INFORMAL FINAL REPORT ON

BLADE FREQUENCY PROGRAM

FOR NONUNIFORM HELICOPTER ROTORS,

WITH AUTOMATED FREQUENCY SEARCH

By S. Gene Sadler

Issued by Originator as RASA Report No. 72-01

Prepared under Contract No. NAS1-11216 by
ROCHESTER APPLIED SCIENCE ASSOCIATES, INC.
Rochester, New York

for Langley Research Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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By S. Gene Sadler
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SUMMARY

A computer program has been implemented which determines the natural frequencies and normal modes of a lumped parameter model of a rotating, twisted beam, with nonuniform mass and elastic properties. The end of the beam near the center of rotation may have one of four types of boundary conditions which are common to helicopter rotor systems; the outboard end has zero forces and moments, i.e., "free" boundary conditions. Six types of motion coupling may be modeled: fully coupled torsional-flatwise-edgewise motion, partially coupled torsional-flatwise motion or flatwise-edgewise motion, and uncoupled torsional motion, flatwise motion, or edgewise motion. Three frequency search methods have been implemented, including an automated search technique which allows the program to find up to the fifteen lowest natural frequencies without the necessity for input estimates of these frequencies by the user.

This report contains computer program documentation information, and is intended to be sufficiently complete to be useful to both engineering users and programmers.

INTRODUCTION

Normal modes and natural frequencies are used frequently in the analysis of elastomechanical systems for determination of instabilities and response to forcing functions. The use of these techniques for analysis of helicopter blades has been done for several years, with increasingly more complex models for both the helicopter rotor system and the aerodynamic load environment in which it operates. The increasing model complexities are tractible because of the implementation of simulation models on large-core high-speed digital computers. The need for more detailed analyses of air loads and blade response is particularly important for helicopters because of their normally complex aerodynamic environment, their elastomechanical characteristics, and because of increasingly more demanding performance requirements. Air loads and blade response are important from the viewpoint of acoustics, blade life, and vibration levels at the helicopter fuselage. Many helicopters have known (or anticipated) forcing function frequencies. In such cases, the blade frequencies and mode shapes themselves are of value in the blade design process. In other cases, the blade normal modes are of use as generalized coordinates in forced response calculations. The predicted response of normal modes is strongly dependent upon the predicted natural frequencies, and for systems such as helicopter blades, coupling between torsional and flatwise or flatwise and edgewise motions can cause significant shift in the predicted natural frequencies, i.e., uncoupled or fully coupled frequencies. The most accurate (presumably the fully coupled) natural frequencies should be used in computing the forced response of such systems. However, there are cases where uncoupled or partially coupled modes and natural frequencies are also of interest.

The theoretical methods and corresponding computer program which are discussed in this report provide an economical, easy-to-use, flexible tool for computing the normal modes and natural frequencies of lumped parameter models of a single helicopter blade (or other similar systems). It was expected that the user of this report and program would have some familiarity with the methods of finding normal modes and natural frequencies of lumped parameter mass-elastic systems, and of their use in the design or analyses of such systems. However, the use of this report with its related references, together with elementary works on vibrating systems, should provide most potential users with sufficient background to make good use of this program.

THEORETICAL FORMULATION

Matrix notation is common in many areas of application. The use of matrix methods in the field of linear elastomechanics has been highly developed. Transfer matrix methods provide a convenient formulation method and an efficient computational procedure for digital computer solution of physical problems which are described as a linear function of two variables, of which one is usually time and another is a spacial variable. Its application is especially well suited for determining the dynamic properties of linear elastic systems which can be considered to be a set of elements linked together end to end, with little or no branching. Beams, shafts, and helicopter blades are examples of systems which are particularly well adapted to analysis by transfer matrix techniques. This technique has been treated extensively in the literature, and the formulation, notation, and methods of solution discussed here are basically those of reference 1. The application of transfer matrix methods to determine the normal modes and natural frequencies of vibrating beams has been developed to the point that these characteristics may be rapidly and accurately determined for essentially any desired range of frequencies. Mode shape and frequency inaccuracies, interdependence of one frequency on previously determined frequencies, and other difficulties which are often encountered in such calculations have been largely overcome by the methods discussed herein. In particular, the basic method discussed herein is referred to as the modified transfer-matrix method, and its development and application are described in detail in Chapters 7 and 11 of reference 1.

Modified Transfer-Matrix Method

Derivation of and examples of the use of general transfer matrices are discussed in detail in reference 1, particularly in Chapter 3. The method utilizes two relatively simple matrices, one which relates dynamical parameters at one point on a structure to those at another point on the structure, and one which is made up of those dynamical parameters of importance for the particular problem which is under consideration; these are referred to as the transfer matrix, [T], and the state vector, (z), respectively. The systems which are considered here have state vectors and transfer matrices as described in the following section.

<u>State vectors and transfer matrices</u>.- The state vector at a point j is a vector, $(z)_j$, whose elements are the deflections (linear and angular) and forces (shears and moments) which exist at that point. The state vector at another point, say j+1, is denoted by $(z)_{j+1}$. The transfer matrix, $[T]_j$, relates to state vector at the point j to that at the point j+1, and the relationship may be stated in the form

$$(z)_{j+1} = [T]_j \, (z)_j \tag{1}$$

The state vector for fully coupled torsion-flatwise-edgewise motion of a helicopter blade has elements given by

$$(z) = (\phi, T, v, \theta, M_z, -V_y, -w, \psi, M_y, V_z)$$

where v and w are deflections in the y and z directions, respectively, ϕ, ψ, and θ are rotations about the x, y, and z axes, respectively, T is the torsional moment about the x axis, M_y and M_z are bending moments about the y and z axes, respectively, and V_y and V_z are shears in the y and z directions, respectively. The right-handed cartesian coordinate system and sign conventions for state vector elements are shown in figure 1 (taken from figure C-1 of reference 1). The transfer matrices for many types of structural elements are given in reference 1 and other literature. Those which are discussed herein are repeated, for convenience, in APPENDIX B, along with a transfer matrix for a point (lumped) inertia and mass in a centrifugal force field.

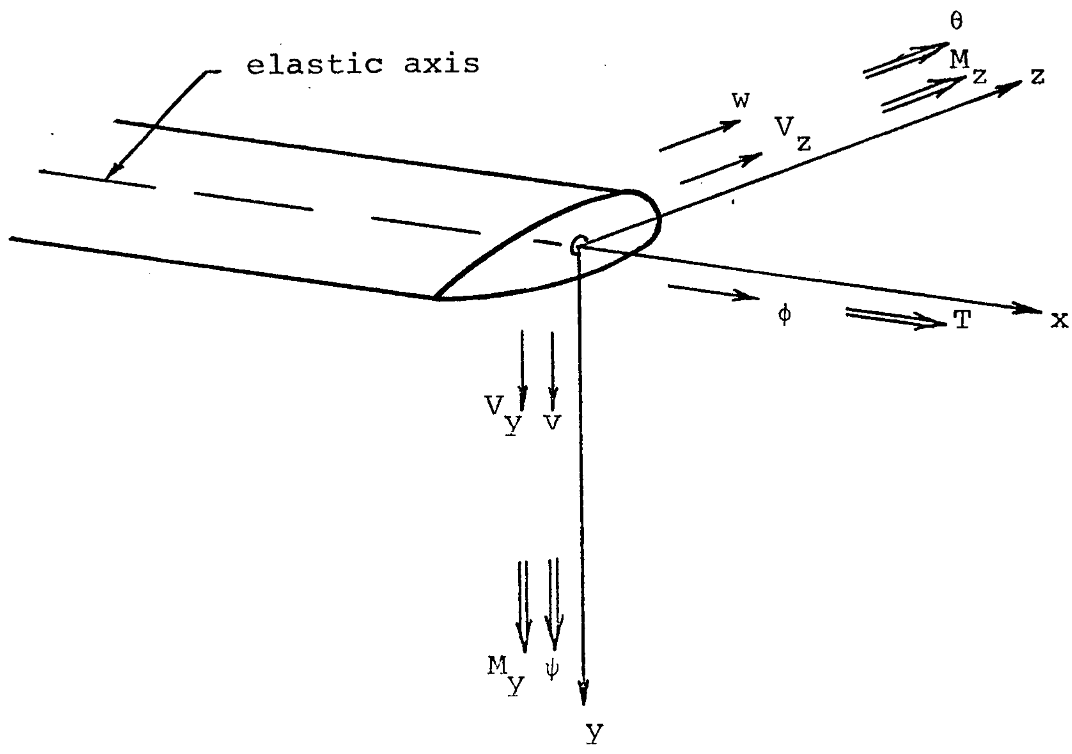


Figure 1. Coordinate system and state vector sign convention

Blade elements and corresponding transfer matrices.- The transfer matrices, [T], are normally a product of field or point matrices which correspond to idealizations of elements of the system which is being analyzed. The typical transfer matrix in this program is the product of the transfer matrices (which may be thought of as occurring in order with increasing radius) for a rigid offset of the elastic axis in the chordwise direction, [C], a point lumped mass and rotary inertia, [M], a massless elastic length, [E], and a rotation about the x-axis, [Φ]. (Detailed definitions of these matrices are given in Appendix B.) That is, in the expression

$$(z)_{j+1} = [T]_j \; (z)_j$$

$$[T]_j = [\Phi]_j \; [E]_j \; [M]_j \; [C]_j \tag{2}$$

Typical elements are sketched in figure 2 for the lumped parameters model, and the signed elements are shown in element number 3 with positive values. That is, ε is positive for the mass center of gravity ahead of the elastic axis, h is positive for the elastic axis ahead of the radial coordinate line, and ℓ_z is positive for an incremental change of elastic axis position which results in increased h with increased r. In the plan view of figure 2 blade twist is neglected, but when it exists the incremental change in twist, $\Delta\phi$, is positive for increased nose-up twist with increased r.

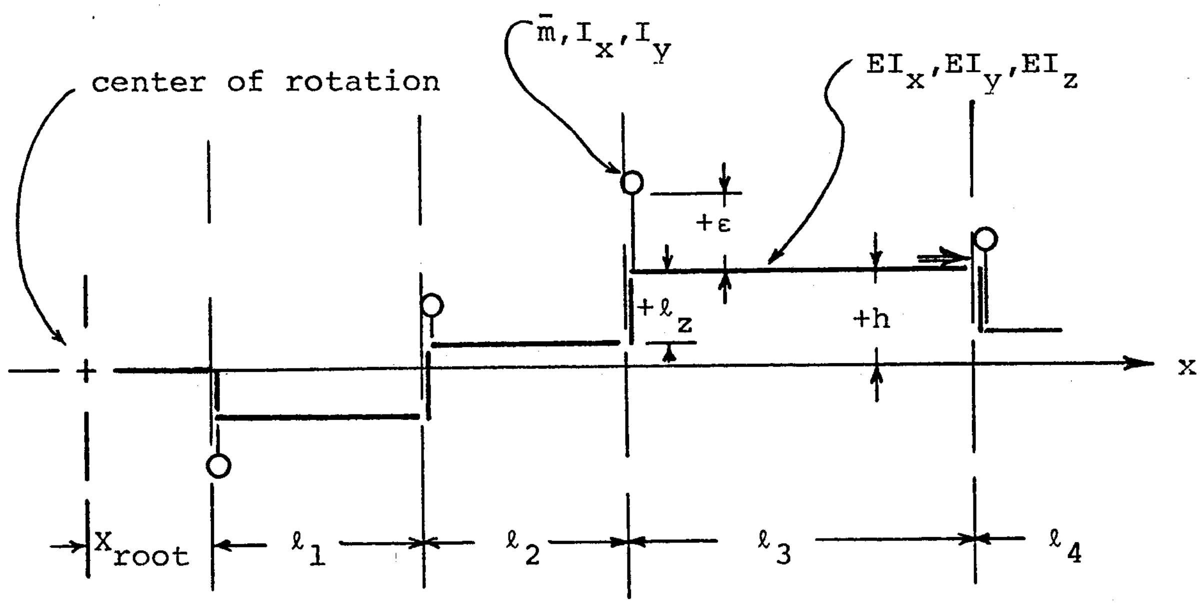


Figure 2. Typical blade elements with signed elements of lumped parameter model indicated

Increasingly more detailed models may be obtained by using more and shorter sections where properties change along the span. For lumped parameter models, it is suggested that at least two or three times as many degrees of freedom be allowed for a particular type of mode than the number of frequencies which are desired for that type of mode. For example, each element corresponds to one degree of freedom in torsion, so if four torsional normal modes and natural frequencies are desired, at least eight to twelve elements should be used in the model. Each element also corresponds to one flatwise degree of freedom and two edgewise degrees of freedom. The use of transfer matrices with distributed mass would reduce the number of elements needed for a given desired number of degrees of freedom. This would result in small savings in core storage and running time. The basis for and a discussion of some of the advantages of the use of such distributed property transfer matrices are contained in reference 2.

Frequency determinant.- The application of transfer matrices in determining normal modes and natural frequencies of a rotating blade is done through their use in defining the characteristic equation for the blade natural frequencies. Consider the blade shown in figure 3, which is made up of N elements, and for which the transfer matrices between adjacent state vectors are known. That is,

$$(z)_1 = [T]_1\,(z)_0,\ (z)_2 = [T]_2\,(z)_1,\ \ldots,\ (z)_N = [T]_N\,(z)_{N-1},$$

and the state vectors at the ends, $(z)_0$ and $(z)_N$ are given for a cantilever - free beam, for example, as

$$(z)_0 = (0,\ T,\ 0,\ 0,\ M_l,\ -V_y,\ 0,\ 0,\ M_y,\ V_z)_0 \tag{3}$$

$$(z)_N = (\phi,\ 0,\ V,\ \theta,\ 0,\ 0,\ -W,\ \psi,\ 0,\ 0,)_N \tag{4}$$

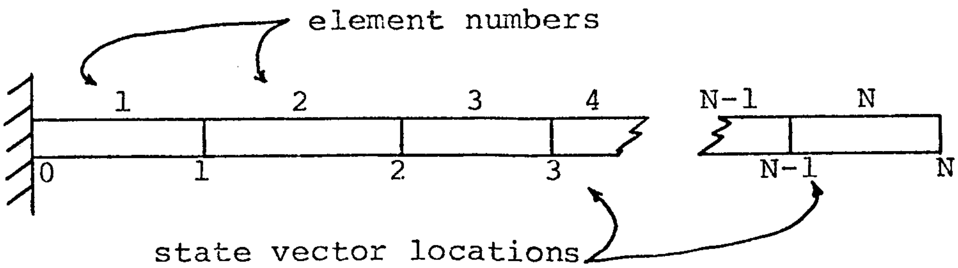


Figure 3. Blade model with N elements

In general, one-half of the elements of the state vectors at the ends of a structure are zero, and it is this fact that is used to obtain the determinental equation. Notice that the intermediate state vectors may be eliminated so that

$$(z)_N = [T]_N \cdot \cdot \cdot [T]_2 \ [T]_1 \ (z)_0 \tag{5}$$

which may be written in the form

$$(z)_N = [P] \ (z)_0 \tag{6}$$

where [P] is the product matrix, which results from the product of the transfer matrices. One of the advatanges of this method over some other methods is that this method can be implemented so that it uses less core storage than most other methods since only two matrices need to be in storage at one time, a matrix which is a product of two or more transfer matrices and the next transfer matrix to be used is defining the product matrix.

Applying the boundary conditions, which are implicit in the state vectors at the ends of the beam, yields the determinant type of characteristic equation. That is, substituting equations (3) and and (4) into (6) results in

$$\begin{pmatrix} \phi \\ 0 \\ v \\ \theta \\ 0 \\ 0 \\ -\omega \\ \psi \\ 0 \\ 0 \end{pmatrix}_N = \begin{pmatrix} P_{11} & P_{12} & P_{13} & \cdots & P_{1,10} \\ P_{21} & P_{22} & P_{23} & & \\ P_{31} & P_{32} & P_{33} & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \\ P_{10,1} & \cdots & \cdots & \cdots & P_{10,10} \end{pmatrix} \begin{pmatrix} 0 \\ T \\ 0 \\ 0 \\ M_1 \\ -V_Y \\ 0 \\ 0 \\ M_Y \\ V_z \end{pmatrix}_0$$

and for a nontrivial solution

$$D(\omega) = | \ Q \ | = 0 \tag{7}$$

where $D(\omega)$ is the determinant of the submatrix, [Q], of the product matrix [P], where [Q] is defined by

$$[Q] = \begin{bmatrix} P_{22} & P_{25} & P_{26} & P_{29} & P_{2,10} \\ P_{52} & P_{55} & P_{56} & P_{59} & P_{5,10} \\ P_{62} & P_{65} & P_{66} & P_{69} & P_{6,10} \\ P_{92} & P_{95} & P_{96} & P_{99} & P_{9,10} \\ P_{10,2} & P_{10,5} & P_{10,6} & P_{10,9} & P_{10,10} \end{bmatrix}$$

The rows of [Q] correspond to the zero elements in $(Z)_N$ and the columns of [Q] correspond to the nonzero elements in $(Z)_0$. This is the standard rule for defining [Q] from [P]. For a vibrating system, the transfer natrices, [T], and therefore the product matrix, [P], the determinant matrix, [Q], and its determinant, D, are all functions of the vibration frequency, ω. Values of ω for which $D(\omega)=0$ are the natural frequencies of the blade. The definition of the determinant in the manner outlined above is straightforward but is often difficult to do with sufficient numerical accuracy that good mode shape quantities may be obtained. Important characteristics of the determinant as defined above are that $D(\omega)$ is a polynomial in ω^2, has only positive roots, and is usually accurate numerically except very close to the natural frequencies of the system.

Reduction of numerical accuracy problems.- A method which utilizes quantities that correspond to an approximate mode shape and mode shape correction terms in addition to an approximate frequency has been developed by Pestel and Leckie, and is explained in detail in reference 1, where it is referred to as the modified transfer-matrix method. In this method, an approximate mode shape, say $(\lambda)_0$, and correction terms, say $[\kappa]$, are used in place of $(Z)_0$; and the product matrix [P], is replaced by an approximate mode shape and correction column matrix, $[\Pi]$, where the first column of $[\Pi]_0$ is $[\lambda]_0$, and the other columns of $[\Pi]_0$ are either zero or κ_i, as discussed in reference 1. That is, it is assumed (again using an N element cantilever - free beam as an example) that

$$[z]_0 = \begin{Bmatrix} 0 \\ T \\ 0 \\ 0 \\ M_z \\ -V_y \\ 0 \\ 0 \\ M_y \\ V_z \end{Bmatrix}_0 = \begin{Bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \lambda_1 \\ \lambda_2 \\ 0 \\ 0 \\ \lambda_3 \\ \lambda_4 \end{Bmatrix}_0 + \kappa_1 \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \kappa_2 \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} + \kappa_3 \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{Bmatrix} + \kappa_4 \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{Bmatrix} \quad (8)$$

and $[\Pi]_0$ is made up of the column matrices which are used in the sum for $(z)_0$. In general, $(z)_0$ is an (n, 1) matrix (or column) and $[\Pi]_0$ is an (n, n/2) matrix, where n is the number of physical quantities which are considered in the state vector.

The transfer matrix process indicated in equation (5) is repeated, but with $(z)_0$ replaced by $[\Pi]_0$, and with the multiplications being done in the order indicated by the following steps

$$
\begin{aligned}
[\Pi]_1 &= [T]_1 \; [\Pi]_0 \\
[\Pi]_2 &= [T]_2 \; [\Pi]_1 \\
&\vdots \\
[\Pi]_5 &= [T]_5 \; [\Pi]_4
\end{aligned}
$$

where it is understood that the columns in $[\Pi]_i$ are to be added to obtain the column matrix $(z)_i$. Since $(z)_N$ has zero elements, it is required that the $[\Pi]_N$ satisfy the appropriate set of homogeneous equations.

For the specific example used in this discussion.

$$
[z]_N = \begin{pmatrix} \phi \\ 0 \\ v \\ \theta \\ 0 \\ 0 \\ -\omega \\ \psi \\ 0 \\ 0 \end{pmatrix}_N = \begin{pmatrix} \Lambda_1 \\ \Lambda_2 \\ \vdots \\ \\ \\ \\ \\ \\ \\ \Lambda_{10} \end{pmatrix}_N + \kappa_1 \begin{pmatrix} K_{11} \\ K_{21} \\ \vdots \\ \\ \\ \\ \\ \\ \\ K_{10,1} \end{pmatrix}_N + \kappa_2 \begin{pmatrix} K_{12} \\ K_{22} \\ \vdots \\ \\ \\ \\ \\ \\ \\ K_{10,2} \end{pmatrix}_N + \kappa_3 \begin{pmatrix} K_{13} \\ K_{23} \\ \vdots \\ \\ \\ \\ \\ \\ \\ K_{10,3} \end{pmatrix}_N + \kappa_4 \begin{pmatrix} K_{14} \\ K_{24} \\ \vdots \\ \\ \\ \\ \\ \\ \\ K_{10,4} \end{pmatrix}_N \qquad (9)
$$

where the columns which the κ_i multiply are the result of multiplying the corresponding columns in $[\Pi]_0$ by the [T] matrices. The requirements that the $[\Pi]_N$ satisfy the boundary conditions as specified by the zero elements in $[z]_N$ is achieved if

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{Bmatrix} \Lambda_2 \\ \Lambda_5 \\ \Lambda_6 \\ \Lambda_9 \\ \Lambda_{10} \end{Bmatrix}_N + \begin{bmatrix} K_{21} & K_{22} & K_{23} & K_{24} \\ K_{51} & K_{52} & K_{53} & K_{54} \\ K_{61} & K_{62} & K_{63} & K_{64} \\ K_{91} & K_{92} & K_{93} & K_{94} \\ K_{10,1} & K_{10,2} & K_{10,3} & K_{10,4} \end{bmatrix}_N \begin{Bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_4 \end{Bmatrix} \tag{10}$$

It is shown in reference 1 that the Λ's in equation (10) are identically zero only if both the frequency, ω, which was used in defining the $[T]_j$, and $[\lambda]_0$, which was part of $[\Pi]_0$, were a natural frequency, ω_n, and its corresponding state vector, $[z]_0$, respectively. (The Λ's in equation (10) are rarely exactly zero in practical cases.) Equation (10) may be thought of as an overdetermined set of equations for the correction factors (κ). Quantities called remainders may be defined from these equations; they are more accurate than the determinant in the neighborhood of the natural frequencies, and their use is vital in overcoming most of the numerical accuracy problems associated with use of the determinant type of characteristic equation which was discussed in the previous section. Their theoretical derivation, examples of their use, and a procedure for their calculation are given in reference 1. The calculation procedure is outlined briefly here. (Chapter 7, section 3 of reference 1 also gives an example.)

If the first row of equation (10) is eliminated, the (κ) may be evaluated. That is

$$\begin{Bmatrix} \kappa_1 \\ \kappa_2 \\ \kappa_3 \\ \kappa_4 \end{Bmatrix} = - \begin{bmatrix} K_{51} & K_{52} & K_{53} & K_{54} \\ K_{61} & K_{62} & K_{63} & K_{64} \\ K_{91} & K_{92} & K_{93} & K_{94} \\ K_{10,1} & K_{10,2} & K_{10,3} & K_{10,4} \end{bmatrix}^{-1} \begin{Bmatrix} \Lambda_5 \\ \Lambda_6 \\ \Lambda_9 \\ \Lambda_{10} \end{Bmatrix} \tag{11}$$

Set b_1 equal to this value of κ_1 (b_1 is called x in eq. 7-10 in ref. 1) define an alternate value for κ_1, say a_1 (called y in reference 1) from

$$a_1 = - (K_{22}\,\kappa_2 + K_{23}\,\kappa_3 + K_{24}\,\kappa_4) / K_{21} \tag{12}$$

Then define the remainder, R_1, as

$$R_1 = b_1 - a_1 \tag{13}$$

This remainder, and others defined by eliminating other rows of equation (10), etc., are the remainders which may be used in determining the natural frequencies of the model. It is shown in reference 1 that the remainders have zeros where the determinent has zeros and that the remainders, R_i, are the sums of small numbers, a_i and b_i, i.e., that a_i and b_i have opposite signs. The determinant is usually the difference of large numbers. This fundamental difference between the determinant and the remainders is the basis of the superior numerical accuracy of the modified transfer-matrix method. The remainders have apparent discontinuities, but this usually tends to be an inconvenience rather than a serious deficiency. Details of how the remainders and determinant may be used in various methods of determining the natural frequency of a system are discussed in a following section. Both the product matrix, [P], and the state vector and correction columns matrix, [Π], are computed and used by these methods.

Mode shape definition.- As the determinant of the remainders approach zero, the approximate frequency and mode shape become more nearly correct, and at some point the frequency becomes sufficiently close to the numerical model's true natural frequency that it can be considered to be the natural frequency. The mode shape may be determined by using the corresponding values of the λ's and κ's, and by adding the columns in the [Π], matrix to obtain $(z)_j$. This method is the least subject to numerical inaccuracies. An alternate method uses the most accurate $(z)_0$, and by successive multiplications by $[T]_1$, $[T]_2$, etc., defines $(z)_1$, $(z)_2$, etc. The combination of all the $(z)_j$'s contains the mode shape of each of the state vector quantities as a function of spanwise position. The zero mode shape quantities in $(z)_0$ are identically zero, but those quantities in the state vector at the other end of the structure, $(z)_N$ which theoretically should be zero are never identically zero in practice. Some indications of the accuracy of a calculated natural frequency and mode shape are the relative changes in magnitude in quantities which should be zero in the $(z)_N$ state vector to the same quantities at the next state vector position, $(z)_{N-1}$. Normally, a $(z)_N$ "zero" type quantity should be at least three orders of magnitude smaller at the end than the same $(z)_{N-1}$ quantity. Two types of mode shapes which are normalized to two different quantities are commonly used in the analysis of helicopter blades. One type of mode shape has a tip deflection quantity (subscript N) which has a value of unity. For shapes which include flatwise deflections, it is common to set v_N equal to unity, otherwise ϕ_N or $(-w)_N$ is usually set equal to unity. The second type of mode shape has a unity generalized mass, and is useful in programs which use normal modes and generalized coordinates to determine the response of the system to forcing functions.

Boundary Conditions

As previously stated, the boundary conditions are inherently contained in the state vectors at the end of the structure. Exceptions are structures which have restraints between the ends of the structure. These may be analyzed by this method also, and such systems are discussed in reference 1. There are boundary conditions which do not automatically result in one-half of the state vector elements being zero, but these may always be written in terms of another transfer matrix and a state vector which does have zero for one-half of its elements.

Choice of various boundary conditions correspond to choice of various rows and columns of [P] to define [Q]. Of the many boundary condition combinations which are possible for the system of equation (5), relatively few are of practical interest. The tip (subscript N) boundary conditions for a helicopter blade correspond to a force-free condition, or to a "free" end. The hub (subscript 0) boundary conditions usually involve hinged or clamped types of systems, and there are occasionally non-coincident hinges used on some helicopter rotor blades. Some rotors are continuous through the hub, e.g. teetering rotors. These may be considered to be symmetrical beams, and one half of them analyzed, with both symmetrical and anti-symmetrical modes and frequencies required to describe the system.

Mode Coupling

The previous discussion has dealt completely with fully coupled torsional-flatwise-edgewise normal modes and natural frequencies. It is sometimes helpful to use partially coupled or uncoupled modes in the analysis of helicopter blades and similar systems. Such data can be obtained by considering only a submatrix of each of the product matrices [P] and [Π], the determinant matrix [Q], and the state vector (z). The types of uncoupled modes which could be obtained from such manipulation of equations (5) through (10), for example, would be uncoupled torsion, flatwise, and edgewise modes. Partially coupled modes would include torsional-flatwise, torsional-edgewise, and flatwise-edgewise.

It should be noted that when uncoupled torsional modes are determined that no remainders are available, and that in general the number of remainders is given by (n/2)-1 where n is the number of state vector elements. The state vector elements corresponding to the various types of mode coupling are given as follows

Mode Types	State Vector Quantities
uncoupled torsional	ϕ, T
uncoupled flatwise	v, θ, M_z, $-V_y$
uncoupled edgewise	-w, ψ, M_y, V_z
coupled torsional-flatwise	ϕ, T, v, θ, M_z, $-V_y$
coupled flatwise-edgewise	v, θ, M_z, $-V_y$, -w, ψ, M_y, V_z
fully coupled	ϕ, T, v, θ, M_z, $-V_y$, -w, ψ, M_y, V_z

It should be noted that partially coupled torsional-edgewise modes are not included as a mode coupling combination. Such modes cannot be determined by using a $\Delta\phi$ of 90° and switching values of EI_y and EI_z on input unless the mass eccentricity is nonzero in the y-direction. Such eccentricities are not included in the present model. Also, the neutral axis cannot be modeled to have a position other than in a plane normal to the rotor shaft which pases through the hub. That is, no precone is allowed in the blade model.

Various types of frequency search methods have been used in connection with eigenvalue type problems in general. The three which are described here are some of several which have been used by the authors for searching for eigenvalues by hand or machine. They are not the best for all cases, but have proven to be useful in connection with the type of problem discussed here. One, which is applicable to elastomechanical systems with positive, real eigenvalues is designed for use with a minimum of required user skill, computational time and cost, and is referred to as the automated frequency search method. The other methods are forerunners of this method, and are referred to as the frequency stepping and initial estimate search method. They have application to some systems for which the automated method should not be assumed to be reliable.

Automated frequency search method.- The natural frequencies of the system are such that the numerically defined determinant, $D(\omega)$ and the remainders, $R_i(\omega)$, are zero when ω is a natural frequency, say ω_n. That is $D(\omega_n)=0$ and $R_i(\omega_n)=0$. The numerical inaccuracy of $D(\omega)$ may prevent it from being used to obtain ω_n with sufficient accuracy to allow definition of mode shapes; the $R_i(\omega_n)$ are not so subject to numerical error, and have always been found to be satisfactory in determining ω_n with sufficient accuracy to result in good mode shape determination. It can be shown that $D(\omega)$ is a polynomial in ω^2, and has no negative (ω^2) roots. That is,

$$D(\omega) = K\ (\omega^2 - \omega_2{}^2)\ (\omega^2 - \omega_2{}^2)(\omega^2 - \omega_3{}^2)\ldots(\omega^2 - \omega_k{}^2)\ldots$$

where K is some constant and the ω_n are natural frequencies, with all $(\omega_n{}^2)$ being positive. The $R_i(\omega)$ are ratios of polynomials in ω^2, and have discontinuities which correspond to roots of the denominator. These characteristics of $D(\omega)$ and $R_i(\omega)$ provide a basis for developing a completely automatic frequency search technique. That is, $D(\omega)$ is a polynomial, and may be used in a Newton iteration to obtain approximate values of ω_n. No values of $\omega_n{}^2$ are negative, so if the iteration is started near zero, the frequency which is found first should be the lowest, and should be approached from the low side, as is indicated in figure 4. Once the lowest frequency is determined, an auxiliary function, say $F_1(\omega)$, may be defined which has had that frequency removed. That is $F_1(\omega)=D(\omega)/(\omega^2-\omega_1{}^2)$.

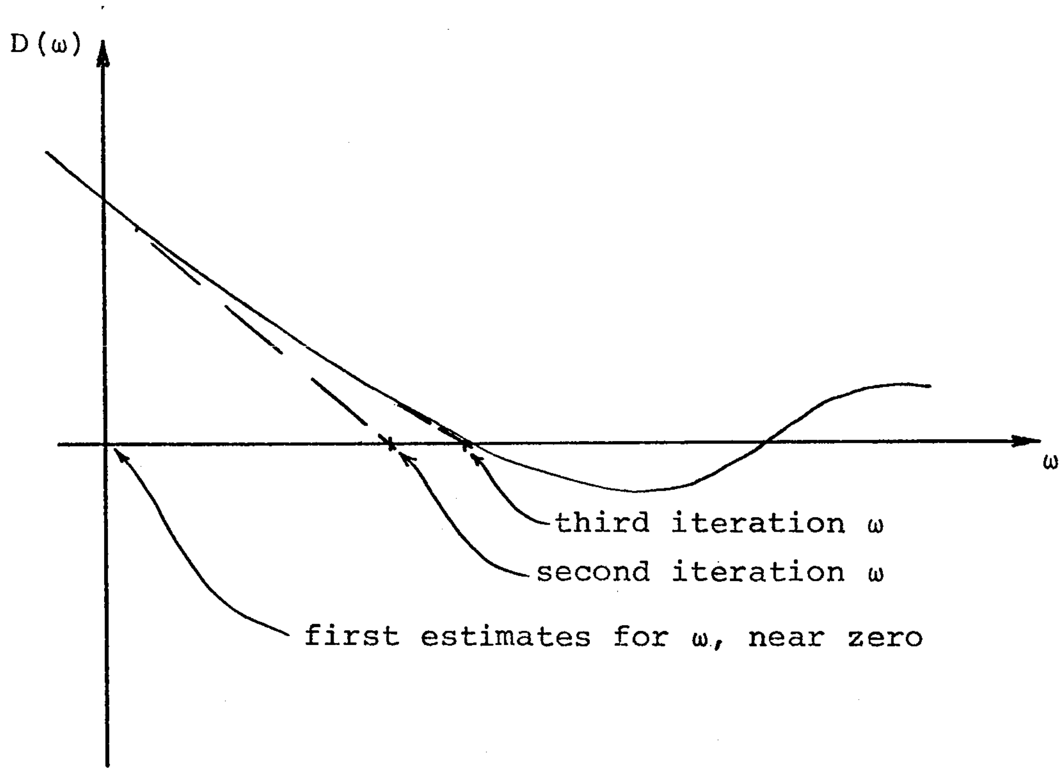
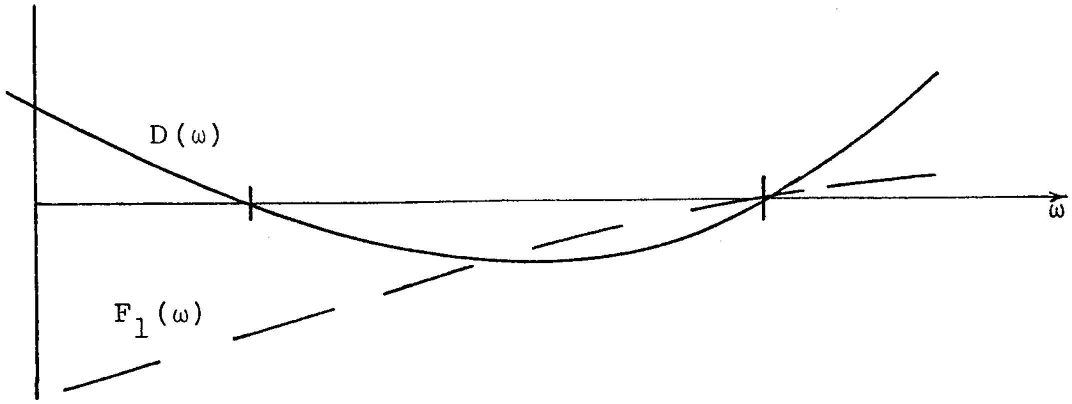
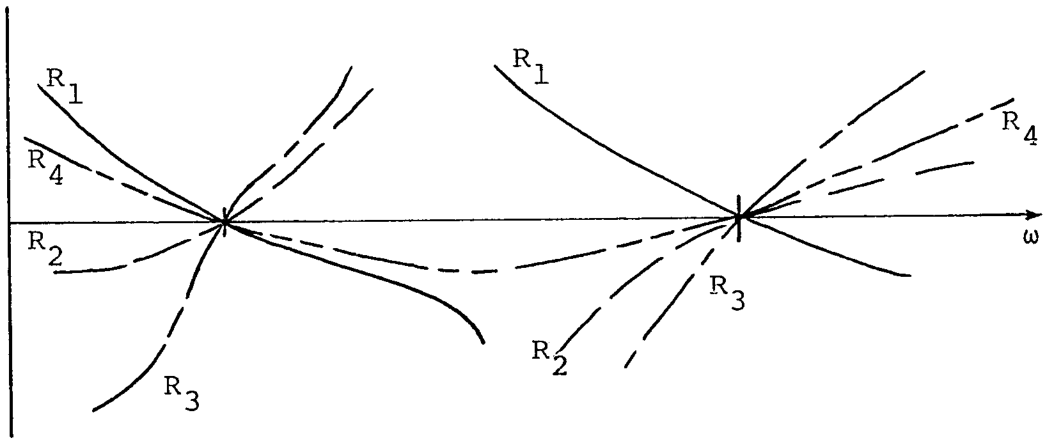


Figure 4. Iteration to lowest natural frequency

The same process should yield the lowest frequency of $F_1(\omega)$. The process may be continued for any desired number of frequencies. Numerical inaccuracy problems and dependence of one frequency (and its mode shape) upon previously determined frequencies may be eliminated by using an appropriate remainder to do the final iterations for the ω_n. The remainders cannot be used during the early iterations because of their discontinuities. Care must be used in choosing one of the $R_i(\omega)$'s which does not have a discontinuity near the desired ω_n. The choice of the best $R_i(\omega)$ is automatic, and depends upon the nearness of the ω predicted by the $R_i(\omega)$'s to that predicted by the $D(\omega)$. Figure 5 shows the determinant, auxilliary function, and remainders for a typical case, for the lowest two natural frequencies.



(a) determinant, D, and auxiliary function, F



(b) remainders

Figure 5. Determinant, auxiliary function, and remainders for a typical automated frequency search.

<u>Frequency stepping</u>.- Frequencies may be determined by this method by specifying a starting value, $\omega^{(0)}$, and a step size factor, f. The determinant and remainders are computed for each ω, and ω is incremented by defining $\omega^{(i)} = \omega^{(i+1)}$ (1 + f) until the determinant changes sign. The Newton iteration technique is then used with an appropriate remainder. After one natural frequency is determined, the frequency stepping continues, beginning with a frequency defined in terms of the frequency just determined, say ω_n, according to

$$\omega^{(1)} = \omega_n \ (1 + f)$$

The method is illustrated in figure 6, and some specific shortcomings are also indicated. While this option can give well-defined $D(\omega)$ and $R_i(\omega)$ curves, it may miss frequencies due to too large a step size (figure 6a) or too high an initial guess (figure 6b), and may be inefficient if the step size is too small (figure 6c). The step factor value, f, may be either positive or negative so the steps may yield increasing or decreasing ω's. The danger of f being too large is more severe for large frequencies than for small, and the inefficiency of f being too small is worsened if the frequency spacing between natural frequencies is large.

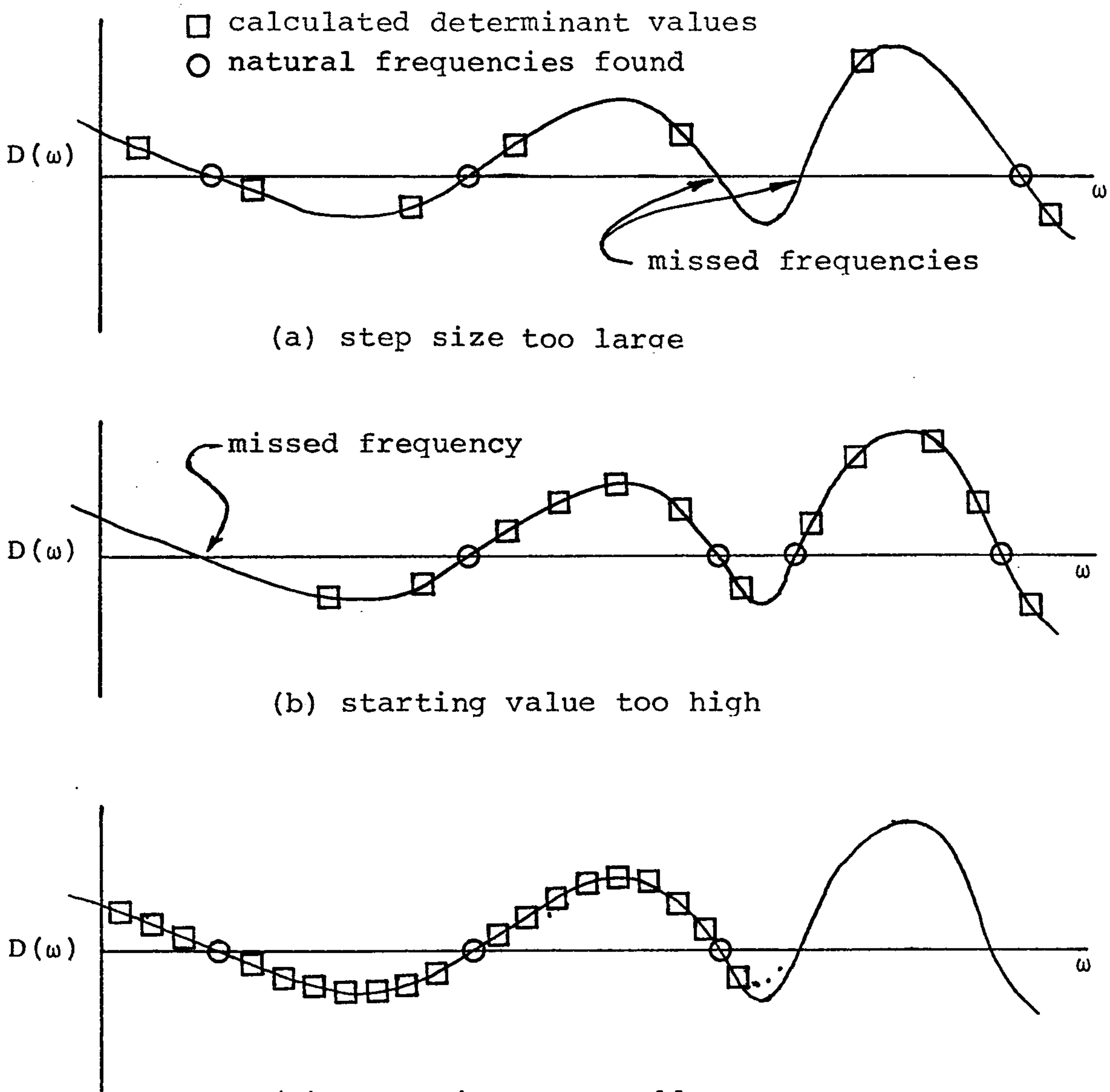
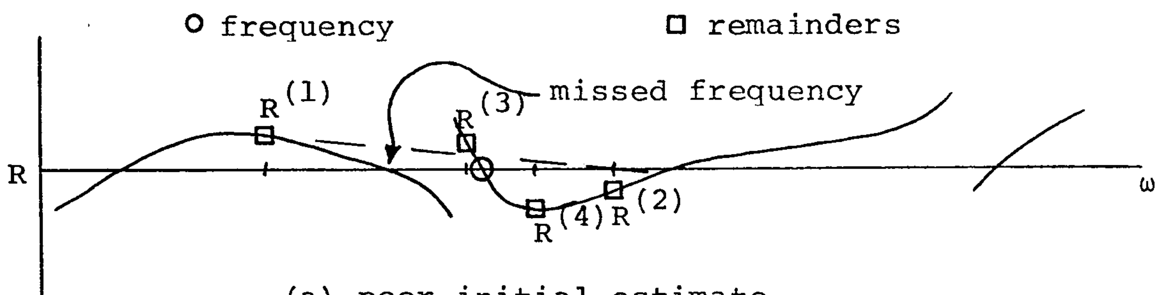


Figure 6. Frequency stepping option

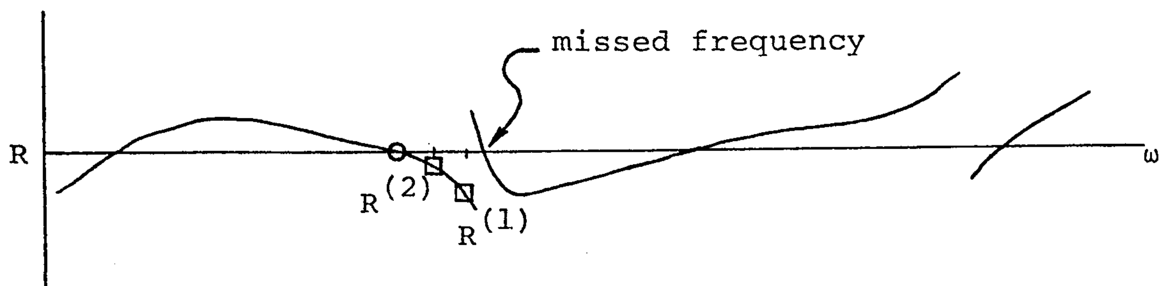
Frequency estimates method.- Frequencies may be determined by this method by defining estimated values of the natural frequencies and using a Newton iteration to search for the actual values. A second value of frequency $\omega^{(1)}$, is defined (in order to use the Newton iteration) in terms of the estimated values, $\omega^{(0)}$, as

$$\omega^{(1)} = \omega^{(0)} (1 + f)$$

where ordinarily f is small. These values of ω are used to start the Newton iteration. Where initial estimates are close to the actual model natural frequency it is possible to use the remainders for the iteration procedure. The primary shortcomings of this method are that the initial estimates may be incorrect, and that the remainder chosen to control the iteration may jump near one or more of the desired natural frequencies. Either of these shortcomings may result in duplication, or omission, of some frequencies. Typical results of this method are illustrated in figure 7. While the use of this method often requires exercise of skill and judgement by the user, it has consistantly been used to find natural frequencies of a number of helicopter rotor and propeller blades. It is less attractive than the automated search option only because of the required user skills and the time spent in obtaining frequencies and mode shapes where the remainders all have discontinuities near a desired frequency.



(a) poor initial estimate



(b) discontinuity in R near a frequency

Figure 7. Initial estimates option

COMPUTER PROGRAM IMPLEMENTATION

A computer program has been developed which implements most of the features of the modified transfer-matrix method for determining frequencies and mode shapes of rotating, twisted, non-uniform beams. The program handles fully coupled, partially coupled, and uncoupled modes, four types of boundary conditions, and has three frequency search options. Details related to the formulation are contained in the section entitled THEORETICAL FORMULATION and in reference 1. Program input and output has been designed for a user who is somewhat familiar with the problem of determining and utlizing normal modes and natural frequencies of lumped parameter models of physical systems. An effort has been made to simplify and minimize input and output for more efficient use in general, and for compatible operation from computer terminals. The program consists of approximately 1500 punched cards, executes (after compilation) in approximately 33000 octal core storage locations, and requires approximately 0.25 CPO seconds per iteration on a CDC 6600 computer when the program is compiled with an optimizing compiler. The number of iterations per frequency varies, but averages approximately 10. The program has accuracy limitations which render the presently implemented program useless for operation on computers that use less than 14 digit numbers in floating point calculations.

It has been anticipated that those portions of the program which a user may be apt to modify, or may want to understand to a greater degree than others include those portions of the program related to (1) the definition of boundary conditions, (2) the definition of the type of mode coupling, and (3) the implementation of the frequency search methods. These areas of probable interest are discussed briefly. It should be expected that a large portion of currently existing and future helicopter rotor systems will be able to be analyzed in terms of single blade natural frequency and mode shape characteristics without modifying the existing program. These brief discussions are intended to help the user better understand the capabilities of the program, and should be useful in the event that program modifications are desired.

The functions of the various program routines are stated briefly, with references to detailed discussions given where they exist. A computer program symbol dictionary and a table of dimensioned variables which defines how dimensions may be changed for different computer simulation models are presented. Finally, a description of computer program input and output are given. Details related to the punched mode shape output are given since this is expected to be one area where users may desire to make changes.

Boundary Condition Options

The program, as presently implemented, has four types of hub boundary condition options available. These are defined in subroutines BNDRY1 or BNDRYX, and are used as specified by the input parameter, IBC.

For the value of IBC indicated they are:

(1) edgewise hinge outboard of flatwise hinge (fully articulated rotor),

(2) clamped flatwise and edgewise (cantilever hub)

(3) pinned flatwise and clamped edgewise, and

(4) clamped flatwise and pinned edgewise.

Of these, (3) and (4) are used to calculate the antisymmetrical and the symmetrical modes, respectively, of a teetering rotor. The cantilever boundary conditions may be used with inboard element stiffness adjusted so as to represent flexures, etc., if desired. The input number, IBC, or the mode shape quantities which are printed at the inboard end of the model may be used to determine the type of boundary conditions which exist.

Torsional deflections have been assumed to be clamped at the hub, with any control system flexibility lumped with the appropriate section's torsional stiffness. The effective section stiffness, EIX_e, is defined in terms of the section length, ℓ, the control spring stiffness k, and the actual section stiffness, EIX_a, as $EIX_e = EIX_a / (1 + EIX_a / \ell k)$.

The inboard blade element has its inboard end located a distance X_{root} from the center of rotation, and $X_{root} = 0$ is allowed. The edgewise hinge of the fully articulated rotor is located at the outboard end of section 1, and that section may have zero length. The outboard state vector has zero force type quantites, i.e., the tip is "free" and these are set automatically by the program.

The boundary conditions subroutines, BNDRY1 and BNDRYX define the hub state and the vectors $(z)_o$ and $(\Pi)_o$ with one quantity, the hub torque, set equal to unity during the iterations. When a natural frequency has been obtained, the hub state vector is multiplied by a factor to yield a tip state vector with one quantity set equal to unity, or with a mode shape generalized mass of unity, as was discussed previously. The boundary conditions at the tip are defined in agreement with the input values of NTYPE, and always correspond to a force-free condition at the tip.

The boundary conditions at the hub as defined in the hub state vector, $(z)_o$, for the different boundary conditions which are presently implemented are given by the following table. (Note that the hub has been assumed to be clamped in torsion in all cases). The pinned flatwise-pinned edgewise boundary conditions are for the flatwise hinge at the inboard end of the first section and the edgewise hinge at the outboard end of the first section. All other boundary conditions are applied at the inboard end of the first section.

TABLE 1.

STATE VECTOR ELEMENTS AT HUB FOR VARIOUS BOUNDARY CONDITIONS

IBC Values	1	2	3	4
flatwise restraint edgewise restraint	pinned pinned	clamped clamped	pinned clamped	clamped pinned
Hub State Vector Quantities				
algebraic names	elements used by computer program			
ϕ	0	0	0	0
T	1.0	1.0	1.0	1.0
v	0	0	0	0
θ	λ_1	0	λ_1	0
M_Z	0	λ_1	0	λ_1
$-V_Y$	λ_2	λ_2	λ_2	λ_2
$-w$	0	0	0	0
ψ	λ_3	0	0	λ_3
M_Y	0	λ_3	λ_3	0
V_Z	λ_4	λ_4	λ_4	λ_4

Mode Coupling Options

The computer program will treat several models, including subsets of the fully coupled torsional-flatwise-edgewise modes. The partially coupled and uncoupled modes are obtained by specifying the appropriate input, NTYPE; the modes and frequencies are computed by using matrix partitioning to obtain the desired data. Uncoupled torsional, flatwise, or edgewise modes and frequencies may be calculated, as may partially coupled torsional-flatwise or flatwise-edgewise modes and frequencies.

The hub and tip boundary conditions for the type of coupling defined by NTYPE are automatically chosen correctly for the boundary conditions of the type which are specified by IBC. The mode shape quantities which are not part of the partially coupled or uncoupled modes are printed or punched as zeros. The values of NTYPE, and resulting type of mode are

NTYPE	type of mode
1	uncoupled torsional
2	uncoupled flatwise
3	uncoupled edgewise
4	coupled torsional-flatwise
5	coupled flatwise-edgewise
6	coupled torsional-flatwise-edgewise

Program control variables for definition of the proper matrix partitioning and tip boundary conditions, are done in the main line of each value of NTYPE. A default for NTYPE > 6 or NTYPE < 1 is implemented, and gives NTYPE = 6 results as the default value. No error message is printed in this case.

It should be noted that partially coupled torsional-flatwise modes are not included as a possible type of mode coupling. The user should be warned against attempting to obtain such modes by making a 90° rotation about the x-axis by inputting a $\Delta\Phi$ of 90 with appropriate changes in EIY and EIZ, etc., since the mass eccentricity in the flatwise direction and the rotary mass moment of inertia about a chord line are assumed to be zero. As a result, the normal type of mass coupling can not be modeled for partially coupled torsional-edgewise modes.

Frequency Search Methods

Three frequency search options are available with the present program. The input variable NFIND determines the option used. Input of NFIND = 1, 0, or -1 results in use of the automated search method, the estimates method or the stepping method, respectively. Of these methods, the automated search technique is expected to be the most useful. All input variables must have some numerical values. However, all those variables defined by NAMELIST are not used for each of the frequency search options. Those not used for a particular option are given below, and required or suggested values for those which are used are given in the section on Description of Input.

Automated frequency search.- The automated frequency search begins with two internally defined initial guesses near $\omega = 1.0$, and uses a Newton's iteration to determine the lowest natural frequency, ω_1. An associated function, $F_1(\omega)$, is then defined in the form

$$F_1(\omega) = D(\omega) / (\omega^2 - \omega_1^2).$$

Thus, the zero of $D(\omega)$ at ω_1 has effectively been removed from $F_1(\omega)$. Two values of $D(\omega)$ which were determined in the search for ω_1 were saved or re-computed and are divided by the appropriate $(\omega^2 - \omega_1^2)$ factor. These provide, by extrapolation, an initial estimate for the next higher natural frequency, ω_2, which is again found by a Newton's iteration procedure.

In the search for the third natural frequency, $F_2(\omega)$ is defined as

$$F_2(\omega) = D(\omega) / [(\omega^2 - \omega_1^2) (\omega^2 - \omega_2^2)]$$

and extensions to higher frequencies are obvious. The process is continued until the desired number of frequencies are determined or until all frequencies below an input cutoff frequency are determined, along with their corresponding mode shapes. This describes the overall approach. In the iteration to find a zero of $D(\omega)$, both the determinant and the associated function, $F(\omega)$, may become inaccurate, but the remainders, $R_i(\omega)$, will not. Also, use of $F_i(\omega)$ to evaluate frequencies above the first would mean that such frequencies would depend on all previous frequencies and numerical inaccuracies. Therefor, once the iteration using $F(\omega)$ closely approximates ω_i the program chooses an appropriate $R_i(\omega)$, and concludes the iteration using that $R_i(\omega)$. The nearness of ω to a natural frequency is measured by the relative change in frequency from one iteration, say the i^{th}, to the next. If $|(\omega^{(i+1)} - \omega^{(i)})^2| / \omega^{(i+1)^2} \leq .316228^{NEXPO}$, a remainder is chosen and used for the subsequent iterations. The $R_i(\omega)$'s, of course, are idependent of previous ω_j's, and are numerically accurate, but they cannot be used during the entire process since they are not polynomials in ω, and display apparent discontinuities. The remainder which is chosen for the subsequent iterations is that which predicts the next frequency value closest to that predicted by the determinant, using remainder and determinant values computed during the previous two iterations.

The iteration sequence for a particular natural frequency will be stopped if the number of iterations, NTIME, exceeds NMAX. In this event, the natural frequency counter NFREQ will be set equal to NUMF, and the program will go to the next model. (This is necessary to avoid incorrect definition of the auxiliary function and subsequent erratic behavior of the automated frequency search.) Therefore, it is suggested that NMAX be set to at least 30 for use of the automated frequency search option.

Input variables not used when NFIND = +1 are NOOR, PRCNT, and, if NPDF$\leq$0, OMINT.

<u>Frequency estimates</u>.- Starting estimates of natural frequencies are input to the program in the OMINT array. A second frequency value factor, PRCNT, for use in starting the Newton iteration, is also input, and should be small, about 1.0 or less. This option is implemented so that one remainder, as specified by the input variable NOOR, is used for the entire frequency range. The values of OMINT need not be in any order, and may have any numerical value. Input variables not used when NFIND = 0 are NPDF and OMLIMT.

<u>Frequency stepping search</u>.- A step factor, PRCNT, and frequency starting value, OMINT(1), are input and the values of ω^2 are stepped according to

$$(\omega^{(i+1)})^2 = (\omega^{(i)})^2 \; (1 + \text{PRCNT} / 100)$$

until the determinant changes sign. The program then begins a Newton iteration, using the remainder specified by the input index, NOOR. The initial value for the next frequency is defined from the natural frequency according to

$$(\omega^{(0)})^2 = \omega_n^{\;2} \; (1 + \text{PRCNT} / 100)$$

The process is repeated until the specified number of frequencies are found. The value of PRCNT to be used for a particular case depends on the intended use of the option. If a scan over a large frequency range is desired, PRCNT is large. If a detailed scan over a limited frequency range is desired, then PRCNT is small. Input variables not used when NFIND = -1 are NPDF, OMINT(I) for I>1, and OMLIMT.

Program Routine Functions

The overall flow of the program is indicated by figure 8. The program BLADE reads and writes input; sets program control constants; calls subroutine BNDRY1 or BNDRYX to establish hub boundary condition; performs transfer matrix multiplications; defines determinants and remainders (using subroutine MATINV to evaluate determinants and solve simultaneous sets of equations for the definition of remainders), does the frequency search options of frequency estimates and frequency stepping (using subroutine INTERP), and calls subroutine FINDM for the automatic search; calls subroutine SHAPE to define mode shapes when a natural frequency is found or when the limit on the number of iterations is reached; and controls the number of frequencies per model and the number of models per run.

Subroutine SNTFR sets program control equivalent to that given by NTYPE = 6 for input values of NTYPE which are not in the accepted range of 1 thru 6.

Subroutine BNDRY1 defines the hub boundary conditions for IBC=1. This option gives modes for a fully articulated rotor with the edgewise hinge at the outboard end of section 1 and with the flatwise hinge at the inboard end of section 1. Subroutine BNDRYX defines the hub boundary conditions for IBC = 2, 3, and 4 respectively. IBC = (2) gives clamped-clamped modes, IBC = (3) gives pinned-flatwise clamped-edgewise modes; and IBC = (4) gives clamped-flatwise pinned-edgewise modes.

Subroutine TEMAT defines the transfer matrix for a typical blade element; see equation (2) and Appendix B for the definition of this matrix. Note that for various values of NTYPE, the blade element is assumed to be rigid (have infinite stiffness) in torsion, edgewise bending, or flatwise bending (as given in the following table) if NX, NY or NZ, respectively, are unity; otherwise, the blade has flexibility. These parameters make the calculation somewhat more efficient for partially coupled or uncoupled modes.

NTYPE	NX	NY	NZ	type of coupling
1	0	1	1	uncoupled torsion
2	1	0	1	uncoupled flatwise
3	1	1	0	uncoupled edgewise
4	0	0	1	coupled torsion-flatwise
5	1	0	0	coupled flatwise-edgewise
6	0	0	0	coupled torsion-flatwise-edgewise

Values of NX, NY, or NZ of unity cause special printed messages after the run title is printed.

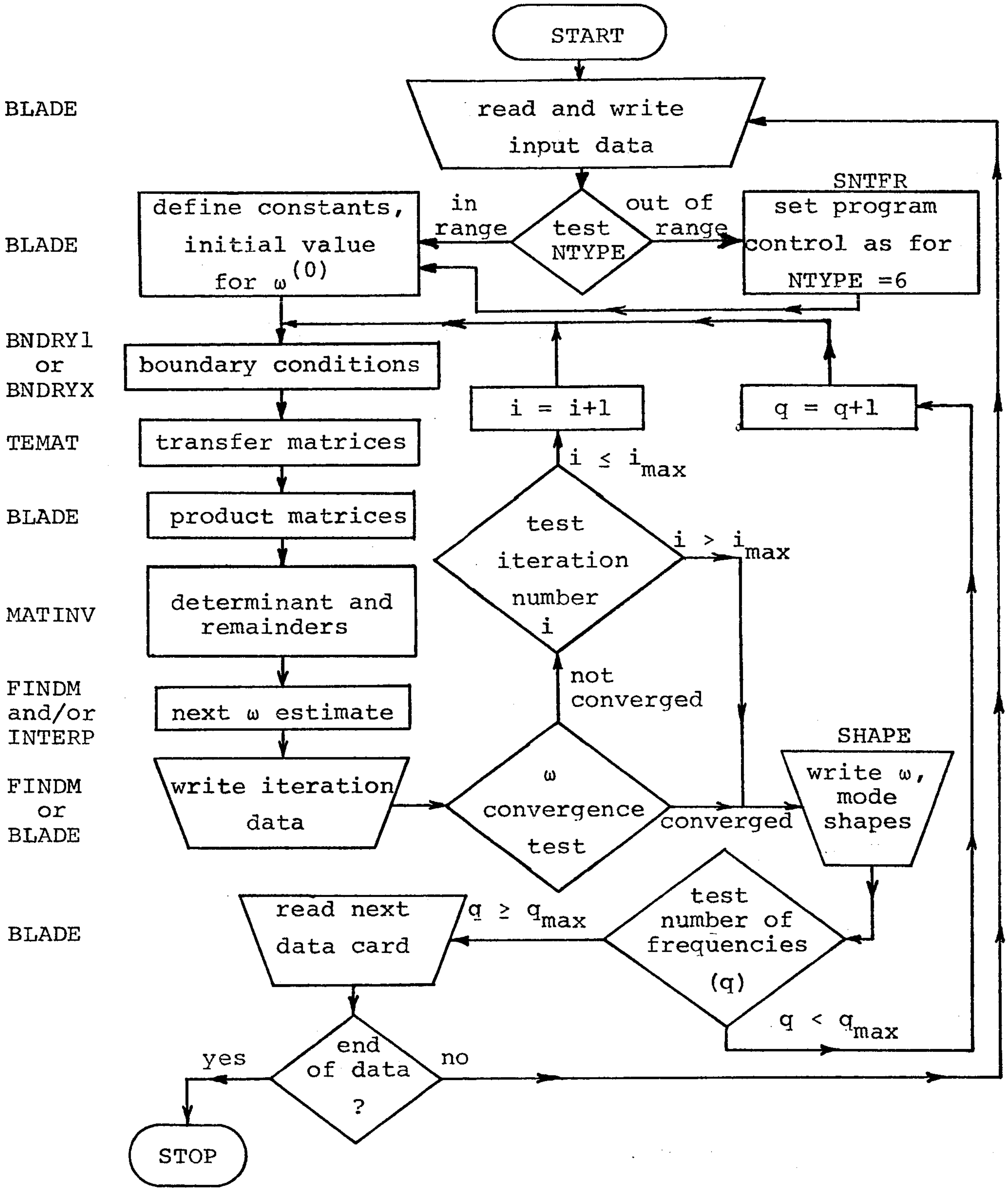


Figure 8. Program Flow

Subroutine MATINV solves sets of simultaneous equations and estimates the determinant of the coefficient matrix. See reference 3 for more details.

Subroutine FINDM controls the automatic frequency search and printout of data during the search. The determinant value D(1.0) is stored during the first calculation for a particular model and is not recomputed. The next frequency value is always 1.001, and this value and the corresponding values of the determinant, auxiliary function, and remainders are printed as iteration number 1. Subsequent frequency estimates and corresponding data are printed until the frequencies of successive iterations satisfy the relationship.

$$(\omega^{(i)2} - \omega^{(i-1)2}) / \omega^{(i-1)2} \leq .316228^{NEXPO}$$

The remainder of the iterations are done by using the "best" available remainder value. (Except in the case of uncoupled torsional modes, for which there are no remainders and the determinant retains good accuracy.) The "best" remainder is that which perdicts the next frequency estimate closest to that predicted by the determinant, using the two previous frequency estimates. The remainder chosen is identified on the output, and subsequent iterations use only that remainder. Values of the frequency determinant and remainders but not the auxiliary function are printed during the subsequent iterations. Usually only one or two iterations are necessary to satisfy the frequency convergence criterion as given by

$$(\omega^{(i)2} - \omega^{(i-1)2}) / \omega^{(i-1)2} \leq 0.1^{NEXPO}$$

Upon satisfaction of this criterion, subroutine FINDM resets program control variables to start searching for the next highest frequency and stores $\omega^{(i)}$ in the OMINT array.

When one or more frequencies are stored in the OMINT array, they are used to define the auxiliary function as described in the THEORETICAL FORMULATION section. (The numerical values stored in OMINT are actually the squares of the frequencies.)

Subroutine INTERP does a linear extrapolation (or interpolation) for the Newton iteration technique.

Subroutine SHAPE controls the calculation, printing, and punching of the mode shapes. (Subroutine SHAPE calls subroutine BNDRY1 or BNDRYX and TEMAT.)

Standard library functions or subroutines which are used include DAYTIM, EXP, SINCOS, SQRT, and TANH.

Computer Program Symbol Dictionary

Computer program symbols are given here in alphabetical order with brief definitions or descriptions. Program routine and function names are described in the previous part of this section. Symbols described as temporary storage quantities may be used for such purposes in more than one location within a program routine and within more than one routine. Details of their use may be obtained from the program listing if necessary. When temporary storage quantities are used in only one routine, that routine is indicated. Input quantities are preceeded by an asterisk. The user may refer to the Description of Input portion of this section for details concerning input quantity physical dimensions and allowable range of values, where appropriate.

Computer name	Algebraic symbol	Definition or Description
A	$[\pi]$	mode shape + correction matrix and temporary storage array in SHAPE
AMAX		temporary storage quantity in MATINV
ASAV		temporary storage for $[\pi]$
AVMAS	$\bar{m}$	lumped mass
A107	$M_{10,7}$	element of [M], see APPENDIX B
Aij	$M_{i,j},$	elements of [M], see APPENDIX B
B		temporary storage vector
BAND		temporary storage array in BNDRY1
BK		temporary storage quantity in BNDRYX
BND		argument equivalent of PRD, in BNDRY1 and BNDRYX
BPL		argument equivalent of A, in BNDRY1 and BNDRYX
B2	a_1	remainder quantity, see equation (12)
C		temporary storage quantity
CAPSI	Ψ	angle between blade element's chord line and plane perpendicular to rotor shaft
CCPS	$\cos \Psi$	
CL		temporary storage quantity in TEMAT
CL2 and CL3		temporary storage quantities in TEMAT
CNVRT		$\pi/180$

Computer name	Algebraic symbol	Definition or Description
*CPOMG	Ω	rotor speed
CPSQ	Ω^2	
CSSI		temporary storage for CCPS in TEMAT
CSSQ	$(\Omega \cos \Psi)^2$	
C1 thru C8		temporary storage quantities in TEMAT
C101		temporary storage quantities for C in BLADE
DATE		run date, defined by calling routine DAYTIM
DET	$D(\omega)$	determinant
DETERM		argument in MATINV, matrix determinant
DET1	$D(1.0)$	determinant
DT	$\|K\|$	determinant of [K] of equation (10)
DTOLD	$D(\omega^{(i-1)})$	storage for determinant from previous frequency iteration
DTSAV		temporary storage for DET in FINDM
EI	EI	temporary storage for EIX, EIY, or EIZ in TEMAT
*EIX	EI_x	torsional rigidity
*EIY	EI_y	chordwise bending stiffness
*EIZ	EI_z	flatwise bending stiffness
*EL	ℓ	length
*ELC	ℓ_c	control link length
*ELZ	$\Delta\ell_z$	elastic axis chordwise offset
*EMAS	m	mass
*EPS	ε	mass eccentricity
	or $\bar{\varepsilon}$	$\bar{\varepsilon}_i = (\varepsilon_i + \varepsilon_{i-1})/2$
EPSEM	$\bar{\bar{\varepsilon}}m$	
EPSN	ε_N	tip section eccentricity
EPST		temporary storage for EPS in TEMAT
EPSX		temporary storage for EPS in BLADE
ERR	$[\kappa]$	kappa vector, see equation (10)
ERR101		temporary storage for ERR in BLADE

Computer name	Algebraic symbol	Definition or Description
FACTR	$(1+f)^{0.5}$	(1 + PRCNT / 1000)
F10		mass load factor
F2		inertia load factor
F6		mass load factor
GAMMA	γ	$\gamma = \ell\ (P_T\ /\ EI)^{0.5}$
GNMS		mode shape generalized mass
G2	γ^2	
H	h	offset of elastic axis from radial coordinate
HI		temporary storage for H in TEMAT
I		matrix index
*IBC		boundary condition control
ICOLUMN		matrix index
ID		argument in MATINV, not used
ID1		argument in MATINV, but used
II		matrix index
IKJ		matrix index
IM1		matrix index
INDEX		matrix index
*INSEC		number of blade sections
IROW		matrix index
*ITAB		blade property table input control
IX		matrix index
J		matrix index
JK		matrix index
JKII		matrix index
JKL		matrix index
JX		matrix index
K		matrix index
KD1		argument in BNDRYX, boundary condition control
KD2		boundary condition control

Computer name	Algebraic symbol	Definition or Description
KD3		boundary condition control
KD4		boundary condition control
L		matrix index
LL		matrix index
L1		matrix index
M		matrix index
M1		argument in MATINV, not used
N		argument in MATINV, not used
NBND		tip boundary condition indices
NC		number of columns in [T], NC2-NC1-1
NC1		(NR1 + 1)/2
NC2		NR2/2
NC3		NC2 - NC1
NC4		NC-1
NC5		NC-2
*NEXPO		convergence criterion limit
NF		matrix index
*NFIND		frequency search option selector
NFLAG		frequency search control
NFLG		frequency search control
NFREQ	q	frequency counter
NGYRO		$\equiv 0$
NIBC		boundary condition control
NINT		matrix index
*NMAX		maximum number of frequency iterations
NN		internal program control
*NOOR		remainder number used for iterations
NOUT		output tape number, = 6
*NPCH		punched output control
*NPDF		number of known frequencies
NPIB		argument in BNDRY1, program control
NR		NR2 - NR1 + 1

Computer name	Algebraic symbol	Definition or Description
NR1		index of first row in [T]
NR2		index of last row in [T]
NR3		NR-1
NR4		NR-2
NSEC		≡INSEC
NSM1		NSEC-1
NSP1		NSEC+1
NSTEP		frequency search control
NTAPE		frequency search control
NTIME		iteration counter
NTRL		frequency search control
*NTYPE		type of mode coupling
*NUMF	q_{max}	number of frequencies to be found
NX		torsional rigidity control
NX1		argument in MATINV, not used
NY		chordwise bending rigidity
NZ		flatwise bending rigidity
N1		argument in MATINV, matrix size
OM		next estimate for ω^2 in INTERP
OMD		auxiliary function factor
OMEG		temporary storage for OMEGA
OMEGA	ω	natural frequency
*OMINT		frequency array
*OMLIMT		limit on ω^2 magnitude
OMLSS	$\omega^{2(i)}$	current iteration value for ω^2
OMLST	$\omega^{2(i+1)}$	previous iteration value for ω^2
OMNXT		temporary storage for OMOR in FINDM
OMOLD		auxiliary function factor
OMOR		temporary storage for OMSQ in FINDM
OMP		OMD*OMOLD
OMQ		temporary storage for OMRD
OMRD		magnitude of difference between values of ω^2 predicted by D and various R's

Computer name	Algebraic symbol	Definition or Description
OMSAV		temporary storage for OMSQ in FINDM
OMSQ	ω^2	
*PHI	$\Delta\Phi$	blade element twist
PHIX		storage for PHI
PI	π	3.1415926
PIVOT		diagonal matrix element in MATINV
*PRCNT		frequency search step size factor
PRD	[P]	product matrix
PTX	P_{T_X}	axial centrifugal force in element
PTZ	P_{T_Z}	chordwise centrifugal force in element
RNEW	$R^{(i)}$	remainders for current iteration
ROLD	$R^{(i-1)}$	remainder for previous iteration
S	$\sinh\gamma/\gamma$ and $(\sinh\gamma-\gamma)/\gamma^3$	
SAV		temporary storage for [P]
SAVXX		temporary storage for SAV
SCSI	$\sin\Psi\cos\Psi$	
*SIZER	θ_o	collective pitch at blade root
SIZERX		storage for SIZER
SL	ℓ/EI	
SL2	ℓ^2/EI	
SL3	ℓ^3/EI	
SNSI	$\sin\Psi$	
SNSQ	$(\Omega\sin\Psi)^2$	
SOC		variable used in determination of $\sinh\gamma$
SUMAS	$\bar{m}$	$\bar{m}_i = (m_i + m_{i-1})/2$
SV1010		temporary storage for SAV in BLADE
SWAP		temporary matrix element storage
T	[T]	transfer matrix
*THETC	θ_c	pitch link control angle
THETCX		storage for THETC

Computer name	Algebraic symbol	Definition or Description
*TITLE		run title
*TMSL		torsional mode punched output control
TMR	$\|Z(1)/Z(3)\|_N$	torsional mode limit ratio
VNORM		mode normalization factor
X	r	radial coordinate
XI		temporary storage for X
*XINR	I_x	torsional inertia
*XROOT	x_{root}	length from shaft center to first flexible blade element
YA		storage for D or R
YB		storage for D or R
*YINR	I_y	chordwise inertia
Z	(z)	state vector
ZERMO		temporary storage for $Z(9)_1$ in BNDRY1
ZSV		temporary storage for Z in SHAPE

Dimensioned Variables

Program variable dimensions may be grouped into three groups: those which depend on the maximum number of elements in the model, MS, those which depend on the maximum number of natural frequencies, MF, and those which have a constant dimension size. Program variables and the appropriate dimensions are given for each routine, in alphabetical order, with dimensions indicated as MS, MF, or the fixed number necessary.

Routine	Variable name (dimension)
BLADE	A (10,5) ASAV (10,5) AVMAS (MS)
	B (10)
	C (10) CAPSI (MS) CCPS (MS) C101 (10,1)
	EIX (MS) EIY (MS) EIZ (MS) EL (MS) ELZ (MS) EMAS (MS) EPS (MS) ERR (10) ERR101 (10,1)
	NBND (5)
	OMINT (MF)
	PHI (MS) PHIX (MS) PRD (10,5)
	RNEW (4) ROLD (4)
	SAV (10,5) SV1010 (10,10)
	T (10,10) TITLE (19)

Routine	Variable	
	XINR (MS	
	YINR (MS)	
BNDRY1 and BNDRYX	same as BLADE except	
	Delete:	A ASAV
		C
		NBND
		OMINT
		PHIX PRD
		RNEW ROLD
		SAV
		T
	Add:	BAND (10,5) BND (10,5) BPL (10,5)
		SAV (10,7)
FINDM	OM (4) OMINT (MF) OMRD (4)	
	RNEW (4) ROLD (4)	
INTERP	none	
MATINV	A (10,10)	
	B (10,1)	
	INDEX (10,3)	

Routine	Variable name (dimension)	
SHAPE	same as BLADE except	
	Delete:	ASAV
		ERR
		NBND
		OMINT
		PHIX
		RNEW ROLD
		SAV
		TITLE
	Add:	Z (10) ZSV (10)
SNTFR	NBND (5)	
TEMAT	same as SHAPE except	
	Delete:	A
		C

Description of Input

Input parameters are provided to the computer on punched cards. A description of the input is given in this section in tabular form, and includes the card number, input format (and FORTRAN statement number), and the computer program variable name, type, definition, and permissible range of values and/or physical dimensions where applicable. Physical dimensions are given and output labeled in English units. Any other system of units is acceptable as long as blade physical properties are consistent. However, output will remain labeled in English units unless changed.

The FORTRAN Statement numbers and formats used to read input parameters are as follows:

```
401   FORMAT(2I1,I2,19A4)

402   FORMAT(I2,F6.5,9E8.0)
```

and other parameters are read using NAMELIST, where variables given in the NAMELIST "IN" are punched in any order or format on cards which begin with a "$IN", which end with a "$" punched after all data, which have no punches in the first column of any card, and with variables separated by commas.

Permissible values or ranges of values for input parameters are given where essential for program operation; other remarks concerning program and model limitations as discussed in the section on PROGRAM USE should also be taken into consideration. All input variables which begin with letters I through N are integer numbers, and are read without decimals and must be right justified in the input field. All other input variables are floating point variables, and are read in F or E format as described, except TITLE, which may contain alphanumeric data, and uses A format. The arrangements of input parameter description is as follows.

(Card No.); Format No., Parameters defined

Column	Computer name	Algebraic symbol	Permissible values, physical dimensions, etc.

(1); 401, Program control and model title

1	IBC		Boundary conditions control =1, pinned-pinned, with lead-lag hinge at outboard end of first element and flap hinge at inboard end of first element; =2, clamped flatwise and edgewise, =3, pinned flatwise and clamped edgewise, or =4, clamped flatwise and pinned edgewise.

2	ITAB		Blade property table input control $\neq 0$, reads blade properties, or $=0$, uses previous model's blade properties. Note that ITAB=0 results in printed output values of PHI and EPS which correspond to the PHI in radians and the averaged values of EPS, $\bar{\varepsilon}$, rather than the "input" numbers as are printed for ITAB=1
3,4	INSEC	N	Number of elements in the blade model (INSEC 30 stops execution), and this may be used as a "stop card" in the data deck.
5-80	TITLE		Title of run (alphanumeric data)

(2 thru INSEC + 1); 402, Blade element lumped parameter values

1-2	J	j	Blade element lumped number, increases with increasing radial coordinate
3-8	EL	ℓ	Blade element length, feet
9-16	EIX	EI_x	Element torsional stiffness, $lb\text{-}ft^2$
17-24	EIY	EI_y	Element edgewise stiffness, $lb\text{-}ft^2$

Column	Computer name	Algebraic symbol	Permissible values, physical dimensions, etc.
25-32	EIZ	EI_z	Element flatwise stiffness, $lb\text{-}ft^2$
33-40	XINR	I_x	Element torsional inertia about c.g., $lb\text{-}sec^2\text{-}ft$
41-48	YINR	I_y	Element chordwise inertia about c.g. ($\equiv$XINR for models where numerical values are unavailable)
49-56	EMAS	m	Element mass, $lb\text{-}sec^2/ft$

57-64	PHI	$\Delta\phi$	Element twist, degrees
65-72	EPS	ε	Mass eccentricity, positive for mass center of gravity forward of elastic axis, ft.
73-80	ELZ	ℓ_z	Change in offset of elastic axis from a radial line drawn through the center of rotation, positive for average elastic axis location of present element ahead of that for previous element, feet

(INSEC + 2); NAMELIST/IN/, Program control and operating conditions

(Order and columnar spacing is not defined; the name and at least one numerical value must be included in the NAMELIST/IN/ data).

NTYPE		Type of mode coupling; =1, uncoupled torsional; =2, uncoupled flatwise; =3, uncoupled edgewise; =4, partially coupled torsional-flatwise; =5, partially coupled flatwise-edgewise; =6, fully coupled torsional-flatwise-edgewise
NMAX	i_{max}	Maximum number of iterations allowed during the search for one natural frequency value, suggested values: 14 for NFIND≠1, 30 for NFIND=1.

Computer name	Algebraic symbol	Permissible values, physical dimensions, etc.
NEXPO		Convergence criteria limit parameter, suggested value: 10 to 12
NUMF	q_{max}	Limit on the number of frequencies found for the current blade model, NUMF≤15 due to dimension sizes (MF parameter in Dimensioned Variables portion of the COMPUTER PROGRAM IMPLEMENTATION section)

NOOR		Remainder number to be used for NFIND= - 1,or 0; Range: 0<NOOR≤ ((number of elements in state vector)/2-1). Values of NOOR out of this range are superceded by the maximum value allowed for the current mode types.
NPCH		Punched output control: =0, no punched output >0, punched output: unity generalized mass type of mode shape and frequency data, using formats 916 and 930 of subroutine SHAPE.
CPOMG	Ω	Rotor speed, rad/sec. Values of CPOMG $\geq$ 0 are allowed.
SIZER	Ψ_o	Blade root collective pitch angle, degrees, any value allowed
THETC	θ_c	pitch link control rod angle with plane perpendicular to rotor shaft, degrees. If no value is known, a suggested value is 0.0.
ELC	ℓ_c	Control link length, ft
XROOT	x_{root}	Length from center of rotation to inboard end of first blade element, XROOT $\geq$ 0 may be used.

Computer name	Algebraic symbol	Permissible values, physical dimensions, etc.
PRCNT		Step factor in frequency search methods, suggested values: not used for NFIND=1; ≤0.01 for NFIND=0; $\overline{1} \leq$ PRCNT $\leq$ 30 for NFIND=1, depending on known or expected frequency spacing. See figure 6. If PRCNT=0 is input, it is revised to PRCNT=10.
NFIND		Frequency search control =1, automated search option =0, initial estimates option =-1, frequency stepping option

NPDF		Number of previously determined frequencies to be eliminated during automated frequency search (NFIND=1). These values of ω_n are stored in the OMINT array.
OMINT	ω	Natural frequency estimates array:

NFIND	No. of estimates
-1	1
0	greater of NUMF and 1
+1	greater of NPDF and 1

TMSL	$(Z_1/Z_3)_{N_{max}}$	Torsional mode size limit. If TMSL$\geq$ |(Z(1)/Z(3)| the punched output is controlled by NPCH, otherwise no punched output will be obtained. Suggested values: TMSL > 1000 for no limit on punched modes TMSL = 1.0 for elimination of punched torsional modes
OMLIMT	ω^2_{max}	Limit on the size of ω_n^2, beyond which no natural frequencies are desired. Program stops after the first value of OMEGA^2 which exceeds OMLIMT.

<u>Alphabetic list of input parameters</u>.- An alphabetic list of the computer program input variable names together with their respective data card group and definition are given here for convenience. The card group is one of the following: (1) title, (2) blade element properties, or (3) NAMELIST /IN/.

Name	Card group	Definition
CPOMG	3	rotor speed
EIX	2	torsional stiffness
EIY	2	edgewise stiffness
EIZ	2	flatwise stiffness
EL	2	length
ELC	3	control link length
ELZ	2	offset of elastic axis from radial coordinate
EMAS	2	mass
EPS	2	mass eccentricity

IBC	1	boundary condition control
INSEC	1	number of elements in blade model
ITAB	1	blade property input control

Name	Card group	Definition
J	2	blade element number
NEXPO	3	convergence criterion limit parameter
NFIND	3	frequency search control
NMAX	3	maximum number of iterations per frequency
NOOR	3	remainder number control
NPCH	3	punched output control
NPDF	3	number of previously determined frequencies
NTYPE	3	type of mode coupling control
NUMF	3	limit on the number of frequencies
OMINT	3	natural frequency estimates array
OMLIMT	3	limit on size of ω_n^2
PHI	2	twist increment
PRCNT	3	frequency search step factor
SIZER	3	blade root collective pitch angle
THETC	3	pitch link angle
TITLE	1	run title (alphanumerical)
TMSL	3	torsional mode limit switch
XINR	2	torsional inertia
XROOT	3	length from center of rotation to first blade element
YINR	2	chordwise inertia

Description of Output

Output is printed in blocks for each model. Punched output of unity generalized mass type of mode shapes results for NPCH $\neq$ 0, and is not obtained for NPCH = 0. (No control over printed output is implemented.) The output for each model is arranged in three groups: (1) program and blade identification, including run title, run date, blade operating conditions and blade properties; (2) program control parameters; and (3) frequency search data and resulting mode shapes. All input parameters are written out, and labels and arrangement are intended to maximize clarity and convenience to the user.

The program and blade model group of input parameters has the arrangement shown in APPENDIX D, page 107. The only variations from that format are those due to special messages concerning rigid models, as controlled by NTYPE, and those noted on page 40 when ITAB=0.

The program control group of input parameters has values of OMINT printed only when they are used, otherwise that output is invarient. Typical output is shown in APPENDIX D on page 111.

The frequency search data and mode shape parameters will be repeated until either NUMF or OMLIMT controls cause the program to stop. This output group normally requires, for each frequency, approximately one page for frequency search data and one or two pages for mode shape data. The data for both the frequency searches and mode shapes is given in columns, with the columns labeled, except that the natural frequency in radians per second and as a multiple of rotor speed and the generalized mass of the mode are printed across the page immediately below each mode shape data set.

The punched output results only if NPCH $\neq$ 0. The punched output corresponds to the printed mode shape data for the mode with unity generalized mass except that: (1) the data at station number "0" is not punched, and (2) a frequency in radians per second, the first 64 alphanumerical characters of the run title, and the number of the frequency, as given by the frequency counting index, NFREQ, is punched on a card preceding the mode shape data. Mode shape data is punched with two cards for each station. The first card contains the state vector elements z(3), z(7), z(1), z(8), and z(4) in that order in format 5(E14.7, 1X) and the second card contains z(2), z(5), z(6), z(9), and z(10) in that order in format 5E15.7. Page 120 of APPENDIX D is a listing of cards corresponding to the printed input given on page 103 of APPENDIX D. The punched data may be used with no changes in the program discussed in reference 5. It is expected that other programs would require different formats, but such changes should be straightforward. All such changes would involve only two existing output statements in subroutine SHAPE. One is located three cards after FORTRAN statement number 17 and punches the natural frequency, title, and frequency number, and the other is given by FORTRAN statement number 51 and the following continuation card, and punches the state vector quantities.

PROGRAM USE

This program was designed to compute the fully coupled normal modes and natural frequencies of helicopter blades, but can be used to analyze other similar physical systems or subsystems. The rotational speed may be positive or zero. The program is applicable to beams with an elastic axis which runs approximately parallel to a radial coordinate. The beams may have principal elastic axes which are twisted, may have nonuniform mass, inertia, and stiffness properties and noncoincident mass center and elastic axes. The neutral axes for torsion and flexure should be approximately coincident. Steady pitch and twist angles are modeled, but steady coning and lagging angles are not modeled. A variety of hub end boundary conditions are available, but the tip end is assumed to be "free". i.e. force type quantities are zero and displacement type quantities are nonzero at the tip. The types of motion coupling which may be modeled are: coupled torsional-flatwise-edgewise motion, partially coupled torsional-flatwise motions and flatwise-edgewise motions, and uncoupled torsional, flatwise, and edgewise motions.

The model used for the beam is a lumped parameter model which has lumped point mass and inertia and uniform massless elastic sections. Twist of the beam is modeled by finite incremental twists at the mass location. The offset of the elastic axis from a radial line is also modeled by finite incremental offsets at the mass location. Centrifugal forces in the offsets are included (i.e. large steady forces in the chordwise direction), but the offset lengths are assumed to be rigid. The elastic axes of the flexible sections are modeled to be parallel to the radial coordinate. Blade precone and prelag are not included in the model.

This type of model does not provide a consistent lower or upper bound to the theoretical continuous model natural frequency, but should provide acceptable values for use in blade design and forced response analysis. A discussion of the errors in frequency estimates which are introduced by lumped parameter models is presented in reference 2.

Rotating or non-rotating beams with the outboard end free, helicopter rotor blades, propellers, fan and turbine blades are examples of the types of systems which can be modeled by this program.

Development of Blade Property Input Data

Blade properties are usually available in graphical or tabular form, and lumped parameter models may be developed from such data in the following manner. The section lengths are chosen so as to permit uniform element subsections to model nonuniform blade properties. Usually the largest spanwise variations occur in mass or stiffness properties. It is suggested that a reasonable minimum number of elements for a uniform blade is ten, but more elements may be needed if more than three of four flatwise modes and frequencies are desired. The blade mass eccentricity, if not available, may be assumed to be zero. The blade element mass, m, inertia, I_x and I_y, and incremental twist, $\Delta\Phi$; are the average values per unit length times the section length. The torsional and bending stiffness EI_x, EI_y, and EI_z, the elastic offset from the previous section ℓ_z, and the eccentricity, ε, are the average values for the element, subject to the following general rules, which have been found to be useful.

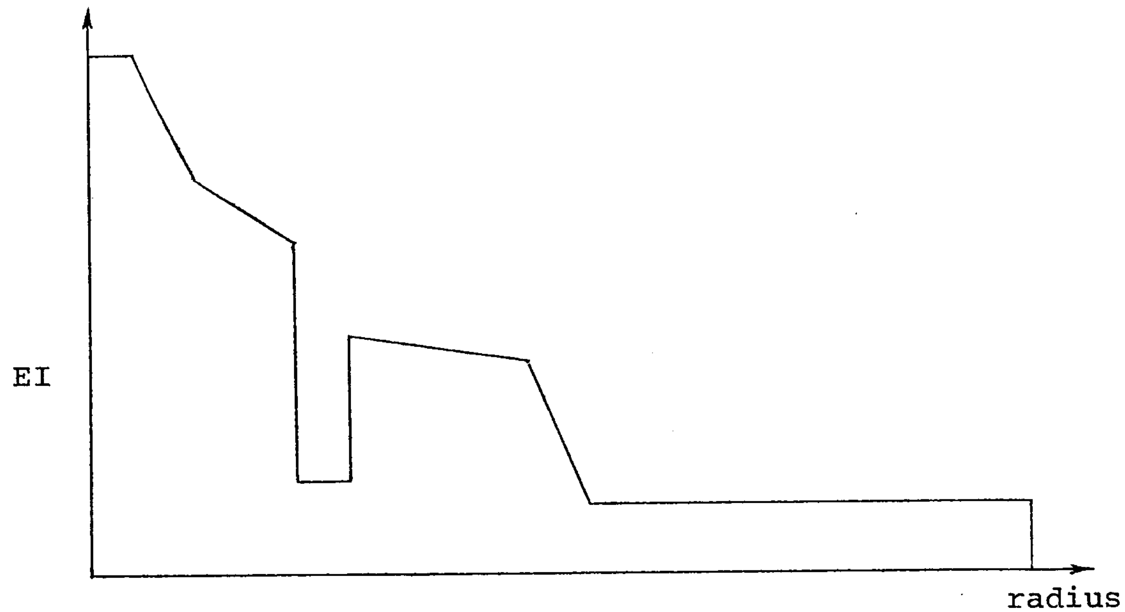
(1) Localized regions of relatively low torsional or bending stiffness, such as that indicated in figure 9(a), should be modeled as accurately as possible, especially if they occur near the center of rotation.

(2) Localized regions of relatively large stiffness may be neglected.

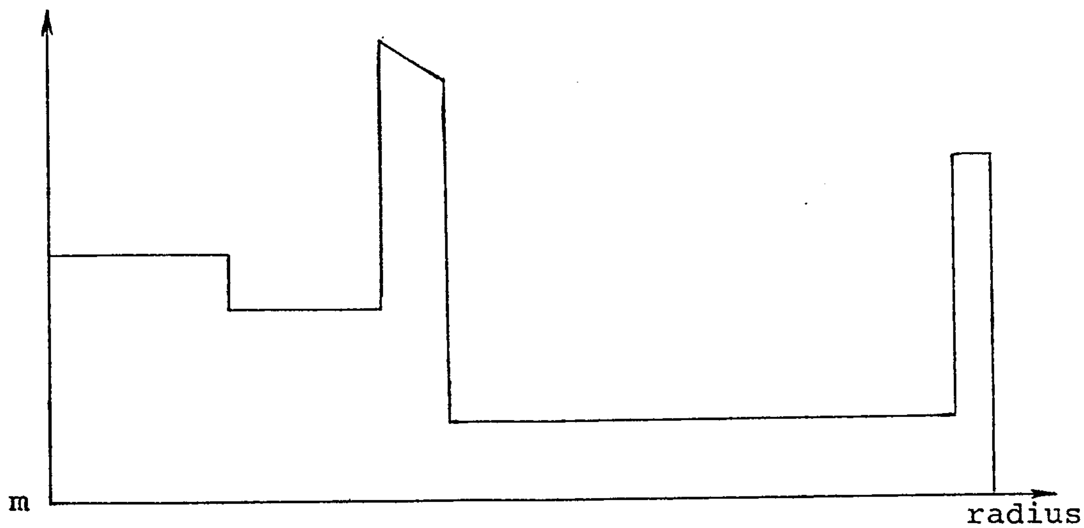
(3) Localized variations in mass and inertia properties, eccentricity, and elastic axis location may be averaged, but should not be ignored. Large, highly localized mass distribution, such as indicated in figure 9(b) should be accurately modeled.

(4) If a choice must be made, it is usually better to have inboard elements shorter than outboard elements.

(5) Chordwise inertia, I_y, are often not given, and in such cases they should be made equal to the torsional inertia, I_x.



(a) system with a localized low stiffness
(EI) region



(b) system with localized large masses
(m) regions

Figure 9. Blade stiffness and mass variation modeling

Boundary Condition Options

The hub boundary conditions should be chosen to most closely approximate the actual system. It should be noted that the fully articulated rotor must have its edgewise hinge "outboard" of the flatwise hinge, even though the length, ℓ, between them may be zero. That is, the flatwise hinge is fixed to the hub and the edgewise hinge "flaps". The location of the torsional pitch bearings is arbitrary, and is determined by the section which has it EI_x value adjusted to account for a point torsional spring. If spanwise location is important, this section length may be made very small.

Pinned boundary conditions result in exactly zero moments at the pin. Bending flexures may be modeled by clamped boundary conditions with appropriate section bending stiffnesses. Other types of boundary conditions may be modeled in a similar manner or new boundary condition options added in subroutine form.

Most of the complications of the BNDRY1 and BNDRYX subroutines arise because of the necessity to be able to model six combinations of motion coupling. It is suggested that a user may implement different boundary conditions, other than those presently implemented, most easily in the following manner.

(1) Define the boundary conditions in terms of zero and nonzero state vector elements.

(2) Define an intermediate transfer matrix composed of zeros and ones which when multiplied by one of the present boundary conditions gives the desired zero and nonzero state vector elements, for fully coupled motion.

(3) Make program changes which perform the necessary multiplication of the existing PRD and A matrices referred to in (2) (which are provided by the BNDRY1 or BNDRYX subroutines) prior to their multiplication by the standard transfer matrix, T, (which is provided by subroutine TEMAT).

Mode Coupling Options

Fully coupled modes and natural frequencies are preferred over partially coupled or uncoupled modes and natural frequencies for the analysis of blade response. This is primarily because the blade response is strongly dependent upon natural frequency values, and natural frequencies of fully coupled modes are generally more accurate than natural frequencies of partially coupled or uncoupled modes. However, even the fully coupled modes and frequencies do not account for mechanical coupling between blades except for teetering rotors, and no torsional coupling between blades is modeled for any boundary conditions. Mechanical coupling between blades and nonuniform swashplate stiffness can be important factors, and can be analyzed by the method discussed in reference 4.

Frequency Search Options

Suggested uses of the frequency search options are discussed here, with a brief summary of their (intended) operating characteristics. Specific details of program input for these options are given in the section on Input.

Automated frequency search.- Ordinarily the lower natural frequencies of a helicopter rotor blade (and many other systems) are of more interest than the higher natural frequencies. When all (or a majority) of the lower natural frequencies (up to a maximum of 15) are of interest, the use of the automated frequency search option provides a method for their determination which required a minimum of user skill in developing program control input. Because of its features, it would usually be the most economical option with regard to both labor and computer costs.

In the event that some natural frequencies of a system are known and it is desired that these not be calculated, they are input in the OMINT array, with the value of NPDF set equal to the number of these previously determined frequencies. The program treats these as roots of $D(\omega)$, and neither these frequencies nor their corresponding mode shapes are determined. (This feature permits the user to obtain, in a "second" run, some of the lowest fifteen natural frequencies which may have been missed in a "first" run due to time limits or which may have had poor mode shapes due to improper convergence limits, etc.)

One of the limitations of this option is that the method as implemented does not allow the user to obtain frequencies above the lowest 15. However, this could be resolved by increasing the dimensions of those variables which depend on the maximum number of frequencies allowed.

Initial estimate option.- When frequencies are not too closely spaced, or where approximate values are known, initial estimates provide a convenient means of obtaining their values. While this option has been somewhat obsoleted by the automated search option, it may be utilized in connection with that option in determining frequencies which the automated option missed due to the limit on the number of frequencies, or which did not yield good mode shapes due to convergence limits. Also, frequencies higher than the lowest 15 may be obtained by using this option.

Frequency stepping option.- This option may be used to compute determinants and remainders as a function of ω^2, or to obtain detailed information over a limited frequency range. It is largely obsolete due to the automated frequency option, but has been retained for possible user needs.

Use of Output

The output from the program consists of printed and punched data. The printed data includes all input quantities. These quantities are useful in checking input data during calculations, and in properly identifying and using previously generated output. The above quantities are labeled and should require no user modifications unless program input is changed. During the iteration process, values of the iteration counter, current frequency estimate, determinant, auxiliary functions, and remainders are printed. Once a natural frequency is obtained or the limit on the number of iterations is reached, a mode shape is printed. The mode shape accuracy can be evaluated from the relative size of the force-type quantities at the tip as compared to those quantities at the next inboard state vector position.

As was previously discussed, two types of mode shapes are printed out. The first has a normalized tip deflection quantity of unity and the second has a unity generalized mass. Where normal modes and natural frequencies are to be used to compute the forced response of blades in programs such as that discussed in reference 5, it is convenient to have this second type of mode. The program has the option to punch the frequency and mode shape for NPCH = 1, for use in such blade response calculations. The mode shape quantities at the inboard end of the first section are not included in the punched outout, and are not normally used in calculations of blade response.

In many design processes, natural frequencies are adjusted by modification of blade properties so as to avoid known forcing function frequencies. Mode shapes and natural frequencies in their printed output form would be useful for such design processes.

The variables read from the title card and NAMELIST /IN/ include most of those which are expected to vary during blade analysis and design processes and for use in forced response calculations. If a user desires to redefine some blade properties, such a property could be changed readily by reading it in the NAMELIST /IN/ group. The only program changes required would be the addition of the appropriate blade property's computer name to the NAMELIST /IN/. Since the NAMELIST is read after the blade properties are defined by the presently implemented read statement, the quantities read from the NAMELIST would replace any previously defined quantities.

Program Limitations

Program limitations fall into two general categories, those related to computer compatibility and those related to limitations of the model and methods of solution. Some of these program limitations have been dealt with in other sections, and are repeated here for emphasis and convenience to the user.

Computer compatibility.- One of the most serious limitations of the program as listed in APPENDIX C is that it is not functional on computers which use less than 14 digits in floating point arithmetic operations, due to numerical inaccuracies in the determination of natural frequencies and mode shapes. That program is compatible with CDC 6600 or CDC 6400 computers at NASA/Langley. Minor changes would be required to make the program compatible with other computers which have at least 14 digit words.

Program changes which resulted in satisfactory program operation on an IBM 360/65, are given in detail in APPENDIX E.

Model limitations.- There are basic limitations of the model or of the computer program as implemented which restrict the types of systems which may be analyzed by the program. The lumped parameter nature of the model requires the assumption of uniform property massless elastic sections and point masses and inertias; stepwise blade twist, and stepwise elastic axis and center of mass location variations, with the elastic axes of all elements assumed to be parallel to a radial coordinate. Chordwise offsets in the elastic axis are assumed to be rigid, but may be large. Shear deformations are neglected, so short, thick beams or other systems for which shear deflections may be important as compared to bending deflections would not be accurately represented by this system. Oscillatory axial force variations and displacements are not included as part of the model, nor rotary inertia about the z-axis. The blade elements may either have an axial tension load as determined by the sum of the centrifugal forces on the masses outboard of the blade element, or may have zero axial load. They may not be in compression, and no spanwise distribution of axial tension load is allowed, other than that which is a stepwise (uniform within each element) approximation to a centrifugal loading distribution. Only those hub and tip boundary conditions which are discussed in the section on PROGRAM IMPLEMENTATION are presently available; others would require program changes. The hub and tip boundary conditions are assumed to be uniform azimuthally. A perfectly rigid hub is assumed with no azimuthal variations in control system stiffness or bending stiffnesses, and no effects of shaft flexibilities or helicopter engine or fuselage mass are included.

Numerical Accuracy Problems

Numerical accuracy problems are inherent in most eigenvalue problems of this type, but have been largely overcome by the modified transfer-matrix approach which is implemented in this program. There are two general areas where numerical inaccuracies may be a problem. One is the use of the frequency search technique to obtain natural frequencies and the other is the definition of acceptable mode shapes with a convergence criterion based on the smallness of the change in the estimated natural frequencies for two successive iterations.

Effects of numerical inaccuracies in the frequency search options.- The types of numerical accuracy problems which are associated with the frequency search methods have been reduced by the automated search method, but are still real. The use in the iteration process of one of the remainders rather than the determinant or auxiliary function may result in convergence to a frequency other than that intended, regardless of the type of search method used. That is, duplicate frequencies may be obtained and frequency skipping may occur. This is less likely with the automated search option than with either of the other methods. A discussion of other types of problems which may be encountered with the frequency search options is given in the THEORETICAL FORMULATION section.

The frequency iterations are now stopped by either a limit on the number of iterations or convergence of the frequency to a natural frequency value. If the iteration limit is reached, the program will print out a mode shape corresponding to that frequency. Usually, this mode shape does not satisfy the tip boundary conditions. A satisfactory mode shape may be obtained in most cases by a subsequent run using the initial estimate option with an initial estimate as obtained from the output of the run which failed to converge. When the automated search method is limited by the number of iterations, the natural frequency counter, NFREQ, is set equal to NUMF, and the program will go to the next model. This is necessary to avoid incorrect definition of the auxiliary function and subsequent erratic behavior of the automated frequency search. Thus, a higher value for NMAX is suggested for use with the automated frequency search option than with the other options.

Effects of numerical inaccuracies on mode shapes.- The use of a convergence criterion which depends on the smallness of the change in successive estimated natural frequency values may not always result in satisfactory mode shapes, as evaluated by the relative orders of magnitude of tip force type quantities as compared to these quantities at the next inboard section. However, use of NEXPO = 12 has resulted in satisfactory mode shapes for all cases which have been run with this program, and use of NEXPO = 10 is usually satisfactory. It is expected that use of NEXPO > 13 is futile for computers which use 14 digit floating-point words.

DISCUSSION OF TEST CASES

Test cases were run at NASA Langley which tested all computer program subroutines and which included all program control options available for frequency search, mode coupling, boundary conditions, and punched output (including suppression of punched output of torsional mode data). The automated frequency search method was used with helicopter blade models which had been made with the previous program to insure consistency of results, and for polynomials with multiple and closely spaced roots to provide a test of the automated search method for possible similarly severe cases which may be encountered in future use. All options perform as expected, and the automated search technique handles multiple and closely spaced roots very well. Details of some of the test case results are given subsequently. A listing of the data deck used to generate the test cases is presented in APPENDIX D, together with selected printed and punched output. The complete printed output has been provided to NASA Langley, and may be duplicated by running the program with the test case data deck. This run was made during March 23, 1972, and required approximately 168 CPU seconds (including 13.4 CPU seconds to load and compile) and 65 PPU seconds for loading, compilation, and execution. The storage required as given by the FWA LOADER and LWA LOAD labels was 041333 and 032750 respectively; 276 O/S CALLS were made during the run; and 5120 lines of printed output were generated.

Test Cases For Helicopter Rotors

Helicopter rotor test cases included runs for an approximately uniform rotor with various boundary conditions, a fully articulated (hinged-hinged) rotor with various types of mode coupling, and one with two nearly coincident frequencies. All frequency options were tested and operated as expected. Most of the cases chosen for discussion used the automated search method, but one frequency stepping case is also presented.

Various boundary conditions.- Test cases for helicopter rotors which had been run with the previous program included that labeled in APPENDIX D as Test Case 1.a. Approximately Uniform Clamped-Clamped Rotor. Three frequencies were determined, and agreed with those determined by previous runs (not using the automated frequency search method). Those three frequencies were 1.0452Ω, 1,5110Ω, and 2.7756Ω. No printed or punched mode shapes corresponding to these frequencies are included in APPENDIX D. The first frequency and mode shape for this same blade but with pinned flatwise-clamped edgewise (IBC=3) and clamped flatwise-pinned edgewise (IBC=4) boundary conditions are presented in APPENDIX D as Test Case 1.b and 1.c, respectively.

Mode coupling variations.- All types of mode coupling were exercised for a fully articulated rotor. Of these, two are presented in APPENDIX D as Test Cases 2.a and 2.b. The rotor parameters and program control parameters are given with Case 2.a, which is for the first frequency for uncoupled torsional motion (NTYPE=1). Only NTYPE is different for Case 2.b, which gives the third frequency for partially coupled torsional-flatwise motion (NTYPE=4), and is the partially coupled equivalent of the first uncoupled torsional mode.

Closely spaced frequencies.- A model of a fully articulated rotor with very little pitch-flap coupling was modified to give two fully coupled (predominantly torsional-flatwise) modes with nearly coincident frequencies. Test Cases 3.a and 3.b present the blade model parameters, program control parameters, frequency search data, and mode shapes for the two coincident frequencies. Note that the option to eliminate previously determined natural frequencies was also used during this run.

Frequency stepping case.- The frequency search data presented in APPENDIX D as Test Case 4 indicates the type of problems which may be encountered if the frequency stepping option is used. The blade data for this case is the same as that in Test Case 3. It should be noted that the choice of NOOR=1 was used, which results in a torsional-motion dominated search. Use of other remainders (2,3, or 4) may have resulted in a search which did not miss the frequencies near 53.95 rad/sec and 70.67 rad/sec, as occurred with the use of NOOR=1.

Polynomial Case

For a model with NSEC=2, the program automatically assumes that the squares of NUMF≡NPDF roots of a polynomial are read from the OMINT array. A polynomial with roots at the square roots of the numbers 5, 6, 6, 10, 10.01, 23.47, 33.8, 999, 1001, and 15000 was used to test the automated frequency search option. In this operation, the numerical value of the polynomial was determined by the program, and that value was treated as a determinant for an uncoupled torsional mode type. The table below gives the squares of the actual and the computed roots as found using the automated search method. As may be seen, the method successfully obtained the multiple root at $\sqrt{6}$, and also distinguished between the closely spaced roots at $\sqrt{10}$ and $\sqrt{10.01}$; and at $\sqrt{999}$ and $\sqrt{1001}$.

Values of Squares of Roots

actual	computed
5	4.9999999997
6	5.99999999998
6	5.99999999997
10	9.999999997
10.01	10.00999996
23.47	23.46999997
33.8	33.7999998
999	998.999999997
1001	1000.999998
15000	14999.547

APPENDIX A

SYMBOLS

The algebraic symbols which are used in this document are given here with their computer equivalent, if any, and definitions.

Symbol Algebraic	Name(s) Computer	Symbol Definition
a		ℓ/EI, length flexibility parameter, 1/foot-pound
a_i	B2	κ_i, as determined from "excluded equation" of equation (10); for example, a_1 as given by equation (12)
b_i	ERR(I)	κ_i, as determined from simultaneous solution of equation (11)
C_1	C1	cosh γ, see APPENDIX B
C_2	C2	$(\cosh \gamma-1)/{}^{2}$, see APPENDIX B
[C]		rigid offset matrix, see equation (2) and APPENDIX B
D	DET or DTOLD	determinant
[E]		elastic field matrix, see equation (2) and APPENDIX B
EI_x, EI_y, EI_z	EIX,EIY,EIZ	torsional, edgewise and flatwise stiffness, respectively, pound-feet^2
F_i	YA or YB	i^{th} auxiliary function
f		frequency step factor
h	H	offset of elastic axis ahead of radial coordinate, in plane perpendicular to shaft, feet
i	NTIME	iteration counter during frequency search
I_x, I_y	XINR,YINR	rotary inertia about x and y axis respectively, pounds feet seconds^2
[K]	part of A,ASAV	mode shape correction coefficient matrix, see equation (10)
k		torsional control system stiffness, foot-pounds/radian

Symbol Algebraic	Name(s) Computer	Symbol Definition
ℓ	ELNTH	section length, feet
ℓ_c	ELC	control link length, feet
ℓ_z	ELZ	offset of elastic axis in chordwise direction, ft
M_y, M_z	Z(9),Z(5)	z axes, respectively, foot pounds
[M]		point mass and inertia matrix, see APPENDIX B
m or $\bar{m}$	EMAS or AVMAS	element mass or lumped parameter point mass, pounds-second^2/foot
N	NR	number of elements in the state vector, (z)
[P]	PRD,SAV	product matrix, see equation (6)
P_{t_x}, P_{t_z}	PTX,PTZ	centrifugal forces is the x and z directions, respectively, pounds
[Q]	SAV	determinant matrix, see equation (7)
q	NFREQ	number of frequencies found, see figure 8
R	ERR	remainder
r	R or X	radial coordinate, feet
S_1	S	sinh γ/γ, see APPENDIX B
S_2	S	(sinh $\gamma-\gamma)\gamma^3$, see APPENDIX B
[T]	T	transfer matrix, see equation (2) and APPENDIX B
T	z(2)	torque, foot pounds
V_y, V_z	-Z(6),Z(10)	shears in the y and z directions, respectively, pounds
v	Z(3)	flatwise lineal deflection, feet
w	Z(7)	edgewise lineal deflection, feet
X_{root}	XROOT	length from center of rotation to inboard end of first section, feet
x,y,z	X,-,-	axial, flatwise, and edgewise local blade coordinates, see figure 1
(z)	Z	state vector, see equation (1)
γ	GAMMA	$\ell(P_{T_x}/EI)^{1/2}$, see APPENDIX B
ε	EPS	mass eccentricity, see APPENDIX B

Symbol Algebraic	Name(s) Computer	Symbol Definition
θ, ϕ, ψ	z(4),z(1),z(8)	angular elastic deformations about z, x, and y axes, respectively, radians
$[\kappa]$	ERR	state vector correction factor matrix
$[\lambda]$	B	estimated hub state vector quantities, see equation (8)
$[\Lambda]$	part of A,ASAV	estimated tip state vector, quantities, see equation (9)
$[\pi]$	A or ASAV	modified matrix transfer matrix
$[\Phi]$		twist transfer matrix, see equation (2) and APPENDIX B
Φ	CAPHI	angle between local blade element z-axis and plane perpendicular to shaft, degrees for input, radians internally
Ω	CPOMG	rotor speed, radians/second
ω	OMINT	frequency estimate, radians/second
ω_n	OMEGA	natural frequency of lumped parmater model, radians/second
Superscripts		
i		iteration number
Subscripts		
a		actual value
e		effective value
i,k		matrix element row and column position indices, respectively
j		blade element number
max		maximum value of parameter
N		tip end quantity
O		hub end quantity
x,y,z		local blade coordinates parallel to the radial coordinate and in the flatwise and edgewise directions respectively, see figure 1
Mathemtical Notations		
Δ		an incremental change or amount
σ		$\frac{d}{dt}$

APPENDIX B

TRANSFER MATRICES

The transfer matrices which are used in defining the element transfer matrix, [T], of equation (2), are given here for convenience. The computer program implementation was done on the basis of the analytically defined products, i.e. the elements of [T] are programmed. For convenience, these matrices are partitioned into submatrices corresponding to uncoupled mode state vectors.

Torsional twist matrix, [Φ], for an incremental twist angle Φ,

$$
\left[\begin{array}{cc|cccc|cccc}
1 & 0 & & & & & & & & \\
0 & 1 & & & 0 & & & & 0 & \\
\hline
0 & & \cos\Phi & 0 & 0 & 0 & -\sin\Phi & 0 & 0 & 0 \\
0 & & 0 & \cos\Phi & 0 & 0 & 0 & -\sin\Phi & 0 & 0 \\
0 & & 0 & 0 & \cos\Phi & 0 & 0 & 0 & -\sin\Phi & 0 \\
0 & & 0 & 0 & 0 & \cos\Phi & 0 & 0 & 0 & -\sin\Phi \\
\hline
 & & \sin\Phi & 0 & 0 & 0 & \cos\Phi & 0 & 0 & 0 \\
 & & 0 & \sin\Phi & 0 & 0 & 0 & \cos\Phi & 0 & 0 \\
0 & & 0 & 0 & \sin\Phi & 0 & 0 & 0 & \cos\Phi & 0 \\
 & & 0 & 0 & 0 & \sin\Phi & 0 & 0 & 0 & \cos\Phi
\end{array}\right]
$$

Massless elastic field with tensile force matrix, [E]:

$$
\left[\begin{array}{c|c|c}
\begin{bmatrix} 1 & 0 \\ 0 & \frac{\ell}{EI} \end{bmatrix}_x & 0 & 0 \\
\hline
0 & \begin{bmatrix} 1 & \ell S_1 & \ell a C_2 & \ell^2 a S_2 \\ 0 & C_1 & \ell a S_1 & \ell a C_2 \\ 0 & P_{T_x}\ell S_1 & C_1 & \ell S_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_z & 0 \\
\hline
0 & 0 & \begin{bmatrix} 1 & \ell S_1 & \ell a C_2 & \ell^2 a S_2 \\ 0 & C_1 & \ell a S_1 & \ell a C_2 \\ 0 & P_{T_x}\ell S_1 & C_1 & \ell S_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_Y
\end{array}\right]
$$

where $a = \ell/EI,$

$C_1 = \cosh\gamma$

$C_2 = (\cosh\gamma - 1)/\gamma^2$

$S_1 = \sinh\gamma/\gamma$

$S_2 = (\sinh\gamma - \gamma)/\gamma^3$

$\gamma = \ell\left\{P_{T_X}/EI\right\}^{1/2}$

$$P_{T_X} = \Omega^2 \sum_{i=j}^{N} m_i r_i$$

where j is the index of the current element being considered and m_i is the lumped mass at a radial position of r_i.

The subscripts x, y, and z refer to the EI, S_1, S_2, C_1, and C_2 quantities within the appropriate brackets. That is the (2,2) element is ℓ/EI_X, and the (4,5) element is $\ell(\ell/EI_Z)(\sinh\gamma_Z/\gamma_Z)$.

Point mass and inertia (in a centrifugal field) matrix, [M]:

$$\left[\begin{array}{cc|cccc|cccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
M_{2,1} & 1 & M_{2,3} & 0 & 0 & 0 & M_{2,7} & M_{2,8} & 0 & 0 \\
\hline
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
M_{5,1} & 0 & 0 & 0 & 1 & 0 & 0 & M_{5,8} & 0 & 0 \\
M_{6,1} & 0 & M_{6,3} & 0 & 0 & 1 & M_{6,7} & M_{6,8} & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
M_{9,1} & 0 & M_{9,3} & 0 & 0 & 0 & M_{9,7} & M_{9,8} & 1 & 0 \\
M_{10,1} & 0 & M_{10,3} & 0 & 0 & 0 & M_{10,7} & M_{10,8} & 0 & 1
\end{array}\right]$$

where $M_{2,1} = I_x\sigma^2 + I_y\Omega^2 (C_\psi^2 - S_\psi^2) + \varepsilon^2 m (\sigma^2 - \Omega^2) + h\varepsilon m\Omega^2 C_\psi$

$$M_{2,3} = -\varepsilon m (\sigma^2 - \Omega^2 S_\psi^2)$$

$$M_{2,7} = -\varepsilon m\Omega^2 S_\psi C_\psi$$

$$M_{2,8}^* = -\Omega\sigma (I_x + I_y + \varepsilon^2 m) S_\psi$$

$$M_{5,1} = r\varepsilon m\Omega^2$$

$$M_{5,8} = -\Omega^2 [(I_x + \varepsilon^2 m) C_\psi + h\varepsilon m] S_\psi$$

$$M_{6,1} = \varepsilon m (\sigma^2 - \Omega^2 S_\psi^2)$$

$$M_{6,3} = -m (\sigma^2 - \Omega^2 S_\psi^2)$$

$$M_{6,7} = -m\Omega^2 S_\psi C_\psi$$

$$M_{6,8}^* = -\Omega\sigma (2\varepsilon m) S_\psi$$

$$M_{9,1}^* = \Omega\sigma (I_x + I_y + \varepsilon^2 m) S_\psi$$

$$M_{9,3}^* = -\Omega\sigma (2\varepsilon m) S_\psi$$

$$M_{9,7}^* = \Omega\sigma (2\varepsilon m) C_\psi$$

$$M_{9,8} = -I_x\Omega^2 S_\psi^2 + I_y\sigma^2 + \varepsilon^2 m (\sigma^2 - \Omega^2 S_\psi^2) + h\varepsilon m\Omega^2 C_\psi$$

$$M_{10,1} = \varepsilon m\Omega^2 S_\psi C_\psi$$

$$M_{10,3} = -m\Omega^2 S_\Psi C_\Psi$$

$$M_{10,7} = -m(\sigma^2 - \Omega^2 C_\Psi^2)$$

$$M_{10,8}{}^* = \Omega\sigma (2\varepsilon m) C_\Psi$$

$$M_{k,k} = 1$$

and all other $M_{i,j} = 0$

Here m is the lumped mass, and is the average of adjacent section masses, and ε is the corresponding averaged eccentricity.

σ indicates $\frac{d}{dt}$

$S_\Psi = \sin \Psi$

$C_\Psi = \cos \Psi$

and Ψ is the angle between the load blade element and the plane perpendicular to the rotor shaft.

Terms with a * superscript are gyroscopic terms, and are not used in the program as presently implemented. (Their use would result in complex mode shapes, which are not orthogonal in the usual sense.) In any blade loads and response program using purely real modes as generalized coordinates, these gyroscopic terms could be used as part of the forcing function.

Rigid offset of the elastic-axis (in the chordwise direction) matrix [C], for an offset of ℓ_z, with a chordwise centrifugal load P_{T_z}:

$$\left[\begin{array}{cc|cccc|cccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_{T_z}\ell_z & 1 & 0 & 0 & 0 & -\ell_z & 0 & 0 & 0 & 0 \\
\hline
-\ell_z & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & P_{T_z}\ell_z & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right]$$

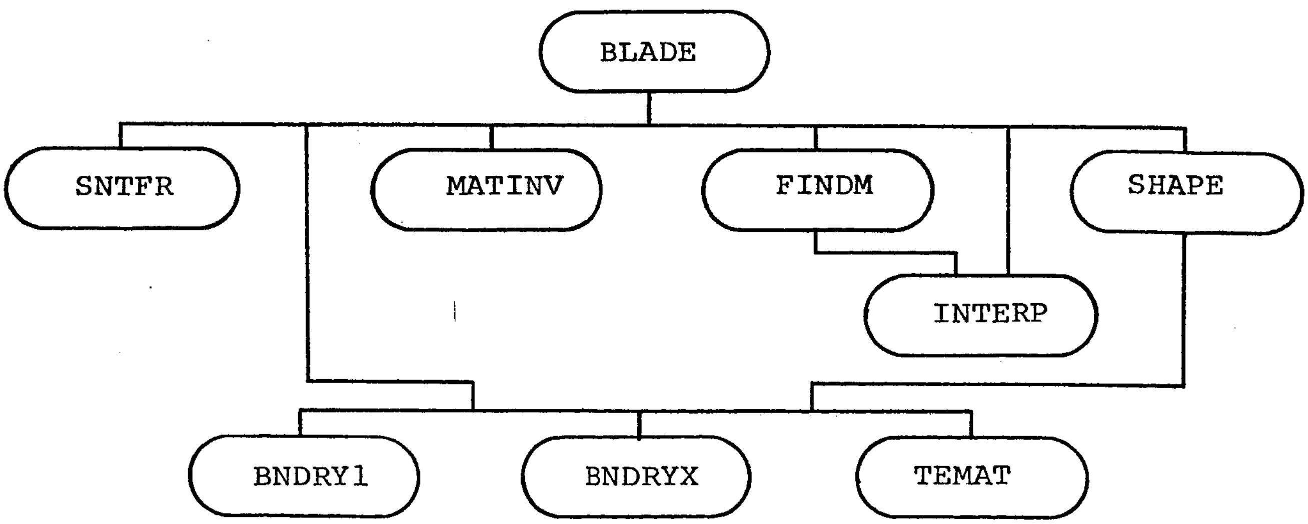
APPENDIX C

PROGRAM LISTING

Computer program listings are given with each subroutine beginning on a new page. Job control cards are not included, as these vary from one facility to another The program does not use OVERLAY or SEGMENT options. Program output format statements give quantities labeled in English units. Input may be in any consistent set of units, but output would be incorrectly labeled unless: (1) English unit input quantities are used or (2) output formats are changed. The listing as given here is identical to that used on CDC 6600 computers at both the NASA Langley facility and at a non-governmental facility. The program should not be expected to operate properly on computers which use fewer than 14 digits for floating point numerical operations.

The main program and subroutines are listed in the approximate order of their first use in the program, with page numbers noted for convenience, as follows.

Program structure is indicated by the following chart.



```
      PROGRAM BLADE(INPUT=1001,OUTPUT=1001,TAPE5=INPUT,TAPE6=OUTPUT,
     1PUNCH=1001,TAPE4=PUNCH,TAPE7=1001,TAPE1,TAPE99)
C     MAIN PROGRAM FOR MATRIX METHOD SOLUTION OF DYNAMIC BLADE PROBLEM
      DIMENSION EL(30),EIX(30),EIY(30),EIZ(30),XINR(30),YINR(30)
      DIMENSION EMAS(30),PHI(30),EPS(30),ELZ(31),CCPS(30),CAPSI(30)
      DIMENSION PRD(10,5),SAV(10,5),T(10,10),ASAV(10,5),AVMAS(31)
      DIMENSION ROLD(4),RNEW(4),ERR(10),A(10,5),B(10),C(10)
      DIMENSION NBND(5),OMINT(15)
      DIMENSION TITLE(19)
      DIMENSION DATE(2),PHIX(30),EPSX(30)
      DIMENSION C101(10,1),ERR101(10,1),SV1010(10,10)
      COMMON CPSQ,OMSQ,XROOT,ZERMO
      COMMON EL,EIX,EIY,EIZ,XINR,YINR,EMAS,PHI,EPS,ELZ,CAPSI,CCPS,EPSN
      COMMON CPOMG,OMEG,SIZER,THETC,ELC,AVMAS
      COMMON NGYRO,NX,NY,NZ,NSEC,NSP1
      COMMON NC,NC1,NC2,NC3,NR,NR1,NR2,NR3,NR4
      COMMON NIBC,B
      COMMON ERR,TITLE,NFREQ
      EQUIVALENCE (C101(1),C(1)),(ERR101(1),ERR(1)),(SV1010(1),SAV(1))
      NAMELIST/IN/NTYPE,NMAX,NEXPO,NUMF,NOOR,NPCH,NFIND,NPDF,CPOMG,
     1SIZER,THETC,ELC,XROOT,PRCNT,OMINT,TMSL,OMLIMT
C     SEE PAGE 3 FOR GEOMETRY
C
      CALL DAYTIM(DATE)
      NOUT=6
C     INPUT
      IBCP=0
  420 READ(5,401) IBC,ITAB,INSEC,TITLE
      IF(INSEC.GT.30) GO TO 1200
C   REPLACE THE NEXT CARD BY A TEST ON END OF DATA OR REMOVE
  430 IF(ITAB.EQ.0) GO TO 440
      NSEC=INSEC
      IF(NSEC.NE.0) GO TO 433
      PRINT 431
  431 FORMAT(33H NSEC=0, CANNOT READ IN NEW TABLE )
      STOP
  433 DO 435 I=1,NSEC
      READ(5,402) J,EL(J),EIX(J),EIY(J),EIZ(J),XINR(J),YINR(J)
     1,EMAS(J),PHI(J),EPS(J),ELZ(J)
      PHIX(J)=PHI(J)
      EPSX(J)=EPS(J)
  435 CONTINUE
      EPSN=EPS(NSEC)
  440 READ(5,IN).
      IF(NFIND.GE.0) GO TO 338
      NSTEP=1
      NTAPE=-1
      GO TO 339
  338 NSTEP=0
      NTAPE=0
```

```
  339 CONTINUE
      NTRL=NPDF
      IF(NFIND.GT.0) NFLG=0
C    DEFAULT FOR NTYPE OUT OF ALLOWABLE RANGE
 4401 IF(NTYPE.GT.0.AND.NTYPE.LT.7) GO TO 4402
      CALL SNTFR ( NTYPE,NR1,NR2,NX,NY,NZ,NBND)
      GO TO 450
 4402 GO TO(441,442,443,444,445,446),NTYPE
  441 NR1=1
      NR2=2
      NX= 0
      NY= 1
      NZ= 1
      NBND(1)=2
      GO TO 450
  442 NR1=3
      NR2=6
      NX =1
      NY =1
      NZ =0
      NBND(1)=5
      NBND(2)=6
      GO TO 450
  443 NR1=7
      NR2=10
      NX =1
      NY =0
      NZ =1
      NBND(1)=9
      NBND(2)=10
      GO TO 450
  444 NR1=1
      NR2=6
      NX= 0
      NY= 1
      NZ= 0
      NBND(1)=2
      NBND(2)=5
      NBND(3)=6
      GO TO 450
  445 NR1=3
      NR2=10
      NX= 1
      NY= 0
      NZ= 0
      NBND(1)=5
      NBND(2)=6
      NBND(3)=9
      NBND(4)=10
      GO TO 450
```

```
  446 NR1=1
      NR2=10
      NX= 0
      NY= 0
      NZ= 0
      NBND(1)=2
      NBND(3)=6
      NBND(2)=5
      NBND(4)=9
      NBND(5)=10
  450 OMEG=OMINT(1)
      SIZERX=SIZER
      THETCX=THETC
      NGYRO=0
      NC1=(NR1+1)/2
      NC2=NR2/2
      NC=NC2-NC1+1
      NC3=NC2-NC1
      NC4=NC-1
      NC5=NC-2
      NR=NR2-NR1+1
      NR3=NR-1
      NR4=NR-2
      IF(NOOR) 5123,5123,5122
 5122 IF(NOOR-NC4) 5124,5124,5123
 5123 NOOR=NC4
 5124 CONTINUE
      WRITE(NOUT,955)
C
C     PRINTOUT OF INPUT
      WRITE (NOUT,950)
      WRITE(6,898) TITLE,DATE(1)
C     SPECIAL MESSAGES IF INFINITE STIFFNESSES OCCUR IN ANY DIRECTION
 481  FORMAT(16I5)
C     LABEL THIS WRITE IF TIME PERMITS
   75 IF (NX) 80,80,77
   77 WRITE (NOUT,956)
   80 IF (NY) 85,85,83
   83 WRITE (NOUT,957)
   85 IF (NZ) 90,90,87
   87 WRITE (NOUT,958)
  90  WRITE(NOUT,961)NSEC,CPOMG,SIZER,THETC,ELC,XROOT
      WRITE (NOUT,962)
      WRITE (NOUT,965) (I,EL(I),EMAS(I),EIX(I),EIY(I),EIZ(I),XINR(I),
     1                 YINR(I),PHI(I),EPS(I),ELZ(I),I=1,NSEC)
  192 CONTINUE
      WRITE (NOUT,955)
      WRITE(NOUT,403) NTYPE,IBC,NFIND,ITAB,TMSL,NUMF
     1,NPCH,NPDF,NMAX,NOOR,NEXPO,CMLIMT,PRCNT
      IF(NFIND.EQ.1.AND.NPDF.GT.0) WRITE(6,405)(OMINT(I),I=1,NPDF)
```

```
      IF(NFIND.EQ.0.AND.NUMF.GT.0) WRITE(6,404)(OMINT(I),I=1,NUMF)
      IF(NFIND.LE.0.AND.NUMF.EQ.0) WRITE(6,4031)
      WRITE (6,981)
      IF(NC4.GT.0) WRITE (6,982) (N,N=1,NC4)
      IF(NC4.EQ.0) WRITE (6,982)
C
C     DEFINE CONSTANTS CONVERT INPUT DEGREES TO RADIANS
      NSP1=NSEC+1
      NFREQ=1
      IF(NFIND.LE.0) GO TO 36
      NFREQ=NPDF
      IF(NFREQ.GT.0) GO TO 36
      OMEG=1.
      GO TO 37
   36 OMEG=OMINT(1)
   37 PI=3.1415926
      CNVRT=PI/180.
C
      DET=0.
      IF (NFIND.GT.0) OMSQ=1.
      NTIME=0
      NFLAG=0
      RN=0.
      IF (NSEC.EQ.2) GO TO 277
C
      SIZER=SIZERX*CNVRT
      THETC=THETCX*CNVRT
      CPSQ=CPOMG*CPOMG
      OMSQ=OMEG*OMEG
      CAPSI(1)=SIZER
      CCPS(1)=COS(SIZER)
   39 DO 40 I=2,NSEC
      IM1=I-1
      EPS(I)=(EPSX(I)*EMAS(I)+EPSX(IM1)*EMAS(IM1))/(EMAS(I)+EMAS(IM1))
      PHI(I)=PHIX(I)*CNVRT
      CAPSI(I)=CAPSI(I-1)+PHI(I)
   40 CCPS(I)=COS(CAPSI(I))
   41 IF (PRCNT) 60,50,60
   50 PRCNT=10.
   60 FACTR=1.+PRCNT/100.
C     INITIALIZATION
   65 DET=0.
      IF(NFIND.GT.0) OMSQ=1.
      NTIME=0
      NFLAG=0
      RN=0.
      DO 92 N=1,4
      B(N)=1.
      C(N)=0.
   92 RNEW(N)=0.
```

```
      B(5)=1.
      C(5)=0.
C
C     DEFINE BOUNDARY MATRIX
  100 I=1
C
      IF (NSEC.EQ.2) GO TO 277
C
      IF(IBC.EQ.1) GO TO 471
      CALL BNDRYX(PRD,A,I,IBC)
      GO TO 475
  471 CALL BNDRY1(PRD,A,I)
  475 NIBC=I
      DO 180 I=NIBC,NSP1
      DO 101 J=1,10
      DO 101 JX=1,5
      ASAV(J,JX)=0.
  101 SAV(J,JX)=0.
      DO 281 J=1,10
      DO 281 JX=1,10
  281 T(J,JX)=0.
      CALL TEMAT(I,T)
C     MODIFY BNDRY SUBROUTINE FOR ANY CHANGE IN BOUNDARY CONDITION AT
C     CENTER END
C     PREMULTIPLY BY NEW T MATRIX
      DO 140 M=NR1,NR2
      DO 140 N=NC1,NC2
      DO 140 NINT=NR1,NR2
      ASAV(M,N)=T(M,NINT)*A(NINT,N)+ASAV(M,N)
  140 SAV(M,N)=T(M,NINT)*PRD(NINT,N)+SAV(M,N)
      DO 180 M=NR1,NR2
      DO 180 N=NC1,NC2
      A(M,N)=ASAV(M,N)
      PRD(M,N)=SAV(M,N)
  180 CONTINUE
  200 CONTINUE
C
C
C
C     COMPUTE SHAPES FOR ITERATION ON FREQUENCIES
C     EXTRACT SUBMATRIX FOR BOUNDARY CONDITIONS AT THE TIP
C     BOUNDARY CONDITIONS AT TIP ARE CONTAINED IN EQUATIONS 2,5,6,9,10
C     GIVING 5X5 SYSTEM TO FIND EIGEN VALUES OF
C
      IF(NR2-2)277,277,249
  249 DO 250 M=1,NC
      J=NBND(M)
      JKL=NC1-1
      DO 250 N=1,NC
      JKL=JKL+1
```

```
      ASAV(M,N)=A(J,JKL)
  250 SAV(M,N)=PRD(J,JKL)
      NX1=1
      DO 276 J=1,NC4
      I=0
      DO 254 M=1,NC
      IF (M-J) 252,254,252
  252 I=I+1
      ERR(I)=-ASAV(M,1)
      DO 253 N=1,NC4
      T(I,N)=ASAV(M,N+1)
  253 CONTINUE
  254 CONTINUE
      CALL MATINV(T,NC4,ERR101,NX1,DT,ID1)
      IF(ABS(DT)-.1E-20)256,257,257
  256 WRITE(NOUT,980)
  257 ROLD(J)=RNEW(J)
      IF(ABS(ASAV(J,J+1))-.1E-20)258,258,259
  258 RNEW(J)=0.
      B2=0.
      GO TO 276
  259 B2=-ASAV(J,1)
      DO 270 N=1,NC4
      IF(J-N)265,270,265
  265 B2=B2-ASAV(J,N+1)*ERR(N)
  270 CONTINUE
      B2=B2/ASAV(J,J+1)
      RNEW(J)=ERR(J)-B2
  275 CONTINUE
  272 IF(J-NOOR)276,273,276
  273 C(1)=B(J)+.5*(ERR(J)+B2)
      DO 274 LL=1,NC4
  274 B(LL)=B(LL)+ERR(LL)
      B(NOOR)=C(1)
  276 CONTINUE
C
C     ENTER MATINV TO
C     COMPUTE DETERMINANT VALUE OF 5X5 SYSTEM
      DTOLD=DET
      CALL MATINV( SV1010,NC,C101,NX1,DET,ID1)
      OMEGA=SQRT(OMSQ)
      IF(NFIND.GT.0) GO TO 721
      WRITE(NOUT,972) NTIME,OMEGA,DET,(RNEW(J),J=1,NC4)
      GO TO 3278
  721 CALL FINDM(OMLST,OMSQ,DET,DTOLD,RNEW,ROLD,NEXPO,NC4,NFREQ,OMINT,NT
     1IME,NFIND,NTRL,NFLG,NPDF)
      GO TO (500,300),NTRL
  277 DTOLD=DET
      DET=PRD(2,1)
C
```

```
      IF (NSEC.NE.2) GO TO 2772
      DET=1.0
      DO 2771 I=1,NUMF
 2771 DET=(OMINT(I)-OMSQ)*DET
 2772 CONTINUE
C
      IF(NFIND.GT.0) GO TO 721
      NOOR=1
      RNEW(NOOR)=DET
      ROLD(NOOR)=DTOLD
 3278 IF (NSTEP) 279,279,278
C
  278 IF (NFLAG) 2781,2781,285
C     CHECK FOR CHANGE IN SIGN OF DETERMINANT FROM PREVIOUS VALUE
 2781 IF(DTOLD*DET) 2782,280 ,280
 2782 NFLAG=1
      GO TO 285
  279 IF (NTIME) 280,280,285
  280 OMLST=OMSQ
      OMSQ=OMSQ*FACTR
      GO TO 290
  285 IF(NR2-2)286,286,287
  286 RNEW(NOOR)=DET
      ROLD(NOOR)=DTOLD
C     INTERPOLATE LINEARLY FOR NEW VALUE OF OMSQ
  287 YA=RNEW(NOOR)
      YB=ROLD(NOOR)
      CALL INTERP(OMLST,OMSQ,YA,YB)
  290 NTIME=NTIME+1
C     CHECK FOR REQUIRED CONVERGENCE ON VALUE OF OMEGA SQUARED
      IF(ABS(OMSQ-OMLST)/OMLST-.1**NEXPO) 500,500,300
  300 IF (NTIME.LT.NMAX) GO TO 100
      WRITE (6,984)
      IF(NFIND.GE.1) NFREQ=NUMF
C     COMPUTE DISPLACEMENTS AND FORCES ALONG THE BLADE
  500 WRITE(NOUT,955)
      WRITE(6,898) TITLE,DATE(1)
      NTIME=0
      IF (NSEC.EQ.2) GO TO 600
      CALL SHAPE(PRD,NTAPE,NPCH,IBC,TMSL)
      IF (NUMF) 800,800,600
  600 IF (NFREQ-NUMF) 6C5,800,800
  605 IF (OMINT(NFREQ).GT.OMLIMT) GO TO 800
      NFREQ=NFREQ+1
      WRITE (6,983)
      IF(NC4.LE.0) WRITE (NOUT,982)
      IF(NC4.GT.0) WRITE (NOUT,982) (J,J=1,NC4)
  607 IF (NFIND.GT.0) GO TO 721
      IF(NFIND.GT.0) GO TO 721
      OMSQ=OMSQ*FACTR
```

```
      IF (NTAPE)620,610,610
C     ALLOWS FOR NEW INITIAL GUESSES FOR EACH FREQUENCY UNLESS NTAPE=-1
  610 OMEG=OMINT(NFREQ)
      OMSQ=OMEG*OMEG
      NSTEP=0
      GO TO 65
  620 NSTEP=1
      GO TO 65
  800 CONTINUE
 1130 CONTINUE
      GO TO 420
 1200 STOP
C
  401 FORMAT(2I1,I2,19A4)
  402 FORMAT(I2,    F6.5,9E8.0)
  898 FORMAT(22X,19A4,10X,A10,/)
 950  FORMAT(50X,23HBLADE FREQUENCY PROGRAM//38X,46HBLADE MODEL AND OPER
     1ATING CONDITION PARAMETERS//)
  403 FORMAT(48X,26HPROGRAM CONTROL PARAMETERS//10X,22HMODE COUPLING, NT
     1YPE =,I3,10X,28HHUB BOUNDRY CONDITION, IBC =,I3,15X,25HFREQUENCY S
     2EARCH, NFIND =,I3/5X,27HBLADE PROPERTY DECK, ITAB =,I3,11X,27HTORS
     3ION MODE SWITCH, TMSL =,F5.1,12X,26HNO. OF FREQUENCIES, NUMF =,
     4I3,/10X,22HPUNCHED OUTPUT, NPCH =,I3,6X,32HNO. OF KNOWN FREQUENCIE
     5S, NPOF =,I3,15X,25HNO. OF ITERATIONS, NMAX =,I3,/9X,23HREMAINDER
     6INDEX, NOOR =,I3,12X,26HCONVERGENCE LIMIT, NEXPO =,I3,/7X,
     725HFREQUENCY LIMIT, OMLIMT =,E11.3,5X,25HSTEP SIZE FACTOR, PRCNT =
     8,F11.6)
  404 FORMAT(4X,28HFREQUENCY ESTIMATES, OMINT =,2X,(7F12.6))
  405 FORMAT(3X,47HPREVIOUSLY DETERMINED FREQUENCIES, OMINT(I) =    ,(7F1
     12.6))
  955 FORMAT (1H1)
  956 FORMAT (43X,37HINFINITE STIFFNESS IN THE X DIRECTION)
  957 FORMAT (43X,37HINFINITE STIFFNESS IN THE Y DIRECTION)
  958 FORMAT (43X,37HINFINITE STIFFNESS IN THE Z DIRECTION)
  961 FORMAT(//43X,20HNUMBER OF SECTIONS =,I3,/35X,
     A                                          28HROTATIONAL SPEED CAP
     1OMEGA =F9.4,12H RADIANS/SEC/40X,23HCONTROL ANGLE SI ZERO =F9.4,
     2  8H DEGREES/40X,23HCONTROL ANGLE THETA C =F9.4,8H DEGREES/
     3  47X,16HCONTROL OFFSET =,F9.4,7H FEET  / 23X,40HRADIAL DISTANCE T
     4O FIRST BLADE ELEMENT =,F9.4//)
  962 FORMAT   (3H  I,4X,6HLENGTH,7X,4HMASS,9X,3HEIX,9X,3HEIY,9X,3HEIZ,
     1          8X,2HIX,10X,2HIY,10X,3HPHI,7X,7HEPSILON,5X,6HOFFSET/7X,
     2          6H FEET ,6X,7HLB-SEC2,5X,6HLB-FT2,2(6X,6HLB-FT2),1X,
     3          2(4X,8HLB-SEC2-),5X,7HDEGREES,6X,4HFEET,8X,4HFEET/19X,
     4          7HPER-FT ,34X,2(8X,4HFEET),2(/))
  965 FORMAT   (I3,10E12.4)
  970 FORMAT (   / ,(5E20.12))
 972  FORMAT( 4X,I3,3X,E20.12,E15.5,15X,4E15.5)
  980 FORMAT (53X,24H4 X 4 SYSTEM IS SINGULAR)
  981 FORMAT(///,43X,34HSEARCH FOR LOWEST FREQUENCY BEGINS,/)
```

```
  982 FORMAT(2X,10HITERATIONS,5X,9HFREQUENCY,6X,11HDETERMINANT,
     16X, 9HAUXILIARY,5X,10HREMAINDERS,/2X, 6HNUMBER,8X, 9H(RAD/SEC),24X
     2, 8HFUNCTION,I13,3I15)
  983 FORMAT(1H1,47X,32HSEARCH FOR NEXT FREQUENCY BEGINS,/)
  984 FORMAT(//,23H WARNING:  NTIME = NMAX,//)
 4031 FORMAT( 1X,27HNO FREQUENCY ESTIMATES USED)
 7123 FORMAT(I5,5X,I5)
 7124 FORMAT(10E8.1)
      END
```

```
      SUBROUTINE SNTFR (NTYPE,NR1,NR2,NX,NY,NZ,NBND)
      DIMENSION  NBND(5)
C     DEFAULT FOR NTYPE OUT OF ALLOWABLE RANGE
C
C     GIVES NTYPE=6 CONTROL, I.E., FULLY COUPLED MODES
      NR1=1
      NR2=10
      NX=0
      NY=0
      NZ=0
      NBND(1)=2
      NBND(2)=5
      NBND(3)=6
      NBND(4)=9
      NBND(5)=10
      RETURN
      END
```

```
      SUBROUTINE BNDRY1(BND,BPL,NPIB)
C     SPECIAL SUBROUTINE TO COMPUTE BOUNDARY CONDITION MATRIX
C     CASE OF FLAP HINGE,LEAD-LAGHINGE,TORSIONAL SPRING
C
      DIMENSION SAVXX(70)
      DIMENSION BAND(10,5)
      DIMENSION BND(10,5),BPL(10,5),B(10),SAV(10,7)
      DIMENSION EL(30),EIX(30),EIY(30),EIZ(30),XINR(30),YINR(30)
      DIMENSION EMAS(30),PHI(30),EPS(30),ELZ(31),CAPSI(30)
      DIMENSION CCPS(30),AVMAS(31)
      DIMENSION TITLE(19),ERR(10)
      COMMON CPSQ,OMSQ,XROOT,ZERMO
      COMMON EL,EIX,EIY,EIZ,XINR,YINR,EMAS,PHI,EPS,ELZ,CAPSI,CCPS,EPSN
      COMMON CPOMG,OMEG,SIZER,THETC,ELC,AVMAS
      COMMON NGYRO,NX,NY,NZ,NSEC,NSP1
      COMMON NC,NC1,NC2,NC3,NR,NR1,NR2,NR3,NR4
      COMMON NIBC,B
      COMMON ERR,TITLE,NFREQ
      EQUIVALENCE (SAV(1),SAVXX(1))
      NOUT=6
      GO TO (1,170,201),NPIB
    1 DO 311 I=1,10
      DO 312 II=1,5
      BPL(I,II)=0.
  312 BND(I,II)=0.
      DO 313 II=1,7
  313 SAV(I,II)=0.
  311 CONTINUE
      L=1
      BND(2,1)=1.
      BND(6,3)=1.
      BND(8,4)=1.
      BND(10,5)=1.
      IF(NX) 2,2,3
    2 BND(1,1)=EL(1)/EIX(1)
    3 NSM1=NSEC-1
    4 PTX=0.
      X=XROOT
      DO 5 J=1,NSM1
      X=X+EL(J)
    5 PTX=PTX+.5*(EMAS(J+1)+EMAS(J))*X
      PTX=PTX+.5*EMAS(NSEC)*(X+EL(NSEC))
      PTX=PTX*CPSQ
      IF(NZ) 8,8,45
    8 EI=EIZ(1)
   10 GAMMA=EL(1)*SQRT(PTX/EI)
      G2=GAMMA*GAMMA
      IF(G2-.00001)12,12,11
   11 SOC=(TANH(GAMMA))**2
      S1=SQRT(SOC/(1.-SOC))/GAMMA
```

```
      C1=SQRT(1./(1.-SOC))
      S2=(S1-1.)/G2
      C2=(C1-1.)/G2
      GO TO 13
   12 S1=1.+G2/6.
      C1=1.+G2*.5
      S2=(1.+G2*.05)/6.
      C2=.5+G2/24.
   13 GO TO (20,50),L
   20 BND(3,2)=EL(1)*S1
      BND(4,2)=C1
      BND(5,2)=PTX*BND(3,2)
   40 BND(4,3)=EL(1)*EL(1)/EIZ(1)
      BND(3,3)=EL(1)*BND(4,3)*S2
      BND(4,3)=BND(4,3)*C2
      BND(5,3)=BND(3,2)
      GO TO 48
   45 BND(3,2)=EL(1)
      BND(4,2)=1.
      BND(5,2)=PTX*EL(1)
      BND(5,3)=EL(1)
   48 IF(NY) 49,49,55
   49 L=2
      EI=EIY(1)
      GO TO 10
   50 BND(9,5)=-S1/C1
      BND(7,5)=(EL(1)**3)/EI*(BND(9,5)*C2+S2)
      BND(9,5)=BND(9,5)*EL(1)
      GO TO 58
   55 BND(9,5)=-EL(1)
   58 IF(ELC-.1E-7) 100,100,60
   60 BND(6,1)=-SIN(THETC)/ELC
      BND(10,1)=-COS(THETC)/ELC
C
  100 ZERMO=BND(9,5)
      BND(9,5)=0.
      DO 61 JK=1,10
      DO 61 JKII=1,5
   61 BAND(JK,JKII)=BND(JK,JKII)
      IF(NC-1)105,105,104
  104 DO 101 N=1,NC
  101 SAV(N,N)=1.
      DO 102 N=2,NC
  102 SAV(N,1)=B(N-1)
  105 SAV(1,1)=1.
  103 DO 111 M=NR1,NR2
      NN=0
      DO 110 N=NC1,NC2
      NN=NN+1
  110 BPL(M,NC1)=BND(M,N)*SAV(NN,1)+BPL(M,NC1)
```

```
      IF(NC-1)113,113,112
  112 NN=NC1+1
      DO 111 N=NN,NC2
  111 BPL(M,N)=BND(M,N)
  113 NPIB=2
      GO TO 400
 170  IF(NC3.EQ.0)GO TO 181
      DO 180 N=1,NC3
  180 SAV(N+1,7)=B(N)
 181  X=1.
      BPL(10,4)=0.
      BPL(10,5)=0.
      GO TO 203
  201 X=BPL(1,1)
      BPL(10,5)=0.0
      IF(NC3.EQ.0) GO TO 203
      DO 202 N=1,NC3
  202 B(N)=SAV(N+1,7)
  203 DO 204 II=1,NR
  204 SAVXX(II)=0.
      I=0
  210 IF(NR1-3)211,212,213
  211 IF(NR2-6)216,217,219
  212 IF(NR2-6)224,224,226
C     EDGEWISE BENDING ONLY   NORMALIZE TO W=1.0
  213 VNORM=(BND(7,4)+BND(7,5)*B(1))/X
      SAV(10,1)=B(1)/VNORM
      SAV(9,1)=SAV(10,1)*ZERMO
      WRITE (NOUT,920)I,(SAV(N,1),N=1,10),XROOT
      SAV(8,1)=1./VNORM
      I=1
      X=XROOT+EL(1)
C     BPL IS RECOMPUTED AS BOUNDARY MATRIX
  214 DO 1214 M=1,10
 1214 B(M)=0.
      DO 215 M=7,10
      DO 215 N=4,5
  215 B(M)=B(M)+BAND(M,N)*SAV(2*N,1)
      WRITE (NOUT,920)I,(B(N),N=1,10),X
      B(8)=SAV(8,1)
      GO TO 300
C     TORSION ONLY   NORMALIZE TO PHI=1.
  216 B(2)=X/BND(1,1)
      B(1)=0.
      WRITE(NOUT,920)I,B(1),B(2),(SAVXX(II),II=3,10),XROOT
      X=XROOT
      GO TO 300
C     FLAP-TORSION   NORMALIZE TO V=1.
  217 VNORM=BND(3,1)+BND(3,2)*B(1)+BND(3,3)*B(2)
      SAV(2,1)=X/VNORM
```

```
      SAV(4,1)=B(1)*SAV(2,1)
      SAV(6,1)=B(2)*SAV(2,1)
      WRITE (NOUT,920)I,(SAV(N,1),N=1,10),XROOT
      DO 218 N=1,6
  218 B(N)=SAV(N,1)
      X=XROOT
      GO TO 300
C     TORSION-FLAP-EDGEWISE   NORMALIZE TO V=1.
  219 VNORM=BND(3,1)+BND(3,2)*B(1)+BND(3,3)*B(2)+BND(3,4)*B(3)+BND(3,5)*
     1B(4)
      SAV(2,1)=X/VNORM
      SAV(4,1)=B(1)*SAV(2,1)
      SAV(6,1)=B(2)*SAV(2,1)
      SAV(10,1)=B(4)*SAV(2,1)
      SAV(9,1)=SAV(10,1)*ZERMO
      WRITE (NOUT,920)I,(SAV(N,1),N=1,10),XROOT
      I=1
      SAV(8,1)=X*B(3)/VNORM
  220 DO 221 M=1,10
      B(M)=0.
      DO 221 N=1,5
  221 B(M)=B(M)+BAND(M,N)*SAV(2*N,1)
      X=XROOT+EL(1)
      B(8)=SAV(8,1)
      WRITE (NOUT,920)I,(B(N),N=1,10),X
      BPL(10,5)=(XINR(1)+AVMAS(2)*EPS(1)*EPS(1))*(B(1)*B(1)+B(8)*B(8))
     1-2.*AVMAS(2)*EPS(1)*B(3)*B(1)+AVMAS(2)*(B(3)*B(3)+B(7)*B(7))
      GO TO 300
C     FLAPWISE BENDING ONLY   NORMALIZE TO V=1.
  224 VNORM=(BND(3,2)+BND(3,3)*B(1))/X
      SAV(4,1)=1./VNORM
      SAV(6,1)=B(1)/VNORM
      WRITE (NOUT,920)I,(SAV(N,1),N=1,10),XROOT
      DO 225 N=3,6
  225 B(N)=SAV(N,1)
      B(1)=0.
      B(2)=0.
      X=XROOT
      GO TO 300
C    FLAP-EDGEWISE   NORMALIZE TO V=1.
  226 VNORM=BND(3,2)+BND(3,3)*B(1)+BND(3,4)*B(2)+BND(3,5)*B(3)
      SAV(4,1)=X/VNORM
      SAV(6,1)=B(1)*SAV(4,1)
      SAV(10,1)=B(3)*SAV(4,1)
       SAV(9,1)=SAV(10,1)*ZERMO
      WRITE(NOUT,920)I,(SAV(N,1),N=1,10),XROOT
      I=1
      X=XROOT+EL(1)
      SAV(8,1)=B(2)*SAV(4,1)
  230 DO 231 M=1,10
```

```
231 B(M)=0.
    DO 232 M=3,10
    DO 232 N=2,5
232 B(M)=B(M)+BAND(M,N)*SAV(2*N,1)
    WRITE (NOUT,920)I,(B(N),N=1,10),X
    B(8)=SAV(8,1)
300 NPIB=I
    BPL(10,4)=X
400 CONTINUE
920 FORMAT(I3,11(E10.3,1X))
    RETURN
    END
```

```
      SUBROUTINE BNDRYX(BND,BPL,NPIB,KD1)
      DIMENSION BND(10,5),BPL(10,5),B(10),SAV(10,7)
      DIMENSION EL(30),EIX(30),EIY(30),EIZ(30),XINR(30),YINR(30)
      DIMENSION EMAS(30),PHI(30),EPS(30),ELZ(31),CAPSI(30)
      DIMENSION CCPS(30),AVMAS(31)
      DIMENSION TITLE(19),ERR(10)
      COMMON CPSQ,OMSQ,XROOT,ZERMO
      COMMON EL,EIX,EIY,EIZ,XINR,YINR,EMAS,PHI,EPS,ELZ,CAPSI,CCPS,EPSN
      COMMON CPOMG,OMEG,SIZER,THETC,ELC,AVMAS
      COMMON NGYRO,NX,NY,NZ,NSEC,NSP1
      COMMON NC,NC1,NC2,NC3,NR,NR1,NR2,NR3,NR4
      COMMON NIBC,B
      COMMON ERR,TITLE,NFREQ
      GO TO(11,12,13,14),KD1
   11 RETURN
   12 KD2=5
      KD3=9
      BK=1.
      GO TO 15
   13 KD2=4
      KD3=9
      BK=1.
      GO TO 15
   14 KD2=5
      KD3=8
      BK=-1.
   15 NOUT=6
      GO TO (1,170,201),NPIB
    1 DO 311 I=1,10
      DO 312 II=1,5
      BPL(I,II)=0.
  312 BND(I,II)=0.
      DO 313 II=1,7
  313 SAV(I,II)=0.
  311 CONTINUE
      BND(2,1)=1.
      BND(KD2,2)=1.
      BND(6,3)=1.
      BND(KD3,4)=1.
      BND(10,5)=1.
      IF(NC-1)  103,103,100
  100 DO 101  I=1,NC
  101   SAV(I,I)=1.
      DO 102  I=2,NC
  102 SAV(I,1)=B(I-1)
  103  SAV(1,1)=1.
C   GO TO BRANCH IS NOT NEEDED
  105 DO 112 M=NR1,NR2
      NN=0
      DO 110 N=NC1,NC2
```

```
      NN=NN+1
  110 BPL(M,NC1)=BND(M,N)*SAV(NN,1)+BPL(M,NC1)
      IF(NC-1)  120,120,111
  111 NN=NC1+1
      DO 112 N=NN,NC2
  112 BPL(M,N)=BND(M,N)
  120 NPIB=1
      GO TO 400
 170  IF(NC3.EQ.0)GO TO 181
      DO 180 N=1,NC3
 180   SAV(N+1,7)=B(N)
 181  X=1.
      GO TO 203
  201  X=BPL(1,1)
      IF(NC3.EQ.0) GO TO 203
      DO 202 N=1,NC3
  202  B(N)=SAV(N+1,7)
  203  DO 204 I=1,NR
  204  SAV(I,1)=0
      I=0
  210 IF(NR1-3)  211,212,213
  211 IF(NR2-6) 215,216,218
  212  IF(NR2-6) 220,220,222
C  EDGEWISE BENDING ONLY   W=1.
  213 B(KD3)=X/(BND(7,4)+BND(7,5)*B(1)*BK)
      B(10)=B(1)*B(KD3)
      KD4=KD3-1
      DO 214 N=1,KD4
  214   B(N)=0.
      IF(KD4  .EQ.7) B(9)=0.
      NN=10
      GO TO 290
C   TORSION ONLY
  215 B(2)=X/BND(1,1)
      B(1)=0
      NN=2
      GO TO 290
C   TORSION FLAP
  216       VNORM=X/(BND(3,1)+BND(3,2)*B(1)+BND(3,3)*B(2))
      SAV(2,1)=VNORM
      SAV(KD2,1)=B(1)*VNORM
      SAV(6,1)=B(2)*VNORM
      DO 217 N=1,6
  217 B(N)=SAV(N,1)
      NN=6
      GO TO 290
  218  VNORM=X/(BND(3,1)+BND(3,2)*B(1)+BND(3,3)*B(2)+BND(3,4)*B(3)+
     1  BND(3,5)*B(4))
      SAV(2,1)=VNORM
      SAV(KD2,1)=B(1)*VNORM
```

```
      SAV(6,1)=B(2)*VNORM
      SAV(KD3,1)=B(3)*VNORM
      SAV(10,1)=B(4)*VNORM
      DO 219 N=1,10
  219   B(N)=SAV(N,1)
      NN=10
      GO TO 290
C    FLAP ONLY
  220   VNORM=X/(BND(3,2)+BND(3,3)*B(1))
      B(KD2)=VNORM
      B(6)=B(1)*VNORM
      DO 221    N=1,4
  221  B(N)=0
      NN=6
      GO TO 290
  222  VNORM=X/(BND(3,2)+BND(3,3)*B(1)+BND(3,4)*B(2)+BND(3,5)*B(3))
      SAV(KD2,1)=VNORM
      SAV(6,1)=B(1)*VNORM
      SAV(KD3,1)=VNORM
      SAV(10,1)=B(3)*VNORM
      DO 223 N=1,10
  223  B(N)=SAV(N,1)
      NN=10
  290   N=NN+1
      IF(N-10)   291,291,293
  291  DO 292  I=N,10
  292   B(I)=0
  293  I=0
      WRITE (NOUT,920) I,(B(N),N=1,10),XROOT
      BPL(10,5)=0
      BPL(10,4)=XROOT
  920 FORMAT (I3,11(E10.3,1X))
      NPIB=I
  400 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE TEMAT (I,T)
C     TRANSFER MATRIX TRANSFERS ACROSS AN OFFSET 60+ ,E+5+LASO+
C     A LUMPED MASS, AN ELASTIC FIELD, AND A TWIST IN THE ELASTIC
C     AXIS. SEE PAGE 13.
C     SEE PAGE 15 FOR TERMS OF T MATRIX
      DIMENSION EL(30),EIX(30),EIY(30),EIZ(30),XINR(30),YINR(30)
      DIMENSION EMAS(30),PHI(30),EPS(30),ELZ(31),CAPSI(30)
      DIMENSION CCPS(30),T(10,10),AVMAS(31)
      DIMENSION B(10)
      DIMENSION TITLE(19),ERR(10)
      COMMON CPSQ,OMSQ,XROOT,ZERMO
      COMMON EL,EIX,EIY,EIZ,XINR,YINR,EMAS,PHI,EPS,ELZ,CAPSI,CCPS,EPSN
      COMMON CPOMG,OMEG,SIZER,THETC,ELC,AVMAS
      COMMON NGYRO,NX,NY,NZ,NSEC,NSP1
      COMMON NC,NC1,NC2,NC3,NR,NR1,NR2,NR3,NR4
      COMMON NIBC,B
      COMMON ERR,TITLE,NFREQ
C
      IM1=I-1
      XI=XROOT
      HI=0.
      PTX=0.
      IF(IM1) 25,25,2
    2 DO 3 J=1,IM1
C     HI IS DISTANCE IN Z DIRECTION BETWEEN PITCH AXIS AND PROJECTION OF
C     MASS ON ROTOR PLANE.  SEE PAGE 6B.
      HI=HI+ELZ(J+1)*CCPS(J)
    3 XI=XI+EL(J)
      X=XI
      H=HI
      IF (I-NSP1) 5,4,4
    4 SUMAS=.5*EMAS(NSEC)
      EPST=EPS(NSEC)
      EPS(NSEC)=EPSN
      GO TO 30
C     PTZ IS A FORCE IN THE Z DIRECTION. SEE PAGE 6B.
    5 PTZ=.5*(EMAS(IM1)+EMAS(I))*(H+EPS(IM1) *CCPS(IM1))
      DO 10 J=I,NSEC
      X=X+EL(J)
      IF(J-NSEC) 8,7,7
    7 SUMAS=.5*EMAS(J)
      ELZ(J+1)=0.
      GO TO 9
    8 SUMAS=.5*(EMAS(J)+EMAS(J+1))
      H=H+ELZ(J+1)*CCPS(J)
    9 PTX=PTX+SUMAS*X
      PTZ=PTZ+SUMAS*(H+(EPS(J)+ELZ(J+1))*CCPS(J))
   10 CONTINUE
      PTX=CPSQ*PTX
      PTZ=CPSQ*PTZ
```

```
      X=XI
      H=HI
   20 SUMAS=.5*(EMAS(IM1)+EMAS(I))
   30 AVMAS(I)=SUMAS
      SNSI=SIN(CAPSI(IM1))
      CSSI=CCPS(IM1)
      SCSI=SNSI*CSSI*CPSQ
      SNSQ=SNSI*SNSI*CPSQ
      CSSQ=CSSI*CSSI*CPSQ
      EPSEM=EPS(IM1)*SUMAS
      H=H+EPS(IM1)*CSSI
      F6=SUMAS*CPSQ*H
      F10=-F6*CSSI
      F6=F6*SNSI
      F2=YINR(IM1)*SCSI+EPS(IM1)*F6
      A51=-EPSEM*X*CPSQ
      A58=-XINR(IM1)*SCSI-EPS(IM1)*F6
      A63=SUMAS  *(OMSQ+SNSQ)
      A67=-SUMAS  *SCSI
      A61=-EPS(IM1)*A63
      A98=-XINR(IM1)*SNSQ-YINR(IM1)*OMSQ-EPSEM*
     1    EPS(IM1)*(OMSQ+CPSQ)-EPS(IM1)*F10
      A107=SUMAS  *(OMSQ+CSSQ)
  970 FORMAT  (//(5E20.12))
C     MASS TERMS
      T(1,1)=1.
      T(2,2)=1.
      T(2,3)=-A61
      T(2,1)=-OMSQ*XINR(IM1)+YINR(IM1)* (CSSQ-SNSQ)-EPS(IM1)*
     1       (T(2,3)+F10)
      T(2,7)=EPS(IM1)*A67
      IF(I.GE.NSP1) GO TO 35
      T(3,3)=COS(PHI(I))
      T(3,7)=-SIN(PHI(I))
      GO TO 36
   35 T(3,3)=1.0
      T(3,7)=0.0
   36 CONTINUE
      T(6,3)=T(3,3)*A63+T(3,7)*A67
      T(6,1)=-EPS(IM1)*T(6,3)
      T(6,6)=T(3,3)
      T(6,7)=T(3,3)*A67+T(3,7)*A107
      T(6,10)=T(3,7)
      T(7,3)=-T(3,7)
      T(7,7)=T(3,3)
      T(10,3)=T(3,3)*A67-T(3,7)*A63
      T(10,1)=-EPS(IM1)*T(10,3)
      T(10,6)=-T(3,7)
      T(10,7)=T( 3,3)*A107-T(3,7)*A67
      T(10,10)=T(3,3)
```

```
   49 IF(I-NSP1) 50,450,450
   25 HI=ELZ(1)
      H=HI
      XI=XROOT
      X=XI
C     PTZ IS THE FORCE IN THE Z DIRECTION
      PTZ=.5*EMAS(1)*H
      DO 29 J=1,NSEC
      X=X+EL(J)
      IF(J-NSEC)27,26,26
   26 SUMAS=.5*EMAS(J)
      ELZ(J+1)=0.
      GO TO 28
   27 SUMAS=.5*(EMAS(J)+EMAS(J+1))
      H=H+ELZ(J+1)*CCPS(J)
   28 PTX=PTX+SUMAS*X
      PTZ=PTZ+SUMAS*(H+(EPS(J)+ELZ(J+1))*CCPS(J))
   29 CONTINUE
      PTX=PTX*CPSQ
      PTZ=PTZ*CPSQ
      X=XI
      H=HI
C
   31 SUMAS=.5*EMAS(1)
      AVMAS(1)=SUMAS
      F10=-SUMAS*CPSQ*H
      A63=SUMAS*OMSQ
      A107=SUMAS*(OMSQ+CPSQ)
      A51=0.
      A58=0.
      A61=0.
      A67=0.
      A98=0.
C     MASS TERMS
      T(1,1)=1.
      T(2,2)=1.
      T(3,3)=COS(PHI(I))
      T(3,7)=SIN(PHI(I))
      T(6,3)=T(3,3)*A63
      T(6,6)=T(3,3)
      T(6,7)=T(3,7)*A107
      T(6,10)=T(3,7)
      T(7,3)=-T(3,7)
      T(7,7)=T(3,3)
      T(10,3)=-T(3,7)*A63
      T(10,6)=-T(3,7)
      T(10,7)=T(3,3)*A107
      T(10,10)=T(3,3)
      GO TO 49
   50 IF(NX) 100,100,150
```

```
  100 T(1,2)=EL(I)/EIX(I)
      T(1,3)=T(1,2)*T(2,3)
      T(1,1)=T(1,1)+T(1,2)*T(2,1)
      T(1,7)=T(1,2)*T(2,7)
C
C
C     Y AXIS TERMS
  150 IF(NY) 160,160,180
  160 L=1
  165 EI=EIY(I)
C     SEE PAGE 68 FOR DEFINITION OF GAMMA
  170 GAMMA=EL(I)*SQRT(PTX/EI)
      G2=GAMMA*GAMMA
      IF(G2-.00001)172,172,171
  171 SOC=(TANH(GAMMA))**2
      S=SQRT(SOC/(1.-SOC))/GAMMA
      C=SQRT(1./(1.-SOC))
      GO TO 173
  172 S=1.+G2/6.
      C=1.+G2*.5
  173 C1=S*T(10,6)
      C2=S*T(10,10)
      C3=C*T(10,6)
      C4=C*T(10,10)
      IF(G2-.00001)174,174,168
  174 S=(1.+G2*.05)/6.
      C=.5+G2/24.
      GO TO 169
  168 S=(S-1.)/G2
      C=(C-1.)/G2
  169 C5=S*T(10,6)
      C6=S*T(10,10)
      C7=C*T(10,6)
      C8=C*T(10,10)
      SL=EL(I)/EI
      SL2=EL(I)*SL
      SL3=EL(I)*SL2
      GO TO (175,178,175,178),L
  175 CL=SL*C1
      CL2=SL2*C7
      CL3=SL3*C5
      GO TO (176,230,332,360),L
C     Y AXIS - SINE TERMS
  176 T(3,1)=+CL3*T(2,7)
      T(3,3)=-CL3*A67+T(3,3)
      T(3,7)=-CL3*A107+T(3,7)
      T(5,10)=-EL(I)*C1
      T(3,8)=-CL2*A98+T(5,10)
      T(3,9)=-CL2
      T(3,10)=-CL3
```

```
      T(4,1)=+CL2*T(2,7)
      T(4,3)=-CL2*A67
      T(4,7)=-CL2*A107
      T(4,8)=-C3-CL*A98
      T(4,9)=-CL
      T(4,10)=-CL2
      T(5,1)=-T(5,10)*T(2,7)
      T(5,3)=T(5,10)*A67
      T(5,7)=T(5,10)*A107
      T(5,8)=PTX*T(5,10)-C3*A98
      T(5,9)=-C3
  178 CL=SL*C2
      CL2=SL2*C8
      CL3=SL3*C6
      GO TO (179,210,335,357),L
C     Y AXIS - COSINE TERMS
  179 T(7,1)=-CL3*T(2,7)
      T(7,3)=CL3*A67+T(7,3)
      T(7,7)=T(7,7)+CL3*A107
      T(9,10)=EL(I)*C2
      T(7,8)=CL2*A98+T(9,10)
      T(7,9)=CL2
      T(7,10)=CL3
      T(8,3)=CL2*A67
      T(8,7)=CL2*A107
      T(8,8)=C4+CL*A98
      T(8,9)=CL
      T(8,10)=CL2
      T(9,3)=T(9,10)*A67
      T(9,7)=T(9,10)*A107
      T(9,8)=C4*A98+PTX*T(9,10)
      T(9,9)=C4
      IF(IM1) 200,200,177
  177 T(8,1)=-T(8,3)*EPS(IM1)
      T(9,1)=-T(9,3)*EPS(IM1)
C     Y AXIS INFINITELY STIFF TERMS
  180 T(3,8)=-EL(I)*T(10,6)
      T(4,8)=-T(10,6)
      T(5,1)=-T(3,8)*T(2,7)
      T(5,3)=T(3,8)*A67
      T(5,7)=T(3,8)*A107
      T(5,8)=PTX*T(3,8)-T(10,6)*A98
      T(5,9)=-T(10,6)
      T(5,10)=T(3,8)
      T(7,8)=EL(I)*T(10,10)
      T(8,8)=T(3,3)
      T(9,3)=T(7,8)*A67
      T(9,7)=T(7,8)*A107
      T(9,8)=T(10,10)*A98+PTX*T(7,8)
      T(9,9)=T(10,10)
```

```
      T(9,10)=T(7,8)
      IF(IM1) 200,200,181
  181 T(9,1)=-EPS(IM1)*A67*T(7,8)
C
C     Z AXIS TERMS
C
  200 IF(NZ) 205,205,250
  205 EI=EIZ(I)
      L=2
      GO TO 170
C     Z AXIS - COSINE TERMS
  210 T(3,1)=CL2*A51-CL3*T(2,3)+T(3,1)
      T(3,3)=T(3,3)+CL3*A63
      T(3,4)=EL(I)*C2
      T(3,5)=CL2
      T(3,6)=CL3
      T(3,7)=CL3*A67+T(3,7)
      T(3,8)=CL2*A58+T(3,8)
      T(4,1)=CL2*A61+CL*A51+T(4,1)
      T(4,3)=CL2*A63+T(4,3)
      T(4,4)=C4
      T(4,5)=CL
      T(4,6)=CL2
      T(4,7)=CL2*A67+T(4,7)
      T(4,8)=CL*A58+T(4,8)
      T(5,1)=C4*A51-T(5,6)*T(2,3)   +T(5,1)
      T(5,6)=T(3,4)
      T(5,3)=A63*T(5,6)+T(5,3)
      T(5,4)=PTX*T(5,6)
      T(5,5)=C4
      T(5,7)=T(5,6)*A67+T(5,7)
      T(5,8)=C4*A58+T(5,8)
      GO TO 175
C     Z AXIS - SINE TERMS
  230 T(7,1)=CL2*A51+CL3*A61+T(7,1)
      T(7,3)=CL3*A63+T(7,3)
      T(7,4)=EL(I)*C1
      T(7,5)=CL2
      T(7,6)=CL3
      T(7,7)=CL3*A67+T(7,7)
      T(7,8)=CL2*A58+T(7,8)
      T(8,1)=CL*A51+CL2*A61+T(8,1)
      T(8,3)=CL2*A63+T(8,3)
      T(8,4)=C3
      T(8,5)=CL
      T(8,6)=CL2
      T(8,7)=CL2*A67+T(8,7)
      T(8,8)=CL*A58+T(8,8)
      T(9,1)=C3*A51+T(7,4)*A61+T(9,1)
      T(9,5)=C3
```

```
      T(9,6)=T(7,4)
      T(9,3)=T(9,6)*A63+T(9,3)
      T(9,4)=PTX*T(9,6)
      T(9,7)=T(9,6)*A67+T(9,7)
      T(9,8)=C3*A58+T(9,8)
      GO TO 280
C     Z AXIS INFINITELY STIFF TERMS
  250 T(3,4)=EL(I)*T(10,10)
      T(4,4)=T(10,10)
      T(5,1)=T(10,10)*(A51+EL(I)*A61)+T(5,1)
      T(5,3)=T(3,4)*A63+T(5,3)
      T(5,4)=PTX*T(3,4)
      T(5,5)=T(10,10)
      T(5,6)=T(3,4)
      T(5,7)=T(3,4)*A67+T(5,7)
      T(5,8)=T(10,10)*A58+T(5,8)
      T(7,4)=EL(I)*T(10,6)
      T(8,4)=T(10,6)
      T(9,1)=T(10,6)*(A51+EL(I)*A61)+T(9,1)
      T(9,3)=T(7,4)*A63+T(9,3)
      T(9,4)=PTX*T(7,4)
      T(9,5)=T(10,6)
      T(9,6)=T(7,4)
      T(9,7)=T(7,4)*A67+T(9,7)
      T(9,8)=T(10,6)*A58+T(9,8)
  280 IF(NGYRO) 400,400,300
C
C     GYROSCOPIC TERMS
  300 OMEGA=SQRT(OMSQ)
      A68=-2.*EMAS(IM1)*CPOMG*EPS(IM1)*OMEGA
      A97=-A68*CSSI
      A68=A68*SNSI
      A28=-(XINR(IM1)+YINR(IM1))*CPOMG*SNSI+A68
      A91=-A28
C
      T(2,8)=A28
      T(6,8)=T(10,10)*A68-T(10,6)*A97
      T(10,8)=T(10,6)*A68+T(10,10)*A97
      IF (I-NSEC) 305,500,500
  305 IF(NX) 310,310,320
  310 T(1,8)=(EL(I)/EIX(I))*A28
  320 IF (NY) 330,330,340
  330 L=3
      GO TO 165
  332 T(3,1)=T(3,1)-CL2*A91
      T(3,3)=T(3,3)-CL2*A68
      T(3,7)=T(3,7)-CL2*A97
      T(3,8)=T(3,8)-CL3*A97
      T(4,1)=T(4,1)-CL*A91
      T(4,3)=T(4,3)-CL*A68
```

```
      T(4,7)=T(4,8)-CL*A97
      T(4,8)=T(4,8)-CL2*A97
      T(5,1)=-C3*A91
      T(5,3)=-C3*A68
      T(5,7)=-C3*A97
      T(5,8)=-EL(I)*C1*A97+T(5,8)
      GO TO 178
  335 T(7,1)=T(7,1)+CL2*A91
      T(7,3)=T(7,3)+CL2*A68
      T(7,7)=T(7,7)+CL2*A97
      T(7,8)=T(7,8)+CL3*A97
      T(8,1)=T(8,1)+CL*A91
      T(8,3)=T(8,3)+CL*A68
      T(8,7)=T(8,7)+CL*A97
      T(8,8)=T(8,8)+CL2*A97
      T(9,1)=T(9,1)+C4*A91
      T(9,3)=T(9,3)+C4*A68
      T(9,7)=T(9,7)+C4*A97
      T(9,8)=T(9,8)+EL(I)*C2*A97
      GO TO 350
  340 T(5,1)=T(5,1)-T(10,6)*A91
      T(5,3)=T(5,3)-T(10,6)*A68
      T(5,7)=T(5,7)-T(10,6)-A97
      T(5,8)=T(5,8)-EL(I)*T(10,6)*A97
      T(9,1)=T(10,10)*A91+T(9,1)
         T(9,3)=T(10,10)*A68+T(9,3)
      T(9,7)=T(9,7)+T(10,10)*A97
      T(9,8)=T(9,8)+EL(I)*T(10,10)*A97
  350 IF(NZ) 355,355,380
  355 EI=EIZ(I)
      L=4
      GO TO 178
  357 T(10,8)=T(10,8)+CL3*A68
      T(4,8)=T(4,8)+CL2*A68
      T(5,8)=T(5,8)+EL(I)*C2*A68
      GO TO 175
  360 T(7,8)=T(7,8)+CL3*A68
      T(8,8)=CL2*A68+T(8,8)
      T(9,8)=EL(I)*C1*A68
      GO TO 400
  380 T(5,8)=T(5,8)+EL(I)*T(10,10)*A68
      T(9,8)=T(9,8)+EL(I)*T(10,6)*A68
C     END GYROSCOPIC TERMS
C
 400  IF(ELZ(I).EQ.0) GO TO 500
C
C     MODIFY T MATRIX FOR RIGID OFFSET BY POST MULTIPLYING BY OFFSET MAT
C     SEE PAGE 10
  410 C=PTZ*ELZ( I )
      DO 420 J=1,10
```

```
      T(J,1)=T(J,1)+C*T(J,2)-ELZ( I )*T(J,3)
      T(J,6)=T(J,6)-ELZ( I )*T(J,2)
  420 T(J,8)=T(J,8)+C*T(J,9)
      GO TO 500
C
C     REMAINDER OF MASS MATRIX AT LAST SECTION
  450 DO 457 J=1,10
  457 T(J,J)=1.
      EPSN=EPS(NSEC)
      EPS(NSEC)=EPST
      T(5,1)=A51
      T(5,8)=A58
      T(9,8)=A98
  500 RETURN
      END
```

```
      SUBROUTINE MATINV (A,N1,B,M1,DETERM,ID)
      DIMENSION A(10,10),B(10,1),INDEX(10,3)
      EQUIVALENCE (IROW,JROW),(ICOLUM,JCOLUM),(AMAX,T,SWAP)
      M=1
      N=N1
   10 DETERM=1.
   15 DO 20 J=1,N
   20 INDEX(J,3)=0.
   30 DO 550 I=1,N
   40 AMAX=0.
   45 DO 105 J=1,N
      IF(INDEX(J,3)-1)60,105,60
   60 DO 100 K=1,N
      IF (INDEX(K,3)-1) 80,100,715
   80 IF (AMAX-ABS(A(J,K))) 85,100,100
   85 IROW=J
   90 ICOLUM=K
      AMAX=ABS(A(J,K))
  100 CONTINUE
  105 CONTINUE
      INDEX(ICOLUM,3)=INDEX(ICOLUM,3)+1
  260 INDEX(I,1)=IROW
  270 INDEX(I,2)=ICOLUM
  130 IF (IROW-ICOLUM) 140,310,140
  140 DETERM=-DETERM
  150 DO 200 L=1,N
  160 SWAP=A(IROW,L)
  170 A(IROW,L)=A(ICOLUM,L)
  200 A(ICOLUM,L)=SWAP
      IF (M) 310,310,210
  210 DO 250 L=1,M
  220 SWAP=B(IROW,L)
  230 B(IROW,L)=B(ICOLUM,L)
  250 B(ICOLUM,L)=SWAP
  310 PIVOT=A(ICOLUM,ICOLUM)
      DETERM=DETERM*PIVOT
  330 A(ICOLUM,ICOLUM)=1.
  340 DO 350 L=1,N
  350 A(ICOLUM,L)=A(ICOLUM,L)/PIVOT
  355 IF (M) 380,380,360
  360 DO 370 L=1,M
  370 B(ICOLUM,L)=B(ICOLUM,L)/PIVOT
  380 DO 550 L1=1,N
  390 IF (L1-ICOLUM) 400,550,400
  400 T=A(L1,ICOLUM)
  420 A(L1,ICOLUM)=0.
  430 DO 450 L=1,N
  450 A(L1,L)=A(L1,L)-A(ICOLUM,L)*T
  455 IF (M) 550,550,460
  460 DO 500 L=1,M
```

```
500 B(L1,L)=B(L1,L)-B(ICOLUM,L)*T
550 CONTINUE
715 ID=2
740 RETURN
    END
```

```
      SUBROUTINE FINDM(OMLST,OMSQ,DET,DTOLD,RNEW,ROLD,NEXPO,NC4,NFREQ,OM
     1INT,NTIME,NFIND,NTRL,NFLG,NPDF)
C
      DIMENSION RNEW(4),ROLD(4),OMINT(15),OM(4),OMRD(4)
C
 900  FORMAT(8(5X,I5))
  910 FORMAT( 4X,I3,3X,E20.12,E15.5,15X,4E15.5)
 920  FORMAT( 4X,I3,3X,E20.12,6E15.5)
C
      NOUT=6
      IF(NFIND.GT.100) GO TO 130
      IF(NFLG.GT.0) GO TO 1
      OMLSS=1.
      NFLG=10
      DET1=DET
      IF(NPDF.LE.0) GO TO 3
      DO 2 I=1,NPDF
    2 OMINT(I)=OMINT(I)*OMINT(I)
  3   NFREQ=NPDF+1
    1 IF(NTIME.NE.0) GO TO 10
      DET=DET1
      OMLST=1.
      OMSQ=1.001
      OMSQ=OMSQ*1.001
      GO TO 6
    5 OMEGA = SQRT(ABS(OMLSS))*OMLSS/ABS(OMLSS)
      IF(NC4.NE.0.AND.NFIND.LT.100)
     1WRITE (6,920) NTIME,OMEGA,DET,YB,(RNEW(J),J=1,NC4)
      IF(NC4.EQ.0.AND.NFIND.LT.100)
     1WRITE (6,920) NTIME,OMEGA,DET,YB
      IF(NC4.EQ.0.AND.NFIND.GE.100)
     1WRITE (6,920) NTIME,OMEGA,DET
      IF(NC4.NE.0.AND.NFIND.GE.100)
     1WRITE(6,910) NTIME,OMEGA,DET,(RNEW(J),J=1,NC4)
    6 NTRL=2
      NTIME=NTIME+1
      RETURN
   10 OMD=1.
      OMOLD=1.
      NF=NFREQ-1
      IF(NF.EQ.0) GO TO 50
      DO 30 I=1,NF
      OMOLD=OMOLD*(OMLST-OMINT(I))
   30 OMD=OMD*(OMSQ-OMINT(I))
      OMP=OMD*OMOLD
      IF(OMP.NE.0) GO TO 50
      OMLST=OMSQ
      OMSQ=OMSQ+1.
      GO TO 5
   50 OMD=1./OMD
```

```
      OMOLD=1./OMOLD
      IF(NFIND.GT.100) GO TO 130
   60 YA=DET*OMD
      YB=DTOLD*OMOLD
      OMOR=OMSQ
      OMLSS=OMLST
      CALL INTERP(OMLSS,OMOR,YA,YB)
      IF (ABS(OMOR-OMLSS)/OMLSS.LT..316228**NEXPO) GO TO 70
      OMSQ=OMOR
      OMLST=OMLSS
      GO TO 5
   70 NFIND=NFIND+100
      DTSAV=DET
      OMSAV=OMSQ
      OMNXT=OMOR
      IF (NC4.EQ.0) GO TO 130
      DO 90 I=1,NC4
      YA=RNEW(I)
      YB=ROLD(I)
      OMOR=OMSQ
      OMLSS=OMLST
      CALL INTERP(OMLSS,OMOR,YA,YB)
      OM(I)=OMOR
   90 OMRD(I)=ABS(OMNXT-OMOR)
      NOOR=1
      OMQ=OMRD(1)
      DO 110 I=1,NC4
      IF (OMQ.LT.OMRD(I)) GO TO 110
      NOOR=I
      OMQ=OMRD(I)
  110 CONTINUE
      OMLST=OMSQ
      OMSQ=OM(NOOR)
      WRITE(6,930) NOOR
  930 FORMAT(/,5X,43HSUBSEQUENT ITERATIONS USE REMAINDER NUMBER ,I3)
      GO TO 5
  130 OMOR=OMSQ
      IF (NTIME.LT.2.AND.NC4.NE.0) GO TO 60
      OMLSS=OMLST
      IF (NC4.NE.0) GO TO 135
      YA=DET
      YB=DTOLD
      GO TO 140
 135  YA=RNEW(NOOR)
      YB=ROLD(NOOR)
  140 CALL INTERP(OMLSS,OMOR,YA,YB)
      IF((OMOR-OMLSS)/OMLSS.LE..1**NEXPO) GO TO 150
      OMLST=OMSQ
      OMSQ=OMOR
      GO TO 5
```

```
150 NFIND=NFIND-100
    OMINT(NFREQ)=OMOR
    NTRL=1
    NTIME=0
    RETURN
    END
```

```
      SUBROUTINE INTERP (OMLST,OMSQ,YA,YB)
      NOUT=6
      OM=OMSQ-YA*(OMSQ-OMLST)/(YA-YB)
      IF(OM) 1,2,2
    1 WRITE (NOUT,900)
      OMLST=OMSQ
      OMSQ=OMSQ*1.1
  900 FORMAT (30H  ITERATION GIVES OM NEGATIVE   )
      GO TO 3
    2 OMLST=OMSQ
      OMSQ=OM
    3 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE SHAPE(PRD,NTAPE,NPCH,IBC,TMSL)
      DIMENSION EL(30),EIX(30),EIY(30),EIZ(30),XINR(30),YINR(30)
      DIMENSION EMAS(30),PHI(30),EPS(30),ELZ(31),CAPSI(30)
      DIMENSION CCPS(30),AVMAS(31)
      DIMENSION PRD(10,5),T(10,10),Z(10),ZSV(10),B(10)
      DIMENSION A(10,10),C(10)
      DIMENSION ERR(10),ASAV(10,5),TITLE(19)
      COMMON CPSQ,OMSQ,XROOT,ZERMO
      CCMMON EL,EIX,EIY,EIZ,XINR,YINR,EMAS,PHI,EPS,ELZ,CAPSI,CCPS,EPSN
      COMMON CPOMG,OMEG,SIZER,THETC,ELC,AVMAS
      CCMMON NGYRO,NX,NY,NZ,NSEC,NSP1
      COMMON NC,NC1,NC2,NC3,NR,NR1,NR2,NR3,NR4
      COMMON NIBC,B
      COMMON ERR,TITLE,NFREQ
C     SHAPE SUBROUTINE FOR PIN AT 0 AND LEAD-LAG HINGE AT POINT 1
      NOUT=6
      TMR=0.
      WRITE(NOUT,910)
C     INITIALIZE STATE VECTOR
      DO 10 II=1,10
      ZSV(II)=0.
   10 Z(II)=0.
      N=2
      IF(IBC.EQ.1) GO TO 401
      CALL BNDRYX(PRD,A,N,IBC)
      GO TO 405
  401 CALL BNDRY1(PRD,A,N)
  405 K=1
      OMEGA= SQRT(OMSQ)
   16 CONTINUE
      GNMS=A(10,5)
      X=A(10,4)
      I=N
      DO 17 J=NR1,NR2
      Z(J)=B(J)
   17 ZSV(J)=0.
      IF (K.EQ.1) GO TO 52
      IF(TMR.GT.TMSL) GO TO 50
      IF(NPCH.NE.0) WRITE (4,916) OMEGA,(TITLE(IKJ),IKJ=1,16),NFREQ
      GO TO 50
C     COMPUTE AN OUTPUT STATE VECTOR AT EACH SECTION, AND COMPUTE
C     GENERALIZED MASS, AND WRITE ON TAPE FOR EACH SECTION ASSOCIATED
C     WITH A CONTRIBUTICN TO THE GENERALIZED MASS
   35 DO 40 J=NR1,NR2
      Z(J)=ZSV(J)
   40 ZSV(J)=0.
   42 C(1)=AVMAS(I+1)*EPS(I)
      C(2)=C(1)*EPS(I)
      GNMS=GNMS+(XINR(I)+C(2))*(Z(1)*Z(1)+Z(8)*Z(8))-2.*C(1)*Z(3)*Z(1)
     1+AVMAS(I+1)*(Z(3)*Z(3)+Z(7)*Z(7))
```

```
      IF(Z(3).EQ.0)  GO TO 49
      IF (K.EQ.2.OR.I.LT.NSEC) GO TO 49
      TMR=ABS(Z(1)/Z(3))
   49 CONTINUE
      X=X+EL(I)
      WRITE (NOUT,920)I,(Z(J),J=1,10),X
   50 IF(K.EQ.1.OR.NPCH.EQ.0.OR.I.EQ.0) GO TO 52
   51 IF (TMR.LE.TMSL) WRITE(4,930)
     1 Z(3),Z(7),Z(1),Z(8),Z(4),Z(2),Z(5),Z(6),Z(9),Z(10)
   52 I=I+1
      IF (I-NSEC)150,150,500
  150 DO 151 II=1,10
      DO 151 IX=1,10
  151 T(II,IX)=0.
      CALL TEMAT(I,T)
      IF (I-NSEC)170,160,500
C     ACCOUNT FOR THIS MASS AT OUTBOARD END
  160 DO 161 M=NR1,NR2
      DO 161 J=NR1,NR2
  161 A(M,J)=T(M,J)
      DO 1611 II=1,10
      DO 1611 IX=1,10
 1611 T(II,IX)=0.
      I=NSP1
      CALL TEMAT (I,T)
      I=NSEC
      DO 163 M=NR1,NR2
      DO 162 J=NR1,NR2
      C(J)=0.
      DO 162 N=NR1,NR2
  162 C(J)=C(J)+T(M,N)*A(N,J)
      DO 163 J=NR1,NR2
  163 T(M,J)=C(J)
C     MULTIPLY STATE VECTOR (I) BY T TO OBTAIN STATE VECTOR (I+1).
  170 DO 180 J=NR1,NR2
      DO 180 N=NR1,NR2
  180 ZSV(J)=ZSV(J)+T(J,N)*Z(N)
      GO TO 35
C     WRITE NATURAL FREQUENCY IN RAD/SEC AND X CAP-OMEGA
  500 X=OMEGA/CPOMG
      GNMS=SQRT(GNMS)
      WRITE (NOUT,915) OMEGA,X,GNMS
      GNMS=1./GNMS
      IF(TMR.GT.TMSL.AND.K.EQ.2.AND.NPCH.EQ.1) WRITE(NOUT,917)
C     COMPUTE MODE SHAPE FOR GENERALIZED MASS IF NEEDED
  501 GO TO (502,510),K
  502 K=2
      X=XROOT
  506 A(1,1)=GNMS
C     COMPUTE GENERALIZED MODE SHAPE
```

```
C     COMPUTE GENERALIZED MASS MODE SHAPES THROUGH NIBC IN BNDRY IF
C     NIBC GT. 2
      N=3
      IF(IBC.EQ.1) GO TO 411
      CALL BNDRYX(PRD,A,N,IBC)
      GO TO 16
  411 CALL BNDRY1(PRD,A,N)
      GO TO 16
  510 CONTINUE
  910 FORMAT(1(/),                                                          1
     121H   I    ANGLE      TORQUE      DISPL       ANGLE      MOMENT      SH
     2EAR        DISPL       ANGLE     MOMENT       SHEAR      RADIAL/      1
     321H         PHI          T          V         THETA        M Z        -
     4VY          -W          PSI        M Y         V Z       COORD.,/)
  920 FORMAT (I3,11(E10.3,1X))
  930 FORMAT (5(E14.7,1X),/,5E15.7)
  915 FORMAT(/ 9X,19HNATURAL FREQUENCY =F10.4,2X,14HRADIANS/SECOND,
     1         3H = ,F10.4,20H X ROTATIONAL SPEED.3X,20HGENERALIZED MASS
     2 = ,F10.4//)
  916 FORMAT(E12.5,16A4,I2)
  917 FORMAT (//3X,107H ** NOTE ** THE ABOVE MODE SHAPE IS NOT INCLUDED
     1IN PUNCHED OUTPUT, PHI(TIP).GT.(V(TIP)*TMSL, TORSION LIMIT)
      RETURN
      END
```

APPENDIX D

TEST CASES AND ANSWER LISTINGS

This appendix contains, in order:

(1) A listing of computer program input which was used to generate various test cases at NASA Langley,

(2) Answer listings, chosen from the printed output listing produced by the above input, and labeled by a Test Case number, and

(3) Answer listings, including all the punched cards in the same order as produced by the above input (part 1).

The portion of the input which corresponds to the enclosed printed output may be identified by comparing blade model and program control parameters. In order to facilitate this comparison, the following observations and table should be useful. Each model has a title card and cards read by the NAMELIST option. There were 19 models in the test case run. If the models are numbered sequentially from 1 to 19, the following table gives the model number which corresponds to the various test cases as chosen from the printed output.

Identification numbers

Test Case	Model
1.a	3
1.b	4
1.c	5
2.a	7
2.b	10
3.a	15
3.b	15
4.	16

The first three modes were punched for model number 2 and the last five modes were punched for model number 14.

APPENDIX D.1, Input for Test Cases

```
1115  UNIFORM BLADE (LUMPED PARAMETER MODEL) HINGED - HINGED
010 .625.1209E+4 2.64E+6 4.76E+4.3644E-2.3644E-2   .0815      0.       0.        0.
02  .875 9.53E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0. .0225       1.0E-4
030 .875 5.03E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0.    .0225        0.
040 .875 5.03E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0.    .0225        0.
050 .875 5.03E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0.    .0225        0.
060 .875 5.03E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0.    .0225        0.
070 .875 5.03E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0.    .0225        0.
080 .875 5.03E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0.    .0225        0.
090 .875 5.03E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0.    .0225        0.
100 .875 5.03E+4 2.64E+6 4.76E+4.5101E-2.5101E-2   .1141      0.    .0225        0.
110 1.75 5.03E+4 2.64E+6 4.76E+4 .010202 .010202   .2282      0.    .0225        0.
120 1.75 5.03E+4 2.64E+6 4.76E+4 .010202 .010202   .2282      0.    .0225        0.
130 1.75 5.03E+4 2.64E+6 4.76E+4 .010202 .010202   .2282      0.    .0225        0.
140 1.75 5.03E+4 2.64E+6 4.76E+4 .010202 .010202   .2282      0.    .0225        0.
150 1.75 5.03E+4 2.64E+6 4.76E+4 .010202 .010202   .2282      0.    .0225        0.
 $IN NTYPE=6, NMAX=20, NUMF=3, NPDF=00, NFIND=-1,  NPCH=0,  PRCNT=10.0,
 NEXPO=11,  NCOR=3,  CPOMG=37.1755, SIZER=10.0, THETC=0.00, ELC=0.0,XROOT=0.25 ,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=10.,40.,80.                         $
1015  UNIFORM BLADE (LUMPED PARAMETER MODEL) HINGED - HINGED
 $IN NTYPE=6, NMAX=20, NUMF=3, NPDF=00, NFIND=01,  NPCH=1,  PRCNT=1.00,
 NEXPO=11,  NCOR=1,  CPOMG=37.1755, SIZER=10.0, THETC=0.00, ELC=0.0,XROOT=0.25 ,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=38.8576,56.1711,103.2065,131.0583 $
2015  UNIFORM BLADE (LUMPED PARAMETER MODEL) CLAMPED - CLAMPED
 $IN NTYPE=6, NMAX=20, NUMF=3, NPDF=00, NFIND=01,  NPCH=0,  PRCNT=1.00,
 NEXPO=11,  NCOR=1,  CPOMG=37.1755, SIZER=10.0, THETC=0.00, ELC=0.0,XROOT=0.25 ,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=38.8576,56.1711,103.2065,131.0583 $
3015UNIFORM BLADE (LUMPED PARAMETER MODEL) TEETERING ANTISYMMETRICAL
 $IN NTYPE=6, NMAX=20, NUMF=3, NPDF=00, NFIND=01,  NPCH=0,  PRCNT=1.00,
 NEXPO=11,  NCOR=1,  CPOMG=37.1755, SIZER=10.0, THETC=0.00, ELC=0.0,XROOT=0.25 ,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=38.8576,56.1711,103.2065,131.0583 $
4015  UNIFORM BLADE (LUMPED PARAMETER MODEL) TEETERING SYMMETRICAL
 $IN NTYPE=6, NMAX=20, NUMF=3, NPDF=00, NFIND=01,  NPCH=0,  PRCNT=1.00,
 NEXPO=11,  NCOR=1,  CPOMG=37.1755, SIZER=10.0, THETC=0.00, ELC=0.0,XROOT=0.25 ,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=38.8576,56.1711,103.2065,131.0583 $
```

```
1116 FULLY ARTICULATED TEST ROTOR,   FOR PROGRAM DOCUMENTATION
01 .89   8.359E+21.0 E+072.722E+42.501E-22.501E-23.602E-10.00 E-2-.280E-3 0.0 E-0
02 .526 3.368E+4.2778E+67.986E+4.8139E-3.8139E-31.037E-1-.233E+0-.0272700.0
03 .41671 8.82E+32.778E+527.77E+3.5999E-3.5999E-36.211E-2-.287E-0-.015870.125 E-2
04 .833 .6528E+4  .271E+6  .868E+41.14 E-31.14 E-3 4.35E-2-.575E+0-.049440-.008
05 .3750 .535E+4  .262E+6  .656E+4  .500E-3  .500E-3  1.61E-2-.259E+0.2335440-.025E-1
06 1.333 .535E+4  .262E+6  .656E+4 2.2 E-3 2.2 E-3 16.1E-2-.920E+0-.006630
07 1.333 .528E+4  .261E+6  .639E+4 1.72E-3 1.72E-3 5.61E-2-.920E+0-.124530-.083E-2
08 1.333 .507E+4  .258E+6  .606E+4 1.71E-3 1.71E-3 5.32E-2-.920E+0.0536620-.025E-1
09 .25    .528E+4  .253E+6  .595E+4  .32 E-3  .32 E-3  .978E-2-.173E+0-.521800-.033E-2
10 .458   .521E+4  .251E+6  .650E+4  .57 E-3  .57 E-3  1.88E-2-.316E+0.3027630.0633E-1
11 1.083 .514E+4  .222E+6  .643E+4 1.58E-3 1.58E-3 12.9E-2-.748E+0-.018920.003 E-0
12 .458   .500E+4  .197E+6  .640E+4  .475E-3  .475E-3 1.79E-2-.316E+0-.088530.0267E-1
13 .583   .458E+4  .174E+6  .590E+4  .556E-3  .556E-3 1.89E-2-.402E+0.029136 -.208E-2
14 1.167 .451E+4  .153E+6  .585E+41.01 E-31.01 E-3 3.70E-2-.806E+0-.056730.833 E-4
15 1.167 .451E+4  .153E+6  .578E+41.01 E-31.01 E-3 3.48E-2-.806E+00.326710-.5   E-3
16 .4580 .556E+4  .239E+61.507E+4.99  E-3.99  E-31.08 E-1-.316E+0-.021540-.008E-0
 $IN NTYPE=6,NMAX=30,          NEXPO=12, NUMF=3,          NCOR=0, NPCH=0,
 NFIND=1, NPDF=0,        CPOMG=50.056, SIZER=17.4,THETC=.0000000, FLC=0.0,
 XRCOT=0.458, PRCNT=1.0, OMINT(1)=20.,TMSL=0.0,OMLIMT=160000.$
1016 FULLY ARTICULATED TEST ROTOR,   FOR PROGRAM DOCUMENTATION
 $IN NTYPE=1,NMAX=30,          NEXPO=12, NUMF=3,          NCOR=0, NPCH=0,
 NFIND=1, NPDF=0,        CPOMG=50.056, SIZER=17.4,THETC=.0000000, ELC=0.0,
 XRCOT=0.458, PRCNT=1.0, OMINT(1)=20.,TMSL=0.0,OMLIMT=160000.$
1016 FULLY ARTICULATED TEST ROTOR,   FOR PROGRAM DOCUMENTATION
 $IN NTYPE=2,NMAX=30,          NEXPO=12, NUMF=3,          NCOR=0, NPCH=0,
 NFIND=1, NPDF=0,        CPOMG=50.056, SIZER=17.4,THETC=.0000000, ELC=0.0,
 XRCOT=0.458, PRCNT=1.0, OMINT(1)=20.,TMSL=0.0,OMLIMT=160000.$
1016 FULLY ARTICULATED TEST ROTOR,   FOR PROGRAM DOCUMENTATION
 $IN NTYPE=3,NMAX=30,          NEXPO=12, NUMF=3,          NCOR=0, NPCH=0,
 NFIND=1, NPDF=0,        CPOMG=50.056, SIZER=17.4,THETC=.0000000, ELC=0.0,
 XRCOT=0.458, PRCNT=1.0, OMINT(1)=20.,TMSL=0.0,OMLIMT=160000.$
1016 FULLY ARTICULATED TEST ROTOR,   FOR PROGRAM DOCUMENTATION
 $IN NTYPE=4,NMAX=30,          NEXPO=12, NUMF=3,          NCOR=0, NPCH=0,
 NFIND=1, NPDF=0,        CPOMG=50.056, SIZER=17.4,THETC=.0000000, FLC=0.0,
 XRCOT=0.458, PRCNT=1.0, OMINT(1)=20.,TMSL=0.0,OMLIMT=160000.$
1016 FULLY ARTICULATED TEST ROTOR,   FOR PROGRAM DOCUMENTATION
 $IN NTYPE=5,NMAX=30,          NEXPO=12, NUMF=3,          NCOR=0, NPCH=0,
 NFIND=1, NPDF=0,        CPOMG=50.056, SIZER=17.4,THETC=.0000000, ELC=0.0,
 XRCOT=0.458, PRCNT=1.0, OMINT(1)=20.,TMSL=0.0,OMLIMT=160000.$
```

```
1016 FULLY ARTICULATED TEST ROTOR,  FOR PROGRAM DOCUMENTATION
 $IN NTYPE=6,NMAX=30,           NEXPO=12, NUMF=1,            NCOR=0, NPCH=0,
 NFIND=1, NPDF=0,          CPOMG=50.056, SIZER=17.4,THETC=.0000000, ELC=0.0,
 XROOT=0.458, PRCNT=1.0, OMINT(1)=20.,TMSL=0.0,OMLIMT=16.00000 $
1118 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED
010.01000.7700E26.4600E65.4170E56.990E-26.990E-22.100E-20.       0.        0.
0201.0738.2640E53.1940E63.4720E66.480E-26.480E-22.199   0.       0.        0.
030.83303.1400E51.6460E64.9100E54.380E-24.380E-29.940E-10.       1.367E-20.
04 1.6671.4580E51.1930E61.1870E52.200E-22.200E-21.650E-1-2.80E-21.367E-26.835E-3
0502.0001.2150E51.1070E61.0210E52.330E-22.330E-22.640E-1-6.81E-11.367E-20.
060.91701.2150E51.1070E61.0210E52.980E-22.980E-21.660E-1-3.12E-11.367E-20.
0702.5831.2150E51.1070E61.0210E53.470E-23.470E-24.260E-1-8.80E-11.367E-20.
0801.5001.2150E51.1070E61.0210E53.530E-23.530E-22.720E-1-5.10E-11.367E-20.
0902.4171.2150E51.1070E61.0210E53.190E-23.190E-23.980E-1-8.23E-11.367E-20.
1001.3331.2150E51.1070E61.0210E53.300E-23.300E-22.420E-1-4.54E-11.367E-20.
1102.3331.2150E51.1070E61.0210E54.110E-24.110E-23.690E-1-7.94E-11.367E-20.
1202.5001.2150E51.1070E61.0210E52.830E-22.830E-23.960E-1-8.52E-11.367E-20.
130.83301.2150E51.1070E61.0210E52.550E-22.550E-21.510E-1-2.83E-11.367E-20.
1402.0001.2150E51.1070E61.0210E53.470E-23.470E-23.300E-1-6.81E-11.367E-20.
1502.0831.2150E51.1070E61.0210E52.930E-22.930E-24.130E-1-7.09E-11.367E-20.
1601.1671.2150E51.1070E61.0210E51.700E-21.700E-22.000E-1-3.97E-11.367E-20.
170.83301.2150E51.1070E61.0210E52.450E-22.450E-22.060E-1-2.84E-11.367E-20.
180.91701.2150E51.1070E61.0210E55.500E-35.500E-31.510E-1-3.12E-11.367E-20.
 $IN NTYPE=6, NMAX=30, NUMF=4, NPDF=04, NFIND=00,  NPCH=0,  PRCNT=1.  ,
 NEXPO=11,  NCOR=3,  CPOMG=20.000,  SIZER=13.6, THETC=0.00, ELC=0.0,XROOT=1.000,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=20.,50.,100.,150.                $
1018 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED
 $IN NTYPE=6, NMAX=30, NUMF=6, NPDF=00, NFIND=01,  NPCH=1,  PRCNT=1.  ,
 NEXPO=11,  NCOR=1,  CPOMG=20.000,  SIZER=13.6, THETC=0.00, ELC=0.0,XROOT=1.000,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=4.9428,20.553,53.95734,70.6665    $
1018 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED
 $IN NTYPE=6, NMAX=30, NUMF=6, NPDF=04, NFIND=01,  NPCH=0,  PRCNT=1.  ,
 NEXPO=11,  NCOR=2,  CPOMG=20.000,  SIZER=13.6, THETC=0.00, ELC=0.0,XROOT=1.000,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=4.9428,20.553,53.95734,70.6665    $
1018 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED
 $IN NTYPE=6, NMAX=30, NUMF=4, NPDF=04, NFIND=-1,  NPCH=0,  PRCNT=10.,
 NEXPO=11,  NCOR=1,  CPOMG=20.000,  SIZER=13.6, THETC=0.00, ELC=0.0,XROOT=1.000,
 TMSL=1.0000,  OMLIMT=990000.0,  OMINT(1)=21.,50.,100.,150.                $
```

```
1102   POLYNOMIAL TEST CASE WITH MULTIPLE AND CLOSELY SPACED ROOTS
01
02
 $IN NTYPE=1, NMAX=300,NEXPO=12, NUMF=10,NOOR=0, NPCH=0, NFIND=1, NPDF=0,
 CPOMG=32.882, SIZER=10., THETC=0., FLC=0., XROOT=0.,PRCNT=1., OMLIMT=99000000.,
  TMSL=1., OMINT(1)=5.,6.,6.,10.,10.01,23.47,33.8,999.,1001.,15000. $
4118  HU-1A  TEETERING ROTOR
01 1.221.1603E 5.2097E 8.1076E 7.4893E-1.4893E-1.7280   0.       .6264E-10.
02.4458 .1875E 7.2097E 8.1076E 7.1787E-1.1787E-1.2658   0.       .3699E-10.
03.7467 .1736E 7.1858E 8.9514E 6.2882E-1.2882E-1.4067   -.06364 .5739E-1-.07500
04.7084 .6875E 6.1458E 8.6944E 6.2776E-1.2776E-1.3089   -.3864  .1620E-1.2585E-2
05 .625 .1250E 6.1163E 8.4965E 6.2476E-1.2476E-1.2178   -.3409  -.01829 .2452E-2
06  .7  .1250E 6.9896E 7.3542E 6.2584E-1.2584E-1.1957   -.3818  .6589E-2.2163E-2
07 .45  .1250E 6.8750E 7.2587E 6.1533E-1.1533E-1.1048   -.2454  -.01665 .2423E-2
08 .65  .1250E 6.7986E 7.1997E 6.2070E-1.2070E-1.1326   -.3545  .3032E-1.1558E-2
09 .95  .1233E 6.7986E 7.1597E 6.2777E-1.2777E-1.1797   -.5182  -.02006 .2250E-2
10 .9   .1198E 6.7847E 7.1567E 6.2582E-1.2582E-1.1685   -.4909  .9951E-2.3288E-2
11 2.3  .1198E 6.6597E 7.1589E 6.5751E-1.5751E-1.4221   -.1255E1.3120E-3.3115E-2
12 2.3  .1198E 6.5278E 7.1632E 6.4605E-1.4605E-1.4029   -.1255E1.2049E-1.7962E-2
13 1.   .1198E 6.5243E 7.1667E 6.1889E-1.1889E-1.1736   -.5454  -.01277 .7962E-2
14  .9  .1198E 6.5514E 7.1675E 6.1860E-1.1860E-1.1898   -.4909  .1225   .3462E-2
15 2.3  .1198E 6.5556E 7.1684E 6.5221E-1.5221E-1.5143   -.1255E1.1294E-1.3115E-2
16 2.25 .1146E 6.5660E 7.1692E 6.5237E-1.5237E-1.5192   -.1227E1.8528E-1.7962E-2
17 1.75 .1076E 6.6701E 7.1771E 6.3849E-1.3849E-1.5322   -.9546  .4027E-1.7788E-2
18 1.8  .1042E 6.6701E 7.1771E 6.4095E-1.4095E-1.5167   -.9818  .5580E-1.6058E-2
 $IN NTYPE=6, NMAX=30, NEXPO=12, NUMF=3, NOOR=0, NPCH=0, NFIND=1, NPDF=0,
 CPOMG=32.882, SIZER=10., THETC=0., FLC=0., XROOT=0.,PRCNT=1., TMSL=1.,
  OMLIMT=160000., OMINT(1)=10.0    $
3018  HU-1A  TEETERING ROTOR
 $IN NTYPE=6, NMAX=30, NEXPO=12, NUMF=3, NOOR=0, NPCH=0, NFIND=1, NPDF=0,
 CPOMG=32.882, SIZER=10., THETC=0., FLC=0., XROOT=0.,PRCNT=1., OMINT=1.,
  TMSL=1., CMLIMT=160000.$
1040   STOP CARD
```

APPENDIX D.2, Printed Output Answer Listings

Test Case 1

a. Model Parameters and Frequencies for IBC =2

BLADE FREQUENCY PROGRAM

BLADE MODEL AND OPERATING CONDITION PARAMETERS

UNIFORM BLADE (LUMPED PARAMETER MODEL) CLAMPED - CLAMPED 03/23/72

NUMBER OF SECTIONS = 15
ROTATIONAL SPEED CAP OMEGA = 37.1755 RADIANS/SEC
CONTROL ANGLE SI ZERO = 10.0000 DEGREES
CONTROL ANGLE THETA C = 0.0000 DEGREES
CONTROL OFFSET = 0.0000 FEET
RADIAL DISTANCE TO FIRST BLADE ELEMENT = .2500

I	LENGTH FEET	MASS LB-SEC2 PER-FT	EIX LB-FT2	EIY LB-FT2	EIZ LB-FT2	IX LB-SEC2-FEET	IY LB-SEC2-FEET	PHI DEGREES	EPSILON FEET	OFFSET FEET
1	6.2500E-01	8.1500E-02	1.2090E+03	2.6400E+06	4.7600E+04	3.6440E-03	3.6440E-03	0.	0.	0.
2	8.7500E-01	1.1410E-01	9.5300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	1.3125E-02	1.0000E-04
3	8.7500E-01	1.1410E-01	5.0300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	2.2500E-02	0.
4	8.7500E-01	1.1410E-01	5.0300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	2.2500E-02	0.
5	8.7500E-01	1.1410E-01	5.0300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	2.2500E-02	0.
6	8.7500E-01	1.1410E-01	5.0300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	2.2500E-02	0.
7	8.7500E-01	1.1410E-01	5.0300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	2.2500E-02	0.
8	8.7500E-01	1.1410E-01	5.0300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	2.2500E-02	0.
9	8.7500E-01	1.1410E-01	5.0300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	2.2500E-02	0.
10	8.7500E-01	1.1410E-01	5.0300E+04	2.6400E+06	4.7600E+04	5.1010E-03	5.1010E-03	0.	2.2500E-02	0.
11	1.7500E+00	2.2820E-01	5.0300E+04	2.6400E+06	4.7600E+04	1.0202E-02	1.0202E-02	0.	2.2500E-02	0.
12	1.7500E+00	2.2820E-01	5.0300E+04	2.6400E+06	4.7600E+04	1.0202E-02	1.0202E-02	0.	2.2500E-02	0.
13	1.7500E+00	2.2820E-01	5.0300E+04	2.6400E+06	4.7600E+04	1.0202E-02	1.0202E-02	0.	2.2500E-02	0.
14	1.7500E+00	2.2820E-01	5.0300E+04	2.6400E+06	4.7600E+04	1.0202E-02	1.0202E-02	0.	2.2500E-02	0.
15	1.7500E+00	2.2820E-01	5.0300E+04	2.6400E+06	4.7600E+04	1.0202E-02	1.0202E-02	0.	2.2500E-02	0.

computed frequencies: 1.0452Ω, 1.5110Ω, and 2.7756Ω

b. First Mode for IBC =3

UNIFORM BLADE (LUMPED PARAMETER MODEL) 03/23/72

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	-3.560E+01	0.	5.749E-02	0.	-1.583E+03	0.	0.	-5.316E+03	6.240E+02	2.500E-01
1	-1.840E-02	-3.560E+01	3.594E-02	5.750E-02	1.646E+00	-1.583E+03	-3.838E-04	-1.213E-03	-4.936E+03	6.240E+02	8.750E-01
2	-1.873E-02	-3.544E+01	8.627E-02	5.754E-02	2.862E+00	-1.578E+03	-2.135E-03	-2.761E-03	-4.420E+03	6.231E+02	1.750E+00
3	-1.934E-02	-3.525E+01	1.366E-01	5.760E-02	3.377E+00	-1.564E+03	-5.166E-03	-4.138E-03	-3.943E+03	6.201E+02	2.625E+00
4	-1.995E-02	-3.476E+01	1.871E-01	5.766E-02	3.573E+00	-1.543E+03	-9.333E-03	-5.357E-03	-3.502E+03	6.149E+02	3.500E+00
5	-2.054E-02	-3.409E+01	2.376E-01	5.773E-02	3.589E+00	-1.513E+03	-1.450E-02	-6.431E-03	-3.094E+03	6.069E+02	4.375E+00
6	-2.112E-02	-3.323E+01	2.881E-01	5.780E-02	3.503E+00	-1.475E+03	-2.055E-02	-7.372E-03	-2.716E+03	5.961E+02	5.250E+00
7	-2.163E-02	-3.219E+01	3.387E-01	5.786E-02	3.358E+00	-1.429E+03	-2.737E-02	-8.188E-03	-2.366E+03	5.820E+02	6.125E+00
8	-2.222E-02	-3.097E+01	3.894E-01	5.792E-02	3.175E+00	-1.375E+03	-3.486E-02	-8.891E-03	-2.042E+03	5.645E+02	7.000E+00
9	-2.273E-02	-2.95[illegible]E+01	4.401E-01	5.799E-02	2.968E+00	-1.312E+03	-4.291E-02	-9.490E-03	-1.742E+03	5.433E+02	7.875E+00
10	-2.322E-02	-2.797E+01	4.908E-01	5.804E-02	2.748E+00	-1.242E+03	-5.145E-02	-9.993E-03	-1.466E+03	5.183E+02	8.750E+00
11	-2.410E-02	-2.531E+01	5.925E-01	5.814E-02	1.517E+00	-1.123E+03	-6.962E-02	-1.069E-02	-9.731E+02	4.749E+02	1.050E+01
12	-2.483E-02	-2.101E+01	6.943E-01	5.820E-02	2.708E-01	-9.327E+02	-8.877E-02	-1.111E-02	-5.720E+02	4.003E+02	1.225E+01
13	-2.533E-02	-1.597E+01	7.962E-01	5.823E-02	-1.218E+00	-7.089E+02	-1.084E-01	-1.131E-02	-2.679E+02	3.085E+02	1.400E+01
14	-2.574E-02	-1.016E+01	8.981E-01	5.822E-02	-2.385E+00	-4.519E+02	-1.283E-01	-1.137E-02	-7.195E+01	1.992E+02	1.575E+01
15	-2.587E-02	1.254E-07	1.000E+00	5.820E-02	-1.002E-04	-5.723E-06	-1.482E-01	-1.138E-02	1.547E-06	9.801E-07	1.750E+01

NATURAL FREQUENCY = 36.6009 RADIANS/SECOND = .9845 X ROTATIONAL SPEED. GENERALIZED MASS = .8739

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	-4.073E+01	0.	6.578E-02	0.	-1.811E+03	0.	0.	-6.082E+03	7.141E+02	2.500E-01
1	-2.105E-02	-4.073E+01	4.112E-02	6.580E-02	1.883E+00	-1.811E+03	-4.391E-04	-1.388E-03	-5.648E+03	7.141E+02	8.750E-01
2	-2.143E-02	-4.055E+01	9.871E-02	6.584E-02	3.275E+00	-1.806E+03	-2.443E-03	-3.159E-03	-5.058E+03	7.130E+02	1.750E+00
3	-2.213E-02	-4.034E+01	1.564E-01	6.591E-02	3.864E+00	-1.790E+03	-5.911E-03	-4.735E-03	-4.512E+03	7.096E+02	2.625E+00
4	-2.282E-02	-3.978E+01	2.141E-01	6.598E-02	4.088E+00	-1.765E+03	-1.068E-02	-6.130E-03	-4.007E+03	7.035E+02	3.500E+00
5	-2.350E-02	-3.901E+01	2.718E-01	6.606E-02	4.106E+00	-1.731E+03	-1.659E-02	-7.359E-03	-3.540E+03	6.945E+02	4.375E+00
6	-2.416E-02	-3.803E+01	3.297E-01	6.613E-02	4.008E+00	-1.688E+03	-2.352E-02	-8.435E-03	-3.108E+03	6.821E+02	5.250E+00
7	-2.480E-02	-3.684E+01	3.876E-01	6.621E-02	3.842E+00	-1.635E+03	-3.132E-02	-9.369E-03	-2.707E+03	6.660E+02	6.125E+00
8	-2.542E-02	-3.544E+01	4.455E-01	6.628E-02	3.633E+00	-1.573E+03	-3.989E-02	-1.017E-02	-2.336E+03	6.459E+02	7.000E+00
9	-2.601E-02	-3.333E+01	5.035E-01	6.635E-02	3.396E+00	-1.502E+03	-4.910E-02	-1.086E-02	-1.994E+03	6.217E+02	7.875E+00
10	-2.657E-02	-3.201E+01	5.616E-01	6.641E-02	3.144E+00	-1.421E+03	-5.887E-02	-1.143E-02	-1.678E+03	5.931E+02	8.750E+00
11	-2.757E-02	-2.896E+01	6.780E-01	6.652E-02	1.735E+00	-1.285E+03	-7.967E-02	-1.224E-02	-1.113E+03	5.434E+02	1.050E+01
12	-2.841E-02	-2.405E+01	7.945E-01	6.660E-02	3.098E-01	-1.067E+03	-1.016E-01	-1.271E-02	-6.545E+02	4.580E+02	1.225E+01
13	-2.905E-02	-1.828E+01	9.110E-01	6.663E-02	-1.394E+00	-8.111E+02	-1.241E-01	-1.294E-02	-3.065E+02	3.530E+02	1.400E+01
14	-2.945E-02	-1.165E+01	1.028E+00	6.662E-02	-2.729E+00	-5.171E+02	-1.468E-01	-1.301E-02	-8.233E+01	2.280E+02	1.575E+01
15	-2.960E-02	1.471E-07	1.144E+00	6.660E-02	-1.145E-04	-6.542E-06	-1.696E-01	-1.302E-02	1.769E-06	1.120E-06	1.750E+01

NATURAL FREQUENCY = 36.6009 RADIANS/SECOND = .9845 X ROTATIONAL SPEED. GENERALIZED MASS = 1.0000

c. First Mode for IBC =4

UNIFORM BLADE (LUMPED PARAMETER MODEL) 03/23/72

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	-3.573E+01	0.	0.	2.264E+03	-1.724E+03	0.	-3.650E-01	0.	9.816E+03	2.500E-01
1	-1.847E-02	-3.573E+01	7.977E-03	2.366E-02	1.407E+03	-1.724E+03	-2.281E-01	-3.650E-01	-1.552E+02	9.816E+03	8.750E-01
2	-1.880E-02	-3.564E+01	3.785E-02	4.255E-02	7.232E+02	-1.719E+03	-5.471E-01	-3.636E-01	-3.620E+02	9.785E+03	1.750E+00
3	-1.942E-02	-3.557E+01	7.980E-02	5.227E-02	3.731E+02	-1.704E+03	-8.649E-01	-3.623E-01	-5.242E+02	9.698E+03	2.625E+00
4	-2.003E-02	-3.515E+01	1.280E-01	5.731E-02	1.936E+02	-1.680E+03	-1.182E+00	-3.611E-01	-6.454E+02	9.560E+03	3.500E+00
5	-2.063E-02	-3.454E+01	1.794E-01	5.993E-02	1.012E+02	-1.647E+03	-1.497E+00	-3.600E-01	-7.252E+02	9.371E+03	4.375E+00
6	-2.122E-02	-3.373E+01	2.325E-01	6.131E-02	5.345E+01	-1.605E+03	-1.812E+00	-3.589E-01	-7.755E+02	9.131E+03	5.250E+00
7	-2.179E-02	-3.272E+01	2.865E-01	6.205E-02	2.849E+01	-1.554E+03	-2.126E+00	-3.579E-01	-8.003E+02	8.840E+03	6.125E+00
8	-2.233E-02	-3.152E+01	3.410E-01	6.245E-02	1.525E+01	-1.494E+03	-2.439E+00	-3.569E-01	-7.955E+02	8.498E+03	7.000E+00
9	-2.286E-02	-3.011E+01	3.958E-01	6.267E-02	8.077E+00	-1.425E+03	-2.751E+00	-3.560E-01	-7.693E+02	8.106E+03	7.875E+00
10	-2.335E-02	-2.852E+01	4.506E-01	6.278E-02	4.045E+00	-1.347E+03	-3.062E+00	-3.551E-01	-7.257E+02	7.663E+03	8.750E+00
11	-2.425E-02	-2.575E+01	5.606E-01	6.286E-02	-8.030E-01	-1.213E+03	-3.681E+00	-3.516E-01	-6.152E+02	6.923E+03	1.050E+01
12	-2.499E-02	-2.137E+01	6.706E-01	6.285E-02	-2.932E+00	-1.009E+03	-4.295E+00	-3.487E-01	-4.248E+02	5.736E+03	1.225E+01
13	-2.556E-02	-1.621E+01	7.806E-01	6.277E-02	-4.332E+00	-7.653E+02	-4.904E+00	-3.464E-01	-2.206E+02	4.350E+03	1.400E+01
14	-2.592E-02	-1.028E+01	8.904E-01	6.267E-02	-4.723E+00	-4.869E+02	-5.510E+00	-3.450E-01	-5.269E+01	2.767E+03	1.575E+01
15	-2.604E-02	1.245E-05	1.000E+00	6.261E-02	-1.559E-01	-5.528E-04	-6.113E+00	-3.444E-01	2.671E-03	1.668E-03	1.750E+01

NATURAL FREQUENCY = 6.0565 RADIANS/SECOND = .1629 X ROTATIONAL SPEED. GENERALIZED MASS = 5.4100

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	-6.605E+00	0.	0.	4.186E+02	-3.187E+02	0.	-6.746E-02	0.	1.814E+03	2.500E-01
1	-3.414E-03	-6.605E+00	1.475E-03	4.373E-03	2.600E+02	-3.187E+02	-4.216E-02	-6.746E-02	-2.869E+01	1.814E+03	8.750E-01
2	-3.475E-03	-6.589E+00	6.997E-03	7.864E-03	1.337E+02	-3.177E+02	-1.011E-01	-6.721E-02	-6.692E+01	1.809E+03	1.750E+00
3	-3.589E-03	-6.574E+00	1.475E-02	9.662E-03	6.896E+01	-3.149E+02	-1.599E-01	-6.697E-02	-9.690E+01	1.793E+03	2.625E+00
4	-3.702E-03	-6.498E+00	2.366E-02	1.059E-02	3.578E+01	-3.105E+02	-2.184E-01	-6.675E-02	-1.193E+02	1.767E+03	3.500E+00
5	-3.813E-03	-6.385E+00	3.316E-02	1.108E-02	1.871E+01	-3.044E+02	-2.768E-01	-6.654E-02	-1.348E+02	1.732E+03	4.375E+00
6	-3.922E-03	-6.235E+00	4.298E-02	1.133E-02	9.879E+00	-2.966E+02	-3.349E-01	-6.634E-02	-1.441E+02	1.688E+03	5.250E+00
7	-4.027E-03	-6.049E+00	5.296E-02	1.147E-02	5.265E+00	-2.872E+02	-3.929E-01	-6.615E-02	-1.479E+02	1.634E+03	6.125E+00
8	-4.128E-03	-5.826E+00	6.303E-02	1.154E-02	2.820E+00	-2.762E+02	-4.508E-01	-6.597E-02	-1.470E+02	1.571E+03	7.000E+00
9	-4.225E-03	-5.567E+00	7.315E-02	1.158E-02	1.493E+00	-2.634E+02	-5.084E-01	-6.580E-02	-1.422E+02	1.498E+03	7.875E+00
10	-4.317E-03	-5.271E+00	8.330E-02	1.161E-02	7.477E-01	-2.491E+02	-5.660E-01	-6.564E-02	-1.341E+02	1.416E+03	8.750E+00
11	-4.482E-03	-4.759E+00	1.036E-01	1.162E-02	-1.484E-01	-2.251E+02	-6.805E-01	-6.499E-02	-1.137E+02	1.280E+03	1.050E+01
12	-4.620E-03	-3.950E+00	1.240E-01	1.162E-02	-5.419E-01	-1.865E+02	-7.939E-01	-6.445E-02	-7.852E+01	1.060E+03	1.225E+01
13	-4.724E-03	-2.997E+00	1.443E-01	1.160E-02	-8.045E-01	-1.415E+02	-9.065E-01	-6.403E-02	-4.078E+01	8.041E+02	1.400E+01
14	-4.790E-03	-1.900E+00	1.646E-01	1.158E-02	-8.729E-01	-8.999E+01	-1.018E+00	-6.376E-02	-1.159E+01	5.114E+02	1.575E+01
15	-4.813E-03	2.301E-06	1.848E-01	1.157E-02	-2.888E-02	-1.022E-04	-1.130E+00	-6.367E-02	4.937E-04	3.083E-04	1.750E+01

NATURAL FREQUENCY = 6.0565 RADIANS/SECOND = .1629 X ROTATIONAL SPEED. GENERALIZED MASS = 1.0000

Test Case 2

Model Parameters

BLADE FREQUENCY PROGRAM

BLADE MODEL AND OPERATING CONDITION PARAMETERS

FULLY ARTICULATED TEST ROTOR, FOR PROGRAM DOCUMENTATION 03/23/72

INFINITE STIFFNESS IN THE Y DIRECTION
INFINITE STIFFNESS IN THE Z DIRECTION

NUMBER OF SECTIONS = 16
ROTATIONAL SPEED CAP OMEGA = 50.0560 RADIANS/SEC
CONTROL ANGLE SI ZERO = 17.4000 DEGREES
CONTROL ANGLE THETA C = 0.0000 DEGREES
CONTROL OFFSET = 0.0000 FEET
RADIAL DISTANCE TO FIRST BLADE ELEMENT = .4580

I	LENGTH FEET	MASS LB-SEC2 PER-FT	EIX LB-FT2	EIY LB-FT2	EIZ LB-FT2	IX LB-SEC2-FEET	IY LB-SEC2-FEET	PHI DEGREES	EPSILON FEET	OFFSET FEET
1	8.9000E-01	3.6020E-01	8.3590E+02	1.0000E+07	2.7220E+04	2.5010E-02	2.5010E-02	0.	-2.8000E-04	0.
2	5.2600E-01	1.0370E-01	3.3680E+04	2.7780E+05	7.9860E+04	8.1390E-04	8.1390E-04	-4.0606E-03	-6.3133E-03	0.
3	4.1670E-01	6.2110E-02	1.8820E+04	2.7780E+05	2.7770E+04	5.9990E-04	5.9990E-04	-5.0071E-03	-2.3000E-02	1.2500E-03
4	8.3300E-01	4.3500E-02	6.5280E+03	2.7100E+05	8.6800E+03	1.1400E-03	1.1400E-03	-1.0036E-02	-2.9697E-02	-8.0000E-03
5	3.7500E-01	1.6100E-02	5.3500E+03	2.6200E+05	6.5600E+03	5.0000E-04	5.0000E-04	-4.5204E-03	2.7004E-02	-2.5000E-03
6	1.3330E+00	1.6100E-01	5.3500E+03	2.6200E+05	6.5600E+03	2.2000E-03	2.2000E-03	-1.6057E-02	1.5204E-02	0.
7	1.3330E+00	5.6100E-02	5.2800E+03	2.6100E+05	6.3900E+03	1.7200E-03	1.7200E-03	-1.6057E-02	-3.7096E-02	-8.3000E-04
8	1.3330E+00	5.3200E-02	5.0700E+03	2.5800E+05	6.0600E+03	1.7100E-03	1.7100E-03	-1.6057E-02	-3.7798E-02	-2.5000E-03
9	2.5000E-01	9.7800E-03	5.2800E+03	2.5300E+05	5.9500E+03	3.2000E-04	3.2000E-04	-3.0194E-03	-3.5700E-02	-3.3000E-04
10	4.5800E-01	1.8800E-02	5.2100E+03	2.5100E+05	6.5000E+03	5.7000E-04	5.7000E-04	-5.5152E-03	2.0600E-02	6.3300E-03
11	1.0830E+00	1.2900E-01	5.1400E+03	2.2200E+05	6.4300E+03	1.5800E-03	1.5800E-03	-1.3055E-02	2.1998E-02	3.0000E-03
12	4.5800E-01	1.7900E-02	5.0000E+03	1.9700E+05	6.4000E+03	4.7500E-04	4.7500E-04	-5.5152E-03	-2.7402E-02	2.6700E-03
13	5.8300E-01	1.8900E-02	4.5800E+03	1.7400E+05	5.9000E+03	5.5600E-04	5.5600E-04	-7.0162E-03	-2.8098E-02	-2.0800E-03
14	1.1670E+00	3.7000E-02	4.5100E+03	1.5300E+05	5.8500E+03	1.0100E-03	1.0100E-03	-1.4067E-02	-2.7698E-02	8.3300E-05
15	1.1670E+00	3.4800E-02	4.5100E+03	1.5300E+05	5.7800E+03	1.0100E-03	1.0100E-03	-1.4067E-02	1.2912E-01	-5.0000E-04
16	4.5800E-01	1.0800E-01	5.5600E+03	2.3900E+05	1.5070E+04	9.9000E-04	9.9000E-04	-5.5152E-03	6.3328E-02	-8.0000E-03

a. Program Control and Frequency Search Data for Uncoupled Torsion

PROGRAM CONTROL PARAMETERS

MODE COUPLING, NTYPE = 1	HUB BOUNDRY CONDITION, IBC = 1	FREQUENCY SEARCH, NFIND = 1
BLADE PROPERTY DECK, ITAB = 0	TORSION MODE SWITCH, TMSL = 0.0	NO. OF FREQUENCIES, NUMF = 3
PUNCHED OUTPUT, NPCH = 0	NO. OF KNOWN FREQUENCIES, NPDF = 0	NO. OF ITERATIONS, NMAX = 30
REMAINDER INDEX, NOOR = 0	CONVERGENCE LIMIT, NEXPO = 12	
FREQUENCY LIMIT, OMLIMT = 1.600E+05	STEP SIZE FACTOR, PRCNT = 1.000000	

SEARCH FOR LOWEST FREQUENCY BEGINS

ITERATIONS NUMBER	FREQUENCY (RAD/SEC)	DETERMINANT	AUXILIARY FUNCTION	REMAINDERS
1	1.00100000000E+00	1.13842E+00	1.13842E+00	
2	1.32110996261 3E+02	1.92539E-01	1.13842E+00	
3	1.449338872241E+02	4.45032E-02	1.92539E-01	
4	1.465775685630E+02	2.76209E-03	4.45032E-02	
5	1.488102088056E+02	4.44299E-05	2.75209E-03	
6	1.488140907323E+02	4.56665E-08		

a. First Uncoupled Torsional Mode Shape

FULLY ARTICULATED TEST ROTOR, FOR PROGRAM DOCUMENTATION 03/23/72

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	6.258E+02	0.	0.	0.	0.	0.	0.	0.	0.	4.580E-01
1	5.663E-01	6.258E+02	0.	0.	0.	0.	0.	0.	0.	0.	1.348E+00
2	6.709E-01	2.911E+02	0.	0.	0.	0.	0.	0.	0.	0.	1.874E+00
3	6.771E-01	2.801E+02	0.	0.	0.	0.	0.	0.	0.	0.	2.291E+00
4	7.117E-01	2.713E+02	0.	0.	0.	0.	0.	0.	0.	0.	3.124E+00
5	7.295E-01	2.546E+02	0.	0.	0.	0.	0.	0.	0.	0.	3.499E+00
6	7.909E-01	2.464E+02	0.	0.	0.	0.	0.	0.	0.	0.	4.832E+00
7	8.442E-01	2.112E+02	0.	0.	0.	0.	0.	0.	0.	0.	6.165E+00
8	8.918E-01	1.809E+02	0.	0.	0.	0.	0.	0.	0.	0.	7.498E+00
9	8.989E-01	1.497E+02	0.	0.	0.	0.	0.	0.	0.	0.	7.748E+00
10	9.115E-01	1.439E+02	0.	0.	0.	0.	0.	0.	0.	0.	8.206E+00
11	9.395E-01	1.330E+02	0.	0.	0.	0.	0.	0.	0.	0.	9.289E+00
12	9.490E-01	1.028E+02	0.	0.	0.	0.	0.	0.	0.	0.	9.747E+00
13	9.609E-01	9.354E+01	0.	0.	0.	0.	0.	0.	0.	0.	1.033E+01
14	9.822E-01	8.253E+01	0.	0.	0.	0.	0.	0.	0.	0.	1.150E+01
15	9.983E-01	6.233E+01	0.	0.	0.	0.	0.	0.	0.	0.	1.266E+01
16	1.000E+00	4.738E-10	0.	0.	0.	0.	0.	0.	0.	0.	1.312E+01

NATURAL FREQUENCY = 148.8141 RADIANS/SECOND = 2.9730 X ROTATIONAL SPEED. GENERALIZED MASS = .1559

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	4.015E+03	0.	0.	0.	0.	0.	0.	0.	0.	4.580E-01
1	4.275E+00	4.015E+03	0.	0.	0.	0.	0.	0.	0.	0.	1.348E+00
2	4.304E+00	1.867E+03	0.	0.	0.	0.	0.	0.	0.	0.	1.874E+00
3	4.344E+00	1.797E+03	0.	0.	0.	0.	0.	0.	0.	0.	2.291E+00
4	4.565E+00	1.741E+03	0.	0.	0.	0.	0.	0.	0.	0.	3.124E+00
5	4.681E+00	1.634E+03	0.	0.	0.	0.	0.	0.	0.	0.	3.499E+00
6	5.075E+00	1.531E+03	0.	0.	0.	0.	0.	0.	0.	0.	4.832E+00
7	5.417E+00	1.355E+03	0.	0.	0.	0.	0.	0.	0.	0.	6.165E+00
8	5.722E+00	1.161E+03	0.	0.	0.	0.	0.	0.	0.	0.	7.498E+00
9	5.767E+00	9.607E+02	0.	0.	0.	0.	0.	0.	0.	0.	7.748E+00
10	5.848E+00	9.234E+02	0.	0.	0.	0.	0.	0.	0.	0.	8.206E+00
11	6.028E+00	8.532E+02	0.	0.	0.	0.	0.	0.	0.	0.	9.289E+00
12	6.089E+00	6.596E+02	0.	0.	0.	0.	0.	0.	0.	0.	9.747E+00
13	6.165E+00	6.002E+02	0.	0.	0.	0.	0.	0.	0.	0.	1.033E+01
14	6.302E+00	5.295E+02	0.	0.	0.	0.	0.	0.	0.	0.	1.150E+01
15	6.406E+00	3.999E+02	0.	0.	0.	0.	0.	0.	0.	0.	1.266E+01
16	6.416E+00	3.049E-09	0.	0.	0.	0.	0.	0.	0.	0.	1.312E+01

NATURAL FREQUENCY = 148.8141 RADIANS/SECOND = 2.9730 X ROTATIONAL SPEED. GENERALIZED MASS = 1.0000

b. Third Partially Coupled Torsion-Flatwise Mode Shape

FULLY ARTICULATED TEST ROTOR, FOR PROGRAM DOCUMENTATION 03/23/72

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	7.844E+03	0.	-2.836E-02	0.	4.400E+02	0.	0.	0.	0.	4.580E-01
1	8.352E+00	7.844E+03	-2.518E-02	-2.814E-02	1.420E+01	4.400E+02	0.	0.	0.	0.	1.348E+00
2	8.408E+00	3.578E+03	-3.597E-02	-2.817E-02	-3.253E+01	3.197E+02	0.	0.	0.	0.	1.874E+00
3	8.484E+00	3.438E+03	-6.229E-02	-2.855E-02	-8.193E+01	3.246E+02	0.	0.	0.	0.	2.291E+00
4	8.909E+00	3.331E+03	-1.710E-02	-2.332E-02	-9.041E+01	5.650E+02	0.	0.	0.	0.	3.124E+00
5	9.128E+00	3.121E+03	-3.293E-03	-2.024E-02	6.881E+01	7.473E+02	0.	0.	0.	0.	3.499E+00
6	9.879E+00	3.015E+03	-5.263E-02	-6.306E-02	6.888E+02	2.460E+02	0.	0.	0.	0.	4.332E+00
7	1.053E+01	2.567E+03	-1.082E-01	-6.645E-02	2.587E+02	-2.323E+02	0.	0.	0.	0.	6.165E+00
8	1.110E+01	2.187E+03	-1.008E-01	3.812E-02	-4.310E+02	1.493E+02	0.	0.	0.	0.	7.498E+00
9	1.119E+01	1.795E+03	-8.323E-02	3.429E-02	-7.731E+01	3.789E+02	0.	0.	0.	0.	7.748E+00
10	1.134E+01	1.720E+03	-1.408E-01	4.970E-02	3.530E+02	4.566E+02	0.	0.	0.	0.	8.206E+00
11	1.167E+01	1.576E+03	-1.152E-01	6.677E-02	7.610E+02	-2.254E+02	0.	0.	0.	0.	9.289E+00
12	1.178E+01	1.191E+03	-1.116E-01	8.267E-02	3.316E+02	-8.941E+02	0.	0.	0.	0.	9.747E+00
13	1.192E+01	1.074E+03	-2.658E-02	1.230E-01	2.807E+02	-7.961E+02	0.	0.	0.	0.	1.033E+01
14	1.216E+01	9.352E+02	1.842E-01	2.520E-01	5.966E+02	-6.022E+02	0.	0.	0.	0.	1.150E+01
15	1.233E+01	6.765E+02	6.452E-01	5.819E-01	2.207E+03	-1.754E+02	0.	0.	0.	0.	1.266E+01
16	1.236E+01	1.241E-09	1.000E+00	5.356E-01	-4.691E-07	2.832E-08	0.	0.	0.	0.	1.312E+01

NATURAL FREQUENCY = 149.9353 RADIANS/SECOND = 2.9954 X ROTATIONAL SPEED. GENERALIZED MASS = 1.9087

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	4.110E+03	0.	-1.486E-02	0.	2.305E+02	0.	0.	0.	0.	4.580E-01
1	4.376E+00	4.110E+03	-1.319E-02	-1.474E-02	7.439E+00	2.305E+02	0.	0.	0.	0.	1.348E+00
2	4.405E+00	1.875E+03	-2.094E-02	-1.476E-02	-1.704E+01	1.675E+02	0.	0.	0.	0.	1.874E+00
3	4.445E+00	1.801E+03	-3.263E-02	-1.496E-02	-4.292E+01	1.701E+02	0.	0.	0.	0.	2.291E+00
4	4.668E+00	1.745E+03	-8.958E-03	-1.222E-02	-4.737E+01	2.960E+02	0.	0.	0.	0.	3.124E+00
5	4.782E+00	1.635E+03	-1.725E-03	-1.060E-02	3.605E+01	3.915E+02	0.	0.	0.	0.	3.499E+00
6	5.176E+00	1.580E+03	-2.757E-02	-3.304E-02	3.609E+02	1.289E+02	0.	0.	0.	0.	4.832E+00
7	5.515E+00	1.345E+03	-5.667E-02	-3.481E-02	1.355E+02	-1.217E+02	0.	0.	0.	0.	6.165E+00
8	5.817E+00	1.146E+03	-5.280E-02	1.997E-02	-2.258E+02	7.822E+01	0.	0.	0.	0.	7.498E+00
9	5.861E+00	9.407E+02	-4.622E-02	1.796E-02	-4.050E+01	1.985E+02	0.	0.	0.	0.	7.748E+00
10	5.940E+00	9.013E+02	-7.378E-02	2.604E-02	1.850E+02	2.392E+02	0.	0.	0.	0.	8.206E+00
11	6.114E+00	8.256E+02	-6.034E-02	3.498E-02	3.987E+02	-1.181E+02	0.	0.	0.	0.	9.289E+00
12	6.171E+00	6.240E+02	-5.847E-02	4.331E-02	1.737E+02	-4.684E+02	0.	0.	0.	0.	9.747E+00
13	6.243E+00	5.625E+02	-1.393E-02	6.447E-02	1.471E+02	-4.171E+02	0.	0.	0.	0.	1.033E+01
14	6.370E+00	4.900E+02	9.649E-02	1.320E-01	3.126E+02	-3.155E+02	0.	0.	0.	0.	1.150E+01
15	6.462E+00	3.545E+02	3.380E-01	3.048E-01	1.156E+03	-9.192E+01	0.	0.	0.	0.	1.266E+01
16	6.474E+00	-1.021E-08	5.239E-01	2.806E-01	-7.025E-07	-2.358E-07	0.	0.	0.	0.	1.312E+01

NATURAL FREQUENCY = 149.9353 RADIANS/SECOND = 2.9954 X ROTATIONAL SPEED. GENERALIZED MASS = 1.0000

Test Case 3

Model Parameters For Model With Nearly Coincident Frequencies

BLADE FREQUENCY PROGRAM

BLADE MODEL AND OPERATING CONDITION PARAMETERS

TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED 03/23/72

NUMBER OF SECTIONS = 18
ROTATIONAL SPEED CAP OMEGA = 20.0000 RADIANS/SEC
CONTROL ANGLE SI ZERO = 13.6000 DEGREES
CONTROL ANGLE THETA C = 0.0000 DEGREES
CONTROL OFFSET = 0.0000 FEET
RADIAL DISTANCE TO FIRST BLADE ELEMENT = 1.0000

I	LENGTH FEET	MASS LB-SEC2 PER-FT	EIX LB-FT2	EIY LB-FT2	EIZ LB-FT2	IX LB-SEC2-FEET	IY LB-SEC2-FEET	PHI DEGREES	EPSILON FEET	OFFSET FEET
1	1.0000E-02	2.1000E-02	7.7000E+01	6.4600E+06	5.4170E+05	6.9900E-02	6.9900E-02	0.	0.	0.
2	1.0730E+00	2.1990E+00	8.2640E+05	3.3940E+06	3.4720E+06	6.4800E-02	6.4800E-02	0.	0.	0.
3	8.3300E-01	9.9400E-01	3.1400E+05	1.6460E+06	4.9100E+05	4.3800E-02	4.3800E-02	0.	4.2556E-03	0.
4	1.6670E+00	1.6500E-01	1.4580E+05	1.1930E+06	1.1870E+05	2.2000E-02	2.2000E-02	-4.8869E-04	1.3670E-02	6.8350E-03
5	2.0000E+00	2.6400E-01	1.2150E+05	1.1070E+06	1.0210E+05	2.3300E-02	2.3300E-02	-1.1886E-02	1.3670E-02	0.
6	9.1700E-01	1.6600E-01	1.2150E+05	1.1070E+06	1.0210E+05	2.9800E-02	2.9800E-02	-5.4454E-03	1.3670E-02	0.
7	2.5830E+00	4.2600E-01	1.2150E+05	1.1070E+06	1.0210E+05	3.4700E-02	3.4700E-02	-1.5359E-02	1.3670E-02	0.
8	1.5000E+00	2.7200E-01	1.2150E+05	1.1070E+06	1.0210E+05	3.5300E-02	3.5300E-02	-8.9012E-03	1.3670E-02	0.
9	2.4170E+00	3.9800E-01	1.2150E+05	1.1070E+06	1.0210E+05	3.1900E-02	3.1900E-02	-1.4364E-02	1.3670E-02	0.
10	1.3330E+00	2.4200E-01	1.2150E+05	1.1070E+06	1.0210E+05	3.3000E-02	3.3000E-02	-7.9238E-03	1.3670E-02	0.
11	2.3330E+00	3.6900E-01	1.2150E+05	1.1070E+06	1.0210E+05	4.1100E-02	4.1100E-02	-1.3858E-02	1.3670E-02	0.
12	2.5000E+00	3.9600E-01	1.2150E+05	1.1070E+06	1.0210E+05	2.8300E-02	2.8300E-02	-1.4870E-02	1.3670E-02	0.
13	8.3300E-01	1.5100E-01	1.2150E+05	1.1070E+06	1.0210E+05	2.5500E-02	2.5500E-02	-4.9393E-03	1.3670E-02	0.
14	2.0000E+00	3.3000E-01	1.2150E+05	1.1070E+06	1.0210E+05	3.4700E-02	3.4700E-02	-1.1886E-02	1.3670E-02	0.
15	2.0830E+00	4.1300E-01	1.2150E+05	1.1070E+06	1.0210E+05	2.9300E-02	2.9300E-02	-1.2374E-02	1.3670E-02	0.
16	1.1670E+00	2.0000E-01	1.2150E+05	1.1070E+05	1.0210E+05	1.7000E-02	1.7000E-02	-6.9290E-03	1.3670E-02	0.
17	8.3300E-01	2.0600E-01	1.2150E+05	1.1070E+06	1.0210E+05	2.4500E-02	2.4500E-02	-4.9567E-03	1.3670E-02	0.
18	9.1700E-01	1.5100E-01	1.2150E+05	1.1070E+06	1.0210E+05	5.5000E-03	5.5000E-03	-5.4454E-03	1.3670E-02	0.

a. Program Control and Frequency Search Data for Smaller of Nearly Coincident Frequencies

PROGRAM CONTROL PARAMETERS

MODE COUPLING, NTYPE = 6
BLADE PROPERTY DECK, ITAB = 0
PUNCHED OUTPUT, NPCH = 0
REMAINDER INDEX, NCOR = 2
FREQUENCY LIMIT, CMLIMT = 9.900E+05

HUB BOUNDRY CONDITICN, IBC = 1
TORSION MODE SWITCH, TMSL = 1.0
NO. OF KNOWN FREQUENCIES, NPDF = 4
CONVERGENCE LIMIT, NEXPO = 11
STEP SIZE FACTOR, PRCNT = 1.000000

FREQUENCY SEARCH, NFIND = 1
NC. CF FREQUENCIES, NUMF = 6
NO. OF ITERATIONS, NMAX = 30

PREVIOUSLY DETERMINED FREQUENCIES, OMINT(I) = 4.942800 20.553000 53.957340 70.666500

SEARCH FOR LOWEST FREQUENCY BEGINS

ITERATIONS NUMBER	FREQUENCY (RAD/SEC)	DETERMINANT	AUXILIARY FUNCTION	REMAINDERS 1	2	3	4
1	1.00100000000CE+00	5.65779E+13	3.94346E+02	1.22511E-04	-1.09524E-01	-9.92635E-03	4.79874E+00
2	5.397026978372E+01	-3.52744E+12	3.94346E+02	1.88004E-07	-2.94568E-06	2.51736E-08	1.64572E-02
3	7.136956356966E+01	2.28117E+14	1.68872E+02	3.56916E-08	-1.20753E-03	1.18141E-04	3.75293E-01
4	7.659617226193E+01	2.27493E+15	4.42080E+01	2.29552E-08	-6.86137E-04	1.43974E-05	1.52871E+00
5	8.465179070995CE+01	4.68969E+15	2.77102E+01	1.04455E-08	-3.17336E-04	4.98409E-06	1.27265E+00
6	8.923535902101E+01	4.28660E+15	1.05372E+01	5.94187E-09	-1.57514E-04	1.63753E-06	8.08158E-01
7	9.286144089834E+01	2.86659E+15	4.77429E+00	3.31673E-09	-6.90357E-05	4.99566E-07	4.32528E-01
8	9.530714150835E+01	1.60680E+15	1.96086E+00	1.91587E-09	-2.97761E-05	1.69352E-07	2.15635E-01
9	9.699613790893E+01	7.84394E+14	8.11316E-01	1.09380E-09	-1.22424E-05	5.92705E-08	9.87112E-02
10	9.81049 429823E+01	3.49210E+14	3.24349E-01	6.09931E-10	-4.88768E-06	2.13500E-08	4.24662E-02
11	9.881337134424E+01	1.43157E+14	1.27154E-01	3.20183E-10	-1.87174E-06	7.66576E-09	1.70922E-02
12	9.924237669848E+C1	5.34480E+13	4.81321E-02	1.49739E-10	-6.70950E-07	2.64403E-09	6.31975E-03
13	9.947868944058E+C1	1.69807E+13	1.71329E-02	5.61633E-11	-2.08474E-07	8.04416E-10	1.99803E-03
14	9.958443700083E+01	3.86155E+12	5.30292E-03	1.39542E-11	-4.69411E-08	1.79434E-10	4.53428E-04
15	9.961507753471E+01	4.42516E+11	1.19197E-03	1.64401E-12	-5.36383E-09	2.04479E-11	5.19303E-05
SUBSEQUENT ITERATIONS USE REMAINDER NUMBER 2							
16	9.961902751114E+01	1.44118E+10		5.37343E-14	-1.74615E-10	6.65427E-13	1.69104E-06
17	9.961916042315E+01	5.59805E+07		2.09644E-16	-6.81177E-13	2.59576E-15	6.59674E-09

a. Mode Shape of Smaller of Nearly Coincident Frequencies

TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED 03/23/72

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	-2.700E+04	0.	1.642E-01	0.	-8.063E+03	0.	0.	9.287E+00	-9.287E+02	1.000E+00
1	-3.506E+00	-2.700E+04	1.642E-03	1.642E-01	-3.257E+01	-8.063E+03	4.792E-11	1.304E-02	0.	-9.287E+02	1.010E+00
2	-3.538E+00	-2.465E+04	1.777E-01	1.637E-01	-3.593E+03	-8.045E+03	1.393E-02	1.287E-02	-6.026E+02	-9.289E+02	2.083E+00
3	-3.598E+00	-2.246E+04	3.113E-01	1.570E-01	-4.273E+03	-5.225E+03	2.449E-02	1.242E-02	-9.206E+02	-7.256E+02	2.916E+00
4	-3.837E+00	-2.091E+04	5.519E-01	1.031E-01	-3.974E+03	-3.203E+03	4.345E-02	1.037E-02	-1.366E+03	-5.965E+02	4.583E+00
5	-4.168E+00	-2.008E+04	6.857E-01	2.899E-02	-4.553E+03	-1.920E+03	5.293E-02	6.580E-03	-1.795E+03	-4.895E+02	6.583E+00
6	-4.312E+00	-1.913E+04	6.942E-01	-1.130E-02	-4.742E+03	-3.353E+02	5.445E-02	5.003E-03	-1.969E+03	-3.843E+02	7.500E+00
7	-4.692E+00	-1.787E+04	5.394E-01	-1.052E-01	-4.333E+03	1.876E+03	5.247E-02	1.185E-03	-2.203E+03	-2.660E+02	1.008E+01
8	-4.893E+00	-1.629E+04	3.416E-01	-1.560E-01	-3.342E+03	3.967E+03	4.891E-02	-5.324E-04	-2.277E+03	-1.293E+02	1.158E+01
9	-5.184E+00	-1.462E+04	-9.095E-02	-1.896E-01	-5.847E+02	5.326E+03	4.293E-02	-2.798E-03	-2.235E+03	-4.732E+01	1.400E+01
10	-5.327E+00	-1.304E+04	-3.419E-01	-1.832E-01	1.143E+03	5.262E+03	4.015E-02	-3.977E-03	-2.195E+03	5.311E+01	1.533E+01
11	-5.546E+00	-1.138E+04	-7.083E-01	-1.177E-01	4.293E+03	4.446E+03	3.561E-02	-6.584E-03	-2.000E+03	1.236E+02	1.767E+01
12	-5.736E+00	-9.234E+03	-8.477E-01	1.744E-02	6.555E+03	2.046E+03	2.673E-02	-1.063E-02	-1.667E+03	2.488E+02	2.017E+01
13	-5.789E+00	-7.714E+03	-8.099E-01	7.376E-02	7.010E+03	-4.229E+01	2.138E-02	-1.215E-02	-1.538E+03	3.364E+02	2.100E+01
14	-5.893E+00	-6.330E+03	-5.293E-01	2.055E-01	6.374E+03	-1.783E+03	1.061E-03	-1.667E-02	-1.095E+03	4.203E+02	2.300E+01
15	-5.968E+00	-4.403E+03	1.662E-02	3.094E-01	3.189E+03	-3.433E+03	-3.543E-02	-2.172E-02	-5.124E+02	4.753E+02	2.508E+01
16	-5.995E+00	-2.732E+03	3.961E-01	3.382E-01	1.355E+03	-3.131E+03	-6.375E-02	-2.438E-02	-2.196E+02	3.836E+02	2.625E+01
17	-6.006E+00	-1.747E+03	6.814E-01	3.464E-01	3.800E+02	-2.166E+03	-8.743E-02	-2.618E-02	-6.969E+01	2.566E+02	2.708E+01
18	-6.009E+00	-1.078E-07	1.000E+00	3.489E-01	-2.466E-05	-7.900E-06	-1.170E-01	-2.811E-02	1.478E-06	4.295E-07	2.800E+01

NATURAL FREQUENCY = 99.6192 RADIANS/SECOND = 4.9810 X ROTATIONAL SPEED. GENERALIZED MASS = 3.9201

0	0.	-6.888E+03	0.	4.190E-02	0.	-2.057E+03	0.	0.	2.369E+00	-2.369E+02	1.000E+00
1	-8.945E-01	-6.888E+03	4.190E-04	4.190E-02	-8.308E+00	-2.057E+03	1.222E-11	3.327E-03	0.	-2.369E+02	1.010E+00
2	-9.026E-01	-6.289E+03	4.532E-02	4.176E-02	-9.165E+02	-2.052E+03	3.554E-03	3.284E-03	-1.537E+02	-2.369E+02	2.083E+00
3	-9.178E-01	-5.730E+03	7.942E-02	4.006E-02	-1.090E+03	-1.333E+03	6.247E-03	3.167E-03	-2.348E+02	-1.851E+02	2.916E+00
4	-9.788E-01	-5.334E+03	1.408E-01	2.631E-02	-1.014E+03	-8.170E+02	1.108E-02	2.644E-03	-3.485E+02	-1.522E+02	4.583E+00
5	-1.063E+00	-5.123E+03	1.749E-01	7.396E-03	-1.162E+03	-4.898E+02	1.350E-02	1.679E-03	-4.578E+02	-1.249E+02	6.583E+00
6	-1.100E+00	-4.881E+03	1.771E-01	-2.882E-03	-1.210E+03	-8.553E+01	1.389E-02	1.276E-03	-5.023E+02	-9.804E+01	7.500E+00
7	-1.197E+00	-4.560E+03	1.376E-01	-2.684E-02	-1.105E+03	4.785E+02	1.338E-02	3.023E-04	-5.621E+02	-6.786E+01	1.008E+01
8	-1.248E+00	-4.155E+03	8.715E-02	-3.978E-02	-8.525E+02	1.012E+03	1.248E-02	-1.358E-04	-5.807E+02	-3.299E+01	1.158E+01
9	-1.322E+00	-3.730E+03	-2.320E-02	-4.837E-02	-1.491E+02	1.359E+03	1.095E-02	-7.139E-04	-5.702E+02	-1.207E+01	1.400E+01
10	-1.359E+00	-3.327E+03	-8.722E-02	-4.673E-02	2.916E+02	1.342E+03	1.024E-02	-1.015E-03	-5.600E+02	1.355E+01	1.533E+01
11	-1.415E+00	-2.902E+03	-1.807E-01	-3.002E-02	1.095E+03	1.134E+03	9.083E-03	-1.680E-03	-5.102E+02	3.152E+01	1.767E+01
12	-1.463E+00	-2.356E+03	-2.163E-01	4.449E-03	1.672E+03	5.220E+02	6.818E-03	-2.713E-03	-4.252E+02	6.346E+01	2.017E+01
13	-1.477E+00	-1.968E+03	-2.066E-01	1.882E-02	1.788E+03	-1.079E+01	5.453E-03	-3.099E-03	-3.924E+02	6.581E+01	2.100E+01
14	-1.503E+00	-1.615E+03	-1.350E-01	5.241E-02	1.626E+03	-4.549E+02	2.706E-04	-4.252E-03	-2.793E+02	1.072E+02	2.300E+01
15	-1.522E+00	-1.123E+03	4.239E-03	7.893E-02	8.136E+02	-8.757E+02	-9.037E-03	-5.540E-03	-1.307E+02	1.212E+02	2.508E+01
16	-1.529E+00	-6.969E+02	1.010E-01	8.626E-02	3.456E+02	-7.987E+02	-1.626E-02	-6.219E-03	-5.602E+01	9.785E+01	2.625E+01
17	-1.532E+00	-4.457E+02	1.738E-01	8.836E-02	9.694E+01	-5.525E+02	-2.232E-02	-6.680E-03	-1.778E+01	6.546E+01	2.708E+01
18	-1.533E+00	-4.029E-09	2.551E-01	8.901E-02	-8.339E-07	-2.972E-07	-2.983E-02	-7.170E-03	5.507E-08	1.604E-08	2.800E+01

NATURAL FREQUENCY = 99.6192 RADIANS/SECOND = 4.9810 X ROTATIONAL SPEED. GENERALIZED MASS = 1.0000

b. Frequency Search Data for Larger of Nearly Coincident Frequencies

SEARCH FOR NEXT FREQUENCY BEGINS

ITERATIONS NUMBER	FREQUENCY (RAD/SEC)	DETERMINANT	AUXILIARY FUNCTION	REMAINDERS 1	2	3	4
1	1.001000000000E+00	5.65779E+13	-3.97407E-02	1.22511E-04	-1.09524E-01	-9.92635E-03	4.79874E+00
2	6.420337888065E+01	-1.18279E+15	-3.97407E-02	6.23988E-08	-8.99775E-04	9.92017E-06	6.76775E+01
3	7.789185258510E+01	2.80654E+15	-1.27426E-02	2.04423E-08	-6.22589E-04	1.30305E-05	1.58351E+00
4	8.926375038094E+01	4.27892E+15	-6.29809E-03	5.91842E-09	-1.56684E-04	1.62436E-06	8.05125E-01
5	9.570570059383E+01	1.40240E+15	-2.42675E-03	1.71192E-09	-2.49401E-05	1.36496E-07	1.85135E-01
6	9.920313618590E+01	6.04271E+13	-8.83103E-04	1.65254E-10	-7.61376E-07	3.01092E-09	7.15098E-03
7	1.004426339954E+02	-2.12425E+13	-2.35199E-04	-5.30075E-10	2.38473E-07	-8.45433E-10	-2.45903E-03
8	1.006667975688E+02	-1.47723E+12	-3.62447E-05	4.80976E-11	1.62469E-08	-5.64903E-11	-1.70503E-04

SUBSEQUENT ITERATIONS USE REMAINDER NUMBER 3

9	1.006794021559E+02	-1.34189E+10		3.85354E-13	1.47361E-10	-5.11822E-13	-1.54803E-06

b. Mode Shape of Larger of Nearly Coincident Frequencies

TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED 03/23/72

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	2.738E+03	0.	1.611E-01	0.	-7.824E+03	0.	0.	7.947E+00	-7.947E+02	1.000E+00
1	3.555E-01	2.738E+03	1.611E-03	1.611E-01	-3.110E+01	-7.824E+03	4.101E-11	1.119E-02	0.	-7.947E+02	1.010E+00
2	3.588E-01	2.494E+03	1.743E-01	1.606E-01	-3.432E+03	-7.806E+03	1.195E-02	1.104E-02	-5.150E+02	-7.949E+02	2.083E+00
3	3.648E-01	2.267E+03	3.055E-01	1.543E-01	-3.975E+03	-4.981E+03	2.101E-02	1.065E-02	-7.857E+02	-6.197E+02	2.916E+00
4	3.894E-01	2.153E+03	5.175E-01	1.039E-01	-3.558E+03	-3.208E+03	3.726E-02	8.897E-03	-1.159E+03	-5.061E+02	4.583E+00
5	4.237E-01	2.084E+03	6.589E-01	3.548E-02	-4.017E+03	-2.097E+03	4.459E-02	5.561E-03	-1.507E+03	-4.072E+02	6.583E+00
6	4.386E-01	2.007E+03	6.751E-01	-8.662E-04	-4.220E+03	-6.739E+02	4.540E-02	4.192E-03	-1.646E+03	-3.149E+02	7.500E+00
7	4.794E-01	1.907E+03	5.559E-01	-9.015E-02	-3.736E+03	1.333E+03	4.218E-02	1.058E-03	-1.811E+03	-2.109E+02	1.008E+01
8	5.012E-01	1.771E+03	3.845E-01	-1.368E-01	-2.851E+03	3.278E+03	3.846E-02	-2.708E-04	-1.855E+03	-1.005E+02	1.158E+01
9	5.334E-01	1.615E+03	1.659E-03	-1.704E-01	-2.259E+02	4.561E+03	3.290E-02	-1.885E-03	-1.804E+03	-4.005E+01	1.400E+01
10	5.493E-01	1.449E+03	-2.231E-01	-1.639E-01	1.289E+03	4.542E+03	3.073E-02	-2.713E-03	-1.769E+03	3.488E+01	1.533E+01
11	5.735E-01	1.262E+03	-5.504E-01	-1.061E-01	4.238E+03	3.828E+03	2.791E-02	-4.686E-03	-1.620E+03	8.541E+01	1.767E+01
12	5.941E-01	1.003E+03	-6.756E-01	1.440E-02	6.478E+03	1.664E+03	2.214E-02	-8.012E-03	-1.368E+03	1.864E+02	2.017E+01
13	5.997E-01	8.130E+02	-6.414E-01	6.773E-02	6.736E+03	-2.318E+02	1.823E-02	-9.295E-03	-1.271E+03	2.620E+02	2.100E+01
14	6.103E-01	6.422E+02	-3.833E-01	1.880E-01	6.062E+03	-1.813E+03	2.270E-03	-1.327E-02	-9.189E+02	3.383E+02	2.300E+01
15	6.174E-01	4.156E+02	1.130E-01	2.789E-01	3.213E+03	-3.284E+03	-2.809E-02	-1.776E-02	-4.355E+02	3.952E+02	2.508E+01
16	6.198E-01	2.437E+ 2	4.554E-01	3.049E-01	1.446E+03	-2.957E+03	-5.217E-02	-2.015E-02	-1.881E+02	3.235E+02	2.625E+01
17	6.208E-01	1.537E+02	7.128E-01	3.125E-01	4.629E+02	-2.035E+03	-7.253E-02	-2.178E-02	-6.019E+01	2.182E+02	2.708E+01
18	6.210E-01	2.189E-06	1.000E+00	3.143E-01	-5.236E-04	1.611E-04	-9.796E-02	-2.351E-02	-2.985E-05	-8.728E-06	2.800E+01

NATURAL FREQUENCY = 100.6795 RADIANS/SECOND = 5.0340 X ROTATIONAL SPEED. GENERALIZED MASS = 1.1479

I	ANGLE PHI	TORQUE T	DISPL V	ANGLE THETA	MOMENT M Z	SHEAR -VY	DISPL -W	ANGLE PSI	MOMENT M Y	SHEAR V Z	RADIAL COORD.
0	0.	2.385E+03	0.	1.404E-01	0.	-6.816E+03	0.	0.	6.923E+00	-6.923E+02	1.000E+00
1	3.097E-01	2.385E+03	1.404E-03	1.404E-01	-2.709E+01	-6.816E+03	3.572E-11	9.745E-03	0.	-6.923E+02	1.010E+00
2	3.125E-01	2.173E+03	1.518E-01	1.399E-01	-2.990E+03	-6.800E+03	1.041E-02	9.620E-03	-4.487E+02	-6.925E+02	2.083E+00
3	3.178E-01	1.975E+03	2.661E-01	1.344E-01	-3.463E+03	-4.339E+03	1.830E-02	9.279E-03	-6.845E+02	-5.399E+02	2.916E+00
4	3.392E-01	1.875E+03	4.508E-01	9.052E-02	-3.100E+03	-2.794E+03	3.246E-02	7.751E-03	-1.010E+03	-4.409E+02	4.583E+00
5	3.691E-01	1.815E+03	5.740E-01	3.091E-02	-3.499E+03	-1.827E+03	3.885E-02	4.845E-03	-1.313E+03	-3.547E+02	6.583E+00
6	3.823E-01	1.748E+03	5.882E-01	-7.546E-04	-3.676E+03	-5.871E+02	3.956E-02	3.652E-03	-1.434E+03	-2.744E+02	7.500E+00
7	4.176E-01	1.661E+03	4.843E-01	-7.853E-02	-3.254E+03	1.162E+03	3.675E-02	9.216E-04	-1.577E+03	-1.837E+02	1.008E+01
8	4.367E-01	1.543E+03	3.349E-01	-1.191E-01	-2.484E+03	2.855E+03	3.351E-02	-2.359E-04	-1.616E+03	-8.752E+01	1.158E+01
9	4.647E-01	1.407E+03	1.454E-03	-1.485E-01	-1.968E+02	3.973E+03	2.866E-02	-1.642E-03	-1.571E+03	-3.489E+01	1.400E+01
10	4.785E-01	1.262E+03	-1.943E-01	-1.428E-01	1.123E+03	3.957E+03	2.677E-02	-2.363E-03	-1.542E+03	3.038E+01	1.533E+01
11	4.996E-01	1.100E+03	-4.795E-01	-9.243E-02	3.692E+03	3.335E+03	2.432E-02	-4.082E-03	-1.411E+03	7.441E+01	1.767E+01
12	5.178E-01	8.737E+02	-5.885E-01	1.255E-02	5.643E+03	1.450E+03	1.929E-02	-6.980E-03	-1.192E+03	1.624E+02	2.017E+01
13	5.225E-01	7.083E+02	-5.588E-01	5.901E-02	5.868E+03	-2.020E+02	1.588E-02	-8.098E-03	-1.107E+03	2.282E+02	2.100E+01
14	5.317E-01	5.595E+02	-3.339E-01	1.638E-01	5.281E+03	-1.579E+03	1.978E-03	-1.156E-02	-8.006E+02	2.947E+02	2.300E+01
15	5.379E-01	3.621E+02	9.844E-02	2.430E-01	2.799E+03	-2.861E+03	-2.447E-02	-1.547E-02	-3.794E+02	3.443E+02	2.508E+01
16	5.399E-01	2.123E+02	3.967E-01	2.657E-01	1.260E+03	-2.576E+03	-4.545E-02	-1.755E-02	-1.639E+02	2.819E+02	2.625E+01
17	5.408E-01	1.339E+02	6.209E-01	2.722E-01	4.032E+02	-1.773E+03	-6.319E-02	-1.897E-02	-5.244E+01	1.901E+02	2.708E+01
18	5.410E-01	1.809E-06	8.712E-01	2.738E-01	-4.790E-04	1.331E-04	-8.534E-02	-2.048E-02	-2.467E-05	-7.211E-06	2.800E+01

NATURAL FREQUENCY = 100.6795 RADIANS/SECOND = 5.0340 X ROTATIONAL SPEED. GENERALIZED MASS = 1.0000

Test Case 4

Program Control and Frequency Search Data Using Frequency Stepping Method

PROGRAM CONTROL PARAMETERS

MODE COUPLING, NTYPE =	6	HUB BOUNDRY CONDITION, IBC =	1	FREQUENCY SEARCH, NFIND =	-1
BLADE PROPERTY DECK, ITAB =	0	TORSION MODE SWITCH, TMSL =	1.0	NO. OF FREQUENCIES, NUMF =	4
PUNCHED OUTPUT, NPCH =	0	NO. OF KNOWN FREQUENCIES, NPDF =	4	NO. OF ITERATIONS, NMAX =	30
REMAINDER INDEX, NOCR =	1	CONVERGENCE LIMIT, NEXPO =	11		
FREQUENCY LIMIT, OMLIMT =	9.900E+05	STEP SIZE FACTOR, PRCNT =	10.000000		

SEARCH FOR LOWEST FREQUENCY BEGINS

ITERATIONS NUMBER	FREQUENCY (RAD/SEC)	DETERMINANT	AUXILIARY FUNCTION	REMAINDERS 1	2	3	4
0	2.100000000000E+01	2.94706E+13		1.89310E-06	1.09606E-03	-2.72727E-05	5.77754E+00
1	2.202408581157E+01	1.05273E+14		1.69853E-06	3.18303E-03	-7.24974E-05	2.95310E+01
2	2.310000000000E+01	1.96694E+14		1.52238E-06	4.83913E-03	-1.00997E-04	1.17300E+02
3	2.422748439273E+01	3.04776E+14		1.36235E-06	6.10529E-03	-1.16910E-04	-4.74177E+02
4	2.541000000000E+01	4.30069E+14		1.21698E-06	7.01884E-03	-1.23497E-04	-1.23201E+02
5	2.665023283200E+01	5.72394E+14		1.08502E-06	7.61465E-03	-1.23322E-04	-8.19561E+01
6	2.795100000000E+01	7.30558E+14		9.65320E-07	7.92597E-03	-1.18396E-04	-6.48646E+01
7	2.931525611520E+01	9.02011E+14		8.56840E-07	7.98514E-03	-1.10287E-04	-5.46919E+01
8	3.074610000000E+01	1.08248E+15		7.58625E-07	7.82403E-03	-1.00204E-04	-4.73380E+01
9	3.224678172672E+01	1.26560E+15		6.69804E-07	7.47437E-03	-8.90678E-05	-4.13489E+01
10	3.382071000000E+01	1.44254E+15		5.89580E-07	6.96793E-03	-7.75664E-05	-3.60991E+01
11	3.547145989940E+01	1.60180E+15		5.17229E-07	6.33651E-03	-6.62011E-05	-3.12936E+01
12	3.720278100000E+01	1.72920E+15		4.52086E-07	5.61184E-03	-5.53244E-05	-2.67917E+01
13	3.901860588933E+01	1.80814E+15		3.93543E-07	4.82529E-03	-4.51707E-05	-2.25323E+01
14	4.092305910000E+01	1.82051E+15		3.41047E-07	4.00747E-03	-3.58820E-05	-1.84977E+01
15	4.292046647827E+01	1.74826E+15		2.94087E-07	3.18770E-03	-2.75292E-05	-1.46932E+01
16	4.501535501000E+01	1.57586E+15		2.52195E-07	2.39338E-03	-2.01294E-05	-1.11347E+01
17	4.721251312609E+01	1.29394E+15		2.14939E-07	1.64933E-03	-1.36600E-05	-7.83938E+00
18	4.951690151100E+01	9.03923E+14		1.81907E-07	9.76992E-04	-8.07075E-06	-4.81642E+00
19	5.193376443870E+01	4.23632E+14		1.52634E-07	3.93821E-04	-3.29282E-06	-2.05550E+00
20	5.446859166210E+01	-1.07092E+14		1.28717E-07	-8.73787E-05	7.51950E-07	4.98180E-01
21	6.647098216088E+01	-9.68986E+14		5.25067E-08	-9.06455E-04	1.07041E-05	-8.03669E+00
22	7.361029865914E+01	1.05022E+15		2.96711E-08	-8.44531E-04	1.78416E-05	1.12337E+00
23	8.196293650647E+01	4.20714E+15		1.38658E-08	-4.31578E-04	8.06035E-06	1.48056E+00
24	8.864481638410E+01	4.43225E+15		6.44104E-09	-1.75251E-04	1.93030E-06	8.71188E-01
25	9.405770540935E+01	2.25934E+15		2.59820E-09	-4.77662E-05	3.06956E-07	3.20808E-01
26	9.754739740425E+01	5.53491E+14		8.48170E-10	-8.18019E-06	3.76178E-08	6.84214E-02
27	9.919455159239E+01	6.19880E+13		1.68649E-10	-7.81677E-07	3.09358E-09	7.33707E-03
28	9.959913522488E+01	2.19978E+12		8.05452E-12	-2.67038E-08	1.01944E-10	2.58228E-04
29	9.961933365103E+01	-2.40455E+10		-8.96882E-14	2.91344E-10	-1.11023E-12	-2.82157E-06

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1.04032E+01  UNIFORM BLADE (LUMPED PARAMETER MODEL) HINGED - HINGED          1
7.5622001E-03 -5.7634434E-05 -3.5860757E-03 -7.0946847E-02  1.2103832E-02
-6.9369048E+00  1.0000247E+00 -3.3205305E+02  0.             1.8727309E+03
1.8163223E-02 -6.2056898E-02 -3.6496053E-03 -7.0675138E-02  1.2128141E-02
-6.9192786E+00  1.6802308E+00 -3.3194085E+02 -6.6000119E+01  1.8725479E+03
2.8789475E-02 -1.2382845E-01 -3.7699045E-03 -7.0426895E-02  1.2160820E-02
-6.9154855E+00  1.9043406E+00 -3.2995593E+02 -1.1779569E+02  1.8618000E+03
3.9445618E-02 -1.8539120E-01 -3.8891067E-03 -7.0198926E-02  1.2195718E-02
-6.8524277E+00  1.9055623E+00 -3.2612289E+02 -1.5676855E+02  1.8405547E+03
5.0132113E-02 -2.4676148E-01 -4.0065037E-03 -6.9988493E-02  1.2229831E-02
-6.7486485E+00  1.8001553E+00 -3.2044742E+02 -1.8424611E+02  1.8088468E+03
6.0847607E-02 -3.0795339E-01 -4.1213886E-03 -6.9793296E-02  1.2261826E-02
-6.6042420E+00  1.6501888E+00 -3.1293420E+02 -2.0152032E+02  1.7667073E+03
7.1589982E-02 -3.6897924E-01 -4.2330562E-03 -6.9611457E-02  1.2291185E-02
-6.4192898E+00  1.4857786E+00 -3.0358755E+02 -2.0985275E+02  1.7141648E+03
8.2356859E-02 -4.2984985E-01 -4.3408023E-03 -6.9441514E-02  1.2317786E-02
-6.1938624E+00  1.3214630E+00 -2.9241147E+02 -2.1048017E+02  1.6512460E+03
9.3145848E-02 -4.9057496E-01 -4.4439239E-03 -6.9282413E-02  1.2341701E-02
-5.9280207E+00  1.1648936E+00 -2.7940976E+02 -2.0462057E+02  1.5779761E+03
1.0395467E-01 -5.5116353E-01 -4.5417189E-03 -6.9133499E-02  1.2363121E-02
-5.6218159E+00  1.0218473E+00 -2.6458594E+02 -1.9347872E+02  1.4943783E+03
1.2562102E-01 -6.7180112E-01 -4.7187451E-03 -6.8475695E-02  1.2393532E-02
-5.0882383E+00  4.9856188E-01 -2.3962208E+02 -1.6177696E+02  1.3535231E+03
1.4733098E-01 -7.9134668E-01 -4.8660615E-03 -6.7929926E-02  1.2412238E-02
-4.2342947E+00  7.8806860E-02 -1.9908936E+02 -1.1146156E+02  1.1246690E+03
1.6906248E-01 -9.0999925E-01 -4.9781028E-03 -6.7507413E-02  1.2417545E-02
-3.2203850E+00 -3.6762944E-01 -1.5136649E+02 -5.8293747E+01  8.5512634E+02
1.9079230E-01 -1.0279865E+00 -5.0493254E-03 -6.7229657E-02  1.2410601E-02
-2.0471427E+00 -6.9206137E-01 -9.6500928E+01 -1.6927659E+01  5.4518924E+02
2.1250671E-01 -1.1455822E+00 -5.0741968E-03 -6.7127705E-02  1.2402797E-02
-5.7528493E-07  3.7869928E-03  2.5553948E-05 -6.4536227E-05 -4.0305626E-05
```

3.76069E+01 UNIFORM BLADE (LUMPED PARAMETER MODEL) HINGED - HINGED 2

4.0349924E-02	1.0621326E-05	-2.1020271E-02	1.3260173E-02	6.5360090E-02
-4.0661516E+01	1.5334216E+00	-1.7997163E+03	0.	-3.4512157E+02
9.0052534E-02	1.1598631E-02	-2.1391824E-02	1.3209959E-02	6.5396198E-02
-4.0472823E+01	2.5500386E+00	-1.7939002E+03	1.5704095E+01	-3.4606287E+02
1.5529524E-01	2.3145024E-02	-2.2092006E-02	1.3165101E-02	6.5445193E-02
-4.0250477E+01	2.8681398E+00	-1.7778774E+03	2.7853462E+01	-3.4506907E+02
2.1258333E-01	3.4654059E-02	-2.2782106E-02	1.3124722E-02	6.5498762E-02
-3.9670891E+01	2.9004565E+00	-1.7526207E+03	3.6856501E+01	-3.4199567E+02
2.6991858E-01	4.6129287E-02	-2.3458513E-02	1.3088077E-02	6.5552327E-02
-3.8883746E+01	2.7840541E+00	-1.7181548E+03	4.3078171E+01	-3.3685022E+02
3.2730007E-01	5.7573658E-02	-2.4117614E-02	1.3054543E-02	6.5603968E-02
-3.7888854E+01	2.5952026E+00	-1.6744711E+03	4.6863828E+01	-3.2964460E+02
3.8472565E-01	6.8989627E-02	-2.4755790E-02	1.3023608E-02	6.5652763E-02
-3.6686032E+01	2.3707554E+00	-1.6215611E+03	4.8542289E+01	-3.2038975E+02
4.4219263E-01	8.0379250E-02	-2.5369423E-02	1.2994870E-02	6.5698263E-02
-3.5275111E+01	2.1277721E+00	-1.5594168E+03	4.8428302E+01	-3.0909578E+02
4.9969801E-01	9.1744274E-02	-2.5954889E-02	1.2968026E-02	6.5740232E-02
-3.3655936E+01	1.8738091E+00	-1.4880307E+03	4.6825036E+01	-2.9577206E+02
5.5723864E-01	1.0308623E-01	-2.6508563E-02	1.2942878E-02	6.5778532E-02
-3.1828364E+01	1.6127842E+00	-1.4073959E+03	4.4026432E+01	-2.8042735E+02
6.7241346E-01	1.2567568E-01	-2.7509926E-02	1.2824344E-02	6.5836792E-02
-2.8782038E+01	3.4219102E-01	-1.2725609E+03	3.6270716E+01	-2.5439107E+02
7.8767134E-01	1.4806766E-01	-2.8340570E-02	1.2725487E-02	6.5869769E-02
-2.3375068E+01	-8.4020188E-01	-1.0557252E+03	2.4824023E+01	-2.1168907E+02
9.0295893E-01	1.7029712E-01	-2.8971428E-02	1.2648414E-02	6.5867375E-02
-1.8132661E+01	-2.1518131E+00	-8.0179678E+02	1.3055351E+01	-1.6112852E+02
1.0182151E+00	1.9240462E-01	-2.9373428E-02	1.2597403E-02	6.5834121E-02
-1.1554635E+01	-2.9900541E+00	-5.1075843E+02	3.9716180E+00	-1.0281293E+02
1.1334156E+00	2.1443992E-01	-2.9517511E-02	1.2578677E-02	6.5813237E-02
1.1341097E-05	-8.4133863E-03	-5.0406621E-04	1.2918240E-04	8.1903685E-05

9.70264E+01 UNIFORM BLADE (LUMPED PARAMETER MODEL) HINGED - HINGED 3

-1.0518477E-01	4.8535285E-06	1.0529437E-01	9.2240782E-04	-1.6748025E-01
2.0484207E+02	1.8909113E+02	4.9434206E+03	0.	-1.5770699E+02
-2.4956942E-01	8.0539090E-04	1.0774185E-01	8.9987793E-04	-1.6167749E-01
2.0121682E+02	4.6531586E+02	4.8461383E+03	-1.1373227E+02	-1.5527040E+02
-3.8672674E-01	1.5700403E-03	1.1110278E-01	8.4072875E-04	-1.5098769E-01
1.9320514E+02	7.4162890E+02	4.5753290E+03	-2.2144808E+02	-1.4751403E+02
-5.1239545E-01	2.2678305E-03	1.1421946E-01	7.4763555E-04	-1.3542740E-01
1.7916489E+02	1.0152088E+03	4.1553480E+03	-3.2000289E+02	-1.3509146E+02
-6.2233243E-01	2.8705690E-03	1.1703656E-01	6.2425692E-04	-1.1504281E-01
1.6194309E+02	1.2840451E+03	3.5996878E+03	-4.0650876E+02	-1.1842195E+02
-7.1235205E-01	3.3536644E-03	1.1950560E-01	4.7513163E-04	-8.9919544E-02
1.4193422E+02	1.5467309E+03	2.9253313E+03	-4.7838710E+02	-9.8046519E+01
-7.7835405E-01	3.6968365E-03	1.2158675E-01	3.0555498E-04	-6.0167735E-02
1.1963643E+02	1.8016589E+03	2.1537792E+03	-5.3346050E+02	-7.4649300E+01
-8.1634610E-01	3.8847233E-03	1.2325062E-01	1.2142572E-04	-2.5923969E-02
9.5649086E+01	2.0472237E+03	1.3109574E+03	-5.7004904E+02	-4.9049890E+01
-8.2246617E-01	3.9073268E-03	1.2447997E-01	-7.0937753E-05	1.2651575E-02
7.0669694E+01	2.2821684E+03	4.2710025E+02	-5.8707087E+02	-2.2194413E+01
-7.9299892E-01	3.7602895E-03	1.2527130E-01	-2.6501307E-04	5.5386757E-02
4.5490254E+01	2.5062227E+03	-4.6339029E+02	-5.8414712E+02	4.8545821E+00
-6.2056189E-01	2.9719595E-03	1.2566574E-01	-6.2813068E-04	1.4328050E-01
1.1337448E+01	2.7760087E+03	-1.7514604E+03	-5.1615185E+02	4.3966221E+01
-2.8878998E-01	1.6016351E-03	1.2464577E-01	-9.2202779E-04	2.3522929E-01
-2.9316977E+01	2.6700205E+03	-3.0967773E+03	-3.8514256E+02	8.4800888E+01
1.9714704E-01	-2.0017104E-04	1.2277481E-01	-1.1172252E-03	3.1637456E-01
-5.3776560E+01	2.0508345E+03	-3.7260696E+03	-2.2170280E+02	1.0453801E+02
8.0256203E-01	-2.2523322E-03	1.2087922E-01	-1.2100980E-03	3.6879042E-01
-5.4484692E+01	9.4125836E+02	-3.3066135E+03	-7.2965243E+01	9.3563094E+01
1.4675774E+00	-4.3971428E-03	1.1998724E-01	-1.2324423E-03	3.8554005E-01
-7.1887894E-10	-5.6377242E-07	-1.4840647E-07	5.4396878E-09	3.7243808E-09

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4.94272E+00 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED        1
6.8396477E-05 -4.1314622E-11 -3.6289458E-04 -3.1093099E-02  6.8396481E-03
-2.7942883E+00  6.5000097E-02 -1.9362701E+02  0.              8.0067752E+02
7.4075866E-03 -3.3311254E-02 -3.6653360E-04 -3.0948667E-02  6.8403000E-03
-2.8026930E+00  3.9580664E+00 -1.9362348E+02 -1.0201113E+02  8.0067058E+02
1.3108447E-02 -5.9067445E-02 -3.7398965E-04 -3.0836205E-02  6.8472075E-03
-2.8105627E+00  4.0279721E+00 -1.8821118E+02 -1.6215424E+02  7.7819382E+02
2.4514696E-02 -1.1017744E-01 -3.9130952E-04 -3.0174404E-02  6.8824383E-03
-1.5148388E+00  3.2024204E+00 -1.8435497E+02 -2.6686274E+02  7.6381829E+02
3.6322422E-02 -1.7054369E-01 -4.1575204E-04 -2.9372027E-02  6.6056518E-03
-1.4848833E+00  4.9474535E-01 -1.7298180E+02 -3.5314186E+02  7.5593772E+02
4.1307537E-02 -1.9775400E-01 -4.2661874E-04 -2.9419429E-02  6.4561138E-03
-1.4398087E+00 -1.7939807E-01 -1.6541142E+02 -3.7028343E+02  7.4133746E+02
5.3817743E-02 -2.7385990E-01 -4.5571896E-04 -2.8215141E-02  6.0660893E-03
-1.3688255E+00 -2.4758779E+00 -1.4887974E+02 -4.1593280E+02  7.1893548E+02
6.0085056E-02 -3.1680880E-01 -4.7127446E-04 -2.8143784E-02  5.8044836E-03
-1.2599954E+00 -2.7343333E+00 -1.3447637E+02 -3.9492356E+02  6.7967351E+02
6.8550855E-02 -3.8555179E-01 -4.9404046E-04 -2.7412238E-02  5.4008687E-03
-1.1444224E+00 -3.4550753E+00 -1.1647080E+02 -3.5721178E+02  6.3643258E+02
7.2381099E-02 -4.2275793E-01 -5.0522375E-04 -2.7416366E-02  5.1568666E-03
-1.0193323E+00 -3.3749333E+00 -1.0228403E+02 -3.1688576E+02  5.8494121E+02
7.7606652E-02 -4.8760933E-01 -5.2239959E-04 -2.6849507E-02  4.7494370E-03
-8.9449842E-01 -3.5864110E+00 -8.5366181E+01 -2.5256547E+02  5.3138971E+02
8.1152801E-02 -5.5565089E-01 -5.3739118E-04 -2.6209231E-02  4.3119669E-03
-7.2859142E-01 -3.2936661E+00 -6.5936932E+01 -1.7100513E+02  4.5327496E+02
8.1881371E-02 -5.7791445E-01 -5.4154534E-04 -2.6237029E-02  4.1601449E-03
-6.0591856E-01 -2.9608208E+00 -5.4641536E+01 -1.4018975E+02  3.8904188E+02
8.2659830E-02 -6.3127135E-01 -5.4973840E-04 -2.5940865E-02  3.8126762E-03
-4.9772802E-01 -2.1972276E+00 -4.2436300E+01 -7.8448214E+01  3.3056752E+02
8.2081982E-02 -6.8621373E-01 -5.5539655E-04 -2.5680221E-02  3.4664690E-03
-3.3003638E-01 -1.1229645E+00 -2.6800823E+01 -2.8746033E+01  2.3134727E+02
8.1157366E-02 -7.1674577E-01 -5.5726574E-04 -2.5646483E-02  3.2803240E-03
-1.9460764E-01 -5.0432744E-01 -1.5473339E+01 -1.0125825E+01  1.4215092E+02
8.0229017E-02 -7.3851222E-01 -5.5799365E-04 -2.5644845E-02  3.1506583E-03
-1.0617123E-01 -1.7927553E-01 -8.3507682E+00 -2.5404766E+00  8.0409715E+01
7.8966985E-02 -7.6246798E-01 -5.5823318E-04 -2.5654443E-02  3.0102774E-03
 4.7344100E-07 -1.1321791E-02 -3.3525240E-05  7.9072593E-04  6.8707781E-05
```

2.09528E+01 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED 2

2.7775246E-04	1.0164196E-11	-2.0534475E-03	7.5491221E-03	2.7775247E-02
-1.5811546E+01	2.7777439E-01	-7.8492191E+02	0.	-1.9698216E+02
3.0081548E-02	8.0875014E-03	-2.0739648E-03	7.5135835E-03	2.7777884E-02
-1.5801979E+01	1.6844574E+01	-7.8478485E+02	2.1751684E+01	-1.9701035E+02
5.3232190E-02	1.4339717E-02	-2.1158616E-03	7.4843006E-03	2.7805687E-02
-1.5793022E+01	1.6082000E+01	-7.6461641E+02	3.4375724E+01	-1.9106756E+02
9.9767386E-02	2.6685987E-02	-2.2359825E-03	7.2999579E-03	2.7981207E-02
-1.0506074E+01	9.7201662E+00	-7.5174845E+02	5.6531832E+01	-1.8687117E+02
1.5636431E-01	3.9310202E-02	-2.4068457E-03	6.7334004E-03	2.8208868E-02
-1.0379937E+01	7.2802462E+00	-7.4499402E+02	7.3506876E+01	-1.7540191E+02
1.8250527E-01	4.4499605E-02	-2.4837018E-03	6.5751909E-03	2.8305862E-02
-1.0183233E+01	6.8955383E+00	-7.3167913E+02	7.5681048E+01	-1.6756262E+02
2.5671248E-01	5.7355973E-02	-2.6934911E-03	5.8256847E-03	2.8531812E-02
-9.8681374E+00	6.2530234E+00	-7.1134794E+02	8.2041223E+01	-1.5066635E+02
3.0014959E-01	6.3436922E-02	-2.8088891E-03	5.5382955E-03	2.8663515E-02
-9.3472346E+00	5.9097689E+00	-6.7484733E+02	7.5223165E+01	-1.3564831E+02
3.7063536E-01	7.1437387E-02	-2.9832084E-03	4.9557762E-03	2.8849008E-02
-8.7628460E+00	5.3305065E+00	-6.3421800E+02	6.4636792E+01	-1.1705051E+02
4.0974008E-01	7.4812940E-02	-3.0717957E-03	4.7187199E-03	2.8950567E-02
-8.0745326E+00	5.0122040E+00	-5.8502376E+02	5.5456531E+01	-1.0231176E+02
4.7853941E-01	7.9158116E-02	-3.2128889E-03	4.2058742E-03	2.9105050E-02
-7.3479736E+00	4.3158212E+00	-5.3343607E+02	4.1868833E+01	-8.4893172E+01
5.5268643E-01	8.1413068E-02	-3.3422346E-03	3.6632809E-03	2.9247294E-02
-6.2862036E+00	3.3996903E+00	-4.5718187E+02	2.6703750E+01	-6.4962163E+01
5.7747086E-01	8.1616846E-02	-3.3793249E-03	3.5213813E-03	2.9291451E-02
-5.4099203E+00	3.1291521E+00	-3.9361046E+02	2.1426522E+01	-5.3493669E+01
6.3711542E-01	8.1077315E-02	-3.4551246E-03	3.1295498E-03	2.9379695E-02
-4.6048322E+00	2.0925753E+00	-3.3546091E+02	1.1388816E+01	-4.1158781E+01
6.9938005E-01	7.8931545E-02	-3.5105640E-03	2.7309003E-03	2.9445801E-02
-3.2337453E+00	8.1395176E-01	-2.3579647E+02	3.9858041E+00	-2.5602677E+01
7.3429949E-01	7.7031182E-02	-3.5296991E-03	2.5213562E-03	2.9470914E-02
-1.9922136E+00	2.6432850E-01	-1.4531892E+02	1.3779155E+00	-1.4607681E+01
7.5923252E-01	7.5368815E-02	-3.5374412E-03	2.3736997E-03	2.9484608E-02
-1.1292561E+00	1.7445928E-02	-8.2364357E+01	3.5082171E-01	-7.8101383E+00
7.8668076E-01	7.3262652E-02	-3.5400353E-03	2.2125026E-03	2.9497417E-02
-3.4339957E-07	6.5923325E-04	2.5053355E-05	-4.5414375E-05	-4.9219969E-06

5.39534E+01 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED 3

-7.7490479E-04	2.2452928E-12	7.2722976E-03	-4.0340400E-04	-7.7490457E-02
5.5996691E+01	3.5256706E+00	2.6199264E+03	0.	-4.3513782E+01
-8.3898750E-02	-4.3565392E-04	7.3433174E-03	-4.1123923E-04	-7.7425515E-02
5.4697797E+01	4.1708753E+02	2.6174035E+03	-5.8994295E+01	-4.3435148E+01
-1.4807949E-01	-7.9262351E-04	7.4851977E-03	-4.4821568E-04	-7.6644659E-02
5.3481912E+01	5.0641545E+02	2.2245969E+03	-9.6216894E+01	-3.3477478E+01
-2.7020210E-01	-1.5380166E-03	7.9010265E-03	-5.8173991E-04	-6.9772852E-02
3.6369425E+01	5.2294346E+02	1.9727903E+03	-1.6168361E+02	-2.8102920E+01
-3.9914955E-01	1.7193190E-03	8.4541323E-03	-2.2638019E-04	-5.8531832E-02
3.3601181E+01	7.2011720E+02	1.8023495E+03	-2.4874523E+02	-4.5322261E+01
-4.4974252E-01	3.8619266E-03	8.6779800E-03	-1.6565320E-04	-5.1675611E-02
2.9659211E+01	8.3256470E+02	1.5505025E+03	-2.9223008E+02	-4.5082118E+01
-5.5596507E-01	1.1003868E-02	9.1810948E-03	-5.0705680E-04	-2.9253623E-02
2.3665680E+01	1.1700944E+03	1.1596601E+03	-3.9839014E+02	-4.7813385E+01
-5.8682915E-01	1.5050737E-02	9.3673288E-03	-9.6748076E-04	-1.1673178E-02
1.5084950E+01	1.3234636E+03	5.9101617E+02	-4.5766782E+02	-2.5008830E+01
-5.7874753E-01	1.9821995E-02	9.4941425E-03	-2.2411455E-03	1.8954874E-02
6.3747904E+00	1.5261732E+03	1.5378073E+01	-5.1507072E+02	6.3609546E+00
-5.4022689E-01	2.0715731E-02	9.4743570E-03	-3.1393997E-03	3.8754593E-02
-1.8034113E+00	1.5952266E+03	-5.2636327E+02	-5.3403480E+02	4.4567497E+01
-4.1001646E-01	1.7899450E-02	9.2977564E-03	-5.1004314E-03	7.2734855E-02
-9.1971595E+00	1.6169607E+03	-1.0081294E+03	-5.1164605E+02	9.0621997E+01
-1.8337475E-01	6.7115590E-03	8.9596062E-03	-7.5284782E-03	1.0790894E-01
-1.6434097E+01	1.4849550E+03	-1.4648144E+03	-4.2484762E+02	1.4483774E+02
-8.8554758E-02	7.5788235E-04	8.8288022E-03	-8.3912792E-03	1.1960931E-01
-1.9078863E+01	1.4133426E+03	-1.6106460E+03	-3.8731222E+02	1.6171371E+02
1.7505618E-01	-1.8611441E-02	8.4913793E-03	-1.0499840E-02	1.4300245E-01
-2.0498436E+01	1.0728591E+03	-1.6708399E+03	-2.6385378E+02	1.8336061E+02
4.9074245E-01	-4.6862826E-02	8.1717271E-03	-1.2653337E-02	1.5858977E-01
-1.8645100E+01	5.1197693E+02	-1.4788277E+03	-1.1626458E+02	1.7554802E+02
6.7812519E-01	-6.6372068E-02	8.0445120E-03	-1.3828824E-02	1.6266080E-01
-1.3244763E+01	2.2053899E+02	-1.0387682E+03	-4.7774999E+01	1.2840677E+02
8.1380079E-01	-8.1934454E-02	7.9890088E-03	-1.4652263E-02	1.6376430E-01
-8.0955938E+00	6.7941395E+01	-6.3649719E+02	-1.4506014E+01	8.1107703E+01
9.6362496E-01	-1.0062179E-01	7.9679347E-03	-1.5547305E-02	1.6398628E-01
4.3374903E-08	-1.9588881E-05	-3.3288916E-06	1.2698379E-06	2.4433621E-07

7.96665E+01 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED 4

6.0353908E-05	-2.8181806E-10	-1.3337144E-03	-9.5886250E-02	8.0353847E-03
-1.0269601E+01	-9.9627443E-01	-3.3474172E+02	0.	5.4616351E+03
8.6961599E-03	-1.0253369E-01	-1.3464870E-03	-9.4901031E-02	8.0182107E-03
-9.8372169E+00	-1.1084791E+02	-3.3429434E+02	2.9296558E+03	5.4616269E+03
1.5291380E-02	-1.8069987E-01	-1.3715103E-03	-9.2449013E-02	7.8090929E-03
-9.4325418E+00	-1.3703460E+02	-2.4969202E+02	4.6040550E+03	4.5810479E+03
2.6565626E-02	-3.2682701E-01	-1.4540320E-03	-8.1247688E-02	5.8119313E-03
-7.2175525E+00	-1.5304912E+02	-1.9366390E+02	7.1909737E+03	4.0178383E+03
2.9246325E-02	-4.7220179E-01	-1.5625222E-03	-6.1439437E-02	1.4332044E-03
-6.5907829E+00	-1.3015067E+02	-1.1538139E+02	1.0191858E+04	3.6423652E+03
2.7111832E-02	-5.2446010E-01	-1.6068260E-03	-5.1790893E-02	-2.1750080E-04
-5.8701281E+00	-1.1710467E+02	-5.8139774E+01	1.1582958E+04	3.0966097E+03
1.2393536E-02	-6.1724198E-01	-1.7113707E-03	-1.7110222E-02	-4.6897502E-03
-4.9176099E+00	-4.3448300E+01	3.0253474E+01	1.4095841E+04	2.2613503E+03
-9.6473976E-04	-6.2797971E-01	-1.7621392E-03	3.5196679E-03	-5.8323413E-03
-4.1122414E+00	1.1766021E+01	7.8686290E+01	1.5148850E+04	1.1019841E+03
-2.3874862E-02	-5.7937590E-01	-1.8342968E-03	3.7051449E-02	-6.0755769E-03
-3.6272876E+00	1.3500301E+02	9.2692706E+01	1.5253040E+04	-3.0858622E+01
-3.5085540E-02	-5.1781697E-01	-1.8748896E-03	5.4726259E-02	-4.2690877E-03
-3.6999424E+00	1.9929428E+02	5.9461072E+01	1.4873758E+04	-1.0284236E+03
-4.5572949E-02	-3.5631089E-01	-1.9517066E-03	8.1893489E-02	4.1396376E-04
-4.0005455E+00	3.1378332E+02	-9.4641248E+00	1.2818327E+04	-1.8791256E+03
-3.8131808E-02	-1.2074245E-01	-2.0485294E-03	1.0373140E-01	8.2173270E-03
-4.7055840E+00	3.7847353E+02	-1.2693412E+02	9.4673455E+03	-2.6101758E+03
-3.0210386E-02	-3.1458793E-02	-2.0836680E-03	1.0993493E-01	1.1665605E-02
-5.1252702E+00	3.8011104E+02	-1.9094844E+02	8.2888292E+03	-2.7865187E+03
1.9707676E-03	2.0001669E-01	-2.1720798E-03	1.1966877E-01	1.9192968E-02
-5.3710140E+00	3.2706251E+02	-2.6041908E+02	5.1247723E+03	-2.8239736E+03
5.3126887E-02	4.5543058E-01	-2.2582168E-03	1.2429462E-01	2.5444510E-02
-5.0243165E+00	1.7310192E+02	-2.9042125E+02	2.0440056E+03	-2.4202937E+03
8.7910288E-02	6.0074620E-01	-2.2936918E-03	1.2523553E-01	2.7706152E-02
-3.6934031E+00	7.7504569E+01	-2.2689953E+02	7.8998294E+02	-1.6673301E+03
1.1468522E-01	7.0466724E-01	-2.3099110E-03	1.2537238E-01	2.8737778E-02
-2.3657117E+00	2.4244408E+01	-1.4791020E+02	2.2706280E+02	-1.0101679E+03
1.4556552E-01	8.1890076E-01	-2.3157732E-03	1.2527503E-01	2.9528939E-02
6.7864547E-07	-2.1712502E-04	-5.0003004E-05	1.3597863E-05	3.1611999E-06

1.00680E+02 TEST BLADE WITH NEARLY COINCIDENT FREQUENCIES, ARTICULATED 6

1.4035484E-03	3.5724132E-11	3.0972247E-01	9.7453446E-03	1.4035468E-01
2.3848630E+03	-2.7092701E+01	-6.8160309E+03	0.	-6.9233381E+02
1.5183607E-01	1.0412092E-02	3.1254405E-01	9.6204294E-03	1.3988889E-01
2.1731170E+03	-2.9896023E+03	-6.8002046E+03	-4.4867318E+02	-6.9247624E+02
2.6614687E-01	1.8299633E-02	3.1778355E-01	9.2790266E-03	1.3443307E-01
1.9750319E+03	-3.4628522E+03	-4.3392444E+03	-6.8447970E+02	-5.3985956E+02
4.5081164E-01	3.2459004E-02	3.3922370E-01	7.7505885E-03	9.0521441E-02
1.8752101E+03	-3.0997357E+03	-2.7944131E+03	-1.0097264E+03	-4.4090734E+02
5.7398923E-01	3.8846447E-02	3.6910862E-01	4.8445917E-03	3.0911813E-02
1.8155086E+03	-3.4991372E+03	-1.8271906E+03	-1.3131328E+03	-3.5474248E+02
5.8815215E-01	3.9554996E-02	3.8230446E-01	3.6518515E-03	-7.5462942E-04
1.7484126E+03	-3.6761101E+03	-5.8707539E+02	-1.4339207E+03	-2.7435721E+02
4.8425482E-01	3.6745513E-02	4.1761628E-01	9.2155103E-04	-7.8532249E-02
1.6610090E+03	-3.2543013E+03	1.1615081E+03	-1.5774915E+03	-1.8374752E+02
3.3493153E-01	3.3506742E-02	4.3666098E-01	-2.3594877E-04	-1.1914200E-01
1.5426211E+03	-2.4835329E+03	2.8554706E+03	-1.6160806E+03	-8.7522992E+01
1.4538123E-03	2.8660342E-02	4.6465792E-01	-1.6419018E-03	-1.4845156E-01
1.4073760E+03	-1.9681391E+02	3.9732362E+03	-1.5713933E+03	-3.4888484E+01
-1.9432550E-01	2.6769793E-02	4.7850865E-01	-2.3633917E-03	-1.4278025E-01
1.2624636E+03	1.1228995E+03	3.9570440E+03	-1.5415061E+03	3.0382452E+01
-4.7945717E-01	2.4315851E-02	4.9962739E-01	-4.0819318E-03	-9.2429644E-02
1.0998398E+03	3.6919621E+03	3.3350975E+03	-1.4109272E+03	7.4407006E+01
-5.8854869E-01	1.9286977E-02	5.1760491E-01	-6.9796148E-03	1.2548014E-02
8.7370777E+02	5.6433119E+03	1.4498097E+03	-1.1917361E+03	1.6241149E+02
-5.5878814E-01	1.5879170E-02	5.2246076E-01	-8.0979184E-03	5.9006058E-02
7.0826554E+02	5.8682763E+03	-2.0195953E+02	-1.1074169E+03	2.2821824E+02
-3.3392556E-01	1.9776953E-03	5.3167034E-01	-1.1559662E-02	1.6378150E-01
5.5948168E+02	5.2807177E+03	-1.5794508E+03	-8.0055128E+02	2.9470743E+02
9.8441664E-02	-2.4473158E-02	5.3787757E-01	-1.5474803E-02	2.4297137E-01
3.6206411E+02	2.7987915E+03	-2.8610845E+03	-3.7937081E+02	3.4426535E+02
3.9669710E-01	-4.5452308E-02	5.3991716E-01	-1.7553760E-02	2.6565161E-01
2.1234795E+02	1.2601259E+03	-2.5757094E+03	-1.6390855E+02	2.8185127E+02
6.2093657E-01	-6.3188772E-02	5.4083497E-01	-1.8969865E-02	2.7222119E-01
1.3386969E+02	4.0328797E+02	-1.7729353E+03	-5.2440496E+01	1.9007539E+02
8.7117086E-01	-8.5341442E-02	5.4098550E-01	-2.0479373E-02	2.7383282E-01
-2.2819089E-04	5.2873188E-02	1.6791123E-02	-3.1116676E-03	-9.0964270E-04

APPENDIX E

PROGRAM CHANGES FOR OPERATION ON AN IBM 360/65

The program as listed in APPENDIX C is not operational on a computer which uses less than 14 digit floating point numbers. Changes to the program listed in APPENDIX C were made, and resulted in satisfactory operation for two test cases. Numerical results did not agree exactly with CDC 6600 resulting and appeared to be more accurate, probably due to the use of 16 digit numbers rather than 14 digit numbers.

The program was compiled on an IBM 360/65 by FORTRAN IV G LEVEL 20 during March and April, 1972, at the University of Rochester, in Rochester, New York. The changes necessary for proper operation with this compiler and computer are given as follows:

1. "Comment" or remove the first two cards in the main program.

2. "Comment" or remove the CALL DAYTIM(DATE) or replace it by an equivalent call to obtain the calendar date. If this call is replaced by an "equivalent" call, the format statement where DATE is written may need to be revised. This is number 898 in the main program.

3. Add the following cards to all subroutines:

   (a) before existing non-executable statements, IMPLICIT REAL*8(A-H, O-Z)

   (b) immediately following existing non-executable statements,
```
ABS(X) = DABS(X)
TANH(X) = DTANH(X)
SIN(X) = DSIN(X)
COS(X) = DCOS(X)
SQRT(X) = DSQRT(X)
```

4. (a) In subroutine MATINV, after the cards specified in 3(b) add the statement: CALL ERRSET (207,256,10)

   (b) In subroutine MATINV, Just before the RETURN and END cards, add the statement:
   740 CALL ERRSET (207,1,1)

5. In subroutine MATINV, before FORTRAN statement number 330, add the statement: IF (PIVOT.EQ.0.) GO TO 740

6. In subroutine FINDM, replace the FORTRAN statement immediately after that numbered 30, i.e., OMP=OMD*OMOLD, by: IF((OMD.EQ.0.).OR.(OMOLD.EQ.0.)) OMP=0.

7. The data must be punched on a compatible keypunch.

8. The data in NAMELIST presently begins with $ and ends with $ symbols, but the IBM 360/65 requires that it begins with $ and ends with $END symbols.

For operation with the University of Rochester's WATFOR, Version 1, Level 1, January 1970 compiler, the cards specified in 4(a) and 4(b) should be replaced by the following:

for 4(a): CALL TRAPS (0,0,1000)

for 4(b): 740 CALL TRAPS (0,0,0)

Items 4 and 5 are required to allow execution with underflow errors which occur when MATINV is called to evaluate a determinant. The underflows occurred when the subroutine was evaluating off-diagonal numbers for use in solving a set of simultaneous equations. These off-diagonal elements are not used in defining the determinant. Thus, use of ERRSET results in no numerical errors, but does allow program execution to continue when such problems occur. Item 6 is required to avoid an overflow in defining OMP. These changes were required on the IBM 360/65 because overflows and underflows result for numbers with smaller magnitude exponents than on the CDC 6600 or CDC 6400. The use of ERRSET or TRAPS is compiler-dependent.

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