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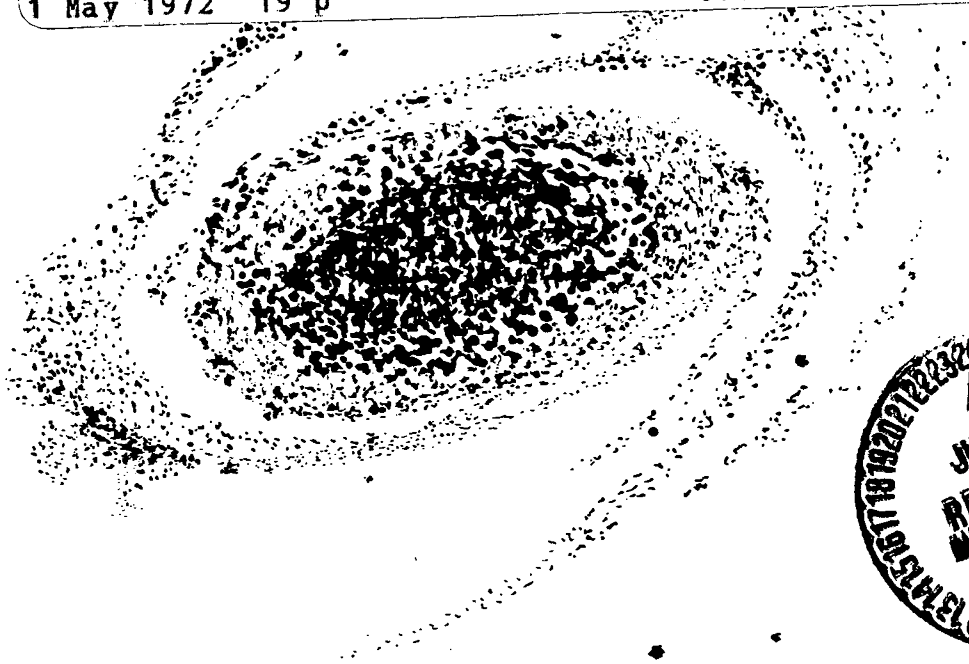
R. F. C. VESSOT



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ABSTRACT

Advances in the development of atomic oscillators having great frequency stability and the progress in space technology now make possible direct measurements of the effects of gravitation on time. Using the earth's gravity these measurements can now be made to 20 parts per million. At present, there is a 1% verification of the equivalence principle for clocks made over a 75-ft vertical distance by use of Mössbauer γ-ray emission and absorption. Measurements made to greater accuracy and spanning distances where appreciable curvature of the metric of spacetime will help verify the equivalence principle, a postulate upon which Einstein developed his General Theory of Relativity.

RÉSUMÉ

Les perfectionnements des oscillateurs atomiques à grande stabilité de fréquence et les progrès en technologie spatiale permettent maintenant de mesurer directement l'effet de la gravitation sur le temps. En utilisant la gravité terrestre, ces mesures peuvent être faites avec une précision de 20.10^{-6}. À l'heure actuelle nous avons une vérification à 1% près du principe d'équivalence pour horloge établie sur une distance verticale de 75 pieds, en se servant de l'émission et de l'absorption du rayonnement γ Mössbauer. Des mesures faites avec une grande précision et des distances d'envergure assez grande pour avoir une courbure appréciable de la métrique de l'espace temps vont aider à vérifier le principe de l'équivalence, postulat sur lequel Einstein développa sa théorie générale de la relativité.

КОНСПЕКТ

Прогресс в развитии атомных осцилляторов имеющих большую частотную устойчивость и прогресс в технике космоса теперь дают возможность провести прямые измерения эффектов гравитации по времени. Пользуясь силой притяжения земли эти измерения могут теперь проводится до миллионных долей. В настоящее время имеется 1% проверка принципа эквивалентности для часов сделанных на вертикальных растояниях превышающих 75 футов пользуясь Моссбауеровским излучением и поглощением гамма-лучей. Измерения проведенные с большей точностью и пролетные растояния со значительной кривизной метрики пространственного времени позволят проверить принцип эквивалентности, постулат на котором Эйнштеин построил свою Общую теорию относительности.

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1. INTRODUCTION

It is well known that direct experimental tests of relativity have been difficult to perform. Appreciable differences in velocity, acceleration, and gravitational potential between two reference frames are required to bring the expected effects to observable levels. Now, however, advances in the technology of space exploration allow us to perform experiments between widely separated frames of reference and over large differences in velocity and acceleration. Thus, only recently has our "laboratory" achieved dimensions where relativistic effects are readily observable.

A great many advances have been made in the last 10 years in the art of keeping time and generating stable frequencies. Today, comparisons of frequency stability are routinely made to better than 1 part in 10^{14} by means of atomic-hydrogen maser oscillators. Substantial progress has also been made in microwave and laser communication systems. The light (or electromagnetic) pulses in the "thought experiments" described in the literature on relativity can be realized by laser signals and by phase-coherent microwave signals.

Traditionally, relativity has been described in terms of systems moving with respect to one another, each containing rods and clocks. Pulsed-light signals connect the systems observationally and provide the basis for comparisons. To make experimental measurements, we can still use rods and clocks. However, the rods are related to the clocks by the velocity of light, and we can describe proper distances in terms of wavelengths of the proper frequency of the clocks, implicitly assuming c (the

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velocity of light) is constant in spacetime. Thus, we can design relativity experiments that require clocks only. Taking advantage of the exceptional stability of hydrogen maser clocks, we can now measure relativistic effects to an accuracy that would have been unthinkable some 20 years ago.

All relativity theories currently under consideration predict that gravitation acceleration will have an effect on the relative rates of proper clocks. This effect is usually described as the gravitational redshift or gravitational time dilation.

The Smithsonian Astrophysical Observatory (SAO) and NASA Marshall Space Flight Center are working on an experiment that will measure the effect of the earth's gravitation on the rate of proper clocks with an accuracy of 20 parts in 10^6.

In establishing the significance of this experiment, we should not be blind to the empirical view that nature shows some relationships between the lengths of rods and the rates of clocks in different parts of space over which gravitational fields exist and that the only way to learn about these relationships is by making direct measurements as accurately as possible so that theories of various kinds can be tested.

2. THE EQUIVALENCE PRINCIPLES

Recently, there have been considerable thought and discussion about the redshift experiment and its significance to relativity. In substance, one can conclude that the SAO experiment tests the principle of equivalence, which asserts that there is no way of distinguishing locally between a gravitational field and an oppositely directed acceleration.

The "Weak" equivalence principle describes the proportionality of inertial and gravitational mass; this has been verified to 1 part in 10^{11} by Roll, Krotkov, and Dicke (1964) and, more recently, by Braginskii and Panov (1972) to 0.9×10^{-12} by using a highly refined version of the Eotvos torsion-balance experiment.

Einstein in 1907 postulated that "all local, freely falling, nonrotating laboratories are fully equivalent for the performance of all physical experiments." This is often called the "Strong" equivalence principle.

There is also a "Semi-Strong" equivalence principle, which asserts that all nonrotating, freely falling laboratories in a given part of the universe are equivalent and that each of these laboratories is subject to the laws of Special Relativity. This view of equivalence does not rule out the possibility of different numerical constants such as c (the velocity of light), g (the gravitational constant), α (the fine-structure constant), etc. at different times and in different parts of the universe. Gradients of nongravitational terms caused by these differing values in space cause nonzero slopes at the origin of a local Lorentz frame and, strictly speaking, are a violation of equivalence.

General relativity is based on the postulate of "Strong" equivalence. Spacetime in General Relativity can be described by a continuum of localities, in each of which the special theory can apply locally. The "Semi-Strong" principle can also be applied; it allows a description of the universe where so-called physical constants can vary with time.

Two consequences of the Strong and Semi-Strong equivalence principles are that 1) light bends in a gravitational field, and 2) light suffers a gravitational redshift when it travels away from a heavy body (or "upward" in a gravitational field).

At this point, one must describe differing views of the significance of the redshift experiment as a test of General Relativity. According to Dicke (1964), one can derive the redshift from the weak equivalence principle by applying the concepts of energy conservation and the equivalence of inertial mass and conserved energy. This amounts to giving the photon an inertial mass

$$\text{"M"} = \frac{h\nu}{c^2} \quad ,$$

where h is Planck's constant, ν is the photon's frequency, and c is the velocity of light, and letting it "fall" with gravitational acceleration through a distance ℓ. The energy difference ΔE given the photon owing to its "fall" over height ℓ is

$$\Delta E = h\Delta\nu = \text{"M"}g\ell \quad ,$$

from which we obtain

$$\Delta\nu = \frac{g\ell}{c^2} \quad .$$

If we relate $g\ell$ to the Newtonian potential difference $\Delta\phi$, we obtain the locally valid expression for the redshift:

$$\frac{\Delta\nu}{\nu} = \frac{\Delta\phi}{c^2} \quad .$$

3. THE SIGNIFICANCE OF REDSHIFT EXPERIMENTS

Thorne, Will, and Ni (1971) question the a priori assumption of energy conservation and point out that some relativity theories violate it. Furthermore, according to Schild (1962), there have been over the years a number of relativity theories that were well accepted in their time but that disagree with the gravitational-redshift experiments.

We can place lightbending and redshift experiments into two categories, local and nonlocal. Local experiments involve a very homogeneous gravitational field such as that seen near the surface of the earth. Nonlocal experiments involve measurements where the field is not uniform over the distances involved – for example, the fields of a massive body measured over distances from its surface that are large compared to its dimensions. If we consider the "Semi-Strong" equivalence principle and make a weak postulate of the conservation of energy and the inertial mass, we can arrive at the two consequences, viz., lightbending* and redshift. The present status of these is shown in the following table.

	Local		Nonlocal	
Lightbending		–	15%	Muhleman, Ekers, and Fomalont (1970)
			12%	Seielstad, Sramek, and Weiler (1970)
Redshift	1%	Pound and Rebka (1960) and Pound and Snider (1965)	5%	Brault (1963)
			0.002%	Proposed SAO probe experiment

*Here, lightbending does not include the curvature effects of space that are in the spatial part of the spacetime metric. This requires a general relativistic treatment as in the Schwartzchild metric.

To the best of the writer's knowledge, no local lightbending experiments have been reported. Results of greater accuracy from nonlocal lightbending experiments are soon likely to be reported.

The accuracies are stated in percentages of the effect predicted, that is, angle of deflection and fractional frequency shift. As can be seen from the table, the most precise experiment related to the redshift has been performed by Pound and Rebka (1960) and Pound and Snider (1965) using the Mössbauer effect of gamma rays from Fe^{57} over a vertical distance of 75 ft. This remarkably elegant experiment has confirmed, in a local sense, the predictions of the equivalence principle to an accuracy of 1%.

The SAO experiment differs in concept from the Pound-Rebka-Snider experiments. Time signals are used rather than incoherent resonance radiation and absorption of Fe^{57} gamma rays. Furthermore, the clocks involved are located in very different gravitational fields; there is considerable inhomogeneity in the field in the path between the two clocks.

We expect that the measurement of redshift to 20 parts in 10^6 with the rocket probe will be an important testing phase for experiments with clocks of better stability interacting with the sun's gravitational field. Here it will be possible to look at terms in $(\Delta\phi/c^2)^2$. Measurement of these terms will test the validity of the concept that the overall value of the interval Δs^2 in spacetime can be obtained by integrating the shift over a continuum of local inertial frames.

For example, to extend the concept of local equivalence, we describe $g\ell$ (the Newtonian potential drop over the small distance of our inertial frame) by $\delta\phi$. Over the large potential difference $\Delta\phi$, we can divide the potential into η equal infinitesimal $\delta\phi = \Delta\phi/\eta$, each causing a frequency ν_η at its extremity. Locally, we have

$$\frac{\nu_1}{\nu_2} = 1 + \frac{\Delta\phi}{\eta c^2} \quad .$$

Overall, we have

$$\frac{\nu_1}{\nu_2} \cdot \frac{\nu_2}{\nu_1} \cdots \frac{\nu_\eta}{\nu_\eta - 1} = \left(1 + \frac{\Delta\phi}{\eta c^2}\right)^\eta .$$

In the limit $\eta \rightarrow \infty$, this becomes

$$\frac{\nu}{\nu_0} = e^{\Delta\phi/c^2} ,$$

which can be expanded as

$$1 + \frac{\Delta\phi}{c^2} + \frac{\Delta\phi^2}{2c^4} + \cdots .$$

In the probe experiment using the earth's gravity, $\Delta\phi/c^2$ is in the order of 5×10^{-10}, and $\Delta\phi^2/2c^4$ is 1.2×10^{-19}. Since the stability of the clock is 7×10^{-15}, the second term is still beyond our reach. However, a solar experiment could perform the test to second order in $\Delta\phi/c^2$ with the present state of the clock. Also, there is a very realistic expectation that the stability of maser clocks can be improved beyond the 7 parts in 10^{15} now available and that an accuracy of 1 part in 10^{15} is possible thus making the second-order test to even better accuracy.

Thorne, Will, and Ni (1971) give considerable importance to the measurement of the gravitational redshift, both in homogeneous fields and in fields that are inhomogeneous such as those encountered over large distances from massive bodies. The following is a synopsis of their description of the relevance of the redshift to the equivalence principle and to the very existence of theories of gravity that can be expressed in terms of a metric.

The two conditions for a theory of gravitation to be a metric theory are the following:

1) There exists a metric of signature -2 (e.g., Minkowski metric +, -, -, -) that governs the proper length and proper time measurements in the usual manner of Special and General Relativity:

$$ds^2 = g_{ij}\, dx^i\, dx^j \quad .$$

2) Stressed matter and fields acted on by gravity respond according to $\nabla \cdot T = 0$, where T is the total stress-energy tensor for all matter and for all nongravitational fields.

These two conditions can be met with the assumption that there exist everywhere local Lorentz frames in which all the laws of Special Relativity apply. (This is the Einstein equivalence principle.)

From the Eotvos-Dicke-Braginskii experiments, we have the assurance to 1 part in 10^{12} of the sun's acceleration at the earth that freely falling test bodies all have the same acceleration regardless of their composition (particle-binding energy, chemical energy, etc.). We can fill spacetime with a manifold of these trajectories. Let us call these special curves in spacetime "test-body trajectories."

Geodesic lines in the spacetime described by the metric appear as straight lines in the local Lorentz frames. This is the same line that would be drawn by a test particle that is not accelerated with respect to the local Lorentz frame.

The question is whether or not local Lorentz frames "fall" in a gravitational field in the same manner as does a particle. If so, it can be established that the particle follows a geodesic.

To date, the best test of this relationship between particles and geodesics is the gravitational redshift experiment. Thorne, Will, and Ni (1971) argue that one need only assume phase conservation of electromagnetic waves and the Minkowski metric ($\Delta s^2 = c^2\, \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$) of local Lorentz frames governing the rate of atomic clocks to show that the downward acceleration of the Lorentz frames in the local sense is the same as that of a free particle. The Pound-Rebka-Snider experiments show this to be true to 1% of the earth's gravitational acceleration. Quoting from

Thorne et al.: "Because this conclusion is crucial to the foundations of metric theories, of the Parametrized Post-Newtonian framework, and the General Relativity it is important that the precision of the redshift experiments be improved as much as possible — both on earth (homogeneous field) and elsewhere in the solar system (inhomogeneous fields)."

4. THE SAO EXPERIMENT

The SAO experiment consists of a rocket probe package attaining an altitude of 10,000 n.mi. The package, containing a hydrogen maser clock and a transmitter and transponder-translation system, weighs 150 pounds and will be sent aloft by a Scout rocket. The engineering effort required to design and package the clocks has been in progress under NASA's sponsorship for 10 years.

The development of a maser for satellite and rocket probe has progressed to the stage where virtually no uncertainties in the technical capabilities of the maser remain to be resolved. The maser has been packaged and successfully transported and used in several applications, including frequency comparisons and very long-baseline interferometry experiments at remote locations. It has survived considerable mishandling. Further engineering to make it spaceworthy is entirely feasible. The microwave transmitter and transponder systems having capabilities critical to the success or failure of the rocket probe experiment have already been developed and require only minor changes for adaptation to the experiment. In evaluating the performance capabilities, etc. of these subsystems, we have made conservative estimates and allowed for systematic biases even though these may not seem important at present.

At the present stage of development of clocks, we have measured stability data of 5 parts in 10^{15} for averaging times of 100 to 1,000 sec. We anticipate the results from a new maser cavity design and from wall coatings of the new Teflon (work done on the present NASA contract NSR 09-015-098) should be in the range of 1 to 2 parts in 10^{15}.

The SAO probe experiment is a very good way of testing concepts for future relativity experiments involving clocks. The accuracy of the redshift experiments can be improved by using an eccentric earth orbit; ultimately, an experiment using the solar gravitational field will give us a look at the second-order terms in $\Delta\phi/c^2$. The experimental measurement of the time delay of laser signals received at a heliocentric-orbiting drag-free satellite during superior conjunction (as proposed by the European Space Research Organization) will also use a clock.

We are confident that the stated accuracy of 20 parts in 10^6 will in fact be realized by this redshift experiment. We believe that when any experimental test of a fundamental physical concept can be improved by a factor of 500, the test should be performed.

5. ACKNOWLEDGMENTS

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BIOGRAPHICAL NOTE

ROBERT F. C. VESSOT received the B.A., M.Sc., and Ph.D. degrees from McGill University, Montreal, Canada, in 1951, 1954, and 1956, respectively.

From 1955 to 1960, Dr. Vessot was a staff member of Massachusetts Institute of Technology, Department of Sponsored Research. From 1960 to 1969, he was manager of maser research and development at Hewlett-Packard. Dr. Vessot joined the staff of Smithsonian Astrophysical Observatory as physicist and the staff of Harvard College Observatory as research associate in 1969.

Dr. Vessot's current research is in atomic frequency standards and the measurement of the gravitational redshift.

NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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