest-frame?	0	$8\Delta - 4\gamma - 4$	$7\Delta_1 + \Delta_2 - 4\gamma - 4$
α_2		0	$\alpha''' - 1$	$\Delta_2 + \zeta - 1$
α_3		0	$4\alpha'' - \alpha''' - 2\gamma - 1$	$4\beta_1 - 2\gamma - 2 - \zeta$
ζ_1	How much and what kind of violation of conservation of total momentum does the theory predict?	0	$\alpha''' - \chi$	ζ
ζ_2		0	$2\beta - \alpha' - 1$	$2\beta + 2\beta_2 - 3\gamma - 1$
ζ_3		0	absent	$\beta_3 - 1$
ζ_4		0	absent	$\beta_4 - \gamma$

[†]Adapted from Thorne, Will, and Ni (1971).

*The metric in the old formalism was valid only in a coordinate frame at rest in the Universe rest-frame (see §II).

TABLE 2

METRIC THEORIES OF GRAVITY AND THEIR PPN PARAMETER VALUES[†]

Theory and Its Parameters[‡]	γ	β	α_1	α_2	α_3	ζ_1	ζ_2	ζ_3	ζ_4
General Relativity	1	1	0	0	0	0	0	0	0
Scalar-Tensor Theories[*] (ω,Λ)	$\dfrac{1+\omega}{2+\omega}$	$1+\Lambda$	0	0	0	0	0	0	0
Vector-Metric Theory (K) (§ III.b; Appendix)	1	1	0	$K^2/(1+\tfrac{1}{2}K^2)$	0	0	0	0	0
Stratified Theories:									
a. Page and Tupper (1968) (a,c)	a	$1+c$	$-4(1+a)$	0	$-2(1+a)$	0	$1+a+2c$	0	$-a$ or $-(1+a)$**
b. Modified Yilmaz (1958, 1962)	1	1	-8	0	-4	0	-2	0	-1 or -2**
c. Papapetrou (1954a,b,c)	1	1	-8	-4	0	0	0	0	0
d. Lagrangian Stratified Theory (Ni 1972)	1	1	-8	0	0	0	0	0	0
e. General Stratified Theory (Ni 1972)(p,q)	1	$1-q$	-8	0	-4	0	$p-2q-2$	0	-1 or -2**
f. Coleman (1971) (p)	1	1	-8	0	-4	0	$p-2$	0	-1 or -2**
g. Rosen (1971a,b) (λ)	λ	$\tfrac{1}{4}(3+\lambda)$	$-4(1+\lambda)$	0	0	0	0	0	0

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Footnotes to TABLE 2

†See Ni (1972) for detailed discussion of all the theories in this table except the Vector-Metric Theory (see § III.b, Appendix), and Coleman's (1971) theory.

‡Several of these theories contain adjustable parameters whose values may be fixed either by fitting to experimental results or by appealing to cosmological or philosophical arguments.

*Brans-Dicke Theory is the special case $\Lambda = 0$.

**The value of ζ_4 for many theories depends on what matter density one chooses as source for the gravitational fields: $\rho = T_{ij} u^i u^j$ = component of stress-energy tensor along four-velocity of matter $[\zeta_4 = -\gamma]$; or ρ = trace (T_{ij}) $[\zeta_4 = -(1+\gamma)]$. See Ni (1972) for discussion of this point.

TABLE 3

PPN PARAMETER DEPENDENCE OF VARIOUS EXPERIMENTAL TESTS

Experimental Test or Observable Effect	PPN Parameter Dependence
Light Deflection, Radar Time Delay	$\frac{1}{2}(1 + \gamma)$
Perihelion Shift	
a) Classical	$\frac{1}{3}(2 + 2\gamma - \beta)$
b) Due to Motion through* Universal rest-frame	$\alpha_1, \ \alpha_2, \ \alpha_3$
Geodetic Precession of a gyroscope	$\frac{1}{3}(1 + 2\gamma)$
Dragging of Inertial Frames	$\frac{1}{2}(1 + \gamma + \frac{1}{4}\alpha_1)$
Equivalence Principle Breakdown $\left(m_{passive} \neq m_{inertial}\right)$	
a) Isotropic	$-(4\beta - \gamma - 3 - \alpha_1 + \alpha_2 - \zeta_1)$
b) Anisotropic	$-(\alpha_2 + \zeta_2 - \zeta_1)$
Perturbations in Earthbound Gravimeter Measurements:	
a) Variation in G	
i) Due to field of Sun and Planets	$4\beta - \gamma - 3 - \zeta_2$
ii) Due to motion through† Universal rest-frame	$\alpha_2 + \alpha_3 - \alpha_1$
iii) Anisotropy due to motion† through rest-frame	α_2

TABLE 3 (continued)

Experimental Test or Observable Effect	PPN Parameter Dependence
b) Other Effects	
i) Due to external field gradients	α_1
ii) Due to internal structure of the Earth	$4\beta - \gamma - 3 - \alpha_2 + \zeta_1 - 2\zeta_2$
Periodic Perturbations in orbits due to Motion through Universe rest-frame[*]	$\alpha_1,\ \alpha_2,\ \alpha_3$

[*]See Nordtvedt and Will (1972) (Paper II).

[†]See Will (1971d), Nordtvedt and Will (1972) (Paper II).

TABLE 4

THE PPN METRIC VALID IN ANY FRAME

Perfect-Fluid Metric

$$g_{00} = 1 - 2U + 2\beta U^2 - (2\gamma + 2 + \alpha_3 + \zeta_1)\,\Phi_1 + \zeta_1\,\alpha$$

$$- 2\Big[(3\gamma - 2\beta + 1 + \zeta_2)\,\Phi_2 + (1 + \zeta_3)\,\Phi_3 + 3(\gamma + \zeta_4)\,\Phi_4\Big]$$

$$+ (\alpha_1 - \alpha_2 - \alpha_3)\,w^2 U + \alpha_2\,w^\alpha w^\beta\,U_{\alpha\beta} - (2\alpha_3 - \alpha_1)\,w^\alpha V_\alpha \quad ,$$

$$g_{0\alpha} = \tfrac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1)\,V_\alpha + \tfrac{1}{2}(1 + \alpha_2 - \zeta_1)\,W_\alpha$$

$$+ \tfrac{1}{2}(\alpha_1 - 2\alpha_2)\,w^\alpha U + \alpha_2\,w^\beta\,U_{\alpha\beta} \quad ,$$

$$g_{\alpha\beta} = -(1 + 2\gamma U)\,\delta_{\alpha\beta} \quad .$$

Point-Mass Metric

$$g_{00} = 1 - 2\,\sum_k \frac{m_k}{r_k} + 2\beta\Big(\sum_k \frac{m_k}{r_k}\Big)^2 - 2(1 - 2\beta + \zeta_2)\sum_k \frac{m_k}{r_k}\sum_{j\neq k}\frac{m_j}{r_{jk}}$$

$$- (2\gamma + 1 + \alpha_3 + \zeta_1)\sum_k \frac{m_k v_k^2}{r_k} + \zeta_1 \sum_k \frac{m_k}{r_k^3}(\underset{\sim}{v}_k \cdot \underset{\sim}{r}_k)^2$$

$$+ (\alpha_1 - \alpha_2 - \alpha_3)\,w^2 \sum_k \frac{m_k}{r_k} + \alpha_2 \sum_k \frac{m_k}{r_k^3}(\underset{\sim}{w} \cdot \underset{\sim}{r}_k)^2 - (2\alpha_3 - \alpha_1)\sum_k \frac{m_k\,\underset{\sim}{w}\cdot\underset{\sim}{v}_k}{r_k} \quad ,$$

$$g_{0\alpha} = \tfrac{1}{2}(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1)\sum_k \frac{m_k v_k^\alpha}{r_k} + \tfrac{1}{2}(1 + \alpha_2 - \zeta_1)\sum_k \frac{m_k}{r_k^3}(\underset{\sim}{v}_k \cdot \underset{\sim}{r}_k)\,r_k^\alpha$$

$$+ \tfrac{1}{2}(\alpha_1 - 2\alpha_2)\,w^\alpha \sum_k \frac{m_k}{r_k} + \alpha_2 \sum_k \frac{m_k}{r_k^3}(\underset{\sim}{w}\cdot\underset{\sim}{r}_k)\,r_k^\alpha \quad ,$$

$$g_{\alpha\beta} = -\Big(1 + 2\gamma \sum_k \frac{m_k}{r_k}\Big)\delta_{\alpha\beta} \quad .$$

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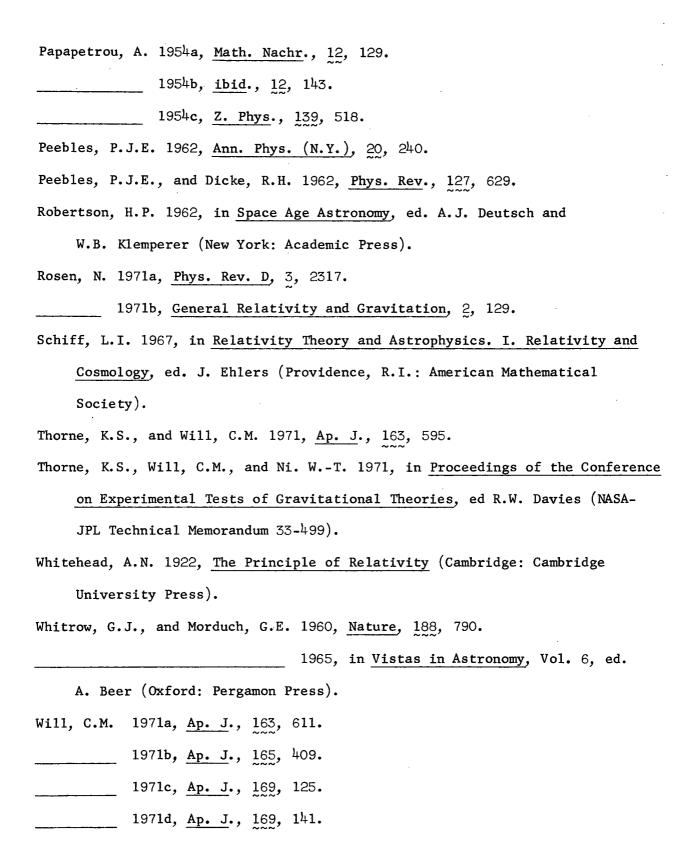Thorne, K.S., and Will, C.M. 1971, Ap. J., 163, 595.

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CONSERVATION LAWS AND PREFERRED FRAMES

IN RELATIVISTIC GRAVITY. II.

EXPERIMENTAL EVIDENCE TO RULE OUT

PREFERRED-FRAME THEORIES OF GRAVITY

KENNETH NORDTVEDT, JR.
Montana State University, Bozeman, Montana

and

CLIFFORD M. WILL
California Institute of Technology, Pasadena, California

I. INTRODUCTION AND SUMMARY

In Paper I of this series (Will and Nordtvedt 1972) we presented a
new version of the Parametrized Post-Newtonian (PPN) Formalism, the theo-
retical tool used to study experimental tests of metric gravitational theories.
An important feature of this new version of the PPN Formalism is a simple
and consistent prescription for analysing "preferred-frame" theories of
gravity. By "preferred-frame" theories we mean metric theories which single
out the frame which comoves with the Universe's smoothed-out distribution of
matter as a "preferred" reference frame. Several such theories are known;
those studied to date fall into two classes: i) "Stratified Theories" (with
"time-orthogonal conformally flat space-slices") whose authors include Page
and Tupper, Yilmaz, Papapetrou, Rosen, Ni, and Coleman; and ii) theories with
additional cosmological vector or tensor fields; one of these, a vector-metric
theory was analysed in Paper I.

Even though these theories single out a preferred Universal rest-frame,
they have until recently been considered to be viable alternatives to general
relativity, since their predictions for the "standard tests" — gravitational
redshift, light bending, radar time delay, and perihelion shift — are all in
agreement with observations. But a closer examination of these theoretical
predictions reveals that they agree with observations <u>only if the solar system
is assumed to be at rest relative to the preferred Universal rest-frame.</u>

A much more reasonable assumption is that the solar system moves through
the Universe with a velocity of around 200 km/sec, resulting from its orbital
motion around the galaxy and from the peculiar motion of the galaxy itself.
Measurements of anisotropies in galactic redshifts and in the cosmic micro-
wave radiation lend some support to this assumption (see Sciama [1971] for

a review of the relevant data; see also §IV). In this paper we will show that such a 200 km/sec motion produces, according to preferred-frame theories of gravity, observable solar-system effects which are in disagreement with observation.

Throughout this paper, we will use the language and notation of the new PPN formalism discussed in Paper I of this series; the reader is urged to consult this paper for details. This formalism treats the post-Newtonian limit of metric theories of gravity in terms of a series of metric parameters, whose values vary from theory to theory. These PPN parameters have a physical or conceptual significance; in particular, the three parameters α_1, α_2, and α_3 measure to what extent a theory of gravity singles out a "preferred universal rest-frame". This can be seen from the form of the PPN metric in Table 4 of Paper I. When written in a coordinate frame which moves with velocity $\underset{\sim}{w}$ relative to some "preferred frame", the PPN metric may contain terms which depend on $\underset{\sim}{w}$; these additional terms are (cf. Table 4, Paper I)[1]

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[1]The units, notation and conventions used in this paper are the same as those used in Paper I.

$$\delta g_{00} = (\alpha_1 - \alpha_2 - \alpha_3)\, w^2\, U + \alpha_2\, w^\alpha\, w^\beta\, U_{\alpha\beta} - (2\alpha_3 - \alpha_1)\, w^\alpha\, V_\alpha \quad ,$$

$$\delta g_{0\alpha} = \frac{1}{2}(\alpha_1 - 2\alpha_2)\, w^\alpha\, U + \alpha_2\, w^\beta\, U_{\alpha\beta} \quad ,$$

$$\delta g_{\alpha\beta} = 0 \quad , \tag{1}$$

where U is the Newtonian gravitational potential. The potentials $U_{\alpha\beta}$ and V_α are defined in Paper I.

These additional metric terms may produce observable solar-system effects,

which depend on the PPN parameters α_1, α_2, and α_3, and on our velocity $\underset{\sim}{w}$ relative to the Universe. In this paper we derive and catalogue these "preferred-frame" effects, and write them in the form

$$\begin{pmatrix} \text{size of} \\ \text{effect} \end{pmatrix} = \begin{pmatrix} \text{combination of} \\ \alpha_1, \ \alpha_2, \ \text{and} \ \alpha_3 \end{pmatrix} \times \begin{pmatrix} \text{amplitude, which} \\ \text{depends on } \underset{\sim}{w} \end{pmatrix} \quad . \qquad (2)$$

In this form, the predicted sizes of "preferred-frame" effects can be compared with experimental or observational data and limits put on the values of the parameters α_1, α_2, and α_3, once we assume a value for $\underset{\sim}{w}$. In this paper, we assume that the solar system's motion through the Universe is due to its (nearly circular) orbital motion around the Galaxy, i.e. we assume $\underset{\sim}{w}$ is 200 km/sec in the direction given by galactic coordinates $\ell^{II} = 90^{\circ}$, $b^{II} = 0^{\circ}$.

The "preferred-frame" effects which we have studied fall into two classes: geophysical (effects which alter the structure of the Earth) and orbital (effects which alter the orbital motions of planets and satellites).

a) Geophysical Effects

One of the authors (Will 1971) has previously used the PPN formalism to show that the Newtonian gravitational constant G as measured by means of Cavendish experiments may depend on the observer's velocity relative to a Universal rest-frame. We have confirmed and generalized this result using the new version of the PPN formalism discussed in Paper I: by considering the Earth's gravitational force on a gravimeter at rest on the surface of the Earth, we obtain for the measured value of G,

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$$G = 1 + \frac{1}{2} \left[(\alpha_3 - \alpha_1) + \alpha_2 \left(1 - \frac{I}{MR^2} \right) \right] w_\oplus^2$$

$$- \frac{1}{2} \alpha_2 \left(1 - \frac{3I}{MR^2} \right) (\underset{\sim}{w}_\oplus \cdot \underset{\sim}{e}_r)^2 \quad , \tag{3}$$

where M is the mass of the Earth, R its radius and I its spherical moment of inertia; $\underset{\sim}{w}_\oplus$ is the Earth's velocity through the preferred Universal rest-frame, and $\underset{\sim}{e}_r$ is a unit vector joining the gravimeter and the center of the Earth. The Earth's velocity $\underset{\sim}{w}_\oplus$ is made up of two parts, a uniform velocity $\underset{\sim}{w}$ of the solar system relative to the preferred frame, and the Earth's orbital velocity $\underset{\sim}{v}$ around the Sun, thus

$$w_\oplus^2 = w^2 + 2\underset{\sim}{w} \cdot \underset{\sim}{v} + v^2 \quad ,$$

$$(\underset{\sim}{w}_\oplus \cdot \underset{\sim}{e}_r)^2 = (\underset{\sim}{w} \cdot \underset{\sim}{e}_r)^2 + 2(\underset{\sim}{w} \cdot \underset{\sim}{e}_r)(\underset{\sim}{v} \cdot \underset{\sim}{e}_r) + (\underset{\sim}{v} \cdot \underset{\sim}{e}_r)^2 \quad . \tag{4}$$

So because of the Earth's rotation (changing $\underset{\sim}{e}_r$) and orbital motion (changing $\underset{\sim}{v}$), there will be variations in the gravimeter measurements of G, given by (we retain only terms which vary with amplitudes larger than 10^{-9} G)

$$\Delta G/G \simeq \left(\frac{1}{2} \alpha_2 + \alpha_3 - \alpha_1 \right) \underset{\sim}{w} \cdot \underset{\sim}{v}$$

$$+ \frac{1}{4} \alpha_2 \left[(\underset{\sim}{w} \cdot \underset{\sim}{e}_r)^2 + 2(\underset{\sim}{w} \cdot \underset{\sim}{e}_r)(\underset{\sim}{v} \cdot \underset{\sim}{e}_r) + (\underset{\sim}{v} \cdot \underset{\sim}{e}_r)^2 \right] \quad , \tag{5}$$

where we have used the fact that, for the Earth,

$$I \simeq \frac{1}{2} MR^2 \quad . \tag{6}$$

The most pronounced effect is the anisotropy in G produced by the $(\underset{\sim}{w}_\oplus \cdot \underset{\sim}{e}_r)^2$ term (eq.[3]). This anisotropy has been shown (Will 1971) to produce "tides of the solid Earth", i.e. variations in the acceleration

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g measured by a gravimeter, which are completely analogous to the tides produced by the Moon and Sun. The most important of these Earth-tides is a 12-hour sidereal-time tide, produced by the $(\underset{\sim}{w} \cdot \underset{\sim}{e}_r)^2$ term of equation (5), with amplitude

$$(\Delta g/g)_{PPN} \approx \alpha_2 (3 \times 10^{-8}) \cos^2(L) \quad , \tag{7}$$

where L is the latitude of the gravimeter's location on the Earth. Will (1971) has examined Earth-tide data and found that any discrepancy between Newtonian theory and experiment for this component of the tides must be less than one part in 10^9 at any latitude for which reliable gravimeter data is available. By comparing equation (7) with this experimental limit, we find that α_2 must satisfy

$$|\alpha_2| < 3 \times 10^{-2} \quad . \tag{8}$$

In §II we summarize other "Earth-tides" of various frequencies produced by the anisotropic terms in equation (5).

As the Earth orbit the Sun, the $(\underset{\sim}{w} \cdot \underset{\sim}{v})$ and $(\underset{\sim}{w} \cdot \underset{\sim}{e}_r)(\underset{\sim}{v} \cdot \underset{\sim}{e}_r)$ terms in equation (5) vary with a period of one sidereal year. Because the Earth is a gravitationally bound object, this variation in G causes the Earth to expand and contract spherically. This "breathing" of the Earth results in a changing moment of inertia, which in turn (by conservation of angular momentum) causes a yearly variation in the Earth's rotation frequency Ω with amplitude, according to the PPN formalism (cf. §II)

$$(\Delta\Omega/\Omega)_{PPN} \approx \left(\frac{2}{3}\alpha_2 + \alpha_3 - \alpha_1\right)(3 \times 10^{-9}) \quad . \tag{9}$$

The observed yearly variation in the Earth's rotation rate (measured by

comparing astronomical time with atomic time standards) has an amplitude of

$$(\Delta\Omega/\Omega)_{\text{OBSERVED}} \approx 4 \times 10^{-9} \qquad (10)$$

(Smith and Tucker 1953). But this variation can be readily understood using Newtonian geophysics: it is produced by an annual variation in the angular momentum of the atmosphere due to seasonal changes in wind patterns and by a long-period (one year) Earth-tide produced by the Sun (Mintz and Munk 1953; see also Melchior 1966). These calculations yield agreement with the observed variation in Ω with uncertainties around 15 percent, hence the PPN variation in the Earth's rotation rate must satisfy

$$(\Delta\Omega/\Omega)_{\text{PPN}} < 6 \times 10^{-10}. \qquad (11)$$

Equations (9) and (11) thus show that the PPN parameter combination $(2/3\ \alpha_2 + \alpha_3 - \alpha_1)$ must satisfy

$$\left| \frac{2}{3}\ \alpha_2 + \alpha_3 - \alpha_1 \right| < 0.2 \quad . \qquad (12)$$

In §II we discuss other, smaller variations in the Earth's rotation rate produced by the velocity-dependent terms in equation (5).

b) Orbital Effects

The solar system's 200 km/sec motion through the Universe may produce, according to the PPN formalism a variety of observable effects, both secular and periodic, in the orbits of planets. The most important of these effects is an anomalous perihelion shift $\Delta\tilde{\omega}$ for the planets, given by (in radians per orbit)

$$\frac{\Delta\tilde{\omega}}{2\pi} = \frac{1}{3}\,(2\gamma + 2 - \beta)\,\frac{3M}{p} - \frac{1}{4}\,\alpha_1\left(\frac{M}{p}\right)^{1/2}\frac{w_Q}{e} - \frac{1}{8}\,\alpha_2\,(w_P^2 - w_Q^2)$$

$$+ \frac{1}{2}\,\alpha_3\,\frac{|\Omega|}{M}\left(\frac{\lambda p^2}{Me}\right)w_Q \quad , \tag{13}$$

where M is the mass of the Sun; p and e ($\neq 0$) are the semi-latus rectum and eccentricity of the planet's orbit; w_P and w_Q are the components of $\underset{\sim}{w}$ in the plane of the planet's orbit, w_P in the direction of the perihelion, w_Q at right angles to w_P; $|\Omega|$ is the gravitational self-energy of the Sun and λ is the Sun's rotational angular velocity. The first term in equation (13) is the "classical" perihelion shift, which depends on the PPN parameters γ and β (see Paper I) and which would be present even if the solar system were at rest in the Universe ($\underset{\sim}{w} = 0$). For Mercury (☿) and the Earth (⊕), equation (13) yields, in seconds of arc per century (§III):

$$(\Delta\tilde{\omega}_{☿})_{\text{PPN}} = 43\left[\frac{1}{3}(2\gamma + 2 - \beta)\right] + 35\,\alpha_1 + 8\,\alpha_2 - 4 \times 10^4\,\alpha_3 \quad ,$$

$$(\Delta\tilde{\omega}_{⊕})_{\text{PPN}} = 4\left[\frac{1}{3}(2\gamma + 2 - \beta)\right] + 57\,\alpha_1 + \alpha_2 - 7 \times 10^5\,\alpha_3 \quad . \tag{14}$$

The measured perihelion shifts are

$$(\Delta\omega_{☿})_{\text{OBSERVED}} = 43 \pm .4 \quad ,$$

$$(\Delta\omega_{⊕})_{\text{OBSERVED}} = 4 \pm .4 \quad , \tag{15}$$

where there may be an additional discrepancy in Mercury's perihelion shift due to a contribution ($< 4''$ per century) of a possible solar quadrupole moment. By combining equations (14) and (15) we can eliminate the term involving γ and β, and obtain a third limit on the parameters α_1, α_2, and α_3: in

order that the PPN perihelion shifts agree with the measured shifts within the experimental error, the parameters α_1, α_2, and α_3 must satisfy

$$|13\,\alpha_1 + 0.07\,\alpha_2 - 170{,}000\,\alpha_3| < .2 \quad . \qquad (16)$$

We now combine the three experimentally determined limits, equations (8), (12), and (16) and obtain individual upper limits on the values of α_1, α_2, and α_3, as listed in Table 1. For comparison, Table 1 also lists the predicted values for α_1, α_2, and α_3 for all the metric theories which our groups at Caltech and Montana State have examined, and which, until now, were considered viable, i.e. agreed with the "classical" tests [see Ni (1972a) and Thorne, Will, and Ni (1971) for examples of "nonviable" theories].

From Table 1, we can see immediately that the stratified theories due to Page and Tupper, Yilmaz, Papapetrou, Ni, Coleman, and Rosen cannot be correct theories of gravitation — they disagree violently with experiment. In fact, we can be more general. We have shown (Paper I) that any Stratified Theory (with time-orthogonal, conformally flat space slices) which agrees with light-deflection and time-delay experiments must have $\alpha_1 \sim -8$, which is forty times larger than our experimental upper limit. Hence no "Stratified Theory" of gravity, past, present, or future, can be the correct theory of gravity.

A second conclusion emerges from Table 1: the squared magnitude of the cosmological vector field (K) in the Vector-Metric Theory of Paper I must satisfy

$$K^2 < 3 \times 10^{-2} \quad , \qquad (17)$$

in order to agree (within the experimental uncertainty) with Earth-tide data.

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From this point of view, the results we have obtained in this paper are complementary to the Hughes-Drever (isotropy of inertial mass) and "ether-drift" experiments. Those experiments put limits on the strengths of cosmological vector or tensor fields which couple to matter's nuclear or electromagnetic energy (Peebles 1962, Peebles and Dicke 1962, Dicke 1964), while our results put limits on vector or tensor fields which couple to matter's gravitational energy. There is a wide class of such vector-metric and tensor-metric theories (Paper I), and the limits on α_1, α_2, and α_3 should be pushed as low as possible in order to put more stringent limits on these cosmological vector and tensor fields. A future paper in this series may study these theories and their predictions in detail.

Recent research by Ni has shown that it may be possible to push the limits on α_1, α_2, and α_3 as low as 10^{-6} by means of studies of the pulsations of white dwarf stars. According to the PPN formalism, motion of pulsating white dwarfs relative to the Universe should produce instabilities, i.e. exponential growth or decay of the amplitudes of their pulsation. Results of observational studies of white dwarf pulsations, proper motion and stabilities may put stringent limits on such velocity-induced effects, which thereby put limits on α_1, α_2, and α_3 (Ni 1972b).

A third conclusion obtained from Table 1 is that this paper's results do not distinguish between general relativity and scalar-tensor theories: these theories are not preferred-frame theories and predict no preferred-frame effects (for a discussion see Paper I).

The remainder of this paper gives detailed calculations of preferred-frame effects. In §II we derive the expression (eq. [3]) for the Newtonian Gravitational Constant as measured by a gravimeter on the Earth, and compute

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the amplitudes of Earth-tides and variations in the Earth's rotation rate produced by the variations in G. We derive in §II the perturbations on planetary motions produced by the additional metric terms (eq. [1]), and calculate the resulting secular changes in planetary orbital elements and the resulting periodic perturbations in the Earth-Moon distance. In §IV we show that other reasonable assumptions for the direction of $\underset{\sim}{w}$ do not significantly affect our limits on α_1, α_2, and α_3. Concluding remarks are made in §V. In an Appendix we compute the amplitude of the spherical expansion and contraction ("breathing") of the Earth produced by the variation in G.

II. GEOPHYSICAL PREFERRED-FRAME EFFECTS

We consider a test body (gravimeter) maintained a constant proper distance R_p from the center of the Earth by a four-acceleration $\underset{\sim}{F}$ (supporting force of the ground). Then the gravitational constant G measured by this gravimeter is related to F_r, the radial component of this force (toward the center of the Earth) by Newton's law of gravitation, written in invariant language (see Will [1971] for discussion)

$$F_r = GM/R_p^2 + F_c \quad , \tag{18}$$

where M is the mass of the Earth, and F_c is an invariant expression for the centrifugal acceleration due to the Earth's rotation. Will (1971) obtained an expression for G by evaluating equation (18) in a coordinate system at rest with respect to the preferred Universal rest-frame. The new version of the PPN formalism discussed in Paper I, allows us to evaluate equation (18) in the momentary rest-frame of the Earth. In this frame, the PPN

metric contains additional terms (eqs. [1]) which depend on $\underset{\sim}{w}_\oplus$, the Earth's velocity relative to the preferred Universal rest-frame. We wish to consider only the effects of these terms; post-Newtonian effects on G due to the Sun and planets and to the Earth's self-gravitational field have been discussed elsewhere (Nordtvedt 1971; Will 1971). Other post-Newtonian effects produce gravimeter-measured forces smaller than 10^{-9} g (Will 1971). These $\underset{\sim}{w}$-dependent terms enter equation (18) only via the equation of motion of the gravimeter,

$$du^i/d\tau + \Gamma^i{}_{jk} u^j u^k = F^i \quad , \tag{19}$$

where u^i is the gravimeter's four-velocity and τ is proper time along the gravimeter's world line. For a gravimeter at rest on the surface of the Earth, equation (19) simplifies to (neglecting forces smaller than 10^{-9} g),

$$F^\alpha = \Gamma^\alpha{}_{00} \simeq \frac{1}{2} g_{00,\alpha} \quad . \tag{20}$$

We note that, to our order of approximation,

$$R_p = R = |\; \underset{\sim}{X}_\oplus - \underset{\sim}{X}_{GRAVIMETER} \;| \quad ,$$

and use equations (1) along with the Newtonian part of g_{00} to obtain

$$F_r = \frac{1}{2}(\underset{\sim}{r}/r) \cdot \underset{\sim}{\nabla} g_{00}$$

$$= - (\underset{\sim}{r}/r) \cdot \underset{\sim}{\nabla}\left[U - \frac{1}{2}(\alpha_1 - \alpha_2 - \alpha_3) w_\oplus{}^2 U - \frac{1}{2} \alpha_2 w_\oplus{}^\alpha w_\oplus{}^\beta U_{\alpha\beta}\right] \quad . \tag{21}$$

For a spherically symmetric Earth (the Earth's oblateness has significant Newtonian effects, but negligible post-Newtonian preferred-frame effects), we have

$$U = M/R \quad , \tag{22}$$

$$U_{\alpha\beta} = MR_\alpha R_\beta/R^3 - \frac{1}{3}(I/R^5)(3 R_\alpha R_\beta - R^2 \delta_{\alpha\beta}) \quad , \tag{23}$$

where I is the spherical moment of inertia of the Earth. Equations (22) and (23) along with (21) give (ignoring the centrifugal acceleration):

$$F_r = \frac{M}{R_p^2}\left\{ 1 + \frac{1}{2}\left[\alpha_3 - \alpha_1 + \alpha_2\left(1 - \frac{I}{MR^2}\right)\right] w_\oplus^2 \right.$$
$$\left. - \frac{1}{2}\alpha_2\left(1 - \frac{3I}{MR^2}\right)(w_\oplus \cdot e_r)^2 \right\} \quad , \tag{24}$$

where

$$e_r = R/R \quad . \tag{25}$$

From equation (24) we obtain the gravimeter-measured value of G as given in equation (3). Except for the contribution due to the Earth's moment of inertia I, this result is in agreement with the result of Will (1971). Because of the Earth's orbital and rotational motion, this measured value of G changes (as $w_\oplus$ and e_r change) according to (cf. Eq. [5])

$$\Delta G/G = \left(\frac{1}{2}\alpha_2 + \alpha_3 - \alpha_1\right) w \cdot v$$
$$+ \frac{1}{4}\alpha_2\left[(w \cdot e_r)^2 + 2(w \cdot e_r)(v \cdot e_r) + (v \cdot e_r)^2\right] \quad , \tag{26}$$

where v is the Earth's orbital velocity, and w is the solar system's uniform velocity through the Universe, which we have assumed to be 200 km/sec in the direction $\ell^{II} = 90°$, $b^{II} = 0°$. In terms of the Geocentric Ecliptic coordinate system (z-axis normal to the Earth's orbit, x-axis directed toward the Sun at vernal equinox) the direction of w is given by $\lambda = 346°$, $\beta = 60°$, and in the Geocentric Equatorial coordinate system (z-axis normal to the Earth's equator, x-axis same as before), it is given by $\alpha = 318°$,

$\delta = 48^\circ$ (see Smart [1960] for definitions of these astronomical coordinate systems, and for equations to transform from one to the other). In obtaining equation (26), we have used the fact that, for the Earth,

$$I \simeq \frac{1}{2} MR^2 \quad , \tag{27}$$

and have assumed a circular Earth orbit (v^2 constant).

In order to compare this variation in G with geophysical data, we must perform a harmonic analysis of the terms in equation (26). The frequencies involved will be the sidereal rotational frequency of the Earth Ω, due to the changing $\underset{\sim}{e}_r$ relative to the fixed direction of $\underset{\sim}{w}$, and its orbital sidereal frequency ω due to the changing direction of $\underset{\sim}{v}$ relative to $\underset{\sim}{w}$, along with harmonics and linear combinations of these frequencies. We work in Geocentric Ecliptic Coordinates, and assume a circular Earth orbit, with the Earth at vernal equinox at t = 0. Then

$$\underset{\sim}{w} = w \left[\cos \beta (\cos \lambda \, \underset{\sim}{e}_x + \sin \lambda \, \underset{\sim}{e}_y) + \sin \beta \, \underset{\sim}{e}_z \right] \quad , \tag{28}$$

$$\underset{\sim}{v} = v(\sin \omega t \, \underset{\sim}{e}_x - \cos \omega t \, \underset{\sim}{e}_y) \quad . \tag{29}$$

For a gravimeter stationed at Earth latitude L,

$$\begin{aligned}
\underset{\sim}{e}_r = {} & \cos L \cos(\Omega t - \epsilon) \, \underset{\sim}{e}_x \\
& + \left[\cos L \sin(\Omega t - \epsilon) \cos \Theta + \sin L \sin \Theta \right] \underset{\sim}{e}_y \\
& - \left[\cos L \sin(\Omega t - \epsilon) \sin \Theta - \sin L \cos \Theta \right] \underset{\sim}{e}_z \quad , \tag{30}
\end{aligned}$$

where ϵ is related to the longitude of the gravimeter on the Earth, and Θ is the "tilt" ($23\ 1/2^\circ$) of the Earth relative to the Earth's orbit (ecliptic). Equations (28), (29), and (30) give

$$\underset{\sim}{w} \cdot \underset{\sim}{v} = wv \cos \beta \sin(\omega t - \lambda) \quad , \tag{31}$$

$$(\underset{\sim}{w} \cdot \underset{\sim}{e}_r)^2 = w^2 \left[\frac{1}{3} + \frac{3}{2}\left(\frac{1}{3} - \sin^2 \delta\right)\left(\frac{1}{3} - \sin^2 L\right) \right.$$

$$+ \frac{1}{2} \sin 2\delta \sin 2L \cos(\Omega t - \epsilon - \alpha)$$

$$\left. + \frac{1}{2} \cos^2 \delta \cos^2 L \cos 2(\Omega t - \epsilon - \alpha) \right] \quad , \tag{32}$$

$$(\underset{\sim}{w} \cdot \underset{\sim}{e}_r)(\underset{\sim}{v} \cdot \underset{\sim}{e}_r) = wv \left\{ \frac{1}{3} \cos \beta \sin(\omega t - \lambda) \right.$$

$$+ \left(\frac{1}{3} - \sin^2 L\right)\left[\frac{1}{2} \cos \beta \sin(\omega t - \lambda) + \frac{3}{2} \sin \delta \sin \Theta \cos \omega t\right]$$

$$+ \frac{1}{4} \sin \delta (1 - \cos \Theta) \sin 2L \sin[(\Omega + \omega) t - \epsilon]$$

$$- \frac{1}{4} \cos \delta \sin \Theta \sin 2L \cos[(\Omega + \omega) t - \epsilon - \alpha]$$

$$- \frac{1}{4} \sin \delta (1 + \cos \Theta) \sin 2L \sin[(\Omega - \omega) t - \epsilon]$$

$$- \frac{1}{4} \cos \delta \sin \Theta \sin 2L \cos[(\Omega - \omega) t - \epsilon - \alpha]$$

$$+ \frac{1}{4} \cos \delta (1 - \cos \Theta) \cos^2 L \sin[(2\Omega + \omega) t - 2\epsilon - \alpha]$$

$$\left. - \frac{1}{4} \cos \delta (1 + \cos \Theta) \cos^2 L \sin[(2\Omega - \omega) t - 2\epsilon - \alpha] \right\} \quad , \tag{33}$$

$$(\underset{\sim}{v} \cdot \underset{\sim}{e}_r)^2 = v^2 \left\{ \frac{1}{3} + \frac{3}{2}\left(\frac{1}{3} - \sin^2 L\right)\left(\frac{1}{3} - \frac{1}{2} \sin^2 \Theta\right) \right.$$

$$- \frac{3}{4} \left(\frac{1}{3} - \sin^2 L\right) \sin^2 \Theta \cos 2\omega t$$

$$+ \frac{1}{4} \sin 2\Theta \sin 2L \sin(\Omega t - \epsilon)$$

$$- \frac{1}{4} \sin \Theta(1 - \cos \Theta) \sin 2L \sin[(\Omega + 2\omega) t - \epsilon]$$

$$+ \frac{1}{4} \sin \Theta(1 + \cos \Theta) \sin 2L \sin[(\Omega - 2\omega) t - \epsilon] \tag{34}$$

$$+ \frac{1}{4} \sin^2 \Theta \cos^2 L \cos 2(\Omega t - \epsilon)$$

$$- \frac{1}{8} (1 - \cos \Theta)^2 \cos^2 L \cos[2(\Omega + \omega) t - 2\epsilon]$$

$$\left. - \frac{1}{8} (1 + \cos \Theta)^2 \cos^2 L \cos[2(\Omega - \omega) t - 2\epsilon] \right\} \quad , \quad (34 \text{ con't.})$$

where we have used both the ecliptic coordinates (λ, β) and the equatorial coordinates (α, δ) corresponding to the direction of $\underset{\sim}{w}$, in order to simplify the various expressions.

Equations (31), (32), (33), and (34) reveal four different types of variations in G.

i) <u>Semi-Diurnal Variations</u>: These are the terms which vary with frequency around 2Ω: 2Ω, $2\Omega + \omega$, $2\Omega - \omega$, $2(\Omega + \omega)$, $2(\Omega - \omega)$; i.e. have periods around twelve hours $(\omega \ll \Omega)$ and vary with latitude according to $\cos^2 L$. These variations are completely analogous to the twelve-hour solid-Earth tides produced by the Sun and Moon, called "Semi-Diurnal Sectorial Waves" by Melchior (1966). The true gravimeter measurements for these tides are affected not only by the variation in G, but also by the displacement of the Earth's surface relative to the center of the Earth, and by the deformation of the Earth. This variation in gravimeter readings is related to the variation in G by

$$(\Delta g/g)_{\text{SEMI-DIURNAL}} = 1.18 (\Delta G/G)_{\text{SEMI-DIURNAL}} \quad , \quad (35)$$

where the factor 1.18 is a combination of so-called "Love Numbers", which depend on the detailed structure of the Earth (Melchior 1966).

ii) <u>Diurnal Variations</u>: These are the terms which vary with a frequency around Ω: Ω, $\Omega + \omega$, $\Omega - \omega$, $\Omega + 2\omega$, $\Omega - 2\omega$; i.e. have periods around 24 hours,

and vary with latitude according to sin 2L. These variations are completely analogous to the 24-hour "Diurnal Tesseral Waves" of the solid Earth (Melchior 1966), and give gravimeter readings related to the variation in G by the same factor:

$$(\Delta g/g)_{\text{DIURNAL}} = 1.18(\Delta G/G)_{\text{DIURNAL}} \quad . \tag{36}$$

iii) <u>Long-Period Zonal Variations</u>: These are the variations with frequencies ω and 2ω, and with latitude dependence $(1/3 - \sin^2 L)$, which are completely analogous to the long-period tides produced by the Sun and Moon, called "Long-Period Zonal Waves" by Melchior (1966). These long-period zonal waves produces variations in the Earth's moment of inertia, which in turn cause variations in the rotation rate of the Earth. These rotation-rate variations are related to the amplitude of the zonal variations by (Mintz and Munk 1953; Melchior 1966)

$$(\Delta\Omega/\Omega)_{\text{ZONAL}} = 0.41 \, A_{\text{ZONAL}} \quad , \tag{37}$$

where A_{Zonal} is related to the zonal variations in G in equations (33) and (34) by

$$(\Delta G/G)_{\text{ZONAL}} = A_{\text{ZONAL}}\left(\frac{1}{3} - \sin^2 L\right) \quad . \tag{38}$$

iv) <u>Long-Period Spherical Variations</u>: These are the variations which have frequency ω, but no latitude dependence; they represent a yearly variation in the <u>strength</u> of G, and have no counterpart in Newtonian tidal theory. These variations produce a purely spherical deformation of the Earth, as opposed to the Sectorial, Tesseral, and Zonal waves which produce purely quadrupole deformations. This yearly spherical "breathing" of the Earth

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as G varies causes a variation in the Earth's moment of inertia, which in turn causes a variation in the rotation frequency, given by

$$(\Delta\Omega/\Omega)_{\text{SPHERICAL}} = -(\Delta I/I) \simeq (1/10)(\Delta G/G)_{\text{SPHERICAL}} \quad . \quad (39)$$

Detailed calculations of this change in the Earth's moment of inertia due to the spherical variation in G are given in an Appendix.

By combining equations (31), (32), (33), and (34) with the expression for $\Delta G/G$, equation (26), substituting numerical values

$$v \simeq 30 \text{ km/sec} , \quad w \simeq 200 \text{ km/sec} , \quad \theta \simeq 23\frac{1^{\circ}}{2} ,$$

$$\lambda \simeq 346^{\circ} , \quad \alpha \simeq 318^{\circ} ,$$

$$\beta \simeq 60^{\circ} , \quad \delta \simeq 48^{\circ} , \quad (40)$$

and using equations (35), (36), (37), and (39), we may compute the amplitudes of all the various components of the Earth tides $(\Delta g/g)$ and of the variations in the Earth's rotation rate $(\Delta\Omega/\Omega)$. These amplitudes are listed in Table 2. The largest predicted Earth-tide components are the sidereal diurnal and semi-diurnal (Ω and 2Ω) tides. Other, smaller components include diurnal and semi-diurnal solar-time tides $[(\Omega - \omega) \text{ and } 2(\Omega - \omega)]$, and tides with frequencies $2\Omega - \omega$ and $\Omega - 2\omega$. We have used the semi-diurnal sidereal tide (2Ω) to put an experimental upper limit on the value of α_2 (eq.[8]) rather than the larger diurnal sidereal tide (Ω), because agreement between Newtonian theory and observation is not as good for the diurnal as for the semi-diurnal tidal components, possibly because of diurnal effects due to heating by the Sun (Harrison et al. 1963).

Although the zonal yearly variation in the Earth's rotation rate

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(Table 2) is apparently larger than the spherical variation, we have ignored it, because the limit set on α_2 by the Earth tides (3×10^{-2}), makes this effect too small to be discernible. Thus we have focused on the spherical variation in Ω in order to set a limit (eq. [12]) on the combination $(2/3 \ \alpha_2 + \alpha_3 - \alpha_1)$.

Future experimental studies of Earth tides should concentrate on separating the various predicted components of the diurnal and semi-diurnal tides. Since the various components in each group are close to each other in frequency, a complete separation of the components would require at least a year of continuous gravimeter data. Such detailed studies should be used not only to justify our rather heuristic discussion of the limit set on α_2 by Earth-tides, but also, perhaps, to improve this limit.

One further geophysical effect produced by motion through the Universe should be mentioned: a yearly precession or "wobble" of the Earth's axis of rotation which depends on the PPN parameter α_1. However, the amplitude of this wobble is too small $(< 10^{-2}$ seconds of arc) to be discerned from the Newtonian Chandler wobble.

III. ORBITAL PREFERRED-FRAME EFFECTS

In this section, we derive a variety of effects on the orbital motions of planets and satellites, which may result from our motion at 200 km/sec through the "preferred Universal rest-frame". We consider a two-body system, one a "test" body, the other a massive body, with self-gravitational energy Ω and spin-angular velocity $\underset{\sim}{\lambda}$, moving through the Universe with velocity $\underset{\sim}{w}$. Using the PPN metric written in the "rest" coordinate system of the two bodies (thus containing the $\underset{\sim}{w}$-dependent metric terms equations [1]), we

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derive those perturbations in the equations of motion for these bodies, which depend on $\underset{\sim}{w}$ and on the PPN parameters α_1, α_2, and α_3. We then specialize to the solar system, and calculate (a) secular changes in the orbital elements of a test body (planet or satellite) orbiting the Sun, and (b) periodic perturbations in the relative distance between the Earth and the Moon.

The perturbation δa^{α}_{T}, in the instantaneous acceleration of the test body due to the $\underset{\sim}{w}$-dependent metric terms (eq. [1]) is calculated from the geodesic equation:

$$\delta a^{\alpha}_{T} = \delta(dv^{\alpha}/dt) = -\delta \Gamma^{\alpha}_{ij} v^{i} v^{j} + \delta \Gamma^{0}_{ij} v^{i} v^{j} v^{\alpha} \quad , \quad (40)$$

where $\delta \Gamma^{i}_{jk}$ is the contribution of δg_{ij} (eq. [1]) to the Christoffel symbols. Equations (1) and (40) yield

$$\delta \underset{\sim}{a}_{T} = -\frac{M\underset{\sim}{r}}{r^{3}} \left\{ \frac{1}{2}(\alpha_2 + \alpha_3 - \alpha_1)w^2 - \frac{1}{2}\alpha_1(\underset{\sim}{w} \cdot \underset{\sim}{v}) - \frac{3}{2}\alpha_2 \left(\frac{\underset{\sim}{w} \cdot \underset{\sim}{r}}{r}\right)^2 \right\}$$

$$- \underset{\sim}{w} \frac{M\underset{\sim}{r}}{r^{3}} \cdot \left(\frac{1}{2}\alpha_1 \underset{\sim}{v} + \alpha_2 \underset{\sim}{w}\right) \quad , \quad (41)$$

where M is the mass of the central body, $\underset{\sim}{r}$ is a vector from the central body to the test body, and $\underset{\sim}{v}$ is the test body's velocity. In calculating δa_{T} we have idealized the central body as a point mass: any contributions due to the central body's finite size (moment of inertia I) while important for geophysical effects at the Earth's surface (§II), have negligible direct effect on the motion of orbiting test bodies. However, the massive body itself, although not affected by the test body (which has negligible mass) is perturbed by its own motion through the Universe. This perturbing acceleration δa_{M} is due to a coupling between the massive body's

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self-gravitational energy, its rotation, and its velocity through the Universe. The test body, with negligible self-gravity, is not affected by this coupling.

We compute δa_M as follows: each element of matter (with mass m_i) in the massive body is perturbed by the body's motion through the Universe according to equation (40), where now the gravitational potentials which appear in the Christoffel symbols are generated all other matter in the massive body. Using equations (1) and (40) along with the expressions U, $U_{\alpha\beta}$ and V_α (Paper I), we get, for the i-th element of mass

$$
\delta \underset{\sim}{a}_i = -\sum_{j\neq i} \frac{m_j \, \underset{\sim}{r}_{ij}}{r_{ij}^3} \left\{ \frac{1}{2}(\alpha_2 + \alpha_3 - \alpha_1)\, w^2 - \frac{1}{2}(\alpha_1 - 2\alpha_2 - 2\alpha_3)\, \underset{\sim}{w} \cdot \underset{\sim}{v}_j \right.
$$

$$
\left. - \frac{1}{2}\alpha_1(\underset{\sim}{w} \cdot \underset{\sim}{v}_i) - \frac{3}{2}\alpha_2(\underset{\sim}{w} \cdot \underset{\sim}{r}_{ij}/r_{ij})^2 - 3\,\alpha_2(\underset{\sim}{w} \cdot \underset{\sim}{r}_{ij})(\underset{\sim}{v}_j \cdot \underset{\sim}{r}_{ij})/r_{ij}^2 \right\}
$$

$$
- \alpha_2 \sum_{j\neq i} \frac{m_j \, \underset{\sim}{r}_{ij}}{r_{ij}^3} \cdot \underset{\sim}{w}\, \underset{\sim}{v}_j
$$

$$
+ \frac{1}{2}\underset{\sim}{w} \sum_{j\neq i} \frac{m_j \, \underset{\sim}{r}_{ij}}{r_{ij}^3} \cdot \left[(\alpha_1 - 2\alpha_2)\, \underset{\sim}{v}_j - \alpha_1 \underset{\sim}{v}_i - 2\alpha_2 \underset{\sim}{w} \right] \quad . \tag{42}
$$

The motion of the center of the mass of the body is given to sufficient accuracy by,

$$
M\underset{\sim}{a}_M \equiv \sum_i m_i\, \underset{\sim}{a}_i \,, \qquad M = \sum_i m_i \,, \tag{43}
$$

so the perturbation $\delta\underset{\sim}{a}_M$ is given by

$$
M\, \delta\underset{\sim}{a}_M = \sum_i m_i\, \delta\underset{\sim}{a}_i \quad . \tag{44}
$$

Combining equations (42) and (44), and simplifying the resulting expressions,

we get

$$M \delta a_M{}^\alpha = - \alpha_3 \, w^\beta \, H_{\alpha\beta} - \alpha_2 \, w^\beta (d \, \Omega_{\alpha\beta}/dt) - (\alpha_1 - \alpha_2) \, w^\alpha (d\Omega/dt) \quad , \qquad (45)$$

where $\Omega_{\alpha\beta}$ and Ω are the massive body's self-gravitational energy tensor and scalar respectively, given by

$$\Omega_{\alpha\beta} = - \frac{1}{2} \sum_{ij} \frac{m_i \, m_j \, r_{ij}{}^\alpha \, r_{ij}{}^\beta}{r_{ij}{}^3} \quad , \qquad \Omega = \sum_\alpha \Omega_{\alpha\alpha} = - \frac{1}{2} \sum_{ij} \frac{m_i \, m_j}{r_{ij}} \quad , \qquad (46)$$

and $H_{\alpha\beta}$ is given by

$$H_{\alpha\beta} = \sum_{ij} \frac{m_i \, m_j \, r_{ij}{}^\alpha \, v_j{}^\beta}{r_{ij}{}^3} \quad . \qquad (47)$$

For a body in static equilibrium, $\Omega_{\alpha\beta}$ and Ω are constant, so we get, from equation (45),

$$M \, \delta a_M{}^\alpha = - \alpha_3 \, w^\beta \, H_{\alpha\beta} \quad . \qquad (48)$$

We now idealize our central massive body as a nearly spherical body, rotating uniformly with angular velocity $\underset{\sim}{\lambda}$. For our purposes, this is a reasonable model for the Sun (oblateness less than three parts in 10^5) or for the Earth (oblateness less than one part in 10^3). Then

$$\underset{\sim}{v}_j = \underset{\sim}{\lambda} \, \underset{\sim}{X} \, (\underset{\sim}{x}_j - \underset{\sim}{x}_c) \quad , \quad \text{or} \quad v_j{}^\alpha = \epsilon^{\alpha\beta\gamma} \, \lambda_\beta \, (x_j - x_c)_\gamma \quad , \qquad (49)$$

where $\underset{\sim}{x}_c$ is the position of the center of mass of the massive body. Then

$$H^{\alpha\beta} = \epsilon^{\beta\gamma\delta} \, \lambda_\gamma \, \sum_{ij} \frac{m_i \, m_j \, r_{ij}{}^\alpha (x_j - x_c)_\delta}{r_{ij}{}^3}$$

$$= \epsilon^{\beta\gamma\delta} \, \lambda_\gamma \, \Omega_{\delta\alpha} \quad . \qquad (50)$$

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For a spherical body, we have

$$\Omega_{\alpha\beta} = \frac{1}{3} \, \delta_{\alpha\beta} \, \Omega \quad , \tag{51}$$

$$H^{\alpha\beta} = \frac{1}{3} \, \epsilon^{\alpha\beta\gamma} \, \lambda_\gamma \, \Omega \quad . \tag{52}$$

Thus, from equations (48) and (52),

$$\delta\underset{\sim}{a}_M = - \frac{1}{3} \, \alpha_3 (\Omega/M) \, \underset{\sim}{w} \underset{\sim}{x} \underset{\sim}{\lambda} \quad . \tag{53}$$

The relative perturbing acceleration δa between the massive body and the test body is defined by

$$\delta\underset{\sim}{a} \equiv \delta\underset{\sim}{a}_T - \delta\underset{\sim}{a}_M \quad ,$$

and from equations (41) and (53) is

$$\delta\underset{\sim}{a} = - \frac{M\underset{\sim}{r}}{r^3} \left\{ \frac{1}{2}(\alpha_2 + \alpha_3 - \alpha_1) \, w^2 - \frac{1}{2} \, \alpha_1 (\underset{\sim}{w} \cdot \underset{\sim}{v}) - \frac{3}{2} \, \alpha_2 \left(\frac{\underset{\sim}{w} \cdot \underset{\sim}{r}}{r} \right)^2 \right\}$$

$$- \underset{\sim}{w} \, \frac{M\underset{\sim}{r}}{r^3} \cdot \left(\frac{1}{2} \, \alpha_1 \, \underset{\sim}{v} + \alpha_2 \, \underset{\sim}{w} \right) + \frac{1}{3} \, \alpha_3 \left(\frac{\Omega}{M} \right) \underset{\sim}{w} \underset{\sim}{x} \underset{\sim}{\lambda} \quad . \tag{54}$$

We now calculate the effects of $\delta\underset{\sim}{a}$ in the solar system.

a) Secular Preferred-Frame Orbital Effects

We consider a planetary orbit with the following instantaneous orbital elements: eccentricity e, semi-major axis a, and angle of perihelion relative to the equinox $\tilde{\omega}$. We assume the orbit has zero inclination i relative to the ecliptic.

Following the standard procedure for computing perturbations of orbital elements (Smart [1953], Robertson and Noonan [1968]), we resolve the perturbing acceleration $\delta\underset{\sim}{a}$ (eq. [54]) into a radial compontent $\mathcal{R}$, a component

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$\mathcal{U}$, normal to the orbital plane, and a component $\mathcal{S}$ normal to $\mathcal{U}$ and $\mathcal{R}$, and calculate the rates of change of the orbital elements using the formulae (in the notation of Robertson and Noonan [1968]):

$$\frac{d\tilde{\omega}}{dt} = -\frac{p\mathcal{R}}{he} \cos \varphi + \frac{\mathcal{S}(p + r)}{he} \sin \varphi \quad , \qquad (55)$$

$$\frac{de}{dt} = \frac{1 - e^2}{h} \left[a \, \mathcal{R} \, \sin \varphi + \frac{\mathcal{S}}{e}\left(\frac{ap}{r} - r\right) \right] \quad , \qquad (56)$$

$$\frac{da}{dt} = \frac{2a^2}{h}\left(\frac{\mathcal{S}p}{r} + \mathcal{R}e \, \sin \varphi\right) \quad , \qquad (57)$$

where h is the angular momentum per unit mass of the orbit, φ is the angle of the planet measured from perihelion, and p is the semi-latus rectum given by

$$p = a(1 - e^2) \quad . \qquad (58)$$

We calculate the perturbations (eqs. [55], [56], and [57]) to first order in δa, retaining only secular terms, and using a Keplerian ellipse as unperturbed orbit, given by

$$r = p(1 + e \cos \varphi)^{-1} \quad , \qquad (59)$$

$$r^2(d\varphi/dt) = h = \text{constant} \quad . \qquad (60)$$

We also assume the Sun's spin axis $(\underset{\sim}{\lambda})$ is normal to the planet's orbital plane. For the secular changes over one orbit, our results are, to zero'th order in the eccentricity $(e \neq 0)$:

$$\Delta\tilde{\omega} = -2\pi\left[\frac{1}{4}\alpha_1\left(\frac{M}{P}\right)^{1/2}\frac{w_Q}{e} + \frac{1}{8}\alpha_2(w_P^2 - w_Q^2)\right.$$

$$\left. -\frac{1}{2}\alpha_3\left(\frac{|\Omega|}{M}\right)\left(\frac{\lambda_P^2}{Me}\right)w_Q\right] \quad , \tag{61}$$

$$\frac{\Delta e}{e} = -2\pi\left[\frac{1}{4}\alpha_1\left(\frac{M}{P}\right)^{1/2}\frac{w_P}{e} - \frac{1}{4}\alpha_2 w_P w_Q\right.$$

$$\left. -\frac{1}{2}\alpha_3\left(\frac{|\Omega|}{M}\right)\left(\frac{\lambda_P^2}{Me}\right)w_P\right] \quad , \tag{62}$$

$$\Delta a = O(e^2) \quad , \tag{63}$$

where w_P and w_Q are the components of $\underset{\sim}{w}$ in the direction of the planet's perihelion (w_P) and in the direction at right angles to this (w_Q), in the plane of the orbit. The perturbation $\delta\underset{\sim}{a}$ of equation (54) can also be shown to produce secular changes in the inclination and angle of nodes of orbits, proportional to the component of $\underset{\sim}{w}$ normal to the orbital plane.

In evaluating the perihelion shift, equation (61) for Mercury and the Earth (cf. eqs. [14]), we have used standard values for the orbital elements (Allen 1963), our adopted value for $\underset{\sim}{w}$ (200 km/sec in the direction $\lambda = 346^{\circ}$, $\beta = 60^{\circ}$), and numerical values for the Sun's gravitational energy and rotational angular velocity:

$$(|\Omega|/M)_{\odot} \approx 4 \times 10^{-6} \quad , \quad \lambda_{\odot} \approx 3 \times 10^{-6} \ \text{sec}^{-1} \quad .$$

b) Periodic Preferred-Frame Orbital Effects

Our motion through the "preferred" Universal rest-frame may produce a variety of periodic perturbations in planetary orbits. Of particular interest are the predicted perturbations in the Earth-Moon distance, since such

perturbations can be studied by means of laser-ranging to the Moon.

We idealize our Earth-Moon system as consisting of a test body (Moon) in orbit around a massive body (Earth) with mass M and self-gravitational energy Ω. The Earth is assumed to rotate uniformly with angular velocity $\underset{\sim}{\lambda}$ with its spin axis inclined at an angle Θ ("tilt" of the Earth) relative to the ecliptic. We assume the Moon's unperturbed orbit is in the plane of the ecliptic (the actual inclination of the orbit is only $\sim 5^{\circ}$), and is given by a nearly Newtonian ellipse with a small perigee advance of $2\pi\epsilon$ per orbit:

$$r_0 = p\left[1 + e \cos(1 - \epsilon)\ \varphi\right]^{-1} \quad , \tag{64}$$

$$h_0 = r_0{}^2\ \dot{\varphi} = \text{constant} = (Mp)^{1/2} \quad , \tag{65}$$

where the subscript $_0$ refers to unperturbed quantities. The perigee advance of $2\pi\epsilon$ per orbit is assumed to be produced by the effects of the Sun and the other planets and corresponds to the Moon's perigee rotation with a period of about 17 years, relative to the fixed stars.

We now compute the effects of the perturbation $\delta\underset{\sim}{a}$ (eq. [54]) on the relative distance between the two bodies. We define

$$u \equiv (1/r) = u_0 + \delta u = (1/r_0) + \delta u \quad , \tag{66}$$

$$h = h_0 + \delta h \quad . \tag{67}$$

Then to first order, the differential equations for the perturbations δu and δh become

$$(d/d\varphi)\ \delta h = (r_0/h_0)^2\ (\underset{\sim}{h}_0 \times \underset{\sim}{r}_0) \cdot \delta\underset{\sim}{a} \quad , \tag{68}$$

$$\frac{d^2}{d\varphi^2} \delta u + (1 - \epsilon)^2 \delta u = -\left(\frac{r_0}{h_0}\right)^2 \frac{\mathbf{r}_0}{r_0} \cdot \delta \mathbf{a} - 2\left(\frac{M}{h_0^2}\right) \frac{\delta h}{h_0}$$

$$+ \frac{1}{h_0 r_0^2} \frac{dr_0}{d\varphi} \frac{d}{d\varphi} (\delta h) \quad . \tag{69}$$

We then substitute equation (54) into equations (68) and (69), and solve these equations, using Geocentric Ecliptic Coordinates (§II) and using the unperturbed expressions for r_0, v_0, h_0 in the right-hand sides of equations (68) and (69). We first integrate equation (68), then substitute our result into equation (69) and solve for δu. We retain terms up to first order in the Moon's orbital eccentricity ($e \sim .055$) and to first order in the quantity e^2/ϵ ($\sim .7$ for the Moon) and neglect terms which produce perturbations smaller than 30 cm, the current experimental limit of laser-ranging technology. Our result for δu is

$$p\delta u = -\left[\frac{1}{2} \alpha_1 (M/p)^{1/2} (e/\epsilon) w_1\right] \sin(\epsilon\varphi - \varphi_1)$$

$$+ \left[\alpha_3 (|\Omega|/M)(\lambda p^2/M)(e/\epsilon) w_2\right] \sin(\epsilon\varphi - \varphi_2)$$

$$+ \left[\frac{1}{8} \alpha_2 (e^2/\epsilon) w_1^2\right] \cos(2\epsilon\varphi - 2\varphi_1)$$

$$- \left[\frac{1}{4} \alpha_1 (M/p)^{1/2} (1/\epsilon) w_1\right] \sin(\varphi - \varphi_1)$$

$$+ \left[\frac{1}{2} \alpha_3 (|\Omega|/M)(\lambda p^2/M)(1/\epsilon) w_2\right] \sin(\varphi - \varphi_2)$$

$$+ \left[\frac{1}{16} \alpha_2 (e/\epsilon) w_1^2\right] \cos\left[(1 + \epsilon)\varphi - 2\varphi_1\right]$$

$$- \left[\frac{1}{12} \alpha_2 w_1^2\right] \cos(2\varphi - 2\varphi_1)$$

$$- \left[\frac{1}{3} \alpha_3 (|\Omega|/M)(\lambda p^2/M) ew_2\right] \sin\left[(2 - \epsilon)\varphi - \varphi_2\right] \quad , \tag{70}$$

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where w_1 is the projection of $\underset{\sim}{w}$ onto the plane of the orbit, given by

$$w_1 = w \cos \beta \quad , \tag{71}$$

and w_2 is given by

$$w_2 = w \cos \delta (\sin^2 \alpha + \cos^2 \alpha \cos^2 \Theta)^{1/2} \quad . \tag{72}$$

The phases φ_1 and φ_2 are related to the direction of $\underset{\sim}{w}$ in a manner which we will not quote here: it is the <u>amplitudes</u> of the terms in equation (70) which we are interested in.

We substitute into equation (70) the standard numerical values for the Moon's orbital elements:

$$e \simeq 5.5 \times 10^{-2} \, , \quad p \simeq 3.8 \times 10^5 \text{ km} \, , \quad \epsilon \simeq 4.4 \times 10^{-3} \quad ;$$

for the Earth's self-gravitational energy, and rotational angular velocity:

$$(|\Omega|/M)_\oplus \simeq 5 \times 10^{-10} \, , \quad \lambda_\oplus \simeq 7 \times 10^{-5} \text{ sec}^{-1} \quad ;$$

and for the velocity $\underset{\sim}{w}$ and the "tilt" of the Earth'

$$w = 200 \text{ km/sec} \, , \quad \Theta = 23 \frac{1}{2}^{\text{o}} \quad ,$$

$$\beta = 60^{\text{o}} \, , \quad \alpha = 318^{\text{o}} \, , \quad \delta = 48^{\text{o}} \quad .$$

The resulting amplitudes of the periodic terms in equation (70) are summarized in Table 3.

Table 3 is by no means a complete list of preferred-frame perturbations in the Earth-Moon distance. Higher order expansions in eccentricity may yield sizable perturbations with different frequencies than those listed in Table 3. Because the Earth orbits the Sun, the Earth's velocity $\underset{\sim}{w}$ is not

precisely constant but various with a period of a year. This variation
will introduce further terms in the list of perturbations, whose frequencies
will be combinations of the Moon's natural sidereal frequency (argument φ)
and the Earth's orbital frequency. Further study of these terms is under-
way.

Our result for the term with argument 2φ (period 13.6 days, amplitude
4 m) is in rough agreement with the results of a heuristic calculation by
Vinti (1971).

IV. THE SOLAR SYSTEM'S VELOCITY THROUGH THE UNIVERSE

Throughout this paper we have assumed that the solar system moves through
the Universe with a velocity $\underset{\sim}{w}$ equal to its nearly circular orbital velocity
around the Galaxy ($\sim$ 200 km/sec in the direction $\ell^{II} = 90^{\circ}$, $b^{II} = 0^{\circ}$). A
more realistic value for $\underset{\sim}{w}$ would be the solar system's velocity through the
cosmic microwave radiation, when it is ultimately measured with confidence.
Current measurements of this velocity, obtained by studying the anisotropy
in the measured temperature of the microwave radiation (caused by the Doppler
shift) are not yet completely reliable (Conklin[1969]; Boughn, Fram, and
Partridge [1971]). Those results which have been obtained, however, are in
rough agreement with measurements of the solar system's velocity relative
to clusters of galaxies, obtained by studies of galactic redshifts (de Vaucouleurs
and Peters 1968), and suggest a net velocity of $\sim$ 200 km/sec in the direction
$\ell^{II} \sim 290^{\circ}$, $b^{II} \sim 24^{\circ}$ (see Sciama [1971] for a discussion). But because of
the experimental uncertainties, this value for our velocity relative to the
Universe should not be given much weight at this time.

For the sake of illustration, however, we have repeated the calculations

of this paper using this new value for $\underset{\sim}{w}$. The resulting limits on the parameters α_1, α_2, α_3 are

$$|\alpha_1| < 0.1 \, , \quad |\alpha_2| < 2 \times 10^{-2} \, , \quad |\alpha_3| < 8 \times 10^{-6} \quad , \quad (73)$$

which are not significantly different than the limits given in Table 1.

V. CONCLUSIONS

The PPN formalism has been used to study possible solar-system effects due to our motion relative to a "preferred Universal rest-frame". Some of these predicted effects were found to be in strong disagreement with experimental data, with the result that the PPN parameters α_1, α_2, and α_3 must satisfy the following experimental constraints:

$$|\alpha_1| < 0.2 \, , \quad |\alpha_2| < 3 \times 10^{-2} \, , \quad |\alpha_3| < 2 \times 10^{-5} \quad . \quad (74)$$

These results rule out the metric theories of gravity due to Page and Tupper, Yilmaz, Papapetrou, Ni, Coleman, and Rosen, and put an upper limit on the strength of a possible cosmological vector gravitational field in a vector-metric theory discussed in Paper I of this series.

APPENDIX

VARIATION IN THE EARTH'S MOMENT OF INERTIA DUE TO

A VARYING GRAVITATIONAL CONSTANT

Here we derive the variation in the Earth's moment of inertia $\Delta I/I$ caused by a variation in the Newtonian gravitational constant $\Delta G/G$. We focus on the part of $\Delta G/G$ which is independent of position on the Earth (eqs. [31] and [33]), and compute the amplitude of the spherical "breathing" of the Earth as G varies. This calculation is similar in spirit and intent to the work of Murphy and Dicke (1964) who studied the effects of a uniformly decreasing gravitational constant on the Earth.

Since G varies so slowly (period of one year), we will assume that the Earth is in hydrostatic equilibrium at each moment of time, and changes only quasistatically. For hydrostatic equilibrium,

$$\frac{dp}{dr} = - \frac{G(t)\rho m(r)}{r^2}, \tag{A1}$$

where p is the pressure, ρ the density, $G(t)$ the time-varying gravitational constant, and $m(r)$ the mass inside radius r, given by

$$m(r) = 4\pi \int_0^r \rho r^2 \, dr . \tag{A2}$$

We will use $m(r)$ instead of r as independent variable, then from equation (A1), we have

$$p(m) = \frac{G(t)}{4\pi} \int_m^M \frac{m \, dm}{r^4} , \tag{A3}$$

where M is the mass of the Earth. Note we have used the boundary condition that the pressure vanish at the surface of the Earth, i.e.,

$$p(M) = 0. \tag{A4}$$

As $G(t)$ changes, the pressure distribution changes, causing a change $\underset{\sim}{\xi} = \xi \underset{\sim}{e}_r$ in the position of each element of matter. For a given shell of matter, the mass inside that shell is constant by conservation of mass. Then if $G(t)$ changes by ΔG, we get from equations (A2) and (A3), following each element of matter

$$\Delta m = 0, \tag{A5}$$

$$\Delta p = p \frac{\Delta G}{G} - \frac{G}{\pi} \int_m^M \frac{m \, dm \, \xi}{r^5} . \tag{A6}$$

But the volume of each element of matter changes, and this change can be related to the change in pressure using the bulk modulus $\varkappa$ (we ignore temperature changes; see Murphy and Dicke [1964]):

$$\Delta p = - \varkappa (\Delta V/V) = - \varkappa \, \underset{\sim}{\nabla} \cdot \underset{\sim}{\xi} = - (\varkappa/r^2)(r^2 \xi)_{,r} . \tag{A7}$$

From equation (A7) we obtain an expression for ξ in terms of Δp:

$$\xi = - \frac{1}{r^2} \int_0^r \frac{r'^2 \Delta p'}{\varkappa'} \, dr' . \tag{A8}$$

The spherical moment of inertia is given by

$$I = \int_0^M r^2 \, dm, \tag{A9}$$

and the change in I caused by the displacement of each shell of matter is

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$$\Delta I = 2 \int_0^M r \, \xi \, dm \, . \tag{A10}$$

Substituting equation (A8) for ξ into equations (A6) and (A10), and re-expressing the equations in terms of r instead of m, we get

$$\Delta I = -8\pi \int_0^R \rho r \, dr \int_0^r \frac{\Delta p'}{\varkappa'} r'^2 \, dr', \tag{A11}$$

$$\Delta p = p \frac{\Delta G}{G} + 4G \int_r^R \frac{\rho \, m(r) \, dr}{r^5} \int_0^r \frac{\Delta p'}{\varkappa'} r'^2 \, dr'. \tag{A12}$$

Since $\Delta G/G$ is small ($\sim 10^{-8}$), equation (A12) can be iterated to obtain Δp, from which ΔI may be computed using equation (A11). We are interested only in a rough estimate for ΔI, so we will take only the first order approximation for equation (A12). Then

$$\Delta I = -8\pi \frac{\Delta G}{G} \int_0^R \rho r \, dr \int_0^r \left(\frac{p'}{\varkappa'}\right) r'^2 \, dr' \, . \tag{A13}$$

The radial distribution of the ratio $(p/\varkappa)$ for the Earth can be approximated by the function

$$(p/\varkappa) = \begin{cases} 0.26 \, [1 - (r/R)^2] \, , & (r/R) \geq 0.2 \, , \\ 0 & , & (r/R) < 0.2 \, , \end{cases} \tag{A14}$$

assuming an incompressible core (Allen 1963). We also use the density distribution

$$\rho(r) = \rho_0 \left[1 - \frac{4}{5} (r/R)\right] \, , \quad \rho_0 = (5/2)(3M/4\pi R^3) \, , \tag{A15}$$

which gives the observed mass and spherical moment of inertia for the Earth.

Equations (A13), (A14), and (A15) then yield

$$\frac{\Delta I}{I} \simeq - (0.10) \frac{\Delta G}{G} \; .$$

Other Earth models which we have tried yield similar results.

TABLE 1

EXPERIMENTAL LIMITS AND THEORETICAL PREDICTIONS

FOR THE PPN PARAMETERS α_1, α_2, AND α_3

EXPERIMENTAL LIMITS ON α_1, α_2, AND α_3

$$|\alpha_1| < 0.2$$

$$|\alpha_2| < 3 \times 10^{-2}$$

$$|\alpha_3| < 2 \times 10^{-5}$$

THEORETICAL PREDICTIONS FOR α_1, α_2, AND α_3

Theory and its adjustible parameters[†]	α_1	α_2	α_3
General Relativity	0	0	0
Scalar-Tensor Theories (ω, Λ)	0	0	0
Vector-Metric Theory (κ)	0	$\kappa^2/(1 + 1/2\ \kappa^2)$	0
Stratified Theories[*]:			
a. Page and Tupper (a,c)	$-4(1 + a)$	0	$-2(1 + a)$
b. Modified Yilmaz	-8	0	-4
c. Papapetrou	-8	-4	0
d. Lagrangian Stratified Theory (Ni)	-8	0	0
e. General Stratified Theory (Ni) (p,q)	-8	0	-4
f. Coleman (p)	-8	0	-4
g. Rosen (λ)	$-4(1 + \lambda)$	0	0

[†]See Ni (1972a) and Will and Nordtvedt (1972) for discussion and references.

[*]"Stratified Theories with time-orthogonal conformally flat space slices" (see Ni 1972a). These theories all have the property that $\alpha_1 = -4(1 + \gamma)$, where γ is the PPN parameter whose value must be $\sim 1.0(\pm\ .1)$ in order to agree with light deflection and radar time-delay experiments.

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TABLE 2

GEOPHYSICAL PREFERRED-FRAME EFFECTS

Effect	Frequency	Amplitude
i) Semi-Diurnal Earth-Tides $(\Delta g/g)$	$2\Omega - 2\omega$	$1 \times 10^{-9} \, \alpha_2 \cos^2 L$
	$2\Omega - \omega$	$1 \times 10^{-8} \, \alpha_2 \cos^2 L$
	$2\Omega^\dagger$	$3 \times 10^{-8} \, \alpha_2 \cos^2 L$
	$2\Omega + \omega$	$< 10^{-9}$
	$2\Omega + 2\omega$	$< 10^{-9}$
ii) Diurnal Earth-Tides $(\Delta g/g)$	$\Omega - 2\omega$	$< 10^{-9}$
	$\Omega - \omega$	$1 \times 10^{-8} \, \alpha_2 \sin 2L$
	Ω	$7 \times 10^{-8} \, \alpha_2 \sin 2L$
	$\Omega + \omega$	$3 \times 10^{-9} \, \alpha_2 \sin 2L$
	$\Omega + 2\omega$	$< 10^{-9}$
iii) Zonal Variations in Earth's Rotation $(\Delta\Omega/\Omega)$	ω	$8 \times 10^{-9} \, \alpha_2$
	2ω	$< 10^{-9} \, \alpha_2$
iv) Spherical Variation in Earth rotation $(\Delta\Omega/\Omega)$	$\omega^\dagger$	$3 \times 10^{-9}(2/3 \, \alpha_2 + \alpha_3 - \alpha_1)$

†Used to put limits on the values of the parameters α_1, α_2, and α_3 (§I).

TABLE 3

PREFERRED-FRAME PERIODIC PERTURBATIONS

IN THE EARTH-MOON DISTANCE — A PARTIAL LIST

Amplitude of distance perturbation in meters	Argument of periodic term	Period
$3 \; \alpha_1$ m	$\epsilon \; \varphi$	17 years
$8000 \; \alpha_3$ m	$\epsilon \; \varphi$	17 years
$4 \; \alpha_2$ m	$2 \epsilon \; \varphi$	8 1/2 years
$25 \; \alpha_1$ m	φ	27.3 days
$70,000 \; \alpha_3$ m	φ	27.3 days
$40 \; \alpha_2$ m	$(1 + \epsilon) \; \varphi$	27.1 days
$4 \; \alpha_2$ m	$2 \; \varphi$	13.6 days
$12 \; \alpha_3$ m	$(2 - \epsilon) \; \varphi$	13.7 days

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