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AN APPROACH TO ATTITUDE DETERMINATION FOR A SPIN-STABILIZED SPACECRAFT (IMP I)

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16. Abstract

This report presents both the analysis and the FORTRAN program for the determination of attitude of a spin-stabilized spacecraft. The use of telemetry data that provide information about two reference vectors and their relation to the spin axis is outlined. A technique for the determination of the spin-axis orientation that employs only simple calculations is described.

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AN APPROACH TO ATTITUDE DETERMINATION FOR A SPIN-STABILIZED SPACECRAFT (IMP I)

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INTRODUCTION

This report describes a simple technique for the determination of attitude of a spin-stabilized spacecraft. Use is made of telemetry data that provide information about two reference vectors and their relation to the spacecraft spin axis. This information indicates that the angle between the spin axis and each of the reference vectors can be obtained. Consider each of these angles as the half-angle of a cone whose axis coincides with the corresponding reference vector; then, the intersection of these two cones determines the position of the spin axis. If the spin axis is represented by its three direction cosines in an inertial coordinate system, two linear equations with three unknowns can be formulated. In order to solve this problem, an additional linear equation containing the same three unknowns is needed. The derivation of the third equation is based on the generation of a new unit vector that makes a known angle with the spin axis. The spacecraft attitude is then computed by solution of these three linear equations. An application of this technique, a determination of the attitude of IMP I, is illustrated in detail, and the associated FORTRAN program is presented.

THE ATTITUDE DETERMINATION TECHNIQUE

Mathematical Considerations

Let the unit vector $\mathbf{W}(W_1, W_2, W_3)$ represent the spacecraft spin axis, and assume that $\mathbf{P}(P_1, P_2, P_3)$ and $\mathbf{Q}(Q_1, Q_2, Q_3)$ are two unit vectors with components known with respect to an inertial coordinate system. The inertial coordinate system employed is an earth-centered, right-handed, orthogonal system whose Z-axis coincides with the North Pole; the X, Y plane is the equatorial plane in which the X-axis passes through the vernal equinox. Consider the coordinate system x, y, z to be a spacecraft-centered coordinate system. If x, y, z are parallel to X, Y, Z, respectively, then the direction cosines of any vector in space are identical in both coordinate systems.

Define the angles:

$$\beta = \text{the angle (in radians) between } \mathbf{W} \text{ and } \mathbf{P} \qquad (0 \leqslant \beta \leqslant \pi)$$

$$\delta = \text{the angle (in radians) between } \mathbf{W} \text{ and } \mathbf{Q} \qquad (0 \leqslant \delta \leqslant \pi)\,.$$

If either β or δ is equal to 0 or π, the position of the spin axis in the inertial system is immediately known without further calculation. Therefore, only those cases for which both β and δ are greater than 0 and less than π will be discussed.

It should be noted that the angles β and δ are quantities measured by sensors aboard the spacecraft. Knowing β and δ, one can determine two possible orientations of the spin axis. Usually, the correct selection is made from a knowledge of the attitude control requirement. Figure 1 shows the relation of the spin axis **W** to the reference vectors **P** and **Q**.

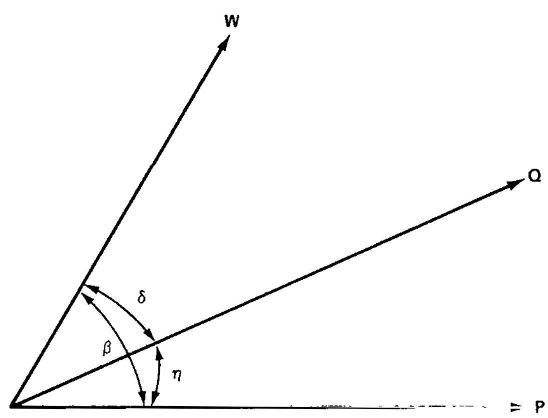

Figure 1—The spin-axis vector **W** and the reference vectors **P** and **Q**.

Since

$$\mathbf{P} \cdot \mathbf{W} = \cos\beta\ , \tag{1}$$

the spin axis must be on a cone with axis **P**. Also, since

$$\mathbf{Q} \cdot \mathbf{W} = \cos\delta\ , \tag{2}$$

the spin axis must lie on another cone with axis **Q**. The intersection of these two cones can be determined by solution of the following three equations:

$$P_1 W_1 + P_2 W_2 + P_3 W_3 = \cos\beta\ , \tag{3}$$

$$Q_1 W_1 + Q_2 W_2 + Q_3 W_3 = \cos\delta\ , \tag{4}$$

$$W_1^2 + W_2^2 + W_3^2 = 1\ . \tag{5}$$

Because Equation 5 is nonlinear, it is obvious that solution of the above three equations will involve the solution of a quadratic equation in one of the three unknowns (W_1, W_2, or W_3). The problem can be simplified if a third linear equation can be derived to replace Equation 5. The spin axis is then determined by the solution of three linear equations.

The Attitude Determination Equations

To obtain the third linear equation, first let η be the angle (in radians) between **P** and **Q**. Then, assume that the two vectors **P** and **Q** are not parallel (i.e., $0 < \eta < \pi$), and define the unit vector $\mathbf{V}(V_1, V_2, V_3)$ perpendicular to both **P** and **Q**:

$$\mathbf{V} = \frac{1}{\sin\eta}(\mathbf{P} \times \mathbf{Q})\ , \tag{6}$$

therefore,

$$V_1 = \frac{P_2 Q_3 - P_3 Q_2}{\sin\eta}\ , \tag{7a}$$

$$V_2 = \frac{P_3 Q_1 - P_1 Q_3}{\sin \eta} , \tag{7b}$$

$$V_3 = \frac{P_1 Q_2 - P_2 Q_1}{\sin \eta} . \tag{7c}$$

Let τ be the angle between **W** and **V**. Then,

$$\mathbf{V} \cdot \mathbf{W} = \cos \tau . \tag{8}$$

Equation 8 yields a third conical surface on which the spin axis must lie. (See Figure 2.) Application of spherical trigonometry to triangle *ABC* of Figure 3 yields

$$\cos \eta = \cos \beta \cos \delta + \sin \beta \sin \delta \cos \alpha ,$$

by law of cosines; hence, by law of sines,

$$\sin \chi = \frac{\sin \alpha \sin \beta}{\sin \eta} .$$

Since

$$\sin \chi = \cos \lambda' ,$$

$$\cos \lambda' = \frac{\sin \alpha \sin \beta}{\sin \eta} .$$

Therefore,

$$\begin{aligned} \cos \tau &= \sin \delta \cos \lambda' \\ &= \frac{\sin \alpha \sin \beta \sin \delta}{\sin \eta} . \end{aligned} \tag{9}$$

Consequently, W_1, W_2, and W_3 are determined by the following three equations:

$$\left. \begin{aligned} P_1 W_1 + P_2 W_2 + P_3 W_3 &= \cos \beta , \\ Q_1 W_1 + Q_2 W_2 + Q_3 W_3 &= \cos \delta , \\ V_1 W_1 + V_2 W_2 + V_3 W_3 &= \cos \tau . \end{aligned} \right\} \tag{10}$$

In matrix form, Equation 10 becomes

$$\begin{pmatrix} P_1 & P_2 & P_3 \\ Q_1 & Q_2 & Q_3 \\ V_1 & V_2 & V_3 \end{pmatrix} \begin{pmatrix} W_1 \\ W_2 \\ W_3 \end{pmatrix} = \begin{pmatrix} \cos \beta \\ \cos \delta \\ \cos \tau \end{pmatrix} . \tag{11}$$

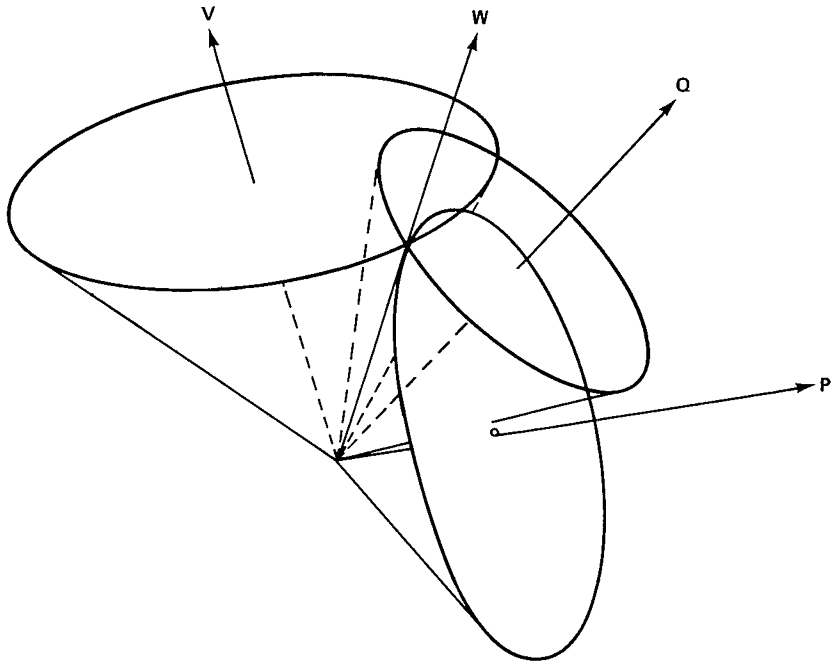

Figure 2—Spacecraft spin axis **W** and the intersection of the three cones.

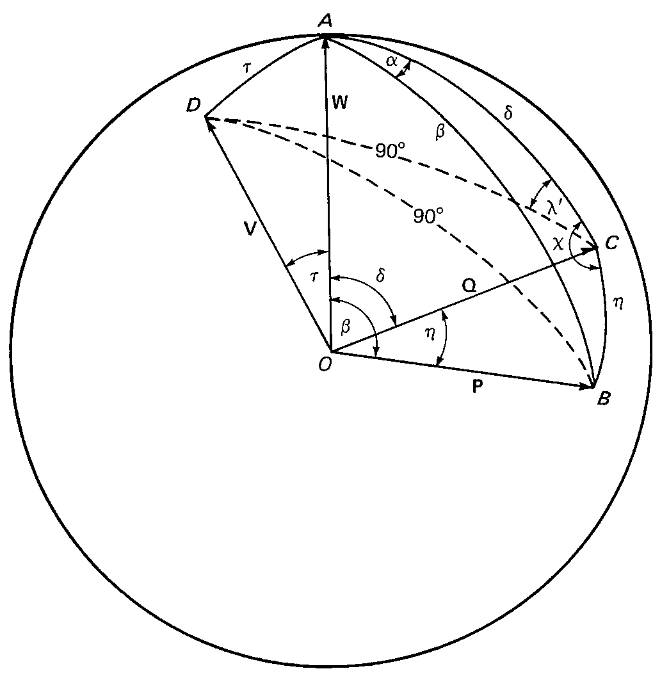

Figure 3—Unit vectors **P, Q, W,** and **V** on the celestial sphere centered at the spacecraft.

Define the determinant Δ by

$$\Delta = \begin{vmatrix} P_1 & P_2 & P_3 \\ Q_1 & Q_2 & Q_3 \\ V_1 & V_2 & V_3 \end{vmatrix} .$$

Since

$$\Delta = \sin \eta$$

$$\neq 0 ,$$

the matrix is nonsingular and possesses an inverse. Therefore,

$$\left. \begin{aligned} W_1 &= \frac{\Delta_1}{\Delta} , \\ W_2 &= \frac{\Delta_2}{\Delta} , \\ W_3 &= \frac{\Delta_3}{\Delta} , \end{aligned} \right\} \tag{12}$$

where

$$\Delta_1 = \begin{vmatrix} \cos \beta & P_2 & P_3 \\ \cos \delta & Q_2 & Q_3 \\ \cos \tau & V_2 & V_3 \end{vmatrix} ,$$

$$\Delta_2 = \begin{vmatrix} P_1 & \cos \beta & P_3 \\ Q_1 & \cos \delta & Q_3 \\ V_1 & \cos \tau & V_3 \end{vmatrix} ,$$

$$\Delta_3 = \begin{vmatrix} P_1 & P_2 & \cos \beta \\ Q_1 & Q_2 & \cos \delta \\ V_1 & V_2 & \cos \tau \end{vmatrix} .$$

If $|W_3| = 1$, the spin axis points toward either the North Pole or the South Pole. For $|W_3| \neq 1$, the right ascension R_s and declination D_s of the spin axis are

$$\left. \begin{aligned} R_s &= \tan^{-1} \frac{W_2}{W_1} \\ D_s &= \sin^{-1} W_3 . \end{aligned} \right\} \tag{13}$$

The Reference Vectors

Two sensing instruments are required for the measurement of β and δ. At present, several types of sensors have been developed for this purpose. Among them are the sun sensor, for measuring the angle between the spin axis **W** and the sun position unit vector **S**; the earth-horizon sensor, which provides the angle between **W** and the downward local vertical unit vector **L**; and the magnetometer, which provides the angle between **W** and the geomagnetic field unit **M**. If the spacecraft possesses these three types of sensing instruments, then **P** and **Q** can represent any two nonparallel vectors chosen from **S**, **L**, and **M**.

However, because of limited space, most spacecraft contain only two types of sensors. Hence, the information given can provide only two of the vectors to be used instead of three. For instance, if a sun sensor and a magnetometer are the two instruments aboard the spacecraft capable of measuring β and δ, then the sun position unit vector **S** and geomagnetic field unit **M** should be represented.

Let

SL = the angular distance of the sun from the vernal equinox, measured eastward in the ecliptic plane (see Figure 4),

i = the inclination of the ecliptic to the Equator.

The right ascension α_s and the declination δ_s of the sun can be obtained from

$$\tan \alpha_s = \cos i \tan \mathrm{SL}$$

$$\sin \delta_s = \sin i \sin \mathrm{SL} .$$

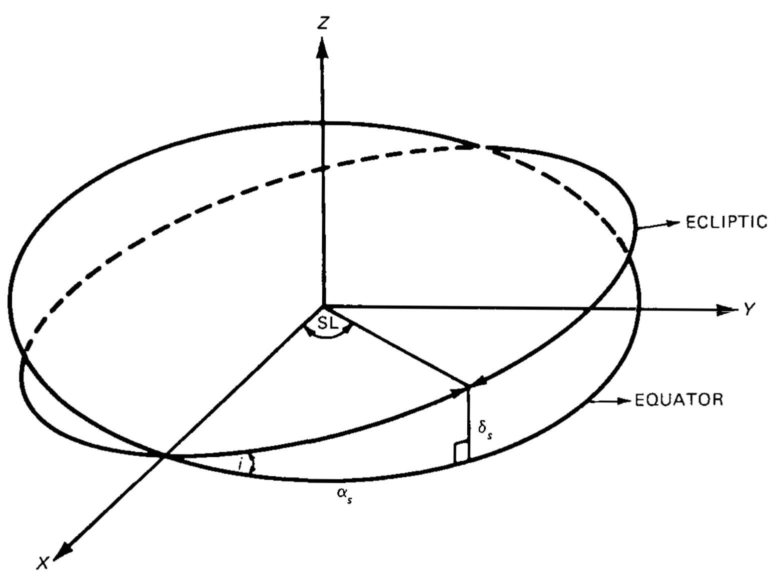

Figure 4—The sun position vector.

The components of the sun position vector **S** are

$$S_x = \cos \delta_s \cos \alpha_s ,$$

$$S_y = \cos \delta_s \sin \alpha_s ,$$

$$S_z = \sin \delta_s .$$

Since **P** = **S**, we have $P_1 = S_x$, $P_2 = S_y$, and $P_3 = S_z$.

If the sun sensor happens to be the only type of sensor providing information, the spacecraft attitude cannot be determined from a single spacecraft telemetry readout. However, if the spin axis changes only slightly, its orientation can be estimated by solution of Equations 10 with **P** as the sun position vector at time T_1 and **Q** as the sun position vector at a different time T_2, where T_1 and T_2 are days.

The spacecraft position unit vector **R** is known from orbit determination. If S_1, S_2, and S_3 are the three direction cosines of the vector **R** at the time when the magnetometer makes the measurements (see Figure 5), then

$$\mathbf{L} = -\mathbf{R} ,$$

$$\cos \sigma = \frac{S_1}{\sqrt{S_1^2 + S_2^2}} ,$$

$$\sin \sigma = \frac{S_2}{\sqrt{S_1^2 + S_2^2}} ,$$

$$\cos \nu = S_3 ,$$

$$\sin \nu = \sqrt{S_1^2 + S_2^2} .$$

Let B_x, B_y, and B_z be the components along the body axes of the earth's magnetic field measured by the magnetometer. If the spacecraft spin axis is designed to coincide with the body z-axis, the angle δ between the spin axis and the earth's magnetic field is given by

$$\delta = \cos^{-1} \frac{B_z}{\sqrt{B_x^2 + B_y^2 + B_z^2}} .$$

Since the position of the spacecraft is known, a local coordinate system with its origin O_l at that position can be defined. In this system, the x_l-axis points north, the y_l-axis points east, and the z_l-axis points down along the local vertical. (See Figure 6.) At the point O_l, the direction cosines M_N, M_E, M_V of the field vector **M** in the x_l, y_l, z_l system can be derived from actual knowledge of the field.* The components of **M** in the geocentric inertial coordinate system can therefore be obtained by the use of a coordinate transformation.

*Cain, Joseph C., Hendricks, Shirley, Daniels, Walter E., and Jensen, Duane C., "Computation of the Main Geomagnetic Field From Spherical Harmonic Expansion," GSFC Data User's Note, NSSDC 68-11, May 1968.

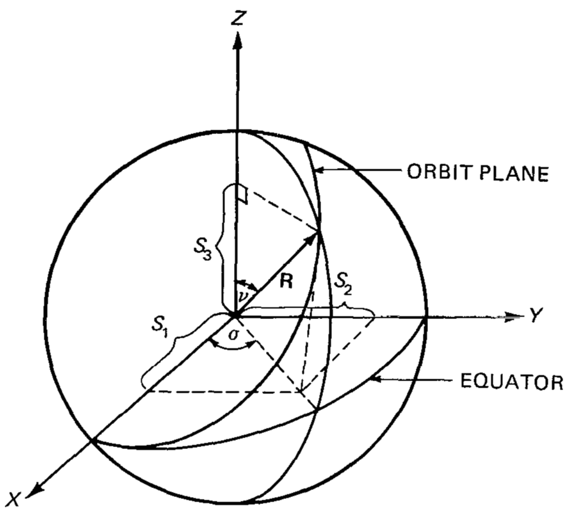

Figure 5—The spacecraft position vector **R**.

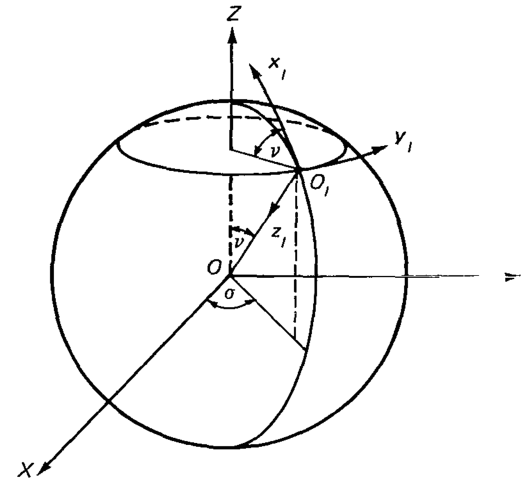

Figure 6—The x_l, y_l, and z_l axes of the local coordinate system.

The first part of the transformation is a rotation $\mathbf{R}_y(\pi - \nu)$ about the y_l-axis through an angle $\pi - \nu$. The second part is a rotation $\mathbf{R}_z(-\sigma)$ about the displaced z_l-axis through an angle $(-\sigma)$. These two rotations bring the x_l, y_l, z_l axes parallel to the X, Y, Z axes, respectively. The matrices for these two rotations are

$$\mathbf{R}_y(\pi - \nu) = \begin{pmatrix} -\cos\nu & 0 & -\sin\nu \\ 0 & 1 & 0 \\ \sin\nu & 0 & -\cos\nu \end{pmatrix}$$

$$\mathbf{R}_z(-\sigma) = \begin{pmatrix} \cos\sigma & -\sin\sigma & 0 \\ \sin\sigma & \cos\sigma & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Therefore, the transformation to obtain the direction cosines of **M** in the inertial coordinate system may be written as

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \mathbf{R}_z(-\sigma)\mathbf{R}_y(\pi - \nu)\begin{pmatrix} M_N \\ M_E \\ M_V \end{pmatrix}$$

$$= \begin{pmatrix} \cos\sigma & -\sin\sigma & 0 \\ \sin\sigma & \cos\sigma & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} -\cos\nu & 0 & -\sin\nu \\ 0 & 1 & 0 \\ \sin\nu & 0 & -\cos\nu \end{pmatrix}\begin{pmatrix} M_N \\ M_E \\ M_V \end{pmatrix}$$

or

$$\begin{pmatrix} M_1 \\ M_2 \\ M_3 \end{pmatrix} = \begin{pmatrix} -\cos\nu\cos\sigma & -\sin\sigma & -\sin\nu\cos\sigma \\ -\cos\nu\sin\sigma & \cos\sigma & -\sin\nu\sin\sigma \\ \sin\nu & 0 & -\cos\nu \end{pmatrix} \begin{pmatrix} M_N \\ M_E \\ M_V \end{pmatrix}.$$

Thus, $Q_1 = M_1$, $Q_2 = M_2$, and $Q_3 = M_3$. However, if an earth-horizon scanner is used instead of the magnetometer, then we would have $Q_1 = -S_1$, $Q_2 = -S_2$, and $Q_3 = -S_3$.

ANALYSIS OF THE IMP I DATA

Sensor Data

The IMP I onboard attitude determination system consists of a sun sensor and an earth-horizon scanner. The sun sensor measures the angle β between the spin axis **W** and the sun position unit vector **P** and triggers a counter that records the time T_S between successive sun pulses. Values of T_S are the spin periods SP. It is to be understood that the spin period cannot be determined when the spacecraft enters earth's shadow.

The horizon scanner is mounted at an angle γ from the spin axis of the spacecraft. As the spacecraft rotates, it scans a cone of half-angle γ. (For IMP I, $\gamma \approx \pi/2$ radians.) During each rotation, the scan may intersect earth. Let E_i be the point where the scan cone enters the sunlit earth disk and E_o be the point where it leaves the sunlit earth disk. (See Figure 7.) At E_i and E_o, the discontinuity between the bright earth disk and the dark background will cause the sensor to produce a positive output pulse. If T_i and T_o represent, respectively, the times at which the E_i and E_o pulses are produced with respect to the previous sun pulse, the earth width μ is defined by

$$\mu = \text{TEW}\,\frac{2\pi}{\text{SP}}, \tag{14}$$

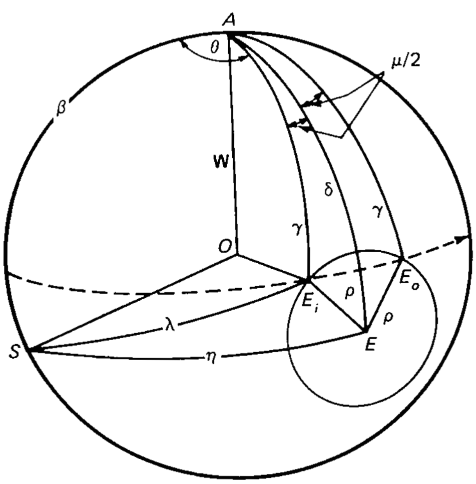

Figure 7—Case I: Full sunlit earth.

where

$$\text{TEW} = T_o - T_i .$$

During the time interval T_i, the spacecraft will rotate through an angle θ measured about the spin axis:

$$\theta = T_i\,\frac{2\pi}{\text{SP}}. \tag{15}$$

The values of TEW and T_i are telemetered to earth and used in the calculation of the nadir angle δ, the angle between the spin axis and the vector **L** from the spacecraft to the center of earth. If ρ is one-half the angle subtended by earth at the spacecraft position, then it is obvious that any telemetry data processing one or more of the following conditions should be rejected:

$$\left.\begin{aligned} &\beta < 0 \, , \\ &\beta > \pi \, , \\ &\mu > 2\rho \, . \end{aligned}\right\} \tag{16}$$

The half-angle ρ is given by the equation:

$$\sin \rho = \frac{R_e}{R_e + h} \, , \tag{17}$$

where R_e is the radius of earth and h is the altitude of the spacecraft orbit.

Since a portion of earth is always dark, a fully illuminated disk may not always be seen by the spacecraft; hence, it is necessary to determine the visibility of the terminator and the sunlit earth horizon before the nadir angle is computed. The situation will be either (1) that both E_i and E_o are at the sunlit horizon (double horizon crossing) or (2) that one of them is at the sunlit earth horizon (single horizon crossing) while the other is at the terminator (terminator crossing).

Determination of Crossings

For the determination of the crossings, the inertial coordinate system will be used. The cosine of the angle φ between the sun and the spacecraft is

$$\cos \varphi = S_x S_1 + S_y S_2 + S_z S_3 \, . \tag{18}$$

A comparison between the angles φ and ρ is required. If

$$\cos \varphi > \cos \rho \, , \tag{19}$$

the spacecraft is in sunlight, and the fully illuminated earth is seen (i.e., both E_i and E_o are sunlit horizon crossings). If

$$-\cos \rho \leqslant \cos \varphi \leqslant \cos \rho \, , \tag{20}$$

the spacecraft is in sunlight, and the terminator is visible. Therefore, one of the crossings is a terminator crossing. Unfortunately, the terminator data are not useful in the calculation of the nadir angle and are rejected. Only when the horizon is sunlit are the data acceptable.

The determination of the sunlit crossing horizon depends upon the rotation angle of the spacecraft θ. If $\theta < \pi$, E_i is at the sunlit horizon. This is because the sun position vector and the vector from earth's center to E_i form an acute angle. If $\theta > \pi$, then E_o is at the sunlit horizon. If

$$\cos \varphi < -\cos \rho \, , \tag{21}$$

the spacecraft is in earth's shadow, and the data are rejected.

Computation of the Nadir Angle δ

Two different techniques are used to obtain δ: one for the case of a fully sunlit earth and the other for the case when the terminator is visible. For both cases, the coordinate system centered at the spacecraft will be employed.

Case I: Solution Procedure for δ When Earth Is Fully Visible

Consider the planes WE, WE_i, and WE_o that are defined by the vector **W** and the points E, E_i, and E_o, respectively. The angle between the planes WE_i and WE_o is μ. If the spacecraft is spinning without precession, the plane WE bisects μ. Thus, the angle δ is related to the measured angles μ and ρ, as can be seen by application of the law of cosines to the spherical triangle AE_iE:

$$\cos \rho = \cos \gamma \cos \delta + \sin \gamma \sin \delta \cos \frac{\mu}{2} \ , \tag{22}$$

since

$$\sin^2 \delta = 1 - \cos^2 \delta \ . \tag{23}$$

If $\gamma = \pi/2$, it follows from Equation 22 that

$$\delta = \sin^{-1} \frac{\cos \rho}{\cos \mu/2} \ . \tag{24}$$

For $\gamma \neq \pi/2$, Equations 22 and 23 can be solved for $\cos \delta$. However, another equation in δ can be obtained if the law of cosines is applied to the spherical triangle ASE:

$$\cos \eta = \cos \beta \cos \delta + \sin \beta \sin \delta \cos \left(\theta + \frac{\mu}{2}\right) . \tag{25}$$

Therefore, for $\gamma \neq \pi/2$, Equations 23 and 25 can also be used to evaluate $\cos \delta$.

It should be noted that Equations 22 and 25 are linear in $\sin \delta$ and $\cos \delta$. Therefore, one can obtain δ by simply solving these two equations simultaneously:

$$\sin \delta = \frac{\cos \beta \cos \rho - \cos \gamma \cos \eta}{\Omega} \tag{26}$$

$$\cos \delta = \frac{\cos \eta \sin \gamma \cos \mu/2 - \cos \rho \sin \beta \cos (\theta + \mu/2)}{\Omega} \ , \tag{27}$$

where

$$\Omega = \cos \beta \sin \gamma \cos \frac{\mu}{2} - \sin \beta \cos \gamma \cos \left(\theta + \frac{\mu}{2}\right) \neq 0 \ ,$$

we have

$$\delta = \tan^{-1} \frac{\cos \beta \cos \rho - \cos \gamma \cos \eta}{\cos \eta \sin \gamma \cos \mu/2 - \cos \rho \sin \beta \cos (\theta + \mu/2)} \ . \tag{28}$$

Equation 28 will satisfy Equation 23 if there are no errors in the values of measured quantities. However, the values of the measured quantities β, ρ, θ, and μ will contain errors, and the measurement errors in ρ and μ might be different from those in β and θ. Consequently, the solution of Equations 22 and 23 might not satisfy Equation 25 (or the solution of Equations 23 and 25 might not satisfy Equation 22). In other words, even with identical input data, the final attitude solutions are expected to have slightly different values if the equations being used in the computations are not exactly the same.

Case II: Solution Procedure for δ When the Terminator Is Visible

If the spacecraft is in a position where the terminator is visible, the angle μ is no longer bisected by the plane *WE*, and the Case I method cannot be employed for the determination of δ. Therefore, another method will be developed for this situation.

Suppose that $0 < \theta < \pi$ and that the point E_i is at the sunlit horizon. Let *WS* be the plane containing the spin axis and the sun. Define a horizon plane WE_i (a plane containing both the spin axis and a point on the sunlit horizon). If ϕ is defined as the smaller of the two angles between planes *WS* and WE_i, then

$$\phi = \theta \; . \tag{29}$$

If λ and η are two great circle arcs extending from the sun *S* to the points E_i and *E*, respectively (see Figure 8), then

$$\cos \lambda = \cos \beta \cos \gamma + \sin \beta \sin \gamma \cos \phi \; . \tag{30}$$

The value of λ must satisfy the following condition:

$$\eta - \rho \leqslant \lambda \leqslant \psi \; , \tag{31}$$

where ψ is the hypotenuse of the right spherical triangle *SEC*, which has legs ρ and η. (See Figure 9.) The value of ψ is determined by the equation

$$\cos \psi = \cos \rho \cos \eta \; . \tag{32}$$

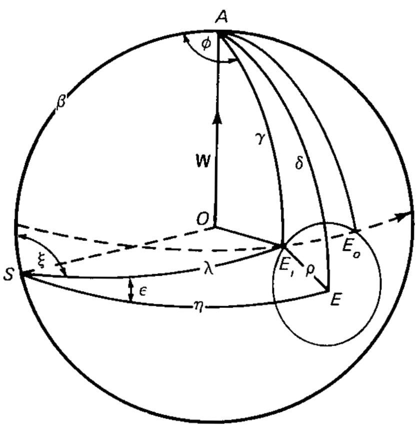

Figure 8—Case II: Terminator visible.

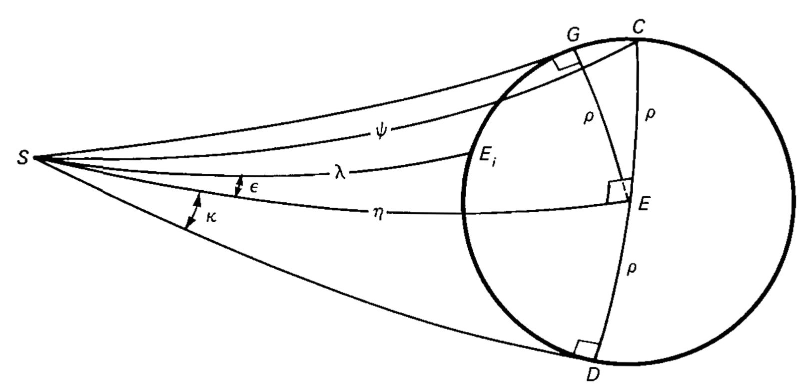

Figure 9—Range of λ and ϵ.

The angle ξ between arcs β and λ of the spherical triangle SE_iA in Figure 8 can be found by application of the law of sines:

$$\sin \xi = \frac{\sin \phi \sin \gamma}{\sin \lambda}\ , \tag{33}$$

where

$$0 \leqslant \xi \leqslant \pi\ .$$

Next, the angle ϵ between arc λ and η of the spherical triangle SE_iE is evaluated by the use of the equation

$$\cos \epsilon = \frac{\cos \rho - \cos \lambda \cos \eta}{\sin \lambda \sin \eta}\ . \tag{34}$$

Since

$$|\cos \lambda| \leqslant 1$$

we have

$$\cos \rho \geqslant \cos \eta\ .$$

Hence, the value of $\cos \epsilon$ determined by Equation 34 can never be negative. The range of ϵ is

$$0 \leqslant \epsilon \leqslant \kappa\ . \tag{35}$$

This can be seen in Figure 9, which shows that the arc SG of the right spherical triangle SGE is tangent to earth. The angle κ is obtained by the use of the equation

$$\kappa = \sin^{-1} \frac{\sin \rho}{\sin \eta} \tag{36}$$

The computed values of the angles λ and ϵ must satisfy Equations 31 and 35, respectively, or the data are rejected, and no further computations are made with them.

The nadir angle δ is determined by the following equations:

$$\cos \delta = \cos \beta \cos \eta + \sin \beta \cos (\xi + \epsilon) \sin \eta \qquad \text{(for } \epsilon \text{ not included in } \xi\text{—see Figure 8)} \tag{37}$$

$$\cos \delta = \cos \beta \cos \eta + \sin \beta \cos (\xi - \epsilon) \sin \eta \qquad \text{(for } \epsilon \text{ included in } \xi\text{—see Figure 10).} \tag{38}$$

The range of δ is

$$\gamma - \rho \leqslant \delta \leqslant \gamma + \rho\ . \tag{39}$$

When θ is in the range $\pi < \theta < 2\pi$, E_o, where the scanning cone crosses the sunlit horizon, becomes the point of interest. The horizon plane in this case would be WE_o, and the viewing angle ϕ is given by

$$\phi = 2\pi - (\theta + \mu)\ . \tag{40}$$

The solution procedure for this case is similar to that already presented and will not be covered here.

Once the nadir angle δ is known, the attitude of the spacecraft can be determined with the aid of the technique described in the first section of this paper.

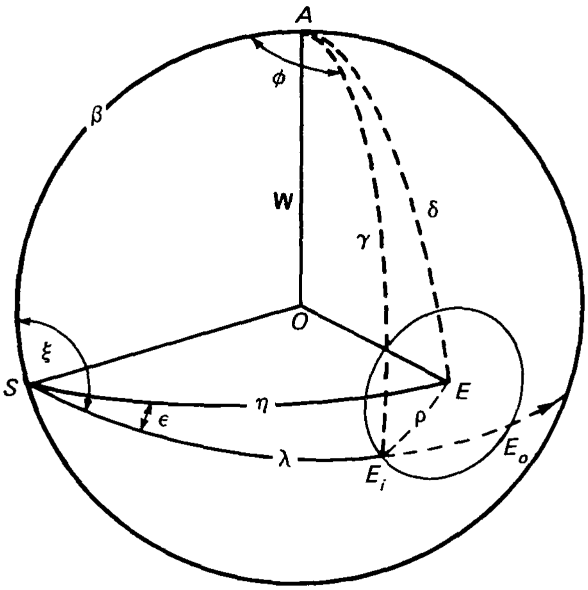

Figure 10—Case II, when ϵ is included in ξ.

FORTRAN PROGRAM*

Subroutine SPIN

Based on the analysis developed in this report, a FORTRAN subroutine, SPIN, was developed for the computation of the components of the spin axis. The FORTRAN listing of this subroutine is given in Appendix B. The subroutine is referenced by the following statement:

CALL SPIN (P, Q, BETA, DETA, W, U, IN) ,

where the input variables are

P = reference vector with components P(1), P(2), P(3),

Q = reference vector with components Q(1), Q(2), Q(3),

BETA = angle β, between spin axis and vector P,

DETA = angle δ, between spin axis and vector Q,

and the output parameters are

W = spin-axis orientation vector with components W(1), W(2), W(3),

U = spin-axis orientation vector with components U(1), U(2), U(3),

IN = 0, if the spacecraft attitude cannot be determined,

= 1, if there is only one solution W for the spin-axis orientation [the components W(1), W(2), W(3) of W are computed],

= 2, if there is another solution U in addition to W for the spin-axis orientation [the components U(1), U(2), U(3) of U are computed].

Generally, there will be more than one attitude solution. The correct solution will be chosen by means of a comparison with an a priori estimate of spin-axis orientation. The solution that is closest to the estimate is chosen.

The main program provides—

(1) The input parameters P, Q, BETA, and DETA before the subroutine SPIN is called

(2) Right ascension and declination, if they are desired, after subroutine SPIN is called when IN $\neq$ 0 and $|W(3)| \neq 1$ [and/or $|U(3)| \neq 1$ if IN = 2]

*See Appendix A for a list of the FORTRAN symbols.

(3) The correct solution for the spin-axis orientation when IN = 2

Attitude Data Reduction and Attitude Determination for IMP I

For the spin-stabilized spacecraft IMP I, a FORTRAN program is used to reduce the input attitude data and compute the nadir angle. The subroutine SPIN (P, Q, BETA, DETA, W, U, IN) is then called to determine the spacecraft attitude. Since the telemetry data obtained from IMP I are quite abundant, it is necessary to select suitable values for the input. In the implementation of this selection procedure, several conditions are imposed that test the available telemetry data. All data that fail to satisfy these conditions would yield spurious attitude solutions and are therefore rejected. The FORTRAN program listing given in Appendix C is intended to conserve computer time and provide the correct spin-axis orientation. The input parameters are

Card deck 1: ID, IDAY, IH, IM, IMS, IET, IEW, SPP, BETAD

FORMAT (I1, I4, 2I3, I6, 5X, 2I5, 5X, 2F8.2)

Card deck 2: SP, SQ, ST, SX, SY, SZ

FORMAT (13X, 3F12.3, 3F10.7) ,

where

BETAD = angle between **W** and **P** (degrees),

ID = option for control to read the input card deck (when ID = 0, the attitude data are read continuously until ID $\neq$ 0,

IDAY, IH, IM, IMS = time (days, hours, minutes, and seconds, respectively),

IET = time of occurrence of E_i-pulse with respect to previous sun pulse,

IEW = time between E_i and E_o pulses,

SPP = spin period of the spacecraft,

SP, SQ, ST = components of spacecraft position vector,

SX, SY, SZ = components of sun position unit vector.

The output parameters are the components of all attitude solutions together with the right ascension and declination of the selected spin-axis orientation. A sample of the output is shown in Appendix D.

CONCLUSION

This report has demonstrated how telemetry data from IMP I are used in a determination of the attitude of that spacecraft. The technique can be applied to any spin-stabilized spacecraft having any two of the following three instruments aboard: sun sensor, magnetometer, and earth-horizon sensor. Examples of such spacecraft would include ISIS 1 and SSS 1.

ACKNOWLEDGMENTS

The author would like to thank Mr. Clyde H. Freeman for valuable discussions during the course of this project, Dr. David L. Blanchard and Mr. Cyrus J. Creveling for reviewing this report, and Mr. Gene A. Smith for assisting in the FORTRAN programming.

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National Aeronautics and Space Administration
Greenbelt, Maryland, November 15, 1971
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Appendix A

FORTRAN Symbols

Mathematical Symbol	FORTRAN Symbol
R_e	RE
γ (degrees)	GAMAD
$\sin \gamma$	SGAMA
$\cos \gamma$	CGAMA
μ (degrees)	EW
$\cos \mu/2$	CEW
β (degrees)	BETAD
β (radians)	BETA
$\sin \beta$	SB
$\cos \beta$	CB
θ (degrees)	ET
$\cos (\theta + \mu/2)$	CETWH
ρ (radians)	ROM
$\sin \rho$	SR
$\cos \rho$	CR
$\sin \phi$	SA
$\cos \phi$	CA
η (degrees)	ETA
$\sin \eta$	SE
$\cos \eta$	CE
ψ (degrees)	XD
$\cos \psi$	CKX

Mathematical Symbol	FORTRAN Symbol
δ (radians)	DETA
$\sin \delta$	SDTA
λ (degrees)	CKD1
$\sin \lambda$	SK1
$\cos \lambda$	CK1
ϵ (degrees)	EK1
Ω	DNUM
$\cos \epsilon$	CEK1
κ (degrees)	SED
ξ (degrees)	BK
$\sin \xi$	SBK
$\cos \varphi$	CSES
φ (degrees)	SESD
$\cos \tau$	CETA
$\cos \alpha$	FA
Δ	DTA
Δ_1	DT1
Δ_2	DT2
Δ_3	DT3

Appendix B

FORTRAN Listing of Subroutine Spin

```
      SUBROUTINE SPIN(P,Q,BETA, DETA,W,U,IN)
C
C     * IF   IN=0,  THERE IS NO SOLUTION FOR ATTITUDE DETERMINATION
C     *             EITHER BECAUSE P IS PARALLEL TO Q OR BECAUSE THERE
C                   IS ERROR IN MEASUREMENT
C     * IF   IN=1   ONE SOLUTION FOR SPIN AXIS ORIENTATION.THE COMPONENTS
C                   OF SPIN AXIS W(1),W(2),AND W(3) ARE THEN COMPUTED.
C     * IF   IN=2   TWO SOLUTIONS FOR SPIN AXIS ORIENTATION . THE SECOND
C                   SET OF COMPONENTS U(1),U(2),AND U(3) ARE COMPUTED
C                   IN ADDITION TO THE FIRST SET W(1),W(2),AND W(3).
C
      DIMENSION P(3),Q(3),W(3),U(3)
      DIMENSION A(3),B(3),C(3),D(3),V(3)
   50 FORMAT (2X, 6F 12.8)
C     ER IS THE ERROR LIMIT WHICH CAN BE MODIFIED.
      ER=0.00001
C
      IN=1
      SBETA=SIN(BETA)
      CBETA=COS(BETA)
      IF(SBETA.LE.ER) GO TO 100
      SDETA=SIN(DETA)
      CDETA=COS(DETA)
      IF(SDETA.LE.ER) GO TO 140
C
      CGEMA=P(1)*Q(1)+P(2)*Q(2)+P(3)*Q(3)
      ACMA=ABS(CGEMA)
      IF(ACMA .GE. 1.0) GO TO 180
      GEMA=(ACOS(CGEMA))/0.01745329
      SGEMA=SQRT(1.0-CGEMA*CGEMA)
      PRINT 52,GEMA
   52 FORMAT(2X,' ANGLE BETWEEN TWO GIVEN VECTORS P AND Q ARE',F12.5)
C
      IF((1.0-ACMA).LE.ER) GO TO 200
      AD=COS(BETA+DETA)
      IF(AD.GT.CGEMA) GO TO 55
      IF((CGEMA-AD).LE.ER) GO TO 65
C
      FA=(CGEMA-CBETA*CDETA)/(SBETA*SDETA)
      IF(ABS(FA).GT.1.0) GO TO 60
      CETA=(SQRT(1.0-FA*FA))*SBETA*SDETA/SGEMA
      IF(ABS(CETA).LE.ER) GO TO 65
      GO TO 78
C
   55 IF((AD-CGEMA).LE.ER) GO TO 65
      PRINT 58,AD,CGEMA
```

```
   58 FORMAT(2X,'COS(BETA+DETA)=',F10.7,', COS(GEMA)=',F9.7,'ERROR')
      GO TO 290
C
   60 IF((ABS(FA)-1.0).GT.ER) GO TO 70
   65 CETA=0.0
      GO TO 80
   70 PRINT 75 ,FA
   75 FORMAT(2X,'ERROR IN COS(ALFA)=', F11.8)
      GO TO 290
C
   78 IN=2
   80 CALL CROSS(P,Q,V)
      DO 90 I=1,2
      A(1)=P(1)
      A(2)=Q(1)
      A(3)=V(1)
      B(1)=P(2)
      B(2)=Q(2)
      B(3)=V(2)
C
      C(1)=P(3)
      C(2)=Q(3)
      C(3)=V(3)
      D(1)=CBETA
      D(2)=CDETA
      D(3)=CETA
C
      DTA=DET(A,B,C)
      DT1=DET(D,B,C)
      DT2=DET(A,D,C)
      DT3=DET(A,B,D)
      W(1)=DT1/DTA
      W(2)=DT2/DTA
      W(3)=DT3/DTA
C
      IF(IN.EQ.1) GO TO 300
      V(1)=-V(1)
      V(2)=-V(2)
      V(3)=-V(3)
      U(1)=W(1)
      U(2)=W(2)
      U(3)=W(3)
C
      PRINT 82,I ,U(1),U(2),U(3)
   82 FORMAT(2X,'SOLUTION ',I2,', COMPONENTS OF SPIN AXIS ARE  ',3F12.5)
   90 CONTINUE
      GO TO 500
C
  100 PRINT 120
  120 FORMAT(2X,'SPIN AXIS IS PARALLEL TO VECTOR  P')
      IF(CBETA.GT.0.0) GO TO 130
C
      W(1)=-P(1)
      W(2)=-P(2)
      W(3)=-P(3)
      GO TO 300
```

```
C
  130 W(1)=P(1)
      W(2)=P(2)
      W(3)=P(3)
      GO TO 300
C
  140 PRINT 150
  150 FORMAT(2X,'SPIN AXIS IS PARALLEL TO VECTOR  Q')
      IF(CDETA.GT.0.0) GO TO 160
      W(1)=-Q(1)
      W(2)=-Q(2)
      W(3)=-Q(3)
      GO TO 300
C
  160 W(1)=Q(1)
      W(2)=Q(2)
      W(3)=Q(3)
      GO TO 300
C
  180 IF ((ACMA-1.0) .LE.ER) GO TO 200
      PRINT 185 ,CGEMA
  185 FORMAT(2X,'ERROR IN  COS(GEMA)=', F11.8)
      GO TO 290
C
  200 PRINT 250
  250 FORMAT(2X,'P AND Q ARE PARALLEL')
  290 IN=0
C
      PRINT 295,IN
  295 FORMAT(2X,'   IN=',I2,'*****NO SOLUTION*****')
      GO TO 500
C
  300 PRINT 82,IN,W(1),W(2),W(3)
  500 RETURN
      END



      SUBROUTINE CROSS(P,Q,W)
C
C     P AND Q ARE DIRECTION COSINES OF TWO UNIT VECTORS
C     W REPRESENTS THE UNIT VECTOR IN THE DIRECTION OF (P)X(Q)
      DIMENSION P(3), Q(3) ,W(3)
      G1=P(2)*Q(3)-P(3)*Q(2)
      G2=P(3)*Q(1)-P(1)*Q(3)
      G3=P(1)*Q(2)-P(2)*Q(1)
      SQ=SQRT (G1*G1+G2*G2+G3*G3)
      W(1)=G1/SQ
      W(2)=G2/SQ
      W(3)=G3/SQ
      RETURN
      END
```

```
      SUBROUTINE DIRC(RA,DE,D)
C
C     RA IS THE RIGHT ASCENSION.
C     DE IS THE DECLINATION.
C     RA AND DE ARE IN RADIANS.
C     DETERMINE THE DIRECTION COSINES
C
      DIMENSION D(3)
      D(1)=(COS(RA))*(COS(DE))
      D(2)=(SIN(RA))* (COS(DE))
      D(3)=SIN(DE)
      RETURN
      END
```

```
      FUNCTION DET(A, B, C)
C
      DIMENSION A(3), B(3), C(3)
C     COMPUTE THE DETERMINANT OF 3X3 MATRIX
      CF1=B(2)*C(3)-B(3)*C(2)
      CF2=B(3)*C(1)-B(1)*C(3)
      CF3=B(1)*C(2)-B(2)*C(1)
      DET=A(1)*CF1+A(2)*CF2+A(3)*CF3
      RETURN
      END
```

Appendix C

FORTRAN Listing of Attitude Determination Program for IMP I

```
C     THIS PROGRAM IS WRITTEN FOR UNIVAC 1108 COMPUTER   BY A. C. FANG
      DIMENSION P(3),Q(3),W(3),U(3)
      DATA RAD,TPI/0.01745329,6.2831853/
      DATA RE/6378.388/
      DATA GAMAD/90.0/
      DATA ERR,BEAM/0.10,3.0/
C     RE IS THE RADIUS OF THE EARTH,ERR IS THE ERROR LIMIT.
C     BEAM--THE FIELD OF VIEW OF THE HORIZON SCANNER.
C
      SGAMA=SIN(GAMAD*RAD)
      CGAMA=COS(GAMAD*RAD)
      SPX=0.0
      SPY=COS(66.55*RAD)
      SPZ=-SIN(66.55*RAD)
C     SPX,SPY,SPZ ARE THE COMPONENTS OF DESIRED SPIN AXIS ORIENTATION.
C
    1 READ 4,ID,IDAY,IH,IM,IMS,IET,IEW,SPP,BETAD
    4 FORMAT(I1,I4,2I3,I6,5X,2I5,5X,2F8,2)
      IF(ID.NE.0) GO TO 150
      TSEC=IH*3600+IM*60+(IMS/1000)
      EW=IEW*360/SPP-BEAM
C     SPP IS THE SPIN PERIOD.
      ET=IET*360/SPP
      BETA=BETAD*RAD
      SB=SIN(BETA)
      CB=COS(BETA)
C
      PRINT 6
    6 FORMAT('1')
      PRINT 8,IDAY,TSEC
    8 FORMAT(//,2X,'TIME OF TELEMETRY DATA IS, DAY= ',I3,' SEC= ',F10.2,
     1' -H M S ')
      PRINT 9,SPP
    9 FORMAT(2X,'THE SPIN PERIOD OF SPACECRAFT IS ',F9.2,' MSEC')
      PRINT 10,SPX,SPY,SPZ
   10 FORMAT(2X,'THE DIRECTION COSINES OF DESIRED SPIN AXIS ARE ',3F9.5)
C
      READ 16,SP,SQ,ST,SX,SY,SZ
      PRINT 14,SP,SQ,ST
   14 FORMAT(2X,'THE COMPONENTS OF SPACECRAFT POSITION VECTOR IN K.M.ARE
     1     ',3F18.5)
   16 FORMAT(13X,3F12.3,3F10.7)
      PRINT 17,SX,SY,SZ
   17 FORMAT(2X,'THE DIRECTION COSINES OF SUN LINE VECTOR ARE  ',3F15.5)
      PRINT 18,BETAD
   18 FORMAT(2X,'THE ANGLE BETWEEN THE SPIN AXIS AND THE SUN POSITION
     1VECTOR IS',F9.3,' DEG.')
```

```
C     COMPUTE THE DIRECTION COSINES OF SPACECRAFT POSITION VECTOR
      SQR=SP*SP+SQ*SQ+ST*ST
      SRR=SQRT(SQR)
      S1=SP/SRR
      S2=SQ/SRR
      S3=ST/SRR
C
      SR=RE/SRR
      ROM=ASIN(SR)
      CR=COS(ROM)
      ROMD=ROM/RAD
      PRINT 19,EW,ROMD
   19 FORMAT(2X,'EARTH WIDTH IS ',F9.4,' DEG.,  HALF ANGLE SUBTENDED BY
     1 EARTH IS  ',F9.4,'  DEG.')
      EWH=EW/2.0
      CEW=COS(EWH*RAD)
      IF(EWH.LE.ROMD) GO TO 22
      PRINT 20
   20 FORMAT(//,2X,'THE EARTH WIDTH IS LARGER THAN THE ANGLE SUBTENDED
     1BY TNE EARTH. NO FURTHER COMPUTATION.')
      GO TO 1
C
C     COMPUTE EQUATION (18)
   22 CSES=S1*SX+S2*SY+S3*SZ
      SESD=ACOS(CSES)/RAD
      Q(1)=-S1
      Q(2)=-S2
      Q(3)=-S3
      PRINT 23,Q(1),Q(2),Q(3)
   23 FORMAT(2X,'THE DIRECTION COSINES OF DOWNWARD LOCAL VERTICAL ARE ',
     13F15.5)
C
C     CHECK THE TWO REFERENCE VECTORS IF THEY ARE PARALLEL
      IF(ABS(CSES).LT.1.0) GO TO 25
      PRINT 24
   24 FORMAT(2X,'NO SOLUTION')
      GO TO 1
C
   25 CE =SX*Q(1)+SY*Q(2)+SZ*Q(3)
      ETA=ACOS(CE)/RAD
      SE=SIN(ETA*RAD)
      PRINT 27,ET
   27 FORMAT(2X,'THE VIEWING ANGLE AT THE SPACECRAFT FROM THE SUN TO THE
     1 EARTH IS ',F9.2,' DEG.')
      PRINT 28,ETA
   28 FORMAT(2X,'THE ANGLE BETWEEN THE SUN AND THE LOCAL VERTICAL IS  ',
     1F9.2,' DEG.')
   29 FORMAT(2X,10F13.8)
C
      P(1)=SX
      P(2)=SY
      P(3)=SZ
C     COMPUTE EQUATION (32)
      CKX=CE*CR
      XD=ACOS(CKX)/RAD
C
C     CHECK THE CONDITION FOR DETERMINATION OF CROSSINGS
      IF(CSES.GE.CR) GO TO 33
```

```
      IF(CSES.GE.(-CR)) GO TO 37
      PRINT 30
   30 FORMAT(//,2X,'THE SPACECRAFT IS IN THE EARTH SHADOW')
      GO TO 1
C
C
   33 PRINT 34
   34 FORMAT(//,2X,'THE FULL EARTH IS SEEN')
      IF(GAMAD.EQ.90.0) GO TO 36
C
C     FOR GAMAD NOT EQUAL TO  90.0,  WHEN FULL EARTH IS SEEN
      ETWH=(ET+EWH)*RAD
      CETWH=COS(ETWH)
      DNUM=CB*SGAMA*CEW-SB*CGAMA*CETWH
C
      IF(DNUM.EQ.0.0) GO TO 49
C     COMPUTE EQUATION (28)
      DETA=ATAN2((CB*CR-CGAMA*CE),(CE*SGAMA*CEW-CR*SB*CETWH))
      GO TO 39
C
C     FOR GAMAD EQUAL TO  90.0, WHEN FULL EARTH IS SEEN
C     COMPUTE EQUATION (24)
   36 SDTA=CR/CEW
      DETA=ASIN(SDTA)
   39 DETAD=DETA/RAD
      JS=1
      GO TO 54
C
   37 PRINT 38
   38 FORMAT(//,2X,'AN EARTH TERMINATOR IS VISIBLE')
C     CHECK THE VIEWING ANGLE,WHEN TERMINATOR IS VISIBLE
      IF(ET.LT.180.0) GO TO 43
C
      EC=(ET+EW)*RAD
      CA=COS(TPI-EC)
      SA=SIN(TPI-EC)
      GO TO 45
C
   43 CA=COS(ET*RAD)
      SA=SIN(ET*RAD)
C     COMPUTE EQUATION (30)
   45 CK1=CB*CGAMA+SB*SGAMA*CA
      ERM=COS((ETA-ROMD)*RAD)
      IF (CK1.LT.ERM)GO TO 48
      IF(ABS(CK1-ERM).LT.ERR) GO TO 50
C
      PRINT 46,CK1,ERM
   46 FORMAT(/,2X,'ERROR IN CK1=',F5.2,' ERM=',F6.2)
   49 PRINT 47
   47 FORMAT(2X,'ERROR , NO SOLUTION')
      GO TO 1
C
   48 IF(CK1.GE.CKX) GO TO 50
      IF(ABS(CK1-CKX).LT.ERR) GO TO 50
      PRINT 47
      GO TO 1
C
   50 SK1=SQRT(1.0-CK1*CK1)
```

```
      CKD1=ACOS(CK1) / RAD
C     COMPUTE EQUATION (34)
      CEK1=(CR-CK1*CE)/(SK1*SE)
      SGL=CEK1/ABS(CEK1)
      IF(ABS(CEK1).GT.1.0) CEK1=SGL
      EK1=ACOS(CEK1)/RAD
C     COMPUTE EQUATION (35)
      SED=ASIN(SR/SE)/RAD
      IF(EK1.LE.SED) GO TO 51
      IF(ABS(EK1-SED).LT.(ERR*5.0)) GO TO 51
      PRINT 47
      GO TO 1
C
C     COMPUTE EQUATION (33)
   51 SBK=SA*SGAMA/SK1
      IF(SBK .GT.1.0) SBK =1.0
      BK=(ASIN(SBK))/RAD
      BEA=(BK+EK1)*RAD
      BEB=(BK-EK1)*RAD
C     COMPUTE EQUATION (37) AND (38)
      CDTA1=CB*CE+SB*SE*COS(BEA)
      CDTA2=CB*CE+SB*SE*COS(BEB)
C
   52 DTA1=ACOS(CDTA1)
      DTA2=ACOS(CDTA2)
      DDA1=DTA1/RAD
      DDA2=DTA2/RAD
      JS=2
      IF(CDTA1.NE.CDTA2) GO TO 53
      JS=1
   53 DETA=DTA1
      PRINT 73,JS
C
   54 TMAX=0.0
      DO 58 I=1,JS
      PRINT 74,I
      PRINT 57
      CALL SPIN(P,Q,BETA,DETA,W,U,IN)
      IF(IN.EQ.0) GO TO 1
      R1=W(1)
      R2=W(2)
      R3=W(3)
C
C     COMPARE EACH SOLUTION OF SPIN AXIS WITH THE DESIRED ORIENTATION.
      DO 56 J=1,IN
      TESTC=SPX*R1+SPY*R2+SPZ*R3
      IF(TESTC.LT.TMAX) GO TO 55
      TMAX=TESTC
      PP1=R1
      PP2=R2
      PP3=R3
   55 R1=U(1)
      R2=U(2)
      R3=U(3)
   56 CONTINUE
C
      PRINT 57
   57 FORMAT(2X,'*')
```

```
      DETA=DTA2
   58 CONTINUE
C     COMPUTE THE RIGHT ASCENSION AND DECLINATION OF SELECTED SOLUTION.
      RA=ATAN2(PP2,PP1)
      DA=ASIN(PP3)/RAD
      IF(RA.GE.0.0) GO TO 70
      RA=RA+6.2831853
   70 RA=RA/RAD
C
      PRINT 72,R1,R2,R3
   72 FORMAT(2X,'FINAL SELECTION OF THE SPIN AXIS ORIENTATION WITH
     1COMPONENTS ',3F12.5)
   73 FORMAT(2X,'THE SCANNER CUTS THE EARTH HORIZON AT  ',I3,' POINT')
   74 FORMAT('0','***************OUTPUT FOR POINT',I3,'***************')
      PRINT 78,RA,DA
   78 FORMAT(2X,'RIGHT ASCENSION AND DECLINATION OF SPIN AXIS ARE ',
     1F14.3,' DEG. , ',F14.3,' DEG.')
      PRINT 57
      GO TO 1
  150 END
```

Appendix D

Sample of Attitude Determination Output

```
TIME OF TELEMETRY DATA IS, DAY=  76 SEC=   61395.00 -H M S
THE SPIN PERIOD OF SPACECRAFT IS  11133.75 MSEC
THE DIRECTION COSINES OF DESIRED SPIN AXIS ARE    .00000   .39795  -.91741
THE COMPONENTS OF SPACECRAFT POSITION VECTOR IN K.M.ARE          47081.58105        30549.70703        10676.79199
THE DIRECTION COSINES OF SUN LINE VECTOR ARE          .99321          -.05646          -.02449
THE ANGLE BETWEEN THE SPIN AXIS AND THE SUN POSITION   VECTOR IS   89.200 DEG.
EARTH WIDTH IS    6.9589 DEG.,  HALF ANGLE SUBTENDED BY  EARTH IS    6.4101  DEG.
THE DIRECTION COSINES OF DOWNWARD LOCAL VERTICAL ARE        -.82410        -.53473        -.18688
THE VIEWING ANGLE AT THE SPACECRAFT FROM THE SUN TO THE EARTH IS    136.22 DEG.
THE ANGLE BETWEEN THE SUN AND THE LOCAL VERTICAL IS     141.60 DEG.


AN EARTH TERMINATOR IS VISIBLE
THE SCANNER CUTS THE EARTH HORIZON AT     2 POINT

***************OUTPUT FOR POINT  1***************
 *
  ANGLE BETWEEN TWO GIVEN VECTORS P AND Q ARE   141.60427
 SOLUTION  1, COMPONENTS OF SPIN AXIS ARE        .01470       .40734      -.91320
 SOLUTION  2, COMPONENTS OF SPIN AXIS ARE        .02297      -.26177       .96489
 *

***************OUTPUT FOR POINT  2***************
 *
  ANGLE BETWEEN TWO GIVEN VECTORS P AND Q ARE   141.60427
 SOLUTION  1, COMPONENTS OF SPIN AXIS ARE        .00323       .23145      -.97296
 SOLUTION  2, COMPONENTS OF SPIN AXIS ARE        .01147      -.43586       .90007
 *
 FINAL SELECTION OF THE SPIN AXIS ORIENTATION WITH      COMPONENTS      .00323      .23145     -.97296
 RIGHT ASCENSION AND DECLINATION OF SPIN AXIS ARE        87.933 DEG.          -65.951 DEG.
 *
 TIME OF TELEMETRY DATA IS, DAY=  76 SEC=   62050.00 -H M S
 THE SPIN PERIOD OF SPACECRAFT IS  11173.75 MSEC
 THE DIRECTION COSINES OF DESIRED SPIN AXIS ARE    .00000   .39795  -.91741
 THE COMPONENTS OF SPACECRAFT POSITION VECTOR IN K.M.ARE          45109.67578        30155.76294        9937.82300
 THE DIRECTION COSINES OF SUN LINE VECTOR ARE          .99322          -.05634          -.02443
 THE ANGLE BETWEEN THE SPIN AXIS AND THE SUN POSITION   VECTOR IS   89.200 DEG.
 EARTH WIDTH IS    6.6333 DEG.,  HALF ANGLE SUBTENDED BY  EARTH IS    6.6398  DEG.
 THE DIRECTION COSINES OF DOWNWARD LOCAL VERTICAL ARE        -.81774        -.54666        -.18015
 THE VIEWING ANGLE AT THE SPACECRAFT FROM THE SUN TO THE EARTH IS    135.77 DEG.
 THE ANGLE BETWEEN THE SUN AND THE LOCAL VERTICAL IS     140.99 DEG.


 AN EARTH TERMINATOR IS VISIBLE
 THE SCANNER CUTS THE EARTH HORIZON AT     2 POINT

***************OUTPUT FOR POINT  1***************
 *
  ANGLE BETWEEN TWO GIVEN VECTORS P AND Q ARE   140.98712
 SOLUTION  1, COMPONENTS OF SPIN AXIS ARE        .01464       .40654      -.91357
 SOLUTION  2, COMPONENTS OF SPIN AXIS ARE        .02492      -.23048       .97281
 *

***************OUTPUT FOR POINT  2***************
 *
  ANGLE BETWEEN TWO GIVEN VECTORS P AND Q ARE   140.98712
 SOLUTION  1, COMPONENTS OF SPIN AXIS ARE        .00132       .20041      -.97985
 SOLUTION  2, COMPONENTS OF SPIN AXIS ARE        .01156      -.43464       .90068
 *
 FINAL SELECTION OF THE SPIN AXIS ORIENTATION WITH      COMPONENTS      .00132      .20041     -.97985
 RIGHT ASCENSION AND DECLINATION OF SPIN AXIS ARE        87.937 DEG. ,         -66.004 DEG.
 *
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