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by Stuart H. Loewenthal and Dennis P. Townsend
Lewis Research Center
Cleveland, Ohio

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by Stuart H. Loewenthal[1] and Dennis P. Townsend[2]

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio

ABSTRACT

A simplified design analysis was conducted on a nutating plate type drive system for a 2500 horsepower helicopter main rotor gear box. A drive system that split the output torque evenly between two nutating plates for the purpose of reducing the load on each nutating plate was analyzed. Needle bearings were used on the nutating plate pins. The results of the analysis indicate that the required load capacity of the pin bearings and the speed of the nutating plate bearings were beyond the state-of-the-art capacity of rolling-element bearings. The analysis further indicates that the nutating plate drive is less efficient, and results in a higher weight per horsepower than a conventional planetary helicopter transmission with similar design specifications.

INTRODUCTION

In V/STOL aircraft the reduction gear box accounts for a large percentage of the total aircraft weight. A typical medium size helicopter might require 2500 horsepower from a 12,000 rpm gas turbine engine and have a rotor speed of approximately 200 rpm. Because of low drive system output speed, the output torque is very high resulting in a large gear box weight to power ratio, typically 0.5 pound per horsepower (1).[3] Consequently, there is always a strong demand for lighter weight transmission systems. Present day helicopters which use bevel gears and two stage planetary systems to obtain the required gear reduction ratio have surprisingly good efficiencies, some better than 98% (1).

There are several devices that will allow large reductions in speed through a single stage reduction system. Some of these are the harmonic drive, the cycloidal cam or planocentric transmission and more recently the nutating plate drive. These systems are similar in several respects. They give a small incremental output rotation for each input rotation and are, therefore, generally compact in size and weight for the speed reduction considered. However, they readily become heavy with increased output torque and may have excessive power losses at high speeds. Because of the way they operate, there usually is a distinct input speed limitation imposed due to vibration and dynamic forces (2). Some have excessive power losses resulting from the sliding contact under high loads on the output stage. For this reason, it has been suggested that rolling motion and/or rolling-element bearings be incorporated into these systems to decrease the power loss.

There has been recent interest in a drive concept utilizing a nutating plate as a means for reducing speed as part of an output transmission for helicopters and other systems. There have been some questions raised regarding the feasibility of such a concept for helicopter application. It therefore becomes the objective of the study reported herein to: (1) perform a simplified design analysis to determine the feasibility of applying the nutating plate type drive to a helicopter and (2) compare the nutating plate transmission weight and efficiency with that of a conventional planetary transmission having similar specifications. No lubrication considerations have been included in this analysis, nor has the effects of dynamic loading to any great extent.

NOMENCLATURE

b_w	bearing width, in.
C	basic bearing dynamic load capacity, lb
C_ℓ	axial half clearance between teeth, in.
c	distance from beam axis of inertia to outside fiber, in.
D_{bi}	inside diameter of pin bearing outer race, in.
D_{bo}	outside diameter of pin bearing outer race, in.
D_i	inside diameter of input shaft, in.
D_o	outside diameter of input shaft, in.
d_e	nutating plate pitch diameter, in.
d_o	outside diameter of output shaft, in.
d_{pi}	inside diameter of pin, in.
d_{po}	outside diameter of pin, in.
E	Young's modulus, 30×10^6, psi
E_1,E_2	Young's modulus of elements 1 and 2, psi

[1]Associate Member ASME - US Army Air Mobility Research & Development Laboratory - NASA/Lewis Research Center, Cleveland, Ohio.
[2]Member ASME - NASA/Lewis Research Center, Cleveland, Ohio.
[3]Numbers underlined designate references at end of paper.

	equivalent Young's modulus, $\left(\frac{}{2E_1} + \frac{}{2E_2}\right)$, psi
E_p	energy stored in single pin bearing outer-race, ft-lb
E_t	total energy stored in pin bearing outer-race, ft-lb
$F_{1,2}$	support bearing forces on output shaft, lb
F_p	frictional force on pins, lb
g	gravitational acceleration, 32.18, ft/sec^2
h	tooth width at working depth line, in.
I	moment of inertia, ft-lb-sec^2
L_o	length of output shaft, in.
L_b	length of pin bearing, in.
ℓ_p	length of pin, in.
$\overline{M}$	bending moment load, in.-lb
M_G	bevel gear moment load, in.-lb
M_p	nutating plate moment load, in.-lb
N	rotational velocity, rpm
N_i	rotational velocity of input shaft, rpm
N_p	rotational velocity of pin bearings, rpm
N_t	total number of pins per nutating plate
n	number of pins per quadrant
P	equivalent bearing load, lb
P_A	axial tooth force, lb
P_L	power loss, hp
P_N	normal tooth force, lb
P_T	tangential tooth force, lb
P_{dp}	pitch diameter of pinion, in.
P_p	total pin load acting on nutating plate, lb
$P_{1,2}$	support bearing forces on input shaft, lb
R_{FG}	bevel gear radial force, lb
R_G	bevel gear radius, in.
r_i	length from pin to centerline of nutating plate, in.
r'	equivalent radius of pin loads, in.
S_b	bending stress, psi
S_h	mean tooth height, in.
S_{max}	maximum Hertz stress, psi
S_s	shear stress, psi
S_s'	maximum design shear stress, psi
T	torque, in.-lb
T_p	torque on pin due to friction, in.-lb
t	time, sec
t_n	nutating plate thickness, in.
t_p	tooth pitch, in.
t_r	tooth root thickness, in.
t_t	average tooth thickness, in.
V_{max}	maximum pin velocity, in./sec
W	weight of needle bearing outer race, lb
w	tooth top land width, in.
y_p	pin shaft deflection, in.
y_m	shaft deflection, in.
y_n	mutating plate deflection, in.
α	nutating plate inclination angle, deg
α_p	angular acceleration of pin bearing outer-race, rad/sec^2
β	tooth face angle, deg
γ	specific weight, lb/ft^3
$\overline{\Delta t}$	nutation cycle time, sec
ν_1,ν_2	Poisson's ratio of elements 1 and 2, 0.3 sec
μ	friction coefficient
ω	angular velocity, rad/sec
ω_{max}	maximum angular velocity of pin bearing, rad/sec
ω_n	nutating plate cyclic frequency, rad/sec

NUTATING PLATE TYPE DRIVE

A schematic of a nutating plate drive is shown in Fig. 1. Essentially the drive comprises an input shaft, a bevel gear set, an output shaft, and various plate members. The bevel gears drive two nutating plates which are mounted on the input shaft. The nutating plates have a plurality of pins which mesh with a plurality of teeth on the outer rotating drive plates and inner stationary reaction plates. The number of teeth on the reaction plate and the number pins on the nutating plate are one less than the number of teeth on the drive plate. For each revolution of the input shaft, the nutating plate makes one nutation and advances the drive plate a distance of one tooth space, due to the presence of an additional tooth on the drive plate. With this arrangement, the reduction ratio is equal to the number of drive plate teeth.

To minimize the amount of loading on the pin-tooth contacts and on the nutating plate bearings, a double nutating plate arrangement has been proposed (fig. 1). Because of the increase in the number of drive components, reduced stress on the structural members may be expected.

SIMPLIFIED ANALYSIS

Several assumptions were necessary in order to conduct a simplified analysis of the nutating plate type drive. In addition, design parameters were selected that would be considered typical for a medium size helicopter main rotor gear box. The following requirements were selected for the analysis:

Input horsepower	2500
Input drive speed	11,200 rpm
Spiral bevel reduction	2:1
Output drive speed	200 rpm
Nutating plate pitch diameter[4]	36 in.

[4]The value chosen for the nutating plate pitch diameter is thought to result in the maximum permissable main rotor gearbox outside diameter which would be compatible with a conventional helicopter of the above specifications.

The following assumptions were made:

1. The nutating plate pins and drive/reaction teeth can be designed to take a uniform load from input mesh to output mesh. That is, a constant load remains on the pins during the loading cycle. This has the effect of reducing the peak loads on the pins.

2. The reaction teeth and drive teeth can be contoured to keep the nutator pin roller rotating in the same direction when leaving the reaction teeth and entering the drive teeth as shown in Fig. 2. This is necessary to prevent excessive sliding of the rollers if reverse rotation were required.

3. The reduction ration is 28:1 with 28 teeth on the drive plate, 27 pins on the nutating plate, and 27 teeth on the reaction plate.

Using the above conditions the total rotor output torque is

$$T = \frac{hp \times 63025}{rpm} = 788{,}000 \text{ in.-lb} \tag{1}$$

Using two nutating plates and assuming a perfect torque split, results in a torque on each nutating plate of 394,000 in.-lb. The drive torque is taken on one quadrant of the nutating plate and the reaction on the directly opposite quadrant.

The resultant force per pin on the nutating plate, noting that 7 pins are engaged per quadrant, will be

$$P_T = \frac{T}{nR} = \frac{394{,}000}{7\times18} = 3127 \text{ lb} \tag{2}$$

Assuming one tooth spacing between teeth, the average tooth thickness on the drive plate is:

$$t_t = \frac{\pi d_e}{2N_t} = \frac{\pi\times36}{2\times28} = 2.02 \text{ in.} \tag{3}$$

or a tooth pitch t_p of 4.04 inches.

In order to determine the proper tooth dimensions and stresses, the nutator plate inclination angle α must be selected. It is desirable to keep α small to minimize the angular acceleration of the nutating plate. However, the selection of too small of an inclination angle will result in large axial loads and consequently large moment loads on the input shaft bearings. For this study an inclination angle of 10° was chosen after preliminary calculations. The characteristic tooth dimensions for both drive and reaction plates are shown in Fig. 3.

The mean tooth height S_h or working depth, i.e., not considering the radius of the root fillet required to accommodate the outside diameter of the nutating plates pins, allowing 0.030 inch for clearance is:

$$S_h = \frac{1}{2} d_e \sin\alpha - C_1$$
$$= 18 \sin 10^\circ - 0.030 = 3.1 \text{ in.} \tag{4}$$

Since the nutator is advancing the drive plate at one tooth pitch per input revolution, the total distance that the output plate advances during each revolution is 4.04 inches. One fourth of this length would be advanced by each tooth face, since each pin-tooth contact is maintained for one fourth of an input revolution. This results in a tooth face angle of

$$\beta = \tan^{-1}\frac{t_p}{4\times S_h} = \tan^{-1}\frac{4.04}{4\times3.1} = 18^\circ \tag{5}$$

assuming a constant tooth face angle.

The maximum pin diameter that could be accommodated at the working depth line would be 2.02 inches assuming a zero top land width. The pin diameter should be somewhat less than this to allow for some minimum top land width, w of 0.3 inch and some backlash, this would leave a maximum nutator pin diameter of 1.7 inches and minimum root tooth thickness, t_r of 2.32 inches. Using this value, the minimum tooth width and maximum Hertz stress can be determined. The force normal to the tooth is

$$P_N = \frac{P_T}{\cos 18^\circ} = 3288 \text{ lb} \tag{6}$$

and the axial tooth force is

$$P_A = P_N \sin 18^\circ = 1016 \text{ lb} \tag{7}$$

The tooth bending stress can be calculated using the well known formula for gear tooth bending stress:

$$S_b = \frac{6P_T S_h}{h t_r^2} \tag{8}$$

With an allowable fatigue stress of 50,000 psi, a factor of safety of 2, and including a conservative factor of 2.5 to account for nonuniform tooth loading, overloads and dynamic loading effects, etc., the width of the tooth required is

$$h = \frac{2\times2.5}{50{,}000} \cdot \frac{6\times3127\times3.1}{(2.32)^2} = 1.08 \text{ in.} \tag{9}$$

The maximum Hertz stress for the above calculated normal load and tooth width is

$$S_{max} = \sqrt{\frac{P_N E'}{\pi h d_{po}}} = \sqrt{\frac{3288\times33\times10^6}{\pi\times1.08\times1.7}} = 138{,}000 \text{ psi} \tag{10}$$

and is within allowable limits. Selecting a pin length ℓ_p of 1.08 inches, the diameter of the nutating plate pins can be determined as follows: The bending stress acting on the pins assuming a hollow cantilevered pin (primarily supported at one end) can be written

$$S_b = \frac{P_N \ell_p}{2} \cdot \frac{d_{po}}{0.0982\left(d_{po}^4 - d_{pi}^4\right)} \tag{11}$$

Also, the maximum deflection of the pin can be written

$$y_p = \frac{5P_N \ell_p^3}{48E\times0.0491\left(d_{po}^4 - d_{pi}^4\right)} \tag{12}$$

Since the size of the pin to insure proper meshing action is more critically determined by the deflection rather than the stress, a pin size was calculated that would result in an arbitrary maximum deflection of 0.002 inch. For a hollow pin, choosing a wall thickness of 15% of the pin's outside diameter to maximize weight savings, results in an outside pin diameter of 1.18 inch and an inside pin diameter of 0.83 inch. Incidently, a solid pin similarly loaded would have an outside diameter of 1.10 inches.

The rolling element bearing that is best suited to the space and capacity requirements in this situation is a needle bearing. The force which acts along the pin axis is considered small in comparison with the radial load acting normal to the pin and, therefore, will be neglected in this discussion. From a typical manufacturer's bearing catalog, a bearing was selected with a bore of 1.25 inches, 1.625 inches outside diameter, and 1.00 inch long which has a basic dynamic capacity C of 5540 pounds. This results in a capacity-load or C/P ratio of 1.7 for this application.

If harmonic motion is assumed for the pin with a half-amplitude of 3.1 inches, then the maximum velocity V_{max} of the pin can be found from

$$V = \omega_N S_h \cos \omega_N t \tag{13}$$

when

$$\cos \omega_N t = 1$$

and where

$$\omega_N = 5600 \text{ rpm} \times 2\pi/60 = 587 \text{ rad/sec}$$

therefore,

$$V_{max} = 587 \times 3.1 \times 1 = 1820 \text{ in./sec}$$

If the pin outside diameter is taken as 1.7 inches, the maximum pin rotational speed is:

$$N_p = \frac{V_{max} \times 60}{\pi D_{bo}} = \frac{1820 \times 60}{\pi \times 1.7} = 20{,}450 \text{ rpm} \tag{14}$$

The average pin rotational speed is approximately 13,000 rpm. If the bearing were able to operate at this speed condition, the life would be approximately 7 hours. However, there is a serious question raised as to whether a needle bearing of this size could operate for a reasonable length of time at speeds of 13,000 rpm even with substantially reduced loading.

NUTATING, DRIVE, AND REACTION PLATE SIZE

The nutating plate will have a moment load imposed on it by the reaction components of the pin load in the axial direction, P_A equal 1016 lb per pin from Eq. (7). There will be seven drive pins loaded on one quadrant of the nutating plate and seven reactions pin loaded on the quadrant 180° from the first, as shown in Fig. 4. The forces which develop on the nutating plate must be supported within small deflection limits. In addition, a moment load will be developed on the nutating plates that must be supported by a bearing rotating at input shaft speed.

The thickness of the nutating plate will be determined by assuming the plate can be properly represented by a cantilevered beam fixed at the center with the axial reaction load of the seven pins acting at a single length as determined from

$$P_p = 7P_A = 7 \times 1016 = 7112 \text{ lb}$$

$$r' = \frac{1}{n}\sum_{i=1}^{n} r_i$$

$$= \frac{d_e}{2 \times 7}\left(2 \sin 50^\circ + 2 \sin 63^\circ + 2 \sin 77^\circ + \sin 90^\circ\right) \tag{15}$$

$r' = 16.1$ inches for $d_e/2 = 18$ inches. The maximum bending stress on the plate is

$$S_b = \frac{MC}{I} = \frac{3P_p r'}{r' t_n^2} \tag{16}$$

from Eq. (16) the plate thickness from stress is

$$t_n = \sqrt{\frac{3P_p}{S_b}} = \sqrt{\frac{21{,}340}{S_b}} \tag{17}$$

The maximum deflection of the plate is

$$y_n = \frac{P_p (r')^3}{3EI} = \frac{2P_p (r')^2}{E t_n^3} \tag{18}$$

From Eq. (18) the plate thickness from deflection is

$$t_n = \sqrt[3]{\frac{2P_p (r')^2}{E y_n}} = \sqrt[3]{\frac{0.123}{y_n}} \tag{19}$$

if y_n is set at 0.020 inch to maintain proper meshing action between teeth and pins, then

$$t_n = 1.83 \text{ in.}$$

and

$$S_b = 6{,}350 \text{ psi}$$

A similar calculation could be made for the drive and reaction plates which would result in a similar thickness.

Figure 1 is a sketch showing the arrangement necessary to split the output torque evenly between the two nutating plates. The input drive will require a spiral bevel gear reduction from either the engine or intermediate shaft. An approximate spiral bevel gear size was selected, assuming a 2:1 reduction, from a typical manufacturer's gear catalog. The pinion pitch diameter for this gear set is 7.666 inches. Using this pitch diameter and, assuming for the present, a bearing length of 1.25 inches to support the input shaft, the length between the two outer support bearings on the output shaft L_o may be determined as follows:

$$L_o = 2\left[\frac{b_w}{2} + 2t_n + 2\left(S_h + C_1 + \frac{d_{po}}{2}\right)\right] + P_{dp}$$
$$= 2\left[0.63 + \overline{3.60} + 8.0\right] + 7.67 = \overline{32.1} \text{ in.} \tag{20}$$

With this dimension and knowing the axial load on the drive plates, the shaft size required to carry the moment loads can be determined. These calculations, given in appendix A, result in an output shaft diameter of 6.6 inches. The input shaft supporting the nutating plates will be considerably shorter and will be required to support the moment load from the nutating plates (see fig. 1).

Nutating Plate Bearing

The nutating plate bearing must operate at the input shaft speed of 5600 rpm and at the same time carry the moment load imposed by the axial reaction loads from the teeth of both the drive and reaction plates. The moment load acting on the nutating plate bearing can be determined as follows:

$$M_p = P_p \times 2r' = 7112 \times 2 \times 16.1 = 230{,}000 \text{ in.-lb} \quad (21)$$

This moment load is very large and must be supported with very little loss due to friction for high efficiency. Therefore, a rolling-element bearing is a necessary requirement. The moment load is such that it requires a bearing to support the nutating plate in both axial directions. In addition, there is a severe axially space restriction of about 6.3 inches between reaction and drive plate members. If, for the moment, the axial space restriction is disregarded, a pair of steep-angle type, single-row tapered roller bearings which are stradle mounted beneath the nutating plate may be considered for this application. The smallest bearing of this type which was thought to meet this load requirement with 3000 hours design life was selected from a typical bearing manufacturer's catalogue. This bearing has a radial load rating of 42,500 lb and a thrust load rating of 58,000 lb at 500 rpm with a bore of 8.00 inches, an outside diameter of 16.00 inches, and is 3.625 inches wide. Assuming that no misalignment will occur, which is exceedingly unlikely due to the large moment loading excerted by the nutating plate, an axial overall length of approximately 11 inches is required to accommodate these bearings. This means that considerable drive and reaction plate overhand would be required with its attendant weight and size penalties. Additionally, it is questionable whether bearings of this type can operate successfully at the required 1.15 million DN (bearing bore in mm × shaft speed in rpm).

The other alternative is to select larger bearings and place them closer together to conserve axial space. Taking this later approach, a double row, steep-angle type, tapered roller bearing was selected from a bearing manufacturer's catalog, which has a radial-load rating of 115,000 lb, a thrust-load rating of 82,000 lb with a bore diameter of 11.25 inches, an outside diameter of 19.75 inches, and is 8.00 inches wide. However, with this approach, the possibility of cup misalignment increases significantly due to the large eccentric load which is present. This condition is to be avoided under all circumstances since misalignment of just a few thousandths of an inch per inch can reduce tapered roller bearing design life by orders of magnitude (3). For this example, the life of the bearing would be approximately 1650 hours, neglecting the life reduction due to misalignment and considering a preload thrust force equal to the induced thrust to partially compensate for the eccentric loading. A considerably shorter bearing life would be expected however, if misalignment effects had been included in this analysis. This bearing would be operating at a speed of 1.6 million DN which is considerably above the current limits for tapered roller bearings. Each bearing will weigh over 200 pounds and will increase the length of the drive system somewhat because of its 8.00 inch width. As a result, the axial length between drive and reaction plates is increased by 1.7 inches. Since two such bearings are required, the drive length is increased by 3.4 inches from those dimensions originally assumed.

Transmission Efficiency and Weight

The efficiency of present day main gear boxes for helicopters are on the order of 98% (1). This figure includes the efficiencies of the input bevel gear (more than 99% (4)) and the two stage planetary reduction. For example, the 33 to 1 main rotor gear box on the S-65 helicopter which transmits approximately 7500 horsepower has an efficiency of more than 98%, as measured by test (1). This implies a loss of only 50 horsepower for a 2500 horsepower state-of-the-art main rotor gear box.

Secondly the weight to power ratio of today's helicopter main gear boxes for low output rotor speeds are rarely greather than 0.5 lb/hp. For example, the weight to power ratio of the 33 to 1, S-65 main rotor gear box described above is 0.36 lb/hp (1). Thus a 2500 hp conventional planetary gear box would not be expected to weigh more than 1250 pounds.

Since transmission weight and efficiency are such important figures of merit in the design of all helicopters, advanced transmission concepts for helicopter application must be strongly competitive with state-of-the-art gear boxes in these areas, if they are to become viable alternatives.

Transmission efficiency. If the nutating plate pins were operating without needle bearings then significant losses would result from the sliding present in the contacts. From Ref. (5), the experimental friction coefficient for steel against steel at sliding speeds above 60 in./sec is approximately 0.04 and is not appreciably dependent upon the type or base viscosity of the particular oil or the magnitude of the load (above a contact pressure of 100,000 psi). Therefore, the frictional load per pin F_p can be calculated from the normal load on the pins as follows:

$$F_p = \mu P_N = 0.04 \times 3288 = 131 \text{ lb} \quad (22)$$

The pin's maximum sliding velocity has been shown from Eq. (13) to be 1820 in./sec. As a result, the average pin's sliding velocity will be approximately 1100 in./sec. Thus, the total sliding power loss, P_L for the 28 pins in engagement (both nutating plates) is

$$P_L = 28 \times \frac{131 \times 1100}{12 \times 550} = 610 \text{ hp} \quad (23)$$

Without considering other losses in the system this would result in a gear box efficiency of 76%, considerably less than that of state-of-the-art planetary gear boxes.

It might be argued that this loss could be reduced with the use of rolling-contact bearings on the pins. The following analysis was conducted to determine what efficiency could be expected with needle bearings mounted on the pins.

Considering a needle bearing which has a bore diameter of 1.25 inches, a width of 1 inch, an outer race inside diameter of 1.5 inches and an outside diameter of 1.7 inches which would contact the plate teeth directly, the outer race's polar moment of inertia is

$$I = \frac{\pi \gamma}{32g}\left(D_{bo}^4 - D_{bi}^4\right) \times L_b = 2.01 \times 10^{-5} \text{ ft-lb-sec}^2 \quad (24)$$

If the pin could reach a maximum rotational speed of 20,310 or 2,120 rad/sec from Eq. (14), then the energy required to rotate the pin bearing up to this speed would be

$$E_p = \frac{1}{2} I\omega^2 = 45 \text{ ft-lb} \quad (25)$$

Each of the 27 pin bearings in the two nutating plates will spin up to speed (absorbing energy) and back down to near zero (releasing energy) under load once each nutation cycle so that the minimum total energy expended for the two nutating plates per cycle, assuming none of this energy is recoverable in usuable form, is

$$E_t = 2N_t \times E_p = 2 \times 27 \times 45 = 2430 \text{ ft-lb} \quad (26)$$

Since the nutating plate and, hence, the pin bearings are cycling at the input shaft speed of 5600 rpm or once in every 0.01075 second, it follows that the power being expended is

$$P_L = \frac{E_t}{\Delta_t} = \frac{2430}{0.01075 \times 550} = 410 \text{ hp} \quad (27)$$

This results in an efficiency, considering just the pin bearing's inertial losses, of 84%.

A question arises as to whether or not the friction forces which are generated in the contact are of sufficient magnitude to enable the pin bearings to accelerate to maximum speed in such a short time period. For example, if the friction coefficient is again taken to be 0.04, the maximum applied torque due to friction would be

$$T_p = 0.04 \times 3288 \times \frac{1.7}{2 \times 12} = 9.0 \text{ ft-lb} \quad (28)$$

The angular acceleration of the pin bearing, assuming a constant traction force would be

$$\alpha_p = \frac{T}{I} = \frac{9.0}{2.01 \times 10^{-5}} = 448{,}000 \text{ rad/sec}^2 \quad (29)$$

Considering that there is constant angular acceleration during the 1/4 cycle when T_p is applied, the maximum angular velocity of the pin bearing would be

$$\omega_{max} = \alpha_p \frac{\Delta_t}{4} = 448{,}000 \times 0.25 \times 0.01075$$
$$= 1200 \text{ rad/sec or } 11{,}500 \text{ rpm} \quad (30)$$

If 11,500 rpm is the actual maximum rotational speed of the pin bearing, then the energy required to accelerate the bearing up to speed is

$$E_p = \frac{1}{2} I\omega^2 = 14.5 \text{ ft-lb} \quad (31)$$

Thus, the total energy required for two nutating plates to spin each of the 27 pin bearing outer races up to speed under load for each nutation cycle is

$$E_t = 2N_t \times E_p = 2 \times 27 \times 14.5 = 795 \text{ ft-lb} \quad (32)$$

Therefore, from Eq. (27), the power being expended is

$$P_L = \frac{E_t}{\Delta_t} = \frac{795}{0.01075 \times 550} = 155 \text{ hp} \quad (33)$$

This results in a nutating plate drive efficiency of approximately 94%, without considering the other significant losses in the system.

It can be concluded that even with the inclusion of pin bearings into this design to improve the overall efficiency over the case with solid pins, significant losses are still present. In either example, the nutating plate drive's efficiency is considerably less than the efficiencies of current state-of-the-art gear boxes.

Transmission weight. A rough estimate of the nutating plate drive weight as shown in Fig. 1 for 2500 hp without considering the weight of a lubrication system is as follows:

2 moment bearings		400 pounds
6 plate members[5]		2000 pounds
2 shafts & 4 bearings & bevel gears		900 pounds
Housing		500 pounds
	Total	3800 pounds

This is a conservative estimate and is approximately three times the weight of present day helicopter main rotor gear boxes.

This figure for weight may be improved by a careful optimization of the system. However, it is not anticipated that this type drive could have weights equal to or less than that of state-of-the-art main rotor gear boxes based upon today's technology.

CONCLUSIONS

From the analysis conducted and reported in this paper the following conclusions can be made regarding the use of a nutating plate type drive as a main rotor gear box for a 2500 hp helicopter. These are:

1. The rolling-element bearings which were selected to support the nutating plate and to provide rolling pin/tooth contact have load and speed requirements which exceed the current state-of-the-art capability of rolling element bearings. In view of the stringent space and weight limitations for helicopter gear box components, rolling-element bearings for a nutating plate type drive would be a major developmental item.

2. The efficiency of the nutating plate drive system is improved with the use of rolling-element bearings on the nutating plate pins. However the efficiency of the nutating plate drive, with or without pin bearings is less than the efficiency of a comparable state-of-the-art transmission.

3. The estimated weight of the nutating plate drive system utilized in this analysis is approximately three times the weight of a comparable state-of-the-art transmission.

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[5] In computing the weight of the plate members, a factor of 2/3 was applied to account for the potential weight reduction benefits attendant with structural ribbing, tapering, hollowing out, and other component weight reducing techniques with may be applied in this case.

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APPENDIX A

Shaft Sizes for Nutating Plate Drive

The output shaft size assuming a shaft length between support bearing of 32.1 inches and 29.1 inches between drive plates to allow for bearings, gears, and various plate members. The force diagram is shown in Fig. 5.

The maximum shear and bending stress can be determined from the following:

$$S_b = \frac{32M}{\pi d^3} \tag{A1}$$

$$S_s = \frac{16T}{\pi d^3} \tag{A2}$$

$$T = \frac{hp \times 63025}{N} = 789{,}000 \text{ in.-lb} \tag{A3}$$

From Mohr's Circle

$$S_{smax} = \sqrt{\left(\frac{S_b}{2}\right)^2 + (S_s)^2} \tag{A4}$$

Using a AISI 4340 steel (115,000 psi tensile strength) with a maximum design sheaving stress S'_s of

$$S'_s = \frac{0.18 \times S_{ult}}{1.5} = 13{,}800 \text{ psi}$$ (ASME Code for the Design of Transmission Shafting, B17c, 1927.)

The shaft diameter, noting that $S_{smax} \leq S'_s$, would then be

$$d_o = \sqrt[6]{\left(\frac{32M}{2\pi S'_s}\right)^2 + \left(\frac{16T}{\pi S'_s}\right)^2}$$

$$= \sqrt[6]{\left(\frac{32 \times 115{,}000}{2 \times \pi \times 13{,}800}\right)^2 + \left(\frac{16 \times 789{,}000}{\pi \times 13{,}800}\right)^2}$$

$$= 6.6 \text{ inches} \tag{A6}$$

and the maximum deflection would be

$$y_m = \frac{ML_o^2}{8EI} = 0.035 \text{ in.} \tag{A7}$$

The shaft, in practice, should be even larger to prevent excessive deflection.

The input shaft size to the nutator plates can be determined in a similar manner. Figure 6 is a force diagram of the shaft.

The maximum bending moment occurs at the point where the spiral bevel gear mates to the input shaft and is equal to

$$M_{max} = 230{,}000 + 1.51\, P_1 \tag{A8}$$

$$P_1 = R_{FG} + P_2$$

$$P_2 = \frac{M_G - 1.51\, P_{FG}}{8.92} = \frac{4780 - 1.51 \times 1290}{8.92} \tag{A9}$$

$$P_2 = 318 \text{ lb}$$

$$P_1 = 1608 \text{ lb}$$

$$M_{max} = 230{,}000 + 2{,}435 + 232{,}435 \text{ in.-lb}$$

For hollow shaft the maximum bending stress and shear stress are:

$$S_b = \frac{M_b\left(\frac{D_o + D_i}{2}\right)}{\frac{\pi}{64}\left(D_o^4 - D_i^4\right)} \tag{A10}$$

$$S_s = \frac{T\left(\frac{D_o + D_i}{2}\right)}{\frac{\pi}{32}\left(D_o^4 - D_i^4\right)} \tag{A11}$$

The inside diameter must be at least 6.8 inches to allow clearance over the output shaft. The power is assumed evenly split between the two ends of the shaft so that

$$T = \frac{\frac{hp}{2} \times 63025}{N} = \frac{1250 \times 63025}{5600} = 14{,}100 \text{ in.-lb} \tag{A12}$$

$$S'_s = 13{,}800 \text{ psi for AISI 4340 steel}$$

From Eqs. (A4), (A10), and (A11)

$$S_{smax} = \sqrt{\left(\frac{16}{\pi}\right)^2\left(M_b^2 + T^2\right)\left(\frac{D_o + D_i}{D_o^4 - D_i^4}\right)^2} \tag{A13}$$

Since

$$S_{smax} \leq S'_s$$

then from Eq. (A13)

$$\frac{D_o^4 - (6.8)^4}{D_o + 6.8} = \frac{\sqrt{25.9\left[(232.5)^2 + (14.1)^2\right]} \times 10^6}{13{,}800} = 87.4$$

$$D_o = 7.6 \text{ in.}$$

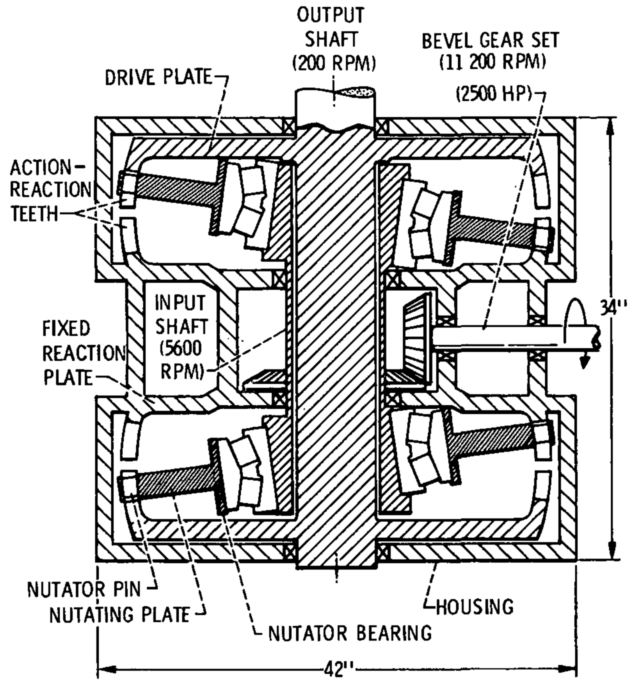


Figure 1. - Sketch showing physical size and arrangement of split nutating drive main rotor gearbox.

E-7050

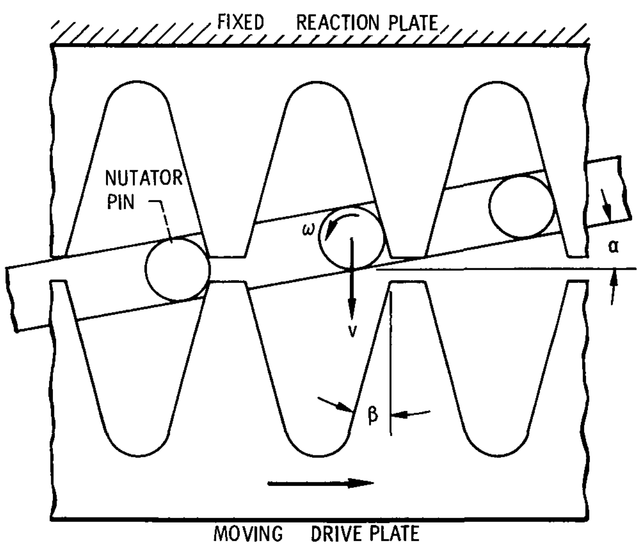


Figure 2. - Rotation of pin bearing on nutating plate.

E-7050

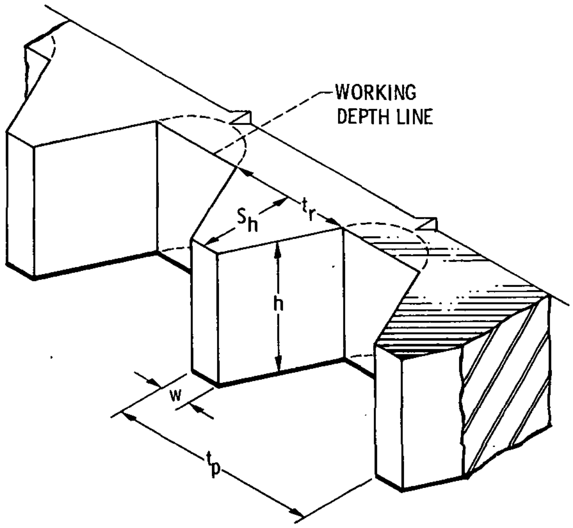
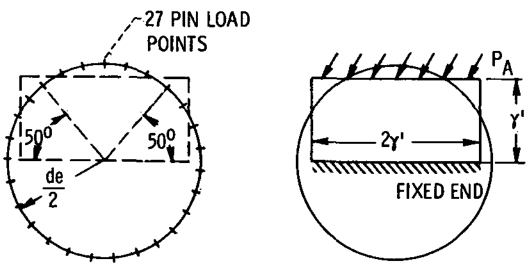


Figure 3. - Characteristic tooth dimensions.



(a) ACTUAL LOADING (b) EQUIVALENT LOADING.

Figure 4. - Equivalent radius for pin loads acting on nutating plate.

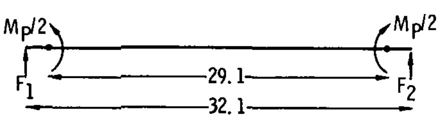


Figure 5. - Force diagram for output shaft.

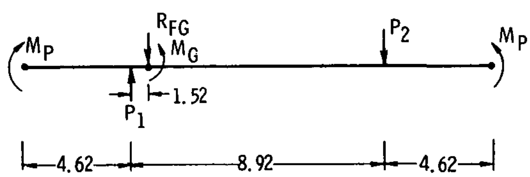


Figure 6. - Force diagram for input shaft.

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