Modulational Instability of Whistlers in Cold Plasmas

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ABSTRACT

The paper obtains the modulational stability spectrum of whistlers in cold plasmas taking into consideration both ion motion and relativistic effects. The unstable band is contiguous to $\Omega_e/4$ and, depending on the plasma density, lies above or below that frequency (Ω_e is the electron cyclotron frequency of the static magnetic field). The relevance of the instability to whistlers in the magnetosphere is discussed.

1. INTRODUCTION

Propagation of finite amplitude waves in nonlinear dispersive media gives rise to self-focusing [Akhmanov et al., 1966] and self-trapping [Chiao et al., 1964] effects caused by the dependence of the dielectric properties on the wave fields. The application of these self-action effects to plasma waves has been primarily [Taniuti and Washimi, 1968; Tam, 1969; Taniuti and Washimi, 1969; Litvak, 1970; Hasegawa, 1970a; Hasegawa, 1970b], but not exclusively [Kakutani et al., 1967; Dysthe, 1968; Tang and Sivasubramanian, 1971; Dewar et al., 1972] concentrated on cyclotron waves.

In this paper we study the modulational instability (self-trapping) of electron cyclotron waves (whistler branch) in cold dense plasmas. Previous work on this problem either neglected ion motion [Tam, 1969; Tang and Sivasubramanian, 1971], or disregarded relativistic effects [Taniuti and Washimi, 1968; Tam, 1969; Hasegawa, 1970a]. It will be demonstrated that the simultaneous consideration of these two factors alters the modulational stability spectrum of the wave trains in a fundamental way. Since the self-action effects in electron cyclotron waves have been contemplated in the context of the magnetospheric [Litvak, 1970; Brinca, 1972] and solar wind [Hasegawa, 1972] plasmas, the determination of the modulational stability spectrum may have more than academic interest.

In Section 2 we derive the equations obeyed by the (complex) amplitude of the wave train envelope, assuming that the (linear and nonlinear) dispersive properties of the medium are known. These equations are used to establish the conditions required for the occurrence of modulational instability. Section 3 applies the results to whistlers. Section 4 discusses the (possible) occurrence of the modulational instability in magnetospheric whistlers, speculating on the relevance of this instability to some observed 'pulsations' [Bell and Helliwell, 1971] when a dilute energetic electron population permeates the cold plasma.

2. THE MODULATIONAL INSTABILITY

We consider one-dimensional propagation of a plasma mode in a nonlinear dispersive medium that can be characterized, to lowest order in the wave amplitude a, by the dispersion relation ($\beta > 0$)

$$\omega = \Omega + \mu\, a^{\beta} , \tag{1}$$

where $\omega = \Omega(k)$ is the linear dispersion relation and

$$\mu(k) = \left(\frac{\partial\omega}{\partial a^{\beta}}\right)_{a=0} \tag{2}$$

characterizes the frequency shift caused by the dependence of the average properties of the medium on the small, but finite, wave amplitude.

We assume the existence of an equilibrium state consisting of the propagation along the z-axis of a wave with amplitude a_0, frequency ω_0 and wavenumber k_0, satisfying

$$\omega_0 = \Omega_0 + \mu_0\, a_0^{\beta} , \tag{3}$$

with $\Omega_0 = \Omega(k_0)$ and $\mu_0 = \mu(k_0)$. To study the stability of this equilibrium with respect to a modulational perturbation, we derive the equation satisfied by the envelope (complex) amplitude of the wave train.

The perturbed wave train may be represented at $t = 0$ by

$$\psi(z,0) = \int_{-\infty}^{\infty} dk\; \Phi(k)\, \exp(-ikz) , \tag{4}$$

where $\Phi(k)$, the spatial Fourier transform of the initial wave, is concentrated about k_0. The subsequent temporal evolution of the wave train is then obtained from

$$\psi(z,t) = \int_{-\infty}^{\infty} dk\; \Phi(k)\, \exp\left[i(\omega t-kz)\right] , \tag{5}$$

with $\omega = \omega(k)$ given by (1). Using the first three terms of the Taylor expansion of $\Omega(k)$ about k_0, we can write (5) as

$$\psi(z,t) = \varphi(z,t) \exp\left[i(\omega_0 t - k_0 z)\right] , \tag{6}$$

where the envelope (complex) amplitude is

$$\begin{aligned} \varphi(z,t) &= a(z,t) \exp i\theta(z,t) \\ &= \int_{-\infty}^{\infty} d\kappa\, \phi(\kappa+k_0) \exp i\left\{\left[\kappa v_{g0} + \frac{1}{2}\kappa^2 v'_{g0} + \mu_0(a^\beta - a_0^\beta)\right] t - \kappa z\right\} , \end{aligned} \tag{7}$$

and

$$v_{g0} = \frac{\partial\Omega(k_0)}{\partial k} , \qquad v'_{g0} = \frac{\partial^2\Omega(k_0)}{\partial k^2} . \tag{8}$$

Differentiation of φ with respect to t and z shows that the envelope amplitude satisfies the equation

$$- i\left(\frac{\partial\varphi}{\partial t} + v_{g0}\frac{\partial\varphi}{\partial z}\right) + \frac{1}{2} v'_{g0} \frac{\partial^2\varphi}{\partial z^2} - \mu_0\, (a^\beta - a_0^\beta)\varphi = 0 , \tag{9}$$

or, going to the packet wave frame with the introduction of new variables, $\xi = z - v_{g0} t$ and $\tau = t$,

$$- i\frac{\partial\varphi}{\partial\tau} + \frac{1}{2} v'_{g0} \frac{\partial^2\varphi}{\partial\xi^2} - \mu_0(a^\beta - a_0^\beta)\, \varphi = 0 . \tag{10}$$

This equation was previously derived through other methods, and for $\beta = 2$, by <u>Karpman and Krushkal'</u> [1969], <u>Taniuti and Yajima</u> [1969], and <u>Dysthe</u> [1970]. It is identical to the Schrödinger equation with a nonlinear potential term (note that $a = |\varphi|$) and, as assumed at the outset, admits $\varphi = a_0$ as a solution. Separating real and imaginary parts, we obtain

$$
-\frac{\partial a^\beta}{\partial \tau} + v'_{g0} \frac{\partial a^\beta}{\partial \xi} \frac{\partial \theta}{\partial \xi} + v'_{g0} \frac{\beta a^\beta}{2} \frac{\partial^2 \theta}{\partial \xi^2} = 0 \ ,
$$

$$
a \frac{\partial \theta}{\partial \tau} + \frac{1}{2} v'_{g0} \frac{\partial^2 a}{\partial \xi^2} - \frac{1}{2} v'_{g0} a \left(\frac{\partial \theta}{\partial \xi}\right)^2 - \mu_0 a(a^\beta - a_0^\beta) = 0 \ . \tag{11}
$$

A modulational perturbation about the equilibrium $a = a_0$ and $\theta = 0$, of the form

$$
a^\beta - a_0^\beta = a_1 \exp i(\overline{\omega}\tau - \overline{k}\xi) \qquad a_1 \ll a_0^\beta \ ,
$$

$$
\theta = \theta_1 \exp i(\overline{\omega}\tau - \overline{k}\xi) \qquad \theta_1 \ll 1 \ , \tag{12}
$$

yields the linear dispersion relation

$$
\overline{\omega}^2 = (\overline{k}\, v'_{g0}/2)^2 \ (\overline{k}^2 - \overline{k}_L^2) \ ,
$$

$$
\overline{k}_L^2 = -2\beta a_0^\beta \mu_0/v'_{g0} \ . \tag{13}
$$

When $\mu_0/v'_{g0} < 0$, i.e., when the potential in the nonlinear Schrödinger equation becomes attractive, we note that the perturbations will be linearly unstable ($\overline{\omega}^2 < 0$) if the modulation wave number satisfies $\overline{k}^2 < \overline{k}_L^2$. The maximal temporal (linear) growth rate is

$$
\overline{\omega}_{iM} = -\frac{\beta}{2} |\mu_0| a_0^\beta \ , \tag{14}
$$

and occurs for

$$
\overline{k} = \overline{k}_M = (\overline{k}_L^2/2)^{1/2} \ . \tag{15}
$$

We thus expect that wave trains in media satisfying $\mu_0/v'_{g0} < 0$ will be unstable when modulated by perturbations of sufficiently large wavelength. The initial (linear) evolution of the instability will tend to increase the depth of modulation, but the subsequent (nonlinear) behavior of the unstable wave train must be followed numerically. This has been done in a few cases for $\beta = 2$ [Karpman and Krushkal', 1969; Hasegawa, 1970a] , and sometimes indicates the breaking up of the original wave into a number of solitary waves (solitons) of stationary (envelope) amplitude in the wave train frame. With respect to this evolution, it is interesting to note that the (soliton-type for the amplitude) expressions

$$a^{\beta} = A^2 \operatorname{sech}^2 \left\{ \beta A \left[- \frac{\mu_0}{(\beta+2) v'_{g0}} \right]^{1/2} (\xi - \xi_0) \right\} ,$$

$$\theta = \mu_0 \left[2A^2/(\beta+2) - a_0^{\beta} \right] \tau + \theta_0 \quad , \tag{16}$$

where A, θ_0 and ξ_0 are real constants, represent exact solutions of (10) or (11). (These solutions were obtained by Chiao et al. [1964], and Karpman and Krushkal' [1969] for the case $\beta = 2$ but, as stressed in Section 4 below, observable modulational instabilities in magnetospheric whistlers will probably occur for $\beta = 1/2$.)

3. THE WHISTLER CASE

For one-dimensional propagation along z in a two-component cold magnetoplasma with constant magnetic field $\underset{\sim}{B}_0 = B_0 \hat{\underset{\sim}{z}}$, the equations of motion, Maxwell and continuity determine the evolution of the system:

$$m_j\left(\frac{\partial}{\partial t} + v_{\parallel j}\frac{\partial}{\partial z}\right)\left(\gamma_j \underset{\sim}{v}_j\right) = q_j \underset{\sim}{E} + q_j \underset{\sim}{v}_j \times (\underset{\sim}{B}_0 + \underset{\sim}{B}) ,$$

$$\nabla \times \underset{\sim}{B} = \frac{1}{\epsilon_0 c^2}\sum_{j=i,e} q_j\, n_j\, \underset{\sim}{v}_j + \frac{1}{c^2}\frac{\partial \underset{\sim}{E}}{\partial t} , \tag{17}$$

$$\nabla \times \underset{\sim}{E} = -\frac{\partial \underset{\sim}{B}}{\partial t} ,$$

$$\frac{\partial n_j}{\partial t} + \frac{\partial}{\partial z}(n_j\, v_{\parallel j}) = 0 .$$

The wave fields are $\underset{\sim}{B}$ and $\underset{\sim}{E}$; subscript j enumerates the electron and ion variables; ϵ_0 and c represent the free space permittivity and speed of light; we use $q_e = -e$, $v_\parallel = v_z$, and $\gamma_j = (1 - v_j^2/c^2)^{-1/2}$.

To proceed, we make several simplifications. First, the displacement current is neglected and charge neutrality is assumed. This amounts to dropping the last term of the second equation in (17), using $n_i = n_e = n$ and (continuity equation) $v_{\parallel i} = v_{\parallel e} = v_\parallel$. The range of validity of these approximations was analyzed by Kakutani et al. [1967] and includes our domain of interest. The displacement current can be disregarded when (dense plasma) $\omega_{pe}^2 \gg \Omega_e^2$, and charge neutrality is a good approximation for $v_A \ll c$; the electron plasma and cyclotron frequencies, ω_{pe} and Ω_e, and the Alfvén velocity, v_A, are defined by

$$\omega_{pe}^2 = \frac{n_0 e^2}{\epsilon_0 m_e} \quad , \quad \Omega_{e,i} = \frac{|q_{e,i}| B_0}{m_{e,i}} \; , \; v_A^2 = \frac{\epsilon_0 B_0^2 c^2}{n_0 m_i} \quad , \tag{18}$$

where n_0 is the unperturbed number density. The expression for v_A^2 makes use of $m_i \gg m_e$; this inequality will be utilized below and, when combined with our interest in right-hand circularly polarized waves of frequencies ω such that $\Omega_e > \omega \gg \Omega_i$, justifies disregarding the relativistic ion mass correction, i.e., we will make $\gamma_i = 1$.

Utilization of these simplifications in (17), and elimination of the wave electric field and the ion velocity, leads to the following system of equations

$$\frac{dH}{dt} + H \frac{\partial v_{\parallel}}{\partial z} - i \frac{v_a}{\Omega_e} \frac{\partial}{\partial z}\left[\frac{d}{dt}(\gamma_e u)\right] - v_A \frac{\partial u}{\partial z} = 0 \; ,$$

$$\frac{du}{dt} - v_A \frac{n_0}{n} \frac{\partial H}{\partial z} + i \frac{v_A}{\Omega_i} \frac{d}{dt}\left(\frac{n_0}{n} \frac{\partial H}{\partial z}\right) = 0 \; ,$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial z}(n \, v_{\parallel}) = 0 \; , \tag{19}$$

$$\frac{dv_{\parallel}}{dt} + \frac{v_A^2}{2} \frac{n_0}{n} \frac{\partial |H|^2}{\partial z} = 0 \; ,$$

with

$$H = (B_x + iB_y)/B_0 \quad , \; u = (v_x + i v_y)/v_A \quad , \; \frac{d}{dt} = \frac{\partial}{\partial t} + v_{\parallel} \frac{\partial}{\partial z} \; .$$

A possible solution of this system is

$$\begin{pmatrix} v_{\parallel} \\ n \end{pmatrix} = \begin{pmatrix} 0 \\ n_0 \end{pmatrix} \quad , \quad \begin{pmatrix} H \\ u \end{pmatrix} = \begin{pmatrix} H_0 \\ u_0 \end{pmatrix} \exp i(\omega_0 t - k_0 z) \quad , \tag{20}$$

provided ω_0 and k_0 satisfy the (relativistic) dispersion relation

$$\left(\frac{\omega_0}{k_0 v_A}\right)^2 = \left(1 + \frac{\omega_0}{\Omega_i}\right)\left(1 - \frac{\gamma_e \omega_0}{\Omega_e}\right) \tag{21}$$

with $\gamma_e = \left[1-(v_A|u_0|/c)^2\right]^{-1/2}$. Monochromatic whistlers of arbitrary amplitude are exact solutions of (19) if their frequency and wavenumber satisfy (21). We have thus found an equilibrium state similar to the one used in Section 2 with $\beta = 2$. In particular, expanding γ_e, $\gamma_e \approx 1 + |u_0|^2 v_A^2/2c^2$, noting from (19) that $|u_0|^2 = (k_0 v_A/\omega_0)^2 (1+\omega_0/\Omega_i)^2 |H_0|^2$ and considering whistlers with frequencies $\omega_0 \gg \Omega_i$, we can write the relativistic dispersion relation (21) as

$$\omega_0 = \Omega(k_0) - \frac{\omega_0^3}{2\omega_{pe}^2} \cdot \frac{|H_0|^2}{(1-\omega_0/\Omega_e)^2} \quad , \tag{21}$$

where the linear dispersion relation ,

$$\omega = \Omega(k) = \frac{k^2 v_A^2}{\Omega_i}\left(1 - \frac{\omega}{\Omega_e}\right) = \frac{c^2 k^2}{\omega_{pe}^2}(\Omega_e - \omega) \quad , \tag{22}$$

defines

$$v_{g0} = 2\frac{\omega_0}{k_0}\left(1 - \frac{\omega_0}{\Omega_e}\right), \quad v'_{g0} = 2\frac{\omega_0}{k_0^2}\left(1 - 4\frac{\omega_0}{\Omega_e}\right)\left(1 - \frac{\omega_0}{\Omega_e}\right). \tag{23}$$

The relativistic frequency shift is characterized by

$$\mu_{0r} = \frac{\partial\omega}{\partial|H_0|^2} = -\frac{\omega_0^3}{2\omega_{pe}^2}(1 - \omega_0/\Omega_e)^{-2} \quad , \tag{24}$$

showing that, if no other nonlinear effects were involved, the whistler mode would be unstable to modulational perturbations $(v'_{g0}/\mu_0 < 0)$ for frequencies $\omega < \Omega_e/4$. This result is at variance with the conclusions of Tang and Sivasubramanian [1971] who did not take into account the dependence of sgn v'_{g0} on ω/Ω_e, and derived a relativistic frequency shift that seems to be incorrect. [For dense plasmas, $\omega_{pe}^2 \gg \Omega_e^2$, it is independent of $(1-\omega_0/\Omega_e)$, ignoring the influence of the cyclotron resonance on the electron velocity, $|u_0| \propto (1-\omega_0/\Omega_e)^{-1}$.]

However, the relativistic correction just considered is not the only nonlinear effect contributing to the total frequency shift experienced by the whistler train of finite amplitude. Although (19) admits (20) as exact solutions subject to the dispersion relation (21), we note [Taniuti and Washimi, 1968] that a superposition of two (or more) waves satisfying (20) [with different frequencies and wavenumbers obeying (21)] is not a solution of (19). We are thus led to look for wave train solutions of the form $\varphi(z,t) \exp i(\omega_0 t - k_0 z)$, where φ is a slowly varying function of time and space. Following the method of Taniuti and Yajima [1969], it is found in the nonrelativistic case ($\gamma_e = \gamma_i = 1$) that the (nonlinear) perturbations in the number density n, and the creation of a nonzero $v_\parallel$ give rise to a further frequency shift characterized by [Taniuti and Washimi, 1968]

$$\mu_{0i} = \frac{\partial\omega}{\partial|H_0|^2} = \frac{k_0 v_A^2}{4\, v_{g0}} = \frac{\Omega_i}{8}\,(1 - \omega_0/\Omega_e)^{-2} \quad . \tag{25}$$

This component of the frequency shift, in contrast to the relativistic one, is positive and becomes zero when the ion motion is neglected ($m_i \to \infty$, $\Omega_i \to 0$). Hence, previous analyses [Tam, 1969] neglecting relativistic effects and ion motion were led to attribute a zero frequency shift, and modulational stability to whistlers propagating along $\mathbf{B}_0$. When ion motion is considered, but relativistic effects are ignored [Taniuti and Washimi, 1968; Hasegawa, 1970a] the resulting positive frequency shift leads to the conclusion that modulational instability occurs for whistler frequencies ω satisfying $\Omega_e/4 < \omega < \Omega_e$. The actual modulational stability spectrum of the whistler train is obtained by combining these two effects (ion motion and relativistic dynamics). The total frequency shift is then characterized by

$$\mu_0 = \mu_{0r} + \mu_{0i} = \frac{1}{2}\,(1 - \omega_0/\Omega_e)^{-2}(\Omega_i/4 - \omega_0^3/\omega_p^2) \ . \tag{26}$$

Recalling the expression given in (23) for v'_{g0} , and defining

$$\bar{\omega}_p^2 = \frac{\Omega_e^3}{16\,\Omega_i} \ , \qquad \omega^* = \left(\frac{\Omega_i\,\omega_{pe}^2}{4}\right)^{1/3} < 1 \ , \tag{27}$$

we find the following modulational stability spectrum for whistler trains of frequency $\omega(\gg \Omega_i)$ in cold dense plasmas $(\omega_p^2 \gg \Omega_e^2)$. The unstable band in media with $\omega_{pe} > \bar{\omega}_p$ is given by $\omega^* > \omega > \Omega_e/4$, whereas for magnetoplasmas having $\omega_{pe} < \bar{\omega}_p$, instability arises in the band $\Omega_e/4 > \omega > \omega^*$. When $\omega_{pe} = \bar{\omega}_p$, no modulational instability occurs.

The importance of considering simultaneously the influence of ion motion and relativistic dynamics when studying the whistler modulational instability is stressed in Figure 1, where the stability spectra obtained using simplifications are contrasted with the actual spectrum.

4. DISCUSSION

The possible observation of the modulational instability in magnetospheric whistlers requires the existence of an unstable band and a growth rate large enough to ensure the development of initial (large wavelength) amplitude perturbations into 'pulsations'. (The instability is most likely to occur in the equatorial portion of the whistler path, due to both homogeneity and slower wave velocity.) Since $\overline{\omega}_p = 11.2\ \Omega_e$ for an electron-proton plasma, most magnetospheric L-shells have $\omega_{pe} < \overline{\omega}_p$: the narrow unstable band will be $\Omega_e/4 > \omega > \omega^*$. Denoting by $\delta\omega (= \mu_0 a_0^\beta)$ the frequency shift experienced by the whistler in the initial value problem, (in CW key-down transmission, the frequency, rather than the wave number, is fixed and thus the shift occurs in k,) we find, from (14) that $|\overline{\omega}_{iM}| = |\delta\omega|\beta/2$. In the cold plasma case μ_0 is defined by (26), and $a_0^\beta = (B/B_0)^2$. Using $\omega_0/\Omega_e \sim 0.25$ and $\omega_p^2/\Omega_e^2 \sim 50$, we obtain $|\delta\omega| \sim 1.6 \times 10^{-4}\ \Omega_e\ (B/B_0)^2$. The growth rate, $|\overline{\omega}_{iM}| = |\delta\omega|$, is negligible since the amplitude of a 'high-field' whistler [Dysthe, 1971] is $B/B_0 \sim 10^{-4}$. It is reasonable to conclude that the whistler modulational instability due to the cold plasma (nonlinear and dispersive) properties may not be observed in the magnetosphere.

However, consideration of the dilute energetic electron population may strongly enhance the growth rate of the whistler modulational instability. Based on the similarity of the evolution of Landau and whistler waves in hot plasmas [Brinca, 1972], and recent results on the Landau modulational instability [Dewar et al., 1972], we expect the whistler frequency shift, $\delta\omega = \mu_0 a_0^\beta$, to have $\beta = 1/2$. The growth rate will now be proportional to $(B/B_0)^{1/2}$.

To facilitate the eventual interpretation of observed 'pulsations' in terms of the modulational instability, we establish a relation between measurable parameters and theoretical growth rates. Utilization of this result may rule out the modulational instability as a source of observed 'pulsations', but cannot ensure the occurrence of the instability. For this latter purpose, a detailed study of the whistler modulational stability in hot plasmas is required and is now under consideration by the author.

The measured spatial and temporal periodicities of the 'pulsations' define spatial and temporal frequencies which, if created by the modulational instability, should approximate $\bar{k}_M$ and $\bar{\omega}_{rM} (= \bar{k}_M v_{g0})$. Using (13)-(15) and (22), we find

$$|\bar{\omega}_{iM}| = \frac{\bar{\omega}_{rM}^2}{4\omega_0} \frac{|1-4\omega_0/\Omega_e|}{(1-\omega_0/\Omega_e)} . \tag{28}$$

As an example, the 'pulsations' reported by Bell and Helliwell [1971] had, near the equator, $\omega_0/\Omega_e \sim 0.5$, $\omega_0/2\pi \sim 15$ kHz and $\bar{\omega}_{rM}/2\pi \sim 8$ Hz, yielding $|\bar{\omega}_{iM}| \sim 1.3 \times 10^{-2}\ \sec^{-1}$. Noting that the time required to cross the (possibly modulational unstable) equatorial region is about ~ 0.3 sec, we conclude that this growth rate is too small to explain the observed pulsations. (Figure 1 shows the modulational stability spectrum for cold plasmas; when an energetic electron population is present, the stability spectrum depends on the characteristics of this population and may be unstable for $\omega_0/\Omega_e \sim 0.5$.)

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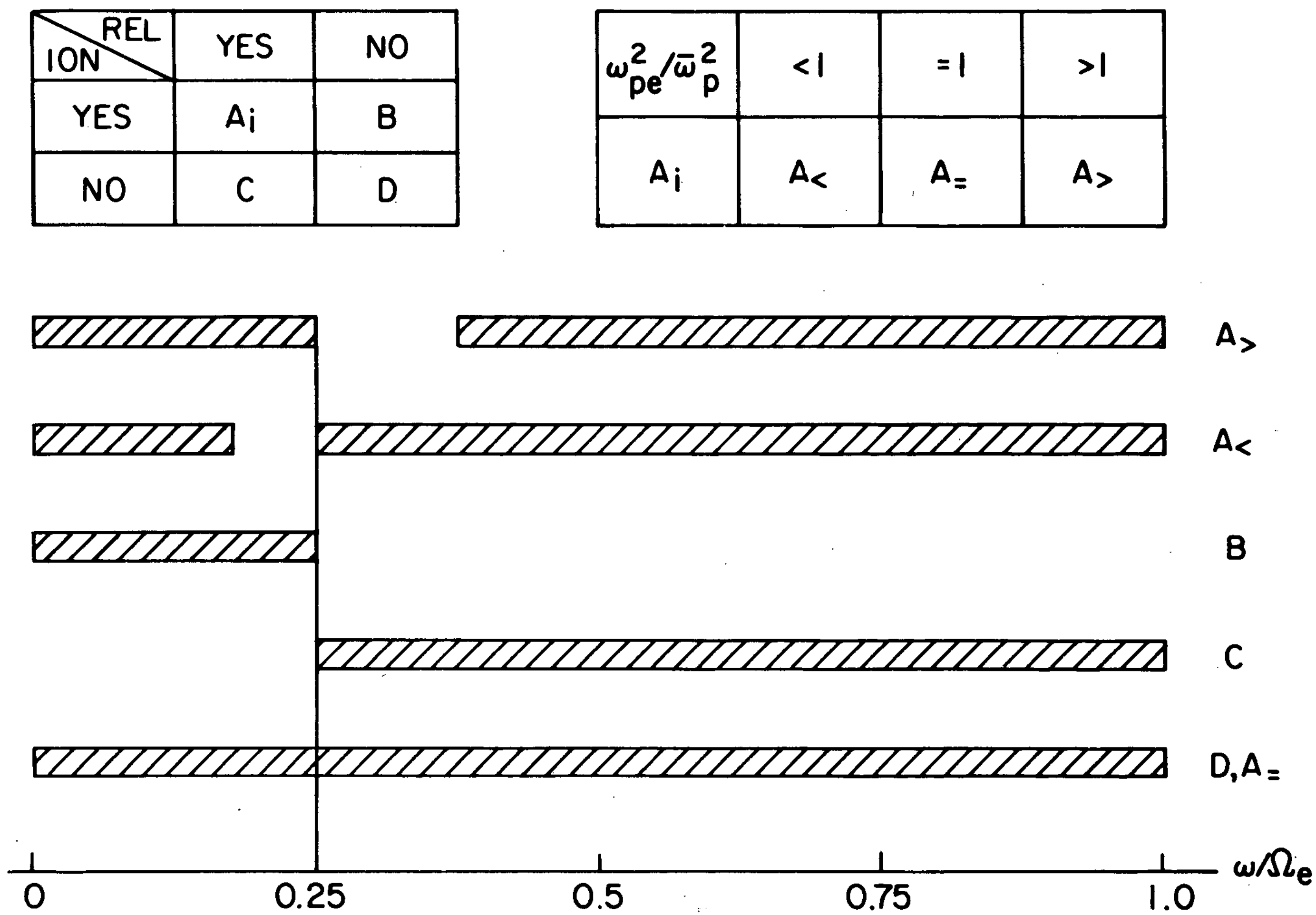

FIG. 1. Whistler modulational stability spectrum in cold plasmas (stable bands are hatched). (REL - Relativistic force equation used; ION - Ion motion considered.) $[\omega \gg \Omega_i\ ,\ \omega_{pe}^2 \gg \Omega_e^2]$.