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ANALYSIS AND TESTING OF HIGH ENTRAINMENT SINGLE-NOZZLE JET PUMPS WITH VARIABLE-AREA MIXING TUBES

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16. Abstract

In this investigation, an analytical model was developed to predict the performance characteristics of axisymmetric single-nozzle jet pumps with variable area mixing tubes. The primary flow may be subsonic or supersonic. The computer program presented in this report uses integral techniques to calculate the velocity profiles and the wall static pressures that result from the mixing of the supersonic primary jet and the subsonic secondary flow.

An experimental program was conducted to measure mixing tube wall static pressure variations, velocity profiles, and temperature profiles in a variable area mixing tube with a supersonic (M = 2.72) primary jet. Static pressure variations were measured at four different secondary flow rates. These test results were used to evaluate the analytical model. The analytical results compared well to the experimental data. Therefore, the analysis is believed to be ready for use to relate jet pump performance characteristics to mixing tube design.

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ANALYSIS AND TESTING OF HIGH ENTRAINMENT SINGLE-NOZZLE JET PUMPS WITH VARIABLE-AREA MIXING TUBES

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SUMMARY

The use of jet pumps is of increasing interest for boundary layer control or control force augmentation in V/STOL aircraft. In typical applications, a small mass flow of primary air at pressures up to 400 psia can be used to entrain a much larger mass flow of secondary air at ambient conditions. The primary nozzle flow is supersonic while the secondary flow is subsonic. The jet pump system design objectives may be maximum entrainment, maximum thrust augmentation, or some combination of the two. Little information is available in the literature to guide the designer of jet pumps for such applications.

In this investigation, an analytical model was developed to predict the performance characteristics of axisymmetric single-nozzle jet pumps with variable area mixing tubes. The primary flow may be subsonic or supersonic. In the region upstream of the section where the central jet reaches the wall, the analysis is based upon the hypothesis that the mixing phenomenon is fundamentally similar to the mixing of a free turbulent jet with the surrounding fluid. The eddy viscosity values used in the analysis are adjusted to allow for the effect of the duct walls on the mixing process. Integral techniques are employed in a computer program to solve the continuity, momentum, and energy equations to determine the variation of flow properties along the mixing tube. Wall boundary layer effects are included in the analysis.

Downstream of the section where the jet reaches the wall, the velocity profile is assumed to approach asymptotically the profile for fully developed turbulent flow in a pipe. Viscous forces are present throughout the flow so no distinct boundary layer analysis is employed. The eddy viscosity is assumed to approach the fully-developed flow value asymptotically. Wall friction forces are calculated from the fully-developed pipe flow friction coefficient. Integral techniques are employed as before to determine the variation of flow properties along the mixing tube.

An experimental program was conducted to measure mixing tube wall static pressure variations, velocity profiles, and temperature profiles in a variable area mixing tube with a supersonic ($M = 2.72$) primary jet. Static pressure variations were measured at four different secondary flow rates. These test results were used to evaluate the analytical model.

The analytical model yields good predictions of wall static pressure distributions, velocity profiles, and temperature profiles along the mixing tube. Therefore, the analysis is believed to be ready for use to relate jet pump performance characteristics to mixing tube design.

Section 1

INTRODUCTION

1.1 Background

A number of STOL aircraft boundary layer control systems now under consideration employ jet pumps to entrain large flows of secondary air and direct them over deflected flaps to achieve lift augmentation. Some proposed VTOL aircraft systems also employ jet pumps for direct lift or control force augmentation. The primary, high-pressure flow for the jet pumps can be provided either by bleed from main engine compressors or by an auxiliary power unit.

The use of jet pumps as primary components of V/STOL aircraft systems makes necessary the development of new design techniques for these devices. In aircraft applications, it is essential to be able to minimize the size of jet pumps for particular primary and secondary flow conditions. Jet pumps for boundary layer control systems generally must have high entrainment ratios--the secondary flow often must be over 10 times larger than the primary flow--but pressure rises of only a few psf are needed. The primary flow may be highly supersonic. Thus, design procedures which have been developed in the past for the more conventional low-entrainment, high-pressure rise industrial jet pumps are not suitable for V/STOL aircraft jet pump design.

1.2 Previous Work

In an earlier program, Dynatech R/D Company carried out an analytical and experimental investigation of high-entrainment ratio air-to-air jet pumps for the Ames Research Center of NASA (reference 1). This investigation was limited to jet pumps with constant-diameter mixing tubes. An analytical procedure and a computer program were developed to predict the performance of such a jet pump over a range of operating conditions. The accuracy of the analysis was confirmed by comparing predicted performance to test results for a number of multiple-nozzle jet pump configurations at different primary flow pressure and temperature levels. Procedures were demonstrated for matching a jet pump to its duct system for maximum entrainment or thrust augmentation.

The selection of the constant-diameter mixing tube configuration allowed considerable simplification of the analysis, design, and construction of the jet pump. However, it is unlikely that constant-area jet pumps give the best performance for all applications. Almost no information is available to indicate the extent of performance improvements which can be achieved with other mixing tube configurations.

A method for predicting the flow behavior in jet pumps with arbitrary mixing tube shapes and incompressible flows was reported by P.G. Hill (reference 2). The method is based upon the hypothesis that the mixing phenomenon in a jet pump has a fundamental similarity to the mixing of a free turbulent jet with the surrounding fluid. Therefore, as in a free jet, the turbulent Reynolds number--

$$Re_T = \frac{\text{jet velocity x duct radius}}{\text{eddy viscosity}}$$

--will remain constant with distance as mixing occurs. This is a rather gross simplifying assumption but the resulting flow predictions are good. Static pressure variations and velocity profiles computed on this basis agreed well with test data for Helmbold's converging-diverging mixing tube. Once the static pressure distribution is known, the jet pump performance can be predicted without further difficulty.

The analytical methods of reference 2 are limited in application to incompressible flow in axisymmetric jet pumps having a single primary jet. These analytical methods must be modified to include compressible flow effects if the methods are to be useful for the designer of V/STOL aircraft jet pump systems.

1.3 Objectives of This Investigation

The specific objectives of this investigation are as follows:

- to develop an analytical procedure for predicting the performance of high-entrainment-ratio compressible flow jet pumps with arbitrary mixing tube geometry.
- to obtain test results with jet pumps having variable-area mixing tubes so that the analytical methods can be checked.

The analytical procedure is formulated to allow prediction of the performance of a particular jet pump nozzle and mixing tube combination over a range of primary and secondary flow conditions. To select the best jet pump design for a particular application, the analysis can be used to predict the performance for a number of different mixing tube shapes. Comparison of the performance characteristics will show which geometry is best. The off-design performance of the jet pump can be determined by using the same analytical procedures.

Section 2

SYMBOLS

A	area, ft^2
b	diameter of jet at which $U = U_o + \frac{U_j}{2}$, ft
C_f	wall friction coefficient
C_w	nozzle flow coefficient
E	dimensionless eddy viscosity = $\frac{\epsilon}{UR}$
f_o	free jet profile value = f_o (η); equation (1)
f_2 (η)	velocity profile at the end of Part 1, equation (7)
g_o	dimensional constant = 32.2, lbm-ft/lbf-sec^2
g_2 (η)	auxilary velocity profile, equation (7)
gg (η)	velocity profile for fully-developed flow in a pipe
H	boundary layer shape factor
k	specific heat ratio
K_L	suction duct loss coefficient
m	entrainment ratio = W_o/W_1
n_s	number of equal-radial-increment annuli used in integral analyses, equation (36)
p	static pressure, lbf/ft^2
P	stagnation pressure, lbf/ft^2
P_{o2}	secondary flow stagnation pressure after correction for suction duct losses, lbf/ft^2
R	tube radius, ft
R_o	radius at nozzle exit section, ft
R_g	gas constant x g_o, ft^2/sec^2-° R

Re_m	Reynolds number based on mean velocity; equation (50)
Re_T	turbulent Reynolds number
R_θ	momentum thickness Reynolds number
S_{oo}, S_{20}	parameters defined by equations (29) and (30)
T_o	stagnation temperature at any radius in mixing zone, °R
T_{oj}	relative stagnation temperature at centerline of jet, °R
T_{oo}	stagnation temperature of flow adjacent to the duct, °R
ΔT_o	difference between stagnation temperature at any radius in the jet and the stagnation temperature of the surrounding flow, °R
$\mathcal{T}$	temperature ratio = T_{oj}/T_{oo}
U	velocity, ft/sec
U_c	velocity at centerline of jet, ft/sec
U_j	velocity at centerline of jet relative to U_o, ft/sec
U_{jo}	relative velocity at centerline of jet at end of transition section, ft/sec
U_{joo}	relative velocity at centerline of jet at beginning of transition section, ft/sec
U_o	velocity of outer stream, ft/sec
U_r	velocity ratio for transition zone = U_{jo}/U_{joo}
V(J)	terms in equation (37)
W_o	mass flow rate, secondary flow, lbm/sec
W_1	mass flow rate, primary flow, lbm/sec
W(J,K)	coefficient matrix; equation (37)
x	axial position along mixing tube, ft
x_{core}	length of the transition zone, ft

y	radius, ft
$Y(K)$	derivatives in equation (37)
γ	velocity profile shape parameter
δ	width of shear layer, ft
δ^*	boundary layer displacement thickness, ft
ϵ	eddy kinematic viscosity, ft^2/sec
η	dimensionless radius = y/δ or y/R
θ	boundary layer momentum thickness, ft
λ	velocity ratio U_o/U_j
ρ	density, lbm/ft^3
τ	shear stress, lbf/ft^2
ν	kinematic viscosity, ft^2/sec

<u>Subscripts</u>

oo	value at top-hat section
1	primary flow
core	dimension at end of transition zone
eff	effective radius or area of mixing tube
m	value at mean area of transition zone
noz	primary nozzle exit area
SD	suction duct upstream of mixing tube

Section 3

ANALYSIS

3.1 Purpose

The purpose of the analysis developed in this section is to predict the performance characteristics of compressible flow jet pumps with variable-area mixing tubes. The jet pumps may have supersonic or subsonic primary flow issuing from a single nozzle located along the axis of an axisymmetric cylindrical mixing tube. The secondary flow and the mixed flow downstream must remain subsonic. The primary and secondary flows are taken to be the same perfect gas.

A particular objective of the analysis is to predict the variation in static pressure along the length of the mixing tube. Knowledge of this pressure variation allows calculation of the thrust augmentation of the jet pump, an essential parameter for jet pump application studies.

3.2 General Description of the Analytical Model

The analysis is based upon the incompressible flow jet pump analytical model developed by Dr. P. G. Hill (reference 2). This analytical model, with its associated computer program, was modified in the present study to account for compressible flow effects. The formulation of the analytical model is described in this section. The computer program which is based upon the compressible flow model is described in Appendix B of this report.

The following initial assumptions are made for the analysis:

1. The primary and secondary flows are the same perfect gas.

2. No heat is transferred across the wall of the jet pump.

3. The jet pump consists of an axisymmetric, cylindrical, variable-area mixing tube with a single primary nozzle located along the axis.

4. The primary and secondary flow conditions and the nozzle geometry are assumed to be such that no normal shocks or moisture condensation shocks occur in the primary flow.

5. The secondary flow and the combined flows after mixing are assumed to remain subsonic throughout the mixing tube.

6. The velocity of the primary jet at the nozzle exit is greater than the velocity of the secondary flow.

7. The static pressure is constant across any section perpendicular to the axis of the jet pump.

Dr. Hill's analysis identifies three distinct flow regimes in a jet pump. These regimes are shown in figure 1; they may be described as follows:

Part 1 - A region in which the jet is approximately self-preserving and is immersed in a potential outer stream which may be accelerating or decelerating, depending on the shape of the duct and the rate of entrainment of mass into the jet.

Recirculation Zone - A possible region in which recirculation occurs, following a deceleration of the outer stream. At the beginning of this zone the "edge" of the jet has not yet diffused to the wall and the secondary fluid recirculates through the jet. The pressure gradient is generally observed to be negligible in this zone.

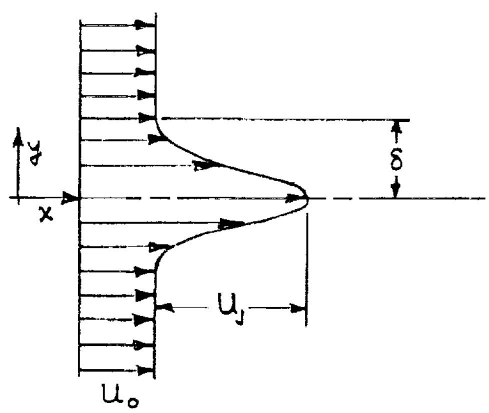
Part 2 - The region downstream of the point (fairly distinct in many cases) at which the jet attaches to the wall. An adverse pressure gradient is generally established but the relatively high shearing forces near the wall tend to accelerate the fluid against the pressure gradient. If there is a zone of recirculation, it is terminated in a short axial distance by these high shearing forces.

In addition to these three regions, there is a relatively short transition zone between the nozzle exit and the section at which a subsequently self-preserving velocity profile is attained.

In Part (1) the jet velocity profile can be approximated well by

$$\frac{U - U_o}{U_j} = f_o \left(\frac{y}{\delta}\right) \quad \text{at any } x \tag{1}$$

where
U = velocity at radius y
U_o = outer stream velocity
U_j = jet relative velocity at centerline
x = axial position along mixing tube
y = radius
δ = width of shear layer (see sketch)



Velocity Profile at Station x

The functional relationship $f_o\ (y/\delta)$ is determined quantitatively from velocity profiles measured in axisymmetric jets discharging into free space.

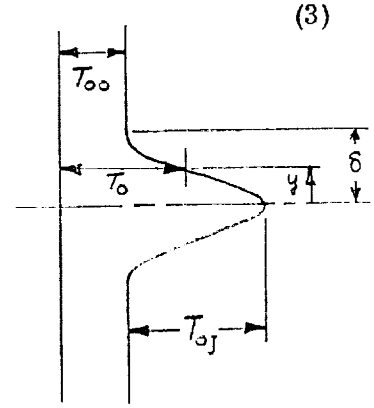
$$f_o(\eta) = 1.0004 - 0.0175\eta - 8.3821\eta^2 + 16.5806\eta^3 - 12.7877\eta^4 + 3.608\eta^5 \qquad (2)$$

where $\eta = y/\delta$ (Part 1)

The same relationship holds in the recirculation zone but the axial pressure gradient in this region is assumed to be zero.

The relationship above is used to describe the velocity profile at a particular axial station in Part 1 of the mixing tube flow. The continuity, momentum, moment-of-momentum, energy, and boundary layer equations are used to determine the changes in U_j, U_o, δ, temperature, and pressure which occur from station to station along the mixing tube. To solve these equations, the temperature profile must be known so that the density variations across the section can be determined. Following Abramovich (reference 3), the stagnation temperature profile is taken to be the square-root of the velocity profile.

$$\frac{T_o - T_{oo}}{T_{oj}} = f_o^{1/2}\left(\frac{y}{\delta}\right) \qquad (3)$$



where T_o = stagnation temperature at any radius in the mixing zone

T_{oo} = stagnation temperature of surrounding secondary flow

T_{oj} = relative stagnation temperature at center of jet

The solution of the moment-of-momentum equation requires shear stress values to be known as a function of radius. These values are obtained as follows:

$$\tau = \epsilon \rho \; \partial U / \partial y \qquad (4)$$

where τ = shear stress

ϵ = eddy kinematic viscosity,

ρ = density

The value of the turbulent Reynolds number is assumed to remain constant across the flow at any axial station in Part 1. This allows calculation of the eddy viscosity from the following equation:

$$\epsilon = U_j \delta / Re_T \qquad (5)$$

where Re_T = turbulent Reynolds number

At the beginning of Part 1, the jet mixing process is not significantly affected by the presence of the mixing tube walls. Therefore, the value Re_{TF} = 147, from incompressible free jet mixing tests, can be used. Further downstream in Part 1, as the jet approaches the walls, the mixing process is altered from a free jet to a free wake type of mixing. The change in the mixing process is accounted for by using the following equation to determine the eddy viscosity at any station in Part 1:

$$\epsilon = \frac{U_j \delta}{Re_{TF}} \left[1 + \frac{3}{2} \left(1 - e^{-1.1\lambda}\right) \right] \qquad (6)$$

where $\lambda = \dfrac{U_o}{U_j}$

Boundary layer growth must be taken into account in order to predict wall static pressure variations with accuracy. Boundary layer displacement thickness variations are obtained in the analysis by using the methods of Moses (reference 4). The equations used are described in Section 3.4 in this report.

In Part 2, the jet has reached the wall. The free jet mixing velocity profile is no longer appropriate. Instead, the velocity profile is assumed to follow the relationship:

$$U/U_c = f_2(\eta) + \gamma\, g_2(\eta) \tag{7}$$

where U_c = jet velocity at centerline

$f_2(\eta)$ = velocity profile at the end of Part 1

$\eta = y/R$

R = mixing tube radius at the axial position considered

$\gamma = \gamma(x)$ adjustable shape parameter

$g_2(\eta)$ = auxilary velocity profile

At the beginning of Part 2, γ is set equal to zero and the velocity profile matches the velocity profile at the end of Part 1. The auxilary profile $g_2(\eta)$ is chosen so that, as γ approaches 1.0, the U/U_c velocity profile approaches the profile for fully-developed flow in a pipe.

$$g_2(\eta) = gg(\eta) - f_2(\eta) \tag{8}$$

where $gg(\eta)$ = velocity profile for fully-developed flow in a pipe

No boundary layer calculations are made in Part 2. Viscous forces are present throughout the flow so no distinct boundary layer exists. Wall friction forces are calculated from turbulent pipe flow correlations.

The continuity, momentum, moment-of-momentum, and energy equations are used to determine the changes in U_c, γ, temperature, and pressure which occur with distance along the mixing tube in Part 2. The solution of the moment-of-momentum equation requires determination of the eddy viscosity as a function of radius and axial position. Because the flow in Part 2 becomes asymptotic to fully-developed pipe flow, the eddy viscosity must be asymptotic to the fully-developed flow value.

$$\tau / \tau_{wall} = y/R = \eta \text{ as } \gamma \text{ (x) approaches } 1.0 \tag{9}$$

$$E_{2f} = - \frac{1}{2} \quad \frac{C_{fd}\,\eta}{\frac{\partial}{\partial\eta}\, gg\,(\eta)} \tag{10}$$

where $E_{2f} = \epsilon_{2f}/U_c R$ = dimensionless eddy viscosity distribution

ϵ_{2f} = eddy kinematic viscosity for fully-developed pipe flow

$C_{fd} = \dfrac{\tau_{wall}}{\frac{1}{2}\rho U_c^2}$ = wall friction coefficient

An arbitrary function is used to make the eddy viscosity distribution in Part 2 continuous with that at the end of Part 1.

$$E_2 = E_1\,(1 - \gamma^2) + \gamma^2 E_{2f} \tag{11}$$

where E_1 = dimensionless eddy viscosity at the end of Part 1 calculated from equation (6)

The paragraphs above have described the basic approaches used for the analysis of flow behavior in the variable-area compressible flow jet pump. The fundamental assumptions for the analysis have been identified. The sections which follow present the sets of equations which must be solved in each of the three regions of the flow; the transition zone, the region upstream of jet attachment to the wall (Part 1), and the region downstream of the point of attachment (Part 2).

3.3 Transition Zone Analysis

The transition zone begins at the primary nozzle exit plane and has a length of approximately 20 jet nozzle diameters. At the nozzle exit plane, the static pressure in the supersonic primary flow may be different from the static pressure in the surrounding secondary flow. We assume that before mixing of the two flows begins, the primary jet expands or contracts isentropically until its static pressure matches that of the secondary flow. At the station where this accommodation is complete, the velocity profile is assumed to resemble a "top-hat" as shown in figure 2. Then mixing of the primary and secondary flows begins.

The transition zone continues downstream to the section where the potential core in the jet ends. At this point, the f_o (y/δ) profile has been attained and the stagnation pressure at the center of the jet begins to fall because of mixing with the secondary flow.

The flow conditions at the end of the transition zone are determined by solving three simultaneous non-linear algebraic equations which are developed from the continuity and momentum equations written for the transition zone as a control volume, and from the condition that the stagnation pressure remains constant along the centerline of the primary jet.

The length of the transition zone is measured from the primary nozzle exit section to the point where the f_o (y/δ) profile is attained. This length is designated as x_{core} and must be specified as input data for the analysis. For incompressible flow, equation (12) may be used (reference 3).

$$x_{core} = 4.08\ \delta_o \quad (1 + \frac{U_{oo}}{U_{joo}}) \tag{12}$$

where δ_o = radius of primary jet at top-hat section

U_{oo} = secondary flow velocity at top-hat section

U_{joo} = primary jet relative velocity

For compressible flow with a supersonic primary jet, the value of x_{core} will depend on whether the jet is under- or over-expanded as it leaves the nozzle. A suitable replacement for equation (12) is not known to be available, so x_{core} was arbitrarily chosen to be equal to the mixing tube inlet diameter. This length is equivalent to about 18 primary nozzle diameters.

The transition from the top-hat profile to the f_o (y/δ) profile is assumed to occur in a control volume of essentially constant area. The effective mixing tube radius at x_{core} is calculated by taking the boundary layer thickness into account.

$$R_{eff} = R_{core} - \Theta H_o \tag{13}$$

where $$\Theta = \Theta_o + 0.001\, x_{core} \tag{14}$$

R_{eff} = effective radius of mixing tube at x_{core}

R_{core} = radius of mixing tube at x_{core}

Θ = boundary layer momentum thickness at x_{core}

Θ_o = inlet boundary layer momentum thickness

H_o = inlet boundary layer shape factor = 1.4 assumed

The flow area available for the secondary flow at the top-hat section is given by equation (15).

$$A_{eff} = \pi R^2_{eff} - A_{noz} \qquad A_{noz} \cong A_{primary\ flow} \tag{15}$$

where A_{eff} = secondary flow area at top-hat section

A_{noz} = area of primary nozzle exit section

The velocity of the secondary flow at the top-hat section is calculated from equation (16).

$$U_{oo} = \frac{W_o}{\rho_o\ A_{eff}} \tag{16}$$

where U_{oo} = secondary flow velocity at top-hat section

W_o = mass flow rate of secondary flow

ρ_o = density of secondary flow

The value of ρ_o in equation (16) is the density corresponding to the local static pressure and temperature. It is computed by an iterative process using the known values of inlet stagnation pressure and temperature and the appropriate perfect gas relationships. The same calculation yields the value of the local static pressure.

The primary flow conditions at the top-hat section are calculated as follows:

$$T_1 = T_{o1} \left(\frac{p_1}{P_{o1}}\right)^{\frac{k}{k-1}} \tag{17}$$

where T_1 = static temperature in primary flow at top-hat section

T_{o1} = specified primary flow stagnation temperature

P_{o1} = specified primary flow stagnation pressure

p_1 = static pressure from secondary flow calculations

$$U_1 = \sqrt{2 \frac{k}{k-1} R_g(T_{o1} - T_1)} \tag{18}$$

where U_1 = primary flow velocity at top-hat section

$$U_{joo} = U_1 - U_{oo} \tag{19}$$

where U_{joo} = primary jet relative velocity

The flow conditions at the end of the transition zone are computed by using the continuity and momentum relationships and the assumption that the stagnation pressure is unchanged at the center of the jet. The stagnation pressure of the secondary flow outside the mixing region is assumed to remain constant during transition. The stagnation temperature of the secondary flow outside the mixing region, and the stagnation temperature at the center of the primary jet, are assumed to remain constant.

The values of U_{oo}, U_{joo}, p_1, W_o, and W_1 are known to begin the analysis which determines the velocity profile at the end of the transition zone. The continuity, momentum, and constant centerline pressure equations at the end of the zone may be written as follows on the next page.

$$\text{Continuity:}\quad 2\pi \int_{o}^{R_{eff}} \rho U y\, dy = W_o + W_1 \tag{20}$$

$$\text{Momentum:}\quad 2\pi \int_{o}^{R_{eff}} \frac{\rho U^2}{g_o} y\, dy + (p - P_{oo}) A_m = (p_1 - P_{oo}) A_m + \frac{W_1 U_1}{g_o} + \frac{W_o U_{oo}}{g_o} \tag{21}$$

$$\text{Constant Stagnation Pressure Along Centerline:}\quad P_{o1} = \text{constant} \tag{22}$$

where $A_m = \pi R^2{}_{eff}$

The velocity profile at the end of the transition zone is given by equations (1) and (2). The temperature profile is given by equation (3).

To permit equations (20), (21), and (22) to be solved simultaneously using standard computer subroutines, these equations were rewritten in terms of the following dependent parameters:

$$U_r = \frac{U_{jo}}{U_{joo}} \tag{23}$$

$$\lambda = \frac{U_o}{U_{jo}} \tag{24}$$

$$\frac{\delta}{R_{eff}} = \frac{\delta}{\sqrt{\frac{A_m}{\pi}}} \tag{25}$$

The continuity, momentum, and constant centerline stagnation pressure equations in final form are as follows:

Continuity:

Let $$C_{mass} = \frac{W_o + W_1}{\pi R^2_{eff}\, \rho_{oo} U_{joo}} \tag{26}$$

It can be shown that

$$C_{mass} = \frac{p}{p_{oo}} \left[U_r \left(\frac{\delta}{R}\right)^2 (Z_1 - S_{20}) + U_r S_{20} \right] \tag{27}$$

where

$$\frac{p}{P_{oo}} = (1 - S_{oo} U_r^2 \lambda^2)^{\frac{k}{k-1}} \tag{28}$$

$$S_{oo} = \frac{k-1}{2} \; \frac{U^2_{joo}}{kR_g\, T_{oo}} \tag{29}$$

$$S_{20} = \frac{\lambda}{1 - S_{oo} U_r^2\, \lambda^2} \tag{30}$$

$$Z_1 = \int_0^1 \frac{(\lambda + f_o)\, 2\eta d\eta}{1 + \mathcal{T} f_o^{1/2} - S_{oo} U_r^2 (\lambda + f_o)^2} \tag{31}$$

$$\mathcal{T} = \frac{T_{oj}}{T_{oo}}$$

Momentum:

Let $$C_{mom} = \frac{g_o(p - P_{oo})\, A_m + W_1\, U_1 + W_o\, U_o}{\pi\, R^2_{eff}\, \rho_{oo} U^2_{joo}} \tag{32}$$

It can be shown that

$$C_{mom} = \left(\frac{p}{P_{oo}} - 1\right) \frac{R_g T_{oo}}{U^2_{joo}} + \left(\frac{p}{P_{oo}}\right) \left[U_r^2 \left(\frac{\delta}{R}\right)^2 (Z_2 - \lambda S_{20}) + U_r^2\; \lambda S_{20} \right] \tag{33}$$

where
$$Z_2 = \int_0^1 \frac{(\lambda + f_o)^2 \, 2\eta \, d\eta}{1 + \pi f_o^{1/2} - S_{oo} U_r^2 (\lambda + f_o)^2} \tag{34}$$

Constant Stagnation Pressure:

$$\frac{1 - S_{oo} U_r^2 \lambda^2}{1 - \dfrac{S_{oo} T_{oo}}{T_{o1}} U_r^2 (1 + \lambda)^2} = \left(\frac{P_{o1}}{P_{oo} - \Delta P_{SD}} \right)^{\frac{k-1}{k}} \tag{35}$$

where ΔP_{SD} = suction duct losses (see section 4.2.3)

The Z integrals are evaluated by using the following summation:

$$Z_k = \frac{1}{n_s} \sum_{i=1}^{n_s} \left(\frac{N_i}{D_i} \right)_k 2\eta_i \tag{36}$$

where n_s = number of summation strips, each of the same $\Delta\eta$

N_i, D_i are defined as follows:

Equation	K	N_i	D_i
Continuity	1	$\lambda + f_{oi}$	$1 + \pi f_{oi}^{1/2} - S_{oo} U_r^2 (\lambda + f_{oi})^2$
Momentum	2	$(\lambda + f_{oi})^2$	same

Equations (27), (33), and (35) are solved simultaneously to yield values of U_r, λ, and δ/R at the end of the transition zone. The value of the static pressure at the end of the zone is then determined from equation (28).

3.4 Flow Analysis Upstream of Jet Attachment

In Part 1, the zone between the end of the transition zone and the section where the jet reaches the wall, seven variables are determined by integral techniques. These dependent variables are U_j, $\lambda = U_o/U_j$, δ, the static pressure p, the relative stagnation temperature at the jet centerline T_{oj}, the boundary layer momentum thickness Θ, and the boundary layer shape factor H.

The values of these variables are obtained by solving seven simultaneous equations of the following general form:

$$\sum_{K=1}^{7} W(J,K) \times Y(K) = V(J) \tag{37}$$

where $W(J,K)$ = a coefficient matrix

$Y(K)$ = the derivatives of the dependent variables with respect to x/R_o

$V(J)$ = a set of terms not containing any of the dependent variables, evaluated at the x/R_o station of interest

The Y(K) values are listed below:

$$\left.\begin{aligned}
Y(1) &= \frac{\partial\left(\frac{U_j}{U_{jo}}\right)}{\partial\left(\frac{x}{R_o}\right)} & \qquad Y(5) &= \frac{\partial\left(\frac{T_{oj}}{T_{oo}}\right)}{\partial\left(\frac{x}{R_o}\right)} \\
Y(2) &= \frac{\partial \lambda}{\partial\left(\frac{x}{R_o}\right)} & \qquad Y(6) &= \frac{\partial\left(\frac{\theta}{R_o}\right)}{\partial\left(\frac{x}{R_o}\right)} \\
Y(3) &= \frac{\partial\left(\frac{\delta}{R_o}\right)}{\partial\left(\frac{x}{R_o}\right)} & \qquad Y(7) &= \frac{\partial H}{\partial\left(\frac{x}{R_o}\right)} \\
Y(4) &= \frac{\partial\left(\frac{p}{P_{oo}}\right)}{\partial\left(\frac{x}{R_o}\right)}
\end{aligned}\right\} \qquad (38)$$

The J simultaneous equations used to evaluate these derivatives are as follows:

$J = 1$: P_{oo} = stagnation pressure in the flow outside the jet = constant

2 : momentum equation for the complete flow

3 : continuity equation

4 : energy equation

5 : moment-of-momentum equation

6 : boundary layer momentum equation

7 : boundary layer moment-of-momentum equation

(39)

These equations and the W(J, K) coefficients are given in detail in Appendix A.

The velocity profile for the jet in Part 1 is given by equations (1) and (2), with the distribution function f_o taken from free jet data (reference 5). The jet temperature profile is given by equation (3). In the jet ($o \leq r \leq \delta$) the shear is obtained from equation (4) with the eddy viscosity given by equation (6).

Outside the jet ($\delta < y < R$), wall shear forces are assumed to be negligible in the momentum equation for the complete flow. The boundary layer momentum thickness Θ and the shape factor H are calculated from the following equations:

$$\frac{d\Theta}{dx} + (2 + H)\frac{\Theta}{U_o}\frac{dU_o}{dx} = \frac{C_f}{2} \tag{40}$$

$$\frac{dH}{dx} = \frac{-H(H+1)(H^2-1)}{2}\frac{1}{U_o}\frac{dU_o}{dx} + \frac{H^2-1}{\Theta}\left[\frac{HC_f}{2} - \frac{0.06\,(H-1)}{(H+3)\,R_\Theta^{\,0.1}}\right] \tag{41}$$

The friction coefficient in these equations is taken from the Ludwieg-Tillman skin friction equation (reference 6):

$$C_f = 0.246\,R_\Theta^{-0.268}\,10^{-.678H} \tag{42}$$

where R_Θ = Reynolds number based on momentum thickness

These equations are based on the assumption that the outer (potential) flow at velocity U_o is incompressible, and use has been made of the relation between the boundary layer and jet mixing parameters given in equation (43):

$$\frac{1}{U_o}\frac{dU_o}{dx} = \frac{1}{\lambda}\frac{d\lambda}{dx} + \frac{1}{U_j}\frac{dU_j}{dx} \tag{43}$$

The equations above allow the boundary layer development to be calculated simultaneously with the main flow mixing. Thus, the boundary layer displacement thickness is taken into account when the momentum, continuity, and energy equations are integrated across the mixing tube cross section.

The seven equations (39) are solved simultaneously to yield the values of the derivatives (38). Then the derivatives are integrated using Runge-Kutta-Merson techniques. This integration yields the desired values of U_j, U_o, δ, p, T_{oj}, Θ, and H at selected values of x/R_o along the mixing tube.

If a region of recirculation is present, the value of U_o becomes negative. The development of an analysis for the flow behavior in a recirculation zone was not included in this investigation.

3.5 Flow Analysis Downstream of Jet Attachment (Part 2)

After the jet reaches the wall, the jet velocity profile is assumed to follow the relationship given in equation (7). At the beginning of Part 2, the value of the shape parameter $\gamma(x)$ is set equal to zero and the velocity profile is given by $f_2(\eta)$. The functional relationship f_2 is defined so as to be identical to the final velocity distribution in Part 1.

$$f_2(\eta) = \frac{f_o(\eta) + \lambda f_{b\ell}(\eta)}{1 + \lambda} \tag{44}$$

In this equation, $f_{b\ell}(\eta)$ is the boundary layer profile at the end of Part 1, estimated by using a power law for the boundary layer:

$$u = U_o \left(\frac{y}{\delta_{b\ell}}\right)^n \tag{45}$$

The exponent n must satisfy the values of momentum and displacement thickness calculated for the boundary layer at the end of Part 1. The resulting equation for $f_{b\ell}$ follows.

$$f_{b\ell} = \frac{R\,(H_1 - 1)}{\Theta_1 H_1 (H_1 + 1)} \left[1 - \eta\right]^{\frac{H_1 - 1}{2}} \qquad (46)$$

where Θ_1 = boundary layer momentum thickness at the end of Part 1

H_1 = boundary layer shape factor at the end of Part 1

The velocity profile in Part 2 includes the auxilary profile $g_2\,(\eta)$. This profile is defined by equation (8) so that, as γ approaches 1.0, the Part 2 velocity profile asympototically approaches the profile for fully-developed turbulent flow in a pipe.

In Part 2, the values of five variables are determined by integral techniques. These dependent variables are the velocity profile values U_c and γ, the static pressure p, the temperature ratio Π, and the temperature of the flow at the outer radius of the mixing tube, T_{oo}. The values of these variables are obtained by solving six simultaneous equations of the general form given by equation (37). The dependent Y(K) variables are listed below:

$$Y(1) = \frac{\partial\,(\frac{Uc}{U_{jo}})}{\partial\,(\frac{x}{Ro})} \qquad Y(4) = \frac{\partial\,(\frac{p}{Poo_i})}{\partial\,(\frac{x}{Ro})}$$

$$Y(2) = \frac{\partial\,(\frac{Uo}{Uc})}{\partial\,(\frac{x}{Ro})} \text{ (not used)} \qquad Y(5) = \frac{\partial\,(\frac{Toj}{Too})}{\partial\,(\frac{x}{Ro})} \qquad (47)$$

$$Y(3) = \frac{\partial\,\gamma}{\partial\,(\frac{x}{Ro})} \qquad Y(6) = \frac{\partial\,(\frac{Too}{T_{ooi}})}{\partial\,(\frac{x}{Ro})}$$

where P_{ooi} and T_{ooi} are the constant values of P_{oo} and T_{oo} in Part 1. The variable Y(2) above remains zero throughout the Part 2 analysis; this variable is a redundant parameter which remains from an earlier version of the computer program.

The equations used to evaluate these derivatives are as follows:

$$
\left.
\begin{array}{lcl}
J = 1 & = & \text{continuity equation} \\
J = 2 & = & \text{energy equation} \\
J = 3 & = & \text{momentum equation for the complete flow} \\
J = 4 & = & \text{moment-of-momentum integral equation} \\
J = 5 & = & \text{centerline velocity-temperature relationship} \\
J = 6 & = & \text{wall velocity} = 0
\end{array}
\right. \qquad (48)
$$

These equations and the W(J, K) coefficients are given in detail in Appendix A.

The form of the stagnation temperature profile must be known in order to solve the first four equations (48). For simplicity, the temperature profile was assumed to be the same as in a free jet.

$$\frac{T_o - T_{oo}}{T_{oj}} = \sqrt{f_o(\eta)} \quad \text{in Part 2} \qquad (49)$$

This approximation is justified by the test results in Section 4.3 of this report.

Wall shear stresses are included in the momentum equation for the complete flow. The wall friction coefficient used in the analysis is based upon pipe flow correlations which yield equation (50).

$$C_{fd_f} = \frac{\tau_{wall}}{\frac{1}{2}\rho U_c^2} = 0.048 \left(\frac{\overline{U}}{U_c}\right)^2 Re_m^{-.20} \qquad (50)$$

where $Re_m = \frac{\overline{U}\,D}{\nu}$ Reynold's number based on mean velocity

$\overline{U}$ = mass-average mean velocity

U_c = centerline velocity

ν = kinematic viscosity

This wall friction coefficient is only an approximation to the actual value because the velocity profile near the wall in Part 2 of the jet pump mixing tube is generally not identical to the fully-developed pipe flow velocity profile. Comparison of analytical predictions to measured wall static pressure values indicated that equation (50) gave values of C_{fd} which were too high. Therefore, the analysis now employs an arbitrarily reduced friction factor.

$$C_{fd} = 1/2\, C_{fd_f} \tag{51}$$

The moment-of-momentum integral equation includes a term which represents axial shear forces between adjacent stream tubes. These shear forces are determined from the eddy viscosity relationship given in equation (11).

The fifth equation in the set, the centerline velocity-temperature relationship, is based upon the test results obtained during this investigation. As shown in Section 4.3 and Figure 16, the following equation may be used to supplement the energy equation in Part 2.

$$\frac{1}{T_j}\frac{dT_j}{dx} = \frac{1}{U_c}\frac{dU_c}{dx} \tag{52}$$

The sixth equation (48) sets Uo, the velocity of the flow along the mixing tube surface, equal to zero. This equation was added to eliminate Y(2), the redundant variable in equation (47), during the solution of the six simultaneous equations (48). The solution of these equations yields the values of the derivatives (47). The derivatives then are integrated using Runge-Kutta-Merson techniques. This integration yields the desired values of Uc, γ, p, Toj, and Too at selected values of x/R_o along the mixing tube in the region after the jet reaches the wall.

Section 4

TEST PROGRAM

The objective of the test program was to provide data which could be used to evaluate the analytical model. The test conditions are summarized below:

<u>Primary Flow</u>

stagnation pressure:	348 psia
stagnation temperature:	807°F
nozzle throat area:	1.587×10^{-4} ft^2
nozzle geometry:	see figure 3
mass flow rate:	6.76 lbm/min

<u>Secondary Flow</u>

inlet stagnation pressure:	laboratory ambient (30.06" Hg)
inlet stagnation temperature:	laboratory ambient (92°F)
mixing tube geometry:	see figure 4
pressure rise:	regulated by discharge throttling device

This section of the report describes the jet pump test arrangement, instrumentation and data reduction procedures, and the results which were obtained.

4.1 Test Arrangement

The jet pump test arrangement is shown in figure 5. The primary flow was supplied by a 2-stage reciprocating compressor. Electrical heaters were used to increase the temperature of the flow up to about 800°F. The primary flow was delivered to a single nozzle directed along the axis of the mixing tube.

The momentum of the primary flow entrains a secondary air flow from the room into the bellmouth inlet and then into the mixing tube. Here, the two streams mix together and the stagnation pressure of the secondary stream is increased. The flow from the mixing tube passes through a conical diffuser and exhausts to the atmosphere through an adjustable throttling cone.

The individual components of the experimental jet pump are described below:

1. Calibrated bellmouth inlet section

   This component consists of a wooden bellmouth, metal connecting tube, and fiberglass primary flow inlet section. The bellmouth differential pressure was calibrated in terms of flow rate by using an orifice and blower available in the laboratory. The calibrated bellmouth permitted direct measurement of secondary mass flow rate for all jet pump tests.

2. Mixing tube

   The mixing tube geometry was chosen rather arbitrarily before the computer program became available as a design guide. The basic Helmbold mixing tube geometry (reference 7) was selected because this geometry has been tested thoroughly in the incompressible flow regime. The incompressible results provide a guide to the flow behavior which may be expected in the compressible flow regime.

   The Helmbold mixing tube was scaled down so that all dimensions were 0.892 times their original values. This scale was selected so that the mixing tube would match an existing discharge diffuser and the mixing tube throat velocity would remain subsonic for all flow rates expected in the test program. The smooth curving profile of the Helmbold tube was approximated with cones and cylinders as shown in figure 4 for ease of fabrication.

3. Discharge diffuser

A conical diffuser with an area ratio of about 2.8 and a total included angle of 7.1° was added at the end of the mixing tube to maximize static pressure recovery and allow high entrainment ratios to be achieved. Changes in the axial positioning of the throttle cone in the diffuser exit produced a variable system resistance so that the jet pump could be tested over a range of secondary flow rates.

4. Nozzle geometry

Figure 3 shows the geometry of the converging-diverging primary flow nozzle. The area ratio from throat to exit section is 3.24, the area ratio corresponding to one-dimensional isentropic expansion from 350 psia to 14.7 psia. When the jet pump was assembled, the exit plane of the nozzle was at x = o where x is defined on the mixing tube drawing, figure 4. The mixing tube diameter at the nozzle exit plane is 5.341 in. The nozzle flow coefficient, according to the definition below, was measured to be 0.929.

$$C_w = \frac{W_1}{W_{1_{ideal}}} \tag{53}$$

where W_1 = measured nozzle flow rate at design pressure and temperature

$W_{1_{ideal}}$ = isentropic flow rate through nozzle throat at design pressure and temperature; based upon one-dimensional flow assumption

4.2 Instrumentation and Data Reduction Procedures

4.2.1 Instrumentation

The instrumentation used to determine the performance of the experimental jet pump is shown on figure 6 and described in table 1.

The jet pump inlet bellmouth was calibrated for use as a flowmeter. The calibration was accomplished by connecting the bellmouth and the suction duct to the inlet of a blower by means of an orifice run and throttling arrangement. The bellmouth flow equation follows:

$$W_o = 229.5 \sqrt{\rho_b \Delta h_b} \tag{54}$$

where Δh_b = p_b differential pressure, in. H_2O gage

ρ_b = inlet density, lbm/ft^3

The mixing tube was provided with 21 static pressure taps along its length. Four additional static pressure taps were located in the discharge diffuser. Provision was made for traverse probe measurements at five of the static pressure tap sections. The location of all of these taps is given in table 2. The exact dimensions were measured at several stations in the mixing tube after its construction; these dimensions are also given in table 2.

The Kiel-temperature probe which was traversed to measure the velocity and temperature profiles had a stem diameter of 1/8". The probe was small enough so that probe blockage effects were negligible during the traversing.

4.2.2 Data Reduction Procedures

The measured data were used to calculate the following jet pump parameters:

$m = \frac{W_o}{W_1}$ - jet pump entrainment ratio

U vs $\left(\frac{y}{R}\right)$ - velocity profiles

T_o vs $\left(\frac{y}{R}\right)$ - temperature profiles

p vs $\left(\frac{x}{R_o}\right)$ - mixing tube static pressure variations

The stagnation pressure and temperature profiles were measured at all traverse locations in a plane perpendicular to the axis of the primary flow feed pipe (see figure 6). At the station in the mixing tube throat ($x/R_o = 9.25$), a traverse also was made in the plane of the feed pipe to confirm that the flow was axisymmetric as desired.

The wall static pressure and the traverse probe stagnation pressure and temperature measurements were used with the appropriate compressible flow equations to allow calculation of the velocity profiles at traverse stations 2 through 6. As a result of a thermocouple failure during the test runs, no temperature data were obtained at traverse station 1. Because the temperature profile is required in order to calculate the velocity profile, it was necessary to prepare an approximate temperature profile for this station. The procedure used is described under Test Results in Section 4.3 of this report.

4.2.3 Suction Duct Losses

Results from previous tests of the bellmouth and suction duct assembly (reference 1) and static pressure data from the present test program indicate that stagnation pressure losses in the suction duct upstream of the mixing tube are of the order of 2 in. H_2O for the tested secondary flow rates. These losses may be accounted for in the jet pump analysis by using equation (55) to calculate the secondary flow stagnation pressure at the primary nozzle exit section in the mixing tube:

$$P_{o2} = P_{oo} - K_L \frac{\rho_{oo} U_{SD}^2}{2 g_o}$$

or

$$P_{o2} = P_{oo} - K_L \frac{W_o^2}{2 g_o \rho_{oo} A_{SD}^2} \qquad (55)$$

where P_{o2} = stagnation pressure at primary nozzle exit plane

P_{oo} = stagnation pressure at suction duct inlet (laboratory ambient)

K_L = suction duct loss coefficient

ρ_{oo} = density corresponding to suction duct inlet stagnation state

U_{SD} = suction duct velocity (assumed uniform)

A_{SD} = suction duct cross-sectional area

For the suction duct in the experimental jet pump, the value of K_L is 0.33.

4.3 Test Results

The jet pump was tested at four values of entrainment ratio, 17.0, 19.4, 21.0, and 23.6. The corresponding values of primary and secondary mass flow rates are given in table 3. The inlet pressures and temperatures were constant throughout the test and were as listed at the beginning of this Section 4.

Wall static pressure values measured along the mixing tube are listed for all four entrainment ratios on table 3. These values are plotted in figure 7.

The operation of the jet pump was reasonably steady (i.e., wall pressure fluctuations were small) when the entrainment ratio was 21.0. Therefore, this condition was selected for the velocity traverse measurements which require long periods of steady operation. The velocity profiles for traverse stations 2 through 6 are shown in figure 8. The associated temperature profiles are shown in figure 9.

Traverses 4 and 5 were taken at the same station in the constant-area throat section of the mixing tube. The axes of the traverse were 90° apart so that any departures from axial symmetry in the flow could be detected. The slight departures which were observed are due to heating of the secondary flow as it passes over the primary nozzle feed pipe upstream of the mixing tube inlet. These departures have a negligible effect on jet pump performance and will not interfere with our comparison of measured and predicted flow behavior through the jet pump.

Because of a thermocouple failure during testing, no temperature data were obtained at traverse station 1. The stagnation pressure measurements at this section cannot be used to determine the velocity profile unless the temperature profile is available. An approximate velocity profile for traverse station 1 was developed by using the analytically-predicted temperature profile together with the measured stagnation pressure values. The resulting velocity profile is given at the end of the next section of this report.

The mass flow rate through the jet pump as determined by the calibrated inlet bellmouth was compared to the mass flow rate obtained by integration of the velocity profiles for stations 4 and 5. Agreement was within 1% (149.8 lbm/min. from integration vs. 148.8 lbm/min. from the bellmouth). The measured velocity profile at station 6 was used for a similar comparison. Integration of this profile gave a mass flow rate of 158.8 lbm/min., about 7% greater than the bellmouth measurement.

The values of $\frac{U_j}{U_{jo}}$ and $\frac{T_j}{T_{jo}}$ calculated from the measured velocity and temperature profiles are plotted in figure 10 to show how the centerline velocities and temperatures vary with distance along the mixing tube. The velocity and temperature ratios are nearly identical over most of the mixing tube length.

Section 5

COMPARISON OF ANALYTICAL AND EXPERIMENTAL RESULTS

5.1 Mixing Tube Wall Static Pressure Variations

The mixing tube wall static pressure measurements are the most valuable results for evaluating the accuracy of the analytical model for use in jet pump design. The prediction of this pressure variation is the primary purpose of this investigation because knowledge of this variation permits calculation of the pressure force on the mixing tube wall. This force must be known in order to solve the momentum equation during jet pump system optimization studies.

The analytical predictions of mixing tube static pressure variations are compared to test results for four entrainment ratios in figures 11, 12, and 13. The analyses were carried out with two different values assumed for x_{core}, the length of the transition region at the primary nozzle exit, and for two values of the secondary flow rates for each test; the values determined from the test results using the bellmouth calibration equation (54), and values 2% lower. A key to the three figures follows:

Figure No.	x_{core}/R_o	Secondary Flow Rates
11	2.5	from (54), reduced by 2 %
12	2.5	from (54)
13	2.0	from (54), reduced by 2%

The mass flow rates given by equation (54) and used to prepare figure 12 cause the analytical predictions of static pressure to fall below the measured values in the throat section of the mixing tube ($\frac{x}{R_o}$ from 7.34 to 10.7). The assumption that the secondary flow rates are 2% lower yields better agreement as shown in figures 11 and 13. The choice of x_{core} to be 2.5 R_o rather than 2.0 R_o causes only a small difference in the predicted static pressure levels. The differences are largest in the diffuser sections downstream of the mixing tube throat.

From these results, it was concluded that further comparisons of analytical and experimental results should be based on the assumption that the true secondary flow rates are 2% lower than the flow rates given by equation (54). An uncertainty of ± 2% in flow rate is not unreasonable for the bellmouth calibration. The 2% flow correction brings the analytical predictions very close to the experimental results except for the static pressures downstream of the mixing tube throat.

5.2 Velocity and Temperature Profiles

The variation of predicted centerline velocity with distance along the mixing tube is shown in figure 14 for three alternative values of x_{core}; 1.0, 2.0, and 2.5. The measured values of centerline velocity at traverse stations 2-6 are also plotted in the figure. A value of x_{core}/R_o between 2.0 and 2.5 appears to make the analytical prediction fit the test data most accurately.

The variation of predicted centerline stagnation temperature with distance along the mixing tube also is shown in figure 14. A value of x_{core}/R_o between 2.0 and 2.5 will make the temperature predictions fit the test data upstream of the throat section of the mixing tube. At traverse stations 4, 5, and 6, the measured temperature levels fall about 30° F below the predicted centerline stagnation temperatures.

The analytical results (U_c, U_o, $\frac{\delta}{R_o}$, f_2, g_2, and ν), together with the known free jet profile $f_o(\eta)$, allow direct comparison of the velocity and temperature profiles predicted by the analysis to the velocity and temperature profiles measured during the test program. The velocity profiles are compared in figure 15, and the temperature profiles are compared in figure 16. The predicted velocity profiles agree reasonably well with the measured profiles. The measured and predicted temperature profiles agree well for traverse stations 2 and 3, but the predicted temperatures near the centerline for stations 4, 5, and 6 are somewhat higher than they should be.

Stagnation pressure measurements only were obtained at traverse station 1. These measurements, coupled with the analytical temperature profiles predicted for this station, can be used to develop an approximate velocity profile.

The procedures used were as follows:

1. The stagnation pressure data from the traverse probe, together with the local static pressure tap reading, were used to determine the Mach number and T/T_o ratio at each y/R position in the mixing tube cross-section. This data is given in table 4.

2. From the analytical solution for $x_{core} = 2.5\ R_o$, the predicted value of $\frac{\delta}{R_o}$ at the traverse station ($\frac{x}{R_o} = 2.5$) was found to be 0.2118. The local value of $\frac{R}{R_o}$ is 0.889. These results allow the y/R positions of the traverse probe near the duct centerline to be interpreted in terms of the y/δ values for the free jet velocity profile of equations (1) and (2). Using the analytically predicted values $U_c = 3019$ ft/sec, $U_o = 268$ ft/sec, $T_{oc} = 1267°$ R, and $T_{oo} = 552°$ R, the free jet velocity and temperature profiles can be used, through equations (1), (2), and (3), to determine the predicted values of velocity and stagnation temperature for each y/R position within the jet mixing region. The corresponding static temperatures can be determined from the T/T_o ratios in table 4. The speed of sound is calculated from the static temperature. The predicted flow velocities and speed of sound values are used to calculate Mach numbers for each y/R position within the jet mixing region. These predicted Mach numbers are compared to the "measured " Mach numbers in table 4. If the predicted and measured numbers agree, the associated velocity and temperature profiles afford a good approximation to the true profiles.

3. The same calculation procedure was followed using the analytical solution for $x_{core} = 2.0\ R_o$. The predicted value of $\frac{\delta}{R_o}$ at the traverse station was 0.287. The other predicted values employed in the analysis were as follows:

$U_c = 2227$ ft/sec. $\qquad T_{oc} = 1041°$ R

$U_o = 261$ ft/sec. $\qquad T_{oo} = 552°$ R

The predicted and "measured" Mach number profiles for traverse station 1 are compared in figure 17. The predicted profile based upon the assumption that $x_{core} = 2.0\ R_o$ is closer to the measured profile than the $x_{core} = 2.5\ R_o$ profile although the predicted centerline velocity is too high. The predicted velocity and temperature profiles for both x_{core} assumptions are given in table 4. The predicted values were obtained using a secondary flow rate which was 2% less than the value given by equation (54).

Section 6
CONCLUSIONS

An analytical method has been developed to predict the performance characteristics of axisymmetric single-nozzle compressible flow jet pumps with variable area mixing tubes. The primary flow may be either subsonic or supersonic. The analysis is divided into two parts. In part 1, the region between the primary nozzle exit and the point where the jet reaches the wall, the analysis is based upon the hypothesis that the mixing phenomena in the jet pump is fundamentally similar to the mixing of a free turbulent jet with the surrounding fluid. The eddy viscosity is adjusted to account for the influence of the duct walls as the jet approaches the walls. In part 2, downstream of the point where the jet reaches the wall, the velocity profile is allowed to vary from the free jet profile at the end of part 1 to a profile which asymptotically approaches the fully-developed turbulent flow profile in a pipe. Integral techniques are employed in both part 1 and part 2 to solve the continuity, momentum, moment-of-momentum, and energy equations to determine the variations of flow properties along the mixing tube.

An experimental program was conducted to measure mixing tube wall static pressure variations, velocity profiles, and temperature profiles in a variable area mixing tube with a supersonic ($M = 2.72$) primary jet. Static pressure variations were measured at four different secondary flow rates. These test results were used to evaluate the analytical model.

Analytical predictions of wall static pressure distributions along the mixing tube generally agreed well with the test results for all four entrainment ratios. The predicted wall static pressure values differed slightly from the measured pressures downstream of the constant-area throat section. The velocity profiles along the mixing tube were predicted accurately by the analysis. The analytical temperature profiles were not as accurate; the predicted centerline temperatures downstream of the throat were too high. These discrepancies are considered to be minor in view of the comparatively extreme mixing tube geometry used for the test case. Thus, the analysis is ready for use to calculate the pressure force on the wall of a variable area mixing tube. This permits the momentum equation to be solved accurately in jet pump-duct system optimization and design studies.

The analysis in part 2 of the jet pump makes the assumption that the temperature profiles are similar to free jet temperature profiles. A very simple and approximate form of the energy equation is employed. A more accurate energy equation, perhaps augmented by assumption of a different form for the temperature profile, might lead to greater accuracy in the prediction of wall static pressures and temperature profiles in this region.

APPENDIX A

Equations for the Flow

A1 - Part 1 - Upstream of Jet Attachment

The general form of the flow equations, as described in Section 3.4, is as follows:

$$\sum_{k=1}^{7} W(J,K) * Y(K) = V(J)$$

The 7 variables are tabulated below, using the convention that the superscript (') represents $\dfrac{\partial}{\partial\left(\dfrac{x}{R_o}\right)}$.

K =	1	2	3	4	5	6	7
Y(K) =	$\left(\dfrac{U_j}{U_{jo}}\right)'$	λ'	$\left(\dfrac{\delta}{R_o}\right)'$	$\left(\dfrac{p}{P_{oo}}\right)'$	$\left(\dfrac{T_{oj}}{T_{oo}}\right)'$	$\left(\dfrac{\theta}{R_o}\right)'$	H'

The W(J,K) coefficients and V(J) terms are determined in this section.

A1-1 Equation for J = 1; Constant stagnation pressure in the flow outside the jet.

$$dp = -\rho \frac{UdU}{g_o}$$

$$R_g T \frac{dp}{p} + \lambda U_j d(\lambda U_j) = 0$$

Normalizing:

$$\frac{R_g T}{U_{jo}^{\;2}} \frac{dp}{p} + \lambda \frac{U_j}{U_{jo}} \left(\lambda \frac{dU_j}{U_{jo}} + \frac{U_j}{U_{jo}}\, d\lambda\right) = 0$$

Let $\quad BP = \dfrac{R_g T_{oo}}{U_{jo}^{\;2}} = \dfrac{k-1}{2kS_o}$

$$S_o = \frac{U_{jo}^{\;2}}{2\,\frac{k}{k-1}\,R_g T_{oo}}$$

Then $\quad T_o = T_{oo}\left[1 - S_o\,\dfrac{U_j^{\;2}}{U_{jo}^{\;2}}\,\lambda^2\right]$ = Static temperature in the flow outside the jet

The final values follow:

$W(1,1) = \lambda^2\,\dfrac{U_j}{U_{jo}}$ $\qquad\qquad$ $W(1,5) = 0$

$W(1,2) = \lambda\left(\dfrac{U_j}{U_{jo}}\right)^2$ $\qquad\qquad$ $W(1,6) = 0$

$W(1,3) = 0$ $\qquad\qquad$ $W(1,7) = 0$

$W(1,4) = \left(1 - S_o\,\dfrac{U_j^{\;2}}{U_{jo}^{\;2}}\,\lambda^2\right)\dfrac{BP}{p}$ $\qquad\qquad$ $V(1) = 0$

A1-2 Equation for J = 2; Momentum equation for the flow

$$-\pi R^2\,\frac{dp}{dx} = \frac{d}{dx}\int_o^R \rho U^2\,2\pi\,y\,dy$$

$$-R^2\,\frac{dp}{dx} = \frac{d}{dx}\left\{\frac{p}{R_g T_{oo}}\,U_j^{\;2}\left[\delta^2\int_o^1 \frac{(\lambda+f_o)^2\,2\eta\,d\eta}{1 + \pi f_o^{\;1/2} - S_o\,\frac{U_j^2}{U_{jo}^{\;2}}(\lambda+f_o)^2} + \frac{(R^2-\delta^2)\lambda^2}{1 - S_o\,\frac{U_j^{\;2}}{U_{jo}^{\;2}}\,\lambda^2}\right]\right\}$$

where $\lambda + f_o = \dfrac{U}{U_j}$

$$1 + \Gamma f_o^{1/2} - S_o \frac{U_j^2}{U_{jo}^2} (\lambda + f_o)^2 = \frac{T_\eta}{T_{oo}} = \frac{\text{Static Temperature @}\eta}{\text{Stagnation Temperature @}\eta = 1.0}$$

$$1 - S_o \frac{U_j^2}{U_{jo}^2} \lambda^2 = \frac{T_o}{T_{oo}} = \frac{\text{Static Temperature}}{\text{Stagnation Temperature}} \quad \text{at } \eta = 1.0$$

Let

$$Z_{12} = \int_0^1 \frac{(\lambda+f_o)^2 \, 2\eta \, d\eta}{1 + \Gamma f_o^{1/2} - S_o \dfrac{U_j^2}{U_{jo}^2} (\lambda+f_o)^2}$$

In the computer analysis, this integration is approximated by a summation across the jet:

Let $$Z_{1J} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{N_{iJ}}{D_{iJ}} \eta_i$$

In this equation N_i and D_i are average values of the numerator and denominator across the i^{th} equal-radius annular segment of the jet. The following additional definitions will be used:

$$Z_{2J} = \frac{2}{n_s} \sum_{i=1}^{n_s} \left(\frac{-N_{iJ}}{D_{iJ}^2}\right) \frac{\partial D_{iJ}}{\partial \left(\dfrac{U_j}{U_{jo}}\right)} \eta_i$$

$$Z_{3J} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{1}{D_{iJ}} \frac{\partial N_{iJ}}{\partial \lambda} \eta_i$$

$$Z_{4J} = \frac{2}{n_s} \sum \left(\frac{-N_{iJ}}{D_{iJ}^{\;2}} \right) \frac{\partial D_{iJ}}{\partial \lambda} \eta_i$$

$$Z_{5J} = \frac{2}{n_s} \sum \frac{1}{D_{iJ}} \frac{\partial N_{iJ}}{\partial \Pi} \eta_i$$

$$Z_{6J} = \frac{2}{n_s} \sum \frac{-N_{iJ}}{D_{iJ}^{\;2}} \frac{\partial D_{iJ}}{\partial \Pi} \eta_i$$

Then the relations below may be used:

$$\frac{\partial Z_{1J}}{\partial \left(\frac{U_j}{U_{jo}} \right)} = Z_{2J}$$

$$\frac{\partial Z_{1J}}{\partial \lambda} = Z_{3J} + Z_{4J}$$

$$\frac{\partial Z_{1J}}{\partial \Pi} = Z_{5J} + Z_{6J}$$

Additional parameters which simplify the equations are defined as follows:

$$S_2 = \frac{\lambda}{1 - S_o \frac{U_j^{\;2}}{U_{jo}^{\;2}} \lambda^2}$$

$$\frac{R}{R_o} = \frac{R_{tube}}{R_o} - \frac{\theta}{R_o} H$$

$$\frac{\delta}{R} = \frac{\delta}{R_o}\ \frac{R_o}{R}$$

Employing these definitions in the momentum equation, the following expression is obtained after reorganizing, normalizing, and differentiating:

$$- \frac{dp}{dx}\ \frac{R_g T_{oo}}{pU_j^{\,2}} = \left[\frac{p'}{p} + 2\frac{U_j'}{U_j}\right]\left[\frac{\delta^2}{R^2}\ Z_{12} + (1-\frac{\delta^2}{R^2})\lambda\ S_2\right] + 2\frac{\delta}{R}\ \frac{\delta'}{R}\ Z_{12}$$

$$+\frac{\delta^2}{R^2}\left[Z_{22}\ \frac{U_j'}{U_{jo}} + (Z_{32} + Z_{42})\ \lambda' + (Z_{52} + Z_{62})\ \Upsilon'\right]$$

$$+\left[\frac{2}{R}\ (R'_{tube} - \Theta' H - H'\Theta) - 2\frac{\delta}{R}\ \frac{\delta'}{R}\right]\lambda\ S_2\ .$$

$$+\left(1-\frac{\delta^2}{R^2}\right)\left(S_2\lambda' + \lambda\frac{\partial S_2}{\partial \lambda}\ \lambda' + \lambda\ \frac{\partial S_2}{\partial U_j}\ U_j'\right)$$

The final values follow:

$$W(2,1) = \left(\frac{\delta}{R}\right)^2\left[\frac{2Z_{12}}{\left(\frac{U_j}{U_{jo}}\right)} + Z_{22}\right] + \left[1-\frac{\delta^2}{R^2}\right]\left[\frac{2\lambda S_2}{\left(\frac{U_j}{U_{jo}}\right)} + \lambda\ \frac{\partial S_2}{\partial\left(\frac{U_j}{U_{jo}}\right)}\right]$$

$$W(2,2) = \left(\frac{\delta}{R}\right)^2\ (Z_{32} + Z_{42}) + \left(1-\frac{\delta^2}{R^2}\right)\left(S_2 + \lambda\ \frac{\partial S_2}{\partial \lambda}\right)$$

$$W(2,3) = 2\ \frac{\delta}{R}\ (Z_{12} - \lambda S_2)$$

$$W(2,4) = \frac{P_{oo}}{p}\left[\frac{\delta^2}{R^2}\; Z_{12} + \left(1 - \frac{\delta^2}{R^2}\right)\lambda\, S_2\right] + \frac{BP * P_{oo}}{p\left(\frac{U_j}{U_{jo}}\right)^2}$$

$$W(2,5) = \frac{\delta^2}{R^2}\;(Z_{52} + Z_{62})$$

$$W(2,6) = -2\,\frac{H}{R}\; S_2\lambda$$

$$W(2,7) = -2\,\frac{\Theta}{R}\; S_2\,\lambda$$

$$V(2) = -\,2\;\frac{R'_{tube}}{R}\; S_2\,\lambda$$

Table A1 lists the values of N_i, D_i, and their derivatives which are required to evaluate the Z parameters in the previous equations.

A1.3 Equation for J = 3; Continuity equation

$$W_o + W_1 = 2\pi\int_o^R \rho\, Uy dy \quad \text{where } R = (\text{Local Duct Radius } -\Theta H)$$

$$W_o + W_1 = \frac{p\,\pi}{R_g T_{oo}}\; U_j\left[\delta^2\int_o^1 \frac{(\lambda + f_o)d\eta^{\,2}}{1 + \Pi f_o^{\,1/2} - S_o\left(\frac{U_j}{U_{jo}}\right)^2 (\lambda + f_o)^2} + \frac{(R^2-\delta^2)\lambda}{1 - S_o\left(\frac{U_j}{U_{jo}}\right)^2 \lambda^2}\right.$$

or

$$W_o + W_1 = \frac{p\,\pi}{R_g T_{oo}}\; U_j\left[\delta^2 Z_{13} + (R^2 - \delta^2) S_2\right]$$

where

$$Z_{13} = \int_o^1 \frac{(\lambda + f_o)\, 2\eta\, d\eta}{1 + \Pi f_o^{\,1/2} - S_o\left(\frac{U_j}{U_{jo}}\right)^2 (\lambda + f_o)^2}$$

The continuity equation is normalized as follows:

$$\frac{W_o + W_1}{\pi R_o^2 U_{jo}} \; \frac{R_g T_{oo}}{P_{oo}} = \frac{U_j}{U_{jo}} \left[\left(\frac{\delta}{R_o}\right)^2 Z_{13} + \left(\frac{R^2}{R_o^2} - \frac{\delta^2}{R_o^2}\right) S_2 \right] \frac{p}{P_{oo}}$$

Taking the derivative with respect to $\frac{x}{R_o}$;

$$o = \left(\frac{U_j}{U_{jo}}\right)' \frac{p}{P_{oo}} \left[\left(\frac{\delta}{R_o}\right)^2 Z_{13} + \left(\frac{R^2}{R_o^2} - \frac{\delta^2}{R_o^2}\right) S_2 \right] + \left(\frac{p}{P_{oo}}\right)' \frac{U_j}{U_{jo}} \left[\left(\frac{\delta}{R_o}\right)^2 Z_{13} + \left(\frac{R^2}{R_o^2} - \frac{\delta^2}{R_o^2}\right) S_2 \right]$$

$$+ \left(\frac{\delta}{R_o}\right)' \left[2 \frac{\delta}{R_o} (Z_{13} - S_2) \right] \frac{U_j}{U_{jo}} \frac{p}{P_{oo}}$$

$$+ \frac{U_j}{U_{jo}} \frac{p}{P_{oo}} \frac{\delta^2}{R_o^2} \left[Z_{23} \left(\frac{U_j}{U_{jo}}\right)' + (Z_{33} + Z_{43}) \lambda' + (Z_{53} + Z_{63}) \Pi' \right]$$

$$+ \frac{U_j}{U_{jo}} \frac{p}{P_{oo}} \left(\frac{R^2}{R_o^2} - \frac{\delta^2}{R_o^2}\right) \frac{\lambda'(1 - S_o (U_j^2/U_{jo}^2) \lambda^2) + \lambda S_o \left[2\lambda\lambda' (U_j^2/U_{jo}^2) + 2\lambda^2 (U_j/U_{jo}) (U_j/U_{jo})' \right]}{\left[1 - S_o \left(\frac{U_j}{U_{jo}}\right)^2 \lambda^2 \right]^2}$$

$$+ 2\frac{R}{R_o} \frac{U_j}{U_{jo}} \frac{p}{P_{oo}} S_2 \left(\frac{R'_{tube}}{R_o} - \frac{\theta}{R_o} H' - \frac{H}{R_o} \theta' \right)$$

Collecting terms and dividing each by $\left(\frac{R}{R_o}\right)^2 \quad \frac{U_j}{U_{jo}} \quad \frac{p}{P_{oo}}$;

$$W(3,1) = \frac{U_{jo}}{U_j} \frac{{R_o}^2}{R^2} \left(\frac{\delta}{R_o}\right)^2 Z_{13} + \left(1 - \frac{R_o^2}{R^2} \frac{\delta^2}{{R_o}^2}\right) S_2 \frac{U_{jo}}{U_j} + Z_{23} \frac{{R_o}^2}{R^2} \frac{\delta^2}{{R_o}^2}$$

$$+ \left(\frac{U_j}{U_{jo}}\right) 2\lambda S_2^2 S_o \left(1 - \frac{{R_o}^2}{R^2} \frac{\delta^2}{{R_o}^2}\right)$$

$$W(3,2) = \frac{{R_o}^2}{R^2} \frac{\delta^2}{{R_o}^2} (Z_{33} + Z_{43}) + \left(1 - \frac{{R_o}^2}{R^2} \frac{\delta^2}{{R_o}^2}\right) \frac{1 + S_o \frac{{U_j}^2}{{U_{jo}}^2} \lambda^2}{\left(1 - S_o \frac{{U_j}^2}{{U_{jo}}^2} \lambda^2\right)^2}$$

$$W(3,3) = 2 \frac{\delta}{R_o} (Z_{13} - S_2) \frac{{R_o}^2}{R^2}$$

$$W(3,4) = \frac{P_{oo}}{p} \left[\frac{\delta^2}{{R_o}^2} \frac{{R_o}^2}{R^2} Z_{13} + \left(1 - \frac{{R_o}^2}{R^2} \frac{\delta^2}{{R_o}^2}\right) S_2 \right]$$

$$W(3,5) = \frac{{R_o}^2}{R^2} \frac{\delta^2}{{R_o}^2} (Z_{53} + Z_{63})$$

$$W(3,6) = -2S_2 \frac{R_o}{R} H$$

$$W(3,7) = -2\, S_2 \; \frac{R_o}{R} \; \frac{\theta}{R_o}$$

$$V(3) = -2\, \frac{R_o}{R} \left(\frac{R}{R_o}\right)' S_2$$

Table A1 lists the values of N_i, D_i, and their derivatives which are required to evaluate the Z parameters in the equations above.

A1-4 Equation for J = 4; Energy Equation

$$W_o\, C_p\, T_{oo} + W_1\, C_p\, T_{oj} = 2\pi \int_o^R \rho\, U\, C_p\, T_o\, y dy \quad \text{where } R = (\text{local duct radius} - \theta H)$$

or

$$W_o\, T_{oo} + W_1\, T_{oj} = 2\pi \int_o^R \rho\, U\, T_o y dy$$

$$W_o T_{oo} + W_1 T_{oj} = \pi \frac{p}{R_g}\, U_j \left[\delta^2 \int_o^1 \frac{(\lambda + f_o)\,(1 + \pi f_o^{\,1/2})\, 2\eta d\eta}{1 + \pi f_o^{\,1/2} - S_o \left(\frac{U_j}{U_{jo}}\right)^2 (\lambda + f_o)^2} + (R^2 - \delta^2) S_2 - 2 S_2\, \theta H R \right]$$

Let

$$Z_{14} = \int_o^1 \frac{(\lambda + f_o)\,(1 + \pi f_o^{\,1/2})\, 2\eta\, d\eta}{1 + \pi f_o^{\,1/2} - S_o \left(\frac{U_j}{U_{jo}}\right)^2 (\lambda + f_o)^2}$$

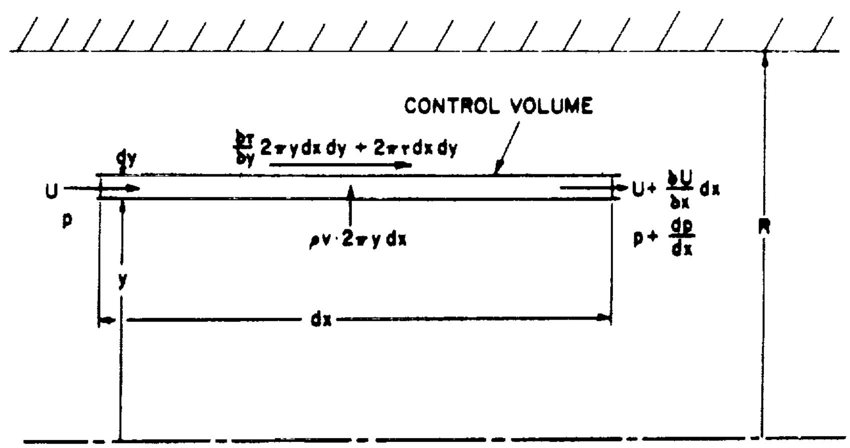
Then, the normalized energy equation may be written as follows:

$$\frac{(W_o\,T_{oo} + W_1\,T_{oj})}{\pi R_o^{\,2}\,U_{jo}}\;\frac{R_g}{P_{oo}} = \frac{p}{P_{oo}}\;\frac{U_j}{U_{jo}}\left[\frac{\delta^2}{R_o^{\,2}}\;Z_{14} + \left(\frac{R^2}{R_o^{\,2}} - \frac{\delta^2}{R_o^{\,2}}\right) S_2 - 2S_2\,\frac{\theta}{R_o}\;\frac{R}{R_o}\;H\right]$$

If this equation is compared to the normalized continuity equation in Section A1-3, it is seen that the right-hand sides are identical except for the substitution of Z_{14} for Z_{13}. This means that all of the W(4, K) coefficients are identical to the W(3, K) coefficients except for the substitution of Z_{i4} for Z_{i3} in all expressions. Table A1 lists the values of N_i, D_i, and their derivatives which are required to evaluate the Z_{i4} parameters.

A1-5 Equation for J = 5; Moment-of-Momentum Integral Equation

The momentum equation for an annular section of the jet can be derived as follows.



$$\tau 2\pi\ dydx - \frac{dp}{dx}\ dx\ (2\pi ydy) + 2\pi y\frac{\partial \tau}{\partial y}\ dydx = \rho\frac{\partial u}{\partial x}\ dx\ u\cdot 2\pi ydy$$

$$+\rho v\ 2\pi\ ydx\frac{\partial u}{\partial y}\ dy$$

or

$$\tau - \frac{dp}{dx}\ y + y\,\frac{\partial \tau}{\partial y} = \rho\ \frac{\partial u}{\partial x}\ uy + \rho vy\,\frac{\partial u}{\partial y}$$

To derive the moment-of momentum integral equation, this momentum equation is multiplied by ydy and integrated across the jet:

$$\int_0^\delta \rho u y \frac{\partial u}{\partial x} \, ydy + \int_0^\delta \rho v y \frac{\partial u}{\partial y} \, ydy = \int_0^\delta \frac{\partial (\tau y)}{\partial y} \, ydy - \int_0^\delta \frac{dp}{dx} \, y^2 dy$$

Noting that $u = U_j(\lambda + f_o)$:

$$\frac{\partial u}{\partial x} = \frac{1}{R_o} \, U_j'(\lambda + f_o) + \frac{U_j}{R_o} \left[\lambda' + \frac{\partial f_o}{\partial \eta} \left(- \frac{\eta \delta'}{\delta} \right) \right]$$

$$\frac{\partial u}{\partial y} = \frac{U_j}{\delta} \, \frac{\partial f_o}{\partial \eta}$$

then

$$\int_0^\delta \rho u y \frac{\partial u}{\partial x} \, ydy = \int_0^1 \frac{p}{R_g T_{oo}} \, \frac{U_j(\lambda + f_o) \frac{\partial u}{\partial x}}{1 + \mathbb{T} f_o^{1/2} - S_o \frac{U_j^2}{U_{jo}^2} (\lambda + f_o)^2} \, \delta^3 \eta^2 d\eta$$

$$= \frac{p U_j^2 \delta^2}{R_g T_{oo}} \int_0^1 \frac{\frac{1}{U_j} (\lambda + f_o) \frac{\partial u}{\partial x} \, \eta^2 \, d\eta}{1 + \mathbb{T} f_o^{1/2} - S_o \frac{U_j^2}{U_{jo}^2} (\lambda + f_o)^2}$$

$$= \frac{p U_j^2 \delta^3}{R_g T_{oo} R_o} \left[\frac{U_j'}{U_j} \, Q_1 + \lambda' Q_2 + \frac{\delta'}{\delta} \, Q_3 \right]$$

in which

$$Q_1 = \int_0^1 \frac{(\lambda + f_o)^2 \eta^2 d\eta}{D}$$

$$Q_2 = \int_0^1 \frac{(\lambda + f_o) \eta^2 \, d\eta}{D}$$

$$Q_3 = - \int_0^1 \frac{\frac{\partial f_o}{\partial \eta} (\lambda + f_o) \, \eta^3 \, d\eta}{D}$$

$$D = 1 + \mathbb{T} f_o^{1/2} - S_o \frac{U_j^2}{U_{jo}^2} (\lambda + f_o)^2$$

In order to evaluate the radial velocity, v, it is necessary to use the continuity relation. Employing a control volume of radius y and length dx, the continuity equation may be written as follows:

$$\rho v \;\; 2\pi y dx = -\int_0^y \frac{\partial}{\partial x} (\rho u \, 2\pi \, y dy) \, dx$$

so

$$\begin{aligned}
\rho v y &= -\int_0^y \frac{\partial}{\partial x} (\rho u y) \, dy \\
&= -\int_0^y \frac{\partial}{\partial x} \left[\frac{p}{R_g T_{oo}} \; \frac{U_j(\lambda + f_o)}{D} \right] y dy
\end{aligned}$$

$$\begin{aligned}
\rho v y = -\int_0^\eta \frac{\delta^2}{R_o R_g T_{oo}} \Bigg\{ & \left[\frac{p' U_j (\lambda + f_o)}{D} + \frac{p U_j' (\lambda + f_o)}{D} + \frac{p U_j}{D} \left(\lambda' - \frac{\partial f_o}{\partial \eta} \eta \frac{\delta'}{\delta} \right) \right] \\
& - \frac{p U_j (\lambda + f_o)}{D^2} \left[\mathbb{T}' \frac{\partial D}{\partial \mathbb{T}} + U_j' \frac{\partial D}{\partial U_j} + \lambda' \frac{\partial D}{\partial \lambda} - \eta \frac{\delta'}{\delta} \frac{\partial D}{\partial \eta} \right] \Bigg\} \eta \, d\eta
\end{aligned}$$

$$\begin{aligned}
-\frac{\rho v y}{\dfrac{p}{R_g T_{oo}} U_j \dfrac{\delta^2}{R_o}} = \Bigg[& \frac{U_j'}{U_j} \int_0^\eta \left[\frac{(\lambda + f_o)}{D} - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial\left(\frac{U_j}{U_{j_o}}\right)} \frac{U_j}{U_{j_o}} \right] \eta \, d\eta \\
& + \lambda' \int_0^\eta \left[\frac{1}{D} - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial \lambda} \right] \eta \, d\eta \\
& + \frac{\delta'}{\delta} \int_0^\eta \left[-\frac{1}{D} \frac{\partial f_o}{\partial \eta} \eta + \frac{(\lambda + f_o)}{D^2} \eta \frac{\partial D}{\partial \eta} \right] \eta \, d\eta \\
& + \frac{p'}{p} \int_0^\eta \frac{(\lambda + f_o)}{D} \eta \, d\eta \\
& + \mathbb{T}' \int_0^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial \mathbb{T}} \eta \, d\eta \Bigg] \\
= \Bigg[& \frac{U_j'}{U_j} \left[V_1 + V_2 \frac{U_j}{U_{j_o}} \right] + \lambda' \left[V_3 + V_4 \right] + \frac{\delta'}{\delta} \left[-V_{10} - V_{11} \right] + \frac{p'}{p} \left[V_1 \right] + \mathbb{T}' \left[V_5 + V_6 \right] \Bigg]
\end{aligned}$$

in which

$$V_1 = \int_o^\eta \frac{(\lambda + f_o)}{D} \eta d\eta \qquad V_5 = 0$$

$$V_2 = \int_o^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial\left(\frac{U_j}{U_{jo}}\right)} \eta d\eta \qquad V_6 = \int_o^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial T} \eta d\eta$$

$$V_3 = \int_o^\eta \frac{1}{D} \eta d\eta \qquad V_{10} = \int_o^\eta \frac{1}{D} \frac{\partial f_o}{\partial \eta} \eta^2 d\eta$$

$$V_4 = \int_o^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial \lambda} \eta d\eta \qquad V_{11} = \int_o^\eta - \frac{(\lambda + f_o)}{D^2} \frac{\partial D}{\partial \eta} \eta^2 \eta$$

With these definitions, the integral

$$\int_o^\delta \rho v y \frac{\partial u}{\partial y} y dy$$

may be evaluated as follows:

$$\frac{\int_o^\delta \rho v y \frac{\partial u}{\partial y} y dy}{\left(\frac{p U_j^2 \delta^3}{R_g T_{oo} R_o}\right)} = \int_o^\delta \frac{\rho v y}{\left(\frac{p U_j}{R_g T_{oo}}\right)\left(\frac{\delta^2}{R_o}\right)} \frac{1}{U_j \delta} \frac{U_j}{\delta} \frac{\partial f_o}{\partial \eta} y dy$$

$$= \int_o^1 \left[\frac{\rho v y}{\frac{p U_j}{R_g T_{oo}} \frac{\delta^2}{R_o}}\right] \frac{\partial f_o}{\partial \eta} \eta d\eta$$

$$= \frac{U_j{}'}{U_j}\left[-\int_o^1 V_1 \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta - \frac{U_j}{U_{jo}}\int_o^1 V_2 \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta\right] + \lambda'\left[-\int_o^1 V_3 \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta - \int_o^1 V_4 \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta\right]$$

$$+ \frac{\delta'}{\delta}\left[\int_o^1 (V_{10} + V_{11}) \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta\right] + \frac{p'}{p}\left[-\int_o^1 V_1 \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta\right] + T'\left[-\int_o^1 (V_5 + V_6) \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta\right]$$

$$= \frac{U_j{}'}{U_j}\left[R_1 + \frac{U_j}{U_{jo}} R_2\right] + \lambda' R_{34} + \frac{\delta'}{\delta}\left[R_{10} + R_{11}\right] + \frac{p'}{p} R_1 + T' R_{56}$$

in which

$$R_1 = -\int_o^1 V_1 \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta \qquad R_{10} = \int_o^1 V_{10} \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta$$

$$R_2 = -\int_o^1 V_2 \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta \qquad R_{11} = \int_o^1 V_{11} \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta$$

$$R_{34} = -\int_o^1 (V_3 + V_4) \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta \qquad R_{56} = -\int_o^1 (V_5 + V_6) \frac{\partial f_o}{\partial \eta}\,\eta\, d\eta$$

The pressure gradient term may be rewritten as follows:

$$\int_o^\delta \frac{dp}{dx}\, y^2 dy = \frac{p U_j{}^2 \delta^3}{R_g T_{oo} R_o}\left[\frac{p'}{p}\ \frac{1}{3}\ \frac{BP}{\dfrac{U_j{}^2}{U_{jo}{}^2}}\right]$$

The shear stress term may be evaluated as follows:

$$\int_o^\delta \frac{\partial (\tau y)}{\partial y}\, y dy = \tau y^2 \Big|_o^\delta - \int_o^\delta \tau y dy$$

$$\tau = \rho\epsilon \frac{\partial u}{\partial y} = \rho\epsilon \frac{U_j}{\delta} \frac{\partial f_o}{\partial \eta}$$ where ϵ is the eddy kinematic viscosity

$$\frac{\int_o^\delta \frac{\partial(\tau y)}{\partial y} \, y dy}{\left(\frac{pU_j^2 \delta^3}{R_g T_{oo} R_o}\right)} = -\int_o^1 \rho\epsilon \frac{1}{U_j^2 \delta} \frac{\partial f_o}{\partial \eta} \eta d\eta \frac{R_g T_{oo} R_o}{p}$$

$$= -\int_o^1 \frac{\epsilon}{U_j \delta} \frac{R_o}{\delta} \frac{1}{D} \frac{\partial f_o}{\partial \eta} \eta d\eta$$

$$= -\left(\frac{\epsilon}{U_j \delta}\right)_{avg} \left(\frac{R_o}{\delta}\right) \int_o^1 \frac{\epsilon}{\epsilon_{avg}} \frac{1}{D} \frac{\partial f_o}{\partial \eta} \eta d\eta$$

$$= \frac{EA_\tau}{(\delta/R_o)}$$

where $A_\tau = -\int_o^1 \frac{\epsilon}{\epsilon_{avg}} \frac{1}{D} \frac{\partial f_o}{\partial \eta} \eta d\eta \cong 0.377$ as in incompressible jet mixing

$E = \left(\frac{\epsilon}{U_j \delta}\right)_{avg}$ is the inverse of the local turbulent Reynolds number.

Assembling all the terms, the final moment-of-momentum integral equation is as follows:

$$\frac{U_j'}{U_j}\left(Q_1 + R_1 + R_2 \frac{U_j}{U_{jo}}\right) + \lambda'(Q_2 + R_{34}) + \frac{\delta'}{\delta}(Q_3 + R_{10} + R_{11}) + \frac{p'}{p}\left(R_1 + \frac{BP}{3\frac{U_j^2}{U_{jo}^2}}\right) + T' R_{56} = \frac{EA_\tau}{(\delta/R_o)}$$

The values of the coefficients follow:

$$W(5,1) = Q_1 + R_1 + R_2 \frac{U_j}{U_{jo}} \qquad W(5,5) = R_{56}$$

$$W(5,2) = Q_2 + R_{34} \qquad W(5,6) = 0$$

$$W(5,3) = Q_3 + R_{10} + R_{11} \qquad W(5,7) = 0$$

$$W(5,4) = R_1 + \frac{BP}{3 \dfrac{U_j^{\,2}}{U_{jo}^{\,2}}} \qquad V(5) = \frac{EA_\tau}{(\delta/R_o)}$$

A1-6 Equation for J = 6; Boundary Layer Momentum Equation

The boundary layer momentum equation, equation (40), is discussed in Section 3.4 of this report.

A1-7 Equation for J = 7; Boundary Layer Moment-of-Momentum Equation

The boundary layer moment-of-momentum equation was used to derive the shape factor equation (41). This equation is discussed in Section 3.4 of this report.

A1-8 Initial Conditions for the Part 1 Analysis

The initial conditions for the Part 1 analysis are established from the transition zone analysis described in Section 3.3. The initial values were set as follows:

$$\frac{U_j}{U_{jo}} = 1 \qquad \frac{T_{oJ}}{T_{oo}} = \frac{T_{o1} - T_{oo}}{T_{oo}}$$ From transition zone Analysis

$\lambda = \lambda$ From transition zone Analysis $\qquad \frac{\theta}{R_o} = \frac{\theta}{R_o}$ Calculated from Equation (14)

$\frac{\delta}{R_o} = \frac{R}{R_o}\left(\frac{\delta}{R}\right)$ From transition zone analysis $\qquad H = 1.4$ Input value to program

$$\frac{p}{P_{oo}} = \left[1 - S_o \frac{k}{k-1} \lambda^2\right] \left[1 - \frac{\Delta P_{sD}}{P_{oo}}\right]$$

A2 - Part 2 - Downstream of Jet Attachment

The general form of the flow equations, as described in Section 3.4, is as follows:

$$\sum_{K=1}^{6} W(J,K) * Y(K) = V(J)$$

The 6 variables employed in Part 2 are tabulated below. The superscript (') represents the derivative $\frac{\partial}{\partial\left(\frac{x}{R_o}\right)}$.

$$\begin{array}{ccccccc} K = & 1 & 2 & 3 & 4 & 5 & 6 \\ Y(K) = & \left(\frac{U_c}{U_{jo}}\right)' & \left(\frac{U_o}{U_c}\right)' & \gamma' & \left(\frac{p}{P_{ooi}}\right)' & \Pi' & \left(\frac{T_{oo}}{T_{ooi}}\right)' \end{array}$$

where P_{ooi} and T_{ooi} are the stagnation pressure and stagnation temperature for the wall streamline at the end of Part 1 just as the jet reaches the duct wall.

The variable Y(2) remains zero throughout the Part 2 analysis; this variable is a redundant parameter which remains from an earlier version of the computer program.

The W(J,K) coefficients and V(J) terms are determined in this section of the appendix.

A2-1 Equation for J = 1; Continuity Equation

$$W_o + W_1 = \int_o^R \rho u \cdot 2\pi y dy$$

$$\frac{W_o + W_1}{\pi g_o} = \frac{p}{R_g T_{oo}} U_c R^2 \int_o^1 \frac{T_{oo}}{T} \frac{u}{U_c} 2\eta d\eta$$

Now $$\frac{u}{U_c} = f_2(\eta) + \gamma g_2(\eta)$$

$$\frac{T}{T_{oo}} = 1 + T\sqrt{f_o(\eta)} - S_o\left(\frac{U_c}{U_{jo}}\right)^2 \left[f_2(\eta) + \gamma g_2(\eta)\right]^2$$

where $$\frac{T_o - T_{oo}}{T_{oJ}} = \sqrt{f_o(\eta)}$$ (Free jet temperature profile) is assumed to hold in Part 2 as a simplification of the analysis.

The value of T_{oo} used in the definitions of BP and S_o, and throughout the Part 2 analysis, is the stagnation temperature for the wall streamline at the axial position selected. T_{oo} varies with x in Part 2.

Let $$D = \frac{T}{T_{oo}}$$

$$BP = \frac{R_g T_{oo}}{U_{jo}^{2}}$$

The continuity equation may be rewritten as follows:

$$\frac{T_{ooi}}{P_{ooi}} \frac{R_g}{R_o^{2}} \frac{W_o + W}{\pi g_o U_{jo}} = \frac{\frac{p}{P_{ooi}}}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \frac{U_c}{U_{jo}} \frac{R^2}{R_o^{2}} Z_{11}$$

where $$Z_{11} = \int_0^1 \frac{(f_2 + \gamma g_2)\, 2\eta\, d\eta}{D}$$

In the computer analysis, the integration is approximated by a summation across the flow:

$$Z_{1J} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{N_{iJ}}{D_{iJ}} \eta_i$$

In this equation, N_i and D_i are average values of the numerator and denominator across the i^{th} equal-radius annular segment of the flow.

The following additional definitions will be used:

$$Z_{2J} = \frac{\partial Z_{1J}}{\partial\left(\frac{U_c}{U_{jo}}\right)} = \frac{2}{n_s} \sum_{i=1}^{n_s} - \frac{N_{iJ}}{D_{iJ}^{2}} \frac{\partial D_{iJ}}{\partial\left(\frac{U_c}{U_{jo}}\right)} \eta i$$

$$Z_{5J} + Z_{6J} = \frac{\partial Z_{1J}}{\partial \mathbb{T}} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{1}{D_{iJ}} \frac{\partial N_{iJ}}{\partial \mathbb{T}} \eta_i + \frac{2}{n_s} \sum_{i=1}^{n_s} - \frac{N_{iJ}}{D_{iJ}^2} \frac{\partial D_{iJ}}{\partial \mathbb{T}} \eta_i$$

$$Z_{7J} + Z_{9J} = \frac{\partial Z_{1J}}{\partial \gamma} = \frac{2}{n_s} \sum_{i=1}^{n_s} \frac{1}{D_{iJ}} \frac{\partial N_{iJ}}{\partial \gamma} \eta_i + \frac{2}{n_s} \sum_{i=1}^{n_s} - \frac{N_{iJ}}{D_{iJ}^2} \frac{\partial D_{iJ}}{\partial \gamma} \eta_i$$

$$Z_{8J} = \frac{\partial Z_{1J}}{\partial \left(\frac{T_{oo}}{T_{ooi}}\right)} = \frac{2}{n_s} \sum_{i=1}^{n_s} - \frac{N_{iJ}}{D_{iJ}^2} \frac{\partial D_{iJ}}{\partial \left(\frac{T_{oo}}{T_{ooi}}\right)} \eta_i$$

Employing these definitions in the continuity equation, the following equation is obtained after differentiating:

$$0 = \left(\frac{\bar{p}}{P_{ooi}}\right)' \frac{\frac{U_c}{U_{jo}}}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \frac{R^2}{R_o^2} Z_{11} + \left(\frac{U_c}{U_{jo}}\right)' \frac{\frac{p}{P_{ooi}}}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \frac{R^2}{R_o^2} Z_{11} + 2 \frac{\frac{p}{P_{ooi}}}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \frac{U_c}{U_{jo}} \frac{R}{R_o} \left(\frac{R'}{R_o}\right) Z_{11}$$

$$- \left(\frac{T_{oo}}{T_{ooi}}\right)' \frac{\frac{p}{P_{ooi}}}{\left(\frac{T_{oo}}{T_{ooi}}\right)^2} \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2} Z_{11} + \frac{\frac{p}{P_{ooi}}}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2} \left[Z_{21} \frac{U_c{}'}{U_{jo}} + (Z_{51} + Z_{61}) \mathbb{T}' \right]$$

$$+ \frac{\frac{p}{P_{ooi}}}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2} \left[(Z_{71} + Z_{91}) \gamma' + Z_{81} \left(\frac{T_{oo}}{T_{ooi}}\right)' \right]$$

Collecting terms and dividing each by $\dfrac{\left(\frac{p}{P_{ooi}}\right)}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \dfrac{U_c}{U_{jo}} \dfrac{R^2}{R_o^2}$;

$$W(1,1) = \frac{Z_{11}}{\left(\frac{U_c}{U_{jo}}\right)} + Z_{21}$$

$$W(1,5) = Z_{51} + Z_{61}$$

$$W(1,2) = 0$$

$$W(1,6) = Z_{81} - \frac{Z_{11}}{\left(\frac{T_{oo}}{T_{ooi}}\right)}$$

$$W(1,3) = Z_{71} + Z_{91}$$

$$V(1) = -2\frac{\left(\frac{R}{R_o}\right)'}{\left(\frac{R}{R_o}\right)} Z_{11}$$

$$W(1,4) = \frac{Z_{11}}{\left(\frac{p}{P_{ooi}}\right)}$$

Table A2 lists the values of N_i, D_i, and their derivatives which are required to evaluate the Z parameters in the previous equations.

A2. 2 Equation for J = 2; Energy Equation

$$\text{constant} = \int_o^R \rho u \, T_o \cdot 2\pi y dy$$

assuming constant specific heat throughout the flow

Using the substitutions for ρ, the velocity profile functions, and η as in Section A2. 1, the energy equation may be rewritten as follows:

$$\text{constant} = \frac{p}{R_g T_{oo}} \; U_c R^2 \int_o^1 \frac{\left[f_2(\eta) + \gamma g_2(\eta)\right]}{D} \; T_o \; 2\eta \, d\eta$$

As in Section A2. 1, the free-jet temperature profile is assumed to hold:

$$\frac{T_o}{T_{oo}} = 1 + \Pi\sqrt{f_o(\eta)}$$

With this, the energy equation becomes:

$$\text{constant} = \frac{p}{R_g} U_c R^2 \int_0^1 \frac{(f_2 + \gamma g_2)(1 + \pi f_o^{1/2})}{D} 2\eta\, d\eta$$

$$\text{constant} = \frac{p}{P_{ooi}} \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2} Z_{12}$$

where $$Z_{12} = \int_0^1 \frac{(f_2 + \gamma g_2)(1 + \pi f_o^{1/2})\, 2\eta\, d\eta}{D}$$

After differentiating with respect to $\frac{x}{R_o}$, the energy equation takes the following form:

$$0 = \left(\frac{p}{P_{ooi}}\right)' \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2} Z_{12} + \left(\frac{U_c}{U_{jo}}\right)' \frac{p}{P_{ooi}} \frac{R^2}{R_o^2} Z_{12} + \left(\frac{R}{R_o}\right)' 2 \frac{p}{P_{ooi}} \frac{U_c}{U_{jo}} \frac{R}{R_o} Z_{12}$$

$$+ \frac{p}{P_{ooi}} \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2} \left[Z_{22} \left(\frac{U_c}{U_{jo}}\right)' + (Z_{52} + Z_{62}) \pi' + (Z_{72} + Z_{92}) \gamma' + Z_{82} \left(\frac{T_{oo}}{T_{ooi}}\right)' \right]$$

Collecting terms and dividing each by $\frac{p}{P_{ooi}} \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2}$;

$$W(2,1) = \frac{Z_{12}}{\left(\frac{U_c}{U_{jo}}\right)} + Z_{22}$$

$$W(2,2) = 0$$

$$W(2,3) = Z_{72} + Z_{92}$$

$$W(2,4) = \frac{Z_{12}}{\left(\frac{p}{P_{ooi}}\right)}$$

$$W(2,5) = Z_{52} + Z_{62}$$

$$W(2,6) = Z_{82}$$

$$V(2) = -2Z_{12} \frac{(R'/R_o)}{(R/R_o)}$$

Table A2 lists the values of N_i, D_i, and their derivatives which are required to evaluate the Z parameters in the equations above.

A2.3 Equation for J = 3; Momentum Equation

$$-\pi R^2 \frac{dp}{dx} - 2\pi R \tau_w = \frac{d}{dx} \int_o^R \rho u^2 \; 2\pi y dy$$

Using the previously-developed substitution for ρ, the velocity profile functions, and η:

$$-R^2 \frac{dp}{dx} - 2 R \tau_w = \frac{d}{dx} \left\{ \frac{p}{R_g T_{oo}} \; U_c^2 \; R^2 \int_o^1 \frac{2\left[f_2(\eta) + \gamma g_2(\eta)\right]^2}{D} \eta d\eta \right\}$$

Let $\quad \tau_w = C_{fd} \frac{1}{2} \frac{\rho U_c^2}{g_o} \quad$ where $\quad \frac{\rho}{g_o} \cong \frac{p}{R_g T_{oo}}$

The momentum equation may be rewritten as follows:

$$-R^2 \frac{dp}{dx} - R C_{fd} U_c^2 \frac{p}{R_g T_{oo}} = \frac{d}{dx} \left\{ \frac{p}{R_g T_{oo}} \; U_c^2 \; R^2 Z_{13} \right\}$$

where
$$Z_{13} = \int_o^1 \frac{(f_2 + \gamma g_2)^2 \; 2\eta d\eta}{D}$$

Normalizing:

$$-\left(\frac{R}{R_o}\right)^2 \left(\frac{p}{P_{ooi}}\right)' - \frac{R}{R_o} C_{fd} \left(\frac{U_c}{U_{jo}}\right)^2 \frac{p}{P_{ooi}} \frac{1}{BP} = \frac{d}{d\left(\frac{x}{R_o}\right)} \left\{ \frac{p}{P_{ooi}} \frac{U_{jo}^2}{R_g T_{oo}} \frac{U_c^2}{U_{jo}^2} \frac{R^2}{R_o^2} Z_{13} \right\}$$

Differentiating:

$$\left[-\left(\frac{R}{R_o}\right)^2 \left(\frac{p}{P_{ooi}}\right)' - \frac{R}{R_o} C_{fd} \left(\frac{U_c}{U_{jo}}\right)^2 \frac{p}{P_{ooi}} \frac{1}{BP}\right] = \frac{Z_{13}}{BP} \left[\left(\frac{p}{P_{ooi}}\right)' \left(\frac{U_c}{U_{jo}}\right)^2 \frac{R^2}{R_o^2} + \left(\frac{U_c}{U_{jo}}\right)' 2 \frac{p}{P_{ooi}} \frac{U_c}{U_{jo}} \frac{R^2}{R_o^2} \right]$$

+ (see next page)

$$+ \frac{Z_{13}}{BP} \left(\frac{R}{R_o}\right)' 2 \frac{p}{P_{ooi}} \frac{U_c^2}{U_{jo}^2} \frac{R}{R_o}$$

$$- \frac{T_{oo}'}{T_{ooi}} \frac{p}{P_{ooi}} \frac{1}{\left(\frac{T_{oo}}{T_{ooi}}\right)} \frac{U_c^2}{U_{jo}^2} \frac{R^2}{R_o^2} \frac{Z_{13}}{BP}$$

$$+ \frac{p}{P_{ooi}} \frac{\frac{U_c^2}{U_{jo}^2}}{BP} \frac{R^2}{R_o^2} \left[Z_{23} \left(\frac{U_c}{U_{jo}}\right)' + (Z_{53} + Z_{63})\mho' + (Z_{73} + Z_{93})\gamma' + Z_{83} \frac{T_{oo}}{T_{ooi}} \right]$$

Collecting terms and dividing each by $\frac{p}{P_{ooi}} \frac{U_c^2}{U_{jo}^2} \frac{R^2}{R_o^2} \frac{1}{BP}$;

$$W(3,1) = \frac{2Z_{13}}{\left(\frac{U_c}{U_{jo}}\right)} + Z_{23}$$

$$W(3,2) = 0$$

$$W(3,3) = Z_{73} + Z_{93}$$

$$W(3,4) = \frac{BP}{\frac{p}{P_{ooi}} \frac{U_c^2}{U_{jo}^2}} + \frac{Z_{13}}{\left(\frac{p}{P_{ooi}}\right)}$$

$$W(3,5) = Z_{53} + Z_{63}$$

$$W(3,6) = \frac{-Z_{13}}{\left(\frac{T_{oo}}{T_{ooi}}\right)} + Z_{83}$$

$$V(3) = - \frac{C_{fd}}{\frac{R}{R_o}} - 2Z_{13} \frac{\left(\frac{R'}{R_o}\right)}{\left(\frac{R}{R_o}\right)}$$

Table A2 lists the values of N_i, D_i, and their derivatives which are required to evaluate the Z parameters in the equations above.

A2.4 Equation for J = 4; Moment-of-Momentum Integral Equation

The moment-of-momentum integral equation is taken from Section A1.5 of this appendix:

$$\int_o^R \rho uy \frac{\partial u}{\partial x} ydy + \int_o^R \rho vy \frac{\partial u}{\partial y} ydy = \int_o^R \frac{\partial (\tau y)}{\partial y} ydy - \int_o^R \frac{dp}{dx} y^2 dy$$

Noting that, in Part 2, $u = U_c (f_2 + \gamma g_2)$

$$\frac{\partial u}{\partial x} = \frac{1}{R_o} U_c{}' (f_2 + \gamma g_2) - \frac{U_c}{R_o}\left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta}\right) \eta \frac{\left(\frac{R'}{R_o}\right)}{\left(\frac{R}{R_o}\right)} + \frac{U_c g_2}{R_o} \gamma'$$

$$\frac{\partial u}{\partial y} = \frac{U_c}{R}\left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta}\right)$$

then $$\int_o^R \rho uy \frac{\partial u}{\partial x} ydy = \int_o^1 \frac{p}{R_g T_{oo}} \frac{U_c (f_2 + \gamma g_2) \frac{\partial u}{\partial x} \eta^2 d\eta R^3}{D}$$

$$= \frac{p}{R_g T_{oo}} U_c{}^2 R^3 \int_o^1 \frac{\frac{1}{U_c} (f_2 + \gamma g_2) \frac{\partial u}{\partial x} \eta^2 d\eta}{D}$$

$$= \frac{p U_c^2 R^3}{R_g T_{oo} R_o} \left[\frac{U_c{}'}{U_c} Q_1 + \frac{\left(\frac{R'}{R_o}\right)}{\left(\frac{R}{R_o}\right)} Q_3 + \gamma' Q_4\right]$$

In which

$$Q_1 = \int_o^1 \frac{(f_2 + \gamma g_2)^2 \eta^2 d\eta}{D}$$

$$Q_3 = -\int_o^1 \frac{\left(\frac{\partial f_2}{\partial \eta}\right) + \gamma\left(\frac{\partial g_2}{\partial \eta}\right)}{D} (f_2 + \gamma g_2)\eta^3 d\eta$$

$$Q_4 = \int_o^1 \frac{(f_2 + \gamma g_2)}{D} g_2 \eta^2 d\eta$$

$$D = 1 + \mathbb{T}\sqrt{f_o(\eta)} - S_o \left(\frac{U_c}{U_{jo}}\right)^2 (f_2 + \gamma g_2)^2$$

Following the analysis in Section A1.5, the second integral is evaluated as follows:

$$\rho vy = -\int_o^y \frac{\partial}{\partial x} \left[\frac{p}{R_g T_{oo}} \quad \frac{U_c(f_2 + \gamma g_2)}{D}\right] ydy$$

$$\rho vy = -\int_o^\eta \frac{R^2}{R_o R_g T_{oo}} \left\{\frac{p' U_c(f_2 + \gamma g_2)}{D} + \frac{pU_c'(f_2 + \gamma g_2)}{D}\right.$$

$$+ \frac{pU_c}{D}\left[-\left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta}\right)\eta \frac{\left(\frac{R}{R_o}\right)'}{\left(\frac{R}{R_o}\right)} + g_2 \gamma'\right]$$

$$\left. - \frac{pU_c(f_2 + \gamma g_2)}{D^2}\left[\mathbb{T}' \frac{\partial D}{\partial \mathbb{T}} + U_c' \frac{\partial D}{\partial U_c} + \gamma' \frac{\partial D}{\partial \gamma} - \eta \frac{\left(\frac{R'}{R_o}\right)}{\left(\frac{R}{R_o}\right)} \frac{\partial D}{\partial \eta}\right]\right\} \eta d\eta$$

$$
\frac{-\rho v y}{\frac{p}{R_g T_{oo}}\, U_c\, \frac{R^2}{R_o}}
\left[
\begin{aligned}
&\frac{U_c{}'}{U_c}\int_o^{\eta}\left[\frac{(f_2+\gamma g_2)}{D} - \frac{(f_2+\gamma g_2)}{D^2}\,\frac{\partial D}{\partial \frac{U_c}{U_{jo}}}\,\frac{U_c}{U_{jo}}\right]\eta\, d\eta \\
&+\gamma'\int_o^{\eta}\left[\frac{g_2}{D} - \frac{(f_2+\gamma g_2)}{D^2}\,\frac{\partial D}{\partial \gamma}\right]\eta\, d\eta \\
&+\frac{p'}{p}\int_o^{\eta}\frac{(f_2+\gamma g_2)}{D}\,\eta\, d\eta \\
&+\Pi'\int_o^{\eta} -\frac{(f_2+\gamma g_2)}{D^2}\,\frac{\partial D}{\partial \Pi}\,\eta\, d\eta \\
&+\frac{\left(\frac{R'}{R_o}\right)}{\left(\frac{R}{R_o}\right)}\int_o^{\eta}\left[\frac{-\left(\frac{\partial f_2}{\partial \eta}+\gamma\frac{\partial g_2}{\partial \eta}\right)}{D} + \frac{(f_2+\gamma g_2)}{D^2}\,\frac{\partial D}{\partial \eta}\right]\eta^2 d\eta
\end{aligned}
\right]
=
\left[
\begin{aligned}
&\frac{U_c{}'}{U_c}\left[V_1+V_2\,\frac{U_c}{U_{jo}}\right] \\
&+\gamma'\left[V_7+V_9\right] \\
&+\frac{p'}{p}\left[V_1\right] \\
&+\Pi'\left[V_5+V_6\right] \\
&+\frac{\left(\frac{R'}{R_o}\right)}{\left(\frac{R}{R_o}\right)}\left[-V_{10}-V_{11}\right]
\end{aligned}
\right]
$$

in which

$$V_1 = \int_o^{\eta}\frac{(f_2+\gamma g_2)}{D}\,\eta\, d\eta \qquad V_7 = \int_o^{\eta}\frac{g_2}{D}\,\eta\, d\eta$$

$$V_2 = \int_o^{\eta} -\frac{(f_2+\gamma g_2)}{D^2}\,\frac{\partial D}{\partial \frac{U_c}{U_{jo}}}\,\eta\, d\eta \qquad V_9 = \int_o^{\eta}\frac{-(f_2+\gamma g_2)}{D^2}\,\frac{\partial D}{\partial \gamma}\,\eta\, d\eta$$

$$V_5 = 0 \qquad V_{10} = \int_o^{\eta}\frac{\left(\frac{\partial f_2}{\partial \eta}+\gamma\frac{\partial g_2}{\partial \eta}\right)}{D}\,\eta^2 d\eta$$

$$V_6 = \int_o^{\eta} - \frac{(f_2 + \gamma g_2)}{D^2} \frac{\partial D}{\partial \Pi} \eta \, d\eta \qquad V_{11} = \int_o^{\eta} \frac{-(f_2 + \gamma g_2)}{D^2} \frac{\partial D}{\partial \eta} \eta^2 d\eta$$

With these definitions, the integral

$$\int_o^R \rho v y \frac{\partial u}{\partial y} y dy$$

may be evaluated as follows:

$$\frac{\int_o^R \rho v y \frac{\partial u}{\partial y} y dy}{\frac{p}{R_g T_{oo}} \frac{U_c^2 R^3}{R_o}} = \int_o^1 \left[\frac{\rho v y}{\frac{p}{R_g T_{oo}} U_c \frac{R^2}{R_o}} \right] \left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta$$

$$= \frac{U_c'}{U_c} \left[- \int_o^1 V_1 \left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta - \frac{U_c}{U_{jo}} \int_o^1 V_2 \left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta \right]$$

$$+ \gamma' \left[- \int_o^1 (V_7 + V_9) \left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta \right] + \frac{p'}{p} \left[- \int_o^1 V_1 \left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta \, d\eta \right]$$

$$+ \Pi' \left[- \int_o^1 (V_5 + V_6) \left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \right] + \frac{\left(\frac{R'}{R_o} \right)}{\left(\frac{R}{R_o} \right)} \left[\int_o^1 (V_{10} + V_{11}) \left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta} \right) \eta d\eta \right]$$

$$= \frac{U_c'}{U_c} \left[R_1 + \frac{U_c}{U_{jo}} R_2 \right] + \gamma' \left[R_{79} \right] + \frac{p'}{p} R_1 + \Pi' R_{56} + \frac{\left(\frac{R'}{R_o} \right)}{\left(\frac{R}{R_o} \right)} (R_{10} + R_{11})$$

in which

$$R_1 = -\int_0^1 V_1\left(\frac{\partial f_2}{\partial \eta} + \gamma\frac{\partial g_2}{\partial \eta}\right)\eta d\eta \qquad R_{79} = -\int_0^1 (V_7+V_9)\left(\frac{\partial f_2}{\partial \eta} + \gamma\frac{\partial g_2}{\partial \eta}\right)\eta d\eta$$

$$R_2 = -\int_0^1 V_2\left(\frac{\partial f_2}{\partial \eta} + \gamma\frac{\partial g_2}{\partial \eta}\right)\eta d\eta \qquad R_{10} = \int_0^1 V_{10}\left(\frac{\partial f_2}{\partial \eta} + \gamma\frac{\partial g_2}{\partial \eta}\right)\eta d\eta$$

$$R_{56} = -\int_0^1 (V_5+V_6)\left(\frac{\partial f_2}{\partial \eta} + \gamma\frac{\partial g_2}{\partial \eta}\right)\eta d\eta \qquad R_{11} = \int_0^1 V_{11}\left(\frac{\partial f_2}{\partial \eta} + \gamma\frac{\partial g_2}{\partial \eta}\right)\eta d\eta$$

The pressure gradient term may be written as follows:

$$\int_0^R \frac{dp}{dx}\, y^2 dy = \frac{p}{R_g T_{oo}}\; \frac{U_c^{\,2} R^3}{R_o}\left[\frac{p'}{p}\;\frac{1}{3}\;\frac{BP}{\dfrac{U_c^{\,2}}{U_{jo}^{\,2}}}\right]$$

The shear stress term may be evaluated as follows:

$$\int_0^R \frac{\partial(\tau y)}{\partial y}\, y dy = \tau y^2 \Big|_0^R - \int_0^R \tau y dy$$

$$\tau = \rho\epsilon \frac{\partial u}{\partial y} = \rho\epsilon \frac{U_c}{R}\left(\frac{\partial f_2}{\partial \eta} + \gamma\frac{\partial g_2}{\partial \eta}\right)$$ where ϵ is the eddy kinematic viscosity

$$\frac{\displaystyle\int_0^R \frac{\partial(\tau y)}{\partial y}\, y dy}{\dfrac{p}{R_g T_{oo}}\;\dfrac{U_c^{\,2} R^3}{R_o}} = -(1/2) C_{fd}\, \frac{R_o}{R} - \int_0^1 \rho\epsilon\, \frac{1}{U_c R^2}\left(\frac{\partial f_2}{\partial \eta} + \gamma\frac{\partial g_2}{\partial \eta}\right)\frac{R_g T_{oo} R_o}{p}\,\eta d\eta$$

$$= -(1/2)C_{fd}\ \frac{R_o}{R} - \frac{R_o}{R}\int_o^1 \frac{\epsilon}{U_c R}\ \frac{1}{D}\left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta}\right)\eta d\eta$$

$$= \frac{R_o}{R}\ A_\tau - \frac{R_o}{R}\ \frac{C_{fd}}{2}$$

$$\text{where } A_\tau = -\int_o^1 E_2\ \frac{\left(\frac{\partial f_2}{\partial \eta} + \gamma \frac{\partial g_2}{\partial \eta}\right)\eta d\eta}{D}$$

$$E_2 = \frac{\epsilon}{U_c R} \text{ in Part 2}$$

Assembling all the terms, the final moment-of-momentum integral equation is as follows:

$$\frac{U_c'}{U_c}\left[Q_1 + R_1 + \frac{U_c}{U_{jo}}\ R_2\right] + \gamma'\left[Q_4 + R_{79}\right] + \frac{p'}{p}\left[R_1 + \frac{1}{3}\ \frac{BP}{\left(\frac{U_c^{\ 2}}{U_{jo}^{\ 2}}\right)}\right] + \mathcal{T}' R_{56}$$

$$= -\frac{\left(\frac{R'}{R_o}\right)}{\left(\frac{R}{R_o}\right)}\left[Q_3 + R_{10} + R_{11}\right] + \frac{R_o}{R}\ A_\tau - \frac{C_{fd}}{2}\ \frac{R_o}{R}$$

The values of the coefficients follow:

$W(4,1) = Q_1 + R_1 + \frac{U_c}{U_{jo}}\ R_2$ $\qquad$ $W(4,5) = R_{56}$

$W(4,2) = 0$ $\qquad$ $W(4,6) = 0$

$$W(4,3) = Q_4 + R_{79} \qquad V(4) = -\frac{\left(\frac{R'}{R_o}\right)}{\left(\frac{R}{R_o}\right)}\left[Q_3 + R_{10} + R_{11}\right]$$

$$+ \frac{R_o}{R} A_\tau - \frac{C_{fd}}{2} \frac{R_o}{R}$$

$$W(4,4) = R_1 + \frac{1}{3} \frac{BP}{\left(\frac{U_c}{U_{jo}}\right)^2}$$

A2.5 Equation for J = 5; Centerline Velocity - Temperature Relationship

The experimental measurements made during this investigation have shown, as in figure 10, that for any value of $\frac{x}{R_o}$ in part 2,

$$\frac{T_j}{T_{jo}} \cong \frac{U_c}{U_{jo}} \text{ x const} \qquad \text{Note } U_c = U_J \text{ in Part 2 Because } U_o = 0 \text{ is assumed}$$

$$\frac{1}{\frac{T_j}{T_{jo}}} \frac{\partial\left(\frac{T_j}{T_{jo}}\right)}{\partial\left(\frac{x}{R_o}\right)} = \frac{1}{\frac{U_c}{U_{jo}}} \frac{\partial\left(\frac{U_c}{U_{jo}}\right)}{\partial\left(\frac{x}{R_o}\right)}$$

now $$\frac{T_j}{T_{jo}} = \Pi \frac{T_{ooi}}{T_{jo}} \frac{T_{oo}}{T_{ooi}}$$

so $$\left(\frac{T_j}{T_{jo}}\right)' = \frac{T_{ooi}}{T_{jo}}\left[\Pi' \frac{T_{oo}}{T_{ooi}} + \Pi\left(\frac{T_{oo}}{T_{ooi}}\right)'\right]$$

$$\frac{\left(\frac{T_j}{T_{jo}}\right)'}{\left(\frac{T_j}{T_{jo}}\right)} = \frac{\mathcal{T}' \frac{T_{oo}}{T_{ooi}} + \mathcal{T}\left(\frac{T_{oo}}{T_{ooi}}\right)'}{\mathcal{T}\,\frac{T_{oo}}{T_{ooi}}} = \frac{\left(\frac{U_c}{U_{jo}}\right)'}{\left(\frac{U_c}{U_{jo}}\right)}$$

finally,

$$\frac{\mathcal{T}'}{\mathcal{T}} + \frac{\left(\frac{T_{oo}}{T_{ooi}}\right)'}{\left(\frac{T_{oo}}{T_{ooi}}\right)} = \frac{\left(\frac{U_c}{U_{jo}}\right)'}{\left(\frac{U_c}{U_{jo}}\right)}$$

The values of the coefficients follow:

$$W(5,1) = \frac{1}{\left(\frac{U_c}{U_{jo}}\right)} \qquad W(5,5) = -\frac{1}{\mathcal{T}}$$

$$W(5,2) = 0 \qquad W(5,6) = -\frac{1}{\left(\frac{T_{oo}}{T_{ooi}}\right)}$$

$$W(5,3) = 0 \qquad V(5) = 0$$

$$W(5,4) = 0$$

A2.6 Equation for J = 6; Wall Velocity = 0

The value of U_o is assumed to be zero throughout Part 2. Therefore,

$$\frac{U_o}{U_c}' = 0$$

and $W(6,1) = W(6,3) = W(6,4) = W(6,5) = W(6,6) = V(6) = 0$

$$W(6,2) = 1$$

Table A1

Values Required to Determine Z Parameters in Part 1

	J = 2 Momentum	J = 3 Continuity	J = 4 Energy
N_{ij}	$(\lambda + f_{oi})^2$	$\lambda + f_{oi}$	$(\lambda + f_{oi})(1 + \Pi f_{oi}^{1/2})$
$\frac{\partial N_{ij}}{\partial \lambda}$	$2(\lambda + f_{oi})$	1	$1 + \Pi f_{oi}^{1/2}$
$\frac{\partial N_{ij}}{\partial \Pi}$	0	0	$(\lambda + f_{oi})\, f_{oi}^{1/2}$
D_{ij}	$1 + \Pi f_{oi}^{1/2} - S_o \frac{U_j^2}{U_{jo}^2} (\lambda + f_{oi})^2$		
$\frac{\partial D_{ij}}{\partial\left(\frac{U_j}{U_{jo}}\right)}$	$-2 S_o \frac{U_j}{U_{jo}} (\lambda + f_{oi})^2$		
$\frac{\partial D_{ij}}{\partial \lambda}$	$-2 S_o \frac{U_j^2}{U_{jo}^2} (\lambda + f_{oi})$		
$\frac{\partial D_{ij}}{\partial \Pi}$	$f_{oi}^{1/2}$		

Table A2

Values Required to Determine Z Parameters in Part 2

	J = 1 Continuity	J = 2 Energy	J = 3 Momentum
N_{ij}	$(f_{2i} + \gamma g_{2i})$	$(f_{2i} + \gamma g_{2i})(1 + \mathcal{T} f_{oi}^{1/2})$	$(f_{2i} + \gamma g_{2i})^2$
$\frac{\partial N_{ij}}{\partial \mathcal{T}}$	0	$(f_{2i} + \gamma g_{2i}) f_{oi}^{1/2}$	0
$\frac{\partial N_{ij}}{\partial \gamma}$	g_{2i}	$g_{2i}(1 + \mathcal{T} f_{oi}^{1/2})$	$2g_{2i}(f_{2i} + \gamma g_{2i})$
D_{ij}	$1 + \mathcal{T}\sqrt{fo(\eta)} - So\left(\frac{U_c}{U_{jo}}\right)^2 \left[f_2(\eta) + \gamma g_2(\eta)\right]^2$		
$\frac{\partial D_{ij}}{\partial \frac{U_c}{U_{jo}}}$	$-2 \frac{U_c}{U_{jo}} S_o \left[f_{2i} + \gamma g_{2i}\right]^2$		
$\frac{\partial D_{ij}}{\partial \mathcal{T}}$	$f_{oi}^{1/2}$		
$\frac{\partial D_{ij}}{\partial \gamma}$	$-2S_o \left(\frac{U_c}{U_{jo}}\right)^2 (f_{2i} + \gamma g_{2i}) g_{2i}$		
$\frac{\partial D_{ij}}{\partial \left(\frac{T_{oo}}{T_{ooi}}\right)}$	$-\left(\frac{U_c}{U_{jo}}\right)^2 (f_{2i} + \gamma g_{2i})^2 \frac{\partial S_o}{\partial \left(\frac{T_{oo}}{T_{ooi}}\right)}$		

Appendix B

THE COMPUTER PROGRAM

B.1 General Description

The computer program has 10 sections. The general functions of each section are described below.

MAIN: The program begins and ends in MAIN. Input data concerning the jet pump geometry, inlet gas flow properties, free jet velocity profile, and stations along the mixing tube where output values are desired, are all read in by MAIN and by two subroutines called by MAIN--DIFFEQ and SUB. MAIN converts the units of the input parameters into other units which are more convenient for subsequent analysis.

After conversion of units, MAIN computes the primary and secondary flow conditions at the top-hat section as described in Section 3.3 of this report. Then MAIN sets up the initial trial values for the velocity profile after transition and calls VBO4A to perform the iterations required to obtain an accurate solution for the profile.

After the transition zone analysis has been completed, MAIN sets up the initial conditions for the flow analysis upstream of the point of jet attachment to the wall. It also defines the stations along the mixing tube for which data will be printed out. MAIN then calls RUNGE to carry out the solution for the remainder of the flow analysis.

SUB: The first section of SUB, called when J = 3, reads in data on the mixing tube geometry--inner diameter vs. length. The diameters are converted to radii and all radii and length values are made non-dimensional by dividing by R_o. The second section of SUB, called when J = 1 or 2, finds the duct radius and slope at any axial position x specified as an input value to the subroutine.

The procedure used is linear interpolation between the nearest upstream and downstream radii which were read as input data by the first section of SUB.

CALXFG: The purpose of CALXFG is to perform the computations required to set up the three transition zone equations (27), (33), and (35) for solution by VBO4A. The three equations and derivatives of each of the three equations with respect to the three variables U_r, λ, and δ/R_{eff} are computed in CALXFG.

VBO4A, VDO2A, and SPNIST: These subroutines are library routines employed to solve the three simultaneous non-linear algebraic equations (27), (33), and (35). A two-page discussion of these subroutines is included at the end of section B.3.

DIFFEQ: The DIFFEQ subroutine is divided into two parts. Part I establishes the 7 simultaneous equations (39) which must be solved to determine the flow conditions upstream of jet attachment. The equations used are outlined in section 3.4 and detailed in appendix A.1. When the simultaneous equations are set up, DIFFEQ calls subroutine SIMQ to solve the equations for the values of the 7 derivatives in equation (38). Then subroutine RUNGE is called to integrate the derivatives using Runge-Kutta-Merson techniques. This integration yields the values of U_j, U_o, δ, p, T_{oj}, Θ, and H at stations closely spaced along the duct.

Part 2 of DIFFEQ establishes the 6 simultaneous equations (48) which must be solved to determine the flow conditions in the mixing tube downstream of jet attachment to the wall. The equations used are outlined in section 3.5 and detailed in appendix A.2. Subroutine SIMQ is called to solve for the 6 derivatives in equation (47). Then subroutine RUNGE integrates the derivatives to find the values of U_c, γ, p, T_{oj}, and T_{oo} at stations closely spaced along the duct.

SIMQ: This is a library subroutine which is called by DIFFEQ to solve simultaneous linear equations to find the values of the Y(K) derivatives in equations (38) and (47).

RUNGE: The RUNGE subroutine performs a Runge-Kutta-Merson integration procedure to integrate the derivatives of the Y(K) quantities which are developed by DIFFEQ and SIMQ as described above. RUNGE also calls the subroutine PRINT to print the desired output values of jet pump flow parameters at each mixing tube station (XOUT) which has been specified by input data and equations in MAIN.

PRINT: This subroutine contains instructions for printing the computer jet pump flow parameters at selected stations along the mixing tube downstream of the transition zone.

B.2 Input Data Format

The input data to the program must be prepared according to the following sequence:

Card No.	Parameters		Format
1	NS		3I2
2	GG(I), I = 1, NS		10F5.4
3	SDLOSS, ASD		2F10.3
4	THETA, SHAPE, VISC, RZERO		4F10.6
5	1-CARD MESSAGE identifying solution desired		80H
6	DELTAX, XTUBE, TURBNO, NSUB, NGAM, XCORE, ANOZ		3F10.4, 2I5, 2F10.5
7	POO, TOO, PO1, T01, AMASS1, AMASSO, AG, RG		8F7.3
8 to 8 + I	X(I), A(I)	omit if NSUB = 2	2F15.4
8 + I + 1	0.0	omit if NSUB = 2	2F15.4

Cards 1 through 7 are required for each solution desired. The cards from 8 on are required to define a new mixing tube geometry for analysis. If the same mixing tube geometry is to be used for additional solutions with altered flow conditions, the cards from 8 on do not have to be included for these additional solutions. The input parameter NSUB tells the computer whether the cards from 8 on are included with a data set, i.e., whether the same mixing tube geometry is to be used for additional solutions.

The input parameters are described below.

NS	number (= 10) of equal-radial-increment strips used to approximate the jet mass flow, momentum, and energy integrals across the jet
GG(I)	average values of $\frac{U}{U_c}$ (η) for I = NS equal-radial-increment strips for the turbulent pipe flow velocity profile
SDLOSS	suction duct loss coefficient; K_L in equation (55)
ASD	suction duct area, ft^2; A_{SD} in equation (55)
THETA	boundary layer momentum thickness at x = 0, ft
SHAPE	boundary layer shape factor at x = 0
VISC	gas kinematic viscosity for secondary flow at inlet, ft^2/sec
RZERO	mixing tube radius at nozzle exit section; x = 0, ft
DELTAX	steps of x/R_o at which data printouts are desired in the mixing tube
XTUBE	mixing tube length, ft
TURBNO	turbulent Reynolds number value = 147
NSUB	control instruction: if 1, a new mixing tube geometry is read in if 2, the tube geometry from the previous solution will be used.
NGAM	control instruction: if 0, incompressible flow solution (not operable) if 1, compressible flow solution
XCORE	length of the transition zone divided by R_o
ANOZ	primary nozzle exit flow area, ft^2
POO	stagnation pressure upstream of the suction duct losses, psia
TOO	stagnation temperature of the secondary flow, °R
PO1	stagnation pressure of the primary flow, psia
TO1	stagnation temperature of the primary flow, °R

AMASS1 mass flow rate of the primary flow, lbm/sec

AMASSO mass flow rate of the secondary flow, lbm/sec

AG specific heat ratio of the gas

RG gas constant, ft-lbf/lbm-°R

X(I) x stations along the mixing tube at which A(I) values are defined, ft

A(I) diameter of the mixing tube at the corresponding $x_{station}$, ft

B.3 Output Data

A complete sample of output data from the computer program is given in section B.5 of this appendix. The first section of the output repeats the input data and thus requires no comment. The remainder of the data is summarized below.

F (I)'s: values of $f_o(\eta)$ at η = 0.05, 0.15, 0.25, ..., 0.95

CONDITIONS AT BEGINNING OF THE TRANSITION SECTION

lists values of U_{oo}, ρ_{oo}, U_{joo}, T_{oo}, p_{oo} (psfa and in. H_2O), $\lambda_{oo} = \frac{U_{oo}}{U_{joo}}$, and primary jet momentum = $W_1 U_1$ where U_1 is the velocity achieved by isentropic expansion of the primary flow to the static pressure at the end of the accomodation process, p_{oo}.

The next portion of the printout monitors the solution by VB04A of equations (27), (33), and (35) for the transition zone. Each iteration employing CALXFG is recorded. The VARIABLES are the values of U_r, λ, and δ/R_{eff} determined during the particular iteration reported. The FUNCTIONS are the values of the functions:

$$\frac{C_{mass_{new}} - C_{mass_{old}}}{C_{mass_{old}}}, \quad \frac{C_{mom_{new}} - C_{mom_{old}}}{C_{mom_{old}}}, \quad \frac{P_{const_{new}} - P_{const_{old}}}{P_{const_{old}}}$$

where C_{mass} is defined by equation (27)

C_{mom} is defined by equation (32)

P_{const} is $\left(\frac{P_{o1}}{P_{oo}}\right)^{\frac{k-1}{k}}$

$(\ )_{new}$ is the value for the current iteration.

$(\ )_{old}$ is the value for the previous iteration.

If these functions are computed to within ERR times the "old" value of C_{mass}, C_{mom}, or P_{const}, VB04A is judged to have converged satisfactorily. In the present program, ERR is set at 10^{-6}, an excessively tight tolerance. As a result, the message "VB04A ACCURACY CANNOT BE ACHIEVED" is often printed out. Following this message, the values of the VARIABLES and FUNCTIONS for the current iteration are printed. These values are used as the first values for subsequent calculations.

Four lines of print follow the end of the VB04A material. The first line restates the values of $XX(1) = U_r$, $XX(2) = \lambda$, and $XX(3) = \delta\ R_{eff}$ in numerical form. The second line compares the values of EN, a dimensionless jet pump parameter developed in reference 2, before and after transition (EN vs. EN2).

$$EN = \frac{W_1 + W_o}{\sqrt{2\pi R_o^2 \rho_{oo}\left[g_o(p - P_{oo})\pi R^2_{eff} + W_1 U_1 + W_o U_o\right]}}$$

The two values should be identical; differences which exist provide a measure of the accuracy of the transition analysis. Following the EN values, the values of S_O and B_P (see appendix A, section A1. 1) are printed. Then the final values of U_{jo} and U_{co} are given.

The printing continues with a tabulation of values along the mixing tube given by Part 1 of the analysis. The parameters listed are as follows:

X/RZERO; values of $\frac{x}{R_o}$ beginning with $\frac{x_{core}}{R_o}$

AREA; local value of $\frac{\pi R^2_{tube}}{\pi R_o^2}$

PH20; wall static pressure, in H_20 relative to P_{oo}

U0; value of U_o, secondary flow velocity

UCENT; value of U_c, velocity of flow at the centerline

UR; value of U_c/U_{co}

LAMBDA; value of $\lambda = \frac{U_o}{U_j}$

DELTA/R; value of δ/R_o

TOCENT; value of stagnation temperature at the duct centerline

TOWALL; value of stagnation temperature in the secondary flow outside the mixing region

THETA/RO; value of θ/R_o

SHAPE; value of H

When the Part 1 analysis indicates that the jet reaches the wall, the message "DELTA/R = 1 -- DIFFERENTIAL EQUATIONS CHANGE" is printed. The local value of U_j, called CL, is also printed. Next, two lines are printed as follows:

F2(I)'s: values of $f_2(\eta)$ at $\eta = 0.05, 0.15, 0.25, \text{------}, 0.95$

G2(I)'s: values of $g_2(\eta)$ at $\eta = 0.05, 0.15, 0.25, \text{------}, 0.95$

The printing concludes with the tabulated results obtained from the Part 2 analysis. The parameters listed are as follows:

X/RZERO, AREA, PH20, UCENT, UR, TOCENT, TOWALL; same as in Part 1

TOWALL/TOO; stagnation temperature of wall streamline divided by secondary flow inlet stagnation temperature, T_{oo}

AUGMENT: the value of the local momentum flux, $\int_o^R \frac{\rho u^2}{g_o} 2\pi y d_y$, divided by the primary jet momentum, $W_1 U_1$, which is printed out earlier.

GAMMA; local value of γ

B. 3 Listing

```
C*****************************************************************************
C  COJET, ANALYSIS OF FLOW BEHAVIOR IN AXISYMMETRIC COMPRESSIBLE FLOW
C  EJECTORS WITH VARIABLE AREA MIXING TUBES
C*****************************************************************************
      COMMON TURBNO,CF,EN,NPART,C,JRUNGE,NGAM
     1,POO,PO1,TOO,TO1,AMACH,AG,RG
     2    ,AMASS1, AMASSO, T
     3 ,SOO, CMAS, CMOM, CENR
     4 ,SO, BP, CM, VISC1
      COMMON ZA, ZB, UJO, DORIN
     6 ,MASRAT,RZERO,AFAC, CFD, HD, F, A1, AAZERO, TF, FP, GP
     7,SDLOSS,ASD
     8,AUG1,U1,UCENT
      DIMENSION Y(10),TOL(10),YMIN(10),XOUT(201),MARK(5)
     1 ,DY(50), F(10)
      DIMENSION FF(3),XX(3), ERR(3), AA(3,10), WORK(10)
     1 ,A1(10), FX(10), TF(10)
     5    ,FP(10), GP(10)
      REAL MASRAT
C      JET MIXING WITH PRIMARY AND SECONDARY STREAMSOF THESAME PERFECT GAS
C     AND CORRECT NOZZLE EXPANSION
C     READ DATA
 100     CONTINUE
      CALL DIFFEQ (1,X,Y,DY)
         READ(5,18,END=200)THETA,SHAPE,VISC,RZERO
C     THETA = INLET B.L. MOMENTUM THICKNESS, FT
C     SHAPE = INLET B.L. SHAPE FACTOR
C     VISC = KINEMATIC VISCOSITY, FT**2/SEC
C     RZERO = INLET DUCT RADIUS, FT
 18   FORMAT (4F10.6)
         READ(5,19)
C        ONE CARD TO IDENTIFY THE SOLUTION
 19      FORMAT(80H
     1                                                                  )
C        DUCT GEOMETRY AT INLET
      A = 3.1416*RZERO*RZERO
C        APPROXIMATE NOZZLE AREA FT**2
      READ(5,15)DELTAX,XTUBE ,TURBNO,NSUB,NGAM,XCORE,ANOZ
C     NGAM=O SUPRESS GAMMA   =1 DO NOT
C     NSUB = 1 READ NEW MIXING TUBE PROFILE DATA    = 2  DO NOT
 15   FORMAT(3F10.4,2I5,2F10.5)
      READ(5,16)POO,TOO,PO1,TO1,AMASS1,AMASSO,AG,RG
C        PO - PSIA
C        TO - DEGR
C     AMASS - LBM/SEC
C        RG - FT-LBF/LBM-DEGR
 16   FORMAT(8F7.3)
      WRITE(6,19)
      WRITE(6,17)DELTAX,SHAPE,RZERO,XTUBE ,NSUB,THETA,TURBNO,NGAM,VISC,
     1AG,RG,POO,TOO,PO1,TO1,AMASSO,AMASS1,ANOZ,XCORE
 17   FORMAT(///,5X,7HDELTAX=,F10.4,3X,6HSHAPE=,F10.6,4X,6HRZERO=,F10.6,
     12HFT,/,5X,6HXTUBE= ,F10.4,4X,5HNSUB=,I5,9X,6HTHETA=,F10.6,2HFT,/,
     25X,7HTURBNO=,F10.4,4X,5HNGAM=,I5,10X,5HVISC=,F10.6,9HFT**2/SEC,/,
     39X,3HAG=,F10.4,26X,3HRG=,F10.4,15HFT-LBF/LBM-DEGR,//,
     48X,4HPOO=,F10.4,4HPSIA,/,8X,4HTOO=,F10.4,4HDEGR,/,
     58X,4HPO1=,F10.4,4HPSIA,/,8X,4HTO1=,F10.4,4HDEGR,/,
     65X,7HAMASSO=,F10.4,7HLBM/SEC,/,5X,7HAMASS1=,F10.4,7HLBM/SEC,/,
```

```
     77X,5HANOZ=,F10.5,5HFT**2,/,6X,6HXCORE=,F10.4,//)
C     THETA = INLET B.L. MOMENTUM THICKNESS, FT
C     SHAPE = INLET B.L. SHAPE FACTOR
C     VISC = KINEMATIC VISCOSITY, FT**2/SEC
C     RZERO = INLET DUCT RADIUS, FT
C        ANOZ = APPROXIMATE NOZZLE AREA FT**2
C        XCORE = LENGTH OF POTENTIAL CORE/RZERO
C        DUCT GEOMETRY AT INLET
      NS = 10
      JPOLY = 5
C        CALCULATION OF F(I) FROM POLYNOMIAL
       AAZERO=  1.0004
        A1(1)= -0.0175
        A1(2)= -8.3821
        A1(3)= 16.5806
        A1(4)= -12.7877
        A1(5)= 3.6058
      CTR = -.05
      DO 807 I=1,NS
      CTR = CTR + 0.1
      F(I) = A1(JPOLY)
      FP(I) = FLOAT(JPOLY)*F(I)
      DO 808 J=2,JPOLY
      JJ = JPOLY - J + 1
      F(I) = F(I)*CTR + A1(JJ)
      FP(I) = FP(I)*CTR + FLOAT(JJ)*A1(JJ)
  808 CONTINUE
      F(I) = F(I)*CTR + AAZERO
      TF(I) = SQRT(F(I))
  807 CONTINUE
      WRITE (6,403) (F(I), I =1,NS)
  403    FORMAT(5X,6HF(I)'S,5X,10F10.4,/)
      A = 3.1416*RZERO*RZERO
      P01 =P01 *144.
      P00 = P00*144.
      RG =RG*32.2
      AMASS1=AMASS1/32.2
       AMASS0=AMASS0/32.2
      XYZ=0.0
      GO TO (10,11),NSUB
   10 CALL SUB(XYZ,R,DR,3,RZERO)
   11 JRUNGE = 4
      NPART=1
C
C
C        TO DETERMINE TOP HAT PARAMETERS
C
C        TO DETERMINE EFFECTIVE DUCT AREA AT END OF CORE
      CALL SUB(XCORE,R,DR,2,RZERO)
      THETA = THETA + XCORE*.0010*RZERO
      RCORE = R*RZERO - THETA*SHAPE
      ACORE = 3.1416*RCORE*RCORE
      AEFF = ACORE -ANOZ
      AM = ACORE
      AFAC = 1.
      AGG = AG/(AG-1.)
      RHO0=P00/(RG*T00)
C        CORRECTION FOR INLET DUCT PRESSURE LOSS
C        P00 LOSS AT INLET OR NOT
      P02=P00-SDLOSS*AMASS0**2./(2.*RHO0*ASD**2.)
```

```
      RHOO = PO2/(RG*TOO)
C     FIRST APPROXIMATION FOR OUTER VELOCITY
      UO = AMASSO/(RHOO*AEFF)
      TO = TOO - (AG-1.)*.5*UO*UO/(AG*RG)
      P = PO2*(TO/TOO)**AGG
      RHO=P/(RG*TO)
C        SECOND APPROXIMATION FOR OUTER VELOCITY
      UO = AMASSO/(RHO*AEFF)
      TO = TOO - (AG-1.)*.5*UO*UO/(AG*RG)
      P = PO2*(TO/TOO)**AGG
      RHO=P/(RG*TO)
C        ASSUME PRIMARY STREAM EXPANDS TO OUTER STREAM PRESSURE
      T1 = TO1*(P/PO1)**(1./AGG)
      RH1=P/(RG*T1)
      U1 = SQRT(2.*AGG*RG*(TO1-T1))
C                                                       ALIT 440
C
C
C
C
C
C
      UJOO=U1-UO
      T = (TO1-TOO)/TOO
      SOO=0.5*(AG-1.)*UJOO*UJOO/(AG*RG*TOO)
      CMAS = AMASS1 +AMASSO
      CMOM = (P-POO)*AM+AMASS1*U1 + AMASSO*UO           ALIT 5
      CM = CMOM/A*2.
      CENR = AMASS1*TO1 + AMASSO*TOO
      EN=CMAS/SQRT(2.*CMOM*A*RHOO)
      C=UJOO*SQRT(A*RHOO*.5/CMOM)
      W2 = ACORE*RHOO*UJOO
      CMAS = CMAS/W2
      CMOM = CMOM/(W2*UJOO)
      CENR = CENR/(W2*TOO)
C      FIRST GUESS OF PARAMETERS AFTER CONSTANT AREA TRANSITION
C      UR = UJ/UJOO
C     Y2 =LAMBDA
C     DCR= DELTA/R
      UR = 1.
      Y2 = UO/UJOO
      PH2O =-(POO-P)*.193
       TMOM= AMASS1*U1
        RHOC= RHO*32.2
       WRITE(6,67) UO,RHOC,UJOO,TO,P,TMOM,Y2,PH2O
 67    FORMAT(5X,49HCONDITIONS AT BEGINNING OF THE TRANSITION SECTION,
     1//,9X,3HUO=,F6.0,6HFT/SEC,6X,4HRHC=,F6.4,9HLBM/FT**3,6X,
     25HUJOO=,F6.0,6HFT/SEC,/,9X,3HTO=,F6.0,4HDEGR,10X,2HP=,F6.0,
     39HLBM/FT**2,6X,13HPRIMARY MOM.=,F6.2,3HLBF,/,5X,7HLAMBDA=,F6.4,12X
     4,5HPH2O=,F6.2,6HIN.H2O)
      XX(1) = UR
      XX(2) = Y2
       XX(3)= SQRT(SQRT(ANOZ/3.1416)/RZERO)
C
      DO 121 J = 1,3
121   ERR(J) =1.E-6*XX(J)
      IP = 1
      MAX = 120
      CALL VB04A(3,3,FF,XX,ERR,AA,300000.,IP,MAX,3,WORK)
C      CHECK CALCULATION OF EN AFTER TRANSITION
```

```
      CMAS2 = (FF(1) + CMAS)*W2
      CMOM2 = (FF(2) + CMOM)*W2*UJOO
      EN2 = CMAS2/SQRT(2.*CMOM2*A*RHOO)
       WRITE(6,68)XX(1),XX(2),XX(3),EN,EN2
   68 FORMAT(///,7X,6HXX(1)=,F15.10,3X,6HXX(2)=,F15.10,3X,6HXX(3)=,F15
     1 .10,//,10X,3HEN=,F10.4,10X,4HEN2=,F10.4)
C
C
      C = C*XX(1)
      SO = SOO*XX(1)*XX(1)
      BP = (AG-1.)/(2.*AG*SO)
      UJO = XX(1)*UJOO
      UCENT= UJO*(1.+XX(2))
       WRITE(6,69)SO,BP,UJO,UCENT
   69  FORMAT(/,10X,3HSO=,F10.4,11X,3HBP=,F10.4,//,9X,4HUJO=,F10.1,6HFT/
     1SEC,2X,6HUCENT=,F10.1,6HFT/SEC)
      VISC1 = VISC/(UJOO*XX(1)*RZERO)
      Y(1) = 1.
      Y(2) =XX(2)
      Y(3) = XX(3)*R
      Y(4) =(1. - Y(2)*Y(2)*SO*AG/(AG-1.))*PO2/POO
      Y(5) = (TO1 - TOO)/TOO
      Y(6) = 1.0
      Y(7) = THETA/RZERO
      Y(8) = SHAPE
C
C
C
C
      D=4.08*Y(3)*(1.+Y(2))
      D = XCORE
      XOUT(1)=D
      XTRO=  XTUBE/RZERO
      M1=  2.* XTRO
      DO 5 I=2,M1
      XOUT(I)=XOUT(I-1)+DELTAX
      IF(XOUT(I).GT.XTRO) GO TO 300
5     CONTINUE
300   CONTINUE
      M1= I-1
      DO 8 K = 1,8
      YMIN(K) = .01
      TOL(K) = .00001
      TOL(K) = .0001
8     CONTINUE
      YMIN(5) = 1.
      YMIN(6) = 1.
      YMIN(7) = 1.
      YMIN(8) = 10.
      MARK(1)=1
      MARK(2)=M1
      MARK(4)=0
      H=.1
      CALL RUNGE (8,D,Y,TOL,YMIN,H,XOUT,MARK)
      GO TO 100
  200  STOP
      END
      SUBROUTINE DIFFEQ (N,X,Y,DY)                                     ALIT
      COMMON TURBNO,CF,EN,NPART,C,JRUNGE,NGAM                          ALIT
     1,POO,PO1,TOO,TO1,AMACH,AG,RG                                     ALIT
```

```
      2  ,AMASS1, AMASSO, T                                             ALIT  40
      3 ,SOO, CMAS, CMOM, CENR                                          ALIT  50
      4 ,SO, BP, CM, VISC1
       COMMON ZA, ZB, UJO, DORIN
      6 ,MASRAT,RZERO,AFAC, CFD, HD, F, A1, AAZERO, TF, FP, GP
      7,SDLOSS,ASD
      8,AUG1,U1,UCENT
       DIMENSION Y(10),DY(50), W(8,8), V(8,1), K(40), XCUT(100)         ALIT
      1 , A1(10), TF(10)
      2 ,A(70),B(10),F(10),G(20),DLF(20),DLG(20)
      3 ,AA(10), GG(10)
      4 ,DLF1(20)
      5   ,FP(10),GP(10)
       EQUIVALENCE (SO,SO)                                              ALIT 120
       REAL MASRAT
       IF (N-1)1,1,10                                                   ALIT 130
1      CONTINUE
       LM = 0
       MM=0                                                             ALIT 520
       MMM=0                                                            ALIT 530
        READ(5,399)NS
  399   FORMAT(3I2)
       JPOLY = 5
       READ (5,401) (GG(I),I=1,NS)
 401   FORMAT(10F5.4)
       READ(5,950)SDLOSS,ASD
 950   FORMAT(2F10.3)
          WRITE(6,405)
 405      FORMAT(1H1)
       WRITE(6,400) NS
 400      FORMAT(5X,5HNS = ,I2)
 404      FORMAT( 5X,7HGG(I)'S,5X,10F10.4,//)
       WRITE(6,404) (GG(I),I=1,NS)
       WRITE(6,951)SDLOSS,ASD
 951   FORMAT(5X,24HDUCT LOSS COEFFICIENT = ,F10.3,/,17X,12HDUCT AREA = ,
      1F10.3,5HFT**2,//)
C
C
       DIVNS = 2./FLOAT(NS)
       DIVDEL = FLOAT(NS)
       RETURN
C                                                                       ALIT 550
C      Y1=UJ/UJZERO                                                     ALIT 560
C      Y2=LAMDA = UO/UJ                                                 ALIT 570
C      Y3= DELTA/RZERO                                                  ALIT 580
C     Y4 = P/POOINITIAL                                                 ALIT 590
C         Y5=(TOCENTER--TOO)/TOO                                        ALIT 600
C.        Y6=TOO/TOCINITIAL                                             ALIT 610
C      Y7 = B. L. MOMENTUM THICKNESS/RZERO
C      Y8 = B. L. SHAPE FACTOR
C                                                                       ALIT 620
10     CONTINUE
402    FORMAT (4F10.4)
       IF (LM) 913,913,914
913    CONTINUE
       DY(1) = 0.
       DY(2) = 0.
914    LM = 1
       E=1./TURBNO                                                      ALIT 640
       Y1=Y(1)
```

```
      Y2=Y(2)
      Y3=Y(3)
      Y4=Y(4)
      Y5=Y(5)
      Y6=Y(6)
      Y7=Y(7)
      Y8=Y(8)
      Y11=Y(1)*Y(1)
      Y22=Y(2)*Y(2)                                                     ALIT 65J
      Y33=Y(3)*Y(3)                                                     ALIT 55J
      DIVY1=1./Y1
      DIVY11 = DIVY1*DIVY1
      DIVY4=1./Y4
      DIVY6 = 1./Y(6)
      S2=Y2/(1.-S0*Y11*Y22)
      DS2Y1=2.*S0*S2*S2*Y1*Y2
      DS2Y2 = 1./(1.-S0*Y11*Y22) + 2.*S0*Y11*S2*S2
      CALL SUB(X,R,DR,2,RZERO)
C
C
      R=R-Y7*Y8
C     NOW BRANCH TO PART 1 OR PART 2
      GO TO (11,20),NPART                                               ALIT 70J
11    CONTINUE                                                          ALIT 710
C                                                                       ALIT 690
C     PART 1                                                            ALIT 720
      IF (Y2) 409,410,411
 409  WRITE(6,1000)
 1000 FORMAT(10X,46HRECIRCULATION PRESENT. CALCULATION NOT CORRECT)
410   DERU = 0
      DY(7) = 0.
      DY(8) = 0. .
      GO TO 412
411   CONTINUE
      Y7 = Y7 + .0001
      RTH = Y2*Y1*Y7/VISC1
      CF=0.246/((RTH)**.268*10.**(.678*Y8))
      CFD = CF
412   CONTINUE
      DIVY3=1./Y3
      E = E
     1      *(1. + 1.5*(1. - 2.718**(-1.1*Y2)))
      HD = E
C                                                                       ALIT 730
C         J = 1        P00 = CONSTANT                                   ALIT 740
C         J = 2        MOMENTUM INTEGRAL                                ALIT 750
C         J = 3        CONTINUITY INTEGRAL                              ALIT 760
C         J = 4        ENERGY INTEGRAL                                  ALIT 770
C         J = 5        MOMENTUM HALF INTEGRAL                           ALIT 780
C         J = 7        B.L. MOMENTUM EQUATION EQN
C         J = 8        B.L. MOMENT-OFMOMENTUM EQUATION
13    CONTINUE                                                          ALIT 790
      DCNR=Y33/(R*R)                                                    ALIT 800
C     CALCULATION OF EN        PART 1
       ZA = 0.
      ZB = 0.
      CTR = -.05
      DO 369 I = 1,NS
      CTR = CTR + .1
      Y2FI = Y2 + F(I)
```

```
      FS = TF(I)
      ANA = Y2FI
      ANB = Y2FI*Y2FI
      D = 1. + Y(5)*FS -SO*Y11*Y2FI*Y2FI
      DIVD = 1./D
      ZA = ZA + DIVNS*ANA*DIVD
     1     *CTR
      ZB = ZB + DIVNS*ANB*DIVD
     1     *CTR
  369 CONTINUE
      S2 = Y2/(1.-SO*Y11*Y2*Y2)
      AMAS = ZA*DONR + (1.-DONR)*S2
      AMOM = ZB*DONR + (1.-DONR)*Y2*S2
      EN = AMAS/SQRT(AMOM + BP*(1.-1./Y(4))*DIVY11)
      EN = EN*.707
      RLOCAL = R*RZERO
      RLOCAL = RLOCAL*RLOCAL
      MASRAT = AMAS*Y4*P00*Y1*UJO*RLOCAL *3.1416/(RG*T00*AMASS1) - 1.
C
C
      W(1,1)=Y22*Y1
      W(1,2)=Y11*Y2
      W(1,3) = 0.                                                       ALIT 83)
      W(1,4)=BP*(1.-SO*Y11*Y22)*DIVY4
      W(1,5) = 0.                                                       ALIT 85)
      W(1,7) = 0.
      W(1,8) = 0.
      V(1,1) = 0.                                                       ALIT 86)
      J = 1                                                             ALIT 91)
   71 J=J+1                                                             ALIT 92)
      Z1 = 0                                                            ALIT 93)
      Z2=0.                                                             ALIT 94)
      Z3=0.                                                             ALIT 95)
      Z4=0.                                                             ALIT 96)
      Z5=0.                                                             ALIT 97)
      Z6=0.                                                             ALIT 93)
      CTR = -.05
      DO 66 I=1,NS
      CTR = CTR + .1
      FS = TF(I)
      FI=F(I)
      Y2FI=Y2+F(I)
      JJ = J-1                                                          ALIT101)
      GO TO (270,70,67) ,JJ                                             ALIT102)
  270  CONTINUE                                                         ALIT103)
      AN=Y2FI*Y2FI
      D2N=Y2FI*2.
      D5N=0
      GO TO 68                                                          ALIT107)
   70 AN=Y2FI
      D2N=1.
      D5N=0.
      GO TO 68                                                          ALIT111)
   67 AN=Y2FI*(1.+Y5*FS)
      D2N=1.+Y5*FS
      D5N=Y2FI*FS
   68 D=1.+Y5*FS-SO*Y11*Y2FI*Y2FI
      DIVD=1./D
      DIVDD=DIVD*DIVD
      D2D=-2.*SO*Y11*Y2FI
```

```
      D1D=-2.*SO*Y1*Y2FI*Y2FI
      D5D=FS
      Z1=Z1+DIVNS*AN*DIVD
     1      *CTR
      Z2=Z2-DIVNS*AN*D1D*DIVDD
     1      *CTR
      Z3=Z3+DIVNS*D2N*DIVD
     1      *CTR
      Z4=Z4-DIVNS*AN*D2D*DIVDD
     1      *CTR
      Z5=Z5+DIVNS*D5N*DIVD
     1      *CTR
      Z6=Z6-DIVNS*D5D*DIVDD*AN
     1      *CTR                                                          ALIT1250
   66 CONTINUE                                                            ALIT1260
      IF (J-2) 210,211,210
  211 W(J,1)=DONR*(2.*Z1*DIVY1+Z2)+(1.-DONR)*(2.*Y2*S2*DIVY1+Y2*DS2Y1)
      W(J,2)=DONR*(Z3+Z4)+(1.-DONR)*(S2+Y2*DS2Y2)
      W(J,3)=2.*DONR*(Z1-Y2*S2)*DIVY3
      W(J,4)=(DONR*(Z1-Y2*S2)+Y2*S2)*DIVY4
     1 + BP*DIVY4*DIVY11
      W(J,5)=DONR*(Z5+Z6)
      W(J,7) =-2.*S2*Y8/R*Y2
      W(J,8) =-2.*S2*Y7/R*Y2
      V(J,1)=-2.*DR/R*Y2*S2                                               ALIT1370
      GO TO 212
  210 W(J,1)=DONR*(Z1*DIVY1+Z2)+(1.-DONR)*(S2*DIVY1+DS2Y1)
      W(J,2)=DONR*(Z3+Z4)+(1.-DONR)*DS2Y2
      W(J,3)=2.*DONR*(Z1-S2)*DIVY3
      W(J,4)=(DONR*(Z1-S2)+S2)*DIVY4
      W(J,5)=DONR*(Z5+Z6)
      W(J,7) =-2.*S2*Y8/R
      W(J,8) =-2.*S2*Y7/R
      V(J,1)=-2.*DR/R*S2                                                  ALIT1490
  212 CONTINUE                                                            ALIT1500
      IF (J-4) 71,69,69                                                   ALIT1510
   69 CONTINUE                                                            ALIT1530
      J = 5                                                               ALIT  10
  226 I = 0                                                               ALIT  20
      Z1 = 0                                                              ALIT  30
      Z2 = 0                                                              ALIT  40
      Z3 = 0                                                              ALIT  50
      Z4 = 0                                                              ALIT  60
      Z5 = 0                                                              ALIT  70
      Z6 = 0                                                              ALIT  80
      Z7 = 0.                                                             ALIT  90
      Z8 = 0.                                                             ALIT 100
      Z9 = 0.                                                             ALIT 110
      Z10 = 0.
      Z11 = 0.                                                            ALIT 120
      Q1 = 0.                                                             ALIT 130
      Q2 = 0.                                                             ALIT 140
      Q3 = 0.                                                             ALIT 150
      Q4 = 0.                                                             ALIT 160
      R1 = 0.                                                             ALIT 170
      R2 = 0.                                                             ALIT 180
      R34 = 0.                                                            ALIT 190
      R56 = 0.                                                            ALIT 200
      R79 = 0.                                                            ALIT 210
      R10 = 0.
```

```
      R11 = 0.
      V1 = 0.
      V2 = 0.
      V3 = 0.
      V4 = 0.
      V5 = 0.
      V6 = 0.
      V7 = 0.
      V9 = 0.
      V10 = 0.
      V11 = 0.
      CTR = -.25*DIVNS
  224 I = I+1
      CTR = CTR + 0.5*DIVNS
      FS = TF(I)
      Y2FI = Y2 + F(I)
      FDER = FP(I)
  221 AN = Y2FI
      D2N=1.
      D5N=0.
  223 D=1.+Y5*FS-S0*Y11*Y2FI*Y2FI
      DIVD=1./D
      DIVDD=DIVD*DIVD
      D2D=-2.*S0*Y11*Y2FI
      D1D=-2.*S0*Y1*Y2FI*Y2FI
      D5D=FS
      CCC = DIVNS*CTR*.5
      DZ1 = CCC*AN*DIVD
      DZ2 = -CCC*AN*D1D*DIVDD
      DZ3 = CCC*D2N*DIVD
      DZ4 = -CCC*AN*D2D*DIVDD
      DZ5 = CCC*D5N*DIVD
      DZ6 = -CCC*AN*D5D*DIVDD
      DZ10 = FDER*CTR*CCC*DIVD
      DZ11 = CCC*CTR*Y2FI*DIVDD*2.*S0*Y11*Y2FI*FDER
      V1 = Z1 + 0.5*DZ1
      V2 = Z2 + 0.5*DZ2
      V3 = Z3 + 0.5*DZ3
      V4 = Z4 + 0.5*DZ4
      V5 = Z5 + 0.5*DZ5
      V6 = Z6 + 0.5*DZ6
      V10 = Z10 + 0.5*DZ10
      V11 = Z11 + .5*DZ11
      Z1 = Z1 + DZ1
      Z2 = Z2 + DZ2
      Z3 = Z3 + DZ3
      Z4 = Z4 + DZ4
      Z5 = Z5 + DZ5
      Z6 = Z6 + DZ6
      Z10 = Z10 + DZ10
      Z11 = Z11 + DZ11
  492 CCC = DIVNS*CTR*CTR
     1   *.5
      ATAU = .377
      BTAU = .0333
      Q1 = Q1 + CCC*Y2FI*DIVD
     1   *Y2FI
      Q2 = Q2 + CCC*Y2FI*DIVD
      Q3 = Q3 - CCC*CTR*Y2FI*FDER*DIVD
      CCC = -CCC/CTR
```

```
      R1 = R1 + CCC*V1*FDER
      R2 = R2 + CCC*V2*FDER
      R34 = R34 + CCC*(V3 + V4)*FDER
      R56 = R56 + CCC*(V5 + V6)*FDER
      R10  = R10 - CCC*V10*FDER
      R11 = R11-CCC*V11*FDER
      IF (I-NS) 224,225,225
225   CONTINUE
      W(J,1) = (Q1 + R1)*DIVY1 + R2
      W(J,2) = Q2 + R34
      W(J,3) = (Q3 + R10 + R11)*DIVY3
      W(J,4) = R1*DIVY4 + BP*DIVY4*DIVY11*.3333
      W(J,5) = R56
      W(J,7) = 0.
      W(J,8) = 0.
      V(J,1) = E*DIVY3*ATAU
      MMM = MMM + 1
245   CONTINUE
246    CONTINUE
247   CONTINUE
947   CONTINUE
C        B.L. MOMENTUM EQUATION
      W(7,1) = (2. + Y8)*Y7*DIVY1*Y2
      W(7,2) = (2. + Y8)*Y7
      W(7,3) = 0.
      W(7,4) = 0.
      W(7,5) = 0.
      W(7,6) = 0.
      W(7,7) = Y2
      W(7,8) = 0.
      V(7,1) = .5*CF*Y2
C        B.L. MOMENT-OF-MOMENTUM EQUATION
      W(8,2) = .5*Y8*(Y8 + 1.)*(Y8*Y8-1.)
      W(8,1) = W(8,2)*DIVY1*Y2
      W(8,3) = 0.
      W(8,4) = 0..
      W(8,5) = 0.
      W(8,6) = 0.
      W(8,7) = 0.
      W(8,8) = Y2
      V(8,1) = (Y8*Y8 -1.)*Y2/Y7*(.5*Y8*CF-.06*(Y8-1.)/((Y8+3. )
     1  *RTH**.1))
      IF (Y(2)) 200,201,201
200   W(1,1) = 0.
      W(1,2) = 0.
      W(7,1) = 0.
      W(7,2) = 0.
      V(7,1) = 0.
      W(8,1) = 0.
      W(8,2) = 0.
      V(8,1) = 0.
201    CONTINUE
      IF (Y8-2.) 482,481,481
481   W(7,1) = 0.
      W(7,2) = 0.
      W(7,7) = 1.
      V(7,1) = 0.
      W(8,1) = 0.
      W(8,2) = 0.
      W(8,8) = 1.
```

```
      V(8,1) = 0.
      W(1,1) = 0.
      W(1,2) = 0.
482    CONTINUE
C         COLLAPSE FROM 8X8 TO 7X7 MATRIX
      DO 687 I=1,8
      W(6,I) = W(7,I)
      W(7,I) = W(8,I)
687   CONTINUE
      V(6,1) = V(7,1)
      V(7,1) = V(8,1)
C
C
      DO 688 J=1,7
      W(J,6) = W(J,7)
      W(J,7) = W(J,8)
688   CONTINUE
C
C
C
C     SOLVING SIMULTANEOUS EQUATIONS BY SIMQ SUBROUTINE
C
      NN = 7
      DO 101 J=1,NN
      B(J)=V(J,1)
      DO 101 I=1,NN
      IJ = I + NN*(J-1)
      A(IJ)=W(I,J)
101   CONTINUE
      CALL SIMQ (A,B,NN,KS)
      DO 18 I=1,NN
18    DY(I)=B(I)
      DY(8) = DY(7)
      DY(7) = DY(6)
      DY(6)=0.
249   CONTINUE
      JRUNGE=JRUNGE +1
C     CHECK FOR EQUATION CHANGE
      IF (JRUNGE-5) 14,15,15
15    JRUNGE=0
      IF (Y(3) -R) 14,16,16
14    RETURN
16    XOUT(100)=X
      CALL PRINT (N,XOUT,Y,DY,100)
      Y(3)=0.
      Y3 = Y(3)
      NPART=2
      BPPT1 = BP
      SOPT1 = SO
      DLSTAR = Y8*Y7
      DELTA = DLSTAR*(Y8+1.)/(Y8-1.)
      POW = 0.5*(Y8-1.)
      R = R + DLSTAR
C        EVALUATE NEW VELOCITY PROFILES
      CTR = -.05
      DO 876 I=1,NS
      CTR = CTR + 0.1
      IF (1.-CTR-DLSTAR/R) 875,875,874
875   F(I) = 0.
      TF(I) = 0.
```

```
      GO TO 840
874   CTR1 = CTR/(1.-DLSTAR/R)
      F(I) = A1(JPOLY)
      DO 873 J=2,JPOLY
      JJ = JPOLY - J + 1
      F(I) = F(I)*CTR1 + A1(JJ)
873   CONTINUE
      F(I) = F(I)*CTR1 + AAZERO
      TF(I) = SQRT(F(I))
840   BL= (R/DELTA*(1.-CTR))**POW
      IF (BL-1.) 860,860,861
861   BL = 1.
860   CONTINUE
      F(I) = (F(I) + Y2*BL)/(1. + Y2)
      G(I) = GG(I) - F(I)
876   CONTINUE
      WRITE(6,403) (F(I),I=1,NS)
403     FORMAT(5X,7HF2(I)'S,5X,10F10.4,//)
      WRITE(6,406) (G(I),I=1,NS)
406     FORMAT(5X,7HG2(I)'S,5X,10F10.4,//)
      WRITE(6,407)
407   FORMAT(          //4X,7HX/RZERO,5X,4HAREA,5X,4HPH2O,5X,35HTOWALL
     2/TOO UCENT(FT/SEC)    UR          7HAUGMENT,6X,5HGAMMA,3X,   25HTOCE
     3NT(DEGR) TOWALL(DEGR))
      L = NS - 1
      DO 877 I=2,L
      FP(I) =.5 *DIVDEL*(F(I+1)-F(I-1))
      GP(I) =.5 *DIVDEL*(G(I+1)-G(I-1))
877   CONTINUE
      FP(1) = DIVDEL*(F(2) - F(1))
      GP(1) = DIVDEL*(G(2) - G(1))
      FP(NS) = DIVDEL*(F(NS) - F(NS-1))
      GP(NS) = DIVDEL*(G(NS) - G(NS-1))
      Y1 = Y1*(1. + Y2)
      Y11 = Y1*Y1
      Y(1) = Y(1)*(1. + Y2)
      DIVY11 = 1./Y11
      E2I = E/(1. + Y2)
      E2I = 2.*E2I
      CFDI = CF/((1. + Y2)*(1. + Y2))
      Y2 = 0.
      Y(2) = 0.
      Y7 = 0.
      Y(7) = 0.
      Y8 = 0.
      Y(8) = 0.                                                        ALIT2450
C                                                                      ALIT2460
C     PART 2                                                           ALIT2470
C                                                                      ALIT2480
C     TURBULENT PRANDTL NUMBER = PR
      PR = 1.                                                          ALIT2500
C     Y3=GAMMA
20    CONTINUE                                                         ALIT2520
      Y23=Y(2)*Y(3)                                                    ALIT2530
      SO = SOPT1/Y(6)                                                  ALIT2540
      DSOY6 = -SO/Y(6)                                                 ALIT2550
      BP = BPPT1*Y(6)                                                  ALIT2560
C
C        PHASE OUT THE B.L. DISPLACEMENT THICKNESS INHERITED FROM PART 1
      DY(7) = 0.
```

```
      DY(8) = 0.
C     CALCULATION OF EN       PART 2
      ZA = 0.
      ZB = 0.
      CTR = -.05
      DO 370 I = 1,NS
      CTR = CTR + .1
      YFGG = Y2 + F(I) + Y3*G(I)
      FS = TF(I)
      ANA = YFGG
      ANB = YFGG*YFGG
      D = 1. + Y5*FS -SO*Y11*YFGG*YFGG
      DIVD = 1./D
      ZA = ZA + DIVNS*ANA*DIVD
     1        *CTR
      ZB = ZB + DIVNS*ANB*DIVD
     1        *CTR
370    CONTINUE
      AMAS = ZA
      AMOM = ZB
      EN = AMAS/SQRT(AMOM + BP*(1.-1./Y(4))*DIVY11)
      EN = EN*.707
      RLOCAL = R*RZERO
      RLOCAL = RLOCAL*RLOCAL
      MASRAT = AMAS*Y4*POO*Y1*UJO*RLOCAL *3.1416/(RG*TOO*Y6*AMASS1)-1.
      UM = AMAS*Y1
      RM = UM*2.*R/VISC1
      CFDF = AMAS*AMAS*.048*RM**(-.20)
      CFDF = 2.*CFDF
       CFD = CFDF
C                                                                ALIT257J
C     J=1     CONTINUITY
C     J=2      ENERGY
C        J=3          MOMENTUM
C        J=4          MOMENT OF MOMENTUM EQUATION
C        J=5          T' /T = UJ'/UJ
C        J=6          D(LAMBDA)/DX = 0.
      J=0
  86  J=J+1
  87  Z1=0
      Z2=0
      Z3=0
      Z4=0
      Z5=0
      Z6=0
      Z7=0
      Z8=0
      Z9 = 0.
      CTR = -.05
      I=0
  82  I=I+1
      CTR = CTR + .1
      FS = TF(I)
      YFGG=Y2+F(I)+Y3*G(I)
      GO TO (81,83,41),J
  81  AN=YFGG
      D2N=1.
      D5N=0
      D3N=G(I)
      GO TO 85
```

```
 83   AN=YFGG*(1.+Y5*FS)
      D2N=1.+Y5*FS
      D5N=YFGG*FS
      D3N=G(I)*(1.+Y5*FS)
      GO TO 85
 41   AN=YFGG*YFGG
      D2N=2.*YFGG
      D3N=2.*YFGG*G(I)
      D5N=0
      IF (F(I)) 720,720,721
720   AN = -AN
      D2N = -D2N
      D3N = -D3N
721   CONTINUE
 85   D=1.+Y5*FS-SO*Y11*YFGG*YFGG
      DIVD=1./D
      DIVDD=DIVD*DIVD
      D1D=-SO*Y1*2.*YFGG*YFGG
      D2D=-SO*Y11*2.*YFGG
      D3D = -SO*Y11*2.*YFGG*G(I)
      D5D=FS
      D6D=-DSOY6*Y11*YFGG*YFGG
      Z1=Z1+DIVNS*AN*DIVD
     1      *CTR
      Z2=Z2-DIVNS*AN*D1D*DIVDD
     1      *CTR
      Z3=Z3+DIVNS*D2N*DIVD
     1      *CTR
      Z4=Z4-DIVNS*D2D*DIVDD*AN
     1      *CTR
      Z5=Z5+DIVNS*D5N*DIVD
     1      *CTR
      Z6=Z6-DIVNS*AN*D5D*DIVDD
     1      *CTR
      Z7=Z7+DIVNS*D3N*DIVD
     1      *CTR
      Z8=Z8-DIVNS*AN*D6D*DIVDD
     1      *CTR
      Z9 = Z9 - DIVNS*DIVDD*AN*D3D
     1      *CTR
      IF(I-NS)82,601,601
601   W(J,1)=Z1*DIVY1+Z2
      W(J,2)=Z3+Z4
      W(J,3) = Z7 + Z9
      W(J,4)=Z1*DIVY4
      W(J,5)=Z5+Z6
      W(J,6)=Z8
      IF (J-2) 602,603,602
602   W(J,6) = W(J,6) - Z1*DIVY6
603   CONTINUE
      V(J,1)=-2.*DR/R*Z1
      IF (J-3) 86,709,709
709   CONTINUE
      W(3,1) = W(3,1) + Z1*DIVY1
      AUG1=Z1
      W(3,4) = W(3,4) + BP*DIVY11*DIVY4
      V(3,1) = V(3,1) - CFD/R
      J = 4
      I = 0                                                    ALIT  40
      Z1 = 0
```

```
      Z2 = 0
      Z3 = 0
      Z4 = 0
      Z5 = 0
      Z6 = 0
      Z7 = 0.
      Z8 = 0.
      Z9 = 0.
      Z10 = 0.
      Z11 = 0.
      Q1 = 0.
      Q2 = 0.
      Q3 = 0.
      Q4 = 0.
      R1 = 0.
      R2 = 0.
      R34 = 0.
      R56 = 0.
      R79 = 0.
      R10 = 0.
      R11 = 0.
      RTAU = 0.
      V1 = 0.
      V2 = 0.
      V3 = 0.
      V4 = 0.
      V5 = 0.
      V6 = 0.
      V7 = 0.
      V9 = 0.
      V10 = 0.
      V11 = 0.
      E2FMAX = -.5*CFD*.45/(FP(5) + GP(5))
      HD = (1.-Y33)*E2I + Y33*E2FMAX
      CTR = -.25*DIVNS
924   I = I + 1
      CTR = CTR + 0.5*DIVNS
      FS = TF(I)
      Y2FI = Y2 + F(I)
     1 + Y3*G(I)
      FDER = FP(I) + Y3*GP(I)
      GGDER = FP(I) + GP(I)
      E2F = -.5*CFD*CTR/GGDER
      E = (1.-Y33)*E2I + Y33*E2F
      AN = Y2FI
      D2N=1.
      D5N=0.
      D6N = G(I)
      D=1.+Y5*FS-S0*Y11*Y2FI*Y2FI
      DIVD=1./D
      DIVDD=DIVD*DIVD
      D2D=-2.*S0*Y11*Y2FI
      D1D=-2.*S0*Y1*Y2FI*Y2FI
      D5D=FS
      D6D = -S0*Y11*2.*Y2FI*G(I)
      CCC = DIVNS*CTR*.5
      DZ1 = CCC*AN*DIVD
      DZ2 = -CCC*AN*D1D*DIVDD
      DZ3 = CCC*D2N*DIVD
      DZ4 = -CCC*AN*D2D*DIVDD
```

```
      DZ5 = CCC*D5N*DIVD
      DZ6 = -CCC*AN*D5D*DIVDD
      DZ7 = CCC*D6N*DIVD
      DZ8 = -CCC*DIVDD*AN*D6D
      DZ9 = -CCC*D6D*AN*DIVDD
      DZ10 = FDER*CTR*CCC*DIVD
      DZ11 = CCC*CTR*Y2FI*DIVDD*2.*SO*Y11*Y2FI*FDER
      RTAU = RTAU-E*FDER*CCC*DIVD
      V1 = Z1 + 0.5*DZ1
      V2 = Z2 + 0.5*DZ2
      V3 = Z3 + 0.5*DZ3
      V4 = Z4 + 0.5*DZ4
      V5 = Z5 + 0.5*DZ5
      V6 = Z6 + 0.5*DZ6
      V7 = Z7 + 0.5*DZ7
      V9 = Z9 + 0.5*DZ9
      V10 = Z10 + 0.5*DZ10
      V11 = Z11 + .5*DZ11
      Z1 = Z1 + DZ1
      Z2 = Z2 + DZ2
      Z3 = Z3 + DZ3
      Z4 = Z4 + DZ4
      Z5 = Z5 + DZ5
      Z6 = Z6 + DZ6
      Z7 = Z7 + DZ7
      Z8 = Z8 + DZ8
      Z9 = Z9 + DZ9
      Z10 = Z10 + DZ10
      Z11 = Z11 + DZ11
      CCC = DIVNS*CTR*CTR
     1   *.5
      Q1 = Q1 + CCC*Y2FI*DIVD
     1   *Y2FI
      Q2 = Q2 + CCC*Y2FI*DIVD
      Q3 = Q3 - CCC*CTR*Y2FI*FDER*DIVD
      Q4 = Q4 + CCC*Y2FI*G(I)*DIVD
      CCC = -CCC/CTR
      R1 = R1 + CCC*V1*FDER
      R2 = R2 + CCC*V2*FDER
      R34 = R34 + CCC*(V3 + V4)*FDER
      R56 = R56 + CCC*(V5 + V6)*FDER
      R79 = R79 + CCC*(V7 + V9)*FDER
      R10 = R10 - CCC*V10*FDER
      R11 = R11-CCC*V11*FDER
      IF (I-NS) 924,925,925
925   CONTINUE
      W(J,1) = (Q1 + R1)*DIVY1 + R2
      W(J,2) = Q2 + R34
      W(J,3) = R79 + Q4
      W(J,4) = R1*DIVY4 + BP*DIVY4*DIVY11*.3333
      W(J,5) = R56
      W(J,6) = 0.
      V(J,1) = RTAU/R
     1   -(Q3 + R10 + R11)*DR/R
     2   -CFD*.5
      W(5,1) = DIVY1
      W(5,2) = 0.
      W(5,3) = 0.
      W(5,4) = 0.
      W(5,5) = -1./Y5
```

```
      W(5,6)= -1./Y6
      V(5,1) = 0.
31    W(6,1) = 0.
      W(6,2) = 1.
      W(6,3) = 0.
      W(6,4) = 0.
      W(6,5) = 0.
      W(6,6) = 0.
      V(6,1) = 0.
30    CONTINUE                                                ALIT4160
C
      MM=MM+1                                                 ALIT2050
168     CONTINUE
169   CONTINUE
C
C     SOLVING SIMULTANEOUS EQUATIONS BY SIMQ SUBROUTINE
C
        NN = 6
      DO 102 J=1,NN
      B(J)=V(J,1)
      DO 102 I=1,NN
      IJ = I + NN*(J-1)
      A(IJ)=W(I,J)
102   CONTINUE
      CALLSIMQ (A,B,NN,KS)
      DO 103 I=1,NN
103   DY(I)=B(I)
165   CONTINUE                                                ALIT2320
166   CONTINUE                                                ALIT2330
      RETURN
      END
      SUBROUTINE PRINT (N,XOUT,YOUT,DY,J)
      DIMENSION YOUT(10),XOUT(200),DY(50),Y(10),YSAVE(10)
     1 ,F(10), A1(10), TF(10)
     5   ,FP(10),GP(10)
      COMMON TURBNO,CF,EN,NPART,C,JRUNGE,NGAM
     1,P00,P01,T00,T01,AMACH,AG,RG
     2    ,AMASS1, AMASS0, T
     3 ,S00, CMAS, CMOM, CENR
     4 ,S0, BP, CM, VISC1
      COMMON ZA, ZB, UJ0, DORIN
     6 ,MASRAT,RZERO,AFAC, CFD, HD, F, A1, AAZERO, TF, FP, GP
     7 ,SDLOSS,ASD
     8 ,AUG1,U1,UCENT
      REAL MASRAT
      X=XOUT(J)
      CALL SUB(X,R,DR,2,RZERO)
C     CHECK FOR INITIAL PRINT
2       IF (J-1) 5,5,100
5     WRITE (6,50)
C
C     PRINT FOR ALL PARTS
100   UJ=YOUT(1)*UJ0
      PH20 = YOUT(4)*P00*.193
     1   - P00*.193
        U0=YOUT(1)*YOUT(2)*UJ0
        SUM=   YOUT(1)*UJ0 +U0
        UR=SUM/UCENT
        TOWALL=YOUT(6)*T00
        TOCENT=TOWALL*YOUT(5)+TOWALL
```

```
      AREA =R**2.
      GO TO (29,30),NPART
 29   WRITE(6,55)X,AREA,PH2O,UO,SUM,UR,YOUT(2),YOUT(3),TOCENT,TOWALL,
     1YOUT(7),YOUT(8)
      GO TO 31
 30   AUG2= 3.1416*((YOUT(4)*POO)/BP)*(YOUT(1)**2.)*AUG1
     1*((R*RZERO)**2.)
      AUG=(AUG2/(AMASS1*U1))
      WRITE(6,56)X,AREA,PH2O,YOUT(6),SUM,UR,AUG,YOUT(3),TOCENT,TOWALL
 31   CONTINUE
      IF (J-100) 40,41,40
 41   WRITE (6,60) UJ
 40   RETURN
 50   FORMAT(///,4X,7HX/RZERO,5X,4HAREA, 5X,4HPH2O,5X,24HUO(FT/SEC) UCE
     1NT(FT/SEC),4X,2HUR,5X,6HLAMBDA,6X,7HDELTA/R,2X,34HTOCENT(DEGR) TOW
     2ALL(DEGR) THETA/RO,4X,5HSHAPE)
 55   FORMAT(3X,F7.3,4X,F6.3,4X,F6.2,5X,F7.1,5X,F7.1,4X,F7.4,4X,F7.3,5X
     1,F7.4,5X,F7.1,5X,F7.1,5X,F7.4,4X,F5.2)
 56   FORMAT(3X,F7.3,4X,F6.3,4X,F6.2,5X,F7.4,5X,F7.1,4X,F7.4,4X,F7.3,5X
     1,F7.4,5X,F7.1,5X,F7.1)
 60   FORMAT (//52H         DELTA/R = 1 -- DIFFERENTIAL EQUATIONS CHANGE,
     115X,5HCL = ,F10.5//)
      END
      SUBROUTINE CALXFG (J,X,FF,GG)
      COMMON TURBNO,CF,FN,NPART,C,JRUNGE,NGAM
     1,POO,PO1,TOO,TO1,AMACH,AG,RG
     2     ,AMASS1, AMASSO, T
     3 ,SOO, CMAS, CMOM, CENR
     4 ,SO, BP, CM, VISC1
      COMMON ZA, ZB, UJO, DORIN
     6 ,MASRAT,RZERO,AFAC, CFO, HO, F, A1, AAZERO, TF, FP, GP
     7,SDLOSS,ASD
     8 ,AUG1,U1,UCENT
      REAL MASRAT
      DIMENSION X(3), FF(3), GG(3), F(10)
     1   ,AN(10) ,D(10) ,D2N(10) ,D1D(10) ,D2D(10)
     1 ,A1(10), TF(10)
     5    ,FP(10),GP(10)
      UR = X(1)
      Y2 = X(2)
      DOR = X(3)
      Z1 = 0.
      Z2 = 0
      Z3 = 0
      Z4 = 0
      Z5 = 0
      Z6 = 0
      CTR = -.05
      S10 = 1./(1.-SOO*UR*UR*Y2*Y2)
      S20 = Y2*S10
      DS20Y2 = S10 +S20*S20*2.*SOO*UR*UR
      DS20UR = S20*S20*Y2*SOO*UR*2.
      DO 89 I=1,10
      CTR = CTR + .1
      FS = TF(I)
      GO TO (60,61,62) ,J
 60   AN(I) = Y2 +F(I)
      D2N(I) = 1.
      GO TO 63
 61   AN(I) = (Y2 + F(I))*(Y2 + F(I))
```

```
      D2N(I) = 2.*(Y2 + F(I))                                          ALIT 380
      GO TO 63                                                         ALIT 390
62    AN(I) = (Y2 + F(I))*(1. + T*FS)                                  ALIT 400
      D2N(I) = 1. + T*FS                                               ALIT 410
63    D(I)=1.+T*FS-SOO*UR*UR*(Y2+F(I))*(Y2+F(I))                       ALIT 420
      D1D(I) = -2.*SOO*UR*(Y2 + F(I))*(Y2 + F(I))                      ALIT 430
      D2D(I) = -2.*SOO*UR*UR*(Y2 + F(I))                               ALIT 440
      Z1 = Z1 + 0.2*AN(I)/D(I)                                         ALIT
     1    *CTR
      Z2 = Z2 -0.2*AN(I)*D1D(I)/(D(I)*D(I))                            ALIT
     1    *CTR
      Z3 = Z3 + 0.2*D2N(I)/D(I)
     1    *CTR
      Z4 = Z4 -0.2*AN(I)*D2D(I)/(D(I)*D(I))
     1    *CTR
89    CONTINUE                                                         ALIT 490
C     THREE EQUATIONS AND THEIR DERIVATIVES WITH RESPECT TO UR, Y2, DOR  ALIT 500
      RHOO = POO/(RG*TOO)
C         POO LOSS AT INLET OR NOT
      PO2 = POO
      PO2 = POO - 9.16*(AMASSO*32.2/2.372)**2.*.072/(32.2*RHOO)
      AGG = AG/(AG-1.)                                                 ALIT 510
      S3 = 1. - SOO*UR*UR*Y2*Y2                                        ALIT 520
      Y4 = S3**AGG*PO2/POO
      DY4UR = -Y4*AGG/S3*2.*SOO*UR*Y2*Y2
      DY4Y2 = -Y4*AGG/S3*2.*SOO*UR*UR*Y2
      GO TO (23,24,241),J                                              ALIT 550
23    FF(J) =(DOR*DOR*UR*(Z1-S20) + UR*S20)*Y4                         ALIT 570
      GG(1) =(DOR*DOR*(Z1-S20) + S20)*Y4                               ALIT 580
     1 +(DOR*DOR*UR*(Z2-DS20UR) + UR*DS20UR)*Y4                        ALIT 590
     2+(DOR*DOR*UR*(Z1-S20) + UR*S20)*DY4UR                            ALIT 600
      GG(2) =(DOR*DOR*UR*(Z3+Z4-DS20Y2) + UR*DS20Y2)*Y4                ALIT 610
     2+(DOR*DOR*UR*(Z1-S20) + UR*S20)*DY4Y2                            ALIT 620
      GG(3) = 2.*DOR*UR*(Z1-S20) *Y4                                   ALIT 630
      DIVDN = 1./CMAS
      GO TO 88                                                         ALIT 640
24    FF(J) =(DOR*DOR*UR*UR*(Z1-S20*Y2)
     1 + UR*UR*Y2*S20)*Y4 + (Y4-1.)*.5/(AGG*SOO)*AFAC
      GG(1) =(DOR*DOR*2.*UR*(Z1-S20*Y2)
     1 + DOR*DOR*UR*UR*(Z2-DS20UR*Y2)
     2 + 2.*UR*S20*Y2 + UR*UR*DS20UR*Y2          )*Y4
     3 + DY4UR*.5/(AGG*SOO)*AFAC
     4 + (DOR*DOR*UR*UR*(Z1-S20*Y2) + UR*UR*Y2*S20)*DY4UR
      GG(2) =(DOR*DOR*UR*UR*(Z3+Z4-DS20Y2*Y2 -S20)                     ALIT
     1 + UR*UR*(Y2*DS20Y2 +S20))*Y4
     3 + DY4Y2*.5/(AGG*SOO)*AFAC
     4 + (DOR*DOR*UR*UR*(Z1-S20*Y2) + UR*UR*Y2*S20)*DY4Y2
      GG(3) = 2.*DOR*UR*UR*(Z1 -S20*Y2)*Y4
      DIVDN = 1./CMOM
      GO TO 88
241   DV1 = 1. - SOO*UR*UR*Y2*Y2                                       ALIT 760
      WV = SOO*TOO/TO1                                                 ALIT 770
      DV2 = 1. -WV*UR*UR*(1. + Y2)*(1. + Y2)                           ALIT 780
      DV2 = 1./DV2                                                     ALIT 790
      FF(J) = DV1*DV2                                                  ALIT 800
      GG(1) = -DV2*SOO*2.*UR*Y2*Y2                                     ALIT 810
     1 +DV1*DV2*DV2*WV*2.*UR*(1.+Y2)*(1.+Y2)                           ALIT 820
      GG(2) = -DV2*SOO*UR*UR*2.*Y2                                     ALIT 830
     1 +DV1*DV2*DV2*WV*UR*UR*2.*(1.+Y2)                                ALIT 840
      GG(3) = 0.                                                       ALIT 850
```

```
      PCONST= (PO1/PO2)**((AG-1.)/AG)
      DIVDN = 1./PCONST
 88    CONTINUE
      GO TO (25,26,27), J
 25   FF(1) = FF(1) -CMAS
      GO TO 28
 26   FF(2) = FF(2) - CMCM
      GO TO 28
 27   CONTINUE
      FF(3) = FF(3) -PCONST
 28   CONTINUE
C        TO NORMALIZE THE FUNCTIONS FF( ) AND ITS DERIVATIVES GG( )
      FF(J) = FF(J)*DIVDN
      DO 111 I=1,3
      GG(I) = GG(I)*DIVDN
111   CONTINUE
C          CALCULATION OF INITIAL VALUE OF DELTA/ R       (APPROXIMATE)
      DDRIN = SQRT ((CMOM + 0.5*Y2*Y2 -Y2*S20)/(Z1-S20*Y2))
      RETURN
      END
      SUBROUTINE SUB(XX,R,DR,J,RZERO)
      DIMENSION A(50), X(50), D(50), DD(50)
      GO TO (2,2,1),J
  1   I=0
C     READ X AND DIAM
C     XX=X/RZERO     ,R=R/RZERO
         WRITE(6,5)
  5   FORMAT(15X,12HPROFILE DATA,//,10X,5HX(FT),9X,7HDIA(FT),/)
 11   I=I+1
      READ (5,10) X(I),A(I)
 10   FORMAT (2F15.4)
      IF (A(I)) 11,12,11
 12   IEND=I-1
      SCALE = 1./RZERO
      DO 50 I=1,IEND
         WRITE(6,10)X(I),A(I)
      A(I)=A(I)*.5*SCALE
 50   X(I)=X(I)*SCALE
      WRITE(6,15)
   15  FORMAT(///)
      I=1
C
C   FIND R AND DR
C
  2   IF (XX-X(I)) 20,23,22
 20   I=I-1
      IF (XX-X(I)) 20,23,23
 23   IB=I
      IA=I+1
      GO TO 24
 22    I=I+1
      IF (XX-X(I)) 25,23,22
 25    IA=I
      IB=I-1
 24    DR=(A(IA)-A(IB))/(X(IA)-X(IB))
      R=A(IB)+(XX-X(IB))*DR
      RETURN
      END
      SUBROUTINE RUNGE (N, X, Y, TOL, YMIN, H, XOUT, MARK)
C          FIRST ORDER DIFF. EQ. ROUTINE--ADJUSTS STEP SIZE
```

```
      DIMENSION Y(10),YMIN(10),TOL(10),SUB(10),XOUT(100),MARK(5)         ALIT 320
      DIMENSION DY(50), YA(50), FA(50), FB(50), FC(50), YKEEP(50)        ALIT 330
      KBTWN = 1                                                          ALIT 340
      KBIG = 1                                                           ALIT 350
      KLOW = 1                                                           ALIT 360
      NCOUNT = 15                                                        ALIT 370
      J = MARK(1)                                                        ALIT 380
      MAX = MARK(2)                                                      ALIT 390
230   DO 250  I = 1, N                                                   ALIT 400
250   SUB(I) = TOL(I)/32.0                                               ALIT 410
10    IF (MAX - J)  20, 30, 30                                           ALIT 420
20    RETURN                                                             ALIT 430
30    A = XOUT(J) - X                                                    ALIT 440
      B = ABS (2.E-6*X)                                                  ALIT 450
      IF (A + B)  40, 35, 35                                             ALIT 460
35    IF (A - B)  50, 50, 60                                             ALIT 470
40    J = J + 1                                                          ALIT 480
      GO TO 10                                                           ALIT 490
 50   CONTINUE                                                           ALIT 500
      CALL PRINT (N, XOUT, Y, DY, J)                                     ALIT 510
      J = J + 1                                                          ALIT 520
      GO TO 10                                                           ALIT 530
60    IF (A - 1.5*H)  70, 70, 80                                         ALIT 540
70    H = A                                                              ALIT 550
      GO TO 1000                                                         ALIT 560
80    IF (A - 3.*H)  90, 1000, 1000                                      ALIT 570
90    H = .5*A                                                           ALIT 580
C                                                                        ALIT 590
C          DO RUNGE-KUTTE-MERSON INTEGRATION                             ALIT 600
C                                                                        ALIT 610
1000  XA = X + H/3.                                                      ALIT 620
      XB = X + .5*H                                                      ALIT 630
      CALL DIFFEQ (N, X, Y, DY)                                          ALIT 640
      X = X + H                                                          ALIT 650
      DO 1030  I = 1, N                                                  ALIT 660
      YKEEP(I) = Y(I)                                                    ALIT 670
      FA(I) = H*DY(I)                                                    ALIT 680
1030  YA(I) = Y(I) + FA(I)/3.                                            ALIT 690
      CALL DIFFEQ (N, XA, YA, DY)                                        ALIT 700
      DO 1040  I = 1, N                                                  ALIT 710
1040  YA(I) = Y(I) + FA(I)/6. + H*DY(I)/6.                               ALIT 720
      CALL DIFFEQ (N, XA, YA, DY)                                        ALIT 730
      DO 1050  I = 1, N                                                  ALIT 740
      FB(I) = H*DY(I)                                                    ALIT 750
1050  YA(I) = Y(I) + .125*FA(I) + .375*FB(I)                             ALIT 760
      CALL DIFFEQ (N,XB, YA, DY)                                         ALIT 770
      DO 1060  I = 1, N                                                  ALIT 780
      FC(I) = H*DY(I)                                                    ALIT 790
1060  YA(I) = Y(I) + .5*FA(I) - 1.5*FB(I) + 2.*FC(I)                     ALIT 800
      CALL DIFFEQ (N,X, YA, DY)                                          ALIT 810
      DO 1130  I = 1, N                                                  ALIT 820
      Y(I) = Y(I) + FA(I)/6. + .666666667*FC(I) + H*DY(I)/6.             ALIT 830
1061  U = Y(I)                                                           ALIT 840
      IF (ABS (U) - YMIN(I))  1130, 1130, 1090                           ALIT 850
1090  KLOW = 2                                                           ALIT 860
      E = .2*ABS (U - YA(I) )                                            ALIT 870
      IF ( E - ABS (TOL(I)*U) )  1110, 1110, 1100                        ALIT 880
1100  KBIG = 2                                                           ALIT 890
      GO TO 1130                                                         ALIT 900
1110  IF ( E - ABS (SUB(I)*U) )  1130, 1120, 1120                        ALIT 910
```

```
1120  KBTWN = 2
1130  CONTINUE
      GO TO (100, 1135), KLOW
1135  GO TO (1180, 1140), KBIG
1140  NCOUNT = NCOUNT - 1
      IF (NCOUNT) 1150, 1150, 1170
1150  PRINT 1160,  X, H
      PRINT 1165, (I, Y(I), DY(I),  I = 1, N)
      RETURN
1160  FORMAT (58H4STEP SIZE HALVED 15 TIMES CONSECUTIVELY SINCE LAST PRI
     1NT  /29H PROGRAM TERMINATED AT    X =  , E16.8, 8H,    H =  , E16.8,
     2//3H   I , 13X, 4HY(I), 16X, 5HDY(I),//)
1165  FORMAT (I3,7X,E16.8,4X,E16.8)
1170  KBIG = 1
      IF (H - B)  1176, 1172, 1172
1172  X = X - H
      H = .5*H
      DO 1174  I = 1, N
1174  Y(I) = YKEEP(I)
      KBTWN = 1
      KLOW = 1
      GO TO 1000
1176  M = 15 - NCOUNT
      PRINT 1178, M, X, H
      PRINT 1165, (I, Y(I), DY(I), I = 1, N)
      RETURN
1178  FORMAT (41H4STEP SIZE BECAME TOO SMALL FOR COMPUTER./20H IT HAS BE
     1EN HALVED , I2, 21H TIMES CONSECUTIVELY./29H PROGRAM TERMINATED AT
     2    X =  , E16.8, 8H,    H =  , E16.8,//3H   I, 13X, 4HY(I), 16X,
     35HDY(I),//)
1180  NCOUNT = 15
      GO TO (1190, 1200), KBTWN
1190  H = 2.*H
1200  KBTWN = 1
      KLOW = 1
C
C           CHECK FOR INTERMEDIATE PRINT OUT
C
 100  XOUT(50)=X
      JK=0
C     CALL PRINT (N,XOUT,Y,DY,JK)
      IF (JK) 10,10,20
 130  FORMAT (5H X = , E16.8, 4X, 4HH = , E16.8, 11X, 1HI, 13X, 4HY(I),
     116X, 5HDY(I)/)
 140  FORMAT (55X, I3, 7X, 2(E16.8, 4X))
      END
      SUBROUTINE SIMQ(A,B,N,KS)
      DIMENSION A(1),B(1)
C
C     ..................................................................
C
C        SUBROUTINE SIMQ
C
C        PURPOSE
C           OBTAIN SOLUTION OF A SET OF SIMULTANEOUS LINEAR EQUATIONS,
C           AX=B
C
C        USAGE
C           CALL SIMQ(A,B,N,KS)
C
```

```
C      DESCRIPTION OF PARAMETERS
C         A - MATRIX OF COEFFICIENTS STORED COLUMNWISE.  THESE ARE
C             DESTROYED IN THE COMPUTATION.  THE SIZE OF MATRIX A IS
C             N BY N.
C         B - VECTOR OF ORIGINAL CONSTANTS (LENGTH N). THESE ARE
C             REPLACED BY FINAL SOLUTION VALUES, VECTOR X.
C         N - NUMBER OF EQUATIONS AND VARIABLES. N MUST BE .GT. ONE.
C         KS - OUTPUT DIGIT
C              0 FOR A NORMAL SOLUTION
C              1 FOR A SINGULAR SET OF EQUATIONS
C
C      REMARKS
C         MATRIX A MUST BE GENERAL.
C         IF MATRIX IS SINGULAR , SOLUTION VALUES ARE MEANINGLESS.
C         AN ALTERNATIVE SOLUTION MAY BE OBTAINED BY USING MATRIX
C         INVERSION (MINV) AND MATRIX PRODUCT (GMPRD).
C
C      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
C         NONE
C
C      METHOD
C         METHOD OF SOLUTION IS BY ELIMINATION USING LARGEST PIVOTAL
C         DIVISOR. EACH STAGE OF ELIMINATION CONSISTS OF INTERCHANGING
C         ROWS WHEN NECESSARY TO AVOID DIVISION BY ZERO OR SMALL
C         ELEMENTS.
C         THE FORWARD SOLUTION TO OBTAIN VARIABLE N IS DONE IN
C         N STAGES. THE BACK SOLUTION FOR THE OTHER VARIABLES IS
C         CALCULATED BY SUCCESSIVE SUBSTITUTIONS. FINAL SOLUTION
C         VALUES ARE DEVELOPED IN VECTOR B, WITH VARIABLE 1 IN B(1),
C         VARIABLE 2 IN B(2),........, VARIABLE N IN B(N).
C         IF NO PIVOT CAN BE FOUND EXCEEDING A TOLERANCE OF 0.0,
C         THE MATRIX IS CONSIDERED SINGULAR AND KS IS SET TO 1. THIS
C         TOLERANCE CAN BE MODIFIED BY REPLACING THE FIRST STATEMENT.
C
C     ..................................................................
C
C
C        FORWARD SOLUTION
C
      TOL=0.0
      KS=0
      JJ=-N
      DO 65 J=1,N
      JY=J+1
      JJ=JJ+N+1
      BIGA=0
      IT=JJ-J
      DO 30 I=J,N
C
C        SEARCH FOR MAXIMUM COEFFICIENT IN COLUMN
C
      IJ=IT+I
      IF(ABS(BIGA)-ABS(A(IJ))) 20,30,30
   20 BIGA=A(IJ)
      IMAX=I
   30 CONTINUE
C
C        TEST FOR PIVOT LESS THAN TOLERANCE (SINGULAR MATRIX)
C
      IF(ABS(BIGA)-TOL) 35,35,40
```

```
   35 KS=1
      RETURN
C
C        INTERCHANGE ROWS IF NECESSARY
C
   40 I1=J+N*(J-2)
      IT=IMAX-J
      DO 50 K=J,N
      I1=I1+N
      I2=I1+IT
      SAVE=A(I1)
      A(I1)=A(I2)
      A(I2)=SAVE
C
C        DIVIDE EQUATION BY LEADING COEFFICIENT
C
   50 A(I1)=A(I1)/BIGA
      SAVE=B(IMAX)
      B(IMAX)=B(J)
      B(J)=SAVE/BIGA
C
C        ELIMINATE NEXT VARIABLE
C
      IF(J-N) 55,70,55
   55 IQS=N*(J-1)
      DO 65 IX=JY,N
      IXJ=IQS+IX
      IT=J-IX
      DO 60 JX=JY,N
      IXJX=N*(JX-1)+IX
      JJX=IXJX+IT
   60 A(IXJX)=A(IXJX)-(A(IXJ)*A(JJX))
   65 B(IX)=B(IX)-(B(J)*A(IXJ))
C
C        BACK SOLUTION
C
   70 NY=N-1
      IT=N*N
      DO 80 J=1,NY
      IA=IT-J
      IB=N-J
      IC=N
      DO 80 K=1,J
      B(IB)=B(IB)-A(IA)*B(IC)
      IA=IA-N
   80 IC=IC-1
      RETURN
      END
      SUBROUTINE VB04A (M,N,F,X,E,A,ESCALE,IPRINT,MAXKFG,IA,W)
      DIMENSION  A(3,10), W(10), F(3), E(3), X(3)
      IC=0
      IKFG=0
      IP=IPRINT
      IPSET=-IPRINT*(IPRINT-1)
      NN=N+N
      NP=N+1
      NNP=N+NP
      ICON=1
      IFS=1
      IS=2
```

```
    1 KK=N
      DO 3 I=1,N
      KK=KK+1
      DO 2 J=1,I
    2 A(I,J)=0.
    3 W(KK)=0.
      FF=0.
      IKFG=IKFG+1
      DO 4 K=1,M
      CALL CALXFG (K,X,F,W)
      KK=N
      DO 5 I=1,N
      KK=KK+1
      DO 6 J=1,I
    6 A(I,J)=A(I,J)+W(I)*W(J)
    5 W(KK)=W(KK)-W(I)*F(K)
    4 FF=FF+F(K)*F(K)
      GO TO (7,8),IS
    8 DC 45 I=2,N
      IK = I - 1
      DC 45 J=I,N
   45 A(IK,J) = A(J,IK)
      CALL SPNIST(A,N,N,IK)
      KK=NP
      DO 9 I=1,N
      W(I)=W(KK)
    9 KK=KK+1
      EM=0.
      DO 10 I=1,N
      W(KK)=0.
      DO 11 J=1,N
   11 W(KK)=W(KK)+A(I,J)*W(J)
      EM = MAX1(EM,ABS(W(KK)/E(I)))
   10 KK=KK+1
      IF (EM-ESCALE) 12,12,13
   13 EM=ESCALE/EM
      KK=NNP
      DO 14 I=1,N
      W(KK)=W(KK)*EM
   14 KK=KK+1
      GO TO 15
   12 IF (EM-1.) 16,16,15
   16 GO TO (27,18),ICON
   27 ICON=2
      GO TO 17
   18 IF (IPRINT) 19,19,20
   19 RETURN
   20 WRITE(6,21)
   21 FORMAT (//5X,18HVB04A FINAL VALUES)
      IFS=2
      GO TO 22
   15 ICON=1
   17 IF (IKFG-MAXKFG) 28,29,29
   29 WRITE(6,30)IKFG
   30 FORMAT (//5X,5HVB04A,I5,17HM CALLS OF CALXFG)
      GO TO 18
   28 IP=IP+IPRINT
      IF (IP) 23,23,22
   22 WRITE(6,26)IC,IKFG
   26 FORMAT (//5X,9HITERATION,I4,I10,17HM CALLS OF CALXFG)
```

```
      WRITE(6,24)(X(I),I=1,N)                                          ALIT2810
   24 FORMAT (/5X,9HVARIABLES/,(5E24.14))                               ALIT2820
      WRITE(6,25)(F(I),I=1,M)                                          ALIT2830
   25 FORMAT (/5X,9HFUNCTIONS/,(5E24.14))                               ALIT2840
      GO TO (38,19),IFS                                                ALIT2850
   38 IP=IPSET                                                         ALIT2860
   23 IC=IC+1                                                          ALIT2870
      ITEST=3                                                          ALIT2880
      FFX=FF                                                           ALIT2890
      XP=0.                                                            ALIT2900
      XC=0.                                                            ALIT2910
      IS=1                                                             ALIT2920
    7 GG=0.                                                            ALIT2930
      KK=NNP                                                           ALIT2940
      DO 31 I=NP,NN                                                    ALIT2950
      GG=GG-W(I)*W(KK)                                                 ALIT2960
   31 KK=KK+1                                                          ALIT2970
      GG=GG+GG                                                         ALIT2980
      CALL VD02A (ITEST,XC,FF,GG,6,0.,0.3,1.)                          ALIT2990
      GO TO (32,33,33,33,33),ITEST                                     ALIT3000
   32 XP=XC-XP                                                         ALIT3010
      KK=NNP                                                           ALIT3020
      DO 34 I=1,N                                                      ALIT3030
      X(I)=X(I)+XP*W(KK)                                               ALIT3040
   34 KK=KK+1                                                          ALIT3050
      XP=XC                                                            ALIT3060
      GO TO 1                                                          ALIT3070
   33 IF (FF-FFX) 39,40,40                                             ALIT3080
   39 IF(ABS(EM*XC) - 1.)36,36,37                                      ALIT3090
   37 ICON=1                                                           ALIT3100
   36 IF (XP-XC) 35,8,35                                               ALIT3110
   35 IS=2                                                             ALIT3120
      GO TO 32                                                         ALIT3130
   40 WRITE(6,41)                                                      ALIT3140
   41 FORMAT (//5X,33HVB04A ACCURACY CANNOT BE ACHIEVED)               ALIT3150
      GO TO 18                                                         ALIT3160
      END                                                              ALIT3170
      SUBROUTINE VD02A (ITEST,X,F,G,MAXFUN,ABSACC,RELACC,XSTEP)        ALIT3180
      ABSF(XYZ)=ABS(XYZ)                                               ALIT3190
      SIGNF(ABC,XYZ)=SIGN(ABC,XYZ)                                     ALIT3200
      SQRTF(XYZ)=SQRT(XYZ)                                             ALIT3210
      GO TO (1,2,2),ITEST                                              ALIT3220
    2 IS=ITEST                                                         ALIT3230
      ITEST=1                                                          ALIT3240
      IINC=1                                                           ALIT3250
      XINC=XSTEP                                                       ALIT3260
      MC=1                                                             ALIT3270
      GO TO (4,4,10),IS                                                ALIT3280
    3 MC=MC+1                                                          ALIT3290
      IF (MC-MAXFUN) 4,4,5                                             ALIT3300
    5 ITEST=4                                                          ALIT3310
   11 F=FA                                                             ALIT3320
      X=XA                                                             ALIT3330
      G=GA                                                             ALIT3340
    4 RETURN                                                           ALIT3350
   10 IS=2                                                             ALIT3360
      MC=0                                                             ALIT3370
    1 IF (G) 6,7,6                                                     ALIT3380
    7 ITEST=5                                                          ALIT3390
      GO TO 4                                                          ALIT3400
    6 GO TO (8,9),IS                                                   ALIT3410
    9 XA=X                                                             ALIT3420
```

```
      FA=F
      GA=G
      IS=1
   12 X=XA-SIGNF(XINC,GA)
      XINC=XINC+XINC
      GO TO 3
    8 IF (F-FA) 13,13,14
   13 DUM=FA
      FA=F
      F=DUM
      DUM=GA
      GA=G
      G=DUM
      DUM=XA
      XA=X
      X=DUM
   14 IF (GA*(X-XA)) 15,16,16
   15 IINC=2
      XINC=X
   16 Z=3.*(FA-F)/(X-XA)+G+GA
      W=Z*Z-G*GA
      IF (W) 20,20,17
   17 W=SIGNF(SQRTF(W),X-XA)
      XP=X-(X-XA)*(G+W-Z)/(G-GA+W+W)
      IF ((XP-XA)*GA) 18,19,20
   18 GO TO (21,22),IINC
   21 IF (ABSF(XP-XA)-ABSF(XINC)) 23,23,12
   22 IF (ABSF(XP-XA)-ABSF(XINC-XP)) 23,23,24
   24 X=0.5*(XINC+XA)
      IF ((X-XA)*(XINC-X)) 25,25,3
   25 ITEST=3
      GC TO 11
   23 X=XP
      IF (ABSF(XP-XA)-ABSF(ABSACC)) 19,19,26
   26 IF (ABSF(XP-XA)-ABSF(XP*RELACC)) 19,19,3
   19 ITEST=2
      GO TO 11
   20 GC TO (12,24),IINC
      END
      SUBROUTINE SPNIST(U,I,J,K)
      DIMENSION U(3,3),V(3,3)
      IF(I - 3) 2,1,2
    2 I = J + K
      WRITE(6,4)
    4 FORMAT(14H0 MODIFY VB04A )
      STOP
    1 V(1,1) = U(2,2)*U(3,3) - U(3,2)**2
      V(2,2) = U(1,1)*U(3,3) - U(3,1)**2
      V(3,3) = U(1,1)*U(2,2) - U(2,1)**2
      V(2,1) = U(3,1)*U(3,2) - U(2,1)*U(3,3)
      V(1,2) = V(2,1)
      V(3,1) = U(2,1)*U(3,2) - U(2,2)*U(3,1)
      V(1,3) = V(3,1)
      V(3,2) = U(3,1)*U(2,1) - U(1,1)*U(3,2)
      V(2,3) = V(3,2)
      DET = U(1,1)*V(1,1) + U(2,1)*V(2,1) + U(3,1)*V(3,1)
      DO 5 L = 1,3
      DO 5 M=1,3
    5 U(L,M) = V(L,M)/DET
      RETURN
      END
```

B. 4 Typical Sets of Input and Output Data

Input Data:

```
10
.9950.9850.9750.9650.9550.9450.9350.9250.9150.9050
     .33      .21
   .000223  1.400000  .000160   .223000
         M=  21.0
    .500         5.2381   147.0         1     1    2.50      .00046
  14.7  552.0  348.0 1267.0  .113   2.375  1.40   53.2
 0.0              .445
 1.6315           .3035
 2.3815           .3037
 3.7686           .4462
 4.2148           .4463
 5.2981           .5796
 0.000000         0.000000
10
.9950.9850.9750.9650.9550.9450.9350.9250.9150.9050
     .33      .21
   .000223  1.400000  .000160   .223000
         M=  17.0 -.02
    .500         5.2981   147.0         2     1    2.00      .00046
  14.7  552.0  348.0 1267.0  .113   1.885  1.40   53.2
```

```
NS = 10
GG(I)'S         0.9950   0.9850   0.9750   0.9650   0.9550   0.9450   0.9350   0.9250   0.9150   0.9050

DUCT LOSS COFFEICIENT =    0.330
            DUCT AREA =    0.210FT**2

   M=  21.0 -.02

DELTAX=    0.5000    SHAPE=  1.400000    RZERO=  0.223000FT
XTUBE=     5.2981    NSUB=   2            THETA=  0.000223FT
TURBNO=  147.0000     NGAM=   1            VISC=   0.000160FT**3/SEC
   AG=     1.4000                           RG=     53.200GFT-LBF/LBM-DEGR

  POO=    14.7000PSIA
  TOO=   552.0000DEGR
  POI=   348.0000PSIA
  TOI=  1269.0000DEGR
AMASSO=    2.3230LBM/SEC
AMASSI=    0.1130LBM/SEC
  ANO7=    0.00046FT**2
 XCORE=    2.5000

F(I)'S         0.9806   0.8589   0.6848   0.5054   0.3482   0.2251   0.1367   0.0768   0.0367   0.0092
CONDITIONS AT BEGINNING OF THE TRANSITION SECTION

   UO=  273.FT/SEC        RHO=0.0698LBM/FT**3        UJOO=  2747.FT/SEC
   TO=  546.DEGR            P= 2026.LBM/FT**2        PRIMARY MOM.=   10.6LBF
LAMBDA=0.0994              PH2O=-17.491N.H2O

ITERATION   0         1M CALLS OF CALXFG

VARIABLES
0.10000000000000E 01      0.99405705929803E-01      0.23294270038605E 00

FUNCTIONS
0.14328718185425E-01     -0.54895021021366E-02     -0.12727778084809E-04

ITERATION   1         2M CALLS OF CALXFG

VARIABLES
```

0.10017890930176E 01 0.97278773784637E-01 0.23825997114187E 00

FUNCTIONS
-0.16507736290905E-04 0.34540961456637E-03 -0.16198988305405E-04

ITERATION 2 3M CALLS OF CALXFG

VARIABLES
0.10017881393433E 01 0.97286164760590E-01 0.23817074298859E 00

FUNCTIONS
0.58956214843420E-06 0.79679296050017E-06 -0.23141410545586E-05

VB04A ACCURACY CANNOT BE ACHIEVED

VB04A FINAL VALUES

ITERATION 3 8M CALLS OF CALXFG

VARIABLES
0.10017862319946E 01 0.97286045551300E-01 0.23817068338394E 00

FUNCTIONS
-0.17686861610855E-05 -0.59759468058473E-05 -0.80994950621971E-05

XX(1)= 1.0017862320 XX(2)= 0.0972860456 XX(3)= 0.2381706834

EN= 0.6539 EN2= 0.6541

SO= 1.1437 BP= 0.1749

UJO= -2751.4FT/SEC UCENT= 3019.1FT/SEC

X/RZERO	AREA	PHZO	UO(FT/SEC)	UCENT(FT/SEC)	UR	LAMBDA	DELTA/R	TACENT(DEGR)	TAWALL(DEGR)	THETA/RO	SHAPE
2.500	0.791	-17.09	267.7	3019.1	1.0000	0.007	0.2118	1267.0	552.0	0.0035	1.40
3.000	0.753	-18.03	275.9	2235.0	0.7403	0.141	0.2857	1041.6	552.0	0.0041	1.38
3.500	0.716	-19.14	285.4	1754.5	0.5811	0.194	0.3580	911.1	552.0	0.0046	1.36
4.000	0.679	-20.52	296.8	1449.3	0.4800	0.257	0.4268	830.1	552.0	0.0049	1.34
4.500	0.644	-22.25	310.5	1245.4	0.4125	0.332	0.4910	776.2	552.0	0.0051	1.32
5.000	0.610	-24.47	327.3	1104.0	0.3657	0.421	0.5490	738.2	552.0	0.0052	1.31
5.500	0.576	-27.31	347.9	1004.2	0.3326	0.530	0.6029	710.4	552.0	0.0051	1.29
6.000	0.544	-30.98	372.7	934.1	0.3094	0.654	0.6494	688.2	552.0	0.0050	1.27

6.500	0.512	-35.72	402.8	887.0	0.2938	0.882	0.6891	672.6	552.0	0.0047	1.26
7.000	0.492	-41.87	439.2	859.0	0.2845	1.046	0.7222	659.3	552.0	0.0045	1.25
7.000	0.492	-41.87	439.2	859.0	0.2845	1.046	0.7222	659.3	552.0	0.0045	1.25

DELTA/R = 1 — DIFFERENTIAL EQUATIONS CHANGE   CL = 419.82104

F2(I)'S	0.9904	0.9301	0.8441	0.7558	0.6789	0.6190	0.5762	0.5474	0.5281	0.4905
G2(I)'S	0.0046	0.0549	0.1308	0.2092	0.2761	0.3360	0.3588	0.3776	0.3869	0.4145

X/RZERO	AREA	PH2O	TOWALL/TOO	UCENT(FT/SEC)	UR	AUGMENT	GAMMA	TOCENT(DEGR)	TOWALL(DEGR)
7.500	0.462	-46.12	1.0054	817.8	0.2709	3.944	0.1875	657.7	555.0
8.000	0.463	-45.98	1.0102	772.0	0.2557	3.924	0.3069	655.0	557.6
8.500	0.463	-46.09	1.0141	734.6	0.2433	3.911	0.4166	652.8	559.8
9.000	0.463	-46.37	1.0172	703.6	0.2331	3.904	0.5169	650.9	561.5
9.500	0.463	-46.77	1.0199	677.9	0.2245	3.901	0.6094	649.3	563.0
10.000	0.464	-47.25	1.0221	655.9	0.2173	3.901	0.6923	647.9	564.2
10.500	0.464	-47.78	1.0239	637.2	0.2110	3.903	0.7694	646.7	565.2
11.000	0.486	-42.13	1.0276	593.9	0.1967	3.692	0.8013	643.5	567.2
11.500	0.523	-34.59	1.0319	543.1	0.1799	3.377	0.8083	638.5	569.5
12.000	0.561	-28.56	1.0353	500.0	0.1656	3.115	0.8130	636.2	571.5
12.500	0.600	-23.66	1.0384	462.9	0.1533	2.836	0.8151	633.3	573.2
13.000	0.640	-19.63	1.0411	430.6	0.1426	2.694	0.8145	630.7	574.7
13.500	0.682	-16.28	1.0435	402.3	0.1333	2.515	0.8112	628.5	576.0
14.000	0.725	-13.46	1.0456	377.4	0.1250	2.349	0.8052	626.5	577.2
14.500	0.769	-11.08	1.0475	355.3	0.1177	2.200	0.7965	624.7	578.2
15.000	0.815	-9.05	1.0491	335.6	0.1112	2.070	0.7854	623.1	579.1
15.500	0.862	-7.30	1.0506	318.0	0.1053	1.952	0.7721	621.7	579.9
16.000	0.911	-5.80	1.0520	302.1	0.1001	1.844	0.7566	620.4	580.7
16.500	0.960	-4.50	1.0532	287.8	0.0953	1.745	0.7394	619.2	591.3
17.000	1.001	-3.59	1.0541	276.7	0.0916	1.679	0.7299	618.3	591.7
17.500	1.001	-3.66	1.0544	274.2	0.0908	1.670	0.7516	618.1	592.0
18.000	1.001	-3.72	1.0546	272.1	0.0901	1.659	0.7724	618.0	592.1
18.500	1.001	-3.79	1.0549	270.1	0.0895	1.659	0.7914	617.8	592.2
19.000	1.014	-3.60	1.0552	265.9	0.0881	1.649	0.8001	617.5	592.5
19.500	1.077	-2.42	1.0563	252.8	0.0837	1.551	0.7699	616.4	593.1
20.000	1.141	-1.41	1.0572	241.0	0.0798	1.461	0.7380	615.4	593.6
20.500	1.208	-0.55	1.0581	230.4	0.0763	1.379	0.7048	614.5	594.1
21.000	1.277	0.20	1.0589	220.7	0.0731	1.304	0.6707	613.7	594.5
21.500	1.347	0.85	1.0597	211.9	0.0702	1.235	0.6361	613.0	584.9
22.000	1.419	1.42	1.0603	203.8	0.0675	1.172	0.6013	612.3	595.3
22.500	1.494	1.91	1.0609	196.3	0.0650	1.113	0.5666	611.7	585.6
23.000	1.570	2.35	1.0615	189.4	0.0627	1.059	0.5323	611.1	596.0
23.500	1.648	2.73	1.0620	182.9	0.0606	1.010	0.4996	610.5	596.3

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TABLE 1

MEASURED PARAMETERS AND INSTRUMENTATION

	Flow Parameter	Instrumentation Used to Measure Parameter	How Recorded	Required for Determining	Data Reduction Procedure
Primary Flow	P_{o1}	Bourdon Tube Gage	Manually	Jet Pump Input Conditions	None Needed
	T_{o1}	Thermocouple and Bridge	Manually	Jet Pump Input Conditions	None Needed
	W_1	Orifice Flow Meter and Panel Gage	Manually	Jet Pump Input Conditions	Standard calibration curves provided by flowmeter manufacturer
Secondary	T_{oo}	Dial Gage in Suction Duct	Manually	Secondary Flow Temperature	None Needed
	$P_{atm} = P_{oo}$	Mercury Barometer	Manually	Atmospheric Pressure	None Needed
	p_b	Manometers	Manually and Photographically	Secondary Flow Rate	See Below
	W_o	Calibrated Bellmouth	Manually	Secondary Flow Rate in lb/min	Equation (48)
Mixing Tube	p vs. length	Manometer Board	Photographically	Mixing Tube and Diffuser Static Pressures	None Needed
	P T_o	Kiel-Temperature Probe Traverse	Manually	Velocity and Temperature Profiles	See Text

TABLE 2

PRESSURE TAP LOCATIONS AND FINAL MIXING TUBE DIMENSIONS

Static Pressure Tap No.	Stagnation Pressure Traverse No.	Pressure Tap Location figure 4 x-inches	Dimensionless Location x/R_o ($R_o = 2.670''$)
1		0.46	0.172
2		2.21	0.828
3		4.71	1.76
4	1	6.71	2.51
5		9.71	3.63
6	2	12.21	4.57
7		14.71	5.51
8	3	17.21	6.45
9		20.46	7.66
10		22.21	8.32
11	4, 5	24.71	9.25
12		27.21	10.19
13		29.71	11.13
14		32.21	12.06
15		34.71	13.0
16		37.21	13.94
17		39.71	14.87
18		42.21	15.81
19		44.46	16.65
20	6	47.21	17.68
21		49.71	18.62
22		51.69	19.36
23		57.69	21.60
24		63.69	23.85
25		69.69	26.09

Measured Mixing Tube Dimensions

x (in)	Dia. (in)	x/R_o
0	5.341	0
19.578	3.643	7.34
28.578	3.645	10.7
45.224	5.355	16.9
50.578	5.356	18.95
63.578	6.956	23.8

TABLE 3

STATIC PRESSURE VALUES MEASURED ALONG THE MIXING TUBE

Entrainment Ratio		17.0	19.4	21.0	23.6
Primary Flow Rate, lbm/min		6.76	6.76	6.76	6.76
Secondary Flow Rate, lbm/min		115.1	131.4	142.0	160.3
Static Pressure Tap No.	x/R_o station	all values in inches of water gage with respect to P_{oo}			
1	0.172	- 7.66	-10.3	-11.8	-15.6
2	0.828	- 8.25	-10.6	-12.4	-16.2
3	1.76	- 8.85	-11.8	-14.15	-18.9
4	2.51	- 9.15	-13.0	-15.6	-20.6
5	3.63	-10.9	-15.0	-18.6	-25.1
6	4.57	-12.1	-17.7	-21.8	-30.1
7	5.51	-14.45	-21.5	-26.8	-37.8
8	6.45	-17.7	-27.4	-34.6	-49.5
9	7.66	-23.0	-36.6	-46.4	-67.5
10	8.32	-22.7	-36.3	-46.4	-68.0
11	9.25	-22.1	-36.3	-46.4	-68.8
12	10.19	-21.8	-36.0	-46.6	-69.3
13	11.13	-17.4	-30.1	-39.6	-62.2
14	12.06	-10.05	-20.3	-28.0	-44.2
15	13.0		-13.55	-20.1	-33.9
16	13.94		- 8.85	-14.45	-26.8
17	14.87	1.4	- 5.4	-10.3	-21.2
18	15.81	3.5	- 2.7	- 7.2	-17.4
19	16.65	5.0	- 0.9	- 5.0	-14.6
20	17.68	5.7	0.3	- 3.6	-12.7
21	18.62	6.0	0.7	- 3.0	-11.4
22	19.36	6.6	1.6	- 2.0	-10.2
23	21.60	9.0	4.6	1.5	- 5.9
24	23.85	10.3	6.4	3.4	- 3.4
25	26.09	11.1	7.5	4.7	- 1.7

TABLE 4

VELOCITY AND TEMPERATURE PROFILES AT TRAVERSE STATION 1

Traverse Probe Position	From Traverse Probe Data			Analytical Predictions for $X_{core}=2.5R_o$			Analytical Predictions for $X_{core}=2.0R_o$		
y/R	p/P	Mach number	T/T_o	To °R	U ft/sec	Mach No.	To °R	U ft/sec	Mach Number
0.949	0.967	0.218	0.991	552	268	.235	552	261	.226
0.837	0.966	0.223	0.990						
0.707	0.965	0.226	0.990						
0.548	0.964	0.229	0.990						
0.447	0.964	0.230	0.990						
0.316	0.949	0.273	0.985				577		.226
0.224	0.830	0.522	0.948	631	296	.245	713	477	.373
0.100	0.544	0.975	0.840	1002	1350	.950	923	1400	1.03
0	0.311	1.45	0.705	1267	3020	2.66	1040	2230	1.80
0.100	0.524	1.01	0.831	1002	1350	.955	923	1400	1.03
0.224	0.897	0.398	0.969	631	296	.245	713	477	.370
0.316	0.962	0.237	0.989	552	268	.235	577	261	.226
0.447	0.962	0.236	0.989				552		.226
0.548	0.963	0.232	0.989						
0.707	0.964	0.229	0.990						
0.837	0.964	0.229	0.990						
0.949	0.966	0.222	0.990						

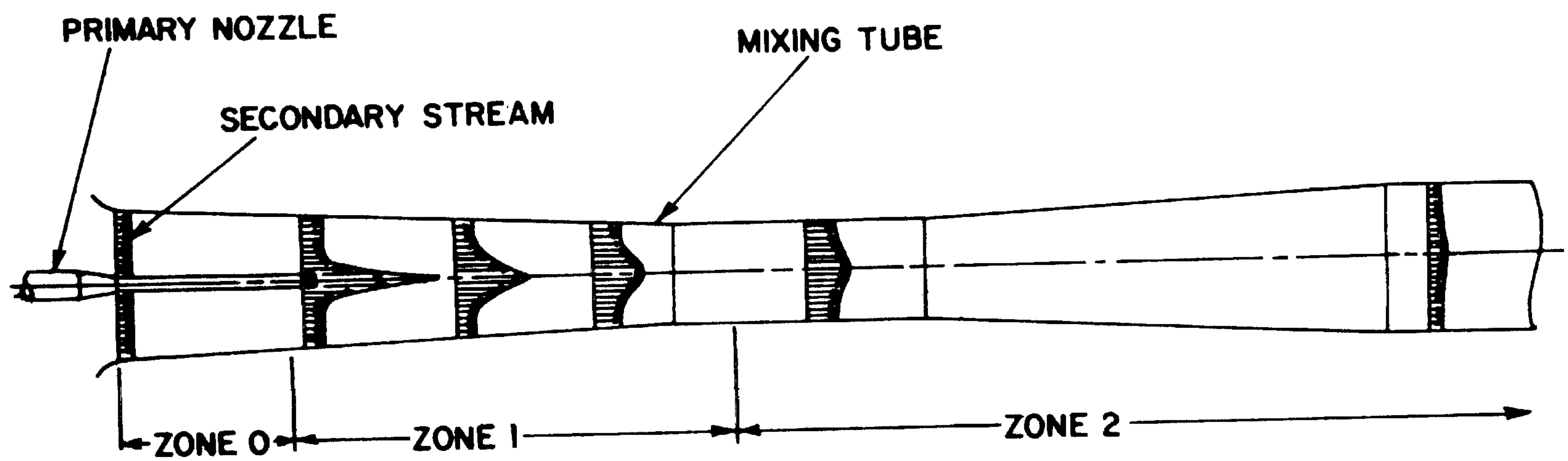

Figure 1 Jet Mixing in a Converging-Diverging Duct

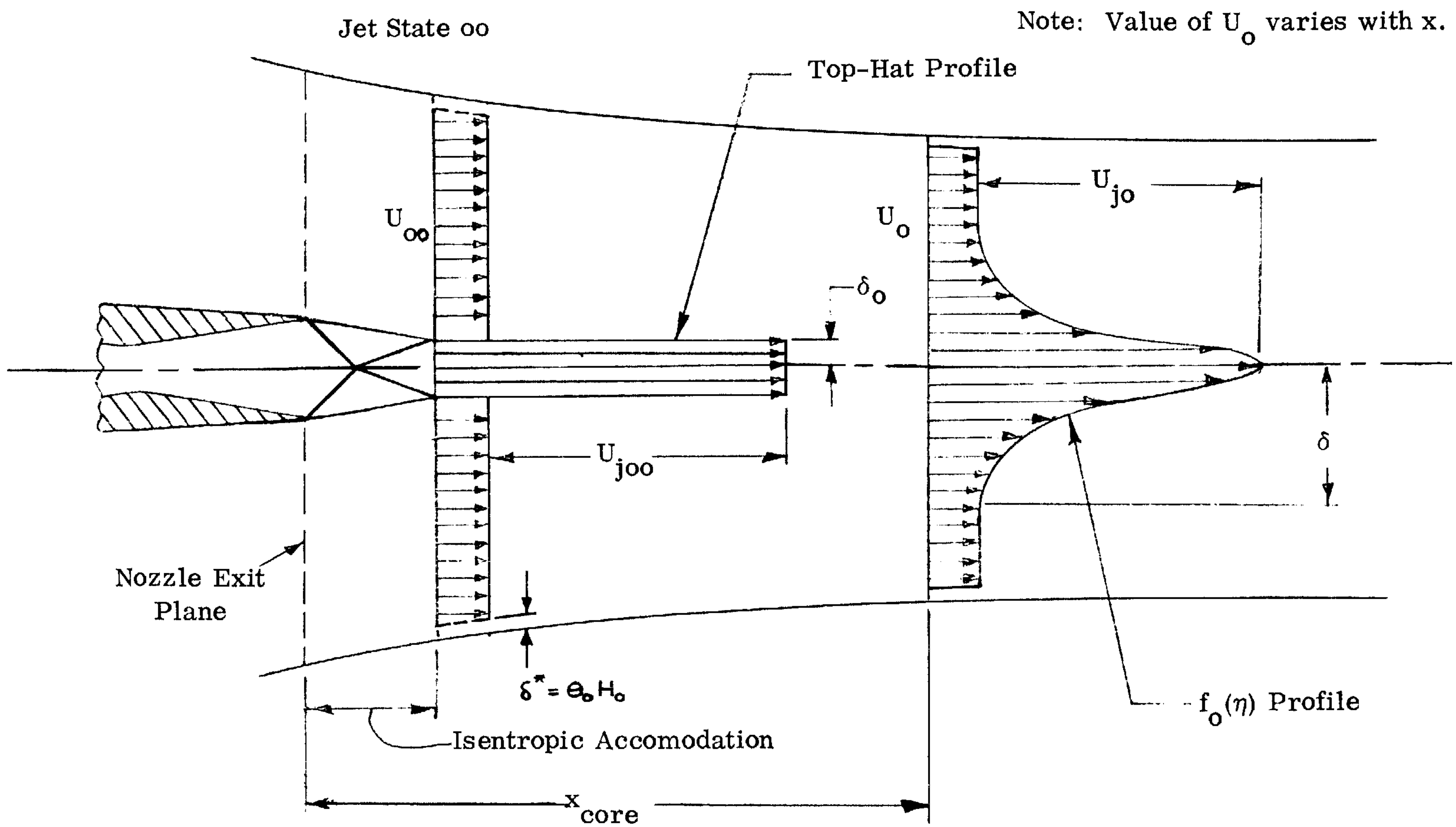

Figure 2. Velocity Profiles in the Transition Zone

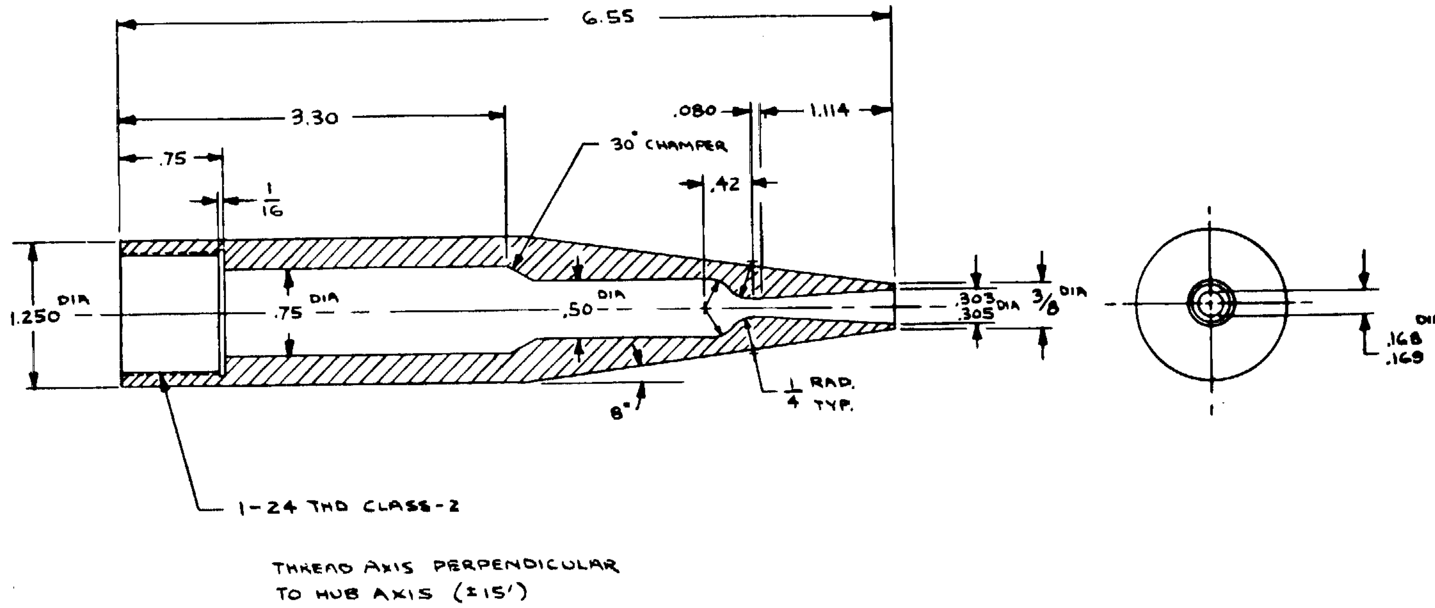

Figure 3 Primary Nozzle Geometry

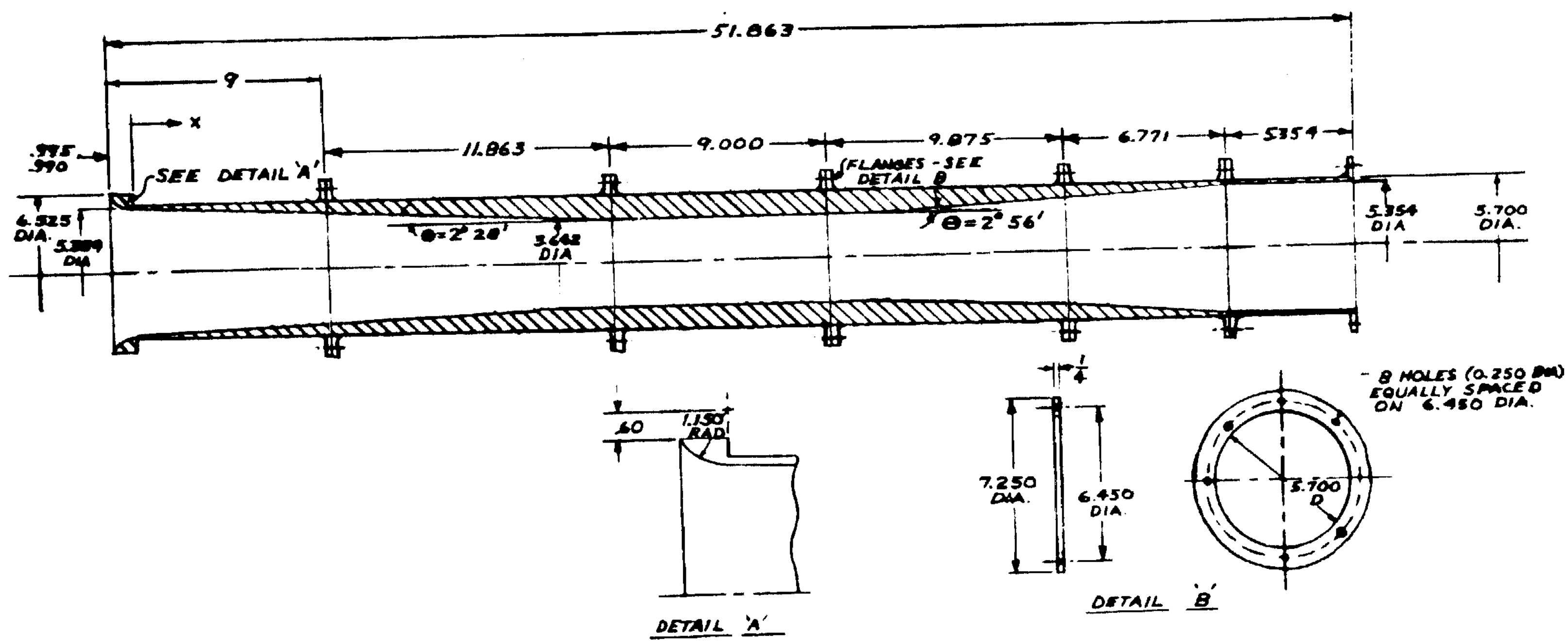

Figure 4 Mixing Tube Geometry

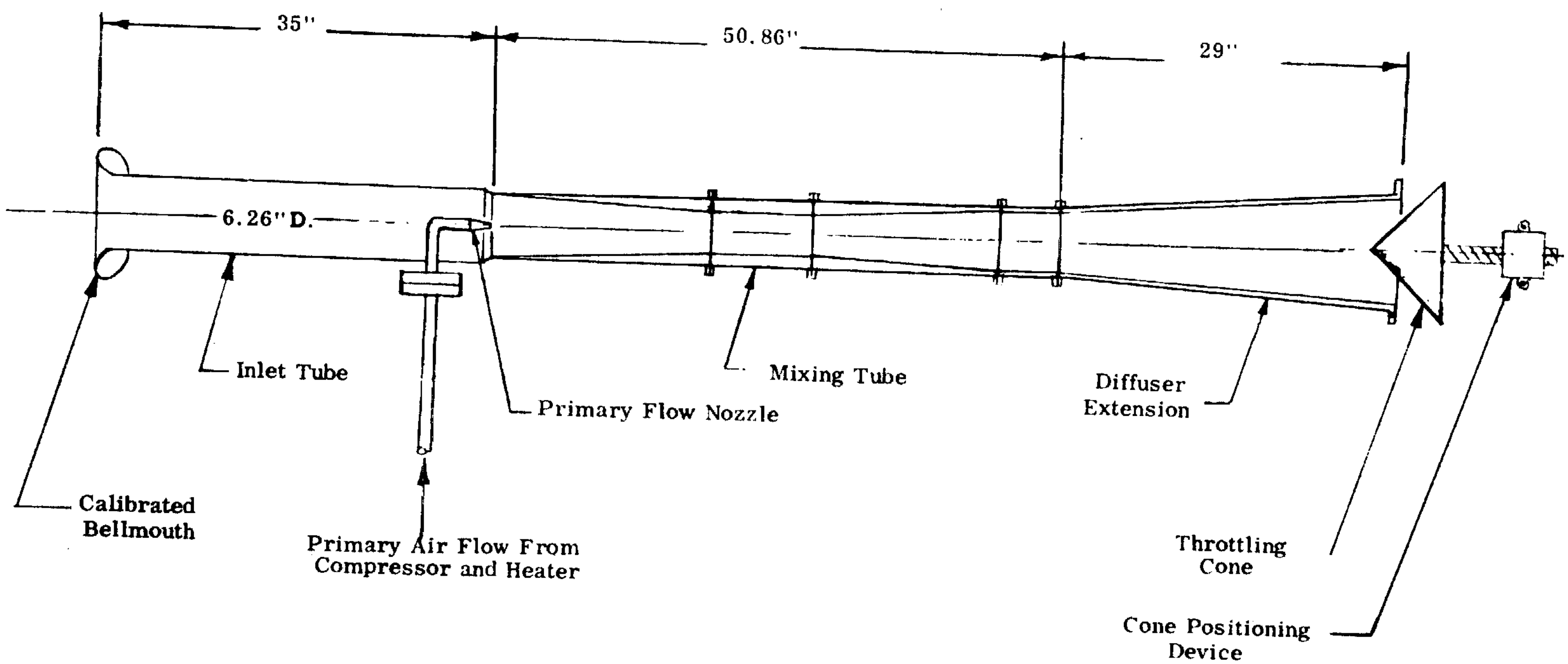

Figure 5 Jet Pump Test Rig

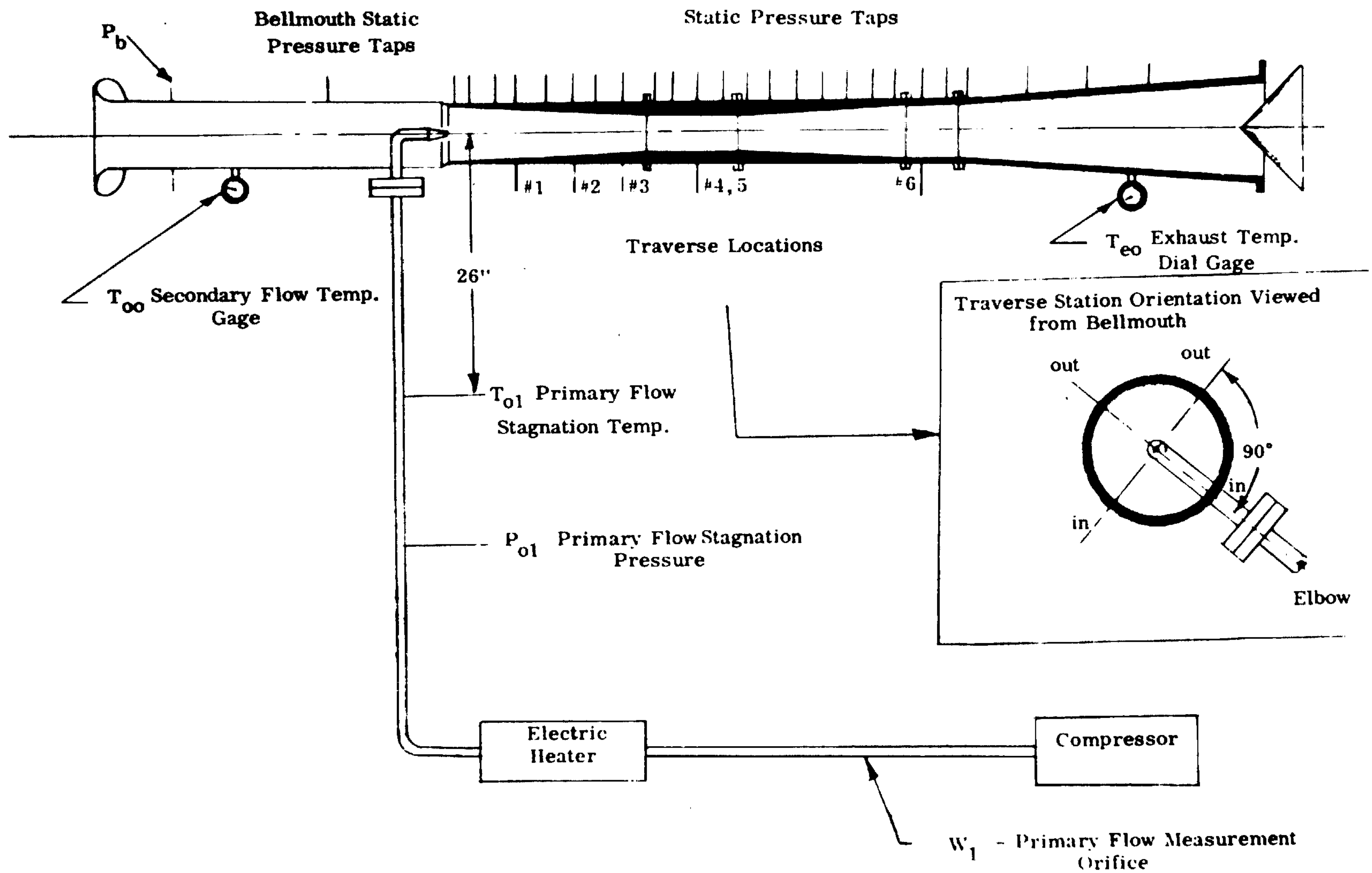

Figure 6 Jet Pump Test Instrumentation

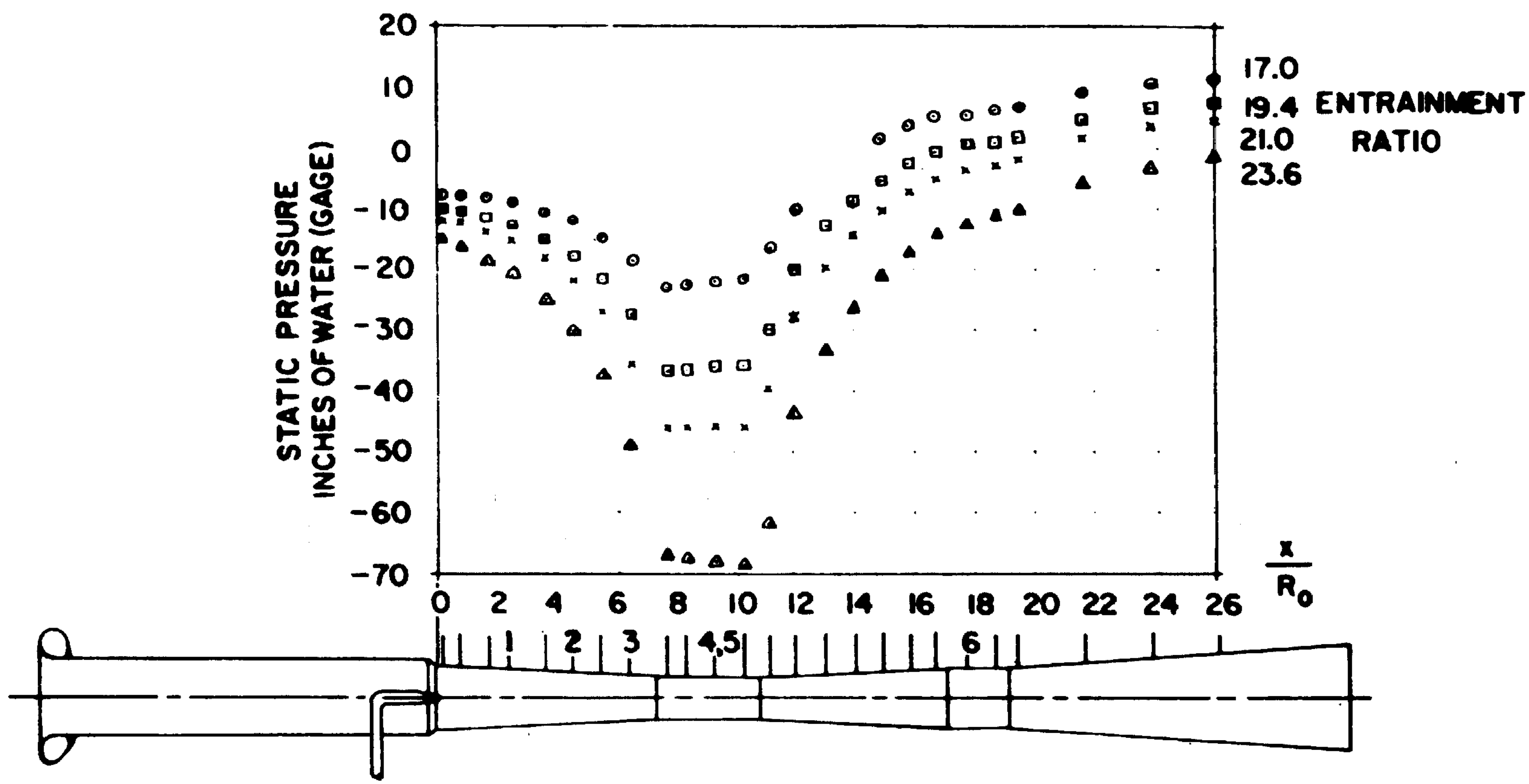

Figure 7 Mixing Tube Static Pressure Variations

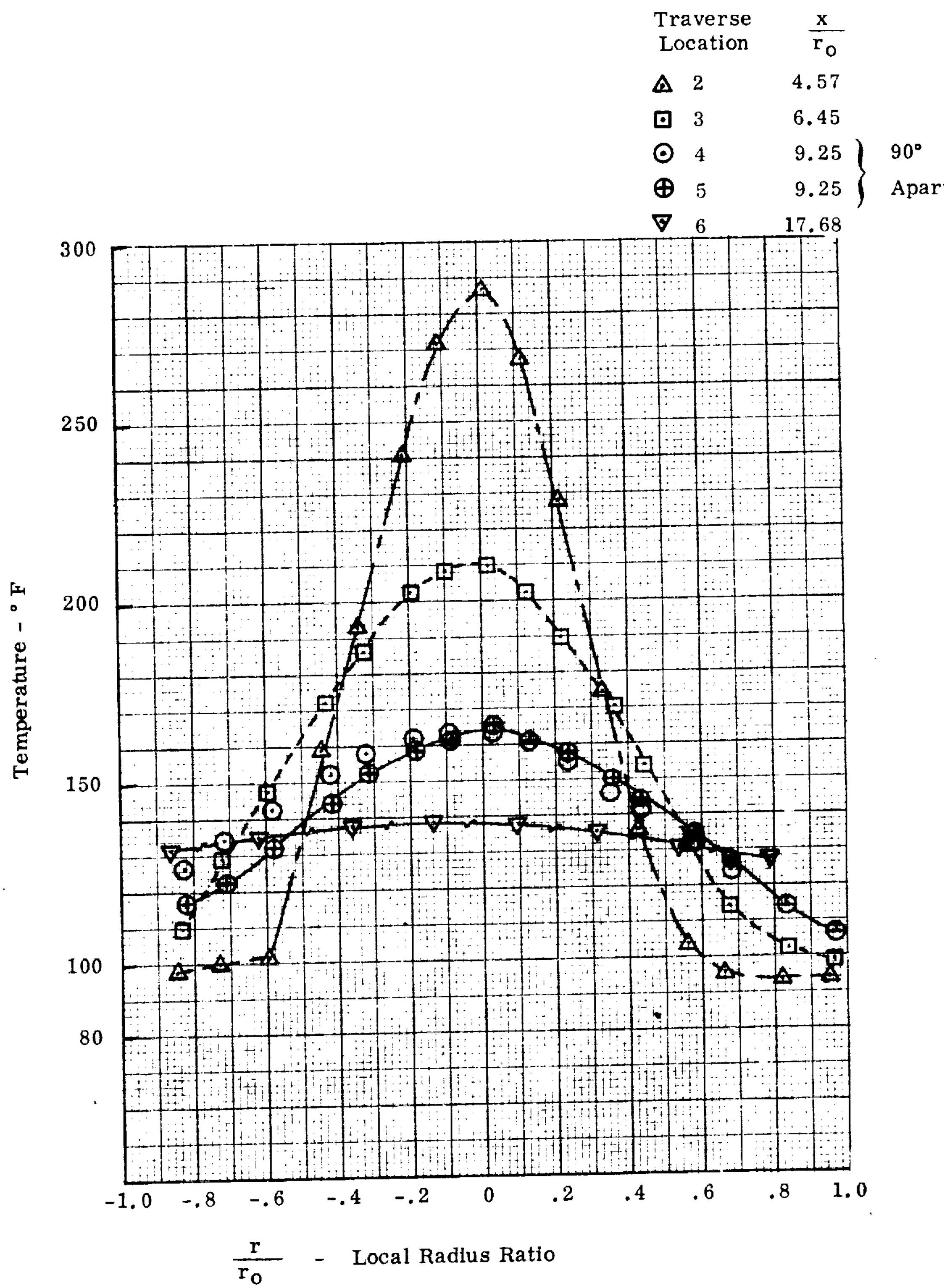

Figure 8 Measured Velocity Profiles in Mixing Tube

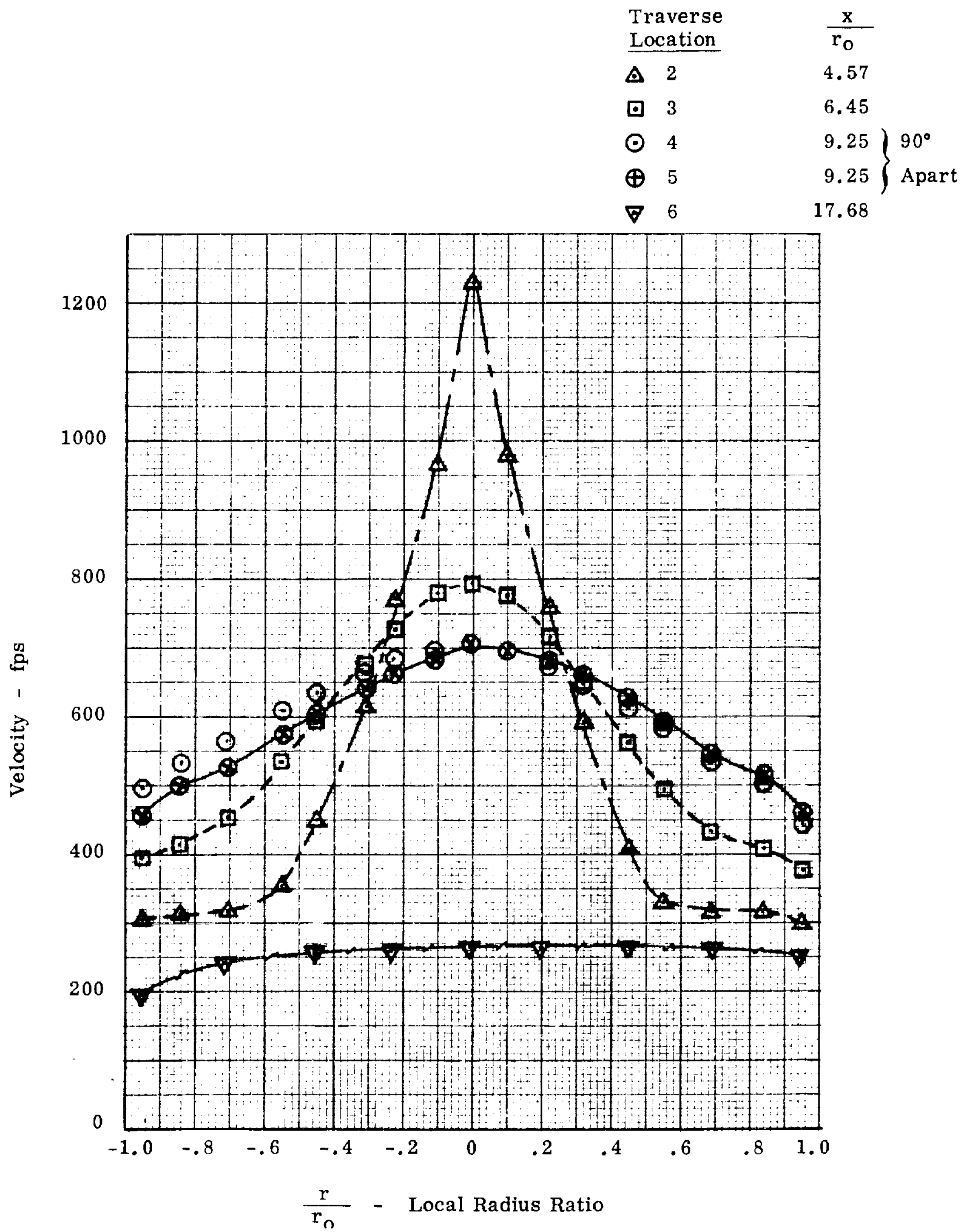

Figure 9 Measured Temperature Profiles in Mixing Tube

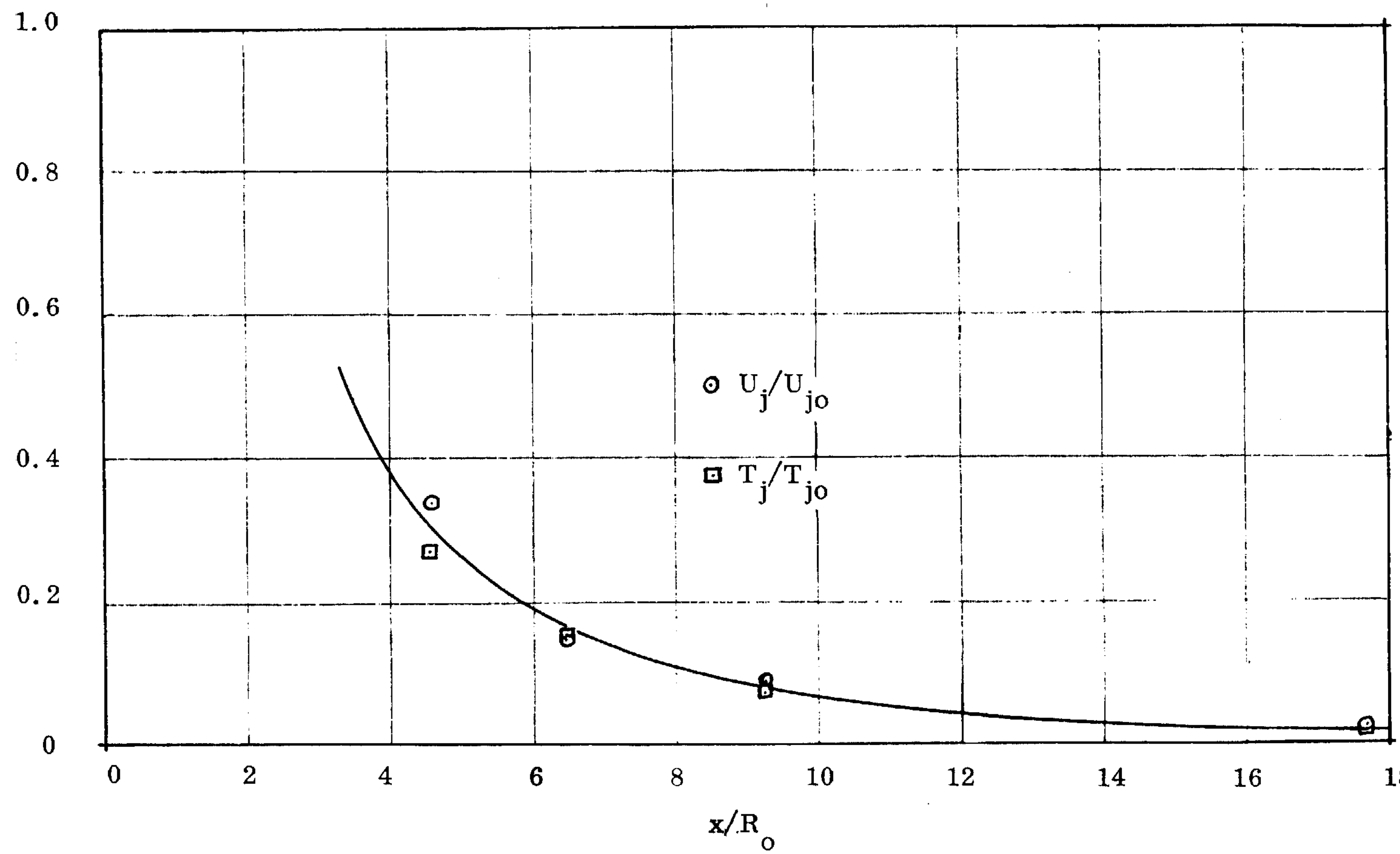

Figure 10. Variation of Centerline Temperature and Velocity along the Mixing Tube (Test Results)

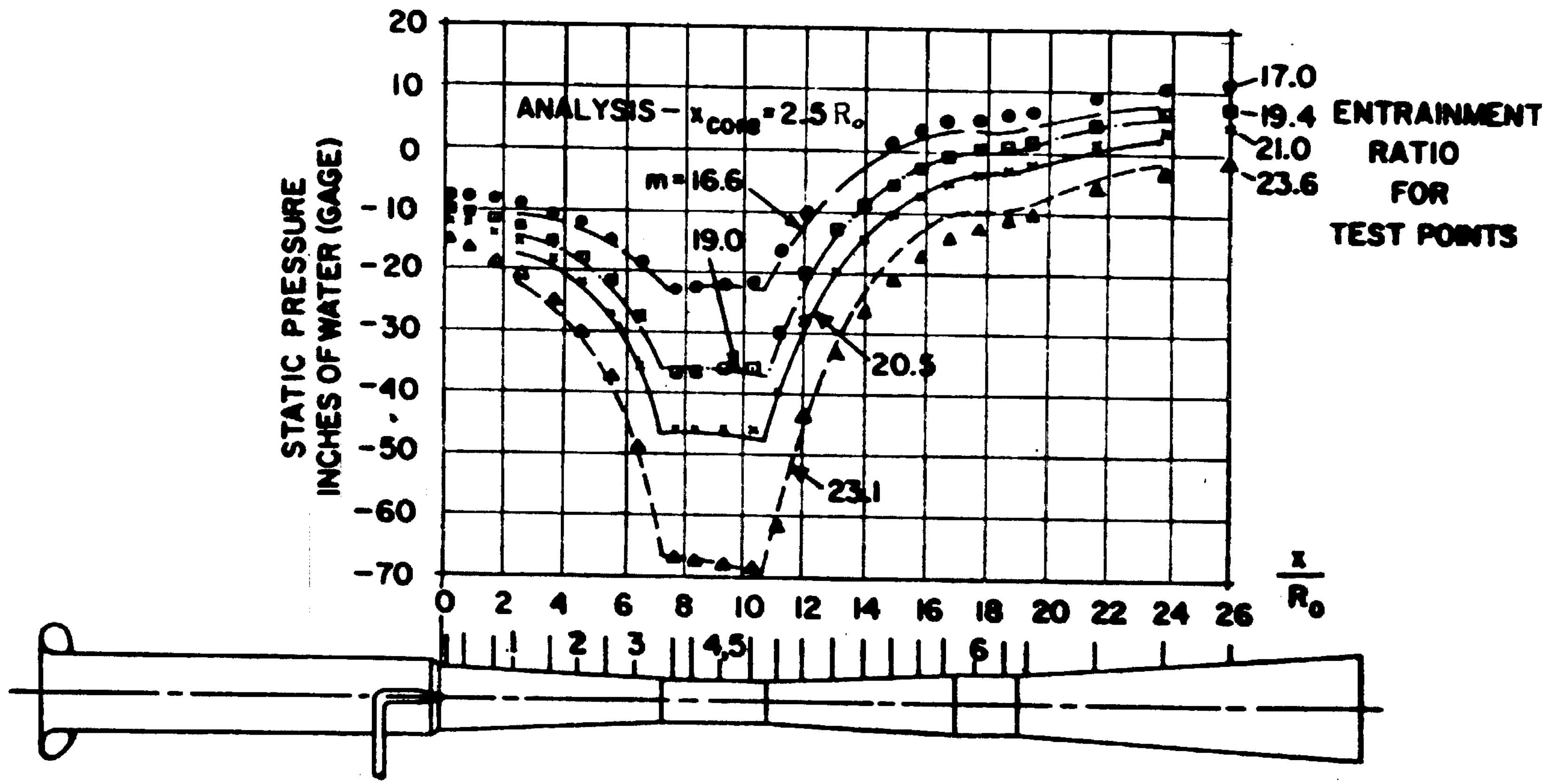

Figure 11 Comparison of Analytical and Experimental Mixing Tube Static Pressure Variations ($\frac{x_{core}}{R_o}$ = 2.5, entrainment reduced by 2%)

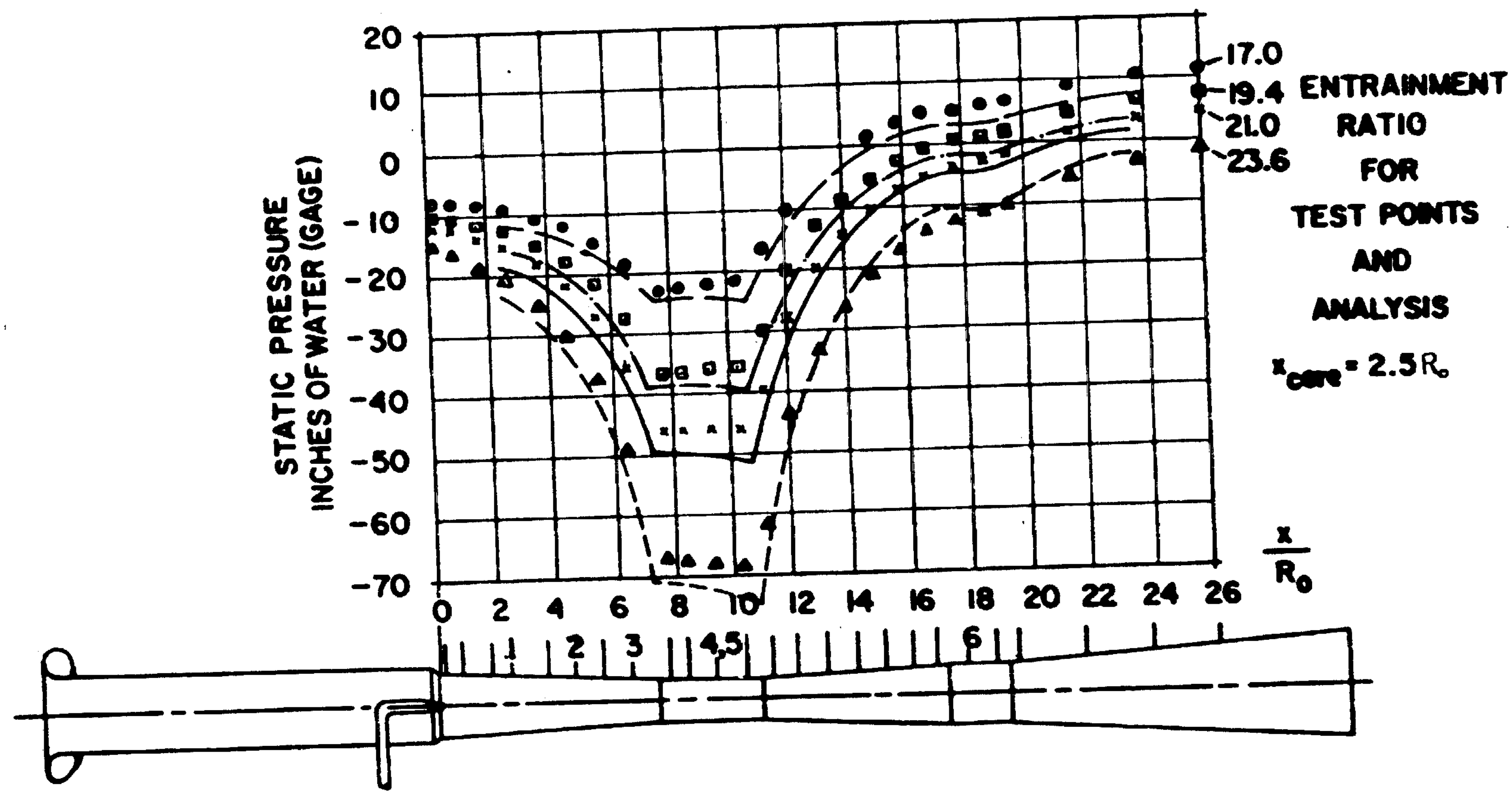

Figure 12 Comparison of Analytical and Experimental Mixing Tube Static Pressure Variations ($\frac{x_{core}}{R_o}$ = 2.5, secondary flow from bellmouth calibration)

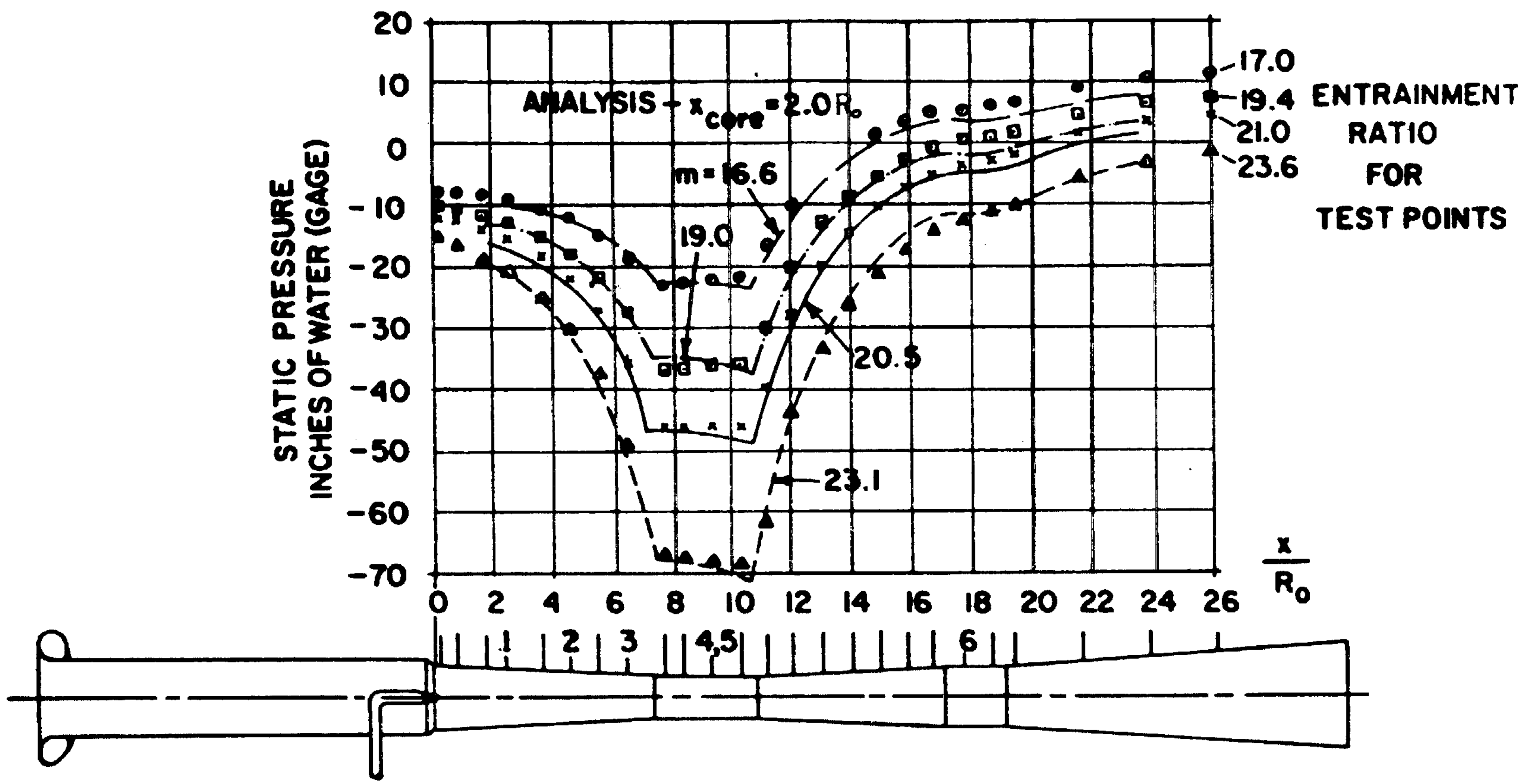

Figure 13 Comparison of Analytical and Experimental Mixing Tube Static Pressure Variations ($\frac{x_{core}}{R_o}$ = 2.0, entrainment reduced by 2%)

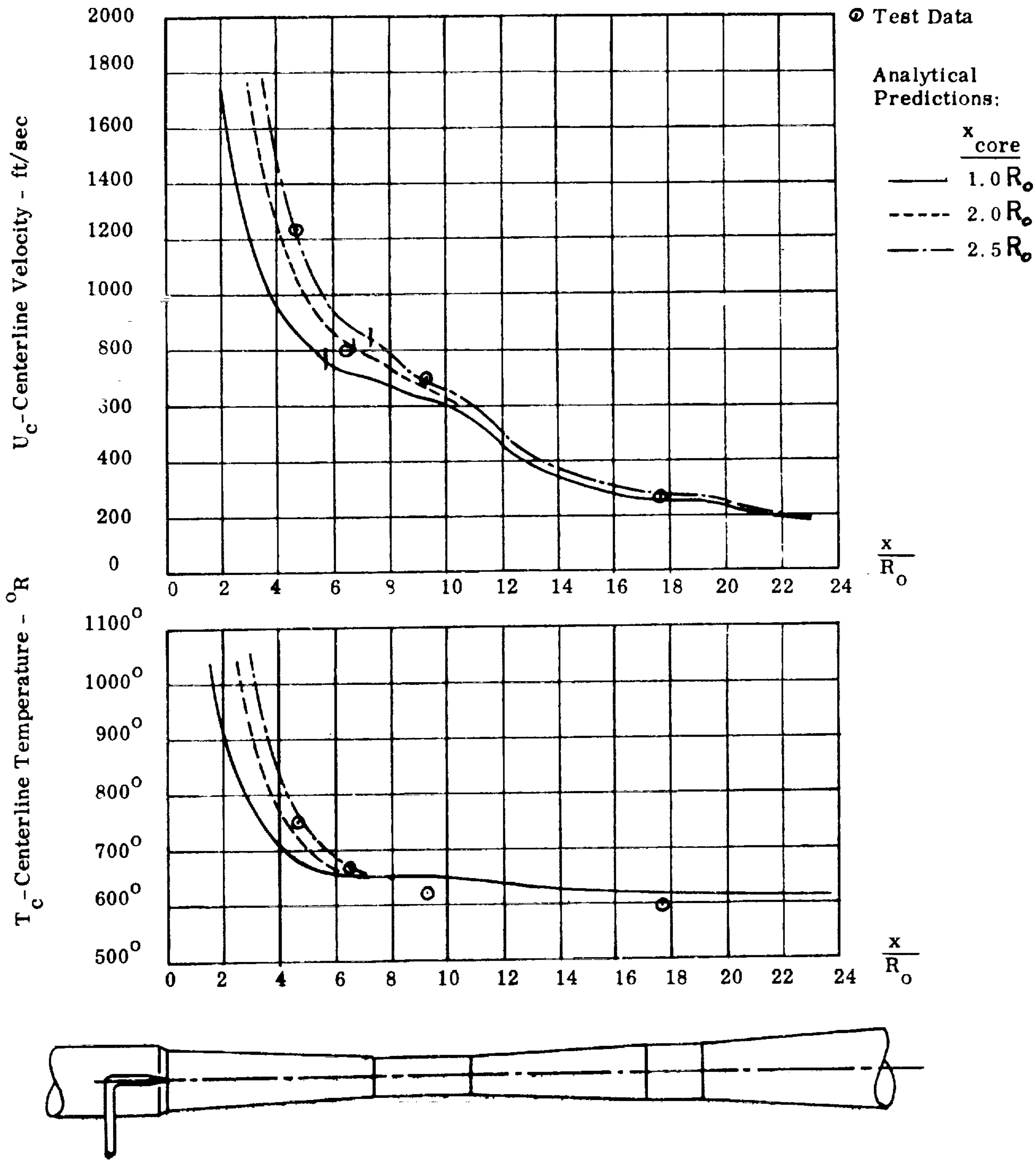

Figure 14 Comparison of Analytical and Experimental Values of Centerline Temperature and Velocity Along the Mixing Tube

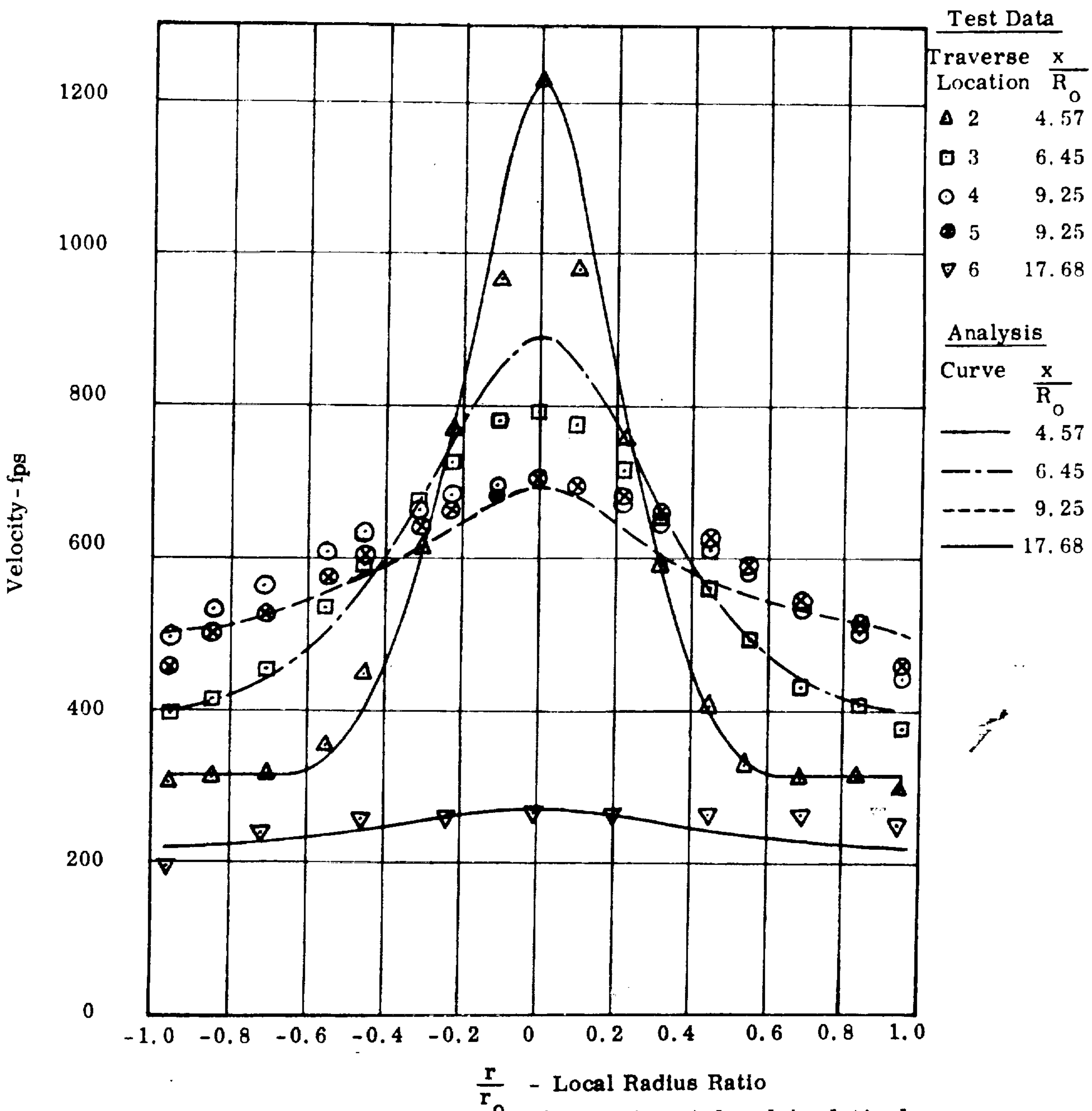

Figure 15 Comparison of Experimental and Analytical Velocity Profiles

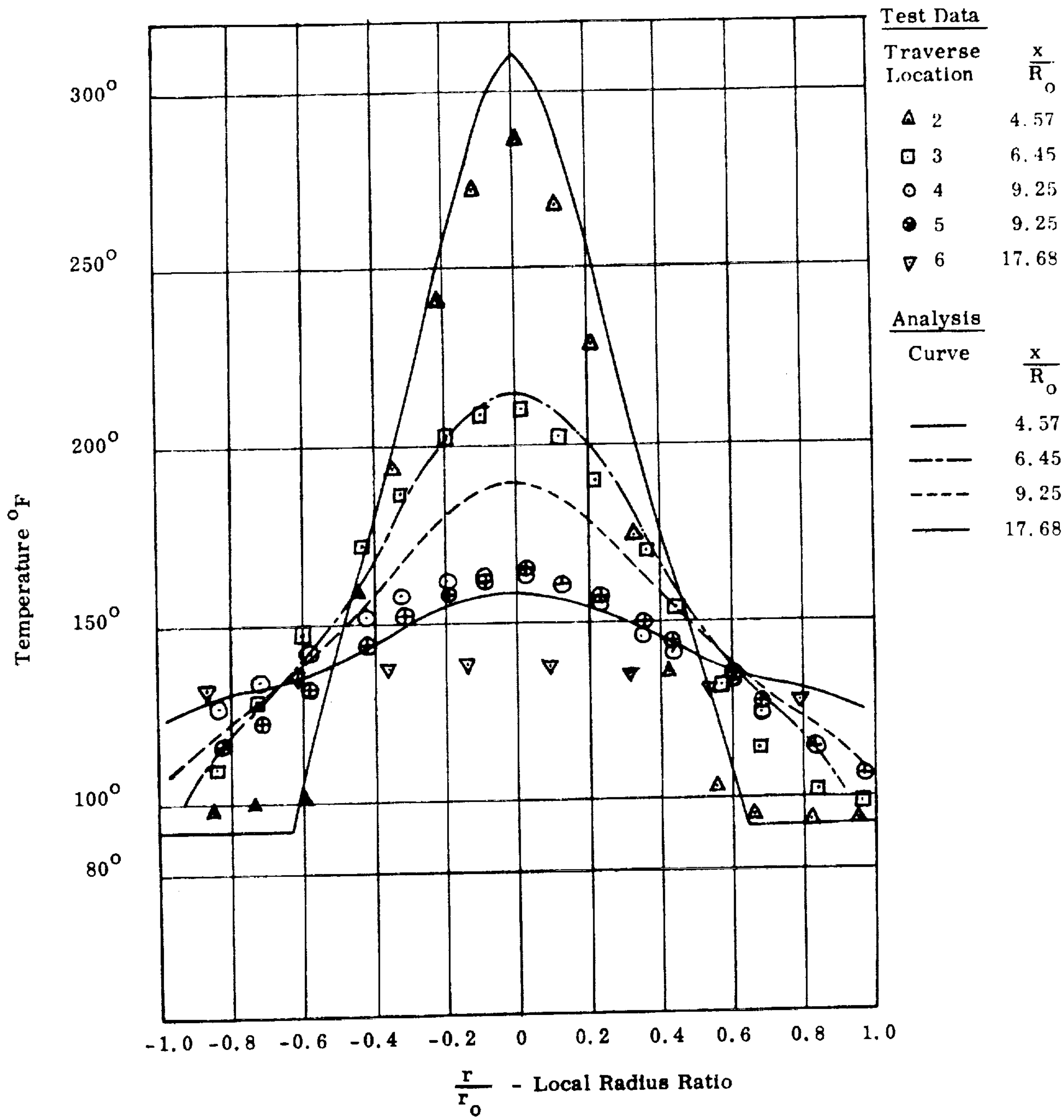

Figure 16 Comparison of Experimental and Analytical Temperature Profiles

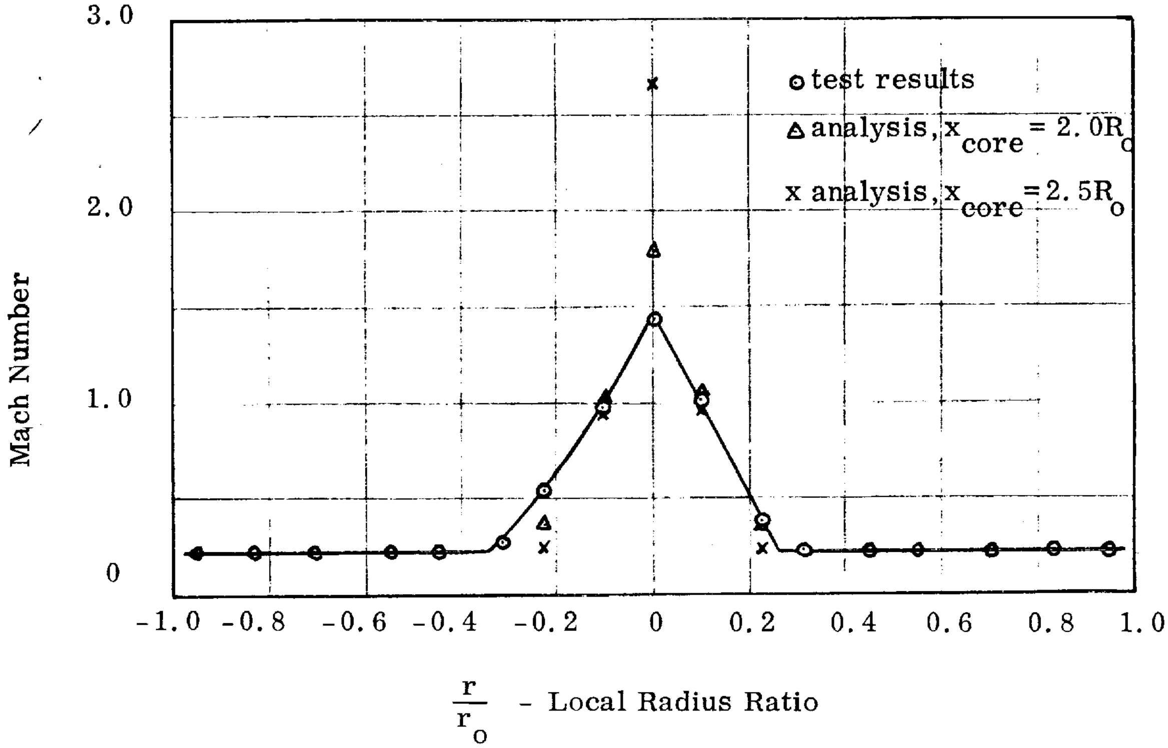

Figure 17. Comparison of "Measured" and Predicted Mach Number Profiles at Traverse Station 1, $\frac{x}{R_o} = 2.51$