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ABSTRACT

Measurements of the microwave emission from the moon and their interpretation in terms of the thermophysics of that body are reviewed. Variations of the brightness temperatures of the moon during a lunation and during eclipses when combined with infrared and radar measurements yield a precise set of physical parameters for the upper few meters of the lunar soil. These parameters include the density, thermal conductivity, dielectric constant, radio absorption coefficients, and the mean temperature gradiant. In particular, the brightness temperatures averaged over a lunation are found to be an increasing function of wavelength, and consequently, an increasing function of depth. The observed quantity is about 0.9 °K per cm of wavelength in the range from 1 to 50 cm. Although the relationship between the true depth of the radio emission layer and wavelength is rather ambiguous we estimate the true temperature gradiant by using the empirical value of the radio absorption length of $\ell_R = 15.2\ \lambda$ which was computed from the lunation data. The resulting estimate of the temperature gradiant is 5.8 °K/meter which should be interpreted as the gradiant in the upper few meters (where the thermal conductivity is very low) averaged over the lunation zone of the subearth point. In all cases where it is possible to compare results from earth-based radio, radar and infrared technique with in situ Apollo measurements the agreement is shown to be surprisingly good. This fact greatly increases our confidence in applying the earth-based techniques to the terrestral planets.

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MICROWAVE EMISSION FROM THE MOON

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Introduction

Measurements of blackbody thermal emission from the moon in the infrared spectral region yield much information concerning the upper few centimeters of the lunar crustal material. (This chapter is intended to be tutorial in nature rather than an exhaustive review of the literature. Consequently, many interesting but perhaps more speculative developments are not discussed. The reader will be referred to more extensive reviews cited in the text.) Obviously, one can study the structure of this material at greater depths from measurements of thermal emission at longer wavelengths, i.e., the radio region of the electromagnetic spectrum. Significant measurements have been obtained over lunation periods and during lunar eclipses for wavelengths ranging from 1 mm to nearly 1 m. The emission mechanism is exclusively thermal in origin, and details of the emission such as polarization, variations with wavelength, lunar phase, etc., are controlled by the thermal and electrical parameters of the lunar surface material to a depth of several meters (at the longest wavelengths).

Investigations of the radio emission from the moon are closely related to the extensive studies of radar reflections from the lunar surface. The latter subject is developed in detail in Chap. 1c. In this chapter, we shall consider only aspects of this work which are directly important in interpreting the radio emission observations.

If we consider the lunar surface to be a smooth semi-infinite slab that is homogeneous in terms of at least electrical properties, we can compute the equivalent blackbody brightness temperature that one would observe at a frequency ν

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and at an angle of incidence θ_o to the surface if the temperature distribution with depth, $T(x)$, is known. The resulting radiative transfer problem was solved by Piddington and Minnett[20] in the form

$$T_B(\nu,p) = \left[1 - R_p(\nu,\theta_o)\right] \int_0^\infty T(x) \exp\left[-k_\nu x/\cos\theta_i\right] k_\nu dx/\cos\theta_i \quad (1)$$

[the integral being the Laplace transform of $T(x)$], where k_ν = the power absorption coefficient at frequency ν; θ_i = the angle of incidence of pencil radiation just below the surface, which is related to the incidence angle of observation θ_o by Snell's law at the surface; p = the state of polarization, either parallel or perpendicular to the plane of incidence; and $R_p(\nu,\theta_o)$ = the Fresnel reflection coefficient of polarization p for radiation emerging at angle θ_o. The Fresnel reflection coefficients for the E vector parallel to the incidence plane $R_\parallel$ and perpendicular to the incidence plane $R_\perp$ are given by

$$R_\parallel(\nu,\theta_o) = \left[\frac{\epsilon\cos\theta_o - \sqrt{\epsilon - \sin^2\theta_o}}{\epsilon\cos\theta_o + \sqrt{\epsilon - \sin^2\theta_o}}\right]^2 \quad (2)$$

$$R_\perp(\nu,\theta_o) = \left[\frac{\cos\theta_o - \sqrt{\epsilon - \sin^2\theta_o}}{\cos\theta_o + \sqrt{\epsilon - \sin^2\theta_o}}\right]^2 \quad (3)$$

where ϵ is the complex dielectric constant. The quantity actually measured in radio astronomy is the flux density (in w/m^2-Hz). For the temperature range of the moon, the Planck radiation function at wavelengths greater than about 1 mm can be replaced by the Rayleigh-Jeans approximation, and the flux density in a pencil of radiation in a solid angle Ω can be written as

$$F = (2kT/\lambda^2)\Omega \quad (4)$$

(λ is the wavelength of the radiation in the material) where k = Boltzmann's constant. Equation (4) was used in the derivation of Eq. (1) and is used to express the measured flux densities in terms of equivalent blackbody brightness temperatures $T_B(\nu,p)$.

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Equations (1-3) contain the complex dielectric constant of the lunar surface material, where the real part ϵ primarily controls the reflection phenomena at the very surface and the imaginary part is expressed in terms of the (power) absorption coefficient k_ν, which supplies the opacity within the material. The absorption coefficient is strongly wavelength-dependent for dielectric materials (ϵ is not), and it is given to sufficient accuracy for lunar conditions by

$$k_\nu = k_o \nu \tag{5}$$

over the frequency range of interest. Troitsky et al.[23] have pointed out that the constant k_o may be temperature-dependent in the lunar application. It can be seen from Eqs. (1) and (5) that with decreasing frequencies, k_ν has the effect of making the temperature distributions at greater depths in the material more important at a given incidence angle.

Equation (1) has been used in all interpretations of the lunar observations known to us. Consequently, it is important to consider carefully all of the assumptions involved. First of all, $T_B(\nu,p)$ is the equivalent temperature that would obtain with an infinitely narrow antenna beam with polarization p. All real observations involve an antenna beam pattern of finite width (often larger than the lunar disk itself) which necessarily averages the emission over a range of incidence angles θ_o and, consequently, over a range of polarization planes relative to the local spherical surface of the moon. The appropriate temperature distribution can no longer be considered just a function of depth in the integration over the beam pattern. The most serious assumption leading to Eq. (1) is the homogeneity of the surface material in the upper few meters of the lunar soil. Both ϵ and k_ν are fairly strong functions of the density of a specified material. Variations in radar cross section of the moon with wavelength from radar measurements strongly suggest a density increase with depth. A rigorous treatment of the problem for a given density profile in x would require a consideration of ray-path bending in the material due to refraction. However, the accuracy of the radio observations probably does not warrant these detailed considerations. The effects of surface roughness on the emission, particularly near grazing incidence, are poorly understood.

Linear Theory

The radio observations of the moon primarily show a sinusoidal variation about a mean brightness with the lunation period. The amplitude and phase of this variation systematically depend on the wavelength, i.e., the amplitude of the variation increases with increasing frequency, and the phase approaches that of the solar insolation. No significant variation can be seen at wavelengths longer than about 5 cm. The observations can be fully exploited only after one has computed a detailed thermophysical model for the moon, i.e., the temperature distribution with longitude and latitude to depths of several meters. Under certain assumptions, the entire problem can be solved analytically as shown by Piddington and Minnett (1949)[20] and Troitsky[21]. Troitsky in particular has greatly developed the method in a distinguished series of papers. (See Troitsky and Tikhonova[24] for complete references and a summary of results.) In this approach, which we call the <u>linear theory</u>, we begin by assuming that the temperature at a given longitude and latitude point on the <u>surface</u> of the moon can be expanded in a Fourier series in time

$$T(0,t) = T_o + T_1 \cos(\omega t - \Phi_1) + T_2 \cos(2\omega t - \Phi_2) + \dots \quad (6)$$

where ω is the angular rotational rate of the moon with respect to the sun and Φ_n is the phase shift of the nth harmonic term relative to the solar insolation. If we assume that the moon is a semi-infinite homogeneous slab with <u>constant</u> thermal properties, the equilibrium solution for the temperature at depth x has been shown to be[2]

$$T(x,t) = T_o + T_1 e^{-\beta_1 x} \cos(\omega t - \Phi_1 - \beta_1 x) + T_2 e^{-\beta_2 x} \cos(2\omega t - \Phi_2 - \beta_2 x) + \dots \quad (7)$$

where β_n is the "thermal absorption coefficient" for the nth harmonic and is given by

$$\beta_n = \left[\frac{n}{2}\frac{\rho c_p}{k}\right]^{1/2} \quad (8)$$

where ρ is the density, c_p the specific heat, and k the thermal conductivity of the material, all of which are constants (with depth and temperature) in this development.

The physical interpretation of Eq. (7) is straightforward. The temperature at any depth x is the sum of a constant term T_o and a series of periodic terms arising from harmonic temperature "waves" that propagate into the material with amplitudes that are attenuated with depth by virtue of the $e^{-\beta_n x}$. The phases of the temperature waves are increasingly retarded with depth by the same factor in the arguments of the cosine terms. The temperature distribution given by Eq. (7) can be utilized in Eq. (1) to compute the radio brightness temperatures at any wavelength for a given set of electrical constants ϵ and k_ν. The angles θ_o and θ_i in Eq. (1) are, of course, determined by the line of sight from the earth to the particular point on the lunar surface for which the temperature distribution with depth is given by Eq. (7). The integral in Eq. (1) becomes an infinite series of integrals, the first two terms of which are the most important. For the constant term in Eq. (7), we get

$$T_{B_o}(\nu,p) = \left[1 - R_p(\nu,\theta_o)\right] \int_o^\infty T_o \exp\left[-k_\nu x/\cos\theta_i\right] k_\nu dx/\cos\theta_i$$

or (9)

$$T_{B_o}(\nu,p) = \left[1 - R_p(\nu,\theta_o)\right] T_o$$

that is, essentially a constant for a given θ_o since R_p is practically frequency-independent. The general nth term becomes (at time t)

$$T_{B_n}(\nu,p) = \left[1 - R_p(\nu,\theta_o)\right] T_n \int_o^\infty \exp\left[-(k_\nu/\cos\theta_i + \beta_n)x\right] \times$$

$$\cos\left[n\omega t - \Phi_n - \beta_n x\right] k_\nu dx/\cos\theta_i \qquad (10)$$

We define the fundamental parameter $\delta_n(\theta_i)$ as

$$\delta_n(\theta_i) = \left(\frac{\beta_n}{k_\nu}\right)\cos\theta_i = \left(\frac{n^{1/2}\beta_1}{k_\nu}\right)\cos\theta_i \qquad (11)$$

The quantity β_1/k_ν is the ratio of the thermal (temperature) absorption coefficient to the radio absorption coefficient, or, introducing the thermal absorption <u>length</u> ℓ_T and the radio absorption <u>length</u> ℓ_R, we see that $\delta_1(\theta_i)$ at normal incidence is the fundamental ratio

$$\delta_1(\nu) = \beta_i/k_\nu = \ell_{R,\nu}/\ell_T \tag{12}$$

We shall see shortly that the quantities $\delta_1(\nu)$ are directly measured by the radio observations during a lunation. The integrals, Eq. (10), are elementary and, after a rather long manipulation, yield the remarkable result (at time t at a specified point on the lunar surface)

$$T_{B_n}(\nu,p) = \left[1 - R_p(\nu,\theta_o)\right]\left\{\frac{T_n \cos\left[n\omega t - \Phi_n - \Psi_n(\theta_i)\right]}{\sqrt{1 + 2\delta_n(\theta_i) + 2\delta_n^2(\theta_i)}}\right\} \tag{13}$$

where the phase factor of the brightness temperature $\Psi_n(\theta_i)$

$$\Psi_n(\theta_i) = \text{arc tan}\left[\delta_n(\theta_i)/(1 + \delta_n(\theta_i))\right] \tag{14}$$

Finally, the complete solution of the thermophysical problem in the linear theory is

$$T_B(\nu,p) = \left[1 - R_p(\nu,\theta_o)\right]\left\{T_o + \sum_{n=1}^{\infty} T_{B_n}(\nu,p)\right\} \tag{15}$$

In all of our equations, we have carried both angles θ_o and θ_i. They are, of course, related by Snell's law applied at the vacuum-surface interface:

$$\sin\theta_o = \sqrt{\epsilon}\,\sin\theta_i \tag{16}$$

where Maxwell's relation is used to write the index of refraction as the square root of the (real) dielectric constant. Since the electrical conductivity of the lunar surface material is small, the use of the real part of the dielectric constant in Eq. (16) and in the Fresnel reflection coefficients, Eqs. (2) and (3), will not cause significant errors in the interpretation of the radio observations.

Equations (13) through (16) formally complete the linear theory. However, it is useful here to recapitulate the development in order to emphasize the simple physical ideas. We started with the knowledge of the <u>surface</u> temperature as a function of time at a specified point on the spherical surface of the moon. Any such point can easily be related to an observer on the earth as a function of time from the motion of the earth-moon-sun system. The temperature variation was then expanded in an infinite Fourier series, Eq. (6), which is always possible mathematically. The lunar surface was then replaced by a semi-infinite homogeneous slab at the local point. This assumption can cause no errors, since the ratio of the thermal skin depth to the radius of the moon is infinitesimal. The resulting temperature distribution with depth below the local point was then given by Eq. (7) under the assumption that the parameters k, ρ, and c_p are independent both of depth and of temperature. We shall see below that these assumptions are serious and that this theory is inadequate to explain some of the details of the observations, although it does accurately predict the major features of the observations. It should be realized that the Fourier expansion approach is not limited to the conditions of lunations. Any surface temperature variation can be treated in this way, including eclipse circumstances, although its usefulness seems to be limited to lunation conditions. The temperature distribution with depth retains the harmonic terms of the surface variation. However, each higher harmonic is increasingly attenuated by a factor of n times the thermal absorption coefficient. Furthermore, each higher harmonic is increasingly delayed in phase relative to the surface variation by the same factor. Only at the shortest wavelengths can harmonics higher than the first be seen in the radio emission, as we shall see below. The thermal (temperature) absorption coefficient is related to the thermal parameters and the rotational rate of the moon by Eq. (8).

The temperature distribution with depth, $T(x,t)$, was then utilized in the formal solution of the radiative transfer problem, Eq. (1), which was derived under the assumptions that electrical parameters are independent of depth and temperature and that the surface is smooth. Departures from these conditions obviously occur on the moon, but they are probably not as serious as the assumptions concerning the thermal parameters. At the time of this writing, the effects of variations in the electrical parameters have not been investigated for the complete lunar thermophysical problem. However, these questions will be considered for the determination of ϵ later in this chapter. The equivalent brightness temperature at a specified frequency (contained in k_ν) at incidence angle θ_o was given by Eqs. (13-15), the constant plus first harmonic terms being

$$T_B(t) = \left[1 - R_p(\theta_o)\right]\left\{T_o + \frac{T_1}{\sqrt{1 + 2\delta_1(\theta_i) + 2\delta_1^2(\theta_i)}} \times \cos\left[\omega t - \Phi_1 - \Psi_1(\theta_i)\right]\right\} \tag{17}$$

where $\delta_1(\theta_i)$ was given by Eq. (11) and $\Psi_1(\theta_i)$ by Eq. (14). Since k_ν is proportional to ν, δ increases with decreasing frequency. The brightness temperature consists of a constant term independent of frequency plus an harmonic term whose amplitude has decreased from the surface values by the division by the factor $[1 + 2\delta + 2\delta^2]^{1/2}$, which increases with increasing wavelength. At long wavelengths, T_B approaches the mean temperature T_o times the emissivity $(1 - R)$. Furthermore, the phase of T_B increases relative to that of the insolation as the wavelength increases, reaching a maximum of $45°$ as δ becomes large, as can be seen from Eq. (14).

Thus far, we have considered the case for observations with an infinitely narrow antenna beam. For a finite beam, Eq. (15) has to be integrated over the beam pattern, which means integrating over a range of θ_o and θ_i as well as over the phase curve, i.e., a range of "time" t. These calculations are complex and will not be considered here (see, e.g., Kaydonovsky et al.[11]). Lunar radio observations can be corrected for the antenna patterns and referred to normal incidence at the sub-earth point with some loss of information. It is these observations that we discuss below. Observations so corrected are easy to interpret, since $\theta_o = \theta_i = 0$ and the Fresnel reflection coefficients become the (polarization-independent) normal power reflection coefficient, which is determined nearly directly by radar cross-section measurements.

Determination of δ from the Observations

A list of radio brightness temperatures over lunations expressed as a mean T_m, amplitude of the first harmonic T_v, and phase delay of the first harmonic Ψ are shown in Table 1 (compiled by Hagfors[8,9]). We have plotted the observed ratios T_m/T_v against the wavelength in Fig. 1. The theoretical value of this ratio from Eq. (17) is (for normal incidence)

$$T_m/T_v = T_o/T_1\left[1 + 2\delta_1 + 2\delta_1^2\right]^{1/2} \tag{18}$$

Following Troitsky[22], we adopt the working hypothesis that δ_1 is proportional to the wavelength over the wavelength range of interest. With this hypothesis, based on the available observations at that time, Troitsky adopted the relationship $\delta = 2\lambda$, where λ is in centimeters. The curve for the relationship is shown in Fig. 1. This curve, as well as the curve for $\delta = 2.5\lambda$, was forced to agree with the observations near 0.1 cm, since the ratio T_o/T_1 is not known. The ratio apparently should be interpreted as the ratio of the mean surface temperature to the amplitude of the first harmonic of the

Table 1 List of results of measurements of lunation variation of thermal emission from the moon

Wavelength, cm	T_o, °K	T_i, °K	ϕ_i, deg	Half-power beam width	References (cited by Hagfors[8,9])
0.10a	229	115	18	3.9'	Low and Davidson [1965]
0.13a	216	120	16	10'	Fedoseev [1963]
0.15b	265	145	...	5'	Sinton [1955]
0.18a	240	115	14	6'	Naumov [1963]
0.32a	210	65	10	9'	Tolbert and Coates [1963]
0.33a	196	70	27	2.9'	Gary et al. [1965]
0.40b	230	73	24	25'	Kislayakov [1961]
0.40a	228	85	27	1.6'	Kislayakov and Salomonvich [1963]
0.40b	204	56	23	36'	Kislayakov and Plechkov [1964]
0.80a	197	32	40	18'	Salomonovich [1958]
0.80a	211	40	30	2'	Salomonovich and Losovskii [1963]
0.86a	225	45	40	6'	Gibson [1958]
1.18a	220	29	48	3.5'	Moran [1965]
1.25a	215	35	45	23'	Piddington and Minnett [1949]
1.37a	220	24	43	4.0'	Moran [1965]
1.6b	208	37	30	44'	Kamenskaya et al. [1962]
1.63b	224	36	34	~26'	Zelinskaya et al. [1959]
1.63b	207	32	10	44'	Dimitrenko et al. [1964]
2.0a	190	20	40	4'	Salomonovich and Koschenko [1961]
3.15a	195	12	44	9'	Mayer [1961]
3.2a	223	17	45	6'	Koschenko et al. [1961]
3.2b	216	16	15	40'	Bondar et al. [1962]
9.4b	220	5.5	5	2°20'	Medd and Broten [1961]
9.6b	230	...	...	19'	Koschenko et al. [1961]
11b	214	...	...	17'	Mezger and Straussl [1959]
20.8b	205	5	49	36'	Waak [1961]
21.0b	232	...	...	10'	Heiles and Drake [1963]

Table 1 (continued)

Wavelength, cm	T_o, °K	T_i, °K	ϕ_i, deg	Half-power beam width	References (cited by Hagfors[8,9])
21.0b	250	5	...	35'	Mezger and Strassl [1959]
21.0b	230	...	...	90'	Razin and Fedorev [1963]
22	270	...	...	15'	Davis and Jenisson [1960]
25.0b	226	...	...	1°17'	Alekseev et al. [1967]
30.2b	227	...	...	1°18'	Alekseev et al. [1967]
31.25b	227	...	...	...	Troitsky et al. [1967]
32.3b	233	...	...	3°00'	Razin and Fedorev [1963]
35.2b	223	...	...	...	Troitsky et al. [1967]
40b	224	...	...	...	Troitsky et al. [1967]
54b	218.5	...	...	...	Troitsky et al. [1967]
60.24b	216.5	...	...	...	Troitsky et al. [1967]
70.16b	217	...	...	1°30'	Krotikov et al. [1964]

[a]Brightness measured at center of disk.

[b]Brightness averaged over disk.

surface temperature. The true surface temperature determined from infrared observations does, of course, contain higher harmonics. The curves of Fig. 1 yield $T_o/T_1 \approx 1.5$. If the mean surface temperature is 240°K, then the maximum and minimum temperatures of the first harmonic would be about 400 and 80°K, respectively. It can be seen from the scatter of the data in Fig. 1 that, although the linear theory agrees rather well with the observations, the hypothesis that δ_1 is proportional to the wavelength is only weakly supported. Consistent with this, we shall adopt the result

$$\delta_1 = (2.4 \pm 0.5)\lambda \tag{19}$$

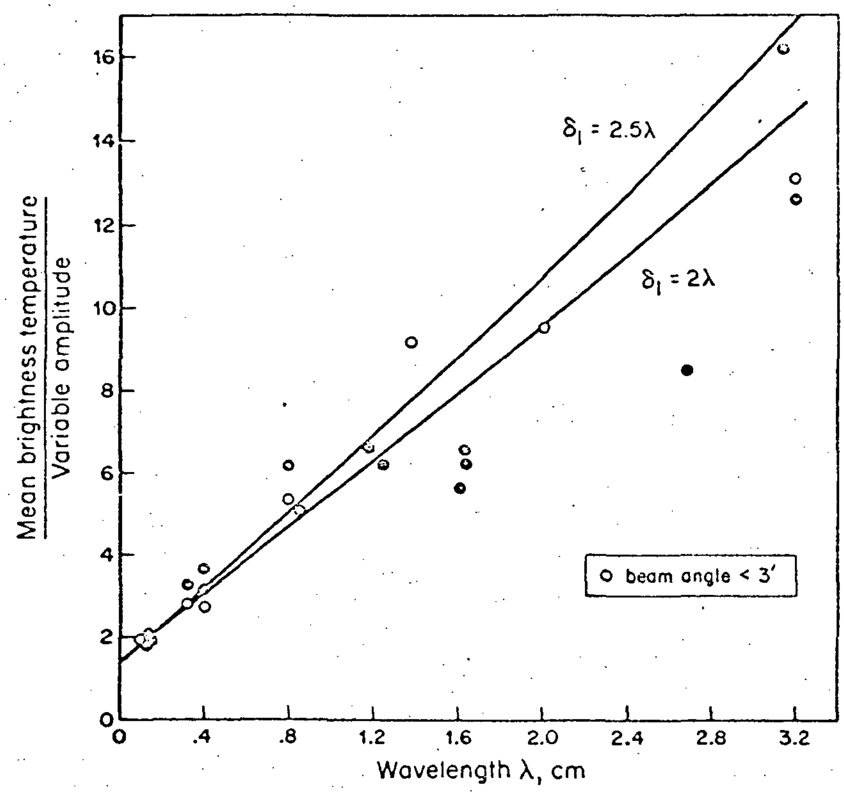

Fig. 1 Ratios of the mean temperatures to the variable temperature amplitudes during a lunation. Circle points are measurements with very narrow beamed antenna systems.

in further discussions. The temperature phase data of Table 1 are also consistent with this result.

Interpretation of δ_1

The value δ_1 which was obtained from the observations is related to the physical parameters of the lunar material through Eq. (12), i.e., it is the ratio of the electrical length at frequency $\nu, \ell_{R,\nu}$, to the thermal length ℓ_T. It is, of course, of considerable interest to separate the physical parameters from the empirical parameter δ_1. This cannot be done from the radio observations alone. A considerable body of information is contained in the infrared and radar measurements of the lunar surface. These data, combined with laboratory measurements of likely lunar materials (see Chap. 3a) and recently with returned lunar samples (see Chap. 3b), allow for a nearly unique separation of the physical parameters.

Before we consider specific numerical values, we shall transform the electrical coefficient k_ν into a more useful form. Since k_ν is not zero for the lunar material, the material possesses a nonzero electrical conductivity. A plane monochromatic electromagnetic wave propagating in the material will have an electric vector $\bar{E}$ harmonic in time which is a solution of the wave equation

$$\nabla^2 \bar{E} + k_c^2 \bar{E} = 0 \tag{20}$$

where the complex propagation constant k_c is given by

$$k_c^2 = (\omega^2 \mu / c^2)\left[\epsilon + i\ 4\pi\sigma/\omega\right] \tag{21}$$

($\omega = 2\pi\nu$, c = light speed in vacuum, σ = specific electrical conductivity, and μ = magnetic permeability). Writing k_c in terms of its real and imaginary parts,

$$\mathrm{Re}(k_c) = \frac{\omega}{c}\sqrt{\frac{\mu\epsilon}{2}}\left\{\sqrt{1 + \frac{4\sigma^2}{\epsilon^2\nu^2}} + 1\right\}^{1/2} \tag{22}$$

$$\mathrm{Im}(k_c) = \frac{\omega}{c}\sqrt{\frac{\mu\epsilon}{2}}\left\{\sqrt{1 + \frac{4\sigma^2}{\epsilon^2\nu^2}} - 1\right\}^{1/2} \tag{23}$$

The electric vector $\bar{E}$ of the plane wave will be absorbed as it propagates through the material by virtue of the imaginary part of k_c. The absorption of the intensity or power in the wave is given by the square of $\bar{E}$ absorption, or

$$\bar{E}^2 \propto \bar{E}_o^2\ e^{-2\mathrm{Im}(k_c)x}$$

for propagation in the x direction. Thus, the power absorption coefficient is twice $\mathrm{Im}(k_c)$:

$$k_\nu = 2\mathrm{Im}(k_c) = \frac{\omega}{c}\sqrt{2\mu\epsilon}\left\{\sqrt{1 + \frac{4\sigma^2}{\epsilon^2\nu^2}} - 1\right\} \tag{24}$$

It has become conventional in the literature on lunar radio emission to express the electrical absorption in terms of the loss tangent, which is defined as

$$\tan\Delta = 2\sigma/\epsilon\nu \tag{25}$$

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In the microwave region of the spectrum, the loss tangents for almost all materials occurring on the earth's surface which are free of liquid water are considerably less than unity, i.e., the materials possess low electrical conductivities at high frequencies. This circumstance undoubtedly is also true for the lunar surface material. With this mild assumption, the absorption coefficient becomes

$$k_\nu = (\omega/c)\sqrt{\mu\epsilon}\,\tan\Delta$$

Except for magnetic materials, which are probably not important on the lunar surface, we can set $\mu = 1$, and the power absorption coefficient becomes

$$k_\nu = (2\pi\nu/c)\sqrt{\epsilon}\,\tan\Delta \tag{26}$$

or, expressed as the radio absorption length (skin depth),

$$\ell_{R,\nu} = \lambda/(2\pi\sqrt{\epsilon}\,\tan\Delta) \tag{27}$$

Now, from Eqs. (12) and (8),

$$\delta_1 = \frac{\ell_{R,\nu}}{\ell_T} = \left\{\left(\frac{\omega\rho c_p}{2k}\right)^{1/2}(2\pi\sqrt{\epsilon}\,\tan\Delta)^{-1}\lambda\right\} \tag{28}$$

Our empirical result, $\delta_1 = (2.4 \pm 0.5)\lambda$, strongly suggests that the loss tangent of the material comprising the upper meter of the lunar surface is frequency independent, which, in turn, by virtue of Eq. (25), suggests that the electrical conductivity σ depends linearly on the frequency. These results conform well to laboratory measurements on dielectric materials. Actually, we anticipated this result with Eq. (5). Equation (28) is the desired factorization of δ_1 into the fundamental physical parameters. The basic result of the linear theory can be written as

$$\sqrt{\frac{\omega\rho c_p}{2k}}\,(2\pi\sqrt{\epsilon}\,\tan\Delta)^{-1} = 2.4 \pm 0.5 \tag{29}$$

Many investigators have attempted to utilize this result with a combination of infrared and radar data to separate the numerical values of the physical parameters (e.g., see Troitsky[22] for a careful analysis of these matters and for complete references). Since we believe that the linear theory is not adequate to interpret the observations fully, we shall not labor over these results but rather shall adopt a somewhat arbitary set of numerical values as an illustration. A more complete discussion will be deferred to later in this chapter, when we consider the variation of the thermal conductivity and the specific heat with temperature. Unfortunately, a careful analysis of the manifold assumptions concerning the electrical parameters does not exist. It appears that the radio observations are not sufficiently accurate at this time to warrant such detail.

A Consistent Set of Constant Parameters

The parameter that is most in doubt is $\tan\Delta$, which we shall adjust to yield the empirical δ_1. Our best estimate of constant parameter values is (illustration only) $\epsilon = 2.0$, $\rho = 1.0\ \mathrm{g/cm^3}$, $c_p = 0.6$ joules/g-°K, and $k = 2.92 \times 10^{-5}$ joules/cm-sec-°K, where the thermal conductivity was computed from the listed values of ρ and c_p and a rather arbitrary value of the thermal parameter $\gamma \equiv (k\rho c_p)^{-1/2} = 1000$. This parameter set is probably valid for the upper meter of the lunar soil averaged over the lunar disk facing the earth at a mean temperature of about 240°K. From Eq. (29), we find

$$\tan\Delta = 0.0074 \pm 0.0022 \tag{30}$$

Winter and Saari[27] have interpreted laboratory data on powders of basaltic material and of pumice in the form

$$\tan\Delta \approx 0.006\left[1 + 0.5\, e^{-(400 - T)/50}\right] \tag{31}$$

The agreement of our result, Eq. (30), with the laboratory measurements is excellent. From Eq. (30), we compute the electrical skin depth (distance over which the power will be attenuated to e^{-1}) as $\ell_{R,\nu} = 15.2\lambda$, and the temperature depth (distance over which the temperature of the first harmonic will be attenuated to e^{-1}) is $\ell_T = 6.3$ cm. It is obvious from these results why no significant variation in the microwave brightness temperatures can be seen beyond a wavelength of about 3 cm.

With this, we conclude our study of the linear theory. A great deal of information concerning the moon has been obtained from the simplified theory. A considerable body of work has been performed beyond that discussed, including investigations of two-layer models, which appear to be required to explain the infrared measurements better. We believe that this work is primarily of historical interest and that the problem is better treated with numerical techniques in which important nonlinear effects in the parameters can be included. Before we review these developments, we shall carefully consider the variations to be expected of the dielectric constant and the thermal properties.

Physical Parameters

Thus far, we have considered a highly simplified model of the moon which is completely homogeneous, with constant parameters independent of both depth and temperature. This model is not sufficient to explain the details of the radio and infrared emission, and it ignores a considerable fraction of our current knowledge of the lunar surface. The basic parameters of lunar thermophysics consist of the dielectric constant, electrical conductivity (or loss tangent), thermal conductivity, specific heat, and the material density. In particular, the density certainly varies with depth and with location on the lunar surface, variations which will affect all of the remaining parameters. These parameters also vary with ambient temperature, although the variations of the electrical parameters are probably negligible within the temperature range of interest. A considerable body of information is available concerning the parameters from diverse lunar observation and laboratory measurements. In this section, we briefly discuss these results; a more thorough discussion of the thermal parameters can be found elsewhere in this volume. We shall begin with the dielectric constant and the loss tangent.

The dielectric constant ϵ has been obtained by two methods: 1) the measurement of radar cross sections of the moon over a range of wavelengths from 0.86 cm to 2 m, and 2) measurements of the polarization of the radio emission from the moon over a range of wavelengths from 0.86 to 21 cm. The methods differ in a number of significant ways. The radar method yields estimates of ϵ near the subearth point averaged over depths of a few wavelengths, whereas the polarization measurements are sensitive to the material from the Brewster angle ($\sim$55 deg from the subearth point) to the limb of the moon over a smaller range of depths. Radar cross-section measurements are very sensitive to the surface roughness (i.e., the radar backscatter law), whereas surface roughness causes a relatively small but significant depolarization of the radio emission. Finally, radar measurements are easily obtained over a large range of wavelengths, whereas polarization measurements must be performed with narrow-beam systems and are technically more difficult.

The radar cross section is defined as the ratio of the measured echo power (for continuous-wave illumination or long pulses) to the power that would be reflected from an equivalent smooth and perfectly conducting sphere. It can be written as

$$S_\lambda = g_\lambda R \tag{32}$$

where R is the power reflection coefficient at normal incidence and g_λ is the backscatter "gain" of the surface or the directivity of the surface. The directivity arises from surface roughness and, in general, is wavelength-dependent. It is formally defined as the ratio of the power scattered back to the radar per unit solid angle to the power scattered into the average unit solid angle. The directivity has never been measured for the lunar surface, since such measurements would require bistatic experiments over the complete range of angles. Backscattering theories that describe lunar radar echoes have been developed by Hagfors[6] and Muhleman[17], and these authors have obtained similar expressions for directivities. Using measured pulse responses of the moon at wavelengths 3.8, 23, and 68 cm and the theoretical results just cited, we have estimated an empirical interpolation expression for the directivities as a function of wavelength

$$g_\lambda \approx 1 + \lambda^{-0.56} \quad (1 \text{ cm} \leq \lambda) \tag{33}$$

where λ is expressed in centimeters. Radar cross-section data from Hagfors[8], are shown in Fig. 2, where we have added a smooth line with error limits. The observations are very difficult for long wavelengths and are probably not reliable beyond 1 m. Using Eq. (33) for g_λ, and either Eq. (2) or (3) for the relationship between $R(\theta_o)$ at $\theta_o = 0$ and ϵ, we obtain the estimates for ϵ shown in Fig. 3, which increase from $\epsilon = 2$ at short wavelengths to $\epsilon \sim 5$ (with a large uncertainty) at the longest wavelengths. This result can be interpreted in terms of the variation in density with depth into the lunar surface. The lunar basalts in a fully consolidated form have a dielectric constant in the range of $\epsilon_s = 5$ to 7. If we take the density of this material to be $\rho_s = 3.0$ g/cm^3 on the basis of Apollo returned samples, the dielectric constant for a porous state of the basalt can be computed from the empirical expression

$$\epsilon = \epsilon_s \left[1 - \frac{3p}{[(2\epsilon_s + 1)/(\epsilon_s - 1)] + p} \right] \tag{34}$$

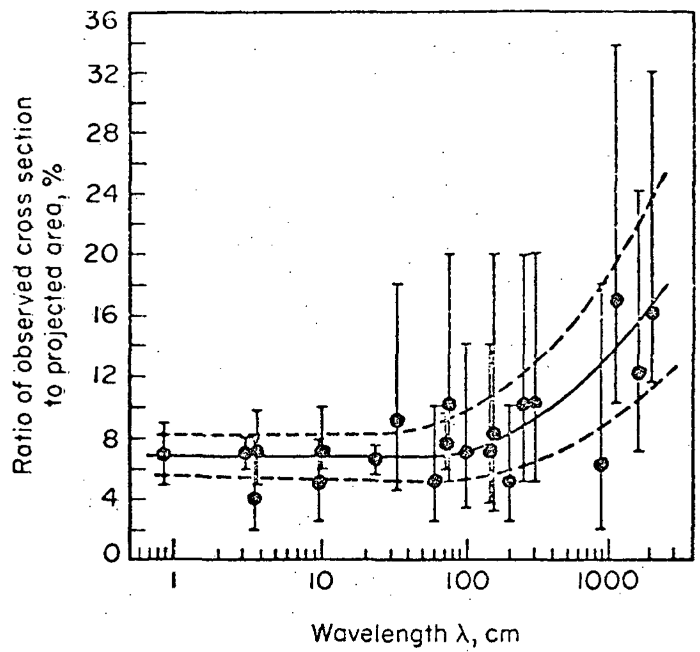

Fig. 2 Radar cross-sectional measurements as a function of wavelength. The smooth line represents moving averages and the dashed lines, approximate error bounds.

where $p = 1 - \rho/\rho_s$ (Troitsky[22]; see also Campbell and Ulrichs[1] for a more recent discussion). The corresponding densities, determined from the radar cross-section data, can be read from the right-hand scale in Fig. 3 (where $\epsilon_s = 6$). It would indeed be an important contribution if we could relate the density-vs-wavelength curve in terms of actual depth into the lunar surface. Although several investigators have attempted this, we believe that their procedures are quite ambiguous.

Estimates of the dielectric constant for four selected sites on the lunar surface have been obtained from the Surveyor radar altimeter data by Muhleman[18]. These measurements were made at a wavelength of 2.3 cm and refer to regions smaller than a square kilometer. The results are shown in Table 2, where we have computed the density from Eq. (34) with $\epsilon_s = 6$ and $\rho_s = 3$ g/cm^3. The uncertainties quoted in the table are almost completely due to the uncertainty in determining the directivity factors. The first three Surveyor sites are mare regions, and the Surveyor VII site is on the Tycho blanket region. The latter region displays a much higher dielectric constant and density, which may be typical of upland regions.

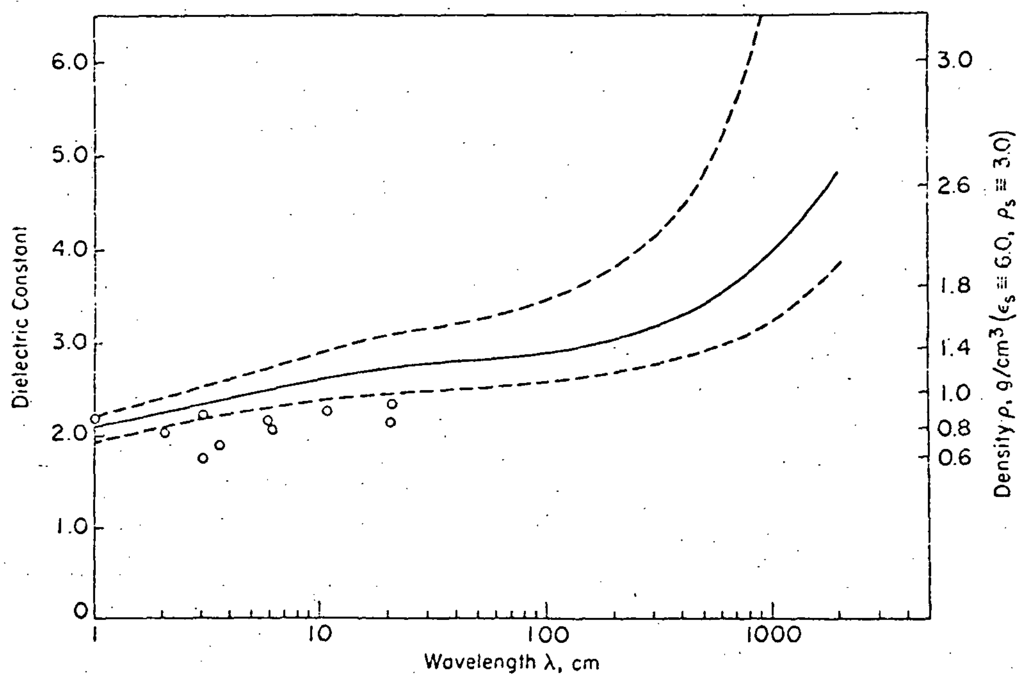

Fig. 3 Estimates of dielectric constant (left-hand scale) and density (right-hand scale) of the effective reflection layer. The circles are measurements of ϵ from the polarization of radio emission.

Table 2 Surveyor radar results

Site	Measured ϵ	Density, g/cm^3
Surveyor I	2.40 ± 0.50	1.0 ± 0.5
Surveyor II	2.07 ± 0.11	0.81 ± 0.1
Surveyor V	2.00 ± 0.16	0.79 ± 0.1
Surveyor VII	3.28 ± 0.40	1.6 ± 0.2

Estimates of ϵ from polarization measurements of radio emission are also shown in Fig. 3 as circled points. We have corrected the values of ϵ obtained by the various investigators for surface roughness by a method developed by Hansen and Muhleman[10]. Radar backscatter observations at a given wavelength are well represented by the function (for the normalized cross section per unit projected surface area)[17]

$$S(\Phi) = \alpha^3/(\sin\Phi + \alpha\cos\Phi)^3 \tag{35}$$

where α is a mean slope parameter that is strongly wavelength-dependent for the moon and Φ is the angle from the subearth point to the reflecting element. Most of the returned power comes from a small range of Φ, and we can approximate $S(\Phi)$ as

$$S(\Phi) = \cos^{-3}\Phi e^{-(3/\alpha)\Phi} \tag{36}$$

It has been shown by Hagfors[7] that the scattering law is in the form of Eq. (36) in the geometrical optics approach if $\exp(-3\Phi/\alpha)$ is the probability density of tilt angles of flat facets; i.e., most of the radar echo can be explained if local tilt angles are exponentially distributed with an angle $\delta = \alpha/3$ corresponding to e^{-1}. Hansen and Muhleman obtained values of δ as a function of wavelength from the radar data. Monte Carlo calculations were then performed over the moon to compute the depolarization for emission by drawing values of Φ from a table of exponentially distributed random numbers with the appropriate δ; i.e., the local values of θ_o in Eqs. (2) and (3) were perturbed by random samples of Φ at each integration grid point. The experimental results so corrected are shown in Fig. 3. Estimates of ϵ as a function of λ do not increase as rapidly as do the radar values. This effect is probably due to the polarization effects which occur near grazing incidence (toward the limb) and the sec θ_o effect which causes the effective emission layer to be nearer to the surface than the effective normal incidence radar reflection layer at the same wavelength.

Finally, the electrical parameters of an Apollo 11 bulk sample have been measured by Gold et al.[5] at a frequency of 450 MHz. They report values of density, dielectric constant, and absorption length $\ell_{R,\nu}$ in wavelengths. Their results are given in the first three columns of Table 3 (as read from their published graphs).

We have computed the loss tangent with Eq. (27) and the ratio $\tan\Delta/\rho$. It can be seen from the table that the value of $\tan\Delta$ for $\rho = 1$ g/cm^3 measured by Gold et al. is in remarkable agreement with our value given in Eq. (30) computed from the linear theory of the lunar radio emission. Values of $\tan\Delta/\rho$ are nearly constant, as is expected from laboratory measurements.

We shall now briefly discuss the range and variation of the thermal parameters. This subject is treated in Chap. 3a and

Table 3 Apollo 11 results

Density, g/cm^3	ϵ	$\ell_{R,\nu}$, wavelengths	$\tan\Delta$	$\tan\Delta/\rho$
1.0	1.8	18.0	0.0066	0.0066
1.22	2.0	13.5	0.0083	0.0068
1.55	2.45	10.0	0.0120	0.0077

3b, and we shall limit our discussion to background information for the review of recent model calculations of radio emission characteristics. It was realized very early that the thermal conductivity of the upper few millimeters of the lunar surface under vacuum conditions must be very low. Apparently, Wesselink[26] was first to point out the importance of the variation of k with temperature. Krotikov and Troitsky[13] developed these ideas further. Watson[25] performed important laboratory experiments with glass beads of various particle sizes and other materials in vacuum conditions. He demonstrated the variation of specific heats with temperature and represented the thermal conductivity in the form

$$k = A + BT^3 \tag{37}$$

where the A term arises from contact conduction and the B term from radiative heat transfer between the grains. The A term was found to be a function of mineral type, and the B term was primarily a function of particle size. Recently, Fountain and West[3] have investigated several materials, including basalts, under conditions closely simulating the lunar surface. Some of these results are shown in Fig. 4, where it can be seen that Watson's form represents the temperature variation very well. Fountain and West also reported the result of introducing a small amount of CO_2 gas (7 mb) in simulating the circumstances on Mars. The temperature variation is then negligible, apparently because of the small amount of CO_2 in the spaces between the grains. (Undoubtedly the same thing would occur for other atmospheric gases, e.g., air.) Figure 4 also demonstrates the effect of the density of the basalt; increasing density increases the contact area of the grains but has little effect on the form of the temperature variation. For the case with $\rho = 0.98$ g/cm^3 at a temperature of 300°K, the thermal conductivity for the conduction and radiation parts is 0.595×10^{-5} and 0.465×10^{-5} w/cm °K, respectively, i.e., essentially the same. Finally, the data

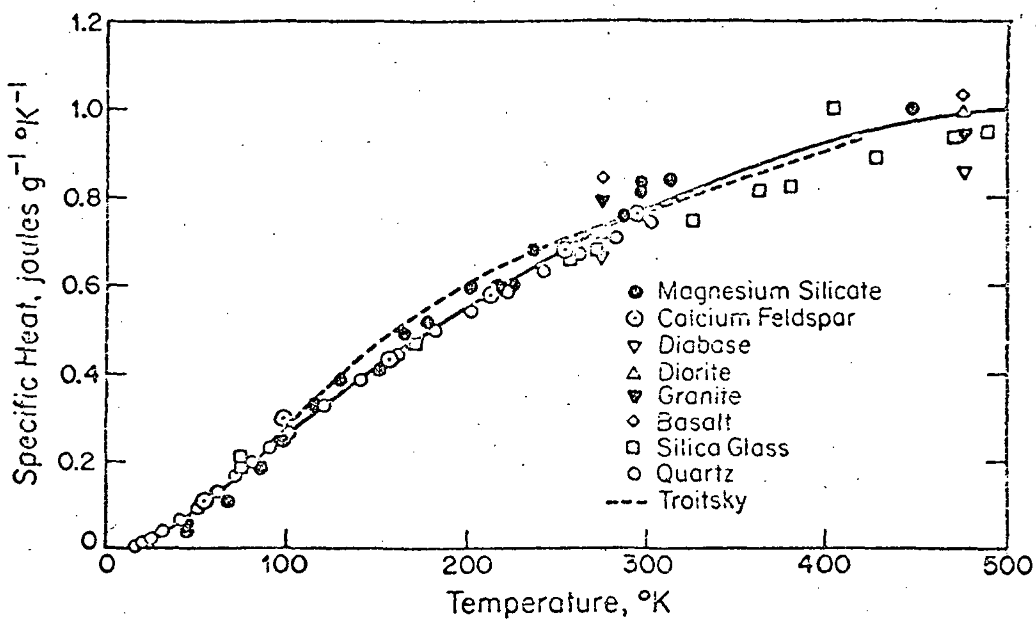

Fig. 5 Temperature variation of specific heats.

increase in the mean temperature with depth into the lunar surface. Krotikov and Troitsky[13] interpreted this temperature gradient in terms of radiogenic heating and estimated quantities of radioactive materials much greater than that found on the earth's crust. On the other hand, Linsky[14] attempted to explain these data in terms of variable thermal parameters. As we have seen, the thermal parameters do vary by factors of 2 or 3 over the lunar temperature range, a complex variation of density with depth certainly exists, and, consequently, all of the fundamental parameters vary sufficiently to affect the observed phenomena. Consequently, a number of people have attempted calculations that more or less include the important variations. None of these calculations can be regarded as definitive for various reasons, and, furthermore, various calculations are difficult to compare, since each worker has begun with a specific set of assumptions.

The temperature distributions with depth at any specified point on the lunar surface can be obtained to any desired accuracy from the numerical integration of the heat equations with appropriate boundary conditions:

$$c_p(x,T)\rho(x)\frac{\partial T(x,t)}{\partial t} = \frac{\partial}{\partial x}\left[k(x,T)\frac{\partial T(x,t)}{\partial x}\right] \qquad (38)$$

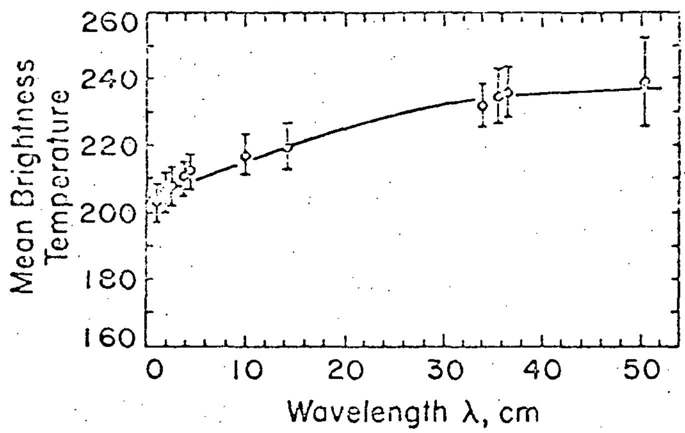

Fig. 6. Increase in mean lunation temperature with wavelength.

subject to the boundary condition

$$k(x,T)\left[\frac{\partial T(x,t)}{\partial x}\right]_{x=0} = (1 - A_I)T^4(0,t) - (1 - A_B)I(p,t) + Q \tag{39}$$

where A_I is the appropriate infrared albedo, A_B the bolometric albedo (i.e., over the solar spectrum), $I(p,t)$ the insolation function of the solar radiation at a specified point on the lunar surface at coordinate p, and Q is the heat flow density from the moon's interior due both to radiogenic heating and the primordal cooling of the moon. Clearly, definitive calculations require a knowledge of parameter variations with x, which are unknown. Furthermore, the temperature curves so obtained must then be integrated over depth using equations analogous to Eq. (1), where the dependence on x of the electrical parameters must be known. Definitive calculations would also require a considerably better understanding of the effects of surface roughness on both the thermal and the radio phenomena. Nevertheless, the computations of Linsky[14] and Troitsky's group do add considerably to our understanding of lunar phenomena.

Linsky, using reasonable values of the thermal parameter γ, evaluated at 350°K and a T^3 variation of k, has shown that differences exist between the mean surface temperature and the asymptotic temperature at depth between 20 and 50°K. This is adequate to explain the observed temperature gradient in the wavelength region from 1 mm to 3.2 cm without interior heating. The higher temperature differences occur for relatively higher ratios of radiative heat transfer to contact conduction, and the variation with temperature of the specific heat is of little importance. In order to explain the temperature gradient at longer wavelengths (3-50 cm), Linsky used a heat flow from the interior of 3.4×10^{-7} cal/cm^2-sec, which is of the same order as the heat flow in the earth's crust. Several of Linsky's models adequately represent the observed mean-temperature data. Expressing the thermal parameter at 350°K as γ_{350} and the ratio of radiative to contact heat transfer as R_{350}, he reports, in part, the following:

1) $\gamma_{350} = 885$, $R_{350} = 1$, $\epsilon = 2.50$, heat flux $= 3.4 \times 10^{-7}$ cal/cm^2-sec.

2) $\gamma_{350} = 1030$, $R_{350} = 0.5$, $\epsilon = 1.50$, heat flux $= 3.4 \times 10^{-7}$ cal/cm^2-sec.

The observed temperature gradient and the heat flow figure cannot extend very deeply into the surface, since the melting temperatures would be exceeded quickly. It is highly unlikely that the figure of the moon could be maintained if a significant fraction of the interior were molten.

Linsky's calculated lunation brightness temperatures agree well with the observations, which are shown in Figs. 7-9. It can be seen that the observations themselves cannot discriminate between his various models, which range in γ_{350} from 625 to 1075. Eclipse observations and model calculations are shown in Fig. 10. In this case, the observations are not very consistent, but the theory does correctly predict the general characteristics. Infrared observations are much more sensitive to γ in the night part of lunations and during eclipses, although these measurements refer to the upper few millimeters of the surface.

All of these matters have been reconsidered by the Soviet investigators (e.g., see the review by Troitsky and Tikhonova[24]). In particular, they have computed models with earth-like quantities of radiogenic heating along with temperature variations in the thermal parameters. Mean temperature gradients with wavelength are shown in Fig. 11 for two-layer models, with an upper layer of thickness d. Troitsky and Tikhonova state that the temperature increase stops at wavelengths from 25-30 cm, apparently because the emission is effectively coming from compacted layers; i.e., the longer wavelength emission arises from essentially the same depth.

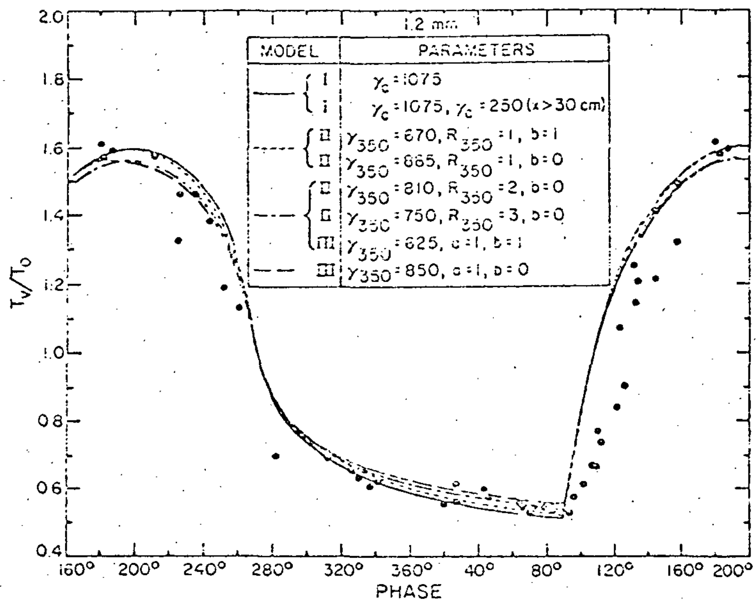

Fig. 7 Computed radio brightness temperatures at 1.2 mm for the center of the disk are compared with the data of Low and Davidson.[15] Figure from Linsky.[14]

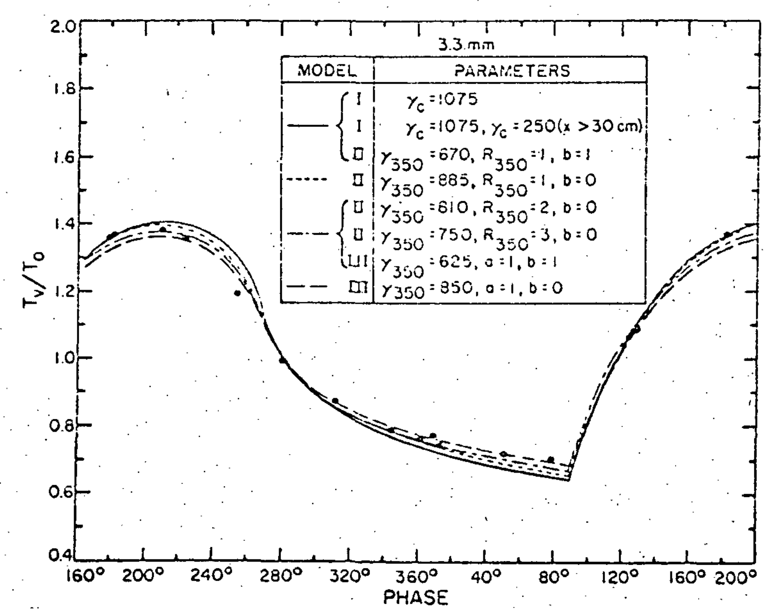

Fig. 8 Computed radio brightness temperatures at 3.3 mm for the center of the disk are compared with temperatures interpolated from the isotherms of Gary et al.[4] Figure from Linsky.[14]

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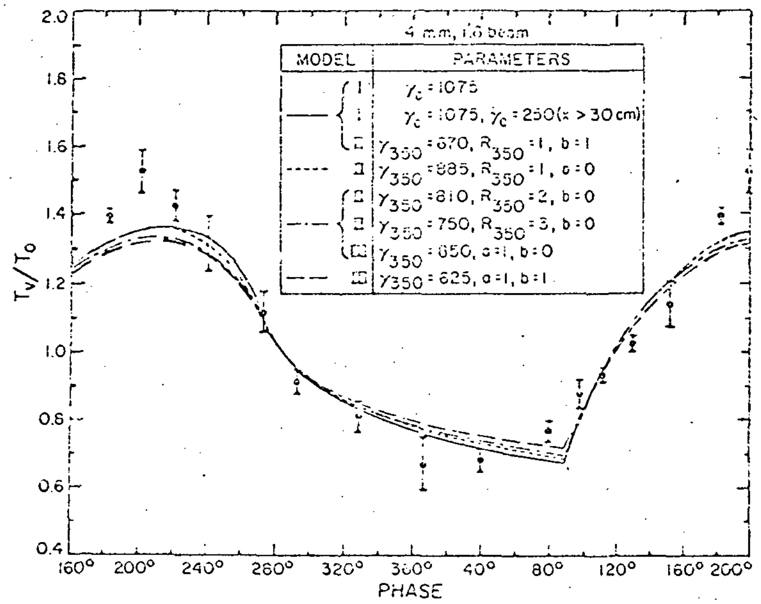

Fig. 9 Computed radio brightness temperatures at 4 mm for the center of the disk are compared with the data of Kislyakov and Salomonovich.[12] Figure from Linsky.[14]

The results of Fig. 11 are in good agreement with Linsky's calculations in that they suggest a similar interior heat flux, which, in turn, is consistent with the earth's flux. Radiogenic heating in the earth's crust arises primarily from the decay of potassium, uranium, and thorium in the mantle. According to MacDonald[16], reasonable values for the earth's crust are K (2.6%), U (2.3×10^{-6} g/g), and Th (8×10^{-6} g/g), which he uses to compute a radiogenic heat flux of 0.8×10^{-6} cal/cm^2-sec, whereas measured values on the earth are about 1.2×10^{-6}. Very recently, some information became available concerning the abundances of radiogenic materials in the lunar Apollo samples. O'Kelley et al.[19] have reported K (0.11%), U (0.49×10^{-6} g/g), and Th (1.92×10^{-6} g/g) from Apollo 11 samples. Preliminary analysis of Apollo 12 samples (see Chap. 3b) gives values for, e.g., sample 12070, of K (0.206%), U (6×10^{-6} g/g), and Th (1.5×10^{-6} g/g). Although it is certainly not possible for us to compute a heat flux for the lunar crust from such meager information, we can state that one would expect values "similar" to that for the earth.

Clearly, it would be of great importance if the temperature gradients on the moon could be accurately measured <u>and</u> precisely interpreted with radio techniques (also see Chap. 2b for in situ measurement of lunar thermal gradients). Perhaps

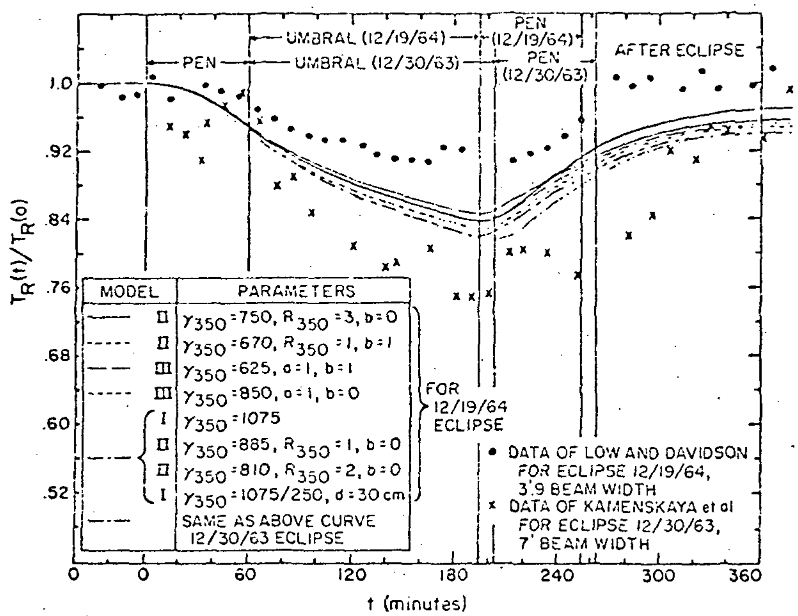

Fig. 10 Computed radio brightness temperatures for 1.2 mm at the center of the disk for the total lunar eclipse of Dec. 30, 1963, and Dec. 19, 1964, are compared with the data of Low and Davidson[15] and Kamenskaya et al. The circumstances of these eclipses are sufficiently similar that the two sets of data should agree. Figure from Linsky.[14]

this alone offers motivation for computing more definitive thermophysical models of the moon.

Conclusions

Thermophysical models of the moon can be constructed which explain the radio emission from the moon to the accuracy of the measurements. The variations during lunations of the radio brightness temperatures measured in the wavelength range from 1 mm to about 5 cm can be reproduced with homogeneous models with a thermal parameter in the range from 800 to 1200 cgs units and electrical parameters $\epsilon \approx 2.0$ and loss tangent ≈ 0.008. Allowing for the temperature variation of the thermal parameters as demonstrated from laboratory measurements with basalts, in particular, has little effect on the lunation temperatures and predicts an increase of the mean lunation brightness temperatures with wavelength, which agrees with observations at short wavelengths. Apparently, the observed increase of mean temperature at longer wavelengths (3-70 cm)

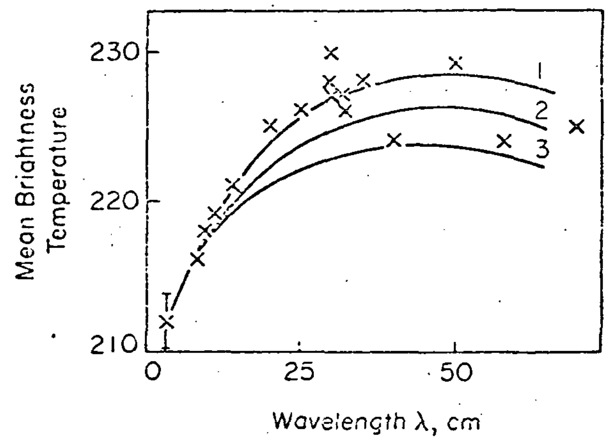

Fig. 11 The mean brightness temperature of the moon as a function of wavelength compared with model calculations with a porous layer of thickness d. The smooth curves represent: 1) d = 500 cm; heat flow = 0.65×10^{-6} cal/cm^2-sec, 2) d = 400 cm; heat flow = 0.72×10^{-6} cal/cm^2-sec, and 3) d = 300 cm; heat flow = 0.80×10^{-6} cal/cm^2-sec (figure from Troitsky and Tikhonova[24]).

can be explained only by hypothesizing a heat flow from the interior of the moon of from 10^{-7} to 10^{-6} cal/cm^2-sec. Such a heat flow, if real, is probably caused by radiogenic materials in the upper kilometers (?) of the lunar crust. Although the long wavelength measurements are poor, the temperature gradient appears to reach zero for wavelengths of about 70 cm. There is no unambiguous way to determine the corresponding depth into the surface, although models by Soviet workers suggest that this depth is on the order of 5 m. Certainly, values considerably larger are possible.

More difficulty is encountered in explaining the millimeter brightness temperature curves during total lunar eclipses. This phenomenon is controlled by the upper few centimeters of the lunar soil (because the time variations are short), and, consequently, the effects are better studied with infrared measurements or, perhaps, submillimeter radio measurements.

The evidence is strong that there is a significant increase of density with depth. More work needs to be done to interpret the radar cross-section data in terms of a unique density-with-depth relationship.

Thermophysical models have been developed from earth-based observations alone. These results have proven to be very accurate for all cases where parameters could be compared with Apollo samples, for example. This success allows one to have considerable confidence in the application of the earth-based techniques to the terrestrial planets.

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