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DAVIS, CALIFORNIA

A COMPUTER PROGRAM FOR THE CALCULATION

OF LAMINAR AND TURBULENT
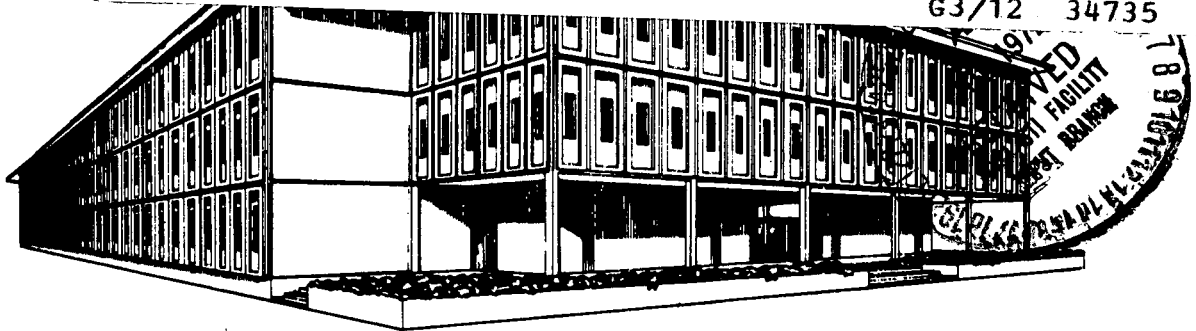
BOUNDARY LAYER FLOWS

NASA REPORT   CR-114366

Prepared by

H. A. Dwyer
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A. Introduction

This report presents the results of a study to produce a computer program to calculate laminar and turbulent boundary layer flows. The program is capable of calculating the following types of flow:

a. incompressible or compressible

b. two dimensional or axisymmetric

c. flows with significant transverse curvature

Also, the program can handle a large variety of boundary conditions, such as blowing or suction, arbitrary temperature distributions and arbitrary wall heat fluxes.

The present program has been specialized to the calculation of equilibrium air flows and all of the thermodynamic and transport properties used are for air. For the turbulent transport properties, the eddy viscosity approach has been used. Although the eddy viscosity models are semi-emperical, the model employed in the program has corrections for pressure gradients, suction and blowing and compressibility.

The basic method of approach is to put the equations of motion into a finite difference form and then solve them by use of a digital computer. The program is written in Fortan IV and requires small amounts of computer time on most scientific machines. For example, most laminar flows can be calculated in less than one minute of machine time, while turbulent flows usually require three or four minutes.

To show the usefulness of the present program, a sample of flow calculations have been carried out and are presented in the report. The calculations exhibit most of the different types of flow situations

mentioned above, and represent a good test of the engineering usefulness of the program. The only major difficulty with the program was the sensitivity of the calculations to the initial conditions for turbulent flow. Therefore, in the report we have stressed the importance of the initial conditions for turbulent flow.

B. Analysis

1. Boundary Layer Equations in the Physical Plane:

The boundary layer equation in a general form for compressible laminar or turbulent flows for two-dimensional and axisymmetric cases can be written as (References [1] and [2])

Continuity:
$$\frac{\partial}{\partial x}(\rho u r^j) + \frac{\partial}{\partial y}[r^j(\rho v + \overline{\rho'v'})] = 0 \tag{1}$$

Momentum:
$$\rho u \frac{\partial u}{\partial x} + (\rho v + \overline{\rho'v'})\frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} \tag{2}$$

$$+ \frac{1}{r^j}\frac{\partial}{\partial y}[r^j(\mu\frac{\partial u}{\partial y} - \rho\overline{u'v'})]$$

Energy:
$$\rho u \frac{\partial h}{\partial x} + (\rho v + \overline{\rho'v'})\frac{\partial h}{\partial y} = -u\rho_e u_e \frac{du_e}{dx} \tag{3}$$

$$+ (\mu + \varepsilon)(\frac{\partial u}{\partial y})^2 + \frac{1}{r^j}\frac{\partial}{\partial y}[r^j(\frac{k}{C_p}\frac{\partial h}{\partial y} - \rho\overline{v'h'})]$$

where the following notation has been used:

- x, y rectangular coordinates
- u, v x- and y- components of velocity
- ρ mass density
- h static enthalpy
- μ dynamic viscosity
- ε eddy viscosity
- k conductivity
- C_p specific heat at constant pressure
- $r = r_0(x) \pm y \cos \alpha$

  where $r_0(x)$ is the body radius and α is the angle between y and r (i.e., slope of body of revolution), (+) for an outside body case and (-) for an inside body case

$j = 0$ for two-dimensional flow and $j = 1$ for axisymmetric flow

primes refer to fluctuating quantities due to turbulence

subscript e - refers to outer edge of the boundary layer

and superscript $\overline{}$ - refers to average quantity.

The boundary conditions are:

Continuity and Momentum:

$$u(x,0) = 0$$

$$v(x,0) = 0 \text{ or } v(x,0) = v_w \text{ (mass transfer)} \tag{4}$$

$$\lim_{y\to\infty} u(x,y) = u_e(x)$$

Energy: $h(x,0) = h_w \text{ or } \frac{\partial h}{\partial y}(x,0) = \left(\frac{\partial h}{\partial y}\right)_w$

$$\lim_{y\to\infty} h(x,y) = h_e(x) \tag{5}$$

where the subscript w refers to the wall.

In order to solve the boundary layer equations subject to the above boundary conditions, we have to relate the time-mean fluctuating quantities $-\rho\overline{u'v'}$ and $-\rho\overline{v'h'}$ to the mean velocity and enthalpy. To do this the concept of eddy viscosity (ε_m) and eddy conductivity (ε_h) is used.

$$-\rho\overline{u'v'} = \rho\varepsilon\frac{\partial u}{\partial y}$$

$$-\rho\overline{v'h'} = \rho\varepsilon_h\frac{\partial h}{\partial y} \tag{6}$$

and introduce a turbulent Prandtl number Pr_t defined as

$$Pr_t = \varepsilon | \varepsilon_h \tag{7}$$

Defining also

$$\overline{\rho v} = \rho v + \overline{\rho'v'} \tag{8}$$

the boundary layer equations (1,2,3) then become

Continuity: $\frac{\partial}{\partial x}(\rho u\, r^j) + \frac{\partial}{\partial y}(\overline{\rho v}\, r^j) = 0$ (1-a)

Momentum: $\rho u \frac{\partial u}{\partial x} + \overline{\rho v} \frac{\partial u}{\partial y} = \rho_e u_e \frac{du_e}{dx} + \frac{1}{r^j} \frac{\partial}{\partial y} [r^j (\mu+\epsilon) \frac{\partial u}{\partial y}]$ \hfill (2-a)

Energy: $\rho u \frac{\partial h}{\partial x} + \overline{\rho v} \frac{\partial h}{\partial y} = -u\rho_e u_e \frac{du_e}{dx} + (\mu+\epsilon)(\frac{\partial u}{\partial y})^2$

$$+ \frac{1}{r^j} \frac{\partial}{\partial y} [r^j (\frac{\mu}{Pr} + \frac{\epsilon}{Pr_t}) \frac{\partial h}{\partial y}]$$

\hfill (3-a)

where $Pr \equiv \frac{C_p \mu}{k}$ is Prandtl number

It is clear that if we put $\epsilon = 0$ and $\overline{\rho v} = \rho v$, the above equations will reduce to the known form of the compressible laminar boundary layer equations.

2. Transformation of Coordinates:

Solving boundary layer equations can be considerably simplified by transforming the dependent and independent variables. The object of the transformations are the following: 1) limit the boundary layer growth due to compressibility and body shape and locate approximately the edge of the boundary layer, 2) decrease the changes and the gradients of velocity and enthalpy in both coordinate directions, and 3) remove the leading edge singularities and starting difficulties. The transformations which were used are (Reference [1])

$$\xi = \int_0^X (\rho\mu)_r u_e r^{2j} dx$$

\hfill (9)

$$\eta = \frac{u_e r^j}{(2\xi)^{1/2}} \int_0^y \rho \, dy$$

\hfill (10)

If we look carefully we find that these transformations include 1) Mangler transformation to take into account the axisymmetric flow, 2) Howarth transformation for compressibility, and 3) Blasius transformation for the changes due to variable u_e and viscous diffusion.

Using the above transformations, the edge of the boundary layer (defined as η at $\frac{u}{u_e}$ = 0.995) is located a priori. For the case of the laminar flow the edge of the boundary layer is around η = 6 while for turbulent flow it varies somewhat between η = 150 and η = 200.

3. <u>Boundary Layer Equations in the Transformed Coordinates</u>:

Using the transformations (9) and (10), the boundary layer equations (1-a, 2-a, 3-a) become:

Continuity:
$$2\xi \frac{\partial f'}{\partial \xi} + \frac{\partial v}{\partial \eta} + f' = 0 \qquad (1\text{-}b)$$

Momentum:
$$2\xi f' \frac{\partial f'}{\partial \eta} + V \frac{\partial f'}{\partial \eta} = \beta(\frac{T}{T_e} - f'^2) + \frac{\partial}{\partial \eta}(\ell\frac{\partial f'}{\partial \eta})$$
$$+ TC \cdot \ell_m \frac{\partial f'}{\partial \eta} \qquad (2\text{-}b)$$

Energy:
$$2\xi f' \frac{\partial \theta}{\partial \xi} + V\frac{\partial \theta}{\partial \eta} = \frac{u_e^2}{h_e}[\ell(\frac{\partial f'}{\partial \eta})^2 - \frac{T}{T_e}\beta f'] + \frac{\partial}{\partial \eta}(\ell_1\frac{\partial \theta}{\partial \eta})$$
$$+ TC \cdot \ell_2 \frac{\partial \theta}{\partial \eta} + \frac{f'u_e^2}{h_e}\beta\theta \qquad (3\text{-}b)$$

where
$$f' = \frac{u}{u_e} \quad , \quad \theta = \frac{h}{h_e}$$

$$V = \frac{2\xi}{(\rho\mu)_r u_e r^{2j}} (\overline{\rho v} \frac{r^j}{(2\xi)^{1/2}} + f' \frac{\partial \eta}{\partial x})$$

$$\beta = \frac{2\xi}{u_e} \frac{du_e}{d\xi} \quad , \quad \ell = \frac{\rho(\mu + \epsilon)}{(\rho\mu)r} \quad , \quad \ell_1 = \frac{\rho(\frac{u}{Pr} + \frac{\epsilon}{Pr_t})}{(\rho u)r}$$

$$\ell_m = \frac{(\mu + \epsilon)}{(\rho\mu)_r} \quad , \quad \ell_2 = \frac{(\frac{\mu}{Pr} + \frac{\epsilon}{Pr_t})}{(\rho\mu)_r}$$

$$TC = \frac{\overset{+}{-}(2\xi)^{1/2} \cos\alpha}{u_e r^{2j}}$$

and $(\rho\mu)_r$ is taken as a reference quantity which is evaluated at the surface.

The boundary conditions become:

Continuity and Momentum:

$$f'(\xi,0) = 0$$

$$V(\xi,0) = 0 \quad \text{or} \quad V(\xi,0) = \frac{(2\xi)^{1/2}(\rho v)_w}{(\rho\mu)_r u_e r^j} \tag{4-a}$$

$$\lim_{\eta\to\infty} f'(\xi,\eta) = 1$$

Energy:

$$\theta(\xi, 0) = \theta_w \quad \text{or} \quad \frac{\partial\theta}{\partial\eta}(\xi,0) = \left(\frac{\partial\theta}{\partial\eta}\right)_w$$

$$\tag{5-a}$$

$$\lim_{\eta\to\infty} \theta(\xi,\eta) = 1$$

C. Transport and Thermodynamic Properties

The following is a listing of all the transport and thermodynamic properties with references which have been used in the program. In general the program can be changed to accept any particular set of properties whether there be laminar or turbulent flow.

Viscosity

For the laminar viscosity coefficient, μ, the empirical formula for air developed by Keyes was chosen to be used. This formula is particularly good at low temperatures (Reference [9]), and this feature was needed for some of the temperatures encountered in the wind tunnel calculations. The formula is

$$\mu = \frac{.0232 \cdot 10^{-6} \, (T)^{1/2}}{(1 + \frac{220}{T \cdot 10^{+9/T}})} \quad \frac{lb - sec}{ft^2} \quad \text{for } T = Temp., \, ^{\circ}R$$

In the program the viscosity is given by the variable name TAB2 and the Keyes formula is given in a separate function subroutine.

Enthalpy Temperature Relationship

The relationship between the air enthalpy and temperature was established by use of the gas tables published by Keenan and Kaye, Reference [5]. The tabulated values were used for temperatures between one hundred and six thousand degrees Rankine. In the program the temperature was calculated from the enthalpy with the use of the function subroutine TAB. This function consisted of a table of temperature versus enthalpy with the enthalpy listed every one hundred degrees.

Turbulent Transport Properties

One of the most important parts of any turbulent boundary layer program is the choice of the eddy viscosity model. At the present time all current models are somewhat unsatisfactory for certain types of flows; however, present models do perform well for a large class of problems of engineering interest. (For proof of this statement, see the results in the later section of the report.)

The eddy viscosity ε_m is defined by the following relationship

$$- \rho \overline{u'v'} = \rho \varepsilon_m \frac{\partial \overline{u}}{\partial y}$$

One of the best semi-empirical models developed for ε_m is that used by Cebeci, Reference [4]. In this model the turbulent boundary layer is broken up into four regions where different expressions are used to model the turbulent flow. The four regions are the laminar sublayer, the transition layer, the fully-turbulent wall region, and the outer wake-like region. A brief description will now be given of the four regions and the expressions used.

Laminar Sublayer - In this region very near the wall the flow is dominated by viscosity and turbulent fluctuations are not important. Experimentally, it has been shown for smooth walls that the velocity and temperature distributions can be calculated by using laminar flow equations. Also, compressibility is important in this region, since viscous dissipation has a maximum in the laminar sublayer.

Transition Layer - Across the transition layer the flow in a turbulent boundary layer goes from one completely dominated by viscosity to one dominated by inertial fluctuations or turbulence. The problem

posed to an eddy viscosity model is to bridge the gap between the laminar sublayer and the fully-turbulent wall region. The best model for treating this region was developed by Van Driest, Reference [7], and extended by Cebeci, Reference [4]. The Van Driest and Cebeci model is based on an unsteady Couette flow analysis of the fluctuating terms and includes the influences of compressibility, mass transfer and pressure gradients. The transition region formula used is

$$\varepsilon_m = \ell^2 \left|\frac{\partial u}{\partial y}\right| [1 - \exp(-\frac{y}{A})]^2$$

The first two terms of the above expression are the Prandtl mixing length formula for the wall region, while the last expression is the Van Driest damping function for the transition region. The unknown semi-empirical constant A of the damping function is determined from the following equation

$$A = \frac{26 \, \nu_w}{v^*} \left(\frac{\rho_w}{\bar{\rho}}\right)^{1/2} \left(\frac{\bar{\mu}}{\mu_w}\right) \frac{1}{N}$$

where

$$N = \{\frac{\bar{\mu}}{\mu_e} \left(\frac{\rho_e}{\rho_w}\right)^{1/2} \frac{p^{+\dagger}}{v_w^+} [1 - \exp(11.8 \frac{\mu_w}{\bar{\mu}}) v_w^+]$$

$$+ \exp(11.8 \frac{\mu_w}{\bar{\mu}} v_w^+)\}^{1/2}$$

v^* is the friction velocity, μ the viscosity, and ρ density. The subscript e refers to quantities evaluated at the edge of the boundary layer, w the wall, and the $^-$ refers to quantities evaluated in the transition region. Also, the following definitions are used:

† Cebeci's paper has a misprint in it at this location.

$$p^+ = \frac{\dfrac{\nu_e}{u_e^2}\dfrac{du_e}{dx}}{(C_f/2)^{3/2}}$$

$$\frac{C_f}{2} = \frac{\mu\left.\dfrac{\partial u}{\partial y}\right|_w}{\rho_e u_e^2}$$

$$v_w^+ = \frac{v_w}{v^*}$$

$$v^* = \left(\frac{\mu\left.\dfrac{\partial u}{\partial y}\right|_w}{\rho_w}\right)^{1/2}$$

The ‾ quantities represent the average value of properties in the transition region, and the average was assumed equal to their value at $y^+ = 11.8$ where

$$y^+ = \frac{v^* y}{\nu_w}$$

Also, there are many specialized forms of the above expression for N which can be found in Reference [4].

Fully Turbulent Wall Region - The Prandtl mixing length model is used to describe the flow in the turbulent wall region. This model is essentially built into the Van Driest model as mentioned previously, and the only necessary description is to define the mixing length. The mixing length ℓ is defined as

$$\ell = ky$$

where k is a constant equal to .4 and y is the distance away from the wall.

Outer Wake-Like Region - In the outer portions of a turbulent boundary layer the eddy viscosity coefficient tends to take on a constant value somewhat like the flow in a jet or wake. To model this behavior the outer eddy viscosity formula developed by Cebeci was employed.

The formula is

$$\varepsilon_m = .0168 \, u_e \delta'$$

where

$$\delta' = \int_0^\delta \left(1 - \frac{u}{u_e}\right) dy$$

The region of the flow, where this model was used, was determined by finding the location in the boundary layer where ε_m from the Prandtl mixing length formula equaled ε_m from the outer region. For values of the boundary layer coordinate larger than this location the outer formula was used.

Cebeci also suggested the use of an intermittancy coefficient on the eddy viscosity model in the outer region. It has been our experience that the influence of intermittancy was usually very small and not noticeable in the calculations.

Prandtl Number

The Prandtl number for laminar flow was assumed constant in the program and to have a value of .723, Reference [5]. For turbulent flow the Prandtl number was also assumed constant and to have a value of .9 . Although this value of turbulent Prandtl number is not completely satisfactory, it does give good results for a wide variety of flow problems and boundary conditions.

D. Numerical Method

The boundary layer equations have a parabolic mathematical nature and pose a problem similar to the unsteady state heat conduction equation. For the numerical solution of the boundary layer equations the most successful schemes are modifications of the Crank-Nicolson and full implicit schemes used with the unsteady heat conduction equation, Reference [1]. In the present program both the Crank-Nicolson and full implicit scheme can be used depending on preference or the problem.

One modification of the present program is that a variable step size may be taken in either the ξ- or η-direction. The variability in the ξ-direction is arbitrary; however, in the η-direction a special type of variability has been employed. Following Cebeci and Smith, Reference [2], a step size in the η-direction with a fixed ratio between adjacent points has been employed. That is, a constant k has been chosen such that

$$\frac{\Delta\eta\ (n+1)}{\Delta\eta\ (n)} = k \qquad \begin{pmatrix} n \text{ is the index for the} \\ \eta \text{ grid location} \end{pmatrix}$$

This type of variation allows for small steps to be taken near the wall and large ones away from the wall for values of k slightly greater than one.

With the above type of numerical scheme employed, the derivatives for various quantities in the boundary layer take the following form:

η-Derivatives

$$\frac{\partial g}{\partial \eta} = a\ (\frac{g_{i+1,\ n+1} - g_{i+1,\ n-1}}{(1+k)\ \Delta\eta(n)}) + b\ (\frac{g_{i,\ n+1} - g_{i,\ n-1}}{(1+k)\ \Delta\eta(n)})$$

$$\frac{\partial^2 g}{\partial \eta^2} = 2a\left(\frac{g_{i+1,\, n+1} - (1+k)\, g_{i+1,\, n} + kg_{i+1,\, n-1}}{\Delta \eta(n)^2\, (1+k)\, K}\right)$$

$$+\ 2b\left(\frac{g_{i,\, n+1} - (1+k)\, g_{i,n} + kg_{i,\, n-1}}{\Delta \eta(n)^2\, (1+k)\, K}\right)$$

ξ-Derivative

$$\frac{\partial g}{\partial \xi} = \frac{g_{i+1,\, n} - g_{i,\, n}}{\Delta \xi}$$

where $a + b = 1.0$ i - ξ index n - η index

By choosing different values of a and b, either the Crank-Nicolson or full implicit methods can be used. For example, if $a = 1$ and $b = 0$ we have the full implicit scheme, while if $a = 1/2$ and $b = 1/2$, we have the Crank-Nicolson method.

Ideally, the Crank-Nicolson scheme would be preferred because of its superior truncation error; however, for turbulent flow, stability requirements forced the use of a full implicit scheme. It was surmised that the stability problem of the Crank-Nicolson method was due to the non-linearity in the equations caused by the turbulent transport properties. For laminar flow, the Crank-Nicolson scheme gave good results for all flow problems. For a full listing of the equations in their finite difference form the reader is referred to the program listing in the latter part of this report.

Both the Crank-Nicolson and full implicit schemes pose similar problems for the solution of the finite difference equations, that is, the solution of a system of simultaneous equations for the unknowns across the boundary layer. Fortunately, these equations always have a tri-diagonal form and they can be solved by a simple algebraic method, Reference [1]. In the present work, all of the non-linear terms were linearized by the use of previous ξ values, and this was

found to be adequate for all laminar flow calculations. However, for turbulent flow an iteration scheme was employed to update the turbulent transport properties in the solution. It was found that three iterations were usually required to obtain good convergence in most turbulent flows.

E. Initial Boundary and Transition Conditions

Since the boundary layer equations are mathematically classified
as being parabolic, they pose a combined initial and boundary value
problem. The boundary conditions are usually obtained from an in-
viscid flow solution at the top of the boundary layer, and from
specified wall conditions. Although two dimensional inviscid flows
are difficult to obtain in some flow regimes, there does exist a
large variety of methods for adequately calculating most of these
flows.

The initial conditions for laminar flows have been thoroughly
studied, and methods of obtaining solutions are given in Reference
[1]. Basically, the two main types of initial conditions are ob-
tained for bodies with sharp leading edges or stagnation type flows.
For turbulent flow, the problem of obtaining initial conditions has
not been adequately treated, and it is currently a difficult
and important problem. The main difficulty is that the fully tur-
bulent boundary layer starts after the transition from laminar to
turbulent flow has occurred. At the present time, the details of
the momentum and energy transport properties in the transition region
are not well understood, and the starting conditions for the turbu-
lent flow are not precisely known. Therefore, the best source of
initial conditions for a turbulent boundary layer is experimental
data.

If experimental data is not available, then approximate tech-
nique must be employed. The technique that has been employed in
the present work is to match up a power law velocity profile with
a linear laminar sublayer. The parameters which must be chosen for
this method are: (1) The boundary layer thickness; (2) the skin

friction coefficient; and (3) the velocity power law. The enthalpy profile was found by assuming a generalized Crocco relationship between the velocity and total enthalpy profiles.

The usefulness of these techniques depends on the problem being solved and the skill of the investigator in estimating the initial parameters. In general, it can be said that the estimation of the initial conditions is more critical for flows with adverse pressure gradients compared to flow with favorable pressure gradients.

F. Program Flow Chart

The flow chart starts with the dimensions, data, boundary conditions and the reference constant.

There are then three possibilities which may occur:

1. Only laminar flow:

In this case, Lamner = 1 and NuX = NuL. Since Lamner = 1, skip will be equal to zero and we go to the right in the flow chart to the laminar flow data and initial conditions and print out these initial conditions. After that, the calculations proceed by calculating the transport properties, and the boundary layer coefficients A, B, C, D, E and F. The solution of the boundary layer equations can then proceed and the new u/ue and h/he are obtained. Since skip = 0, there are no iterations and the calculation of the V - velocity component from the continuity equation directly follows. Then we calculate the boundary layer characteristics and print out the results. This procedure continues until L = Nul = NUL = NUX and the program stops.

2. Only turbulent flow:

In this case, Lamner = 2 and NuL, NuX are given. Since Lamner = 2, skip ≠ 0 and we go to the left in the flow chart to turbulent flow data and initial conditions and point them out. The calculations then proceed exactly as explained in 1. except that we iterate (ITER = ITERT = 3 in this program) with the transport properties to update their values. In this case the calculations stop when L = NU1 = NUX.

3. Both Laminar and Turbulent:

For this case Lamner = 1 as in the laminar case 1., but NUX > NUL, where NUL will be the number of laminar steps while NUX is the total number of forward steps. We start with Lamner = 1 and skip = 0, and we proceed with the laminar calculations exactly as explained in case 1. When L = NU1 = NUL, laminar calculations stop and Lamner takes the value 2 and skip will not equal zero. Then calculations start from the turbulent flow data as in case 2. and the turbulent calculations proceed until L = NUX.

PROGRAM FLOW CHART
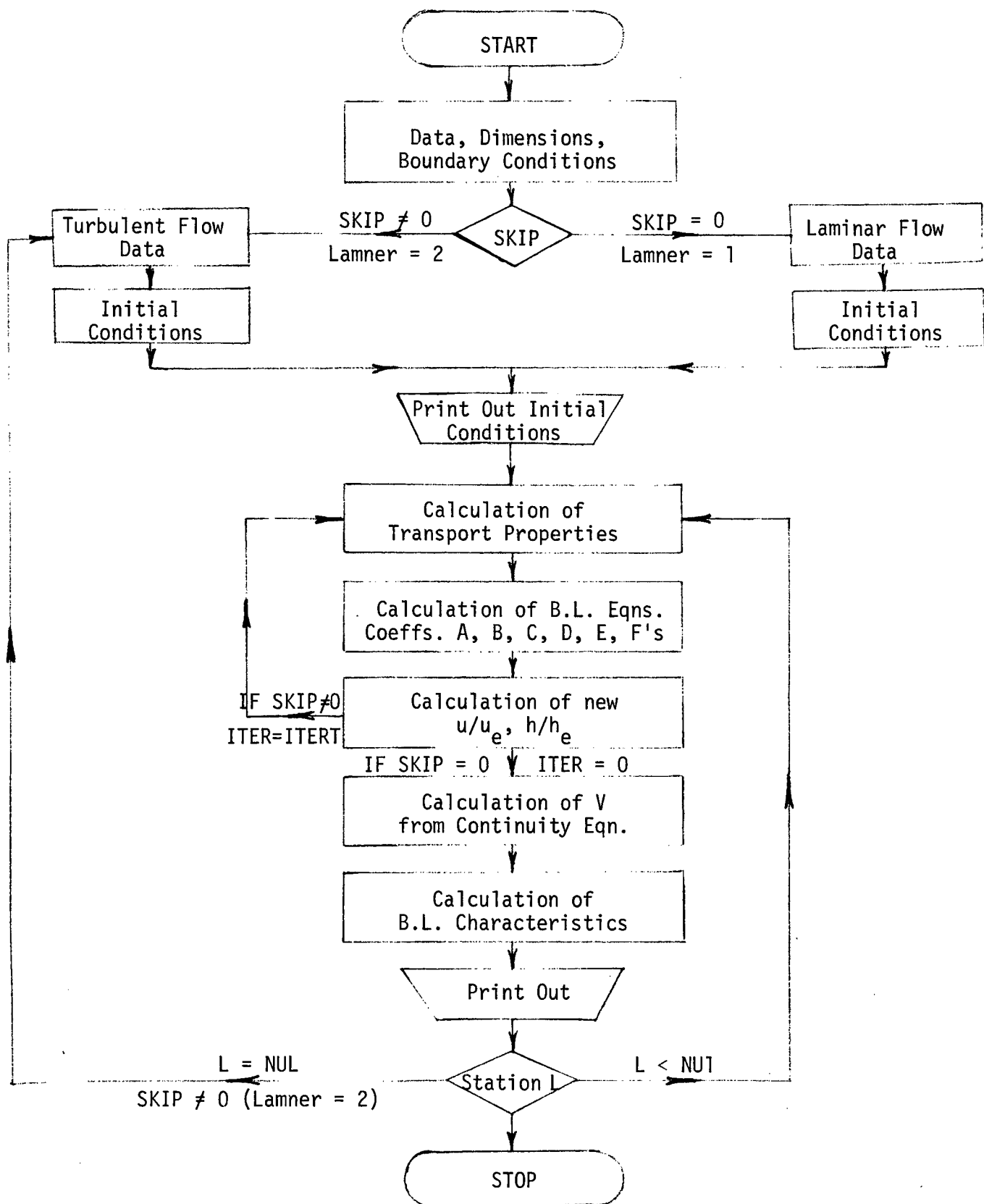
G. Program Inputs

This section of the report details the information and inputs necessary to use the program. Also included in this section are some frequently encountered program errors and their meaning. The information presented here is in an order most logical to its understanding, and not necessarily in its order of appearance in the program.

Definition:

A station is a point on the body at which the boundary layer equations are to be solved. The stations are numbered consecutively starting from one (1) at the leading edge, stagnation point, or origin of the body up to station N where the calculations end.

The boundary layer is divided into a grid for solution by the finite-difference method, where the nodes of the grid are the points where the solution is given. A step constitutes movement from one node to an adjacent node.

Program Inputs:

XD(N) - The physical distance from station 1 to station N (always expressed in feet). For the case of a flat plate XD(1) = 0. , XD(2) = 1. , ...

LAMINAR ← ——————— | ——————→ TURBULENT

1 ←— 1' —→ 2   3   4   5   N
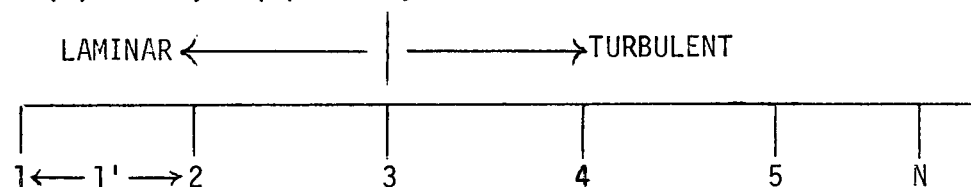
NUL - Index which governs the number of steps which are calculated as laminar flow. In the case of a flat plate, if it is desired to have laminar flow starting at the leading edge up to station 3, and turbulent flow thereafter, then NUL = 2.

NUX - Index which governs the total number of steps taken, i.e. both laminar and turbulent. For the above example if the calculations are to be carried out to station 5, then

$$NUX = 4$$

Be sure to note that NUL and NUX refer to the number of steps taken, and not to a particular station number.

LAMNER - Index which allows deletion of the laminar flow calculations. With

$$LAMNER = 1$$

the program calculates the first NUL stations as laminar flow, but if

$$LAMNER = 2$$

the program skips these steps and begins the calculations at the first turbulent station. In the example program at the end of this report these indices are put on data cards at the beginning of the program.

Initial Profiles:

<u>Laminar Flow</u> - The program requires an initial velocity and enthalpy profile to get started. The profiles used are in the transformed plane, i.e.

$$u/U_e = f' (\eta, \xi)$$

The program also requires that this profile be divided into 60 equally spaced steps between $U/U_e = 0$ and $U/U_e = 1$. If LAMNER = 2 this information can be omitted as the program skips all operations involved in laminar flow.

The initial profile will be different for each type of flow, i.e. flat plate, stagnation flow, wedge flow etc. In

the example program the initial velocity is read in (see

LAMINAR FLOW --- READING THE UFVE - VELOCITY PROFILE).

TURBULENT FLOW:

Power - To begin the turbulent flow calculations the program provides a power law profile

$$u/U_e = (y/\delta)^{1/power}$$

The operator must supply the value of power, but for many problems

$$POWER = 7$$

Energy Equation:

Just as an initial velocity profile is needed for the momentum equations, an initial energy profile is needed for the energy equation. Such a profile is automatically supplied in both the laminar and turbulent calculations.

CO1 - The initial energy profile (static enthalpy) is given by

$$\frac{H - Hw}{H_e - H_w} = (u/U_e)^{CO1}$$

By changing CO1 the shape of the initial profile can be changed. For turbulent calculations other information is required.

NUT - The number of steps across the boundary layer. In laminar flow this is a constant set equal to 61 (the wall value is always 1 thus giving 60 steps across the boundary layer). In the turbulent portion this quantity is variable and may be set as high as 501, but usually is set at 350.

DN1T - The laminar portion of the boundary layer is divided into 60 equal steps, each step being DN = DN1 = .1. The turbulent boundary layer is divided into NUT steps, but they are of unequal size. DN1T is the size of the first step, i.e. the distance from the wall to the first station in the y-direction across the boundary layer. Typically DN1T = .005.

KT - The ratio of step size of any two successive steps across the boundary layer:

$$KT = \frac{DN\ (i\ +\ 1)}{DN\ (i)}$$

```
        •    • i + 2 ⎫
                     ⎬ DN (i + 1)
        •    • i + 1 ⎧
                     ⎬ DN (i)
        •    • i     ⎭
        N    N + 1
```

Typically, KT = 1.021. For laminar flow this number is always 1 since the boundary layer is divided into equal steps.

ITERT - Iteration index. To calculate the velocity and energy profiles at station N + 1 requires the turbulent transport properties at station N + 1. Since these properties depend upon the velocity and temperature at station N+1, which are exactly the unknown in the problem, an iterative procedure is used. The transport properties are calculated based upon information at station N, then the boundary layer equations are solved to obtain velocity and energy profiles at station N + 1. This information is used to update the values of transport properties, and the equations are resolved. The number of times this process of recalculation takes place is given by ITERT. Satisfactory results are obtained with ITERT = 3.

CFT, DELTIT, DELTT - The initial turbulent velocity profile is calculated automatically in the program by matching a linear profile in the sublayer with a power law profile outside the sublayer.
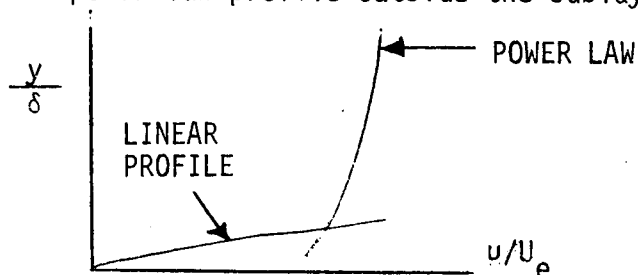
To carry out these calculations the program requires an initial estimate of the boundary layer thickness (DELTIT), skin friction (CFT), and

displacement thickness (DELTT).
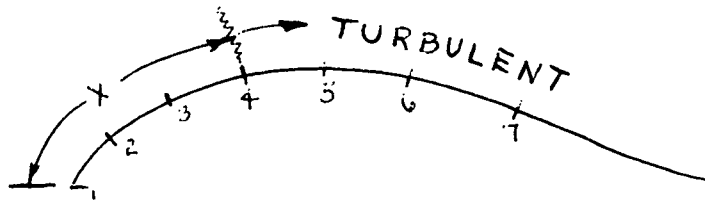
As an example, look at the case of flow over an airfoil. Turbulent flow starts at station 4. One way of estimating the needed quantities is by using flat plate data and formulations:

$$C_f = .0592 \; R_x^{-1/5}$$

$$\delta = .37 \; x \; R_x^{-1/5}$$

$$\delta^* = (1/8) \; \delta$$

$$R_x = U\infty \; x \; /\nu$$

Note: These quantities are used only at the first turbulent station to begin the calculations; thereafter new values of these quantities are calculated by the program consistent with the solution of the boundary layer equations.

ARJ(N), TCFACT - These quantities give the curvature of the body for axisymmetric flow, and take into account the transverse curvature terms in the boundary layer equations.

For any 2-D flow ARJ(N) = 1. (N refers to a station number) For axisymmetric flow, i.e. flow through a nozzle ARJ(N) gives the local value of the body radius, the distance from the center line of the body to the surface of the body.
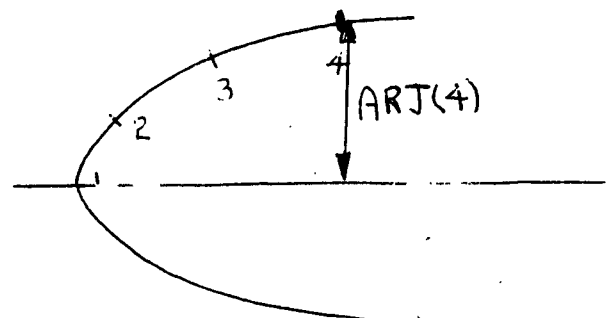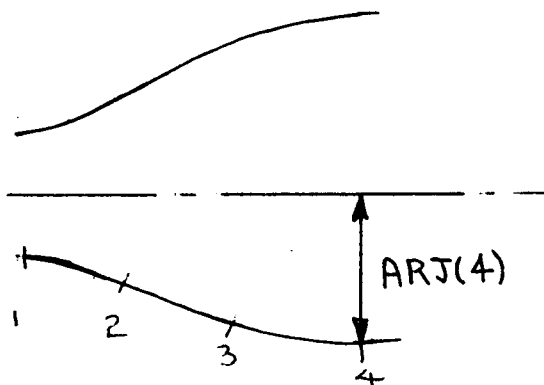
TCFACT is used to correct for transverse curvature. For 2-D flows.

$$TCFACT = 0$$

Internal Axisymmetric Flows (Nozzle)

$$TCFACT = -1$$

External Axisymmetric Flows (Blunt body)

$$TCFACT = +1$$

Boundary conditions: UE(N), PE(N), ENTO, ENTTW(N), VWALP(N)

ENTO - Total enthalpy of the free stream.

UE(N) - The free stream velocity (1 value is required at each station on the body).

PE(N) - The free stream pressure (again a value at each station).

Both UE and PE may be read in, but since the boundary layer equations reduce to

$$\rho_e \, U_e \, \frac{dU_e}{dx} = - \frac{dP}{dx}$$

at the edge of the boundary layer, one quantity can be calculated from the other.

ENTTW(N) - The enthalpy of the wall at each station.

VWALP(N) - If there is blowing or suction at the wall VWALP is its value at each station.

Prandtl Number: The Prandtl Number (PR) is set by the operator at the beginning of the program. The turbulent Prandtl Number is given by a subroutine in the program called TAB3. At present, TAB3 returns a constant value of .9 for the turbulent Prandtl Number, but a subroutine is set up so that the operator can change to a variable if so desired.

H. Print Out of Boundary Layer Characteristics

The output is divided into two types of quantities, those which are a single value at the station, i.e., free stream or wall quantities; and those which resulted from the solution of the boundary layer equations, i.e., the velocity and temperature at each point across the boundary layer.

The first type of quantity may be given by a number followed by another number in brackets

$$X = 5.0(1.518)$$

The first number is the quantity expressed in English units, and the number in brackets is the quantity expressed in metric units.

$$X = \text{Feet (meters)}$$

All boundary layer thicknesses: inches (cm)

Pressure: lb_f/ft^2 $(Newtons/cm^2)$

Temperature: °R (°K)

Velocity: ft/sec (M/sec)

Any quantity in this group of variables given by a single number (no brackets following number) is a dimensionless quantity.

The second set of variables is given in columns headed by:

U/UE   - velocity ratio

M/Me   - Mach No. ratio

T/TE   - Static temperature ratio

TT/TTE - Total temperature ratio

TB   - Temperature ratio $\dfrac{T_t - T_w}{T_{t_e} - T_w}$

HB   - Enthalpy Ratio $\dfrac{h_t - h_w}{h_{t,e} - h_w}$

PT/PTE   - Total Pressure Ratio

Y/THET   - Dimensionless distance (THET - Momentum thickness)

VP-N   - Velocity Profile Exponent

AL1   - Transport Property -

In the print out of the second type of quantities, there will be one row which is underlined. The underline is to show the point where reference quantities are calculated.

$$y^* = 11.8$$

One of the output quantities, the average skin friction (CFA) needs special explanation.

The average skin friction is given by

$$C_{fa} = \frac{1}{x} \int_0^X C_f(x) \, dx$$

The integral is evaluated numerically using the trapezoid rule i.e.

$$\int_{x_i}^{x_{i+1}} \phi(x)dx = [\frac{\phi(i+1) + \phi(i)}{2}] + [x_{i+1} - x_i]$$

In some cases though C_f at $X = 0$ is undefined, making it impossible to evaluate the integral in the above manner. For the case of a flat plate although C_f is undefined, it is a known function of X and the integral may be explicitly evaluated. This has been done in the sample program at the end of this report.

$$C_f = .332(R_x)^{-1/2}$$

$$C_{fa} = \frac{1}{x} \int_0^X C_f \, dx = .664 \, R_x^{-1/2}$$

In the body of the program in the section headed by "Calculation of Initial Laminar Profiles" CF1 (1) is the initial value of the integral over the distance between stations 1 and 2; and was obtained as

shown above. Where C_f is not a known function of x, but is singular at some point, it is required to exclude such a point from the calculations.

This means that each time the program is used for a different problem, it must be taken into account and possibly changed.

One final word about print outs. Although there are 60 stations across the boundary layer in laminar and NUT stations in turbulent flow, all of the information at every one of the points is not needed. Therefore, the program prints out more points close to the wall where quantities change quickly and few points out near the free stream where quantities change slowly. The method used is to print every step over the first twenty, every other step for the next 40 steps, every third step over the next 80 steps, and so on to the edge of the boundary layer.

Subroutine for Calculation of Pressure Ratio

The ratio of the total pressure to the total pressure at the edge of the boundary layer is calculated in subroutine TAB1. The formulations used are:

$$\frac{P_{t_2}}{P_1} = [1 + 1/2(\gamma-1)M^2]^{\frac{\gamma}{\gamma-1}} \qquad\qquad M < 1$$

$$\frac{P_{t_2}}{P_1} = 1.8929 \qquad\qquad M = 1$$

$$\frac{P_{t_2}}{P_1} = [1/2(\gamma+1)M^2]^{\frac{\gamma}{\gamma-1}} [(\gamma+1)/(2\gamma M^2 - (\gamma-1))]^{\frac{1}{\gamma-1}} \qquad M > 1$$

The main program reports the Mach number at each point across the boundary layer, and is used to calculate the total pressure by the above formulations. The last step in the subroutine is to form the pressure ratio so that all that is necessary to get this information is to call TAB1 in the main program.

I. Results of the Computations

In this section of the report will be presented some results of our calculations of flows with the present computer program. Only results for turbulent flow will be shown since the laminar flow case has already exhibited very good accuracy, Reference [1]. In general, it can be said that the results are very good with the most critical drawbacks being the initial conditions and the breakdown of the eddy viscosity models near separation. Also, it should be mentioned that the present program does not include any provisions for the inclusion of rough walls into the present program.

a. Equilibrium Boundary Layer at Constant Pressure: Figure 1 gives a comparison of experimental and calculated values of skin friction vs. Reynolds Number for flow over a flat plate. Experimental results are for incompressible flow with a free-stream velocity of 33 m/sec. The experiments were performed by Wieghardt and Tillmann as cited in Reference [6]. As is expected, since an exact profile was used for the initial starting profile, the comparison of results is very good.

b. Compressible Flow Over a Flat Plate: Two cases were carried out numerically and were compared with the experimental data of Hopkins, Keener and Dwyer, Reference [8].

For the first case, the following data were given

$$M_e = 6.43$$

$$U_e/\nu_e = 13.68 \cdot 10^6 \ (1/\text{meter})$$

$$T_{t,e} = 711°K$$

$$T_w = 307°K$$

$$T_w/T_{aw} = 0.47$$

$$P_e = 31.1 \ \text{psf}$$

$$U_e = 1139 \ \text{m/sec}$$

For the second case

$$M_e = 7.4$$

$$u_e/\nu_e = 4.58 \cdot 10^6 \, (1/\text{meter})$$

$$T_{t,e} = 1092°K$$

$$T_w = 304°K$$

$$T_w/T_{aw} = 0.3$$

$$P_e = 12.5 \text{ psf}$$

$$u_e = 1453 \text{ m/sec}$$

To start the turbulent calculations in both cases, one of the experimental points was chosen at which C_f $R_{e\theta}$ and x were given. Knowing $R_{e\theta}$, θ is calculated $(R_{e\theta} = \frac{u_e}{\nu_e} \theta)$. Using the 1/7 power law as an initial condition δ and δ_1 can be calculated and the numerical calculations can then proceed.

Figures (2) and (3) show the comparison between the calculated and experimental local skin friction coefficient versus momentum thickness Reynolds number. The agreement is good.

c. <u>Incompressible Flow Over Flat Plate with Uniform Blowing</u>: The experimental data used for comparison with the numerical one were obtained by Mickley and Davis, Reference [10]. The blowing rate $V_w/u_e = 0.002$ was studied and a flat plate flow was calculated for a Reynolds number of 3.125 x 10^5 per foot (using the values of μ and δ corresponding to atmospheric temperature and pressure) with $u_e = 50$ ft/sec. An effective length that matches the momentum thickness at the station where blowing started was determined, and so the corresponding ξ_o was known. The boundary conditions (4-a) in this case are

$$V_w = 0 \qquad \text{for } \xi < \xi_0$$

$$\frac{V_w}{U_e} = .002 \qquad \text{for } \xi \geqslant \xi_0$$

Two methods are used to start the numerical calculation. The first is to start the turbulent flow with the usual 1/7 power law at a distance behind where blowing starts and then turn on the blowing. The second method is to start the calculation at the location where blowing starts. In this case, other power laws are used, such as 1/5 and 1/4. The same c_f value given by experiment and the values of δ and δ_1 corresponding to the power law are used.

Figure (4) shows the calculated values of local skin friction in coefficient along the plate, together with the experimental data.

The trend in all cases is the same although all the numerical calculations overpredicted the coefficient of friction. The best result is obtained starting with a 1/5 power law and not including the correction of blowing on the eddy viscosity formula.

Finally, it should be noted that still more experience is needed at this stage with the initial conditions to have satisfactory results with experiments.

d. <u>Equilibrium Boundary Layer in Mild Negative Pressure Gradient</u>: Experiments were conducted by Herring and Norbury as cited in Reference [6] for a value of

$$\beta = \frac{\delta^*}{\tau_w} \frac{dP}{dx} = -.35$$

Figure 5 presents the comparison of the experimental and calculated data. Again, flat plate correlations were used for starting data, and although this creates oscillations about the experimental data, it is seen that the calculations tend toward the experimental data after only a few steps.

The use of flat plate data to start all problems is not always acceptable, and some modifications must be made. The modification

chosen was to change to the power law.

e. <u>Equilibrium Boundary Layer in Positive Pressure Gradient</u>: Experimental results were tabulated by Clauser as cited in Reference [6]. The data is for the Clauser flow P. D. 1 with fluid properties taken at 75°F. Figure 6 shows the experimental data compared to the calculated data for several different initial turbulent velocity profiles. As can be seen, the choice of the power law can radically effect the results.

However, with a little experience, the starting profile is readily chosen which will produce good results, as can be seen in Figure 6.

f. <u>Airfoil Boundary Layer Flow Proceeding Toward Separation</u>: Figure 7 gives a comparison of the experimental and calculated results for flow over an airfoil. Data was taken by Newman as cited in Reference [6] for a flow with a free-stream velocity of 120 fps. The machine calculations were made using a starting (1/4) power law velocity profile, and as can be seen, the results are in good agreement with experiment.

g. <u>Axisymmetric Nozzle Flow</u>: Numerical calculations were carried out for the NASA AMES 3.5 ft. wind tunnel. The information given was the area ratio at different locations, stagnation conditions and wall temperature.

Five runs were carried out and the data given for them are:

Run	M_e	$(\frac{u_e}{\nu_e}) \times 10^{-6} (ft^{-1})$	$T_{t,e}(°R)$	$T_w(°R)$	$P_e(psf)$
1	7.4	2.95	1359	543	13.99
2	7.4	3.86	1340	555	18.13
3	7.4	1.88	1363	567	8.97
4	7.4	2.98	1564	568	17.84
5	7.4	1.66	1879	558	13.52

The calculations were carried out in the following manner:

i. From the area ratio using supersonic tables for air in isentropic flow the pressure ratio, temperature ratio and Mach number can be obtained along the nozzle.

ii. For the initial conditions at the throat of the nozzle an approximation is used, since experimental data was not available. This approximation is to locate a virtual origin which is specified at a distance equal to the diameter of the throat from the throat location. The value of C_f corresponding to this virtual origin was obtained from the known flat plate data. Using the 1/7 power law, the values of δ and δ_1 are obtained. Since the flow is accelerating at the throat, it is expected that the values of δ and δ_1 will be smaller than those obtained. Therefore, half these values are assigned to start the calculation, and the numerical calculations can then proceed.

It should be noted that there was no fluctuation in the numerical results because of the initial guess of C_f, δ and δ_1. This could be due to the strong favorable pressure gradient, since the flow can possibly adjust itself when using a reasonable guess of the initial conditions.

iii. Since the boundary layer is thick in this flow, we have to correct the calculation by subtracting the displacement thickness at every location from the radius of the nozzle to get the new area ratios. The previous steps (i) and (ii) are then repeated as discussed above.

Figure 9 shows the calculated values of Mach number and displacement thickness along the nozzle with and without corrections for run (1). It should be noted that with one iteration, the exit Mach number gave good agreement with the experimental results.

Figure 8, corresponding to run (1), shows that the Crocco rela-
tionship is not satisfied almost everywhere across the boundary layer.
This is believed to be due to the strong pressure gradients in the
flow.

Figures 10 to 17 are the same as Figure 8 and 9 repeated for the
other runs.

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2. Cebeci, T. and Smith, A. M. O., "Calculation of Compressible Adiabatic Turbulent Boundary Layers", AIAA, No. 69-687, June 1969.

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6. Coles, D. E. and Hirst, E. A.,"Proceedings Computation of Turbulent Boundary Layers - 1968", Vol. II, Published by Dept. of Mech. Eng., Stanford University, Calif.

7. Van Driest, E. R., "On Turbulent Flow Near a Wall", J. of Aerospace Sci., Vol. 23, No. 11, Nov. 1956, pp. 1007-1011.

8. Hopkins, E. J., Keener, E. R., and Dwyer, H. A., "Turbulent Skin Friction and Boundary Layer Profiles Measured on Non-adiabatic Flat Plates at Hypersonic Mach Numbers", AIAA Paper No. 71-167, New York, Jan. 1971

9. Bertram, M. H., "Comment on Viscosity of Air", Journal of Spacecraft and Rockets, Vol. 4, No. 2, Feb. 1967, pp.287-288

10. Mickley, H. S. and Davis, R. S., "Momentum Transfer for Flow Over a Flat Plate with Blowing", NACATN 4017, Nov. 1957
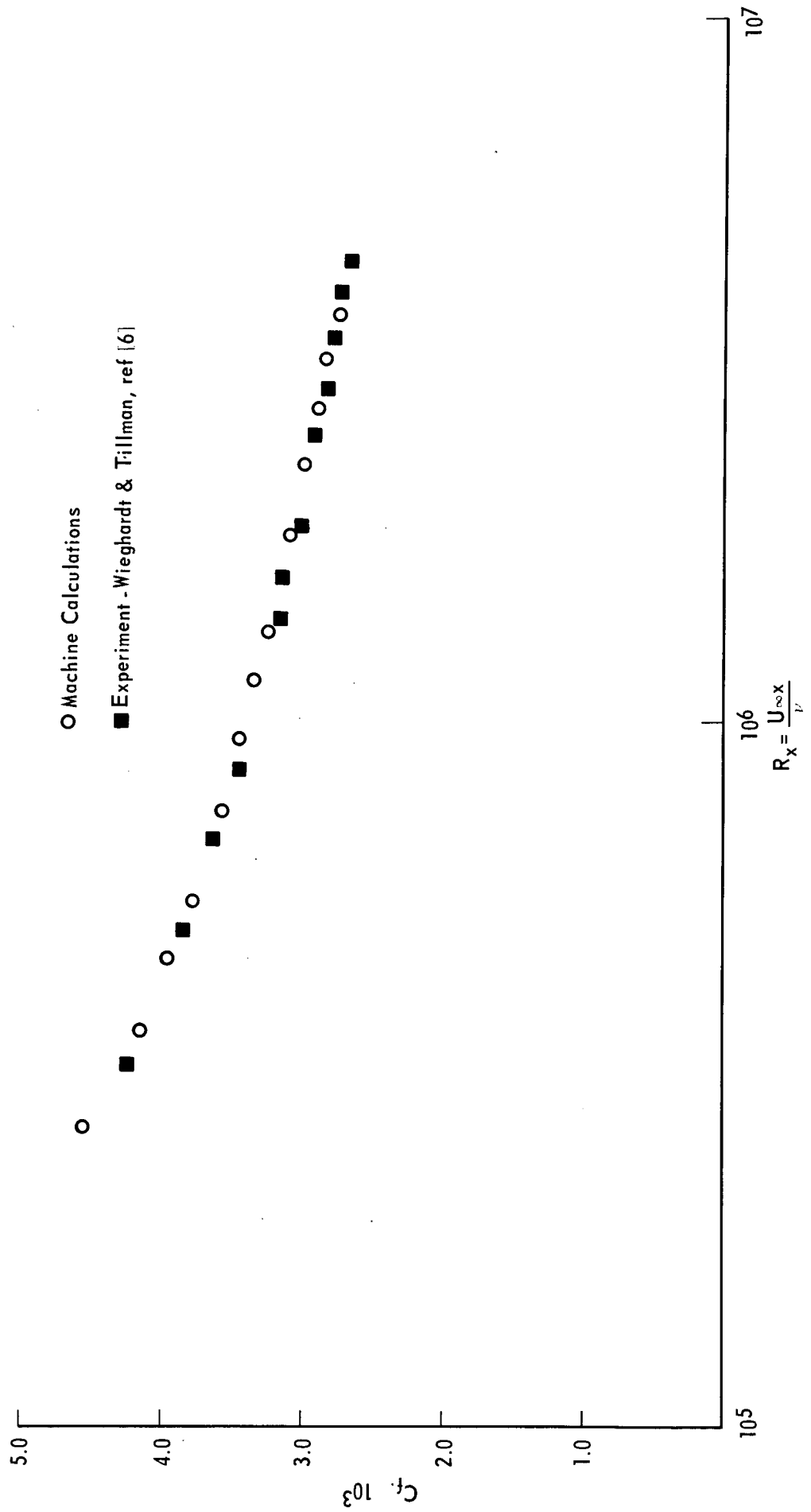
Figure 1 EQUILIBRIUM BOUNDARY LAYER AT CONSTANT PRESSURE – FLAT PLATE

O Machine Calculations

■ Experiment - Wieghardt & Tillman, ref [6]

$R_x = \dfrac{U_\infty x}{\nu}$

$C_f \cdot 10^3$

38
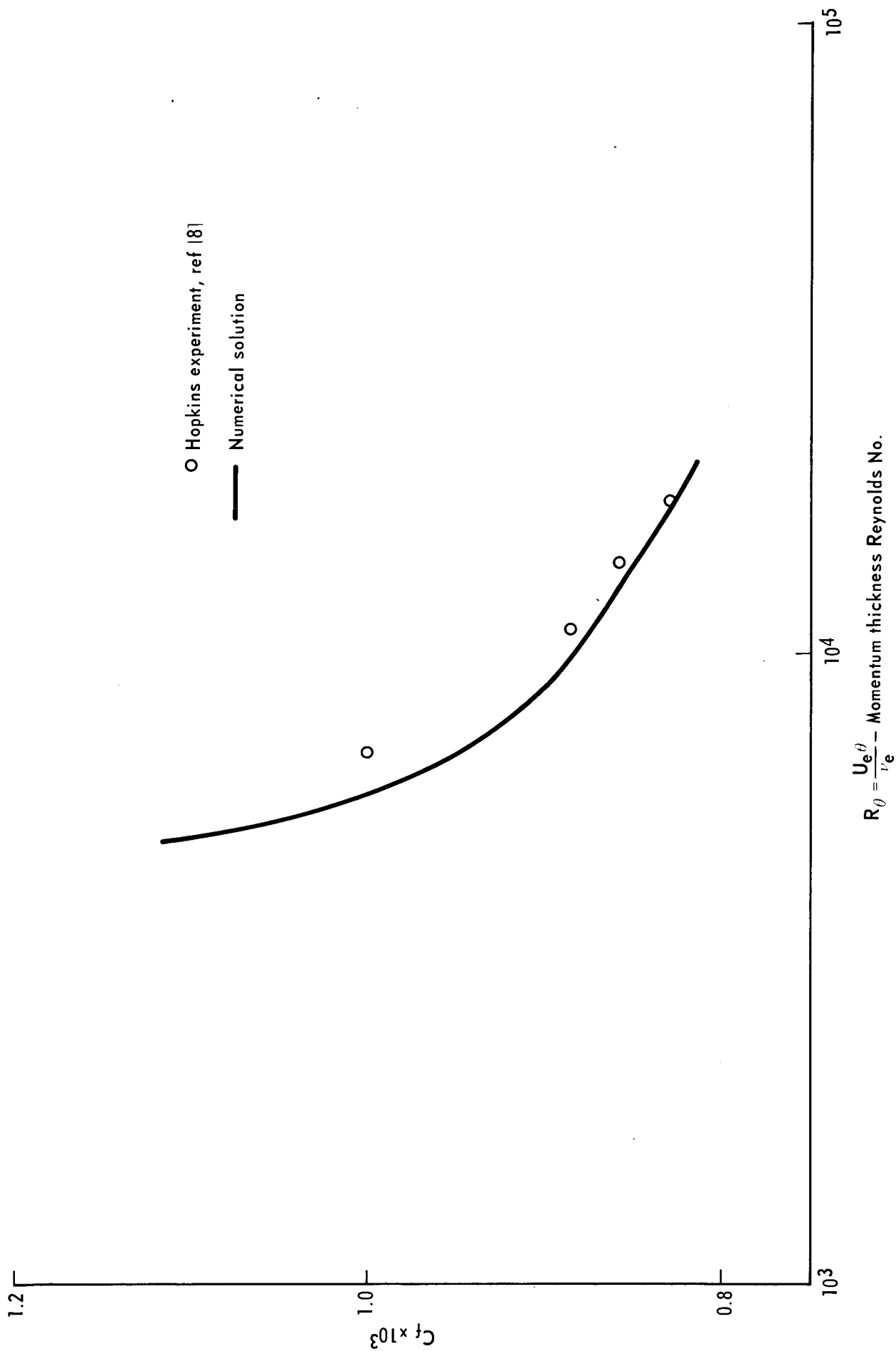
Figure 2 COMPRESSIBLE FLAT PLATE $M_e = 6.43$, $T_w/T_{aw} = 0.47$

$R_\theta = \dfrac{U_e \theta}{\nu_e}$ - Momentum thickness Reynolds No.

O Hopkins experiment, ref [8]

—— Numerical solution

$C_f \times 10^3$

39
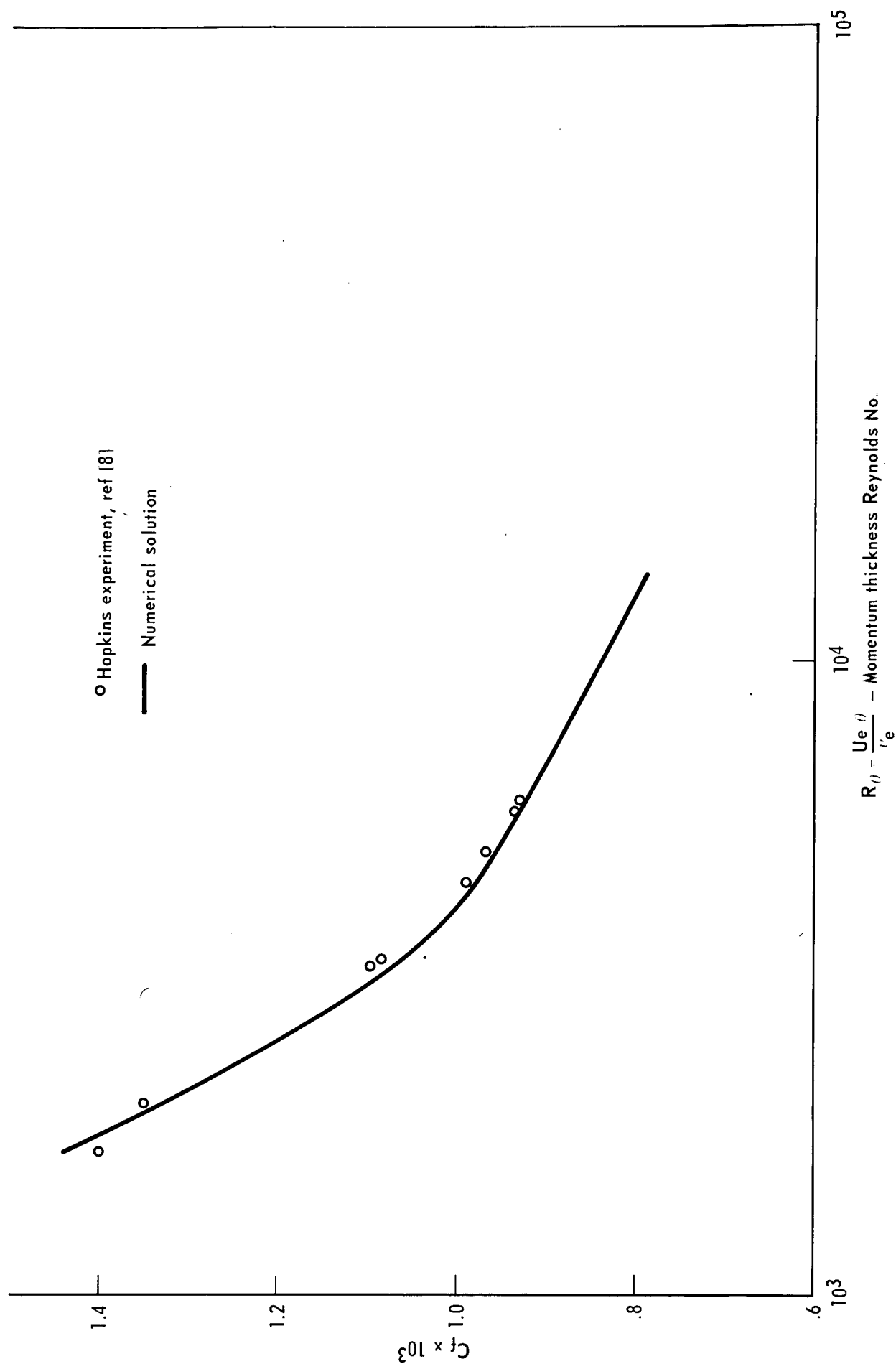
Figure 3 COMPRESSIBLE FLAT PLATE $M_e = 7.4$, $T_w/T_{aw} = 0.3$

$R_\theta = \dfrac{U_e \theta}{\nu'_e}$ – Momentum thickness Reynolds No.

o Hopkins experiment, ref [8]

—— Numerical solution

$C_f \times 10^3$

40
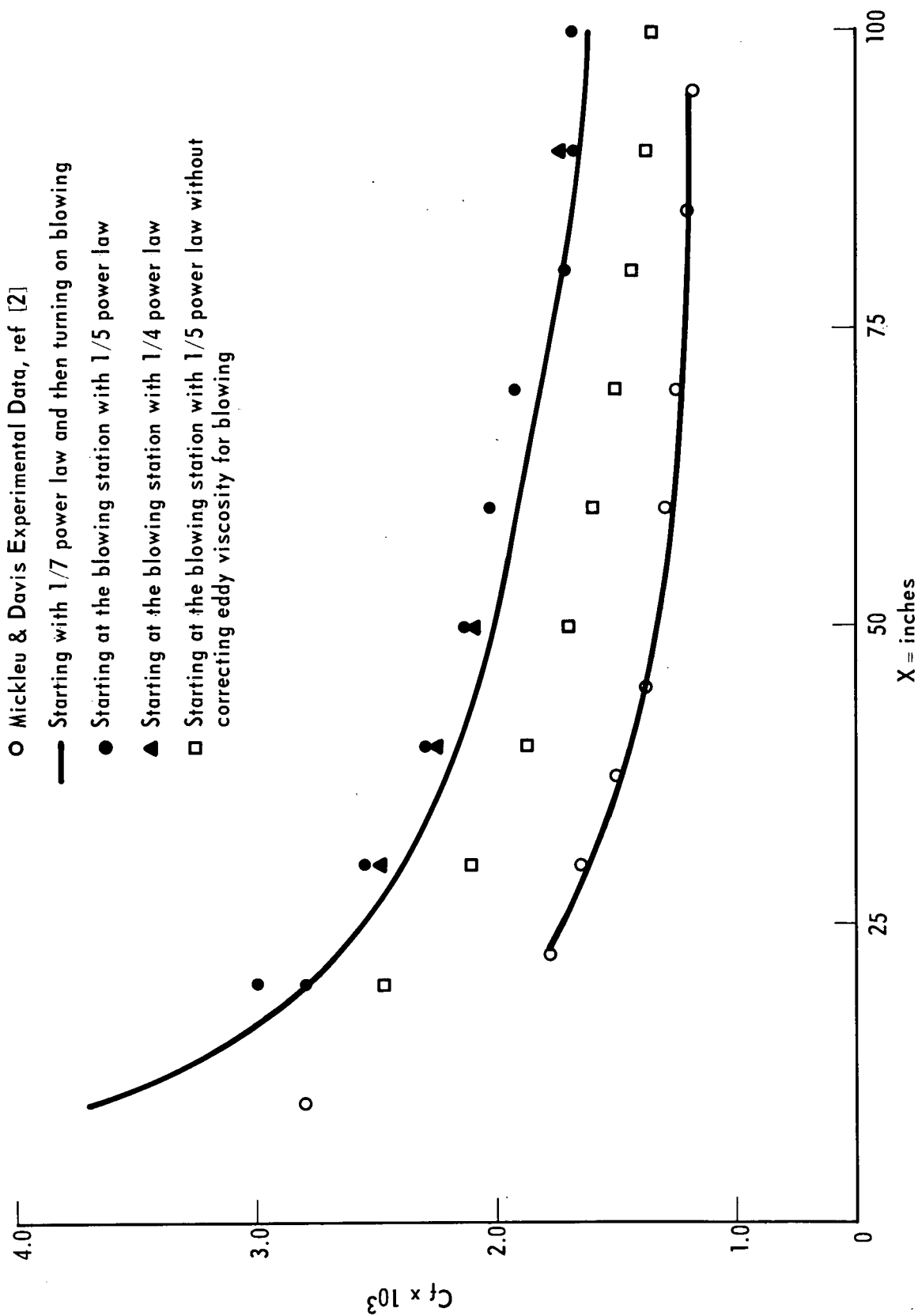
Figure 4 FLAT PLATE WITH BLOWING ($V_w/U_e = .002$)

Legend:

○ Mickleu & Davis Experimental Data, ref [2]
— Starting with 1/7 power law and then turning on blowing
● Starting at the blowing station with 1/5 power law
▲ Starting at the blowing station with 1/4 power law
□ Starting at the blowing station with 1/5 power law without correcting eddy viscosity for blowing

X = inches

$C_f \times 10^3$

41
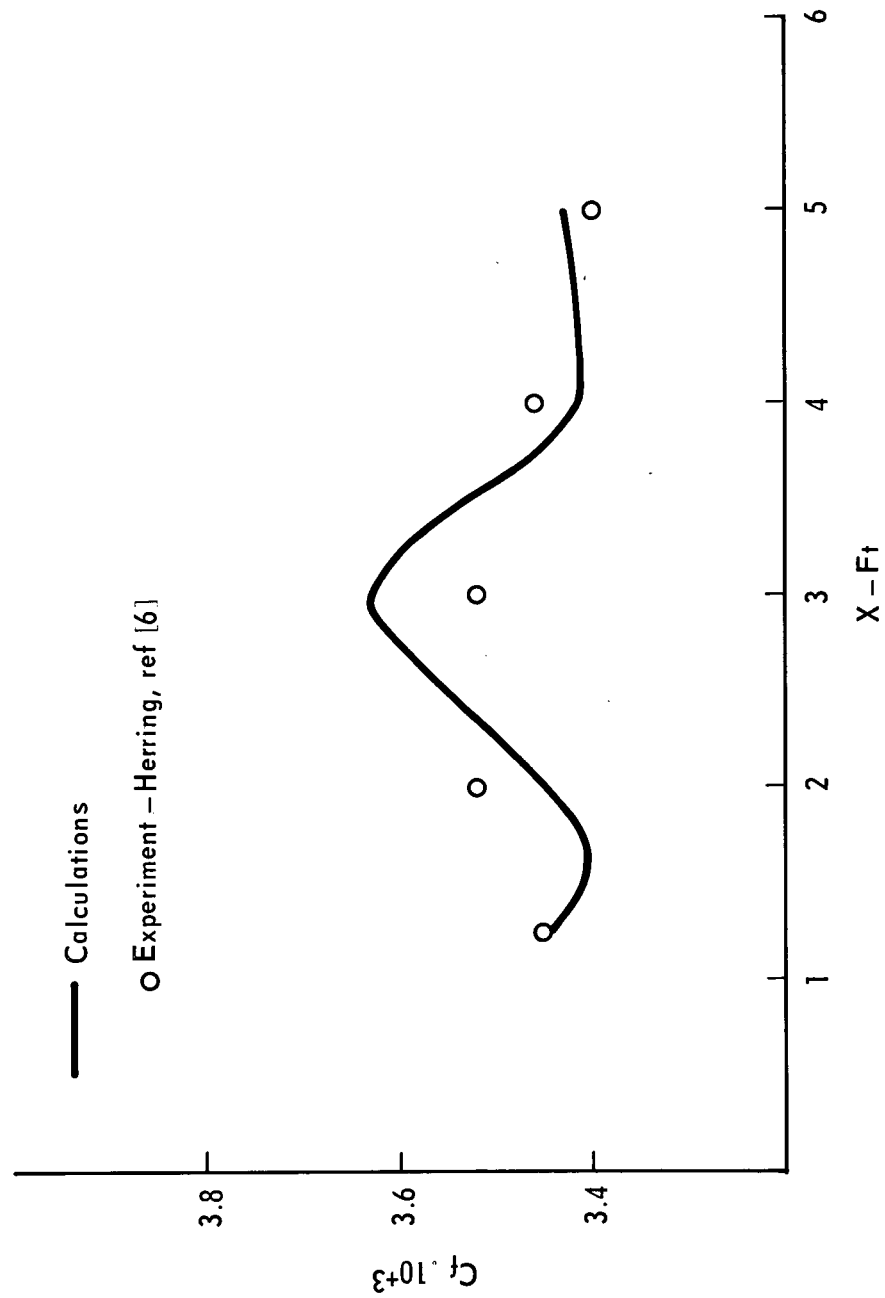
Figure 5 EQUILIBRIUM BOUNDARY LAYER IN MILD NEGATIVE PRESSURE GRADIENT

42
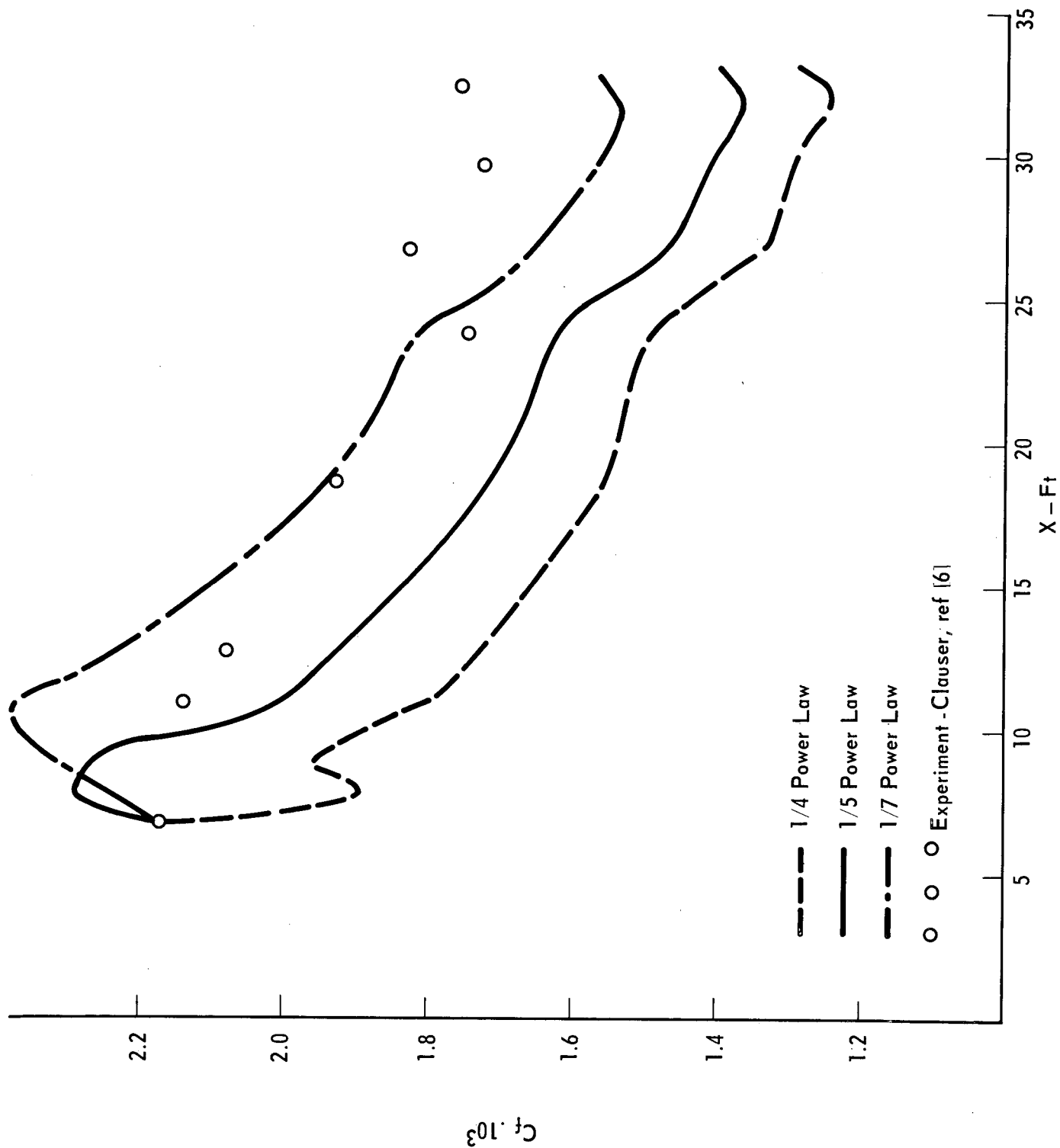
Figure 6 EQUILIBRIUM BOUNDARY LAYER IN POSITIVE GRADIENT

1/4 Power Law

1/5 Power Law

1/7 Power Law

O O O Experiment - Clauser, ref [6]

$C_f \cdot 10^3$

X — Ft

43
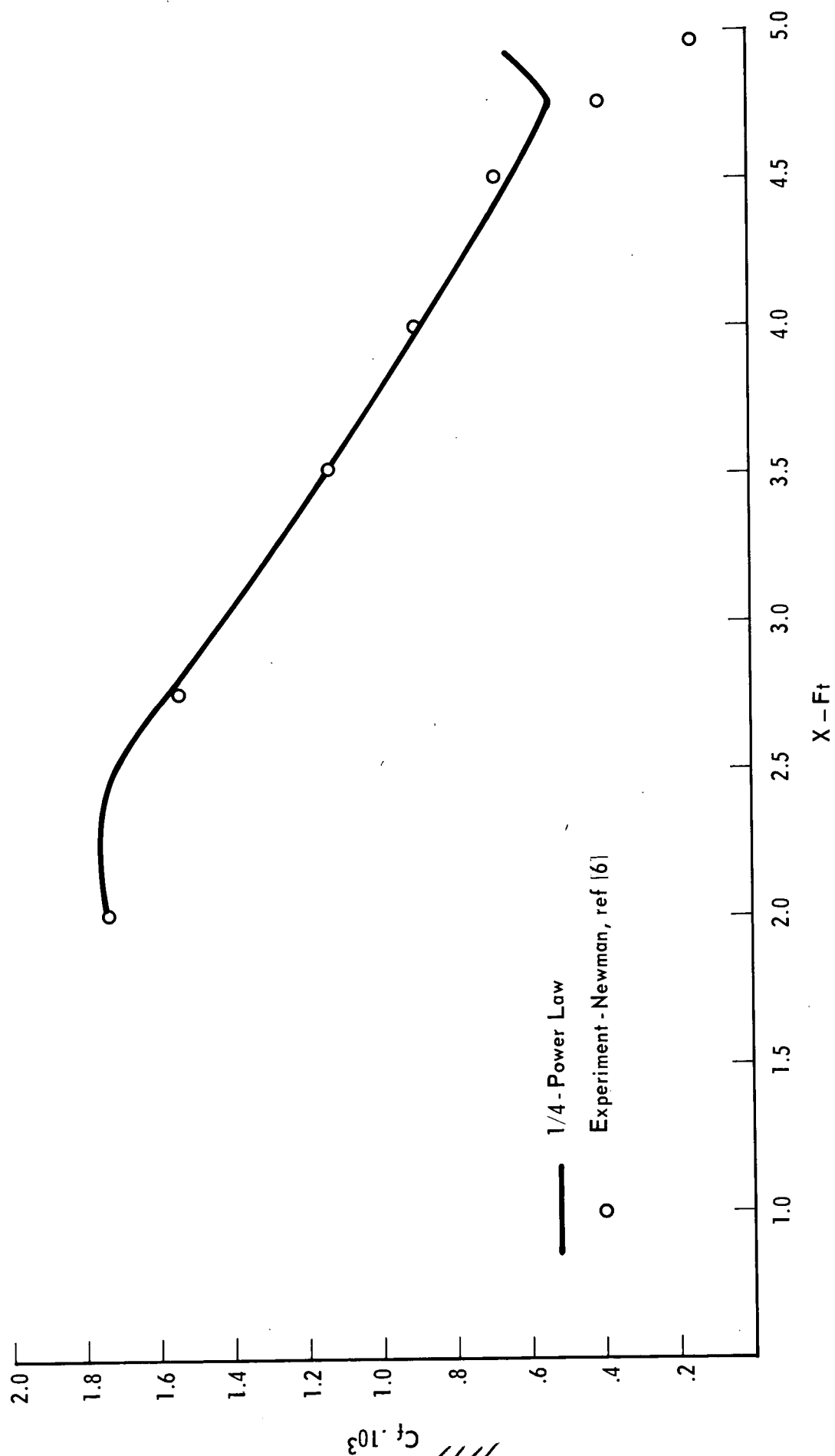
Figure 7 AIRFOIL BOUNDARY LAYER FLOW PROCEEDING TOWARD SEPARATION

X – Ft

$C_f \cdot 10^3$

1/4 - Power Law

Experiment - Newman, ref [6]

44
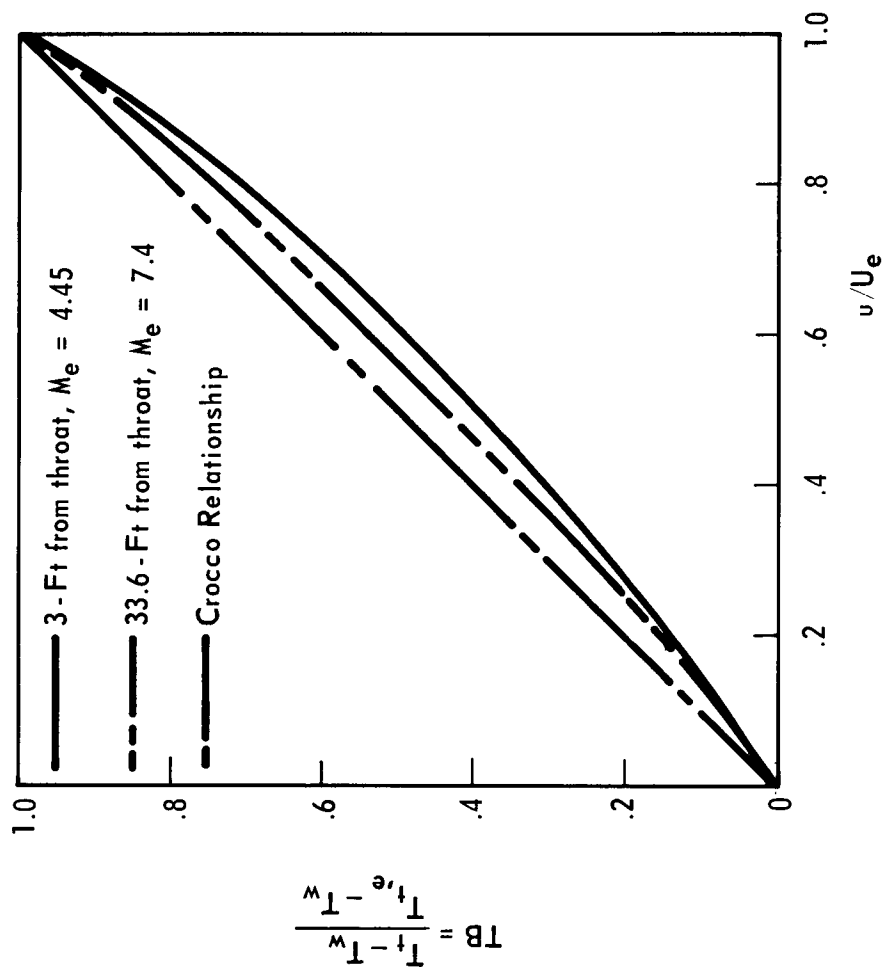
Figure 8 CHECK OUT OF CROCCO RELATIONSHIP FOR AXISYMMETRIC NOZZLE FLOW
Ames – 3.5 - Ft Wind Tunnel
$M_e = 7.4$, $T_{t,e} = 1359°R$
$T_w = 543°R$, $P_e = 13.99$ psf

45
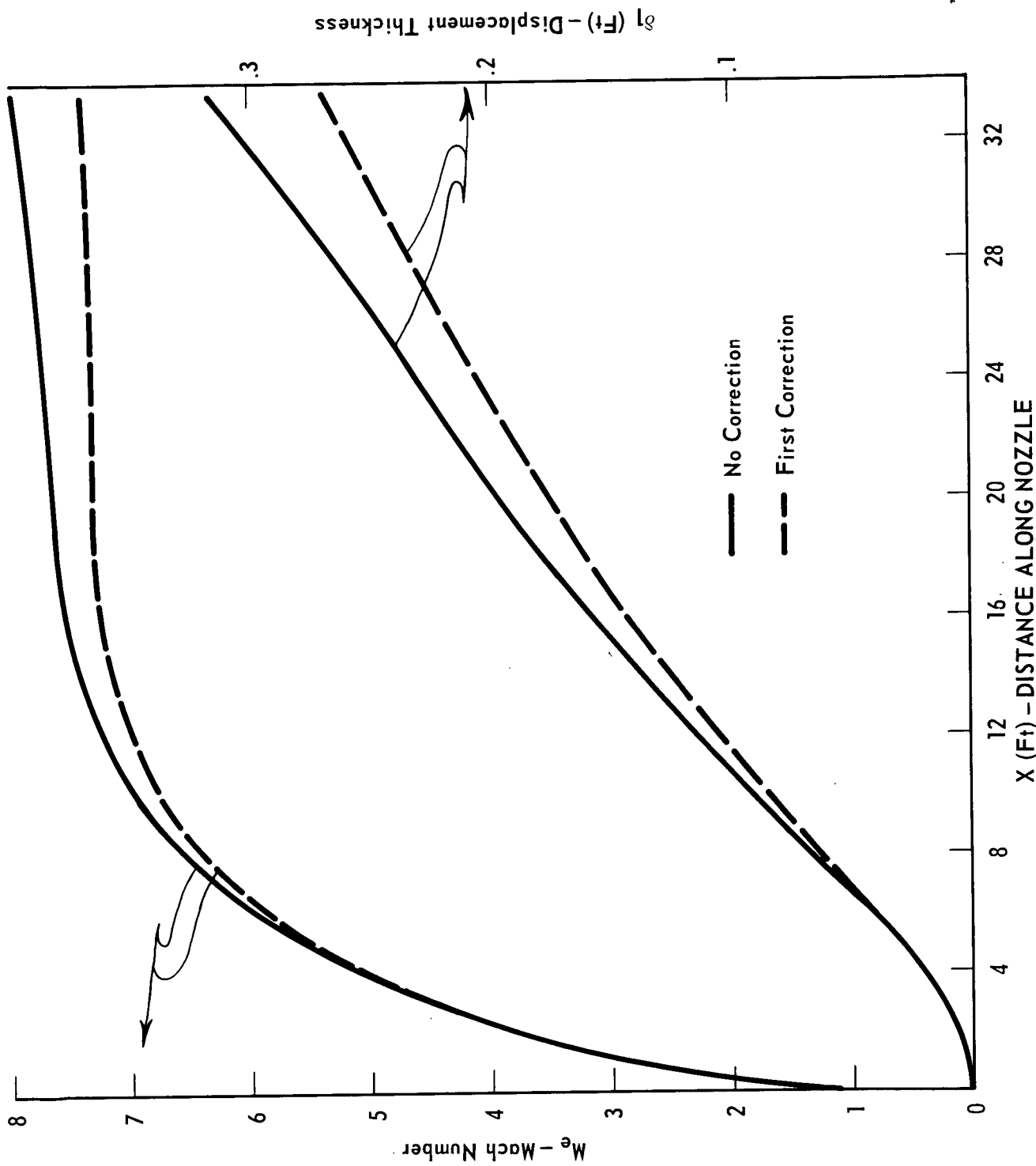
Figure 9 BOUNDARY LAYER INFLUENCES ON WIND TUNNEL FLOWS
$M_e = 7.4$, $T_{t,e} = 1359°R$ $T_w = 543°R$, $P_e = 13.99$ psf
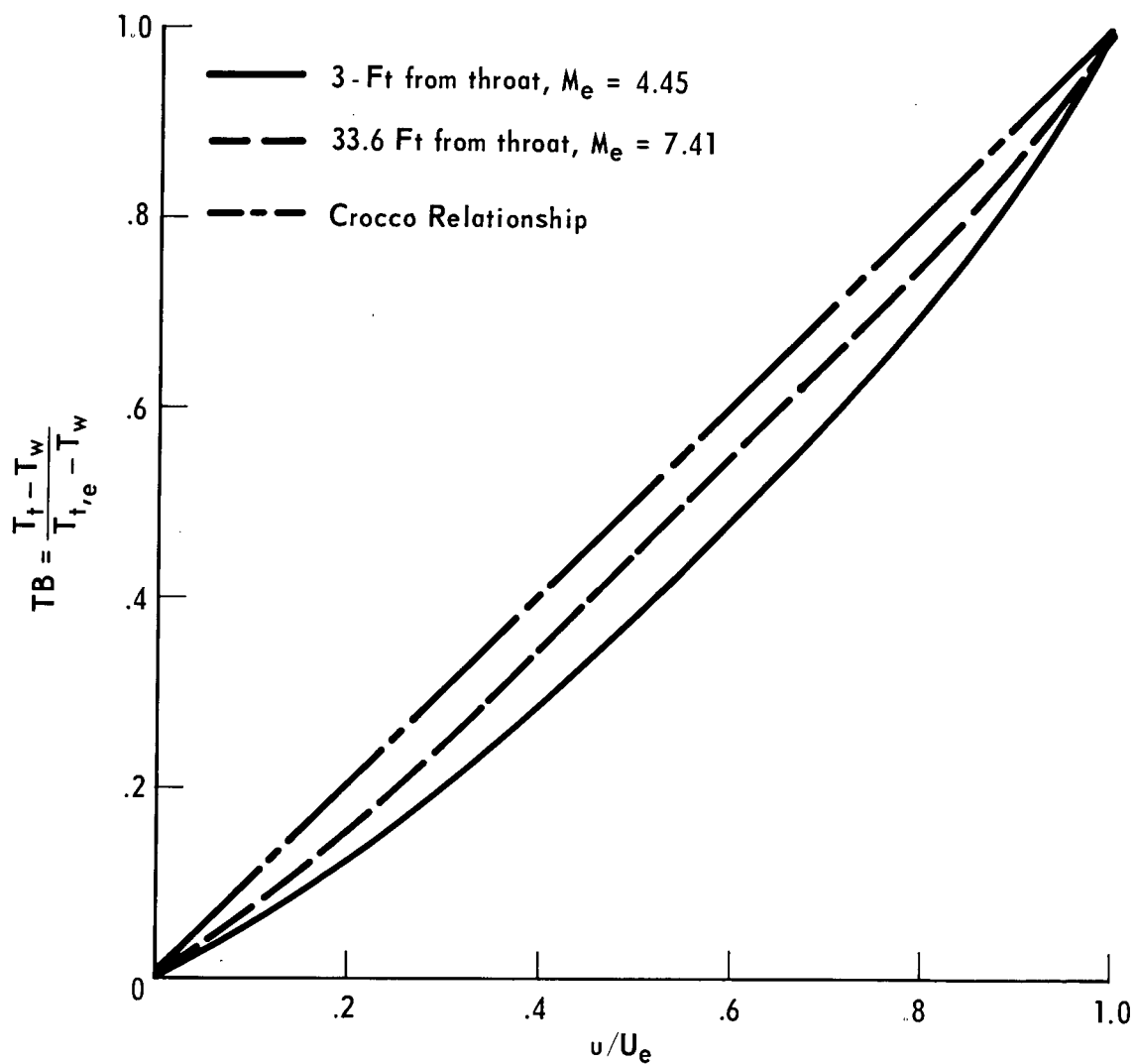
Figure 10 CHECK OF CROCCO RELATIONSHIP FOR AXISYMMETRIC
NOZZLE FLOW
Ames 3.5 Ft Wind Tunnel
$M_{e_,} = 7.4, T_{t_{,e}} = 1340°R$
$T_w = 555°R, P_e = 18.13$ psf

47
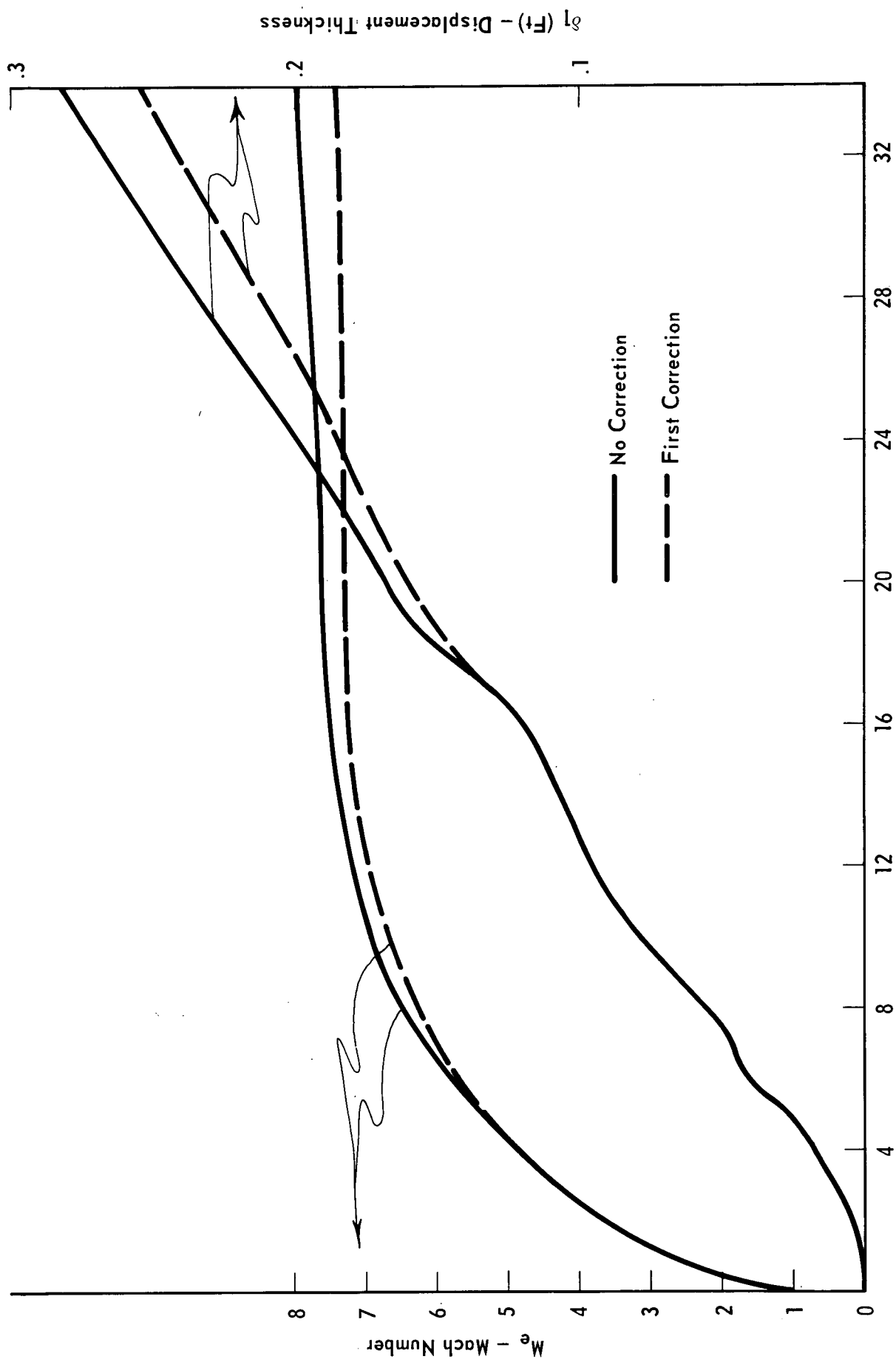
Figure 11 BOUNDARY LAYER DIFFERENCES ON WIND TUNNEL FLOW
$M_e = 7.4$, $T_{t,e} = 1340°R$ $T_w = 555°R$, $P_e = 18.13$ psf

48
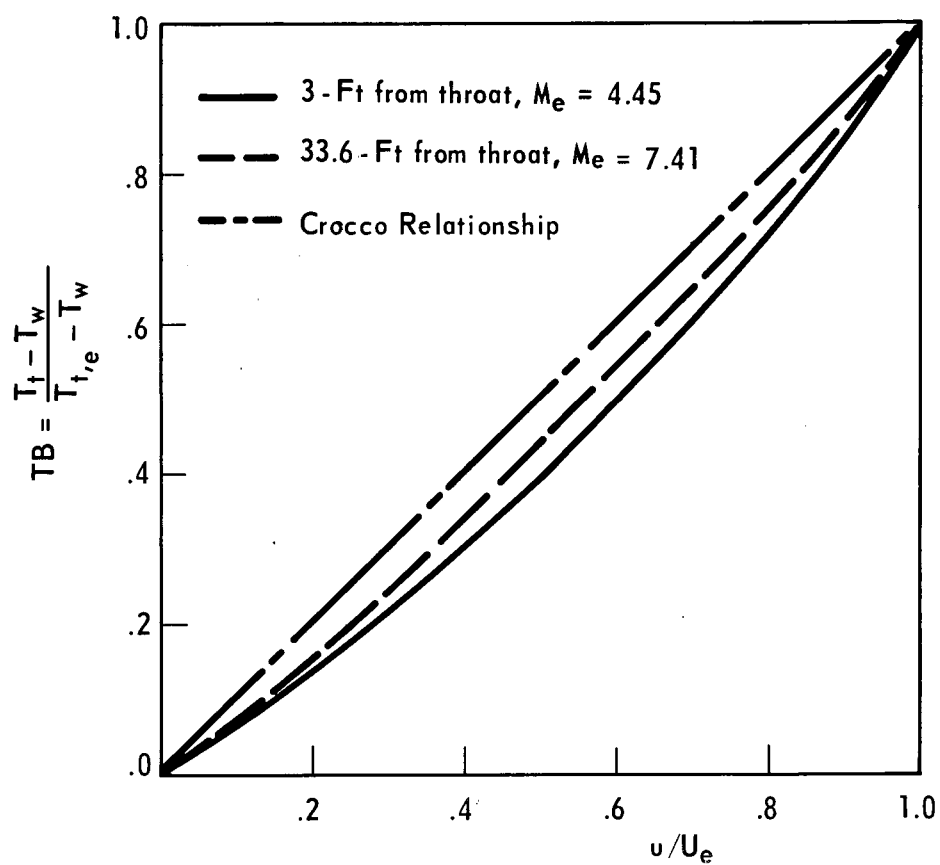
Figure 12 CHECKOUT OF CROCCO RELATIONSHIP FOR
AXISYMMETRIC NOZZLE FLOW
Ames 3.5 - Ft Wind Tunnel
$M_e = 7.4, T_{t_{/e}} = 1363°R$
$T_w = 567°R, P_e = 8.97$ psf

49
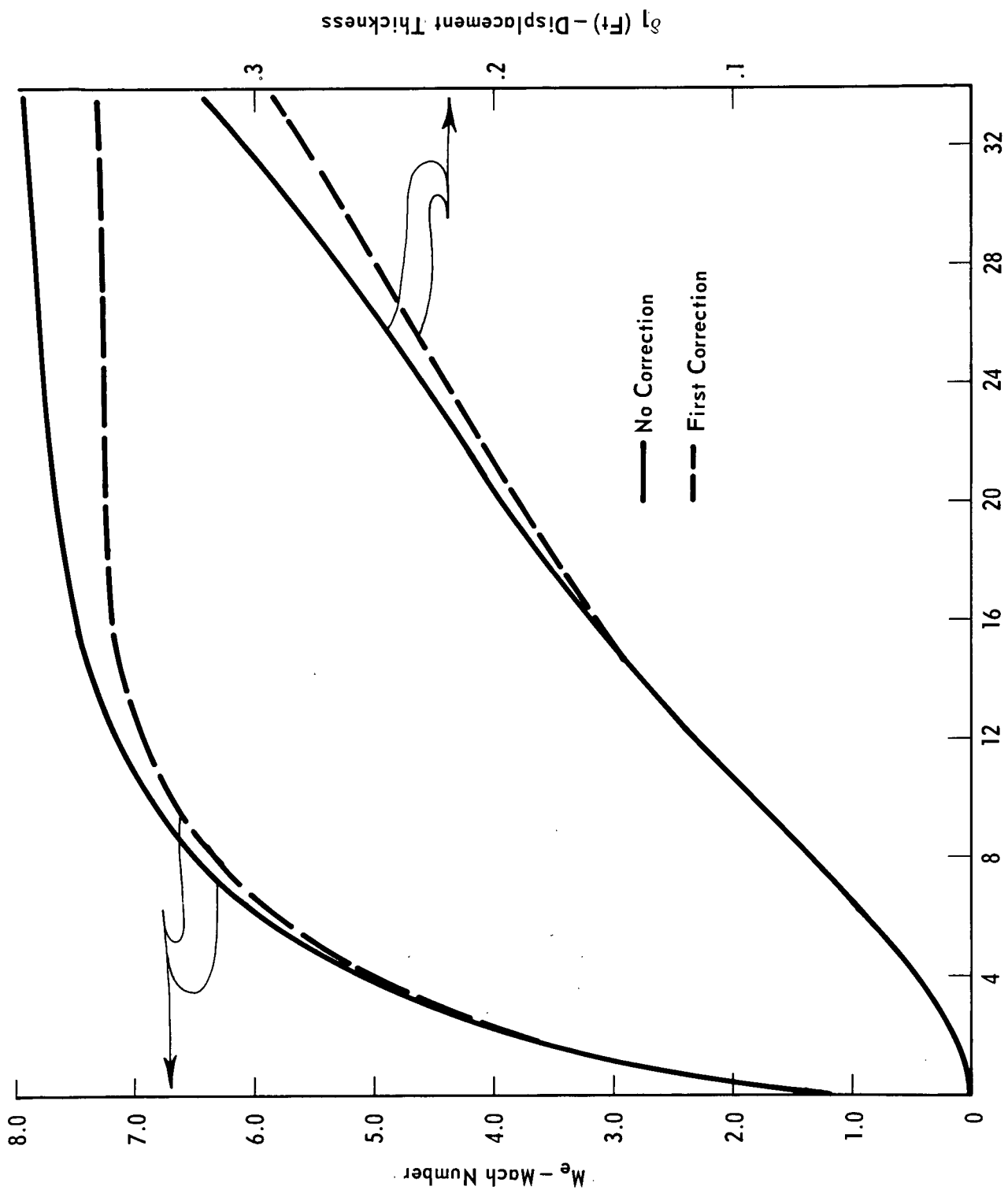
Figure 13 BOUNDARY LAYER INFLUENCES ON WIND TUNNEL FLOWS
$M_e = 7.4$, $T_{t_{r_e}} = 1363°R$ $T_w = 567°R$, $P_e = 8.97$ psf

X (Ft) – DISTANCE ALONG NOZZLE

δ_1 (Ft) – Displacement Thickness

M_e – Mach Number

No Correction
First Correction
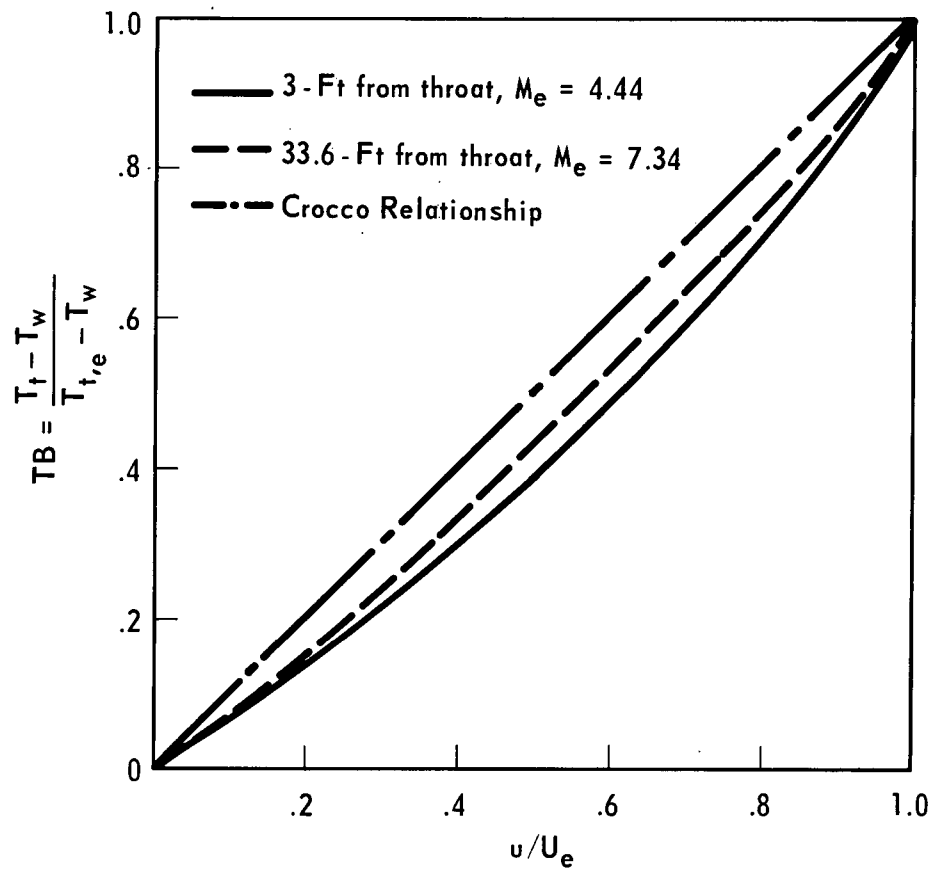
Figure 14 CHECK OUT OF CROCCO RELATIONSHIP FOR
AXISYMMETRIC NOZZLE FLOW
Ames 3.5 - Ft Wind Tunnel
$M_e = 7.4$, $T_{t_{,e}} = 1564°R$
$T_w = 568°R$, $P_e = 17.84$ psf

51
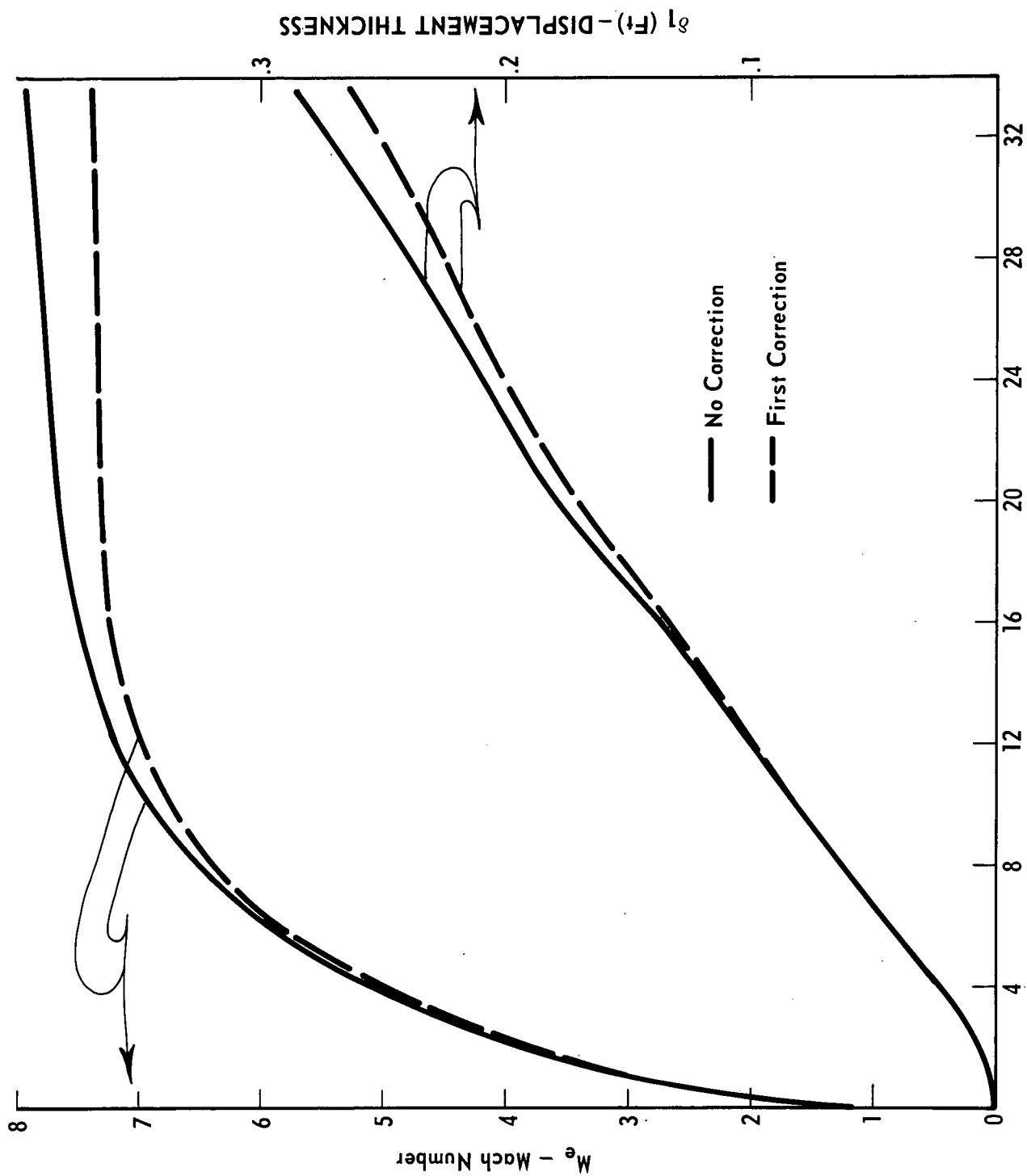
Figure 15 BOUNDARY LAYER INFLUENCES ON WIND TUNNEL FLOWS
$M_e = 7.4$, $T_{t,e} = 1564°R$ $T_w = 568°$, $P_e = 17.84$ psf

δ_1 (Ft) — DISPLACEMENT THICKNESS

X (Ft) — DISTANCE ALONG NOZZLE

M_e — Mach Number

No Correction

First Correction
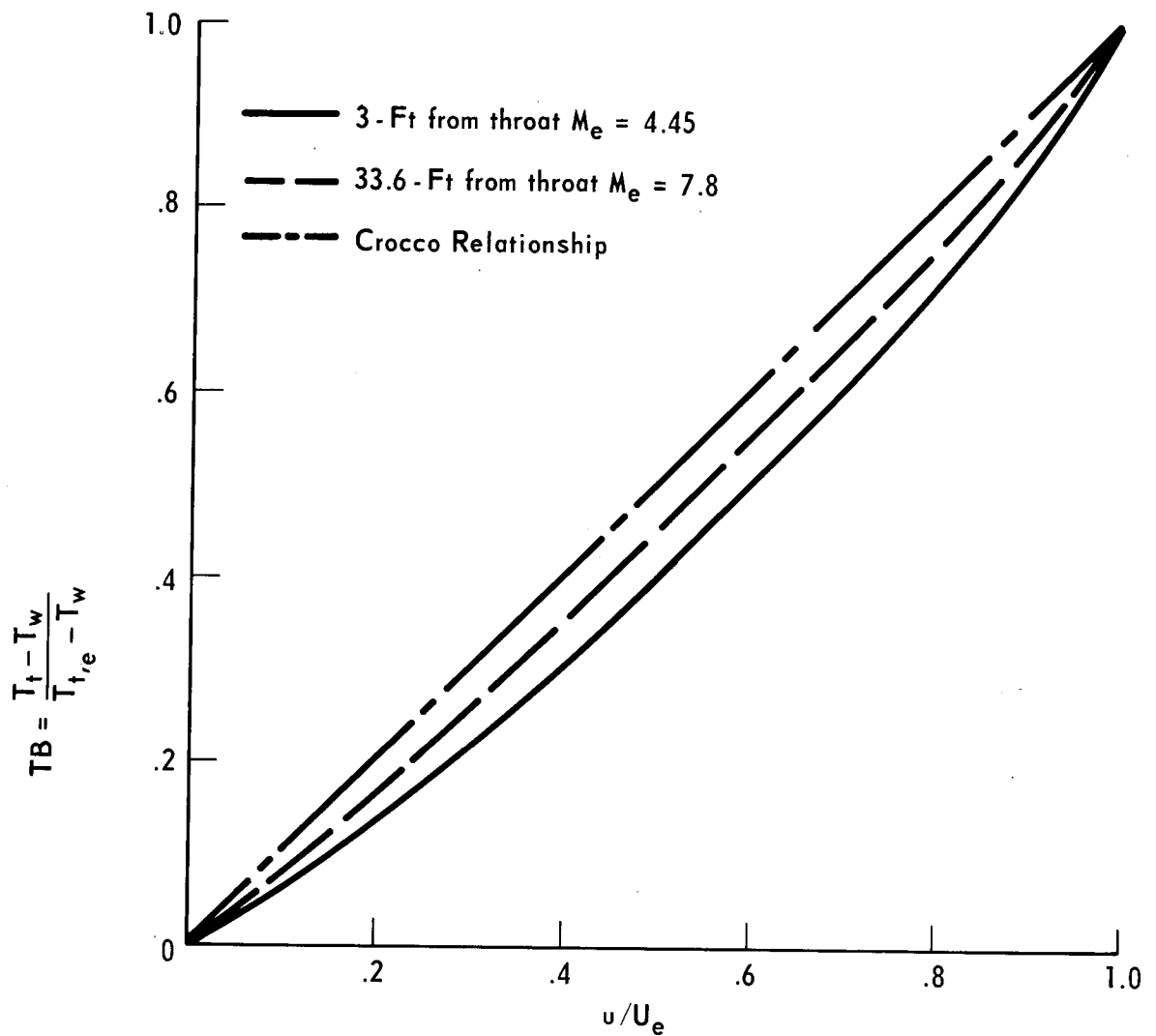
Figure 16 CHECK OF CROCCO RELATIONSHIP FOR
AXISYMMETRIC NOZZLE FLOW

Ames 3.5 - Ft Wind Tunnel
$M_e = 7.4$, $T_{t,e} = 1881°R$
$T_w = 572°R$ $P_e = 17.57$ psf
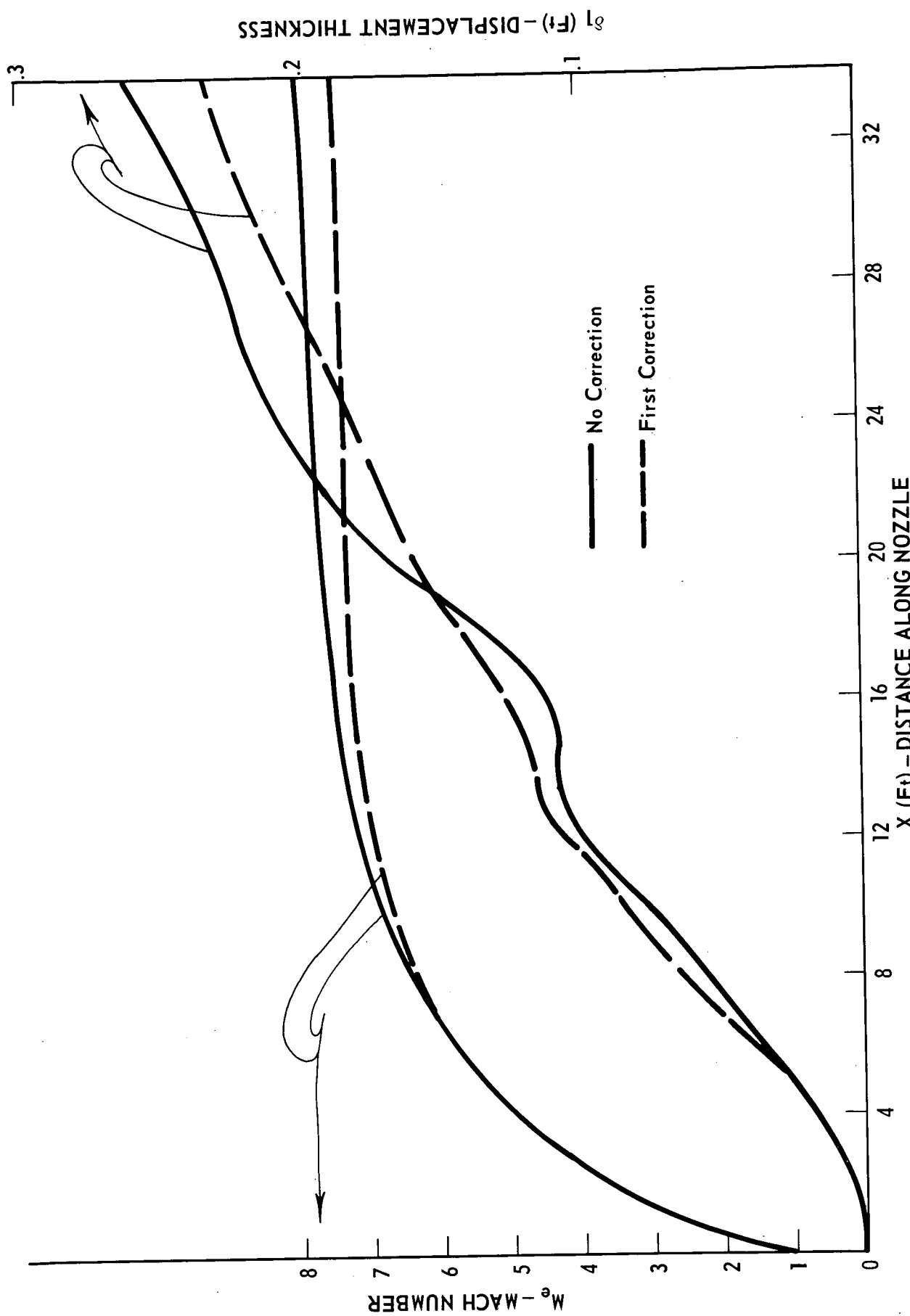
Figure 17 BOUNDARY LAYER DIFFERENCES ON WIND TUNNEL FLOW

$M_e = 7.4$, $T_{t,e} = 1881°R$ $T_w = 572°R$, $P_e = 17.57$ psf

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List of Symbols

$ACAL = A* (ACAL1)^{1/2}$.....for eddy viscosity

$ACAL1 = \nu^2 (\frac{\tau}{\rho})^{-1} = \frac{\mu}{\rho} / \frac{du}{dy}$....for eddy viscosity

ACO = Boundary layer equation coefficient

$AL = \frac{\rho (\mu + \varepsilon)}{(\rho\mu)_r}$, r - refers to the reference

$ALP = \frac{AL}{\rho} = \frac{(\mu + \varepsilon)}{(\rho\mu)_r}$

$AL1 = \frac{\rho (\frac{\mu}{Pr} + \frac{\varepsilon}{Prt})}{(\rho\mu)_r}$

$AL2 = \frac{AL1}{\rho} = \frac{(\frac{\mu}{Pr} + \frac{\varepsilon}{Prt})}{(\rho\mu)_r}$

$ALPHA$ = Slope of the body

$AMACH$ = Mach Number

$AMUR = (\rho\mu)_r$

AR = 1716 - gas constant

ARJ = Radius of curvature of the surface

$ASTAR = A* = 26$.....for eddy viscosity

$B = T_0/T_e \cdot \beta_x$, accelerating parameter

$BIG = Jg = 25040$

BCO = Boundary layer equation coefficient

$BETX = \beta_x = \frac{2\xi}{U_e} \frac{dU_e}{d\xi}$, pressure gradient coefficient

$CF = 2\tau_w/\rho_e U_e^2$, skin friction coefficient

List of Symbols

CFT = Initial value of CF to begin turbulent calculations

CF1 = Starting value for calculating CF

CF2 = Starting value for calculating CF

CFAV = Average skin friction coefficient (2 - D bodies)

CH   = $\dfrac{q_w}{\rho_e U_e C_p (T_w - T_r)}$ - Stanton Number

CNS = Index = 0 - implicit, = 1 - Crank Nicolson

CNS1 = Index = 2 - implicit, = 1 - Crank Nicolson

CO1 = Exponent in relation $\theta = (\dfrac{u}{U_e})^{CO1}$, for initial total

temperature profile guessing

CCO = Boundary layer equation coefficient

DCO = Boundary layer equation coefficient

DELBAR= $\int_0^\infty [1 - \dfrac{u}{u_e}] [1 \pm \dfrac{y}{r}]$ dy - transformed displacement thickness in

inches (cm), (+) for outside body case, (-) for inside body case

DELBM = Transformed displacement thickness in centimeters

DELIN = y/δ - point of intersection between the laminar sublayer and

1/7 power low for initial turbulent velocity profile

DELM = Displacement thickness in centimeters

DELT = $\delta_1 = \int_0^\infty [1 - \dfrac{\rho u}{\rho_e u_e}] [1 \pm \dfrac{y}{r}]$ dy, displacement thickness in inches (cm),

(+) for outside body case and (-) for inside body case

DELT1 = δ - boundary layer thickness (at $\dfrac{u}{u_e}$ = 0.995)

DELT1T= δ - boundary layer thickness (initial value to start turbulent

calculations)

List of Symbols

DELTT = Initial value of δ_1 to start turbulent calculations

DN = $\Delta\eta$ - step size in transformed plane

DN1 = Initial step size in transformed plane (laminar)

DN1T = Initial step size in transformed plane (turbulent)

DNN = η - distance across the B. L. in transformed plane

DX = Δx - forward step size in physical plane

DXE = $\Delta\xi$ - forward step size in physical plane

ECO = Boundary layer equation coefficient

ENT = h - static enthalpy

ENTE = h_e - free stream static enthalpy

ENTO = H_e - free stream total enthalpy

ENTR = h/h_e - static enthalpy ratio

ENTT = $\theta = \dfrac{H - H_w}{H_e - H_w}$ - dimensionless total enthalpy

ENTTW = H_w - wall total enthalpy

EPS = $\varepsilon = \rho \ell^2 \left|\dfrac{\partial u}{\partial y}\right|$ - eddy viscosity

EPSK = $\dfrac{\varepsilon}{\rho}$ - kinematic eddy viscosity

EPSO = $\varepsilon_0 = 0.0168\, \rho_e\, U_e\, \delta$ - outer region eddy viscosity

FACT = $\text{EXP}\left(11.8\, \dfrac{\mu_w}{\bar{\mu}}\, V_w^+\right)$.....for eddy viscosity

FACTOR= $\left\{\dfrac{\bar{\mu}}{\mu_e}\left(\dfrac{\rho_e}{\rho_w}\right)^{1/2} \dfrac{p^+}{V_w^+}[1 - \text{FACT}] + \text{FACT}\right\}^{1/2}$.....for eddy viscosity

C.2.

List of Symbols

FCO   = Boundary layer equation coefficient

G   = C_p/C_v

H   = Shape factor = δ^*/θ

HB   = Transformed shape factor (DELTT/THETT)

ITER = Number of iterations between two x - locations (= 1- laminar)

ITERT = Number of iterations between two x - locations (= 3 - turbulent)

JSTOP = Writing parameter

K   = Growth factor between two steps across B. L. (laminar)

KT   = Growth factor between two steps across B. L. (turbulent)

LAMNER= See p. 22

Me   = Freestream Mach number

MRAT = $\dfrac{M}{M_e}$ - Mach Number ratio

MXL   = ℓ - mixing length

NSUB = Number of steps across B. L. corresponding to y^+ = 11.8

NU   = Total number of stations across B. L. (laminar)

NU2   = NU-2, number of unknown steps across B. L.

NU3   = NU-1

NUL   = Number of laminar forward steps

NUL1 = NUL+1

NUS   = An index given for the first forward step in laminar or turbulent calculations

NUT   = Total number of stations across B. L. (turbulent)

NUX   = Total number of forward steps

NUX1 = NUX+1

List of Symbols

PE $\quad$ = Free stream pressure $\quad$ lb/ft^2 (Newtons/cm^2)

PEM $\quad$ = Free stream pressure $\quad$ Newtons/cm^2

PERAT = P_e/P_t

POWER = Initial N for $u/U_e = (y/\delta)^{1/N}$

PPLUS = $p^+ = \dfrac{(\nu_e/U_e^2)\ (dU_e/dx)}{[Cf/2]^{3/2}}$

PR $\quad$ = PRANDTL Number

PRAT $\quad$ = Pt_2/Pt_{2_e} - total pressure ratio

PT2 $\quad$ = Total pressure (after the shock)

q_w $\quad$ = Rate of heat transfer from the surface per unit area

QE $\quad$ = Free stream dynamic pressure lb$_f$/ft^2 (Newtons/cm^2)

QEM $\quad$ = Dynamic pressure Newtons/cm^2

RE $\quad$ = $\dfrac{T_R - T_e}{T_o - T_e}$ - recovery factor

REX $\quad$ = Reynolds Number = $\dfrac{\rho_e U_e X}{\mu_e}$

RMU $\quad$ = $\dfrac{\rho\mu}{(\rho\mu)_r}$

RTHET = Momentum Reynolds Number = $\dfrac{\rho_e U_e \theta}{\mu_e}$

S $\quad$ = 2 ST/CF

List of Symbols

SKIP = Index = 0 - laminar, = 1 - turbulent flow

TAW   = Adiabatic wall temperature ^{O}R (^{O}K)

TAWM = Adiabatic wall temperature ^{O}K

TC   = $\dfrac{(2\xi)^{1/2} \, Cos\alpha}{U_e \, r_b^{2j}}$ - transverse curvature term

TCFACT= Transverse curvature factor (= 0 - 2 - D, = +1 for external

axisymmetric, -1 for internal axisymmetric

TE   = Free stream temperature ^{O}R (^{O}K)

TEM   = Temperature

TEME = Free stream temperature

TEMP = Temperature used in TAB2

TEQ   = Free stream temperature ^{O}K

TERAT = T_e/T_t

THEBM = Transformed momentum thickness in centimeters

THET = $\int_0^\infty (1 \pm \dfrac{y}{r}) \, [1 - \dfrac{u}{u_e}] \, \dfrac{\rho}{\rho_e} \, \dfrac{u}{u_e}$ dy - momentum thickness in inches (cm),

(+) for outside body case and (-) for inside body case

THETM = Momentum thickness in centimeters

THETT = $\int_0^\infty (1 \pm \dfrac{y}{r}) \, [1 - \dfrac{u}{u_e}] \, \dfrac{u}{u_e}$ dy - transformed momentum thickness in

inches (cm), (+) for outside body case and (-) for inside body case

TRAT = T/T_e - temperature ratio

TTE   = Total free stream temperature ^{O}R (^{O}K)

List of Symbols

TTEM = Total free stream temperature $^{\circ}K$

TW = Wall temperature $^{\circ}R$ ($^{\circ}K$)

TWM = Wall temperature $^{\circ}K$

TWRAT = T_w/T_{Aw} - wall temperature ratio

UE = U_e - free stream velocity fps (mps)

UEM = U_e - free stream velocity in meters/sec

UFVE = $f' = \dfrac{u}{u_e}$ - dimensionless velocity

VP-N = $\dfrac{VPE1}{VPE2}$ - velocity profile exponent for $u/U_e = (y/\delta)^{1/N}$

VPE1 = $\log (y_{n+1}/y_n)$

VPE2 = $\log (u_{n+1}/u_n)$

VPLUS = $v^+ = \dfrac{v}{u^*}$ .....for eddy viscosity

VSTAR = $u^* = (\tau/\rho)^{1/2}$

VSTAR2 = $u^{*2} = (\tau/\rho)$

VVE = Dimensionless velocity across the B. L.

VWALP = Vertical velocity at the wall in physical plane in ft/sec

VWAL = Vertical velocity at the wall in transformed plane

WDUDY = $\partial u/\partial y\big|_w$

XD = x - forward distance in physical plane in feet (m)

XDM = x - forward distance in physical plane in meters

XE = ξ - forward distance in transformed plane

List of Symbols

XED   = XEN

$$XEN = \frac{XE\ (N + 1) + XE\ (N)}{2}$$

Y   = y - vertical distance across B. L. in physical plane

YB   = y - corresponding to y^+ = 11.8.....for eddy viscosity

YDD   = y/δ - dimensionless distance across B. L.

YRAT   = y/δ_1 - dimensionless distance across B. L.

List of Tables and Subroutine

All tables and subroutine are for air

TAB   = To compute the temperature from enthalpy

TAB1   = Subroutine to calculate PRAT (pressure ratio)

TAB2   = To compute the viscosity from temperature (Keye s' formula)

TAB3   = To compute the turbulent Prandtl Number

B 6 7 0 0   F O R T R A N   C O M P I L A T I O N   M A R K   1.09.00.06.

APPENDIX

CON
CON
CON
CON

```
FILE   5=DWYER,UNIT = READER
FILE   6=DWYER/F210F001,UNIT = PRINTER
C
C      *************************************************************** ****
C      *                                                                 *
C      **********   H.A.DWYER-UNIVERSITY OF CALIF.,DAVIS,MARCH 1971 ****** ****
C      ** PROGRAM FOR CALCULATING THE LAMINAR AND TURBULENT BOUNDARY LAYERS  **
C      ******* THE METHOD IS A BASIC FINITE DIFFERENCE SCHEME-IMPLICIT *** ****
C      ****  THE PROGRAM IS BASICALLY A NUMERICAL EXPERIMENT WITH EDDY  *** ****
C      **    VISCOSITY AND TURBULENT PRANDTL NUMBER ********************** ****
C      *                                                                 *
C      *************************************************************** ****
C
C
C      -------- ALL DIMENSIONS ARE IN SLUG,FOOT,SECOND,DEGREE RANKINE  ---- ----
C
C      **********   SOME OF THE KEY SYMBOLS USED IN THIS PROGRAM   ******** ****
C
C              UEVE,VVE ................. DIMENSIONLESS VELOCITIES IN STREA MWISE
C                                         AND CROSS DIRECTIONS
C              UE,PE,ENTE ............... FREE STREAM VELOCITY,PRESSURE
C                                         AND ENTHALPY
C              ENT,ENTT,ENTO,ENTR ....... STATIC,TOTAL AND DIMENSIONLESS
C                                         STATIC ENTHALPY
C              VWAL,ENTTW,WALT .......... WALL VERTICAL VELOCITY,ENTHALPY
C                                         AND TEMPERATURE
C              XD,Y,YD .................. DISTANCES IN PHYSICAL PLANE
C              XE,XEN,DNN ............... DISTANCES IN TRANSFORMED PLANE
C              AL,ALP,AL1,AL2 ........... TRANSPORT COEFFICIENTS
C              DXE,DN ................... STEP SIZES IN TRANSFORMED PLANE
C              NU,NU2,NU3 ............... NO. OF STEPS ACROSS B.L.
C              NUX,NUL .................. NO. OF TOTAL AND LAMINAR STEPS
C              NUS,NU1 .................. STARTING AND ENDING STATIONS IN
C                                         B.L. CALCULATION
C              ACO,BCO,CCO,DCO,ECO,FCO .  COEFFICIENTS OF THE MOMENTUM AND
C                                         ENERGY FINITE DIFFERENCE EQUATION S
C              VRAT,TRAT,PRAT,PRAT ...... DIMENSIONLESS VERTICAL DISTANCE,
C                                         TEMPERATURE,PRESSURE AND MACH NO.
C              CNS,CNS1 ................. FULL IMPLICIT AND CRANK-NICLSON
C                                         FACTORS
C
C      ******************   LIST OF TABLES AND SUBROUTINE   *************** ****
C
C              TAB ....... TO COMPUTE THE TEMPERATURE FROM ENTHALPY
C              TAB1 ...... SUBROUTINE TO CALCULATE PRESSURE RATIO
C              TAB2 ...... TO COMPUTE VISCOSITY FROM TEMPERATURE
C              TAB3 ...... TO COMPUTE TURBULENT PRANDTL NUMBER
C
```

```
C
      INTEGER SO,SKIP
      REAL K,KT,MXL,MRAT
C
C
C     *************************     DIMENSIONS     ************************* ****
C
      DIMENSION UFVE(502,2),VVE(501,2),UE(100),PE(100),ENTE(100)
     1          ,ENT(501,2),ENTT(501,2),ENTTW(100),TEM(501,2),ENTR(501,2)
     2          ,ACO(501,2),BCO(501,2),CCO(501,2),DCO(501,2),ECO(501,2)
     3                                                       ,FCO(501,2),
     4          AL(501),AL1(501),AL2(501),AMUR(100),EPS(501),ALP(501),
     5          XE(100),XEN(100),XD(100),Y(501),DN(501),ARJ(100),
     6          PRAT(501),VPE(501),
     7          PT2(501),AMM(501),CF1(100),JSTOP(501),VWAL(100)
     8          ,VWALP(100)
     9          ,AMACH(100),PERAT(100),TERAT(100)
C
C
C     ----------------------------------------------------------------- ----
C
      DATA                                                          CHANG
     1    AL(1),AR,ASTAR,BIG,POWER/1.,1716.,26.,2.504E4,7./           CHANG
     2    ,CO1,ENTO/1.,471.32/                                        CHANG
     3    ,PR,TCFACT/.723,-1.0/                                       CHANG
     4    ,NUL,NUX,LAMNER/1,21,2/                                     CHANG
     5    ,NUT,ITERT,DN1T,KT/350,3,.005,1.021/                        CHANG
     6    ,CFT,DELT1T,DELTI,YPLUSS/.00212,.00178,.0002225,0.0/        CHANG
      DATA                                                          CHANG
     1    (XD(N),N=2,22)/.0,.25,.499,.739,1.009,1.249,1.489,1.999,2.989,  CHANG
     2    4.,5.,6.25,7.5,10.,12.5,15.83,19.167,26.49,29.82,33.636,34./   CHANG
C
C
C     ----------------------------------------------------------------- ----
C
C
C     ******  READ THE BOUNDARY CONDITIONS AND REFERENCE CONSTANTS  ******
      CF2 = 0.0
      NUX1 = NUX + 1
      NUL1 = NUL+1
      READ(5,931)(ARJ(N),AMACH(N),PERAT(N),TERAT(N),N = 2,22)     CHANG
  931 FORMAT(F15.8,19X,F5.3,2X,F8.7,2X,F6.5)                      CHANG
      DO 1000 N=2,NUX1
      XD(N) = XD(N)+.273                                         CHANG
C     ........  FREE STREAM BOUNDARY CONDITIONS
      PE(N) = PERAT(N)*83302.                                     CHANG
      UE(N) = 49.1*AMACH(N)*SQRT(1879.*TERAT(N))                  CHANG
      ENTE(N) = ENTO - UE(N)**2/(2.*BIG)
C     ........  WALL BOUNDARY CONDITIONS
      ENTTW(N) = 133.38                                          CHANG
      WALT=TAB(ENTTW(N))
      VWALP(N) = 0.0                                             CHANG
C     ........  REFERENCE CONSTANTS
      AMUR(N)=PE(N)/(AR*WALT)*TAB2(WALT)
 1000 CONTINUE
C     ........  X-DISTANCES IN THE TRANSFORMED PLANE
      XE(1) = 0.0
      DO 1250 N = 2,NUX1
      XE(N)=XE(N-1)+(XD(N)-XD(N-1))/2.*((AMUR(N-1)*UE(N-1)*ARJ(N
     1-1)**2)+(AMUR(N)*UE(N)*ARJ(N)**2))
      XEN(N) = XE(N)-(XD(N)-XD(N-1))*(3.*AMUR(N)*ARJ(N)**2*UE(N) +
     1          AMUR(N-1)*ARJ(N-1)**2*UE(N-1))/8.
      VWAL(N) = SQRT(2.*XE(N))*PE(N)/(AR*TAB(ENTTW(N))*AMUR(N)*ARJ(N))*
     1          VWALP(N)/UE(N)
```

```
 1250 CONTINUE
C
C          ********** THE MAIN DO LOOP OF THE PROGRAM **********
C
      DO 8000 MAIN = LAMNER,2
      SKIP = MAIN - 1
C          SKIP ........  = 0 - LAMINAR , =1 - TURBULENT
      IF ( SKIP .NE.0 ) GO TO 1
C          ********** LAMINAR FLOW **********
C
      XD(1) = 0.0
      NUS = 1
      NU1 = NUL
      CNS = 1.0
      CNS1 = 1.0
      NU = 61
      ITER = 1
      DN1 = .1
      K = 1.0
      WRITE(6,150)
      WRITE(6,550)
C ...... CALCULATION OF INITIAL LAMINAR PROFILES
      DN(1) = DN1
      VVE(1,1) = VWAL(1)
C ...... READING THE UFVE-VELOCITY PROFILE                          CHANG
C ......  THIS IS THE INITIAL LAMINAR VELOCITY PROFILE FOR FLAT PLATE  .....
      READ(5,100)(UFVE(N,1),N = 1,61,2)                            CHANG
  100 FORMAT (9X,F8.6)
      DO 1500 N = 1,30
      UFVE(2*N,1) = (UFVE(2*N-1,1) + UFVE(2*N+1,1))/2.
 1500 CONTINUE
      DO 1750 N = 2,NU
      DN(N) = K*DN(N - 1)
C .......... TEMPERATURE PROFILE
      ENTT(N,1) = UFVE(N,1)**CO1                                  CHANG
      ENT(N,1) = ENTT(N,1)*(ENTO-ENTTW(1))+ENTTW(1)-(UFVE(N,1)*UF(1))
     1**2/(2.*BIG)
      TEM(N,1) = TAB(ENT(N,1))
      ENTR(N,1) = ENT(N,1)/ENTE(1)
C .......... VVE-VELOCITY PROFILE
      VVE(N,1) = VVE(N-1,1)-DN(N-1)*(UFVE(N,1)+UFVE(N-1,1))/2.
 1750 CONTINUE
C .......... B.L. THICKNESS FOR FLAT PLATE                        CHANG
      THET = 1.
      TEME = TAB(ENTE(1))
      ISTOP = 61
      UFVE(NU+1,1) = 1.0
      CF1(1) = UE(2)**1.5*UFVE(1,1)/DN1*.5/SQRT(XD(2)*TAB2(TEM(1,1))*  CHANG
     1PE(2)/(AR*TEM(1,1)))
      GO TO 2
C
C          ********** TURBULENT FLOW **********
C
C ........ TURBULENT FLOW CONSTANTS
    1 NUS = NUL+1
      NU1 = NUX
      CNS = 0.0
      CNS1 = 2.0
      NU = NUT
      ITER = ITERT
```

```
      DN1 = DN1T
      K = KT
      WRITE(6,200)
      WRITE(6,550)
C
C     ......  CALCULATION OF INITIAL TURBULENT B.L. THICKNESS
C             AND PROPERTIES AFTER TRANSITION
      TEM(1,1)=TAB(ENTTW(NUL1))
      CF = CFT
      DELT1 = DELT1T
      DELT = DELTT
      TEME = TAB(ENTE(NUL1))
      VSTAR2=CF*UE(NUL1)**2*TEM(1,1)/(2.*TEME)
      VSTAR=SQRT(VSTAR2)
      DELIN=1./(DELT1*VSTAR2*PE(NUL1)/(TAB2(TEM(1,1))*AR*TEM(1,1
     1)*UE(NUL1)))**(POWER/(POWER-1.))
C
C     ......  CALCULATION OF INITIAL TURBULENT PROFILES                    CHANG
C     ....   THE INITIAL VELOCITY PROFILE IS THAT OF LAMINAR SUBLAYER   ... .
C     ..................         AND 1 / POWER TURBULENT FLOW     ............... ....
      DN(1)=DN1
      DNN=0.
      Y(1)=0.
      UFVE(1,1)=0.
      UFVE(NU+1,1) = 1.0
      VVE(1,1) = VWAL(NUL1)
      ENTR(1,1) = ENT(1,1)/ENTE(NUL1)
C     ..........   UFVE-VELOCITY PROFILE
      DO 2000 N = 2,NU
      DNN = DNN + DN(N - 1)
      Y(N) = DN(N-1)*(SQRT(2.*XE(NUL1))*AR*TEM(N-1,1)/(UE(NUL1)*
     1        ARJ(NUL1)*PE(NUL1))) + Y(N-1)
      DN(N) = K*DN(N - 1)
      YDD = Y(N)/DELT1
      IF (YDD.LE.DELIN) UFVE(N,1)=VSTAR2*Y(N)*PE(NUL1)/(AR*TEM(1
     1,1)*TAB2(TEM(1,1)))/UE(NUL1)
      IF (YDD.GT.DELIN) UFVE(N,1)=(YDD)**(1./POWER)
      IF ( UFVE(N,1) .GE.1.0 ) UFVE(N,1) = 1.0
C     ..........   TEMPERATURE PROFILE
      ENTT(N,1) = UFVE(N,1)**C01                                           CHANG
      ENT(N,1)=ENTT(N,1)*(ENTO-ENTTW(NUL1))+ENTTW(NUL1)-(UFVE(N,1
     1)*UE(NUL1))**2/(2.*BIG)
      TEM(N,1) = TAB(ENT(N,1))
      ENTR(N,1) = ENT(N,1)/ENTE(NUL1)
C     ..........   VVE-VELOCITY PROFILE
      VVE(N,1) = VVE(N-1,1)-DN(N-1)*(UFVE(N,1)+UFVE(N-1,1))/2.
 2000 CONTINUE
      DO 2250 N = 125,NU
      IF(UFVE(N-1,1) .LT. 1. .AND. UFVE(N + 1,1) .EQ. 1.0)ISTOP = N + 1
 2250 CONTINUE
C
C     ***************   PRINT OUT THE INITIAL CONDITIONS   *************** ****
C
    2 DO 2500 N = 1,NU
      AMM(N) = UE(NUL1)*UFVE(N,1)/SQRT(1.4*AR*TEM(N,1))
      IF(Y(N) .LE. 0.0 .OR. Y(N+1) .LE. 0.0 ) GO TO 2500
      VPE1 = ALOG10(Y(N+1)/Y(N))
      VPE2 = ALOG10(UFVE(N+1,1)/UFVE(N,1))
      IF( VPE2 .EQ. 0.0 ) GO TO 2500
      VPE(N) = VPE1/VPE2
```

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```
 2500 CONTINUE
      CALL TAB1(NU,AMM,PRAT)
      JSTOP(1) = 0
      DO 2750 JK = 1,5
      JSTOP(JK + 1) = JSTOP(JK) + 20*JK
      LZ = JSTOP(JK)
      IF(JK .EQ. 1) LZ = 1
      JQ = JSTOP(JK + 1)
      DO 2750 JL = LZ,JQ,JK
      IF(JL .EQ. JSTOP(JK)) GO TO 2750
      IF( JL .GT. ISTOP) GO TO 3
      YRAT = Y(JL)/THET
      TRAT = TEM(JL,1)/TEM(NU,1)
      MRAT = UFVE(JL,1)/SQRT(TRAT)
      WRITE(6,600)JL,UFVE(JL,1),MRAT,TRAT,TTRAT,TB,HB,PRAT(JL),YRAT,
     2VPE(JL),AL1(JL)
 2750 CONTINUE
C
    3 NU3 = NU - 1
      NU2=NU-2
C ...... NU - TOTAL NO. OF STEPS ACROSS THE B.L.
C ...... NU2 - NO. OF UNKNOWN STEPS ACROSS THE B.L.
C
C ********************* THE HEART OF THE PROGRAM ***************** ****
C
C
C ********** START OF ONE FORWARD STEP CALCULATIONS **********
C
      DO 7750 L=NUS,NU1
C
C ......... FREE STREAM PRESSURE GRADIENT FACTOR-BETX
      DX = XD(L+1) - XD(L)
      DXE=XE(L+1)-XE(L)
      BETX = 2.*XEN(L+1)*(UE(L+1)-UE(L))/(DXE*((UE(L+1)+UE(L))/2.))
      XED = XEN(L+1)
      ALPHA =ARSIN((ARJ(L+1)-ARJ(L))/DX)
      TC1 = (SQRT(2.*XED)*COS(ALPHA)/UE(L+1))*TCFACT
      IF( SKIP .EQ. 0 ) GO TO 22
      PPLUS=TAB2(TEMF)*AR*TEME/PE(L+1)*(UE(L+1)-UE(L))/((XD(L+1)-XD(L))
     1         *((UE(L+1)+UE(L))/2.)**2*(CF/2.)**1.5)
   22 CONTINUE
C
C ........ LOADING THE BOUNDARY CONDITIONS
C
      ECO(1,1)=0.
      ECO(1,2)=0.
      FCO(1,1)=0.
      FCO(1,2)=ENTTW(L+1)/ENTE(L+1)
      UFVE(1,1)=0.
      VVE(1,2) = VWAL(L+1)
      UFVE(NU,2)=1.0
      ENTR(NU,2) = 1.0
C
C ........ (1) CALCULATION OF THE TRANSPORT PROPERTIES ............... ....
C
      AL1(1)=AL1(1)/PR
      IF(SKIP .NE. 0) GO TO 4
C
C .......... LAMINAR FLOW
C
```

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```
      ALP(1) = AL(1)*AR*TEM(1,1)/PE(L+1)
      AL2(1) = AL1(1)*AR*TEM(1,1)/PE(L+1)
      DO 3000 N = 1,NU2
      EPS(N+1)=0.0
      RMU=TEM(1,1)*TAB2(TEM(N+1,1)))/(TEM(N+1,1)*TAB2(TEM(1,1)))
      AL(N+1) = RMU
      ALP(N+1) = AL (N+1)*AR*TEM(N+1,1)/PE(L+1)
      AL1(N+1)=RMU/PR
      AL2(N+1)=AL1(N+1)*AR*TEM(N+1,1)/PE(L+1)
 3000 CONTINUE
C
C     **********    START OF THE ITTERATION LOOP FOR TURBULENT FLOW
    4 DO 6000 JJ = 1,ITER
      IF (SKIP .EQ. 0) GO TO 9
C
C     ..........   TURBULENT FLOW
      M = 1
C
      IF(JJ .GT. 1) M=2
      ALP(1) = AL(1)*AR*TEM(1,M)/PE(L+1)
      AL2(1) = AL1(1)*AR*TEM(1,M)/PE(L+1)
C     ........   EDDY VISCOSITY CALCULATION
      EPSO=.0168*UE(L+1)*DELT
C     ........   CORRECTION FACTOR FOR SUBLAYER AND TRANSITION
      WDUDY = (-K*(2.+K)*UFVE(1,M)+(K*(2.+K)+1.)*UFVE(2,M)-UFVE(3,M))
     1       *UE(L+1)/(K*(1.+K)*Y(2))
      ACAL1 =AR*TEM(1,M)*TAB2(TEM(1,M))/(PE(L+1)*WDUDY)
      ACAL=ASTAR*SQRT(ACAL1)
      YB=11.8*SQRT(ACAL1)
      DO 3250 N=2,100
      IF (YB.LT.Y(N).AND.YB.GT.Y(N-1)) GO TO 5
 3250 CONTINUE
    5 NSUB=N-1
      ACAL=ACAL*TAB2(TEM(NSUB,M))/TAB2(TEM(1,M))*SQRT(TEM(NSUB,M)/
     1  TEM(1,M))
      IF(VWALP(L+1).EQ.0.0) GO TO 6
      VPLUS =VWALP(L+1)/SQRT(AR*TEM(1,M)+TAB2(TEM(1,M))*WDUDY/PE(L+1))
      FACT = EXP(11.8*VPLUS*TAB2(TEM(1,M))/TAB2(TEM(NSUB,M)))
      FACTOR = SQRT(TAB2(TEM(NSUB,M))/TAB2( TEME )  *SQRT(TEM(1,M)/TEME)
     1         *PPLUS/VPLUS*(1.-FACT) + FACT)
      GO TO 7
    6 FACTOR =(1.-11.8*TAB2(TEM(1,M))/TAB2(TEME)*(TEM(1,M)/TEME)**.5
     1         *PPLUS)**.5
    7 ACAL = ACAL/FACTOR
      DO 3500 N = 1,NU2
C     ........   INNER REGION
      MXL=Y(N+1)*.4*(1.-EXP(-Y(N+1)/ACAL))
      EPSK = MXL**2*ABS(UFVE(N+2,M)-UFVE(N,M))*UE(L+1)/(Y(N+2)-Y(N))
      EPS(N+1)=EPSK*PE(L+1)/(AR*TEM(N+1,M))
      IF (EPSK .GT. EPSO) GO TO 8
 3500 CONTINUE
C     ........   OUTER REGION
    8 I = N-1
      DO 3600 N = I,NU2
      EPS(N+1) = EPSO*PE(L+1)/(AR*TEM(N+1,M))
 3600 CONTINUE
      DO 3700 N = 1,NU2
      RMU=TEM(1,M)*TAB2(TEM(N+1,M))/(TEM(N+1,M)*TAB2(TEM(1,M)))
      AL(N+1) = RMU+PE(L+1)*EPS(N+1)/(AR*TEM(N+1,M)*AMUR(L+1))
      ALP(N+1) = AL (N+1)*AR*TEM(N+1,M)/PE(L+1)
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      AL1(N+1)=RMU   /PR+PE(L+1)*EPS(N+1)/(AR   *TEM(N+1,M)*AMUR(L+1)
     1*TAB3(YD))
      AL2(N+1)=AL1(N+1)*AR*TEM(N+1,M)/PE(L+1)
 3700 CONTINUE
C
    9 AL(NU) = AL(NU-1)
      ALP(NU) = ALP(NU-1)
      AL1(NU) = AL1(NU - 1)
      AL2(NU) = AL2(NU - 1)
C
C  ......... (2) CALCULATION OF THE B.L. EQUATIONS COEFFICIENT  ......... ....
C
      DO 4000 N = 1,NU2
      TC = TC1/(ARJ(L+1)+TCFACT*Y(N+1)*COS(ALPHA))**2
      ACO(N,1)=(VVE(N+1,1)/(2.*DN(N)*(1.+K))-(AL(N+2)-AL(N))/(2.*(DN(N)*
     1(1.+K))**2)-AL(N+1)/(DN(N)**2*(1.+K)*K)-TC*ALP(N+1)/(2.*DN(N)*
     2(1.+K)))*CNS1
      BCO(N,1)=(AL(N+1)*(1.+K)/(DN(N)**2*(1.+K)*K))*CNS1+BETX*UFVE(N+1,1
     1) + 2.*XED*UFVE(N+1,1)/DXE
      CCO(N,1)=(-VVE(N+1,1)/(2.*DN(N)*(1.+K))+(AL(N+2)-AL(N))/(2.*(DN(N)
     1*(1.+K))**2)-AL(N+1)*K/(DN(N)**2*(1.+K)*K)+TC*ALP(N+1)/(2.*DN(N)*
     2(1.+K)))*CNS1
      DCO(N,1)=BETX*TEM(N+1,1)/TEM(NU,1)+2.*XED*UFVE(N+1,1)**2/DXE+CNS*(
     1-VVE(N+1,1)*(UFVE(N+2,1)-UFVE(N,1))/(2.*DN(N)*(1.+K)) + (AL(N+2)-
     2AL(N))*(UFVE(N+2,1)-UFVE(N,1))/(2.*(DN(N)*(1.+K))**2) + AL(N+1)*(
     3UFVE(N+2,1)-(1.+K)*UFVE(N+1,1)+K*UFVE(N,1))/(DN(N)**2*(1.+K)*K)+
     4TC*ALP(N+1)*(UFVE(N+2,1)-UFVE(N,1))/(2.*DN(N)*(1.+K)))
C
      ACO(N,2)=(VVE(N+1,1)/(2.*DN(N)*(1.+K))-(AL1(N+2)-AL1(N))/(2.*(DN(N
     1)*(1.+K))**2)-AL1(N+1)/(DN(N)**2*(1.+K)*K)-TC*AL2(N+1)/(2.*DN(N)*
     2(1.+K)))*CNS1
      BCO(N,2) = 2.*XED*UFVE(N+1,1)/DXE+(AL1(N+1)*(1.+K)/(DN(N)**2*
     1(1.+K)*K))*CNS1
      CCO(N,2)=(-VVE(N+1,1)/(2.*DN(N)*(1.+K))+(AL1(N+2)-AL1(N))/
     1(2.*(DN(N)*(1.+K))**2)-AL1(N+1)*K/(DN(N)**2*(1.+K)*K)+TC*AL2(N+1)
     2/(2.*DN(N)*(1.+K)))*CNS1
      DCO(N,2)=2.*XED*UFVE(N+1,1)*ENTR(N+1,1)/DXE+(UF(L+1)**2/ENTE(L+1)
     1         +UF(L)**2/ENTE(L))/(2.*BIG)+(AL(N+1)*((UFVE(N+2,1)-
     2         UFVE(N,1))/(DN(N)*(1.+K)))**2 -  BETX*TEM(N+1,1)/TEM(NU,1
     3         )*UFVE(N+1,1)+BETX*UFVE(N+1,1)*ENTR(N+1,1))+(-VVE(N+1,1)*
     4        (ENTR(N+2,1)-ENTR(N,1))/(2.*DN(N)*(1.+K))+(AL1(N+2)-AL1(N))
     5        *(ENTR(N+2,1)-ENTR(N,1))/(2.*DN(N)*(1.+K))**2 +AL1(N+1)*(ENT
     6        R(N+2,1)-(1.+K)*ENTR(N+1,1)+K*ENTR(N,1))/(DN(N)**2*(1.+K)
     7        *K)+TC*AL2(N+1)*(ENTR(N+2,1)-ENTR(N,1))/(2.*DN(N)*(1.+K)
     8        ))*CNS
C
      DO 3750 II = 1,2
      ECO(N+1,II)=-ACO(N,II)/(BCO(N,II)+CCO(N,II)*ECO(N,II))
      FCO(N+1,II)=(DCO(N,II)-CCO(N,II)*FCO(N,II))/(BCO(N,II)+
     1CCO(N,II)*FCO(N,II))
 3750 CONTINUE
 4000 CONTINUE
C
C  .......... (3) SOLUTION TO THE BOUNDARY LAYER EQUATIONS  .......... ....
CC                 BY INVERTING THE MATRIX
C
      DO 4250 I = 1,NU3
      SO=NU-I
      UFVE(SO,2)=ECO(SO,1)*UFVE(SO+1,2)+FCO(SO,1)
      ENTR(SO,2)=ECO(SO,2)*ENTR(SO+1,2)+FCO(SO,2)
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      ENT(SO,2)=ENTR(SO,2)*ENTE(L+1)
      TEM(SO,2)=TAB(ENT(SO,2))
 4250 CONTINUE
      TEM(NU,2) = TEM(NU3,2)
      TEME=TEM(NU,2)
C
C  ......... THE NEW VALUES OF Y
      DO 5000 N = 2,NU
      Y(N)=DN(N-1)/2.*(SQRT(2.*XE(L+1))*AR*TEM(N-1,2)/(UE(L+1)*ARJ(L+1)
     1*PE(L+1))+SQRT(2.*XE(L+1))*AR*TEM(N,2)/(UE(L+1)*ARJ(L+1
     1)*PE(L+1)))
     1+Y(N-1)
 5000 CONTINUE
C  ...... THE NEW INCOMPRESSIBLE DISPLACEMENT THICKNESS
      DELT=0.
      DO 5500 N = 2,NU3
      DELT=(Y(N)-Y(N-1))/2.*((1.-UFVE(N-1,2))+(1.-UFVE(N,2)))+DELT
 5500 CONTINUE
 6000 CONTINUE
C     *********     ENDT OF THE ITTERATION LOOP FOR TURBULENT FLOW
C
C  ......... (4) CALCULATION OF THE V-VELOCITY FROM THE CONTINUITY EQN.   ...
C
      DO 6250 N = 1,NU2
      VVE(N+1,2)=VVE(N,2)-VVE(N+1,1)+VVE(N,1)-2.*DN(N)*XED*(UFVE(N+1,2)
     1-UFVE(N+1,1)+UFVE(N,2)-UFVE(N,1))/DXE-DN(N)*(UFVE(N+1,2)+UFVE(N,2)
     1+UFVE(N+1,1)+UFVE(N,1))/2.
 6250 CONTINUE
C
C  ********************     THE HEART IS FINISHED   ****************** ****
C
C  ************     CALCULATION OF SOME B.L. CHARACTERISTICS   ********** ****
C
C  ...... 1. THE NEW VALUES OF Y
C          CALCULATED IN THE 5000 DO LOOP
C
C  ...... 2. THE EDGE OF THE BOUNDARY LAYER
      IF (SKIP .EQ. 0) I=30
      IF (SKIP .NE. 0) I=150
      DO 6500 N = I,NU
      KSTOP = N + 2
      IF ( UFVE(N,2) .GT. .995 ) GO TO 10
 6500 CONTINUE
   10 DELT1 = Y(N)
C
C  ...... 3. DISPLACEMENT AND MOMENTUM THICKNESS
      DELT=0.
      THET=0.
      DELBAR = 0.0
      THEBAR = 0.0
      DO 6750 N=2,NU3
      ACORR =(1.+TCFACT*Y(N)/ARJ(L+1))*(Y(N)-Y(N-1))
      DELT = ACORR       /2.*((1.-TEM(NU3,2)*UFVE(N-1,2)/TEM(N-1,2))
     1+(1.-TEM(NU3,2)*UFVE(N,2)/TEM(N,2)))+DELT
      THET = ACORR       /2.*((1.-UFVE(N-1,2))*UFVE(N-1,2)*TEM(NU3,2)/
     1TEM(N-1,2)+(1.-UFVE(N,2))*UFVE(N,2)*TEM(NU3,2)/TEM(N,2))+THET
      IF( N .GT. KSTOP) GO TO 6750
      DELBAR = .5*((1.-UFVE(N,2))+(1.-UFVE(N-1,2)))*   ACORR       +DELBAR
      THEBAR=.5*((1.-UFVE(N,2))*UFVE(N,2)+(1.-UFVE(N-1,2))*UFVE(N-1,2))*
     1   ACORR + THEBAR
```

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 6750 CONTINUE
      SHFA = DELT/THET
      TRSHFA = DELBAR/THEBAR
C
C   ......   4. REYNOLDS NUMBERS
      REX=PE(L+1)/(AR*TEM(NU,2))*UE(L+1)*XD(L+1)/TAB2(TEM(NU,2))
      RTHET = UE(L+1)*PE(L+1)/(AR*TEM(NU,2))*THET/TAB2(TEME)
C   ......   5. SKIN FRICTION COEFFICIENT
      CF = 2.*TAB2(TEM(1,2))*ARJ(L+1)**1*TEM(NU2,2)*(-K*(2.+K)*UFVE(1,2)
     1    +(K*(2.+K)+1.)*UFVE(2,2)-UFVE(3,2))/(K*(1.+K)*DN(1))*1./((2.*
     2     XE(L+1))**.5*TEM(1,2))
      IF( ARJ(L) .NE. 1.0 ) GO TO 11
      CF1(L+1) = CF
      CF2 = CF2 + (CF1(L+1) + CF1(L))*(XD(L+1) - XD(L))/2.
      CFAV = CF2/XD(L+1)
C
C   ......   6. HEAT TRANSFER COEFF. , STANTON NO.
   11 RE = SQRT(PR)
      IF( SKIP .NE. 0 ) RE = PR**(1./3.)
      TR=(TAB(ENTO)-TEM(NU3,2))*RE*TEM(NU3,2)
      ST =-TAB2(TEM(1,2))*TEM(NU2,2)*ARJ(L+1)*(-K*(2.+K)*TEM(1,2)+(K*(2.
     1    +K)+1.)*TEM(2,2)-TEM(3,2))/(K*(1.+K)*DN(1))*1./(PR*TEM(1,2)*
     2    (2.*XE(L+1))**.5*(TEM(1,2)-TR))
      S = 2.*ST/CF
C
C   ......   7. VELOCITY PROFILE EXPONENT
      DO 7100 N = 2,NU
      IF( Y(N) .LE. 0.0 .OR. Y(N+1) .LE. 0.0 ) GO TO 7100
      VPE1 = ALOG10(Y(N+1)/Y(N))
      VPE2 = ALOG10(UFVE(N+1,2)/UFVE(N,2))
      IF( VPE2 .EQ. 0.0 ) GO TO 7100
      VPE(N) = VPE1/VPE2
 7100 CONTINUE
C
C   ......   8. PRESSURE RATIO
C
C   ......   9. TEMPERATURE AND MACH NO. RATIO
      ENT(NU,2) = ENT(NU3,2)
      DO 4500 N = 1,NU
      ENTT(N,2) = ENT(N,2) + (UFVE(N,2)*UE(L+1))**2/(2.*BIG)
      AMM(N)=UE(L+1)*UFVE(N,2)/SQRT(1.4*AR*TEM(N,2))
 4500 CONTINUE
      CALL TAB1(NU,AMM,PRAT)
      QE = PE(L+1)*UE(L+1)**2/(2.*AR*TAB(ENT(NU,2)))
      AMACH(L+1) = UE(L+1)/(49.1*SQRT(TAB(ENT(NU,2))))
      DELT = DELT*12.
      THET = THET*12.
      DELBAR = DELBAR*12.
      THEBAR = THEBAR*12.
      DELTM = DELT/.3937
      THETM = THET/.3937
      DELBM = DELBAR/.3937
      THEBM = THEBAR/.3937
      XDM = XD(L+1)/3.281
      UEM = UE(L+1)/3.281
      TE = TAB(ENT(NU,2))
      TEQ = TE*5./9.
      TTE = TAB(ENTO)
      TTEM = TTE*5./9.
      TW = TAB(ENTTW(L+1))
```

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```
         TWM = TW*5./9.
         TAW = TAB(ENT(NU,2))*(1. + RE*AMACH(L+1)**2*.4/2.)
         TAWM = TAW*5./9.
         PEM = PE(L+1)/208.9
         QEM = QE/208.9
         B = TAB(ENTO)*BETX/TEM(NU,2)
         TWRAT = TW/TAW
C
C     ........................... OUTPUT     ........................... ....
C
         WRITE(6,250)L
         WRITE(6,300)AMACH(L+1),UE(L+1),UEM,PE(L+1),PEM,QE,QEM,TE,TEQ,TTE,
        1 TTEM,TW,TWM,TAW,TAWM
         WRITE(6,350)DELT,DELTM,THET,THETM,SHFA,B,DELBAR,DELBM,THEBAR,THEBM
        1,TRSHFA,TWRAT,RTHET,REX,CF,CFVA,ST,S,XD(L+1),XDM
         WRITE(6,550)
         JSTOP(1) = 0
         DO 7250 JK = 1,5
         JSTOP(JK + 1) = JSTOP(JK) + 20*JK
         LZ = JSTOP(JK)
         IF(JK .EQ. 1) LZ = 1
         JQ = JSTOP(JK + 1)
         DO 7250 JL = LZ,JQ,JK
         IF(JL .EQ. JSTOP(JK)) GO TO 7250
         IF( JL .GT. KSTOP ) GO TO 12
         YRAT = Y(JL)/THET * 12
         TRAT = TEM(JL,2)/TEM(NU,2)
         MRAT = UFVE(JL,2)/SQRT(TRAT)
         TTRAT = TAB(ENTT(JL,2))/TAB(ENTO)
         TB = (TAB(ENTT(JL,2))-TAB(ENTTW(L+1)))/(TAB(ENTO)-TAB(ENTTW(L+1)))
         HB = (ENTT(JL,2) - ENTTW(L+1))/(ENTO - ENTTW(L+1))
         WRITE(6,600)JL,UFVE(JL,2),MRAT,TRAT,TTRAT,TB,HB,PRAT(JL),YRAT,
        2VPE(JL),AL1(JL)
         JKZ = JL + JK
         IF( Y(JL) .LT. YB .AND. Y( JKZ ) .GT. YB ) WRITE(6,650)
    7250 CONTINUE
C
C     ...... INTERCHANGE INDICES 1 AND 2 TO START NEW FORWARD CALCULATIONS
      12 DO 7500 N = 1,NU
         UFVE(N,1)=UFVE(N,2)
         VVE(N,1)=VVE(N,2)
         ENTT(N,1)=ENTT(N,2)
         TEM(N,1)=TEM(N,2)
         ENT(N,1)=ENT(N,2)
         ENTR(N,1) = ENTR(N,2)
         UFVE(N,2)=0.
         VVE(N,2)=0.
         ENTR(N,2)=0.
         TEM(N,2)=0.
         ENT(N,2)=0.
    7500 CONTINUE
         DELT = DELT/12.
         THET = THET/12.
C
C              **********  END OF FORWARD STEP CALCULATIONS   **********
    7750 CONTINUE
C
C              **********  END OF MAIN DO LOOP   **********
         IF (NUX .EQ. NUL) GO TO 9000
    8000 CONTINUE
```

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```
C
C                    *************    FORMAT    **********
  150 FORMAT(6X,112H*********************************************** LAMINAR FLOW .
     1..... INITIAL PROFILES   ****************************************/)
  200 FORMAT(////8X,110H*********************************** TURBULENT FLOW
     1 ...... INITIAL PROFILES   *************************************/)
  250 FORMAT(///1X,80H********************************************** FLOW
     1QUANTITIES .......... STATION = ,I2,47H  **************************
     2*******************)
  300 FORMAT(//1X,4HME =,E11.4,18X,4HUE =,E11.4,1H(,E11.4,1H), 6X,4HPE =
     1,E11.4,1H(,E11.4,1H), 5X,4HQE =,E11.4,1H(,E11.4,1H)//1X,4HTE =,
     2 E11.4,1H(,E11.4,1H),5X,5HTTE =,E11.4,1H(,E11.4,1H),5X,4HTW =,E11.
     3 4,1H(,E11.4,1H),5X,5HTAW =,E11.4,1H(,E11.4,1H))
  350 FORMAT(/1X,6HDELT =,E11.4,1H(,E11.4,1H),5X,6HTHET =,E11.4,1H(,E11.
     14,1H),5X,3HH =,E11.4,5X,3HB =,        E11.4//1X,7HDELTT =,E11.4,1H(,E
     211.4,1H),4X,7HTHETT =,E11.4,1H(,E11.4,1H),4X,4HHT =,E11.4,5X,8HTW/
     3TAW =,E11.4//1X,6HRTHET=,E11.4,3X,4HREX=,E11.4,3X,3HCF=,E11.4,3X,4
     4HCFA=,E11.4,3X,3HCH=,E11.4,3X,2HS=,E11.4,2X,2HX=,E11.4,1H(,E11.4,1
     5H))
  550 FORMAT(/3X,1HN,7X,4HU/UE,8X,4HM/ME,8X,4HT/TE,7X,6HTT/TTE,8X,2HTB,
     110X,2HHB,8X,6HPT/PTE,6X,6HY/THET,7X,4HVP-N,8X,3HAL1)
  600 FORMAT(2X,I3,3X,10(E11.4,1X))
  650 FORMAT(2X,122H-------------------------------------------------
     1-------------------------------------------------------
     2----)
C
 9000 STOP
      END
```

REENTRANT FORMAT
NONREENTRANT F

CON
CON

```
      FUNCTION TAB(ENT)
      DIMENSION X(60)
      DATA ( X(J), J = 1,60 ) /23.74,47.67,71.61,95.53,119.48,143.47,
     1 167.56,191.81,216.26,240.98,265.99,291.30,316.94,342.90,369.17,
     2 395.74,422.59,449.71,477.09,504.71,532.55,560.59,588.82,617.22,
     3 645.78,674.49,703.35,732.23,761.45,790.68,820.03,849.48,879.02,
     4 908.66,938.40,968.21,998.11,1028.09,1058.14,1088.26,1118.46,
     5 1148.72,1179.04,1209.42,1239.86,1270.36,1300.92,1331.51,1362.17,
     6 1392.87,1423.62,1454.41,1485.26,1516.14,1547.07,1578.03,1609.04,
     7 1640.09,1671.17,1699.18/
      IF( ENT .GT. X(60) ) GO TO 63
      N=2
   60 IF (ENT .LE. X(N)) GO TO 61
      N=N+1
      GO TO 60
   61 TAB=100.*(FLOAT(N-1)+(ENT-X(N-1))/(X(N)-X(N-1)))
      GO TO 64
   63 TAB = 2000.0
   64 RETURN
      END
```

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```fortran
      SUBROUTINE TAB1(N,M,P)
      REAL M
      DIMENSION M(501),P(501),PT2(501)
      G = 1.4
      DO 1000 J = 1,N
      IF( M(J) .LT. 1.0 ) GO TO 1
      IF( M(J) .EQ. 1.0 ) GO TO 2
      PT2(J)=((.5*(G+1.)*M(J)**2**(G/(G-1.)))*((G+1.)/(2.*G*M(J)**2-(G
     1 -1.)))**(1./(G-1.))
      GO TO 1000
    1 PT2(J) = (1. + .5*(G - 1.)*M(J)**2**(G/(G - 1.))
      GO TO 1000
    2 PT2(J) = 1.8929
 1000 CONTINUE
      DO 2000 J = 1,N
      P(J) = PT2(J)/PT2(N)
 2000 CONTINUE
      RETURN
      END
```

##

```fortran
      FUNCTION TAB2(TEMP)
C
C     ******  TAB2(TEMP)  ******   IS KEYES FORMULA TO CALCULATE THE VISCOSITY
C                                  OF AIR FROM THE TEMPERATURE
C
      TAB2      = 0.0232*1.E-6*SQRT(TEMP)/(1.+220./TEMP*10.**
     1            (-9./TEMP))
      RETURN
      END
```

##

```fortran
      FUNCTION TAB3(YD)
      TAB3 = 0.9
      KPR = 2
      GO TO (1234,81),KPP
 1234 IF (YD .LE. .05) GO TO 80
      IF (YD .GT. .05) TAB3=2.
      GO TO 81
   80 TAB3=.9 + YD/ .05*1.1
   81 RETURN
      END
```

##

CON

THE FOLLOWING ROUTINES WERE REQUESTED FROM THE FORTRAN SYMBOLIC LIBRARY:

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