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ON BACKSCATTER FROM SPATIALLY VARYING SURFACES

NORMAN C. GRODY

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Norman C. Grody

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ABSTRACT

This paper deals with the theory of radar backscatter from rough (spatially varying) surfaces. An integral equation developed by Kerr to describe the radar backscatter from a perfectly conducting surface is applied to non-smooth surfaces. The results are compared with those obtained by Beckmann as well as those obtained by Wright. Differences between the three solutions are discussed.

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INTRODUCTION

The current interest in remote sensing of the environment has focused attention on the interaction of radiation with rough land and ocean surfaces. In recent years a number of theories have been developed with different approximations to determine the scattering from rough surfaces[1-9]. Of the various approaches, the work of Beckmann et al[1,2] and Wright[3,4] have been most successful in explaining the radar backscatter measurements from sea and land surfaces[10,11], respectively.

Beckmann[2] applied the Kirchhoff method using the tangent plane approximation to obtain the radar cross section, while Wright[3] employed a zeroth order perturbation technique. An alternate approach is presented here for a perfectly conducting surface which agrees basically with Beckmann's result, however a more accurate radar cross section is obtained by adjusting the end term contributions to be exactly zero. Also, the small perturbation solution exhibits different angular dependence than Wright's result, although both methods agree under specific conditions.

THEORY

The relationship between transmitted power P_t, and received power P_r, of a radar system is given by the equation

$$\frac{P_r}{P_t} = \frac{G^2 \lambda^2}{(4\pi)^3 R^4} \sigma \tag{1}$$

where G is the antenna gain, λ the operating wavelength, and R is the distance between receiver to target. The quantity σ is called the radar cross section and is given by the equation

$$\sigma = \frac{4\pi}{\lambda^2} \left| \int_S \epsilon^{-i2\kappa z} \, dA \right|^2 \tag{2}$$

where $\kappa = 2\pi/\lambda$.

These two equations were developed by Kerr[12] for a uniform plane wave incident on a perfect conducting surface placed in a vacuum environment. Equation (2) is written for either horizontal or vertical polarizations, where effects due to other polarizations enter in the form of a multiplicative factor as given in Reference 12. The z axes (refer to Fig. 1) corresponds to the direction of transmitted and received power and the differential area dA is that of the scattering surface projected on a plane transverse to the z axes.

The radar cross section was derived using the "tangent plane" approximation, which physically assumes small wavelengths compared with the radius of curvature of the scattering surface. Hence, scattering from sharp discontinuities are not adequately treated using this formulation. Also, multiple scattering and shadowing effects are neglected in this approximation. These latter effects are especially important near grazing incidence so that the theory developed here does not apply for angles approaching grazing.

Equation (2) is applied to the case of a rough conducting surface resulting from perturbations of a flat surface. The geometry used is shown in Figure 2,

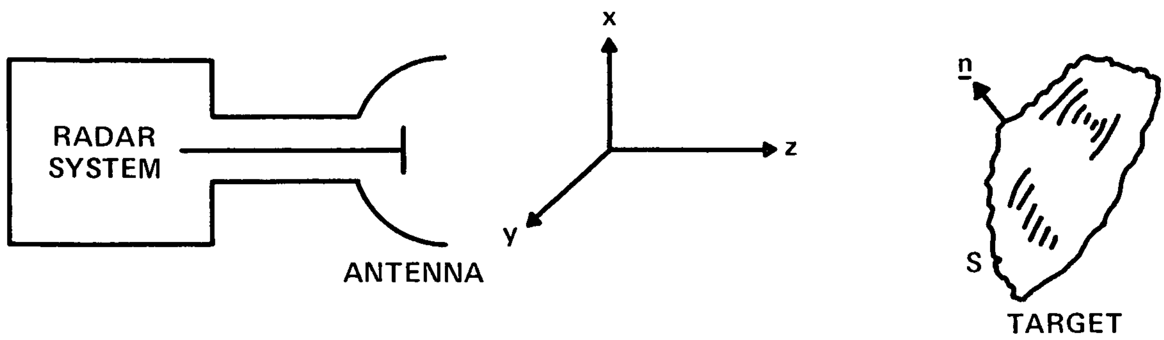

Figure 1. Schematic Diagram of Radar System in Free Space

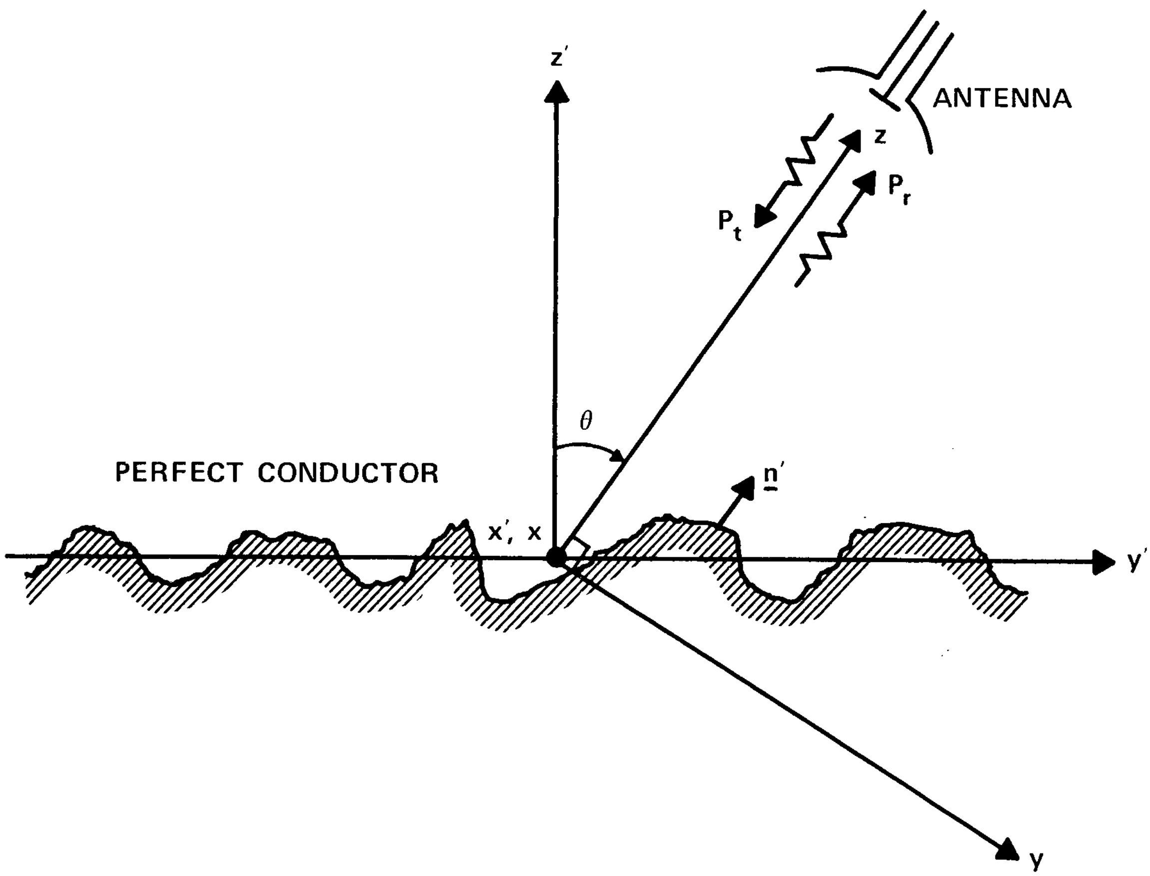

Figure 2. Backscatter from a Perfectly Conducting Rough Surface

where the x, y, z coordinate system is defined by the direction of transmitted-received radiation (z axes) and the transverse plane xy on which the scattering surface is projected. The second set of coordinates x', y', z' is defined such that the x'y' plane contains the unperturbed conducting surface whose normal direction is z'.

Let γ (x', y') be the equation of the surface height at the location x', y'.

The differential surface area dA is computed as follows:

$$dA = \underline{a}_z \cdot \underline{n}' dS', \quad \underline{n}' = \frac{\nabla' [z' - \gamma(x', y')]}{|\nabla' [z' - \gamma(x', y')]|}$$

so that,

$$\underline{n}' = \frac{\underline{a}_z' - \underline{a}_x' \frac{\partial \gamma}{\partial x'} - \underline{a}_y' \frac{\partial \gamma}{\partial y'}}{\sqrt{1 + \left(\frac{\partial \gamma}{\partial x'}\right)^2 + \left(\frac{\partial \gamma}{\partial y'}\right)^2}}$$

also,

$$dS' = \sqrt{dx'^2 + dz'^2} \sqrt{dy'^2 + dz'^2} = \sqrt{1 + \left(\frac{\partial \gamma}{\partial x'}\right)^2} \sqrt{1 + \left(\frac{\partial \gamma}{\partial y'}\right)^2} \; dx' dy'$$

Hence,

$$dA = \left(\cos\theta - \frac{\partial \gamma}{\partial y'} \sin\theta \right) \frac{\sqrt{1 + \left(\frac{\partial \gamma}{\partial x'}\right)^2} \sqrt{1 + \left(\frac{\partial \gamma}{\partial y'}\right)^2}}{\sqrt{1 + \left(\frac{\partial \gamma}{\partial x'}\right)^2 + \left(\frac{\partial \gamma}{\partial y'}\right)^2}} dx' dy'$$

where for small slopes, i.e., $\left\{ \frac{\partial \gamma}{\partial x'}; \frac{\partial \gamma}{\partial y'} \right\} \ll 1$,

$$dA = \left(\cos\theta - \frac{\partial \gamma}{\partial y'} \sin\theta \right) \; dx' \, dy' \tag{3}$$

It is observed that equation (3) is exact for one dimensional surface variations. Substituting (3) into (2), together with $z = \gamma(x', y') \cos\theta + y' \sin\theta$, we obtain

$$\sigma = \frac{4\pi}{\lambda^2}\left|\iint_{-\infty}^{\infty}\left(\cos\theta - \frac{\partial\gamma}{\partial y'}\sin\theta\right)\epsilon^{-i2\kappa\,(y'\sin\theta + \gamma\cos\theta)}\,dx'dy'\right|^2 \tag{4}$$

Equation (4) is the radar cross section for a rough surface whose height variation is given by the function γ (x', y'). The function γ is a measure of the perturbation from a flat surface and may be considered a deterministic quantity (eg; sinusoidal reflecting grating) or a random variable (eg; sea surface).

For large reflecting surfaces in comparison to the antenna coverage, the integrand in (4) must contain the gain function of the antenna. For simplicity, we will consider uniform coverage for $y' = \pm L/2$, $x' = \pm L/2$ and zero coverage outside this range. Hence, the integration limits are reduced as follows:

$$\sigma = \frac{4\pi}{\lambda^2}\left|\iint_{-L/2}^{L/2}\left(\cos\theta - \frac{\partial\gamma}{\partial y'}\sin\theta\right)\epsilon^{-i2\kappa\,(y'\sin\theta + \gamma\cos\theta)}\,dx'dy'\right|^2 \tag{5}$$

where this result is simplified by integrating by parts the term

$$\iint_{-L/2}^{L/2}\frac{\partial\gamma}{\partial y'}\,\epsilon^{-i2\kappa\,(y'\sin\theta + \gamma\cos\theta)}\,dx'dy' \tag{6}$$

which can be written as,

$$\tan\theta \iint_{-L/2}^{L/2} (1 - \epsilon^{-i2\kappa\gamma\cos\theta})\, \epsilon^{-i2\kappa y'\sin\theta}\, dx'dy' \tag{7}$$

$$+ \int_{-L/2}^{L/2} \gamma\left(\frac{\sin(\kappa\gamma\cos\theta)}{\kappa\gamma\cos\theta}\right) \epsilon^{-i2\kappa(y'\sin\theta + \frac{1}{2}\gamma\cos\theta)} \Big]_{y'=-L/2}^{y'=L/2} dx'$$

where the above result was obtained by assuming γ can be expressed as $\gamma = f(x')g(y')$. If we also assume $g(\pm L/2) = 0$, then the end term contributions are identically zero (for all angles) and equation (5) simplifies to

$$\sigma = \frac{4\pi}{\lambda^2}\left| \frac{1}{\cos\theta} \iint_{-L/2}^{L/2} \epsilon^{-i2\kappa(y'\sin\theta + \gamma\cos\theta)}\, dx'dy' \right. \tag{8}$$

$$\left. - \frac{\sin^2\theta}{\cos\theta} \iint_{-L/2}^{L/2} \epsilon^{-i2\kappa y'\sin\theta}\, dx'dy' \right|^2$$

Beckmann et al[1] applied the Kirchhoff method using the tangent plane approximation to obtain the scattering from rough surfaces. By refering to Beckmann's solution[2] of the radar cross section, it is found that the second integral term in (8) is missing. The reason for the missing term in Beckmann's result arises from the manner in which he evaluated the integral equation (6). His result, obtained by integration by parts, is[1]

$$- \tan\theta \iint_{-L/2}^{L/2} \epsilon^{-i2\kappa\,(y'\sin\theta + \gamma\cos\theta)}\,dx'dy' \tag{9}$$

$$+ \int_{-L/2}^{L/2} \left. \frac{\epsilon^{-i2\kappa\,(y'\sin\theta + \gamma\cos\theta)}}{-i2\kappa\cos\theta} \right]_{y'=-L/2}^{y'=L/2} dx'$$

where the end point contributions were simply neglected in comparison with the first term for $\kappa L > 1$. In order to obtain a more accurate result we utilized equation (7) which can be deduced from (9) by adding and subtracting the term

$$\tan\theta \int_{-L/2}^{L/2} \epsilon^{-i2\kappa y'\sin\theta}\,dx'dy'$$

As mentioned previously, equation (7) unlike (9) has zero end-term contributions when $\gamma = 0$ at $y' = \pm L/2$.

The importance of the second integral term in (8) is easily seen by considering a flat surface ($\gamma = 0$) so that we obtain the familiar result

$$\sigma = \frac{4\pi}{\lambda^2} \left| \cos\theta \iint_{-L/2}^{L/2} \epsilon^{-i2\kappa y'\sin\theta}\,dx'dy' \right|^2 = 4\pi \left(\frac{A}{\lambda}\right)^2 \left(\frac{\sin u}{u}\right)^2 \cos^2\theta \tag{10}$$

where $A = L^2$ is the area illuminated by the antenna and $u = \kappa L \sin\theta$. Had the second term been eliminated in (8), i.e., Beckmann's result, the $\cos^2\theta$ factor would be replaced by $\sec^2\theta$, resulting in an erroneous solution.

To determine the effects due to perturbations of a flat surface, the exponential term in (8) containing γ is expanded as follows:

$$\epsilon^{-i2\kappa\gamma\cos\theta} = \sum_{n=0}^{\infty} \frac{(-i2\kappa\cos\theta)^n}{n!} \gamma^n \tag{11}$$

which converges for the condition, $2\kappa\gamma \cos\theta < 1$. Substituting (11) into (8) we find

$$\sigma = \frac{4\pi}{\lambda^2} \left| \sum_{n=0}^{\infty} a_n \iint_{-L/2}^{L/2} \gamma^n \epsilon^{-i2\kappa y'\sin\theta} dx'dy' \right|^2 \tag{12}$$

where

$$a_0 = \cos\theta \text{ and } a_{n\neq 0} = \frac{(-i2\kappa\cos\theta)^n}{n!\cos\theta}.$$

This final form of σ appears as a series of Fourier transforms of γ^n. The restrictions, as mentioned previously, are summarized below,

(a) Tangent Plane Approximation

(b) $\gamma = f(x')\,g(y')$ where $g(\pm L/2) = 0$ (13)

(c) $2\kappa\gamma \cos\theta < 1$

Equation (12) is useful for both small and large perturbations as long as the conditions (13) are met. The last part of this paper will consider small perturbations and compare the resulting equation with that obtained by Wright.

SMALL PERTURBATIONS

For small perturbations, i.e., $2\kappa\gamma \cos\theta << 1$, keeping only the first two terms of the series expansion (12) we obtain

$$\sigma = \frac{4\pi}{\lambda^2} \left| \cos\theta \iint_{-L/2}^{L/2} \epsilon^{-i2\kappa y' \sin\theta}\, dx' dy' - i2\kappa \iint_{-L/2}^{L/2} \gamma \epsilon^{-i2\kappa y' \sin\theta}\, dx' dy' \right|^2 \tag{14}$$

The first term is the contribution due to the unperturbed (flat) surface (Eq. 10) and is negligable for the condition

$$|\pm\theta| > \sin^{-1} \lambda/2L \tag{15}$$

where the right half of the inequality corresponds to the first zero crossing of equation (10). Hence, for angles sufficiently far from the zenith direction, the second term of (14) predominates so that we can write approximately

$$\sigma \cong \frac{4\kappa^4}{\pi} \left| \iint_{-L/2}^{L/2} \gamma \epsilon^{-i2\kappa y' \sin\theta}\, dx' dy' \right|^2$$

This equation is similar to Wright's result[3] when applied to a perfectly conducting surface, except for the $\cos^4\theta$ multiplicative factor in Wright's solution. This difference can only be attributed to the different approximation made in Wright's analysis, i.e., the use of the zeroth order perturbation solution for the scattered field rather than the direct use of the tangent plane approximation. It is interesting to note that the $\cos^4\theta$ term can be obtained starting with equation (5) by retaining only the first order term $\cos\theta$, disregarding the derivative term $\partial\gamma/\partial y' \sin\theta$ and repeating the above analysis.

CONCLUSIONS

The simple theory developed is useful for analyzing both small and large perturbations of a flat surface, under the restrictions enumerated above. It is found that this result is more accurate in describing the backscatter from rough surfaces than Beckmann's solution. Upon comparing the small perturbation result with Wright's formulation it is seen that they generally display different angular characteristics, although both agree under certain conditions.

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