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ANALYTICAL PREDICTION OF THE HEAT TRANSFER FROM A BLOOD VESSEL NEAR THE SKIN SURFACE WHEN COOLED BY A SYMMETRICAL COOLING STRIP

by

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Technical Report No. ME-TR-344
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ABSTRACT

An analytical method has been developed to estimate the amount of heat extracted from an artery running close to the skin surface which is cooled in a symmetrical fashion by a cooling strip.

The results indicate that the optimum width of a cooling strip is approximately three times the depth to the centerline of the artery. The heat extracted from an artery with such a strip is about 0.9 w/m-°C which is too small to affect significantly the temperature of the blood flow through a main blood vessel, such as the carotid artery.

The method is applicable to veins as well.

NOMENCLATURE

A through H	locations in Fig. 1
a	width of tissue, (L)*
$2a\eta$	cooled width of skin, (L)
b	depth to centerline of artery, (L)
c_b	specific heat of blood, (E/M - T)
c_p	specific heat of tissue, (E/M - T)
d	depth to body core, (L)
F_1, F_2, F_3	constant heat fluxes, (E/L^2t)
k	thermal conductivity of tissue, (E/t - L - T)
L	length of cooled section, (L)
q	local volumetric heat generation rate, ($E/t - L^3$)
q_a	heat loss from 1/2 artery per unit length, (E/t - L)
q_1	heat extracted by cooling strip per unit length, (E/t - L)
Q	q/k, (T/L^2)
R	radius of artery, (L)
t	time, (t)
T	temperature, (T)
T_1	temperature of cooled skin, (T)
T_2	temperature of uncooled skin, (T)
T_a	arterial temperature, (T)
T_m	mean temperature, (T)
w	$(w_b c_b/k)^{1/2}$, (L^{-1})
w_b	blood perfusion rate, ($M/t - L^3$)
x	coordinate perpendicular to centerline of artery, parallel to skin surface, (L)

*Units in parentheses are: E, energy; M, mass; L, length; T, temperature; t, time.

y	coordinate perpendicular to centerline of artery, perpendicular to skin surface, (L)
β	length ratio AA'/AF, approaching zero in the limit
γ	length ratio AG/AF
ζ_n	$(w^2 + \lambda_n^2)^{1/2}$, (L^{-1})
η	length ratio CD/CE
θ	$(T - T_1)/\theta_o$
θ'	$T - T_a$, (T)
$\theta_o = -\theta_1$	$T_a - T_1$, (T)
$\theta_1 = -\theta_o$	$T_1 - T_a$, (T)
θ_2	$T_2 - T_a$, (T)
λ_n	$n\pi/a$, (L^{-1})
ρ	density of tissue, (M/L^3)

SUBSCRIPTS

a	arterial
o	reference condition

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INTRODUCTION

Experiments on localized cooling of the neck skin above the carotid artery indicated a significant effect of this cooling on the thermal comfort sensation of the individual [1]*. An analytical study of the thermal interaction between an artery near the skin and a cooling patch on the skin surface was undertaken to estimate the amount of heat that can be extracted from a blood vessel transcutaneously. The results should help to determine if the effect on the thermal comfort is due to indirect hypothalamic cooling by a reduction of the arterial blood supply temperature or due to some other influence such as that of a local cold receptor.

It is to be understood that the method is applicable to veins as well even though the term "artery" is used throughout this work.

*Numbers in brackets refer to entries in REFERENCES.

ANALYSIS

The analysis depends on the solution of the "bio-heat" equation [2,3]:

$$\rho c_p \frac{\partial T}{\partial t} = k\nabla^2 T + w_b c_b (T_a - T) + q \qquad (1)$$

The following assumptions were made in developing this equation [3]:

1. The thermophysical properties of the tissue are constant and the tissue is homogeneous and isotropic.
2. Blood enters the tissue at a constant arterial temperature, T_a, and leaves at the local temperature of the tissue, T. (This last assumption is based on the almost perfect heat exchange occurring in the capillary bed.)
3. Blood perfusion, w_b, and metabolic heat production rates, q, are uniform and constant everywhere in the tissue.

The boundary conditions depend on the assumed geometry as well as on considerations which make the solutions easier to obtain while retaining a reasonable similarity with the physical situation.

Figure 1 depicts the basic tissue geometry considered. It is assumed that the cooling patch runs parallel and symmetrically to the artery. If the temperature gradients along the artery are neglected in comparison to those perpendicular to the artery, the problem becomes two dimensional as shown.

The line of symmetry, through the centerline of the artery A-C, must be adiabatic. Along the skin surface, C-D and D-E, temperatures or heat fluxes can be specified. Since the temperature measurements are much more reliable than flux measurements at the skin surface,

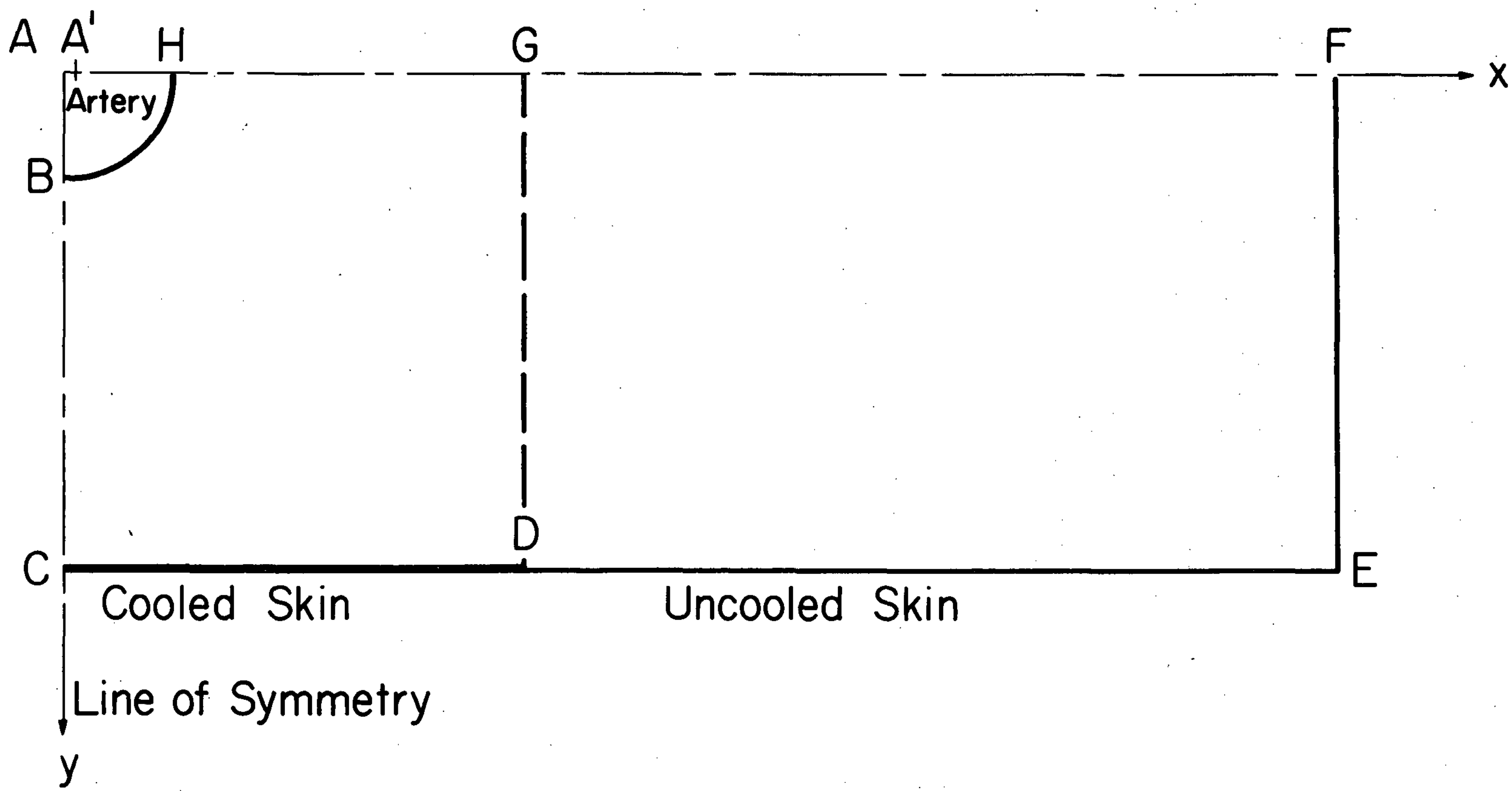

Figure 1 Geometrical configuration of the cooling strip and blood vessel.

temperatures are selected as boundary conditions. If the boundary, E-F, is far enough away from the artery as described below, it may be considered adiabatic. This assumption is equivalent to saying that at long distances from the artery and from the cooling patch, the heat flow is one dimensional, corresponding to a temperature difference between the uncooled skin and the body core located at some fixed distance under the skin. By numerical computations it was found that these two conditions are essentially identical if η = CD/CE < 1/3. For all calculations $\eta = 0.25$ was used thereafter. At the surface of the artery, B-H, the temperature should be T_a. Since it is difficult to solve a basically rectangular problem with conditions specified along a curved boundary, an approximation was developed which depended on the specification of the heat flux along the inner boundary, A-F. The approximation is based on the well established fact that the two-dimensional isotherms (constant temperature lines) around a point heat source are circular, (cf. [4]). Consequently, specifying a heat flux of some appropriate intensity at the centerline of the artery, A, will produce circular temperature lines, one of which can be made to coincide as nearly as possible with the surface of the artery. Near the other end of the inner boundary, F, a uniform heat flux can be specified corresponding to a temperature difference between the uncooled skin and the body core. Between A and F, the flux can be somewhat arbitrary, but it must be zero near A to keep the isotherms circular. As will be seen later, the flux distribution is expressed in terms of Fourier series which become more involved as the flux function becomes more complicated.

It has been found, however, that the most important single result, i.e., the total heat lost from the artery, is rather insensitive to the exact shape of the flux function. Therefore, in addition to the heat source at A, zero flux is assumed along A-G and an appropriate, uniform flux is assumed along G-F. G is arbitrarily located under the outer edge of the cooling strip, D. Finally, deep body temperature is assumed to be equal to T_a.

If the origin of the coordinate system is located at the center of the artery, the problem in mathematical terms becomes as follows:

$$\nabla^2\theta' - w^2\theta' = -Q \qquad (2)$$

where

$$\theta' = T - T_a \qquad (3)$$

$$Q = \frac{q}{k} \qquad (4)$$

$$w^2 = \frac{w_b c_b}{k} \qquad (5)$$

The boundary conditions are

$$\frac{\partial\theta'}{\partial x} = 0 \text{ at } x = 0 \qquad (6a)$$

$$\frac{\partial\theta'}{\partial x} = 0 \text{ at } x = a \qquad (6b)$$

$$\frac{\partial\theta'}{\partial y} = -\frac{F_1}{k} \text{ at } 0 < x < \beta a,\ y = 0 \qquad (6c)$$

where

$$\beta = \frac{AA'}{AF} \to 0$$

$$\frac{\partial\theta'}{\partial y} = 0 \text{ at } \beta a < x < \gamma a,\ y = 0 \qquad (6d)$$

$$\frac{\partial\theta'}{\partial y} = -\frac{F_2}{k} \text{ at } \gamma a \leq x \leq a,\ y = 0 \qquad (6e)$$

where $\gamma = \frac{AG}{AF}$

$$\theta' = \theta_1 \text{ at } 0 < x < \eta a,\ y = b \tag{6f}$$

$$\theta' = \theta_2 \text{ at } \eta a < x < a,\ y = b \tag{6g}$$

where $\eta = \frac{CD}{CE}$, and

$$\theta_1 = T_1 - T_a \tag{7}$$

$$\theta_2 = T_2 - T_a \tag{8}$$

To non-dimensionalize the problem and to utilize the fact that T_1 is the lowest temperature in the system, define

$$\theta_o = -\theta_1 = T_a - T_1 > 0 \tag{9}$$

$$\theta = \frac{T - T_1}{\theta_o} = 1 - \frac{\theta'}{\theta_1} \tag{10}$$

The solution can be obtained by appropriate transformations of the variables and by Fourier series expansion of the temperature and flux functions [3]. The result is:

$$\begin{aligned}\theta = 1 &- \left[\frac{Q}{w^2\theta_o} + \left(1 - \frac{\theta_2}{\theta_1}\right) + \frac{\theta_2}{\theta_1}\right]\frac{\cosh (wy)}{\cosh (wb)} + \frac{Q}{w^2\theta_o} \\ &+ \frac{F_1\beta + F_2(1 - \gamma)}{kw\theta_o}\left[\frac{\sinh [w(b - y)]}{\cosh (wb)}\right] \\ &+ 2\sum_{n=1}^{\infty}\left\{\frac{F_1\beta}{k\theta_o\zeta_n}\sinh [\zeta_n(b - y)] - \frac{F_2}{k\theta_o\zeta_n}\right. \\ &\left.\cdot \frac{\sin (n\pi\gamma)}{n\pi}\sinh [\zeta_n(b - y)] - \frac{\left(1 - \frac{\theta_2}{\theta_1}\right)\sin (n\pi\eta)\cosh (\zeta_n y)}{n\pi}\right\} \\ &\cdot \frac{\cos (\lambda_n x)}{\cosh (\zeta_n b)}\end{aligned} \tag{11}$$

where

$$\lambda_n = \frac{n\pi}{a} \tag{12}$$

$$\zeta_n^2 = w^2 + \lambda_n^2 \tag{13}$$

F_2 can be evaluated from the one-dimensional temperature distribution [3,5] calculated for a depth, d, from the skin surface to the body core and a temperature difference of $(T_a - T_2)$.

$$\frac{F_2}{kw\theta_o} = \frac{F_3}{kw\theta_o}\,\frac{\cosh\,[w(d-b)]}{\cosh\,(wd)} - \frac{Q}{w^2\theta_o}\,\frac{\sinh\,(wb)}{\cosh\,(wd)} \tag{14}$$

where

$$\frac{F_3}{kw\theta_o} = \frac{\dfrac{\theta_2}{\theta_1} + \dfrac{Q}{w^2\theta_o}\left[1 - \dfrac{1}{\cosh\,(wd)}\right]}{\tanh\,(wd)} \tag{15}$$

F_1 must be selected to provide $\theta = 1$ at the circumference of the artery at a distance R from the center. Mathematically, this condition can be satisfied only at one point, which was arbitrarily chosen to be

$$\theta = 1 \text{ at } x = R,\ y = 0 \tag{16}$$

Thus, from Eq. (11)

$$\begin{aligned}\frac{F_1\beta}{kw\theta_o} = \Bigg\{&\left[\frac{Q}{w^2\theta_o} + \left(1 - \frac{\theta_2}{\theta_1}\right)\eta + \frac{\theta_2}{\theta_1}\right]\frac{1}{\cosh\,(wb)} - \frac{Q}{w^2\theta_o} \\ &- \frac{F_2(1-\gamma)\tanh\,(wb)}{kw\theta_o} \\ &+ 2\sum_{n=1}^{\infty}\left[\frac{F_2}{k\theta_o\zeta_n}\,\frac{\sin\,(n\pi\gamma)}{n\pi}\tanh\,(\zeta_n b) + \frac{\left(1 - \dfrac{\theta_2}{\theta_1}\right)\sin\,(n\pi\eta)}{n\,\cosh\,(\zeta_n b)}\right] \\ &\cdot \cos\,(\lambda_n R)\Bigg\} \Bigg/ \left\{\tanh\,(wb) + 2w\sum_{n=1}^{\infty}\frac{\tanh\,(\zeta_n b)}{\zeta_n}\cos\,(\lambda_n R)\right\}\end{aligned} \tag{17a}$$

An alternate selection was

$$\theta = 1 \text{ at } x = 0,\ y = R$$

Then, from Eq. (11)

$$\frac{F_1 \beta}{kw\theta_o} = \left\{ \left[\frac{Q}{w^2\theta_o} + \left(1 - \frac{\theta_2}{\theta_1}\right)\eta + \frac{\theta_2}{\theta_1} \right] \frac{\cosh (wR)}{\cosh (wb)} - \frac{Q}{w^2\theta_o} \right.$$

$$- \frac{F_2(1 - \gamma)}{kw\theta_o} \frac{\sinh [w(b - R)]}{\cosh (wb)}$$

$$+ 2 \sum_{n=1}^{\infty} \left[\frac{F_2}{k\theta_o \zeta_n} \frac{\sin (n\pi\gamma)}{n\pi} \sinh [\zeta_n (b - R)] \right.$$

$$\left. \left. + \frac{\left(1 - \frac{\theta_2}{\theta_1}\right) \sin (n\pi\eta) \cosh (\zeta_n R)}{n\pi} \right] \frac{1}{\cosh (\zeta_n b)} \right\} \Bigg/$$

$$\left\{ \frac{\sinh [w(b - R)]}{\cosh (wb)} + 2w \sum_{n=1}^{\infty} \frac{\sinh [\zeta_n (b - R)]}{\zeta_n \cosh (\zeta_n b)} \right\} \quad (17b)$$

The two alternatives, Eqs. (17a) and (17b), gave similar results as long as the $\theta = 1$ isotherm was nearly circular, i.e., as long as a constant arterial surface temperature was closely approximated. Equation (17) generally gave slightly higher heat extraction rates from the artery due to the slightly higher temperature gradients occurring between B and C (see Fig. 1) with this equation.

For evaluating all the series on a digital computer, at least 30 terms were used to obtain good convergence.

The heat lost per unit length from the quarter artery shown in Fig. 1 is equal to

$$\frac{q_a}{2} = F\ \beta a + \frac{q\pi R^2}{4} + \frac{w_b c_b (T_a - T_m)\ \pi R^2}{4} \quad (18)$$

where q_a is the heat loss per unit time per unit length of the skin-side half of the artery.

The first term on the right-hand side is the heat produced by the source at the centerline. The second term is the heat generated within the quarter circle. The third term represents an approximation of the heat supplied by the assumed "blood flow" within the quarter circle. The mean tissue temperature, T_m, was calculated as the average of 12 temperatures representing 12 equal areas within the quarter circle. This last term was usually negative. The heat removed from the deeper half of the artery must be less than q_a; it was estimated to be generally less than 15 percent of q_a. Because of the uncertainties in establishing the boundary conditions for the other half of the artery, only q_a was considered for all numerical work and comparisons.

The heat extracted by the cooling strip per unit length can be found by integrating the heat flux at y = b from x = 0 to x = ηa and doubling the result. In dimensionless form this expression becomes

$$\frac{q_l b}{k\theta_o a} = 2wb\left\{\left[\frac{Q}{w^2\theta_o} + \left(1 - \frac{\theta_2}{\theta_1}\right)\eta + \frac{\theta_2}{\theta_1}\right]\tanh(wb) + \frac{F_1\beta + F_2(1-\gamma)}{kw\theta_o \cosh(wb)}\right\}\eta + 4wb\sum_{n=1}^{\infty}\left\{\frac{F_1\beta}{kw\theta_o} - \frac{F_2}{kw\theta_o}\frac{\sin(n\pi\gamma)}{n\pi} + \frac{\left(1 - \frac{\theta_2}{\theta_1}\right)\sin(n\pi\eta)\,\zeta_n \sinh(\zeta_n b)}{wn\pi}\right\}\frac{\sin(n\pi\eta)}{n\pi\cosh(\zeta_n b)} \qquad (19)$$

DISCUSSION

Although the heat lost from the artery is the most significant result, it is worth while to investigate other aspects of the problem as well. Examination of the analytical results indicates that the following dimensionless parameters occur:

$$\theta, \frac{Q}{w^2\theta_o} \frac{\theta_2}{\theta_1}, \gamma, \eta, wb, wd$$

In addition, other geometrical parameters such as $b_o/2\eta a$, b/b_o, and R_o/R can be considered.

The influence of η was discussed earlier. It was always taken as 0.25. Variations of γ and wd were found to affect the heat lost from the artery only minimally. Therefore, γ was made equal to η and d was set equal to b.

For the sake of comparisons, the following additional quantities were selected for "normal" or "reference" conditions [3,6]:

Width of cooling strip • • • • • • $2\eta a$ = 0.0264 m

Depth to centerline of artery• • • b_o = 0.01 m = 1 cm

Radius of artery • • • • • • • • • • R_o = 0.0025 m = 0.25 cm

Temperature ratio • • • • • • • • θ_2/θ_1 = 0.3 and 0.15

Blood heat capacity rate • • • • $w_b c_b$ = 1746 $w/m^3\,{}^oC$

Heat generation rate • • • • • • • • • q = 660 w/m^3

Thermal conductivity of tissue • • • k = 0.54 w/m^oC

Figure 2 shows the dimensionless temperature distribution in the vicinity of the artery under the assumed "normal" conditions. The θ = 1.0 line is virtually circular, as is required for good modeling of the arterial wall temperature. The effect of the temperature

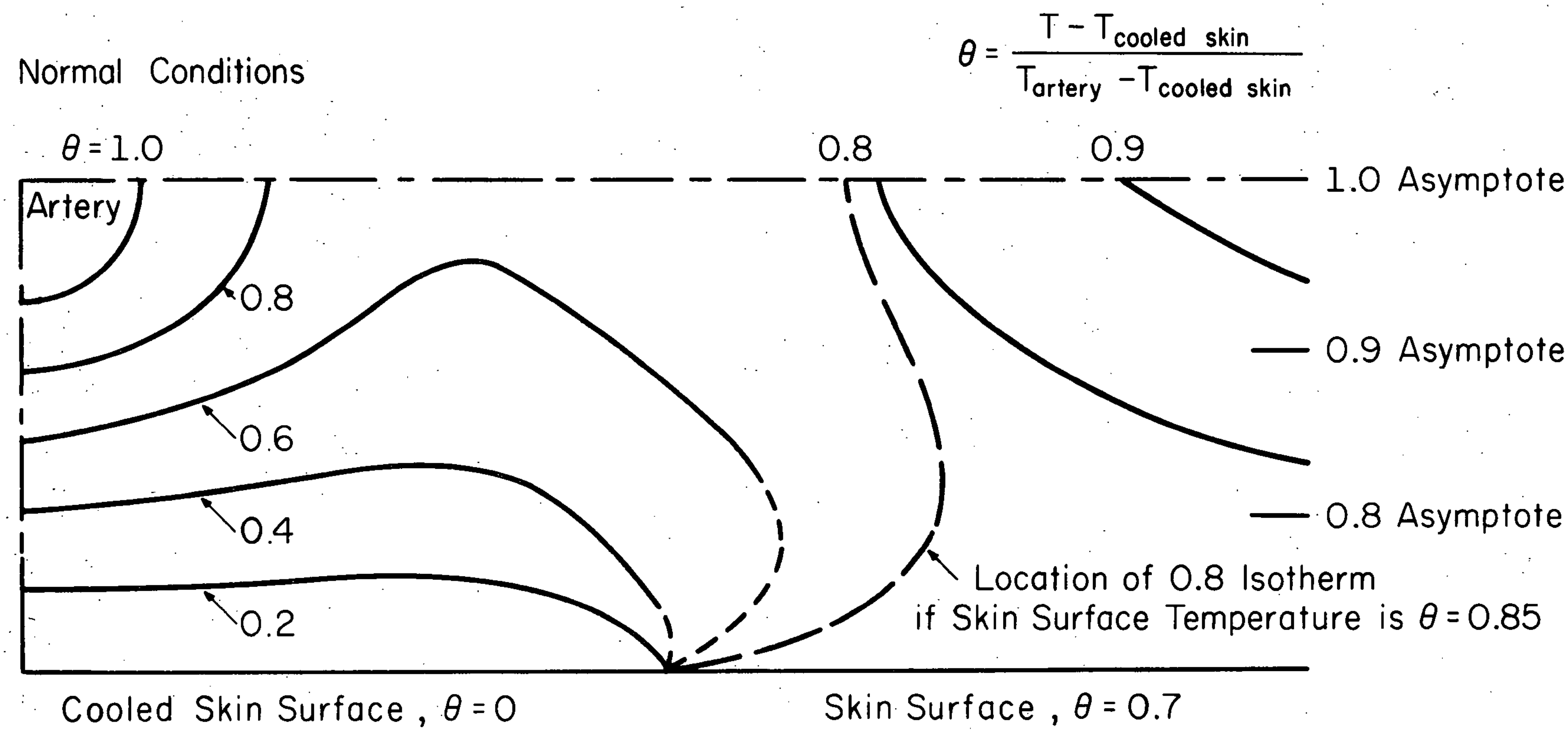

Figure 2 Dimensionless temperature distribution in the vicinity of a cooled artery under the assumed "normal" conditions.

ratio, θ_2/θ_1, is significant only in the relatively unimportant area, DEFG (see Fig. 1), as is demonstrated by the $\theta = 0.8$ isotherm. The heat loss from the artery changed little for the two temperature ratios used because all this heat was extracted through the cooled portion of the skin as calculated from Eqs. (18) and (19).

Figure 3 should be compared with Fig. 2 to realize the effects of blood perfusion and internal heat generation rates on the temperature distributions. Increasing both of these variables shifts the isotherms toward the skin surface. The effects of the high heat generation rate are relatively small, whereas the increased blood perfusion rate changes the temperature field very significantly, as shown most dramatically by the $\theta = 0.8$ isotherm.

Figure 4 shows the heat extraction rate from an artery per unit length and unit temperature difference, $\theta_o = 1$, as a function of five dimensionless parameters, all of which have significant influence. One of the curves, showing the effect of $b_o/2\eta a$, behaves anomalously below $b_o/2\eta a = 0.36$. Physical considerations suggest an asymptotic leveling off of the curve as this parameter approaches zero, i.e., as the width of the cooling strip becomes very wide compared to the depth of the artery. Instead, the calculations show a decrease of heat extraction rates. Investigation of the temperature profiles, however, revealed that in this regime ($b_o/2\eta a < 0.36$) the $\theta = 1$ isotherm ceased to be circular (it flattened out) the deviation becoming more pronounced as the parameter decreased. In some instances, indications of a possible numerical instability in this regime also occurred. Thus, the model presented here should be considered inappropriate in this region. Instead, it should be assumed that the curve asymptotically

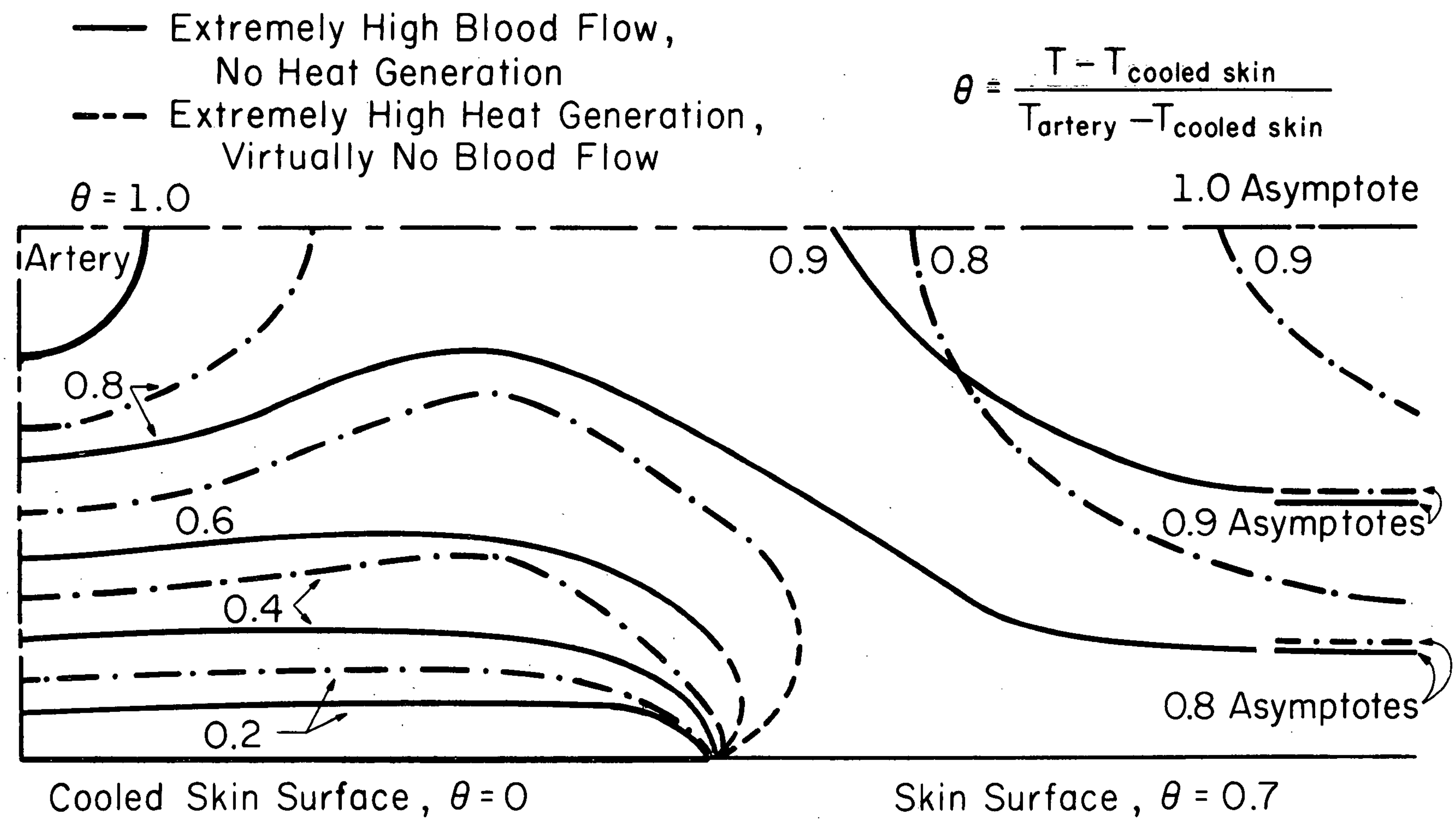

Figure 3 Dimensionless temperature distributions in the vicinity of a cooled artery with extremely high blood perfusion rate (wb = 2) and with extremely high internal heat generation rate ($Q/w^2\theta_o$ = 250) within the tissue.

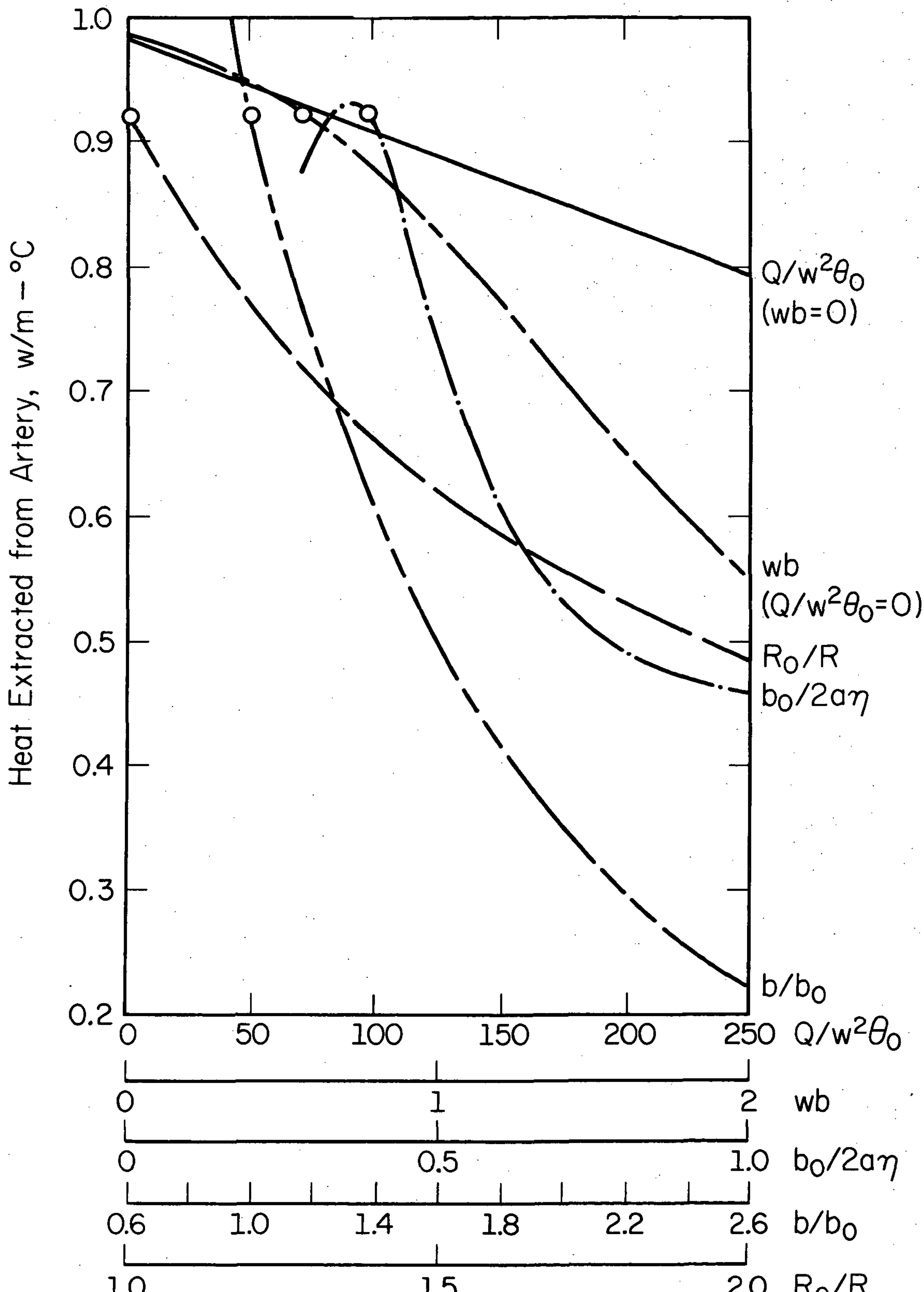

Figure 4 Heat extraction rates per unit length of artery, per unit temperature difference between artery and cooled skin. $\theta_2/\theta_1 = 0.15$. Circles indicate "normal" conditions.

reaches a maximum as $b_o/2\eta a$ approaches zero. This maximum should be just slightly higher than the maximum of the curve shown. Thus, the "normal" conditions are close to optimal as far as the width of the cooling strip is concerned. The optimum, i.e., minimum width for close to maximum heat extraction rate, is around a cooling width to arterial depth ratio of 3.

The remaining curves indicate that the heat extraction rate increases with decreasing internal heat generation rate ($Q/w^2\theta_o$), with decreasing blood flow rate (wb), with decreasing arterial depth (b/b_o), and with increasing arterial radius (i.e., decreasing R_o/R). These are, however, physiological parameters which usually cannot be controlled arbitrarily. It is to be noted, however, that if the cooling strip temperature could be lowered enough to cause local vasoconstriction, then the heat extraction rate from the artery would increase not only because of the increased temperature difference, but also as a result of decreased blood perfusion of the tissue.

The results indicate that under the "normal" conditions a square cooling patch with an area of 7 cm^2 (sides, $2a\eta$ = 2.64 cm) will extract about 0.025 watts from the artery for each 1°C difference between the artery and the cooled skin surface. On the other hand, the blood requires about 0.06 watts to change its temperature by 1°C for each cc/min of flow. Thus, if θ_o = 20°C and the arterial blood flow is 200 cc/min, the arterial blood temperature is expected to decrease only about 0.04°C. Even if allowance is made for additional heat loss from the deep side of the artery, this is indeed a very small temperature change. The temperature drop is too small to have any effect on the hypothalamic temperature regulator; consequently, the pronounced effects on thermal comfort found by Williams and Chambers [1] were probably due to some local thermal receptor action rather than to reduced arterial temperatures reaching the hypothalamus.

To check the magnitude of these results, the maximum possible heat extraction rate from the artery can be calculated by assuming an equivalent one-dimensional conduction problem from an area equal to that of the skin-side surface of the artery, $2\pi RL$, across a distance equal to the depth of the point on the artery closest to the skin, $b - R$.

$$\frac{q_a}{\theta_1} = \frac{2\pi Rk}{b-R} \tag{20}$$

Substituting the values for the "normal" condition yields a value of 1.13 w/m°C, which corresponds to about 0.03 w/°C for a patch with an area of 7 cm^2. Thus, the previously estimated value of 0.025 w/°C is quite reasonable.

CONCLUSIONS AND RECOMMENDATIONS

An analytical method has been developed to estimate the amount of heat extracted from an artery running near and parallel to the skin surface, such as the carotid artery in the neck, when the skin surface is cooled in a symmetrical fashion above the artery.

The results indicate that the optimum width of the cooling strip is approximately three times the depth to the centerline of the artery. However, even at the optimum size the amount of heat removed from a main artery with reasonable skin temperatures is too small to affect the temperature of the blood significantly.

A three-dimensional, finite difference method could be attempted to obtain a more accurate model of the actual, circular cooling patch used in the work reported on in [1]. However, the results presented in this report indicate that the additional accuracy is not going to change the values sufficiently to alter the conclusions drawn.

ACKNOWLEDGEMENTS

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