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SIMULATION STUDY OF THE LIFT-ROLL COUPLING PROBLEM FOR HOVERING VTOL AIRCRAFT

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16. Abstract

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NOTATION

d	distance from roll axis to outboard engine, m (ft)
F	force, N (lb)
g	gravitational acceleration, m/sec^2 (ft/sec^2)
h	altitude, m (ft)
I_x	moment of inertia about x axis, kg-m^2 (slug-ft^2)
K_{sp}	stick sensitivity in roll, N/cm (lb/in.)
L	rolling moment, N-m (lb-ft)
T	thrust, N (lb)
$\frac{T}{W}$	commanded total thrust-to-weight ratio of the aircraft
$\frac{T_m}{W}$	maximum total thrust-to-weight ratio of the aircraft
$\frac{T_{oh}}{W}$	ratio of outboard engine hover thrust to aircraft gross weight
$\frac{T_{om}}{T_{oh}}$	ratio of maximum thrust to hover thrust for an outboard engine
W	gross weight of aircraft, N (lb)
δ_p	lateral stick deflection, cm (in.)
Δ	incremental change
ϕ	bank angle, deg
$(\ddot{\ })$	$\frac{d^2}{dt^2}$ ( ), 1/sec^2

Subscripts

C	center engine
h	hover condition
L	left engine

m	maximum
o	outboard engine
R	right engine
u	uncoupled
x	longitudinal axis
z	vertical axis

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SUMMARY

The effects of lift-roll coupling on the handling qualities of hovering VTOL aircraft using differential thrust for roll control were assessed in a piloted simulation study in the Ames six-degrees-of-freedom motion simulator. The configuration tested had three vertical thrust sources, one on the roll axis and two laterally displaced from the roll axis, with a thrust distribution of 25%/50%/25%. The outboard thrust sources were modulated to provide roll control whereas all three provided height control. Maximum thrust-to-weight ratio was varied together with a coupling parameter that combined roll-inertia, weight, and engine location. Results showed that handling qualities are affected not only by the occurrence of lift-roll coupling (dependent on both variables) but also by the severity of the coupling (dependent on the coupling parameter alone). However, the advantages of differential thrust for control can be retained with careful design.

INTRODUCTION

Vertical take-off aircraft in hover and low-speed flight cannot obtain sufficient forces for lift and control from aerodynamic surfaces. Consequently, these forces must be generated in some other manner. The lift requirement can be satisfied in several ways, all of which employ an oversized propulsion system with some means to deflect its thrust in a vertical direction. Control forces can also be generated in several ways, but usually they are derived from the same thrust source used to create lift. Whatever the arrangement, there is always the possibility of undesirable cross coupling between height control and attitude control. The extent of this cross coupling is closely associated with such factors as the amount of thrust in excess of that necessary for supporting the aircraft, the location and direction of the control force vectors, and whether the control forces are derived independently from the thrust source or are obtained by some manipulation of the lift vectors themselves. Coupling can occur in combinations of lift-roll, lift-pitch, and even lift-yaw depending on the control configuration.

Some of the early jet-lift VTOL aircraft, such as the Short SC-1, the Bell X-14A, and the Lockheed XV-4, use continuous engine compressor bleed air for attitude control. This method employs continuous discharge air through the attitude control nozzles in a way that produces no net moments on the aircraft when it is not necessary to correct the aircraft attitude. Because of the small quantities of bleed air available, such schemes led to minimal height-control–attitude-control cross coupling, but this advantage was often offset by limitations in efficiency and control power. Some of the more recent jet-lift VTOL airplanes, particularly the larger ones such as the Dornier Do-31 and the EWR VJ-101, use direct jet-lift for roll-attitude control by differentially modulating the thrust of one or more pairs of laterally displaced lifting engines. This scheme makes the height-control–attitude-control cross-coupling problem potentially more severe.

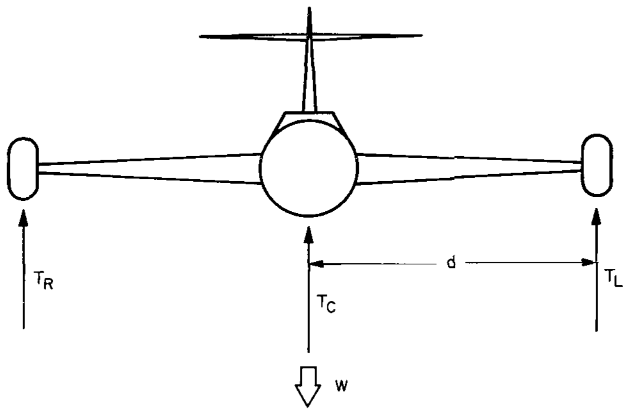

Figure 1.–Typical jet-lift VTOL aircraft.

This report describes a study to determine the effects of height-control–attitude-control cross coupling on vehicle handling qualities during maneuvers associated with hovering flight. The investigation was conducted on the Ames six-degrees-of-freedom motion simulator. All degrees of motion were activated but the study was limited to coupling of the lift-roll type, using a simplified representation of a typical jet-lift VTOL configuration (fig. 1).

DESCRIPTION

Simulator

The Ames six-degrees-of-freedom motion simulator (ref. 1) is shown in figure 2, and its physical specifications are given in table 1. The linear travel limits of ±2.75m (±9 ft) provide a 5.5m (18 ft) cube of space in which the simulated aircraft could maneuver. The angular travel limits on the machine are ±45°.

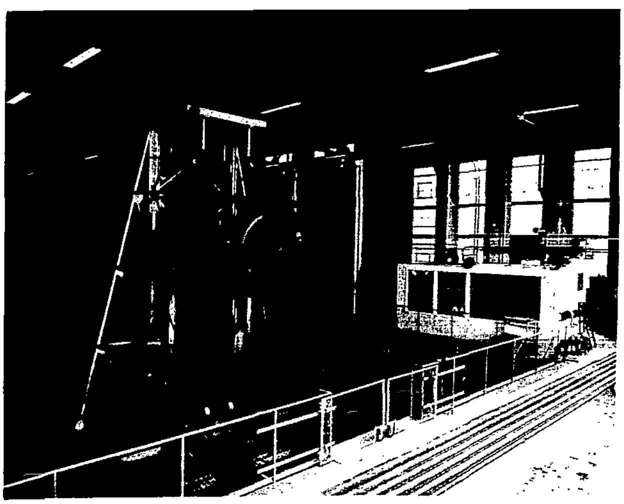

Figure 2.–Six-degrees-of-freedom motion simulator.

Since this was essentially a hovering simulation, no motion scaling was necessary. Consequently the cockpit was left open so the pilot could use an actual outdoor scene that was available by opening the large doors of the building housing the simulator. Basic cockpit instruments were provided but the pilot's primary visual cue was the outdoor scene.

The control system consisted of a center control stick with undamped force gradients of about 1.75 N/cm (1 lb/in.) in both the pitch and roll axes, rudder pedals with no centering and no force gradients, and a left-hand throttle quadrant. For both pitch and roll, the stick had travel limits of ±12.7 cm (±5 in.) and a control sensitivity of 0.236 rad/sec^2/cm (0.6 rad/sec^2/ in.). (As explained later, roll-control sensitivity was cut in half during conditions of lift-roll coupling.) The rudder pedal travel limits were ±7 cm. (±2.75 in.). The maximum throttle travel was 15.2 cm (6 in.) with a sensitivity of 0.08 g/cm (0.2 g/in.). A variable bias was used to change the maximum thrust-to-weight ratio.

TABLE 1.– SPECIFICATIONS OF SIX–DEGREES–OF–FREEDOM MOTION SIMULATOR

Axis	Acceleration, m/sec^2 (ft/sec^2)	Rate, m/sec (ft/sec)	Displacement, m (ft)
Longitudinal	±2.29 (±7.5)	±2.75 (±9.0)	±2.75 (±9.0)
Lateral	±2.80 (±9.2)	±2.44 (±8.0)	±2.75 (±9.0)
Vertical	±2.68 (±8.8)	±2.29 (±7.5)	±2.75 (±9.0)
Axis	rad/sec^2	rad/sec	deg
Roll	±10.0	±1.3	±40
Pitch	±4.5	±1.7	±40
Yaw	±3.0	±3.0	±40

The simulator was driven from a general purpose analog computer programmed with equations of motion applicable to a hovering VTOL aircraft. Since aerodynamic reactions are negligible in hover, the program did not include any forces or moments derived through aerodynamic means. Approximations were made for small angles that would be appropriate in the hover situation.

Task

The pilot, always the same man, had to perform precision hovering and maneuvering within the limits of simulator travel. He also did lateral quick-stops and roll reversals to help assess the vehicle's handling qualities.

On the six-degrees-of-freedom simulator the lateral quick-stop is performed by starting from a steady hover, translating approximately 4.57m (15 ft) and reestablishing a steady hover. For this investigation, the period of the quick-stop maneuver was about 3.5 sec. The roll reversal was a commanded roll oscillation of about 3 cycles with a period of approximately 1.5 sec.

Vehicle Simulated

A simplified aircraft configuration was used with three lift engines. One engine was mounted in the fuselage on the aircraft's roll axis and one was mounted on each wing tip a distance, d, from the roll axis. The engines were considered as single units, but they could also represent three sets of engines with each set acting collectively as if it were a single engine. The distribution of thrust among the engines was such that in trimmed hovering flight with a thrust-to-weight ratio of one, 50 percent of the total thrust was provided by the center engine and 25 percent by each outboard engine. The ratio of maximum thrust to hover thrust was the same for all engines.

Height control was obtained by modulating the thrust of all three engines in unison. Roll control was achieved by modulating the thrust of the outboard engines differentially; that is, by raising the thrust level of one outboard engine and lowering the thrust level of the opposite outboard engine so that a rolling moment was generated but the total lift force remained constant.

Engine thrust limits are involved with this method of control; consequently there can be a cross coupling between roll control and height control. For example, any time a commanded roll acceleration requires a thrust level in excess of the maximum possible for an outboard engine, the thrust of that engine will simply saturate at its limit. Meanwhile, the thrust of the opposite engine will continue to decrease, causing a loss of net lift as well as a decrease in the rate at which control power is developed. The reverse condition is also possible. If enough vertical acceleration is commanded while the outboard engines are producing differential thrust to satisfy a roll-control command, one outboard engine will reach its thrust limit before the other. Beyond that point, height control is not only less effective, it has a direct reducing effect on rolling moment (fig. 3). The effect of lift-roll coupling on roll-control sensitivity is illustrated in figure 4. As lateral stick displacement is increased, the roll acceleration is determined by following the uncoupled roll-control line in figure 4. The point at which an outboard engine reaches its thrust limit determines the maximum uncoupled roll acceleration, $\ddot{\phi}_{mu}$. As the lateral stick displacement continues to increase, the roll acceleration increases, but at half the rate because only one engine is changing thrust. Consequently, the roll-acceleration trace parallels the coupled roll-control line shown in the figure. As a result of the mechanization used in this investigation, the maximum roll acceleration could never exceed 3.0 nor fall below 1.5 rad/sec^2.

This investigation was limited to a study of the problems of height loss due to roll control. The limited vertical travel of the simulator made it impossible to examine the problem in its reverse sense where available roll control is reduced because of the command of high thrust-to-weight ratios.

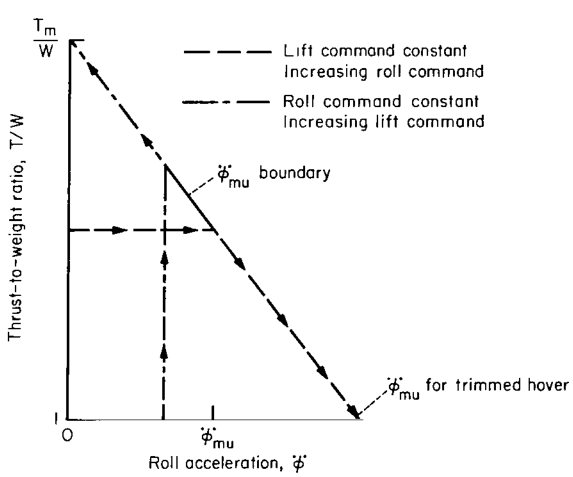

Figure 3.— The two ways lift-roll cross coupling can occur.

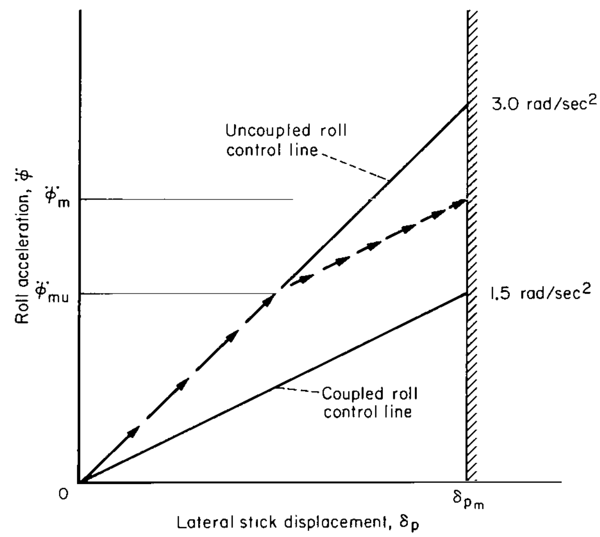

Figure 4.— Effect of lift-roll coupling on roll-control sensitivity.

To relieve the pilot's workload and allow him to concentrate on the control system's lift-roll cross-coupling characteristics, the simulated vehicle was provided with rate and attitude stabilization. For this investigation the undamped natural frequency of the stabilization system was 2 rad/sec, and the damping ratio was 0.7. These were found to be optimum values in an earlier investigation (ref. 2).

DERIVATION OF COUPLING PARAMETERS

A relationship (eq. (1)) was developed for the propulsion concept studied to understand the mechanism by which the lift-roll cross coupling occurs. This equation gives the maximum roll-control power available without introducing lift-roll cross coupling (for the derivation see appendix A).

$$\ddot{\phi}_{mu} = 2\frac{Wd}{I_x}\frac{T_{o_h}}{W}\left(\frac{T_{o_m}}{T_{o_h}} - 1\right)\left[1-\frac{\left(\frac{T}{W} - 1\right)}{\left(\frac{T_m}{W} - 1\right)}\right] \tag{1}$$

In this report $T_{oh}/W = 0.25$ and $T_{om}/T_{oh} = T_m/W$. Consequently, the control power relationship reduces to

$$\ddot{\phi}_{mu} = 0.5\,\frac{Wd}{I_x}\left(\frac{T_m}{W} - \frac{T}{W}\right) \tag{2}$$

This relationship indicates that the parameter I_x/Wd is fundamental to the uncoupled control power boundary. Values of this parameter ranging from 0 to 0.5 were investigated.

From equation (1) it is seen that the effect of altering T_{oh}/W, which, in essence, establishes the thrust distribution among the engines, can be compensated for by altering T_{om}/T_{oh} (the outboard engine maximum thrust to hover thrust ratio) so as to maintain $T_{oh}/W\ (T_{om}/T_{oh} - 1)$ constant. Therefore, the same maximum uncoupled roll-control power would be available and pilot opinion concerning roll would not be largely affected. On the other hand, it is recognized that altering T_{oh}/W and T_{om}/T_{oh} in this manner could cause some changes in the height control characteristics. That is, the maximum thrust-to-weight ratio will change unless the center engine maximum thrust-to-hover thrust ratio, T_{cm}/T_{ch}, is changed in a compensatory manner. This second-order effect was considered negligible in this study. As a result, neither T_{oh}/W nor T_{om}/T_{oh} was considered as an independent parameter. This argument is valid provided thrust response time lags are negligible. As T_{om}/T_{oh} increases, however, it is characteristic for the thrust response time lags of turbine engines to increase to a point where they are no longer negligible. Since engines with ideal response time (zero lag) were simulated in this study, a further study with realistic values for thrust response time constants should include T_{O_m}/T_{O_h} and T_{O_h}/W as variables.

As developed in appendix B, the relationship between loss of lift (in terms of vertical acceleration) and roll-control power once thrust saturation occurs is indicated by

$$\frac{d\ddot{h}}{d\ddot{\varphi}} = -\frac{I_x}{Wd} g \tag{3}$$

As shown by this equation and equation (1), the parameter I_x/Wd is fundamental to the height-control–attitude-control configuration, and it is instrumental in two aspects of the lift-roll coupling problem. Not only does it affect the point at which coupling occurs (the maximum uncoupled control power), it also indicates the severity of the loss of lift once thrust saturation takes place.

RESULTS AND DISCUSSION

The results of the investigation are presented in a way to show the effects of maximum thrust-to-weight ratio and the coupling parameter I_x/Wd on the handling qualities of an attitude-stabilized vehicle in the hover mode. Pilot opinion is used to assess the handling qualities of the vehicle, and pilot comments are given when they provide additional information. A summary of pilot opinion and comments is presented in table 2.

The data are presented in figure 5 in the form of Cooper pilot ratings (ref. 3) vs. the coupling parameter I_x/Wd for constant values of T_m/W from 1.05 to 1.3. The boundaries between the satisfactory, unsatisfactory, and unacceptable handling qualities regions are indicated by lines of constant pilot ratings of 3-1/2 and 6-1/2.

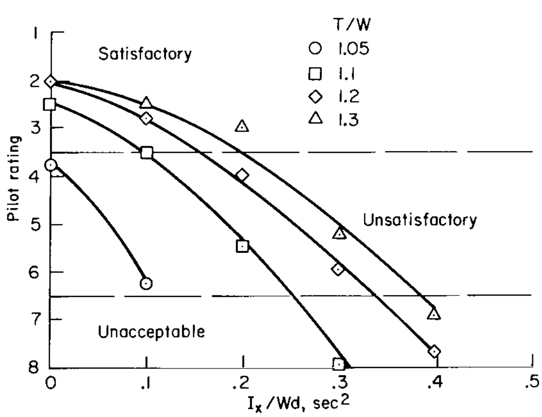

Figure 5.– Pilot ratings vs I_x/Wd.

The results can be discussed more meaningfully if replotted as shown in figure 6 where curves represent the boundaries between the satisfactory, unsatisfactory, and unacceptable handling qualities regions as a function of maximum thrust-to-weight ratio and the coupling parameter I_x/Wd. These results are repeated in figure 7 along with loci of constant uncoupled control power, which are represented by radial lines. The slopes of these lines are proportional to the maximum roll-control power available before encountering loss of lift. The slopes are calculated from equation (2) for a commanded thrust-to-weight ratio of unity and are given by $2\ddot{\phi}_{mu}$ for the aircraft under consideration. In addition to imparting a quantitative appreciation of the lift-roll coupling effect, the radial lines provide a means of comparing coupled systems

TABLE 2.–PILOT COMMENTS

T_m/W	I_x/Wd	Pilot rating	Comments
1.05	0	3-1/2 - 4	Limited by T/W.
1.05	.1	6 - 6-1/2	Can't check sink rate due to lack of T/W.
1.10	0	2-1/2	No problem.
1.10	.1	3-1/2	Using full thrust to recover from quick-stops. Roll reversals limited.
1.10	.2	5-1/2	Can't perform roll reversals. Can't maintain altitude in even mild quick-stops.
1.10	.3	8	Can't perform roll reversal. Must use full thrust for any maneuver.
1.20	0	2	No problem.
1.20	.1	2-1/2 - 3	Lift loss noted during maximum roll reversals.
1.20	.2	4	Losing roll response and gaining lift loss. Notice lift loss more during roll reversals than during quick-stops.
1.20	.3	6	Using full thrust to arrest sink rate from quick-stops. Lift loss noticeable during standard maneuvering. Not bad for mild maneuvering.
1.20	.4	7-1/2 - 8	Can't perform roll reversal; need full power to stop descent. Maneuverability very limited.
1.20	.6	8	Nearly unflyable.
1.3	.0	2-1/2	No problem.
1.3	.1	2-1/2	No loss of lift evident in roll reversal or quick-stop.
1.3	.2	3	Lift loss noted in roll reversal. Can adequately check lift loss in quick-stops with throttle.
1.3	.25	5	
1.3	.3	5 - 5-1/2	Getting harder to perform roll reversal. Have to overcontrol in quick-stop somewhat. Still operating within throttle limits.
1.3	.4	7	Reaching vertical acceleration limits of simulator.

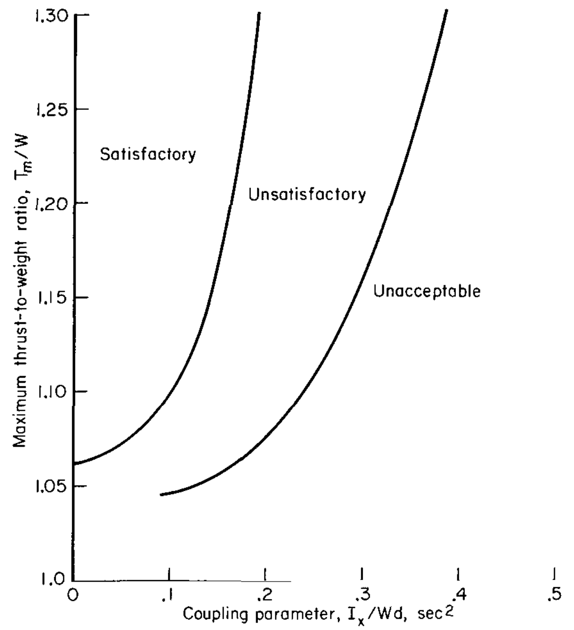

Figure 6.– Pilot rating boundaries as a function of maximum thrust-to-weight ratio and coupling parameter for a 25%/50%/25% thrust distribution.

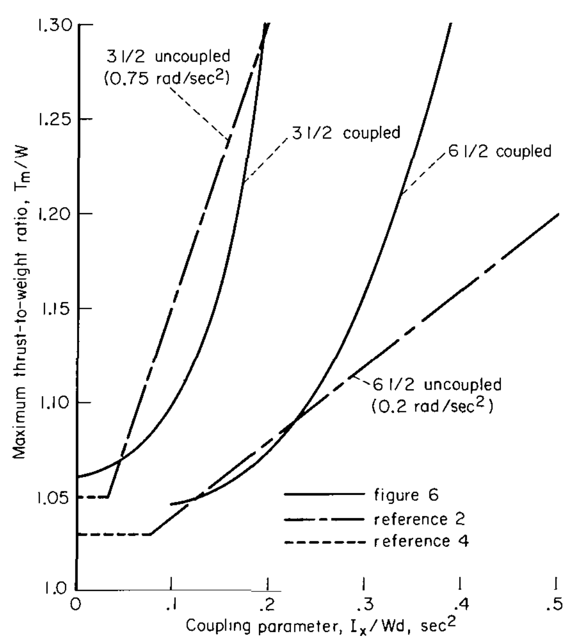

Figure 7.– Comparison of pilot rating boundaries between lift-roll coupled systems and uncoupled systems.

with uncoupled systems. The dashed lines in figure 7 are lines of constant maximum uncoupled roll-control power and also represent the boundaries between satisfactory, unsatisfactory, and unacceptable handling qualities for uncoupled systems as determined by the investigation of reference 2. The lower horizontal clipping of these boundaries represents the minimum height control requirements indicated in reference 4. Comparison of the two sets of results indicates that, as long as I_x/Wd is less than about 0.2, lift-roll coupled systems are not downrated because of any lift loss effects. Except for very low values of I_x/Wd, where height control requirements predominate, the ratings follow those of the uncoupled systems quite closely. If anything, they are slightly uprated, especially along the boundary for satisfactory operation, probably as a reflection of pilot appreciation for the large control power reserves offered by the coupled systems.

For values of I_x/Wd greater than 0.2, the advantage disappears. The reserve control power is still there, but pilots are reluctant to use it because of the accompanying severity of lift loss. In other words, the pilot is influenced not only by coupling but by the rate at which the loss of lift occurs. That rate is a direct function of I_x/Wd. When $I_x/Wd = 0.1$, the vertical acceleration is +0.1 g/rad/sec^2 of commanded roll acceleration once thrust saturation takes place. The pilot comments indicate that above this level the loss of lift in a moderately brisk lateral quick-stop maneuver becomes quite noticeable. At the larger values of I_x/Wd the pilot commented about the difficulty of maintaining precise altitude control during even mild roll reversals.

This sensitivity to I_x/Wd is illustrated in figure 8, which shows vertical displacement, lateral displacement, bank angle, and roll control displacement for two values of I_x/Wd while T_m/W = 1.2. Performance is shown for both roll reversal and lateral quick-

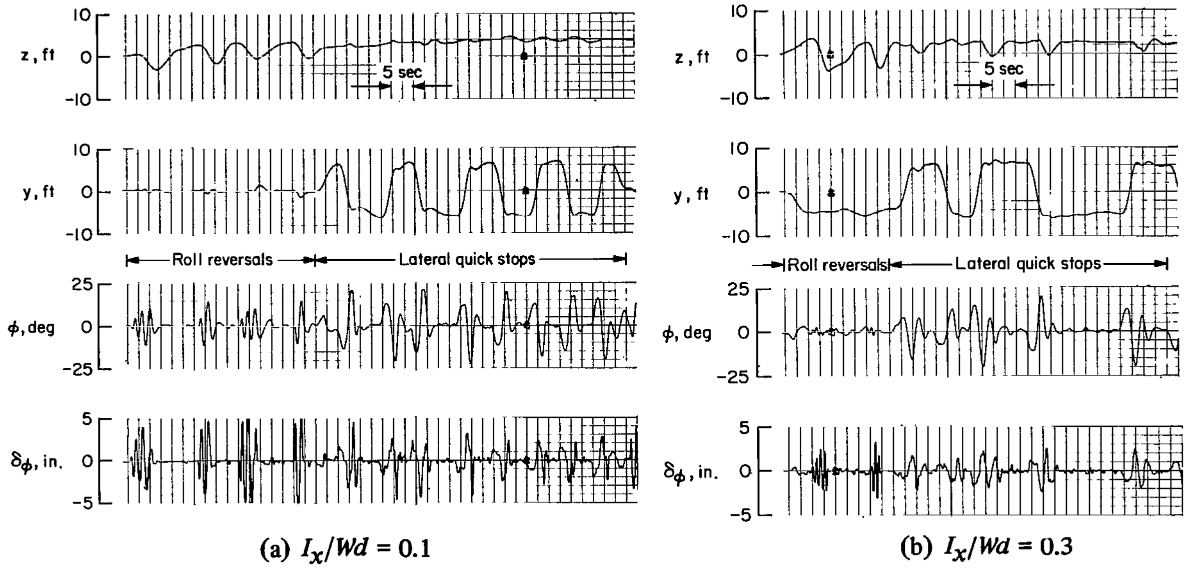

Figure 8.— Time history of vertical displacement (z), lateral displacement (y), bank angle (ϕ), and roll control displacement (δ_ϕ) for T/W = 1.2

stop maneuvers. Case (a) (I_x/Wd = 0.1) demonstrates a situation which the pilot rated 2-1/2 - 3. Under these conditions the pilot was able to perform roll reversals of ±12° with a corresponding altitude loss of about 1.22m (4 ft). Lateral quick-stop maneuvers resulted in essentially no altitude loss. In contrast, the pilot rated case (b) (I_x/Wd = 0.3) at 6. In that situation a roll reversal of ±2° was accompanied by an altitude loss of about 2.13m (7 ft). Full thrust was required to arrest the sink rate during lateral quick-stop maneuvers.

Values of I_x/Wd were calculated for two aircraft to demonstrate that the investigation had covered realistic values. The EWR VJ-101C has a value of I_x/Wd near 0.1, whereas the Dornier Do-31 has a value near 0.2. These two aircraft are not the same configuration as the one investigated, but they are very close. Thrust distributions are 33%/33%/33% for the VJ-101C and 25%/50%/25% for the Do-31. Both have attitude-stabilized control systems. Unfortunately, no pilot ratings are available over a range of gross weights or maximum T/W, so no data points could be shown to support the simulator results.

CONCLUDING REMARKS

The Ames six-degrees-of-freedom motion simulator was used to investigate lift-loss effects due to height-control—attitude-control cross coupling caused by engine-thrust saturation. The configuration studied was an attitude-stabilized vehicle with three engines thrusting vertically; one on the aircraft center line and two displaced left and right. Height control was achieved through collective thrust modulation of all engines. Roll control was derived through differential thrust modulation of the two outboard engines. The thrust distribution considered in the investigation was 50 percent from the center engine and 25 percent from each of the outboard engines when flying in a trimmed hover ($T/W = 1$). The ratio of maximum thrust to hover thrust was the same for all engines. The configuration studied was one with ideal thrust response to throttle commands. Further investigations are needed to include characteristic engine thrust time lags because the ratio of hover thrust to maximum thrust would then become an important variable. (When the thrust of a typical jet engine becomes a smaller percentage of its maximum thrust, the engine response time constant becomes longer.) The results of this investigation can be stated as follows:

1. Lift-roll cross coupling can be a problem in the hover situation. Its nature appears to depend strongly on two parameters: The coupling parameter, I_x/Wd, of the configuration; and the maximum thrust-to-weight ratio, T_m/W, of the propulsion system. In a trimmed hover, both of these parameters determine the point at which coupling occurs, but I_x/Wd alone determines the severity of lift loss after coupling occurs.

2. Handling qualities deteriorate significantly with increasing I_x/Wd, but for values less than about 0.2, good handling qualities can be restored by increasing T_m/W.

3. For values of I_x/Wd greater than 0.2, the increasing severity of lift-loss compounds the problem to the extent that disproportionate and eventually impractical increases in T_m/W are required to restore good handling qualities.

4. Analysis of some existing VTOL designs shows that practical values of I_x/Wd are in a range where lift/roll coupling can be a problem.

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Moffett Field, Calif., 94035, April 13, 1972

APPENDIX A

DERIVATION OF LINEARITY BOUNDARY IN CONTROL POWER THRUST TO WEIGHT RATIO PLANE

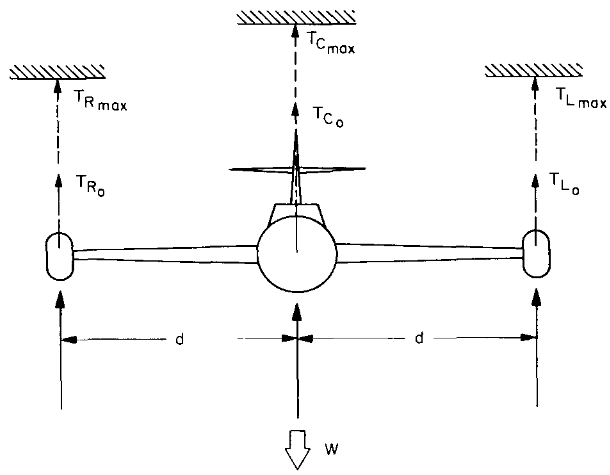

Figure 9.— Aircraft configuration.

When one outboard engine is at its maximum thrust limit, the other has an equally and oppositely displaced thrust value. Therefore, the maximum uncoupled rolling acceleration is

$$\ddot{\phi}_{mu} = \frac{2d}{I_x} \frac{T_{o_h}}{W} \left(T_m - T\right)$$

where

T_m	maximum vehicle thrust limit
T	trimmed vehicle thrust level for a given throttle setting
I_x	moment of inertia in the roll axis
d	the outboard engine displacement from the roll axis

Multiplying and dividing by W

$$\ddot{\phi}_{mu} = \frac{2Wd}{I_x} \frac{T_{o_h}}{W} \left(\frac{T_m}{W} - \frac{T}{W}\right)$$

$$= \frac{2Wd}{I_x} \frac{T_{o_h}}{W} \left(\frac{T_m}{W} - 1\right) - \left(\frac{T}{W} - 1\right)$$

which can be written

$$\ddot{\phi}_{mu} = \frac{2Wd}{I_x} \frac{T_{o_h}}{W} \left(\frac{T_m}{W} - 1\right) \left[1 - \frac{(T/W - 1)}{(T_m/W - 1)}\right]$$

But

$$\frac{T_m}{W} = \frac{T_{o_m}}{T_{o_h}}$$

therefore,

$$\ddot{\phi}_{mu} = 2 \frac{Wd}{I_x} \left(\frac{T_{o_h}}{W}\right)\left(\frac{T_{o_m}}{T_{o_h}} - 1\right) \left(1 - \frac{T/W - 1}{T_m/W - 1}\right)$$

APPENDIX B

DEVELOPMENT OF CROSS–COUPLING RELATIONSHIP

Assume that the right engine is at its maximum thrust limit. The change in thrust that provides the rolling moment will also cause a vertical acceleration.

Let

$\Delta\ddot{h}$ change in vertical acceleration due to roll control

$\Delta\ddot{\phi}$ change in roll acceleration

The change in thrust is related to the vertical acceleration by

$$\Delta T = \left(\frac{W}{g}\right)\left(-\Delta\ddot{h}\right)$$

and to the roll acceleration by

$$\Delta T = \frac{\Delta\ddot{\phi}\, I_x}{d}$$

Therefore,

$$\frac{\Delta\ddot{h}}{\Delta\ddot{\phi}} = -\frac{I_x}{Wd}\, g$$

and, in the limit at the point in question,

$$\frac{d\ddot{h}}{d\ddot{\phi}} = -\frac{I_x}{Wd}\, g$$

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