"Page missing from available version"

(i & ii)

THE EFFECT OF THE NONUNIFORM DC FIELD ON CARRIER WAVES IN NEGATIVE DIFFERENTIAL MOBILITY SEMICONDUCTORS†

by

M. Bini* and G. R. Bisio*

Microwave Laboratory
Stanford University
Stanford, California 94305

ABSTRACT

The growth of a carrier wave propagating through a negative differential mobility semiconducting sample is discussed under nonuniform dc bias conditions. A simple analytical expression for the overall numerical gain is given in terms of current density and of the input and output carrier wave velocities only. Applications to n-type GaAs are discussed.

†This work was supported by the National Aeronautics and Space Administration.

*On leave from Istituto di Ricerca sulle Onde Elettromagnetiche, Consiglio Nazionale delle Ricerche, Florence, Italy.

THE EFFECT OF THE NONUNIFORM DC FIELD ON CARRIER WAVES IN NEGATIVE DIFFERENTIAL MOBILITY SEMICONDUCTORS

It is well-known that growing carrier waves can be obtained in a material such as GaAs which exhibits negative differential mobility. The rates of growth of such carrier waves have been calculated by many authors (see, for example, reference [1]), and the theory used to predict the performance of a unilateral amplifier and to predict the region where the domains are formed. Most of these theories assumed that the dc fields in the sample are uniform, although Shockley [2] showed some years ago that in a semiconductor which exhibits negative differential mobility the dc electric field is nonuniform. Therefore, an rf perturbation theory based on the assumption of a uniform dc field is not necessarily accurate. Recently, Williamson [3] took account of the nonuniform dc fields in a n-type GaAs diode and carried out a computer calculation to determine the total gain of an rf space charge wave excited at the cathode. He noticed that the loss in the positive differential mobility region exceeds the gain in the negative differential mobility region, unless the cathode field is above about half threshold.

We shall show in this paper, from a purely analytic theory, that when a space charge wave is excited at a perfect cathode there is no net gain. We shall determine from a general analysis, conditions under which net gain can be obtained.

We consider first the dc field variation in a semiconductor of infinite cross section and finite length in the x direction. The semiconductor is extrinsic, let us say n-type with uniform donor charge density ρ_0 . The carrier velocity v vs electric field E characteristic is of the type shown in the insert of Fig. 1; i.e., the differential mobility $\mu_d = dv/dE$ is > 0 for $E < E_t$ and is < 0 for $E > E_t$; E indicates the absolute value of the electric field, and the subscript t stands for threshold. The case in which the high electric field tail of the v(E) curve displays a positive differential mobility is not considered here only for simplicity sake, but the final formula, Eq. (5), is valid also for that case.

From Poisson's and continuity equation it can be shown that the dc electric field must satisfy the following equation

$$\frac{dE}{dx} = \frac{\rho_0}{\epsilon}\left[\frac{J}{\rho_0 v[E(x)]} - 1\right] \tag{1}$$

where ϵ is the dielectric permittivity of the medium and J is the dc current density. We are interested in determining the regions in which the electric field is above or below threshold. To do this it is sufficient to know the value of the field at the cathode E(0) . The sign of the spatial derivative of the electric field dE/dx is given by the right hand side of Eq. (1). We can distinguish two cases: (a) when the current density is above its threshold value $J > \rho_0 v_t$ and (b) when it is below the threshold value $J < \rho_0 v_t$. For case (a), dE/dx is positive everywhere, the electric field increases indefinitely starting from E(0); if the sample is sufficiently long it will always reach threshold (see Fig. 1,dotted lines). For case (b), we have three possibilities depending on the value of $E(0)$ with respect to E_1 and E_2, where E_1 and E_2 are the values of the electric field at which $v = J/\rho_0$ in the positive and negative differential mobility region respectively, as is shown in the insert of Fig. 1. If $E(0) < E_1$ dE/dx is positive at the cathode, the electric field starts to increase but cannot go above E_1, where dE/dx would become negative. The electric field is below threshold everywhere. If $E_1 < E(0) < E_2$ dE/dx is negative at the cathode, the electric field starts decreasing but cannot go below E_1, where dE/dx would become positive. In this case, the electric field is above threshold only in a region near the cathode. If $E(0) > E_2$, dE/dx is positive at the cathode and the electric field increases indefinitely remaining above threshold everywhere. These three subcases are summarized in Fig. 1 (full lines).

We now consider the propagation of a carrier wave in the same semiconductor. Using Poisson's and continuity equation for the rf fields (time dependence $\exp(i\omega t)$), one finds that the rf electric field must satisfy a differential equation of this form

$$\frac{dE_{rf}}{dx} + i[\beta_r(x) - i\beta_i(x)]\, E_{rf} = I_{rf}/\epsilon v(x) \qquad (2)$$

where $\beta_r(x) = \omega/v(x)$, $\beta_i(x) = [J/\epsilon v^2(x)]\,(dv/dE)$, and I_{rf} is the total rf current density.

The total rf current density I_{rf} is different from zero only where interaction with an external circuit is present. In a negative-differential-mobility unilateral amplifier, interaction takes place only where the rf signal is injected or extracted [4,5].

The solution of (2) can be written in the following form:

$$E_{rf}(x) = E_{rf}(0)\exp[-F(0,x)] + \int_{\Gamma} [I_{rf}/\epsilon v(x_a)]\exp[-F(x_a,x)]\,dx_a \qquad (3)$$

where $F(x_a,x)$ is a function such that

$$F(x_a,x) = i\int_{x_a}^{x} [\beta_r(\xi) - i\,\beta_i(\xi)]\,d\xi \qquad (4)$$

and Γ indicates the space intervals from 0 to x where $I_{rf} \neq 0$. In writing (3) we used the identity $F(x_a,x) = F(0,x) - F(0,x_a)$, which can be easily derived from (4).

The solution (3) has a straightforward physical interpretation. The electric field of a carrier wave at a given x is the superposition of a contribution due to the electric field E_{rf} at $x = 0$ (surface excitation) and of contributions due to elementary excitations $I_{rf}\,dx_a/\epsilon v(x_a)$ generated at each x_a (bulk excitation). The function $\exp[-F(x_a,x)]$ is the transfer function between the input x_a and the output x . If β_i and β_r were constant with x , $\exp[-F(x_a,x)]$ would describe a wave going from x_a to x . Nevertheless, it is apparent from (3) and (4) that $\exp[-F(x_a,x)]$ can still be regarded as the propagation factor of a carrier wave also under nonuniform dc bias conditions.

To compute the terminal gain of a device one should perform the integration in (3). The value of the terminal gain will depend on the actual length and location of the regions of interaction with the input and output circuits. In this letter we will confine our analysis to the general problem of the growth of elementary carrier waves through nonuniformily biased semiconductors.

The overall numerical gain of a carrier wave traveling from x_a (rf input) to x_b (rf output) is $G(x_a,x_b) = \mathrm{Re}\left|\exp[-F(x_a,x_b)]\right| = \exp[-\int_{x_a}^{x_b} (J/\epsilon v^2)(dv/dE)dx]$. Using (1) to eliminate dE/dx , the integral can be performed analytically and the gain results

$$G(x_a,x_b) = \frac{v(x_a)}{v(x_b)}\left[\frac{J - \rho_0 v(x_b)}{J - \rho_0 v(x_a)}\right] \quad . \tag{5}$$

We can discuss this equation making use of the above analysis on the dc field distributions (see Fig. 1 for a summary). The results are given in the following table.

	$J > \rho_0 v_t$	$J < \rho_0 v_t$		
		$E(0) < E_1$	$E_1 < E(0) < E_2$	$E(0) > E_2$
$v(x_b) < v(x_a)$	gain	----	loss	gain
$v(x_b) > v(x_a)$	loss	loss	gain	----

The use of Eq. (5) does not give a complete picture of the variation of the gain with current, as $v(x_b)$ cannot be chosen arbitrarily, for $v(x_b)$ is a function of $E(x_a)$, J and the distance $\ell = (x_b - x_a)$. Nevertheless without solving the probelm in detail, it is possible to draw some interesting conclusions on this point. We will limit ourselves to consider only cases for which the dc field at the input is such that $E(x_a) > E_t$ and the dc current density J is $\geq \rho_0 v(x_a)$. These conditions are often satisfied in practice, for the electric field goes above threshold in a short distance from the cathode and the injection of the carrier wave can be over comparatively wide region [4] or at a point sufficiently far from the cathode [1,6] to have gain. We take $v(x_a)$ fixed and look at the variation of $G(x_a,x_b)$ with J and ℓ . When $J \to \rho_0 v(x_a)$ the dc fields tend to become uniform (see Eq. (1)), β_i becomes constant along the sample and, with a finite value of ℓ , $G(x_a,x_b) \to \exp[-(dv/dE)_{x=x_a} \rho_0 \ell/\epsilon v(x_a)]$. The gain is bounded for any finite ℓ . The maximum of $G(x_a,x_b)$ does not have to occur at $J = \rho_0 v(x_a)$ but depends on the detailed shape of the v(E) curve. For any given dc current density such that

$J > \rho_0 v(x_a)$, the gain is an increasing function of ℓ reaching a saturation value $G_{sat} = [v(x_a)/v_\infty][J-\rho_0 v_\infty]/[J-\rho_0 v(x_a)]$ where v_∞ is the high electric field limit of the velocity (see the insert of Fig. 1).

To illustrate these results we will consider the case of the n-type GaAs. Following reference [1] we assume that $v = v_t/[0.45 + 0.55\,(E/E_t)]$ for $1 < (E/E_t) < 3$ and $v = 0.5\, v_t$ for $(E/E_t) > 3$. Figure 2 shows the computed dependence of $G(x_a,x_b)$ on ℓ when the rf is injected at threshold point, i.e., where $v(x_a) = v_t$, for different values of $(J/\rho_0 v_t)$. As might be expected, the gain saturates both with J and ℓ because $v(x_b) \rightarrow v_t/2$ for $E \gg E_t$. The dotted line, which is the envelope to the $G(x_a,x_b)$ vs ℓ curves, gives the maximum gain achievable for any given ℓ by changing J .

Finally, we can note that the gain does not depend on frequency, since Eq. (5) contains only dc quantities. We can also write Eq. (5) in terms of the dc free charge density at the input $\rho(x_a)$ and at the output $\rho(x_b)$, $G(x_a,x_b) = [\rho(x_b) - \rho_0]/[\rho(x_a)-\rho_0]$. In this way $G(x_a,x_b)$ assumes the form of what can be called dc gain: the ratio of the net dc charge density at the rf output to the net dc charge density at the rf input.

Let us now consider an implication of these results. They indicate that for a unilateral carrier wave amplifier biased above threshold, i.e., $J > \rho_o v_t$, the input signal must be injected at a point where the carrier drift velocity is larger than the dc velocity at the output of the diode. This implies likewise that the dc charge density at the input is less than the dc charge density at the output.

We are indebted to Professor G. S. Kino for suggesting the problem and discussing the results of this note.

REFERENCES

1. P. N. Robson, G. S. Kino, and B. Fay, IEEE Trans. Electron Devices (Correspondence), 14, 612 (1967).
2. W. Shockley, Bell Sys. Tech. J., 33, 799 (1954).
3. W. J. Williamson, Appl. Phys. Letters 18, 389 (1971).
4. B. Fay, Microwave Laboratory Report No. 1908, Stanford University (1970) (to be published).
5. K. Kumabe, H. Kanbe, and S. Ohara, Proceedings of the 1st Conference on Solid State Devices, Tokyo 1969, pp. 39-44.
6. R. H. Dean, A. B. Dreeben, J. F. Kaminski, and A. Triano, Electronics Letters 6, 775 (1970).

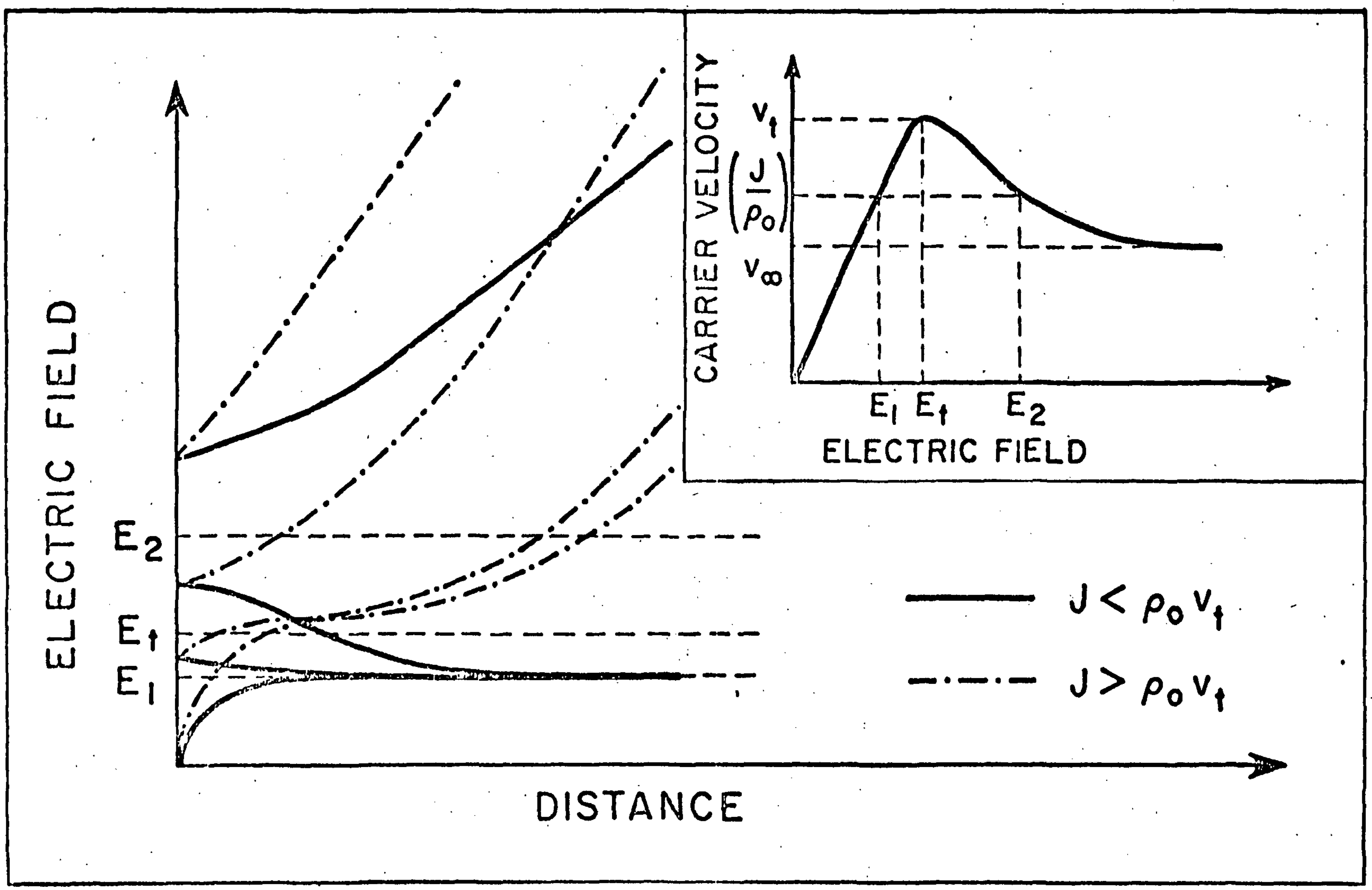

FIG. 1--Electric field E vs distance x for different values of boundary condition $E(0)$, with current density $J > \rho_0 v_t$ (dotted lines) and $J < \rho_0 v_t$ (full lines). The insert shows the $v(E)$ curve of a medium displaying negative differential mobility at electric field values larger than the threshold field E_t . For the meaning of E_1 and E_2 see the text.

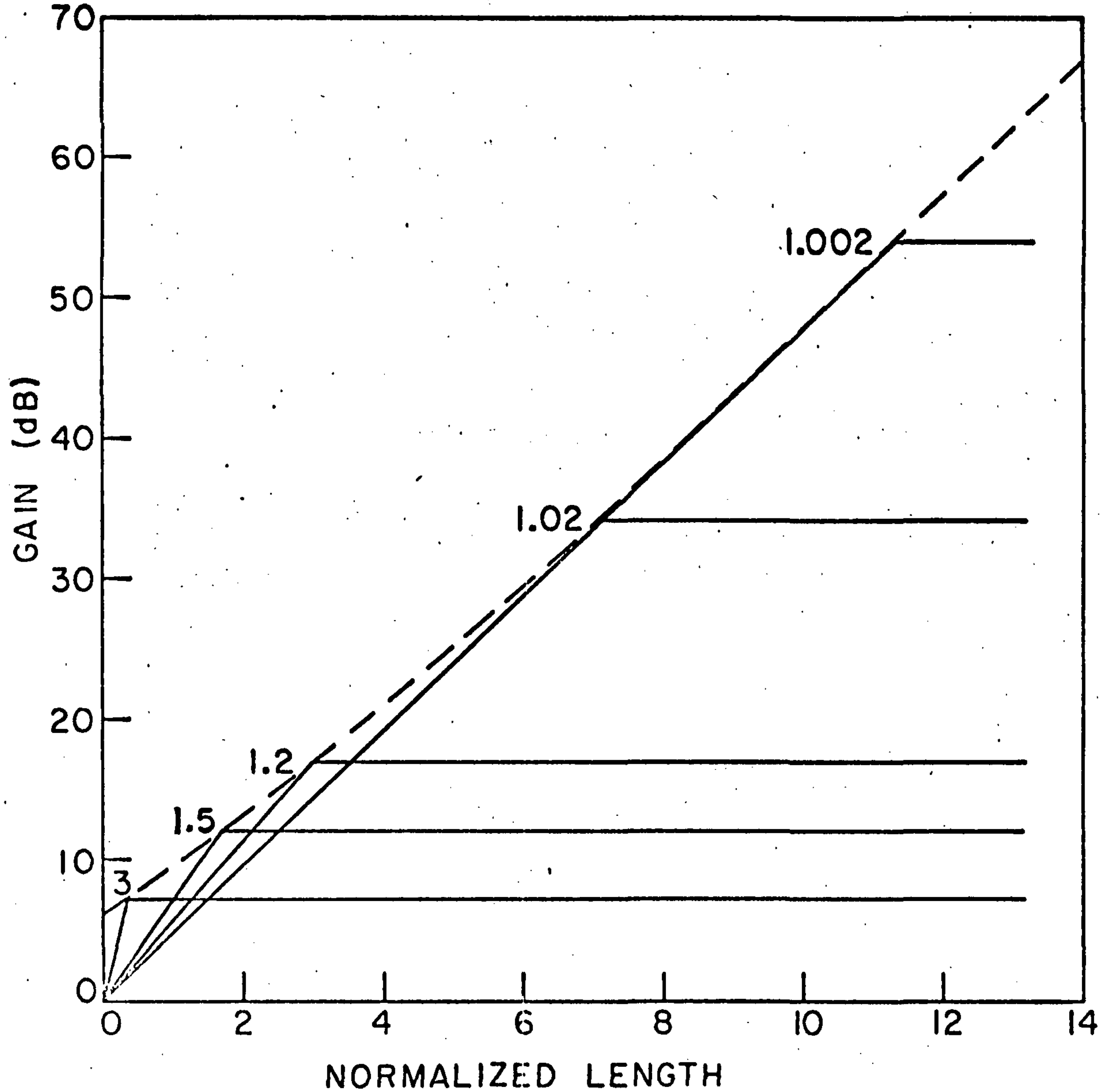

FIG. 2--Gain vs normalized length $\ell/(\epsilon E_t/\rho_0)$ of a n-type GaAs diode for different values of the normalized current density $J/\rho_0 v_t$.