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NOTE ON THE ORIGIN OF JUPITER'S MAGNETIC FIELD

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ABSTRACT

This paper examines the effect of the Coriolis force for the stability of the rotating hydromagnetic systems. It is shown that the effect of this force is to inhibit the onset of instability which leads to convective motion in these systems. Since it is thought of that this motion is closely related to the hydromagnetic dynamo process which generates magnetic fields of stars and planets, the result as mentioned above must be considered in the study of this process. A possibility of the dynamo process in Jupiter's interior is discussed by taking into account the effect of the Coriolis force.

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1. INTRODUCTION

At present, radio astronomical observations at microwave frequencies suggest that the planet Jupiter has a dipole type magnetic field (e.g., Berge, 1966; Branson, 1968). It is estimated that the axis of this dipole is tilted from the rotation axis by about 10 degrees and that the strength of the magnetic field is ~10 gauss at the equator of the planet's surface. Although Warwick (1967, 1970) proposed a different model of the magnetic field distribution based on his decametric radio observations for Jupiter, it seems certain that Jupiter has its own magnetic field, the origin of which is seated in the interior of the planet.

The physical state of the interior of Jupiter has been theoretically investigated by many authors (e.g., Wildt, 1961; Öpik, 1962; Peebles, 1964; Moroz, 1968). We, however, do not know as yet this state on the basis of direct observations as done for the earth. If the magnetic field of Jupiter is generated by the hydromagnetic dynamo process, this planet must have the conducting core inside, where there may exists convective motion.

It is believed currently that the magnetism of the earth and some magnetic stars like the sun are generated and maintained by the hydromagnetic self-exciting dynamo mechanism in their interiors (e.g., Bullard and Gellman, 1954; Herzenberg, 1958; Babcock, 1961; Parker, 1970). Furthermore, this mechanism is thought of as being closely related to the rotating motion of such planets and stars. This idea assumes that the convective motion inside of these objects is necessarily generated as a result of their rotating motion. We, however, do not know whether this assumption is automatically fulfilled in the case of the rotating stars and planets.

In this paper, we shall, therefore, investigate conditions for the onset of convective motion and then consider their relation to the origin of the magnetic field of stars and planets like Jupiter.

2. THE ORIGIN AND MAINTENANCE OF JUPITER'S MAGNETISM

Hydromagnetic equilibrium condition in the interior of Jupiter is expressed by

$$-\nabla P + \frac{1}{c} \vec{j} \times \vec{B} + \rho\vec{g} = \rho\vec{\Omega} \times (\vec{\Omega} \times \vec{r}), \tag{2-1}$$

$$\mathrm{rot}\ \vec{B} = \frac{4\pi}{c} \vec{j} \tag{2-2}$$

and

$$\mathrm{div}\ \vec{B} = 0, \tag{2-3}$$

where $\vec{B}, \vec{j}, P, \rho, \vec{g}, \vec{\Omega}$ and $\vec{r}$ are respectively the magnetic field, the electric current, the pressure, the mass density, the gravity force, the angular velocity and the position vector from the center of Jupiter. In writing these equations, we have assumed that, in the interior, there exists an electrically conducting core where magnetic fields are originated. In the above equations, the velocity of convective motions does not explicitly appear, but the electric current is directly connected with this velocity as follows:

$$\vec{j} = \sigma \left(\vec{E} + \frac{1}{c} \vec{v} \times \vec{B}\right), \tag{2-4}$$

where $\vec{v}$, σ and $\vec{E}$ are the velocity, the electrical conductivity and the electric field, respectively.

By using these four equations and the electromagnetic induction equation, we derive the well-known equation for magnetic dissipation as follows:

$$\frac{\partial \vec{B}}{\partial t} = \mathrm{rot}\ (\vec{v} \times \vec{B}) + \frac{c^2}{4\pi\sigma} \nabla^2 \vec{B} \tag{2-5}$$

where t is the time.

When we assume $\partial\vec{B}/\partial t = 0$ in this equation, we obtain the well-known dynamo equation which is currently used to study the hydromagnetic self-exciting dynamo process (e.g., Elsasser, 1946a,b, 1947, 1950; Cowling, 1957). In this case, we must remark that the velocity field as given by $\vec{v}$ plays an essential role for this process.

If we assume that $\vec{v} = 0$, the magnetic field necessarily decay due to the Joule heating because of finiteness of the electrical conductivity in the medium. The time for this decay is given by

$$\tau = \frac{4\pi\sigma}{c^2} \ell^2,$$

(e.g., Cowling, 1946, 1953), where ℓ is the characteristic length of the conducting core. In this case, this is expressed by the radius of this core in Jupiter's interior. For instance, $\ell \simeq \frac{1}{2} r_J$ (r_J: Jupiter's radius). When we assume that σ is numerically given as $\sim 10^{12}$ e.s.u., for instance, the above time is calculated as $\sim 10^6$ years. Even if we are likely to apply the idea of "fossil" magnetism to the case of Jupiter, the decay time just obtained is too short to explain the Jupiter magnetic field as currently estimated.

The property of solutions of equations, (2-1) - (2-3) was investigated by Ferraro (1954) and Roberts (1955) in greater detail. According to Ferraro (1954), these equations give a solution that magnetic field is of dipole type. In this case, he considered the effect of the centrifugal force which appears on the right hand side of eq. (2-2). Since the rotation of Jupiter cannot be neglected when we consider the equilibrium condition, his solution may have expressed a possible configuration of Jupiter's field in hydromagnetic equilibrium. As has been discussed, however, the magnetic field would freely decay with the time constant τ. Thus some amplification mechanism for the magnetic field must inevitably exist in order for the existence and maintenance of the magnetic field of Jupiter to be explained. This requirement would also be applied to many rotating magnetic stars like the sun and the earth.

3. NECESSITY OF DYNAMO PROCESSES

We have so far considered the case which there exists no convective motion, i.e., $\vec{v} = 0$. In reality, even if $\vec{v} \neq 0$ at first, the system under consideration becomes stable as far as this motion diminishes with time. Unless this motion is maintained or amplified, the hydromagnetic dynamo action cannot be considered as a possible source for the maintenance of the magnetism of Jupiter.

At present, it is thought of that the dynamo action is associated with the convective motion inside of Jupiter and other magnetic stars and planets. Thus important step is to examine the stability of equations (2-1) - (2-4) with respect to some perturbations which are able to produce convective motion.

We shall here examine this problem by using the method of the energy principle as developed by Bernstein et al.(1958). Since the present case includes the effect of the Coriolis force, the method of Bernstein et al. need be slightly modified (e.g., Steinitz, 1965; Sakurai, 1972). We shall now define a perturbed displacement vector by $\vec{\xi}$. Thus the perturbed velocity is given by

$$\vec{v} = \frac{\partial \vec{\xi}}{\partial t}.$$

By applying the method of Bernstein et al. (1958), we obtain the equation of motion as follows:

$$\rho \frac{\partial \vec{v}}{\partial t} + 2\rho\vec{\Omega} \times \vec{v} = \vec{F}(\vec{\xi}) \tag{3-1}$$

and

$$\vec{F}(\vec{\xi}) = \text{grad}\,[\gamma\ P\ \text{div}\ \vec{\xi} + (\vec{\xi}\cdot\text{grad})\ P] + \vec{j} \times \vec{Q}$$

$$-\vec{B} \times \text{rot}\ \vec{Q} + [\text{div}\ (\rho\vec{\xi})]\ \text{grad}\ \phi \tag{3-2}$$

where ϕ and γ are the gravitational potential including that of the centrifugal force and the ratio of two specific heats. Here $\vec{Q}$ is given by

$$\vec{Q}(\xi) = \text{rot}\ (\vec{\xi} \times \vec{B}). \tag{3-3}$$

It is known that the contribution of the centrifugal force to ϕ is generally negligibly small (e.g., Chandrasekhar, 1961). In the case which is rotating so fast, however, force $\vec{F}(\vec{\xi})$ necessarily becomes a function of Ω^2 as suggested by Steinitz (1965). In the case of Jupiter, this contribution is very small.

We here assume that $\vec{\xi}$ is expressed as

$$\vec{\xi} = \vec{\xi}_o(\vec{r})\ e^{i\omega t}, \tag{3-4}$$

where $\vec{\xi}_o(\vec{r})$ is a complex positional vector. By substituting (3-4) into (3-1), we obtain

$$-\omega^2 \rho \vec{\xi} + 2i\omega\rho \ (\vec{\Omega} \times \vec{\xi}) = \vec{F}(\vec{\xi}) \tag{3-5}$$

By scalarly multiplying $\vec{\xi}^*$ (a complex conjugate of $\vec{\xi}$) with (3-5), we further obtain

$$-\omega^2 \rho \vec{\xi}^* \vec{\xi} + 2i\omega\rho \ \vec{\xi}^* \ (\vec{\Omega} \times \vec{\xi}) = \vec{\xi}^* \cdot \vec{F}(\vec{\xi}). \tag{3-6}$$

It is known that $\vec{F}(\vec{\xi})$ is self-adjoint and therefore the right hand side of this equation is always real (Bernstein et al., 1958). This equation is furthermore integrated over the volume, in which the system under consideration is involved, with the boundary conditions as

$$\vec{n} \cdot \vec{\xi} = 0,$$

where $\vec{n}$ is the unit vector normal to the surface which encloses the above volume. Thus we obtain

$$-\omega^2 \int \rho \vec{\xi}^* \vec{\xi} \ dv + 2\omega \int \rho i \vec{\xi}^* \ (\vec{\Omega} \times \vec{\xi}) \ dv = \int \vec{\xi}^* \cdot \vec{F}(\vec{\xi}) \ dv. \tag{3-7}$$

It is clear that the coefficient of the first term on the left hand side of the above equation is real and positive. Since the coefficient of the second term on the same side is also real (Sakurai, 1972), we can solve the above equation algebraically with respect to ω.

The solution for ω is given by

$$\omega = \frac{1}{\int \rho \vec{\xi}^* \vec{\xi} \, dv} \Big\{ \int i\rho \vec{\xi}^* \, (\vec{\Omega} \times \vec{\xi}) \, dv$$

$$\pm ([\int i\rho \vec{\xi}^* (\vec{\Omega} \times \vec{\xi}) \, dv]^2 - [\int \rho \vec{\xi}^* \vec{\xi} dv][\int \vec{\xi}^* \cdot \vec{F}(\vec{\xi}) \, dv]^{1/2} \Big\}$$

$$(3\text{-}8)$$

The system under consideration is stable as far as ω is real. In order for ω to be real, it is necessary that

$$\int \vec{\xi}^* \cdot \vec{F}(\vec{\xi}) \, dv < 0. \qquad (3\text{-}9)$$

Even if this inequality is not fulfilled, the system is also stable when

$$[\int i\rho \vec{\xi}^* (\vec{\Omega} \times \vec{\xi}) \, dv]^2 - [\int \rho \vec{\xi}^* \vec{\xi} \, dv] \; [\int \vec{\xi}^* \cdot \vec{F}(\vec{\xi}) \, dv] > 0. \qquad (3\text{-}10)$$

Insofar as either (3-9) or (3-10) is fulfilled, the system is, therefore, always stable for any perturbation. Thus, in these cases, no convective motion is generated in the interior of Jupiter.

In consequence, the system is always unstable when inequality

$$[\int i\rho \vec{\xi}^* \; (\vec{\Omega} \times \vec{\xi}) \; dv]^2 \Big/ [\int \rho \vec{\xi}^* \vec{\xi} \; dv] < \int \vec{\xi}^* \cdot \vec{F} \; (\vec{\xi}) \; dv \qquad (3\text{-}11)$$

is fulfilled. It is clear from this equation that the lowest value for the onset of instability increases in proportion with Ω^2. This means that the effect of the

Coriolis force is to inhibit the onset of convective motion. In the case of the non-rotating system, the necessary condition for instability is given by (Bernstein et al., 1958)

$$0<\int \vec{\xi}^{*}\cdot\vec{F}(\vec{\xi})\ dv. \tag{3-12}$$

We can say, therefore, that the non-rotating system becomes unstable much easier than the case of the rotating system. In general, however, the convective motion thus produced is axisymmetric in the non-rotating system (Chandrasekhar, 1956). Hence no hydromagnetic dynamo process would be produced (e.g., Cowling, 1933). This result suggests that, even if Venus has a conducting core inside, it does not generate magnetic field.

As has been shown above, the onset of instability is restricted to some energy states of the system as determined by (3-11). Once this inequality is fulfilled, the system moves to an overstable state because ω becomes complex. In this case, the convective motion is set up, and is oscillatory in nature. Furthermore, this motion seems to become turbulent due to further growth of this overstability. Important is that such motion in the rotating system is non-reflection-symmetric by the effect of the Coriolis force (e.g., Braginskii, 1965; Steenbeck and Krause, 1969). Thus the onset of turbulent motion in this system seems to be very important from the view point of the dynamo process for the maintenance of the magnetic field of Jupiter.

Since the effect of the Coriolis force is to inhibit the onset of convective motion, we cannot say that every rotating star or planet has its own magnetic field as a result of hydromagnetic dynamo process. In order for this process to be built up, inequality (3-11) must be fulfilled by such a star or planet. This results, therefore, suggests that, although many stars and planets are more or less rotating, all of them do not necessarily have their own magnetic fields.

4. CONCLUDING REMARKS

In this paper, we have shown that the effect of the Coriolis force is to inhibit the onset of instability in the rotating systems as Jupiter. As shown in (3-11), the criterion of this onset is related to the Coriolis force.

At present, the maintenance and amplification of the magnetic fields of planets and stars are believed to be associated with their rotating motions because it seems that the latters are closely connected with the hydromagnetic self-exciting dynamo process. As shown in this paper, this process, however, does not necessarily occur in all of the rotating planets and stars; those which only fulfill condition (3-11) may have their own magnetic fields. If only the dynamo process is able to explain the origin of the magnetic fields of planets and stars, the conducting core must exist in the interior of Jupiter and then fulfill condition (3-11) from energetical point of view.

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