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RELATIVISTIC TIME CORRECTIONS FOR APOLLO 12 AND APOLLO 13

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16. Abstract

Results are presented of computer calculations on the relativistic time corrections relative to a ground-based clock of on-board clock readings for a lunar mission, using simple Newtonian gravitational potentials of Earth and Moon and based on actual trajectory data for Apollo 12 and Apollo 13. Although the second order Doppler effect and the gravitational "red shift" give rise to corrections of opposite sign, the net accumulated time corrections, namely a gain of 560 ($\pm$1.5) microseconds for Apollo 12 and gain of 326 ($\pm$1.3) microseconds for Apollo 13, are still large enough that with present day atomic frequency standards, such as the rubidium clock, they can be measured with an accuracy of about $\pm$0.5 percent.

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INTRODUCTION

Interest in a determination of the relativistic time shift as manifested on board a spacecraft has been sharpened by the recent publication of an estimate by C. O. Alley indicating that a blue shift of some 300 μs occurred for the Apollo 8 spacecraft versus a ground-based clock (ref. 1) and of a description of a proposed orbiting clock experiment to determine the gravitational red shift by Kleppner, Vessot, and Ramsey (ref. 2). However, precise computations using actual tracking data for specific space missions have yet to be published. To fill this gap, this report presents precise frequency and time shift computations with tight error bounds for one long-duration lunar mission (Apollo 12) and one short-duration lunar mission (Apollo 13). Because no precision frequency standards were carried on these flights, no comparison of these computations with experimental data can be attempted. However, the error bounds achieved in the computations presented here are to be taken as indicative of the accuracy achievable in future computations using the methods presented in this report.[1]

THEORETICAL DEVELOPMENT

Introduced in the theory of relativity is the concept of "proper time" (see, for example, ref. 3), which is the time indicated by a clock rigidly connected with a moving material frame of reference. If $x_\alpha(\alpha = 1, 2, 3)$ are the space coordinates and t the time coordinate of this moving frame with respect to a given inertial frame of reference, it can be shown that the interval of proper time $d\tau$ is given by

$$d\tau^2 = \left(\frac{1}{c^2} \sum_{\alpha,\beta=1}^{3} g_{\alpha\beta} \frac{dx_\alpha}{dt} \frac{dx_\beta}{dt} + \frac{1}{c} \sum_{\alpha=1}^{3} g_{\alpha4} \frac{dx_\alpha}{dt} + g_{44} \right) dt^2 \tag{1}$$

where $g_{\alpha\beta}$ are the components of the metric tensor involving the spatial coordinates x_α, the subscript 4 labels the time coordinate, and dt is the interval of coordinate time. (Coordinate time is the time

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[1] All computations presented here were carried out by program E00032 of the Goddard Space Flight Center Computer Program Library. This program can be used without modification for calculations for any near-Earth mission. With small modifications it can be used for interplanetary missions.

indicated by a clock at rest with respect to the coordinate system and under no gravitational potential.)

For a weak static gravitational field having potential ϕ, the components of the metric tensor can be taken to be

$$g_{\alpha\beta} = -\delta_{\alpha\beta}$$

$$g_{\alpha 4} = 0$$

and.

$$g_{44} = 1 + \frac{2\phi}{c^2}$$

where $\delta_{\alpha\beta} = 1$ if $\alpha = \beta$ and $\delta_{\alpha\beta} = 0$ if $\alpha \neq \beta$. Hence

$$d\tau = \sqrt{1 + \frac{2\phi}{c^2} - \frac{v^2}{c^2}} \; dt \tag{2}$$

where ϕ is the total gravitational potential at the clock, v is the speed of the clock with respect to the given inertial frame, and $c = 2.997\,925 \times 10^8$ m/s (the speed of light). Let ϕ_A and ϕ_B represent the gravitational potential at clock A and the gravitational potential at clock B, respectively, and v_A and v_B represent the speed of clock A and the speed of clock B, respectively. If we compare clock A to clock B, both in the same inertial frame, the relative frequency of clock A with respect to clock B is given by

$$\frac{d\tau_A}{d\tau_B} = \frac{\sqrt{1 + \dfrac{2\phi_A}{c^2} - \dfrac{v_A^2}{c^2}} \; dt}{\sqrt{1 + \dfrac{2\phi_B}{c^2} - \dfrac{v_B^2}{c^2}} \; dt}$$

$$= \frac{\sqrt{1 + \dfrac{2\phi_A}{c^2} - \dfrac{v_A^2}{c^2}}}{\sqrt{1 + \dfrac{2\phi_B}{c^2} - \dfrac{v_B^2}{c^2}}} \tag{3}$$

$$\approx 1 + \frac{\phi_A}{c^2} - \frac{v_A^2}{2c^2} - \frac{\phi_B}{c^2} + \frac{v_B^2}{2c^2} \tag{3a}$$

Hence, to the first order, the frequency shift of clock A with respect to clock B is

$$\frac{\phi_A}{c^2} - \frac{v_A^2}{2c^2} - \frac{\phi_B}{c^2} + \frac{v_B^2}{2c^2} \tag{4}$$

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Thus, the total time difference of clock A with respect to clock B accumulated from (coordinate) time t_1 to (coordinate) time t_2 is

$$\Delta t = \int_{t_1}^{t_2} \left[\frac{\phi_A(t)}{c^2} - \frac{v_A^2(t)}{2c^2} - \frac{\phi_B(t)}{c^2} + \frac{v_B^2(t)}{2c^2} \right] dt \tag{5}$$

Now, for points in the neighborhood of the Earth-Moon system, the gravitational potentials ϕ_A and ϕ_B may be computed with sufficient precision from the expression

$$\phi = - \frac{GM_S}{r_S} - \frac{GM_E}{r_E} \left[1 - \frac{J_2}{2} \left(\frac{R_E}{r_E} \right)^2 (3 \sin^2 l_E - 1) - \frac{J_3}{2} \left(\frac{R_E}{r_E} \right)^3 (5 \sin^3 l_E - 3 \sin l_E) \right.$$
$$\left. - \frac{J_4}{8} \left(\frac{R_E}{r_E} \right)^4 (35 \sin^4 l_E - 30 \sin^2 l_E + 3) + 3(C_{22} \cos 2\lambda_E + S_{22} \sin 2\lambda_E) \left(\frac{R_E}{r_E} \right)^2 \cos^2 l_E \right]$$
$$- \frac{GM_M}{r_M} \left[1 - \frac{K_2}{2} \left(\frac{R_M}{r_M} \right)^2 (3 \sin^2 l_M - 1) + 3K_{22} \left(\frac{R_M}{r_M} \right)^2 \cos^2 l_M \cos 2\lambda_M \right] \tag{6}$$

where

$GM_S = (1.327\ 124\ 99 \pm 0.000\ 000\ 15) \times 10^{20}\ \text{m}^3/\text{s}^2$ = gravitational parameter of the Sun

r_S = distance from the clock to the center of mass of the Sun

$GM_E = (3.986\ 012 \pm 0.000\ 004) \times 10^{14}\ \text{m}^3/\text{s}^2$ = gravitational parameter of the Earth

r_E = distance from the clock to the center of mass of the Earth

$J_2 = (1.0827 \pm 0.0001) \times 10^{-3}$

$R_E = 6.378\ 166 \times 10^6\ \text{m}$ = equatorial radius of the Earth

l_E = geocentric latitude (declination) of the clock

$J_3 = (-2.56 \pm 0.1) \times 10^{-6}$

$J_4 = (-1.58 \pm 0.2) \times 10^{-6}$

$C_{22} = (1.57 \pm 0.01) \times 10^{-6}$

$S_{22} = (-0.897 \pm 0.01) \times 10^{-6}$

λ_E = geocentric longitude of the clock

$GM_M = (4.902\ 78 \pm 0.000\ 06) \times 10^{12}\ \text{m}^3/\text{s}^2$

r_M = distance from the clock to the center of mass of the Moon

$K_2 = (2.071\ 08 \pm 0.05) \times 10^{-4}$

3

$R_M = (1.738\ 09 \pm 0.07) \times 10^6$ m = mean lunar radius

l_M = selenocentric latitude (declination) of the clock

$K_{22} = (2.0716 \pm 0.5) \times 10^{-5}$

λ_M = selenocentric longitude of the clock (positive eastward with respect to the Moon's prime meridian)

These constants are taken from pages 6-2 to 7-1 of reference 4. Equation (6) is derived from the gravitational-potential equations for the Earth and for the Moon on these same pages.

CALCULATIONS

Calculations of the frequency shift and resulting time difference of a spacecraft clock (clock A) versus an Earth-based clock (clock B) were made by equations (4) and (5), respectively, where the inertial frame of reference is a nonrotating Cartesian coordinate system with origin at the center of mass of the Earth. The spacecraft clock was assumed to be on the command module (CM) of the Apollo 12 and Apollo 13 spacecraft and the Earth-based clock was assumed to be at NASA's Network Test and Training Facility (NTTF) (38°59'56.7" N, 76°50'22.7" W, 52 m above the Earth model ellipsoid). To an accuracy of 10^{-14}, the frequency of a clock at NTTF is the same as the frequency of an identical clock anywhere on the Earth model ellipsoid. Hence the results presented in the tables and figures in this report are typical of the results expected for a comparison of a spacecraft clock to

Table 1.—Frequency and Time Corrections for a Clock on Apollo 12 for Selected Values of Ground Elapsed Time (GET)

Event	GET, s	Distance of CM from center of Earth r_E, m	Distance of CM from center of Moon r_M, m	Speed of CM v, m/s	Frequency shift of CM standard versus Earth standard, 10^{-10}					Time correction (advance) of CM standard versus Earth standard, μs
					Due to gravity of–			Due to velocity	Total shift	
					Sun	Earth	Moon			
Begin TLC	10 380.0	6 724 000	370 883 000	10 805	-0.0061	0.3633	-0.0000	-6.4873	-6.1301	0.0
TLC	247 080.0	347 003 000	63 883 000	674	.0822	6.8248	-.0071	-.0180	6.8819	153.797
In orbit around Moon	425 580.0	378 498 000	1 846 000	2 660	.1570	6.8354	-.2941	-.3865	6.3120	271.328
In orbit around Moon	514 380.0	385 457 000	1 851 000	613	.2012	6.8376	-.2934	-.0136	6.7318	329.228
TEC	671 280.0	346 213 000	64 183 000	905	.2099	6.8245	-.0071	-.0383	6.9890	434.334
Reentry	879 769.6	6 464 000	401 185 000	11 033	-.0056	.0975	.0000	-6.7650	-6.6731	570.430
Splashdown	880 557.6	6 377 000	402 727 000	[a]442	-.0063	.0032	.0000	-.0036	-.0067	570.345

[a]This speed is due mainly to the rotational speed of the Earth, not the slight downward speed of the CM.

Table 2.—Frequency and Time Corrections for a Clock on Apollo 13 for Selected Values of GET

Event	GET, s	Distance of CM from center of Earth r_E, m	Distance of CM from center of Moon r_M, m	Speed of CM v, m/s	Frequency shift of CM standard versus Earth standard, 10^{-10}					Time correction (advance) of CM standard versus Earth standard, μs
					Due to gravity of—			Due to velocity	Total shift	
					Sun	Earth	Moon			
Begin TLC	11 197.0	13 361 000	387 728 000	7 631	−0.0065	3.6375	−0.0000	−3.2327	0.3983	0.0
Oxygen tank explosion	201 420.0	336 373 000	91 536 000	993	.0599	6.8208	−.0046	−.0475	6.8286	122.841
TLC	226 620.0	360 077 000	63 750 000	930	.0637	6.8295	−.0072	−.0408	6.8452	140.074
Closest approach to Moon	278 470.0	406 425 000	2 002 000	1 481	.0702	6.8435	−.2715	−.1148	6.5274	175.499
TEC	325 020.0	355 207 000	63 771 000	1 170	.0573	6.8278	−.0072	− 0689	6.8090	207.174
Begin reentry	513 645.7	6 495 000	404 547 000	11 037	.0045	.1325	.0000	−6.7696	−6 6326	327.559

any clock near sea level on the Earth. The Earth model ellipsoid referred to has equatorial radius R_E and polar radius $(1 - 1/298.3)R_E$. Tables 1 and 2 give frequency and time differences at selected times during translunar coast (TLC), near the Moon, during trans-Earth coast (TEC), and during reentry for the flights of Apollo 12 and Apollo 13, respectively. Figures 1 and 2 are the plots for Apollo 12 of the frequency shift of the CM standard versus the Earth standard and the time advance of the CM standard relative to the Earth standard. Figures 3 and 4 are the same plots for Apollo 13.

From the columns in tables 1 and 2 giving the frequency shift of the spacecraft standard versus the Earth standard, it is clear that the main factor in the blue shift of the spacecraft clock with respect to the Earth clock is that the spacecraft standard is far away from the Earth's relatively strong gravitational field. From table 1, we see that the total time advance of a hypothetical standard on the CM of Apollo 12 versus an Earth standard accumulated during TLC, lunar orbit, TEC, and reentry (up to splashdown) is 570.3 μs. From table 2, the time advance of a clock on Apollo 13 accumulated from TLC up to reentry is 327.6 μs.

ERROR ANALYSIS

The errors in the calculation of these numbers are (1) error due to the use of the approximation (3a) for $d\tau_A/d\tau_B$; (2) error in equation (6); (3) errors in the raw data, particularly the ranging and velocity data; (4) integration errors in equation (5) due to integrating over GET rather than coordinate time t and calculating the integral by the trapezoidal rule; and (5) roundoff errors in the values 570.3 and 327.6 μs. The error bounds which will now be calculated are not optimum bounds. They are, however, sufficiently sharp for our calculations.

The error due to the use of expression (3a) to approximate $d\tau_A/d\tau_B$ can be estimated by means of the Taylor-series remainder terms. It is bounded by 7.0×10^{-19} (resulting in a time-difference
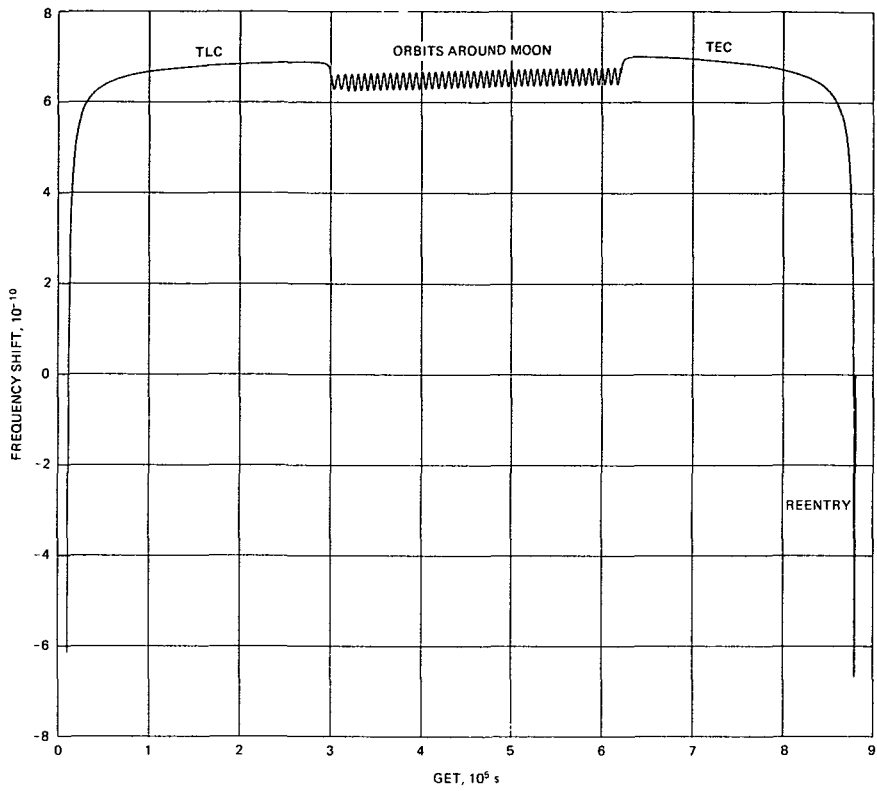
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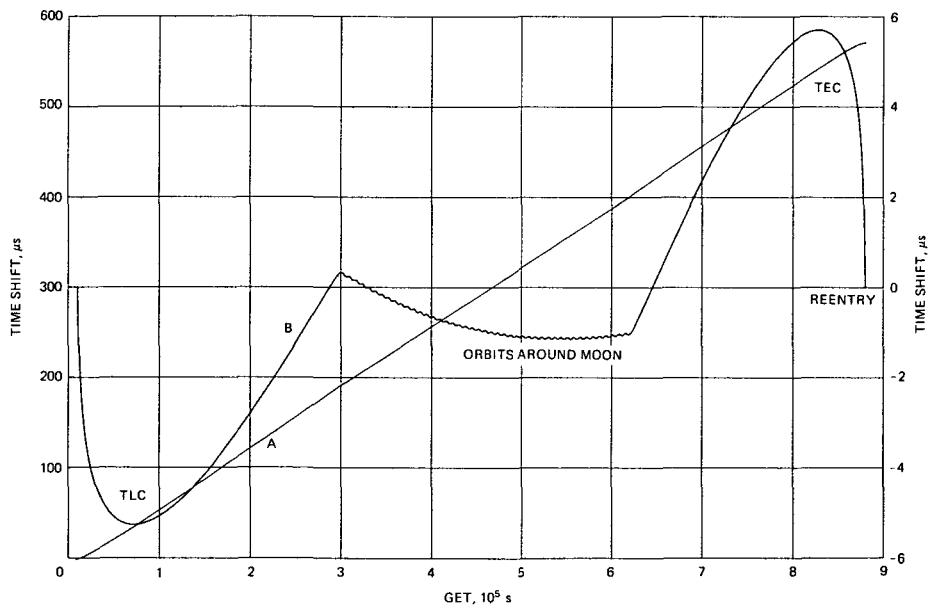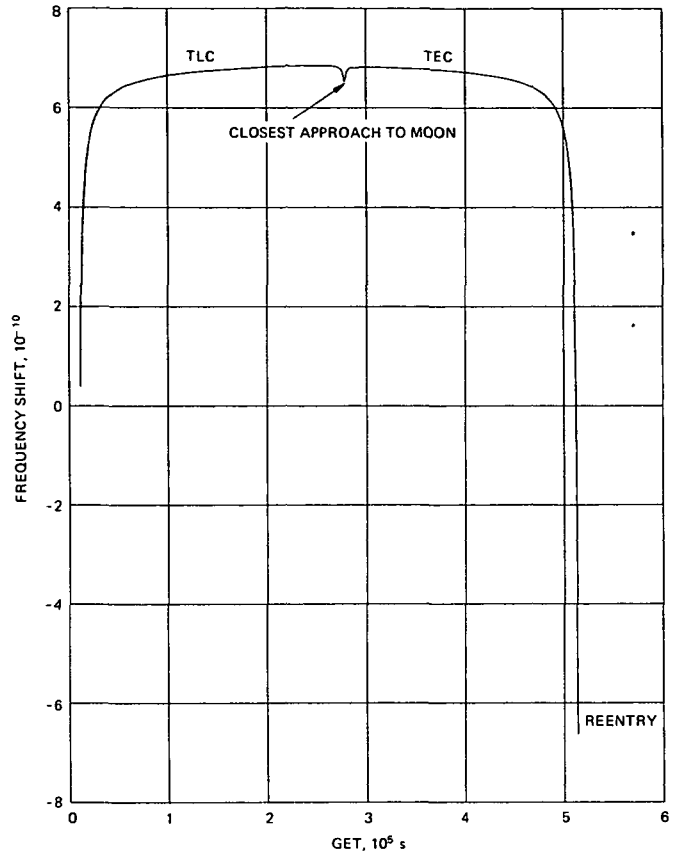Figure 1.—Frequency shift of clock on Apollo 12 CM versus Earth clock.

Figure 2.—Time shift of clock on Apollo 12 CM versus Earth clock (curve A, left-hand time shift scale); the results of subtracting out the average offset frequency (6.5544×10^{-10}) are given in curve B (using the right-hand time shift scale).

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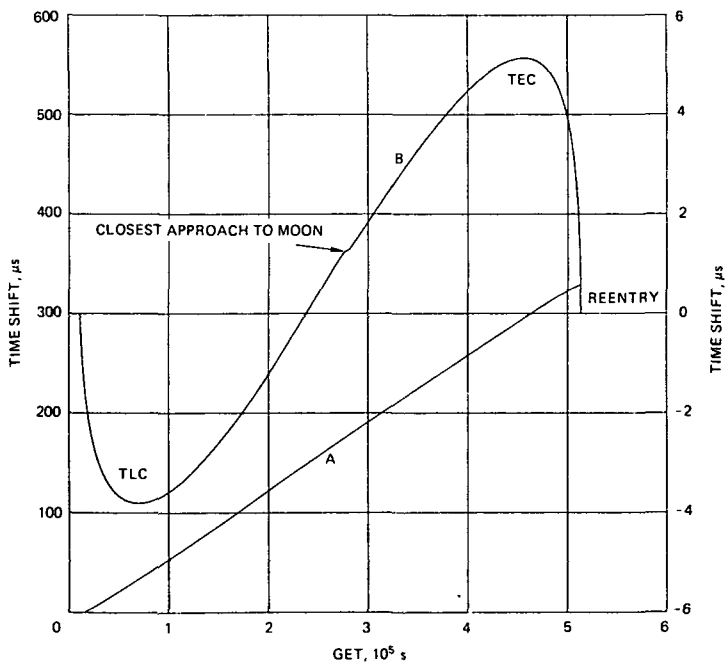Figure 3.—Frequency shift of clock on Apollo 13 CM versus Earth clock.

Figure 4.—Time shift of clock on Apollo 13 CM versus Earth clock (curve A, left-hand time shift scale); the results of subtracting out the average offset frequency (6.5193×10^{-10}) are given in curve B (using the right-hand time shift scale).

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error of less than 1 ps for both Apollo 12 and Apollo 13). The error resulting from errors in equation (6) can be bounded by using the error bounds on the constants given after equation (6).

Although error estimates for the internal consistency of the tracking and ephemeris data used in the calculations are available (ref. 5), no absolute error bounds for the range and velocity data are available. However, the following error bounds, based on the error estimates for the internal consistency of the tracking and ephemeris data will be assumed. The ranging errors of both the CM standard and the Earth standard to the center of the Sun are bounded by 200 km. The ranging error of the CM standard to the center of the Earth is bounded by 2 km when the range is less than 100 000 km and by 20 km when the range is greater than 100 000 km. The increased error bound for ranges greater than 100 000 km is due to large ranging errors near the Earth-Moon interface. The ranging error of the Earth standard to the center of the Earth is bounded by 10 m. The ranging error of the CM standard to the center of the Moon is bounded by 2 km when the range is less than 10 000 km and by 20 km when the range is greater than 10 000 km. Again, the increased error bound for larger ranges is to account for large ranging errors near the Earth-Moon interface. The ranging error of the Earth standard to the center of the Moon is bounded by 2 km. The velocity error of the CM standard is bounded by 2 m/s and the velocity error of the Earth standard is bounded by 10^{-3} m/s.

Let $f(t)$ denote the integrand in equation (5). The error in integrating $f(t)$ over GET (τ_B) rather than over coordinate time t is

$$\left| \int_{\tau_B(t_1)}^{\tau_B(t_2)} f[t(\tau_B)] \, d\tau_B - \int_{t_1}^{t_2} f(t) \, dt \right| \tag{7}$$

and is bounded by

$$\left| (t_2 - t_1) - [\tau_B(t_2) - \tau_B(t_1)] \right| \max_{t_1 \leqslant t \leqslant t_2} \left| f(t) \right| \tag{8}$$

which is less than 1 ps for both Apollo 12 and Apollo 13. For our data $f(t)$ is piecewise twice continuously differentiable. Then, if $n + 1$ represents the number of equispaced points on a twice continuously differentiable segment of the curve $f(t)$ from t_3 to t_4, which are used to calculate

$$\int_{t_3}^{t_4} f(t) \, dt$$

by the trapezoidal rule, the error in the numerically calculated value of the integral is bounded by

$$\left[\frac{(t_4 - t_3)^3}{12n^2} \right] \max_{t_3 \leqslant t \leqslant t_4} \left| f''(t) \right| \tag{9}$$

Summing these error bounds over all segments of each flight gives an error bound for the final time corrections.

Table 3 gives the magnitudes of the error bounds for the final time corrections listed in tables 1 and 2. All error bounds in table 3 have been rounded upward to the nearest nanosecond except those less than 0.1 ns, which are entered as "negligible." As a result, the relativistic blue shift from

Table 3.—Error Bounds for Final Time Corrections

Error source	Error bound, μs	
	Apollo 12	Apollo 13
Approximation (3a)	Negligible	Negligible
Equation (6)	0.015	0.007
CM standard.		
Ranging to Sun	0.012	0.007
Ranging to Earth	0.009	0.006
Ranging to Moon	0.017	0.006
Velocity	0.065	0.039
Earth standard:		
Ranging to Sun	0.012	0.007
Ranging to Earth	0.001	0.001
Ranging to Moon	Negligible	Negligible
Velocity	Negligible	Negligible
Integration over GET instead of coordinate time	Negligible	Negligible
Trapezoidal integration error	0.084	0.038
Roundoff in final value	0.045	0.041
Total	0.260	0.152

10 380.0 s GET to 880 557.6 s GET of a standard on Apollo 12 versus an Earth standard is 570.3 ± 0.3 μs and the relativistic blue shift from 11 197.0 s GET to 513 645.7 s GET of a standard on Apollo 13 versus an Earth standard is 327.6 ± 0.2 μs.

It remains now only to indicate the precision to which an actual experiment could measure these relativistic time corrections. The only atomic frequency standards of sufficiently small size and weight to be carried on Apollo missions are cesium and rubidium standards. For the intervals of time used to make the theoretical calculations for both flights, the standard deviation of the frequency[2] of a cesium standard is bounded by 5.0 X 10^{-13}, and that of a rubidium standard is bounded by 2.0 X 10^{-12}. Multiplying these standard deviations by the duration of the flights and rounding them off to the nearest 0.1 μs yields the experimental error estimates of 0.5 μs (cesium) and 1.8 μs (rubidium) for Apollo 12 and 0.3 μs (cesium) and 1.1 μs (rubidium) for Apollo 13. Hence, if an experiment were performed, one could expect a fractional statistical error of 0.1 percent with a cesium standard and 0.33 percent with a rubidium standard.

ACKNOWLEDGMENTS

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[2]Operational conditions such as those on a space flight are considered (See J. E. Lavery· *Operational Frequency Stability of Rubidium and Cesium Frequency Standards.* NASA Technical Note, to be published.) It is assumed, however, that the spacecraft standard is sufficiently rugged to be operationally unaffected by acceleration and shock.

Goddard Space Flight Center and Donald J. Incerto of Manned Spacecraft Center for their help in obtaining the trajectory tapes for Apollo 12 and Apollo 13.

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where $g_{\alpha\beta}$ are the components of the metric tensor involving the spatial coordinates x_α, the subscript 4 labels the time coordinate, and dt is the interval of coordinate time. (Coordinate time is the time

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