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Spatial Cyclotron Damping

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Abstract

To examine spatial electron cyclotron damping in a uniform Vlasov plasma, we note that the plasma response to a steady-state transverse excitation consists of several terms (dielectric-pole, free-streaming, and branch-cut), but that the cyclotron-damped pole term is the dominant term for $z > \ell = c/\omega_{ce}$ provided $(\omega_{pe}/\omega_{ce})^2(c/a) >> 1$. If the latter inequality does not hold, then the free-streaming and branch-cut terms persist well past $z = c/\omega_{ce}$ as ω_1 approaches ω_{ce}, making experimental measurement of cyclotron damping essentially impossible. Considering only $(\omega_{pe}/\omega_{ce})^2(c/a) >> 1$, we show how collisional effects should be estimated and how a finite-width excitation usually has little effect on the cyclotron-damped part of the response. We establish criteria concerning collisional damping, measurable damping length sizes, and allowed uncertainty in the magnetic field $\underline{B}_o$. Results of numerical calculations, showing the regions in the appropriate parameter spaces that meet these criteria, are presented. From these results, one can determine the feasibility of, or propose parameter values for, an experiment designed to measure spatial cyclotron damping. It is concluded that the electron temperature T_e should be at least 1 ev., and preferably 10 ev. or higher, for a successful experiment.

SPATIAL CYCLOTRON DAMPING

Although it is an important fundamental process, spatial cyclotron damping in a uniform, hot, collisionless plasma has not yet been measured experimentally. A recent attempt by Lee, Fessenden, and Crawford[1] proved unsuccessful because of dominant collisional damping. In this paper we consider the theoretical response of a Vlasov plasma to a steady-state (spatial) transverse excitation. Without loss of generality we neglect the ions and consider only electron cyclotron damping. We note that the plasma response consists of several terms but that the expected cyclotron-damped term is the dominant term at distances sufficiently far away from the excitation. We examine effects of collisions and effects of a finite-width excitation on this cyclotron-damped term. Then, from experimental considerations, we determine ranges of the appropriate parameter values (density n_e, temperature T_e, ratio of plasma frequency to cyclotron frequency ω_{pe}/ω_{ce}, uniformity and precision of the zero-order magnetic field $\underline{B}_0$, and length of experimental apparatus) for which experimental measurements of electron cyclotron damping rates, and their comparison with theory, would be feasible.

Theoretical Considerations

We consider a steady-state external electric field excitation of the form

$$\underline{E}_{ext}(z,t) = E(z)\left[\hat{e}_x \cos(\omega_1 t) + \hat{e}_y \sin(\omega_1 t)\right] \tag{1}$$

$$E(z) = \Phi\,\delta(z) \tag{2}$$

through which particles are allowed to pass freely. If this excitation is applied to an infinite, hot, collisionless plasma, which is described by the Vlasov equation

$$\frac{\partial f}{\partial t} + \underline{v} \cdot \frac{\partial f}{\partial \underline{x}} + \frac{q}{m}\left[\underline{E} + \frac{1}{c} \underline{v} \times \underline{B} \right] \cdot \frac{\partial f}{\partial \underline{x}} = 0 \tag{3}$$

then the first-order (linear) current response is found to be[2]

$$\underline{J}_1(z,t) =$$

$$\mathcal{R}e\left[\hat{e}_+ \frac{\omega_{pe}^2 \Phi}{a} \left\{ \frac{\omega_{pe}^2 \omega_1 \, e^{-3\left[\frac{(\omega_1-\omega_{ce})\alpha z}{2a}\right]^{2/3}\left[\frac{1}{2} - i\alpha\frac{\sqrt{3}}{2}\right] - i\omega_1 t}}{\sqrt{3}\left(k_{se}^2 c^2 - \omega_1^2\right)\left(\omega_1 - \omega_{ce}\right) \epsilon_{T_+}(k_{se},\omega_1)\, \epsilon_{T_-}(k_{se},\omega_1)} \, Z_+\!\left(\frac{\omega_1-\omega_{ce}}{k_{se} a}\right) \right.\right.$$

BRANCH-CUT TERM

$$+ \frac{e^{-3\left[\frac{(\omega_1-\omega_{ce})\alpha z}{2a}\right]^{2/3}\left[\frac{1}{2} - i\alpha\frac{\sqrt{3}}{2}\right] - i\omega_1 t}}{\sqrt{3}\, \frac{V_s}{a}\, \epsilon_{T_\alpha}\!\left(\frac{\omega_1-\omega_{ce}}{V_s}, \omega_1\right)} \tag{4}$$

FREE-STREAMING TERM

$$\left.\left. + \frac{e^{ik_1 z - i\omega_1 t}}{k_1 \left.\frac{\partial \epsilon_{T_+}(k,\omega_1)}{\partial k}\right|_{k=k_1}} \left[Z_+\!\left(\frac{\omega_1-\omega_{ce}}{k_1 a}\right) + 2\sqrt{\pi} i \, S(\omega_1)\, e^{-\left(\frac{\omega_1-\omega_{ce}}{k_1 a}\right)^2} \right] \right\} \right]$$

DIELECTRIC-POLE TERM

FREE-STREAMING POLE-ENHANCED TERM

for z > 0, where we have assumed an isotropic zero-order Maxwellian velocity distribution

$$f_0(\underline{v}) = \frac{1}{\pi^{3/2} a^3} e^{-(v_z^2 + v_\perp^2)/a^2} .$$

The transverse dielectric function is defined by

$$\epsilon_{T_\pm}(k,\omega) = 1 - \frac{\omega}{(k^2c^2 - \omega^2)} \frac{\omega_{pe}^2}{ka} Z_\pm\left(\frac{\omega - \omega_{ce}}{ka}\right)$$

where

$$Z_\pm(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} \frac{dt\, e^{-t^2}}{t - \zeta} ,$$

the plasma dispersion function[3], is defined for Im$\zeta \gtrless 0$, and as the analytic continuation thereof for Im$\zeta \lesseqgtr 0$. The saddle points used in (4) are

$$k_{se} = \left[\frac{2\{(\omega_1 - \omega_{ce})\alpha\}^2}{z a^2}\right]^{\frac{1}{3}} e^{i\left(\frac{\pi}{2} - \alpha\frac{\pi}{3}\right)}$$

$$\frac{V_s}{a} = \left[\frac{(\omega_1 - \omega_{ce})\alpha z}{2a}\right]^{\frac{1}{3}} e^{-i\alpha\frac{\pi}{6}}$$

where

$$\alpha = \mathrm{SGN}(\omega_1 - \omega_{ce}) .$$

Also

$$S(\omega_1) = \begin{cases} 1 & \text{for} \quad \omega_1 < 0 , \ \omega_1 > \omega_{ce} \\ 0 & \text{for} \quad 0 < \omega_1 < \omega_{ce} \end{cases}$$

and the root(s) k_1 are defined by

$$\epsilon_{T_+}(k_1, \omega_1) = 0 .$$

Note that the response (4) consists of three types of terms, whose characteristics we briefly discuss:

(i) The branch-cut term, which is not readily explainable physically, has a significant amplitude only for $z \approx 0$ and $\omega_1 \approx \omega_{ce}$.

(ii) The free-streaming term represents phase mixing of the free-streaming waves created by those particles that free stream through the excitation region at $z = 0$. Free-streaming waves have many unique characteristics[2] (negative phase velocities for $0 < \omega_1 < \omega_{ce}$ and $z > 0$, absence of cyclotron damping, etc.) but these features will not concern us. The free-streaming term is always significant, has a characteristic length $a/(\omega_1 - \omega_{ce})$ for $\omega_1 \approx \omega_{ce}$, and is divergent at $z = 0$ due to the delta-function excitation (2).

(iii) The dielectric-pole term(s) represent the collective plasma oscillations. Actually there are an infinite number of pole terms because the transverse dispersion relation

$$1 - \frac{\omega}{(k^2c^2-\omega^2)} \frac{\omega_{pe}^2}{ka} Z_{\pm}\left(\frac{\omega - \omega_{ce}}{ka}\right) = 0 \qquad (5)$$

has an infinite number of roots (for real ω and complex k, the case we are considering). The free-streaming pole-enhanced term is due to the excitation of a collective mode by a particular free-streaming wave (i.e., a wave formed from particles which all have velocity v_z where $v_z = v$ and v, a complex number, solves $\epsilon_T([\omega_1 - \omega_{ce}]/v,\ \omega_1) = 0$). Typically this term is negligible because the exponential factor in (4), $\exp -([\omega_1 - \omega_{ce}]/[k_1 a])^2$, is usually very small.

The least-damped pole term in (4) represents the cyclotron-damped response we are interested in. But, of course, all of the terms in (4) would be seen in an experiment. Thus to examine only cyclotron damping, we must verify that a distance ℓ exists, beyond which all of the terms in (4), except the least-damped pole term, will have damped away. Then for $z > \ell$ the least-damped pole term will truly be the dominant term. Provided

$$\frac{\omega_{pe}^2}{\omega_{ce}^2} \frac{c}{a} \gg 1 \tag{6}$$

the various terms in (4), and upper-bound estimates of their maximum characteristic penetration lengths, are[2]

Branch-cut term	c/ω_{ce}	
Free-streaming term	c/ω_{ce}	(7)
Infinite sequence of pole terms (All pole terms except the least-damped one)	$< c/\omega_{ce}$	

It follows that a useful estimate for ℓ is $\ell = c/\omega_{ce}$, provided (6) holds. If (6) does not hold, the estimates in (7) do not hold and all of the terms in (4) can extend well past $z = c/\omega_{ce}$. Henceforth we restrict our attention to those cases in which (6) holds.

Note that the least-damped pole term and the free-streaming pole-enhanced term in (4) concern the same least-damped collective model being excited by two different means: (i) directly by the excitation, and (ii) indirectly by a free-streaming wave. However, all that is measured experimentally is Im k for these least-damped terms, so it is not necessary to know what fraction of the total least-damped response is caused by each of these two methods. (In fact, for $0 < \omega_1 < \omega_{ce}$, the free-streaming waves do not excite any collective modes as indicated by the $S(\omega_1)$ factor in (4): this is due to the fact that for $0 < \omega_1 < \omega_{ce}$, all free-streaming waves have negative phase velocities while all collective modes have positive phase velocities.[2]) In any case for $z > \ell = c/\omega_{ce}$, the cyclotron damping rate of the response is determined simply by Im k of the least-damped root of the transverse dispersion relation (5).

In Figures 1a, b, c we present some numerical results concerning this cyclotron-damped root for values of ω slightly below ω_{ce} . Note that Im k is essentially zero until ω is very near ω_{ce} and that when $\omega = \omega_{ce}$, Im k is typically large enough that it could not be measured easily. Thus for cyclotron-damping measurements our interest is confined to a small region where $\omega \approx \omega_{ce}$ but $\omega < \omega_{ce}$. (For a complete analysis of all the roots of the transverse dispersion relation (5), including the cyclotron-damped root for $\omega > \omega_{ce}$, see reference 2.) We comment in passing that the cyclotron-damped root can only be obtained numerically: an asymptotic expression analagous to Landau damping for longitudinal waves does not exist.[2]

Having determined that the cyclotron-damped response to the transverse excitation (1)-(2) shows simple exponential damping for $z > \ell = c/\omega_{ce}$ provided (6) holds, and that the damping rate can be determined only by numerically solving for Im k of the least-damped root of (5), we proceed to examine the effects of collisions and the effects of a finite-width excitation mechanism (in place of the delta-function excitation (2)).

To determine when collisional damping is significant, it is sufficient to use the Krook, Bhatnager, Gross model for the collision term in the Landau-Boltzmann equation. Then in place of (3) we have

$$\frac{\partial f}{\partial t} + \underline{v}\cdot\frac{\partial f}{\partial \underline{x}} + \frac{q}{m}\left[\underline{E} + \frac{1}{c}\,\underline{v}\times\underline{B}\right]\cdot\frac{\partial f}{\partial \underline{v}} = -\nu\left(f - f_0\right) \tag{8}$$

where ν is an effective self-collision frequency[4]

$$\begin{aligned} \nu &\approx 2.9 \cdot 10^{-6} \left(n_e / T_e^{\frac{3}{2}} \right) \ln_{10} \Lambda \\ \Lambda &\approx 1.54 \cdot 10^{10} \left(T_e^{\frac{3}{2}} / n_e^{\frac{1}{2}} \right) \end{aligned} \tag{9}$$

We neglect electron-neutral collisions because their effect can be negligibly small (as may easily be verified for any particular experiment by consulting Brown's book[5]). When collisions are included as per (8)-(9), the plasma response (4) is appropriately modified. Our chief concern is how collisions affect the cyclotron-damped pole term, or more specifically, how Im k of the least-damped root of (5) is modified. When collisions are included, the collisionless transverse dispersion relation (5) is replaced by

$$1 - \frac{\omega \omega_{pe}^2}{(k^2c^2 - \omega^2) ka} Z_{\pm}\left(\frac{\omega + i\nu - \omega_{ce}}{ka} \right) = 0 \tag{10}$$

Clearly Im k of the least-damped root of (10) is due to cyclotron damping <u>plus</u> collisional damping, whereas Im k of the least-damped root of (5) is due only to cyclotron damping. We will use these facts shortly to determine when collisional damping may be neglected.

The effects of a finite-width excitation on (4) have been studied in detail[2], but for our purposes it is sufficient to know the effect on the cyclotron-damped pole term. Qualitatively, the main result is that if the effective width of the excitation region is much smaller than the wavelength of the cyclotron-damped pole term, then that pole term will be essentially unaltered. More specifically, if in place of (2) we have

$$E_1(z) = \Phi \frac{e^{-\frac{z^2}{\Delta^2}}}{\sqrt{\pi}\, \Delta} \tag{11}$$

(where Δ is the effective width of this Gaussian-shaped excitation), then the pole term in (4) acquires an additional amplitude factor of

$$e^{-k_1^2\Delta^2/4} \tag{12}$$

where k_1 is as defined in (4). Since Δ may typically be of the order of a Debye length ($\lambda_d = \{T_e/[4\pi n_e e^2]\}^{1/2}$), and since the cyclotron-damped root k_1 is of order $2\pi(\omega_{ce}/c)$, we have $k_1^2\Delta^2/4 \ll 1$ and the finite-width excitation has little effect on the cyclotron-damped pole term.

Experimental Considerations

Experimental measurements of cyclotron damping rates are limited by the following three important considerations.

(i) Collisional effects: Prompted by the above discussion on collisions, we let

L = Im k of the least-damped root of (5)

M = Im k of the least-damped root of (10)

then L represents cyclotron damping whereas M represents cyclotron damping plus collisional damping. To establish where collisional effects are negligible, we numerically calculate the locus of points where L/M = .9 on a T_e vs. ω_1/ω_{ce} diagram as shown in Figure 2a. Then along the locus there is roughly 90% cyclotron damping and 10% collisional damping. Above the locus cyclotron damping is even more dominant and collisional damping can be ignored. [We note that Lee, Feesenden, and Crawford[1] constructed a similar locus, but that their's erroneously turns upward as ω_1 approaches ω_{ce} because it represents the locus where hot plasma cyclotron damping equals cold plasma collisional damping. When ω_1 is near ω_{ce} we find

$|\zeta| \leq 3$ (where ζ is the argument of the Z function in (10)), which means the cold plasma collisional dispersion relation, which may be derived from (10) for $|\zeta| \geq 3$, is no longer valid. The correct procedure, as performed above, is to compare hot plasma cyclotron damping without collisions to hot plasma cyclotron damping with collisions.]

(ii) Damping length size: The size of Im k that can be measured experimentally is restricted by two limits. First, the cyclotron-damping length $\delta = (\text{Im } k)^{-1}$ must not be less than c/ω_{ce} because, as discussed above, only for $z > \ell = c/\omega_{ce}$ is the cyclotron-damped term the dominant term in the total response (4). Second, the wave must show damping within the length of the experimental apparatus. These requirements limit observable values of Im k to a range of the form

$$.1 \lesssim \frac{\text{Im } k c}{\omega_{ce}} \lesssim 1 \tag{13}$$

say, which corresponds to

$$1 \lesssim \frac{z\, \omega_{ce}}{c} \lesssim 10 \tag{14}$$

(The limits in (13), (14) correspond to one e-folding length of the cyclotron-damped term.) Using (13) we have numerically calculated the restricted region shown in Figure 2b wherein cyclotron damping should be readily observable.

(iii) Magnetic field uniformity: Since ω_1 must be very close to $_{ce}$ for a cyclotron damping measurement, it is imperative that $\underline{B}_0$ be extremely uniform and that its value be known accurately. As evidenced by Figure 1, even a 1% uncertainty in $\underline{B}_0$ would render any experimental comparisons with theoretical damping rates essentially impossible. Ideally we must meet a criterion like

$$\left| \frac{\Delta \text{Im } k}{\text{Im } k} \right| \lesssim .1 \tag{15}$$

where

$$\Delta \mathrm{Im}\, k = \mathrm{Im}\, k \Big|_{\frac{\omega}{\omega_{ce}}\left(1+\frac{1}{2}\left|\frac{\Delta B_0}{B_0}\right|\right)} - \mathrm{Im}\, k \Big|_{\frac{\omega}{\omega_{ce}}\left(1-\frac{1}{2}\left|\frac{\Delta B_0}{B_0}\right|\right)} \tag{16}$$

and $\Delta B_0/B_0$ represents the per cent uncertainty in B_0. Assuming $\Delta B_0/B_0$ equals .001 (i.e., a .1% uncertainty in B_0), and using (15) we have numerically obtained the restricted region shown in Figure 2c.

Only the region that includes all three of the above restricted regions is suitable for cyclotron damping measurements. This is shown in Figure 2d. It is apparent that experimental parameters will have to be chosen carefully to insure a sizeable region over which experiment and theory can be compared.

Feasibility

The restrictions adopted in the previous sections,

(a) $\frac{\omega_{pe}^2}{\omega_{ce}^2}\frac{c}{a} \gg 1$ (restricts penetration of undesired terms in (4)).

(b) $z \gtrsim \ell \approx \frac{c}{\omega_{ce}}$ (insures cyclotron-damped pole term is dominant, provided restriction (a) holds)

(c) $\frac{\mathrm{Im}\, k_{\text{without collisions}}}{\mathrm{Im}\, k_{\text{with collisions}}} \geq .9$ (restricts collisional damping to a negligible level)

(d) $.1 \lesssim \frac{\mathrm{Im}\, k\, c}{\omega_{ce}} \lesssim 1$ (combines restriction (b) with requirement that damping occur within length of machine)

(e) $\left|\frac{\Delta \mathrm{Im}\, k}{\mathrm{Im}\, k}\right| \lesssim .1$ (restricts error in Im k, given the % uncertainty in B_0

serve as guidelines for determining whether an experiment designed to measure cyclotron damping would be feasible or not.

If restriction (a) holds, then the remaining restrictions are met only in limited regions of parameter space, as was illustrated for a sample case in Figure 2. In Figure 3 we have repeated the numerical calculations of Figure 2 for a wide range of parameter values. From these results one should be able to roughly estimate the feasibility of a cyclotron damping experiment for parameter values in the ranges

T_e	.1	$\rightarrow$	100 ev.
ω_{pe}/ω_{ce}	.1	$\rightarrow$	10
n_e	10^8	$\rightarrow$	10^{14} cm.$^{-3}$

Additional information concerning $f_{ce} = \omega_{ce}/2\pi$, c/ω_{ce}, and the range of z that corresponds to the range Im $kc/\omega_{ce} = 1 \rightarrow .1$ are given in Table 1.

Discussion and Conclusions

It is apparent from Figure 3 that T_e should be of the order of 10 ev. or higher to attain a useful measurement range. Values of T_e as low as 1 ev. might be used but only if ω_{pe}/ω_{ce} is large ($\gtrsim 4$) and the density is low ($n_e \lesssim 10^{10}$ cm.$^{-3}$). In no case could a successful cyclotron damping experiment be performed at T_e = .1 ev. (as evidenced by Lee, Fessenden, and Crawford[1]).

It should be noted that the dashed loci in Figure 3 are based on an assumed .1% uncertainty in $\underline{B}_0$. If the uncertainty in $\underline{B}_0$ is greater than .1%, then the dashed loci would be raised. This would further restrict the lowest temperature at which an experiment could be performed, especially for small values of ω_{pe}/ω_{ce} (see Figure 3a). For large values of ω_{pe}/ω_{ce}, collisions determine the lower temperature limit (see Figure 3c).

Thus even under ideal conditions, a successful cyclotron damping experiment could be performed with T_e only as low as about 1 ev. In general, it would be preferable to have $T_e \sim 10$ ev. or higher. In any event, with the aid of Figure 3 and Table 1, one can readily establish the frequency range and distance over which cyclotron damping could be seen and measured, for many typical experimental values of the plasma parameters (T_e, ω_{pe}/ω_{ce}, and n_e).

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* Present address: Sandia Laboratories, Albuquerque, New Mexico. This paper is based in part on a dissertation submitted to the University of California, Los Angeles in partial fulfillment of the requirements for the Ph.D. degree.

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FIGURE CAPTIONS

Figure 1

The cyclotron-damped root [the least-damped root of (5)] is shown for several values of c/a (200, 750, 1500, ∞) and ω_{pe}/ω_{ce} (.25, .67, 1.5).

Figure 2

Restrictions imposed on experimental measurement of cyclotron damping (ω_{pe}/ω_{ce} = .4).

(a) Dotted line: locus where the total damping is 10% collisional damping and 90% cyclotron damping ($n_e = 10^{10}$ cm.$^{-3}$). Above the locus collisional effects may be neglected.

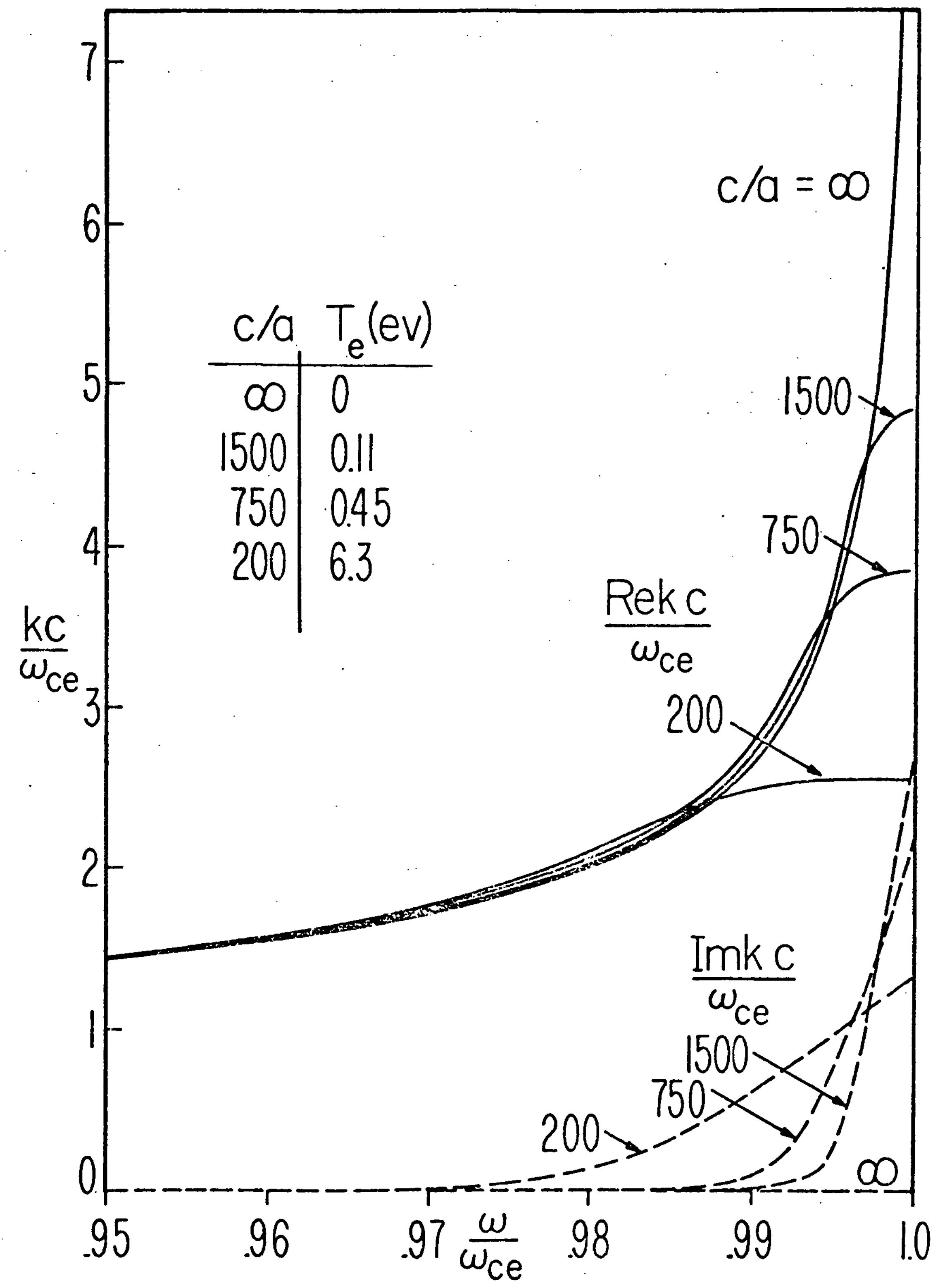
(b) Solid line: loci where Im kc/ω_{ce} = .1 and 1, between which the cyclotron damping length $\delta = (\mathrm{Im}\ k)^{-1}$ is neither too short [in which case the cyclotron-damped response would be masked by other terms in the total response (4)] nor too long [in which case the damping could not be observed within the length of a typical experimental apparatus].

(c) Dashed line: locus where the inherent error in measuring Im k is 10% [as per (15), (16)] for an assumed uncertainty in $\underline{B}_0$ of .1%. Above the locus the error is less than 10%.

(d) A superposition of (a), (b), (c). The shaded region is where the shaded regions of (a), (b), (c) all overlap. Only in this region is experimental measurement of cyclotron damping feasible.

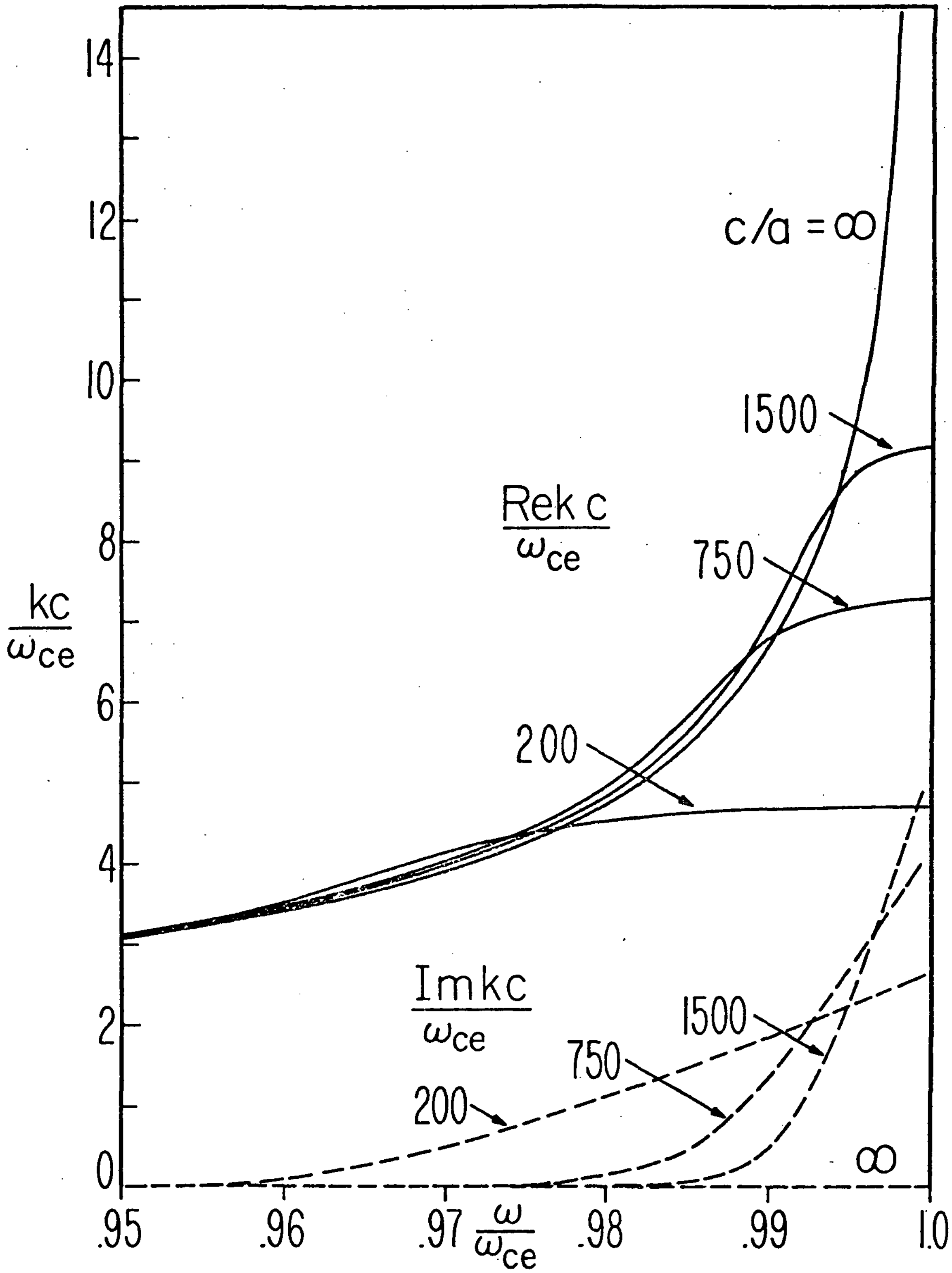
Figure 3

Regions wherein experimental measurement of cyclotron damping is feasible are shown for a wide range of parameter values. The loci (dotted, solid, and dashed lines) all have the same meaning as in Figure 2. In (c) the dashed locus does not appear because it occurs for T_e < .1 ev., and therefore the collision (dotted) loci determine the lower boundary of the feasibility region. Also in (c) note the change of scale along the abscissa axis.



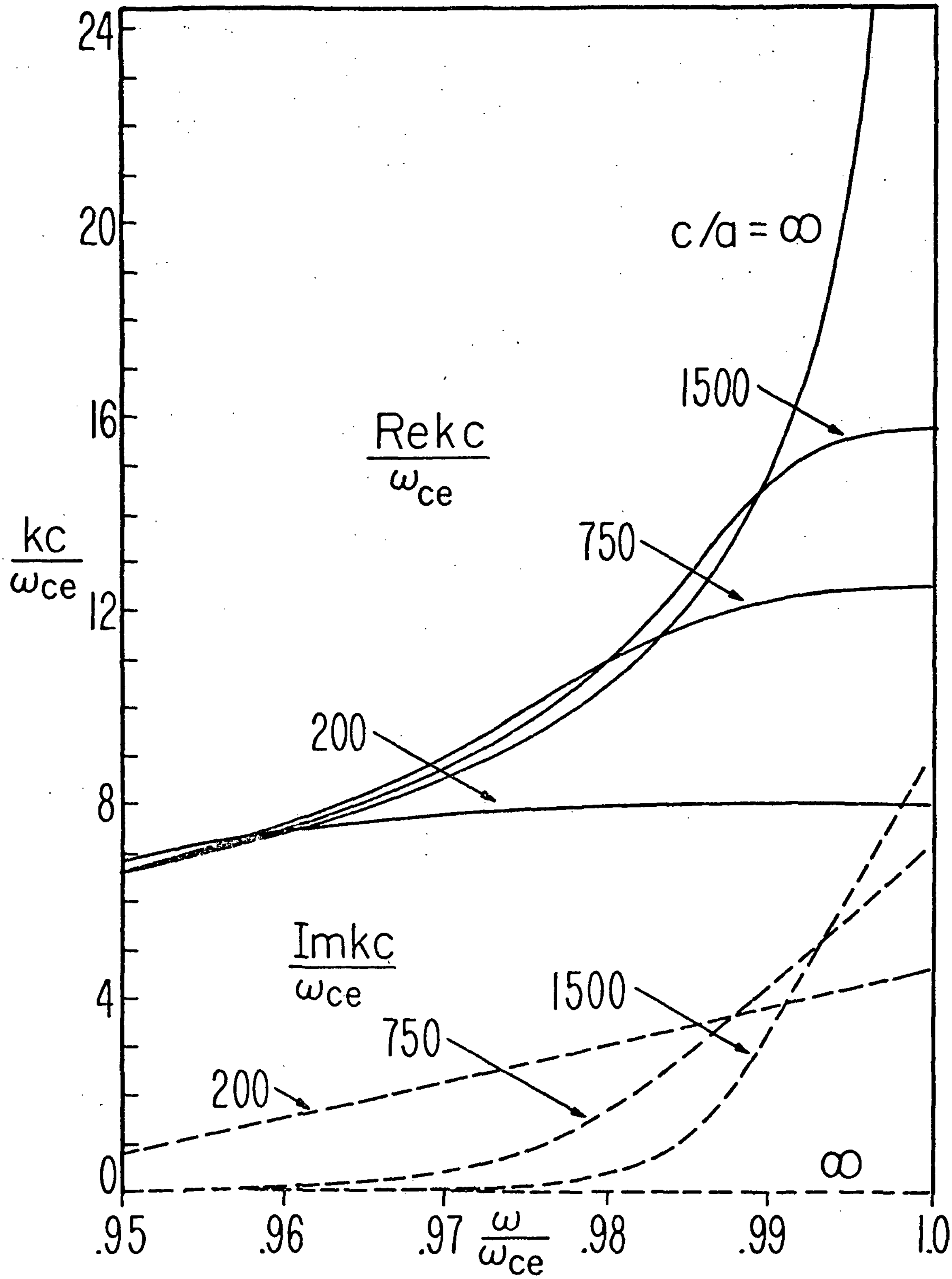
$\omega_{pe}/\omega_{ce} = .25$

Fig. 1a



$\omega_{pe}/\omega_{ce} = .67$

Fig. 1b



$\omega_{pe}/\omega_{ce} = 1.5$

Fig. 1c

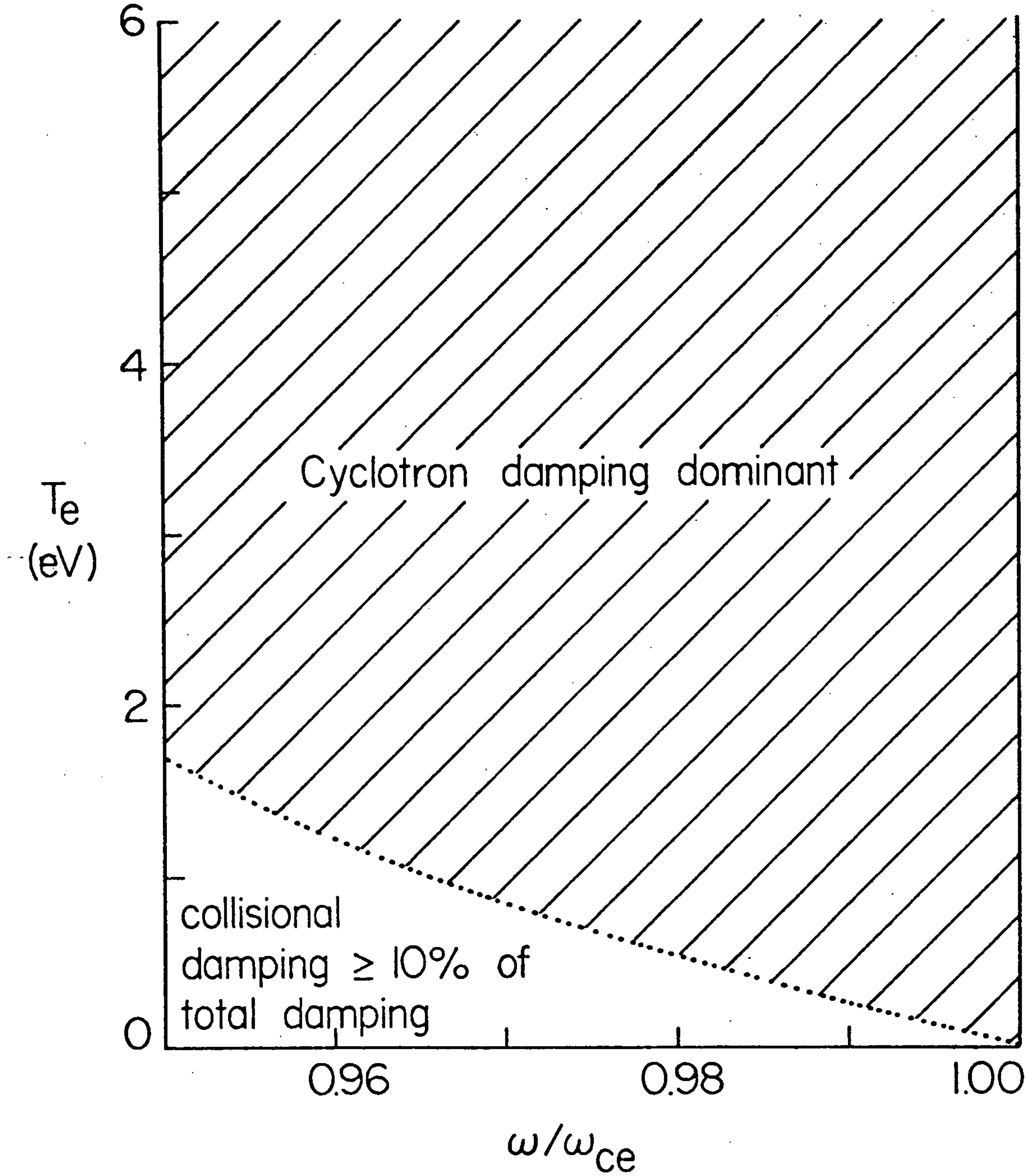

6
4
2
0
T_e
(eV)
Cyclotron damping dominant
collisional
damping ≥ 10% of
total damping
0.96
0.98
1.00
ω/ω_{ce}

Fig. 2a

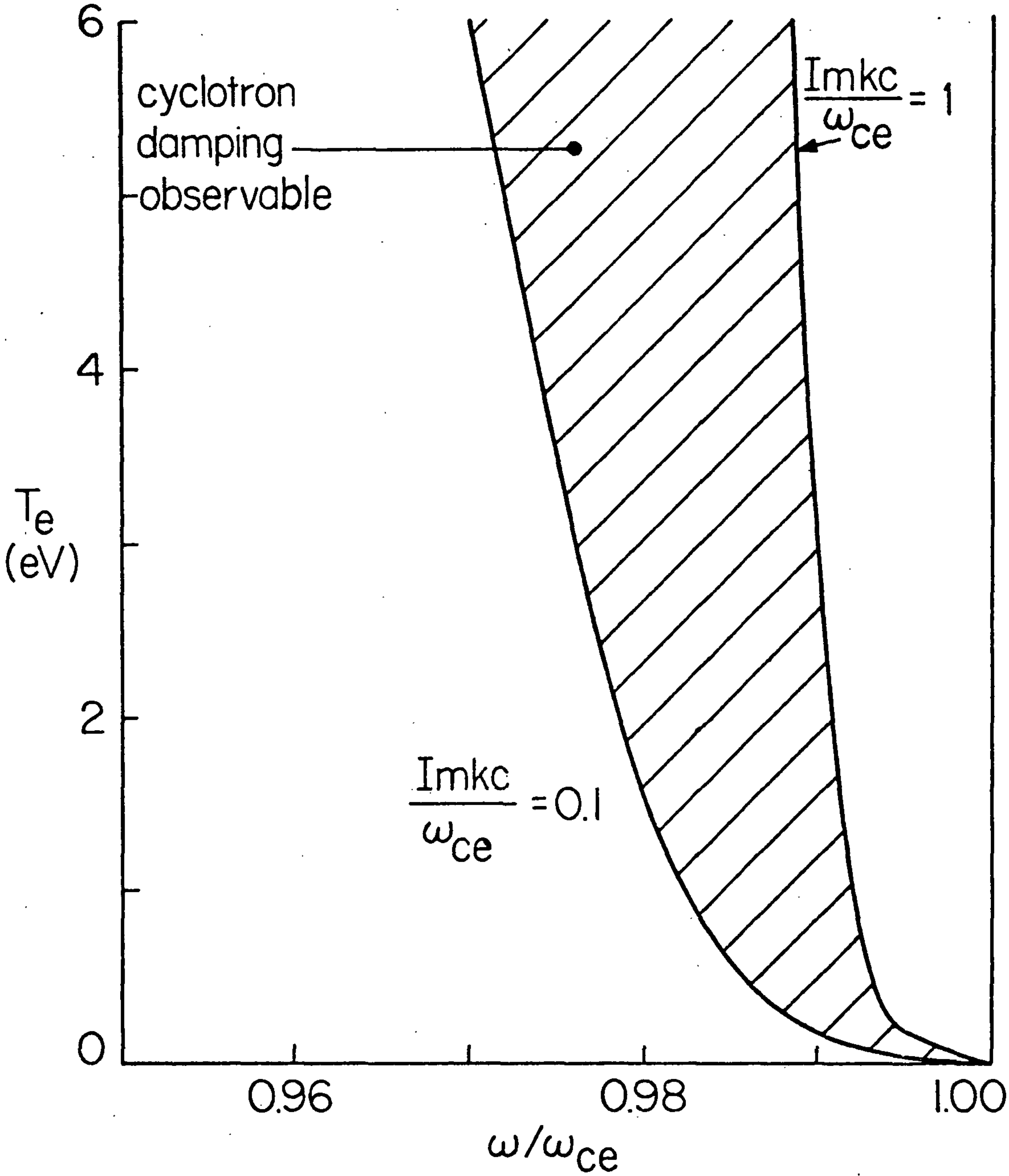

6
4
2
0
cyclotron
damping
observable
$\frac{\mathrm{Im}\,kc}{\omega_{ce}} = 1$
$\frac{\mathrm{Im}\,kc}{\omega_{ce}} = 0.1$
T_e
(eV)
0.96
0.98
1.00
ω/ω_{ce}

Fig. 2b

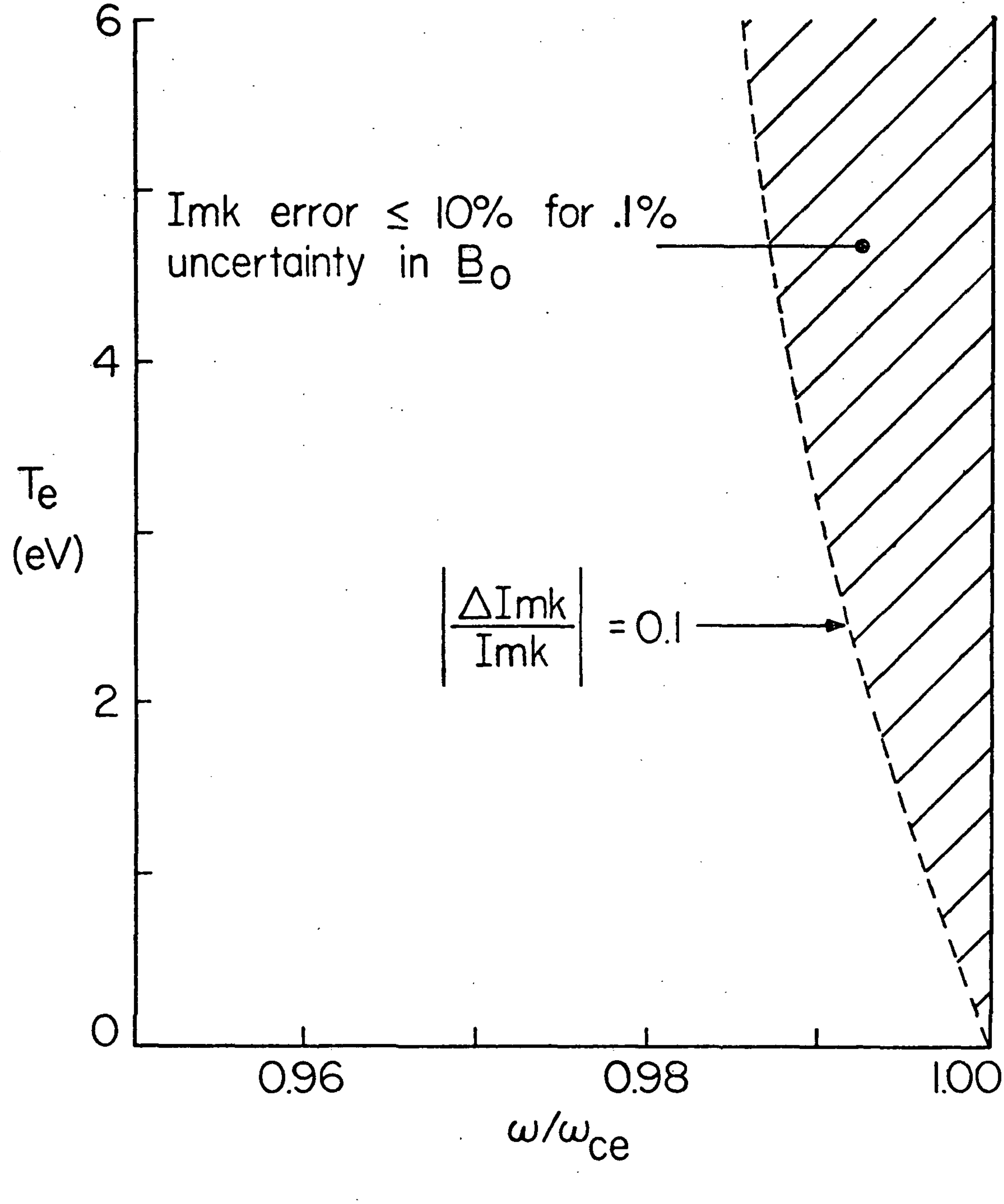
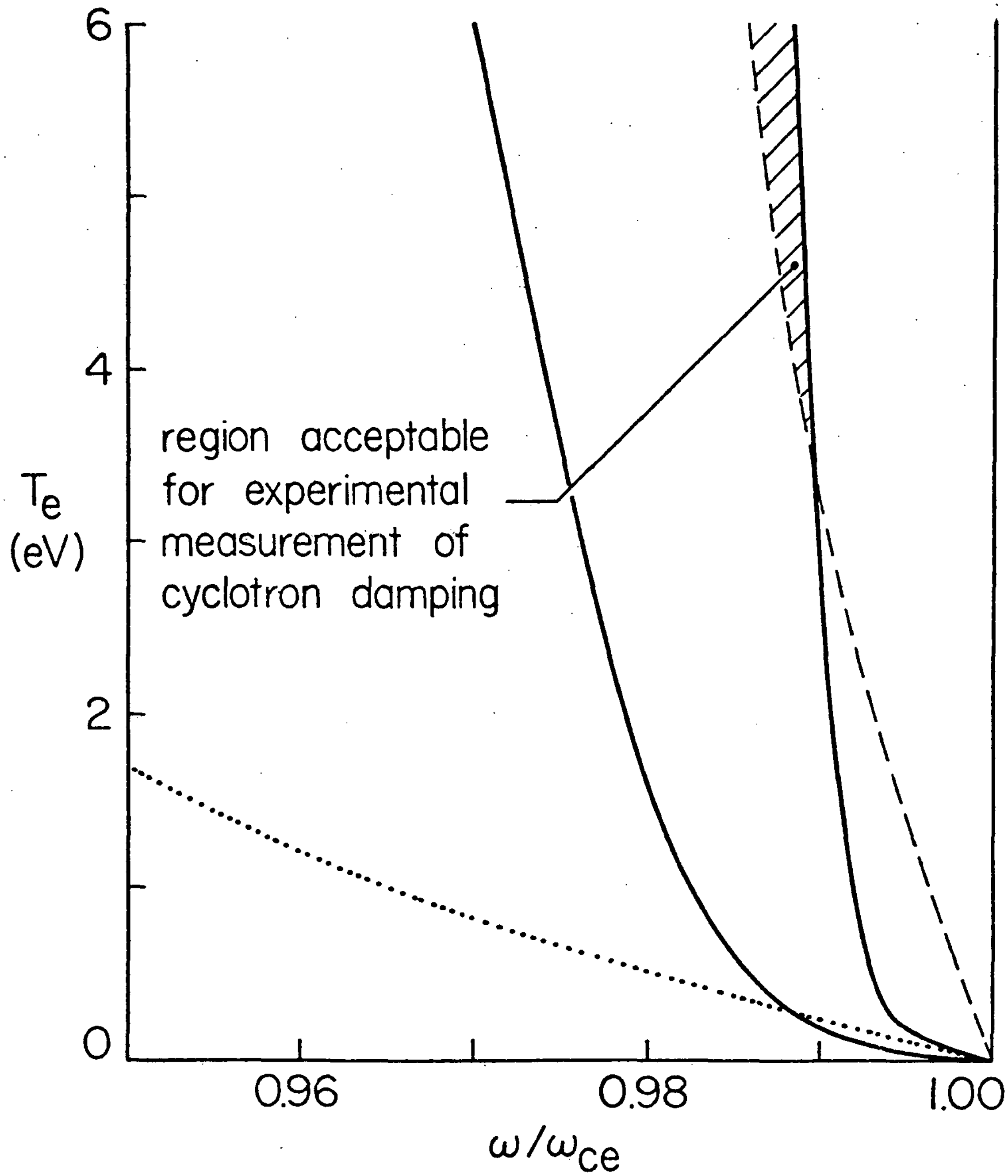
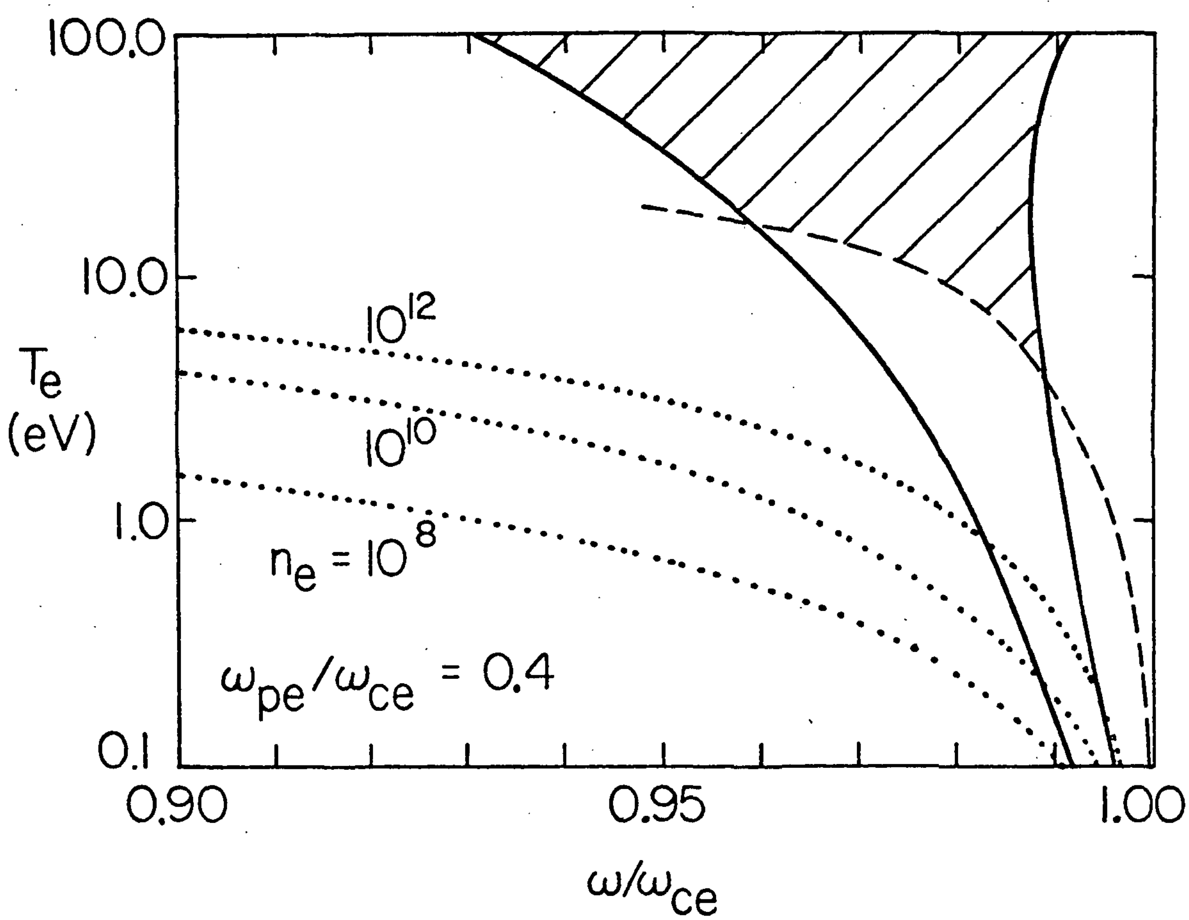

6
4
2
0
T_e
(eV)
Imk error ≤ 10% for .1%
uncertainty in $\underline{B}_0$
$\left|\frac{\Delta \mathrm{Imk}}{\mathrm{Imk}}\right| = 0.1$
0.96
0.98
1.00
ω/ω_{ce}

Fig. 2c



6
4
2
0
T_e
(eV)
region acceptable
for experimental
measurement of
cyclotron damping
0.96
0.98
1.00
ω/ω_{ce}

Fig. 2d



100.0
10.0
1.0
0.1
T_e
(eV)
10^{12}
10^{10}
$n_e = 10^8$
$\omega_{pe}/\omega_{ce} = 0.4$
0.90
0.95
1.00
ω/ω_{ce}

Fig. 3a

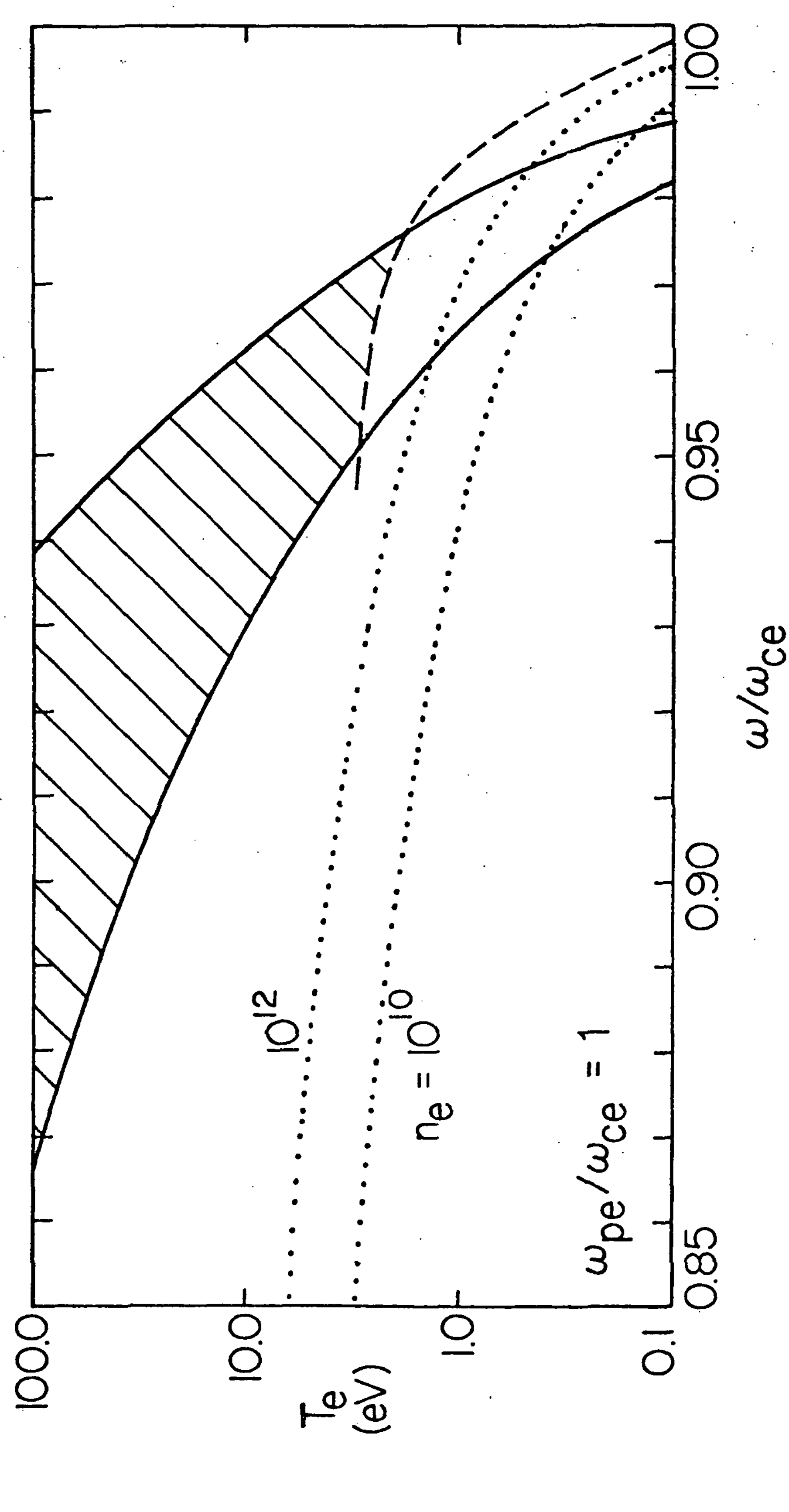

100.0
10.0
1.0
0.1
T_e (eV)
0.85
0.90
0.95
1.00
ω/ω_{ce}
10^{12}
$n_e = 10^{10}$
$\omega_{pe}/\omega_{ce} = 1$

Fig. 3b

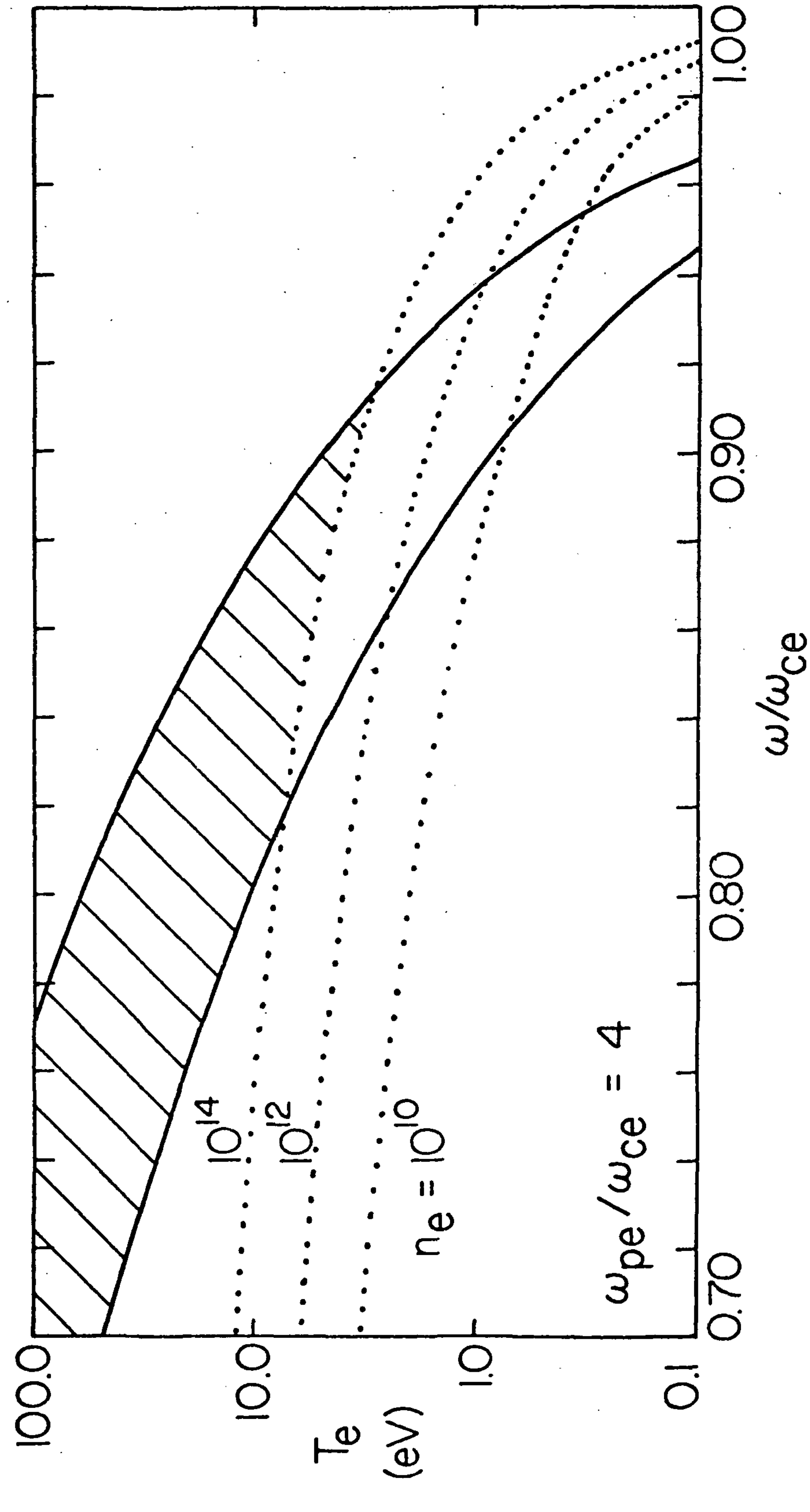

100.0
10.0
1.0
0.1
T_e
(eV)
0.70
0.80
0.90
1.00
ω/ω_{ce}
10^{14}
10^{12}
$n_e = 10^{10}$
$\omega_{pe}/\omega_{ce} = 4$

Fig. 3c

ω_{pe}/ω_{ce}	$n_e(\text{cm.}^{-3})$	$f_{ce}=\omega_{ce}/2\pi$ (GHz.)	c/ω_{ce} (cm.)	z(cm.)
.4	10^8 10^{10} 10^{12}	.225 2.25 22.5	20 2 .2	20 → 200 2 → 20 .2 → 2
1	10^{10} 10^{12}	.9 9	8 .8	8 → 80 .8 → 8
4	10^{10} 10^{12} 10^{14}	.225 2.25 22.5	20 2 .2	20 → 200 2 → 20 .2 → 2

Table 1.

Values of the cyclotron frequency f_{ce} and length c/ω_{ce} for the parameter values used in Figure 3. The "z" column refers to the range $z = c/\omega_{ce} \rightarrow 10c/\omega_{ce}$ which corresponds to the range $\text{Im}\, kc/\omega_{ce} = 1 \rightarrow .1$.